THE USE OF LOGARITHMS IN ANALYZING TRAP COLLECTIONS[1]

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INTRODUCTION

Increasingly, mosquito control directors are relying upon sampling to evaluate the effects of their control program. A major problem is interpretation of sampling results. Often these results are confusing, or even misleading, although if properly handled, the same data could have provided valuable information. Logarithms have long been used for analyzing trap collections by a small number of investigators. The purpose here is to acquaint field workers with this technique.

This paper will involve statistics only to the extent of discussing the average, or mean, of a series of numbers. The mean is a measure of central tendency; to be reliable, it must be located near the middle of the series with about the same number of values above and below it. If this requirement is not met, then it is necessary to change the numbers to another scale. While different mathematical transformations are possible, in insect sampling a logarithmic transformation is most frequently required.

The mosquito population is not uniformly distributed within even a small area. Furthermore, the effectiveness of sampling varies with environmental and biological factors. When this area is sampled by a single trap, the trap site may be characterized by low, medium or high densities. Since the relationship between the size of the trap collection and the mosquito population of the entire area is always unknown, *changes* in the numbers collected from day to day are of greater importance than the actual size of the sample.

The Rationale of Logarithms. If a city, Alpha (population 10,000), was found to have increased since the last census by 1,000 persons (to 11,000), and Beta (population 100,000) grew at the same rate, it is immediately realized that Beta's increase was 10,000, not 1,000, i.e., in both cities an increase of 10 percent.[2] The increase is proportional to the population. If these populations are expressed as logarithms, the proportional relationship becomes direct. Thus, Alpha had a population of 10,000 whose log is 4.00 (two decimal places are sufficient), and increased by 1,000 to 11,000 (log 4.04). Beta's population was 100,000 (log 5.00) and a similar increase to 110,000 has a logarithm of 5.04. The mantissa, .04, is unchanged whether the increase is 1,000 or 10,000. Its value is determined by the value of the characteristic (the number to the left of the decimal). Thus if the population is small, it adds a small value; if large, a large value.

In regard to mosquito sampling, a change in one trap from 500 to 250 mosquitoes is the same as from 50 to 25 in another, i.e., both populations declined 50 percent. But unless compensated for, changes in areas with larger populations will dominate when traps from different locations are averaged.

The Geometric Mean. The following examples will illustrate the application of logarithms to different aspects of mosquito sampling.

A. If a mosquito control district had

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[2] The writer is indebted to Dr. E. T. Nielsen for this example.

TABLE 1.—Effect of logarithmic transformations on means of insect collections.

PART	DATE	TRAPS										TOTAL		
		Numbers (X) taken												Mean
1	1	4	7	11	2	1	8	5	450	9	3	500		50
	2	10	8	23	18	12	35	6	100	18	20	250		25
		Logarithm of X											Mean log	$Mean_G$
2	1	.60	.85	1.04	.30	.00	.90	.70	2.65	.95	.48	8.47	.85	7.1
	2	1.00	.90	1.36	1.26	1.08	1.54	.78	2.00	1.26	1.30	12.48	1.25	17.8
		Numbers (X) taken											Mean	$Mean_G$
3	1	4	7	11	2	1	8	5	450	9	3	500	50	7.1
	2a	8	14	22	4	2	16	10	900	18	6	1000	100	14.1
		Logarithm of X+1											Mean log	$Mean_W$
4	1	.70	.90	1.08	.48	.30	.95	.78	2.65	1.00	.60	9.44	.94	7.7
	2	1.04	.95	1.38	1.28	1.11	1.56	.85	2.00	1.28	1.32	12.77	1.28	18.1

10 traps and the director learned that the average collection the previous night was 50 mosquitoes, his initial concern is lessened by an inspection of the individual trap results (Table 1, Part 1). Obviously the arithmetic mean of 50 has been inflated by a single large collection. In this series, nine collections are smaller than the mean and only one is larger. The mean is strongly skewed toward the large collection. The second night's results are also shown in Part 1.

The mean (M) is still skewed as only two trap collections were larger than the mean. Even worse, while the mean of 25 indicates the general population has dropped 50 percent since the preceding day, nine traps had increased collections and only the large one showed a decline. It is evident these means are unreliable, having been disproportionately affected by one trap

Let us now transform the trap collections into logarithms (Part 2). They are added together, then averaged, and the anti-log of this number is looked up. This is a geometric mean (M_G). Now the director finds he has an average of 7.1 mosquitoes instead of 50. Not only does he feel better, but he has more confidence in it since the mean log is more centrally located: 4 values are larger and 5 are smaller. Note that 4 of the trap collections (Part 1) are larger than the geometric mean and 5 are smaller. The second night's trap catches are similarly treated. The new mean is 17.8 and again the mean log is close to the center. An inspection of the actual collections (Part 1) shows 2 collections of 18 while 4 are larger and 4 smaller. The new mean shows that the population estimate has more than doubled since the previous day. This would indicate a probable increase of mosquitoes in the entire district.

B. Some traps consistently take much larger collections than others. As a result, most of the day to day change in the trap totals reflects the day to day changes of these particular traps. If on the second day, all traps had taken exactly twice as many as on the first (Part 3), both arithmetic and geometric means would be twice as large. As the population ratio between the two dates did not vary, the relationship between their respective means was unchanged. When different traps are sampling different sized populations, the *range* of variation for each trap will be proportional to its mean. For example (Table 2), if collections from trap X varied from 2 to 18 (mean=10; range=16), while trap Y collections varied from 20 to 180 (mean=100; range=160), the daily variations in trap Y will swamp trap X, e.g., if trap X goes up by 4 but Y goes down by 40. Thus, even in these instances where collections made by a single

Table 2.—Effect of logarithms on traps sampling different sized populations.

			Trap X		Trap Y	
	Trap X	Trap Y	log.	a-log.	log.	a-log.
	2	20	.30		1.30	
	4	40	.60		1.60	
	6	60	.78		1.78	
	8	80	.90		1.90	
	10	100	1.00		2.00	
	10	100	1.00		2.00	
	12	120	1.08		2.08	
	14	140	1.15		2.15	
	16	160	1.20		2.20	
	18	180	1.26		2.26	
Total	100	1000	9.27		19.27	
Mean	10	100	.93	8.5	1.93	85.1
Range	16	160	.96		.96	

trap can be accurately represented by an arithmetic mean, it is not possible to make comparisons with other traps sampling different sized populations. However, transforming these collections into logarithms shows collections from trap X varied from .30 to 1.26 (range .96), while trap Y collections varied from 1.30 to 2.26 (range .96). If both traps have similar ranges the fluctuations that occur will receive nearly equal weight.

From these examples, we can conclude that the geometric mean is the appropriate mean to interpret these trap data, and the use of an arithmetic mean would be in error.

WILLIAMS' MEAN. One problem associated with the use of logarithms is that there is no logarithm of zero. This is overcome by adding 1 to all collections, and when converting back to the antilogarithm, subtracting 1. For example, for a collection of zero mosquitoes, use the logarithm of 1 (which is .00); for 1, the logarithm of 2; for 2 the logarithm of 3; etc. If the mean log subsequently obtained is .81, the anti-log is 6.5; subtracting 1 gives a mean of 5.5. Expressed as the logarithm of X+1, the two series of trap collections in Table 1 now become as shown in Part 4.

The mean logs are still centrally located—the new means (called Williams' mean—M_W) are slightly larger than their respective geometric means. The differences between geometric and Williams' mean will be greatest at low values (adding 1 to 2 is proportionately a much greater increase than adding 1 to 20). However, in view of the many influences other than population changes that affect the size of trap collections, this error can be ignored.

DETERMINING WILLIAMS' MEAN BY MEASUREMENT. It is also possible to determine Williams' mean by plotting each trap collection, plus 1, directly upon 3 cycle semi-logarithmic paper (Fig. 1). This method makes the use of logarithmic tables unnecessary. The trap collections, plus 1 (from Table 1, Part 1), are plotted vertically. Note from the distribution of these points it is now apparent that the mean for day 2 will be greater than for day 1. The distance each point lies above the base line is now determined by measurement with a ruler. These distances are summed, and the mean distance calculated. In Figure 1, for days 1 and 2 the total distances were 801.7 and 1083.8 mm respectively. Mean distances were 80.2 mm and 108.4 mm.

A mean distance of 80.2 mm lies opposite a numerical value of 8.8; as 1 had been added to each trap collection, 1.0 must now be subtracted to furnish a $mean_W$ of 7.8 for day 1. The same procedure is followed for day 2 to provide a $mean_W$ of 18.0.

ARRANGING TABULAR DATA. While, in general, it is preferable to transform each trap collection into the logarithm of X+1, when a large volume of data is to be analyzed, it will be easier to transform subtotals. This procedure is demonstrated in Table 3 (although for such small samples it would not be employed). Here 4 traps were operated 5 nights (Part A). The daily totals are transformed, summed, and the mean log and Williams' mean (=90.2 mosquitoes per night) obtained. The same procedure with the trap totals shows a Williams' mean of 117 mosquitoes per trap.

However, if we wish to determine individual Williams' means, such as the $mean_W$ number per night for each trap, the $mean_W$ number per trap for each night, or the $mean_W$ number per trap-night, it is necessary to transform each collection into the logarithm of X+1 as shown in Part B. Because of the nature of logarithms, it is not possible to reconcile the totals by summing trap and night means.

The writer has found it convenient to make tables to 2 decimal places of both the logarithms of X+1 and of the antilogarithm −1. This makes it possible to have all numbers up to 1000 handy on one page, and also to save time and reduce

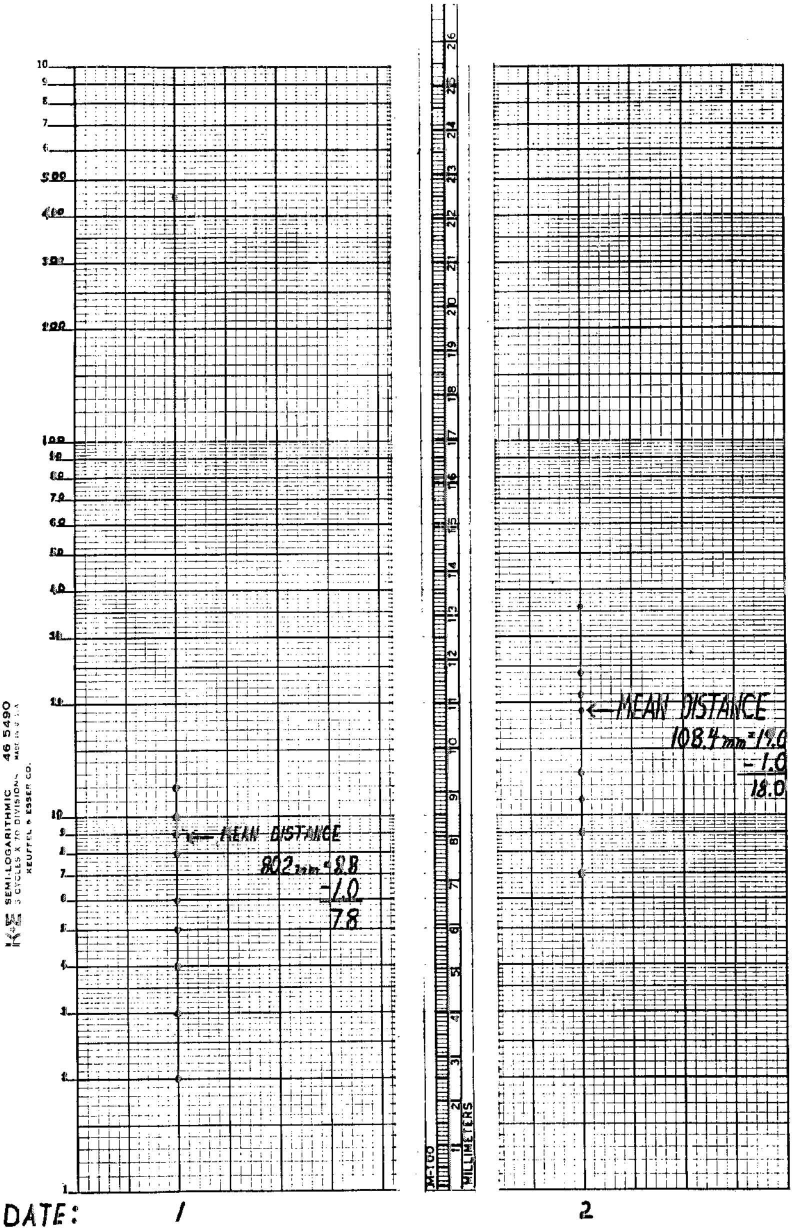

Fig. 1.—Determining Williams' mean (M_w) by measurement.

TABLE 3.—The use of logarithms on tabular data.

A. DATE	TRAPS A	B	C	D	Total	Total as log X+1
1	39	61	48	7	155	2.19
2	21	15	31	13	80	1.91
3	8	7	9	18	42	1.63
4	17	43	21	12	93	1.97
5	30	57	28	15	130	2.12
Total	115	183	137	65	500	
Total as log X+1	2.06	2.26	2.14	1.82	8.28	9.82
N					4	5
Mean log					2.07	1.96
Meanw					117/trap	90.2/night

B. DATE	A	B	C	D	Total	Total	N	Mean log	Meanw
1	1.60	1.79	1.69	.90		5.98	4	1.50	30.6
2	1.34	1.20	1.51	1.15		5.20	4	1.30	19.0
3	.95	.90	1.00	1.28		4.13	4	1.03	9.7
4	1.26	1.64	1.34	1.11		5.35	4	1.34	20.9
5	1.49	1.76	1.46	1.20		5.91	4	1.48	29.2
Total	6.64	7.29	7.00	5.64	26.57				
N	5	5	5	5	20				
Mean log	1.33	1.46	1.40	1.13	1.33				
Meanw	20.4	27.8	24.1	12.5	20.4/trap night				

errors as it is unnecessary to be continuously adding and subtracting one.

SUMMARY. The rationale for using the logarithm of X+1 to analyze trap collections was presented. Because *changes* in the size of trap collections are of greater importance than the actual numbers, logarithms can correct two major problems encountered in analyzing data: (1) when the arithmetic mean of a series of numbers is not centrally located; and (2) when one number series, much larger than the others, exerts a disproportionate effect. A failure to employ logarithms when required will lead to incorrect conclusions as inevitably as if mistakes in arithmetic were made.

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