Solar Records. By Pliny Earle Chase, LL.D., Professor of Philosophy in Haverford College.

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I. *Harmonies of Lockyer's "Basic Lines."*

From the third law of motion it follows, as a necessary consequence, that cosmical and molecular bodies act and react upon one another in accordance with laws of perfect elasticity. Hence, by introducing formulas of undulatory motion, results can often be speedily reached which would, otherwise, require the use of long and difficult analytical processes.

In previous communications I have shown :

1. That some of the most striking indications of nodal aggregation in the planetary system, are connected, by the laws which govern the relations between density and altitude in elastic atmospheres, with the nodal indications of the Fraunhofer lines.*

2. That the collisions of subsiding particles, from opposite diametral extremities of a condensing spherical nebula, tend to form shells or rings of nodal aggregation, at $\frac{2}{3}$ of the radial distance from the centre of the nebula.†

3. That centres of linear and of spherical oscillation, exert an important influence, both upon molar and upon molecular arrangements.‡ Professor Stephen Alexander had previously pointed out some instances of the results of spherical oscillation in the solar system.

4. That the nodal resistance of large cosmical bodies tends to form other nodal aggregations, at harmonic intervals, in accordance with the laws of musical rhythm which govern the vibrations of elastic media.§

5. That there are reasons for anticipating, in the fundamental oscillations of terrestrial elementary bodies, symmetrical harmonic evidences of the same laws as govern the harmonic nodes of elastic media and the harmonic grouping of planetary systems.‖

I have also shown, both from independent considerations and as corollaries from the foregoing laws :

6. That in paraboloidal aggregation,¶ there are three wave systems, with tendencies to nodal collisions and orbital aggregations in which the major axes have successive differences of 4 x_0.

* *Ante*, xvii, 109, *sq*.; 1877 : 294, *sq*; 1878.

† Ib. xvii. 100 ; 1877.

‡ Ib , x, 103, ; 1869.: xiii, 140 *sq*.; 1873.

§ xiii., 140, 193, 237 ; 1873.

‖ Ib., xii., 392 *sq*.; 1872.

¶ Ib., xvi, 507 ; 1877.

7. That centripetal energy $\left(f \propto \frac{1}{r^2}\right)$ varies as the fourth power of tangential energy in a circular orbit* $\left(v = \sqrt{fr} \propto \sqrt{\frac{1}{r}}\right)$.

Lockyer has published eight† "basic lines," which furnish illustrations of all these laws, or established harmonies.

The mean *vis viva* of the æthereal sphere of which Earth is the centre, tends (law 3) to form a node at .4 of Sun's distance from Earth, or at .6 of the same distance from Sun. Having already seen that the Fraunhofer line A is the exponential correlative of the planet Neptune, we readily find that this node is represented by a wave length of 4215.8 ten millionths of a millimetre. For (Laws 1, 5):

	Neptune.		Earth.		A.		
Log.	6442.985	: log.	$214.524 \times .6$	: :	7612	:	4215.8

If we regard this value as a fundamental wave-length for terrestrial chemical elements, we may also (Laws 6, 7), regard $(\frac{1}{4})^4$ of $4215.8 = 16.468$ as a fundamental increment, for such harmonic undulations as may be excited in the elastic æther by inertial resistance.

The "theoretical" column in the following table, is constructed by simple combinations of the fundamental wave length and the fundamental increment.

Theoretical.	"Basic Lines."
$5269.8 + 3^2 \times 16.468 = 5418.0$	5416
$4215.8 + 8^2 \times 16.468 = 5269.8$	5269
	5268
$5170.9 + 2^2 \times 16.468 = 5236.8$	5235
$5022.7 + 3^2 \times 16.468 = 5170.9$	b_3 b_4
$4215.8 + 7^2 \times 16.468 = 5022.7$	5017
$4^4 \times 16.468 = 4215.8$	4215

Lockyer does not give the wave lengths of b_3 and b_4. Gibbs‡ gives 5177 as the wave length of the *b* line. Law 2 is illustrated in the third theoretical line (5236.8), which represents $\frac{2}{3}$ of the interval between 5170.9 and 5269.8. These are both double lines in Lockyer's system. The doubling may, perhaps, be owing to the modification of the other activities by Law 2. Lines 2 and 5 (5269.8 and 5022.7) are directly connected with the fundamental line. All the incremental multipliers are integral squares. The difference between line 2 and line 5 is 15×16.468. The greatest square in 15 is 3^2, and the greatest square in $15 - 3^2$ is 2^2. These squares are the in-

* Ib., xiii, 245; 1873.

† Proc. Roy. Soc. Jan. 1879.

‡ Am. Jour. Sci. [2] xliii, 4.

cremental coefficients for lines 4 and 3. The difference between lines 1 and 2 is the same as that between lines 4 and 5.

The greatest difference between the theoretical and measured values (5022.7—5017 = 5.7) is only $\frac{1}{40000000}$ of an inch. The closeness of the accordance may be more readily seen by dividing each of the theoretical values by 1.00028.

Reduced Theoretical.	Measured.
5416	5416
5268	5269
5268	5268
5235	5235
5021	5017
4215	4215

In some respects this symmetry seems even more remarkable than those which I found, more than eighteen months ago, in many of the chemical elements. They were, however, directly harmonic, being based on centrifugal relations to the centres of wave systems (Law 5). These are reciprocally harmonic, being based on centripetal relations to the surface of Sun's chromosphere.

Multiples of the fundamental increment often appear in the differences between the wave lengths of elementary spectra. The following instances, in elements for which I have already shown harmonic relations,* will serve as examples. The left hand columns contain exact multiple differences ; the right-hand columns, measured wave-lengths :

Mercury.	
546.09	546.13
542.80	542.80

Lead.	
537.78	537.71
439.07	439.07

Lithium.	
479.69	479.48
459.93	459.93

Ruthenium and Iridium.	
545.34	545.44
530.52	530.52

Copper.	
578.67	578.67
529.27	529.30
522.21	522.24
465.75	465.64

Arsenic.	
617.54	617.54
533.55	533.55
611.67	611.67
578.73	578.73

Zinc.	
636.99	636.99
610.64	610.64
472.31	472.25

In the copper lines, the first theoretical difference is 30 $\times$ 1.6468 ; the second is $\frac{1}{7}$ as much ; the third is the sum of the other two. In arsenic, the second line is $2^2 \times 3^4 \times 1.6468$.

* *Ante* xvii, 297 ; 1878.

II. *Spectral Estimates of Sun's Distance.*

I have further shown :

8. That the harmonic undulations of our atmosphere are such as to furnish a simple method of estimating Sun's distance, by means of barometric fluctuations.*

9. That approximate estimates of Sun's distance, may also be made from the harmonic disturbances of magnetism,† (chemical energy, light, sound, gt, gt^2, simultaneous attraction of Sun, Earth and other planets upon elastic fluids),‡ lunar distance and orbital time.§

10. That there are evidences of paraboloidal nucleation, connecting the Sun, each of the planets, the asteroidal belt, and the star *Alpha Centauri.*‖

11. That planetary rotation is merely retarded orbital revolution, through the collision of particles near paraboloidal or ellipsoidal foci.¶

12. That gt, when t is the time of cosmical or molecular semi-rotation, represents the limiting velocity between complete dissociation and incipient aggregation.**

13. That gt, for the principal planets in the supra-asteroidal and in the infra-asteroidal belt (Jupiter and Earth), is determined†† by Sun's orbital influence ($\sqrt{gr}$); while gt, for the Sun, is the velocity of light.

14. That Jupiter is at the centre of the Neptuno-Uranian nebula ; Earth is at the centre of the belt of greatest density ; Sun is at the nucleal centre of the entire system.‡‡

15. That the frequency of oscillations in the violet rays, and the superficial gravitating energy of the Sun, are indicative of reciprocal action and reaction.§§

16. That successful predictions may be made from simple considerations of the principles which are involved in harmonic undulation.‖‖

All of these laws were found by means of the hypothesis that the undulations of an æthereal medium, when intercepted by inert bodies, tend to produce harmonic undulations (Law 4).

The discovery of the foregoing "basic" harmony, therefore, led me to look with confident expectation for such evidences of undulatory collision, between solar and terrestrial waves, as would furnish satisfactory grounds for new estimates of the Sun's mass and distance.

Beginning with the most far-reaching of all the indications (Law 10), and taking Earth's half radius as the unit and focal abscissa of a primitive

**Ante*, ix, 287; 1863.
†Ib., ix, 356, 367, 427, 487 ; 1864.
‡ Ib., xi, 103 ; 1869 : xii, 392 ; 1872 : xiii, 142 ; 1873.
§ xiii, 398-400 ; 1872.
‖ Ib., xii, 519 ; 1872.
¶ Ib. xii, 406 ; 1872 : xiv, 112 ; 1874.
**Ib. xiv, 111 ; 1874 : xvi, 298, 496 ; 1876–7.
††Ib. xii, 406 ; 1872.
‡‡Ib. xvi, 497 ; 1877.
§§Ib. xiii, 149 ; 1873.
‖‖Ib. ix, 288 ; 1863 : xiii, 238 ; 1873 : xviii, 34 ; 1878.

paraboloid, the focal ordinate would be equal to radius. I then found (Laws 6, 7, 11, 12), that by comparing the *vis viva* of satellite revolution at Earth's surface ($\propto v_0^2 = gr$), with the *vis viva* of rotation at the primitive focal abscissa ($\propto v_1^2 = \frac{1}{4}$ of the square of the velocity of equatorial superficial rotation), we may obtain the equation :

$$\left(\frac{v_0}{v_1}\right)^2 = \frac{r}{y}, \qquad (1)$$

in which y represents the distance traversed by a ray of light (compare Laws 13, 15), while a body, at the equator, would fall through the "fundamental increment" of the foregoing tabular comparison ($\frac{1}{256}$ of 4215.8 ten millionths of a millimetre). For,

$$v_0 = \sqrt{gr} = \sqrt{\frac{385}{12} \times \frac{3963}{5280}} = 4.907\ m.$$

$$v_1 = \pi \times 3963 \div 86165 = .1445\ m.$$

$$y = r\left(\frac{v_1}{v_0}\right)^2 = 3.436\ m.$$

$$t = \sqrt{.0000000016468 \div 4.8894} = .000018353\ sec.$$

Light traverses Earth's mean radius-vector in 497.825 *sec.* Therefore, according to this estimate, Sun's mean distance is

$$\frac{497.825y}{t} = 93{,}203{,}000 \text{ miles} \qquad (2)$$

A second approximation may be made by remembering that the basic lines are the reciprocals of harmonic lines, and comparing the æthereal volumes, or the reciprocals of the ratio of variability in tidal influence, $\left(\frac{1}{d}\right)^3$, at the points where the disturbing forces are greatest (the surfaces of the disturbing bodies). By the laws of elasticity, the æthereal undulations that are set up at any point, are propagated with uniform velocity. If we take the theoretical fundamental wave length as our fundamental unit, and if we call the mean orbital distance which Earth traverses in the time ($t = .000018353$ *sec.*) of falling through the fundamental increment, the "orbital unit," we find that

$$\frac{\text{Orbital unit}}{\text{Fundamental unit}} = \left(\frac{\text{Sun's radius}}{\text{Earth's radius}}\right)^3 \qquad (3)$$

For, representing Earth's mean radius-vector by x ;

Orbital unit $= 2\pi\, x \times .000018353$ *sec.* $\div$ 1 *year.*

$$\text{Fundamental unit} = \frac{4215.8 \times .0000000039371}{63360}\ m.$$

Sun's radius $= x \div 214.524$.

Earth's radius $= 3963$ *m.*

Substituting these values in equation (3), we get

$$\frac{2\,\pi\, x \times .000018353 \times 63360}{365.256 \times 86400 \times 4215.8 \times .0000000039371} = \left(\frac{x}{214.524 \times 3963}\right)^3$$

$$\therefore x = 92{,}579{,}000 \text{ miles} \qquad (4)$$

It will be readily seen that equations (2) and (4) are entirely independent of each other. The true unknown quantity, or common unit of comparison, in each case, is the velocity of light. The comparison is drawn, in the first instance, between Earth's centripetal and centrifugal forces; in the second, between Sun's orbit-controlling influence upon Earth, and Earth's reaction upon Sun. That reaction must be exerted, either through an elastic medium, or by means of *quasi*-elastic forces. The elimination of the comparative unit, shows that the hypothesis of a luminiferous æther, or "æthereal spirit" as Newton termed it, accounts for inter-stellar, planetary, chemical, electrical, cosmical and molecular action. I do not, however, regard this fact as conclusive of the existence of such a medium, although it seems to lend the hypothesis a higher degree of probability than any previous investigations, and it requires, at least, *quasi*-elastic action.

The difference between the two results is less than one-half of one per cent. It would have been easy to assume values for the constants, which are within the limits of probable errors of observation, and which would have made the accordance exact. The value of Sun's radius ($x \div 214.524$) is deduced from Dr. Fuhg's estimate of Sun's apparent diameter. Three other estimates, which do not make so large an allowance for irradiation, are also included in the following table:

	Apparent Diam.		$x \div$ Sun's r.	x.
Dr. Fuhg.*	32′	2.″99	214.524	92,579,000
British Naut. Al........	32	3. 64	214.451	92,531,700
American " "	32	4. 00	214.412	92,506,500
Lockyer's Astron........	32	4. 205	214.388	92,491,000

Among the numerous previous mechanical estimates that I have given, the one which accords most nearly with the two present determinations, was the one which was based upon thermo-dynamical considerations derived from the "heating energy of flames,"† and which gave

$$x = 92,639,500 \text{ miles} \ldots\ldots \quad (4.)$$

The intimate connection between Sun, Jupiter and Earth, which is indicated by Laws‡ 13 and 14, should lead to many other relations, no less interesting than the foregoing.

If we take $\frac{1}{256}$ of the cosmical distance which corresponds to the fundamental wave-length, we find

$$\tfrac{1}{256} \text{ of } .6 \text{ of } 214.524 = .5028 = 1.0056 \times .5 \ldots\ldots \quad (5)$$

But .5 is the focal abscissa of the primitive paraboloid, of which Sun's radius is the focal ordinate.

III. *Relations of Mass.*

According to Professor Peirce's meteoric hypothesis, it may be reasonably presumed that each planet receives meteoric increments, or suffers

* Deduced from 6827 measurements; Astron. Nach. 2040, cited in Am. Jour. Sci., x, 159, Aug. 1875.

† *Ante*, xii, 394; Am. Jour. Sci., iii, 292; 1872.

‡ I call all these harmonies "laws," because they exhibit pre-established purposes, though some of them are more special than others.

changes from meteoric influences, in proportion to its mass, so as to maintain a permanency of relative mass among the principal members of our system.

I have already pointed out various harmonic mass relations (Law 3), including the following equation involving figurate powers of the supra-asteroidal masses, as well as of their distances.*

$$\text{Saturn}^{10} = \text{Neptune}^{1} \times \text{Uranus}^{3} \times \text{Jupiter}^{6} \ldots\ldots (6)$$

I have also called attention to the fact that these four planets, together with Earth and Sun, represent important centres of nebular or *quasi*-nebular influence, viz:

Neptune, centre of primitive annular condensation.
Earth, centre of belt of greatest density.
Sun, centre of nucleal condensation.
Uranus, centre of primitive "subsidence" collision (Law 2).
Jupiter, centre of Neptuno-Uranian nebula.
Saturn, nebular centre of mean planetary inertia. Saturn is also the centre of paraboloidal subsidence when Neptune was focal and Sun was at the vertex.

The report of Professor Pierce's lecture led me to look for some equation to connect the masses at the two remaining centres (Earth and Sun) with those of the two chief planets, and I soon found that

$$\text{Jupiter}^{3} = \text{Sun} \times \text{Earth} \times \text{Saturn} \ldots\ldots (7)$$

This equation gives

$$\left.\begin{array}{lll} \text{Sun's mass} \ldots\ldots & = & 328{,}600. \\ \text{" parallax} \ldots\ldots & = & 8.''832 \\ \text{" distance} \ldots\ldots & = & 92{,}549{,}000 \text{ miles.} \end{array}\right\} \ldots\ldots (7_2)$$

Combining (6) and (7), we find

$$\text{Saturn}^{9} = \text{Jupiter}^{3} \times \text{Uranus}^{3} \times \text{Sun} \times \text{Earth} \times \text{Neptune} \ldots\ldots (8)$$

The masses of Neptune and Uranus seem to be so related as to give them equal ratios between their present orbital momentum and the orbital momentum at their respective abscissas in the solar-stellar paraboloid ($\frac{8}{7}$ Neptune and $\frac{7}{8}$ Uranus).

$$\left.\begin{array}{rl} 7 \times \text{Neptune} = & 8 \times \text{Uranus} \\ \therefore \sqrt{\tfrac{7}{8}} \times \text{Neptune} = & \sqrt{\tfrac{8}{7}} \times \text{Uranus} \end{array}\right\} \ldots\ldots (9)$$

Equation (8) may be stated under the form

$$\left(\frac{\text{Sat.}}{\text{Sun.}} \times \frac{\text{Sat.}}{\text{Ear.}} \times \frac{\text{Sat.}}{\text{Nep.}}\right)\left(\frac{\text{Sat.}}{\text{Jup.}} \times \frac{\text{Sat.}}{\text{Ura.}}\right)^{3} = 1 \ldots\ldots (10)$$

Here the equation of planetary stability groups the centres in two sets, as in equation (7), the first introducing the first powers, the other the cubes, of the relative masses. The same exponential grouping also occurs in (3), but with linear factors instead of mass factors. If we consider that, in a rotating nebula, the time of rotation varies inversely as the square of the

* *Ante*, xiv, 652.

radius, and also inversely as the disturbing mass, the first group leads to the equation

$$\frac{\text{Earth} \times 1 \text{ year} \times (\text{Neptune's r. vec.})^2}{\text{Sun} \times 1 \text{ day} \times (\text{Earth's r. vec.})^2} = 1 \dots\dots\dots\dots\dots\dots (11)$$

This equation gives

$$\left.\begin{array}{ll} \text{Sun's mass} \dots\dots\dots & = 330{,}375. \\ \text{" parallax} \dots\dots & = 8.''816 \\ \text{" distance} \dots\dots & = 92{,}717{,}000 \text{ miles.} \end{array}\right\} \dots\dots\dots\dots (12)$$

In considering this and other relations of mass to æthereal disturbance, it is well to remember that the simple disturbance varies as the mass; the *vis viva*, or radius of consequent oscillation, as the square of the mass; and the consequent orbital period, as the cube of the mass.

By introducing the vector-radii also into the cubical factor of (10) and designating secular perihelion, mean perihelion, mean aphelion, secular aphelion, respectively, by subscript 1, 2, 3, 4, 5, we find

$$\left.\begin{array}{l} \dfrac{\text{Sat.}_3 \times \text{Sat.}_4}{\text{Jup.}_2 \times \text{Ura.}_3} = 1 \\[2ex] \dfrac{\text{Sat.}_3 \times \text{Sat.}_4}{\text{Jup.}_3 \times \text{Ura.}_2} = 1 \end{array}\right\} \dots\dots\dots\dots\dots\dots\dots\dots (13)$$

The greatest deviation from exactness, in the first of these equations, is less than $\frac{1}{8}$ of one per cent.; in the second, less than $\frac{1}{16}$ of one per cent. The mean deviation, in the square root of the product of the two equations, is only $\frac{1}{36}$ of one per cent.

We see by (5) and (13), as well as by ordinary astronomical investigations, that questions of relative mass are intimately connected with those of orbital eccentricity. One of the most interesting evidences of such connection, in this special line of investigation, is to be found in the position of the mean fulcrum of the system, or centre of gravity of Sun and Jupiter, together with the significance which it lends to equations (5), (6), (8), (13), as well as to the fundamental increment which is the ground of equation (3). The orbital *vis viva* has lengthened the radius-vector of simple equilibrium by $\frac{1}{16}$ of its value. For $5.2028 \times 214.524 = 1116.125$; $\frac{16}{17}$ of $1116.125 = 1050.471$. The limit of synchronous radial and circular oscillations is at $2\,r$. Deducting 2 from 1050.471 we find

$$\frac{\text{Sun's mass}}{\text{Jupiter's mass}} = 1048.471 \dots\dots\dots\dots\dots\dots\dots\dots (14)$$

Equations 7, 8 and 9 give the following theoretical values, for Uranus and Neptune, which I compare with Newcomb's:

Sun ÷	Theoretical.	Newcomb.
Uranus....................	22116	22600 ± 100
Neptune..................	19352	19380 to 19700

Newcomb gives two estimates for Neptune, one (19380 ± 70) from satellite, the other (19700) from perturbations of Uranus. The latter agrees precisely with the theoretical ratio (9) between the two planets, while the former is presumably more accurate.

The uncertainty in regard to all the planetary masses, except that of Jupiter, is still so great, that it is impossible to tell how closely they are represented by the equations for the combined central activities (6, 7, 8). The latest investigations of Leverrier and Newcomb, however, show a closeness of approximation which is remarkable, in view of the wide discrepancy in some of the values. Leverrier's mass-denominators, based on the old parallax (8.″57), are: Neptune, 14400; Uranus, 24000; Jupiter, 1050; Saturn, 3512; Earth, 354936. The accordance with the combined equation (8) is within $\frac{3}{8}$ of one per cent. if we deduce Earth from the other masses; within $\frac{1}{14}$ of one per cent. if we deduce Saturn.

If we look to the partial equations, (6, 7), we find that Saturn's mass, as deduced from Neptune, Uranus and Jupiter, (6), is about $\frac{3}{4}$ of one per cent. greater than Leverrier's assumption, and about the same amount less than Bessel's, which was adopted by Newcomb. *The mean of the two results shows an exact accordance*, as follows:

	Deduced.	Assumed.
Leverrier............	3488.3	3512.0
Newcomb............	3525.0	3501.6
Mean..................	3506.6	3506.8

The results of the second partial equation, (7_2), may confidently await the verdict of the observations upon the last transit of Venus. No other estimate can now claim a greater degree of probability. It may be, as Leverrier suggests, that a small portion of the mass may belong to a group of minute asteroids, near Earth's orbit, but there is no present likelihood that any material inaccuracy will ever be found in the equation which connects the two principal intra-asteroidal centres with the two principal extra-asteroidal centres.

E. Wiedemann's experiments upon the illumination of gases by electricity,* have convinced him that the electric discharge may excite a considerable increase of the *vis viva* of oscillation in æthereal envelopes, without increasing the *vis viva* of the enclosed molecules. Peirce's meteoric hypothesis opens an immense field for new physical speculation and investigation. If the æther is material, where shall we draw the boundary between æthereal and meteoric influences? If cosmical masses have been formed by paraboloidal aggregation, may not radiation also be paroboloidal? The solar forces of association and dissociation seem to be almost exactly balanced, and the law of equal action and reaction may, perhaps, free the science of thermodynamics from the opprobrium of its apparent tendencies to universal aggregation, stagnation and death.

* Wied. Ann., vi, p. 301.