Reference: *Biol. Bull.* **179:** 186–190. (October, 1990)

Drag Coefficients of Swimming Animals: Effects of Using Different Reference Areas

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Abstract. The drag coefficient (C_D) is useful for comparing the hydrodynamic drag among different swimming animals. However, C_D is calculated using an arbitrary reference area for which there is no uniform convention; both total surface area ("wetted area") and maximum cross-sectional area ("frontal area") are widely used. The choice of reference area can have a profound effect on calculations of drag coefficient. To illustrate this problem, drag measurements from two isopod crustacean species were used to calculate C_D based on both wetted and frontal areas. *Idotea wosnesenskii* had a higher mean C_D based on wetted area (0.084) than *Idotea resecata* (0.059), but a lower mean C_D based on frontal area (0.95) compared to *I. resecata* (1.22); both differences are statistically significant. Given that there is no powerful hydrodynamic basis for choosing either reference area, and that conversions between wetted area C_D and frontal area C_D cannot accurately be made for complex shapes, I suggest reporting both wetted area and frontal area C_D's wherever practical.

Introduction

Students of animal swimming often find it useful to measure the hydrodynamic drag experienced by an animal. In steady swimming, thrust equals drag, and data on thrust production are requisite to study such topics as swimming biomechanics (*e.g.*, Webb, 1975; Wu, 1977) and energetic costs of transport (*e.g.*, Hargreaves, 1981; Daniel, 1983). To facilitate comparisons among individuals or among different species, many investigators have borrowed, from engineers, the concept of "drag coefficient" (C_D) defined by:

$$C_D = 2D/\rho v^2 S \qquad (1)$$

where ρ = fluid density, v = speed, S = reference area, and D = drag force (Fox and McDonald, 1978)[1]. The drag coefficient is dimensionless and is typically used to compare the effects of drag on objects of different configurations or morphologies. For a given shape, the drag coefficient is a function of the Reynolds number (Re):

$$Re = \rho v l/\mu \qquad (2)$$

where l = reference length (usually the length parallel to movement or fluid flow), and μ = dynamic viscosity (Hoerner, 1965; Fox and McDonald, 1978). The Reynolds number may be interpreted as a dimensionless index of the relative importance of pressure (inertial or form) drag *versus* viscous (friction) drag (Fox and McDonald, 1978; Vogel, 1981). For simple shapes at Re $\leq$ 1, viscous drag predominates, and the drag coefficient is a simple log-linear decreasing function of the Reynolds number. However, at higher Reynolds numbers, pressure drag is most important, and the behavior of C_D with increasing Re is very complex and may be strongly influenced by turbulence (although C_D changes very gradually at Re $\geq 10^6$) (Hoerner, 1965). Most swimming animals have "intermediate" Reynolds numbers (10^2–10^5), where neither viscous nor inertial forces dominate the flow (Hoerner, 1965), and it may be difficult to predict what shapes will give the lowest drag (Vogel, 1981). Biologists have thus found it convenient to compare drag coefficients of a variety of animals to determine which morphologies generate the least drag. For example, Blake (1985) used drag coefficient measurements to show that an actively swimming decapod crab species had a lower drag morphology than two other benthic species. Similarly, Gal and Blake (1987)

Received 11 April 1990; accepted 20 July 1990.

[1] Note that this equation, often given as $D = \frac{1}{2}\rho v^2 S C_D$ [*e.g.*, Hargreaves, 1981; Blake, 1985], defines the "drag coefficient," not the "drag."

compared drag coefficients of a frog species that is entirely aquatic with one that is more amphibious.

Comparisons based on drag coefficients can be complicated by the choice of reference area. The drag coefficient in the form of equation (1) is derived from dimensional analysis (Fox and McDonald, 1978), and the reference area is an arbitrary scale factor with dimensions of $(\text{length})^2$. The choice of reference area can have a significant effect on the magnitude of the drag coefficients. For example, Webb (1975) reported a C_D of 0.015 for a small trout, whereas Nachtigall's beetles had C_D's from about 0.3 to 0.4 (Nachtigall, 1977). Some difference might be expected between fish and beetles on morphological grounds, but the major reason is that different reference areas were used to compute the drag coefficients: the fish drag coefficients were based on total surface area or "wetted" area, C_{D_w} (Webb, 1975), but the drag coefficients of the beetles were based on maximum cross-sectional or "frontal" area, C_{D_f} (Nachtigall, 1977). Both reference areas are commonly used in engineering practice (*e.g.*, Hoerner, 1965; Fox and McDonald, 1978; Bertin and Smith, 1979). Hoerner (1965) and Fox and McDonald (1978) generally use frontal area for the drag coefficients of simple shapes (spheres, cylinders, etc.) and wetted area for streamlined objects and whole vehicles or vehicle models (but without an explicit statement of conventions). Frontal area is typically much easier to measure than wetted area (see below), but wetted area is probably more appropriate in most cases, as animals rarely have simple shapes (if they did, there would be little point in measuring their drag coefficients!). Two other reference areas do have explicit usage conventions: vertically projected area, or "planform" area, is used for the drag coefficient of wings (or other lifting surfaces) and $\text{volume}^{2/3}$ is used for airship drag coefficients (Vogel, 1981).

In the present paper, data from two species of swimming isopods (Alexander, 1988; Alexander and Chen, 1990) are used to show how the choice of reference area can have a profound effect on drag comparisons. Choosing the appropriate reference area in different situations is also discussed.

Materials and Methods

All drag measurements and wetted area measurements are taken from Alexander and Chen (1990). Briefly, specimens of *Idotea resecata* and *Idotea wosnesenskii* (Isopoda: Crustacea) were preserved, fixed in a life-like swimming posture, and mounted on a force transducer; they were placed in a flow tank at a flow speed equal to a realistic swimming speed, and the drag was measured with the force transducer. Wetted area was estimated by approximating animals as oblate spheroids, with body lengths used for major axes, and the means of maximum body height and width used for minor axes (Alexander and Chen, 1990).

The frontal area of these same animals was measured as follows. The preserved isopods were mounted directly head-on in the field of view of a closed-circuit, solid-state Sony video camera equipped with a Nikon 55mm macro lens. The image was displayed on a Burle high-resolution television monitor (38 cm diagonal screen). A 1-cm^2 graph paper grid was also in the field of view of the camera. Each isopod's frontal image was traced onto a plastic transparency sheet, along with the 1-cm^2 grid. The isopod tracing and the area grid were cut out and weighed to the nearest 0.1 mg on an electronic balance, and the weight of the area grid was used to calculate the area of the isopod tracing. Each isopod was traced and cut out three times, and the average weight used to calculate the frontal area; typically, tracings for one individual varied by about 3%, never more than 8%. Drag coefficients were calculated using equation (1). Statistical analyses were based on procedures given in Zar (1984) and the microcomputer version of "Minitab" software.

Results and Discussion

Figure 1a shows the drag coefficients based on wetted area for the two species. *Idotea resecata* had significantly lower C_{D_w}'s than *I. wosnesenskii* ($P < 0.001$, $F_{[1,18]} = 19.38$); the mean C_{D_w} for *I. resecata* was 0.059, *versus* 0.084 for *I. wosnesenskii*. Using the same drag data, but recalculating the drag coefficients using frontal area, Figure 1b shows that the situation is reversed; in this case, *I. wosnesenskii* has a significantly lower mean C_{D_f} (0.95) relative to *I. resecata* (mean $C_{D_f} = 1.22$) ($P < 0.01$, $F_{[1,18]} = 12.6$). The data in Figure 1a and b are from the same animals in the same orientation and posture. The only difference is the choice of reference area.

How can such a seemingly trivial change cause such a drastic reversal in the results? Vogel mentioned that when frontal area is used, "stubbier" (shorter, blunter) shapes should have the lowest drag coefficient, but if wetted area is used, elongate shapes will generally have lower drag coefficients (Vogel, 1981, p. 112). Consider two elongate (not bluff) objects with the same frontal area but differing substantially in length: the short object will typically have lower drag, and thus, lower C_{D_f}. In contrast, if two objects have the same wetted area, but one is shorter, the short one will necessarily be more bulbous or "bluff" and, hence, generally have a larger wake. Where $Re > 1$, the wider wake of the short object is likely to cause more drag and hence, a larger C_{D_w}. As Figure 2 shows, the two *Idotea* species exactly fit these descriptions: *Idotea resecata* is more elongate and has a lower C_{D_w}, whereas *Idotea wosnesenskii* is blunter and shorter, and has a lower C_{D_f}. The startling aspect is that the differences between the species

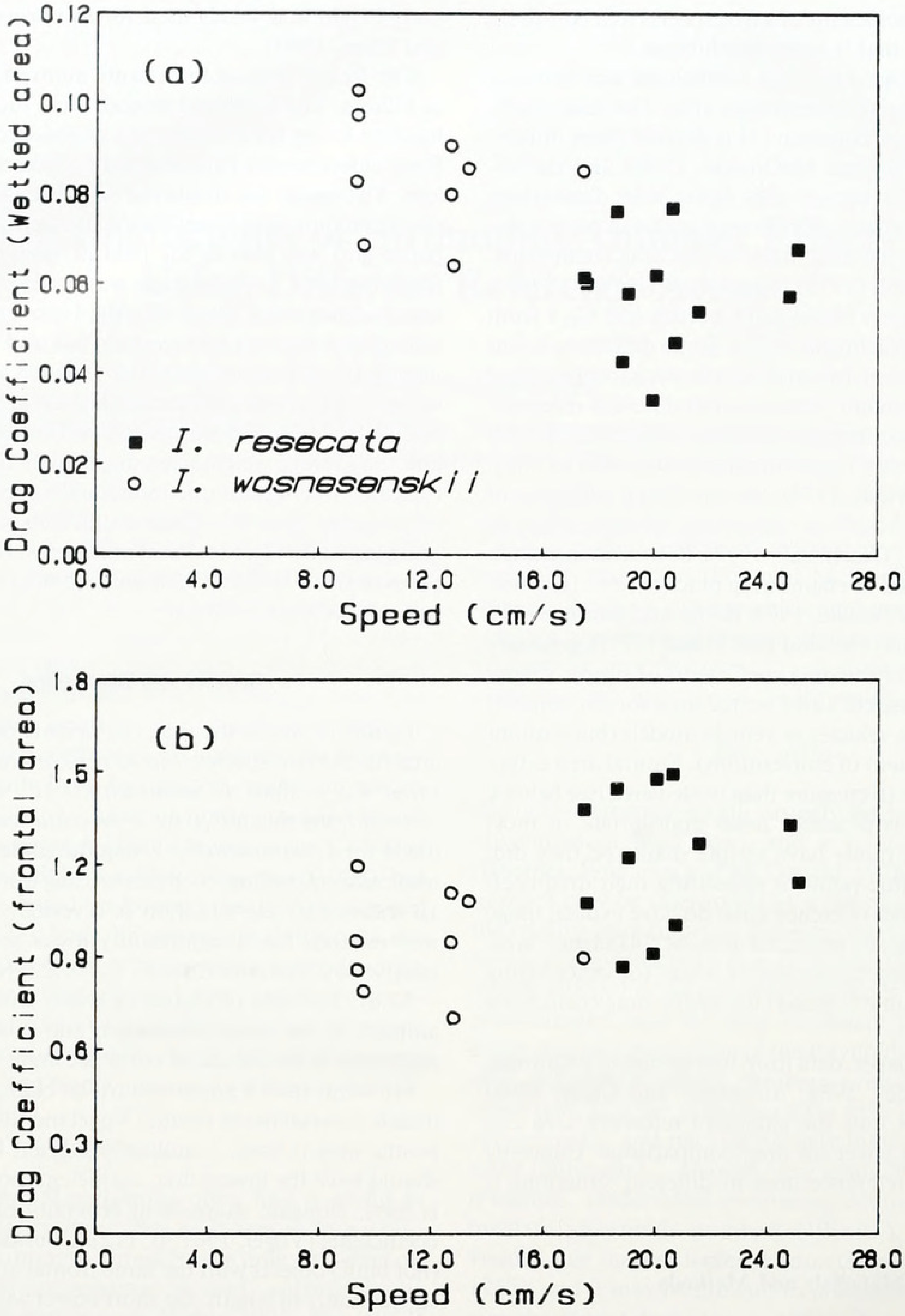

Figure 1. The relationship between swimming speed and drag coefficients for two species of *Idotea*. Each symbol represents a different individual, and shows the mean of 6 to 12 swimming speed trials and 3 drag measurements. The same individuals are represented on both graphs. (a) The drag coefficient calculated using wetted area. (b) The drag coefficient using the same data as in (a) but with drag coefficient calculated using frontal area.

are statistically significant in both cases, even though reversed.

Engineers tend to use the frontal area for drag coefficients where the viscous drag is important, and wetted area where pressure drag is more important (Fox and McDonald, 1978). This is reasonable: at low Reynolds numbers details of shape and orientation have little influence on drag, so frontal area is an adequate scale factor;

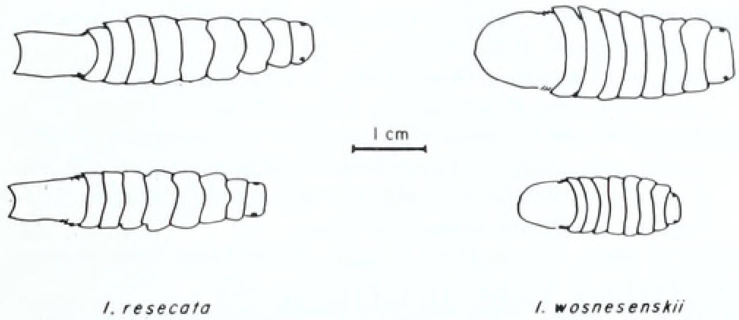

Figure 2. Dorsal views of the bodies of male (upper) and female (lower) individuals of *Idotea resecata* and *I. wosnesenskii* (traced from video images). The mean fineness ratio (length/width) for the individuals in this study was 4.5/1 for *I. resecata* and 2.9/1 for *I. wosnesenskii*.

at high Reynolds numbers, streamlining and orientation are important in that the length of an object in the direction of flow (or movement) affects boundary layer separation and wake size. Thus, at high Reynolds numbers, frontal area would be a poor scale factor, as it would be the same for a sphere and a well-streamlined object. However, the choice of a reference area is ultimately arbitrary, and the typical types of objects to which engineers apply C_{D_f} or C_{D_w} may be as much a matter of convenience, as due to fluid mechanical principles.

The problem for biologists is that many (if not most) macroscopic swimming animals operate at intermediate Reynolds numbers. In such cases, the relationship between drag coefficient and Reynolds number is strongly dependent on the geometry of the object and the presence or amount of turbulence in the fluid (Hoerner, 1965: p. 16.6). Furthermore, one cannot be sure *a priori* whether viscous or pressure drag is most important. Therefore, as Vogel (1981) noted, it is not clear what shapes will give the lowest drag.

Frontal area is attractive simply because it is much easier to measure accurately. Wetted area is difficult to measure on any but the simplest shapes, and virtually impossible on an object as morphologically complex as an arthropod. Thus, for all practical purposes, an estimate for wetted area must be used, as in Cowles *et al.* (1986) or Alexander and Chen (1990). Such wetted area data are very likely to be underestimates for arthropods, as they do not include appendages or surface irregularities due to segmentation; an underestimate of the reference area would lead to an overestimate of the drag coefficient.

Because frontal area will typically be more accurate, its use might seem to be preferable. But, as most animals do not have simple shapes, and as typical swimming animals are elongate in the direction of swimming, wetted area is the reference area of choice. However, the wetted area will necessarily be an estimate (and probably a slight underestimate) for animals with complex shapes, so comparisons among animals with different shapes must be made with due caution. Frontal area is appropriate in some situations, primarily for sessile organisms, or motile organisms with no preferred directionality or which do not move with their longest dimension parallel to the direction of travel.

If a set of drag coefficients is meant as an index for comparing groups of animals, then both C_{D_w} and C_{D_f} should be provided. If a study is meant to investigate the relationship between C_D and some other variable (*e.g.*, Reynolds number or speed) where complete presentation of C_{D_w} and C_{D_f} would be redundant, then future researchers may find the data most useful if major relationships are presented using C_{D_w}, with average or typical C_{D_f} values being included for reference. Drag coefficient data presented without an explicit statement of the choice of reference area are useless; biologists have no clear conventions for choosing reference area.

Other possible reference areas ignored so far in this discussion are planform area and $(\text{volume})^{2/3}$. Planform area is appropriately used when investigating drag on an object that is also generating a significant amount of lift. Such an object will have an additional drag component, induced drag, produced by the same process that generates lift. Indeed, to correctly determine the lift-drag ratio, the planform area must be used to calculate the drag coefficient (Hoerner and Borst, 1975; Blake, 1985). Finally, Vogel (1981) suggests using $\text{vol}^{2/3}$ because, as with lighter-than-air vehicles, the internal volume of a swimming animal is likely to be more biologically relevant than other measures of surface area; also, $\text{vol}^{2/3}$ can be measured as accurately as desired. However, C_D's based on $\text{vol}^{2/3}$ will be lower for blunt shapes than elongate ones (for the same reasons as for frontal area) (Vogel, 1981), which de-emphasizes streamlining. Also, drag coefficients based on $\text{vol}^{2/3}$ are exceedingly rare in the biological literature, so for comparative purposes it may be advantageous to include C_{D_w} data in such studies as well.

Acknowledgments

I thank T. Chen for help with data collection, and S. Vogel and H. M. Alexander for useful comments on the manuscript. This work was supported in part by a Biomedical Sciences Support Grant from the University of Kansas.

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