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ON THE MECHANISM AND PREVENTION OF CAVITATION

by
Phillip Eisenberg

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Errata - Figure 34, page 58

1. Change label of curve marked K_1 to σ_1
2. Change label of other curve and vertical scale
from $-\left[\frac{\Delta p}{2\rho u}\right]_{\min}$ to $-\left[\frac{\Delta p}{2\rho U}\right]_{\min}$

Equation [31], page 46

Divide right-hand side by $\sqrt{1 + \sigma_2}$

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PREFACE

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ON THE MECHANISM AND PREVENTION OF CAVITATION

by

Phillip Eisenberg

ABSTRACT

A brief survey of available information on the mechanism of cavitation is presented with the emphasis on those aspects which are not only of theoretical interest but of interest to the design engineer and naval architect for technical applications.

The "types" of cavitation are distinguished according to their description by the hydrodynamical equations rather than the thermodynamical and physical-chemical processes involved. This leads to a distinction between small, individual cavities which oscillate and eventually disappear, and large, steady-state cavities whose average envelope does not vary with time. The inception of cavitation in pure and in contaminated liquids is discussed. For pure liquids, some recent applications of kinetic theory formulations to the prediction of tensile strength of liquids are presented. Factors in inception that are of more immediate engineering interest are then discussed. These include the roles of surface-active materials and other impurities, air content, turbulence, and pressure gradients in laminar and turbulent boundary layers as related to inception of cavitation.

Results of analytical and experimental studies of the dynamics of transient, oscillating cavities are discussed. Tentative descriptions of the mechanism of steady-state cavities in real liquids are given. In connection with the analytical description of such cavities, the results of free streamline theory for two-dimensional cavities with finite cavitation members, and the conclusions obtainable for three-dimensional flows are summarized. A treatment of cavities in venturi-type nozzles is proposed in which it is assumed that the vapor phase plays no significant role. Computations and experimental results are presented for the latter problem.

The mechanism of cavitation damage is discussed in terms of both mechanical and electrochemical origins. A few remarks are then made on scaling of cavitating systems and designing for cavitation prevention. In connection with design, some methods that have been proposed for reducing the adverse effects of cavitation are discussed, and some methods available for the prediction of low pressure regions are mentioned. Finally, it is shown how cavitation itself may be used for certain cases in designing non-cavitating systems.

INTRODUCTION

In recent years, requirements in the design of hydrodynamic systems for use at increasingly greater speeds and lower pressures have emphasized the problem of prevention of cavitation and its associated damage to materials, losses in efficiency and objectionable noise, as well as increase of drag of submerged bodies with cavitation onset. Although many questions remain which must be answered before the mechanism of cavitation phenomena can be more completely understood, much progress has been made toward rational descriptions of the processes in cavitating systems. This work includes not only the hydrodynamics of flows with both liquid and gas or vapor phases, but also the physical-chemistry of such two-phase flows in relation to the formation, collapse, and maintenance of cavities. It is the purpose of this discussion to present a brief survey of the available information that is not only of theoretical interest but of interest to the design engineer and naval architect for technical application. At the same time, those areas in which much work remains to be done will be apparent. No attempt will be made to be exhaustive and, of necessity, the material discussed will reflect the immediate interests of the writer. However, it is hoped that, although little detail is given on several aspects of the cavitation problem, no significant portions have been overlooked. An attempt will also be made to point out the various systems which may be characterized as cavitating-systems from either the hydrodynamical or the physical-chemical point of view or both.

The possibility of cavitation in hydraulic machines was recognized as long ago as 1754 in a memoir of Euler in connection with the theory of turbines.¹ However, significant work on cavitation in engineering applications can probably be dated from the experiments of Reynolds reported in 1894² and the observations of Thornycroft and Barnaby.^{3,4} The latter works on screw propellers were especially of importance in the identification of cavitation as a major problem in the field of naval architecture, and gave impetus to investigations of the effects of cavitation on drag and efficiency not only on propellers but other ships' appendages as well.

A resume of some of the early work of this century is given in Reference 5, and a rather complete bibliography of cavitation researches especially of interest to the hydraulic engineer and naval architect is given in Reference 6. A survey of the cavitation problems considered of importance in hydraulic machines and large hydraulic structures will be found in Reference 7.

¹References are listed on page 63 of this report.

The present discussion will, in general, proceed according to the following outline: Description of "types" of cavitation ordinarily encountered in technical applications, the effects of cavitation in such applications, ideas relating to inception of cavitation, dynamics of "transient" cavitation bubbles, mechanism of "steady-state" cavities, the mechanism of collapse of cavities and associated damage, some remarks on scaling of cavitating systems, and some methods available in design for prevention of cavitation and cavitation damage.

TYPES OF CAVITATION

It is clear that cavitation will occur in a liquid when the pressure is reduced to a certain critical value without change in the ambient temperature, or, conversely, when the temperature is raised above a critical value at constant pressure. Thus, from a purely physical-chemical point of view, there is no difference between boiling of a liquid and cavitation in a flowing liquid, at least in so far as the question of inception is concerned. However, a certain distinction must be made depending upon whether the cavitation is a purely two-phase, one-component, phenomenon with a vapor cavity or whether the cavity is formed by outgassing of dissolved or entrained gases at pressures well above the pressure of the vapor phase of the liquid. Although the thermodynamical conditions for the formation and maintenance of such types of cavities may differ considerably, from a hydrodynamical point of view no distinctions need be made. Thus, for cavitation in a flowing liquid, it is of interest to point out the conditions under which low pressures may occur.

It will be sufficient for illustrating the "types" of cavitation to consider the phenomena from an elementary point of view. In steady, irrotational flow of an incompressible fluid, the pressure equation may be written (neglecting effects of gravitational acceleration):

$$p = P + \frac{1}{2}\rho(U^2 - u^2) \quad [1]$$

where p is the local pressure,

P is a reference pressure, usually taken as the pressure in the undisturbed fluid,

u is the local velocity,

U is a reference velocity taken in the undisturbed fluid, and

ρ is the mass density of the fluid.

In dealing with cavitating flows, the pressures must, of course, be referred to the absolute scale. It is clear from Equation [1] that the pressure p may drop to very small values depending upon the velocity u . The liquid of particular interest to the present discussion is water; the difference in air

content and particle content of fresh and sea water is of importance, however. For most applications, in which the water contains a fairly high air-content, cavitation in the form of air bubbles may begin at pressures well above the vapor pressure. In general, however, cavitation will occur at or near the vapor pressure with the cavities containing primarily vapor, some initial air and possibly some gas diffused from the liquid into the cavity during the life of the cavity. Although these statements are approximately correct for engineering applications, the role of gas or solid nuclei in inception is of a very complicated nature and is discussed in somewhat more detail in subsequent sections.

Familiar examples of cavitation in flows which obey Equation [1] on individual streamlines are shown in Figures 1, 2, 3, and 4. Figure 1, from

Figure 1 - Cavitation in a Venturi Section—After Föttinger, Reference 8

Reference 8, shows the cavitation developed in a venturi section. Figures 2 and 3 show cavitation on the backs of propeller models. Cavitation on a body of revolution is shown in Figure 4. The latter three photographs were taken during tests of models in the TMB variable-pressure water tunnels. It will be noted that the character of the cavities in the venturi section and near the top of the uppermost blade in Figure 2 is different from that of the cavities in Figures 3 and 4. The first type has been characterized as "sheet" cavitation and the second as "bubble" cavitation by naval architects working on the problems of cavitation of propellers and other ships' appendages. As will be shown in the subsequent discussion, however, the first type behaves, on a time-average, as a steady cavity and the second type behaves as a transient oscillating system. The writer therefore prefers the terms "steady-state" and "transient" cavities, respectively.

Another type of flow in which low pressures may occur is the vortical flow which may be approximated in a real fluid by a rectilinear vortex. The equation of motion for such flows is

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{u^2}{r} \quad [2]$$

Figure 2 - Cavitation on a Model Propeller Showing "Sheet" or "Steady-State" Cavitation Near the Tip of the Uppermost Blade

which on integration and evaluation at the center of the vortex gives the pressure (see, e.g., Reference 9).

$$p_c = P - \frac{\rho \Gamma^2}{4\pi r_1^2} \quad [3]$$

where p_c is the pressure at the center of the vortex,
 Γ is the circulation, and
 r_1 is the radius of the vortex core.

Here, again, the pressure may drop to very low values depending upon the strength of the vortex. In the case of lifting surfaces, the conditions in the tip vortices whose strengths are related to the lift distribution, may be described approximately by Equation [3]. An example of cavitation in such a flow is shown in Figure 5 in which the tip vortices of a model propeller are clearly defined.

Figure 3 - "Transient" or "Bubble" Cavitation on a Model Propeller

Figure 4 - Cavitation on a Body of Revolution

Figure 5 - Cavitation in the Tip Vortices of a Model Propeller

Finally, following an example given by Ackeret,¹⁰ consider the conditions in an unsteady flow. The contribution to the pressure due to a time rate of change of a velocity potential ϕ is

$$p = P - \rho \frac{\partial \phi}{\partial t} \quad [4]$$

For a receding piston, in a tightly fitting cylinder, the velocity potential is

$$\phi = xu(t)$$

where x is measured from the beginning of the motion.

Substituting this into [4], the pressure is

$$p = P - \rho x \frac{du}{dt} \quad [5]$$

From [5] it is seen that, even for small accelerations, the pressure may reach low values as x increases. An example of cavitation resulting in such a flow is shown in Figure 6, which was taken from Reference 11, although here there are additional terms due to the displacement flow about the rod. In the experiments of Figure 6 an aluminum rod was moved upward in corn syrup. The same type of phenomenon may be produced by magnetostriction oscillations of a bar.¹²

Another type of cavitation is that produced by the entry of a missile from air into water. Although such cavities exhibit the properties of "steady-state" cavities during parts of the life cycle, the dynamics of the motion is somewhat different and will be described below. Such a cavity formed behind an aluminum sphere dropped through an air-water interface is shown in Figure 7. A final example of a steady-state cavity, which might be considered as associated with water-to-air entry, is shown in Figure 8. This figure shows a towed float in the shape of a blunt, symmetrical airfoil of

Figure 6 - Cavitation Produced in a Highly Viscous Syrup by a Receding Piston—After Harvey et al, Reference 11

very low aspect ratio during emergence from a submerged condition to a planing condition.¹³ In Figure 8a the float is running in a cavity near the free surface; this is an unstable condition and a small perturbation causes breakdown of the flow over the float to a more stable type associated with planing in which the water is thrown forward and to the sides. From a theoretical standpoint, these two types of flow are closely related.^{14, 15, 60}

Figure 7 - Cavity Formed During Air-Water Entry of an Aluminum Sphere

Figure 8a

Figure 8b

Figure 8 - TMB Planing Float in the Shape of a Symmetrical Airfoil of Low Aspect Ratio in Submerged and Planing Conditions

The forward speed of the float is the same in each photograph (see Reference 13)

SOME EXAMPLES OF THE EFFECTS OF CAVITATION

The effects of cavitation on efficiency of hydraulic machines and the damage associated with cavitation are well known. It will be sufficient here to give only an example or two of these effects. The loss of efficiency in pumps and turbines with cavitation onset is discussed in many publications (see e.g., References 16 and 17). Cavitation damage on turbine runners and pump impellers, on baffle piers in the stilling basins below dams, on the face of dams, and on large needle-valve installations are described in Reference 7. Illustrations of the effects of cavitation on the efficiency of marine propellers will be found in References 18, 19, and 20. An example of data taken with a propeller model during a characterization test in a TMB water tunnel is shown in Figure 9, in which the thrust coefficient K_T and torque coefficient K_Q are plotted as functions of an "advance" coefficient J with the cavitation number σ as parameter.

Here,

$$K_T = \frac{T}{\rho n^2 d^4}$$

$$K_Q = \frac{Q}{\rho n^2 d^5}$$

$$J = \frac{U}{nd}$$

$$\sigma = \frac{P - p_v}{\frac{1}{2} \rho U^2}$$

where T is thrust,

Q is torque,

n is rotation rate,

d is propeller diameter, and

p_v is vapor pressure.

Figure 9 - Propeller Thrust and Torque as Functions of Speed Coefficient with Cavitation Number as Parameter

The cavitation number, which is a similitude parameter, will be discussed somewhat further in connection with modelling of cavitating systems. The magnitude of the cavitation number is

an indication of the degree of cavitation or of the tendency to cavitate. Thus, the lower the cavitation number, in general, the greater the volume of cavitation. It is clear from Figure 9, then, that the greater the "amount" of cavitation, the greater the loss in thrust and torque.

An example of damage to a marine propeller subjected to cavitation for a few days of service is shown in Figure 10. In this case there is actually erosion of the metal. Figure 11 shows cavitation on several concrete specimens tested by the Bureau of Reclamation. The form of the damage will, however, depend upon the material; lead, for example, exhibits deformations of the surface without appreciable loss of material. This point will be discussed further in connection with the mechanism of damage.

SOME IDEAS ON THE INCEPTION OF CAVITATION

PRELIMINARY REMARKS

As mentioned in the foregoing, for most engineering applications, inception of cavitation may be expected when the local pressure drops to the vapor pressure corresponding to the temperature of the liquid. However, this is only an approximation which, while useful for design purposes, requires careful qualification. Depending upon the condition of the liquid (air or solid matter content, etc.), cavitation may begin above or below the vapor pressure. It is well known that clean, degassed liquids can withstand large tensions or may be superheated considerably without formation of cavities or inception of boiling. It is now generally accepted that the inception of cavitation in a flowing liquid or boiling in a heated liquid originates with the formation of gas or vapor nuclei. Consequently, it is of interest to examine the kinetics of formation of such nuclei in connection with cavitation resulting from pressure reduction. For discussions of inception of boiling in heated liquids, reference may be made to References 21 and 22, for example.

TENSILE STRENGTH OF LIQUIDS - RESULTS OF RECENT ATTEMPTS TO FORMULATE A KINETIC THEORY OF CONDENSED SYSTEMS

Since the concepts which are of interest to the present discussion are an outgrowth of attempts to determine the tensile strength of liquids

Figure 10 - Cavitation Damage on a Marine Propeller

Figure 11 - Cavitation Erosion on Concrete Specimens—From
Bureau of Reclamation Photographs

and to explain the wide discrepancies between theoretical and observed results, the question of cavitation in a liquid will first be discussed without consideration as to whether the liquid is flowing or at rest. On the assumption that a liquid is fractured when the gap between molecular layers is of the order of one molecular radius, computation of the work required to bring about and maintain this separation gives values of the order of 10,000 atmospheres as the breaking strength of water.²³ This result is known to be at least an order of magnitude too high. Various experimenters have reported values ranging from 4 atmospheres to estimates as high as 1,000 atmospheres. However, it is now recognized that these discrepancies are associated with the air and particle contents of the specimens, the treatment of the liquid before testing, and the experimental methods used. For a critical examination of the various methods and the results reported, reference may be made to Reference 24.

Although a completely satisfactory theory of the liquid state still remains to be formulated, it seems worth while to mention some of the results of recent attempts to develop a kinetic theory of liquids. Recent work in the "theory of nucleation" in condensed systems has led to clearer concepts of the processes accompanying the formation of cavities in liquid, and leads to more realistic estimates of the tensile strength of liquids. Although the methods of nucleation theory apply both to the problems of cavity formation as well as condensation, i.e., to phase transformations in general, these two aspects of the cavitation process are here discussed separately. It is the view of recent kinetic theories of the liquid state that the formation of vapor nuclei proceeds from thermal or density fluctuations* in the interior of a liquid, and that cavity growth takes place one molecule at a time as a result of these statistical thermal fluctuations. If these holes or cavities reach a critical size, they will continue to grow. The statistical conditions for such growth have been investigated by Turnbull, Fisher, and Holloman^{25, 26, 27} among others, specifically for condensed systems, although similar methods have been used to a much greater extent in the problem of condensation of supersaturated vapors. The latter work is mentioned later. A more recent formulation of a theory of condensed systems is given in Reference 28. The results given here on cavity formation are taken primarily from the work of References 23, 25, 26, and 27.

To establish the critical size of cavity required for growth, we compute the work associated with the reversible formation of a vapor bubble. The work of formation of a cavity of volume V is pV . The work required for the formation of the liquid-vapor interface of area A bounding the bubbles is γA where γ is the liquid-vapor surface tension. Finally, the work required to fill the bubble reversibly with vapor at pressure p_v is $-p_v V$. Thus, the net-work of formation of a spherical bubble of radius r is

$$W = 4\pi r^2 \gamma + \frac{4}{3} \pi r^3 (p - p_v)$$

For high negative pressures, the vapor pressure p_v may be neglected in comparison with p , so that the expression for the work simplifies to $W = 4\pi r^2 \gamma + \frac{4}{3} \pi r^3 p$. The work is a maximum at

$$W_{\max} = \frac{16\pi\gamma^3}{3p^2}$$

*To be precise, one should speak of "energy fluctuations." However, for the present purpose, the above terminology is believed to give a clearer physical picture.

for bubbles with radius $r^* = -2\gamma/p$ (see Figure 12 - after Reference 25). In order to grow, cavities with radii less than r^* require free energy of activation† for the motion of an individual molecule of liquid past other molecules into or away from the surface. However, those with radii larger than r^* grow

Figure 12 - Work Required for Reversible Formation of a Vapor Cavity in a Liquid under Tension

with decreasing free energy. Since the growth of such cavities is a statistical process, cavities with radii less than r^* will usually disappear without reaching the critical size. On a statistical basis, however, there will be a certain (small) number of cavities for which the thermal fluctuations are such as to produce a cavity exceeding the critical radius. When this happens, the supercritical cavity will grow until the pressure of the system rises to the vapor pressure of the liquid. It has been shown^{23, 26, 29}, that the rate of formation of such cavities is proportional to $e^{-W_{\max}/kT}$ where k is Boltzmann's constant and T is the absolute temperature of the process. The proportionality factor is estimated from the theory of absolute reaction rates²⁹ to be $\frac{NkT}{h} e^{-\Delta f_o^*/kT}$ where n is the number of molecules in the liquid, Δf_o^* is the free energy and h is Planck's constant. With these results, Fisher²⁵ writes the equation for the rate of formation of bubbles of vapor in a mole of liquid subjected to negative pressure p :

$$\frac{dn}{dt} = \frac{NkT}{h} e^{-(\Delta f_o^* + W_{\max})/kT} = \frac{NkT}{h} \exp \left\{ -\left(\Delta f_o^* + \frac{16\pi}{3} \frac{x^3}{p^2} \right) / kT \right\}$$

† "Free energy of activation" is associated with the formation of a region of a new phase of a substance and is proportional to the surface energy, the bulk free-energy in the absence of stress or surfaces, and the strain energy.²⁶

where N is Avogadro's number. Assuming for the fracture pressure that pressure which gives one bubble in t seconds, i.e., $\frac{dn}{dt} = \frac{1}{t}$, the following pressure is found upon integration:

$$p_t = - \left[\frac{16\pi}{3} \frac{\gamma^3}{kT \ln NkT t/h - \Delta f_0^*} \right]^{1/2}$$

Values of the fracture pressure p_t corresponding to a number of waiting times and several values of Δf_0^* indicate insignificant differences for rather large waiting periods. For computations of the rupture pressure, Fisher states that the free energy of activation away from or toward a surface may be estimated from the free energy of activation for viscosity since the two should be approximately equal. However, since computations of this quantity for viscosity indicate that the value lies between 0 and 5000 cal./mole for most liquids at room temperature,²⁵ the error in putting $\Delta f_0^* = 0$ gives a value of p_t "generally less than 5 percent and almost certainly less than 10 percent too small for room-temperature liquids." Thus, Fisher finally obtains the result (with $t = 1$, $\Delta f_0^* = 0$):

$$p = - \left[\frac{16\pi}{3} \frac{\gamma^3}{kT \ln NkT/h} \right]^{1/2}$$

For water, this gives a value of -1320 atmospheres at 300° K, which, although still greater than observed results, is an order of magnitude lower than the value predicted on the basis of molecular forces. Fisher has carried the computation further to investigate the formation of cavities at liquid-solid interfaces, and shows that much lower tensions are required for rupture.

Although the free energy is proportional to the viscosity of the liquid, Fisher found that the pressure for rupture did not vary significantly with this quantity and so neglected it in his numerical computations. On the other hand, Zeldovich³⁰ found that the rate of formation is determined by the viscosity, but that it is not important for water and gives values somewhat higher than Fisher. That the breaking strength does vary with viscosity was demonstrated by Briggs, Johnson, and Mason.³¹ For the range of variation of the viscosity of water, however, this variation in strength may be neglected.

The details of the theory leading to the above results are beyond the intended scope of this survey. However, despite the fact that the results were carried out for liquids at rest and are still of little value for engineering use, they have been included to illustrate a type of analysis which may eventually lead to more precise estimates of the pressures for cavitation in flowing liquids. Although, as pointed out above, the assumption that cavitation begins at the vapor pressure corresponding to the temperature of the

liquid is sufficiently precise for most engineering applications, in the laboratory and in some technical applications, it has become important to be able to predict more precisely the pressure at which cavitation may begin. It is clear that the problem of inception in a flowing liquid may be much more complicated than that in a fluid at rest because of the uncertainties both of the role of turbulence and of the processes in the presence of dissolved and entrained air and solid particles.

SOME REMARKS ON THE ROLE OF TURBULENCE IN CAVITATION INCEPTION

With regard to the role of turbulence in cavitation inception, it is of some interest to know whether the velocity fluctuations in a turbulent flow can give rise to pressure fluctuations that are large enough to bring about momentary inception at a mean pressure higher than that required by a prediction based on average velocity. An estimate of the effect of turbulence can be made in the following way. Suppose that for a given system, it has been predicted that cavitation will begin when the cavitation number

$$\sigma = \frac{P - p}{\frac{1}{2} \rho U^2} \quad [6]$$

drops to a predetermined value. Here p is the local pressure. With a turbulent pressure fluctuation, the instantaneous cavitation number will be

$$\sigma' = \frac{P - (p + p')}{\frac{1}{2} \rho U^2} = \sigma - \frac{p'}{\frac{1}{2} \rho U^2} \quad [7]$$

Thus, depending on the value of p' , inception may occur momentarily at a value of σ higher than that predicted as critical. It remains, then, to estimate the magnitude of $p' / \frac{1}{2} \rho U^2$.

From computations based on models of isotropic turbulence,* G.I. Taylor³² found that

$$\sqrt{p'^2} = \frac{3}{2} K \rho u'^2 \quad [8]$$

where u' is the local velocity fluctuation. The value of K as given by Taylor and by Green³³ for a number of models, is of the order of one. Now for a Gaussian distribution, p' may be expected to be about $3\sqrt{p'^2}$ for a small percentage of the time and $2\sqrt{p'^2}$ for a rather large percentage of the time.

*The writer is indebted to Dr. J.V. Wehausen for his suggestion of these results in estimating the pressure fluctuations in preference to a method originally used by the writer. On the basis of similar considerations, Wehausen had also reached the conclusions given herein.

Taking the smaller value, the instantaneous fluctuation may be

$$p' = 3\rho \overline{u'^2}$$

or

$$\frac{p'}{\frac{1}{2}\rho U^2} = 6 \frac{\overline{u'^2}}{U^2} \quad [9]$$

The instantaneous cavitation number is then

$$\sigma' = \sigma - 6 \frac{\overline{u'^2}}{U^2} \quad [10]$$

The effect of turbulence in the boundary layer of a submerged form can be estimated from flat-plate results. In Reference 34 values of $\sqrt{\overline{u'^2}}/U$ are given which, near the plate, may be of the order of 0.15 in a region of adverse pressure gradient and at Reynolds number of the order of 10^7 . For this case, the instantaneous cavitation number may be 0.14 lower than the average value. Thus, in a highly turbulent region, a "microscale" cavitation might occur in a random fashion well before it can be observed, and suggests the possibility of hearing cavitation before it is seen.

THE ROLE OF "SURFACE-ACTIVE" MATERIALS

The further problem of liquids contaminated with materials which change the effective surface tension may be treated for liquids at rest by the methods described above.^{23, 25 - 30} Such "surface-active" materials (see, e.g., Reference 35) may change the surface tension at surfaces in the interior of liquids or at boundaries by large amounts for extremely small concentrations (of the order of less than 1 percent). On the other hand, there are surface-active materials which are adsorbed on the surface of materials and act to increase the interfacial tension. A discussion of surface activity from the point of view of the mechanism of adsorption is given in Reference 23.

The presence of such surface-active materials complicates the question of cavitation inception in the presence of a solid boundary or of suspended materials. It has generally been assumed that small solid particles promote the formation of cavities and, on this basis, such particles or "chips" have been used in boilers to prevent super-heating of water. It has further been assumed that the greater the number of points on these chips, the greater the ability to promote boiling. On the basis of the mechanism of cavity formation discussed above there is little reason to suppose that such chips should

promote boiling, unless there is already adsorbed air or surface-active materials on these chips which have little or nothing to do with the number of points. That this conclusion is valid has been demonstrated in the experiments of Dean³⁶ and Jakob²² among others. On the other hand, deep crevices with sharply pointed bottom contours are positions where vapor phase may easily occur. Harvey, et al¹¹ showed by a series of experiments that "hydrophobic" surfaces cavitate more easily than "hydrophilic" surfaces; these experiments evidently demonstrate the role of surface-active materials. The latter experiments also showed very high tensile strength of water saturated with air but completely de-nucleated.

TENSIONS IN FLOWING WATER AND EFFECTS OF AIR-CONTENT ON CAVITATION INCEPTION

Although it is clear that the pressure for fracture of a liquid may be raised toward vapor pressure rather easily, the presence of large amounts of entrained air and of surface-active agents may raise the cavitation pressure to well above the vapor pressure. In the oceans, for example, the presence near the surface of plankton and other marine growth and of a large amount of entrained as well as dissolved air will promote cavitation. Experiments by Crump³⁷ have shown cavitation inception at pressures corresponding to as much as 5 feet of water above vapor pressure, Figure 13.

Figure 13 - Cavitation Inception in Sea Water—From Reference 37

The effects of air content on cavitation inception in flowing water has been the subject of many investigations. It will be sufficient here to discuss the experiments carried out by Crump³⁷ and Numachi and Kurokawa.^{38, 39} Using a smooth nozzle with a very gradual diffusor, Crump was able to obtain tensions as high as 60 feet of water for an air content of 5 cm³ per liter of water, and showed a general increase of tension with decreasing air content, Figure 14. On the other hand, Numachi and Kurokawa showed somewhat different results although a careful analysis of their results indicated that they also obtained tensions.^{37, 39} Whereas Crump's experiments showed a decrease in critical pressure with increased absolute velocity, Numachi's results showed an opposite trend. An explanation of Numachi's results is rather easily formulated, but the reasons for Crump's results are quite obscure. It was at first thought that the trend observed in Crump's experiments was related to the time

Figure 14 - Cavitation Inception in Fresh Water of Varying Air Content—From Reference 37

of exposure of the liquid to pressures below vapor pressure. However, an examination made by the writer and Mr. Crump of the original data of Reference 37 reveals that the liquid was exposed to tensions for times that increased with increasing absolute velocity. Furthermore, the times of exposure were of the order of 10^{-3} seconds—apparently sufficient, in any case, for nuclei to reach critical size before being carried into the high-pressure regions. Thus, the apparently anomalous result was obtained that the higher the tension the longer the water could withstand the tension before rupture. It would appear that the water was well de-nucleated even with relatively high air content although no special methods were used in treatment—the only treatment being settling of the water for several hours before each experiment. However, these trends can often be obtained in this nozzle even after the water has been disturbed although under these conditions they can be maintained for only short periods. It would be of interest to conduct similar experiments in which the time of exposure is held constant but the velocity and, thus, the pressure reduction varied.

In the case of the experiments of Numachi and Kurokawa, a nozzle with an abrupt expansion was used so that a vortex was induced in the separation zone just downstream of the nozzle. Although the ambient pressure rises at this location, the pressures at the core of the vortex might fall well below the pressure in the minimum section of the nozzle. Thus, with increasing stream velocity, the strength of the vortex may be expected to increase with resultant decreasing core pressure.^{8, 40} Since the fluid in the separation zone is not swept away as rapidly as that in the main stream, the results obtained by Numachi are quite tenable as to trend despite other objections as to the size of nozzle used and to his method of analysis. If the actual pressure at the vortex core could have been measured and used as the inception pressure rather than the pressure in the constriction, the apparent discrepancy might have been resolved. Mr. Crump has agreed with this explanation (see Reference 37), and, at the writer's suggestion, has conducted experiments with a model of the Numachi nozzle (to be reported) to test this hypothesis. He has found that, although the numerical results differ somewhat, the trend is as predicted above. The possibility of cavitation occurrence in separation zones at pressures above those predicted by analysis of the boundary is discussed more fully on page 47 in connection with designing for cavitation prevention.

CONDITIONS IN A BOUNDARY LAYER AS RELATED TO INCEPTION OF CAVITATION

Although some remarks have been made as to the effects of viscosity on cavitation, both in a stationary fluid and as manifested in the separation zone of a flowing fluid, a small point as to the effect of viscosity—in connection with the boundary layer in the flow of real liquids over a surface without separation—will be cleared up here. In the Prandtl theory of the boundary layer, it is shown that the pressure change across the boundary layer is a second-order effect and that the pressure at the "edge" of the boundary layer is very nearly that at the solid surface, and as a matter of fact, this is confirmed by experiment. However, the question has arisen from time to time whether the actual pressure increases or decreases across the boundary layer and, thus, whether cavitation will occur at the surface always or whether it might begin just outside the boundary layer. This question is especially of interest for thick boundary layers either from the standpoint of the surface itself or of an appendage fixed to the surface and, thus, inside its boundary layer. The question can be answered for laminar boundary layers from an examination of the Blasius solution for the laminar boundary layer on a flat plate. Taking the Navier-Stokes equation for two-dimensional, incompressible flow

$$\left. \begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \end{aligned} \right\} \quad [11]$$

and the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad [12]$$

where u is the component of velocity in the x -direction,

v is the component in the y -direction, and

ν is the kinematic viscosity,

and using an order-of-magnitude argument, Prandtl⁴¹ obtains the boundary layer equations:

$$\left. \begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \end{aligned} \right\} \quad [13]$$

Blasius obtained a solution of Equations [13] for a flat plate, i.e., $\frac{\partial p}{\partial x} = 0$, in the form (see Reference 41):

$$u = Uf'(\eta) \quad [14]$$

$$v = \frac{1}{2} \sqrt{\frac{\mu U}{x}} (\eta f' - f) = \frac{\nu}{2} \frac{\eta}{y} (\eta f' - f) \quad [15]$$

$$\eta = y \sqrt{\frac{U}{\nu x}} \quad [16]$$

where $f(\eta)$ is the solution of the equation

$$ff'' + 2f''' = 0 \quad [17]$$

Here, x is the distance from the leading edge of the plate, and the primes indicate differentiation with respect to η . Since the pressure distribution in the neighborhood of a region likely to cavitate (at least on bodies of fairly high length-diameter or length-beam ratios) is such as to approximate the boundary conditions assumed above, the pressure across the boundary layer should behave at least qualitatively as predicted from this solution. Substituting the results [14] through [17] into the second of Equations [11], the pressure gradient across the boundary layer is

$$\frac{\partial p}{\partial y} = \frac{1}{2} \rho U^2 R_x^{-1/2} f'' \quad [18]$$

where

$$R_x = \sqrt{\frac{Ux}{\nu}}$$

From the asymptotic expansion for the solution of [17] Blasius gives the value $f'' = 0.332$ for $\eta = 0$. Thus,

$$\frac{\partial p}{\partial y} > 0 \quad [19]$$

across the boundary layer. It follows, therefore, that the pressure decreases across the boundary layer toward the body. This result indicates that in unseparated flows with a laminar boundary layer, cavitation will always occur at the body and not outside the boundary layer. While the above result applies to two-dimensional flows, and, thus, approximately, to thin struts, the same result is obtained for bodies of revolution. In the latter case, it can be shown that the pressure gradient across the boundary layer is not second order

and that one of the boundary layer equations is

$$\kappa u^2 = \frac{1}{\rho} \frac{\partial p}{\partial y} \quad [20]$$

where κ is the curvature of the surface and y is the direction normal to the surface.⁴² That the same result is obtained here is seen directly from Equation [20].

Although satisfactory theoretical formulations of the value of the pressure gradient across turbulent boundary layers are not available, measurements made by Lyon⁴³ in the turbulent layer of streamline bodies give the desired results. In regions of negative pressure, it was found that $\frac{\partial p}{\partial y} > 0$ and in regions of positive pressure, $\frac{\partial p}{\partial y} < 0$. Thus, here, again, for unseparated turbulent boundary layers, cavitation will occur at the body and not outside the boundary layer. This result must, of course, be qualified in view of Equation [10], since the maximum turbulent velocity fluctuations occur at a small distance from the wall or body.³⁴

NOTE ON THE PROPAGATION OF CAVITATED REGIONS

So far, an attempt has been made to give an accurate qualitative description of the process involved in the inception of cavitation. Nothing has been said as to the propagation of a cavitated region. It is not intended to discuss this problem in detail. However, an analysis of this problem has been made by Kennard⁴⁴ and the results are summarized here. Assuming irrotational motion of an ideal liquid, Kennard assumed that the pressure falls to a given rupture-pressure and then rises immediately to a fixed cavity pressure. He found that "the boundary of the cavitated region either advances as a breaking-front, moving with supersonic velocity, or remains stationary as a free surface, or recedes toward the cavitated region as a closing-front." The various conditions for these motions and the pertinent formulas are given in Reference 38.

DYNAMICS OF TRANSIENT CAVITATION BUBBLES

THE MOTION OF SMALL TRANSIENT CAVITIES

Cavitation nuclei which grow beyond the critical size, reach a maximum, collapse and then undergo oscillations while maintaining their identity as small, individual cavities before disappearing, have been characterized herein as "transient" cavities. Studies of the motion of such cavities have been made by various investigators. Mueller⁴⁵ published motion picture frames in 1928 showing cavitation on a hydrofoil. These photographs showed clearly the growth and collapse of individual cavities, but evidently no oscillations

were observed. Subsequently, oscillations of such cavities were reported by Briggs et al,³¹ Kornfeld and Suvorov,¹² Harvey¹¹ and Osborne.⁴ The first two groups of experimenters used sound fields and magnetostriction oscillators, respectively, so that it is not clear that the oscillations observed were associated with a self-excitation. Harvey and his colleagues, however, using rods withdrawn rapidly from a liquid, observed several oscillations of the resultant cavity and attributed them to energy storage in air entrained in the bubble.

The first definitive results on the oscillations of such cavities were obtained by Knapp and Hollander,⁴⁷ who succeeded in photographing cavities in a flowing liquid through several cycles of the motion. A strip from the films obtained by these investigators is reproduced in Figure 15, which clearly shows two cycles of the motion—the initial growth and collapse and the subsequent rebound, growth, and collapse. Figure 16 from Reference 47

Figure 15 - Oscillation of a Cavitation Bubble in a Flowing Liquid—From Knapp and Hollander, Reference 47

shows the time history of the life of a bubble. It will be seen that the bubble radius is greater than the average radius for the largest part of each cycle. Knapp and Hollander showed by a rather convincing argument, that in their experiments the initial air content of the cavities was extremely small for cavities in the range of their observations and that the cavities contain mostly water vapor. They attribute the rebound of the cavity primarily to the storage of energy in the liquid in elastic compression, with this stored energy subsequently producing the outward radial velocity. As has been suggested by Harvey and others,⁴⁸ however, an oscillating cavity may grow by diffusion of gas into the cavity from the liquid with the cavity continually gaining gas molecules, because of the larger surface area and longer time available during that part of the cycle for which the radius of the cavity is greater than the average radius.

Figure 16 - Time History of the Life of a Cavitation Bubble—From Knapp and Hollander, Reference 47

DIFFUSION OF AIR INTO AN OSCILLATING CAVITY

The diffusion of air into a cavity of the type considered here has been studied analytically by Herzfeld and Pode and some results are given by Pode in a dissertation, Reference 49. Assuming that the dynamics of the motion of the cavity is not materially affected by the diffusion process, that the gas cycle is adiabatic, that the gas density is uniform, and that the diffusion process is confined to a thin surface layer, Pode shows that, for motions in which the maximum amplitude is double the equilibrium radius, the order of the average rate of change of number of air molecules within the cavity is

$$\frac{\overline{dN}}{N_1 dt} \approx \frac{10^{-6} \text{ cm}^2 \text{ sec}^{-1}}{\delta R_1}$$

Here, δ is the thickness of the diffusion layer,

R_1 is the average radius of the cavity,

N is the number of air molecules in the bubbles, and

N_1 is the initial number of air molecules.

A further result is that the diffusion into the cavity increases with increasing ratio of maximum amplitude to equilibrium radius. From the above result it is seen that, on the average, the cavity continues to gain air molecules,

i.e., that the average diffusion into the cavity is one-sided. The significant conclusion that may be reached from the experimental and theoretical work cited above is evidently that air or gas diffused into a transient cavity plays an increasingly important role as the number of cycles increases, and is of especial importance in determining the maximum pressures that can be developed at the time the minimum radius is reached. For sea water, with its high air content (both dissolved and entrained—especially near the surface), the presence of air in large quantities in even the initial stages of the motion is a distinct possibility. Furthermore, the change in the properties of the water through action of surface-active agents may assist the diffusion of gases into the bubbles.

ANALYTICAL DESCRIPTION OF THE MOTION

The hydrodynamics of the motions of individual cavities have been analyzed for parts of the cycle by various writers. Lord Rayleigh⁵⁰ considered the problem of the collapse of a spherical cavity in a homogeneous, incompressible fluid, the cavity supposed empty and the pressure at an infinite distance remaining constant. Plesset⁵¹ extended these computations to include the case of a single empty cavity moving in a field in which the external pressure is a function of the time (corresponding to the cavities of Knapp and Hollander's experiments). Since Plesset's computation includes the Rayleigh case, only the former will be discussed here. The radius of the cavity at any time t is taken as R , and the distance to any point in the liquid as r , the origin of coordinates being chosen at the center of the sphere. The velocity potential ϕ for the motion of a liquid with spherical symmetry is given by Lamb⁵² as:

$$\phi = \frac{R^2 \dot{R}}{r}$$

where $\dot{R} = \frac{dR}{dt}$. The equation of motion for irrotational, incompressible flow is

$$-\frac{\partial \phi}{\partial t} = \frac{1}{2} (\Delta \phi)^2 + \frac{p(r)}{r} = \frac{P(t)}{\rho}$$

where $p(r)$ is the pressure at r ,

$P(t)$ is the pressure at infinity, and

$\Delta \phi$ is the gradient of ϕ .

Using these equations, the following are obtained:

$$(\Delta\phi)^2 = \frac{R^4 \dot{R}^2}{r^4}$$

$$\frac{\partial\phi}{\partial t} = \frac{1}{r} (2R\dot{R}^2 + R^2\ddot{R})$$

Evaluating the last equation at $r = R$,

$$\left(\frac{\partial\phi}{\partial t}\right)_{r=R} = 2\dot{R}^2 + R\ddot{R}$$

$$(\Delta\phi)_{r=R}^2 = \dot{R}^2$$

With these results, Plesset obtains the equation of motion of the surface of the cavity as follows:

$$\frac{1}{\rho} [p(R) - P(t)] = \frac{3}{2} \dot{R}^2 + R\ddot{R}$$

Rayleigh's case is obtained by putting $P(t) - p(R) = P_0 = \text{constant}$. Plesset takes $p(R) = p_v - \frac{2\gamma}{R}$, and thus includes the effects of surface tension. Taking as boundary conditions, $R = R_{\text{MAX}}$ at $\dot{R} = 0$, Plesset integrated the equation of motion numerically, for the pressure field corresponding to the conditions of the experiments of Knapp and Hollander. The results of the analysis for one cavity are shown in Figure 17 which is taken from Reference 51. It is understood that Plesset has attributed the discrepancies on the right side of Figure 17 to wall proximity and has successfully accounted for them on this basis.

It is clear that analyses based on empty cavities in an incompressible liquid cannot give rise to oscillations. Furthermore, with constant external pressure, the velocity $\dot{R}$ increases without limit as the bubble collapses. To obviate this result, Rayleigh extended his computations to include the case of a cavity filled with a gas which is expanded and compressed isothermally and showed that the boundary oscillates between two positions, of which one is the initial position. Although the motion of the oscillating cavity in a real liquid is evidently complicated by the diffusion

Figure 17 - Comparison of Theoretical and Measured Cavity Radii from Plesset's Analysis—Reproduced from Reference 51

and vaporization processes and the problems of energy dissipation, such solutions, which are clearly over-simplifications, nevertheless give a clear quantitative picture of the hydrodynamics of the motion as long as the cavity radius is large compared with the minimum radius. Further extensions to include the problems of energy dissipation associated with the compressibility of the vapor and gas mixture and of the liquid would be of great practical as well as theoretical interest. For a bubble filled with a permanent gas being compressed and expanded adiabatically rather than isothermally, Rayleigh's case coincides with the pulsations of a gas-globe following an underwater explosion.⁵³ Although, in the latter problems, much progress has been made in describing an oscillatory motion with energy dissipation, the problem of the vapor condensation and formation prevents a complete analogy to gas-globe theory and a satisfactory description of the motion of cavities on the basis of this theory.

An accurate knowledge of the processes at collapse of cavities and of the associated pressures is still lacking, and is of importance in connection with cavitation damage. (This point is discussed somewhat further in subsequent sections.)

MECHANISM OF STEADY-STATE CAVITIES AND THEIR ANALYTICAL DESCRIPTION STEADY-STATE CAVITIES IN REAL LIQUIDS

It is characteristic of some cavitating flows that the cavity appears as a large, smooth surface stationary in space such as that shown in Figure 2. Such cavities often appear to be filled only with vapor or gas, and, except for oscillations near the end of the cavity, the shape of the surface does not vary with time. On the other hand, a cavitating region made up entirely of small "transient" bubbles may exhibit the properties of a steady cavity in that the average envelope of such a region does not vary with time.* Another case of such "steady-state" cavities in which the average envelope remained unchanged, but in which violent surface fluctuations were observed without evidence of individual bubbles, was studied by the writer and Mr. H.L. Pond.⁴⁰ These studies, which were an extension of work carried out by H. Reichardt,⁵⁴ will be discussed herein. Finally, the cavity illustrated in Figure 7 is included in this part of the discussion since, as will be shown,

*It is not possible, at present, to predict with certainty whether cavitation on a curved surface will occur in the form of a large steady state cavity or a mass of small, oscillating cavities. It seems clear that the appearance of the cavitation is associated with the pressure gradients, but no satisfactory criteria are available as to the initial appearance or possible transition from transient cavities to a large, steady state cavity.

such a cavity behaves at a given instant of its motion in the same way as the steady-state cavities mentioned above. A photograph obtained by Worthington in 1908—of the air-water entry cavity of a sphere at a later stage than that of Figure 7, i.e., after the upper cavity has been detached—is shown in Figure 18, which is taken from Reference 55. In this case, the cavity has closed at the surface and a jet is seen to be formed inside the cavity. It will be observed that the jet is of sufficiently high velocity to penetrate the wall of the cavity, the edge of the position of the jet in the liquid being delineated evidently by entrapped air. Since the life history of air-water entry cavities is not of direct interest to the present discussion, details are not pertinent here. Descriptions of the motion in the entry phase, with the associated splash, the subsequent formation of a cavity extending to the surface, first growing and then closing up, and finally the condition in which a portion of the cavity may become detached with the missile continuing to fall with an attached cavity, are given in References 14 and 56.

An examination of the cavitation number defined previously

$$\sigma = \frac{P - P_v}{\frac{1}{2} \rho U^2}$$

indicates that its magnitude is a function of the vapor pressure. However, for steady-state cavities, it makes little difference whether the pressure within the cavity is vapor pressure or some other pressure. On this basis, in order to obtain cavities of very low cavitation number, Reichardt injected air into the region behind a disc, thus forming cavities with internal pressures well above vapor pressure. Sketches of these cavities were made by Reichardt and are reproduced in Figure 19 from Reference 54. In these experiments, Reichardt used discs and cones which were about 3/8-inch in diameter. Such small models were necessary to avoid distortion due to the buoyancy of the air-filled cavity. A photograph taken in a TMB water tunnel of the cavity

Figure 18 - Air-Water Entry Cavity
After Surface Closure—After
Worthington, Reference 55

Figure 19 - Cavities Formed by the Emission of Air Behind a Disc—After Reichardt, Reference 54

formed behind a disc 1 1/2 inch in diameter when air is injected is shown in Figure 20. The distortion of the cavity due to buoyancy is marked. It will be observed that the surface of the cavity is quite smooth and transparent. Cavities which are maintained by vaporization, on the other hand, and look essentially smooth to the eye, reveal a very distorted, almost opaque surface that is oscillating rapidly.* In Figure 21a is shown a photograph taken in a TMB water tunnel of a cavity formed behind a disc taken with an exposure time of about 2 seconds. The same cavity, with an exposure time of 1/10,000 second, is shown in Figure 21b. The latter photographs were taken during the studies reported in Reference 40.

It appears from the results so far

available that there are essential differences between the air-filled and vapor cavities in the details of the mass flows within the cavity surface. But there is little difference in the average resultant shape of the cavity surface.

The studies of Reference 40 revealed that some care must be exercised in the definition of the flow configuration that is to be considered a steady-state cavity flow for cavitation about blunt obstacles. On such forms, cavitation may occur in the vortices induced in the zone of separation of flow or along the boundary of this zone before it occurs at the point of minimum pressure itself (as pointed out previously). Thus, at high cavitation numbers, cavitation may be observed in the cores of vortices which are shed periodically from the model, and which appear to form a well-defined cavity when viewed by the eye or seen in a photographic exposure of long duration. This situation is illustrated in Figure 22.

*There is a difference of experience on this point among various investigators. Whereas the writer's experience with experiments in the TMB tunnels is that the bubble surface is clear only when the cavity contains large amounts of air, others have reported that when it is clear it contains only water vapor. The writer has also observed clear cavities in the California Institute of Technology tunnel which, in the opinion of Dr. R.T. Knapp, the Director of the CIT laboratory, were completely vapor filled. The reasons for this anomaly are not clear.

Figure 20 - Air-Filled Cavity Formed Behind a Disc

Figure 21a - $\sigma = 0.188$, Exposure Time: 2 Seconds

Figure 21b - $\sigma = 0.188$, Exposure Time: 1/10,000 Second

Figure 21 - Cavity Behind a Disc

It is possible to define the limits of the flow configuration that behaves as a steady-state cavity on the basis of the correlation between average cavity shape and cavitation number and of results showing that cavity shape is a single-valued function of the cavitation number.⁵⁴ The relation between the maximum ordinate of bubbles formed behind a disc and the cavitation number is shown in Figure 23. It is seen that above a certain cavitation number, the data show a large scatter with no apparent correlation, while below this region the data fall on a single curve. It is difficult to determine the process that takes place during the transition. However, it appears from an examination of photographs of the region titled "vortex cavitation" in Figure 23, that cavitation in this region is intermittent as a result of the shedding process. In the region of the lower cavitation numbers, however, there is evidence that the zone of cavitation is more or less continuous although violent oscillations may still be observed.

Figure 22a - $\sigma \approx 0.5$, Exposure Time: 2 Seconds Figure 22b - $\sigma \approx 0.5$, Exposure Time: 1/10,000 Second

Figure 22 - Vortex Cavitation Behind a Disc

Figure 23 - Relation Between Maximum Cavity Ordinate and Cavitation Number for Cavitation Behind a Disc—From Reference 40

Pressure measurements taken along the center line of various cavities are shown in Figure 24. The measurements in the upper part of Figure 24 were made in cavities within the region of "vortex" cavitation. The remaining measurements were made in cavities of progressively lower maximum cavitation number. It will be observed that for the highest cavitation number, the

Figure 24 - Pressure Distribution Along Axis of Cavities Formed Behind 1 1/2-Inch Disc

minimum pressure is well above vapor pressure. For the lower cavitation numbers, the pressure falls below vapor pressure becoming progressively lower until for the smallest cavitation number it begins to rise again toward vapor pressure. The steep rise in pressures appeared to coincide roughly with the "average" end of the cavities. The form of the pressure distribution for the measurements in the second set from the top of Figure 24 indicates the presence of a stagnation point. However, the absolute magnitude of the pressures are probably not correct because of the manometric system used (see Reference

40). Nevertheless, this is of interest in connection with the mathematical theory of such cavities discussed below. Although a satisfactory quantitative description of the processes involved in the motion and maintenance of such cavities is still lacking, the tentative conclusions reached in Reference 40 are summarized below.

If it is assumed that the temperature of the vapor will be the same as the temperature at the surface supplying the heat of vaporization, the pressure of the vapor will be lower than that corresponding to the temperature in the interior of the liquid. On this assumption, the minimum pressure measured requires a surface-temperature drop of the order of 2° to 3° C. However, that this situation can actually occur has no well founded theoretical basis. For systems in equilibrium, results of kinetic theory of gases indicate that the molecules escaping into the vapor phase will be slowed down into a maxwellian distribution of velocities corresponding to the temperature of the liquid from which they came.⁵⁷ However, the violent oscillations at the end of the cavity appeared to indicate a high rate of entrainment and, thus, a correspondingly high rate of evaporation at the surface, so that the possibility that static equilibrium is not established could not be overlooked. As it turned out, temperature measurements within the cavity and in the surface of the cavity failed to disclose the requisite temperature difference. Furthermore, this idea does not explain the pressure rise just behind the model. That no temperature differences were observed at the cavity surface may be indicative of highly fluctuating temperatures at frequencies beyond the response of the thermocouples used.

The conclusion finally reached was that the entire flow might be considered as purely a wake phenomenon in which the vapor phase plays only a minor role. Here, the term "wake" is applied in the ordinary sense of a wake resulting from boundary layer thickening or separation in a viscous liquid. Thus, it is postulated that the motion is governed primarily by the hydrodynamics of the liquid phase. Even if the pressure were constant across the wake, it is clear that cavitation will first occur in the small scale eddies formed at the boundary of the wake due to the high viscous shear at this boundary or in the core of large vortices shed from the model as in Figure 22. For the more fully developed cavities, the pressures lower than vapor pressure measured near the center line might be explained on the basis of an essentially vortical flow within the cavity. This flow is associated with the downstream motion at the surface of the cavity and the motion of the liquid reentering at the end of the cavity in a manner similar to that described in the air-water entry cavity. Because of the small mass, a vortex flow of vapor

alone does not account for the lowest pressures.⁴⁰ Thus, it appears that liquid phase is present throughout the cavity. That vapor phase is not present throughout the entire wake, even with the pressures lower than vapor pressure, may then be explained by establishment of a system in which equilibrium is maintained between the liquid and vapor phases in such a way that the evaporation and entrainment of vapor is just balanced by reentrant liquid. Because of the large volume ratio between vapor and liquid, the presence of unevaporated liquid at pressures somewhat smaller than vapor pressure is quite possible despite the high rate of evaporation, and, as a matter of fact, there is photographic evidence that this conclusion is valid.⁴⁰ Furthermore, because of the high rate of condensation, there are no difficulties in accounting for the vapor.

It seems clear, then, that the actual composition of the cavities in the experiments of Reference 40 is a mixture of liquid and vapor phases and that the surface oscillations are associated with a high rate of turbulent mixing at the boundary of the cavitating zone and the general instability of wakes. The resultant motion when considered as a wake is then a function not only of the cavitation numbers but also the Reynolds number of the flow. It was also pointed out in Reference 40 that the finite angle of separation of the cavities from the surfaces of the discs, hemispheres, semi-ellipsoids and ogive section used in the experiments are, on the basis of the above descriptions, merely a manifestation of the bounding streamline of a separated boundary layer. It is interesting that the same conclusions as to treatment of the cavitation as a wake phenomenon and as to the appearance of the separation angles were reached by Wayland and White⁵⁸ in experiments on spinning spheres, although the dependence on Reynolds number of pressure distributions on spheres was already well known.

Subsequent to the experiments described in Reference 40, high speed motion pictures were taken of the cavity behind a disc. These showed that not only does the surface of the cavity oscillate, but that the entire cavity appears to grow and collapse although it is difficult to show definite periodicity or simultaneous collapse over the entire surface. Such oscillations were also observed in the experiments of Hunsaker and Spannhake⁵⁹ in their experiments in venturi tubes. The latter experiments are discussed in somewhat more detail in a later section.

Measurements of the drag of cavitating bodies were made by Reichardt and also by the writer and Mr. Pond. These measurements will be discussed in connection with the results of analytical descriptions of steady-state cavities.

ANALYTICAL DESCRIPTION OF STEADY-STATE CAVITIES AND DRAG OF CAVITATING MODELS IN REAL AND IDEAL LIQUIDS

The conditions in steady-state cavities are such that the velocity magnitude (but not necessarily the direction) along the boundary streamline may be taken as constant. Thus, the flow about such cavities falls in the class of problems which compose the free-streamline theory in theoretical hydrodynamics. The earliest interest in flows with free streamlines was in connection with the drag of bodies in a real fluid. The treatment of the wake of bodies as free-streamline flows resulted in the now classical conformal mapping methods of Kirchhoff and Helmholtz.⁶⁰ The Kirchhoff flow, in which it is assumed that the pressure in the wake is the same as that at infinity corresponds to a cavity flow with cavitation number zero. Work in the mathematical theory of wakes, jets, and cavities has resulted in the accumulation of a voluminous literature on free streamline theory. A bibliography of work on two-dimensional wakes will be found in Reference 61, and a critical examination of the progress made during the last 25 years has been carried out by Weinstein.⁶² The problem of two-dimensional free-streamline theory is to find the complex potential $w(z) = \phi + i\psi$ where ϕ is the velocity potential, ψ is the stream function, and $z = x + iy$, with the condition

$$P + \frac{1}{2}\rho U^2 = p + \frac{1}{2}\rho \left\{ \left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right\}$$

where $\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 = \text{constant}$ on the bounding streamline of the cavity. The nonlinearity of the boundary conditions has restricted successful solutions to the two-dimensional case. Much work remains to be done for three dimensions, and, so far, useful results for three-dimensional flow are still lacking. However, the results of two-dimensional treatments and approximations to axisymmetric three-dimensional problems are of immediate usefulness in design. The results that are of especial interest in engineering applications are the drag associated with cavitating flows and the extent of the cavitating regions, the latter data being of interest in the prediction of regions where collapse may be expected and, thus, where damage may occur.

The extensive mathematical analysis associated with obtaining drag and cavity shape will not be reproduced here; the details will be found in the references cited. However, the results of these analyses will be discussed. There are two mathematical models of two-dimensional cavities that are of special interest to the present discussion. The first, which is a flow suggested by Riabouchinsky,⁶³ artificially introduces an after plate in tandem with the forward one—with the cavity formed between them, Figure 25. It should be

noted that a given distance between plates corresponds to one, and only one, cavitation number, and vice versa. The second model, attributed to Prandtl and Wagner, utilizes the single plate perpendicular to the flow but is characterized by the reversal of the free streamlines at the end of the cavity with the resultant reentrant jet, Figure 26, The essential features of the

Figure 25 - The Two-Dimensional
Riabouchinsky Cavitation Model

Figure 26 - The Reentrant Jet
Cavitation Model

latter model were given in 1945 by Kreisel,¹⁴ and in 1946 by Efros.⁶⁴ The Riabouchinsky model was also studied by Weinig⁶⁵ and Zoller.⁶⁶ The results of the two theories have been examined by Gilbarg and Rock⁶⁷ and Gurevich.⁶⁸ A comparison of various approximations of the drag and cavity shapes derived from these theories has been made by Wehausen in an addendum to his translation of Gurevich's paper. An analysis of the two-dimensional finite cavity has been carried out by Shiffman⁶⁹ from a somewhat different point of view from that of the above writers.

To avoid the zero drag (d'Alembert's paradox) which would result if the pressures were integrated over both plates in the Riabouchinsky model, only the drag of the forward plate is computed. To avoid this difficulty in the Prandtl-Wagner model, the reentrant jet is assumed to go to infinity on the second sheet of a Riemann surface, and the pressure over the downstream face is taken to be the cavity pressure. Although this is a reasonable method of computing the drag for this model on the assumption that in a real fluid the jet is dissipated by the turbulent mixing and oscillations at the end of the cavity, the jet has been observed to reach and penetrate the forward surface of the cavity, as in Figure 18, and in the closure of the attached cavity the jet has been observed to deflect the missile from its course.⁵⁶ Although the origin of the jet in a real liquid is evidently in the transverse momentum, the jet behaves as the theoretical one which originates through the requirement of constant pressure.

For both models, it turns out that the drag coefficient obtained by integrating the pressures in the manner described above may be approximated by the relation

$$C_D(\sigma) = C_D(0)(1 + \sigma)$$

This result was obtained independently by several of the writers cited above. The coefficient $C_D(\sigma)$ is the drag coefficient of the plate at cavitation number σ , and $C_D(0)$ is the drag coefficient for cavitation number zero, or $\frac{2\pi}{\pi+4} = 0.88$ (Kirchhoff flow). Here,

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 l}$$

where D is the drag, and l is the breadth of the plate. This expression holds with excellent approximation for cavitation numbers less than one. At $\sigma = 1$, Wehausen found that the error between this approximation and the exact calculation is less than 0.8 percent (see Wehausen's commentary⁶⁸).

It is interesting that Reichardt⁵⁴ obtained precisely the above result for axisymmetric flow on the basis of a reasonable choice of assumption of the variation of the pressures on the face of the model. Reichardt assumed that, for small cavitation numbers, the pressure coefficient on the face of the model σ_f at cavitation number σ will vary from the pressure coefficient σ^* for cavitation number zero according to the equation

$$1 - \sigma_f = (1 + \sigma)(1 - \sigma^*)$$

or

$$\sigma_f + \sigma = (1 + \sigma)\sigma^*$$

The drag is given by

$$D = \int_A (p_f - p_c) dA$$

where p_f is the pressure on the face,

p_c is the cavity pressure, and

A is the area of the model.

Dividing by $\frac{1}{2}\rho U^2 A$, this equation may be written

$$C_D = \frac{1}{A} \int_A (\sigma_f + \sigma) dA$$

which with the assumption above, becomes

$$C_D = \frac{1+\sigma}{A} \int_A \sigma^* dA$$

so that

$$C_D(\sigma) = (1 + \sigma)C_D(0) \quad [21]$$

Another simple drag law which was proposed by Betz⁷⁰ is obtained by assuming that the drag given for Kirchhoff flow is increased by the product of area times the pressure reduction in the finite cavity. This leads to the result $C_D = \frac{2\pi}{\pi+4} + \sigma$ which agrees well with the above approximation for very small cavitation numbers.

The results of drag measurements made by Reichardt⁵⁴ and the TMB drag measurements⁴⁰ on various axisymmetric obstacles are shown in Figure 27.

Figure 27 - Drag of Various Models at Different Cavitation Numbers

In this figure, the curves which are extrapolated down to $\sigma = 0$ are of the form of Equation [21]. The correlation of the TMB data and Reichardt's data with Equation [21] is very good over a rather wide range of cavitation numbers. It will further be seen that Reichardt's data for cones appear to be on a line defined by Equation [21]. However, the slope of drag curves for the forms with surface curvature is somewhat greater. This difference might be attributed to the fact that for the disc and cones the point of separation is fixed at the downstream edges for all cavitation numbers; for the curved surfaces, on the other hand, the separation point was observed in the TMB tests to shift with changing cavitation number. A part of this shift may be attributed to the dependence of the pressure distribution and boundary-layer growth on Reynolds number,⁷¹ but the largest part is probably associated with the curvature of the surface. The extension of free-streamline theory with finite cavitation numbers for curved surfaces will be required to throw more light on this point. (The two-dimensional cavity for $\sigma = 0$ behind curved forms has been treated by Levi-Civita, see Reference 60.)

An extension of the Riabouchinsky model to two-dimensional wedges, Figure 28, has been carried out by Plesset and Shaffer.⁷² As an approximation to cones, they assumed that the pressure distribution is nearly the same as

Figure 28 - Extension of Riabouchinsky Model to Two-Dimensional Wedges by Plesset and Shaffer—Reference 72

Figure 29 - Comparison of Theory of Plesset and Shaffer with Reichardt's Data for Discs and Cones—From Reference 72

that found from the two-dimensional flow about a wedge of the same included angle. Their comparison of the results obtained using this assumption with Reichardt's data is shown in Figure 29. As might have been anticipated, the theoretical drag coefficients are somewhat high, but, nevertheless, the agreement is reasonably good for all cases except the 14° included angle. The writers offer no explanation for this large discrepancy. However, it might be pointed out that, for such small entrance angles, boundary layer effects may become of importance and separation may occur somewhat forward of the downstream edge with a resultant alteration in the boundary conditions. It is also of interest that the curves of Plesset and Shaffer are fitted very well by Equation [21].

The length of the two-dimensional cavity formed behind a plate and the breadth at the maximum section have been given in various forms by the writers cited. The results of the exact solution for the Riabouchinsky model, stated in terms

of parameters k and α , in the form given by Gurevich are as follows:

The cavitation number is

$$\sigma = \frac{2\sqrt{1 - k^2}}{k^2} (1 + \sqrt{1 - k^2})$$

The length of the cavity a is

$$a = \frac{2\alpha}{U} \left\{ \frac{1}{k^2} E - \frac{1 - k^2}{k^2} K \right\}$$

The width of the cavity b is

$$b = l + \frac{2\alpha}{U} \frac{\sqrt{1 - k^2}}{1 + \sqrt{1 - k^2}}$$

The width of each flat plate is

$$l = \frac{2\alpha}{U} \left\{ \frac{1 - k^2}{k^2} E' - K' \right\}$$

These equations fix k , α , a , and b in terms of σ and l .

In the above formulas K and E are the complete elliptic integrals of the first and second kind, respectively, with modulus k , and K' and E' are those with modulus $k' = \sqrt{1 - k^2}$. The drag coefficient of a single plate is:

$$C_D = 2 \left\{ 1 - \frac{2\alpha}{Ul} \frac{1 - k^2}{k^2} \right\} (1 + \sigma)$$

Zoller gives the following approximate formulas for the Riabouchinsky model (see Wehausen's commentary, Reference 68):

$$\frac{b}{l} = \frac{4}{\pi + 4} \left\{ \frac{1}{\sigma'} + \frac{\pi}{4} \right\}$$

$$\frac{a}{l} = \frac{4}{\pi + 4} \left\{ \frac{1}{\sigma'^2} + \frac{3}{4} + \frac{1}{2} \ln \frac{\sigma'}{4} \right\}$$

where

$$\sigma' = \frac{\sigma}{2 + \sigma}$$

For the reentrant-jet model, Gilbarg and Rock give the following approximation for the cavity length

$$\frac{a}{l} = 3.5 \sigma^{-1.85}$$

Here, the length a is taken to be the distance between the plate and the rear stagnation point. Despite the extreme differences in the models, the lengths and widths based on the definitions given here agree fairly well.

The usefulness of the two-dimensional drag data for engineering problems has been demonstrated to some extent by the approximation to the three-dimensional case made by Plesset and Shaffer. In connection with cavity width the correspondence between two and three dimensions might be expected to be in the area ratios rather than the width ratios, on the basis that the "wake" is related to the drag. A comparison of the squares of the data of Figure 23 with the corresponding data for the two mathematical models is shown in Figure

30. The theoretical curves are computed from the exact theories and are taken from Reference 68. It is seen that for small cavitation numbers this method of comparison may be used as an approximation. It should be pointed out, how-

Figure 30 - Comparison of Area Ratios for Two-Dimensional Mathematical Models and Three-Dimensional Cavities Behind the Discs of Figure 23

ever, that the two-dimensional theory does not give values for the length of the cavities which may be used to estimate the length in the three-dimensional case, the two-dimensional cavities being much larger than three-dimensional ones for the same cavitation number. The question of the length of cavitating regions in venturi-shaped nozzles is discussed in the following section.

The effect of wall or free surface proximity on finite cavities has been considered by Simmons.⁷³ This problem is of interest in the design of water tunnels.

More exact results for engineering use must await development of the three-dimensional free-streamline theory for bodies with fixed separation points and both two- and three-dimensional theory for curved surfaces.

MECHANISM OF STEADY-STATE CAVITATION IN VENTURI-TYPE NOZZLES

In the case of the cavities studied at the Taylor Model Basin, it has been suggested that the oscillations are associated with the pressure and velocity fluctuations in an unstable wake. Although, on the average, such cavities have been successfully described (in two dimensions) from purely hydrodynamical considerations, the processes in the actual cavity are still not very clear.

In the cavitating venturi of Pöttinger⁸ and Hunsaker⁵⁹ the conditions are somewhat clearer. The conditions at the end of the cavitating region may be likened to the conditions in a nozzle with an abrupt expansion since there is apparently always a central jet of liquid which flows through the cavitating region. In this case, the conditions at the "jump" may be such that its position is stable or the downstream pressures may be altered in such a way that this point may move upstream in analogy to the moving hydraulic jump.

If this is the situation, the oscillations in the direction of motion can be expected. On the other hand, the oscillations of the jet in the transverse direction (see, e.g., Reference 8) might be associated with an underpressure near the point where the jet leaves the boundary. Here, the viscous drag of liquid on gas can give rise to underpressures which are sufficient to pull the jet toward the conduit wall (the so-called "Coanda effect;" see for example, Reference 74). With a curving boundary and with the unstable conditions in such vortex flows, the underpressures can be expected to change in a rather random fashion around the periphery of the jet with consequent oscillations of the type observed. (This effect has also been proposed to explain the finite angles of separation of the cavity surface from the obstacles used in the TMB tests, the underpressure being assumed to be of importance only at the point of separation. However, though this may be a contributing cause, the conditions in a separated wake alone would give rise to this discontinuity.)

As pointed out in the foregoing, it is of interest for engineering applications to be able to predict the position of the end of a cavitating region, and thus where damage may be expected to occur. For the three-dimensional cavity in an unlimited liquid, practical results are not as yet available. An approximate analysis of the point of cavity termination in a nozzle can, however, be carried out along the lines suggested above. Although no attempt will be made to give results suitable for design, an analysis will be carried out to show that the problem can be treated purely as a hydrodynamic one in which the vapor phase plays no significant role in determining the motion. The analysis is carried out for a nozzle with a conical diffuser of small angle, and pressures are referred to positions which coincide with piezometers located in a nozzle of this type available at the Taylor Model Basin for tests.

In the computations, we assume that the diameter of the jet passing through the cavitating region remains constant and equal to the diameter of the throat. Actually, this jet may contract or, because of the underpressure effects at the throat, may expand. For the purpose of this calculation, it will be sufficient to assume that, on the average, the diameter is approximately equal to the throat diameter. An essential difficulty in such an analysis stems from a lack of knowledge of the velocity distribution and, hence, the longitudinal momentum distribution in the region just downstream from the end of the cavity. Here, the mixing of the jet with the slower moving fluid produces a highly turbulent zone of nonuniform and fluctuating velocity distribution—the characteristics of which are but imperfectly known. Thus, if the average velocity in this region is used in writing the momentum equation, the term containing this velocity must be modified by a coefficient which is

a function of the actual distribution. For nozzles having diffusers of small angle, this coefficient would be expected to be very close to one for cavities of small length since, for such cavities, the expansion from the jet diameter to diffuser diameter is small and the motion as observed in tests is not particularly disturbed. As the cavitating zone increases, the ratio of diffuser diameter to jet diameter at the "jump" increases, the jet oscillates, and the motion becomes progressively more violent in this region, with greater nonuniformity in the velocity distribution, so that the coefficient would be expected to increase.

Referring to Figure 31, let r_t be the radius of the jet (assumed equal to the throat radius), r_l the radius of the diffuser at the position of

Figure 31 - Notation for Computation of Cavity Length in a Diffuser

the end of the cavitating region, r_2 the radius at a reference position downstream of the "jump", u_j the velocity of the jet, u_l the average velocity just downstream of the "jump", U_2 the velocity at the positions of r_2 , p_v the pressure across the jet (which may be taken as vapor pressure), p_l and P_2 the pressures at the positions of r_l and r_2 , respectively, l the length of the cavity (this length is defined more precisely later when discussing the experiments), and k the momentum-correction coefficient.

The momentum and continuity relations across the jump are

$$k\rho r_l^2 u_l^2 - \rho r_t^2 u_j^2 = -r_l^2(p_l - p_v) \quad [22a]$$

and

$$r_t^2 u_j = r_l^2 u_l \quad [22b]$$

Eliminating r_t and r_l ,

$$p_l - p_v = -\rho u_l(ku_l - u_j) \quad [23]$$

The pressure p_l and the velocities u_l and u_j may be referred to the reference position at radius r_2 by the pressure and continuity relations (neglecting friction):

$$p_l + \frac{1}{2}\rho u_l^2 = p_2 + \frac{1}{2}\rho U_2^2 \quad [24]$$

and

$$r_l^2 u_l = r_2^2 U_2 \quad [25]$$

Furthermore, for a conical diffuser, the radius r_l , expressed in terms of the throat radius r_t , the cavity length l , and the diffuser half-angle α , is

$$r_l = r_t + l \sin \alpha \quad [26]$$

Thus, from [25] and [26],

$$u_l = \frac{r_2^2 U_2}{(r_t + l \sin \alpha)^2} = \frac{U_2}{\xi^2} \quad [27]$$

where

$$\xi = \frac{r_t}{r_2} + \frac{l}{r_2} \sin \alpha \quad [28]$$

Combining [23], [24], and [27] to eliminate p_l and u_l the result may be written in the following form

$$\frac{p_2 - p_v}{\frac{1}{2}\rho U_2^2} + 1 - \frac{1}{\xi^4} = -\frac{2}{\xi^2} \left[\frac{k}{\xi^2} - \left(\frac{r_2}{r_t} \right)^2 \right] \quad [29]$$

Defining the cavitation number σ_2 as

$$\sigma_2 = \frac{p_2 - p_v}{\frac{1}{2}\rho U_2^2}$$

and letting

$$r = \frac{r_2}{r_t}$$

Equation [29] becomes

$$\xi^4(1 + \sigma_2) - 2r^2\xi^2 + (2k - 1) = 0 \quad [30]$$

Solving for ξ ,*

$$\xi = \sqrt{r^2 + \sqrt{r^4 - (2k - 1)(1 + \sigma_2)}} \quad [31]$$

Expressed in terms of a dimensionless cavity length

$$l_o = \frac{l}{r_t}$$

Equation [31] becomes, using [28],

$$l_o = \frac{1}{\sin \alpha} \left\{ r \sqrt{r^2 + \sqrt{r^4 - (2k - 1)(1 + \sigma_2)}} - 1 \right\} \quad [32]$$

Solving for k from [30]

$$k = \frac{1}{2} \left\{ 1 + 2r^2 \xi^2 - (1 + \sigma_2) \xi^4 \right\} \quad [33]$$

Experiments to check the equations were made using a nozzle with a diffuser of 5° included angle (the same nozzle used in the experiments of Reference 37). In an actual case, there is some difficulty in defining the cavity length. For this comparison, however, the length of the cavity was taken to be the farthest point downstream at which the major portion of the vapor apparently disappeared. The cavitating region in these experiments was fairly stable, and the length used was of the order of 5 percent greater than what appeared to be the center of the highly disturbed region.

Figure 32 - Variation of Momentum Correction Coefficient with Cavity Length for 5° Diffuser

To check the hypothesis of the variation of k , the experimental values were used in Equation [33], and the results are shown in Figure 32. It is clear from this figure that the above analysis gives good results for cavities of ratio l_o less than about 10 without any correction for the mixing process (i.e., $k = 1$). The increasing values of k with increasing l_o are as expected, and the actual values give some clue as to the order of magnitude of k for diffusers of angle near 5° . (It should

*It will be noted that the solution containing the negative sign has been discarded. This was done to eliminate the possibility of imaginary solutions, and, as will be seen, appears to be correct. Nevertheless, the existence of the alternative solution raises the interesting question of two possible configurations in an actual case. Whether this can occur, and thus account for the longitudinal oscillations sometimes observed, would require a detailed stability analysis.

be noted that the value of k computed from the experimental results includes the effect of nonuniform jet, if any, and the (small) effect of friction.) The important result is evidently that once the mixing process can be accurately characterized for diverging conduits, an analysis of this type should lead to quite accurate predictions of the points at which damage might be expected for such systems.

THE MECHANISM OF COLLAPSE AND THE ASSOCIATED DAMAGE

INTRODUCTORY REMARKS

Among all the many aspects of the cavitation phenomenon, perhaps the greatest amount of controversy has been in connection with the origin of the damage associated with cavitating liquids. Although there is general agreement among investigators that, both from theoretical and experimental observations, cavitation damage may be attributed to purely mechanical action leading to eventual fracture of brittle materials and fatigue failure of ductile materials, there is still a small group who are convinced that the damage is chemical or electrochemical in nature associated with or hastened by the cavitation. It will not be possible in this survey to examine critically all the arguments pro and con the latter theories. The discussion will be concerned primarily with the results which indicate that the pressures developed in the collapse of a cavitating region are sufficient to cause damage, while only a few remarks will be made concerning the ideas of associated chemical action. A reference will also be made to more recent ideas on cavitation damage which associate damage with the corrosive action of electrical currents within the solid, itself, generated by crystal deformation under repeated impact. It should be noted that the earliest observers attributed the observed corrosion to chemical action. However, it should be emphasized that a great deal of confusion arose on this point since damage on propellers after service at sea might be associated not only with mechanical damage due to cavitation but also with chemical corrosion associated with sea water. However, controlled experiments have demonstrated that mechanical erosion is the only tenable explanation for the majority of cases. This point will be elaborated further. The hydrodynamic and thermodynamic processes involved in the collapse of cavitated regions and the origin of damage from this point of view will be discussed first. Secondly, a description of the possible mechanism which may account more clearly for some anomalous results will be proposed. Finally, some remarks will be made on the role of electrochemical phenomena in the cavitation process. Any discussion of metallurgical aspects of the problem are beyond the intended scope of the discussion.

In discussing the collapse phases, care must be taken to distinguish the "type" of cavitation under consideration. There is much discussion in the literature of the pressures developed by a single "transient cavity" (see, e.g., References 12, 31, 46, 47, 50, 75, 76) and of a mass of such bubbles, 10, and 75. However, the writer has found no parallel analysis for the collapse of the large steady-state cavities corresponding to the observed oscillations of such cavities in the MIT venturi studies⁵⁹ and in the DTMB results mentioned in the foregoing. Descriptions of the motion and, consequently, the collapse process in such cavities have been proposed herein in the previous discussion.

ARGUMENTS FOR THE MECHANICAL ORIGIN OF DAMAGE

In the case of the small, transient cavities, collapse begins upon entrance to a high pressure region. For the Rayleigh (empty) cavity, the motion is governed by the inertial effects of the liquid and the ambient pressures. In Plesset's extension, the additional effect of "surface tension" is included but he did not compute collapse pressures. Rayleigh's computation⁵⁰ gives for the velocity of the cavity wall,

$$U = \sqrt{\frac{2}{3} \frac{P}{\rho} \left(\frac{R_0^3}{R^3} - 1 \right)}$$

where R_0 is the initial cavity radius, and R the radius corresponding to U . The pressure in the vicinity of the cavity has a maximum at $1.57 R$ from the center equal to

$$p_{\max} = 0.163 P \left(\frac{R_0}{R} \right)^3$$

It is clear that the pressure may rise indefinitely depending on the radius of the cavity. Assuming that the cavity collapses concentrically onto a rigid sphere and that the kinetic energy of the fluid is converted into elastic energy of compression, Cook (see Rayleigh's paper) found for the pressure

$$p = \sqrt{\frac{2}{3}} Pk \left(\frac{R_0^3}{R} - 1 \right)$$

where k is the volume modulus of compressibility.

Rayleigh extended the computation for the case of isothermal compression of a permanent gas in the cavity, and showed that the cavity oscillates between the initial radius R_0 and a limiting radius which may be obtained from the equation

$$P \left(1 - \frac{R^3}{R_0^3} \right) + 3 P_c \frac{R}{R_0} = 0$$

where P_c is the initial pressure in the cavity. Since the actual conditions within the cavity are not accurately represented by the Rayleigh problem, this will not be discussed further here. The results are, of course, of some use in estimating pressures for cavities which are completely air-filled, and do show that very high pressures may arise. Osborne,⁴⁶ assuming adiabatic compression, obtained results which agreed well with his experiments in which the cavities contained large quantities of air. These solutions neglect surface tension, however, and the possibilities of retardation of the motion by the finite rate of vaporization and condensation in an actual cavity. Nevertheless, the wall velocity given by these results are fairly good approximations in the beginning of the motion, as shown by the experimental results of Mueller⁴⁵ and of Knapp and Hollander.⁴⁷

A similar computation of bubble collapse was carried out by Ackeret¹⁰ for densely packed, spherical bubbles, assuming a polytropic law of compression. However, the same objections apply here that are mentioned above. Since these theories lead to pressures which are obviously too high, both Ackeret¹⁰ and Knapp and Hollander,⁴⁷ as well as a number of other writers reporting between the appearance of these two references, propose that the pressures at collapse may be estimated by the usual theory of water-hammer, providing the impact velocity is known, i.e., $P = \rho cU$ where c is the velocity of sound in the liquid. Even if the velocity could be obtained accurately for cavities which contain no permanent gas, the result would not correspond with most cases since entrained air and air diffused during several cycles would result in smaller pressures than predicted by such a theory.

Another analysis was carried out by Silver.⁷⁵ In Silver's computation, the rate of collapse is assumed to be governed solely by the rate at which the vapor in the cavity can be condensed. Thus, although there is the serious objection that inertial effects are ignored, this computation might be taken to represent a lower limit for a cavity that is completely vapor filled—the Rayleigh (empty) cavity representing an upper limit. For this reason, the results are given here. (Although Plesset recognized that the rate of condensation might become of importance in the final stages of the collapse, it was not important over the range of his numerical computations, and he did not include it.) Assuming that the rate of condensation is proportional to the rate of thermal conduction away from the surface of the cavity, and that the rate of thermal conduction is proportional to the difference between the saturation temperature T at vapor pressure p_v and the ambient temperature T_0 , Silver related the rate of bubble collapse to the pressure difference using

the Clapeyron equation for equilibrium between liquid and vapor phases.* Supposing, then, that the total energy (surface and gas) is given up to the liquid as potential energy of compression, Silver found for the maximum pressure

$$p_m = \sqrt{\frac{kPV_s}{V_w} \left\{ 1 + \frac{3\gamma}{PR_0} \left[1 - \left(\frac{V_w}{V_s} \right)^{2/3} \right] \right\}}$$

where k is the bulk modulus of the liquid,

V_s and V_w are the specific volume of vapor and liquid, respectively, and γ is the surface tension.

Assuming further that the pressure is propagated as a spherical wave (i.e., varies inversely with the distance from the source), Silver computes the pressure which would be produced at a solid surface by a bubble which is initially tangent to this surface and obtains the result

$$p_s = \left(\frac{V_w}{V_s} \right)^{1/6} p_m$$

In spite of the objection that can be raised that Silver did not take into account the effect of wall proximity, the result is of interest in that it shows that the pressure pulse of a single bubble touching a surface cannot be less than $\sqrt{k\rho(V_s/V_w)^{1/3}}$. Silver extended the computation for a mass of bubbles, but the above result is sufficient to show the order of magnitude of the pressures. Where the Rayleigh theory including surface tension (worked out by Beeching⁷⁶) gives pressures of the order of hundreds of tons per square inch, Silver's result predicts pressures of the order of tens of tons, which are of the proper order for fatigue failure of cast metals.

The preceding results have been presented as indications that the pressures developed at collapse of isolated cavities are sufficient to account for cavitation damage. On the other hand, a rather interesting hypothesis was outlined by Poulter⁷⁷ in 1942, in which he attributed damage to pressure release rather than pressure application. In a series of experiments on glass and quartz rods, Poulter showed that breakage would occur on release of pressure slowly applied through several different liquids, and that the susceptibility to breakage increased with increased time of pressure application. Also, since the specimens were found to withstand greater pressures with oil or glycerine as compared with water, alcohol or ether, Poulter concluded that

*A further objection to Silver's analysis is that he used differences rather than complete partial differential equations. However, judging from a similar computation made by the writer,⁴⁰ Silver's results are probably not too greatly in error.

the breakage depended upon the ability of the liquid to penetrate the specimen. He further conducted cavitation tests with a magnetostriction oscillator and reported similar results—that damage was greater with water and alcohol. There are objections, however, to comparing the damage from the two different experiments. As already pointed out by several writers, the entire life cycle of the transient cavities is well within the minimum time required for pressure application before breakage would occur upon pressure release in Poulter's experiments. In connection with Poulter's oscillator experiments, the lesser damage with oils as compared with water might be attributed to the much slower collapse rates which would be associated with the higher viscosity of the oil, and, thus, lower collapse pressures. On the other hand, Poulter showed that a specimen first treated with oil under high pressure and then cavitating in water showed less damage than an untreated specimen. This result might be accounted for on the basis of Petracchi's results cited below.

Further evidence that cavitation damage is associated with impact phenomena is the damage produced by small drops impinging on a specimen. Ackeret and de Haller⁷⁸ showed that mechanical damage produced by small water drops closely resembles cavitation damage and that the drops need not be travelling at exceptionally high speeds to cause damage as long as there were a sufficient number of impacts.

On the basis of the spherical collapse of cavities, there is sufficient theoretical evidence that the pressures developed are high enough for damage. However, Ackeret has pointed out⁷⁹ on the basis of Mueller's experiments⁴⁵ that the collapse is not spherical, but that the "bubbles are collapsed by the upstream face being forced against the downstream face," and, further, that such collapse "can be shown to result in a pressure intensity of only a few hundred atmospheres," which are too low to account for most damage. Unfortunately, even the much more definitive photographs of Knapp and Hollander⁴⁷ do not throw further light on this question since the behavior of the bubbles near minimum radius is somewhat obscured.* Until such work as that of the latter investigators is extended, the following tentative hypothesis is proposed as an additional source of damage associated with cavitation: The unsymmetrical collapse of cavitation bubbles in a pressure gradient may give rise to the same type of jet as observed in the steady-state and air-water entry cavities described in the foregoing; in view of Ackeret and de Haller's experiments, such high speed jets could cause damage. However, the

*Recent experiments at the David Taylor Model Basin showed clearly, in some cases, collapse of such cavities by contact of upstream and downstream faces and repetition of this manner of collapse over several cycles.

answer to whether such jets actually arise on the scale of the small transient cavities must await further development of experimental techniques.

Although much thought has been given to the mechanism of collapse of the small, transient cavitation bubbles, there seems to be no parallel developments for the collapse of large, steady-state cavities such as the one shown in Figure 21. As pointed out previously, although the mean motion appears to be correctly described by free streamline theory, and attempts have been made to explain the surface appearance of actual bubbles,⁴⁰ the growth and collapse observed recently at the Taylor Model Basin, and in the tests of Reference 59, suggest other mechanisms for the oscillation.

In the case of the cavities studied at the Taylor Model Basin, it has been suggested that the oscillations are associated with the pressure and velocity fluctuations in an unstable wake. Although, on the average, such cavities have been successfully described (in two dimensions) from purely hydromechanical considerations, the processes in the actual cavity are still not too clear. If the cavity is considered as a wake, however, in which the reentrant fluid is alternately swept away from various sectors, the collapse of the cavitating boundary may easily be forced by the pressure variations during this shedding process. Although it is known that the shedding of eddies in a wake gives rise to sufficiently large pressure fluctuations to induce vibration (even at very high Reynolds numbers), it is not likely that such oscillations can produce damage of the type associated with cavitation collapse. Since, as mentioned above, such cavities do not appear to be made up of individual, spherical cavities, the requisite pressures for damage cannot be attributed to the collapse of groups of such individual cavities. Thus, although damage has been observed near the end of such steady-state cavities when they terminate on a solid surface, it is not clear what the origin of the requisite pressure may be. In the latter case of termination on a solid surface the conditions at the point where the end of the cavity meets the surface are also not clear. At this point, the pressure would tend to rise to stagnation pressure—which upsets the conditions of constant pressure on the solid and liquid boundaries. This pressure increase may be the origin of the fluctuations observed near the end of such cavities, with a forced condensation resulting in a decrease in cavity size and subsequent readjustment, growth and repetition of the cycle. Here again, it is not clear that such oscillations can give rise to pressures that are so much greater than the order of stagnation pressure as to cause damage; as a matter of fact, judging from the noise and induced vibrations, the conditions in such cavities appear to be much less violent than in the case of a number of individual cavities. A possible source of damage in this case may be the separation of individual water

droplets during the oscillation which cause damage on impact. Before definite statements as to the origin of damage in this case can be made, however, it will be necessary to investigate the accelerations that can be produced in the liquid during the oscillation and whether detached water droplets can acquire sufficient momentum toward the solid material.

It has been suggested to the writer that the collapse of such cavities might be associated with condensation phenomena within the vapor itself. Such conditions might be likened to the conditions leading to condensation shocks observed in high-speed air or steam flows.^{80, 81, 82, 83} However, such condensation is associated with conditions of supersaturation in the vapor phase and it is not likely, in cavitating flows of the type discussed here, that the expansions required for supersaturation can occur. It is possible that at various points throughout the vapor phase, supersaturated conditions may exist momentarily, but equilibrium would be quickly established without condensation throughout the vapor phase.

In the case of the cavitating venturi a possible mechanism of the observed oscillations, and thus of the collapse process, has been described in the foregoing—both for the transverse and for the longitudinal motion. Here, again, however, it is not clear that the longitudinal and transverse oscillations are of sufficient strength to cause damage. If they were, damage might be expected over the entire length of the nozzle, whereas, to the writer's knowledge, the damage is usually observed near the end of the cavity, at least for these experiments in which the position of the end of the cavity is fairly stable. Whether the damage in this case is again associated with the small jets and drops which must be present when the central core of liquid impinges on the filled portion of the diffuser remains to be investigated.

IDEAS ON ELECTROCHEMICAL ORIGIN OF DAMAGE

There has been some discussion recently of cavitation damage due to electrochemical phenomena associated with the rapid formation of new surfaces in a liquid. This theory associates the formation of surface double-electric layers with a subsequent spark discharge and dissociation which supposedly contributes to damage.⁸⁴ It should be noted, when considering these theories, that the mechanical theory of corrosion is supported by experiments in which cavitation damage was obtained on plate glass and on a lead plate without loss of weight.⁵⁹ It is not clear how a spark can originate during cavitation in the interior of a liquid since it would seem that the orientation of dipoles at the cavity surface would lead to no electrical unbalance. In the case of cavitation at a solid, it is again not clear that a current would flow across the "gap" when a path of much lower resistance from the water to the solid is

available. However, visible light has been observed in the interior of liquids cavitated by high-frequency acoustic pressures⁸⁵ so that the above objection may not be valid. When the results of Reference 85 were brought to the writer's attention, tests were made in the same nozzle used by Crump³⁷ wherein the high tensions mentioned previously were obtained. Observations were made under conditions of tension with individual bubbles of the type which formed occasionally and disappeared so that the cavitation was intermittent. The rate of growth of the bubble is very high and this rate is certainly toward the upper limit of rate of growth which might occur in practice. Despite extreme precautions as to darkness of the laboratory and darkness adaptation, no light flashes were visible to the eye nor could any be recorded on a photographic plate. In view of the results of Reference 85, perhaps the emission of light is associated with the frequency of formation of cavities. Nevertheless, that damage is somehow associated with this occurrence seems untenable since damage appears to occur in the regions of collapse rather than formation.

A much more tenable proposal relating electrical phenomena to damage is outlined by Petracchi.⁸⁶ Petracchi indicates that additional corrosion not accounted for entirely by fatigue is associated with the corrosive effects of the electrical currents generated in adjacent crystals of the solid as a result of the alternating mechanical stresses and deformations produced by the forces associated with the collapse of cavitating regions. This would account for the lack of correlation between hardness and ductility of metals and resistance to corrosion.⁸⁷ Petracchi points out that the presence of the oil in Poulter's experiments may prevent crystal deformation and, thus the associated electrical phenomena and damage. However, this problem is evidently one of the physics of crystal lattices and will not be elaborated here.

The conclusion which may be drawn from the above discussion is that the origin of cavitation erosion is in the high collapse pressures, whether the damage is directly a result of fatigue caused by impacts or by corrosive action due to electrical currents generated by crystal deformation in the solid under these impacts.

SOME REMARKS ON SCALING OF CAVITATING SYSTEMS

The use of scaled models in reproducing flows with cavitation is well known. However, it might be well to point out a few precautions which must be observed in such tests. The fundamental parameter relating the model and prototype is the cavitation number $\sigma = \frac{P - P_v}{\frac{1}{2}\rho U^2}$. This parameter establishes the condition of similarity of pressures and is, thus, merely the Euler number or another form of pressure coefficient. Although the vapor pressure

p_v is indicated above, it is clear from the discussion of steady-state cavities that, for such cavities, the pressure p_v should be replaced by the actual pressure within the cavity. For the small cavities, the question of scaling becomes even more complicated. Although the assumption of vapor pressure is usually adequate, errors may arise when attempting to establish critical cavitation numbers from model tests. For complete scaling, it would be necessary to investigate the question of nucleus content and formation on the two scales, and, thus, the thermodynamic parameters in relation to growth of nuclei and heat conduction. Furthermore, additional work must be done on surface tension under dynamic conditions. For most engineering applications, however, it is sufficient to test under reduced pressure, as in the variable-pressure water tunnel, using the vapor pressure corresponding to the temperature of the water.

In the case of tests of model propellers, discrepancies between model tests and sea trials have led several laboratories to adopt a correction of about 15 percent pressure reduction on the model scale. This discrepancy might be attributed to several causes. These are discussed in the following paragraphs.

The greater number of air and solid nuclei in sea water than in fresh water raises the pressure for inception. On the basis of his experiments in sea water and in the water used in the TMB tunnels, Crump³⁷ reached the conclusion that the tunnel water should be about 30 percent supersaturated to duplicate inception. However, tests for which the correction is made have almost wholly been on rather small propellers running at low Reynolds numbers.

The effect of Reynolds number on airfoils is well known—with changes in the maximum lift being observed even over the range of full-scale Reynolds numbers. It is not surprising, therefore, that the characteristics of model propellers should be quite variable with Reynolds numbers. This conclusion has been substantiated in part by recent tests* at the David Taylor Model Basin with 8- and 16-inch propellers operated under high pressures at speeds of 30 knots, i.e., at much higher Reynolds numbers than obtained previously. It was found that no corrections are necessary for these results. It should be noted, furthermore, that the difference between the flow about the model and on the prototype, due to absence on model scale of a heterogeneous wake, may be another contributing cause, as has been pointed out many times.

It is clear that in any model test, the cavitation number and the Reynolds number, which are compatible parameters, should be modelled. Usually, it will not be possible to run at prototype Reynolds number. However, care

*These results were made available to the writer by Mr. W. Bowers of the water tunnel staff of the David Taylor Model Basin.

must be taken that the Reynolds number is high enough so that similarity in the flow patterns is approximately obtained. This is especially of importance in tests of models on which separation may occur at the lower Reynolds numbers. In such cases, cavitation in separated zones on the model may be misleading since this zone may be eliminated at the higher Reynolds numbers. Usually, it is possible to determine whether this will be the case by running the model test over a range of Reynolds numbers to determine the minimum value for which changes in the resistance, lift, or pressure distribution disappear.

In cavities in which the gravitational acceleration plays an important role as in air-water entry of missiles, the Froude number is an additional parameter of scaling. For this type of cavitation, it can be shown that for dynamical similarity, the ratio of the mass of gas to liquid must be modelled as well. Thus, for similarity in the test of an air-to-water entry, the tests must be performed under reduced pressure but with a gas heavier than air. A comprehensive discussion of such modelling is given in Reference 56.

SOME REMARKS ON DESIGNING FOR CAVITATION PREVENTION

METHODS OF REDUCING ADVERSE EFFECTS OF CAVITATION

Although it is generally agreed that attempts should be made to eliminate the possibility of cavitation in hydraulic systems, this is not always possible without excessive costs and difficulties. In such cases, methods have been developed for reducing the adverse effects of cavitation and much work has been done on selection of materials which are resistant to cavitation damage (see, for example, References 76, 87, and 88). To prevent excessive damage in hydraulic systems, the use of air injection into the cavitating region to "cushion" the impacts at collapse has been adopted in many cases.^{7, 88} A striking illustration of the effect of small air contents on cavitation damage to concrete is shown in tests made by the Bureau of Reclamation, Figure 33. It is seen that the damage is considerably reduced with increasing air content. Another method proposed for amelioration of cavitation damage is the use of "water cushions"⁷ on the assumption that if a layer of water is maintained between the cavitation and the solid, damage will be minimized.

PREDICTION OF MINIMUM PRESSURE POINTS

In designing hydraulic systems, the most important information needed in investigating the possibility of cavitation is a knowledge of the distribution of pressures. Primarily, it is required to know at what points on the boundary minimum pressures occur and, thus, at what points cavitation may be expected. For most engineering applications, the minimum pressure

Figure 33 - The Effect of Air Content on Cavitation—From
Bureau of Reclamation Photographs

coefficient may be taken as the critical cavitation number for cavitation inception by changing the sign of the coefficient and replacing the local pressure by the vapor pressure. For flows with very high air contents, it may be appropriate to replace the vapor pressure by some higher pressure—perhaps that given by Henry's law for equilibrium saturation of the gas in the liquid.

An equally important reason for a knowledge of the pressures is the possibility of occurrence of cavitation in zones of separation (as discussed in the foregoing). Although, in general, it will not be possible to predict pressures in such zones, the possible existence of such zones can be recognized from the form of the pressure distribution on the boundary. Since the occurrence of separation is associated with positive pressure gradients, those systems which show sharply rising gradients may be subject to cavitation even though the minimum pressures at the boundary do not fall below the vapor

pressure. Thus, positions of abrupt changes in boundary geometry may be points at which cavitation will occur in the separation zone even though the ambient pressure is well above vapor pressure. Similarly, care must be exercised in evaluating systems in which the influence of walls may lead to large pressure reductions.

Experiments in which the effect of local zones of separation on the pressure distribution, before visible cavitation was observed but as the cavitation number of the experiment was varied, have been reported by Rouse and McNown.^{8,9} Their experiments showed that below a certain cavitation number the pressure distribution started changing, although no visible cavitation was observed. They attributed this change to onset of cavitation in microscale eddies in a zone of separation. Defining the cavitation number at which the pressure distribution starts changing as the critical cavitation number, σ_1 , they compared this value with the minimum pressure coefficient for noncavitating conditions. Their experiments showed that the greater the adverse pressure gradients and, thus, the stronger the separation zone, the earlier the change in minimum pressure coefficient occurred, Figure 34. The critical

cavitation number for change in minimum pressure coefficient approached more closely the negative of the minimum pressure coefficient with decreasing adverse pressure gradient.

Analytic methods presently available for computation of pressure distributions are evidently restricted in practical use to two-dimensional flows and axisymmetric three-dimensional flows, although in principle any flow field might be determined by a proper distribution of mathematical singularities (sources, sinks, doublets, vortices, etc.). Since these methods are all based on potential-flow theory, the results must be used with care for bodies of small length-diameter ratios. However, in the majority of cases, these methods will be adequate. Although no details can be given here, it seems worth while

Figure 34 - Critical Cavitation Number for First Change in Minimum Pressure Coefficient and Minimum Pressure Coefficient versus Caliber of Rounding—From Reference 89

to mention the various methods and the problems they are designed to solve. The classical theory of two-dimensional flows about airfoils and strut shapes and axisymmetric flows (ellipsoids, etc.) is given in Lamb⁵² and Milne-Thompson.⁶⁰ A method of computing the pressure distributions on arbitrary airfoil shapes has been outlined by Theodorsen and Garrick⁹⁰ in which conformal mapping methods are used to compute the pressure distribution on prescribed shapes; various improvements in the methods for the successive approximations have since been reported but without essential change in the theory.

The inverse problem of specifying the pressure distribution and then finding the corresponding shape has been treated for the two-dimensional case by Mangler⁹¹ and Peebles.⁹² These methods should be particularly useful to the designer since the pressure distribution can be specified for the lift distribution and the allowable pressure gradients for drag as well as for the minimum pressures for cavitation conditions, depending, of course, on whether the conditions specified are compatible and lead to a closed surface.

For bodies of revolution, if they cannot be replaced by ellipsoids with sufficient accuracy, the pressures can be determined from distributions of singularities on the axis which give the body outline as a closed streamline. One procedure for such computations has been outlined by von Kármán.⁹³ Another procedure in which the flow field is obtained by approximations using the Legendre functions is outlined by Kaplan;⁹⁴ other results are given in Reference 95. These and other available methods are adequate for bodies of rather large length-diameter ratios, but may lose accuracy for bodies with rapid changes in curvature near the nose and afterbody. For such cases, the body may be approximated by the flow about singularities distributed over a ring or a disc placed perpendicular to the stream direction. Discussions of such flows will be found in References 96 and 97. Estimates of the pressure reduction due to wall proximity may also be obtained by the methods of potential theory (see, e.g., Reference 98).

For those problems which are not conveniently handled by analytic methods recourse may be had to analogy methods of solution. A most useful one is that of the analogy of the potential-flow field to the electrostatic field—the velocity potential and electrostatic potential both satisfying Laplace's equation with corresponding boundary conditions (see, e.g., References 99 and 100).

The methods outlined above will be useful in only a limited number of cases so that recourse must often be had to tests of models in the variable-pressure water tunnels using the modelling laws discussed previously. However, it will often be possible to use cavitation, itself, as an analogue method

for the solution of the hydrodynamical equations when the critical cavitation numbers are specified.

The cavitation method for developing forms having specified critical cavitation numbers is based on the approximation of the minimum-pressure coefficient as the negative of the critical cavitation number:

$$\left(\frac{p}{\frac{1}{2} \rho U^2} \right)_{\min} = -\sigma_{\text{critical}}$$

In this method, which was evidently first used by Reichardt⁵⁴ for designing body shapes with essentially constant pressure surfaces for high speed air-flight, the shape of a cavity formed behind an obstacle is used as the body (see Figure 21a). It is clear that the method gives a restricted class of forms—those having essentially constant pressures. The method has since been used at the Taylor Model Basin for designing bodies with specified critical cavitation numbers.¹⁰¹ In the experiments of Reference 101 only axisymmetric bodies were developed, but it is clear that other three- as well as two-dimensional shapes may be developed depending upon the shape of the obstacles. In actual design, the desired volume, length-diameter ratio, and general shape will govern the selection of the obstacle. For example, the body shape derived from cavities formed behind discs have larger length-diameter ratios than those derived from hemispheres at the same cavitation number.

A cavity with cavitation number 0.116 formed behind a disc is shown in Figure 35, and the body conforming to the cavity shape is shown in Figure 36, which was taken from Reference 101. The pressure distribution on the resulting body is shown in Figure 37. A discussion of the method of "closing" the cavity to obtain the solid body, as well as the precautions which must be

Figure 35 - Cavity Formed Behind a Disc at Cavitation Number 0.116

observed in using the method is given in References 71 and 101. It will be seen that the magnitude of the minimum cavitation number agrees quite well with the cavitation number of the cavity except for a point near the nose. The low value of the pressure coefficient near the nose of the model was attributed to inaccurate determination of cavity shape at this point. In view

Figure 36 - Model Constructed to the Shape of the Cavity in Figure 35

Figure 37 - Pressure Distribution on the Model of Figure 36

of the results shown in Figure 24, it is necessary to measure the actual pressures within the cavity for defining the cavitation number. In general, it will be more satisfactory to use plane, sharp-edged obstacles for such experiments. In using forms having surface curvature care should be taken to fair out the discontinuity in cavity shape due to finite angle of separation before constructing the body.

CONCLUDING REMARKS

Although the foregoing discussion has emphasized the progress made in understanding the mechanism of cavitation, the gaps in the present knowledge appear clearly. A great deal of research remains to be done on the prediction of cavitation inception in "engineering" liquids, the mechanism of maintenance of steady-state cavities in real liquids and the analytic description of such cavities (especially in three dimensions), and the mechanism of collapse and the associated damage.

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