TWO-STAGE AUTOMATIC RADAR DETECTION

Theoretical analysis of a binomial sequential detection system
with coarse-fine range resolution

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PROBLEM

Apply statistical decision theory to the design of automatic radar detection systems. The specific phase of the problem reported here is the theoretical investigation of a system based on a two-stage binomial test which uses low resolution data in the first stage and high resolution data in the infrequently employed second stage.

RESULTS

1. Detection probabilities for binomial two-stage detection systems with coarse-fine range resolution have been calculated; they indicate substantial saving over conventional systems.
2. The use of sequential rather than fixed-sample-size testing in the first stage has been found, in the special cases analyzed, to improve the detection probability only slightly.
3. In certain clutter-free situations, doubling the number of first-stage range bins used to cover the detection zone (by halving the first-stage pulse length and doubling the instantaneous power) has been shown to cause a loss in detection probability equivalent to roughly 0.3 to 0.5 dB/pulse in signal-to-noise ratio (assuming that the dwell-time, false-alarm rate, and second-stage range resolution are held constant).
4. The optimum length of the second stage has been shown to vary considerably with other design parameters and quantities.

RECOMMENDATIONS

1. Extend the analysis of two-stage systems with fixed-sample stages to cases where the radar cross section is slowly fluctuating.
2. Calculate the cumulative detection probability (the probability that an approaching target is detected before it reaches a given range) for cases of practical interest.
3. Consider two-stage systems for applications in which the power capability of the radar dictates that the average number of pulses per beam position be large.

4. Consider two-stage systems having both doppler and range resolution.
5. Investigate the effect of clutter and interference on the performance of the two-stage system.
6. Implement a two-stage system as soon as a suitable antenna is available; perform two-stage and conventional tests with live targets to verify performance predictions.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

A theoretical analysis was made of the performance of some binomial two-stage radar detectors which use low range resolution in the first stage and high range resolution in the second stage. In the first stage of the detection procedure, one or more pulses are transmitted while the antenna beam is in a given position, and a tentative decision is made for each of the wide range bins as to whether or not a target is present. If no alarm occurs in the first stage, the antenna beam is moved to the next position; otherwise, the beam is held fixed and pulses of the second-stage type are transmitted. In the latter case, the regions corresponding to first-stage alarms are then re-examined by collecting and processing the high-range-resolution data, after which the antenna beam is moved to the next position for a new first stage.

The two-stage detectors reported on here belong to a class of multiple-stage radar detectors requiring beam agility and versatile transmission and reception capabilities. After a brief description of this class of multiple-stage detectors and a mention of particular versions considered by others, the binomial high-resolution two-stage detectors are investigated and their performances compared with that of a conventional detector.

HISTORICAL BACKGROUND

The large body of literature on sequential analysis reveals a wide variety of hypothesis-testing situations in which a statistical test of the sequential type will require fewer observations, on the average, than a fixed-sample-size test with the same probabilities of error. With the development of electronic scan antennas, the application of sequential testing procedures to radar detection has become practical; in many detection situations it is possible to design sequential procedures which promise appreciable energy savings over procedures limited by the constant rotation rate of a mechanically scanning antenna.

In the study of sequential radar detection the usual assumption has been that successive observations (detected returns from a particular range gate) are obtained under nearly identical circumstances (identical transmitted waveforms, same receiver characteristics, etc.) and the testing procedures considered have typically consisted of operating on the acquired sequence of measurements of a matched-filter receiver output with the same rule of procedure after each transmission. In particular, earlier work was generally with Wald's¹ sequential probability-ratio test of two statistical hypotheses, the optimum nature of which can be realized in cases with a single resolution cell (or a single decision) per antenna-beam position. Later applications to radar involved modifications of the classical Wald test and various schemes for using sequential tests in multiple-resolution element situations.

¹See list of references at end of report.

Recently some attention has been given to a type of sequential detection in which the statistical testing procedure and the radar transmission and reception characteristics are programmed to change between pulses or groups of pulses. The general form of this type of multiple-stage detection is outlined in figure 1.

Figure 1. Multiple-stage detection with a maximum of m stages.

An electronically scanned antenna points, once each on every scan, in a number of different directions. For any given stay of the beam, stage- $(i+1)$ testing occurs only if stage- i testing indicates that a signal might be present. The antenna beam stays in a given position until a final decision is made as to the presence of any targets in that sector. Note that in this type of multiple-stage test, final signal decisions can be made only in the final stage, while a final decision that no signal is in any resolution element can be made in any stage. Some of the many possible degrees of freedom in the design of each stage are the following.

1. Range resolution, number and placement of range bins
2. Doppler resolution, number and frequency coverage of filters

3. Energy per transmission
4. General class of statistical testing procedure (e.g., fixed-sample-size tests, Wald sequential tests,¹ etc.)
5. Number or average number of transmissions
6. Choice of not quantizing data or quantizing data into any number of levels
7. Decision statistic
8. Decision thresholds and, if used, quantization thresholds

A two-stage detection system having the same radar transmission characteristics in both stages was investigated by Helstrom.² In the system he describes, square-law detected returns from a number of pulses would be processed by superposing them and comparing the accumulation at the end of each stage with the threshold of that stage to see if it exceeds it for any delay, that is, for any range. The second-stage decision would be based on both first-stage and second-stage returns. For convenience in analyzing the process, he supposes that each return is sampled regularly in range, with the number of samples, or increments, equal to twice the product of the bandwidth and the sampling period. Letting the test statistic for each of the range increments be the sum of sampled returns over successive pulses is then analogous to integrating video returns. Two of Helstrom's conclusions as to the performance of this system were that its advantage over the ordinary detection system (a fixed-sample single-stage system) decreases as the SNR increases and that at best it achieved about twice the average scanning rate attained by an ordinary detection system. His investigation was not extended to cases with three or more stages since it appeared that little further improvement could be expected and the computations would be much more difficult.

H. M. Finn analyzed an "Energy Variant Sequential Detector" (EVSD)³⁻⁵ and, in a later report,⁶ a "Resolution-Variant EVSD." For rapidly fluctuating targets (Swerling's⁷ fluctuating target types 2 and 4) the decision statistic at each stage is the sum of square-law detected outputs, generally for three pulses in the first stage and six in the second, while for slowly fluctuating (types 1, 3) and nonfluctuating targets a linearly detected matched-filter output is used, generally with two or three stages of one pulse each. In some of the proposed versions of an EVSD the sum of both first-stage and second-stage outputs is used in the second stage. References 4 and 6 include analytical results on the optimization and performance of some of the EVSD's described, a verification of predicted performance by simulation on a digital computer, and descriptions of the general implementation of EVSD's and RV-EVSD's. One of his conclusions was that in many applications the increased efficiency which would result from employing more than two stages would not compensate for the inconveniences in implementation.

A more thorough analysis of an EVSD having single-pulse stages is made by Brennan and Hill.^{8,9} They present optimum energy levels and cumulative detection probabilities for various false-alarm probabilities and numbers of range

bins, and for each of the five standard target types. Their results indicate that the energy-variant two-stage radar has a power saving of several dB over a conventional, uniformly scanning radar.

In a brief note in an *IEEE Proceedings*, R. F. Baum gives a comparison of some two-stage and three-stage tests and of a sequential probability-ratio test with a conventional detection mode.¹⁰

BINOMIAL HIGH-RESOLUTION TWO-STAGE DETECTION

Glossary of Symbols

M_A	number of stage-A range bins per beam position
M_B	number of stage-B range bins in a stage-A range bin
m	$M_A M_B$, number of range bins searched by the single-stage system used as a comparison
R	range to target
$R_{\max}$	range to outer circumference of surveillance zone
r	$R/R_{\max}$, normalized range
S	mean power signal-to-noise ratio (SNR) per pulse, expressed in decibels
S_d	SNR in dB/pulse corresponding to a "design" target at range $R_{\max}$
$S(r)$	SNR in dB/pulse as a function of target range, with $S(1) = S_d$
S_A	SNR in dB/pulse for stage A when it differs from that for stage B, S then denoting the latter
FAR	false-alarm rate (average number per hour)
$P_D, P_D(S)$	single-scan detection probability
PRF	pulse repetition frequency — number of pulses used per second by the two-stage mode only, if time-shared with other modes
a, θ	parameters of Rice and Rayleigh distributions, respectively
N, N_A, N_B	number of pulses in a stage

$\bar{N}$	average number of pulses per beam position when noise alone is present, for a two-stage detector
$\bar{N}(S)$	average number of pulses per beam position occupied by a target, for a two-stage detector
$E(n), E_o(n)$	average number of pulses in a sequential stage-A, $E_o(n)$ in the noise-alone case
$\alpha, \alpha_A, \alpha_B$	false-alarm probability per test – same for every range bin in that stage; α_A is stage-A false-alarm probability when $M_A = 1$
β, β_A, β_B	miss probability per test (therefore per stage)
q, q_A, q_B	quantization level against which sample voltage x is compared; level may vary with range
p	$\text{prob } \{x \geq q\}$ for the sample voltage x corresponding to any particular stage and range bin
p_o	value of p when noise alone is present
p_A, p_B	specific cases of p_o
$p(S), p_A(S), p_B(S)$	value of p for particular values of the SNR
K, K_A, K_B	decision threshold of a fixed-sample-size test
C	parameter specifying a sequential test

General Description

The detection scheme to be investigated here is a binomial two-stage system featuring coarse resolution in stage A and fine resolution in stage B. The properties of the stages are listed below. Binomial quantization is performed by comparing each of the sample voltages with a quantization level chosen for that particular stage and range bin.

STAGE A

1. *Long pulses, generally few range bins.* The long pulses result in radially wide range bins; thus the surveillance zone is divided in range into a relatively small number M_A of bins per beam position.

2. *One stage-A decision per stage-A bin per beam position.* There are M_A stage-A tests per beam position, each having the two possible decisions, *alarm*

and no alarm. Computations will be for both binomial sequential testing and binomial fixed-sample-size testing.

3. *Very few pulses.* In a given beam position, a small number N_A of pulses are transmitted for stage-A fixed-sample-size testing, or a variable number for sequential testing. $E_o(n)$ denotes the mean number of pulses in a sequential stage-A when only noise is present.

STAGE B

1. *Stage B used infrequently.* In a given beam position, a stage B is used only if one or more alarms occur in stage A.

2. *Short pulses or pulse compression, many range bins.* The M_A wide bins of stage A are each subdivided into M_B radially narrow bins. (The equivalent single-stage system must therefore examine $M_A M_B$ bins.) The term *pulse* will refer to the entire waveform transmitted each pulse repetition period.

3. *Not all bins examined.* Stage-B tests are performed for those stage-B bins contained in a stage-A bin which alarmed.* Since the probability of a false alarm in stage A is small, the probability of multiple false alarms in stage A is very small; thus the number of stage-B bins examined when a stage B is used is usually M_B in noise-only cases.

4. *Same pulse repetition frequency as in stage A.*

5. *Many pulses.* Stage-B testing is the performance of a number of binomial fixed-sample-size tests, one for each bin being examined. The sample size N_B of these tests is considerably larger than N_A or $E_o(n)$.

6. *Same SNR as for stage A in results shown.* However, the equations given throughout are also valid when the SNR (signal-to-noise ratio per pulse) is different in stage B.

Distribution of Signal and Noise

The graphical results presented assume pulse-to-pulse independence and are for the Rayleigh and Rice cases, which correspond respectively to Swerling's rapidly fluctuating target type 2 and nonfluctuating target type. Computations for the rapidly fluctuating target of type 4 were made but for brevity are not included, since in every situation the type 4 curve fell almost entirely between the curves for type 2 and the nonfluctuating type.

*It would be better to examine, for each stage-A bin that alarms, the stage-B bins contained in a somewhat larger range interval (covering the alarming stage-A bin). An analysis of this more complicated case is unnecessary, since the higher threshold(s) required to provide the same false-alarm rate generally result in only a very slight loss in detection probability.

For target types with only scan-to-scan independence (slowly fluctuating types 1 and 3 for example) it would be necessary to calculate the single-scan detection probability

$$\bar{P}_D(\bar{S}) = \int P_D(S) g(S; \bar{S}) dS$$

where $g(S; \bar{S})$ is the density function of the SNR with mean $\bar{S}$ and $P_D(S)$ is the single-scan detection probability for a given value of the SNR.

The equations below are stated in the form for linear detection, but the results based on these may be used for square-law detection as well.

When the output voltage x of an envelope detector has the Rayleigh probability density function

$$f(x) = \frac{x}{\sigma^2} \exp(-x^2/2\sigma^2) \quad (x \geq 0, 2\sigma^2 = E(x^2))$$

where $\sigma^2 = 1$ when only noise is present and $\sigma^2 > 1$ when a signal is present, then the probability that x exceeds a quantization level q is

$$p = p_0 = \exp(-q^2/2)$$

in the noise-only case and is

$$p = p(S) = \exp(-q^2/2\sigma^2)$$

in the signal case, where

$$S = 10 \log_{10}(2\sigma^2 - 1)$$

is the mean power SNR in dB/pulse.

For a nonfluctuating target the normalized output voltage x (in units of rms i-f voltage) after linear envelope detection is assumed to have the well-known Rice density function

$$f(x) = x \exp\left[\frac{-x^2 - a^2}{2}\right] I_0(ax)$$

where a is the ratio of peak i-f signal voltage to rms i-f noise voltage and $I_0(\cdot)$ is the modified Bessel function of first kind, zero order. The probability that x exceeds q is

$$p = p_0 = \exp(-q^2/2)$$

in the noise-only case and is (ref. 7, page 159)

$$p = p(S) = \exp\left[\frac{-q^2 - a^2}{2}\right] \sum_{j=1}^{\infty} (q/a)^j I_j(aq)$$

in the signal case, where

$$S = 10 \log_{10} a^2/2$$

is the mean power SNR in dB/pulse.

Let S_d be the SNR corresponding to a "design" target (a target with specified radar cross section) at range $R_{\max}$. The fourth-power relation between range and received echo-power gives as the SNR of this design target at range r

$$S = S(r) = S_d - 40 \log_{10} r$$

where r is the normalized range $R/R_{\max}$.

Binomial Fixed-Sample-Size Testing

In a stage of the fixed-sample type, a binomial fixed-sample-size test is performed for each range bin. An individual test is performed by comparing, for each of N successive independent pulses, the sample voltage (the amplitude of the envelope at the time delay corresponding to the given range bin) with the quantization level q , and deciding "alarm" if at least K of the N voltages exceed q and "noise only" otherwise. If p is the probability for a particular range bin that the sample voltage for any given pulse will exceed the threshold q , the probability for that range bin that K out of N independent samples exceed q is the cumulative binomial probability

$$D(N, K, p) = \sum_{i=K}^N \binom{N}{i} p^i (1-p)^{N-i}$$

(For convenience the subscripts designating stage are omitted here.) For fixed N and α each possible value of the threshold K determines a value of p_o , from $\alpha = D(N, K, p_o)$, and therefore determines a value of q . Then each K , from its q , also determines a value of $p(S)$ for any given value of S . The optimum value of K for the given value of S is that which minimizes the miss probability $\beta(K, S) = 1 - D(N, K, p(S))$. Whenever convenient, $\beta(K, S)$ will be written $\beta(S)$ or β , and since the optimum values of the thresholds are used for each S in all calculations, $\beta(S)$ and β will be the minimum $\beta(K, S)$ for that particular N , α , and S . Ideally, for each range-bin test the threshold K should be chosen to optimize detection for the SNR of the design target at that range, i.e., for $S(r)$, where arbitrarily r could be the center of the bin. Generally, however, the use of a different quantization level q for each bin is impractical in implementation. It is fortunate, therefore, that using the same value of K for all bins results in miss probabilities which do not differ appreciably from those in the ideal case. In figure 2, curves of $\beta(K, S)$ are given for a typical stage-B test ($N=12$; $\alpha=6 \times 10^{-6}$; Rice distribution); note that $K=7$ is optimum over effectively the entire range of $\beta(S)$, as S varies, and that the curves are relatively flat over several values of K .

Figure 2. These curves for a binomial fixed-sample-size test typical of those employed in stage B indicate that using the same threshold for all range bins would result in little or no loss in optimality.

False-Alarm Rate (FAR)

For a fair comparison of the detection capabilities of various testing procedures, all should have the same FAR. The FAR of the fixed-sample single-stage system is proportional to the number m of range bins per beam position and to the false-alarm probability α of an individual test and is inversely proportional to the number N of pulses per beam position.

$$\text{FAR} = (3600 \text{ sec/hr}) \times \text{PRF} \times m \alpha / N$$

The FAR of the two-stage system (sequential or fixed stages) has no simple exact formulation but is closely approximated by

$$\text{FAR} = 3600 \times \text{PRF} \times M_A M_B \alpha_A \alpha_B \sqrt{N}$$

where $\alpha_A \alpha_B$ is the per-bin false-alarm probability for all $M_A M_B$ range bins and $\bar{N}$ is the average number of pulses per beam position when only noise is present. When comparisons are made where the number of range bins and the PRF are fixed, the FAR is made the same for all cases by holding $\alpha_A \alpha_B$ constant if $\bar{N}$ is fixed or $\alpha_A \alpha_B / \bar{N}$ constant if $\bar{N}$ varies.

Sample Sizes

When applying two-stage detection to a particular detection situation, it probably will not be immediately evident what values of $\bar{N}$, N_A (or $E_0(n)$), N_B and α_A are best to use. A relation among these quantities is

$$\bar{N} = N_A + \left[1 - (1 - \alpha_A)^{M_A} \right] N_B$$

for fixed-sample stages, and

$$\bar{N} = E_0(n) + \alpha_A N_B$$

when the first stage is sequential and involves a single range bin. (Sequential tests are discussed later; calculations were not made for the multiple-resolution-element type.)

The nature of binomial tests makes it necessary to use numerical rather than analytical methods to find the optimum values of the unspecified quantities, but the amount of computation time required on a high-speed computer is moderate.

When values of $\bar{N}$, M_A , N_A (or $E_0(n)$), $S - S_A$,* S , and $\alpha_A \alpha_B$ (or $\alpha_A \alpha_B / \bar{N}$) are specified, the optimum N_B is the value yielding the maximum single-scan detection probability

$$P_D(S) = \left[1 - \beta_A(S_A) \right] \left[1 - \beta_B(S) \right]$$

For each value of N_B considered, the values of α_A and α_B must be determined before calculating $P_D(S)$; the value of α_A is determined from the equation for $\bar{N}$ (an additional relation between α_A and $E_0(n)$ must be satisfied in the sequential case), and α_B is found from fixed $\alpha_A \alpha_B$. It is important to keep in mind that the optimum value of N_B varies with S and therefore varies with range for a target of given cross section. For simplicity it is assumed that the same number N_B of pulses are transmitted no matter which stage-A bin alarms.

The optimization of N_A , given $\bar{N}$, and the optimization of $\bar{N}$ are discussed later.

Another sample size of interest is the mean number of pulses when a signal is present. When $M_A = 1$, this quantity is $\bar{N}(S) = N_A + [1 - \beta(S)] N_B$ for the fixed-sample case and is $\bar{N}(S) = E(n) + [1 - \beta(S)] N_B$ for the sequential case, where $E(n)$ is a function of S .

* S_A and S are the per-pulse SNR's in stage A and stage B, respectively; their difference is assumed constant with range and cross section, and is zero for all results presented.

Resolution

Large range bins for stage A can be produced by transmitting pulses of long duration. When only one range bin is used in stage A, the minimum range of the bin must be at least half of $R_{\max}$, because of propagation time. The two-stage mode would then be an acquisition mode, and a single-stage high-resolution mode could be included for close targets; in fact, for any application in which each stage-A pulse is very long, detection for the resulting blind range must be provided for in some other way. If each stage-A pulse is a very long uncoded waveform, the width of the signal frequency spectrum might be small compared with the doppler frequency shift. Then, to cover the range of expected doppler frequency, it would be necessary to use in stage A either a bank of doppler filters or a receiver bandwidth considerably larger than the width of the signal frequency spectrum. The analysis of a two-stage system with doppler filters used in this manner is more complicated and will not be considered here. If the doppler shift is provided for by making the receiver bandwidth large or if sampling long pulses results in a loss in SNR, then the decrease in the stage-A SNR must be taken into account. As the comparisons presented in this paper are in terms of a per-pulse SNR common to both stages, power saving can be measured only when assuming that both the transmitted energy and the resulting SNR are the same for stage-A pulses as for stage-B pulses. Otherwise, one must compute for each case the average amount of transmitted energy required per beam position.

Even though the resolution cells will be referred to as range bins, the results hold for any type of resolution such that (1) each of the M_A cells is subdivided (in range and/or doppler frequency or other) into M_B cells, and (2) a stage-B cell is tested if and only if the one stage-A cell containing it alarms.

Accepting the fact that the resolution-varying property of two-stage detection contributes to its efficiency, the logical conclusion is that the range of surveillance (in a clutter-free environment) should be covered with as few stage-A range bins as possible under the limitations on minimum range and doppler shift mentioned above. This is supported by the P_D -versus- S curves given in figure 3 for fixed-sample stages, a Rice distribution, and $\bar{N} = 1.2$. Since the optimum value of N_B decreases with increase in M_A , each curve was computed using the N_B which was optimum for P_D about 0.5; these values of N_B were 14, 10, 7, 5, and 4, respectively, for $M_A = 1, 4, 16, 64$, and 256. Note that quadrupling the number of stage-A range bins causes an increase of roughly 0.6 to 1.0 dB/pulse in the SNR required for a single-scan detection probability of 0.5. For a fair comparison, the M_A bins would cover the same search zone in each of the cases; then the product $M_A M_B$ must be the same for each case in order that the FAR and final resolution be the same. ($\text{FAR} = 3 \text{ PRF} \times M_A M_B \times 10^{-5}$) The correspondence of range bins (bin centers) to values of S is determined by whatever value is assigned to S_d . The P_D -versus- S curves are valid for any S_d (in the range of the graph) as long as the threshold assigned to each range-bin test is the optimum one for the value of S corresponding to that range (for that S_d) and the target is indeed of the design cross section. For any other situation, where S is the actual rather than design value of the SNR, the curves give only upper bounds to P_D , generally close bounds, though, since (as was suggested by fig. 2) the thresholds would still be at least nearly optimum.

Figure 3. Single-scan detection probability vs required SNR, where the final resolution remains constant but the coarseness of the stage-A bins varies. Rice distribution; $\bar{N} = 1.2$, optimum N_B for each M_A for $P_D = 0.5$; FAR = $3 \text{ PRF} \times M_A M_B \times 10^{-5}$, where $M_A M_B$ may vary.

The case of a single range bin in stage A will be treated in detail since (1) the detection zone to be searched by a two-stage system will in some applications be sufficiently narrow radially that the pulse length necessary to bridge it with a single range bin is not inefficiently long, and for these applications $M_A = 1$ is optimum; (2) from the analysis of the $M_A = 1$ case, many general conclusions can be drawn concerning the nature of two-stage systems having any M_A ; and (3) it is the simplest case to analyze.

$\bar{N}$ and the Average Scanning Rate

The average scanning rate in revolutions per minute for a two-stage system is $60 \times \text{PRF} / (\text{no. of beam positions}) \times \bar{N}$ when no target is present and is approximately that when the target density is low. Suppose first that the PRF

is predetermined by equipment restraints or other considerations. The following properties of the two-stage system are pertinent to the problem of selecting a suitable value of $\bar{N}$.

1. The single-scan detection probability P_D increases with $\bar{N}$. Figure 4 illustrates this and also indicates how the optimum value of N_A changes with $\bar{N}$. (Fixed-sample stages; Rice distribution; $S = 5$ dB/pulse; $M_A = 1$; $\alpha_A \alpha_B = \bar{N} \times 10^{-6}$; K_A, N_B, K_B optimum; $\text{FAR} = 3.6 \times M_B \times \text{PRF} \times 10^{-5}$, where M_B may vary.)

2. The average number of scans across a target staying in the surveillance zone a relatively long time is approximately inversely proportional to $\bar{N}$ if the number of beam positions is large and the target density is low.

3. The cumulative probability of detection (probability of at least one detection before a given range) tends to increase slightly with $\bar{N}$ but in an irregular manner. Property 3 is the resultant of properties 1 and 2. It has not been verified, but is supported by calculations of the quantity $1 - (1 - P_D)^{\text{constant}/\bar{N}}$, where P_D is a function of $\bar{N}$ and of the optimum test parameters for that $\bar{N}$. This quantity is an indicative measure of the cumulative detection probability; it is based on the fact that the probability of detection is equal to $1 - (1 - P_D)^L$ for a target scanned L times while at the same range, with L approximately inversely proportional to $\bar{N}$ by property 2. Curves of $1 - (1 - P_D)^{4/\bar{N}}$ versus $\bar{N}$ are given in figure 5 for the values of P_D plotted in figure 4.

4. The length of an optimized two-stage test in the case of a detection (or at least a stage-A alarm) tends to increase with N . Figure 6 illustrates this with a plot of $N_A + N_B$ versus $\bar{N}$, where N_B is optimized for each N_A and $\bar{N}$. Although $N_A + N_B$ is an upper bound to $\bar{N}(S)$, it varies with $\bar{N}$ in a similar manner and is approximately equal to $\bar{N}(S)$ for signals somewhat stronger than that for which the N_B 's are optimized here.

Properties 3 and 4 suggest that a small $\bar{N}$ is at least nearly as efficient as a large $\bar{N}$, so that choosing the most suitable average scanning rate is largely a matter of determining (and this involves examining the corresponding single-scan detection probability curve) the most desirable value of the number of scans across a target (of particular velocity) while it travels through the surveillance zone.

Consider now the case where the PRF is adjustable in that it may be set at one of a wide range of values. The optimum choice of the combination of PRF, average scanning rate, and $\bar{N}$ then depends on the particular application. For a situation in which the radar is peak-power limited and the cost of the power can be disregarded, the PRF might be best set at the highest rate allowing the desired maximum unambiguous range. If, on the other hand, the major limitation is that of average power, then the most efficient procedure would probably be to maximize the transmitted energy per pulse by using a small $\bar{N}$ and, for that $\bar{N}$, the lowest PRF that permits the antenna to scan across the target a reasonable number of times.

It is fair to conclude that a small $\bar{N}$ is suitable in many applications. Also, note in figure 4 that for $\bar{N}$ as large as 2.6 the $N_A = 1$ case is still better (in the particular situation considered and in other typical situations) than the $N_A = 2$ case. The subsequent analysis is restricted to cases with $N_A = 1$ and $E_o(n) < 2$ for these two reasons and because computationally they are the most tractable.

Figure 4. Single-scan detection probability vs mean number of pulses (noise-only case) per beam position. Rice distribution; $S = 5$ dB/pulse; $M_A = 1$; K_A, N_B, K_B optimum for each point; $\text{FAR} = 3.6 M_B \times \text{PRF} \times 10^{-5}$, where M_B may vary.

Figure 5. An approximation to the probability of detecting a constant-range target when the mean number of scans across the target is $4/\bar{N}$; an indication of how the cumulative detection probability would vary with $\bar{N}$. The values of P_D used are those plotted in figure 4.

Figure 6. Number of pulses in an optimized two-stage test when a target is detected vs the mean number when noise only is present, for the cases plotted in figures 4 and 5.

ANALYSIS OF SPECIAL CASES

General Description

Sequential Cases. STAGE A: Binomial Wald¹ sequential test, single-range-bin type ($M_A = 1$), with $E_o(n)$ less than about two. STAGE B: M_B range bins (radially narrow). Returns from N_B pulses used in M_B binomial fixed-sample-size tests.

Fixed-sample Cases. STAGE A: One pulse, one range bin ($N_A = 1, M_A = 1$). STAGE B: M_B range bins. Returns from N_B pulses used in M_B binomial fixed-sample-size tests.

The detection capabilities of these two-stage detectors will be compared with that of a conventional single-stage detector, one that employs a binomial fixed-sample-size test for each of $m = M_B$ range bins.

A Sequential Test For Stage A

When there is only one resolution element to be tested and $E_o(n)$ is to be less than about two, the "best" binomial sequential test is a very simple form of Wald's test, one which may be performed in the following manner. A random variable y_i is assigned the value 1 if the sample voltage x for the i th pulse exceeds q , and the value (-1) otherwise. After each pulse, say the n th, one of three possible actions is taken. (C is an integer larger than two and is specified in accordance with desired test characteristics.)

If $0 < 1 + y_1 + \dots + y_n < C$, testing is continued with another pulse.

If $C \leq 1 + y_1 + \dots + y_n$, testing is terminated with an alarm.

If $1 + y_1 + \dots + y_n \leq 0$, testing is terminated without an alarm.

A test of this form is "best" in the following sense.

1. It is a probability-ratio test for (and only for) any p_o and $p(S)$ such that $p(S) = 1 - p_o$. (The probability that y_i equals 1 is p_o when noise only is present and $p(S)$ when the SNR has value S .) Since the SNR varies widely with range and cross section, the choice of the value S for which $p(S) = 1 - p_o$ as the value for which to optimize the test is often as fair as any.

2. For any binomial sequential test having an $E_o(n) < 2$, there is some C and q such that a test of the above form has the same $E_o(n)$, approximately the same α , and a $\beta(S)$ which is not appreciably larger for any S (assuming any particular functional form of the noise and signal-plus-noise distributions and assuming q adjustable for each test). This was apparent from an examination, for a wide variety of binomial sequential tests, of the trade-offs in detection characteristics that occur from test to test; the nature of binomial tests does not permit direct comparisons.

3. A test of this form is much simpler to analyze than any other binomial sequential test. Note that the sum $1 + y_1 + \dots + y_n$ will equal one of the bounds

at termination. Since Wald's approximation equations yield true values for tests in which there can be no excess over either bound, exact results can be obtained without resorting to unwieldy iterative computational methods. The necessary equations can be put in terms of quotients of polynomials;* those for the $C = 6$ case, for example, are

$$E(n|p) = \frac{1 - 2p + 3p^2 + 3p^4}{1 - 4p + 7p^2 - 6p^3 + 3p^4}, \quad P\{\text{alarm}|p\} = \frac{p^5}{1 - 4p + 7p^2 - 6p^3 + 3p^4}$$

$E_o(n) \equiv E_o(n|p_o)$ is assigned a value, and the computer solves for $p_A \equiv p_o$ and $\alpha_A \equiv P\{\text{alarm}|p_o\}$ and, given a value of S , for $p(S)$, $\beta_A(S) \equiv 1 - P\{\text{alarm}|p(S)\}$, and $E(n|p(S))$.

Detector Optimization

Since $M_A = 1$, the mean number of pulses per beam position in the noise-only situation is $\bar{N} = 1 + \alpha_A N_B$ for a system with a single-pulse stage A, and $\bar{N} = E_o(n) + \alpha_A N_B$ for a system with a sequential stage A.

For each N_B there is a value of $\bar{N}$ for which P_D is maximized when the SNR and FAR are fixed and optimum thresholds are used. The value of $\bar{N}$ at which this maximum P_D occurs is close to 1 for small N_B but in typical cases reaches $\bar{N} = 1 + N_B$ for N_B greater than about 6 or 8. As $\bar{N}$ is increased while holding S , $\alpha_A \alpha_B / \bar{N}$ and N_B constant, α_A increases, and therefore β_A decreases and (since α_B decreases for fixed $\alpha_A \alpha_B / \bar{N}$) β_B increases. For the sequential case the increase of α_A with $\bar{N}$ depends on the fact that α_A increases with $E_o(n)$ (see fig. 7) for tests (fixed C) of the class considered. The effect of the opposite behaviors of the per-stage miss probabilities on $P_D = (1 - \beta_A)(1 - \beta_B)$ accounts for the existence of a maximum P_D . Graphs of P_D versus $\bar{N}$ are given as figure 8 for a system with fixed-sample stages and as figure 9 for a system with a sequential first-stage. (Rice distribution; $S = 8$ dB/pulse; $M_A = 1$; $\alpha_A \alpha_B = \bar{N} \times 10^{-8}$; optimum K_B for each N_B and $\bar{N}$.) Since a larger $\bar{N}$ results in fewer opportunities to detect the target, the actual optimum values of $\bar{N}$ (those which maximize the cumulative detection probability) are smaller than those for which P_D is maximized. One conclusion from these results is that when N_B is limited to a relatively small value by certain factors in a particular application – possibly by the length of time the target remains in the resolution cell or by high-target density, clutter, or other conditions occasionally present which cause a large proportion of the stage A's to alarm – then increasing $\bar{N}$ beyond a certain value will not improve even the *single-scan* detection probability. In fact, $\bar{N}$ can often be set considerably smaller than that value with little loss in P_D and perhaps no loss in the cumulative detection probability.

*These are obtained from equations (5:20), (5:19) and (5:23) in reference 1 by letting $p_1 = 1 - p_0 > 1/2$ and $\log \frac{1 - \beta_1}{\alpha} = (C - 1) \log \frac{1 - \alpha}{\beta_1} = (C - 1) \log \frac{1 - p_0}{p_0}$, where β_1 is the value of β when $p(S) = p_1$. They are also derivable by the method of difference equations ("the classical ruin problem,"¹¹) or by using Markov chain theory.

Figure 7. False-alarm probability vs average sample size (noise-only case) for sequential tests of the type employed in stage A.

Figure 8. Single-scan detection probability vs mean number of pulses (noise-only case) per beam position, for a two-stage test with fixed-sample stages. Rice distribution; $S = 8$ dB/pulse; $M_A = 1$; $N_A = 1$; optimum K_B for each N_B and $\bar{N}$; $\text{FAR} = 3.6 M_B \times \text{PRF} \times 10^{-5}$, where M_B may vary.

Figure 9. Single-scan detection probability vs mean number of pulses (noise-only case) per beam position, for a two-stage test with a sequential ($C = 4$) stage A. Rice distribution; $S = 8$ dB/pulse; $M_A = 1$; optimum K_B for each N_B and $\bar{N}$; $\text{FAR} = 3.6 M_B \times \text{PRF} \times 10^{-5}$, where M_B may vary.

For fixed $\bar{N}$, FAR, and SNR, there is an optimum N_B and, therefore, an optimum α_A . The existence of an optimum N_B has an explanation similar to that for $\bar{N}$ above: as N_B is increased while holding $\bar{N}$, $\alpha_A \alpha_B / \bar{N}$, and S constant, α_A decreases and therefore β_A increases and β_B decreases. Figure 10 illustrates this for a typical fixed-sample case (Rice distribution; $S = 8$ dB/pulse; $M_A = 1$; $\bar{N} = 1.2$; $\alpha_A \alpha_B = 10^{-8}$; optimum K_B for each N_B). The optimum N_B tends to gradually increase with $\bar{N}$ (as can be seen in figs. 8 and 9); however, because of the nature of binomial tests it conceivably can dip, as it did for the $N_A = 2$ case in figure 6. For most sequential cases, the N_B giving the maximum P_D is too large to be practical; while the improvement of P_D with N_B is noticeable up to about $N_B = 8$, further improvement is not enough to compensate for the disadvantages (when clutter or many targets are present, etc.) of having long second stages.

The value 1.2 is used for $\bar{N}$ in many of the figures primarily because it is small enough that rough comparisons with the $\bar{N} = 1$ single-stage system are possible and also because the higher P_D that would be achieved by using a larger $\bar{N}$ might not compensate for the smaller average number of scans across the target and for the longer second stage required for optimality.

Figure 10. For fixed M_A , $\bar{N}$, FAR, and SNR, the manner in which the per-stage miss probabilities change with N_B produces an optimum N_B .

Even when the number N_B of stage-B pulses is not constrained by other considerations, it can be optimized for at most only a few adjacent range bins for a target of given cross section. For the particular case graphed in figure 11

Figure 11. Single-scan detection probability vs normalized range, for a target of design size.

(fixed-sample stages; Rice distribution; $S_d = 0$ dB/pulse; $S(r) = -40 \log_{10} r$; $M_A = 1$; $\bar{N} = 1.2$; $\alpha_A \alpha_B = 10^{-6}$; optimum K_B for each N_B), the dashed curve is a bound for any curve with N_B fixed — a curve for an N_B somewhat larger than 12 would be closer to the dashed curve only at outer ranges. Probably the more practical procedure is to optimize the test for an SNR that corresponds to a chosen value of P_D , rather than to optimize it for the SNR that corresponds to an arbitrarily chosen range and design cross section. Figure 12 (fixed-sample stages; $M_A = 1$; $\bar{N} = 1.2$; optimum K_B for each point; P_D varied by changing the SNR) shows how the optimum N_B changes considerably with P_D . The corresponding optimum values of the stage-A false-alarm probability (optimum $\alpha_A = 0.2 / [\text{optimum } N_B]$ when $\bar{N} = 1.2$) for these cases are plotted in figure 13. These

Figure 12. Optimum number of stage-B pulses vs single-scan detection probability. Optimum K_B for each point; $M_A = 1$; $\bar{N} = 1.2$; P_D varied by changing the SNR.

Figure 13. Stage-A false-alarm probability (corresponding to optimum N_B in figure 12) vs single-scan detection probability.

results indicate that the percentage of times a second stage is necessary in the noise-only case is between about 0.5 percent and 4 percent when the two-stage test is optimized for a P_D between about 0.2 and 0.8. It can be seen in figures 14 and 15 that the ratio of per-stage miss probabilities of an optimized test tends to fall between about 0.04 and 0.1 (a large variance results from the binomial

Figure 14. Ratio of per-stage miss probabilities vs single-scan detection probability for the optimized tests of figures 12 and 13.

and small-sample nature of the tests) even as the SNR (and therefore P_D), the FAR, the underlying distribution, and $\bar{N}$ vary. (Fig. 15 is for fixed-sample stages, $M_A = 1$, $N_A = 1$, $\alpha_A \alpha_B = \bar{N} \times 10^{-8}$, optimum N_B and K_B per point. Successive segments correspond to increasing values of N_B ; the ratio increases fairly smoothly with $\bar{N}$ for constant N_B .)

Figure 15. Ratio of per-stage miss probabilities vs mean number of pulses (noise-only case) per beam position, for optimized tests. Optimum N_B and K_B for each point; $M_A = 1$; $N_A = 1$; $\alpha_A \alpha_B = \bar{N} \times 10^{-8}$.

Comparison of Systems

In figures 16 through 19, single-scan detection probabilities are plotted for a single-stage system, a two-stage system with fixed-sample stages, and two cases of two-stage systems with sequential first stages. In each figure, the two sequential cases ($C = 4$, $N_B = 10$) and ($C = 6$, $N_B = 8$), require approximately the same average dwell-time in the signal case, and for large signals the values of $\bar{N}(S)$ for the sequential cases and for the fixed-sample case are all approximately 13. A comparison of figure 16 with figure 17 will show that the power savings of the two-stage systems over the single-stage system increase if the FAR is reduced by decreasing $\alpha_A \alpha_B$ and α . Figures 16 through 18 do not allow a completely fair comparison of the single-stage case with the two-stage cases because of the larger number of opportunities to detect the target in an $N = 1$ single-stage case than in $\bar{N} = 1.2$ cases, so a comparison for $N = \bar{N} = 2$ was included as figure 19.

As was seen in figure 7, the sequential test with $C = 6$ is capable of providing a smaller α_A than the $C = 4$ test with approximately the same $E_0(n)$. However, since a very small α_A is not efficiently utilized in a two-stage system, the detection probability of a system employing the $C = 6$ test is only slightly better than that of a system employing the $C = 4$ test and the same N_B . The $C = 4$ test is generally a good choice since tests with larger C result in larger sample sizes in clutter or signal situations. The tendency of the sequential system toward a long first stage when clutter is present or the signal is small is evident in figure 20.

Figure 16. Single-scan detection probability vs SNR. Rice distribution; $M_A = 1$; $FAR = 3 M_B \times PRF \times 10^{-8}$, where M_B may vary.

Figure 17. Single-scan detection probability vs SNR. Rice distribution; $M_A = 1$; $FAR = 3 M_B \times PRF \times 10^{-7}$, where M_B may vary.

Figure 18. Single-scan detection probability vs SNR. Rayleigh distribution; $M_A = 1$; $FAR = 3 M_B \times PRF \times 10^{-5}$, where M_B may vary.

Figure 19. Single-scan detection probability vs SNR. Rice distribution; $M_A = 1$; $\text{FAR} = 1.8 M_B \times \text{PRF} \times 10^{-6}$, where M_B may vary.

Figure 20. The mean number of pulses in a sequential stage-A vs the probability that the receiver output voltage exceeds the quantization level.

MULTIPLE-RANGE-BIN FIRST-STAGE

In most of the practical applications of two-stage systems, it will be necessary to use at least several first-stage bins to cover the search zone. Figure 21 presents plots of P_D vs S for a Rice distribution for various combinations of M_A and N , where $\text{FAR} = 3 \times \text{PRF} \times M_A M_B \times 10^{-8}$. As mentioned in the discussion of figure 3, for a fair comparison of cases having different values of M_A , the M_A bins must cover the same search zone and $M_A M_B$ must have the same value. From a comparison of these figures the following observations can be made.

1. An increase of about 0.6 to 1.0 dB/pulse in the SNR (the SNR required for a single-scan detection probability of 0.5) is necessary as M_A is quadrupled (under the assumptions made in the earlier discussion under "Resolution"). This same observation was mentioned in connection with figure 3. The amount of this increase tends to be smaller for the larger values of M_A .
2. For fixed N (and $P_D = 0.5$), the decrease in the optimum value of N_B with an increase in M_A is often considerable.
3. For fixed M_A , the optimum value of N_B increases with $\bar{N}$; this is consistent with the results for $M_A = 1$ shown in figure 6.
4. The optimum value of N_B decreases with an increase in P_D (equivalently, with an increase in S), as was shown for the $M_A = 1$ case in figure 12.

One important conclusion drawn from these figures is that often an N_B considerably smaller than the optimum N_B can be used without significant loss in detectability; this is fortunate, since a small value of N_B is desirable in situations involving occasional clutter or high target density.

Figure 21. Single-scan detection probability vs SNR, for various values of $\bar{N}$ and M_A . Rice distribution; $\text{FAR} = 3 \text{ PRF} \times M_A M_B \times 10^{-5}$, where M_B may vary.

Figure 21 (Continued)

Figure 21 (Continued)

Figure 21 (Continued)

Figure 21 (Continued)

Figure 21 (Continued)

APPLICATIONS INVOLVING CHANGES IN ENERGY

When employing a two-stage procedure with any particular radar, the full power of the radar must, of course, be used in both stages if the best possible detection probability is to be achieved. This policy generally results in using the same per-pulse energy in both stages if the radar is average-power limited but is not (for the waveforms used) peak-power limited. If continual operation at maximum power is impractical, as when the source of energy is so limited by its cost or weight that only a certain average energy can be transmitted per beam position, then the optimum division of energy between the two stages (equivalently, the optimum per-pulse energy for each stage) must be determined for each detection situation under consideration. The determination of these optimum energy levels may involve boundary conditions imposed by peak-power and/or average-power limitations.

CONCLUDING REMARKS

Two-stage procedures promise considerable saving over conventional procedures when the target density is low and the radar video is clutter-free. In other environments, two-stage detection systems, like all sequential detection systems, spend too much time in too many sectors. This can be alleviated to some extent by automatically switching to a fixed dwell-time mode in cluttered or jammed sectors.

For cases where the average number of pulses per beam position is less than about two and a single range-bin is examined in the first stage, calculations showed that the detector with a sequential first stage has a slightly higher detection probability than the detector with fixed-sample stages; however, this saving hardly seems enough to compensate for the more complex implementation required by the sequential system and for its slowness in clutter situations. For applications which call for a large average number of pulses per beam position, it seems likely that a detector employing a carefully designed sequential test in the first stage would have a substantially better detection capability than a detector with fixed-sample stages, especially if only one or a very few range bins are used in the first stage. The sequential test becomes less suitable as the number of first-stage range bins increases since, as a number of people have demonstrated for a variety of multiple-resolution-element sequential tests, the saving of sequential tests over fixed-sample-size tests decreases as the number of resolution elements increase. This latter property of sequential tests is the reason for considering their incorporation into two-stage detection procedures when high resolution is required; the benefits of sequential detection may be realized in the detection function of the coarse-resolution first stage, while the fine-resolution requirements are fulfilled in the second stage.

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13. ABSTRACT Multiple-stage detection systems which allow stage-to-stage changes in the radar characteristics and in the statistical decision process are described, and the literature on such systems is briefly reviewed. A binomial two-stage detection system with coarse-fine resolution is then outlined and the optimization of some of its design parameters considered, with appropriate curves given. Some special cases of these binomial two-stage detectors are investigated in detail; for these, detection probabilities and optimum design parameters are plotted as functions of other design parameters and of the signal strength, and comparisons made with a conventional detection method.		

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