TP 81-5

Design of Riprap Revetments for Protection Against Wave Attack

by

John P. Ahrens

TECHNICAL PAPER NO. 81-5

DECEMBER 1981

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U.S. ARMY, CORPS OF ENGINEERS
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RESEARCH CENTER

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TP 81-5	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DESIGN OF RIPRAP REVETMENTS FOR PROTECTION AGAINST WAVE ATTACK		5. TYPE OF REPORT & PERIOD COVERED Technical Paper
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John P. Ahrens		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of the Army Coastal Engineering Research Center (CERRE-CS) Kingman Building, Fort Belvoir, Virginia 22060		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS D31616
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Army Coastal Engineering Research Center Kingman Building, Fort Belvoir, Virginia 22060		12. REPORT DATE December 1981
		13. NUMBER OF PAGES 31
14. MONITORING AGENCY NAME & ADDRESS (If different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Armor overlays Revetments Riprap Wave runup		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Basic information on the design of riprap revetments for protection against wave attack is presented. The topics covered include the selection of armor and filter layers, zero damage and reserve stability, design wave height, wave runup, and the use of armor overlays. Example problems are worked to illustrate the concepts presented.		

PREFACE

This report provides information and specific guidance on the design of stone riprap revetments exposed to wave attack, including several examples to illustrate the concepts presented. It supplements Sections 7.21 and 7.37 of the Shore Protection Manual (SPM).

The report was prepared by John P. Ahrens, Oceanographer, under the general supervision of Dr. R.M. Sorensen, Chief, Coastal Processes and Structures Branch, Research Division.

The author acknowledges the numerous contributions by various reviewers to an early draft of this report, and especially the comprehensive and helpful review by D.D. Davidson, Chief, Wave Dynamics Branch, Hydraulics Laboratory, U.S. Army Engineer Waterways Experiment Station (WES).

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

TED E. BISHOP

Colonel, Corps of Engineers
Commander and Director

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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins ¹

¹To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9) (F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9) (F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

C	overlay stone weight per square meter of embankment surface (kilograms per square meter)
D	typical dimension of a stone (meters)
d	water depth (meters)
d _s	water depth at toe of structure (meters)
g	acceleration of gravity (9.80 meters (32.2 feet) per second squared)
H	wave height at toe of structure (meters)
H _{max}	maximum wave height at toe of structure (meters)
H _O ¹	deepwater unrefracted wave height (meters)
H _s	significant wave height at toe of structure (meters)
K _{RR}	stability coefficient for riprap (eq. 6)
L _O	deepwater wavelength, $L_O = gT^2/2\pi$ (meters)
ℓ _ℓ	lower limit of damage (meters)
ℓ _u	upper limit of damage (meters)
N _s	stability number (eq. 4)
R	wave runup (meters)
R _{max}	maximum wave runup for irregular wave conditions (meters)
R _s	runup of a wave with the significant height and period of maximum energy density (meters)
r	thickness of the armor layer when used with respect to runup; the ratio of the runup on riprap to the runup on a smooth surface for the same slope and wave conditions
T	wave period of a monochromatic wave (seconds)
T _p	wave period of maximum energy density of the spectrum (seconds)
$\bar{W}$	average stone weight (kilograms)
W ₅₀	median stone weight (kilograms)
w _r	unit weight of stone (kilograms per cubic meter)
w _w	unit weight of water (kilograms per cubic meter)
θ	angle between the embankment slope and the horizontal
σ	standard deviation

DESIGN OF RIPRAP REVETMENTS FOR PROTECTION AGAINST WAVE ATTACK

by
John P. Ahrens

I. INTRODUCTION

Quarrrystone is the most commonly used material for protecting earth embankments from wave attack because, where high-quality stone is available, it provides a stable and unusually durable revetment armor material at relatively low cost. This report provides information and specific guidance on the design of stone riprap revetments, including several examples to illustrate the concepts presented. It supplements Sections 7.21 and 7.37 of the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977).

II. RIPRAP DESIGN CONSIDERATIONS

The discussion in this section draws heavily on laboratory studies of riprap stability. Currently, there is little well-documented information available on the field performance of riprap. In the design of a riprap revetment, a careful evaluation of the performance of riprap or other revetments near the design site or at similar sites is an important adjunct to the guidance given in this report. Information on the design of armor and filter layers, zero-damage and reserve stabilities of the armor layer, selection of overlay armor to upgrade existing revetments, and wave runup is given in this section. Two design aspects which are particularly difficult to study in the laboratory include the toe design of a riprap revetment and tying the ends of the revetment into a nonrevetted embankment. Consequently, these aspects are not discussed in this report since little information is available on them.

A definition sketch for some terms used in this section is shown in Figure 1.

Figure 1. Definition sketch.

1. Armor Layer.

Stone used in the armor layer should be hard and durable. Experience is the best guide in choosing a durable stone. Whenever possible, stone which has proven to be satisfactory on earlier, similar projects should be used. Persons familiar with local quarries can often provide information on stone quality. Esmiol's (1968) study of rock used to protect the upstream slope of earth dams concluded that granite or granitic-type rock is the best for riprap and that the best means to evaluate durability before use are by a specific gravity test, an absorption test, and a petrographic analysis. A recent survey of riprap stone quality by M.L. Giles (Research Hydraulic Engineer, U.S. Army Engineer District, Kansas City, personal communication, 1979) indicates that there are, at present, no foolproof tests which can give assurance of rock durability, but that the specific gravity test is the single, most reliable method.

Thomsen, Wohlt, and Harrison (1972) found that the gradation of stone used in riprap had little influence on stability when the median weight, W_{50} , was used to characterize the stone size. Following Thomsen, Wohlt, and Harrison (1972), this report uses W_{50} to characterize stone size. Their laboratory tests of riprap stability included both narrow and wide stone gradations but only a few tests were conducted with a gradation ratio, W_{85}/W_{15} , greater than 8.0 (W_{85} is the weight of an armor stone where 85 percent of the total weight of the gradation is contributed by stones of lesser weight; W_{15} is the corresponding weight for the 15-percentile stone). Prototype-scale riprap stability tests conducted by Ahrens (1975) used the stone gradation specified in EM 1110-2-2300 (U.S. Army Corps of Engineers, 1971) and referred to as the "EM" gradation. Portions of EM 1110-2-2300 have been superseded by ETL 1110-2-222 (U.S. Army Corps of Engineers, 1978). The EM gradation specifications for the maximum and minimum stone weights are

$$W_{\max} = 4W_{50}$$

$$W_{\min} = 0.125W_{50}$$

Ahrens established the following approximate empirical relations for the EM gradation:

$$\bar{W} \approx 0.75W_{50}$$

$$\frac{W_{85}}{W_{15}} \approx 4.9$$

and

$$W_{15} \approx 0.4W_{50} \quad (1)$$

where $\bar{W}$ is the average weight of the riprap armor stone. Fully mixed, wide gradations are probably as stable to wave attack as narrow gradations with the same W_{50} ; however, gradations where the ratio W_{85}/W_{15} exceeds 8.0 are not recommended due to the shortage of data on their performance. The advantages of a wide gradation over a narrow gradation are that a larger percentage of the quarry-run stone can be used and that the filter layer-size criteria can be met

easier (discussed in the next subsection); the disadvantage is that the stone may become segregated and some areas of the revetment can be unusually vulnerable to wave attack.

The thickness of the armor layer should be great enough to accommodate the largest stone in the gradation. To do this, the thickness of the layer must be slightly greater than a typical dimension of the largest stone. A typical dimension may be computed using the cube root of the volume of the stone. For the EM gradation, the typical dimension of the largest stone is

$$\left(\frac{W_{\max}}{w_r}\right)^{1/3} = \left(\frac{4W_{50}}{w_r}\right)^{1/3} = 1.59 \left(\frac{W_{50}}{w_r}\right)^{1/3}$$

where w_r is the unit weight of the stone in kilograms per cubic meter. The recommended minimum armor layer thickness, $r_{\min}$, was set at twice the typical dimension of the median stone, i.e.,

$$r_{\min} = 2.0 \left(\frac{W_{50}}{w_r}\right)^{1/3} \quad (2)$$

Equation (2) provides sufficient thickness to accommodate the largest stone in the EM gradation. EM 1110-2-2300 also recommends that $r_{\min}$ be at least 0.30 meter (1 foot).

Flat and rod-shaped stones should not be used in the riprap armor gradation. The lift and drag forces on flat stones and the drag forces on rod-shaped stones are greater in proportion to their weight than the more desirable angular and blocky shapes. Flat and rod-shaped stones may also require a greater armor layer thickness to accommodate them and they do not key in well with the other stones. Stones with a maximum dimension greater than three times their minimum dimension are not recommended for the armor gradation.

2. Underlayers.

The stone used in the layer just beneath the armor layer (i.e., the filter layer) should be large enough to prevent removal of stone through the voids in the armor layer by wave action. To describe the required stone-size relationship between the armor and filter, it is convenient to use the concept of a typical stone dimension again. Let the typical stone dimension be given by

$$D_x = \left(\frac{W_x}{w_r}\right)^{1/3}$$

where the subscript x indicates the percent of the weight of the total gradation contributed by stones of lesser weight. The proper size relationship between the 15-percentile size of the armor and the 85-percentile size of the filter is given by

$$\frac{D_{15} \text{ (armor)}}{D_{85} \text{ (filter)}} \leq 4.0 \quad (3)$$

The filter criterion given by equation (3) is somewhat more conservative (i.e., requires larger stone in the filter layer) than the criteria accepted by Thomsen, Wohlt, and Harrison (1972) and given in the SPM, EM 1110-2-2300, and ETL 1110-2-222 (U.S. Army Corps of Engineers, 1978), but it appears necessary based on the riprap stability tests conducted by Ahrens (1975).

If the armor stone is large, it may be necessary to have a second underlayer of stone beneath the first underlayer. The stone-size relationship between the first and second underlayers is also given by equation (3). The thickness of the underlayers should be at least three median stone diameters (i.e., $3D_{50}$) and not less than 0.23 meter (9 inches) (see ETL 1110-2-222). Sometimes it is economical to replace the smallest size underlayer with a geotextile fabric; however, because of unsatisfactory experience, Corps policy currently does not permit the use of geotextile fabrics beneath riprap on embankment dams and navigation channels.

3. Zero-Damage Stability.

The usual method to evaluate riprap stability is by use of Hudson's (1959) stability number, N_s . The stability number is defined by the equation

$$N_s = \frac{H}{\left(\frac{W_{50}}{w_r}\right)^{1/3} \left(\frac{w_r}{w_w} - 1\right)} \quad (4)$$

where H is the local wave height and w_w is the unit weight of water (1,000 and 1,026 kilograms per cubic meter or 62.4 and 64 pounds per cubic foot for freshwater and for seawater, respectively). Normally, the wave height used in equation (4) would be the height at the toe of the structure; however, in some situations, particularly on deep reservoirs, where there is no clearly defined toe for the structure, the deepwater wave height may be used in equation (4). The use of the significant wave height in equation (4) is discussed in subsection 5.

When the stability number is used to define the zero-damage stability condition, the symbol N_{sz} is used, and the corresponding wave height is the local zero-damage wave height, H_z . For zero-damage stability, the relation between the stability number and the slope of the embankment to be protected is

$$N_{sz} = 1.45(\cot \theta)^{1/6} \quad (5)$$

where θ is the angle between the embankment face and the horizontal. Equation (5) is intended for use with armor stone placed by dumping and is considered to be conservative enough to account for wave period effects (Ahrens and McCarthy, 1975), for both breaking and nonbreaking wave conditions, and for naturally occurring irregular wave conditions (discussed in the next two subsections).

***** EXAMPLE PROBLEM 1 *****

GIVEN: An earth embankment (to be protected from wave attack) located on a freshwater lake has a slope of 1 on 3, i.e., $\cot \theta = 3.0$; the design wave height at the toe of the embankment is 1.52 meters (5.0 feet). The unit weight of the stone to be used in the armor and filter layers is 2,644 kilograms per cubic meter (165 pounds per cubic foot).

FIND: The zero-damage median riprap weight, the minimum armor layer thickness, and the minimum W_{85} for the filter layer stone.

SOLUTION: Solving equation (5) gives

$$N_{sz} = 1.45(3.0)^{1/6} = 1.74$$

Next, using equation (4)

$$N_{sz} = \frac{H}{\left(\frac{W_{50}}{W_r}\right)^{1/3} \left(\frac{W_r}{W_w} - 1\right)}$$

and solving for W_{50} , gives

$$W_{50} = \frac{2,644(1.52)^3}{(1.74)^3 \left(\frac{2,644}{1,000} - 1\right)^3} = 397 \text{ kilograms (875 pounds)}$$

The minimum armor layer thickness given by equation (2) is

$$r_{min} = 2.0 \left(\frac{397}{2,644}\right)^{1/3} = 1.06 \text{ meters (3.49 feet)}$$

To compute W_{85} for the filter stone, first use equation (1) to compute W_{15} for the riprap, i.e.,

$$W_{15} \text{ (riprap)} = 0.4 \times 397 = 159 \text{ kilograms (350 pounds)}$$

Since the riprap and filter stone have the same unit weight, equation (3) can be written as

$$\frac{D_{15} \text{ (riprap)}}{D_{85} \text{ (filter)}} = \left[\frac{W_{15} \text{ (riprap)}}{W_{85} \text{ (filter)}}\right]^{1/3} = \left[\frac{159}{W_{85} \text{ (filter)}}\right]^{1/3} \leq 4.0$$

which gives a minimum W_{85} (filter) of 2.48 kilograms (5.5 pounds). If the riprap had a gradation narrower than the EM gradation, the minimum W_{85} (filter) would have had to have been greater than 2.48 kilograms, since W_{15} (riprap) would have been greater than 159 kilograms.

4. Wave Period Effects.

Some laboratory studies of riprap stability conducted with monochromatic waves (i.e., waves of constant height and period) show a strong influence of wave period (e.g., see Thomsen, Wohlt, and Harrison, 1972; Ahrens and McCartney, 1975); other studies such as Hudson and Jackson (1962) do not. A comprehensive laboratory study conducted at the Hydraulic Research Station (HRS) (1975) in Wallingford, England, for the Construction Industry Research and Information Association (CIRIA) of the United Kingdom, concluded that there was little influence of wave period on riprap stability for tests with irregular waves. The tests at HRS included a wide range of irregular wave conditions considered to be typical of naturally occurring conditions.

Wave period is not considered in this analysis of riprap stability because (a) the monochromatic test results were inconsistent, (b) the HRS tests with natural wave conditions do not indicate any period effects, and (c) there is no accepted method, at present, to account for the influence of wave period on riprap stability.

5. Zero-Damage Conservatism and the Design Wave Height.

The equation recommended for calculating the zero-damage stability numbers (eq. 5) is more conservative than some other design equations; e.g., the equation given in the SPM is

$$K_{RR} = \frac{N_s^3}{\cot \theta} = 2.2 \quad (6)$$

where K_{RR} is the stability coefficient for riprap. The additional conservatism is intended to account for the most severe wave breaking conditions and the effects of irregular wave attack. Equations (5) and (6) are compared in Figure 2 which shows that they give about the same stability number on a steep slope (1 on 2) but diverge considerably for flatter slopes. The reason for the divergence is that equation (5) is based on a small absolute measure of damage, while equation (6) is based on a 5-percent allowable damage which causes it to be more slope dependent. Since a percent-damage equation is useful in evaluating the progress of damage toward failure, the following equation was developed for a 5-percent level of damage (also shown in Fig. 2)

$$N_s = 1.37(\cot \theta)^{1/3} \quad (7)$$

Equation (7) is consistent with equation (5) since both equations were developed primarily from large wave tank tests of riprap stability conducted by Ahrens (1975) and both were based on the most damaging wave conditions. Equation (7) is equivalent to $K_{RR} = 2.37$ and can be used to compute the median riprap weight in situations where some damage could be tolerated. In Figure 3, equation (7) is used to give perspective on the concept of reserve stability discussed in the next subsection.

Ahrens (1975) and ETL 1110-2-222 indicate that stability coefficients as high as 4.37 can be used if damage to the riprap can be accepted. Using $K_{RR} = 4.37$ necessitates consideration of maintenance costs and safety factors.

Figure 2. Comparison of stability equations.

Figure 3. Reserve stability as a function of the reserve stability parameter.

Normally the significant wave height should be used as the design wave height for riprap, e.g., in equation (4). The guidance provided in Section 7.12 of the SPM should be followed in the selection of the design wave. Research underway (1980) at CERC is expected to provide improved guidance on the choice of the design wave for irregular wave attack on riprap.

6. Reserve Stability.

The ability of riprap to provide protection to an embankment when it is exposed to waves greater than the zero-damage wave height is well known and constitutes an important advantage in this type of revetment. This is referred to as *reserve stability*. Reserve stability increases with the thickness of the armor layer and the flatness of the embankment slope; these characteristics are quantified in Figure 3 which is based on tests by Ahrens (1975). The reserve stability in the figure is indicated by H/H_z , the ratio of the wave height to the zero-damage wave height. This ratio is equivalent to the ratio of the stability number to the zero-damage stability number given by equation (5). Reserve stability is plotted in Figure 3 versus the parameter

$$\left[\frac{r}{(W_{50}/W_r)^{1/3}} \right] (1 + \cot^2 \theta)^{1/2}$$

where the quantity inside the bracket is the armor layer thickness in terms of the typical stone dimension. In Figure 3, the zero-damage criterion (eq. 5) is represented by the horizontal line where $H/H_z = 1.0$; there is no damage below this line. In the wedge-shaped region above this line, damage would be expected but not failure. Failure, as used here, indicates that wave action will remove filter stone from the damaged slope, but does not necessarily mean the embankment will be destroyed. The dashline through the wedge-shaped region is the 5-percent damage level given by equation (7) using the recommended minimum armor layer thickness defined by $[r/(W_{50}/W_r)^{1/3}] = 2.0$.

***** EXAMPLE PROBLEM 2 *****

This example, which is a continuation of example 1, illustrates the concept of reserve stability and the use of Figure 3.

GIVEN:

$$\cot \theta = 3.0$$

$$H = 1.52 \text{ meters (5.0 feet) (design wave height)}$$

$$W_r = 2,644 \text{ kilograms per cubic meter (165 pounds per cubic foot)}$$

$$W_w = 1,000 \text{ kilograms per cubic meter (62.4 pounds per cubic foot)}$$

$$W_{50} = 397 \text{ kilograms (875 pounds) (computed in example 1)}$$

In addition, it is specified that the armor be two layers thick, i.e., the minimum thickness is given by equation (2).

$$r_{\min} = 2.0 \left(\frac{W_{50}}{W_r} \right)^{1/3}$$

This is required to determine the reserve stability parameter.

FIND: The maximum wave height above the design value which will not cause riprap failure and the smallest median weight riprap which will not fail for the design wave height.

SOLUTION: The reserve stability parameter is

$$\left[\frac{r}{(W_{50}/W_r)^{1/3}} \right] (1 + \cot^2 \theta)^{1/2} = 2\sqrt{10} = 6.32$$

and using Figure 3 gives

$$\frac{H}{H_z} = 1.31$$

Therefore, $H = 1.31 \times 1.52 = 1.99$ meters or 2.0 meters (6.5 feet). Thus, a wave height as great as 2.0 meters will not cause failure; for wave heights between 1.5 and 2.0 meters, some damage would be expected but not failure. No damage would be expected below $H = 1.5$ meters; failure could occur for $H > 2.0$ meters.

From Figure 3 and recalling from example 1 that $N_{sz} = 1.74$, gives

$$\frac{N_s}{N_{sz}} = \frac{N_s}{1.74} = 1.31$$

or

$$N_s = 1.31(1.74) = 2.28$$

Then, using equation (4)

$$N_s = \frac{1.52}{\left(\frac{W_{50}}{2,644} \right)^{1/3} \left(\frac{2,644}{1,000} - 1 \right)} = 2.28$$

and solving for W_{50} gives,

$$W_{50} = \frac{(1.52)^3 (2,644)}{(2.28)^3 \left(\frac{2,644}{1,000} - 1 \right)^3} = 176 \text{ kilograms (389 pounds)}$$

Example 1 showed that $W_{50} = 397$ kilograms was necessary for no damage; for W_{50} between 176 and 397 kilograms, damage could be expected but no failure. However, for $W_{50} < 176$ kilograms, failure could occur.

7. Location of Damage.

Damage to the armor layer can extend over a surprisingly large extent of the revetment face. Generally, the worst damage is above the stillwater level (SWL) on steep slopes and below the SWL on flat slopes. Table 1 quantifies the findings of Ahrens (1975) regarding the upper limit of damage, l_u , and the lower limit of damage, l_l . In the table, l_u and l_l are divided by the wave

Table 1. Average values of ℓ_u/H and ℓ_d/H and the standard deviations, σ , for slopes of 1 on 2.5, 3.5, and 5.0.

Slope	ℓ_u/H	σ	ℓ_d/H	σ
1 on 2.5	1.20	0.38	-0.65	0.33
1 on 3.5	0.56	0.24	-0.76	0.29
1 on 5.0	0.48	0.29	-0.85	0.34

height, H , which caused the damage. The parameters ℓ_u and ℓ_d are measured in the vertical from the SWL. Table 1 indicates that typically the vertical range of damage was about 1.8 wave heights on a 1 on 2.5 slope and 1.3 wave heights on slopes of 1 on 3.5 and 1 on 5. When inspecting for damage, it is necessary to consider the water level which may have existed during a storm.

8. Wave Runup.

Wave runup on riprap may be estimated using the method in Stoa (1979). Stoa indicates that runup on riprap ranges from 60 to 72 percent of the value for smooth embankments with similar slopes and wave conditions. An alternative method has been developed using the runup data from Ahrens (1975). Runup, R , is given by the general equation

$$\frac{R}{H} = \frac{a}{b + (H/L_0)^{1/2} \cot \theta} \quad (8)$$

where a and b are the dimensionless coefficients, H the wave height at the toe of the structure, and L_0 the deepwater wavelength, given by

$$L_0 = \frac{gT^2}{2\pi}$$

where T is the wave period and g the acceleration of gravity. The best fit coefficients for predicting runup on riprap in equation (8) are $a = 0.956$ and $b = 0.398$; these coefficients were rounded off to 1.0 and 0.4, respectively, for the runup prediction method given in ETL 1110-2-221 (U.S. Army Corps of Engineers, 1976). Equation (8) has been determined to give reliable estimates of monochromatic wave runup for $d_s/H \geq 3.0$ and for slopes from 1 on 2 to 1 on 10. If there is no clearly defined toe, equation (8) may still be used as shown in the following example.

***** EXAMPLE PROBLEM 3 *****

This example illustrates how to compute the maximum runup for situations where there is little truncation of the wave height distribution due to depth-limit breaking. Three different methods are used to illustrate the runup calculations and to show comparative answers.

GIVEN: An earth dam is being constructed to form a deep reservoir. The upstream face of the dam will have a 1 on 3 slope which will require riprap protection. The design wave has a significant height of 1.52 meters and a period of 4.7 seconds. No wave refraction is assumed for the design condition.

FIND: The height to which the riprap must extend above the design water level to prevent being exceeded by the runoff.

SOLUTION: It is necessary to compute d_s/H'_0 to determine which figure to use in Stoa (1979). Since there is no clearly defined toe for this structure, a water depth of one-half the deepwater wavelength will be used (this is the depth where the waves first "feel" the bottom)

$$d_s = 0.5L_0 = \frac{0.5 \times 9.80 \times (4.7)^2}{6.28} = 17.24 \text{ meters (56.5 feet)}$$

therefore,

$$\frac{d_s}{H'_0} = \frac{17.24}{1.52} = 11.3$$

which leads to using Figure 4 (Fig. B-3 in Stoa, 1979). To use Figure 4, the wave steepness parameter is required, so

$$\frac{H'_0}{gT^2} = \frac{1.52}{9.80 \times (4.7)^2} = 0.0070$$

Figure 4. Relative runoff for riprap slopes; $d_s/H'_0 = 8.0$; $H'_0/K_T \approx 2.8$. Use this figure also for $d_s/H'_0 > 8.0$ (from Stoa, 1979).

and from Figure 4

$$\frac{R}{H_0} = 0.88$$

and

$$R = 0.88(1.52) = 1.34 \text{ meters (4.39 feet)}$$

As a check, the runup will be calculated using equation (8). Assuming that the toe of the structure is in a water depth of 17.24 meters (56.5 feet), the required local wave height is the incident deepwater height of 1.52 meters.

Using equation (8) with the best fit coefficients gives

$$\frac{R}{H} = \frac{0.956}{0.398 + (1.52/34.47)^{1/2} (3.0)} = 0.93$$

and

$$R = 0.93 \times (1.52) = 1.41 \text{ meters (4.64 feet)}$$

Using equation (8) with the ETL 1110-2-221 coefficients gives

$$\frac{R}{H} = \frac{1.0}{0.4 + (1.52/34.47)^{1/2} (3.0)} = 0.97$$

and

$$R = 0.97 \times 1.52 = 1.47 \text{ meters (4.82 feet)}$$

Agreement among the three methods shown above is good, and since the significant wave height was used in the computations the runup will be referred to as the significant runup, R_S . Since some waves will produce runup greater than R_S , one way to estimate the maximum runup, R_{max} , is to assume that the ratio of R_{max} to R_S is the same as the ratio of the maximum wave height at the toe of the structure, H_{max} , to the significant wave height at the toe of the structure, H_S . For the deepwater conditions of this example, Goda (1975) gives

$$\frac{H_{max}}{H_S} \approx 1.64$$

where H_{max} represents the average highest wave in a group of about 250 waves. For wave breaking in shallow water, the ratio of the maximum to significant wave height is lower than shown above and can be calculated using a model developed by Goda (illustrated in example 4). The value $H_{max}/H_S = 1.64$ is consistent with the limiting value for deep water in Goda's model. Thus, the maximum runup for Stoa's method is

$$R_{max} = R_S \left(\frac{H_{max}}{H_S} \right) = 1.34(1.64) = 2.20 \text{ meters (7.22 feet)}$$

and the maximum runup using the best fit coefficients in equation (8) gives

$$R_{\max} = R_s \left(\frac{H_{\max}}{H_s} \right) 1.41(1.64) = 2.31 \text{ meters (7.58 feet)}$$

The method used in ETL 1110-2-221 to compute the maximum runup assumes a constant 50 percent greater than the significant runup; therefore,

$$R_{\max} = R_s(1.5) = 1.47(1.5) = 2.20 \text{ meters}$$

Table 2 summarizes the results of this example problem.

Table 2. Example problem 3 summary.

Method	$R_{\max}$	
	(m)	(ft)
Stoa (1979)	2.20	7.22
This report	2.31	7.58
ETL 1110-2-221	2.20	7.22

The three methods yield similar results and possibly the highest value of $R_{\max}$ should be chosen to be conservative.

In computing the maximum runup, the assumption is that

$$\frac{R_{\max}}{R_s} = \frac{H_{\max}}{H_s}$$

This assumption is not intended to suggest that the maximum runup is caused by the maximum wave but only to provide a reasonable factor by which to obtain $R_{\max}$ from a typical value of runup such as R_s . If relatively shallow water fronts the structure there will be truncation of the wave height distribution due to depth-limited and steepness-induced breaking which should cause a corresponding truncation in the runup distribution. Using a constant factor, such as 1.5, to estimate the maximum runup from the significant runup (by the method in ETL 1110-2-221) may overestimate $R_{\max}$ for shallow-water conditions. In example 4, a shallow-water situation where there is truncation of the wave height distribution due to wave breaking will be considered. The three methods used in example 3 are also used in example 4 to show comparative answers; the problem requires the use of Table 3 which gives the ratios $H_{\max}/H_s$ and H_s/H_0 based on the Goda (1975) model.

Table 3. Local wave conditions for various offshore slopes and water depths based on Goda's (1975) model.

d_s H_0'	$H_0'/L_0 = 0.002$			$H_0'/L_0 = 0.005$			$H_0'/L_0 = 0.010$			$H_0'/L_0 = 0.020$			$H_0'/L_0 = 0.040$			$H_0'/L_0 = 0.080$		
	H_s/H_0'	H_{max}/H_0'	H_{max}/H_s	H_s/H_0'	H_{max}/H_0'	H_{max}/H_s	H_s/H_0'	H_{max}/H_0'	H_{max}/H_s	H_s/H_0'	H_{max}/H_0'	H_{max}/H_s	H_s/H_0'	H_{max}/H_0'	H_{max}/H_s	H_s/H_0'	H_{max}/H_0'	H_{max}/H_s
Offshore slope = 1 on 10																		
0.5	0.83	1.49	0.76	1.38	0.70	1.32	0.63	1.30	0.58	1.31	0.51	1.24						
1.0	1.26	1.35	1.18	1.29	1.11	1.26	1.02	1.26	0.94	1.25	0.80	1.26						
1.5	1.67	1.30	1.59	1.27	1.45	1.30	1.26	1.38	1.06	1.44	0.89	1.42						
2.0	2.04	1.28	1.85	1.35	1.49	1.52	1.16	1.61	0.99	1.62	0.91	1.54						
2.5	2.90	1.32	1.74	1.46	1.31	1.60	1.06	1.61	0.95	1.61	0.93	1.58						
3.0	1.87	1.39	1.56	1.54	1.18	1.62	1.03	1.59	0.96	1.58	0.95	1.59						
3.5	1.73	1.46	1.41	1.59	1.11	1.61	0.99	1.61	0.95	1.58	0.95	1.61						
4.0	1.60	1.53	1.30	1.61	1.06	1.61	0.97	1.62	0.94	1.58	0.97	1.61						
Offshore slope = 1 on 20																		
0.5	0.60	1.50	0.56	1.39	0.51	1.34	0.48	1.29	0.44	1.26	0.40	1.23						
1.0	0.95	1.36	0.91	1.28	0.86	1.26	0.82	1.24	0.75	1.24	0.67	1.24						
1.5	1.31	1.29	1.25	1.26	1.18	1.26	1.08	1.30	0.95	1.34	0.81	1.33						
2.0	1.64	1.28	1.55	1.28	1.36	1.38	1.13	1.50	0.98	1.53	0.88	1.44						
2.5	1.90	1.31	1.67	1.41	1.29	1.58	1.06	1.61	0.94	1.61	0.91	1.51						
3.0	1.87	1.39	1.56	1.54	1.18	1.62	1.03	1.59	0.94	1.61	0.94	1.56						
3.5	1.73	1.46	1.41	1.59	1.11	1.61	0.99	1.61	0.94	1.60	0.96	1.57						
4.0	1.60	1.53	1.30	1.61	1.06	1.61	0.97	1.62	0.94	1.58	0.96	1.60						
Offshore slope = 1 on 50																		
0.5	0.50	1.51	0.46	1.41	0.43	1.35	0.40	1.29	0.38	1.25	0.35	1.24						
1.0	0.80	1.35	0.77	1.28	0.74	1.25	0.70	1.24	0.65	1.24	0.59	1.22						
1.5	1.12	1.28	1.08	1.25	1.03	1.25	0.96	1.26	0.87	1.28	0.75	1.29						
2.0	1.44	1.25	1.37	1.26	1.25	1.31	1.08	1.41	0.94	1.45	0.84	1.38						
2.5	1.68	1.29	1.54	1.35	1.27	1.49	1.04	1.58	0.94	1.57	0.89	1.46						
3.0	1.83	1.36	1.53	1.51	1.18	1.61	1.02	1.61	0.94	1.60	0.92	1.51						
3.5	1.73	1.46	1.41	1.59	1.11	1.61	1.00	1.59	0.93	1.61	0.94	1.55						
4.0	1.60	1.53	1.30	1.61	1.06	1.61	0.97	1.62	0.93	1.61	0.97	1.57						
Offshore slope = 1 on 100																		
0.5	0.48	1.50	0.44	1.40	0.41	1.35	0.38	1.29	0.36	1.25	0.33	1.24						
1.0	0.77	1.34	0.74	1.28	0.71	1.25	0.68	1.24	0.63	1.23	0.58	1.21						
1.5	1.07	1.28	1.03	1.25	1.00	1.24	0.92	1.26	0.84	1.28	0.73	1.28						
2.0	1.36	1.26	1.31	1.26	1.21	1.30	1.06	1.38	0.93	1.42	0.83	1.37						
2.5	1.62	1.28	1.50	1.32	1.25	1.47	1.04	1.56	0.94	1.55	0.88	1.44						
3.0	1.78	1.35	1.52	1.47	1.18	1.60	1.02	1.61	0.94	1.61	0.92	1.50						
3.5	1.73	1.46	1.41	1.59	1.11	1.61	0.99	1.60	0.93	1.61	0.94	1.53						
4.0	1.60	1.53	1.30	1.61	1.06	1.61	0.97	1.62	0.94	1.60	0.97	1.55						

***** EXAMPLE PROBLEM 4 *****

GIVEN: A riprap revetment with a slope of 1 on 2.5 is to be built where the design water depth at the toe is 4.57 meters (14.99 feet). Seaward of the toe, the offshore slope is 1 on 100. The deepwater, unrefracted, significant wave height is 3.05 meters (10.01 feet) and the design wave period is 7.0 seconds. Assume no wave refraction from deep water to the structure site.

FIND: The elevation above the design water level to which the riprap must extend to prevent being exceeded by the runup.

SOLUTION: The first method follows the procedure of Stoa. For $d_s/H_0' = 1.5$, Table 4 and Figure 5 (App. A in Stoa, 1979) indicate that the smooth-slope reduction factor, r , for runup on riprap on a 1 on 2.5 slope is $r = 0.63$. To find the smooth-slope runup, Figure 6 (Fig. 10 in Stoa, 1978) is used with

$$\frac{H_0'}{gT^2} = \frac{3.05}{9.80(7.0)^2} = 0.0063$$

which yields $R/H_0' = 2.05$. According to Stoa (1979), there is no scale correction for this condition, so the runup is

Table 4. Values of r for application
at $d_s/H'_0 < 3$ (from Stoa, 1979).

Slope ($\cot \theta$)	H/k_r^1	r
1.5	3 to 4	0.60
2.5	3 to 4	0.63
3.5	3 to 4	0.60
5.0	3	0.60
5.0	4	0.68
5.0	5	0.72

¹ H'_0 was used to derive these values from experiments with $d_s/H'_0 > 3$; for application at $d_s/H'_0 < 3$, use H , where H is the wave height at the proposed structure location.

Figure 5. Sketch of quarrystone (riprap) embankment (from Stoa, 1979).

Figure 6. Relative runoff for smooth slopes on a 1 on 10 bottom;
 $l/L \geq 0.5$; $d_s/H'_0 = 1.5$ (from Stoa, 1978).

$$R_S = r \left(\frac{R}{H'_O} \right) (H'_O) = 0.63(2.05)(3.05) = 3.94 \text{ meters (12.93 feet)}$$

This runup is regarded as the significant runup since it was computed from the deepwater significant wave height. The maximum runup is estimated by multiplying R_S by the ratio $H_{\max}/H_S$. The value of $H_{\max}/H_S$ is derived from Table 3 by using the parameters

$$\frac{H'_O}{L_O} = \frac{3.05}{76.46} = 0.040$$

where

$$L_O = \frac{g(7.0)^2}{2\pi} = 76.46 \text{ meters (251 feet)}$$

and

$$\frac{d_S}{H'_O} = 1.50$$

With an offshore slope of 1 on 100, Table 3 shows

$$\frac{H_{\max}}{H_S} = 1.28$$

Therefore, Stoa's method yields

$$R_{\max} = R_S \left(\frac{H_{\max}}{H_S} \right) = 3.94(1.28) = 5.04 \text{ meters (16.54 feet)}$$

The second method uses equation (8) with the best fit coefficients. To use this equation it is necessary to have the local significant wave height at the toe of the structure, obtained from Table 3 recalling that $H'_O/L_O = 0.040$, $d_S/H'_O = 1.50$, and the slope is 1 on 100. Therefore, from Table 3, $H_S/H'_O = 0.84$ and $H_S = H'_O \times H_S/H'_O = 3.05(0.84) = 2.56 \text{ meters (8.40 feet)}$. Equation (8) gives

$$\frac{R_S}{H_S} = \frac{0.956}{0.398 + (2.56/76.46)^{1/2} (2.5)} = 1.12$$

and

$$R_S = 1.12 \times 2.56 = 2.87 \text{ meters (9.42 feet)}$$

then

$$R_{\max} = R_S \left(\frac{H_{\max}}{H_S} \right) = 2.87(1.28) = 3.67 \text{ meters (12.04 feet)}$$

where the value for $H_{\max}/H_S$ was previously determined for Stoa's method.

The third method based on ETL 1110-2-221 uses equation (8) with the rounded-off coefficients, i.e.,

$$\frac{R_s}{H_s} = \frac{1.0}{0.4 + (2.56/76.46)^{1/2} (2.5)} = 1.17$$

and $R_s = 2.56 (1.17) = 3.00$ meters (9.84 feet). Increasing the significant runoff by 50 percent gives

$$R_{\max} = 3.00(1.5) = 4.50 \text{ meters (14.76 feet)}$$

Table 5 provides a summary of methods used in this problem.

Table 5. Example problem 4 summary.

Method	$R_{\max}$	
	(m)	(ft)
Stoa (1979)	5.04	16.54
This report	3.67	12.04
ETL 1110-2-221	4.50	14.76

The rather wide range of estimates for $R_{\max}$ shown in the example 4 summary (Table 5) is partly due to the inherent difficulty in estimating extreme values and the specific difficulty of adapting the results of monochromatic wave tests to irregular wave conditions in relatively shallow water. To evaluate which of the three methods would produce the best estimates of $R_{\max}$, a comparison was made with observed values from the laboratory tests of Ahrens and Seelig (1980). These tests measured the maximum wave runoff on a riprap-protected dike using various irregular wave conditions. The dike had a slope of 1 on 2 and a submerged fronting slope of 1 on 15; some of the water levels tested had wave conditions similar to those in example 4. All three methods overpredicted the observed maximum runoff on an average, and overpredicted for most of the individual conditions compared. Stoa's method overpredicted $R_{\max}$ by an average of 38 percent, the method of this report by 29 percent, and the method of ETL 1110-2-221 by 38 percent. Since data were available only for one slope with which to compare predicted and observed values, it is not clear how general the tendency to overpredict is. Based on the comparison, the method of this study is regarded as the best estimate of maximum runoff; however, the value from another method might be selected in order to be conservative. Laboratory tests to improve the existing guidelines for estimating the characteristics of irregular wave runoff are now underway at CERC.

9. Overlays.

Overlays are single layers of larger stone placed on top of existing riprap which is too small to provide adequate protection to the embankment. The concept of an overlay as a simple and logical method to upgrade existing revetment was developed by the U.S. Army Engineer Division, Missouri River (see McCartney and Ahrens, 1976). Overlays using 100-percent coverage are recommended to upgrade existing riprap; this means that all stones touch adjacent stones. Photos in McCartney and Ahrens show 100-percent coverage.

A more quantifiable means to estimate the amount of stone required for an overlay is given by the coverage fraction, C.F., where

$$C.F. = \frac{C}{(\bar{W}/W_T)^{1/3} W_T} \quad (9)$$

where C is the overlay stone weight per square meter of embankment surface. McCartney and Ahrens (1976) found that the coverage fraction of 100-percent coverage varied by stone shape when C.F. = 0.42 (typical for a relatively blocky quarystone) and C.F. = 0.55 (typical for rounded boulders). The minimum W_{50} weight for the overlay stone should be computed using equation (5). A wide gradation in the overlay stone is not recommended since each stone is exposed to wave action and receives little support or shelter from adjacent stones. The prototype-scale overlay tests (discussed by McCartney and Ahrens) used an overlay with the following maximum, minimum, and average overlay weights:

$$W_{\max} = 3.1 W_{50}$$

$$W_{\min} = 0.4 W_{50} \quad (10)$$

$$\bar{W} \approx 0.87 W_{50}$$

where W_{50} is the median weight of the overlay gradation; an overlay gradation wider than denoted above is not recommended.

***** EXAMPLE PROBLEM 5 *****

This example reviews concepts discussed throughout the text, introduces a few new ideas, and develops several possible alternate designs to present advantages and disadvantages of each design.

GIVEN: A low bluff composed of bank-run gravel is eroding due to wave attack. Behind the bluff is a large industrial park and further erosion cannot be permitted. A riprap revetment is to be built with a design freshwater depth at the toe of 1.83 meters (6.0 feet); no overtopping should be permitted, however, the consequences of overtopping would not be life threatening. The offshore slope is 1 on 100; the design deepwater, unrefracted, significant wave height is 1.52 meters and the design wave period is 5.0 seconds. There is no wave refraction between deep water and the structure site. The unit weight of the armor and filter stone is 2,644 kilograms per cubic meter and the EM-size gradation should be assumed for the armor stone.

FIND: Consider slopes of 1 on 1.5, 1 on 2, 1 on 3, and 1 on 5. For each slope, compute the zero-damage median riprap armor weight, the minimum armor layer thickness, the minimum W_{85} for the filter layer, and the elevation above the design water level to which the riprap must extend to prevent overtopping. Compare the advantages and disadvantages of the various slopes.

As a second part of this example, assume there is existing riprap protecting the bluff but the stone is too small for the design wave conditions. Compute the weight of overlay stone required to upgrade the existing riprap

to the design wave height for both blocky quarystone and rounded boulders. Also compute the overlay weight per meter of revetment length based on the selected maximum runup.

SOLUTION: To compute the zero-damage median weight, use Table 3 to calculate the local significant wave height at the toe of the structure. To use Table 3, compute

$$\frac{H'_O}{L_O} = \frac{H'_O}{(gT^2/2\pi)} = \frac{1.52}{9.8(5.0)^2/6.28} = 0.039$$

$$\frac{d_S}{H'_O} = \frac{1.83}{1.52} = 1.20$$

and the offshore slope = 1 on 100.

Use Table 3 for $H'_O/L_O = 0.040$, since interpolation H'_O/L_O would not change values of H_S/H'_O or H_{max}/H_S appreciably, and then interpolate on d_S/H'_O to get

$$\frac{H_S}{H'_O} = 0.71 \text{ (to be used to calculate } H_S)$$

and

$$\frac{H_{max}}{H_S} = 1.25 \text{ (to be used for runup calculations)}$$

The local significant height is

$$H_S = 0.71(H'_O) = 0.71(1.52) = 1.08 \text{ meters (3.54 feet)}$$

The considerable reduction in the significant height from the deepwater value is due to breaking of the larger and steeper waves over the shallower parts of the 1 on 100 offshore slope. Solving equation (5), using $\cot \theta = 1.5$, gives

$$N_{sz} = 1.45(1.5)^{1/6} = 1.55$$

and using this value in equation (4) with $H_S = 1.08$ meters gives

$$\left(\frac{W_{50}}{2,644} \right)^{1/3} = \frac{1.08}{(1.55) \left(\frac{2,644}{1,000} - 1.0 \right)} = 0.424$$

and $W_{50} = 202$ kilograms (445 pounds).

The minimum armor layer thickness for this stone size is computed using equation (2)

$$r_{min} = 2.0 \left(\frac{201}{2,644} \right)^{1/3} = 0.85 \text{ meter (2.79 feet)}$$

Equation (1) is used to compute W_{15} (armor)

$$W_{15} \text{ (armor)} = 0.4 \times W_{50} = 0.4 \times 201 = 80 \text{ kilograms (179 pounds)}$$

which is used to compute the minimum W_{85} (filter) using equation (3)

$$\frac{D_{15} \text{ (armor)}}{D_{85} \text{ (filter)}} = \frac{(80/2,644)^{1/3}}{(W_{85}/2,644)^{1/3}} = \left(\frac{80}{W_{85}}\right)^{1/3} \leq 4.0$$

which gives the minimum W_{85} (filter) = 1.25 kilograms (2.76 pounds).

The maximum runoff is computed using the three methods given in examples 3 and 4. Taking Stoa's (1979) method first, for $d_s/H'_0 = 1.2$, the smooth-slope reduction factor for runoff on riprap, r , is given in Table 4. For a 1 on 1.5 slope, $r = 60$. The smooth-slope runoff is computed by interpolating between Figures 7 and 6 (Figs. 9 and 10 in Stoa, 1978). To use the figures, calculate

$$\frac{H'_0}{gT^2} = \frac{1.52}{9.8(5)^2} = 0.0062$$

which gives

$$\frac{d_s}{H'_0} = 1.0 \text{ and } \frac{R}{H'_0} = 2.63 \text{ (Fig. 7)}$$

$$\frac{d_s}{H'_0} = 1.5 \text{ and } \frac{R}{H'_0} = 2.43 \text{ (Fig. 6)}$$

therefore, for $d_s/H'_0 = 1.2$, $R/H'_0 = 2.55$. Following the procedures illustrated in example 4, the maximum runoff may be computed

$$R_{\max} = R_S \left(\frac{H_{\max}}{H_S} \right) = (r) (H'_0) \left(\frac{R}{H'_0} \right) \frac{H_{\max}}{H_S} = (0.60) (1.52) (2.55) (1.25)$$

$$= 2.91 \text{ meters (9.55 feet).}$$

Computing the maximum runoff by the method developed in this report requires using $a = 0.956$ and $b = 0.398$ in equation (8), thus

$$\frac{R_S}{H_S} = \frac{0.956}{0.398 + (H_S/L_0)^{1/2} \cot \theta} = \frac{0.956}{0.398 + (1.08/39.01)^{1/2} (1.5)} = 1.48$$

and the maximum runoff is

$$R_{\max} = R_S \left(\frac{H_{\max}}{H_S} \right) = (H_S) \left(\frac{R_S}{H_S} \right) \left(\frac{H_{\max}}{H_S} \right) = (1.08) (1.48) 1.25$$

$$= 2.00 \text{ meters (6.56 feet).}$$

Computing the maximum runoff by the ETL 1110-2-221 method requires using $a = 1.0$ and $b = 0.40$ in equation (8), therefore

Figure 7. Relative runup for smooth slopes on a 1 on 10 bottom;
 $l/L \geq 0.5$; $d_s/H_0^1 = 1.0$ (from Stoa, 1978).

$$\frac{R_s}{H_s} = \frac{1.0}{0.40 + (H_s/L_0)^{1/2} \cot \theta} = \frac{1.0}{0.40 + (1.08/39.01)(1.5)} = 1.54$$

and the maximum runup is

$$R_{\max} = R_s (1.5) = (H_s) \left(\frac{R_s}{H_s} \right) (1.5) = (1.08)(1.54)(1.5) \\ = 2.49 \text{ meters (8.17 feet).}$$

Computations shown above were performed for the other slopes and are tabulated in Table 6. Table 6 also shows some additional data (e.g., the length of the revetment) to provide information for comparing the advantages of the various slopes. The length of the revetment is the slant length distance from the toe to the top of the riprap as determined by the chosen value of $R_{\max}$; i.e., length of revetment = $(d_s + R_{\max}) (1 + \cot^2 \theta)^{1/2}$.

Table 6 shows that the 1 on 1.5 slope has the shortest length and requires the smallest quantity of armor per meter. The length for each slope was calculated using $R_{\max}$ as estimated by the method of this report. The weight of stone per meter is the product of $r_{\min}$, the slope length, the unit weight, and 1.0 minus the porosity. The unit weight is 2,644 kilograms per cubic meter and the porosity is assumed to be 0.40. Since the 1 on 1.5 slope needs the least armor stone per meter it may have the lowest first costs; however, in some locations it might be cheaper to purchase smaller stone for a flatter slope. Problems with the 1 on 1.5 slope include the

Table 6. Example problem 5 comparison data.

Slope	N _{sz}	Zero damage W ₅₀ kg(lb)	r _{min} m(ft)	Min. W ₈₅ filter kg(lb)	R _{max} (Stoa 1978 method) m(ft)	R _{max} ¹ (this report method) m(ft)	R _{max} (ETL 1110-2- 221 method) m(ft)	Length of revetment m(ft)	Armor weight ² kg/m(lb/ft)	Reserve stability factor ³ (H/H _z)
1 on 1.5	1.55	201 (443)	0.85 (2.79)	1.25 (2.76)	2.91 (9.55)	2.00 (6.56)	2.49 (8.17)	6.90 (22.64)	9,304 (6,253)	1.12
1 on 2	1.63	173 (381)	0.81 (2.66)	1.08 (2.38)	2.72 (8.92)	1.77 (5.81)	2.20 (7.22)	8.05 (26.41)	10,344 (6,952)	1.18
1 on 3	1.74	142 (313)	0.75 (2.46)	0.89 (1.96)	2.02 (6.63)	1.44 (4.72)	1.80 (5.91)	10.34 (33.92)	12,303 (8,269)	1.31
1 on 5	1.90	109 (240)	0.69 (2.26)	0.69 (1.52)	1.15 (3.77)	1.05 (3.44)	1.31 (4.30)	14.69 (48.20)	16,080 (10,807)	1.59

¹Used to compute length of revetment.²Void space in the riprap armor is assumed to be 40 percent of the total volume.³From Figure 3.

lack of riprap stability and runup data for this condition, and its anticipated low reserve stability. These factors indicate that a 1 on 1.5 slope is useful to consider as an example, but it would not be the most acceptable design.

In Table 6 the height of the revetment was chosen to be the value of R_{max} calculated by the method developed in this report. If overtopping might cause a life-threatening situation, then a more conservative estimate of R_{max} should be used due to the uncertainty in predicting extreme values of runup and model studies to determine R_{max} should be considered. Additional conservatism could also be used in the riprap weight and armor layer thickness. Since the riprap weight is proportional to the cube of the wave height, an uncertainty of ±15 percent in the wave height becomes ±52 percent in the riprap weight. It may be assumed that the uncertainty about the incident wave height is compensated for by the reserve stability; however, for steep slopes there may not really be enough compensation so that use of a larger W₅₀ might have to be considered.

A complete analysis would have to weigh the first costs against maintenance costs and the possibility of other losses if the design conditions were exceeded. These considerations are beyond the scope of this report.

Since the weight of overlay stone required to upgrade an existing revetment is the same as the weight of armor stone required for stability (eq. 5), the overlay stone weight is the same as given in Table 6. Using the slope of 1 on 3 and blocky-shaped stone as an example, the average overlay stone weight and weight of overlay per square meter can be calculated using equation (10) and (9), respectively

$$\bar{W} = 0.87 W_{50} = 0.87(142) = 124 \text{ kilograms (273 pounds)}$$

and

$$C = C.F. \left(\frac{\bar{W}}{W_R} \right)^{1/3} (W_R) = 0.42 \left(\frac{124}{2,644} \right)^{1/3} (2,644)$$

$$= 400 \text{ kilograms per square meter (82 pounds per square foot)}$$

The weight of overlay stone per linear meter is the product of the weight per square meter times the length of the revetment. For this example, overlay stone weight per linear meter = $400 \times 10.34 = 4,136$ kilograms per meter or 1.4 tons per foot. Table 7 shows the results of the overlay computations for each of the four slopes using both blocky quarrystone and rounded boulders as overlay stones. Overlay would normally be used to repair a damaged revetment and the reserve stability would be partly a function of the thickness and size of the original armor. The overlay layer itself will have little reserve stability as is suggested by comparing the weight of overlay per linear meter in Table 7 with the weight of armor per linear meter in Table 6.

Table 7. Overlay stone data.

Slope	W ₅₀ kg(lb)	W̄ kg(lb)	Blocky quarrystone			Rounded boulders		
			C.F.	C kg/m ² (lb/ft ²)	Armor weight kg/m(lb/ft)	C.F.	C kg/m ² (lb/ft ²)	Armor weight kg/m(lb/ft)
1 on 1.5	201 (443)	175 (386)	0.42	449 (92)	3,098 (2,082)	0.55	588 (120)	4,057 (2,727)
1 on 2	173 (381)	151 (333)	0.42	428 (88)	3,445 (2,315)	0.55	560 (115)	4,508 (3,030)
1 on 3	142 (313)	124 (273)	0.42	400 (82)	4,136 (2,780)	0.55	524 (107)	5,418 (3,641)
1 on 5	109 (240)	95 (209)	0.42	366 (75)	5,377 (3,614)	0.55	480 (98)	7,051 (4,739)

III. SUMMARY AND CONCLUSIONS

A number of design considerations relating to riprap stability to wave attack and maximum runup elevations are discussed; examples are worked to illustrate techniques. The information presented is primarily the result of laboratory studies. Equally important to the development of a good design are considerations difficult to quantify, such as a careful evaluation of the performance of other revetments near the design site or in similar sites. It is extremely important to utilize the experience of others and when this is coupled with the guidance provided in the literature, many alternative designs can hopefully be reduced to a few good ones. The best design may have to be selected on the basis of model tests.

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