842

NAVY DEPARTMENT
THE DAVID W. TAYLOR MODEL BASIN
WASHINGTON 7, D.C.

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS
OF CAVITATION

An Addendum to DTMB Report 712,
On the Mechanism and Prevention of Cavitation

by

Phillip Eisenberg

Revised Edition

June 1953

Report 842

NS 715-102

GC
1
.D3
no. 842

NAVY DEPARTMENT

THE DAVID W. TAYLOR MODEL BASIN

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS
OF CAVITATION

As Addendum to DTMB Report 712,
On the Mechanisms and Prevention of Cavitation

by

Philip Glassman

Report 842

MBL/WHOI

5 6622600 1030 0

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS OF CAVITATION

An Addendum to DTMB Report 712,

On the Mechanism and Prevention of Cavitation

by

Phillip Eisenberg

First Printing, October 1952

Revised Edition

June 1953

Report 842

NS715-102

PREFACE

This report was prepared for presentation to the Chesapeake Section of the Society of Naval Architects and Marine Engineers. It was presented in preliminary form at the Meeting of October 18, 1952.

TABLE OF CONTENTS

	Page
ABSTRACT	1
INTRODUCTION	1
TYPES OF CAVITATION	3
THE INCEPTION OF CAVITATION	
Introductory Remarks	3
Further Remarks on the Role of Turbulence and Boundary Layer Effects in Cavitating Flows	4
Cavitation in Separated Flows	6
Effects of Scale on the Inception of Cavitation	7
A Remark on the Question of Gas Content and Nuclei Content	10
DYNAMICS OF TRANSIENT CAVITIES	
The Motion of Small Transient Cavities	11
Analytical Description of the Motion	11
MECHANISM OF STEADY-STATE CAVITIES AND THEIR ANALYTICAL DESCRIPTION	
Introductory Remarks	15
Some Remarks on Steady-State Cavities in Real Liquids	15
The Analytical Description of Steady-State Cavities	17
A Remark on the Drag of Cavitating Bodies	19
SOME RECENT WORK ON NONSTATIONARY CAVITIES.....	20
CONCLUDING REMARKS	21
REFERENCES.....	21

ABSTRACT

This report is intended to bring up to date the discussions of TMB Report 712 which summarized briefly the status of cavitation problems up to 1950. An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. Thus, the earlier discussions of the role of turbulence and of boundary layer effects in inception are extended. Some recent work on scale effects in inception is reviewed and some further thoughts on the influence of nuclei content are presented. Recent results of theoretical analysis by various writers on transient cavities including the effects of compressibility and viscosity are briefly mentioned. Further remarks are included on steady-state cavities in real liquids, and recent theoretical works on this type of cavitation are cited. In particular, mention is made of a recent linearized theory by Tulin, which will be of considerable interest for technical applications. Some recent observations on the drag of cavitating bodies are summarized. Finally, it is observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.

INTRODUCTION

Although the significance of cavitation phenomena in technical applications has been clearly recognized for many years, investigations of the various aspects were, until recent years, rather limited. In general, earlier researches were restricted to rather isolated problems with little evidence of recognition of the relationships between cavitation and other hydrodynamic phenomena or even between the various types of cavitation. However, modern requirements in the design of hydrodynamic systems for use at increasingly greater speeds, higher temperatures, and lower pressures have emphasized the need for prevention of cavitation and the associated damage to materials, objectionable noise and induced vibration, losses in efficiency and increase in drag with cavitation onset. Moreover, there has been increasing interest in the design of systems in which normal operation occurs under fully cavitating conditions. These problems require a clear understanding of the physical and thermodynamical processes involved and not merely the assembling of systematic, empirical data; the phenomena are such that without this understanding even seemingly straightforward tests may be invalid or subject to misinterpretation when attempting to apply these data. As a result of these requirements and realizations, there has been within the past ten years, a significant increase in research activities on the mechanics of cavitation and in the analytic descriptions of the various types of cavitation.

In addition to these motivations for cavitation research, the use of cavitated systems for accelerating chemical processes has been of interest for some years, and more recently,

the recognition in the medical field of phenomena which may be included in the field of cavitation has provided yet another incentive for research. For example, interest in the inception and subsequent behavior of small, individual cavitation bubbles arises in connection with the general medical problem of gas separation in living tissue (see, e.g., Reference 1*) of which a special problem is the phenomenon of deep-sea divers' "bends" in which gas separates from the blood stream during too-rapid decompression. Again, there is interest in the medical field in the type of cavity formed during the air-water entry of a projectile since the wounds produced by missiles evidently behave as such cavities during the entry phases (see, e.g., Reference 2).

Although many questions remain to be investigated before the mechanism of cavitation phenomena can be more completely understood, much progress has been made toward rational descriptions of the processes in cavitating systems. This work includes not only the hydrodynamics of flows with both liquid and gas or vapor phases, but also the physical chemistry of such multiphase flows in relation to the formation, collapse, and maintenance of cavities. In a previous report,³ the writer attempted to outline very briefly and approximately the status of knowledge of various cavitation problems up to the year 1950. In that report, an attempt was also made to indicate some of the relationships between cavitation and other hydrodynamic phenomena and between the various types of cavitation in a somewhat more systematic way than had been done previously. Because of the constantly growing need among naval architects and engineers for more precise data on cavitation, it seems worthwhile to bring up to date the discussions of Reference 3 with the purpose not of providing design data for specific problems but, primarily, of providing background information necessary in the development of rational design criteria, procedures, and evaluations.

Although the present work is intended primarily as a survey of recent progress, an attempt will also be made to indicate the various problems and factors that, in the writer's opinion, require further study and to indicate how these problems arise by following an outline based on the sequence of events in the establishment of cavitating flows. This requires definitive terms for the "types" of cavitation, discussion of inception and of the processes in these various types of cavitation as well as of the types to be expected in any given system. Of necessity, some of the material in Reference 3 will be repeated, but, in general, liberal reference will be made to that work in preference to repetition. In this short paper, it will not be possible, of course, to give many details of recent investigations. However, it is hoped that not too many available references have been overlooked.

*References are listed on page 21.

TYPES OF CAVITATION

In Reference 3, brief discussions were given and several illustrations shown of the types of multiphase, single or multicomponent flows which can be considered cavitation flows from either a hydrodynamical or thermodynamical point of view or both. For the present work, it will only be necessary to define the terms preferred by the writer in referring to the various types of cavitation:

1. **TRANSIENT CAVITIES.** This term will be applied to the small, individual cavitation bubbles which grow, sometimes oscillate, and eventually collapse and disappear. This type has been characterized as "bubbling" or "bubble" cavitation by naval architects working on problems of cavitation of propellers and other ships' appendages.

The following types may be considered in the general class of free streamline flows and characterized as Kirchhoff flows or Helmholtz motions:

2. **STEADY-STATE CAVITIES.** This term is applied to the large, stationary cavities observed behind blunt bodies and very often on hydrofoil profiles having relatively sharp leading edges. The analytic description is not a function of time.* Such cavitation is referred to among naval architects as "laminar" and, more recently, as "sheet" cavitation. It should be noted that stationary vortex cavitation may be included here, being a time-independent flow with circulation.

3. **NONSTATIONARY CAVITIES.** This term is applied to cavities resembling the steady-state cavity but varying with time as in the air-water entry of an air-dropped missile or of an initially submerged but accelerating body. The term "unsteady" is also often used for this type of flow.

These terms provide a nonambiguous set both for physical descriptions and for the associated mathematical formulations. Although all three are free-boundary flows, in the first, the pressure at the boundary varies with time; in the second, the boundaries are free streamlines (constant velocity magnitude and, therefore, pressure); and in the third, the boundaries are such that the material lines are not necessarily free streamlines.

THE INCEPTION OF CAVITATION

INTRODUCTORY REMARKS

Although it is usual to assume, in most engineering applications, that cavitation will begin when the local pressure reaches the vapor pressure corresponding to the temperature of the liquid, the actual conditions are much more complex and very often the errors resulting

*It must be recognized here that when applying the term "steady-state" to cavities in real liquids reference is made to the average description taken over a relatively long time interval. When examined in detail,³ such cavities are very often observed to oscillate both in the streamwise direction and locally in the cavity surface.

from this assumption cannot be tolerated. Depending upon the condition of the liquid (air content and air or vapor nuclei which may exist in a stable condition or be stabilized on solid particles, etc.), cavitation may begin above or below the vapor pressure. The role of air and vapor nuclei in the inception of cavitation in both pure and contaminated liquids was discussed briefly in Reference 3. Some effects associated with turbulence in a free stream and in a boundary layer, as well as conditions in a boundary layer that are of some importance in cavitation problems were also discussed. In the present report, these remarks will be elaborated and extended somewhat, and some recently reported work relating to scaling of inception will be reviewed.

The problem of the limiting tensions which can be developed in liquids will not be discussed in this report—some of the recent ideas and methods having been reviewed briefly in Reference 3. Although these ideas, based on so-called "hole" and "nucleation" theories, lead to predictions that are of the order of magnitude of available experimental results, the theoretical models and approximations are not complete and, at best, can only result in probable predictions (in the statistical sense). In view of the experimental complications, e.g., the ever-present uncertainties associated with the behavior of the liquid at a wall, and of the properties of the wall itself, and the lack of sufficiently detailed theory needed for design of critical experiments, a completely satisfactory prediction procedure cannot as yet be established. Nevertheless, these ideas and methods are of immediate importance in helping to gain insight into the processes involved in cavitation even in "technical" liquids, and of continuing interest from the standpoint of eventual contribution to a unified theory of the mechanism of inception in any liquid, pure or contaminated.

FURTHER REMARKS ON THE ROLE OF TURBULENCE AND BOUNDARY LAYER EFFECTS IN CAVITATING FLOWS

In Reference 3, a rough calculation was carried out to illustrate the possibility of cavitation associated with the pressure fluctuations in a turbulent flow. It was shown, that in a boundary layer, the velocity fluctuations are such that the instantaneous pressures may vary sufficiently from the average value to result in a "microscale" cavitation which would occur in a more-or-less random fashion well before it could be observed visually. Thus, it is of some interest to be able to estimate the magnitude of the pressure fluctuations in turbulent flows of both the boundary layer type and wake and jet type. In these cases, however, the complete relations of turbulence phenomena to cavitation inception cannot be established until more is known of the turbulence mechanism itself. Although a great deal of work has been done in describing the velocity fluctuations, little effort has been devoted to investigations of the pressure fluctuations. The extreme difficulty of the latter problem has restricted the work so far to isotropic turbulence and only statistical averages can so far be obtained. For the cavitation problem, it will probably be important to know in somewhat more detail the values of the pressure fluctuations, the scale(length), and ultimately a time scale.

The estimates made in Reference 3 were based on results of Taylor⁴ and Green⁵ who found from computations based on models of isotropic turbulence that the pressure fluctuation p' may be written

$$\sqrt{p'^2} = \frac{3}{2} K \rho u'^2$$

where ρ is the mass density of the fluid and u' is the velocity fluctuation. The value of K given by Taylor and by Green for a number of theoretical models is of the order of one. In 1950, Batchelor⁶ considered this problem in the light of modern theories of turbulence and was able to derive the following relation for the fluctuating pressures in homogeneous, isotropic turbulence:

$$\sqrt{p'^2} = 0.34 \rho^2 (\overline{u'^2})^2$$

In terms of Taylor's formula, this corresponds to a value of K of 0.39. Since this magnitude is essentially of the order assumed in estimating the effects on cavitation, the previous computation need not be modified. Batchelor found further that the pressure scale is of the order of half the velocity scale. In addition to such computations, actual measurements of pressure fluctuations will be required.* Such measurements should include the temporal as well as spatial correlations with the ultimate aim of estimating the time available for formation of cavities.

The average conditions in both laminar and turbulent boundary layers are also of some interest with respect to analysis of behavior of nuclei within the boundary layer. It was shown in Reference 3 that, aside from low pressures associated with turbulence which occur at a small distance from the boundary, the minimum pressures may be expected to occur at the boundary rather than elsewhere in the boundary layer. Furthermore, nuclei which are exposed to low pressures in the slowly moving fluid of the boundary layer have a longer time in which to grow beyond a critical size and start the cavitation process.

Further work on boundary layer and wake flows will, of course, be necessary before the importance of the processes in such flows in relation to cavitation can be fully evaluated. Recent studies by Townsend⁷ have shown that the outer regions of the turbulent boundary layer resemble wake flows with a rather well-defined boundary between the turbulent and non-turbulent regions but that at any point in this outer region, the turbulence is intermittent. The picture presented is that of a wake with jets or fingers of turbulent fluid extending into the nonturbulent fluid. This phenomenon will be mentioned again in connection with the discussion of steady-state cavities in real liquids.

*Such measurements are being taken as part of a program of studies of jet cavitation at the Iowa Institute of Hydraulic Research. Preliminary results were reported by Dr. H. Rouse at the Eighth International Congress on Theoretical and Applied Mechanics, Istanbul, Turkey, August, 1952. However, published results, which will be of considerable interest in relation to these questions, are not, as yet, generally available.

CAVITATION IN SEPARATED FLOWS

The foregoing remarks were concerned with the conditions near a hydraulically smooth surface having unseparated boundary layers. It was pointed out in Reference 3 that care must be exercised in the design of hydrodynamic systems to insure that separated regions do not occur. In such regions, the very low pressures which can be developed in the essentially vortex flows can lead to cavitation in spite of relatively high ambient pressures. This was illustrated for wake flows from experiments at the Taylor Model Basin and for locally separated boundary layer flows on bodies of revolution from experiments at the Iowa Institute of Hydraulic Research. An example of cavitation in a separated boundary layer in the vicinity of the stagnation point is shown in Figure 1. In this case, the (probably laminar) boundary

Figure 1 - Cavitation in the Separated Boundary Layer on the Upstream Side of a Toroidal Ring

A half-toroid is fastened to a lucite plate through which this view was taken. The flow is from left to right. The two horseshoe-like vortices are swept downstream, one leg of each being clearly seen outside the ring and the other leg passing through the hole but obscured by the cavitation on the ring itself.

This photograph was obtained by Mrs. E. A. Sykes of the David Taylor Model Basin.

layer separated in the region of the adverse pressure gradients approaching the toroidal ring. The cavitating region was swept downstream in the form of two horseshoe vortices with legs outside and inside the toroid.

Conditions similar to these are evidently associated with cavitation about surface roughnesses. It is clear that the flow about roughness elements with sharply varying contours will separate and that cavitation may occur in the core of the separated regions. The strength of the vortices will evidently depend upon the velocity near the top of the roughness element so that the cavitation characteristics will depend upon the boundary layer configuration in the vicinity of the elements considered. Although much work has been done on the noncavitating flows about rough surfaces, results are not available in sufficient quantity to enable design criteria to be established for roughness limits acceptable from the standpoint of cavitation prevention. Experiments have been made by Shalnev⁸ for roughness elements in a restricted flow but lack of complete correlation with boundary layer characteristics make these results of limited usefulness. Nevertheless, they are the only published results known to the writer, and some empirical formulas are given which can be used when the roughness height is large compared with the boundary layer thickness.

Other separation phenomena may be observed when fluid issues from a nozzle with an abrupt expansion. This was discussed in Reference 3 in connection with the discrepancies between the results of Crump in a nozzle with a gradually expanding diffuser and of Numachi in a nozzle with an abrupt expansion. The trends observed by Numachi were confirmed by Crump in a subsequent TMB report⁹ although the numerical values differ somewhat. The observations of increasing tensions with increasing time of exposure to low pressures in Crump's first nozzle have not as yet been explained.

EFFECTS OF SCALE ON THE INCEPTION OF CAVITATION

Only a few remarks were made in Reference 3 on the question of scaling cavitation phenomena. However, this problem is of much importance in view of the need for reliance on results from model experiments. It is desired to mention here only the problem of scaling of the inception of cavitation and, in particular, to review some very interesting results obtained at the California Institute of Technology during the last two years on the effects of geometrical scale on inception of cavitation. Before discussing the latter results, however, it seems worthwhile to review the physical picture of the role of nuclei in cavitation inception and to point out the expected consequences of this concept.

It is now generally accepted that cavitation in a fluid under reduced pressure or boiling in a heated liquid begins with the growth of microscopic nuclei containing gas phase (air or vapor, etc.) It is well known that the absence of such nuclei requires very large forces for rupture since the surface tension forces become very large. Thus in well degassed liquids one expects rupture forces of the order of those predicted by kinetic theoretical formulations. Experimental evidence has also been obtained that water saturated with air, but denucleated by application of very high pressures, exhibits high tensile strength (of the order of several hundred atmospheres).¹⁴ Thus, the presence of nuclei is evidently necessary for the inception of cavitation at pressures of the order of vapor pressure. In many engineering applications, it is usually sufficient to assume that cavitation will occur at the vapor pressure corresponding to the temperature of the liquid. This assumes that there are sufficient nuclei of large enough initial size to grow to observable size during the time of application of reduced pressure. In supersaturated liquids, it is easy to account for the presence and stability of such nuclei, but in saturated and undersaturated liquids it is necessary to account for such nuclei on the basis that they are stabilized on particles suspended in the liquid (see, e.g., Reference 3 and the discussion of Reference 10). As a consequence, depending upon the size and number of these nuclei, cavitation may be expected to begin above as well as below the vapor pressure, as was shown in Crump's experiments with sea water and fresh water. A further consequence of this concept is that inception will depend upon the time of exposure of these nuclei to low pressures. Thus, for nuclei of a given size, the longer the time of exposure the higher inception pressures that would be expected.

In addition to the effect of actual time of exposure to low pressure, another factor which

must be considered when examining the effects of scale on inception is the dynamical behavior of the nuclei as a function of the rate of application of pressure, i.e., the pressure gradients. Thus, in scaling experiments in which the pressures are determined by the geometrical boundary conditions, not only will the actual time of exposure to low pressures vary but also the pressure gradients.* Since, in general, with geometrically similar boundaries and the same velocity, the pressure gradients will increase with decreasing model size, the dynamical response of the nuclei will be initially retarded and the initial appearance of an observable inception will be delayed in the experiment of smaller size if the properties of the liquid are independent of scale, i.e., if nuclei size remains unchanged. From a most elementary viewpoint* it would appear, therefore, that in order to carry out experiments at reduced scale, the nuclei size must vary inversely with the model scale, i.e., in order for the nuclei to grow to an "observable" size (visual, aural, etc.), larger nuclei must be present in the small scale experiment than in the prototype experiment. This discussion has assumed that surface tension effects are always important in the inception of cavitation. If the nuclei are initially so large that surface tension is not important in determining the rate of growth, then the effects of pressure gradients will not be controlling and only the time of exposure need be considered. However, if one further considers that only certain nuclei will grow to observable size, then the number of exposed nuclei must also be considered. Thus, on a reduced scale, the total number of nuclei in the liquid must be greater for the smaller scale than the larger since the region of low pressure is also reduced. For such considerations, the total number of nuclei must vary inversely as the cube of the linear ratio.**

The experiments on the effects of geometrical scale on cavitation inception carried out at the California Institute of Technology are of particular interest in relation to the foregoing remarks. It will be observed that in the type of systems investigated in these experiments it was not possible to scale nuclei size and content to correspond to model scaling.

Kermeen¹¹ has conducted a large number of experiments on bodies of revolution with hemispherical noses and cylindrical afterbodies having diameters ranging from 1/4 to 2 inches. He has observed a definite dependence of the cavitation number for inception on the model size and upon the absolute flow velocity. Furthermore, as will be seen from his results, Figure 2, the data cannot be correlated on the basis of Reynolds number alone. Further studies have been conducted on hydrofoils by Parkin who also attempted an analysis of the observations based on the growth of nuclei assumed to be stabilized on solid particles in the fluid.¹²

The results shown in Figure 2, which is only a sample of the data obtained by Kermeen, represent average values of a large number of data. The methods used in obtaining these data

*Effects of viscosity, diffusion, etc., have been neglected in this discussion.

**Investigations of the various factors discussed in the foregoing are now getting underway at the Taylor Model Basin.

Figure 2 - Cavitation Number K for Incipient Cavitation as a Function of Reynolds Number for Bodies with Hemispherical Noses

These results were reported by R. W. Kermeen, California Institute of Technology, Reference 11.

are of some interest in connection with the remarks made in the subsequent section of this report. The curves shown were obtained by first allowing cavitation to become fully developed at cavitation numbers well below the inception value and then raising the ambient pressure until cavitation just disappeared. This point was defined as the "inception point." Attempts to observe inception by the direct procedure of approaching the critical point starting with a noncavitating system evidently resulted in so large a scatter that it was difficult to obtain accurate correlation of the results. Kermeen remarks that a so-called "hysteresis" could often be observed, i.e., when approaching the inception point from a noncavitating condition, much lower critical cavitation numbers were observed than those at which cavitation disappeared. Nevertheless, the results evidently indicated the trends clearly shown in Figure 2. These results are of further importance in that they tend to substantiate the physical picture of the role of nuclei in cavitation inception since the observed trends coincide with the expected consequences of this concept.

Although the results obtained by Kermeen and Parkin are consistent with the present ideas of the role of nuclei, there still remains the anomaly of the results obtained by Crump in which simultaneously increasing time of exposure to low pressures and decreasing pressure resulted in decreasing critical cavitation numbers.* On the other hand, it will be observed that in Crump's experiments, the inception point was obtained by approaching from a noncavitating condition, and that the same effect characterized as "hysteresis" was obtained when approaching from the fully cavitating condition. (The latter method was not considered as a criterion for inception in his experiments and no quantitative results were recorded.)

*Whether Crump's results can be reconciled in terms of the effects of the pressure gradients remains to be investigated.

Thus, some care must be exercised in relating the results of these different types of experiments to practical systems in which the actual onset is of interest. It would seem to be of rather significant interest to know the maximum possible critical cavitation number for any given system and, further, to know a "critical" cavitation number at which adverse effects (noise, loss in performance, etc.) first begin. (These numbers are not, of course, necessarily the same. As a matter of fact, this is only the question of the definition of cavitation onset since, presumably, nuclei will be affected by any pressure field whether or not they are seen or, as a result of the motion, radiate noise.)

A REMARK ON THE QUESTION OF GAS CONTENT AND NUCLEI CONTENT

It seems clear from the foregoing remarks and the discussions in Reference 3 that the inception of cavitation in technical liquids is intimately associated with the presence of nuclei whether as free gas or vapor bubbles or bubbles trapped on solid particles. That the role of gases in liquids is well recognized is reflected in the now almost standard procedure of measuring air-content in experiments in which cavitation phenomena are involved. However, the measurements that have been reported so far have all been of the total air and gas content, i.e., both dissolved, and entrained as individual bubbles. On the basis of the previous discussion, it seems unlikely that completely dissolved gases can play an important role in the inception of cavitation* since, in this situation, very high tensions would be expected. Thus, it is the opinion of the writer that further progress on inception may depend, to a considerable extent, on the development of theoretical and experimental methods for the characterization and measurement of the nuclei content of liquids.** These studies should eventually result in descriptions of the size distribution and quantity of undissolved gas nuclei—in other words, a "spectrum" of nuclei.

With such information available, it would then be possible to undertake correlation studies of inception of cavitation as a function of the nuclei spectrum. Judging from the experiments that have been made and remembering the many factors which must be considered (turbulence, physical-chemical properties of the liquid, other foreign materials, etc.), it seems not unlikely that under a given set of conditions, it will not be possible to prescribe a unique critical cavitation number but only a "most probable" critical cavitation number.

*Except insofar as the equilibrium conditions between dissolved gas and undissolved nuclei are concerned.

**Such studies are now underway at the David Taylor Model Basin.

DYNAMICS OF TRANSIENT CAVITIES

THE MOTION OF SMALL TRANSIENT CAVITIES

Since, in Reference 3, a brief outline of the experimental observations on transient cavities up to 1950 was given, only a few remarks will be made here in connection with an apparent difference in experience among different investigators on the rebound of such cavities. For this discussion, we exclude observations in which magnetostriction oscillators or ultrasonic fields were used to cavitate the liquid. Also excluded are experiments in which small air bubbles were introduced to assist the formation of the cavity.

In 1928, Mueller¹³ published prints of motion picture frames showing cavitation on a hydrofoil. These photographs showed clearly the growth and collapse of individual cavities, but no oscillations were observed. Harvey et al,¹⁴ using rods withdrawn rapidly from a liquid, obtained several oscillations of the resultant cavity and attributed them to energy storage in air entrained in the bubble. Knapp and Hollander,¹⁵ in a now classical series of photographs, showed several cycles in the oscillations of cavities in a flowing liquid. They argued that in their experiments the initial air content of the cavities was extremely small and attributed the rebound of the cavity primarily to the storage of energy in the liquid in elastic compression, with this stored energy subsequently producing the outward radial velocity.

More recently, experiments by Harrison¹⁶ have shown that cavities formed in water of low air content do *not* rebound, and the conclusion was reached that only cavities containing a large amount of gas will oscillate. In a private communication, Dr. M.S. Plesset informed the writer that the same results were recently obtained at the California Institute of Technology. It may, therefore, be concluded that in liquid of low air-content, the effects of compressibility and viscosity in both the liquid and gas phases in dissipating energy are such as to allow no rebound if there is only a small amount of permanent gas in the bubble. However, no consideration appears to have been given as to what this lower limit of initial air content in the nucleus must be to prevent rebounds.

ANALYTICAL DESCRIPTION OF THE MOTION

Work on the motions of a spherical cavity in an incompressible fluid in which the role of the gas within the bubble is neglected was reviewed in Reference 3. These theories give adequate descriptions of the motion for the largest part of the cycle of such bubbles but are inadequate toward the end of the collapse stage where compressibility and viscosity effects in both the liquid and the gas phase become of importance. This was shown by the computations of Plesset as compared with the experimental results of Knapp and Hollander. Discrepancies noted between theory and observation for the collapse part of the cycle were attributed to wall effects--the bubbles having been formed near a model and these effects neglected. This was clearly shown by Rattray¹⁷ in a dissertation in which he computed the motion of such cavities when near a plane wall. The analysis is too lengthy to give details here,

being carried out to several terms in an expansion of the velocity potential in Legendre polynomials. However, Rattray showed that for the above bubbles, the time of collapse could be increased by as much as 20 percent for the assumed distance of approximately one-half to one diameter from the wall.

It is clear that analyses based on empty cavities in an incompressible liquid cannot give rise to oscillations. Furthermore, with constant external pressure, the velocity of the bubble wall increases without limit as the bubble collapses. To obviate this result, Lord Rayleigh, who gave the first complete solution of the collapse of an empty, spherical cavity in an incompressible liquid¹⁸ extended his computations to include the case of a cavity filled with a gas which is expanded and compressed isothermally and showed that the boundary oscillates between two positions, of which one is the initial position. Although the motion of the oscillating cavity in a real liquid is evidently complicated by the diffusion and vaporization processes and the problems of energy dissipation, such solutions, which are clearly oversimplifications, nevertheless, give a clear, quantitative picture of the hydrodynamics of the motion as long as the cavity radius is large compared with the minimum radius.

In Reference 3, the writer pointed out that further extensions to include the problems of energy dissipation associated with the compressibility of the vapor and gas mixture and of the liquid would be of great practical as well as theoretical interest. For a bubble filled with a permanent gas being compressed and expanded adiabatically rather than isothermally, Rayleigh's case corresponds to the case of pulsation of a gas globe following an underwater explosion (see, e.g., Reference 19). Although, in the latter problems, much progress had been made (up to the time Reference 3 was written) in describing an oscillatory motion with energy dissipation, the problem of the vapor condensation and formation prevents a complete analogy to gas-globe theory and a completely satisfactory description of the motion of cavities based on this theory had not been formulated. However, if one neglects the condensation problems, which are important only insofar as determining the conditions under which the vapor begins to act as a permanent gas, many of the results and methods first developed in the field of explosion hydrodynamics may be applied to the present problem.

This has been done within the last two years, first by Trilling²⁰ and later by Gilmore.²¹ Trilling derived the velocity and pressure fields about a bubble in a slightly compressible liquid using the acoustic approximation (i.e., only velocities that are first order small compared with the velocity of sound are considered) and obtained results which coincides with those of C. Herring^{22, 23} who carried out the same computation some years earlier, but only for the conditions at the bubble surface. Since these results are included in the later extension of Gilmore,²¹ only one result of interest will be mentioned in connection with Trilling's work. Computations of the collapse of a bubble supposed to be filled with a perfect gas showed that a series of shock waves are propagated into the gas. However, it was shown that the average variation at the bubble wall is very nearly the same as if the gas were compressed uniformly and isentropically.

Gilmore²¹ extended these computations to include higher order compressibility terms, as well as the effects of viscosity and surface tension. Instead of using the acoustic approximation (i.e., all disturbances propagated with the velocity of sound c), Gilmore assumes the Kirkwood-Bethe hypothesis,²⁴ which assumes that disturbances are propagated with the velocity $c + u$, u being the local fluid velocity. Gilmore then derives the equation of motion of the bubble wall in the form:*

$$RU \frac{dU}{dR} \left(1 - \frac{U}{C}\right) + \frac{3}{2} U^2 \left(1 - \frac{U}{3C}\right) = H \left(1 + \frac{U}{C}\right) + \frac{RU}{C} \frac{dH}{dR} \left(1 - \frac{U}{C}\right)$$

where R is the radius of the bubble,

C is the sonic velocity in the liquid at the bubble wall,

H is the enthalpy difference $\left(= \int_{p_\infty}^P \frac{dp}{\rho}\right)$ between the liquid at pressure P at the bubble

wall and at pressure p_∞ at an infinite distance from the bubble, and

U is the velocity of the bubble wall.

The values of H and C are derived from the experimentally developed formula for isentropic compression of liquids.

$$\frac{p + B}{p_\infty + B} = \left(\frac{\rho}{\rho_\infty}\right)^n$$

where B and n are constants for each liquid (for water, $B \approx 3000$ atm and $n \approx 7$). Thus, he finds that

$$C = C_\infty \left(\frac{P + B}{p_\infty + B}\right)^{\frac{n-1}{2n}}$$

and

$$H = \frac{n(p_\infty + B)}{(n-1)\rho_\infty} \left[\left(\frac{P + B}{p_\infty + B}\right)^{\frac{n-1}{n}} - 1 \right]$$

The effects of surface tension and viscosity are included in the boundary conditions by writing the pressure at the bubble wall as

$$P = P_i - \frac{2\sigma}{R} - 2\mu \left(\frac{\partial u}{\partial r}\right)_{r=R}$$

where P_i is the pressure of the internal gas,

σ is the surface tension, and

μ is the dynamic viscosity.

*Only a very few results of Gilmore's paper are abstracted here. For full details see Reference 21.

Using the equation of continuity for a compressible fluid and assuming that the viscosity and compressibility of the liquid are small, he writes, finally,

$$P = P_i - \frac{2\sigma}{R} - \frac{4\mu U}{R}$$

If the internal pressure is constant and the surface tension and viscosity negligible,* the equation of motion can be solved analytically if only terms in dU/dR and dH/dR are retained, thus, Gilmore finds

$$\left(\frac{R_0}{R}\right)^3 = \left(1 - \frac{U}{3C}\right)^4 \left(1 + \frac{3\rho_\infty U^2}{2(p_\infty - P_i)}\right)$$

Neglect of the term $U/3C$ yields the Rayleigh results. As $R \rightarrow 0$, the Rayleigh theory (incompressible) gives $U \sim R^{-3/2}$, whereas Gilmore's results gives $U \sim R^{-1/2}$.

Figure 3 - The Theoretical Wall-Velocity of a Bubble in Water Collapsing under a Constant Pressure Difference of 0.517 Atmospheres

These curves are reproduced from Gilmore, Reference 21. "Present Theory" refers to Gilmore's results, see text.

Figure 3 shows a comparison of Gilmore's theory with the results of Rayleigh, Herring, and a numerical integration of the complete equation of motion carried out by Schneider.²⁵ It will be seen that the solutions approach each other for small ratios of the wall velocity to sound velocity but diverge rapidly as the sonic velocity is approached and finally exceeded. Gilmore also gives the equations of pressure and velocity throughout the fluid but did not compute numerical values.

More recently, computations have been carried out by Poritsky²⁶ and Shu²⁷ to determine specifically the effects of viscosity and surface tension, but assuming the fluid to be incompressible. In these papers, large effects of viscosity in retarding both the growth and

*Although Gilmore only carried through the computation with surface tension and viscosity neglected, he examined the effects of these variables and found bounds for the ratio R/R_0 within which the effects could be neglected without affecting the motion except in the very last period of collapse (which will usually be of the order of a few microseconds).

collapse were found, while for surface tension, some cases are worked out to illustrate the effects in retarding growth and accelerating the collapse.

MECHANISM OF STEADY-STATE CAVITIES AND THEIR ANALYTICAL DESCRIPTION

INTRODUCTORY REMARKS

A complete review and analysis of the experimental and analytical work on steady-state cavities would require a much more detailed and voluminous discussion than is intended here. Some results of more immediate interest in technical applications were presented in Reference 3. In this report, the discussion will again be confined to a few more-immediately applicable results and only mention made of the direction of other researches.

The recognition of the applicability of free streamline theory (classical wake theory) to the steady-state cavity problem has greatly stimulated the mathematical work on such flows and has resulted in a rapidly growing literature in this field. The two-dimensional problem is now well understood and the theory is available for the solution of flows about a large class of solid boundaries. However, very few numerical results are, as yet, available and will be required before application in engineering problems can be made. These results would be of much interest in connection with so-called "supercavitating" propellers, for example, as well as other problems requiring knowledge of the forces on fully cavitating hydrofoils. The successful treatment of the two-dimensional cases has been possible through the very powerful conformal mapping techniques. However, the extreme difficulty of the general problem (arising from the nonlinearity of the boundary conditions) has precluded general treatments, and in the three-dimensional case, progress has been made only in the treatment of flows with axial symmetry.

In the following paragraphs, some additional thoughts on cavities in real liquids will be outlined and some recent results on the analytical treatment will be presented.

SOME REMARKS ON STEADY-STATE CAVITIES IN REAL LIQUIDS

The degree to which the theoretical models of steady-state cavities represent such flows in real liquids was discussed to some extent in Reference 3. The added complications of the properties of the liquid and of the surface conditions of the solid boundaries require further clarification and much work remains to be done in this direction. Furthermore, the question of the maintenance of such cavities in real liquids requires further investigation—for example, the vaporization process at the cavity wall, the processes in removal of all liquid phase from cavities in which cavitation first occurs only in the small scale eddies on the boundary of a viscous wake, and the processes of entrainment and condensation at the tail of cavities which have been observed to oscillate rather violently. The latter questions were also discussed to some extent in Reference 3 and, in this report, only a few additional

remarks on these problems will be made.

So far, the transient cavities and the steady-state cavity flows have been discussed as entirely unrelated problems from a hydrodynamical point of view. One of the interesting and practically important questions which has not been considered to any extent is that of the conditions (both geometrical and physical) under which cavitation will occur in the form of a large steady-state cavity or a mass of small, oscillating bubbles. In Reference 3, the writer expressed the view that the appearance of the cavitation is associated in part with the pressure gradients, but no satisfactory criteria are available as to the initial appearance or transition from transient cavities to a large, steady-state cavity. A cavitated region made up entirely of transient bubbles may exhibit the properties of a steady cavity in that the average envelope of such a region does not vary with time. Another case of such "steady-state" cavities in which the average envelope remained unchanged, but in which rapid surface oscillations were observed without clear evidence of individual bubbles was discussed in Reference 3 in connection with the study of the development of cavitation in wakes.

This case of cavitation in wakes raised the question of the processes involved in removing the liquid phase from the cavity and the spread of cavitation from the wake boundaries into the interior. It was pointed out in Reference 28 (and also discussed in Reference 3) that in the studies reported therein the cavities contained large quantities of liquid phase and that the liquid was not removed even at the lowest cavitation number reached (0.116). An example of such a flow is shown in Figure 4, which was obtained during the experiments reported in Reference 28. Subsequent experiments have disclosed certain cases in which the liquid phase was removed from the cavity. Under conditions not yet defined or even understood the liquid could be seen to move rearward out of the cavity. This process occurred at a speed slow enough to be followed by eye, leaving a fairly transparent cavity wall. This phenomenon has also been observed in the water tunnel of the Ordnance Research Laboratory at the Pennsylvania State College. Just what conditions determine the point at which equilibrium is upset between vaporization and possibly shearing motion at the boundary and the replenishment with re-entrant fluid from the tail of the cavity are not clear and require further investigation.

Figure 4 - "Steady-State" Cavity
behind a Hemisphere

The photograph was taken with an exposure time
of 1/10,000 second.

It will be observed in Figure 4 that cavitation appears to be intermittent even though this case is one in which the cavitation number is well below the values associated with the shedding of well defined vortices (see Reference 3). This intermittency further suggests the close connection with the turbulent wake flow mentioned previously. Furthermore, the "billowing" nature of the surface resembles the descriptions of the turbulent wake as given by Townsend. Nevertheless, the average envelopes of such regions evidently behave as steady-state cavities. Thus, the problem of cavitation in such turbulent regions and adequate descriptions of the processes remain to be investigated and reconciled with available theory. Otherwise stated, the problem is one of describing the transition from a cavitating wake flow to free streamline flow.*

THE ANALYTICAL DESCRIPTION OF STEADY-STATE CAVITIES

Some results of free streamline theory for two-dimensional flows were given in Reference 3. Further work continues on these flows based on the Riabouchinsky and the re-entrant jet models but the results have not been applied in a sufficient number of technically important cases. Discussions of these flows will be found in References 3, 29, 30, and 31 which are cited here not only for their own content but also for the bibliographies and references given therein.

Theoretical work on three-dimensional cavities has been concerned almost exclusively with existence and uniqueness of solutions for axially symmetric flows. Recent work in this direction is given in References 32 and 33 which also summarize previous work on this problem. Various attempts to apply numerical procedures to the solution of specific problems have been made but have evidently been unsuccessful in producing accurate or even physically realistic results and will not be discussed here (a brief account of some computations will be found in Reference 29).

In addition to the above attempts to develop exact solutions of free streamline flows, a recent theory which avoids the necessity for artificial mathematical models deserves special mention. M.P. Tulin, in a recent report,³⁴ has developed a linearized theory for two-dimensional cavity flows about slender, symmetric bodies. An important feature of the results is that the calculation of drag and cavity shapes of arbitrary slender bodies is reduced to quadratures with the resultant attractiveness for use in actual applications. In this report, only some comparisons between Tulin's results and the exact results for the Riabouchinsky model will be presented to indicate the range of applicability of this theory. The method of linearization is similar to that of the linearized airfoil theory. The results chosen for illustration are from computations for wedge profiles. Tulin gives the following results for the

*From another point of view, such cavitating turbulent wakes might be used in studying the pressure fluctuations associated with turbulence, as has been pointed out by others, as well.

relation between cavitation number and cavity length (any profile),

$$\frac{\sigma}{1 + \frac{1}{2}\sigma} = \frac{4}{\pi l} \int_{-c}^0 \frac{dy_0}{dt} \frac{\sqrt{l-t}}{\sqrt{-t}} dt$$

the origin of coordinates being chosen at the trailing edge (base) of the profile: Here, σ is the cavitation number, l is the cavity length, c is the body length, and y_0 is the body ordinate. His result for the drag coefficient as a function of wedge half-angle γ is

$$C_D = \frac{8\gamma}{\pi}$$

General results for cavity shape and drag are given in Reference 34. Figure 5 shows the comparison between the drag coefficients from Tulin's theory and the exact theory for various wedge angles at zero cavitation number. Figure 6 shows the comparison of cavity lengths from Tulin's theory and the exact Riabouchinsky model for various cavitation numbers for a 30-degree wedge. The success of this theory promises to be of much importance in engineering applications since Tulin has been able to extend the method to the computation of cavitating flows about lifting surfaces in both the steady and unsteady cases. The computations for the latter problems are now being carried out. He has also developed a linearized theory for axisymmetric, three-dimensional flows, but the results are not yet in a form suitable for numerical computation.

Other results of immediate interest in technical application include work on wall effects in steady cavitation flows. In Reference 35, the authors consider the two-dimensional cavity flow about tandem laminae situated normal to the flow direction in a straight-sided channel of finite width and in a free jet of finite width. Results are given for finite as well as zero cavitation numbers. These results are of much interest in the design of water tunnels for studies of finite cavities of very small cavitation number, it being shown that the "wall" effects are much less severe with free jets than with rigid boundaries. Moreover, for a given cavitation number, there is a limiting value of the ratio of lamina width to channel width for rigid channels which cannot be exceeded (a "choking" phenomenon).

Another problem which arises in flows with circulation is that of the stability of a cavitating vortex. This question is of some interest in connection with the effects of cavitation in the tip vortices of hydrofoils and propellers. Although no changes in the flow conditions at the hydrofoil or propeller blade will occur as long as the cavitated region remains detached, the behavior of such cavitating vortices will be of interest in gaining an understanding of the changes and effects that result when the cavitated region becomes attached to the blade edge. An analysis of standing waves of infinitesimal amplitude on a vortex core has been carried out by Ackeret³⁶ and his results verified, at least qualitatively, in experiments by Lerbs.³⁷ More recently, Binnie³⁸ has considered the problem in a more general way and has derived the properties of traveling waves, as well, giving numerical results for several cases. It will be recalled, as Binnie points out, that this is a classical problem of

Figure 5 - Cavity Drag Coefficients for Wedge Profiles at Zero Cavitation Number

(After Tulin, Reference 34)

W. Thomson (Lord Kelvin).³⁹ Interest in these results extend beyond their intrinsic content since such oscillations may lead to vibrations of the propeller blades. Actual investigations of such interactions remain to be carried out, however.

A REMARK ON THE DRAG OF CAVITATING BODIES

Although the drag of bodies with fixed points of separation of the cavity (flat plates, discs, cones) can evidently be approximated by an equation of the form (see Reference 3)

$$C_D(\sigma) = C_D(0) (1 + \sigma)$$

where $C_D(\sigma)$ is the drag coefficient for cavitation number σ , this does not appear to be the case in actual flows for bodies with longitudinal curvature. The linear increase of drag with cavitation number shown in experiments is in good agreement with theory, but the slope of the curve evidently depends on the form. For results available so far, it appears that the drag can be closely approximated by the formula

$$C_D(\sigma) = C_D(0) (1 + \alpha \sigma)$$

Figure 6 - Cavity Length as a Function of Cavitation Number for a 30-Degree Wedge

(After Tulin, Reference 34)

Cavity Drag Coefficients for Various Bodies

Model	$C_D(0)$	α	Reynolds Number
Disc	0.805	1.0	---
Hemisphere	0.241	2.024	---
2:1 Semiellipsoid and 2 Caliber Ogive	0.114	3.65	---
Circular Cylinder	≈ 0.55	0.81	2.72×10^5
		0.68	1.75×10^5
		0.73	$2 \text{ to } 6 \times 10^5$

The accompanying table summarizes the results examined by the writer. The value of $C_D(0)$ for the disc is the result obtained by Plesset and Shaffer⁴⁰ by assuming that the pressure distribution in the meridian plane is the same as that of the two-dimensional computation. The values of $C_D(0)$ for the hemisphere, ellipsoid, and ogive are extrapolated from the experimental data of Reference 28, from which the results for α for these bodies and the disc were also obtained. The value of $C_D(0)$ for the circular cylinder is from the computation of Brodetsky.⁴¹ The value $\alpha = 0.73$ for the circular cylinder is given by Birkhoff²⁹ based on the experiments of Martyrer. The other values of α for the circular cylinder are based on Konstantinov's experiments,⁴² which show differences depending on Reynolds number (based on cylinder diameter). For comparison, the range of Reynolds numbers in Martyrer's tests is also shown. It should be noted that Konstantinov's results are for constant Reynolds number, whereas in Martyrer's tests the Reynolds number varied as the cavitation number was varied. There may be some question, however, as to the accuracy of Konstantinov's results since the forces were found by integrating pressure distributions rather than by direct measurement.

SOME RECENT WORK ON NONSTATIONARY CAVITIES

In addition to the steady-state theory for lifting surfaces, it will eventually be of considerable interest to have results for nonstationary cavities for such cases (the interest for nonlifting bodies is already well established). For example, the cavity on a blade of a surface ship propeller of large diameter will grow and contract depending on the position of the blade during rotation. Results for two-dimensional unsteady motions without circulation have been obtained by Gilbarg⁴³ for polygonal obstacles. This problem differs from the steady-state case in that the free boundary is a material line and not, in general, a streamline. Gilbarg replaced the latter requirement by the approximating condition that the free boundary is a streamline, and then used standard conformal mapping techniques to obtain solutions for cavities behind a flat plate normal to the flow direction (symmetric cavities). He showed that these solutions are exact for unsteady flows whose free boundaries are of constant shape. In particular, two classes were distinguished: one in which the cavities have a cusped end

or a stagnation point (the latter found also by von Kármán⁴⁴ and included in the more general class of cavities of constant shape found by Gilbarg); the second comprises cavities for which the free streamlines cross the axis of symmetry and are, thus, not physically realistic. In addition to the above detailed computations, he briefly indicated the method of extension of these results to arbitrary polygonal shapes and also to asymmetric polygonal shapes.

CONCLUDING REMARKS

An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. It is hoped, however, that some old problems which still require investigation and some new ones have been more clearly pointed out in the course of this survey. Although no direct mention of damage or cavitation prevention has been made, the remarks of Reference 3 regarding the mechanisms need no material modification. However, direct work on damage of specific materials is still of much importance and programs are underway in various laboratories.

As a final remark, it might be observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.

REFERENCES

1. Harvey, E. Newton, Barnes, K.D., McElroy, W.D., Whiteley, A.H., Peese, D.C., and Cooper, K.W., "Bubble Formation in Animals," *Jour. of Cellular and Comp. Physiology*, Vol. 24, No. 1, August 1944, pp. 1-34.
2. Harvey, E. Newton, Whiteley, Arthur A., Grundfest, H., and McMillen, J.H., "Piezo-electric Crystal Measurements of Pressure Changes in the Abdomen of Deeply Anaesthetized Animals during Passage of a High-Velocity Missile," *The Military Surgeon*, Vol. 98, No. 1, January 1946, pp. 509-528.
3. Eisenberg, Phillip, "On the Mechanism and Prevention of Cavitation," TMB Report 712, July 1950.
4. Taylor, G.I., "The Mean Value of the Fluctuations in Pressure and Pressure Gradient in a Turbulent Fluid." *Proc. Camb. Phil. Soc.*, 1936, Vol. 32, pp. 380-384.
5. Green, A.E., "The Mean Value of the Fluctuations in Pressure and Pressure Gradient in a Turbulent Fluid," *Proc. Camb. Phil. Soc.*, 1938, Vol. 34, pp. 534-539.
6. Batchelor, G.K., "Pressure Fluctuations in Isotropic Turbulence," *Proc. Camb. Phil. Soc.*, Vol. 47, April 1951, pp. 359-374.
7. Townsend, A.A., "The Structure of the Turbulent Boundary Layer," *Proc. Camb. Phil. Soc.*, Vol. 47, Pt. 2, April 1951, pp. 375-395.

8. Shalnev, K.K., "Cavitation of Surface Roughnesses," Jour. of Theoretical Physics, U.S.S.R., Vol. 21, No. 2, 1951, pp. 206-220 (in Russian).
9. Crump, S. F., "Critical Pressures for the Inception of Cavitation in a Large-Scale Numachi Nozzle as Influenced by the Air Content of the Water," TMB Report 770. July 1951.
10. Epstein, P.S., and Plesset, M.S., "On the Stability of Gas Bubbles in Liquid-Gas Solutions," Jour. Chem. Phys., Vol. 18, No. 11, November 1950, pp. 1505-1509.
11. Kermeen, R.W., "Some Observations of Cavitation on Hemispherical Head Models," Hydrodynamics Laboratory, California Inst. of Tech. Report No. E-35.1, June 1952.
12. Parkin, Blaine R., "Scale Effects in Cavitating Flow," Hydrodynamics Laboratory, California Inst. of Tech. Report No. 21-8, July 31, 1952.
13. Mueller, J., "Über den gegenwertigen Stand der Kavitationsforschung," Die Naturwissenschaften, Vol. 22, June 1928, pp. 423-426.
14. Harvey, E. Newton, McElroy, Wm. D., and Whiteley, A.H., "On Cavity Formation in Water," Jour. of Appl. Phys., Vol. 18, February 1947, pp. 162-172.
15. Knapp, R.T., and Hollander, A., "Laboratory Investigations of the Mechanism of Cavitation," Trans., A.S.M.E., Vol. 70, No. 5, July 1948, pp. 419-435.
16. Harrison, Mark, "An Experimental Study of Single Bubble Cavitation Noise," TMB Report 815, June 1952.
17. Rattray, Maurice, Jr., "Perturbation Effects in Bubble Dynamics," California Inst. of Tech. Doctoral dissertation, issued as Hydrodynamics Laboratory Report under ONR contract N6one-24420 (NR-062-059), undated but issued in 1951.
18. Lord Rayleigh, "On the Pressure Developed in a Liquid during the Collapse of a Spherical Cavity," Phil. Mag., Vol. 34, 1917, pp. 94-98.
19. Kennard, E.H., "Report on Underwater Explosions," TMB Report 480, October 1941.
20. Trilling, Leon, "The Collapse and Rebound of a Gas Bubble," Jour. of Appl. Phys., Vol. 23, No. 1, January 1952, pp. 14-17.
21. Gilmore, Forrest R., "The Growth or Collapse of a Spherical Bubble in a Viscous Compressible Liquid," California Inst. of Tech., Hydrodynamics Laboratory Report No. 26-4, April 1, 1952.
22. Herring, C., "Theory of the Pulsations of the Gas Globe Produced by an Underwater Explosion," OSRD (U.S.A.) Report 236, 1941.
23. Cole, R.H., "Underwater Explosions," Princeton Univ. Press, 1948, pp. 305-307.
24. Kirkwood, J.G., and Bethe, H.A., "The Pressure Wave Produced by an Underwater Explosion," OSRD (U.S.A.) Report 588, 1952. (A brief account is given in Reference 23, pp. 28-45, 102-109, 114-126, 425-426.)

25. Schneider, A.J.R., "Some Compressible Effects in Cavitation Bubble Dynamics," Doctoral dissertation, California Inst. of Tech., 1949.
26. Poritsky, H., "The Collapse or Growth of a Spherical Bubble or Cavity in a Viscous Fluid," Proc. First U.S. Nat. Cong. Appl. Mech., A.S.M.E., 1952, pp. 813-821.
27. Shu, S.S., "Note on the Collapse of a Spherical Cavity in a Viscous Incompressible Fluid," Proc. First U.S. Nat. Cong. Appl. Mech., A.S.M.E., 1952, pp. 822-825.
28. Eisenberg, Phillip, and Pond, Hartley L., "Water Tunnel Investigations of Steady-State Cavities," TMB Report 668, October 1948.
29. Birkhoff, Garrett, "Hydrodynamics," Princeton University Press, 1950.
30. Gilbarg, David, and Serrin, James, "Free Boundaries and Jets in the Theory of Cavitation," Jour. Math. and Phys., Vol. 29, No. 1, April 1950, pp. 1-12.
31. Serrin, James B., Jr., "Existence Theorems for Some Hydrodynamical Free Boundary Problems," Jour. Rational Mech. and Analysis, Vol. 1, No. 1, January 1952.
32. Gilbarg, David, "Uniqueness of Axially Symmetric Flows with Free Boundaries," Jour. Rational Mech. and Analysis, Vol. 1, No. 2, April 1952.
33. Garabedian, P.R., Lewy, H., and Schiffer, M., "Axially Symmetric Cavitation Flow," Applied Math. and Statistics Lab., Tech. Report No. 10, Stanford Univ., April 25, 1952.
34. Tulin, M.P., "Steady Two-Dimensional Cavity Flows about Slender Bodies," TMB Report 834, May 1953.
35. Birkhoff, G., Plesset, M., and Simmons, N., "Wall Effects in Cavity Flow - I and II," Quart. Jour. of Math., Pt. I: Vol. VIII, No. 2, July 1950, pp. 151-168; Pt. II: Vol. IX, No. 4, January 1952, pp. 413-421.
36. Ackeret, J., "Über stationäre Hohlwirbel," Ingenieur-Archiv, Vol. 1, 1930, pp. 399-402.
37. Lerbs, H., "Photographische Aufnahmen der Kavitation im Randwirbel eines Tragflügels," Werft-Reederei-Hafen, Heft 1, 1 January 1932, pp. 1-2.
38. Binnie, A.M., "The Theory of Waves Traveling on the Core of a Swirling Liquid," Proc. Roy. Soc. Lond., Ser. A, Vol. 205, No. 1083, 7 March 1951, pp. 530-540.
39. Thomson, W., "Vibrations of a Columnar Vortex," Phil. Mag., Vol. 10, No. 5, 1880, pp. 155-168.
40. Plesset, Milton S., and Shaffer, Philip A., "Drag in Cavitating Flow," Rev. Mod. Phys., Vol. 20, No. 1, January 1948, pp. 228-231.
41. Brodetzky, S., "Discontinuous Fluid Motion Past Circular and Elliptic Cylinders," Proc. Roy. Soc. Lond., Ser. A, Vol. 102, No. A718, February 1923, pp. 542-553.

42. Konstantinov, W.A., "Influence of the Reynolds Number on the Separation (Cavitation) Flow," *Izvestiia Akademii Nauk SSSR, Otdelenie Tekhnicheskikh Nauk*, 1946, No. 10, pp. 1355-1373, TMB Translation 233, November 1950.

43. Gilbarg, David, "Unsteady Flows with Free Boundaries," *Zeit. für angewandte Math. und Phys.*, Vol. III, 1952, pp. 34-42.

44. von Kármán, Theodor, "Accelerated Flow of an Incompressible Fluid with Wake Formation," *Annali di Matematica*, Ser. 4, Tom. 29, Bologna, 1949.

INITIAL DISTRIBUTION

Copies

- 19 Chief, Bureau of Ships, Technical Library (Code 327), for distribution:
- 5 Technical Library
 - 1 Civilian Consultant to Chief (Code 106)
 - 2 Research and Development (Code 300)
 - 1 Applied Science (Code 370)
 - 1 Noise, Shock, and Vibration Branch (Code 371)
 - 1 Ship Design (Code 410)
 - 2 Preliminary Design and Ship Protection (Code 420)
 - 1 Preliminary Design (Code 421)
 - 1 Model Basin Liaison (Code 422)
 - 1 Submarines (Code 515)
 - 1 Minesweeping (Code 520)
 - 1 Torpedo Countermeasures (Code 520I)
 - 1 Propellers and Shafting (Code 554)
- 3 Chief, Bureau of Ordnance, Underwater Ordnance
- 2 Code Re 6
 - 1 Code Re 3
- 3 Chief, Bureau of Aeronautics, for distribution:
- 2 Aero and Hydrodynamics Branch (DE-3)
 - 1 Applied Mathematics Branch (RS-7)
- 7 Chief of Naval Research, for distribution:
- 3 Fluid Mechanics Branch (Code N426)
 - 1 Mathematics Sciences Division (Code 430)
 - 1 Mathematics Branch (Code 432)
 - 1 Naval Sciences Division (Code 460)
 - 1 Undersea Warfare Division (Code 466)
- 1 Director, Office of Naval Research, Branch Office, 346 Broadway, New York 13, N.Y.
- 1 Officer in Charge, Office of Naval Research, Branch Office, London, England, Navy 100, Fleet Post Office, New York, N.Y.
- 1 Director, Office of Naval Research, Branch Office, 1030 E. Green Street, Pasadena 1, Calif.
- 1 Director, Office of Naval Research, Branch Office, 1000 Geary Street, San Francisco 9, Calif.
- 1 Director, Office of Naval Research, The John Crerar Library Building, Tenth Floor, 86 E. Randolph Street, Chicago 1, Ill.
- 1 Director, Office of Naval Research, Branch Office, 150 Causeway Street, Boston 14, Mass.
- 1 Commander, Operational Development Force, U.S. Atlantic Fleet, U.S. Naval Base, Norfolk 11, Va.
- 1 Commander, Portsmouth Naval Shipyard, Portsmouth, N.H.

Copies

- 1 Commander, Puget Sound Naval Shipyard, Bremerton, Wash.
- 1 Commander, Philadelphia Naval Shipyard (Code 263), Naval Base, Philadelphia 12, Pa.
- 1 Commander, Pearl Harbor Naval Shipyard (Code 264), Navy No. 128, Fleet Post Office, San Francisco, Calif.
- 1 Commander, New York Naval Shipyard, Material Laboratory, Technical Libraries (Code 158), Brooklyn 1, N.Y.
- 1 Commander, Boston Naval Shipyard (263a), Boston 29, Mass.
- 1 Commanding Officer, U.S. Navy Mine Countermeasures Station, Panama City, Fla.
- 1 Commanding Officer, Surface Anti-Submarine Development Detachment, U.S. Atlantic Fleet, Key West, Fla.
- 2 Commander, U.S. Naval Ordnance Laboratory, White Oak, Silver Spring 19, Md.
- 1 Director, U.S. Naval Research Laboratory, Anacostia, Washington 25, D.C.
- 2 Commander, U.S. Naval Ordnance Test Station, Pasadena Annex, 3202 E. Foothill Blvd., Pasadena, Calif.
- 2 Commander, U.S. Naval Ordnance Test Station, China Lake, Inyokern, Calif.
- 1 Commanding Officer and Director, U.S. Navy Electronics Laboratory, San Diego, Calif.
- 1 Commanding Officer and Director, U.S. Navy Underwater Sound Laboratory, Fort Trumbull, New London, Conn.
- 1 Director, U.S. Naval Engineering Experiment Station (Wave Mechanics), Annapolis, Md.
- 1 Commanding Officer, U.S. Naval Underwater Ordnance Station, Newport, R.I.
- 1 Superintendent, U.S. Naval Postgraduate School, Monterey, Calif.
- 1 Superintendent, U.S. Naval Academy, Annapolis, Md.
- 1 Chairman, Research and Development Board, Department of Defense Building, Washington 25, D.C.
- 1 The Director, Waterways Experiment Station, Corps of Engineers, U.S. Army, Vicksburg, Miss.
- 1 U.S. Atomic Energy Commission, Technical Information Service, 1901 Constitution Avenue, Washington 25, D.C. Attn: Chief Librarian
- 1 Chief Service Division, Armed Services Technical Information Agency, Document Service Center, Knott Building, Dayton 2, Ohio
- 1 Office of Secretary of Defense, Weapons Systems Evaluation Group, Department of Defense Building, Washington 25, D.C.
- 1 Director, National Bureau of Standards, Washington 25, D.C.

Copies

- 1 Commandant, U.S. Coast Guard Headquarters, 1300 E Street, N.W. Washington 25, D.C.
- 1 Chief, Division of Preliminary Design, Maritime Administration, Washington 25, D.C.
- 1 Director, U.S. Coast and Geodetic Survey, Washington 25, D.C.
- 1 Director, Woods Hole Oceanographic Institution, Woods Hole, Mass.
- 1 Commanding Officer, Transportation Research and Development Station, Fort Eustis, Va.
- 1 Director, Technical Information Branch, Aberdeen Proving Grounds, Aberdeen, Md.
- 1 Library of Congress, Technical Information Division, Washington 25, D.C.
- 2 Director, Langley Aeronautical Laboratory, Langley Air Force Base, Va.
- 1 Director, Aeronautical Research, National Advisory Committee for Aeronautics, 1724 F Street, N.W., Washington 25, D.C.
- 1 Director, Oak Ridge National Laboratory, P.O. Box P, Oak Ridge, Tenn.
- 1 Director, Applied Physics Division, Sandia Laboratory, Albuquerque, N.M., via Headquarters Field Commander, AFSWP
- Chief, Hydraulic Data Branch, Tennessee Valley Authority, Knoxville, Tenn.
- 1 Head, Technical Reference Section, U.S. Department of Interior, Bureau of Reclamation, Denver Federal Center, Denver, Colo.
- 1 Editor, Bibliography of Technical Reports, Office of Technical Services, U.S. Department of Commerce, Washington 25, D.C.
- 1 Editor, Technical Data Digest, Armed Services Technical Information Agency, Document Service Center, U.B. Building, Dayton 2, Ohio
- 1 Editor, Applied Mechanics Reviews, Midwest Research Institute, 4049 Pennsylvania, Kansas City 2, Mo.
- 1 Editor, Aeronautical Engineering Reviews, 2 East 64th Street, New York 21, N.Y.
- 1 Librarian, American Society of Mechanical Engineers, 20 West 39th Street, New York 18, N.Y.
- 1 Librarian, American Society of Civil Engineers, 33 West 39th Street, New York 18, N.Y.
- 1 Librarian, American Chemical Society, 1155 16th Street, N.W., Washington 6, D.C.
- 1 Technical Library, Consolidated Vultee Aircraft Corporation, San Diego 12, Calif.
- 1 Main Library, Carnegie Institute of Technology, Pittsburgh, Pa.
- 1 Editor, Engineering Index, 29 West 39th Street, New York 18, N.Y.

Copies

- 1 Librarian, Franklin Institute, Parkway at 20th Street, Philadelphia, Pa.
- 1 Technical Library, Glenn L. Martin Company, Baltimore 3, Md.
- 1 Technical Library, Grumman Aircraft Engineering Corporation, Bethpage, Long Island, N.Y.
- 1 Librarian, Institute of Aeronautical Sciences, 2 East 63rd Street, New York 21, N.Y.
- 1 Director, Armour Research Foundation, 35 West 33rd Street, Chicago 16, Ill.
- 1 Director, Alden Hydraulic Laboratory, Worcester Polytechnic Institute, Worcester 2, Mass.
- 2 Director, Applied Physics Laboratory, Johns Hopkins University, 8621 Georgia Avenue, Silver Spring, Md.
- 1 Head, Aeronautical Engineering Department, Catholic University, Washington, D.C.
- 1 Director, Cornell Aeronautical Laboratory, Cornell Research Foundation, Box 235, Buffalo 21, N.Y.
- 1 Director, Daniel Guggenheim Aeronautical Laboratory, California Institute of Technology, Pasadena 4, Calif.
- 1 Director, Fluid Mechanics Laboratory, Columbia University, New York 27, N.Y.
- 1 Director, Fluid Mechanics Laboratory, University of California, Berkeley 4, Calif.
- 1 Director, Hydraulic Laboratory, Carnegie Institute of Technology, Pittsburgh 13, Pa.
- 1 Director, Hydraulics Laboratory, University of Colorado, Boulder, Colo.
- 1 Director, Hydraulic Research Laboratory, University of Connecticut, Box U-37, Storrs, Conn.
- 1 Hydrodynamics Laboratory, Attn: Executive Committee, California Institute of Technology, 1201 E. California Street, Pasadena 4, Calif.
- 1 Director, Scripps Institute of Oceanography, University of California, La Jolla, Calif.
- 1 Department of Civil Engineering, Colorado A and M College, Fort Collins, Colo.
- 1 Director, Acoustics Laboratory, Harvard University, Cambridge, Mass.
- 1 Harvard University, School of Engineering, Cambridge, Mass.
- 1 Hudson Laboratory, Dobbs Ferry, New York, N.Y.
- 1 Head, Department of Aeronautical Engineering, Johns Hopkins University, Baltimore 18, Md.
- 1 Director, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pa.

Copies

- 1 Director, Acoustics Laboratory, Massachusetts Institute of Technology,
Cambridge 39, Mass.
- 1 Director, Department of Civil Engineering, Missouri School of Mines and Metal-
lurgy, Rolla, Mo.
- 1 Director, Experimental Naval Tank, Department of Naval Architecture and Marine
Engineering, University of Michigan. Ann Arbor, Mich.
- 1 Director, Institute for Fluid Dynamics and Applied Mathematics, University of
Maryland, College Park, Md.
- 1 Director, Midwest Research Institute, 4049 Pennsylvania, Kansas City 2, Mo.
- 1 Head, Department of Naval Architecture and Marine Engineering, Massachusetts
Institute of Technology, Cambridge 39, Mass., via INSMAT, Boston
- 1 Commanding Officer, U.S. Naval Administrative Unit, Massachusetts Institute of
Technology, Cambridge 39, Mass.
- 2 Chairman, Department of Aeronautical Engineering, New York University, New
York 53, N.Y. 1 for Consolidated Engineering Wind Tunnel
- 1 Director, Fluid Mechanics Laboratory, New York University, New York 53, N.Y.
- 1 Director, Institute for Mathematics and Mechanics, New York University, 45
Fourth Avenue, New York 53, N.Y.
- 1 Director of Research, The Technological Institute, Northwestern University,
Evanston, Ill.
- 1 Director, Robinson Hydraulic Laboratory, Ohio State University, Columbus, Ohio
- 1 Head, Department of Aeronautical Engineering and Applied Mechanics, Polytech-
nic Institute of Brooklyn, 99 Livingston Street, Brooklyn 2, N.Y.
- 1 Director, Hydraulics Laboratory, Pennsylvania State College, State College, Pa.
- 1 Department of Mechanical Engineering, Stanford University, Calif.
- 6 Director, Experimental Towing Tank, Stevens Institute of Technology, 711 Hud-
son Street, Hoboken, N.J.
- 1 Dean, School of Engineering, University of Texas, Austin, Texas
- 1 Director, Engineering Experiment Station, University of Tennessee, Knoxville
16, Tenn.
- 1 Chairman, Graduate Division of Applied Mathematics, Brown University, Provi-
dence, R.I.
- 1 Cornell Aeronautical Laboratory, Inc., 4455 Genesee Street, Buffalo 21, N.Y.
- 1 The Forrestal Research Center, Princeton University, Princeton, N.J.
Attn: Mr. Maurice H. Smith, Research Associate
- 1 Rensselaer Polytechnic Institute, Troy, N.Y. Attn: Librarian

Copies

- 1 New York State Maritime College, State University of New York, Fort Schuyler,
N.Y. Attn: Librarian
- .1 Administrator, Webb Institute of Naval Architecture, Crescent Beach Road, Glen
Cove, Long Island, N.Y.
- 1 Director, Hydraulic Laboratory, University of Wisconsin, Madison 6, Wis.
- 1 Director, Hydraulic Laboratory, University of Washington, Seattle 5, Wash.
- 2 Development Contract Administrator, Ordnance Research Laboratory, for
Director, Ordnance Research Laboratory, Pennsylvania State College,
State College, Pa.
- 1 Goodyear Aircraft Corporation, Akron, Ohio
- 1 Inspector of Machinery, USN, General Electric Company, Schenectady 1, N.Y.,
for General Electric Company
- 1 Supervisor of Shipbuilding, USN, and Naval Inspector of Ordnance, 118 E.
Howard Street, Quincy 69, Mass.
- 1 Supervisor of Shipbuilding, USN, and Naval Inspector of Ordnance, Bath Iron
Works Corporation, Bath, Maine
- 1 Supervisor of Shipbuilding, USN, and Naval Inspector of Ordnance, New York
Shipbuilding Corporation, Camden, N.J.
- 1 Director, Westinghouse Electric Corporation, Research Laboratories, E. Pitts-
burgh, Pa.
- 2 Newport News Shipbuilding and Dry Dock Company, Newport News, Va., for
distribution:
1 Senior Naval Architect
1 Supervisor, Hydraulic Laboratory
- 1 Reed Research, Inc., 1048 Potomac Street, N.W., Washington 7, D.C.
Attn: Librarian
- 1 Carbide and Carbon Chemical Company, UCC, K-25 Plant, P.O. Box P, Oak
Ridge, Tenn. Attn: Miss Evelyn Levine, Librarian
- 1 Vickers, Incorporated, 14544 Third Avenue, Highland Park 3, Mich.
- 1 Sparkman and Stephens, Inc., 11 E. 44th Street, New York, N.Y.
- 1 General Electric Company, River Works, 920 Western Avenue, W. Lynn, Mass.
Attn: Miss Ruth E. Butrick, Librarian, Accessory Turbine Bldg. 63 NG
- 1 Instrument Development Laboratories, Inc., 163 Highland Avenue, Needham
Heights 94, Mass. Attn: Mr. C.A. Logan, Engineer
- 1 General Motors Corporation, Research Laboratory Division, Detroit 2, Mich.
Attn: Miss Caroline Lutz, Librarian
- 1 Carter Oil Company, 1133 N. Lewis, Tulsa, Okla. Attn: Librarian

Copies

- 1 Hydrofoil Corporation, 69-A West Street, P.O. Box 861, Annapolis, Md.
- 1 Aeroprojects, Inc., 323 S. Matlack Street, W. Chester, Pa. Attn: Norma F.A. Fisher
- 1 Bell Telephone Laboratories, Inc., Murray Hill, N.J. Attn: Lera E. Smith, Librarian
- 1 Vitro Corporation, 962 Wayne Avenue, Silver Spring, Md.
- 1 Union Oil Company of California, Research Department, Wilmington, Calif.
- 1 Worthington Corporation, Harrison, N.J. Attn: Librarian
- 1 Chemical Engineering Progress Company, 120 E. 41st Street, New York 17, N.Y.
- 1 Shell Development Company, Emeryville, Calif. Attn: Thelma Hoffman, Librarian
- 1 Fairbanks Morse and Company, Beloit, Wis.
- 1 Eclipse-Pioneer Division of Bendix Aviation Corporation, Teterboro, N.J. Attn: Miss Mildred M. Baker.
- 1 Standard Oil Company (Indiana), Engineering Research Department, 910 S. Michigan Avenue, Chicago 80, Ill.
- 1 Babcock and Wilcox Company, Research and Development Department, Alliance, Ohio Attn: Librarian
- 1 Allis-Chalmers Manufacturing Corporation, Milwaukee, Wis. Attn: Mr. W.J. Rheingans
- 1 Rustless Iron and Steel Corporation, Baltimore, Md. Attn: Mr. J.M. Mousson
- 1 American Shipbuilding Company, Foot of West 54th Street, Cleveland, Ohio Attn: Mr. Kent C. Thornton, Naval Architect
- 1 Friede and Goldman, Inc., Naval Architects and Marine Engineers, 1420 National Bank of Commerce, New Orleans, La.
- 2 Prof. M.L. Albertson, Head of Fluid Mechanics Research, Colorado A and M College, Fort Collins, Colo., 1 for Prof. A. Yih
- 1 Prof. M.A. Abkowitz, Massachusetts Institute of Technology, Cambridge 39, Mass.
- 1 Mr. J.P. Breslin, Gibbs and Cox, 21 West Street, New York 6, N.Y.
- 1 Prof. G. Birkhoff, Department of Mathematics, Harvard University, Cambridge 38, Mass.
- 1 Prof. R.C. Binder, Department of Mechanical Engineering, Purdue University, Lafayette, Ind.
- 1 Prof. N.W. Conner, North Carolina State College, Raleigh, N.C.

Copies

- 1 Dr. E.O. Cooper, U.S. Naval Ordnance Test Station, 3202 E. Foothill Blvd.,
Pasadena Annex, Pasadena, Calif.
- 1 Dr. F.H. Clauser, Chairman, Department of Aeronautics, Johns Hopkins University,
Baltimore 18, Md.
- 1 VADM E.L. Cochrane, USN (Ret.), Member, Panel on the Hydrodynamics of Sub-
merged Bodies, Massachusetts Institute of Technology, Cambridge, Mass.
- 1 Prof. K.J. DeJuhasz, Pennsylvania State College, State College, Pa.
- 1 CAPT Walter S. Diehl, USN, (Ret.), Associate Member, Panel on the Hydrody-
namics of Submerged Bodies, 4501 Lowell Street, N.W. Washington 25, D.C.
- 1 Mr. Hollinshead de Luce, Chairman, Hydrodynamics Committee, c/o Bethlehem
Steel Company, Shipbuilding Division, Quincy 69, Mass.
- 1 Prof. R.A. Dodge, Engineering Mechanics Department, University of Michigan,
Ann Arbor, Mich.
- 1 Dr. D. Gunther, Head, Department of Mathematics, Cornell University, Ithaca,
N.Y.
- 1 Dr. D. Gilbarg, Department of Mathematics, Indiana University, Bloomington, Ind.
- 1 Prof. L.M. Grossman, College of Engineering, University of California, Berkeley
4, Calif.
- 1 Dr. N.J. Hoff, Polytechnic Institute of Brooklyn, Brooklyn 2, N.Y.
- 1 Mr. E.V. Hobbs, National Bureau of Standards, Washington, D.C.
- 1 Prof. W.S. Hamilton, Technical Institute, Northwestern University, Evanston, Ill.
- 1 Prof. A.D. Hay, School of Engineering, Princeton University, Princeton, N.J.
- 1 Dr. A.T. Ippen, Director, Hydrodynamics Laboratory, Department of Civil and
Sanitary Engineering, Massachusetts Institute of Technology, Cambridge
39, Mass.
- 1 Dr. A. Kantrowitz, Cornell University, Ithaca, N.Y.
- 1 Dr. G.H. Keulegan, National Hydraulic Laboratory, National Bureau of Standards,
Washington, D.C.
- 1 Prof. B.V. Korvin-Kroukovsky, Stevens Institute of Technology, Hoboken, N.J.
- 1 Dr. Th. von Kármán, 1051 South Marengo Street, Pasadena, Calif.
- 1 Inspector of Naval Material, 1206 South Santee Street, Los Angeles, Calif., for
Dr. R.T. Knapp, Hydrodynamics Laboratory, California Institute of Tech-
nology, Pasadena 4, Calif.
- 1 Dr. C. Kaplan, Langley Aeronautical Laboratory, Langley Air Force Base, Va.
- 1 Mr. C.A. Lee, Research and Development Laboratories, Kimberly-Clark Corp.,
Neenah, Wis.

Copies

- 1 Dr. C.C. Lin, Department of Mathematics, Massachusetts Institute of Technology, Cambridge 39, Mass.
- 1 Dr. L. Lees, Aeronautical Engineering Department, Princeton University, Princeton, N.J.
- 1 RADM P.F. Lee, USN (Ret.), Member, Panel on the Hydrodynamics of Submerged Bodies and Vice President, Gibbs and Cox, Inc., New York, N.Y.
- 1 Mr. Walter Daskin, Instructor, Mechanical Engineering Department, Johns Hopkins University, Baltimore, Md.
- 1 Prof. M.B. McPherson, Hydraulics Laboratory, Lehigh University, Bethlehem, Pa.
- 1 Prof. Walter L. Moore, Department of Civil Engineering, University of Texas, Austin 12, Texas
- 1 Prof. Carl E. Kindsvater, School of Civil Engineering, Georgia Institute of Technology, Atlanta, Ga.
- 1 Prof. Lydik S. Jacobsen, Department of Mechanical Engineering, Stanford University, Calif.
- 1 Prof. N.M. Newmark, College of Engineering, University of Illinois, 111 Talbot Laboratory, Urbana, Ill.
- 1 Dr. Harold Wayland, Hydrodynamics Laboratory, California Institute of Technology, Pasadena 4, Calif.
- 1 Prof. A. Hollander, Hydrodynamics Laboratory, California Institute of Technology, Pasadena 4, Calif.
- 1 Prof. Charles A. Shreeve, Department of Mechanical Engineering, University of Maryland, College Park, Md.
- 1 Dr. J. Younger, Head, Mechanical Engineering Department, University of Maryland, College Park, Md.
- 1 Dr. A. Miller, Bureau of Ordnance, Navy Department, Washington 25, D.C.
- 1 Mr. Richard M. Dailey, Virginia Technical Station, Box 3539, Blacksburg, Va.
- 1 Mr. W.L. Schneider, College of Engineering, New York University, University Heights, New York 53, N.Y.
- 1 Mr. A.C. Daskocil, Jr., 45-34 41st Street, Sunnyside 4, N.Y.
- 1 Mr. Arthur B. Laufer, Asst. Prof. of Physics, University of Missouri, Columbia, Mo.
- 1 Prof. Maxwell Gensamer, School of Mines, Columbia University, New York 27, N.Y.
- 1 Prof. John Kaye, Massachusetts Institute of Technology, Cambridge 39, Mass.
- 1 Mr. Paul F. Ruff, 13216 Thornhurst Avenue, Cleveland 5, Ohio

Copies

- 1 Mr. J.O. Maloney, Executive Director, University of Kansas, Research Foundation, Lawrence, Kansas
- 1 Mr. Carrol F. Merriman, 1404 Bolton Street, Baltimore, Md.
- 1 Dr. G.H. Hickox, University of Tennessee, Knoxville, Tenn.
- 1 Prof. L.J. Hooper, Alden Hydraulic Laboratory, Worcester Polytechnic Institute, Worcester, Mass.
- 1 Mr. R.E.B. Sharp, S. Morgan Smith Company, York, Pa.
- 1 Mr. John G. Waugh, 479 E. Del Mar Street, Pasadena 5, Calif.
- 1 Mr. Allyn C. Vine, Woods Hole Oceanographic Institution, Woods Hole, Mass.
- 1 Mr. R.L. Rather, Woods Hole Oceanographic Institution, Woods Hole, Mass.
- 1 Mr. Von Arx, Woods Hole Oceanographic Institution, Woods Hole, Mass.
- 1 Dr. J.H. McMillen, National Science Foundation, Washington, D.C.
- 1 Prof. J.W. Miles, University of California, Los Angeles, Calif.
- 1 Dr. A. May, Aerodynamics Division, U.S. Naval Ordnance Laboratory, White Oak, Silver Spring 19, Md.
- 1 Dr. George C. Manning, Prof. Naval Architecture, Massachusetts Institute of Technology, Cambridge 39, Mass.
- 1 RADM A.I. McKee, USN (Ret.), Member, Panel on the Hydrodynamics of Submerged Bodies, and Assistant General Manager, Electric Boat Company, Groton, Conn.
- 1 Prof. C.J. Peirce, Headquarters, USAFE, APO 633, c/o Postmaster, New York, N.Y.
- 1 Mr. J.B. Parkinson, Langley Aeronautical Laboratory, Langley Air Force Base, Va.
- 1 Dr. M.S. Plesset, Hydrodynamics Laboratories, California Institute of Technology, Pasadena 4, Calif.
- 1 Dr. H. Rouse, Director, Iowa Institute of Hydraulic Research, State University of Iowa, Iowa City, Iowa
- 1 Mr. Vito L. Russo, Chief, Division of Preliminary Design, U.S. Maritime Administration, Department of Commerce, Washington 25, D.C.
- 1 Dr. J.M. Robertson, Ordnance Research Laboratory, Pennsylvania State College, State College, Pa.
- 1 Dr. H.A. Wagner, U.S. Naval Air Missile Center, Ft. Mugu, Port Heuneme, Calif.
- 1 Prof. A. Weinstein, Department of Mathematics, University of Maryland, College Park, Md.

Copies

- 1 Dr. J.V. Wehausen, Executive Editor, Mathematical Reviews, 80 Waterman Street, Providence, R.I.
- 1 Inspector of Naval Material, 401 Water Street, Baltimore 2, Md., for Dr. G.F. Wislicenus, Mechanical Engineering Department, Johns Hopkins University, Baltimore 18, Md.
- 1 Prof. L.I. Schiff, Department of Physics, Stanford University, Calif.
- 1 Dr. L.G. Straub, Director, St. Anthony Falls Hydraulic Laboratory, University of Minnesota, Minneapolis 14, Minn.
- 1 Dr. V.L. Streeter, Director, Fundamental Fluid Research, Illinois Institute of Technology, Technology Center, Chicago 16, Ill.
- 1 Dr. F.B. Seely, Fluid Mechanics and Hydraulics Laboratory, University of Illinois, Urbana, Ill.
- 1 Dr. C.R. Soderberg, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge 39, Mass.
- 1 Dr. K.E. Schoenherr, College of Engineering, University of Notre Dame, Notre Dame, Ind.
- 1 Prof. W. Sears, Graduate School of Engineering, Cornell University, Ithaca, N.Y.
- 1 Dr. Stanley Corrsin, Department of Aeronautics, Johns Hopkins University, Baltimore 18, Md.
- 1 Dr. C.A. Truesdell, Department of Mathematics, University of Indiana, Bloomington, Ind.
- 1 Mr. F.L. Thompson, Member, Panel on the Hydrodynamics of Submerged Bodies and Director of Research, Langley Aeronautical Laboratory, Langley Air Force Base, Va.
- 1 Prof. J.K. Vennard, Director, Hydraulic Laboratory, Stanford University, Calif.
- 1 Mr. W. Vermeulen, General Electric Company, Schenectady, N.Y.
- 1 Prof. E.V. Laitone, University of California, Berkeley, Calif.
- 1 Dr. L. Trilling, Massachusetts Institute of Technology, Cambridge 39, Mass.
- 1 Mr. C.H. Hancock, Newport News Shipbuilding and Dry Dock Company, Newport News, Va.
- 1 Mr. J.P. Comstock, Newport News Shipbuilding and Dry Dock Company, Newport News, Va.
- 1 Mr. J.W. Thompson, Naval Architect (Design), New York Shipbuilding Corporation, Camden 1, N.J.
- 1 Pacific Aeronautical Library, 7660 Beverly Blvd., Los Angeles 36, Calif.
- 1 Technical Library, Lockheed Aircraft Corporation, 2555 N. Hollywood Way, Burbank, Calif.

Copies

- 1 Technical Library, McDonnell Aircraft Corporation, Box 516, St. Louis 3, Mo.
- 1 Technical Library, North American Aviation, Inc., 12241 Lakewood Blvd., Downey, Calif.
- 1 Technical Library, North American Aviation, Inc., El Segundo, Calif.
- 1 Technical Library, Northrop Aircraft Company, 1017 E. Broadway, Hawthorne, Calif.
- 1 Technical Library, Pratt and Whitney Aircraft Division, United Aircraft Corporation, East Hartford 8, Conn.
- 1 Librarian, Mechanics Research Library, Illinois Institute of Technology, Technology Center, Chicago 16, Ill.
- 9 British Joint Services Mission (Navy Staff), P.O. Box 165, Benjamin Franklin Station, Washington, D.C.
- 3 Canadian Joint Staff, 1700 Massachusetts Avenue, N.W., Washington 6, D.C.
- 1 Australian Scientific Liaison Office, 1800 K Street, N.W., Washington, D.C.
- 1 Director, Hydrodynamics Laboratory, National Research Council, Ottawa, Canada
- 1 Directeur, Laboratoire Dauphinois d'Hydraulique des Ateliers Neyrpic, Avenue de Beauvert, Grenoble (Isere), France
- 1 The University of Liverpool, Mathematical Institute, Department of Applied Mathematics, Grace Library, Liverpool, England
- 1 Laboratorium voor Scheepsbouwkunde, Technical University, Nieuwe Laan 76, Delft, The Netherlands. Attn: dr ir J. Balhan
- 1 Prof. K. Seetharamiah, Civil and Hydraulics Engineering Section, Indian Institute of Science, Bangalore 3, India
- 1 Prof. F. Numachi, Tohoku University, Gendai, Japan
- 1 Jan-Olof Traung, Naval Architect, Fisheries Development, Food and Agriculture Organization of United States, Viale delle Terme di Caracalla, Rome, Italy
- 1 Norges Tekniske Hogskole Hovedbiblioteket, Technical Institute of Norway, Trondheim, Norway
- 1 Spanish Naval Attaché, 1629 Columbia Road, N.W., Washington, D.C.
- 1 Dr. H.H. Rust, University of Hamburg, Jungiusstr. 4, (24a) Hamburg 36, Germany
- 1 Gen. Ing. U. Pugliese, Presidente, Istituto Nazionale per Studi ed Esperienze di Architettura Navale, via della Vasca Navale 89, Rome, Italy
- 1 Sr. M. Acevedo y Campoamor, Director, Canal de Experiencias Hidrodinámicas, El Pardo, Madrid, Spain
- 1 Prof. J. Ackeret, Institut für Aerodynamik der Eidgenössischen, Technischen Hochschule, Zurich, Switzerland

Copies

- 1 Prof. J.M. Burgers, Laboratorium Voor, Aero-En Hydrodynamica, Nieuwe Laan 76, Delft, The Netherlands
- 1 Capt. R. Brard, Directeur, Bassin d'Essais des Carenes, 6 Boulevard Victor, Paris XV, France
- 1 Prof. G.K. Batchelor, Trinity College, Cambridge University, Cambridge, England
- 1 Dr. J. Dieudonné, Directeur, Institut de Recherches de la Construction Navale, 1 Boulevard Haussmann, Paris (9e), France
- 1 Dr. S. Goldstein, Haifa Institute of Technology, Haifa, Israel
- 1 L. Escande, Ingenieur I.E.T., Professeur a la Faculte Des Sciences, Directeur, de l'Ecole Nationale Superieure, D'Electrotechnique et d'Hydraulique, 4 Boulevard Requit, Toulouse, France
- 1 Prof. T.H. Havelock, 8 Westfield Drive, Gosforth, Newcastle-on-Tyne 3, England
- 1 Prof. Harold Jeffreys, St. John's College, Cambridge, England
- 1 Prof. J. Kampe de Fariet, Faculte des Sciences, Universite de Lille, Lille (Nord) France
- 1 Prof. J.K. Lunde, Skipsmodelltanken, Tyholt, Trondheim, Norway
- 1 Dr. L. Malavard, Office National d'Etudes et de Recherches Aéronautique, 25 Avenue de la Division - LeClerc, Chatillon, Paris, France
- 1 Prof. H. Nordstrom, Director, States Skeppsprovvningsanstalt. Göteborg 24, Sweden
- 1 Dr. F.C. Roesler, Physikalisches Institut, Tech. Hochschule Graz, Rechbauerstrasse 12, Graz, Austria
- 1 Prof: L. Rosenhead, University of Liverpool, Liverpool, England
- 1 Prof. J.L. Synge, School of Theoretical Physics, National University of Ireland, Dublin, Ireland
- 1 Sir R.V. Southwell, 9 Lathburg Road, Oxford, England
- 1 Superintendent, Nederlandsh Scheepsbouwkundig Proefstation, Haagsteeg 2, Wageningen, The Netherlands
- 1 Prof. Dr. G. Weinblum, Ingenieur School, Berliner Tor 21, Z260, Hamburg, Germany
- 1 Mr. C. Wigley, 6-9 Charterhouse Square, London EC-1, England
- 1 Dr. J.F. Allan, Superintendent, Ship Division, National Physical Laboratory, Teddington, Middlesex, England
- 1 Director, British Shipbuilding Research Association, 5 Chesterfield Gardens, Curzon Street, London W.1, England
- 1 Admiralty Research Laboratory. Teddington, Middlesex, England

Copies

- 1 Editor, Bulletin of the British Hydromechanics Research Association, Netteswell Road, Harlow, Essex, England
- 1 Head, Aerodynamics Division, National Physical Laboratory, Teddington, Middlesex, England
- 1 Office of Scientific Attaché of Netherlands Embassy, 1470 Euclid Street, N.W., Washington, D.C.
- 1 Canadian National Research Establishment, Halifax, Nova Scotia, Canada
- 1 Armament Research Establishment, Fort Halstead, Hants, England

David W. Taylor Model Basin. Rept. 842.

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS OF CAVITATION - An Addendum to DTMB Report 712, On the Mechanism and Prevention of Cavitation, by Phillip Eisenberg. Washington, June 1953.

iii, 39 p. incl. figs., refs. (Rev. ed.)

UNCLASSIFIED

An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. Thus, the earlier discussions of the role of turbulence and of boundary layer effects in inception are extended. Some recent work on scale effects in inception is reviewed and some further thoughts on the influence of nuclei content are presented. Recent results of theoretical analysis by various writers on transient cavities including the effects of compressibility and viscosity are briefly

1. Cavitation
2. Flow
3. Turbulence
4. Boundary layer
- I. Eisenberg, Phillip
- II. NS 715-102

David W. Taylor Model Basin. Rept. 842.

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS OF CAVITATION - An Addendum to DTMB Report 712, On the Mechanism and Prevention of Cavitation, by Phillip Eisenberg. Washington, June 1953.

iii, 39 p. incl. figs., refs. (Rev. ed.)

UNCLASSIFIED

An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. Thus, the earlier discussions of the role of turbulence and of boundary layer effects in inception are extended. Some recent work on scale effects in inception is reviewed and some further thoughts on the influence of nuclei content are presented. Recent results of theoretical analysis by various writers on transient cavities including the effects of compressibility and viscosity are briefly

1. Cavitation
2. Flow
3. Turbulence
4. Boundary layer
- I. Eisenberg, Phillip
- II. NS 715-102

David W. Taylor Model Basin. Rept. 842.

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS OF CAVITATION - An Addendum to DTMB Report 712, On the Mechanism and Prevention of Cavitation, by Phillip Eisenberg. Washington, June 1953.

iii, 39 p. incl. figs., refs. (Rev. ed.)

UNCLASSIFIED

An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. Thus, the earlier discussions of the role of turbulence and of boundary layer effects in inception are extended. Some recent work on scale effects in inception is reviewed and some further thoughts on the influence of nuclei content are presented. Recent results of theoretical analysis by various writers on transient cavities including the effects of compressibility and viscosity are briefly

1. Cavitation
2. Flow
3. Turbulence
4. Boundary layer
- I. Eisenberg, Phillip
- II. NS 715-102

David W. Taylor Model Basin. Rept. 842.

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS OF CAVITATION - An Addendum to DTMB Report 712, On the Mechanism and Prevention of Cavitation, by Phillip Eisenberg. Washington, June 1953.

iii, 39 p. incl. figs., refs. (Rev. ed.)

UNCLASSIFIED

An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. Thus, the earlier discussions of the role of turbulence and of boundary layer effects in inception are extended. Some recent work on scale effects in inception is reviewed and some further thoughts on the influence of nuclei content are presented. Recent results of theoretical analysis by various writers on transient cavities including the effects of compressibility and viscosity are briefly

1. Cavitation
2. Flow
3. Turbulence
4. Boundary layer
- I. Eisenberg, Phillip
- II. NS 715-102

mentioned. Further remarks are included on steady-state cavities in real liquids, and recent theoretical works on this type of cavitation are cited. In particular, mention is made of a recent linearized theory by Tulin, which will be of considerable interest for technical applications. Some recent observations on the drag of cavitating bodies are summarized. Finally, it is observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.

mentioned. Further remarks are included on steady-state cavities in real liquids, and recent theoretical works on this type of cavitation are cited. In particular, mention is made of a recent linearized theory by Tulin, which will be of considerable interest for technical applications. Some recent observations on the drag of cavitating bodies are summarized. Finally, it is observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.

mentioned. Further remarks are included on steady-state cavities in real liquids, and recent theoretical works on this type of cavitation are cited. In particular, mention is made of a recent linearized theory by Tulin, which will be of considerable interest for technical applications. Some recent observations on the drag of cavitating bodies are summarized. Finally, it is observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.

mentioned. Further remarks are included on steady-state cavities in real liquids, and recent theoretical works on this type of cavitation are cited. In particular, mention is made of a recent linearized theory by Tulin, which will be of considerable interest for technical applications. Some recent observations on the drag of cavitating bodies are summarized. Finally, it is observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.

David W. Taylor Model Basin. Rept. 842.

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS OF CAVITATION - An Addendum to DTMB Report 712, On the Mechanism and Prevention of Cavitation, by Phillip Eisenberg. Washington, June 1953.

iii, 39 p. incl. figs., refs. (Rev. ed.)

UNCLASSIFIED

An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. Thus, the earlier discussions of the role of turbulence and of boundary layer effects in inception are extended. Some recent work on scale effects in inception is reviewed and some further thoughts on the influence of nuclei content are presented. Recent results of theoretical analysis by various writers on transient cavities including the effects of compressibility and viscosity are briefly

1. Cavitation
2. Flow
3. Turbulence
4. Boundary layer
- I. Eisenberg, Phillip
- II. NS715-102

David W. Taylor Model Basin. Rept. 842.

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS OF CAVITATION - An Addendum to DTMB Report 712, On the Mechanism and Prevention of Cavitation, by Phillip Eisenberg. Washington, June 1953.

iii, 39 p. incl. figs., refs. (Rev. ed.)

UNCLASSIFIED

An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. Thus, the earlier discussions of the role of turbulence and of boundary layer effects in inception are extended. Some recent work on scale effects in inception is reviewed and some further thoughts on the influence of nuclei content are presented. Recent results of theoretical analysis by various writers on transient cavities including the effects of compressibility and viscosity are briefly

1. Cavitation
2. Flow
3. Turbulence
4. Boundary layer
- I. Eisenberg, Phillip
- II. NS715-102

David W. Taylor Model Basin. Rept. 842.

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS OF CAVITATION - An Addendum to DTMB Report 712, On the Mechanism and Prevention of Cavitation, by Phillip Eisenberg. Washington, June 1953.

iii, 39 p. incl. figs., refs. (Rev. ed.)

UNCLASSIFIED

An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. Thus, the earlier discussions of the role of turbulence and of boundary layer effects in inception are extended. Some recent work on scale effects in inception is reviewed and some further thoughts on the influence of nuclei content are presented. Recent results of theoretical analysis by various writers on transient cavities including the effects of compressibility and viscosity are briefly

1. Cavitation
2. Flow
3. Turbulence
4. Boundary layer
- I. Eisenberg, Phillip
- II. NS715-102

David W. Taylor Model Basin. Rept. 842.

A BRIEF SURVEY OF PROGRESS ON THE MECHANICS OF CAVITATION - An Addendum to DTMB Report 712, On the Mechanism and Prevention of Cavitation, by Phillip Eisenberg. Washington, June 1953.

iii, 39 p. incl. figs., refs. (Rev. ed.)

UNCLASSIFIED

An attempt has been made to present a more unified outline of the various cavitation processes and to indicate, approximately, the progress on the various topics in this outline. Thus, the earlier discussions of the role of turbulence and of boundary layer effects in inception are extended. Some recent work on scale effects in inception is reviewed and some further thoughts on the influence of nuclei content are presented. Recent results of theoretical analysis by various writers on transient cavities including the effects of compressibility and viscosity are briefly

1. Cavitation
2. Flow
3. Turbulence
4. Boundary layer
- I. Eisenberg, Phillip
- II. NS715-102

mentioned. Further remarks are included on steady-state cavities in real liquids, and recent theoretical works on this type of cavitation are cited. In particular, mention is made of a recent linearized theory by Tulin, which will be of considerable interest for technical applications. Some recent observations on the drag of cavitating bodies are summarized. Finally, it is observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.

mentioned. Further remarks are included on steady-state cavities in real liquids, and recent theoretical works on this type of cavitation are cited. In particular, mention is made of a recent linearized theory by Tulin, which will be of considerable interest for technical applications. Some recent observations on the drag of cavitating bodies are summarized. Finally, it is observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.

mentioned. Further remarks are included on steady-state cavities in real liquids, and recent theoretical works on this type of cavitation are cited. In particular, mention is made of a recent linearized theory by Tulin, which will be of considerable interest for technical applications. Some recent observations on the drag of cavitating bodies are summarized. Finally, it is observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.

mentioned. Further remarks are included on steady-state cavities in real liquids, and recent theoretical works on this type of cavitation are cited. In particular, mention is made of a recent linearized theory by Tulin, which will be of considerable interest for technical applications. Some recent observations on the drag of cavitating bodies are summarized. Finally, it is observed that perhaps the greatest progress in this field in recent years has been in the recognition and more clear definition of the problems requiring investigation.