TECHNICAL REPORT

SPLINE INTERPOLATION ALGORITHMS
FOR TRACK-TYPE SURVEY DATA WITH
APPLICATION TO THE COMPUTATION OF
MEAN GRAVITY ANOMALIES

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A B S T R A C T

Cubic spline interpolation is a mathematical procedure which is an analog of the draftsman's plastic spline. The advantage of this interpolation procedure over the more commonly used methods such as Lagrange lies in its ability to not only fit each given data value exactly but to maintain continuity of the first and second derivatives.

The relative accuracy of the cubic spline interpolation procedure for generating gridded data values and estimates of mean gravity anomalies from track-type geophysical surveys is shown to be excellent when applied to properly designed surveys. Techniques for interpreting the two-dimensional Fourier transform in terms of track spacing, track orientation, and down-track sampling rate are presented to demonstrate the effect of these parameters on interpolation accuracy. A procedure for utilizing closed form integration of the bicubic spline surface to produce mean gravity anomalies is shown to yield accuracies comparable to the method of averaging cubic spline grid values.

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FOREWORD

The U. S. Naval Oceanographic Office, as part of its mission to provide deflection of the vertical information in ocean areas, is currently engaged in studies concerning the accuracy of computing deflections of the vertical from gravity survey data. The deflection of the vertical calculation is the last step in a process involving many stages; consequently, the overall accuracy will be dependent on the accuracy of each individual stage. This report is concerned with the interim step of determining mean anomalies for use in the Vening Meinesz formulation. Utilization of the numerical method presented in this report is expected to result in a significant improvement in the accuracy of interpolating mean anomalies from original survey data.

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INTRODUCTION

Many of the analytical techniques which are currently in use in geophysics involve some form of mathematical operation on digitized survey data. The two-dimensional character of some of these techniques requires that the survey data, usually collected along tracks, be reduced to values of the surveyed parameters on an equally spaced grid. Examples of these techniques include upward and downward continuation, derivative calculations, regional trend removal, two-dimensional trend filtering, model studies, deflection of the vertical and geoidal undulation calculations, and automated contouring techniques. The interpolation procedures which are discussed in this report are applicable to any measurements obtained by track-type surveys. Emphasis, however, is placed on the problem of gridding gravity survey data and the determination of mean values of the gravity field within specified areas.

In general there are two approaches available for gridding track-type survey data. One approach is that of fitting a least-squares surface, usually of polynomial form, to all of the available survey data within a predetermined area and using the resulting formula to generate the required grid values. Examples of this procedure, applied to irregularly spaced data, may be found in Crain and Bhattacharyya (1967) and Krumbein (1959). The advantages of this least-squares method are that it tends to operate as a low-pass filter, which reduces the effects of noisy or erroneous data and it can be applied directly to unequally-spaced data points. The chief disadvantages are:

1. The wavelengths of the features which will be accurately fit is a function not only of the degree of the polynomial but also of the size of the area which is selected. This causes some difficulty in controlling the technique in a production type operation.

2. The method is quite sensitive to round-off errors and, except in cases where orthogonalization techniques are used, can become unstable for high degree surfaces. Recent work by Wampler (1969) discusses the problems associated with inverting ill-conditioned matrices using standard procedures.

3. The technique requires a rather large amount of computer time, although this problem can be minimized through the utilization of recursive linear-regression techniques (Fagin, 1964) when additional data points are added to the set.

The second general approach which is used to grid survey data is based on interpolation techniques which fit each data value exactly. In this case, it is assumed that any large errors or noise in the data have been removed prior to application of the gridding process, and that the survey coverage is sufficient to minimize aliasing in the data. Historically, this approach has consisted of connecting each pair of data points in such a way as to partition the survey area into non-overlapping triangular sections (Rankin, 1963) and fitting a plane to each of these sections, or by partitioning the survey area into a preliminary rectangular grid and interpolating within each of these areas by an exact-fitting two-dimensional polynomial (Crain and Bhattacharyya, 1967). The principal disadvantage of these methods is that no conditions are imposed on the derivative of the interpolating functions. Except in very smooth areas, this results in the generation of non-realistic gradients. The problem is somewhat reduced by the use of a two-dimensional weighting function of the type developed by Shepard (1968). Shepard's technique generates the data value at a desired grid location by applying a weighting function based upon the distance and direction to the neighboring data points. In addition, estimates of the derivatives at each data point are included. This removes the requirement that the partial derivatives be zero at each data point thus allowing the interpolating function to have extrema anywhere and not just at the original data points.

In order to overcome some of the difficulties associated with these existing techniques and to effectively utilize the computational efficiency afforded by track-type survey data, an interpolation procedure was developed by Bhattacharyya (1969) which makes use of cubic and bicubic splines. The advantage of the cubic spline method, which is a mathematical analog of the draftsman's plastic spline, lies in its ability to not only fit the observed data values but to maintain continuity of the first and second derivatives. This paper concerns the development and testing of a cubic spline technique as applied to the computation of mean gravity anomalies. By utilizing simulated survey data obtained from a model gravity field, it is shown that the spline technique will produce accurate estimates of the mean gravity anomalies provided the original survey is properly designed. The use of a two-dimensional Fourier transform for a solution to the survey design problem is also presented.

DEVELOPMENT OF THE CUBIC SPLINE ALGORITHM FOR GRIDDING TRACK TYPE SURVEY DATA

The algorithm presented here is based upon the technique developed by Bhattacharyya (1969) with some modifications to the spline formulation in order to use the boundary conditions and development presented by Pennington (1965).

Given data values $y_1 y_2 \dots y_m$ located at the points $x_1, x_2 \dots x_m$ where $x_k < x_{k+1}$, we first assume that the second derivative of $f(x)$ is a linear function of x between any pair of adjacent data points. Thus, if $z_1, z_2 \dots z_m$ are the values of the second derivative of $f(x)$ at the given data points then for the interval $x_k \leq x \leq x_{k+1}$ we have

$$f''(x) = \frac{(x - x_k)(z_{k+1} - z_k)}{d_k} + z_k \quad (1)$$

where $d_k = x_{k+1} - x_k$.

This is simply the equation for a straight line passing through the points (x_k, z_k) and (x_{k+1}, z_{k+1}) .

Integrating (1) once yields

$$f'(x) = \frac{-z_k(x_{k+1} - x)^2}{2d_k} + \frac{z_{k+1}(x - x_k)^2}{2d_k} + c_1 \quad (2)$$

where c_1 is the constant of integration.

Integrating (1) a second time yields

$$f(x) = \frac{z_k(x_{k+1} - x)^3}{6d_k} + \frac{z_{k+1}(x - x_k)^3}{6d_k} + c_1 x + c_2. \quad (3)$$

With the requirement that $f(x)$ match the observed data at the points x_k and x_{k+1} , the constants c_1 and c_2 may be evaluated from equation (3).

Thus, at the point x_k ,

$$y_k = f(x_k) = \frac{z_k d_k^2}{6} + c_1 x_k + c_2 \quad (4)$$

and at the point x_{k+1} ,

$$y_{k+1} = f(x_{k+1}) = \frac{z_{k+1} d_k^2}{6} + c_1 x_{k+1} + c_2. \quad (5)$$

Solving equations (4) and (5) for the values of c_1 and c_2 , and substituting these into equation (3), yields the following formula for the cubic spline in the interval $x_k \leq x \leq x_{k+1}$,

$$f(x) = \frac{z_k (x_{k+1} - x)^3}{6d_k} + \frac{z_{k+1} (x - x_k)^3}{6d_k} + \frac{(y_{k+1} - y_k) x}{6d_k} - \frac{(z_{k+1} - z_k) x d_k}{6} + \frac{(y_k x_{k+1} - y_{k+1} x_k)}{d_k} - \frac{(z_k x_{k+1} - z_{k+1} x_k) d_k}{6} \quad (6)$$

In order to solve equation (6), the second derivatives (z_k, z_{k+1}) must be determined at the end points of the interval. This is accomplished by requiring that the first derivative, equation (2), be continuous at each given data point, i.e.,

$$\lim_{x \rightarrow x_k} f'(x) = f'(x_k).$$

This means that when equation (2) is written for the intervals (x_{k+1}, x_k) and (x_k, x_{k+1}) , the two values obtained for $f'(x_k)$ must agree. Thus, from equation (2), we have for the interval (x_{k+1}, x_k) ,

$$f'(x_k) = \frac{z_k (x_k - x_{k+1})^2}{2d_{k+1}} + \frac{(y_k - y_{k+1})}{d_{k+1}} - \frac{(z_k - z_{k+1}) d_{k+1}}{6} \quad (7)$$

and for the interval (x_k, x_{k+1}) ,

$$f'(x_k) = -\frac{z_k (x_{k+1} - x_k)^2}{2d_k} + \frac{(y_{k+1} - y_k)}{d_k} - \frac{(z_{k+1} - z_k) d_k}{6} \quad (8)$$

By equating (7) and (8), the formula for determining the values of the unknown second derivatives becomes

$$\frac{z_{k+1} d_{k+1}}{6} + \frac{z_k (d_{k+1} + d_k)}{3} + \frac{z_{k+1} d_k}{6} = \frac{(y_{k+1} - y_k)}{d_k} - \frac{(y_k - y_{k+1})}{d_{k+1}} \quad (9)$$

Now as k takes on the values $2 \rightarrow m-1$, equation (9) will produce $m-2$ equations in the m unknowns $(z_1, z_2 \dots z_m)$. In order to solve this system, two additional equations are needed. These additional equations are obtained through utilization of an appropriate set of boundary conditions. There are several reasonable boundary conditions which may be selected. Bhattacharyya (1969) used the assumption that the second derivative is zero at the points (x_1, x_m) . In the present development we will use the boundary conditions proposed by Pennington (1965). These conditions state that the second derivatives at the two end points are a linear extrapolation of those associated with the adjoining interval. Thus, from equation (1) we have

$$z_{k-1}'' = f''(x_{k-1}) = \frac{(x_{k-1} - x_k)(z_{k+1} - z_k)}{d_k} + z_k. \quad (10)$$

For $k = 2$ this reduces to

$$\frac{-z_1}{d_1} + z_2 \left(\frac{1}{d_2} + \frac{1}{d_1} \right) - \frac{z_3}{d_2} = 0, \quad (11)$$

and for $k = m-1$ we have

$$\frac{-z_{m-2}}{d_{m-2}} + z_{m-1} \left(\frac{1}{d_{m-2}} + \frac{1}{d_{m-1}} \right) - \frac{z_m}{d_{m-1}} = 0. \quad (12)$$

Combining equations (11) and (12) with the $m-2$ equations obtained from (9) for $k = 2, 3 \dots m-1$, yields a set of m equations in m unknowns $(z_1, z_2 \dots z_m)$. With the exception of one additional element in the first and last row, this set of equations forms a tridiagonal matrix which is easily solved using the standard Gauss reduction method. In matrix form, the set is given by $AZ = B$ where A is $m \times m$, Z is $m \times 1$ and B is $m \times 1$ thus,

$$A = \begin{bmatrix} -\frac{1}{d_1} & \left(\frac{1}{d_1} + \frac{1}{d_2}\right) & -\frac{1}{d_2} & 0 & \dots & 0 \\ \frac{d_1}{6} & \frac{(d_1 + d_2)}{d_3} & \frac{d_2}{6} & 0 & \dots & 0 \\ 0 & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \frac{d_{m-2}}{6} & \frac{(d_{m-2} + d_{m-1})}{3} & \frac{d_{m-1}}{6} & 0 \\ 0 & \dots & \frac{d_{m-2}}{6} & \frac{d_{m-2} + d_{m-1}}{3} & -\frac{1}{d_{m-1}} & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix}, \text{ and } B = \begin{bmatrix} 0 & & & & \\ \frac{(y_3 - y_2)}{d_2} & -\frac{(y_2 - y_1)}{d_1} & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{(y_m - y_{m-1})}{d_m} & -\frac{(y_m - y_{m-1})}{d_{m-1}} & & & \\ 0 & & & & \end{bmatrix}$$

Utilizing this solution in equation (6) yields an interpolation formula which fits each given data value exactly, has continuous first and second derivatives and is a simple cubic polynomial in x within the interval between each pair of data points.

Since the cubic spline is a function of only one independent variable, the data obtained along a survey track must be adjusted to lie on a straight line. The interpolation may then be accomplished by utilizing distance along track as the independent variable. The procedure presented by Bhattacharyya

(1969) is followed for this operation. The steps in this technique are as follows:

1. Taking the data from one track at a time, the position of the data points are converted into x , y coordinates and a least-squares straight line is fitted to these locations. Since there is no statistical significance attached to this operation, either x or y may be considered the independent variable. The computer program listed in the appendix considers x , or equivalently longitude, as the independent variable. If the survey tracks happen to run exactly north-south, the program should be modified to consider y as the independent variable. With the coordinates of the data points given as $(x_i, y_i, i = 1, N)$, the least squares line is $f(x) = a_0 + a_1 x$ where

$$a_0 = \frac{\left(\sum_i y_i \right) \left(\sum_i x_i^2 \right) - \left(\sum_i x_i y_i \right) \left(\sum_i x_i \right)}{N \left(\sum_i x_i^2 \right) - \left(\sum_i x_i \right)^2} \quad (13)$$

and

$$a_1 = \frac{\left(\sum_i y_i \right) - N a_0}{\left(\sum_i x_i \right)} \quad (14)$$

2. The perpendicular distance between the straight line and each data point is determined and utilized to project the points orthogonally onto the line with an adjusted data value determined by applying an estimate of the local gradient, assumed to be a function of distance only. If (x_k, y_k) are the coordinates, and V_k the value of the K th data point, then the perpendicular distance between this point and the least squares line is given by

$$B_k = \left| \frac{y_k - a_1 x_k - a_0}{(a_1^2 + 1)^{1/2}} \right| \quad (15)$$

The mapped coordinates (x_k^*, y_k^*) of this point on the least-squares straight line are given by

$$x_k^* = \frac{a_0 - \frac{x_k}{a_1} + y_k}{-(\frac{1}{a_1} + a_1)} \quad (16)$$

and

$$y_k^* = a_0 + a_1 x_k^* . \quad (17)$$

If B_k is less than a predetermined pivot distance, the value associated with the data point is unchanged. If B_k is greater than this pivot distance then the adjusted value V_k^* associated with the mapped coordinates (x_k^*, y_k^*) is computed by

$$V_k^* = \frac{(V_k - V_i) \left[(y_k^* - y_i^*)^2 + (x_k^* - x_i^*)^2 \right]^{1/2}}{\left[(y_k - y_i)^2 + (x_k - x_i)^2 \right]^{1/2}} + V_i \quad (18)$$

with $i = \left\{ \begin{matrix} k+1 \\ k-1 \end{matrix} \right\}$ as appropriate.

In the computer program for the cubic spline algorithm contained in the appendix, the pivot distance is selectable via a control card. This distance is usually set equal to the maximum distance which one would be willing to move a data point without changing its value. In order to minimize the propagation of the error associated with the assumption that the gradient correction in equation (18) is independent of direction, continuous survey tracks which deviate appreciably from a straight line should be broken up into smaller segments with each segment treated as a separate track.

The mapped coordinates and adjusted data values may now be considered as irregularly-spaced digital samples from a function with the independent

variable being distance along track from some arbitrary starting point, and the dependent variable being of course the adjusted data values.

3. Utilizing the mapped data, the cubic spline, equation (6), is determined for each track. This equation may then be used to interpolate data values at the intersections of the straight least-squares track lines and a set of parallel lines whose spacing is equal to the desired final grid spacing. If the direction of the survey tracks is predominately east-west then the direction of the set of parallel lines is north-south. Similarly, for north-south tracks, the lines are run east-west. The computer program, contained in the appendix, is designed to operate on tracks in any direction except exactly north-south. The direction of the set of parallel grid lines is controlled by the direction of track line number one. Since the track number designation is arbitrary, this feature allows the user to determine the desired orientation (N-S or E-W) of the parallel grid lines in order to obtain as many intersections as possible.

4. The interpolated data values generated in step 3 may be regarded as unequally-spaced digital samples from a function whose independent variable is distance along each of these parallel lines. Application of the spline procedure in this cross-track direction produces the final interpolated values at the desired grid points. If mean anomalies are desired, grid points are generated at one-half the final grid spacing and the resulting nine points are averaged to produce the mean value for each grid cell.

THE BICUBIC SPLINE ALGORITHM FOR THE GENERATION OF MEAN ANOMALIES

In the previous development (Step 4) the mean value of the function is obtained by averaging the nine interpolated values located within the region in which the mean is desired. The accuracy of this method will, of course, depend on the spacing of the points used in the averaging process relative to

the frequency content of the measured field. We now consider an alternative approach of determining mean anomalies which utilizes the bicubic spline method of de Boor (1962). In this approach, a two-dimensional cubic polynomial surface is obtained by use of the bicubic spline, and the mean value is generated directly by integration. The algorithm presented here is based on de Boor's development, modified to utilize the preceding cubic spline formulation and to compute mean anomalies by integration of the resulting bicubic surface. The development of the algorithm proceeds as follows.

Assume that we have available a set of data points with coordinates (x_i, y_j) , $i = 0 \dots I$, $j = 0 \dots J$, and with data value U_{ij} at the intersections of a rectangular grid covering an area R (Figure 1).

FIGURE 1. THE R_{ij} GRID OF DATA VALUES FOR INPUT INTO THE BICUBIC SPLINE ALGORITHM.

Utilizing this data set with an appropriate set of boundary conditions specified around the perimeter of R , a two-dimensional cubic polynomial of the form

$$U(x,y) = R_{ij}(x,y) = \sum_{m=0}^3 \sum_{n=0}^3 \alpha_{mn}^{ij} (x - x_{i-1})^m (y - y_{j-1})^n \quad (19)$$

where $x_{i-1} \leq x \leq x_i$ and $y_{j-1} \leq y \leq y_j$,

may be determined for each R_{ij} rectangle within R . In analogy with the cubic spline, the unknowns α_{mn}^{ij} for this two-dimensional bicubic surface are computed by requiring that the surface fit each data point exactly, i.e., $R(x_i, y_j) = U_{ij}$, and that the partial derivatives $\frac{\partial U(x,y)}{\partial x}$ and $\frac{\partial U(x,y)}{\partial y}$ be continuous. In order to accomplish this, the first step is to compute estimates of the partial derivatives at each data point by utilizing the cubic spline formulation. Let

$$P_{ij} = \frac{\partial U_{ij}}{\partial x}, \quad q_{ij} = \frac{\partial U_{ij}}{\partial y}, \quad \text{and} \quad S_{ij} = \frac{\partial^2 U_{ij}}{\partial x \partial y} = \frac{\partial P_{ij}}{\partial y}.$$

In order to compute P_{ij} at each grid point, equations (9,11,12) are utilized to obtain values of the second derivative z_j from the U_{in} data values along the n th row. Equation (8) is then used with these z_j to produce values for P_{in} , $i=0 \dots l-1$ with P_{ln} being computed from equation (7). This process is repeated for each row of input data to generate the full P_{ij} matrix. In identical fashion, q_{ij} and S_{ij} are computed by operating on columns of U_{ij} and P_{ij} respectively. Utilizing the given data values and these computed derivatives at each grid point, the unknown, α_{nm}^{ij} , may be determined for each R_{ij} rectangle covering R by solving the matrix equation

$$A(x_i - x_{i-1}) K_{ij} A^T (y_j - y_{j-1}) = (\alpha_{mn}^{ij}) \quad (20)$$

The proof for this equation is given in de Boor (1962).

Normally, the U_{ij} are given on a square grid, in which case we let $h=(x_i - x_{i-1})=(y_j - y_{j-1})$. The elements of the matrices in equation (20) are then

$$A(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{h^2} & \frac{-2}{h} & \frac{3}{h^2} & -\frac{1}{h} \\ \frac{2}{h^3} & \frac{1}{h^2} & \frac{-2}{h^3} & \frac{1}{h^2} \end{bmatrix}, K_{ij} = \begin{bmatrix} U_{i-1,j-1} & q_{i-1,j-1} & U_{i-1,j} & q_{i-1,j} \\ p_{i-1,j-1} & s_{i-1,j-1} & p_{i-1,j} & s_{i-1,j} \\ U_{i,j-1} & q_{i,j-1} & U_{i,j} & q_{i,j} \\ p_{i,j-1} & s_{i,j-1} & p_{i,j} & s_{i,j} \end{bmatrix}$$

$$\text{and } \alpha_{mn}^{ij} = \begin{bmatrix} \alpha_{00}^{ij} & \alpha_{03}^{ij} \\ \alpha_{30}^{ij} & \alpha_{33}^{ij} \end{bmatrix}$$

Substituting the α_{mn}^{ij} elements into equation (19) produces a two-dimensional cubic polynomial formula representing the data within each of the R_{ij} subareas covering R . The mean value of the field $(\bar{C}_{ij})$, within the R_{ij} rectangle, is computed from equation (19) by

$$\bar{C}_{ij} = \frac{1}{h^2} \int_{x_{i-1}}^{x_i} \int_{y_{i-1}}^{y_i} \sum_{m=0}^3 \sum_{n=0}^3 \alpha_{mn}^{ij} (x - x_{i-1})^m (y - y_{i-1})^n dy dx. \quad (21)$$

The bicubic spline computer program given in the appendix contains the option of evaluating this integral between arbitrary limits within each R_{ij} rectangle. This feature allows computation of the mean anomalies at a grid spacing either equal to that of the U_{ij} input data, or one-half or one-third the spacing of the input data. Setting the (x_{i-1}, y_{i-1}) origin equal to $(0,0)$,

and letting the limits of integration be $C \leq x \leq D$ and $A \leq y \leq B$, and with $x_{\max} = y_{\max} = h$, integration of equation (21) yields

$$\bar{c}_{ij} = \frac{1}{(B-A)(D-C)} \sum_{m=0}^3 \sum_{n=0}^3 \frac{\alpha_{mn}^{ij}}{(n+1)(m+1)} (D^{m+1} - C^{m+1})(B^{n+1} - A^{n+1}). \quad (22)$$

The computer program for the bicubic spline algorithm uses this formula to determine estimates of the mean anomaly on a specified grid interval.

GENERATION OF MODELS FOR TESTING THE ACCURACY OF INTERPOLATION PROCEDURES

Two essential criteria for determining the accuracy of an interpolation technique are (1) the existence of some reference surface with which the interpolated values can be compared, and (2) all errors due to sources unrelated to the interpolation process be minimal. We are concerned with testing interpolation methods for use with data collected by systematic surveys. For this specific application, the above criteria can be met by properly designed simulated surveys of a mathematically-derived reference surface. Proper design implies that the survey parameters viz., track spacing, track orientation, and down-track sampling interval be chosen so as to define the essential characteristics of the reference surface. In the following material we discuss the theory and application of the fast Fourier transform for the design of surveys. In addition, the method of determining the reference surface for use in obtaining quantitative estimates of interpolation accuracy is given.

A. The Two-Dimensional Fast Fourier Transform as a Tool for Simulated Survey Design

Simulated surveys must be designed in such a way as to minimize aliasing of the spectral components of the data. At the present time, the sampling theory that we have available is based primarily on the Shannon

sampling theorem. This theorem states that a continuous band-limited function of time may be reconstructed from equally spaced digitized samples provided that you have at least two samples per shortest period. The reconstruction is accomplished by convolving the digital values with a Sinc function

$$\left(\frac{\sin x}{x} \right) \text{ i.e.,}$$

$$f(t) = \sum_{n=-\infty}^{\infty} f\left(\frac{n}{2A}\right) \frac{\sin 2\pi A \left(t - \frac{n}{2A}\right)}{2\pi A \left(t - \frac{n}{2A}\right)} \quad (23)$$

where the band limits are $\pm 2\pi A$ and the sampling rate is $2A$. See Papoulis (1962) for a proof of this theorem. Although it is theoretically impossible for a function of finite length to be band limited, a reasonably accurate estimate of A may be obtained for a small area by application of a two-dimensional Fourier transform.

In one dimension, the direct Fourier transform of an arbitrary function, $g(x)$, is defined as

$$G(f_x) = \int_{-\infty}^{\infty} g(x) e^{-i 2\pi f_x x} dx, \quad (24)$$

with f_x equal to the frequency. By means of this formula, it is possible to transform information from the space domain into the frequency domain. $G(f_x)$ is, in general, a complex number of the form $a(f_x) + ib(f_x)$ or equivalently $(a^2 + b^2)^{1/2} e^{i \arctan \frac{b}{a}}$ where $(a^2 + b^2)^{1/2} = |G(f_x)|$ is called the amplitude spectrum and $\arctan \frac{b}{a} = \theta(f_x)$ is termed the phase spectrum. For our application of the Fourier transform, we will be primarily concerned with the amplitude spectrum. In the context in which it is used here, the term frequency refers to spatial frequency with the units being either cycles per unit distance or normalized to cycles per data interval.

For a two-dimensional function $g(x,y)$, equation (24) is expanded to

$$G(f_x, f_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) e^{-i 2\pi(f_x x + f_y y)} dx dy. \quad (25)$$

With $g(x,y)$ consisting of digitized values on a uniform grid covering a finite area R , an estimate of $G(f_x, f_y)$ may be obtained for the normalized frequency range $0 \leq f_x, f_y \leq 0.5$ cycles/data interval, by use of the discrete two-dimensional fast Fourier transform (FFT) of Cooley and Tukey (1965). This discrete approximation to $G(f_x, f_y)$ must be interpreted to provide estimates of the three survey parameters; track spacing, sampling rate, and track orientation. The specification of track orientation requires a knowledge of what happens to trends or lineations in the (x,y) domain when they are mapped into the (f_x, f_y) plane. It has been determined by Fuller (1967) that a trend in the (x,y) plane maps into a trend in the (f_x, f_y) plane in an orthogonal direction. Fuller shows this simply in the following way.

Let $g(x,o)$ be an arbitrary function of x defined for $y = o$. Using this definition, we can construct a function of both x and y , i.e., $g(x,y)$, by shifting $g(x,o)$ a distance Sy in the x direction, with $S = \frac{\Delta x}{\Delta y}$ as shown in Figure 2.

FIGURE 2. THE FUNCTION $g(x,y)$.

We will use what is usually called the "shift theorem" from transform theory which states that, if $G(f_x)$ is the Fourier transform of $g(x)$ then, from equation (24), $e^{-i2\pi f_x A} G(f_x)$ is the Fourier transform of $g(x-A)$. With $A = Sy$, this theorem is used to integrate out the x dependence in equation (25) resulting in the equation

$$F(f_x, f_y) = \int_{-Y}^Y G(f_x) e^{-i2\pi y(f_x S + f_y)} dy \quad (26)$$

and, by symmetry,

$$F(f_x, f_y) = 2G(f_x) \int_0^Y \cos [2\pi y(f_x S + f_y)] dy. \quad (27)$$

Integrating over y yields

$$F(f_x, f_y) = 2G(f_x) \frac{\sin [2\pi Y(f_x S + f_y)]}{2\pi (f_x S + f_y)} \quad \text{or}$$

$$F(f_x, f_y) = 2Y G(f_x) \text{Sinc} [2\pi Y (f_x S + f_y)] . \quad (28)$$

Since the Sinc function has its maximum value when the argument is zero, equation (28) defines a function in the (f_x, f_y) plane possessing a trend with $S = \frac{-f_y}{f_x}$. Thus we see that, since trends in the (x, y) plane are mapped into trends in the (f_x, f_y) plane in the orthogonal direction, survey track direction may be determined from the two-dimensional FFT by aligning the tracks parallel to the trends in the (f_x, f_y) plane. With this direction specified, the shape of the two-dimensional amplitude spectrum is used in conjunction with the Shannon sampling theorem to estimate the band limits of $g(x, y)$ which, in turn, define the required track spacing and sampling rate for the survey. A general two-dimensional FFT computer program utilizing equally spaced gridded data and a fast Fourier

transform subroutine called NLOGN, written by Robinson (1967), is contained in the appendix. This program was used to design the simulated surveys for testing the cubic spline algorithms.

B. Development of a Point-Mass Model Field

To provide a reference surface for determining quantitative estimates of interpolation accuracy and to ensure that the data used in the interpolations are error free, gravity model data were generated from a random distribution of point masses as follows.

Consider a point (P) on the surface of a sphere (Figure 3) with spherical coordinates (θ, λ) , and a point mass located at depth d beneath the surface with coordinates (θ', λ') . The gravitational potential at P(θ, λ) due to the point mass is

$$V_P = \frac{-km}{r} \quad (29)$$

where m is mass, k is the universal gravitational constant, and r is the distance between the point mass and the point (P).

FIGURE 3. GEOMETRY FOR COMPUTING THE GRAVITATIONAL POTENTIAL DUE TO A POINT MASS.

For a sphere of radius R , the distance r is

$$r = (R^2 + R'^2 - 2RR' \cos \psi)^{1/2}$$

where $R' = R - d$

and

$$\cos \psi = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\lambda' - \lambda) .$$

The radial (vertical component) $\vec{g}_R$ of the gravitational force at P is

$$\vec{g}_R(\theta, \lambda) = - \frac{\partial V_P}{\partial R} = \frac{km}{r^3} (R - R' \cos \psi) .$$

For a finite number (N) of point masses situated at various depths and locations beneath the surface of the sphere, we can generate gravity anomalies on the sphere by summing the radial components due to the N point masses, i.e.,

$$\vec{g}_R(\theta, \lambda) = \sum_{i=1}^N \frac{km_i}{r_i^3} (R - R'_i \cos \psi_i) . \quad (30)$$

A computer program to calculate the radial, easterly, and northerly components, in addition to the deflection of the vertical components with respect to a spherical earth of a set of point masses is given in the appendix.

COMPARISON OF GRIDDING TECHNIQUES AND THE COMPUTATION OF MEAN ANOMALIES USING MODEL DATA

The accuracy of the spline interpolation method, relative to several other techniques, is now considered in terms of gridding survey data and the computation of mean gravity anomalies by utilizing simulated survey data.

A set of gravity anomalies, for use as a reference surface, were calculated from equation (30), at one-minute intervals, over an area of 80×80 square minutes. The anomalies, given in Figure 4, result from 16 point

FIGURE 4. CONTOUR CHART OF THE GRAVITY ANOMALIES GENERATED BY 16 POINT MASSES WITH CONTOUR INTERVAL OF 5 mgls.

masses located at depths ranging from 7 to 12 kilometers beneath the surface of a spherical earth. Amplitudes of the anomalies range from -17 mgls to +83 mgls. For purposes of comparing the various models and residual charts to follow, a transparency of Figure 4 is contained in a pocket attached to the rear cover of this report.

A. Gridding of Survey Data

To design a simulated survey for obtaining data to test the interpolation procedures, a two-dimensional FFT was computed using all of the model field data (Figure 4) available on a one-minute grid interval. Contours of the amplitude spectrum resulting from this computation are shown in Figure 5. Since only the shape of the spectrum is used to design the simulated survey, the contour values are arbitrarily normalized, and the frequency range is normalized to the units of cycles per data interval. Utilizing the result embodied in equation (28), Figure 5 shows that the trends in the model field are predominately in the NE-SW direction indicating that the preferred survey track direction is NW-SE. With the track direction specified, the flattening of the amplitude spectrum at a frequency of approximately 0.17 cycles per data interval in both the along-track and cross-track directions indicates that a track spacing and sampling interval of three nautical miles would be sufficient to minimize aliasing of the data. Figure 6 shows the simulated survey, designated as model 1, which was designed from this interpretation of the two-dimensional amplitude spectrum. By overlaying this survey with the transparent copy of Figure 4, it is apparent that no attempt was made to sample the peak values of the anomalies. Simulated deviation of the survey vehicle from the planned survey tracks was included, but measurement errors were excluded by computing the exact value of the gravity field at each survey point by using equation (30). The cubic spline algorithm was utilized with this survey data to compute interpolated values of the gravity field at one minute grid intervals. The interpolated

FIGURE 5. THE TWO-DIMENSIONAL FOURIER TRANSFORM OF THE ENTIRE 80 X 80 MINUTE MODEL FIELD WITH ONE DATA INTERVAL EQUAL TO ONE MINUTE AND CONTOURS IN NORMALIZED UNITS.

FIGURE 6. MODEL 1 - SIMULATED SURVEY TRACKS BASED ON THE FOURIER TRANSFORM OF THE 80 X 80 MINUTE MODEL FIELD. NOMINAL TRACK SPACING AND SAMPLING INTERVAL EQUAL TO 3 nm.

value at each grid point was then subtracted from the true value obtained via equation (30) and the result was machine contoured at a contour interval of 0.5 mg/l. This residual contour chart is shown in Figure 7. Figure 8 is a histogram of the model-1 errors with the standard deviation and range noted.

In order to compare the accuracy of the spline interpolation to that which could be achieved through the use of a two-dimensional least-squares polynomial surface, a twelfth-degree surface was fitted to the simulated survey data obtained from model 1. The contoured field values generated by this surface are shown in Figure 9. Comparison of this surface with the true field values in Figure 4 indicates that the least-squares surface reproduces the general shape of the low-frequency components but does not depict the higher frequency features. As stated previously, the least-squares surface can be made to fit the high frequency components by reducing the size of the area over which the surface is computed but this is not considered to be an appropriate approach to interpolation when adequate survey data is available.

By overlaying Figure 7 with the contour chart of the model field, it is readily apparent that the errors in the interpolated values are correlated with those regions of the model field which contain a large amount of energy at the higher frequencies. This correlation is interpreted as arising from the fact that the two-dimensional FFT is essentially a least-squares operation which produces an estimate of the average amplitude spectrum over the entire region in which the computation is carried out. This effect, coupled with the obvious non-stationarity (in space) of the model field, caused the amplitude spectrum to flatten out at an unrealistically low frequency (0.17 cycles/data interval). In turn, this led to an estimate of track spacing and sampling interval which was too large to adequately define the higher frequencies. In order to determine the effect of a shorter sampling interval, a second model, designated model 2, was constructed with track spacing and direction similar to model 1 but with a sample interval of 1 nm. The histogram of the errors resulting from the

FIGURE 7. CONTOUR CHART OF MODEL-1 POINT-VALUE RESIDUALS (SPLINE INTERPOLATION VS. TRUE VALUE) WITH A CONTOUR INTERVAL OF 0.5 mg/l.

FIGURE 8. HISTOGRAM FOR MODEL-1 SPLINE POINT-VALUE RESIDUALS.

FIGURE 9. CONTOURS OF THE 12° LEAST-SQUARES POLYNOMIAL SURFACE GENERATED FROM MODEL-1 DATA. CONTOUR INTERVAL IS 5 mgls. (SURFACE COMPUTED FROM PROGRAM SUPPLIED BY GULF RESEARCH AND DEVELOPMENT COMPANY.)

interpolated values determined from this survey is shown in Figure 10. It is apparent that only a slight decrease in error was achieved by use of the increased sampling rate. In order to more effectively reduce this error, the FFT was computed for that anomalous portion of the model field contained within 10' and 70' latitude and 0' and 60' longitude. Contours of the normalized amplitude spectrum for this part of the model field are shown in Figure 11. With an estimate of 0.25 cycles per data interval as the frequency at which the spectrum for this smaller area flattens out, a track spacing and sampling interval of 2 nm was selected. Although the lack of a definite trend in the high frequency components indicates that the track direction is not a particularly significant factor for this model field, the distinct trending of the low-frequency components indicates that some control of this factor is required. The model 3 simulated survey, based on the amplitude spectrum in Figure 11, is shown in Figure 12. Figure 13 is a contour of the differences between the spline interpolated data values obtained from the model-3 survey and the true values from the model field. The histogram of these errors, as shown in Figure 14, indicates that the model-3 survey produced a significant decrease in the overall errors in the interpolated data values.

In order to determine whether the model-3 errors were caused by the cubic spline interpolation algorithm or by the simulated survey design, a two-dimensional, high-pass filter was applied to the one-minute grid values of the model field. This digital filter, which was designed by Fuller (1967) as a general high-pass filter, has the two-dimensional amplitude response shown in Figure 15. The weight function for this filter is symmetric, thus, no phase shifting is involved in the operation. The amplitude response is such that those frequencies which were not adequately sampled with the model-3 survey are retained while the lower frequencies are removed. The contoured result of this filtering operation is shown in Figure 16. By comparing this result with the residual contours in Figure 13, it is apparent that at least some of the errors in the interpolated values from the model-3 survey are generated by locally inadequate sampling.

FIGURE 10. HISTOGRAM FOR MODEL 2 SPLINE POINT-VALUE RESIDUALS.

FIGURE 11. THE TWO-DIMENSIONAL FOURIER TRANSFORM OF THE MODEL FIELD FROM LONGITUDE 0' TO 60' AND LATITUDE 10' TO 70' WITH ONE-DATA INTERVAL EQUAL TO ONE MINUTE AND CONTOURS IN NORMALIZED UNITS.

FIGURE 12. SIMULATED SURVEY TRACKS BASED ON THE FOURIER TRANSFORM OF THE 60 X 60 MINUTE MODEL FIELD. NOMINAL TRACK SPACING AND SAMPLING INTERVAL EQUAL TO 2 nm.

FIGURE 13. CONTOUR OF MODEL-3 POINT-VALUE RESIDUALS (SPLINE INTERPOLATION VS. TRUE VALUE) WITH A CONTOUR INTERVAL OF 0.5 mg/l.

FIGURE 14. HISTOGRAM FOR MODEL-3 SPLINE POINT-VALUE RESIDUALS.

FIGURE 15. AMPLITUDE RESPONSE OF THE TWO-DIMENSIONAL HIGH-PASS FILTER APPLIED TO THE 80 X 80 MINUTE MODEL FIELD. CONTOURS ARE IN NORMALIZED UNITS.

FIGURE 16. CONTOURS OF POINT-MASS ANOMALY DATA AFTER APPLICATION OF HIGH-PASS FILTER. CONTOUR INTERVAL IS 0.1 mgls.

These tests lead to the conclusion that, given an adequate track type survey design based on the frequency characteristics of the data, the cubic spline algorithm is a reliable interpolation technique. The computational efficiency of the algorithm is demonstrated by the fact that the 81×81 grid values interpolated from the model-1 survey required only one minute and ten seconds of computer time on a CDC-3800.

As a qualitative test of the cubic spline gridding procedure applied to actual shipboard survey data, the algorithm was used to grid total magnetic intensity data obtained from a well-controlled marine magnetic survey consisting of E-W tracks spaced nominally 3 nm apart. Figure 17 is the hand-drawn contour chart of this area and Figure 18 is the machine contoured result using the cubic spline generated grid points. Aside from the fact that the cubic spline generated what is considered to be a more realistic position for the peaks of the anomalies, the two contour charts are nearly identical. The computer time required to generate the gridded data and plotter tape was less than three minutes on a CDC 3800 with an actual plotter time of twenty minutes. This is compared to an estimate of approximately 100 man-hours to plot the data values and produce the hand-drawn contour chart.

B. Mean Gravity Anomaly Calculations

In order to test the algorithms for computing mean gravity anomalies, a survey was designed which simulated a conventional shipboard gravity survey. This survey plan, designated model 4, consists of E-W tracks spaced 3 nm apart with a sampling interval of $1/3$ nm. Figure 19 is a plot of this survey showing the random deviation from the planned track which was added to simulate a normal survey track. Figure 20 is a histogram of the errors in the interpolated data points resulting from the model-4 survey. Comparing this result with that obtained from the model-3 survey (Figure 14) indicates that the increased sampling rate (six times that of model 3) was sufficient to overcome the effect of increasing

TOTAL MAGNETIC INTENSITY CONTOUR CHART

FIGURE 17. HAND-DRAWN CONTOURS OF TOTAL-MAGNETIC INTENSITY FROM A SHIPBOARD MAGNETIC SURVEY WITH E-W TRACKS SPACED 3 NM APART AND CONTOUR INTERVAL OF 50 GAMMAS.

FIGURE 18. MACHINE DRAWN CONTOURS OF TOTAL-MAGNETIC INTENSITY GENERATED BY THE CUBIC SPLINE ALGORITHM FROM ORIGINAL SURVEY DATA WITH CONTOUR INTERVAL OF 50 GAMMAS.

FIGURE 19. MODEL 4 – SURVEY PLAN APPLIED TO POINT-MASS MODEL DATA TO SIMULATE A NORMAL SHIPBOARD GRAVITY SURVEY, i.e., TRACK DIRECTION E-W, TRACK SPACING 3 nm, SAMPLING INTERVAL $1/3$ nm.

FIGURE 20. HISTOGRAM FOR MODEL-4 SPLINE POINT-VALUE RESIDUALS.

the track spacing and surveying at an angle to the trends in the data. Obviously this will not be true in the general case, since the effect of changing any of the survey design parameters will be controlled by the characteristics of the field being measured.

Survey models 3 and 4 were used to produce data for testing three algorithms for computing mean gravity anomalies. The technique for generating mean anomalies using both a nine point averaging of the gridded data produced by the cubic spline and the two-dimensional integration of the bicubic spline surface have been discussed previously. In addition to these methods, a technique was tested which utilized a one-dimensional Lagrange interpolation (see e.g., Hamming, 1962). This procedure generated mean gravity anomalies on a uniform grid by averaging all of the survey data points falling within a grid interval to form a mean for that interval. Following this, a one-dimensional fourth-degree Lagrange interpolation was applied to these mean values to obtain an estimate of the mean gravity value for each of the grid cells for which no survey data was available. The results of these mean anomaly tests are shown in Figures 21-27. Figure 21 is a contour chart of the differences between the true mean anomalies computed for each one-minute square of the model field via equation (30), and the mean values computed by averaging the $1/2$ minute grid values produced by applying the cubic spline procedure to the survey data from model 3. Figure 22 is a contour chart showing the differences between the true one-minute means and those generated by application of the Lagrange procedure to the model-3 data. The contour interval is the same in these two figures. The errors produced by the lack of control of the derivatives in the Lagrange interpolation method is apparent in Figure 22. It is probable that these errors could be reduced somewhat by using a third-degree Lagrange polynomial instead of the fourth-degree but, in light of the obvious advantages of the spline formulation, no additional testing of this approach was undertaken.

FIGURE 21. CONTOURS OF MEAN-ANOMALY RESIDUALS GENERATED FROM MODEL-3 SURVEY BY UTILIZATION OF A CUBIC SPLINE INTERPOLATION PROCEDURE WITH CONTOUR INTERVAL OF 0.5 mgl.

FIGURE 22. CONTOURS OF MEAN ANOMALY RESIDUALS GENERATED FROM THE MODEL-3 SURVEY BY UTILIZATION OF A LAGRANGE INTERPOLATION PROCEDURE WITH CONTOUR INTERVAL OF 0.5 mgl.

Figure 23 compares the histograms of the errors generated by the cubic spline and Lagrange algorithms. Figures 24–26 show that similar results are obtained with the cubic spline and Lagrange methods when they were applied to a conventional shipboard gravity survey simulated by the test model 4 previously described.

In order to determine if the increased computing time necessary for producing mean anomalies by integrating the bicubic spline surface was justified, the algorithm was applied to the one-minute grid values generated by application of the cubic spline to the model-3 survey data. A histogram of the errors resulting from this technique is shown in Figure 27. A comparison of Figures 23 and 27 clearly indicates that the unique capabilities of the bicubic spline approach are wasted in so far as the computation of mean anomalies is concerned.

CONCLUSIONS

After evaluating the results obtained from this series of tests, the following conclusions may be drawn:

1. The cubic spline algorithm for gridding track-type survey data and for computing mean gravity anomalies is accurate and computationally efficient in comparison with the other techniques which were tested.

2. The significant improvement achieved by the model-3 survey in comparison to the model-1 survey illustrates the concept of the sampling theorem embodied in equation (23), i.e., the accuracy of mathematical interpolation, and in particular the spline method, is highly dependent on the density of the survey data points relative to the frequency content of the anomalies. The results obtained by models 3 and 4 show that, if a survey is well designed, highly accurate interpolation may be anticipated by use of the spline procedure. Implicit in this conclusion is the requirement that the survey data itself be of high quality.

FIGURE 23. MODEL 3 HISTOGRAMS SHOWING COMPARISON OF ACCURACIES OF SPLINE AND LAGRANGE ALGORITHMS FOR COMPUTING MEAN GRAVITY ANOMALIES.

FIGURE 24. CONTOURS OF MEAN ANOMALY RESIDUALS GENERATED FROM MODEL-4 SURVEY BY UTILIZATION OF A CUBIC SPLINE INTERPOLATION PROCEDURE WITH CONTOUR INTERVAL OF 0.5 mgal.

FIGURE 25. CONTOURS OF MEAN ANOMALY RESIDUALS GENERATED FROM MODEL-4 SURVEY BY UTILIZATION OF A LAGRANGE INTERPOLATION PROCEDURE WITH CONTOUR INTERVAL OF 0.5 mgal.

FIGURE 26. MODEL-4 HISTOGRAMS SHOWING COMPARISON OF ACCURACIES OF SPLINE AND LAGRANGE ALGORITHMS FOR COMPUTING MEAN GRAVITY ANOMALIES.

FIGURE 27. MODEL-3 HISTOGRAM SHOWING ACCURACY OF THE BICUBIC SPLINE ALGORITHM FOR COMPUTING MEAN GRAVITY ANOMALIES.

3. The results obtained by computing mean anomalies using a two-dimensional integration of the bicubic spline surface do not justify the additional computing time required for this technique.

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APPENDIX A

FORTRAN PROGRAM FOR GRIDDING AND COMPUTATION OF
MEAN ANOMALIES USING THE CUBIC SPLINE


```

PROGRAM SPLINT
C
C PROGRAM TO GRID RANDOM TRACK TYPE SURVEY DATA WITH OPTION TO
C COMPUTE MEAN ANOMALIES, GRIDDING METHOD IS ONE DIMENSIONAL CUBIC
C SPLINES, REF, BHATTACHARYYA, GEOPHYSICS V34, NO. 3, JUNE 69. MEAN
C ANOMALIES COMPUTED BY AVERAGING 9 PTS. IN EACH GRID INTERVAL
C POSITION OF MEAN FOR EACH GRID CELL IS THE UPPER LEFT HAND
C CORNER OF THE CELL
C
C NOTE=INPUT DATA IS ASSUMED TO BE ERROR FREE. ALAT, ALONG=LOCATION
C OF ORIGIN IN DEGREES AND TENTHS, SET AT LOWER LEFT CORNER, GRID=
C DESIRED GRID INTERVAL IN MINUTES, PLAT, PLONG= COORDINATES(DEGREES
C AND TENTHS) OF UPPER RIGHT HAND CORNER, MEAN=BLANK, NO MEAN ANOMALY
C COMPUTED, =1-MEAN COMPUTED AT CENTER OF EACH GRID, NTRK=TRACK NUMBER
C FROM 1 TO ITOT, LAST CARD IN EACH TRACK IS BLANK, 1ST CARD IS NTRK
C CARD, NUMBER TRACKS IN ANY ORDER ACROSS AREA, PROGRAM REQUIRES
C SPLINE, SPLICEN AND SORTY SUBROUTINES
C
C LAT, LONG OF INPUT DATA IS IN DEG, MIN, SEC(FORMAT 70) OR IN
C DECIMAL X,Y (FORMAT 90)
C
C IF ITYPE=BLANK GRID WILL BE EQUAL INTERVALS OF LAT AND LONG, IF
C ITYPE=1 GRID WILL BE IN MERCATOR PROJECTION
C
C ITYPE=-1 GRID, ORIGIN AND LIMITS OF INPUT DATA ARE IN X,Y UNITS
C WITH X= LONG AND Y= LAT, PIVOT= MAX DISTANCE(X-Y UNITS) FROM
C TRACK FOR PIVOT TEST
C
C REF, FOR CUBIC SPLINE IS PENNINGTON, R, H, INTRODUCTORY COMPUTER
C METHODS FOR NUMERICAL ANALYSIS, MACMILLAN, 1965
C
C EXCEPT FOR ICT, VARIABLES WITH 1 DIMENSION SHOULD BE DIMENSIONED
C TO THE MAX NO. OF POINTS ON ANY LINE OR THE MAX NO. OF GRID
C INTERVALS IN THE LONGEST DIMENSION OF THE AREA WHICHEVER IS
C GREATER, IN THE MAIN PROGRAM, VARIABLES WITH 2 DIMENSIONS
C HAVE THE 2ND DIMENSION = TO ITOT AND THE 1ST DIMENSION = TO THE
C MAX NO. OF GRID INTERVALS IN THE LONGEST DIMENSION OF THE AREA,

```



```

C THE LARGEST DIMENSION OF 2 DIMENSIONAL VARIABLES IN THE
C SUBROUTINES SHOULD BE AT LEAST EQUAL TO THE SIZE OF THE 1
C DIMENSIONAL VARIABLES IN THE MAIN PROGRAM,
C DIMENSION X(660),Y(660),Z(660),DIST(660),AX(660),AY(660),RX(086,
130),RY(066,30),BZ(086,30),IC(30),AZ(660),AVE(660)
C READ IN CONTROL CARD AND CONVERT ORIGIN AND LIMIT TO X=Y
READ (60,10),ALAT,ALONG,PLAT,PLONG,GRID,MEAN,ITOT,ITYPE ,PIVOT
10 FORMAT(5F10.0,I1,I2,I2,I2,F5.0)
IF (ITYPE)91,71,72
71 BLAT= ALAT *60.0
BLONG=ALONG *60.0
PLAT=(PLAT*60.0)=BLAT
PLONG=BLONG *(PLONG*60.0)
CHANGE APPROPRIATE SIGN IF AREA IS IN EAST LONG OR SOUTH LAT
GO TO 31
91 BLAT=ALAT
BLONG=ALONG
PLAT=PLAT-BLAT
PLONG=PLONG-BLONG
GO TO 31
72 BLAT=ALAT *.0174533
AP*.705398
TEMP=SIN(AP+BLAT/2.0)/COS(AP+BLAT/2.0)
BLAT=7915.7045*ALOG10(TEMP)-23.268932*SIN(BLAT)
BLONG=ALONG*60.0
PLAT=PLAT*.0174533
TEMP=SIN(AP+BLAT/2.0)/ COS(AP+BLAT/2.0)
PLAT=(7915.7045*ALOG10(TEMP)-23.268932*SIN(PLAT))-BLAT
PLONG=BLONG-PLONG*60.0
READ TRACK NO,AND DATA ,CONVERT TO X-Y
31 READ(60,20) NTRK
20 FORMAT(I2)
ATER= 9999.99

```

```

I=1
99 IF (ITYPE) 92,3,3
92 READ(60,90) HLONG,HLAT,Z(1)
90 FORMAT(3F20,0)
   HLAT= HLAT* .00029089
   GO TO 93
3 READ(60,70) DLAT,ELAT,FLAT,ELONG,ELONG,FLONG,Z(1)
70 FORMAT(2(F4,0,F4,0,F2,0),F10,0)
   HLAT=((FLAT/60,0)+ELAT+DLAT*60,0)*.00029089
   HLONG=(FLONG/60,0)+FLONG+ELONG*60,0
C
CHECK IF LAST CARD THIS TRACK
93 IF (HLAT+HLONG) 2,4,2
2 X(1)=BLONG-HLONG
  IF (ITYPE) 94,95,95
94 X(1)= -1,0*X(1)
95 IF(ITYPE) 73,73,75
73 Y(1)=(HLAT/.00029089)=BLAT
   GO TO 74
75 TEMP= SIN(AP+HLAT/2,0)/ COS(AP+HLAT/2,0)
   Y(1)=(7915,7045*ALOG10(TEMP)-23,268932*SIN(HLAT))=BLAT
74 I=1*1
   GO TO 99
C
NOTE PROGRAM ASSUMES LONGITUDE IS WEST, CHANGE STATEMENT 2 IF
C DESIRED ,NOW FIT LEAST SQUARES LINE TO POSITIONS
4 N=1*1
3006 AN=N
   A=0,0
   B=0,0
   C=0,0
   D=0,0
   DO 5 I=1,N
     A=A+X(I)
     B=B+X(I)**2

```

```

C=C*Y(I)
5 D=B*Y(I)*X(I)
  A1=(C*B-D*A)/(A*B+A**2)
  A2=(C-A1*A*N)/A
  IF (ABS(A2),LT,0.00001) A2 = A2 + 0.00001
  WRITE (61,101) NTRK,A1,A2
101 FORMAT(* TRACK NO *,I3,* TRACK LINE IS Y=,F8.2,*,*,F8.2,*X*)
C LEAST SQUARES LINE IS Y=A1+A2X
C NOW SEARCH FOR A POINT LESS THAN PIVOT DISTANCE FROM TRACK LINE
C TO USE FOR 1ST PIVOT AND MAP PTS. ONTO TRACK WITH CORRECT Z VALUE
  DO 6 I=1,N
6 DIST(I)=ABS((Y(I)-A2*X(I)-A1)/(SQRT(A2**2+1.0)))
  DO 7 I=1,N
7 IF (DIST(I) .LT. PIVOT) 8,8.7
  7 CONTINUE
8 A3= -1.0/A2
  AX(I)=(A1+A3*X(I)-Y(I))/(A3-A2)
  AY(I)= A2*AX(I) +A1
  IA=I+1
C NOW WORK BACKWARDS ON TRACK TO PICK UP POINTS THAT FAILED
C PIVOT TEST
11 J=I-1
  IF (J)12,12,9
9 DELZ=(Z(J)-Z(I))/SQRT((X(I)-X(J))**2 +(Y(I)-Y(J))**2)
  AX(J)=(A1+A3*X(J)-Y(J))/(A3-A2)
  AY(J)=A2*AX(J)+A1
  Z(J)=(DELZ *SQRT((AX(I)-AX(J))**2+(AY(I)-AY(J))**2))+Z(I)
  I=J
  GO TO 11
C NOW WORK FORWARD ON TRACK TO PICK UP REMAINING PTS.
12 DO 13 I=1A,N
  IF (DIST(I) .LT. PIVOT) 14,14,15
14 AX(I)=(A1+A3*X(I) -Y(I))/(A3-A2)

```

```

      AY(I) = A2*AX(I) +A1
      GO TO 13
15  J = I+1
      DELZ = (Z(I)-Z(J))/SQRT((X(I)-X(J))**2 + (Y(I)-Y(J))**2)
      AX(I) = (A1+A3*X(I)-Y(I))/(A3-A*2)
      AY(I) = A2*AX(I) +A1
      Z(I) = (DELZ *SQRT((X(I)-X(J))**2 + (AY(I)-AY(J))**2))*Z(J)
13  CONTINUE
      WRITE(61,103) PIVOT      Y      Z      INPUT DATA MAPPED ONTO TRAC
103  FORMAT(* X
1K PIVOT DISTANCE =*,F4.1,*UNITS*)
      IF THESE DATA ARE DESIRED REMOVE C FROM NEXT CARD
      WRITE(61,102) (AX(I),AY(I),Z(I), I=1,N)
102  FORMAT(3F10.1)
      AT THIS POINT WE HAVE MAPPED ALL INPUT PTS. ONTO TRACK WITH
      CORRECT Z VALUES, NOW USE CUBIC SPLINE TO INTERPOLATE FOR GRID PTS.
      INDEPENDENT VARIABLE IS DISTANCE DOWN TRACK FROM 1ST PT.
      CHECK QUADRANT TO DETERMINE IF INTERPOLATION IS IN X OR Y
      DIRECTION, THIS IS CONTROLLED BY TRACK NO.1 IN STATEMENT 32 AND 30
      IF INTRK=1)83,83,30
      A4=A2
      83  A4=A2
      121  IF(MEAN) 122,30,122
      122  GRID=GRID/2.0
      30  IF(ABS(A4)=1.0)16,16,17
      16  DELD=SQRT((A2*GRID)**2 *GRID **2)
      KX=AX(1)/GRID
      AKX=KX
      AKX=AKX*GRID
      AKY=A1+A2*AKX
      START=SQRT((AX(1)=AKX)**2 + (AY(1)=AKY)**2)
      IF(AX(2)-AX(1))88,88,21
      21  IF(AX(1))98,98,22
      22  START =DELD*START

```

```

      AKX=AKX*GRID
      AKY=A1+A2*AKX
      SIGN=1,0
      GO TO 18
19  DELD=SQRT((GRID/A2)**2 +GRID**2)
      KY=AY(1)/GRID
      AKY=KY
      AKX=AKX*GRID
      AKY=AKY*GRID
      AKX=(AKY-A1)/A2
      START=SQRT((AX(1)-AKX)**2 + (AY(1)-AKY)**2)
      IF (AY(2)-AY(1))88,23,23
23  IF (AY(1)-98,98,24
24  START=DELD-START
      AKX=AKX*GRID
      AKY=(AKY-A1)/A2
      SIGN= 1,0
      GO TO 18
88  SIGN= -1,0
18  DO 19 I=1,N
19  DIST(I)=SQRT((AX(I)-AX(1))**2+(AY(I)-AY(1))**2)
      ICT(NTRK)=(DIST(N)/DELD)+1.0
      JCT= ICT(NTRK)
      WRITE(61,104)
104  FORMAT(*      X      Y      Z      INTERPOLATED VALUES ALONG Y
      TRACK AT EQUAL GRID SPACING*)
      IK=0
      DO 25 I=1,JCT
      AJ#I=1
      XINT=(AJ*DELD)+START
      CALL SPLINE( DIST,Z,N,XINT,ZINT,ATER)
      NOW COMPUTE X AND Y VALUE FOR THIS INTERPOLATED VALUE OF Z
      IF (ABS(A4)-1,0) 26,26,27
26  TP=AKX-AJ*GRID*SIGN

```

C

```

      IF(TP,GE,0.0,AND,TP,LE,RLONG) GO TO 591
      GO TO 25
591  IK=IK+1
      BX(IK,NTRK)=TP
      BY(IK,NTRK)=A1+A2*TP
      GO TO 28
27  TR=AKY+AJ*GRID*SIGN
      IF(TP,GE,0.0,AND,TP,LE,RLAT) GO TO 592
      GO TO 25
592  IK=IK+1
      BX(IK,NTRK)=TP
      BX(IK,NTRK)= (TP-A1)/A2
28  BZ(IK,NTRK)=ZINT
      C  IF THESE DATA ARE DESIRED REMOVE C FROM NEXT CARD
      C  WRITE(61,102)(BX(I,NTRK),BY(I,NTRK),BZ(I,NTRK))
25  CONTINUE
      IF(IK,EQ,0 ) GO TO 593
      GO TO 594
593  IK=1
      BY(1,NTRK)= -1.0
      BX(1,NTRK)= -1.0
      BZ(1,NTRK)= -1.0
594  JCT=IK
      ICT(NTRK)=IK
      C  INTERPOLATED VALUES OF Z HAVE NOW BEEN COMPUTED AT EQUALLY SPACED
      C  VALUES OF X OR Y DEPENDING ON TEST IN STATEMENT 32 AND 30 AND
      C  STORED WITH 2ND INDEX = TRACK NO., NOW COMPLETE ABOVE PROCESS
      C  FOR ALL TRACKS ON THIS RUN
      IF(NTRK-ICT) 31,32,32
      C  NOW COMPUTE GRID VALUES, IF ABOVE INTERPOLATION WAS IN X, SORT
      C  POINTS INTO INCREASING Y AND INTERPOLATE FOR GRID VALUES IN Y
      C  DIRECTION , IF ABOVE PROCESS WAS IN Y DIRECTION, INTERCHANGE X AND Y
32  IF(ABS(A4)=1.0) 33,33,53

```



```

AX(IA)= DX
AZ(IA)= ZINT
45 IF(MEAN) 112,112,124
124 DO 125 IA=1,ITEMP
125 AVE(IA)=AVE(IA)*(AZ(IA)/9.0)
IF(KTEMP=3) 126,127,127
127 DO 128 IA=3,ITEMP,2
128 AVE(IA)=AVE(IA)+AVE(IA)+AVE(IA)
WRITE(61,80)
80 FORMAT(* X Y Z MEAN ANOMALY DATA*)
1 Y Z
DO 129 IA=1,ITEMP,2
AX(IA)=AX(IA)-GRID *2.0
129 AY(IA)=AY(IA)+GRID *2.0
WRITE(61,50) (AX(I),AY(I),AVE(I), I=1,ITEMP,2)
PUNCH 1000, (AVE(I), I=1,ITEMP,2)
DO 131 J=1,ITEMP
131 AVE(J)=0.0
KTEMP=1
GO TO 124
126 KTEMP=KTEMP+1
GO TO 109
C OUTPUT THIS GRIDDED DATA
112 WRITE(61,40)
40 FORMAT(* X Y Z FINAL GRIDDED DATA*)
1 Y Z
37 WRITE(61,50) (AX(I),AY(I),AZ(I), I=1,ITEMP)
PUNCH 1000, (AZ(I), I=1,ITEMP)
1000 FORMAT(7F11,2)
50 FORMAT(1X,3(F7,1,F8,1,F11,2))
GO TO 109
Z X
Z X

```

```

108 WRITE(61,110) DX
110 FORMAT(= NOT ENOUGH PTS FOR VALID INTERPOLATION ALONG X=,F7,1)
109 IF(IRLONG>DX) 67,67,46
46 DX=DX+GRID
GO TO 39
53 DY=0,0
ITEMP=(RLONG/GRID)*1,0
DO 132 I=1,ITEMP
132 AVE(I)=0,0
KTEMP=1
59 K=0
ATER= 9999,99
DO 54 J=1,ITOT
JCT= ICT(J)
DO 55 I=1,JCT
IF(BY(I,J).LT,(DY+.001).AND,BY(I,J).GT,(DY-.001)) GO TO 56
55 CONTINUE
GO TO 54
56 K=K+1
Y(K)=DY
X(K)=BX(I,J)
Z(K)=BZ(I,J)
54 CONTINUE
IF(K,EQ,0) GO TO 111
WRITE(61,107) DY
107 FORMAT(= X Y Z INPUT DATA FOR FINAL INTERP
10 LATION IN X DIRECTION FOR Y = ,F7,1)
C IF THESE DATA ARE DESIRED REMOVE C FROM NEXT CARD
C WRITE(61,102) (X(I),Y(I),Z(I),I=1,K)
C CHECK IF ENOUGH PTS TO INTERPOLATE
IF(K=3) 111,111,58
C NOW SORT DATA INTO INCREASING X FOR THIS VALUE OF Y
58 CALL SORTY(Y,X,Z,AY,AX,AZ,K,1,=1,GRID)

```

```

61 D0 65 IA=1,ITEMP
   AJ*IA=1
   XINT=AJ*GRID
   CALL SPLINE(X,Z,K,XINT,ZINT,ATER)
   AX(IA)= XINT
   AY(IA)= DY
65  A2(IA)= ZINT
   IF(MEAN) 113,113,133
133 D0 134 IA=1,ITEMP
134 AVE(IA)= AVE(IA)+(A7(IA)/ 9.0)
   IF (KTEMP=3) 135,136,136
136 D0 137 IA=3,ITEMP,2
   I8*IA=1
   IC*IA=2
137 AVE(IC)= AVE(IA)+AVE(IB)+AVE(IC)
   WRITE (61,80)
   D0 138 IA=1,ITEMP,2
   AX(IA)= AX(IA)
   AY(IA)= AY(IA)
138  WRITE(61,50) (AX(I),AY(I),AVE(I), I=1,ITEMP,2)
   PUNCH 1000, (AVE(I), I=1,ITEMP,2)
   D0 139 J=1,ITEMP
139  AVE(J)=0,0
   KTEMP=1
   GO TO 133
135 KTEMP= KTEMP+1
   GO TO 114
C   OUTPUT THIS GRIDDED DATA
113 WRITE(61,40)
57  WRITE(61,50) (AX(I),AY(I),AZ(I), I=1,ITEMP)
   PUNCH 1000,(AZ(I),I=1,ITEMP)
   GO TO 114
111 WRITE(61,115) DY

```

```

115 FORMAT(* NOT ENOUGH PTS FOR VALID INTERPOLATION ALONG Y=*,F7,1)
114 IF(RLAT=DY)67,67,66
66 DY=DY +GRID
GO TO 59
67 STOP
END
SUBROUTINE SPLINE (X,Y,M,XINT,YINT,ATER)
SEE PENNINGTON REF, FOR DESCRIPTION OF THIS SUBROUTINE
DIMENSION X(20),Y(20),C(4,660)
IF(X(1)*Y(M)+Y(M+1)+X(M+1)+Y(M+2)-ATER) 10,3,10
10 CALL SPLICON(X,Y,M,C)
ATER= X(1)+Y(M)+Y(M+1)+X(M+1)+Y(M+2)
K=1
3 IF(XINT=X(1)) 70,1,2
70 K=1
GO TO 7
1 YINT=Y(1)
RETURN
2 IF(XINT-X(K+1))6,4,5
4 YINT=Y(K+1)
RETURN
5 K=K+1
IF(M=K) 71,71,3
71 K=M+1
GO TO 7
6 IF(XINT-X(K))13,12,11
12 YINT=Y(K)
RETURN
13 K=K+1
GO TO 6
14 YINT=(X(K+1)+XINT)*(C(1,K)+X(K+1)-XINT)+2*(C(3,K)+
YINT+YINT+(X(1)-X(K))*(C(2,K)+X(K)))+2*(C(4,K))
RETURN

```

C

```

7 PRINT 101, XINT
101 FORMAT(* CAUTION VALUE AT POSITION=F10.2,* WAS EXTRAPOLATED*)
GO TO 11
END
SUBROUTINE SPLCON(X,Y,M,C)
DIMENSION X(20),Y(20),C(4,660),D(660),P(660),E(660),A(660,3),
1B(660),Z(660)
MM=M-1
DO 2 K=1,MM
D(K)=X(K+1)-X(K)
P(K)=D(K)/6.
2 E(K)=(Y(K+1)-Y(K))/D(K)
3 B(K)=E(K)=E(K-1)
A(1,2)=-1,-D(1)/D(2)
A(1,3)=D(1)/D(2)
A(2,3)=P(2)-P(1)*A(1,3)
A(2,2)=2,*(P(1)+P(2))-P(1)*A(1,2)
A(2,3)=A(2,3)/A(2,2)
B(2)=B(2)/A(2,2)
DO 4 K=3,MM
A(K,2)=2,*(P(K-1)+P(K))-P(K-1)*A(K-1,3)
B(K)=B(K)-P(K-1)*B(K-1)
A(K,3)=P(K)/A(K,2)
4 B(K)=B(K)/A(K,2)
Q=D(M-2)/D(M-1)
A(M,1)=1,*Q+A(M-2,3)
A(M,2)=-Q-A(M,1)*A(M-1,3)
B(M)=B(M-2)-A(M,1)*B(M-1)
Z(M)=B(M)/A(M,2)
MM=M-2
DO 6 I=1,MM
K=M-I

```

```

6 Z(K)=B(K)-A(K,3)*Z(K+1)
  Z(1)=A(1,2)*Z(2)-A(1,3)*Z(3)
  DO 7 K=1,MM
    Q=1./(6.*D(K))
    C(1,K)=Z(K)*Q
    C(2,K)=Z(K+1)*Q
    C(3,K)=Y(K)/D(K)=Z(K)*P(K)
    C(4,K)=Y(K+1)/D(K)=P(K)
  7 C(4,K)=Y(K+1)/D(K)=P(K)
  END
  SUBROUTINE SORTY(X,Y,Z,AX,AY,AZ,K,KODE,JCODE,GRID)
  Y=INPUT VARIABLE TO BE SORTED, X,Z=VALUES ASSOCIATED WITH Y
  K=LENGTH OF Y,IF KODE=1,VALUES OF Y WHICH ARE WITHIN 1 GRID INT OF
  PREVIOUS VALUE ARE REMOVED,IF KODE=0 ALL VALUES OF Y ARE
  RETAINED,IF JCODE=1,Y IS SORTED IN INCREASING ORDER,IF JCODE=1,
  Y IS SORTED IN DECREASING ORDER,OUTPUT IS SORTED VALUES OF
  Y WITH ASSOCIATED X AND Z
  DIMENSION Y(20),X(20),Z(20),AX(20),AY(20),AZ(20)
  KB = K
  CODE=JCODE
  J = 1
  I = 1
  JCT=0
  AY(J) = Y(I)
  129 TEMP= CODE*(AY(J)-Y(I+1))
    IF(ABS(TEMP))-GRID)122,122,120
  120 IF(TEMP)121,122,123
  121 I = I + 1
    IF(I = 1) = KB) 132,132,125
  123 JCT = 1
    AY(J) = Y(I + 1)
    AX(J) = X(I + 1)
    AZ(J) = Z(I + 1)
    KT = I + 2

```

```

GO TO 121
122 IF (KODE) 121,121,136
136 KD=I+2
137 IF (KD=KB) 124,124,139
139 K=K+1
      KB=KB+1
GO TO 125
124 DO 126 JD=KD,KB
      JF = JD + 1
      Y(JF) = Y(JD)
      X(JF) = X(JD)
126 Z(JF) = Z(JD)
      KB = KB + 1
      K = K + 1
GO TO 132
125 IF (JCT) 127,127,128
127 AY(J) = Y(1)
      AX(J) = X(1)
      AZ(J) = Z(1)
      KT = 2
128 J = J + 1
      IF (J - K) 131,133,133
131 DO 134 KA = KT,KB
      JT = KA + 1
      Y(JT) = Y(KA)
      X(JT) = X(KA)
134 Z(JT) = Z(KA)
      KB = KB + 1
GO TO 129
133 IF (JCT) 137,137,138
137 KB=KB+1
138 AY(K) = Y(KB+1)
      AX(K) = X(KB+1)

```



```

      AZ(K) = Z(KB=1)
      DO 135 I = 1,K
        Y(I) = AY(I)
        X(I) = AX(I)
        Z(I) = AZ(I)
      RETURN
      END

```

135

APPENDIX B

FORTRAN PROGRAM FOR GENERATING MEAN ANOMALIES
USING THE BICUBIC SPLINE


```

C
C PROGRAM BISPLI
C GRID DATA ,OUTPUT IS 16 BICUBIC COEFFICIENTS FOR EACH GRID
C INTERVAL WITH ASSOCIATED MEAN ANOMALY COMPUTED BY CLOSED FORM
C INTEGRATION ,REF,DE BOOR,J.MATH.AND PHYS,V.41,1962,DERIVATIVES
C COMPUTED BY MODIFICATION OF PENNINGTON TECHNIQUE=INTRODUCTION
C COMPUTER METHODS AND NUMERICAL ANALYSIS,MACMILLAN,1965
C OUTPUT OF COEFFICIENTS MAY BE OBTAINED AFTER STATEMENT 22
C IX=NO,OF COL AND JY=NO,OF ROWS OF INPUT DATA U,P=PARTIAL WRT X
C Q WRT Y, S WRT X AND Y,ALFA =BICUBIC COEFF,AVE=MEAN ANOMALY
C JCODE=1,MEAN INTERVAL=GRID,JCODE=2,MEAN INTERVAL=1/2 GRID,JCODE
C =3,MEAN INTERVAL=1/3 GRID,INPUT DATA COLUMNWISE STARTING AT
C LOWER LEFT CORNER,NEED SUBROUTINE DERIV
C DIMENSION U(61,61),P(61,61),Q(61,61),S(61,61),ALFA(4,4),AVE(3,3),
C 1Y(61),C(61),A(4,4),BK(4,4),AT(4,4)
C READ IN CONTROL CARD
C READ(60,10) IX,JY,JCODE,GRID
C 10 FORMAT(3I2,F6.0)
C DO 2 I=1,IX
C DO 2 J=1,JY
C READ(60,20) (U(I,J),J=1,JY)
C 20 FORMAT(7F11,2)
C 2 CONTINUE
C 3 COMPUTE PARTIAL DERIVATIVES WRT X
C DO 3 J=1,JY
C DO 4 I=1,IX
C 4 Y(I)=U(I,J)
C CALL DERIV (GRID,Y,IX,C)
C DO 5 I=1,IX
C 5 P(I,J)=C(I)
C 5 CONTINUE
C NOW COMPUTE PARTIAL WRT Y
C DO 6 I=1,IX

```

```

      DO 7 J=1,JY
      7 Y(J)= U(I,J)
      CALL DERIV (GRID,Y,JY,C)
      DO 8 J=1,JY
      8 Q(I,J)= C(J)
      6 CONTINUE
      C      NOW COMPUTE 2ND PARTIAL BY COMPUTING PARTIAL OF P WRT Y
      DO 9 I=1,IX
      DO 11 J=1,JY
      11 Y(J)= P(I,J)
      CALL DERIV (GRID,Y,JY,C)
      DO 12 J=1,JY
      12 S(I,J)=C(J)
      9 CONTINUE
      C      ALL REQUIRED DERIVATIVES COMPUTED ,NOW SOLVE MATRIX EQUATION FOR
      C      BICUBIC COEFF, FOR EACH GRID SQUARE AND INTEGRATE FOR MEANS
      C      SET UP(A) MATRIX
      DO 13 J=1,4
      A(1,J)=0,0
      13 A(2,J)=0,0
      A(1,1)=1,0
      A(2,2)=1,0
      B=1,0/ (GRID**3)
      A(3,1)= *3,*GRID*B
      A(3,2)= *2,*B*(GRID**2)
      A(3,3)= *A(3,1)
      A(3,4)= A(3,2)/2,
      A(4,1)= 2,*B
      A(4,2)= A(3,3)/3,
      A(4,3)= *A(4,1)
      A(4,4)= A(4,2)
      DO 25 I=1,4
      DO 26 J=1,4

```

```

26 AT(I,J) = A(J,I)
25 CONTINUE
    WRITE (61,40)
40 FORMAT (*,X,Y,MEAN,POSITION IS UPPER LEFT COR
1NER OF AREA*)
ACODE=JCODE
C NOW SOLVE FOR ALFA MATRIX USING COLUMNWISE GRID BLOCKS
DO 15 I=2,IX
DO 15 J=2,JY
C SET UP BK MATRIX
IA=I-1
AIA=IA-1
JA=J-1
AJA=JA-1
BK(1,1) = U(IA,JA)
BK(1,2) = Q(IA,JA)
BK(1,3) = U(IA,J)
BK(1,4) = Q(IA,J)
BK(2,1) = P(IA,JA)
BK(2,2) = S(IA,JA)
BK(2,3) = P(IA,J)
BK(2,4) = S(IA,J)
BK(3,1) = U(I,JA)
BK(3,2) = Q(I,JA)
BK(3,3) = U(I,J)
BK(3,4) = Q(I,J)
BK(4,1) = P(I,JA)
BK(4,2) = S(I,JA)
BK(4,3) = P(I,J)
BK(4,4) = S(I,J)
C MULTIPLY BK ON THE LEFT BY A AND STORE PRODUCTS IN ALFA
DO 17 M=1,4
DO 17 N=1,4

```

```

ALFA(M,N)=0.0
DO 18 IR=1,4
DO 19 M=1,4
DO 20 N=1,4
18 CONTINUE
19 CONTINUE
20 CONTINUE
21 BK(M,N)= ALFA(M,N)
22 CONTINUE
23 CONTINUE
24 ALFA(M,N)= ALFA(M,N) * (BK(M,K) * AT( K,N))
25 CONTINUE
26 CONTINUE
27 CONTINUE
28 CONTINUE
29 CONTINUE
30 CONTINUE
31 CONTINUE
32 CONTINUE
33 CONTINUE
34 CONTINUE
35 CONTINUE
36 CONTINUE
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78 CONTINUE
79 CONTINUE
80 CONTINUE
81 CONTINUE
82 CONTINUE
83 CONTINUE
84 CONTINUE
85 CONTINUE
86 CONTINUE
87 CONTINUE
88 CONTINUE
89 CONTINUE
90 CONTINUE
91 CONTINUE
92 CONTINUE
93 CONTINUE
94 CONTINUE
95 CONTINUE
96 CONTINUE
97 CONTINUE
98 CONTINUE
99 CONTINUE
100 CONTINUE

```



```

ANY=NY
  AVF(M,N)=AVE(M,N)*(ALFA(MX,NY)*(D=MX=G*MX)
1*(B=NY-E*NY))/(AMX*ANY)
27 CONTINUE
  AVE(M,N)=AVE(M,N)/(SGRID**2)
  CX=(AJA*GRID)+(AM=1,0)*SGRID
  CY=(AJA*GRID)*(AN*SGRID)
28 WRITE(61,30) CX,CY,AVE(M,N)
30 FORMAT(3F10,2)
15 CONTINUE
  STOP
END
SUBROUTINE DERIV(GRID,Y,M,C)
  MODIFICATION OF PENNINGTON SPLICON TO OBTAIN FIRST DERIVATIVES
  Y=INPUT EQUID-SPACED AT GRID INT, M= NO. OF Y PTS., C=DERIVATIVE
  AT EACH INPUT PT,
  DIMENSION Y(300),D(300),P(300),E(300),C(300),A(300,3),B(300),
1Z(300)
  MM=M-1
  DO 2 K=1,MM
    D(K)=GRID
    P(K)=D(K)/6,
2 E(K)=(Y(K+1)-Y(K))/D(K)
    DO 3 K=2,MM
      B(K)=E(K)=E(K-1)
      A(1,2)=*1,-D(1)/D(2)
      A(1,3)=D(1)/D(2)
      A(2,3)=P(2)-P(1)*A(1,3)
      A(2,2)=2,*(P(1)+P(2))-P(1)*A(1,2)
      A(2,3)=A(2,3)/A(2,2)
      B(2)=B(2)/A(2,2)
      DO 4 K=3,MM
        A(K,2)=2,*(P(K-1)+P(K))-P(K-1)*A(K-1,3)

```

```

B(K)=B(K)*P(K+1)*B(K-1)
A(K,3)=P(K)/A(K,2)
4 B(K)=B(K)/A(K,2)
  Q=D(M-2)/D(M-1)
  A(M,1)=1.+Q*A(M-2,3)
  A(M,2)=Q*A(M,1)*A(M-1,3)
  B(M)=B(M-2)*A(M,1)*B(M-1)
  Z(M)=B(M)/A(M,2)
  MN=M-2
  DO 6 I=1,MN
    K=M-I
    6 Z(K)=B(K)*A(K,3)*Z(K+1)
      Z(1)=A(1,2)*Z(2)*A(1,3)*Z(3)
      DO 7 K=1,MM
        7 C(K)=(Z(K)*U(K)/2.)*E(K)+((Z(K)*Z(K+1))*B(K)/6.)
      C(M)=(Z(M)*D(MM)/2.)*E(MM)+((Z(MM)-Z(M))*D(MM)/6.)
      RETURN
      END

```

APPENDIX C

FORTRAN PROGRAM FOR POINT-MASS MODEL DATA


```

PROGRAM GMASS
  DIMENSION PMASS(50),DEPTH(50),PLATH(50),PLONGM(50),PLONG(500),VERT
    1(500),VERTOTAL(500),CNORTH(500),EAST(500),XSI(500),ETA(500),THETA(
    2500),COSPSI(50),RAD1(50),RAD2(50),ANGLE(50),FLONG(500),RPRIME(50),
    3JTITLE(80),X(1000),Y(1000)
  C PLATH AND SLAT ARE COLATITUDES
  C LONGITUDE RUNS FROM 0 - 360 RELATIVE TO PRIME MERIDIAN
  C PMASS INCORPORATES THE CONSTANT AND MASS OF POINT MASS FORMULA
  C DEPTH IS IN METERS
  C IF NSPACE IS ONE, CALCULATE AT EQUALLY SPACED POINTS IN DISTANCE
  C IF NSPACE IS ZERO CALCULATE AT EQUALLY SPACED INTERVALS OF LAT,=LONG
  C FOR NSPACE OF ONE, DISTANCE IS IN METERS
  C FOR NSPACE OF ZERO, DISTANCE IS IN MILES
  C NCODE IS 8888 IF CALCULATIONS ARE FOR A RANDOM SET OF POINTS
  C NCODE IS 9999 IF CALCULATIONS ARE FOR A GRIDDED ARRAY OF POINTS
  C NMASS IS THE TOTAL NUMBER OF MASSES
  C NLAT IS THE NUMBER OF ROWS OF GRID ARRAY
  C NLONG IS THE NUMBER OF COLUMNS OF GRID ARRAY
  C GRAVZSUM IS THE SUM OF THE VERTICAL COMPONENTS DUE TO POINT MASSES
  C GRAVXSUM IS THE SUM OF THE NORTHERLY COMPONENTS DUE TO POINT MASSES
  C GRAVYSUM IS THE SUM OF THE EASTERLY COMPONENTS DUE TO POINT MASSES
  C FOR RANDOM DATA SET DISTANCE = 0, AND NSPACE = NLONG = NLAT = 1
  C FOR RANDOM DATA TERMINATE DATA DECK WITH CARD FULL OF NINES
  DOUBLE PRECISION R,PI,PLATH,PLONGM,SLAT,SLONG,DELAT,DELONG,PL
    1AT,PLONG,EARTHG,COSPSI,RAD1,RAD2,GRAVRAD,GRAVZSUM,ANGLE,GRAVLAT,
    2GRAVXSUM,GRAVLONG,GRAVYSUM,VERT,VERTOTAL,XSI,ETA,THETA,RPRIME,PC
  C ***** READ IN TITLE
    READ 2,(JTITLE(I),I=1,80)
    2 FORMAT(80A1)
    PRINT 2,(JTITLE(I),I=1,80)
    READ 99,NCODE,NMASS
    99 FORMAT(2I5)
    R = 6371229.32

```

```

PI= 3.14159265358979
C ***** READ POINT MASS PARAMETERS
DO 3 I=1,NMASS
  READ 4,PMASS(I),DEPTH(I),PLATH(I),PLONGM(I)
  4 FORMAT(4F10.0)
  3 CONTINUE
  PRINT 7
  7 FORMAT(1H0,3X,* COLATITUDE LONGITUDE
1S *)
  DO 9 I=1,NMASS
    PRINT 8,PLATH(I),PLONGM(I),DEPTH(I),PMASS(I)
    8 FORMAT(1H0,5X,F10.5,3X,F10.5,7X,F7,0,7X,F7,0)
    9 CONTINUE
    DO 12 I=1,NMASS
      PLATH(I)=PLATH(I)*(PI/180.)
      PLONGM(I)=PLONGM(I)*(PI/180.)
    12 CONTINUE
    555 READ 1,SLAT,SLONG,DISTANCE,NSPACE,NLAT,NLONG
      IF(SLAT=999999999.)16,888,888
      1 FORMAT(3F10.0,3I5)
      16 SLAT=SLAT*(PI/180.)
      SLONG=SLONG*(PI/180.)
      K1=0
      200 IF(NSPACE)10,11,10
      10 K1=K1+1
C DELAT IN RADIANS
      DELAT =DISTANCE/R
      PLAT = SLAT +(K1-1)*DELAT
      DELONG= DISTANCE/( R*DSIN(PLAT))-
      DO 5 I=1,NLONG
        PLONG(I)= SLONG +(I-1)*DELONG
      5 CONTINUE
      GO TO 101

```

DEPTH MAS

```

11 K1=K1+1
   DELTA=(DISTANCE/60.)*(PI/180.)
C DELAT IN THIS CASE IS ENTERED IN MINUTES SO WE CONVERT TO RADIANS
   PLAT=SLAT-(K1-1)*DELAT
   DELONG=(DISTANCE/60.)*(PI/180.)
   DO 6 I=1,NLONG
     PLONG(I)=SLONG +(I-1)*DELONG
6 CONTINUE
   PC=PI/2
101 EARTHG = 980.63227 -2.586157 *DCOS(2.*(PC=PLAT))*0.002885*DCOS( 4.
     1*(PC=PLAT))
100 DO 68 N=1,NLONG
   GRAVXSUM=0.0
   GRAVYSUM=0.0
   GRAVZSUM=0.0
   DO 40 I=1,NMASS
     COSPS(I)=DCOS(PLAT)*DCOS(PLATH(I))*DSIN(PLAT)*DSIN(PLATH(I))*DCOS
       1(PLONG(I)-PLONG(N))
     RPRIME(I)= R-DEPTH(I)
     RAD1(I)=DSQRT(R*R+RPRIME(I)*RPRIME(I))-2.*R*RPRIME(I)*COSPS(I))
     RAD1(I)=RAD1(I)*RAD1(I)*RAD1(I)
     RAD2(I)=PMASS(I)/RAD1(I)
     GRAVRAD= RAD2(I)*(R+RPRIME(I)*COSPS(I))
     GRAVXSUM= GRAVZSUM+GRAVRAD
     ANGLE(I)=DSIN(PLAT)*DCOS(PLATH(I))-DCOS(PLAT)*DSIN(PLATH(I))*DCOS(
       1PLONG(I)-PLONG(N))
     GRAVLAT = RAD2(I)*RPRIME(I)*ANGLE(I)
     GRAVXSUM= GRAVLAT+GRAVXSUM
     GRAVLONG =RAD2(I)*RPRIME(I)*DSIN(PLATH(I))*DSIN(PLONG(I)-PLONG(N))
1) GRAVYSUM=GRAVYSUM+GRAVLONG
40 CONTINUE
   VERT(N)=GRAVZSUM +100000.

```



```

CNORTH(N)=GRAVXSUM*100000,
EAST(N)=GRAVYSUM*100000,
VERTOTAL(N)=VERT(N)*EARTHG*1000,
XS(N)=DATAN2(CNORTH(N),VERTOTAL(N))*206264.81*(=1,)
ETA(N)=DATAN2(EAST(N),VERTOTAL(N))*206264.81*(=1,)
THETA(N)=DSORT(XS(N)*XS(N)+ETA(N)*ETA(N))
68 CONTINUE
PRINT 734
734 FORMAT(1H0,*,.....*)
1.....*
C CHANGE RADIAN'S TO DEGREES
DO 76 I=1,NLONG
  FLONG(I)=PLONG(I)*(180./PI)
76 CONTINUE
  FLAT=PLAT*(180./PI)
  IF(NCODE=99999)31,32,32
31 PRINT 50,FLAT
50 FORMAT(1H0,20X,* COMPONENTS AT COLATITUDE *,F10,5)
  GO TO 33
32 PRINT 70,FLAT
70 FORMAT(1H0,20X,* COMPONENTS ALONG COLATITUDE *,F10,5)
33 PRINT 71
71 FORMAT(1H0,* LONGITUDE ETA XS! RADIAL NORTHERLY EASTERLY
1 G XS! THETA *)
DO 72 N=1,NLONG
  PRINT 73,FLONG(N),VERT(N),CNORTH(N),EAST(N),VERTOTAL(N),XS(N),
  1ETA(N),THETA(N)
73 FORMAT(7X,F10,5,4F12,2,3F10,2)
72 CONTINUE
C ***** PUNCH DESIRED OUTPUT
PUNCH 702,(VERT(N),N=1,NLONG)
702 FORMAT(10F8,2)
  IF(NCODE=99999)555,39,39

```

```
39 IF (K1-NLAT)200,888,888  
888 STOP  
END
```


APPENDIX D

FORTRAN PROGRAM FOR THE TWO-DIMENSIONAL FAST FOURIER TRANSFORM


```

C      PROGRAM FFTW0
C      PROGRAM TO COMPUTE 2D FOURIER TRANSFORMS USING FFT
C      USES SUBROUTINE NLOGN FROM ROBINSON,1967,MULTICHANNEL TIME
C      SERIES ANALYSIS WITH DIGITAL COMPUTER PROGRAMS,HOLDEN=DAY,
C      DATA MUST BE EVENLY SPACED IN X AND Y
C      NX= NO, OF POINTS IN X DIRECTION, NY= NO, OF POINTS IN Y DIRECTION
C      IT IS NOT NECESSARY THAT NX=NY
C      DELR=NORMALIZED FREQUENCY INTERVAL IN CYCLES PER DATA INTERVAL
C      NMAX=MAX OF NX,NY,LIMIT IS 127 FOR NMAX
C      INPUT DATA HAS ORIGIN AT LOWER LEFT, +Y UP, +X RIGHT, AND INPUT
C      SHOULD BE IN FORM Y WITHIN X, IE PROGRAM WILL READ COLUMNS OF DATA
C      FROM LEFT TO RIGHT IN +Y DIRECTION
C      ONLY CONTROL CARD REQUIRED IS NX AND NY SPECIFICATION
C      DIMENSION DATA(128),ADATA(256),DATA(128,128)
      READ(60,10)NX,NY
10    FORMAT(2I5)
      IF(NX=NY)63,63,62
63    NMAX=NY
      GO TO 64
62    NMAX=NX
C      NOW CHECK IF NMAX IS A POWER OF TWO
64    N=NMAX
      DO 5 M=1,7
      I=2**M
      IF(I=N)5,6,6
5    CONTINUE
6    NMAX=I
      ANMAX=NMAX
      DELR=1.0/ANMAX
C      NMAX IS NOW THE TOTAL NUMBER OF DATA PTS, TO A POWER OF TWO
C      DELR = NORMALIZED FREQUENCY INTERVAL FOR FOURIER TRANSFORM
      WRITE(61,50)DELR,NMAX
50    FORMAT(/28H NORMALIZED FREQ INTERVAL = ,F8.5,/52H NO, OF DATA PTS

```

```

1, REQUIRED TO MAKE NF{ A HARMONIC = ,16)
IX=1
67 READ(60,20){DATA(J),J=1,NY)
20 FORMAT(10F8,5)
NP=NY+1
DO 9 J=NP,NMAX
8 DATA(J)=0,0
C NOW STORE DATA IN COMPLEX FORM IN ADATA
DO 2 J=1,NMAX
I=(J+2)-1
ADATA(I)=DATA(J)
IA=I+1
2 ADATA(IA)=0,0
C CALL NLOGN (M,ADATA,-1,0)
C AT THIS POINT ADATA IS A FUNCTION OF FREQUENCY IN THE Y DIRECTION
C AND TIME OR LOCATION IN THE X DIRECTION AND IS IN COMPLEX FORM OUT
C TO TWICE THE NYQUIST FREQUENCY
C NOW STORE ADATA IN BDATA 2D MATRIX OUT TO THE 1ST FOLDING FREQ, IN
C COMPLEX FORM 1ST INDEX=X DIRECTION, 2ND=Y DIRECTION
DO 65 J=1,NMAX
65 BDATA(IX,J)=ADATA(J)
IF(IX=NX)66,68,300
66 IX=IX+1
GO TO 67
68 JX=NX+1
DO 70 J=JX,NMAX
DO 69 I=1,NMAX
69 BDATA(J,I)=0,0
70 CONTINUE
C AT THIS POINT WE HAVE COMPLETED THE TRANSFORM IN THE Y DIRECTION
C AND STORED THE REQUIRED NO. OF PTS. IN THE X DIRECTION
C NOW TRANSFORM IN X DIRECTION AND PUNCH OUT 2D AMPLITUDE SPECTRUM,
JMAX=NMAX+2

```



```

D0 72 I=1,NMAX,2
IA=I+1
D0 71 J=1,JMAX,2
JA=(J+1)
JB=JA/2
ADATA(J)=BDATA(JB,I)
71 ADATA(JA)=BDATA(JB,IA)
CALL NLGN (M,ADATA,-1,0)
IB =IA/2
D0 201 N=1,NMAX,2
JX=(N+1)/2
J=N+1
201 BDATA(JX,IB)=SQRT(ADATA(N)**2+ADATA(J)**2)
72 CONTINUE
C BDATA IS NOW 2D AMPLITUDE SPECTRUM AT FREQ INTERVAL DELR,1ST INDEX
C IS THE X FREQUENCY, THE 2ND IS THE Y FREQUENCY, INCREASING INDEX
C EQUALS INCREASING FREQUENCY, NOTE THAT THIS THE REVERSE OF NORMAL
C MATRIX NOTATION
MMAX=NMAX/2
D0 202 J=1,MMAX
A=J+1
A=A*DELR
D0 82 JA=1,MMAX
82 BDATA(J,JA)=(4.0* BDATA(J,JA))/ NMAX**2
WRITE(61,80)A
80 FORMAT(10H X FREQ = ,F8.5)
WRITE(61,90)(BDATA(J,I),I=1,MMAX)
PUNCH 21, (BDATA(J,I),I=1,MMAX)
90 FORMAT(10F10.5)
21 FORMAT(10F7,2)
202 CONTINUE
300 STOP
END

```

```

C
C
C
C
SUBROUTINE NL0GN(N,X,SIGN)
  NMAX = LARGEST VALUE OF N TO BE PROCESSED
  NONDUMMY DIMENSION M(NMAX)
  FOR EXAMPLE, IF NMAX = 15 THEN
    DIMENSION M(15)
    DIMENSION X(2**N)
    DIMENSION X(2)
    COMPLEX X,WK,HOLD,Q
    LX = 2**N
    DO 1 I = 1,N
      1 M(I) = 2**(N-I)
    DO 4 L = 1,N
      NBLOCK = 2**(L-1)
      LBLOCK = LX / NBLOCK
      LBHALF = LBLOCK / 2
      K = 0
      DO 4 IBLCK = 1,NBLOCK
        FK = K
        FLX = LX
        V = SIGN * 6.2831853 * FK / FLX
        WK = CMPLX(COS(V),SIN(V))
        ISTART = LBLOCK * (IBLOCK - 1)
        DO 2 I = 1,LBHALF
          J = ISTART + I
          JH = J + LBHALF
          Q = X(JH) * WK
          X(JH) = X(J) * Q
          X(J) = X(J) + Q
        2 CONTINUE
        DO 3 I = 2,N
          II = I
          IF(K,LT,M(I)) GO TO 4
        3 K = K + M(I)

```

```

4 K = K + M(I)
  K = 0
  DO 7 J = 1, LX
    IF (K,LT,J) GO TO 5
    HOLD = X(J)
    X(J) = X(K + 1)
    X(K + 1) = HOLD
5 DO 6 I = 1, N
  II = I
  IF (K,LT,M(I)) GO TO 7
6 K = K + M(I)
7 K = K + M(II)
  IF (SIGN,LT,0,0) RETURN
  DO 8 I = 1, LX
    X(I) = X(I) / FLX
  RETURN
  END

```


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Cubic spline interpolation is a mathematical procedure which is an analog of the draftsman's plastic spline. The advantage of this interpolation procedure over the more commonly used methods such as Lagrange lies in its ability to not only fit each given data value exactly but to maintain continuity of the first and second derivatives. The relative accuracy of the cubic spline interpolation procedure for generating gridded data values and estimates of mean gravity anomalies from track-type geophysical surveys is shown to be excellent when applied to properly designed surveys. Techniques for interpreting the two-dimensional Fourier transform in terms of track spacing, track orientation, and down-track sampling rate are presented to demonstrate the effect of these parameters on interpolation accuracy. A procedure for utilizing closed form integration of the bicubic spline surface to produce mean gravity anomalies is shown to yield accuracies comparable to the method of averaging cubic spline grid values.			

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KEY WORDS

LINK A

LINK B

LINK C

ROLE

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Cubic Splines

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Geophysical Survey Design

Bicubic Splines

Fourier Transforms

Mean Free-air Gravity Anomalies

Physical Geodesy

Gravity