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DIRECTIONAL WAVE MEASUREMENT AND ANALYSIS

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by

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Results have agreed well with other wave measuring buoys as detailed in other sources.

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ABSTRACT

A directional wave analysis program has been developed to analyze the measured directional wave data from an ENDECO Wave-Track buoy. Due to the nature of the buoy, the direction components must be adjusted in order to apply a conventional wave slope analysis. The choice of a Wave-Track buoy stems from the requirement of a light-weight, easy to handle directional sensing wave buoy. The results can be used to validate the directional spread of wave energy produced by the Spectral Ocean Wave Model (SOWM). Also, the resulting directional spread of energy, as produced by the buoy, may now be used with ship motions to produce ship response amplitude operator (RAO's), or combined with ship RAO's to estimate ship motions.

The equations are developed to account for buoy motions in a current and in shallow water. Although electronic phase shifts have been adjusted, it is still necessary to develop some analytical solution to compensate for hydrodynamic responses and phase shifts.

The analyzed results include frequencies, energy densities, Fourier coefficients, first and second harmonic mean wave directions and root mean square (rms) spreading. Also included is the directional spreading of energy calculated by four different methods.

Results have agreed well with other wave measuring buoys as detailed in other sources.

ADMINISTRATIVE INFORMATION

This report was prepared under the sponsorship of the Naval Sea Systems Command (NAVSEA), Code 3213 Work Request Number WR91589, the Ship Performance and Hydromechanics Program funded under Program Element 62543N and Block Number ZF 43 421 001, and the Surface Wave Spectra for Ship Design Program under Program Element 62759N and Project Number SF 59 557 695. It is identified by Work Unit Numbers 1568-823, 1500-104, 1568-838, and 1500-300 at David W. Taylor Naval Ship Research and Development Center (DTNSRDC).

INTRODUCTION

The measurement of directional waves has taken on an increased significance over the last decade. The U.S. Navy has become concerned with the directional spreading in addition to the energy of waves as they affect ships. It is no longer sufficient to determine only the frequency distribution of the wave energy, but rather the directional distribution of each component is required.

A ship responds to wave conditions depending on the ship heading angles to the encountered waves. A wave train coming directly toward the bow (head seas) will cause a ship to pitch with little roll motion, whereas, a wave train approaching the beam of a ship (beam seas) will cause a ship to roll with little pitch motion. Since ship response is a function of encountered wave angle and wave energy distribution, it is clear that applying just the wave height spectrum to the ship response amplitude operators (RAO's) will not accurately predict or estimate the ship motions. It thus becomes necessary to determine the distribution of wave energy for each direction and frequency. The product of the wave energy and RAO for each direction and frequency will yield a ship motion for each given wave encounter angle and its associated frequency. Linear superposition of the motions at each direction and frequency will yield a total ship response for a specific motion. The assumption of linear superposition of ship response is valid provided the seaway and motions are not extreme.^{1*}

The measurement of directional waves also has been used to validate the Navy's hindcast/forecast ocean wave model. The Spectral Ocean Wave Model (SOWM), based on work by Pierson, et al.,² has been operational at the Fleet Numerical Oceanographic Center (FNOC).³ This model hindcasts directional wave spectra based on the input wind velocities derived from local and global pressure fields. The model also generates directional wave forecasts based on current meteorological forecasts. Some of the analyzed results have been applied in ship design and operations and have been reported by Bales, Cummins and associates.^{4,5,6,7}

Until the last decade, U.S. Navy ships rarely were designed to consider the natural environment.⁸ It became necessary to account for not only the man-made threat and calm water characteristics, but also the environmental conditions affecting the ship, primarily ocean waves. Since ships seldom operate in calm water, it is important to design ships to operate well in the natural environment. This provides a greater percentage of operability and a more habitable condition for the crew.

The data produced by the SOWM are being implemented in the Navy ship design process. However, prior to utilizing the data, some degree of confidence should be developed for it. The energy spectrum has been validated using various techniques, such as radar altimeters on satellites, wave buoy measurements, and observations on

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*A complete listing of references is given on page 25.

ships at sea.^{7,9} However, the wave directionality of the model has only been indirectly validated. The question has risen whether the adequate verification of the point spectra can imply that the wave energy has been distributed and propagated in the proper directions. A more direct method of validating the directional aspects of the model is necessary. The measurement of the directions, heights and frequencies of the waves provides an important step in the verification of the model. This can be accomplished in a variety of ways, as will be discussed shortly.

COMPLEXITY OF WAVE ENVIRONMENT

Waves in the ocean and coastal environment reveal three dimensional characteristics. They can be found to be long-crested or short-crested. The complexity of these waves stems from the fact that they reflect, diffract, refract, become standing or trapped partially or totally, and interact with other physical parameters such as wind, other waves, and currents. Therefore, data measured from the field must be properly interpreted to represent the given sea conditions (Figure 1).

Long-crested seas are not common, and may be locally wind generated or decaying swell in an otherwise benign sea. Short-crested seas occur for most of the time. Waves of varying height, frequency and direction interact continually in the ocean. The processes and results of these interactions produce confused seas and make the observation of these conditions difficult.

Another seaway condition difficult to represent is wave-current interaction. Currents can refract, reflect, attenuate and amplify waves, depending on their relative magnitudes and directions, see Figure 2. Further discussions are presented by Forristall¹⁰ and Lai.^{11,12}

TECHNIQUE OF WAVE MEASUREMENT

Because of the nature and complexity of ocean waves, many techniques for measuring waves and their directions have been developed to take advantage of different wave characteristics. The instruments used to obtain these measurements operate on principles based on either pressure, orbital wave velocity, wave slope, any combination of these, or the reflection of radar.

Some systems that lend themselves to measurement of nearshore directional waves include: pressure sensor biaxial current meter combinations, triaxial current meters, arrays of pressure sensors and shore-based radars, such as X-band

surface imaging radar, and coastal HF radar (CODAR). An intercomparison of these sensors was performed during the Atlantic Remote Sensing Land Ocean Experiment (ARSLOE) and has been reported by Grosskopf, et al.¹³ Several offshore wave measurement systems were also compared during ARSLOE. These systems included wave buoys and airborne radars. The pitch-roll-heave buoys used were the MET buoy, Cloverleaf, XERB and the ENDECO Wave-Track. The Surface Contouring Radar (SCR) measured wave height, frequency and direction from the air. Two systems that did not measure all three characteristics were also present. The Datawell Waverider Buoy was deployed as a standard for wave height and frequency, while the Side Looking Airborne Radar (SLAR) determined only wave direction, not height and frequency. The preliminary comparisons have been published by Szabados.¹⁴

A hull mounted radar has also been developed to measure wave height, frequency, and direction. This radar, developed at the Naval Research Lab (NRL), has shown some potential in the measurement of high frequency wind waves, as reported by Schuler, et al.¹⁵

CHOICE OF SENSOR

Given the wide variety of sensors available, it was difficult to select a system that would best validate the Spectral Ocean Wave Model and provide more appropriate wave data during ship motion trials. It was clear from the outset that onshore and bottom mounted/moored sensors were impractical to operate in the deep open ocean. Airborne mounted radars would be prohibitively expensive to maintain lengthy on-station times for the many flights necessary over a period of time. Hull mounted radars may provide quality data in limited fetch areas, but require considerable time and expertise to set up and operate. This leaves deployable wave buoys as the optimum instrument to be used from naval ships of opportunity where there may be little deck handling area. Also, there may not be the usual cranes and A-frames found on most oceanographic research ships. At most, a davit may be available. This restriction points to the selection of a small, lightweight pitch-roll-heave buoy which would survive rough sea conditions and provide a sufficient first order measurement of wave directionality.

Therefore, an ENDECO Wave Track Buoy was selected (see Figure 3). This buoy relies on a different concept to measure the wave directionality than does a conventional slope following buoy (see Figure 4). Subsequently, the analysis of the data requires a modification to the conventional slope analysis.

The method of analysis of Wave Track Buoy data developed by DTNSRDC is introduced here. Some theoretical background is provided which includes parametric models for directional waves and analysis based on slope. Measurements of the directional waves by an orbital following buoy are presented. Finally, a discussion of assumptions, practical considerations, and errors is included.

ANALYSIS OF THE DIRECTIONAL WAVE SPECTRUM

METHOD OF ANALYSIS

The expression of the two-dimensional spectrum, $E(f, \theta)$, as given by Longuet-Higgins, et al.¹⁶ is

$$E(f, \theta) = F(f) \cdot H(f, \theta) \quad (1)$$

where f is wave frequency, θ is approaching wave direction, $F(f)$ is the one-dimensional wave spectrum component, and $H(f, \theta)$ is the directional spreading function with

$$\int_0^{2\pi} H(f, \theta) d\theta = 1 \quad (2)$$

Spreading functions can be developed by either deterministic methods or parametrical models. A conventional method to analyze the spreading function has been proposed by Longuet-Higgins and associates.¹⁶ The spreading function is given by the Fourier series as

$$H(f, \theta) = \frac{1}{\pi} \left[\frac{1}{2} + \sum_{n=1}^2 W_n (a_n \cos n\theta + b_n \sin n\theta) \right] \quad (3)$$

where W_n is a weighting parameter, a_n and b_n are the Fourier coefficients in the expansion of $H(f, \theta)$.

Field instruments which follow the surface slope, such as cloverleaf buoys^{17,18} and discus buoys,^{19,20} also referred to as pitch-roll buoys, measure the surface elevation, η , and surface slopes, η_x and η_y (where $\eta_x = \frac{\partial \eta}{\partial x}$ and $\eta_y = \frac{\partial \eta}{\partial y}$).

Analysis of the measured data yield the six cross-spectra as expressed by Longuet-Higgins et al.¹⁶ and Ewing and Pitt²¹. Based on Equations (1) and (3), it can be shown that

$$\begin{aligned}
 C_{\eta\eta}(f) &= \int_0^{2\pi} (2\pi f)^4 E(f, \theta) d\theta \\
 C_{\eta_x\eta_x}(f) &= \int_0^{2\pi} k^2 \cos^2 \theta E(f, \theta) d\theta \\
 C_{\eta_y\eta_y}(f) &= \int_0^{2\pi} k^2 \sin^2 \theta E(f, \theta) d\theta \\
 Q_{\eta\eta_x}(f) &= \int_0^{2\pi} k(2\pi f)^2 \cos \theta E(f, \theta) d\theta \\
 Q_{\eta\eta_y}(f) &= \int_0^{2\pi} k(2\pi f)^2 \sin \theta E(f, \theta) d\theta \\
 Q_{\eta_x\eta_y}(f) &= \int_0^{2\pi} k^2 \sin \theta \cos \theta E(f, \theta) d\theta
 \end{aligned} \tag{4}$$

where k is the wave number and C is the co-incident spectrum and Q is the quadrature spectrum of wave surface elevation (η) and surface slopes (η_x and η_y). By using the following dispersion relation,

$$\frac{(2\pi f)^2}{gk} = \tanh kd = \left[\frac{C_{\eta\eta}}{C_{\eta_x\eta_x} + C_{\eta_y\eta_y}} \right]^{1/2} \tag{5}$$

the normalized Fourier coefficients can be estimated, as demonstrated by Ewing and Pitt²¹ by

$$A_1 = Q_{\eta\eta_x} / [C_{\eta\eta}(C_{\eta_x\eta_x} + C_{\eta_y\eta_y})]^{1/2} \tag{6}$$

$$B_1 = Q_{\eta\eta_y} / [C_{\eta\eta}(C_{\eta_x\eta_x} + C_{\eta_y\eta_y})]^{1/2} \tag{7}$$

$$A_2 = (C_{\eta_x\eta_x} - C_{\eta_y\eta_y}) / (C_{\eta_x\eta_x} + C_{\eta_y\eta_y}) \tag{8}$$

$$B_2 = 2 C_{\eta_x\eta_y} / (C_{\eta_x\eta_x} + C_{\eta_y\eta_y}) \tag{9}$$

All of these normalizations are dependent on Equation (5). The validity of this relation has been examined from measured data.^{20,21} It is interesting to observe that the variation of $C_{\eta\eta}/(C_{\eta_x\eta_x} + C_{\eta_y\eta_y})$ with $\frac{(2\pi f)^2}{gk}$ is frequency and water depth dependent. The deviations increase as the frequency increases.

THE DIRECTIONAL SPREADING FUNCTION

Fourier Coefficient with Weight Parameter

Wave directionalities are determined by the directional spreading functions. A conventional spreading function is given by Longuet-Higgins et al.¹⁶ in Equation (3) with W_1 and W_2 equal to $2/3$ and $1/6$, respectively. For the discus buoy, the spreading function with second order harmonics is given by

$$H_1(f, \theta) = \frac{1}{\pi} \left[\frac{1}{2} + \frac{2}{3} (A_1 \cos \theta + B_1 \sin \theta) + \frac{1}{6} (A_2 \cos 2\theta + B_2 \sin 2\theta) \right] \quad (10)$$

The results of angular spreading from Equation (10) indicate a wide spreading of energy and large angular resolution as shown by Kinsman.²²

Values of the weight parameter, W_i , have been proposed by other investigators. Mitsuyasu and his colleagues¹⁸, based on their field data obtained by a cloverleaf buoy data, proposed

$$H_2(f, \theta) = \frac{1}{\pi} \left[\frac{1}{2} + \sum_{n=1}^4 W_n (A_n \cos n\theta + B_n \sin n\theta) \right] \quad (11)$$

with $W_1 = 8/9$, $W_2 = 28/45$, $W_3 = 56/165$ and $W_4 = 14/99$. Here, the cloverleaf buoy not only measured surface slope but also surface curvature. The Fourier coefficients can be evaluated up to the fourth order harmonics. However, when adapted to second order Fourier expansions, only the first two weighting parameters can be used. This will slightly alter the spreading. Recently, LeBlanc and Middleton²³ proposed a special weight function to improve the resolution, based on some field measurements. They proposed $W_1 = 0.781$ and $W_2 = 0.348$, with the angular resolution of 110 degrees. They indicated these weight parameters can eliminate some of the minor and negative lobes in the angular spreading.

Parametrical Models

Another technique for developing the spreading function is based on the parameters which are determined from the measured data. Several directional wave models are then proposed based on these parameters. For a wave system of uni-modal directional spreading, Longuet-Higgins et al.¹⁶ assumed the form

$$H_3(f, \theta) = G(s) \cos^{2s(f)} \frac{1}{2} (\theta - \bar{\theta}) \quad (12)$$

where G is a normalization factor such that Equation (2) holds and which implies that

$$G(s) = 2^{2s-1} \cdot \frac{\Gamma(s+1)}{\Gamma(2s+1)} \quad (13)$$

where Γ is a gamma function and where s and $\bar{\theta}$ are the exponential spreading factor and mean wave approach angles, respectively.¹⁸ These two parameters are determined from the Fourier coefficients. The mean direction, $\bar{\theta}$, is defined by

$$\theta_1 = \tan^{-1} (B_1/A_1) \quad (14)$$

or

$$\theta_2 = \frac{1}{2} \tan^{-1} (B_2/A_2) = \theta_1 \text{ with an ambiguity of } \pm \frac{\pi}{2} \quad (15)$$

Long¹⁹ showed that the values of s are determined from

$$s_1 = K_1/(1 - K_1) \quad (16)$$

or

$$s_2 = \left[\frac{1 + 3K_2 + (K_2^2 + 14K_2 + 1)^{1/2}}{2(1 - K_2)} \right] = s_1 \quad (17)$$

where $K_1 = (A_1^2 + B_1^2)^{1/2}$ and $K_2 = (A_2^2 + B_2^2)^{1/2}$. Another commonly used parameter is the root mean square (rms) spreading angle, θ_p , which is defined by

$$\theta_p = (2 - 2K_1)^{1/2} \quad (18)$$

The values of s_1 and θ_1 are equal to s_2 and θ_2 in single peak spectra and are different in bi-modal spectra. Large values of s tend to narrow the directional distribution. The values of s have been correlated to local wind and wave frequency by Mitsuyasu et al.¹⁸ and Hasselmann et al.²⁴

Other Spreading Functions

For a fully developed sea, Pierson and his colleagues²⁵ proposed the spreading function

$$H_4(\theta) = \frac{2}{\pi} \cos^2(\theta - \bar{\theta}) \quad (19)$$

Furthermore, the spreading function has been modified and applied in the Spectral Ocean Wave Model (SOWM) of Fleet Numerical Oceanography Center (FNOC).^{9,26} The spreading function is related to the mean wind direction, θ_* , as

$$H_5(f, \theta) = \frac{1}{\pi} \{ [1 + [0.5 + 0.82 e^{-1/2 f^4}] \cos 2(\theta - \theta_*) + 0.32 e^{-1/2 f^4} \cos 4(\theta - \theta_*)] \} \quad (20)$$

for $-\frac{\pi}{2} < (\theta - \theta_*) < \frac{\pi}{2}$ and zero otherwise, and where f is the dimensionless frequency, defined by

$$f = \frac{\omega u}{g} \quad (21)$$

where ω is radian wave frequency and u is local wind speed. For small values of ω , H_5 becomes

$$H_5(0, \theta) = \frac{1}{\pi} [1 + 1.32 \cos 2(\theta - \theta_*) + 0.32 \cos 4(\theta - \theta_*)] \quad (22)$$

Equation (22) can also be approximated for low frequencies as

$$\begin{aligned} H_5(0, \theta) &\approx \frac{8}{3\pi} \cos^4(\theta - \theta_*) \\ &\approx \frac{8}{3\pi} \left[1 + \frac{4}{3} \cos 2(\theta - \theta_*) + \frac{1}{3} \cos 4(\theta - \theta_*) \right] \end{aligned} \quad (23)$$

Another interesting observation is that Equation (23) also approaches Equation (12) with the values of $2s = 16$.

Based on some field data, Forristal and his co-workers¹⁰ proposed a Fourier series of the spreading function as

$$H_6(f, \theta) = \frac{1}{\pi} \left[\frac{1}{2} + \sum_{n=1}^{\infty} c_n \cdot \cos(n\theta) \cdot \cos(n\theta) + \sum_{n=1}^{\infty} c_n \cdot \sin(n\theta) \cdot \sin(n\theta) \right] \quad (24)$$

where

$$c_n = r^2(s+1)/[\Gamma(s+n+1)\Gamma(s-n+1)] \quad (25)$$

The function $H_6(f, \theta)$ is intended to be fitted to Equations (12) and (13). In practice, the values of c_1 and c_2 can be computed from:

$$c_1 = s/(s+1) = \pi(A_1^2 + B_1^2)^{1/2} \quad (26)$$

$$c_2 = s(s-1)/(s+1)(s+2) = \pi(A_2^2 + B_2^2)^{1/2} \quad (27)$$

This model has been used to increase the statistical reliability of the results by combining several instruments measured in the same location.

MEASUREMENT OF DIRECTIONAL WAVES BY AN ORBITAL FOLLOWING BUOY

The development of directional wave spectra entails four efforts: deploying the buoys; collecting the data; editing the data; and running the data through the analysis routines, resulting in spectral densities, mean directions and spreading.

MEASURING PROCEDURE

Upon deployment of a buoy, it may be allowed to free drift, it may be moored, or it may be tethered to the ship. In the free drift mode, a navigational fix is made at the position of deployment. The buoy then free floats completely isolated from the ship with only wind, wave, and current forces acting on it. In this mode, the ship is free to perform other activities, such as maneuvering and seakeeping trials. When these activities are completed, the buoy is retrieved and another fix is determined. These two fixes allow one to calculate the average velocity of the buoy due to the wind and current.

In the moored mode, the buoy is limited in travel to its watch circle, or horizontal radius of its mooring line around the anchor. The buoy is moored via a special mooring accumulator obtained from ENDECO and an anchor chain. The accumulator consists of an elastic cord encased by braided nylon rope. The accumulator can elongate up to three times its static length before the rope becomes taut. Therefore the abrupt forces acting on a buoy at the end of its line are reduced. In this mode, a ship is also free to perform other activities while collecting wave data. Mooring is limited to relatively shallow water and is not practical in the open ocean. However, DTNSRDC has moored these buoys, in conjunction with other investigators' instruments, in water depths of up to 600 meters.

In the third mode, the buoy is tethered to the ship using the mooring accumulator and any additional line necessary. This method provides for an easy retrieval of the buoy. It is not necessary to maneuver close to the buoy and try to grab its flag line during retrieval. When the seas are rough, this may be the only way of practically deploying the buoy. The ship would not have to track down and maneuver close to the buoy while battling rough weather. However, in this case as well as in more benign conditions, it is desired that the ship try to maintain a slack line between the buoy and ship.

The three modes of deployment provide some latitude in collecting wave directional data for various conditions. However, it may be necessary to interpret the results to account for mooring or drifting.

DATA COLLECTION

When the buoy is deployed, the wave height and direction data are collected via telemetry and onboard solid state memory. An overview of this procedure of data collection is shown in Figure 5.

The data collected through telemetry is transmitted via FM radio link from the buoy to the receiver onboard the ship. Once the data reaches the receiver, it is output in digital and analog form. The analog data is recorded on analog tape for a backup and in case the user wants to redigitize at different sampling rates or wants to apply analog filters. The receiver also digitizes the data at a rate of one hertz, which is then transferred to floppy disk via a PDP 11/23 mini-computer. An advantage of collecting the data on the computer is that it is then possible to analyze the data in near real time while still on board the ship.

Telemetry is the standard method of data collection provided the ship, i.e., the receiver, is within transmission range. In situations where the ship is out of range, the primary collection medium is the solid state memory located with the other instrumentation inside the buoy sphere. The data is sampled at 1 hertz, which allows 42 runs of approximately 17 minutes each to be collected before filling the memory modules. Likewise, 20 runs of about 34 minutes or 10 runs of approximately 68 minutes will fill the memory. The different run lengths cannot be combined without resetting the electronics, but the electronics can be set to collect runs back to back or with intervals of up to 68 hours.

Under most circumstances, the simultaneous collection of data via the solid state memory and telemetry provides the redundancy to ensure satisfactory quality of data.

DATA EDITING

Once collected, whether through telemetry or solid state memory, it becomes necessary to transform and edit the data. Because of the nature of the buoy, the two direction channels measure the vertical tilt of the buoy. Since conventional analyses require buoy slope, the vertical tilt data must be converted to slope data.

At this stage of development, the conversion from tilt to slope is based on water particle displacements and the wave profile. It does not consider the hydrodynamic response of the buoy to wave motion. This will be presented at a later date.

The horizontal particle displacement at depth z is represented by:

$$\zeta = \frac{a \cosh k(d-z)}{\sinh(kd)} \sin(kx - \omega t) \quad (28)$$

where a is the wave amplitude

d is the water depth

z is the depth below the surface

ζ is the horizontal displacement due to the wave particle velocity at the depth of z

The change in the horizontal displacement due to the wave particle velocity with respect to depth, which is related to tilt angle, is

$$\frac{\partial \zeta}{\partial z} = ak \frac{\sinh k(d-z)}{\sinh(kd)} \sin(kx - \omega t) \quad (29)$$

Now, the wave profile is represented by

$$\eta = a \cos(kx - \omega t) \quad (30)$$

and the wave slope follows as:

$$\frac{\partial \eta}{\partial x} = -a k \sin(kx - \omega t) \quad (31)$$

If we assume linear wave theory, then Equations (29) and (31) can be combined to relate the measured tilt angle to the desired wave slope as

$$\frac{\partial \zeta}{\partial z} = - \left(\frac{\partial \eta}{\partial x} \right) \frac{\sinh k(d-z)}{\sinh(kd)}$$

or rearranging,
$$\frac{\partial \eta}{\partial x} = - \frac{\partial \zeta}{\partial z} \frac{\sinh(kd)}{\sinh k(d-z)} \quad (32)$$

where $\frac{\partial \zeta}{\partial z} = \tan \phi$ for the Wave Track buoy and where ϕ is the vertical tilt angle.

Initially, the tilt angles are converted to surface slope in the raw data file. Once the auto and cross spectra have been computed for the spectral analysis, the depth effect, i.e., hyperbolics, are applied if necessary.

The phase correction and the magnetic declination are also applied in this process. Phase corrections must be applied since wave buoys do not follow waves precisely, nor do their electronics respond instantly. Hydrodynamic and electronic lags are inherent. At this time there are no hydrodynamic phase corrections to the buoy known to the authors. However, the manufacturer, ENDECO, has provided the authors with electronic phase corrections that were developed from rotating arm tests. Ideally, the corrections consist of electronic phase lags of the heave

along with the effect of buoy tilt on the heave. The magnetic declination is a user input value that describes the compass deviation from true north at the global position during actual buoy measurements.

ANALYSIS PROGRAM

As mentioned earlier, the analysis scheme offered on these pages deals with buoy transformations, corrections and declinations prior to the directional computations, but following the calculation of auto and cross spectra. The software, which uses a Fast Fourier Transformation (FFT) to calculate the auto and cross spectra, was developed by Pierce at DTNSRDC. These details of the procedure will not be presented here, but some of its characteristics include: (i) the data is broken down into segments of a power of two, based on user input, (ii) a cosine window is applied to each segment, and (iii) the segments are overlapped by 50 percent.

The flow chart of the directional wave program can be found in the Appendix. The program has two basic options. The first is to calculate the spectral density, the Fourier coefficients of the directional distribution, mean directions based on the first and second harmonic components, and rms spreading, among other useful information. The second option is to calculate the directional spreading of wave energy of each frequency. In this option, a choice of methods of spreading is offered between $\cos^2 \frac{1}{2} (\theta - \bar{\theta})$, a cosine Fourier series and the Longuet-Higgins method as mentioned earlier, three different weighting parameters are available to minimize or eliminate leakage in the Longuet-Higgins method. Further assumptions, corrections and errors will be discussed presently.

COMPUTED RESULTS - A SAMPLE

In accordance with the two basic options, two displays of the results are outputted. The first display, as seen in Table 1, presents basic information, spectral components, and directions.

The Fourier components, A_0 , A_1 , B_1 , A_2 , B_2 ; the mean direction and principal angle, i.e., first and second harmonics, respectively; and the root mean square spreading about the mean direction are computed for each frequency. Furthermore, the significant wave height, the spectral bandwidth parameter, the broadness parameter, and the average period are derived from the spectrum. The significant wave height, $(\tilde{H}_w)_{1/3}$ is defined as

$$(\bar{n}_w)_{1/3} = 4.0 \sqrt{m_0} \quad (33)$$

where m_0 , the area under the curve of the wave spectrum, is the zero-moment of the spectrum,

$$m_0 = \int_0^\infty S_\eta(\omega_w) d\omega_w$$

The spectral bandwidth parameter and the broadness parameter are two methods of depicting the broadness of the spectrum. The spectral bandwidth parameter,

$$v_0^2 = \frac{m_0 m_2}{m_1^2} - 1 \quad (34)$$

and the broadness parameter

$$\epsilon^2 = \frac{m_0 m_4 - m_2^2}{m_0 m_4} \quad (35)$$

each depend on the moments of the spectrum, where m_n is the n th moment of the spectral density

$$m_n = \int_0^\infty S_\eta(\omega_w) \omega_w^n d\omega_w$$

The average zero crossing period is defined as

$$T_z = \left(\frac{m_0}{m_2} \right)^{1/2} \quad (36)$$

If current effects, which will be discussed later, are brought into consideration, the spectrum is modified. The Jacobian transformation associated with current changes the spectral densities, while the frequency becomes stationary rather than encountered. Only the frequencies and the spectral densities have been changed, the other spectral components, the directions, and rms spreading have not. If the current effects are accounted for, the frequency bandwidths no longer are necessarily constant.

Besides the spectral information, statistical information pertaining to the spectrum is presented. Segments refers to the number of individual segments processed. The run length (sec) is the total time length of the run in seconds. Iskip refers to the number of data points skipped between selected data points, e.g., 1 means every data point is analyzed, 2 means every other data point is analyzed, etc. This option allows for a choice in effective sample rate. The effective sample rate is output next. The segment size is the number of data points in each segment based on the effective sample rate. The number of degrees of freedom is useful in calculating the confidence level of the results. The statistical bandwidth is also presented. Finally, the mean and standard deviation of the time history are presented for each channel. Also, the standard deviation of the spectral data for each channel accompany those of the time history.

The second display, as seen in Table 2, presents the spreading, based on any of the three earlier mentioned techniques. The energy is distributed by direction and frequency. The data are variances relative to direction, but are densities relative to frequency. This gives the units of length-squared/hertz or length-squared-seconds. An optional method of viewing a portion of the results is available in graphic form, as seen in Figure 6. This is operational for frequency versus energy densities, mean direction and rms spreading. The graphic output of energy spreading of Table 2 is not yet operational.

DISCUSSION

LINEAR WAVE THEORY AND NONLINEAR EFFECTS

One of the assumptions made during the process of data analysis of the Wave-Track buoy is the linear wave theory. The buoy responds to the wave particle velocities beneath the water surface and traces the particle displacement. From the linear wave theory, the rate of change of tilt angle can be related to the surface elevation as indicated in Equation (32).

The verification of linear theory of wave particle velocities has been reported in papers based on the results of field investigations.^{10,27,28} According to a recent paper by Battjes and Heteren,²⁸ the particle velocity, based on linear theory predictions from measured surface waves and the direct field measurements of particle velocity agree within ± 5 percent in both amplitude and phase. The agreements are validated for various sea states such as wind waves from young seas to old swells. However, no agreement in range has been found for other sea states

in which the nonlinear characteristics become stronger. Several ocean environments are linked to this condition. Common phenomena related to strong nonlinear interactions in the field are breaking waves, waves encountering a strong current, or waves entering a shoaling zone in a shallow water zone.

Breaking waves are caused by strong wind, by wave-current interaction, and by interaction with the bottom topography in the nearshore zone. When waves break, the particle velocity at the peak increases and overtakes the propagating wave. The surface elevation forms a discontinuous profile and generates a strong turbulent vortex under the water. Since breaking is such a nonlinear event, there is no clear way to describe this process. The effect of buoy response to the breaking has not been taken into account in the analysis. If the breaking is caused by a strong and steady wind while the wind and waves follow the same direction, the Wave-Track buoy's accuracy of the measurement of wave heights may suffer but not the wave approach angles.

In general, wave characteristics show weak nonlinearities in open water. The assumption of a linear model applied to the computation of wave direction generates small percentages of error. As the waves approach shallow water, or strong currents, the characteristics of nonlinearity increases. The effects of water depth, currents and other wave-wave interaction systems will be discussed in the following paragraphs.

Current Effects

When the buoy is deployed in a free drift mode, it may drift with the current or wind, provided either exist. When this occurs, the buoy is measuring waves in a moving coordinate system. If it is desired to know the wave environment in a fixed coordinate system, a spectral transformation must be performed. In transforming the ocean wave energy from a moving, or encountered, spectrum to a fixed, or ordinary, wave spectrum, the principle of conservation of wave energy must be applied. That is to say,

$$\phi(\omega_e, \theta) d\omega_e = \phi(\omega, \theta) d\omega \quad (37)$$

Given
$$\left(\frac{\partial \omega_e}{\partial \omega} \right) d\omega = d\omega_e \quad (38)$$

and substituting this into (37), we get,

$$\phi(\omega_e, \theta) \left(\frac{\partial \omega_e}{\partial \omega} \right) d\omega = \phi(\omega, \theta) d\omega \quad (39)$$

where $\phi(\omega_e, \theta)$ is the wave spectrum in the encountered frequency domain, $\phi(\omega, \theta)$ is the wave spectrum in the fixed frequency domain, ω_e is the encountered frequency expressed in radians per second, θ is the angle between the buoy drift and wave propagating directions, and ω is the angular frequency in the fixed coordinate system.

The Jacobian of the transformation, $\frac{\partial \omega_e}{\partial \omega}$, can be determined from the dispersion relation

$$\omega_e = \omega - kU \cos \theta \quad (40)$$

and the angular wave frequency,

$$\omega^2 = kg \tanh(kd) \quad (41)$$

where U is the relative velocity between the moving and fixed coordinate systems, and d is the water depth. Now, if we substitute equation (41) into (40) and differentiate with respect to ω , we get the following

$$\omega_e = \omega - \frac{\omega^2 U \cos \theta}{g \tanh(kd)} \quad (42)$$

and

$$\frac{\partial \omega_e}{\partial \omega} = 1 - \left[\frac{2\omega U \cos \theta}{g \tanh(kd)} - \frac{\omega^2 d \cos \theta}{g \sinh^2(kd)} \cdot \frac{\partial k}{\partial \omega} \right] \quad (43)$$

The inverse of the last term on the right hand side of equation (43) can be found by implicitly differentiating equation (41) with respect to k ,

$$2\omega \frac{\partial \omega}{\partial k} = g \tanh(kd) + kgd \frac{1}{\cosh^2(kd)} \quad (44)$$

Inverting both sides of the equation, applying a little algebra and a trigonometric identity yields,

$$\frac{\partial k}{\partial \omega} = \frac{2\omega}{g} \left[\frac{2 \cosh^2(kd)}{\sinh(2kd) + 2kd} \right] \quad (45)$$

Substituting equation (45) back into equation (43) gives us,

$$\frac{\partial \omega_e}{\partial \omega} = 1 - \left[\frac{2\omega U \cos \theta}{g \tanh(kd)} - \frac{\omega^2 U \cos \theta}{g \sinh^2(kd)} \cdot \frac{2\omega}{g} \cdot \frac{2 \cosh^2(kd)}{\sinh(2kd) + 2kd} \right] \quad (46)$$

Finally, by substituting equation (41) into equation (46) and manipulating (46) algebraically, we get the Jacobian, for any water depth as

$$\frac{\partial \omega_e}{\partial \omega} = 1 - \frac{2\omega U \cos \theta}{g \tanh(kd)} \left[1 - \frac{2kd}{\sinh(2kd) + 2kd} \right] \quad (47)$$

For deep water, $kd > \pi$, the term in the bracket approaches unity, as does the hyperbolic tangent. The Jacobian of the transformation in deep water then becomes,

$$\frac{\partial \omega_e}{\partial \omega} = 1 - \frac{2\omega U \cos \theta}{g} \quad (48)$$

The angular wave frequency, in the fixed coordinate system, can be determined from the encountered angular wave frequency. Rearranging equation (42), we can find ω in terms of ω_e by

$$b\omega^2 + \omega - \omega_e = 0 \quad (42b)$$

where $b = -\frac{U \cos \theta}{g \tanh(kd)}$

The quadratic of ω is

$$\omega = \frac{-1 \pm \sqrt{1 + 4b\omega_e}}{2b} \quad (49)$$

As mentioned earlier, current effects may be included in the first analysis section with mean wave directions, but has not yet been implemented in the second

section of energy spreading. However, this method has been applied in other investigations of wave-current interaction¹² and some of the computed results are shown in Figure 7.

Effect of Water Depth

Waves in finite water depths affect wave measuring platforms such as the ENDECO Wave-Track buoy. Also known as an orbital following buoy, it derives wave directional measurements from tilts induced by the wave orbits. Since wave orbits are affected by water depth, it becomes necessary to account for the depth effect when converting the tilt data to slope form.

In deep water, the depth effect is neutralized when the Fourier coefficients become normalized, as per equations (6) and (7). Meanwhile, the dispersion relation of equation (5) indicates that $(2\pi f)^2/gk$ is equal to $\tanh(kd)$ and approaches unity. In a finite water depth the normalized Fourier coefficients also neutralized the depth effect of hyperbolic functions in (32). However, the relation of $(2\pi f)/gk$ no longer approaches unity and the correction becomes necessary. The correction will not change the values of mean direction, but will reduce the rms values of spreading angle. This result is consistent with the general observation that the waves become narrow banded as they approach the shore.

BIMODAL AND MULTIMODAL WAVE SPECTRA

The ocean is a dynamic system. As the wind blows along the ocean surface, waves are generated at different locations and propagate in various directions. The combination of two or more wind wave systems generate bimodal or multimodal wave spectra. Bimodal sea systems are enhanced by the passage of a storm or a rotating wind^{29,30} (see Figure 8). The appearance of bimodal wave spectra have been estimated to occur in about 20 to 30 percent of the measured field data.³¹ The multimodal wave spectra also represent an important surface wave mechanism.

The superposition of two wave systems, which originate from two different storms, is a common technique applied to estimate the bimodal wave spectrum.³¹ Based on field data, several models have been developed to fit the bimodal wave spectra and to investigate the wave mechanisms.^{20,32} The mechanisms are complex and must take into account the wind-wave and wave-wave interactions of two-dimensional waves. The results have not been conclusive.

When the buoy is operating in the environment of a bimodal wave system, it responds to the dynamics of the whole system. The analysis of the measured data in this environment requires the knowledge of the interaction mechanisms of the two wave systems. The corresponding hydrodynamic response of the buoy can then be determined and the results can be estimated. Currently, bimodal wave spectra are not immediately analyzed as such. The data are analyzed without any assumption of the specific wave model. The procedure can then be modified once the interaction mechanisms of bimodal wave systems have been developed.

ERROR CONSIDERATIONS

Errors from mechanical and electrical configurations and components in the measurement of wave height are considered to be manageable or negligible compared to errors attributed to statistical errors and environmental effects. However, the same cannot be said of wave direction measurement. The combined effects of the inclinometers, compass, their alignments and sensitivities produce a wave direction accuracy of ± 10.0 degrees for the buoy associated with this analysis, as specified by the manufacturer, ENDECO.

Statistical errors develop during the analysis of the data. The receiver and the buoy's solid state memory sample the data at a rate of one hertz. This sampling rate provides an un-aliased upper frequency limit of 0.5 hertz, though our interest lies in frequencies below 0.33 hertz. When digitized from analog tape, the sample rate is not limited to one hertz, but a practical limit of four hertz has been imposed. The resolution of the receiver and digitizer in the solid state memory yields less than an 0.4 percent uncertainty.

Ultimately, the statistical uncertainty, or confidence limits, can be determined from the number of degrees of freedom. The degrees of freedom is a function of the number of segments overlapped, the sample rate and the total time length of the run. The number of degrees of freedom increase with sample rate and with the length of the run, but decrease with the size of the individual segments. Given the number of degrees of freedom, the confidence limits is determined from a chi-square graph.

Perhaps the greatest errors that can develop occur from environmental effects, such as shallow water depth, current, and mooring. The depth effect is eliminated through normalization using the dispersion relation, as previously mentioned. The current effect was also discussed earlier. However, at the time of this

writing, there has been no quantification to evaluate the current effect on the waves. The theoretical transformation of the waves is applied in the frequency domain in this analysis.

During mooring conditions, buoy motions are limited by a mooring cable connected to a weight and anchor. The abrupt limiting motion that normally occurs with a synthetic rope is reduced when using an accumulator cable, such as that supplied with the Wave-Track buoy.

The effects of an accumulator type cable on the motions of a buoy such as the Wave-Track buoy was evaluated using a computer model. Some results of the model, presented by Brainard and Wang³³, indicate that the maximum difference of the buoy surge and heave motions relative to the wave motion are about 10 percent and 1 percent of the wave heights, respectively. Buoy peak-to-peak pitch angles range from 1.20 to 1.86 times the double wave slope and depend on current velocity, depth, length of mooring line, wave height and period. There is an indication of a mean pitch angle offset which tends to be greater in stronger currents. This bias, though, can be eliminated in analysis.

Since the model deals in regular waves, no single correction can adequately be applied to a buoy in random seas. However, these values indicate that the buoy closely follows waves in the heave motion.

In the future, a comparison of a buoy free-floating and a buoy moored is intended to quantify the effects of a mooring on a buoy in the field under specified conditions.

CONCLUDING REMARKS

The adequate verification of the wave directionality produced by the SOWM, and the application of measured directional spreading to ship motions during sea trials is now possible given the ability to measure the directional spreading of wave energy. A lightweight, easy-to-handle directional wave sensing ENDECO buoy has been employed during dedicated trials and ships of opportunity to measure the sea conditions. The Wave-Track buoy measures the directional aspects of the wave by sensing the orbital velocities, as opposed to the wave slope measured by a conventional slope following buoy. The nature of the buoy measurements require that the data be converted into a convention of wave slope directional analysis. Therefore, the analysis as outlined in this report was developed to determine the directional characteristics of ocean waves as measured through the wave orbits.

Uncertainties exist in the low frequency range. A closer look at the electro-nic phases and possibly hydrodynamic phases may yield greater confidence in this range. Also, the transition of wave orbits in shallow water may affect the tilt angles and phases. This effect is currently under study and will be reported on at a later date.

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APPENDIX

FLOWCHART OF DIRECTIONAL WAVE ANALYSIS PROGRAM

Figure 1 - Wave measurement in finite water depths

Figure 2 - Waves crossing a front during the SEBEX experiment

Figure 3 - ENDECO Type 956 Wave Track buoy

ORBITAL FOLLOWING BUOY

SLOPE FOLLOWING BUOY

Figure 4 - Principle of the ENDECO buoy

Figure 5 - Data collection procedure

Figure 6 - Graphic Form of Energy Densities, Mean Directions and RMS Spreading of Table 1

Figure 7 - Wave/current interaction during the SEBEX experiment
Waves/in (a) fixed coordinate and (b) moving coordinate systems

Figure 8 - Bimodal wave spectrum in shallow water during storm driven seas.

TABLE 1 - Sample Output of Directional Wave Spectrum

DIRECTIONAL WAVE SPECTRA AND FOURIER COEFFICIENTS

DATE AND TIME OF RUN : 14-JUL-82 AT 11:45

DATE PROCESSED : 28-MAY-85

SEBEX RUN NO. X7D
 SEGMENTS = 16. RUN LENGTH (SEC) = 1088.000 ISKIP = 1 EFFECTIVE SAMPLE RATE = 2.000 HERTZ
 SEGMENT SIZE = 256 NO. DEGREES OF FREEDOM = 32 STATISTICAL BANDWIDTH = 0.016

COMMENTS : BUOY IN FREE DRIFT MODE

SUMMARY

HEAVE	TH. MEAN	TH. STD. DEV.	PSD. STD. DEV.
	-4.83348E-02	7.27473E-01	7.06175E-01
N-S SLOPE	1.26573E-03	6.59533E-02	6.59914E-02
E-W SLOPE	5.26705E-03	4.02144E-02	3.86603E-02

SPECTRAL COEFFICIENTS

FREQ	A0	A1	B1	A2	B2	MEAN DIR (deg)	PRINC ANG (deg)	SPREAD (deg)	PERIOD (sec)
0.047	8.628E-02	-3.116E-01	1.100E-01	7.084E-01	7.309E-02	327.6	-10.1	66.3	21.3
0.055	1.111E-01	-1.886E-01	1.154E-01	7.102E-01	-1.415E-01	315.5	-18.6	71.5	18.2
0.063	4.932E-02	1.486E-02	-7.053E-02	7.756E-01	-1.242E-02	88.9	166.5	78.1	15.9
0.070	2.169E-01	2.890E-01	-3.987E-02	8.491E-01	2.564E-02	159.1	167.9	68.2	14.3
0.078	3.821E-01	2.751E-01	-4.793E-02	8.254E-01	-5.658E-02	157.1	165.0	68.8	12.8
0.086	2.109E+00	1.971E-01	-4.955E-02	7.832E-01	-7.867E-02	152.9	164.1	72.3	11.6
0.094	3.139E+00	1.978E-01	-3.332E-02	8.785E-01	-6.661E-02	157.4	164.8	72.4	10.6
0.102	3.107E+00	1.054E-01	-1.923E-02	9.205E-01	-9.581E-02	156.7	164.0	76.6	9.8
0.109	1.834E+00	1.381E-01	-4.349E-02	8.140E-01	-6.032E-02	149.5	164.9	74.9	9.2
0.117	1.951E+00	-1.983E-02	-1.026E-01	6.935E-01	6.618E-02	66.1	-10.3	76.7	8.5
0.125	1.771E+00	6.498E-02	6.314E-02	6.507E-01	2.383E-01	211.2	177.1	77.3	8.0
0.133	1.693E+00	3.230E-01	3.731E-02	6.825E-01	1.110E-01	173.6	171.6	66.6	7.5
0.141	2.574E+00	2.376E-01	2.856E-02	6.018E-01	-1.167E-01	173.9	161.5	70.7	7.1
0.148	3.104E+00	2.610E-01	6.021E-02	6.569E-01	1.967E-01	180.0	175.3	69.3	6.8
0.156	4.902E+00	3.818E-01	1.794E-01	6.222E-01	4.377E-01	192.2	184.6	61.6	6.4
0.164	6.466E+00	3.291E-01	3.232E-01	2.982E-01	3.837E-01	211.5	193.1	59.5	6.1
0.172	7.900E+00	4.445E-01	3.628E-01	1.859E-01	2.088E-01	206.2	191.2	52.9	5.8
0.180	5.064E+00	5.835E-01	3.022E-01	3.816E-01	2.709E-01	194.4	184.7	47.5	5.6
0.188	2.454E+00	4.862E-01	3.028E-01	4.303E-01	1.056E-01	198.9	173.9	53.0	5.3
0.195	1.855E+00	6.095E-01	3.089E-01	4.835E-01	2.025E-01	193.9	178.4	45.6	5.1
0.203	2.447E+00	6.454E-01	3.539E-01	4.034E-01	3.485E-01	195.7	187.4	41.6	4.9
0.211	1.901E+00	6.069E-01	3.288E-01	5.222E-01	2.331E-01	195.4	179.0	45.1	4.7
0.219	1.858E+00	5.227E-01	4.994E-01	2.416E-01	4.370E-01	210.7	197.5	42.7	4.6
0.227	1.157E+00	3.568E-01	4.389E-01	2.935E-01	3.183E-01	217.9	190.7	53.4	4.4
0.234	6.744E-01	3.821E-01	3.144E-01	3.845E-01	4.066E-01	206.4	190.3	57.6	4.3
0.242	5.866E-01	3.834E-01	2.476E-01	4.215E-01	1.729E-01	199.9	178.2	59.7	4.1
0.250	6.748E-01	4.362E-01	2.220E-01	5.130E-01	-9.168E-02	194.0	161.9	57.9	4.0
0.258	5.610E-01	3.519E-01	2.053E-01	2.959E-01	-2.265E-03	197.3	166.8	62.4	3.9
0.266	4.024E-01	2.399E-01	1.023E-01	4.249E-01	-1.057E-01	190.1	160.0	69.7	3.8
0.273	4.424E-01	3.780E-01	1.549E-01	6.201E-01	2.090E-01	189.3	176.3	62.3	3.7
0.281	3.714E-01	3.716E-01	4.451E-02	7.347E-01	-4.542E-02	173.8	165.2	64.1	3.6
0.289	3.539E-01	8.899E-02	3.174E-02	5.297E-01	-1.455E-01	186.6	159.3	77.1	3.5
0.297	4.539E-01	-3.518E-02	1.019E-01	3.309E-01	1.645E-02	276.0	-11.6	76.5	3.4

SIGNIFICANT WAVE HEIGHT IS 2.8 FEET
 THE SPECTRAL BANDWIDTH PARAMETER IS 0.080
 THE AVERAGE PERIOD IS 6.1 SEC

NOTE : WAVES ARE COMING FROM THE MEAN DIRECTION.
 EPSILON IS 0.233

TABLE 2 - Sample Output of Directional Spreading

DIRECTIONAL SPREADING (FEET SQ-SEC)												
SEBEX . RUN NO. X7D												
WEIGHTING COEFFICIENTS : W1 = 0.889 W2 = 0.622												
DATE AND TIME OF RUN : 14-JUL-82 AT 11:45												
COMMENTS - In : BUOY IN FREE DRIFT MODE												
FREQUENCY (hertz)												
0.047 ;	0.016	0.011	0.003	0.000	0.000	0.006	0.011	0.010	0.006	0.004	0.008	0.014
0.055 ;	0.021	0.017	0.007	0.001	0.003	0.010	0.014	0.010	0.004	0.001	0.007	0.016
0.063 ;	0.008	0.007	0.003	0.000	0.001	0.005	0.008	0.007	0.003	0.000	0.001	0.005
0.070 ;	0.029	0.027	0.014	0.005	0.013	0.032	0.044	0.036	0.012	0.000	0.000	0.014
0.078 ;	0.053	0.053	0.029	0.013	0.024	0.055	0.074	0.058	0.018	0.000	0.000	0.025
0.086 ;	0.308	0.299	0.167	0.068	0.119	0.275	0.373	0.298	0.099	0.000	0.000	0.161
0.094 ;	0.487	0.457	0.229	0.066	0.159	0.426	0.591	0.465	0.139	0.000	0.000	0.253
0.102 ;	0.538	0.486	0.225	0.035	0.117	0.386	0.558	0.438	0.122	0.000	0.000	0.303
0.109 ;	0.284	0.265	0.135	0.040	0.084	0.227	0.321	0.261	0.091	0.000	0.002	0.158
0.117 ;	0.288	0.256	0.119	0.011	0.041	0.184	0.303	0.288	0.159	0.048	0.065	0.189
0.125 ;	0.214	0.155	0.041	0.000	0.069	0.222	0.319	0.283	0.159	0.066	0.083	0.169
0.133 ;	0.179	0.161	0.083	0.052	0.131	0.269	0.340	0.272	0.113	0.000	0.000	0.100
0.141 ;	0.331	0.325	0.216	0.147	0.213	0.354	0.416	0.307	0.100	0.000	0.012	0.189
0.148 ;	0.319	0.258	0.105	0.053	0.211	0.475	0.620	0.513	0.246	0.043	0.052	0.210
0.156 ;	0.321	0.157	0.000	0.007	0.409	0.916	1.152	0.949	0.495	0.153	0.120	0.269
0.164 ;	0.303	0.056	0.000	0.210	0.728	1.176	1.256	0.968	0.585	0.386	0.403	0.438
0.172 ;	0.311	0.125	0.168	0.584	1.174	1.540	1.427	0.954	0.489	0.310	0.379	0.436
0.180 ;	0.183	0.092	0.077	0.310	0.741	1.099	1.119	0.785	0.342	0.076	0.070	0.170
0.188 ;	0.166	0.121	0.087	0.163	0.339	0.492	0.489	0.320	0.108	0.000	0.037	0.131
0.195 ;	0.093	0.061	0.041	0.113	0.273	0.414	0.423	0.285	0.098	0.000	0.000	0.066
0.203 ;	0.061	0.004	0.000	0.134	0.375	0.572	0.584	0.408	0.175	0.036	0.029	0.070
0.211 ;	0.098	0.057	0.027	0.101	0.273	0.431	0.447	0.302	0.103	0.000	0.002	0.073
0.219 ;	0.033	0.000	0.000	0.086	0.286	0.422	0.408	0.278	0.145	0.092	0.104	0.100
0.227 ;	0.060	0.008	0.000	0.047	0.149	0.226	0.224	0.154	0.080	0.055	0.074	0.088
0.234 ;	0.032	0.006	0.000	0.018	0.075	0.129	0.142	0.109	0.061	0.033	0.033	0.041
0.242 ;	0.042	0.029	0.017	0.030	0.069	0.106	0.112	0.081	0.036	0.011	0.017	0.036
0.250 ;	0.066	0.062	0.046	0.052	0.088	0.124	0.123	0.076	0.015	0.000	0.000	0.040
0.258 ;	0.043	0.039	0.035	0.046	0.071	0.092	0.089	0.060	0.025	0.009	0.017	0.035
0.266 ;	0.045	0.043	0.032	0.029	0.040	0.056	0.059	0.041	0.016	0.001	0.009	0.030
0.273 ;	0.038	0.029	0.012	0.012	0.041	0.079	0.094	0.072	0.031	0.003	0.005	0.026
0.281 ;	0.045	0.044	0.026	0.018	0.033	0.061	0.073	0.054	0.015	0.000	0.000	0.021
0.289 ;	0.050	0.047	0.031	0.019	0.024	0.040	0.048	0.036	0.014	0.001	0.010	0.033
0.297 ;	0.055	0.045	0.027	0.019	0.028	0.045	0.053	0.045	0.030	0.024	0.034	0.049
TOTALS ;	5.121	3.801	2.002	2.490	6.402	10.947	12.314	9.222	4.133	1.353	1.573	3.958
	26	56	86	116	146	176	206	236	266	296	326	356
DIRECTION - FROM (degrees)												

DTNSRDC ISSUES THREE TYPES OF REPORTS

1. DTNSRDC REPORTS, A FORMAL SERIES, CONTAIN INFORMATION OF PERMANENT TECHNICAL VALUE. THEY CARRY A CONSECUTIVE NUMERICAL IDENTIFICATION REGARDLESS OF THEIR CLASSIFICATION OR THE ORIGINATING DEPARTMENT.

2. DEPARTMENTAL REPORTS, A SEMIFORMAL SERIES, CONTAIN INFORMATION OF A PRELIMINARY, TEMPORARY, OR PROPRIETARY NATURE OR OF LIMITED INTEREST OR SIGNIFICANCE. THEY CARRY A DEPARTMENTAL ALPHANUMERICAL IDENTIFICATION.

3. TECHNICAL MEMORANDA, AN INFORMAL SERIES, CONTAIN TECHNICAL DOCUMENTATION OF LIMITED USE AND INTEREST. THEY ARE PRIMARILY WORKING PAPERS INTENDED FOR INTERNAL USE. THEY CARRY AN IDENTIFYING NUMBER WHICH INDICATES THEIR TYPE AND THE NUMERICAL CODE OF THE ORIGINATING DEPARTMENT. ANY DISTRIBUTION OUTSIDE DTNSRDC MUST BE APPROVED BY THE HEAD OF THE ORIGINATING DEPARTMENT ON A CASE-BY-CASE BASIS.