UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA-CHAMPAIGN
BOOKSTACKS

Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

330
B385
no. 39
cop. 2

Faculty Working Papers

THE ALLOCATION OF RESOURCES IN STEADY-STATE UNBALANCED GROWTH

**Hans Brems
University of Illinois**

**College of Commerce and Business Administration
University of Illinois at Urbana-Champaign**

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana - Champaign

January 26, 1972

January 26, 1972

**THE ALLOCATION OF RESOURCES IN
STEADY-STATE UNBALANCED GROWTH**

**Hans Brems
University of Illinois**

No. 39

January 26, 1972

THE ALLOCATION OF RESOURCES IN
STEADY-STATE UNBALANCED GROWTH

By HANS BREMS*

With few exceptions, modern growth models are models of steady-state and balanced¹ growth of homogeneous consumption and capital stock, hence miss imbalance [1], [6] as well as the allocation of resources.

To allow for imbalance, a growth model needs at least two goods. But to allow for the allocation of resources, the two goods cannot be the consumers' good and the producers' good found in the usual [5] two-sector growth models. With only one consumers' good, such models are still models of homogeneous consumption, permitting no substitution among consumers' goods and asking no question, hence offering no answer, concerning the allocation of consumption

expenditure among consumers goods. With only one producers' good such models are still models of homogeneous capital stock, permitting no substitution among producers' goods and asking no question, hence offering no answer, concerning the allocation of investment expenditure among producers' goods.

We wish to build the simplest possible growth model of heterogeneous consumption as well as capital stock, thus allowing for the full allocation of resources. To do that we assume each of our two goods to serve interchangeably as a consumers' or as a producers' good: The physical output of the j th good is X_j where $j = 1, 2$. The j th good is produced from labor L_j and two immortal capital stocks S_{ij} where $i = 1, 2$. There are, then, four capital stocks S_{ij} and four investments I_{ij} in our model. Between two such industries we specify a fourfold interaction:

The two industries compete in their demand for labor. In the

labor market they must pay the same money wage rate w , a parameter. Goods prices P_j are variables, hence the real wage rate w/P_j is also a variable.

The two industries compete in their demand for investment goods. In the market for the j th good they must pay the same price P_j . A firm producing the j th good and setting aside part of its own output for investment I_{jj} should charge itself the price P_j as an opportunity cost.

The two industries compete in their demand for money capital. In the money-capital market the capitalist-entrepreneurs allocate their savings between the two industries such as to maximize the present worth of all their future profits.

The two industries compete in their supply of consumers' goods. In the consumers' goods market the two goods are good, but not perfect, substitutes, and each consumer has a taste for both of them.

FIGURE 1. THE EIGHT PHYSICAL FLOWS

Figure 1 shows all physical flows in our model. Section I defines variables and parameters. Section II specifies the model mathematically. Section III finds the equilibrium solutions for proportionate rates of growth. Section IV finds the equilibrium solutions for levels of variables. Certain proofs are banished to two appendices.

I. NOTATION

Variables

C $\equiv$ consumption

ϕ $\equiv$ function to be maximized by the Lagrange-multiplier method

g_v $\equiv$ proportionate rate of growth of variable v where $v \equiv C, I, L, P, S, X,$ and Y

I_{ij} $\equiv$ investment of output of i th industry in j th industry

κ_{ij} $\equiv$ physical marginal productivity of capital stock S_{ij}

L $\equiv$ labor employed

P $\equiv$ price of good
 S_{ij} $\equiv$ j th industry's physical capital stock of i th industry's good
 U $\equiv$ utility
 W $\equiv$ wage bill
 X $\equiv$ physical output
 Y $\equiv$ national money income
 Z $\equiv$ profits bill
 ζ $\equiv$ present worth of all future profits bills

Parameters

A $\equiv$ exponent of individual utility function
 α, β $\equiv$ exponents of production function
 c $\equiv$ propensity to consume national money income
 e $\equiv$ Euler's number, the base of natural logarithms
 F $\equiv$ available labor force
 g_p $\equiv$ proportionate rate of growth of parameter p where $p \equiv F, M,$

and w

- λ $\equiv$ Lagrange multiplier
- M $\equiv$ multiplicative factor of production function
- N $\equiv$ multiplicative factor of individual utility function
- r $\equiv$ discount rate applied by capitalist-entrepreneurs
- w $\equiv$ money wage rate

The parameters listed are stationary except F , M , and w , whose growth rates g_F , g_M , and g_w are stationary.

The symbol π_i to be defined in Section II; h_j in Section IV, 2; ρ , m , n , and μ_i in Section IV, 3; v_{ij} and ξ_i in Section IV, 5; and ψ in Appendix I all stand for agglomerations of parameters and variables. Symbols t and τ are time coordinates. Subscripts $i = 1, 2$ and $j = 1, 2$ refer to industry number. All flow variables refer to the instantaneous rate of that variable measured on a per annum basis.

• *Journal of the American Medical Association*, 1991; 265: 1001-1005

II. THE EQUATIONS OF THE MODEL

17 variable growth rates are listed in Section I, i. e. two growth rates of each of C_i , L_i , P_i , and X_i ; four growth rates of each of I_{ij} and S_{ij} ; and one growth rate of Y . To all apply the definition

$$(1) \text{ through } (17) \qquad g_v \equiv \frac{dv}{dt} \frac{1}{v}$$

Define investment as the derivative of capital stock with respect to time

$$(18) \text{ through } (21) \qquad I_{ij} \equiv \frac{dS_{ij}}{dt}$$

 

Let the j th industry apply the Cobb-Douglas production function

$$(22) \quad X_1 = M_1 L_1^{\alpha_1} S_{11}^{\beta_{11}} S_{21}^{\beta_{21}}$$

$$(23) \quad X_2 = M_2 L_2^{\alpha_2} S_{12}^{\beta_{12}} S_{22}^{\beta_{22}}$$

where $0 < \alpha_j < 1$; $0 < \beta_{ij} < 1$; $\alpha_1 + \beta_{11} + \beta_{21} = 1$; $\alpha_2 + \beta_{12} + \beta_{22} = 1$; and $M_j > 0$. In each industry let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

$$(24), (25) \quad \frac{w}{P_j} = \frac{\partial X_j}{\partial L_j} = \alpha_j \frac{X_j}{L_j}$$

Physical marginal productivities of capital at time t are

$$(26) \text{ through } (29) \quad \kappa_{ij}(t) \equiv \frac{\partial X_j(t)}{\partial S_{ij}(t)} = \beta_{ij} \frac{X_j(t)}{S_{ij}(t)}$$

Multiply (26) through (29) by price of output of j th industry $P_j(t)$ to find value marginal productivities of capital at time t . Define money profits earned at time t on each physical unit of capital stock $S_{ij}(t)$ as its value marginal productivity. Then multiply by $S_{ij}(t)$ to find money profits earned at time t on capital stock $S_{ij}(t)$. Sum over $i = 1, 2$ and define the outcome as money profits earned at time t on whatever capital stock exists at that time in the entire j th industry:

$$(30), (31) \quad Z_j(t) \equiv \sum_{i=1}^2 \kappa_{ij}(t) P_j(t) S_{ij}(t) = P_j(t) X_j(t) \sum_{i=1}^2 \beta_{ij}$$

- 11 -

Sum over $j = 1, 2$ and define the outcome as money profits earned at time t on whatever capital stock exists at that time in the entire economy:

$$(32) \quad Z(t) \equiv \sum_{j=1}^2 Z_j(t)$$

As seen from the present time τ this profits bill is $Z(t)e^{-r(t - \tau)}$ where e is Euler's number, the base of natural logarithms, and r is the discount rate applied by the capitalist-entrepreneurs. Finally integrate this over $t = \tau$ through ∞ and define the outcome as the present worth of all future profits bills

$$(33) \quad \zeta(\tau) \equiv \int_{\tau}^{\infty} Z(t)e^{-r(t - \tau)} dt$$

Now let capitalist-entrepreneurs use their control variable I_{ij} to optimize the allocation of their capital stock

100

... ..

$$\log_{10} \frac{1}{100} = -2 \quad (100)$$

... ..

$$\log_{10} \frac{1}{100} = -2 \quad (100)$$

... ..

S_{ij} within as well as between industries. Within the j th industry they act as stockholders optimizing S_{ij} where $i = 1, 2$ by appointing the right managers. Between industries they act as stockholders optimizing S_{ij} where $j = 1, 2$ by purchasing stock in the right industry. "Optimizing" in what sense? In the sense that

$$(34) \quad \zeta(\tau) = \text{maximum}$$

Under full employment, available labor force must equal the sum of labor employed by the two industries:

$$(35) \quad F = \sum_{i=1}^2 L_i$$

Define the wage bill as the money wage rate times employment:

Page 1

It is also well known that the industry
they are working in is very different from
the other side of the street. Between the two
there is a great difference in the way of
thinking and feeling. In the course

of the day, the mind is very different from
the body, and the body is very different from
the mind.

Page 2

It is also well known that the industry

$$(36) \quad W \equiv w \sum_{i=1}^2 L_i$$

Define national money income as the sum of the wage bill and the profits bill:

$$(37) \quad Y \equiv W + Z$$

Let all persons have the same utility function. Let the utility function of the k th person be

$$U_k = N C_{1k}^{A_1} C_{2k}^{A_2}$$

where $0 < A_i < 1$ and $N > 0$. Let there be s persons, and let the k th person's money income be Y_k where

1941

1941

1941

1941

1941

1941

1941

1941

1941

1941

1941

1941

1941

1941

1941

- 14 -

$$\sum_{k=1}^S Y_k = Y$$

Let all persons spend the fraction c , where $0 < c < 1$, of their money income. Then the budget constraint of the k th person is

$$cY_k = P_1C_{1k} + P_2C_{2k}$$

Maximize the k th person's utility subject to his budget constraint and find his two demand functions. Then add the s individual demand functions for each good and find the two Graham [2] aggregate demand functions

$$(38), (39) \quad C_i = \pi_i Y / P_i$$

where

$$- \frac{1}{2} \frac{d}{dt} \left(\frac{1}{\rho} \right)$$

$$\frac{1}{\rho} = \frac{1}{\rho_0} \left(\frac{V}{V_0} \right)^{\frac{1}{\gamma}}$$

where ρ_0 is the initial density, V_0 is the initial volume, and γ is the adiabatic index. The equation of state for an ideal gas is given by

$$p = \frac{1}{\beta} \left(\frac{1}{V} \right)^{\frac{1}{\gamma}}$$

where β is a constant. The equation of state for a real gas is given by

$$p = \frac{1}{\beta} \left(\frac{1}{V} \right)^{\frac{1}{\gamma}} \left(1 + \frac{a}{V} + \frac{b}{V^2} + \dots \right)$$

where a and b are constants. The equation of state for a real gas is given by

$$p = \frac{1}{\beta} \left(\frac{1}{V} \right)^{\frac{1}{\gamma}} \left(1 + \frac{a}{V} + \frac{b}{V^2} + \dots \right)$$

$$p = \frac{1}{\beta} \left(\frac{1}{V} \right)^{\frac{1}{\gamma}}$$

$$\pi_i = \frac{cA_i}{A_1 + A_2}$$

Industry output equilibrium requires the output of the i th industry to equal the sum of consumption and investment demand for it, or inventory would either accumulate or be depleted:

$$(40), (41) \quad x_i = C_i + \sum_{j=1}^2 I_{ij}$$

III. SOLUTIONS FOR PROPORTIONATE RATES OF GROWTH

Our system (1) through (41) possesses the following set of steady-state solutions for its equilibrium proportionate rates of growth:

$$(42), (43) \quad g_{Ci} = g_{Xi}$$

$$(44) \text{ through } (47) \quad g_{Iij} = g_{Xi}$$

$$(48), (49) \quad g_{Li} = g_F$$

$$(50) \quad g_{P1} = g_w - \frac{(1 - \beta_{22})g_{M1} + \beta_{21}g_{M2}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}}$$

$$(51) \quad g_{P2} = g_w - \frac{(1 - \beta_{11})g_{M2} + \beta_{12}g_{M1}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}}$$

$$(52) \text{ through } (55) \quad g_{Sij} = g_{Xi}$$

Journal of Management Studies, 19(6), 701-718.

$$(56) \quad g_{X1} = \frac{(1 - \beta_{22})g_{M1} + \beta_{21}g_{M2}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}} + g_F$$

$$(57) \quad g_{X2} = \frac{(1 - \beta_{11})g_{M2} + \beta_{12}g_{M1}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}} + g_F$$

$$(58) \quad g_Y = g_F + g_W$$

To see that it does, the reader should take derivatives with respect to time of all equations (18) through (41) except (26) through (29) and (33), (34). He should then use definitions (1) through (17), insert solutions (42) through (58), and convince himself that each equation is satisfied.

1. The first part of the report
 2. The second part of the report
 3. The third part of the report
 4. The fourth part of the report
 5. The fifth part of the report
 6. The sixth part of the report
 7. The seventh part of the report
 8. The eighth part of the report
 9. The ninth part of the report
 10. The tenth part of the report

11. The eleventh part of the report
 12. The twelfth part of the report
 13. The thirteenth part of the report
 14. The fourteenth part of the report
 15. The fifteenth part of the report
 16. The sixteenth part of the report
 17. The seventeenth part of the report
 18. The eighteenth part of the report
 19. The nineteenth part of the report
 20. The twentieth part of the report

21. The twenty-first part of the report
 22. The twenty-second part of the report
 23. The twenty-third part of the report
 24. The twenty-fourth part of the report
 25. The twenty-fifth part of the report
 26. The twenty-sixth part of the report
 27. The twenty-seventh part of the report
 28. The twenty-eighth part of the report
 29. The twenty-ninth part of the report
 30. The thirtieth part of the report

We defined balanced growth as identical proportionate rates of growth of physical output for all goods. According to our solutions (42) through (58), is our steady-state growth balanced or unbalanced?

Growth does spill over from one industry to the other. For example, according to (44) through (47) a more rapidly growing industry i would transmit some of its growth to a more slowly growing industry j investing in the i th industry's good. But the spillover is normally not enough to generate balanced growth. Use (56), (57), and the assumptions that $\alpha_1 + \beta_{11} + \beta_{21} = 1$ and $\alpha_2 + \beta_{12} + \beta_{22} = 1$ to find that

$$g_{X1} \geq g_{X2} \text{ implies } g_{M1}/g_{M2} \geq \alpha_1/\alpha_2,$$

respectively. Or in English: The first industry's physical output may grow more rapidly than that of the second industry for two and only² two reasons, i. e., first if everything else being equal the

1. The first of these is the fact that the
2. second of these is the fact that the
3. third of these is the fact that the
4. fourth of these is the fact that the
5. fifth of these is the fact that the
6. sixth of these is the fact that the
7. seventh of these is the fact that the
8. eighth of these is the fact that the
9. ninth of these is the fact that the
10. tenth of these is the fact that the

...and the other is the fact that the ...

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

first industry has more rapid technological progress g_{M_i} than the second industry, second, if everything else being equal the physical output of the first industry has a lower labor elasticity α_i than that of the second industry: The less labor-sensitive industry is less hampered by the fact that under technological progress labor force is growing less rapidly than physical capital stocks.

It does, however, follow from (50), (51), (56), and (57) that unlike physical outputs X_i , industry revenues $P_i X_i$ will grow at the same proportionate rate $g_F + g_W$.

IV. SOLUTIONS FOR LEVELS

So much for proportionate rates of growth. Let us now turn to the allocation of resources and solve for the allocation of savings between industries; the levels of industry revenues; employments; national money income; physical outputs; prices; physical

...the ... of ...
...the ... of ...
...the ... of ...

...the ... of ...
...the ... of ...
...the ... of ...

...the ... of ...
...the ... of ...
...the ... of ...

...the ... of ...
...the ... of ...
...the ... of ...

...the ... of ...
...the ... of ...
...the ... of ...

...the ... of ...
...the ... of ...
...the ... of ...

capital stocks and their physical marginal productivities;
consumption; and income distribution.

1. Saving Equals Investment

Use (24), (25), and (36) to see that $W = \alpha_1 P_1 X_1 + \alpha_2 P_2 X_2$, and
(30) through (32) to see that $Z = (\beta_{11} + \beta_{21})P_1 X_1 + (\beta_{12} + \beta_{22})P_2 X_2$,
hence national income equals national output:

$$(59) \quad Y = P_1 X_1 + P_2 X_2$$

Multiply (40) and (41) by P_1 and P_2 , respectively, insert (38),
(39), and (59) and find saving to equal investment:

$$(60) \quad (1 - c)Y = P_1(I_{11} + I_{12}) + P_2(I_{21} + I_{22})$$

$$- 12 -$$

... ..

... ..

... ..

$$\frac{1}{X} = \frac{1}{Y} + \frac{1}{Z}$$

... ..

$$\frac{1}{X} = \frac{1}{Y} + \frac{1}{Z}$$

2. Present-Worth Maximization

Subject to the constraint (60) let the capitalist-entrepreneurs use their control variable I_{ij} to optimize the allocation of their capital stock S_{ij} within as well as between industries. "Optimize" in what sense? In the sense of maximizing the present worth $\zeta(\tau)$ of all future profits bills in accordance with (34). Using (30) through (33) we write present worth as

$$\zeta(\tau) = \int_{\tau}^{\infty} [(\beta_{11} + \beta_{21})P_1(t)X_1(t) + (\beta_{12} + \beta_{22})P_2(t)X_2(t)]e^{-r(t - \tau)}dt$$

Let it be foreseen by the capitalist-entrepreneurs that prices are growing in accordance with our steady-state solutions (50) and (51), hence

... in the ...

... the ...
... the ...
... the ...
... the ...
... the ...
... the ...

... the ...
... the ...

... the ...
... the ...

... the ...
... the ...
... the ...

- 22 -

$$P_j(t) = e^{g_{Pj}(t - \tau)} P_j(\tau)$$

and that outputs are growing in accordance with our steady-state solutions (56) and (57), hence

$$X_j(t) = e^{g_{Xj}(t - \tau)} X_j(\tau)$$

Consequently we may take prices and outputs outside the integral sign and write present worth as

$$\begin{aligned} \zeta(\tau) = & (\beta_{11} + \beta_{21})P_1(\tau)X_1(\tau) \int_{\tau}^{\infty} e^{(g_{P1} + g_{X1} - r)(t - \tau)} dt + \\ & (\beta_{12} + \beta_{22})P_2(\tau)X_2(\tau) \int_{\tau}^{\infty} e^{(g_{P2} + g_{X2} - r)(t - \tau)} dt \end{aligned}$$

Since in this expression all variables refer to the same time

τ , we may purge it of τ . Use (50), (51), (56), and (57) to see that $g_{pj} + g_{Xj} = g_F + g_W$. Assume that $g_F + g_W < r$, then integrate:

$$\zeta = \frac{(\beta_{11} + \beta_{21})P_1X_1 + (\beta_{12} + \beta_{22})P_2X_2}{r - (g_F + g_W)}$$

Inserting (30) through (32) into this we find the simple relationship between profits and present worth under steady-state growth:

$$(61) \quad Z = [r - (g_F + g_W)]\zeta$$

Maximizing present worth ζ subject to the constraint (60) is most easily done by using a Lagrange multiplier: Define a new function to be maximized

189

Let $f(x) = (x^2 + 1)^2$. Then $f'(x) = 4x(x^2 + 1)$. We have $f'(1) = 4(1)(1^2 + 1) = 8$. The tangent line to the graph of f at $(1, 4)$ is $y - 4 = 8(x - 1)$, or $y = 8x - 4$.

$$\frac{d}{dx} \left(\frac{x^2 + 1}{x^3 + 1} \right) = \frac{(2x)(x^3 + 1) - (x^2 + 1)(3x^2)}{(x^3 + 1)^2} = \frac{2x^4 + 2x - 3x^4 - 3x^2}{(x^3 + 1)^2} = \frac{-x^4 - 3x^2 + 2x}{(x^3 + 1)^2}$$

Let $f(x) = \sin(x)$. Then $f'(x) = \cos(x)$. We have $f'(0) = \cos(0) = 1$. The tangent line to the graph of f at $(0, 0)$ is $y - 0 = 1(x - 0)$, or $y = x$.

$$\frac{d}{dx} \left(\frac{x^2 + 1}{x^3 + 1} \right) = \frac{2x(x^3 + 1) - (x^2 + 1)(3x^2)}{(x^3 + 1)^2} = \frac{2x^4 + 2x - 3x^4 - 3x^2}{(x^3 + 1)^2} = \frac{-x^4 - 3x^2 + 2x}{(x^3 + 1)^2}$$

Let $f(x) = \sin(x)$. Then $f'(x) = \cos(x)$. We have $f'(0) = \cos(0) = 1$. The tangent line to the graph of f at $(0, 0)$ is $y - 0 = 1(x - 0)$, or $y = x$.

$$\phi \equiv \zeta + \lambda[(1 - c)Y - P_1(I_{11} + I_{12}) - P_2(I_{21} + I_{22})]$$

What to do with Y? Insert (61) into (37), insert the outcome into ϕ and write the latter

$$(62) \quad \phi = \{1 + \lambda(1 - c)[r - (g_F + g_W)]\}\zeta +$$

$$\lambda(1 - c)W - \lambda[P_1(I_{11} + I_{12}) + P_2(I_{21} + I_{22})]$$

The first four first-order conditions for a maximum ϕ are

$$(63) \quad \frac{\partial \phi}{\partial I_{ij}} = h_j \frac{\partial X_j}{\partial I_{ij}} - \lambda P_i = 0$$

where

$$h_j \equiv \frac{\{1 + \lambda(1 - c)[r - (g_F + g_w)]\}(\beta_{1j} + \beta_{2j})P_j}{r - (g_F + g_w)}$$

Now according to the production functions (22) and (23), output X_j is a function of capital stock S_{ij} rather than of investment I_{ij} . But according to (1) through (21)

$$(64) \quad S_{ij} \equiv I_{ij}/g_{Sij}$$

where our steady-state growth, as specified by (52) through (57), permits us to express g_{Sij} solely in terms of parameters. Inserting (64) into the production functions (22) and (23) we find

100

[illegible]

1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 26

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

$$(65) \quad \frac{\partial X_j}{\partial I_{ij}} = \beta_{ij} \frac{X_j}{I_{ij}}$$

and write the first-order conditions as

$$\begin{aligned} (66) \text{ through } (69) \quad & \beta_{11}(\beta_{11} + \beta_{21})X_1/I_{11} = \beta_{12}(\beta_{12} + \beta_{22})P_2X_2/(P_1I_{12}) \\ & = \beta_{21}(\beta_{11} + \beta_{21})P_1X_1/(P_2I_{21}) = \beta_{22}(\beta_{12} + \beta_{22})X_2/I_{22} \\ & = \frac{\lambda[r - (g_F + g_w)]}{1 + \lambda(1 - c)[r - (g_F + g_w)]} \end{aligned}$$

That the second-order conditions are satisfied is demonstrated

Figure 1

(1)

The reaction network is described by the following set of equations:

$\frac{dA}{dt} = -k_1 A + k_2 C$
 $\frac{dB}{dt} = -k_3 B + k_4 C$
 $\frac{dX}{dt} = k_1 A + k_3 B - k_5 X$
 $\frac{dC}{dt} = k_5 X - k_6 C$

where A , B , X , and C are the concentrations of the species, and $k_1, k_2, k_3, k_4, k_5, k_6$ are the rate constants.

The initial conditions are $A(0) = A_0$, $B(0) = B_0$, $X(0) = 0$, and $C(0) = 0$. The system is analyzed for its stability and dynamics.

The Jacobian matrix of the system is given by:

$$J = \begin{pmatrix} -k_1 & 0 & k_2 & 0 \\ 0 & -k_3 & 0 & k_4 \\ k_1 & k_3 & -k_5 & 0 \\ 0 & 0 & k_5 & -k_6 \end{pmatrix}$$

The eigenvalues of the Jacobian matrix are calculated to determine the stability of the steady state.

The results show that the system is stable for all parameter values considered.

in Appendix I.

3. Solving for Industry Revenues $P_j X_j$

Use the first-order conditions (66) through (68) to express I_{12} in terms of I_{11} and I_{21} in terms of I_{22} . Insert the results into (40) and (41). Insert (59) into (38) and (39). Insert the results into (40) and (41). Divide (40) by $\beta_{11}(\beta_{11} + \beta_{21})X_1$ and (41) by $\beta_{22}(\beta_{12} + \beta_{22})X_2$, deduct (41) from (40), and again use the first-order conditions (66) through (68). Now define

$$(70) \quad \rho \equiv (P_1 X_1 / (P_2 X_2))$$

rearrange, and write the quadratic

$$(71) \quad \rho^2 + m\rho + n = 0$$

where m and n are the following agglomerations of taste and

technology parameters

$$m \equiv \frac{(\beta_{12} + \beta_{22})[\beta_{12}\pi_2 + \beta_{22}(1 - \pi_1)] - (\beta_{11} + \beta_{21})[\beta_{11}(1 - \pi_2) + \beta_{21}\pi_1]}{(\beta_{11} + \beta_{21})[\beta_{11}\pi_2 + \beta_{21}(1 - \pi_1)]}$$

$$n \equiv - \frac{(\beta_{12} + \beta_{22})[\beta_{12}(1 - \pi_2) + \beta_{22}\pi_1]}{(\beta_{11} + \beta_{21})[\beta_{11}\pi_2 + \beta_{21}(1 - \pi_1)]}$$

The quadratic has the two roots

$$\rho = -m/2 \pm \sqrt{(m/2)^2 - n}$$

We have assumed that $0 < A_i < 1$, $0 < \beta_i < 1$, and $0 < c < 1$, hence $n < 0$. Now regardless of the sign of m , $0 \leq (m/2)^2$, hence

1907

1907

1907

1907

1907

1907

$$0 \leq (m/2)^2 < (m/2)^2 - n$$

Two things follow. First, from $0 < (m/2)^2 - n$ it follows that both roots are real. Second, from $(m/2)^2 < (m/2)^2 - n$ it follows that regardless of the sign of m , the first root is positive and the second negative. We reject the latter and are left with

$$(72) \quad p = -m/2 + \sqrt{(m/2)^2 - n}$$

Use (24), (25), (35), and (36) to find

$$\alpha_1 P_1 X_1 + \alpha_2 P_2 X_2 = wF$$

Take this together with (70) and find

— 200 —

[illegible]

1. The first step is to identify the problem. In this case, the problem is that the company is not meeting its sales targets.

End of test for 0001, 0002, 0003, 0004

Number of hauls	<i>P. setiferus</i> (%)	<i>P. setiferus</i> + <i>P. setiferus</i> + <i>P. setiferus</i> (%)	<i>P. setiferus</i> + <i>P. setiferus</i> + <i>P. setiferus</i> (%)
1	~10	~20	~70
2	~15	~25	~60
3	~20	~30	~50
4	~25	~35	~40
5	~30	~40	~30
6	~35	~45	~20
7	~40	~50	~10
8	~45	~55	~5
9	~50	~60	~2
10	~55	~65	~1

1. 1990年12月，在《中国环境报》上刊登了“中国环境报”的创刊号，这是中国环境报创刊以来的第一份报纸。

$$(73), (74) \quad P_i X_i = \mu_i w F$$

where

$$\mu_1 \equiv \rho / (\alpha_1 \rho + \alpha_2)$$

$$\mu_2 \equiv 1 / (\alpha_1 \rho + \alpha_2)$$

4. Solving for Employments L_i and Income Y

Use (24), (25), (73), (74) to solve for employments

$$(75), (76) \quad L_i = \alpha_i \mu_i F$$

Insert (73) and (74) into (59) and solve for national money income

- 62 -

1. 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

$$(77) \quad Y = (\mu_1 + \mu_2)wF$$

5. Solving for Physical Outputs X_j

Let us begin by finding four investment-output ratios. Again use the first-order conditions (66) through (68) to express I_{12} in terms of I_{11} . Insert the result into (40), insert (59) into (38), and insert the result into (40). Divide (40) by X_1 . Use a similar procedure upon (41) and find the four ratios

$$(78) \quad I_{11}/X_1 = v_{11} \equiv \frac{1 - \pi_1 - \pi_1/\rho}{1 + \beta_{12}(\beta_{12} + \beta_{22})/[\rho\beta_{11}(\beta_{11} + \beta_{21})]}$$

$$(79) \quad I_{12}/X_1 = v_{12} \equiv \frac{1 - \pi_1 - \pi_1/\rho}{1 + \rho\beta_{11}(\beta_{11} + \beta_{21})/[\beta_{12}(\beta_{12} + \beta_{22})]}$$

$$T^2 = \frac{4\pi^2}{g} \left(\frac{1}{\sin \theta} \right)$$

(10)

$$T^2 = \frac{4\pi^2}{g} \left(\frac{1}{\sin \theta} \right)$$

Let θ be the angle between the vertical and the string. The forces acting on the mass are the tension T along the string and the weight mg vertically downwards. The weight can be resolved into two components: one along the string, $mg \cos \theta$, and one perpendicular to the string, $mg \sin \theta$. The component $mg \cos \theta$ balances the tension T , so $T = mg \cos \theta$. The component $mg \sin \theta$ provides the centripetal force for the circular motion, so $mg \sin \theta = \frac{mv^2}{r}$, where r is the radius of the circular path. The radius r is the horizontal distance from the pivot to the mass, which is $L \sin \theta$, where L is the length of the string. Thus, $mg \sin \theta = \frac{mv^2}{L \sin \theta}$. The speed v can be related to the angular speed ω by $v = r\omega = L \sin \theta \omega$. Substituting this into the previous equation gives $mg \sin \theta = \frac{m(L \sin \theta \omega)^2}{L \sin \theta}$, which simplifies to $g \sin \theta = L \omega^2$. The period T is the time for one complete revolution, so $T = \frac{2\pi}{\omega}$. Substituting $\omega = \sqrt{\frac{g \sin \theta}{L}}$ into this equation gives $T = 2\pi \sqrt{\frac{L}{g \sin \theta}}$.

$$T^2 = \frac{4\pi^2}{g} \left(\frac{1}{\sin \theta} \right)$$

Let θ be the angle between the vertical and the string. The forces acting on the mass are the tension T along the string and the weight mg vertically downwards. The weight can be resolved into two components: one along the string, $mg \cos \theta$, and one perpendicular to the string, $mg \sin \theta$. The component $mg \cos \theta$ balances the tension T , so $T = mg \cos \theta$. The component $mg \sin \theta$ provides the centripetal force for the circular motion, so $mg \sin \theta = \frac{mv^2}{r}$, where r is the radius of the circular path. The radius r is the horizontal distance from the pivot to the mass, which is $L \sin \theta$, where L is the length of the string. Thus, $mg \sin \theta = \frac{mv^2}{L \sin \theta}$. The speed v can be related to the angular speed ω by $v = r\omega = L \sin \theta \omega$. Substituting this into the previous equation gives $g \sin \theta = L \omega^2$. The period T is the time for one complete revolution, so $T = \frac{2\pi}{\omega}$. Substituting $\omega = \sqrt{\frac{g \sin \theta}{L}}$ into this equation gives $T = 2\pi \sqrt{\frac{L}{g \sin \theta}}$.

$$T^2 = \frac{4\pi^2}{g} \left(\frac{1}{\sin \theta} \right)$$

Let θ be the angle between the vertical and the string. The forces acting on the mass are the tension T along the string and the weight mg vertically downwards. The weight can be resolved into two components: one along the string, $mg \cos \theta$, and one perpendicular to the string, $mg \sin \theta$. The component $mg \cos \theta$ balances the tension T , so $T = mg \cos \theta$. The component $mg \sin \theta$ provides the centripetal force for the circular motion, so $mg \sin \theta = \frac{mv^2}{r}$, where r is the radius of the circular path. The radius r is the horizontal distance from the pivot to the mass, which is $L \sin \theta$, where L is the length of the string. Thus, $mg \sin \theta = \frac{mv^2}{L \sin \theta}$. The speed v can be related to the angular speed ω by $v = r\omega = L \sin \theta \omega$. Substituting this into the previous equation gives $g \sin \theta = L \omega^2$. The period T is the time for one complete revolution, so $T = \frac{2\pi}{\omega}$. Substituting $\omega = \sqrt{\frac{g \sin \theta}{L}}$ into this equation gives $T = 2\pi \sqrt{\frac{L}{g \sin \theta}}$.

$$(80) \quad I_{21}/X_2 = v_{21} \equiv \frac{1 - \pi_2 - \pi_2 \rho}{1 + \beta_{22}(\beta_{12} + \beta_{22})/[\rho \beta_{21}(\beta_{11} + \beta_{21})]}$$

$$(81) \quad I_{22}/X_2 = v_{22} \equiv \frac{1 - \pi_2 - \pi_2 \rho}{1 + \rho \beta_{21}(\beta_{11} + \beta_{21})/[\beta_{22}(\beta_{12} + \beta_{22})]}$$

Apply (64) to (78) through (81) and find

$$(82) \quad S_{ij} = X_i v_{ij} / g_{Sij}$$

Insert (82) and our solutions (75) and (76) into the production functions (22) and (23), arrive at two equations in the two unknowns X_j , solve them, and find

— 27 —

1. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

2. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

3. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

4. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

5. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

$$(83) \quad x_1 = (\xi_1^{1 - \beta_{22} \xi_2^{\beta_{21}}})^{\frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12} \beta_{21} F}}$$

$$(84) \quad x_2 = (\xi_1^{\beta_{12} \xi_2^{1 - \beta_{11}}})^{\frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12} \beta_{21} F}}$$

where

$$\xi_1 \equiv M_1(\alpha_1 \mu_1)^{\alpha_1 (v_{11}/g_{S11})^{\beta_{11}} (v_{21}/g_{S21})^{\beta_{21}}}$$

$$\xi_2 \equiv M_2(\alpha_2 \mu_2)^{\alpha_2 (v_{12}/g_{S12})^{\beta_{12}} (v_{22}/g_{S22})^{\beta_{22}}}$$

The reader may convince himself that (83) and (84) are indeed growing at the rates (56) and (57) said they should be.

— 38 —

1. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$ (1)

2. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$ (2)

3. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

4. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

5. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

6. Solving for Prices P_j

Divide our revenue solutions (73) and (74) by our physical output solutions (83) and (84), respectively, and find

$$(85) \quad P_1 = (\xi_1^{1 - \beta_{22}} \xi_2^{\beta_{21}})^{-\frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}w\mu_1}}$$

$$(86) \quad P_2 = (\xi_1^{\beta_{12}} \xi_2^{1 - \beta_{11}})^{-\frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}w\mu_2}}$$

Similarly the reader may convince himself that (85) and (86) are indeed growing at the rates (50) and (51) said they should be.

Let $\mathcal{A}$ be a $\mathcal{C}^*$ -algebra and let $\mathcal{B}$ be a $\mathcal{C}^*$ -subalgebra of $\mathcal{A}$.

For any $a \in \mathcal{A}$ and $b \in \mathcal{B}$, we have $ab \in \mathcal{B}$ and $ba \in \mathcal{B}$.

Let $\mathcal{C}$ be a $\mathcal{C}^*$ -algebra and let $\mathcal{D}$ be a $\mathcal{C}^*$ -subalgebra of $\mathcal{C}$.

$$\begin{aligned} \text{Let } \mathcal{A} \text{ be a } \mathcal{C}^*\text{-algebra and let } \mathcal{B} \text{ be a } \mathcal{C}^*\text{-subalgebra of } \mathcal{A}. \\ \text{For any } a \in \mathcal{A} \text{ and } b \in \mathcal{B}, \text{ we have } ab \in \mathcal{B} \text{ and } ba \in \mathcal{B}. \\ \text{Let } \mathcal{C} \text{ be a } \mathcal{C}^*\text{-algebra and let } \mathcal{D} \text{ be a } \mathcal{C}^*\text{-subalgebra of } \mathcal{C}. \end{aligned}$$

$$\begin{aligned} \text{Let } \mathcal{A} \text{ be a } \mathcal{C}^*\text{-algebra and let } \mathcal{B} \text{ be a } \mathcal{C}^*\text{-subalgebra of } \mathcal{A}. \\ \text{For any } a \in \mathcal{A} \text{ and } b \in \mathcal{B}, \text{ we have } ab \in \mathcal{B} \text{ and } ba \in \mathcal{B}. \\ \text{Let } \mathcal{C} \text{ be a } \mathcal{C}^*\text{-algebra and let } \mathcal{D} \text{ be a } \mathcal{C}^*\text{-subalgebra of } \mathcal{C}. \end{aligned}$$

(1) Let $\mathcal{A}$ be a $\mathcal{C}^*$ -algebra and let $\mathcal{B}$ be a $\mathcal{C}^*$ -subalgebra of $\mathcal{A}$.
 (2) Let $\mathcal{A}$ be a $\mathcal{C}^*$ -algebra and let $\mathcal{B}$ be a $\mathcal{C}^*$ -subalgebra of $\mathcal{A}$.
 (3) Let $\mathcal{A}$ be a $\mathcal{C}^*$ -algebra and let $\mathcal{B}$ be a $\mathcal{C}^*$ -subalgebra of $\mathcal{A}$.

7. Capital Stocks S_{ij} and their Marginal Productivities κ_{ij}

With (83) and (84) inserted into it, (82) will be a solution for physical capital stocks S_{ij} . With (82) inserted into them, (26) through (29) will be solutions for the physical marginal productivities of capital

$$(87) \quad \kappa_{ij} = \beta_{ij} \varepsilon_{S_{ij}} / v_{ij}$$

8. Consumption C_i and Income Distribution W and Z

With (77), (85), and (86) inserted into them, (38) and (39) will be solutions for consumption. With (35) inserted into it, (36) will be a solution for the wage bill

$$(88) \quad W = wF$$

With (73) and (74) inserted into them, (30) through (32) will generate the profits bill

$$(89) \quad Z = [(\beta_{11} + \beta_{21})\mu_1 + (\beta_{12} + \beta_{22})\mu_2]wF$$

With (89) inserted into it, (61) will be a solution for present worth.

9. Properties of Solutions

We have now solved for the levels of all variables. Our solutions (78) through (81) for the investment-output ratios and (87) for the physical marginal productivities of capital are stationary. All other solutions for levels are nonstationary, because they contain one or more of our three nonstationary

1. The first of the three main points of the report is that the Commission has found that the Government of the United Kingdom has failed to comply with its obligations under the European Convention on Human Rights in relation to the treatment of persons in custody.

2. The second point is that the Commission has found that the Government of the United Kingdom has failed to provide adequate information to the Commission regarding the treatment of persons in custody.

3. The third point is that the Commission has found that the Government of the United Kingdom has failed to take adequate steps to ensure that persons in custody are treated in accordance with the European Convention on Human Rights.

4. The Commission's findings are based on the following evidence:

- (a) The Commission received information from the European Court of Human Rights that the Government of the United Kingdom had failed to comply with its obligations under the European Convention on Human Rights in relation to the treatment of persons in custody.
- (b) The Commission received information from the European Court of Human Rights that the Government of the United Kingdom had failed to provide adequate information to the Commission regarding the treatment of persons in custody.
- (c) The Commission received information from the European Court of Human Rights that the Government of the United Kingdom had failed to take adequate steps to ensure that persons in custody are treated in accordance with the European Convention on Human Rights.

parameters, i. e. available labor force F , the multiplicative factor M_i of the production functions, and the money wage rate w .

Are our solutions real and positive? Section IV, 3 found both roots ρ to be real and found one to be positive, the other negative. All solutions (73) through (89), then, are real. Rejecting the negative root we find solutions (73) through (77), (88), and (89) to be obviously positive. Less obviously, so are solutions (78) through (87), as demonstrated in our Appendix II.

A P P E N D I X I

SECOND-ORDER CONDITIONS FOR A MAXIMUM OF EQUATION (62)

Write the bordered Hessian

$$(90) \quad H \equiv \begin{vmatrix} \frac{\partial^2 \phi}{\partial I_{11}^2} & \frac{\partial^2 \phi}{\partial I_{11} \partial I_{12}} & \frac{\partial^2 \phi}{\partial I_{11} \partial I_{21}} & \frac{\partial^2 \phi}{\partial I_{11} \partial I_{22}} & -P_1 \\ \frac{\partial^2 \phi}{\partial I_{12} \partial I_{11}} & \frac{\partial^2 \phi}{\partial I_{12}^2} & \frac{\partial^2 \phi}{\partial I_{12} \partial I_{21}} & \frac{\partial^2 \phi}{\partial I_{12} \partial I_{22}} & -P_1 \\ \frac{\partial^2 \phi}{\partial I_{21} \partial I_{11}} & \frac{\partial^2 \phi}{\partial I_{21} \partial I_{12}} & \frac{\partial^2 \phi}{\partial I_{21}^2} & \frac{\partial^2 \phi}{\partial I_{21} \partial I_{22}} & -P_2 \\ \frac{\partial^2 \phi}{\partial I_{22} \partial I_{11}} & \frac{\partial^2 \phi}{\partial I_{22} \partial I_{12}} & \frac{\partial^2 \phi}{\partial I_{22} \partial I_{21}} & \frac{\partial^2 \phi}{\partial I_{22}^2} & -P_2 \\ -P_1 & -P_1 & -P_2 & -P_2 & 0 \end{vmatrix}$$

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

NAME	ADDRESS	CITY	STATE	COUNTRY
J. D. Smith	123 Main St	Chicago	Ill.	U.S.A.
A. B. Jones	456 Oak St	New York	N.Y.	U.S.A.
C. D. Brown	789 Elm St	Los Angeles	Calif.	U.S.A.
E. F. Green	101 Pine St	San Francisco	Calif.	U.S.A.
G. H. White	202 Cedar St	Boston	Mass.	U.S.A.
I. J. Black	303 Maple St	Philadelphia	Pa.	U.S.A.
K. L. Gray	404 Birch St	Seattle	Wash.	U.S.A.
M. N. Hall	505 Spruce St	Portland	Maine	U.S.A.

The first derivatives $\partial\phi/\partial I_{ij}$ have already been taken and were of the form (63). It follows from that form that a good many of the second derivatives contained in our Hessian are zero: After inserting (64) into our production functions (22) and (23) we realize that X_j is a function of neither I_{ii} nor I_{ji} where $i \neq j$, hence

$$(91) \quad \frac{\partial X_j}{\partial I_{ii}} = \frac{\partial X_j}{\partial I_{ji}} = \frac{\partial^2 X_j}{\partial I_{ii} \partial I_{ij}} = \frac{\partial^2 X_j}{\partial I_{ji} \partial I_{ij}} = \frac{\partial^2 X_j}{\partial I_{ii} \partial I_{jj}} = \frac{\partial^2 X_j}{\partial I_{ji} \partial I_{jj}} = 0$$

$$(i \neq j)$$

But X_j is a function of I_{ij} and I_{jj} , hence

$$(92) \quad \frac{\partial^2 X_j}{\partial I_{ij} \partial I_{jj}} = \frac{\partial^2 X_j}{\partial I_{jj} \partial I_{ij}} = \frac{\beta_{ij} \beta_{jj} X_j}{I_{ij} I_{jj}} \quad (i \neq j)$$

$$(93) \quad \frac{\partial^2 X_j}{\partial I_{ij}^2} = \beta_{ij}(\beta_{ij} - 1) \frac{X_j}{I_{ij}^2} \quad (i = j \text{ or } i \neq j)$$

Apply (91), (92), and (93) to the Hessian (90). Then try to produce even more zero elements, making the Hessian easier to evaluate. Factor out $\beta_{11}h_1X_1/I_{11}$ from first row; $\beta_{12}h_2X_2/I_{12}$ from second row; $\beta_{21}h_1X_1/I_{21}$ from third row; and $\beta_{22}h_2X_2/I_{22}$ from fourth row, where h was defined as part of (63). Thereby the first four elements of the fifth column become

$$- P_1 I_{11} / (\beta_{11} h_1 X_1),$$

$$- P_1 I_{12} / (\beta_{12} h_2 X_2),$$

$$- P_2 I_{21} / (\beta_{21} h_1 X_1),$$

$$- P_2 I_{22} / (\beta_{22} h_2 X_2)$$

Journal of Management Inquiry, Vol. 17 No. 4, December 2008
DOI: 10.1177/1056492608322800
© The Author(s) 2008

© 2006 The Authors
Journal compilation © 2006 Blackwell Publishing Ltd

... ..

1. *Phragmites australis* (Cav.) Trin. ex Steud.

1. *Phragmites australis* (Cav.) Trin. ex Steud.

Journal of Management Education 30(6)

But according to the first-order conditions (66) through (69) those four values are all equal to $-1/\lambda$. Now factor out $1/I_{11}$ from first column, $1/I_{12}$ from second column, $1/I_{21}$ from third column, $1/I_{22}$ from fourth column, and $1/\lambda$ from fifth column.

If to each element of a row is added the corresponding element of another row, the determinant remains unchanged. So factor out (-1) from the first row and add to each element of it the corresponding element of the third row. Factor out (-1) from the second row and add to each element of it the corresponding element of the fourth row.

If to each element of a column is added the corresponding element of another column, the determinant remains unchanged. So add to each element of the third column the corresponding element of the first column. Add to each element of the fourth column the corresponding element of the second column.

By now the Hessian has been transformed into the following very tractable form:

- 42 -

$$H = \frac{\beta_{11}\beta_{12}\beta_{21}\beta_{22}h_1^2h_2^2x_1^2x_2^2}{I_{11}^2I_{12}^2I_{21}^2I_{22}^2\lambda} \times$$

1	0	0	0	0
0	1	0	0	0
β_{11}	0	$-\alpha_1$	0	-1
0	β_{12}	0	$-\alpha_2$	-1
$-P_1I_{11}$	$-P_1I_{12}$	$-P_1I_{11}-P_2I_{21}$	$-P_1I_{12}-P_2I_{22}$	0

$$= \frac{\beta_{11}\beta_{12}\beta_{21}\beta_{22}h_1^2h_2^2x_1^2x_2^2}{I_{11}^2I_{12}^2I_{21}^2I_{22}^2\lambda} [\alpha_2(P_1I_{11} + P_2I_{21}) + \alpha_1(P_1I_{12} + P_2I_{22})]$$

— 17 —

Вопросы, касающиеся
взаимоотношений между
государством и обществом

0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

Вопросы, касающиеся
взаимоотношений между
государством и обществом

Вопросы, касающиеся
взаимоотношений между
государством и обществом

Is our Hessian positive, then? Appendix II will demonstrate that all solutions, including those for $P_i I_{ij}$, are positive. To see if λ is positive, write the fifth first-order condition $\partial\phi/\partial\lambda = 0$ and find it to be the constraint (60). Use (66) through (69) to write

$$P_1(I_{11} + I_{12}) + P_2(I_{21} + I_{22}) =$$

$$\frac{1 + \lambda(1 - c)[r - (g_F + g_w)]}{\lambda[r - (g_F + g_w)]} [(\beta_{11} + \beta_{21})^2 P_1 X_1 + (\beta_{12} + \beta_{22})^2 P_2 X_2]$$

Insert (59) and this into the constraint (60), rearrange, and

(1) The function $f(x)$ is defined on the interval $[0, 1]$ by the formula $f(x) = x^2 + 2x + 1$. Find the value of the definite integral $\int_0^1 f(x) dx$.

$$f(x) = x^2 + 2x + 1$$

$$\int_0^1 (x^2 + 2x + 1) dx = \left[\frac{x^3}{3} + x^2 + x \right]_0^1 = \frac{1}{3} + 1 + 1 = \frac{7}{3}$$

The value of the definite integral is $\frac{7}{3}$.

write the latter

$$\lambda = \frac{\psi}{(1 - c)(1 - \psi)[r - (g_F + g_w)]}$$

where

$$\psi \equiv \frac{(\beta_{11} + \beta_{21})^2 P_1 X_1 + (\beta_{12} + \beta_{22})^2 P_2 X_2}{P_1 X_1 + P_2 X_2}$$

It follows from $0 < \psi < 1$ that $\lambda > 0$, hence the Hessian (90) is positive. And now for its principal minors.

1957

$$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right)$$

1957

$$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right)$$

1957

From the Hessian (90) remove successively fourth, third, and second column and row and obtain the bordered 4×4 , 3×3 , and 2×2 principal minors. Their values are respectively

$$- \frac{\beta_{11}\beta_{12}\beta_{21}h_1^2h_2^2X_1^2X_2^2}{I_{11}^2I_{12}^2I_{21}^2\lambda} [(1 - \beta_{12})(P_1I_{11} + P_2I_{21}) + (1 - \beta_{11} - \beta_{21})P_1I_{12}]$$

$$\frac{\beta_{11}\beta_{12}h_1h_2X_1X_2}{I_{11}^2I_{12}^2\lambda} [(1 - \beta_{12})P_1I_{11} + (1 - \beta_{11})P_1I_{12}]$$

$$- \frac{\beta_{11}h_1X_1}{I_{11}^2\lambda} P_1I_{11}$$

The three values are negative, positive, and negative, respectively.

A P P E N D I X I I

SIGN OF SOLUTIONS (78) THROUGH (87)

Solutions (78) through (87) contain one of the factors v_{ij} . Could those factors be nonpositive? To show that they cannot we prove that our positive root ρ has the following bounds:

$$(94) \quad \pi_1/(1 - \pi_1) < \rho < (1 - \pi_2)/\pi_2$$

Take the first inequality of (94), insert (72), move the term $-(m/2)$ to the other side, and write the inequality

$$\sqrt{(m/2)^2 - n} > m/2 + \pi_1/(1 - \pi_1)$$

Square the inequality, multiply it by $(1 - \pi_1)^2$, and write it

$$- \pi_1^2 - m\pi_1(1 - \pi_1) - n(1 - \pi_1)^2 > 0$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Let $\mathcal{H}$ be the Hilbert space of all functions $f: \mathbb{N} \rightarrow \mathbb{C}$ such that $\sum_{n=0}^{\infty} |f(n)|^2 < \infty$. For $f, g \in \mathcal{H}$, define the inner product $\langle f, g \rangle = \sum_{n=0}^{\infty} f(n) \overline{g(n)}$. Let $\{e_n\}_{n=0}^{\infty}$ be the standard orthonormal basis of $\mathcal{H}$, where $e_n(k) = \delta_{nk}$. Let T be the operator on $\mathcal{H}$ defined by $Tf = Af + Bf$, where A and B are the operators defined above.

$$\|Tf\|_2^2 = \langle Tf, Tf \rangle = \langle Af + Bf, Af + Bf \rangle = \langle Af, Af \rangle + \langle Bf, Bf \rangle + 2\langle Af, Bf \rangle \quad (1)$$

Since A and B are self-adjoint operators, we have $\langle Af, Bf \rangle = \langle Bf, Af \rangle$. Therefore, $\langle Af, Bf \rangle = \langle Bf, Af \rangle = \langle B(Af), f \rangle = \langle (BA)f, f \rangle = \langle C f, f \rangle$, where $C = BA$. Since C is a self-adjoint operator, we have $\langle C f, f \rangle = \langle f, C f \rangle = \langle f, C^2 f \rangle = \langle C^2 f, f \rangle = \langle C^2 f, f \rangle = \langle C^2 f, f \rangle$.

$$\|Tf\|_2^2 = \|Af\|_2^2 + \|Bf\|_2^2 + 2\langle C f, f \rangle = \|Af\|_2^2 + \|Bf\|_2^2 + 2\langle C f, f \rangle$$

Since A and B are self-adjoint operators, we have $\langle Af, Bf \rangle = \langle Bf, Af \rangle$. Therefore, $\langle Af, Bf \rangle = \langle Bf, Af \rangle = \langle B(Af), f \rangle = \langle (BA)f, f \rangle = \langle C f, f \rangle$, where $C = BA$. Since C is a self-adjoint operator, we have $\langle C f, f \rangle = \langle f, C f \rangle = \langle f, C^2 f \rangle = \langle C^2 f, f \rangle = \langle C^2 f, f \rangle = \langle C^2 f, f \rangle$.

$$\|Tf\|_2^2 = \|Af\|_2^2 + \|Bf\|_2^2 + 2\langle C f, f \rangle = \|Af\|_2^2 + \|Bf\|_2^2 + 2\langle C f, f \rangle$$

Now insert the definitions of m and n attached to (71), recall that $\pi_1 + \pi_2 = c$, rearrange, and find

$$(1 - c)[(\beta_{11} + \beta_{21})\beta_{11}\pi_1 + (\beta_{12} + \beta_{22})\beta_{12}(1 - \pi_1)] > 0$$

which it is under our assumptions about β_{ij} and π_i .

Then take the second inequality of (94), insert (72), move the term $-(m/2)$ to the other side, and write the inequality

$$\sqrt{(m/2)^2 - n} < m/2 + (1 - \pi_2)/\pi_2$$

Square the inequality, multiply it by π_2^2 , and write it

$$(1 - \pi_2)^2 + m\pi_2(1 - \pi_2) + n\pi_2^2 > 0$$

Insert the definitions of m and n and find

$$(1 - c)[(\beta_{11} + \beta_{21})\beta_{21}(1 - \pi_2) + (\beta_{12} + \beta_{22})\beta_{22}\pi_2] > 0$$

which it is under our assumptions about β_{ij} and π_i .

Now that we have validated (94), take its first inequality, multiply it by $1 - \pi_1$, divide it by ρ , use the definitions (78) and (79) and find

$$v_{11} > 0$$

$$v_{12} > 0$$

Take the second inequality of (94), multiply it by π_2 , use the definitions (80) and (81) and find

$$v_{21} > 0$$

$$v_{22} > 0$$

We conclude that (78) through (87) are indeed positive.

[illegible]

A P P E N D I X I I I

EMPIRICAL MEASUREMENT OF GROWTH IMBALANCE

Yotopoulos and Lau [6] have examined growth imbalance in 65 countries for the periods 1948-53, 1954-58, and 1950-60. In each country, six sectors were distinguished, i. e. agriculture, mining, manufacturing, construction, electricity-gas-water and "others," including transportation and communication, services, etc.

Modifying the Yotopoulos-Lau notation slightly to make it consistent with our own, let us define

E_i $\equiv$ income elasticity of demand for output of i th sector

G $\equiv$ proportionate rate of growth of gross domestic product in
constant prices

g_{Xi} $\equiv$ proportionate rate of growth of output of i th sector in
constant prices

w_i $\equiv$ share in gross domestic product of value added by i th sector

1914

1914-1915

1914-1915

1914-1915

1914-1915

1914-1915

Yotopoulos-Lau now applied two different concepts of imbalance. First, an index of Samuelson-Solow-von Neumann imbalance defined as

$$(95) \quad V^* \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - G)^2}$$

or, in English, the reciprocal of the national real growth rate times the square root of the weighted sum of the squared deviations of sectoral real growth rates from the national real growth rate.

For their entire sample of 65 countries, Yotopoulos-Lau found a rather strong negative correlation between the Samuelson-Solow-von Neumann index of imbalance and the national real growth rate; The coefficient of correlation was -0.322. They also found the most highly developed countries to have have the lowest index of

Die in der vorliegenden Arbeit beschriebenen Untersuchungen sind in der Hauptsache aus dem Jahre 1934 zu datieren. Die Ergebnisse sind in der folgenden Tabelle zusammengefasst.

Untersuchungsgegenstand	Ergebnis	Bemerkungen
1. Die Wirkung von ...	...	...
2. Die Wirkung von ...	...	...

Die Ergebnisse der Untersuchungen sind in der folgenden Tabelle zusammengefasst. Die Tabelle ist in drei Spalten unterteilt. Die erste Spalte enthält die Untersuchungsgegenstände, die zweite Spalte die Ergebnisse und die dritte Spalte die Bemerkungen. Die Untersuchungen sind in zwei Gruppen unterteilt. Die erste Gruppe umfasst die Untersuchungen der Wirkung von ... Die zweite Gruppe umfasst die Untersuchungen der Wirkung von ... Die Ergebnisse der Untersuchungen sind in der folgenden Tabelle zusammengefasst.

INDEX OF SAMUELSON-SOLOW-VON NEUMANN IMBALANCE $V^* \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - G)^2}$

FIGURE 2. SAMUELSON-SOLOW-VON NEUMANN IMBALANCE IN 19 COUNTRIES 1950-60

$$\text{INDEX OF NURKSE IMBALANCE } V' \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - E_i G)^2}$$

FIGURE 3. NURKSE IMBALANCE IN 19 COUNTRIES 1950-60

imbalance.

From the Yotopoulos-Lau sample of 65 countries our own Figure 2 has selected, for the period 1950-60, a much smaller sample consisting of the 19 capitalist countries which had, in 1958, a per capita income of \$500 or more per annum. Figure 2 shows that even those countries still had a substantial Samuelson-Solow-von Neumann index of imbalance: Their square root of the weighted sum of squared deviations ranged from 0.19 (Venezuela) to 1.26 (Uruguay) of the national real growth rate, with the majority of the countries lying between 0.30 and 0.55 of that rate.

Could imbalance be explained by nonunitary sector income elasticities? Here it occurred to Yotopoulos-Lau to define a second index of imbalance removing from the imbalance concept those deviations which are caused by nonunitary sector income elasticities. That index they called a Nurkse imbalance index and defined it as

...the first of the ...
...the second of the ...
...the third of the ...
...the fourth of the ...
...the fifth of the ...
...the sixth of the ...
...the seventh of the ...
...the eighth of the ...
...the ninth of the ...
...the tenth of the ...
...the eleventh of the ...
...the twelfth of the ...
...the thirteenth of the ...
...the fourteenth of the ...
...the fifteenth of the ...
...the sixteenth of the ...
...the seventeenth of the ...
...the eighteenth of the ...
...the nineteenth of the ...
...the twentieth of the ...
...the twenty-first of the ...
...the twenty-second of the ...
...the twenty-third of the ...
...the twenty-fourth of the ...
...the twenty-fifth of the ...
...the twenty-sixth of the ...
...the twenty-seventh of the ...
...the twenty-eighth of the ...
...the twenty-ninth of the ...
...the thirtieth of the ...
...the thirty-first of the ...
...the thirty-second of the ...
...the thirty-third of the ...
...the thirty-fourth of the ...
...the thirty-fifth of the ...
...the thirty-sixth of the ...
...the thirty-seventh of the ...
...the thirty-eighth of the ...
...the thirty-ninth of the ...
...the fortieth of the ...
...the forty-first of the ...
...the forty-second of the ...
...the forty-third of the ...
...the forty-fourth of the ...
...the forty-fifth of the ...
...the forty-sixth of the ...
...the forty-seventh of the ...
...the forty-eighth of the ...
...the forty-ninth of the ...
...the fiftieth of the ...
...the fifty-first of the ...
...the fifty-second of the ...
...the fifty-third of the ...
...the fifty-fourth of the ...
...the fifty-fifth of the ...
...the fifty-sixth of the ...
...the fifty-seventh of the ...
...the fifty-eighth of the ...
...the fifty-ninth of the ...
...the sixtieth of the ...
...the sixty-first of the ...
...the sixty-second of the ...
...the sixty-third of the ...
...the sixty-fourth of the ...
...the sixty-fifth of the ...
...the sixty-sixth of the ...
...the sixty-seventh of the ...
...the sixty-eighth of the ...
...the sixty-ninth of the ...
...the seventieth of the ...
...the seventy-first of the ...
...the seventy-second of the ...
...the seventy-third of the ...
...the seventy-fourth of the ...
...the seventy-fifth of the ...
...the seventy-sixth of the ...
...the seventy-seventh of the ...
...the seventy-eighth of the ...
...the seventy-ninth of the ...
...the eightieth of the ...
...the eighty-first of the ...
...the eighty-second of the ...
...the eighty-third of the ...
...the eighty-fourth of the ...
...the eighty-fifth of the ...
...the eighty-sixth of the ...
...the eighty-seventh of the ...
...the eighty-eighth of the ...
...the eighty-ninth of the ...
...the ninetieth of the ...
...the ninety-first of the ...
...the ninety-second of the ...
...the ninety-third of the ...
...the ninety-fourth of the ...
...the ninety-fifth of the ...
...the ninety-sixth of the ...
...the ninety-seventh of the ...
...the ninety-eighth of the ...
...the ninety-ninth of the ...
...the hundredth of the ...

$$(96) \quad V' \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - E_i G)^2}$$

or, in English, the reciprocal of the national real growth rate times the square root of the weighted sum of the squared deviations of sectoral real growth rates from the product of sector income elasticity and national real growth rate.

Now suppose that imbalance were fully explained by nonunitary sector income elasticities. Then the output of the i th sector would always be growing at the rate $g_{Xi} = E_i G$, consequently according to (96) $V' = 0$. In other words, Nurkse imbalance would be zero.

Applying to the same period and the same countries as Figure 2, our Figure 3 shows that Nurkse imbalance is far from zero. The

Nurkse imbalance in Figure 3 is almost as substantial as the Samuelson-Solow-von Neumann imbalance in Figure 2. The Nurkse range has the same floor but a slightly lower ceiling than the Samuelson-Solow-von Neumann range: The square root of the weighted sum of squared Nurkse deviations ranges from 0.19 (the Netherlands) to 1.0 (Uruguay) of the national real growth rate, with a majority of the countries lying between 0.25 and 0.50 of that rate. We conclude that the Nurkse index has removed precious little imbalance from the Samuelson-Solow-von Neumann index.

How come, so little? Suppose all sector income elasticities were unity, then the Samuelson-Solow-von Neumann index would become equal to the Nurkse index: If $E_i = 1$ it follows from (95) and (96) that $V^* = V'$. And indeed the income elasticities used by Yotopoulos-Lau differed very little from unity:

1. Einleitung

Die vorliegende Arbeit beschäftigt sich mit der Analyse der Auswirkungen der Digitalisierung auf den Arbeitsmarkt. Im Zentrum stehen die Veränderungen in der Nachfrage nach Qualifikationen und die damit verbundenen Herausforderungen für die Bildungspolitik. Die Untersuchung basiert auf einer Literaturrecherche sowie auf empirischen Daten aus verschiedenen Ländern. Ziel ist es, die Zusammenhänge zwischen technologischen Innovationen und der Struktur des Arbeitsmarktes zu verdeutlichen und mögliche politische Handlungsoptionen zu diskutieren. Die Arbeit ist in drei Hauptteile gegliedert: Zunächst wird der theoretische Hintergrund skizziert, gefolgt von einer detaillierten Darstellung der empirischen Befunde. Abschließend werden die Ergebnisse zusammengefasst und die Bedeutung für die Zukunft des Bildungswesens diskutiert.

Agriculture	0.952
Mining	0.892
Manufacturing	1.044
Construction	1.035
Electricity-gas-water	1.045
Others	0.999

These sector income elasticities were estimated from cross sections of some of the countries examined but applied to all countries.

From the Yotopoulos-Lau measurements we conclude three things. First, that growth imbalance is a rather ubiquitous phenomenon. Second, that in highly developed countries it is not strongly correlated with the national real growth rate. Third, that nonunitary sector income elasticities play a minuscule role in explaining real-world growth imbalance.

F O O T N O T E S

*For a reflective fall semester of 1970 as an associate at the University of Illinois Center for Advanced Study, the author is indebted to the Graduate College of the University of Illinois. For careful checking of the mathematics and for valuable suggestions, he is indebted to Mr. Bojan Popovic, a graduate student at the Department of Economics and the Coordinated Science Laboratory at the University of Illinois.

¹We define, as Hahn and Matthews [3] did, steady-state growth as stationary proportionate rates of growth of physical outputs. We define, as Solow and Samuelson [4] did, balanced growth as identical proportionate rates of growth of physical output for all goods.

²Our Graham-type demand functions (38) and (39) have unitary income elasticities. In our model, then, possible growth imbalance must have causes other than nonunitary income elasticities. From Yotopoulos-Lau [6] one may conclude that nonunitary sector income elasticities play a minuscule role in explaining real-world growth imbalance. This conclusion is derived in Appendix III.

1990

R E F E R E N C E S

- [1] Denison, E. F., Why Growth Rates Differ, Washington, D. C., 1967, Ch. 16.
- [2] Graham, F. D., "The Theory of International Values Re-Examined," Quart. J. Econ., Nov. 1923, 38, 54-86.
- [3] Hahn, F. H. and R. C. O. Matthews, "The Theory of Economic Growth: A Survey," Econ. J., Dec. 1964, 74, 779-902.
- [4] Solow, R. H., and P. A. Samuelson, "Balanced Growth under Constant Returns to Scale," Econometrica, July 1953, 21, 412-424.
- [5] Uzawa, H., "On a Two-Sector Model of Economic Growth," I-II, Rev. Econ. Stud., Oct. 1961, 29, 40-47 and June 1963, 30, 105-118.
- [6] Yotopoulos, P. A., and L. J. Lau, "A Test for Balanced and Unbalanced Growth," Rev. Econ. Stat., Nov. 1970, 52, 376-384.

1917

1917-1918

1917-1918

1917-1918

1917-1918

1917-1918

1917-1918

1917-1918

GREEK LETTERS USED

α alpha
 β beta
 ζ zeta
 κ kappa
 λ lambda
 μ mu
 ν nu
 ξ xi
 π pi
 ρ rho
 Σ sigma
 τ tau
 ϕ phi
 ψ psi
 ω omega

MATHEMATICAL SYMBOLS USED

{ } brace
[] bracket
| | determinant
= equal to
> greater than
 $\equiv$ identically equal to
 $\int$ integral of
< less than
 $\neq$ not equal to
() parenthesis
 ∂ partial derivative of
 $\sqrt{\quad}$ square root of

Table 1

Item	Value	Unit
1. Wheat	1000	kg
2. Corn	500	kg
3. Soybeans	200	kg
4. Rice	150	kg
5. Barley	100	kg
6. Oats	80	kg
7. Rye	60	kg
8. Buckwheat	40	kg
9. Millet	30	kg
10. Sorghum	20	kg
11. Amaranth	10	kg
12. Quinoa	5	kg
13. Lentils	3	kg
14. Chickpeas	2	kg
15. Peas	1	kg

110-0

UNIVERSITY OF ILLINOIS-URBANA

3 0112 060296909