EXPERIMENTS IN WAVE RECORD ANALYSIS

Jose Luis Goncalves Cardoso

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THESIS

EXPERIMENTS IN WAVE RECORD ANALYSIS

by

Jose Luis Goncalves Cardoso

September 1980

Thesis Advisor:

W. C. Thompson

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Experiments in Wave Record Analysis

by

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Lieutenant Commander, Portuguese Navy
Portuguese Naval Academy, 1965

Submitted in partial fulfillment of the
requirements for the degree of

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September 1980

ABSTRACT

By examining the relationship of the individual waves in an ocean wave record and the power density spectrum (PDS), it is demonstrated that the PDS conventionally obtained using spectral analysis can be reasonably approximated, except at the very low frequency end, by a pseudo-power density spectrum constructed from the individual heights and periods contained in the record. The very low frequencies seen in the side bands of the PDS are examined as products of amplitude modulation by analogy with simulated wave records and as an indication of the presence of wave groups. In addition, wave groups are identified and their properties determined in 19 selected ocean wave records by integrating the wave record using the procedure of Funke and Mansard (1979).

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I. INTRODUCTION

The computer era and the Fast Fourier Transform (FFT) analysis that is associated with it permits the manipulation of wave records in digital form to produce a power density spectrum (PDS) with great efficiency.

The PDS gives a presentation of the distribution of the total wave energy contained in successive frequency bands or sometimes in period bands. In this way, the analyst and interpreter of wave records has a process that describes the more or less random individual wave heights and periods present in a wave record with a deterministic approach, where the peak spectral frequency or period and its relationship to the total energy content are the salient features.

However, the PDS does not fully describe all practical characteristics of the wave field. For example, to synthesize a realistic wave field in a laboratory channel, the matching of amplitude spectra between the prototype and the simulated wave field is no longer sufficient for the reason that different wave fields may have basically the same frequency spectrum (Funke and Mansard, 1979).

The particular sequence of high and low waves is an additional relevant factor. Therefore, attention to grouping of waves and their successive individual properties must be included as an additional objective of the analysis of wave records.

Also, it will be shown that the two evident parameters which can be detected by an observer when studying sequences of individual waves, their heights and periods (see definition in Section III.A.2), can be further manipulated and utilized to generate a so-called pseudo-spectrum. This pseudo-spectrum may be considered another descriptor of energy in relation to the individual waves that are seen in the field.

The general objectives of this thesis, therefore, are to inquire into the feasibility of producing a pseudo-frequency spectrum from the height and period measured from the individual waves constituting the wave record, and to analyze sequences of heights and periods.

With regard to the relation between spectrum form and group properties, it is also an objective of the thesis to inquire into their detection if possible through consideration of apparent peaked symmetrical side bands occurring in the PDS which may represent the effect of low-frequency amplitude modulation.

Finally, it is our intention in the course of the study presented by Funke and Mansard (1979) to examine wave group properties, namely their "degree" of presence in the wave record, their duration and their variability.

II. WAVE DATA

A. DATA SOURCE

The body of this work was accomplished with data from the Point Santa Cruz waverider buoy obtained during the period from June 1978 to February 1979. The location, shown in Figure 1, is in a water depth of 60 meters. This is deep to intermediate water for all ordinary gravity waves (sea and swell) as defined by the relative depth.

The wave buoy was installed as part of the California Coastal Engineering Data Network (CEDN) which is sponsored jointly by the California Department of Boating and Waterways and the U.S. Army Corps of Engineers. Additional funding for the wave buoy system is supplied by the University of California Sea Grant Program with Scripps Institution of Oceanography providing overall direction for the actual data collection program.

The data from the buoy received several times a day is recorded on a magnetic tape compatible with the IBM 360/67 computer system. Each record consists of 1024 discrete points of instantaneous surface elevation at 1 second interval. Thus each wave record has a duration of 17 minutes and 4 seconds.

The individual wave records are spectrally analyzed by CEDN and the results are published monthly. Tabular data,

an example of which is shown in Figure 2, include:

Significant wave height

Total energy

Percentage of total energy in period bands of 2 seconds.

Also a visual presentation of the daily energy spectra is given so as to provide an indication of the overall daily energy pattern for a one month period (Figure 3). This presentation enables one to visualize on what days of the month the energy content is high, how the energy is partitioned between the shorter and longer period waves, and how broad or narrow is the spectrum.

B. WAVE RECORD ANALYSIS

For this study wave records were provided on a magnetic tape by the CEDN. Spectral and other analysis of the data were performed independently of the CEDN computations. The published analyses made by CEDN were used only for purposes of comparison with certain of our own analyses. The wave records selected for analysis in this thesis were chosen for their high total energy and for the dominantly unimodal character of their spectra.

C. FILE NUMBER

For convenience in discussing individual wave records and the products of all analyses, file numbers were assigned to all records in agreement with those given by CEDN. It should be noted that the figures are arranged in a logical order but that they are not necessarily discussed in the

following sections in the order in which they appear. Because of the large number of printouts contained in the 19 files analyzed the decision was made to include only one complete file in the thesis for illustration (File Number 228). The wave record for this file, which was considered a good representation of a typical wave field, is shown in Figure 4.

D. CHARACTERISTICS OF WAVE RECORDS ANALYZED

The significant wave characteristics of the 19 wave records analyzed are given in Table I. The wave records are classified in the order of the wave type as defined by their significant steepness (after Thompson and Reynolds, 1978), where steepness is defined as

$$\frac{2\pi H_s}{g T_p^2}$$

by analogy with linear theory, H_s is significant wave height and T_p is peak spectral period. The defining steepness limits adopted were the following:

Sea	1/12 - 1/40
Young swell	1/40 - 1/100
Moderate swell	1/100 - 1/250
Old swell	>1/250

III. INDIVIDUAL WAVE ANALYSIS

This chapter considers the question of whether the frequency spectrum of a wave record can be approximately constructed from the H and T measures of the individual waves constituting the record. If this is shown to be feasible, it is felt that it will permit a fuller interpretation of the frequency spectrum in terms of the waves composing it.

The approach taken in addressing this question was to produce a true spectrum from a given wave record using the FFT technique and also a so-called pseudo-spectrum using the H and T measures from the same record.

For various reasons, spectra are difficult to compare. Accordingly, both spectra were converted into cumulative energy distributions and these were then compared. These procedures and the principal results obtained are described below.

A. SPECTRAL ANALYSIS

1. True Spectrum¹

¹ Although the values are "spectral estimates" of the "true spectrum", with uncertainty described by the Chi-squared probability density function with eight degrees of freedom, the name true spectrum was chosen as contrasted to the "pseudo-spectrum" derived from the individual H_i and T_i .

a. Power Density Spectrum (PDS)

With a basic program that utilizes the International Mathematical and Statistical Library subroutine RHARM, the power density was found via the Fast Fourier Transform performed on each record of length 1024 points with one second interval.

The data were divided into four "windows" which results in spectral estimates distributed according to a Chi-squared density with eight degrees of freedom, while maintaining sufficient resolution to detect the low-frequency components for periods as long as four minutes. Thus the width of each window is 256 seconds, the maximum period that can be detected. Consequently, the resolution is $1/256$ Hz or 0.0039 Hz.

It is worthwhile to note that for more "windows" of shorter duration, say 8 with 128 seconds width, the number of frequency components in the PDS decreases. This increases the stability of the spectral estimates, since in the fewer but broader spectral bands the number of degrees of freedom is increased. For comparison, Figures 10 and 11 show PDS produced for the same wave record (File Number 228) using four and eight windows, respectively (note that the energy density scales differ).

b. True Energy Histogram

Also using a class interval of $\Delta f = 1/256$ Hz a histogram of the distribution of energy in percentage of

the total was computed (Figure 12) from the PDS (Figure 10). This true energy histogram represents the distribution of energy centered about the same frequencies that define the PDS. In a sense each value represents the area of a "slice" resulting from the sum of two pieces with base width $\Delta f/2$ extending on each side of the central frequency.

For comparison, an energy histogram with class interval 1/128 Hz produced from the PDS obtained using eight windows (Figure 11) is shown in Figure 13. In both figures, these histograms are labelled true energy. The values are connected by a line as an aid to visual inspection.

c. Cumulative True Energy Distribution

Early trials showed that comparison of the true energy with our pseudo-energy, not described yet, was better done with a cumulative curve, summed from low to high frequency.'

The normalized cumulative energy distribution computed from the energy histogram in Figure 12 is shown in Figure 15 (labelled true spectrum), with ordinates of percentage of the total energy.

A cumulative energy description in energy units of length squared was also produced. Figure 14 presents this curve also labelled true spectrum.

2. Pseudo-Spectrum

a. Determination of H_i and T_i

Using the zero-upcrossing method, the period, T_i , and the height, H_i , of successive individual waves were measured in a wave record. The still water level was obtained from the mean of the 1024 instantaneous surface elevation digital points (the zero level reference). The individual period, T_i , was taken as the time difference between two successive zero-upcross points. The individual wave height, H_i , was taken as the crest to trough distance where the crest and trough were represented by the maximum positive and minimum negative values, respectively, within each pair of zero-upcross points.

b. Pseudo-Energy Histogram

The energy histogram that results from a power density spectrum (PDS) is a histogram of energy versus frequency, where the energy is interpreted as energy per unit area of sea surface per unit weight, with dimensions of length squared. This energy is associated with the central frequency of the band.

It is also possible to treat a sequence of waves so that individual H_i and T_i are presented in terms of H_i^2 and $1/T_i$, the same dimensions as above. It is logical that the average of the H_i^2 in the wave record should represent a parameter that is directly related to the energy per unit area because those are the waves in that space.

Thus if each H_i^2 divided by the total number of waves, N , in the record lends its contribution to the average total energy per unit area, we can use it as an energy parameter for waves within a class interval as well. Then, the same class intervals previously used to produce the true histogram can produce a pseudo-energy histogram.

Thus, in our approach, the individual wave period was converted to its reciprocal. This parameter, with the dimensions of frequency, was then organized into class groups. Then the summation of $\frac{H_i^2}{N}$ in each class interval was computed and this value was found in percentage of the total sum in all classes. With the central frequency taken as reference, a plot of the summed values in each class interval yielded a histogram. Figures 12 and 13 present histograms, labelled pseudo-energy, plotted using class intervals of 1/256 Hz and 1/128 Hz, respectively. These are superimposed for comparison on the true energy histograms derived from the PDS. Both sets of curves are produced from the same wave record shown in Figure 4 (File Number 228). A comparison of the two curves is discussed later.

c. Cumulative Pseudo-Energy Distribution

In order to compare more objectively the pseudo-energy histogram derived from H_i and T_i values with the true energy histogram derived from the PDS, the histograms were transformed into cumulative energy distributions. This was

done by cumulating the energy in each class interval (frequency band) from low to high frequency in the same manner as with the true energy histogram. Figure 15 provides an example of the cumulative pseudo-energy distribution in percentage for File Number 228, derived from the pseudo-energy histogram with a 1/256 Hz class interval (Figure 12).

With the objective of standardizing the total energy in the pseudo-spectrum to the total energy in the true spectrum in energy units of length squared, an attempt was made to obtain a conversion parameter. This was accomplished by dividing the summation of H_i^2 by the number of waves in the record, N , and by the variance, V , obtained from statistical analysis,

$$\frac{\frac{1}{N} \sum_{i=1}^N H_i^2}{V}$$

The interesting result is that a mean value of 6.70 with a standard deviation of 0.37 was obtained for the 19 records analyzed. These individual values are presented in Table II. The reason that the total energy factor appears to be a constant and to have the value 6.70 is not clear.

In view of this apparent constant it was decided to apply a proportionality factor of 6.7 to convert our individual energy parameter, $\frac{H_i}{N}$, to a new parameter

$$\frac{H_i^2}{6.7 \times N}$$

Following this, each individual wave was put in ascending order by its apparent frequency (the inverse of its period) and each individual $\frac{H_i^2}{6.7 \times N}$ was summed in a cumulative way. The results of File Number 228 and of four typical records chosen out of 19 records are presented in Figures 14, 32, 42, 52, and 62 in comparison with the cumulative true energy distribution.

3. Comparison of Pseudo and True Spectra

Since the presentation of the cumulative energy distribution in energy units depends on the proportionality factor chosen, the cumulative curves were compared in percentage (see examples in Figures 15, 32A, 42A, 52A and 62A).

The significant points that result from comparing the two cumulative energy curves derived from the 19 wave records analyzed are presented below:

The two curves agree quite closely about the central part of the distribution, but differ most toward both ends. Thus the slope of the central portion of the pseudo-spectrum identifies the true spectrum peak and also characterizes the proper spectral width.

The true spectrum in all of the records analyzed identifies a significant amount of energy in frequencies lower than the frequency equivalent to the reciprocal of the maximum wave period detected by the zero-upcrossing method. At this frequency it was found that the differences in

percentage of cumulated energy between the true and pseudo-spectra among the 19 wave records ranges from 1.75 to 18. Thus the lower frequencies of the true energy distribution are poorly represented in the pseudo-energy distribution.

It is also evident that the pseudo-spectrum differs substantially from the true spectrum, in the sense that it is less smooth. This means that the same quantity of energy is described with differently weighted components by the pseudo-spectrum.

B. WAVE HEIGHT, PERIOD, AND STEEPNESS

As already described, the production of the pseudo-cumulative spectrum starts with the computation of the successive wave heights and periods. It was of some interest for the purpose of visually comparing wave heights and periods with both the true and the pseudo-spectra produced, and also in regard to the examination of wave groups occurring in the wave record (described later), to print out in graphical form the sequence of H_i and T_i occurring in the record. From our 19 files we present the sequences of heights and periods for five wave records in Figures 5, 6 (File Number 228), 24, 25 (File Number 579), 34, 35 (File Number 600), 44, 45 (File Number 446), and 54, 55 (File Number 677).

In addition, the individual wave steepness derived from H_i and T_i was also computed for each wave and the sequence of

steepness values was also printed out graphically for each wave record (examples are shown in Figures 7, 26, 36, 46, and 56).

The individual wave steepness, defined by analogy with the linear wave theory, is given by

$$S_i = \frac{H_i}{L_i} = \frac{2\pi H_i}{g T_i^2}$$

We are aware that in real water waves the length of an individual wave may be different from the length given by linear theory. Nevertheless, we reasonably assume that the quantity S_i defined here is proportional to the height-to-length ratio in real waves.

Finally, we present in Figures 8, 27, 37, 47, and 57 graphs of H_i versus T_i ; these can be interpreted as a representation of the individual wave steepness distribution in the record. Also plotted on each graph is the significant wave steepness,

$$S_s = \frac{2\pi H_s}{g T_p^2}$$

which characterizes the wave record.

1. Sequences of Individual Height and Period (H_i, T_i)

The following is a summary of the most significant points that were found from examination of the sequence graphs of H_i and T_i :

The sequence of wave heights gives a first indication about the presence of wave groups and permits one to count the number of waves in a group (see Figures 35, 45, and 55). It is interesting to note that the number of successive waves that exceeds the significant wave height is normally one to two and rarely three to four or more waves. It would be interesting to compare these observations with the theory of, e.g., Ewing (1973) on the length of runs of high waves as a function of spectral bandwidth, but that is beyond the scope of this thesis.

In the sequence of periods, the spectral peak is consistently higher than the mean period, as expected. However, in records that can be characterized as "sea", the difference is small (Figure 24), and the sequence of periods appears to be random; whereas, when looking at records of young and moderate swell, such as shown in Figures 44 and 54, well-defined wave groups can actually be distinguished by their higher values and patterned sequence.

2. Sequence of Individual Steepness, S_i

It was found that records classified as "sea" (Table I) display more uniform values of steepness (Figure 26), whereas steepness values in young and moderate swell have higher variability (see Figures 36 and 46; note the different vertical scales). Figure 56, as expected with old swell, presents a preponderantly constant steepness. All sequence graphs of S_i characteristically show steepness spikes.

3. Individual Height Versus Period

Each plotted point in these graphs represents the steepness S_i of an individual wave in the wave record. The points are seen, in most of the graphs, to fall within a generally well-defined steepness envelope. The envelope is associated with large steepness values for sea and young swell (Figures 27 and 37) and low values for old swell (Figure 57), as expected. In most graphs the width of the envelope, representing the scatter of the S_i values, is relatively narrow (Figure 27) but in several it is broad (Figure 66).

In some graphs the data points show a pattern suggesting more than one steepness envelope. In Figure 8, for example, the data points associated with the highest and steepest waves are contained within the steepness values of $1/40$ and $1/100$, whereas a set of lower, less steep waves lies within the envelope bounded approximately by the steepness values of $1/100$ and $1/250$. In Figure 57, the higher waves have the lower steepness, their envelope being centered about $S_i = 1/250$, whereas, a second set of waves having low heights but high steepness appear to occupy an envelope contained between the steepness values of about $1/40$ to $1/50$.

The significant steepness of the wave record which is also shown on each graph is generally representative of the waves of greatest height and period in the record, as

may be expected. However, it was found that for a few records, significant steepness is not a good representation of the waves observed (Figure 66). This situation may be due to the presence of a young sea superimposed on an old swell, as Figure 63 suggests.

C. AUTOCORRELATION OF SURFACE ELEVATION

The application of the autocorrelation function to a wave record and the analysis of its periodicity is, as is known, another way of finding the main spectral frequency of the record (see Figures 9, 28, 38, 48, and 58). However, this process can also provide an indication of the presence of wave groups.

As an example, compare the autocorrelation of a sea record (Figure 28) with an old swell record (Figure 58). The small exponential decay in the swell correlogram is in correspondence with the form of the envelope of the wave record (Figure 53), and is a distinct "signature" of the presence of well-defined wave-groups, in contrast with the rapid decay observed in the sea type wave record.

D. COMPARISON OF TOTAL ENERGY PARAMETERS

In the analysis of wave records to obtain the cumulative true-energy and cumulative pseudo-energy distributions several estimates of the total energy were obtained. Later another function designated SIWEH, but not described yet, was

obtained whose mean value is also an indication of the total energy. With the objective of comparison, Table III presents for the 19 wave records analyzed the total energy computation obtained using the following procedures:

- Statistical variance
- Integration of the PDS
- Mean of the Smooth Instantaneous Wave Energy History or SIWEH (Section IV-B)
- Cumulative pseudo-energy using a factor of proportionality of 6.7 (Section III-A-2(c))
- Total energy presented by CEDN

From this table it can be said that the first three of these methods of computing the variance from a wave record present almost equal results, the differences being due to the differences in the methods of numerical computation. It is of interest that the cumulative pseudo-energy computation using a factor of 6.7 gives a value for the total energy with an average error of plus or minus 5.3 percent in relation to the variance.

IV. WAVE GROUP ANALYSIS

The question examined here about what process may be interpreted as operating in the peak spectral frequency was due to the graphs showing the sequence of H_i and T_i that were produced for the identification of individual waves and also from certain characteristics of the PDS.

Both sequence graphs show high variability and if the sequence of wave height as manifested by wave groups can be interpreted as the consequence of amplitude modulation, the sequence of periods suggests that some kinds of frequency modulation may also be present. We have to say to the reader that we did not find conclusive evidence for a simple and easily identifiable frequency modulation in our wave records examined, but the question is raised because it seems that more could be done in describing the wave record through a more deterministic approach than has been taken before. Thus we are assuming that if the PDS of real wave records show peaks on either side of the central peak, this should be due to amplitude modulation instead of frequency modulation.²

² The possibility should not be ignored that secondary peaks can arise as artifacts of the analysis procedure due to "leakage". If so, they appear at intervals related to the resolution Δf .

It was felt that a simple mathematical approach to modulation phenomena will help in the interpretation of narrow band PDS and in probing the nature of wave groups in real wave records. Accordingly, this is presented in some detail in the next three sections.

A. MODULATION AND WAVE GROUPS

1. Amplitude Modulation

In this type of modulation the amplitude of the carrier wave, or carrier, is changed in a proportional way to the instantaneous value of the modulating signal. We can represent this concept by:

Transport wave carrier

Modulating signal

Wave modulated in amplitude

For simplification we assume that the modulating wave and the carrier are sine waves. In this way we can produce something with this form:

We can manipulate expression [3] in this form:

$$\begin{aligned}
 e &= (E_C + e_m) \sin 2\pi f_c t \\
 &= (E_C + E_m \sin 2\pi f_m t) \sin 2\pi f_c t \\
 &= E_C \sin 2\pi f_c t + E_m \sin 2\pi f_c t \sin 2\pi f_m t
 \end{aligned}$$

Keeping in mind that

$$\sin a \sin b = \frac{\cos (a-b)}{2} - \frac{\cos (a+b)}{2}$$

we get:

$$\begin{aligned}
 e &= E_C \sin 2\pi f_c t + \frac{E_m}{2} \cos 2\pi (f_c - f_m) t \\
 &\quad - \frac{E_m}{2} \cos 2\pi (f_c + f_m) t
 \end{aligned}$$

Now if we define as an index of modulation the value $m = \frac{E_m}{E_c}$, we get:

$$e = E_c \sin 2\pi f_c t + \frac{mE_c}{2} \cos 2\pi(f_c - f_m)t - \frac{mE_c}{2} \cos 2\pi(f_c + f_m)t$$

This is the general expression for a wave train modulated in amplitude by a single frequency.

What can be learned from this is that the spectrum of such a modulated wave should present three distinct peaks at the frequencies f_c , $(f_c + f_m)$, and $(f_c - f_m)$.

In Figures 68 (lower graph) and 69, we have a simple wave form and its spectrum, with $E_c = 100$ and $f_c = 0.1$ Hz ($T = 10$ sec) modulated by an $E_m = 80$ with $f = 0.016$ Hz ($T = 60$ sec). The spectrum for this simple case can be interpreted in this way:

The diagram shows a central peak, two side bands equally spaced from the central peak, and a low frequency component.

As far as we can understand, the modulating wave in true wave records is not as simple as presented, but can be visualized by a complex of frequency components.

In a visual sense, it seems possible that the characterization of the low-frequencies, f_m , may be achieved by extrapolating to the following schematic concept:

In this representation it can be understood that the existence of symmetrical side bands can be an indication of a well-defined low-frequency modulating signal, and thus the presence of symmetrical side bands in the PDS may be expected to indicate the occurrence of wave groups. So the wave group frequency might be deduced from the frequency separation between the side bands and the spectral peak.

2. Frequency or Phase Modulation

In frequency modulation the instantaneous value of the carrier frequency is changed by the modulating signal, but the amplitude of the carrier is kept constant. Simple modulation of sinusoidal waves is illustrated as follows:

Here the instantaneous difference between the carrier frequency and the frequency of the modulated signal is proportional to the instantaneous amplitude of the modulator signal, but is independent of the frequency of the modulator signal. However, the number of times per second that the frequency changes around the central frequency is the frequency of the modulator signal. Thus the maximum excursion of the frequency from the central or peak frequency, Δf is:

$$\Delta f = K E_m$$

where: Δf = maximum change in frequency

E_m = amplitude of the modulating signal

K = constant of proportionality

To understand the relationship between side bands and carrier, let us consider that the initial carrier can be written as:

$$e = E \sin \theta(t)$$

where: e = instantaneous amplitude

E = maximum amplitude

$\theta(t)$ = angular value at time (t)

The instantaneous angular velocity is by definition the rate of change of $\theta(t)$ with time,

$$\omega_i = \frac{\partial \theta(t)}{\partial t} = 2\pi f_i$$

Thus we can say that our carrier with constant frequency f_c has $\omega_c = 2\pi f_c$ and our $\theta(t) = \omega_c t + \psi$, where ψ is the initial phase angle at $t = 0$.

Here, as the frequency is constant

$$\omega_i = \frac{\partial \theta(t)}{\partial t} = \omega_c$$

which corresponds to the f_c (i.e., $\omega_c = 2\pi f_c$).

Now, according to the previous definition, a sinusoidal signal modulated in frequency by another sinusoidal signal has an instantaneous value of frequency, expressed by:

$$f_i = f_c + \Delta f \cos \omega_m t ,$$

where: $\Delta f = K E_m$

and the instantaneous angular velocity should be (multiplying both terms by 2π)

$$\omega_i = \omega_c + 2\pi \Delta f \cos \omega_m t$$

where: ω_i = instantaneous angular velocity

ω_c = angular velocity of the carrier

ω_m = angular velocity of the modulating signal

Δf = maximum change in frequency

Now if
$$\omega_i = \frac{\partial \theta(t)}{\partial t}$$

then
$$\theta(t) = \int_0^t \omega_i(t) dt$$

which gives
$$\theta(t) = \omega_c t + \frac{2\pi \Delta f}{\omega_m} \sin \omega_m t$$

therefore the amplitude of our initial wave when under the influence of the modulating signal can be expressed by:

$$e = E \sin \left(\omega_c t + \frac{2\pi \Delta f}{\omega_m} \sin \omega_m t \right)$$

where $J_{(n)}(m_f)$ are the Bessel functions of order n with argument m_f .

The objective for this approach is not only to show that a typical frequency modulation has these kinds of components from a spectral point of view, but also to take note of the fact that for an $m_f = 0.6$ the lateral bands of second and higher order are approximately zero because the Bessel function with that argument is almost zero. This means that only the fundamental and the first harmonic are important in regard to the spectrum of such a signal, so that modulation of amplitude and frequency from the spectral point of view look the same.

3. Modulation in Real Waves

To exemplify the special equivalence described in the previous section and with the objective to better interpret real waves, the reader is asked to compare Figures 69 and 73, derived from the wave records shown in Figures 68 and 71, respectively (lower graphs). As stated, the PDS in Figure 69 was produced from amplitude modulation with $f_c = 1/10$ Hz and $f_m = 1/60$ Hz having $E_c = 100$ and $E_m = 80$, whereas the PDS in Figure 73 was produced from frequency modulation with $m_f = 0.6$ where the $f_c = 1/10$ Hz and $f_m = 1/60$ Hz. As can be seen, the two spectra are almost identical. Both include side bands centered at the same frequencies about the fundamental.

What can be learned from this discussion that may be applicable to real ocean waves is that side bands should be expected in narrow band wave records. The presence of side bands is probably due to some kind of amplitude modulation rather than frequency modulation because the latter interpretation implies only a very small change in the wave periods (see Figure 72) and a constant amplitude, factors normally not expected in ocean wave records. On the other hand, larger variability in the periods can be interpreted as the consequence of a larger m_f , and necessarily a larger number of components should be present in the spectrum.

Thus it seems more interesting to confirm whether the side bands that can be identified in a wave spectrum may be associated with a more or less well-defined amplitude modulation which appears in the wave record as wave-groups.

B. SIWEH AND WAVE GROUPS

When our research on the side bands of the spectrum had reached this point a very interesting report by Funke and Mansard (1979) on the subject of synthesizing wave records was brought to our attention.

For the purpose of isolating wave groups in a wave record, these authors note that "a more meaningful description of group activity could be obtained by computing the distribution of wave energy along the time axis where wave energy would be defined as the square of the water surface elevation

averaged over a period which is a function of the peak frequency." In a sense this operation represents a process of smoothing the time series; that is, if we look in the frequency domain, what they have done is to perform a filtering process with a low-pass band filter having a cut-off frequency equal to the peak spectral frequency.

The name that they give to the smoothed wave record, the Smooth Instantaneous Wave Energy History (SIWEH), seems to us to be well chosen and we will use it here.

1. SIWEH Analysis

The Smooth Instantaneous Wave Energy History (SIWEH) is produced by operating on the wave record using the Bartlett window, which is a triangular weighting function defined in mathematical terms by the following:

$$\text{SIWEH} = E(t)$$

$$\text{where: } E(t) = \frac{1}{T_p} \int_{\tau = -T_p}^{T_p} \eta^2(t + \tau) \cdot Q_1(\tau) d\tau$$

$$\text{for } T_p \leq t \leq T_n - T_p$$

$$\text{and } Q_1(\tau) = 1 - |\tau|/T_p \text{ for } -T_p < \tau < T_p$$

$$= 0 \text{ everywhere else}$$

where: T_n = duration of the finite wave record

T_p = peak spectral period.

For the beginning and end segments of the record we have:

$$E(t) = \frac{2}{(T_p + t)} \int_{\tau = -t}^{T_p} \eta^2(t + \tau) \cdot Q_1(\tau) d\tau$$

for $0 \leq t \leq T_p$

$$\text{and } E(t) = \frac{2}{T_p + (T_n - t)} \int_{\tau = -T_p}^{T_n - t} \eta^2(t + \tau) \cdot Q_1(\tau) d\tau$$

$$T_n - T_p \leq t \leq T_n$$

The product of the SIWEH operation on the wave record is a time series that reflects the envelope of the record, and thereby identifies the larger waves that constitute wave groups. Examples of SIWEH are shown in Figures 4, 22, 23, 33, 43, 53, and 63 (upper graphs). Another comment by these authors is necessary at this point. In their analysis, the authors make the important comment that their work was done with "paper waves" which means simulated data. Therefore it was considered worthwhile here to analyze our thoughts about low frequency amplitude modulation utilizing experiments with real wave data.

Also, some parameters were introduced by Funke and Mansard to describe wave groups that had not been evaluated using real wave data. We took the opportunity here to produce such data for our 19 wave records. These parameters and the results obtained are described in the following sections.

2. SIWEH Wave Group Parameters

The identification of the low-frequency components of our wave records was done by the application of the already defined SIWEH.

This analysis of wave groups is based on the concept expressed by Funke and Mansard (1979) that it is important to characterize at least four parameters, each derived from the SIWEH, in order to fully describe wave group activity.

The first parameter, the groupiness factor, indicates whether there are wave groups present and in what degree. This factor is defined by

$$GF = \sqrt{\frac{1}{T_n} \int_0^{T_n} (E(t) - \bar{E})^2 dt / \bar{E}}$$

where: $E(t) = \text{SIWEH}$

In words, the groupiness factor is the standard deviation of the SIWEH about its mean and normalized with respect to this mean.

The other three parameters describe the average group duration, the group repetition period, and the variability of the group repetition period.

To calculate the group duration the autocorrelation function was applied to the SIWEH, and compared with the group length measured in the following manner:

- The mean energy $\bar{E}$ of SIWEH was computed.

- Using $\bar{E}$ as a threshold the intersections between SIWEH and $\bar{E}$ were determined.
- The slope of SIWEH at the points of intersection on both sides of each wave group was projected to the zero-energy line.
- These intersections with the zero-energy line define the extent of the group.

For the group repetition period and its variability the SIWEH spectral density function was computed using again four "windows", which means a maximum detectable period of 256 seconds with a resolution of 1/256 Hz and eight degrees of freedom.

The results of these computations for the 19 wave records tabulated by wave type are presented in Table IV and are discussed in the following sections.

3. Wave Group Properties

Based on the criterion of significant wave steepness the 19 wave records analyzed were divided into groups by wave type: sea, young swell, moderate swell, and old swell, according to the definitions given earlier.

For illustrative purposes, one wave record of each type was chosen as follows:

File Number 579	Sea
File Number 600	Young swell
File Number 446	Moderate swell
File Number 677	Old swell

Other illustrations from File Numbers 228, 178, and 451 are also presented as examples of particular characteristics that will be pointed out later.

a. Groupiness Factor

In Table IV we present the values of the groupiness factor found. The values vary widely, but show a weak relationship to the wave type (see Table I) with sea having the highest median value and old swell the lowest value.

Figure 4 (File Number 228) and Figure 22 (File Number 451) represent wave records having high groupiness factors. It can be seen that the higher values correspond to wave records where a high concentration of energy appears after relatively long periods of lower activity close to the mean value. It is interesting to note that we did not find the high values that are associated with the synthesized waves described by Funke and Mansard (1979), for example $GF = 1.35$.

b. Duration of Wave Groups

The duration of wave groups, as expected, acquires some meaning only with swell-like records. Here wave groups are usually easily identified visually, but examination of the SIWEH shows that a typical mean duration of wave groups is not easy to define. In sea-like wave records the wave groups are not defined and the interpretation of the SIWEH has no apparent meaning other than to signify an essentially random variation of wave heights.

The general conclusion from these 19 wave records analyzed is that the zero-crossing time lag from the autocorrelation graphs (see Figures 16, 30, 40, 50, and 60) does

not describe the mean duration of wave groups, but that a rough estimative for this parameter can be obtained from the SIWEH.

c. Group Repetition Period and Variability

The spectrum of SIWEH for swell records gives an indication of the periodicity of wave groups (Figures 51 and 61). The values found and presented in Table IV are conditioned by our 256 second "window" and the consequent resolution of $1/256$ Hz. This implies that the peak frequency should be a multiple of the fundamental frequency $1/256$ Hz such as that shown in Figure 20, and this explains the periods presented. It may be noted that these periods generally coincide with the periods of surf beat commonly seen in coastal wave records.

As discussed above, the presence of wave groups may be indicated by the presence of side bands that appear in the power density spectrum according to the theory of amplitude modulation. Thus Figure 59 of File Number 677 shows two side bands present in the PDS that are symmetrically distributed about the peak spectral period of 15 seconds. This low-frequency signal, also visualized in the envelope of the wave record, should have a spectral profile identical to the side bands. The PDS of the SIWEH, Figure 61, confirms this characteristic. A similar situation is shown in Figures 10 and 17 of File Number 228.

We recognize that it is not evident from the PDS of the sea-surface elevation which spectral component of the side bands represents the peak frequency such as was produced with the simulated waves (Figures 68 and 69). Thus it is important in defining the group repetition period to identify this low frequency signal, and the application of the SIWEH is one process for doing this.

Thus, as the PDS of the SIWEH of the simulated waves (Figure 70) shows the group repetition period of the simulated wave groups, the PDS of the SIWEH of File Number 677 (Figure 61) presents a group repetition period that agrees with the occurrence of wave groups visualized on Figure 53 (upper graph).

The variability of the wave group period can be seen in the bandwidth of the spectral representation of the SIWEH. A broad bandwidth is an indication of large variability and a narrow one an indication of small variability. In Table IV we categorize the variability only as large or small because of the difficulty of establishing some quantified parameter. It may be seen that the variability of wave group period tends to be large with young swell and small with old swell. The form of the PDS of the SIWEH presented in Figures 17, 31, 41, 51, and 61 is instructive when compared with the simulated example presented in Figure 70 where theoretically the variability is zero.

V. CONCLUSIONS

This thesis presents the results of experiments in the analysis of ocean wave records. For this purpose 19 digitized records obtained from a wave buoy in 60 meters depth off the California coast were analyzed. Two general approaches were taken: (1) examination of the interrelationship of the properties, H and T , of the individual waves in a wave record, and (2) study of the properties of the larger sets of waves in a wave record, termed wave groups, and consideration of wave groups as a manifestation of amplitude modulation. The principal findings are as follows.

Wave heights and periods, determined using the zero-upcross method and treated as a function of H^2 versus $1/T$ can be manipulated to produce a cumulative pseudo-energy distribution that approximates the cumulative true power density. Thus it is possible to reasonably approximate some important properties of the PDS by the use of the H and T values obtained from a wave record.

Sequences of H and T for each wave record were produced graphically in order to provide different ways of looking at the wave records visually. The presence of groups of successive large waves is readily seen in the wave height sequence plots. Examination of period sequences plots show these larger waves to have almost constant and longer periods.

Plots of H versus T were also produced. Those give an indication of how the individual wave steepness is scattered about the significant record steepness.

As a manifestation of amplitude modulation, the side bands in the PDS of the sea surface elevation give for narrow band spectra (swell type waves) an indication of wave group activity and periodicity.

The Smooth Instantaneous Wave Energy History (SIWEH), developed by Funke and Mansard (1979) from work with synthesized waves, and the power density spectrum of the SIWEH were found to provide a good additional means of characterizing wave groups in narrow band ocean wave records and to give information on the frequency of occurrence, variability, and duration of wave groups. They proved less useful for studying wide band spectra (sea waves).

The groupiness factor derived from the SIWEH was found to be a fair descriptor of the degree of development of wave groups. The application of the autocorrelation function to the SIWEH was found to give inconclusive results about the duration of wave groups.

TABLE I: CHARACTERISTICS OF WAVE RECORDS ANALYZED

File Number	Sig. Height (m)	Peak Period (s)	Sig. Steepness	Wave Type
579	2.85	7.0	0.090	Sea
683	3.19	8.0	0.077	Sea
336	1.68	6.0	0.072	Sea
686	2.29	8.2	0.053	Young swell
547	2.60	9.0	0.050	Young swell
228	2.27	10.2	0.034	Young swell
339	2.25	10.2	0.033	Young swell
023	1.62	9.0	0.031	Young swell
600	3.33	13.5	0.028	Young swell
611	3.13	13.5	0.026	Young swell
536	2.75	13.2	0.024	Young swell
612	2.91	13.5	0.024	Young swell
451	2.62	14.0	0.020	Moderate swell
410	2.71	14.4	0.020	Moderate swell
607	0.92	10.0	0.014	Moderate swell
446	2.29	16.0	0.013	Moderate swell
370	1.69	16.0	0.010*	Sea
677	1.47	14.5	0.010	Old swell
178	1.61	17.0	0.008	Old swell

* Peak period not representative of wave record

TABLE II

TOTAL ENERGY PROPORTIONALITY FACTOR
BETWEEN PSEUDO AND TRUE SPECTRA

File Number	Number Waves-N	$\sum_{i=1}^N H_i^2$ (cm ²)	Statistical Variance (cm ²)	Factor of Proportionality
579	151	5238301	5102	6.80
683	144	6639113	6384	7.22
336	192	2091401	1768	6.16
686	156	3507162	3294	6.83
547	169	4813661	4242	6.71
228	123	2863876	3242	7.18
339	144	3129877	3182	6.83
023	146	1659909	1656	6.87
600	108	5161522	6949	6.88
611	118	5067237	6168	6.96
536	119	3847429	4749	6.81
612	116	4124692	5333	6.67
451	128	3369897	4307	6.11
410	146	4494133	4597	6.70
607	128	463045	528	6.85
446	99	2178222	3292	6.68
370	192	2042953	1791	5.94
677	97	925187	1360	7.01
178	138	1361507	1632	6.05
				Mean 6.70
				Standard Deviation
				0.37

TABLE III: COMPARISON BETWEEN DESCRIPTORS OF TOTAL ENERGY (cm²)

File Number	Statistical Variance	Area Under PDS	Mean of SIVEH	Cumulative Pseudo-Energy Factor 6.7	Total Energy by CEDN
579	5102	5095	5101	5177	4780
583	6384	6377	6400	6881	6142
336	1768	1766	1772	1625	1552
586	3294	3290	3305	3355	2913
547	4246	4241	4243	4251	3974
228	3242	3230	3244	3475	3189
339	3182	3178	3182	3244	3042
023	1656	1654	1657	1696	1592
600	6949	6939	6959	7133	6828
611	6168	6160	6169	6409	5927
536	4749	4741	4757	4825	4654
612	5333	5322	5338	5307	5247
451	4307	4296	4316	3929	4173
410	4597	4591	4615	4594	4415
607	528	527	528	539	507
446	3292	3287	3297	3079	3204
370	1791	1789	1793	1588	1552
677	1360	1358	1366	1423	1275
178	1632	1630	1633	2094	1504

TABLE IV: WAVE GROUP PROPERTIES

File Number	Groupiness Factor	Average Group Duration	SIWEH correlation		Group Peak Period (sec)	Group Period Variability
			(sec)	(sec)		
579	.71	10	10	10	45	Large
683	.75	20	20	18	128	Large
336	.74	20	20	20	Not defined	Large
686	.56	25	25	30	128	Large
457	.62	40-50	40-50	30	85	Large
228	.81	45	45	80	64	Large
339	.68	30-50	30-50	20	128	Large
023	.55	25-30	25-30	10	42	
600	.58	40-100	40-100	40	Not defined	Large
611	.67	40-50	40-50	16	128	Large
536	.68	30-50	30-50	30	85	Large
612	.63	30-40	30-40	20	85	Small
451	.80	40	40	30	64	Small
410	.54	35-40	35-40	65	128	Small
607	.55	20-50	20-50	17	64	Large
446	.61	60-100	60-100	50	128	Small
370*	.38	50	50	40	Not defined	Large
677	.61	50	50	35	128	Small
178	.46	80-100	80-100	28	85	Small

NOTE: See significant steepness in Table I

* Anomalous record; see Table I

FIG 1. LOCATION OF MONTEREY BAY WAVERIDER BUOY.

SANTA CRUZ PIER WAVERIDER
OCT. 1978

PERCENT ENERGY IN BAND
(TOTAL ENERGY INCLUDES RANGE 2048-4 SECS)

LOCAL DAY/TIME	SIG HT (CM)	TOT EN (CM.SQ)	BAND PERIOD LIMITS (IN SECS)									
			22+	22-18	18-16	16-14	14-12	12-10	10-8	8-6	6-4	
1 0648	138.9	1078.2	8.2	3.7	4.7	8.8	1.4	18.8	38.8	23.8	12.5	
1 1648	123.3	958.1	8.2	9.3	21.3	8.4	8.9	11.7	19.2	25.6	11.5	
2 0248	138.7	1281.5	8.4	8.6	13.6	2.3	1.5	18.7	38.8	17.6	14.4	
2 1248	138.3	1195.9	8.4	2.3	13.8	7.9	2.1	9.7	23.5	24.8	19.3	
2 2248	126.9	1886.9	8.2	8.6	14.2	28.8	2.5	2.8	28.4	24.8	14.7	
3 0842	113.6	886.6	8.2	1.3	5.8	25.9	4.3	2.8	18.3	25.1	17.1	
3 1848	118.8	767.8	8.4	1.9	2.7	12.9	14.6	2.5	16.1	38.6	18.3	
4 0448	118.8	767.8	8.9	8.9	4.5	18.2	13.7	9.7	15.6	26.1	18.4	
4 1736	133.8	1104.9	8.1	1.2	2.9	2.8	7.6	11.5	21.3	33.2	19.3	
5 0333	123.1	947.3	8.1	8.7	2.7	5.2	5.4	19.3	31.1	28.1	16.4	
5 1347	112.8	784.5	8.2	21.7	2.2	3.6	5.3	7.4	25.9	21.2	12.4	
6 0022	122.8	938.5	8.1	1.8	28.2	29.6	3.6	8.7	11.7	15.4	8.8	
6 1207	112.9	796.1	8.2	1.4	7.9	33.6	29.9	4.9	5.9	9.6	6.7	
6 2207	114.3	816.7	1.1	8.5	5.5	26.4	24.5	11.5	6.5	9.7	14.3	
7 0807	118.7	881.3	1.8	1.2	4.8	6.9	35.8	14.6	2.8	15.8	16.2	
7 1807	126.9	1086.9	8.6	1.5	3.2	7.4	23.1	21.9	5.8	5.5	31.8	
8 0407	125.8	977.2	8.2	11.4	1.5	6.2	18.3	21.2	12.6	18.5	18.1	
8 1407	122.8	943.8	8.3	28.9	8.9	8.1	9.5	17.8	11.3	9.7	14.3	
9 0007	111.3	773.8	8.3	20.6	1.3	6.3	11.5	22.1	16.2	9.1	12.6	
9 1157	129.9	1854.3	8.4	9.6	15.3	36.8	6.8	18.2	7.9	6.9	6.9	
9 2158	185.6	2153.1	8.2	1.6	11.7	18.6	22.1	14.3	19.2	9.2	4.8	
10 0756	151.8	1425.3	8.3	1.5	14.7	15.3	19.8	28.2	12.7	3.8	8.3	
10 1648	152.3	1458.8	8.3	1.3	3.8	15.2	14.2	11.7	13.3	17.5	22.6	
11 0248	161.2	1624.6	8.7	8.9	4.6	13.3	17.9	23.3	13.3	16.8	9.2	
11 1248	135.5	1146.7	1.3	3.9	3.1	5.7	15.3	17.3	19.4	24.1	18.8	
11 2248	138.6	1886.5	1.8	4.8	2.8	12.8	9.5	27.4	18.4	14.7	9.5	
12 0848	125.6	954.8	1.3	6.3	5.8	7.8	18.4	28.8	22.7	18.8	8.6	
12 1848	139.2	1218.8	8.5	11.1	24.8	12.6	8.3	13.6	14.9	3.2	6.8	
13 0448	122.3	935.3	8.5	8.8	7.1	19.7	21.5	18.1	9.5	9.3	7.4	
13 1514	122.3	935.2	8.4	9.2	12.1	8.1	23.9	19.9	12.7	3.2	5.6	
14 0113	121.5	922.6	8.6	6.3	15.4	7.2	24.1	15.6	15.3	18.8	7.4	
14 1113	97.5	594.6	8.4	5.8	19.8	7.5	18.7	16.6	28.9	9.6	18.3	

FIG 2. EXAMPLE OF CEDN TABULAR SPECTRA PRESENTATION.

FIG 3. EXAMPLE OF CEDN GRAPHICAL SPECTRA PRESENTATION.

FIG 4 . INSTANTANEOUS SEA-SURFACE ELEVATION AND S I W E H. FILE NBR 228 .

FIG 5. SEQUENCE OF WAVE PERIODS. FILE NBR 228.

FIG 6. SEQUENCE OF WAVE HEIGHTS. FILE NBR 228.

FIG 7. SEQUENCE OF WAVE STEEPNESSES. FILE NBR 228.

FIG 8. INDIVIDUAL WAVE STEEPNESS AND SIGNIFICANT WAVE STEEPNESS. FILE NBR 228.

FIG 9 . AUTOCORRELATION OF SURFACE ELEVATION . FILE NBR 228.

FIG 10. POWER DENSITY SPECTRUM OF SURFACE ELEVATION. FILE NBR 228. (4 windows)

FIG 11. POWER DENSITY SPECTRUM OF SURFACE ELEVATION. FILE NBR 228. (8 windows)

FIG 12. HISTOGRAMS OF TRUE AND PSEUDO SPECTRA WITH 1/256 HZ CLASS INTERVAL. FILE NBR 228.

FIG 13 . HISTOGRAMS OF TRUE AND PSEUDO SPECTRA WITH 1/128 HZ CLASS INTERVAL. FILE NBR 228 .

FIG 14 . COMPARISON OF CUMULATIVE ENERGY IN TRUE AND PSEUDO SPECTRA. FILE NBR 228.
(IN ENERGY UNITS)

FIG 15 . COMPARISON OF CUMULATIVE ENERGY IN THE TRUE AND PSEUDO SPECTRA. FILE NBR 228.
(IN PERCENTAGE)

FIG 16 . AUTOCORRELATION OF SIWEH . FILE NBR 228.

FIG 17 . POWER DENSITY SPECTRUM OF S I W E H. FILE NBR 228. (4 windows)

FIG 18 . POWER DENSITY SPECTRUM OF S I W E H. FILE NBR 228. (8 windows)

SIWEH

[illegible]

FIG 21. POWER DENSITY SPECTRUM OF SURFACE ELEVATION AND SIWEH IN TABULAR FORMAT. (8 windows)
FILE NBR 228

FIG 22 . INSTANTANEOUS SEA-SURFACE ELEVATION AND S I W E H. FILE NBR 451.

FIG 23 . INSTANTANEOUS SEA-SURFACE ELEVATION AND S I W E H. FILE NBR 579 .

FIG 24. SEQUENCE OF WAVE PERIODS. FILE NBR 579.

FIG 25 . SEQUENCE OF WAVE HEIGHTS. FILE NBR 579 .

FIG 26. SEQUENCE OF WAVE STEEPNESSES. FILE NBR 579.

FIG 27 . INDIVIDUAL WAVE STEEPNESS AND SIGNIFICANT WAVE STEEPNESS. FILE NBR 579.

FIG 28 . AUTOCORRELATION OF SURFACE ELEVATION . FILE NBR 579 .

FIG 29. POWER DENSITY SPECTRUM OF SURFACE ELEVATION. FILE NBR 579.

FIG 30 . AUTOCORRELATION OF SIWEH . FILE NBR 579 .

FIG 31. POWER DENSITY SPECTRUM OF S I W E H. FILE NBR 579.

FIG 32. COMPARISON OF CUMULATIVE ENERGY IN TRUE AND PSEUDO SPECTRA. FILE NBR 579

(IN ENERGY UNITS)

FIG 32A. COMPARISON OF CUMULATIVE ENERGY IN THE TRUE AND PSEUDO SPECTRA. FILE NBR 579.
(IN PERCENTAGE)

FIG 33. INSTANTANEOUS SEA-SURFACE ELEVATION AND S I W E H. FILE NBR 600 .

FIG 34. SEQUENCE OF WAVE PERIODS. FILE NBR 600.

FIG 35 . SEQUENCE OF WAVE HEIGHTS. FILE NBR 600 .

FIG 36. SEQUENCE OF WAVE STEEPNESSES. FILE NBR 600.

FIG 37. INDIVIDUAL WAVE STEEPNESS AND SIGNIFICANT WAVE STEEPNESS. FILE NBR 600.

FIG 38 . AUTOCORRELATION OF SURFACE ELEVATION . FILE NBR 600 .

FIG 39. POWER DENSITY SPECTRUM OF SURFACE ELEVATION. FILE NBR 600.

FIG 40 . AUTOCORRELATION OF SIWEH . FILE NBR 600.

FIG 41. POWER DENSITY SPECTRUM OF SIWEH. FILE NBR 600.

FIG 42 . COMPARISON OF CUMULATIVE ENERGY IN TRUE AND PSEUDO SPECTRA. FILE NBR 600.
(IN ENERGY UNITS)

FIG 42A. COMPARISON OF CUMULATIVE ENERGY IN THE TRUE AND PSEUDO SPECTRA. FILE NBR 600.
(IN PERCENTAGE)

FIG 43. INSTANTANEOUS SEA-SURFACE ELEVATION AND S I W E H. FILE NBR 446.

FIG 44. SEQUENCE OF WAVE PERIODS. FILE NBR 446.

FIG 45 . SEQUENCE OF WAVE HEIGHTS. FILE NBR 446.

FIG 46. SEQUENCE OF WAVE STEEPNESSES. FILE NBR 446.

FIG 47. INDIVIDUAL WAVE STEEPNESS AND SIGNIFICANT WAVE STEEPNESS. FILE NBR 446.

FIG 48. AUTOCORRELATION OF SURFACE ELEVATION . FILE NBR 446 .

FIG 49. POWER DENSITY SPECTRUM OF SURFACE ELEVATION. FILE NBR 446.

FIG 50. AUTOCORRELATION OF SIWEH . FILE NBR 446 .

FIG 51. POWER DENSITY SPECTRUM OF SIWEH. FILE NBR 446.

FIG 52. COMPARISON OF CUMULATIVE ENERGYIN TRUE AND PSEUDO SPECTRA. FILE NBR 446.
 (IN ENERGY UNITS)

FIG 52A. COMPARISON OF CUMULATIVE ENERGY IN THE TRUE AND PSEUDO SPECTRA. FILE NBR 446.
(IN PERCENTAGE)

FIG 53 . INSTANTANEOUS SEA-SURFACE ELEVATION AND S I W E H. FILE NBR 677.

FIG 54. SEQUENCE OF WAVE PERIODS. FILE NBR 677.

FIG 55. SEQUENCE OF WAVE HEIGHTS. FILE NBR677 .

FIG 56. SEQUENCE OF WAVE STEEPNESSES. FILE NBR 677.

FIG 57 . INDIVIDUAL WAVE STEEPNESS AND SIGNIFICANT WAVE STEEPNESS. FILE NBR 677.

FIG 58. AUTOCORRELATION OF SURFACE ELEVATION . FILE NBR 677.

FIG 59 . POWER DENSITY SPECTRUM OF SURFACE ELEVATION. FILE NBR 677.

FIG 60 . AUTOCORRELATION OF SIWEH . FILE NBR 677 .

FIG 61. POWER DENSITY SPECTRUM OF S I W E H. FILE NBR 677.

FIG 62. COMPARISON OF CUMULATIVE ENERGYIN TRUE AND PSEUDO SPECTRA. FILE NBR 677.
(IN ENERGY UNITS)

FIG 62A. COMPARISON OF CUMULATIVE ENERGY IN THE TRUE AND PSEUDO SPECTRA. FILE NBR 677.
(IN PERCENTAGE)

FIG 63 . INSTANTANEOUS SEA-SURFACE ELEVATION AND S I W E H. FILE NBR 178 .

FIG 64 . POWER DENSITY SPECTRUM OF SURFACE ELEVATION. FILE NBR J78.

FIG 65. SEQUENCE OF WAVE PERIODS. FILE NBR 178.

FIG 66 . INDIVIDUAL WAVE STEEPNESS AND SIGNIFICANT WAVE STEEPNESS. FILE NBR J78.

FIG 67 . SEQUENCE OF WAVE HEIGHTS. FILE NBR J78.

FIG 68. WAVE FORM MODULATED IN AMPLITUDE AND SIWEH.

FIG 69. POWER DENSITY SPECTRUM OF WAVE FORM MODULATED IN AMPLITUDE.

FIG 70. POWER DENSITY SPECTRUM OF SIWEH OF WAVE FORM MODULATED IN AMPLITUDE.

FIG 71. WAVE FORM MODULATED IN FREQUENCY AND SIWEH.

FIG 72. SEQUENCE OF PERIODS OF WAVE FORM MODULATED IN FREQUENCY.

FIG 73. POWER DENSITY SPECTRUM OF WAVE FORM MODULATED IN FREQUENCY.

FIG 74. POWER DENSITY SPECTRUM OF SIWEH OF WAVE FORM MODULATED IN FREQUENCY.

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