DUDLE
NAVAL PO
MONTERE

TE SCHOOL
IRNIA 93943

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

THE IMPLICIT FINITE-DIFFERENCE (IFD)
ACOUSTIC MODEL
IN A SHALLOW WATER ENVIRONMENT

by

Mark E. Kosnik

June 1984

Thesis Co-advisors:

J.V. Sanders
C.R. Dunlap

Approved for public release; distribution unlimited

T222197

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) The Implicit Finite-Difference (IFD) Acoustic Model in a Shallow Water Environment		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis June 1984
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Mark E. Kosnik		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		12. REPORT DATE June 1984
		13. NUMBER OF PAGES 128
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Pressure Amplitude, Trapped Normal Mode Propagation, Shallow Water Acoustics, Ocean Modeling		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An implicit finite-difference (IFD) computer model was developed by Jaeger to solve the parabolic equation. The model preserves continuity of pressure and the normal component of particle velocity at the ocean bottom where there is an interface between media with different sound speeds and densities. This feature was implemented to make the model more accurate		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

in a shallow water environment. The IFD performance in a shallow water environment is analyzed. The IFD results are compared with those of two other models and analyzed in light of basic physical reasoning. In addition, a single sloping ocean bottom is modeled in an experimental tank so that the measured pressure field can also be compared to IFD model results.

Approved for public release; distribution unlimited.

The Implicit Finite-Difference (IFD)
Acoustic Model
in a Shallow Water Environment

by

Mark E. Kcsnik
Lieutenant, United States Navy
E.B.A., University of Notre Dame, 1973

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN METEOROLOGY AND OCEANOGRAPHY

from the

NAVAL POSTGRADUATE SCHOOL
June 1984

ABSTRACT

An implicit finite-difference (IFD) computer model was developed by Jaeger to solve the parabolic equation. The model preserves continuity of pressure and the normal component of particle velocity at the ocean bottom where there is an interface between media with different sound speeds and densities. This feature was implemented to make the model more accurate in a shallow water environment. The IFD performance in a shallow water environment is analyzed. The IFD results are compared with those of two other models and analyzed in light of basic physical reasoning. In addition, a simple sloping ocean bottom is modeled in an experimental tank so that the measured pressure field can also be compared to IFD model results.

TABLE OF CONTENTS

I.	INTRODUCTION	10
II.	THE IFD COMPUTER MODEL	12
	A. BACKGROUND	12
	B. THE COMPUTER MODEL	13
	C. MODEL PROBLEMS/MODIFICATIONS	14
	D. MODEL VERIFICATION	18
	1. Comparison with Jaeger Model Run	18
	2. Comparison with Jensen and Kuperman Model Run	19
	3. Comparison with Coppens, Humphries and Sanders Model Run	28
	4. Comparison with Physical Reasoning	30
	5. Verification Summary	40
III.	LABORATORY MEASUREMENTS	42
	A. BACKGROUND	42
	B. EXPERIMENTAL DESIGN	42
	1. The Task	42
	2. Signal Generating/Receiving Equipment	44
	C. MEASUREMENT PROCEDURES	47
IV.	MODEL RESULT COMPARISONS WITH LABORATORY MEASUREMENTS	53
	A. INTRODUCTION	53
	B. THE GENERAL ANALYSIS	53
	C. THE DETAILED ANALYSIS	64
V.	CONCLUSIONS/RECOMMENDATIONS	75
	A. CONCLUSIONS	75

1. Performance of the IFD Model	75
2. Modeling/Measurement Procedures	76
E. RECOMMENDATIONS	76
APPENDIX A: REVISED IFD PROGRAM LISTING	78
APPENDIX E: TL CONTCUR PLOT PROGRAM LISTING	113
APPENDIX C: RUNNING THE CONTCUR PLOT ON THE NPS COMPUTER	115
A. INTRODUCTION	115
E. COPYING THE FILE FOR USE	115
C. RUNNING THE PROGRAM	115
APPENDIX D: SOURCE DEPTH SENSITIVITY ANALYSIS	117
BIBLIOGRAPHY	125
INITIAL DISTRIBUTION LIST	127

LIST OF FIGURES

2.1	Jaeger's Deep-to-Shallow Water Case	19
2.2	Jensen and Kuperman Sloping Bottom Case	20
2.3	IFL and JKM Comparison at a Range of 2.5 Km	22
2.4	IFL and JKM Comparison at a Range of 5.0 Km	23
2.5	IFL and JKM Comparison at a Range of 7.5 Km	24
2.6	IFL and JKM Comparison at a Range of 10.0 Km	25
2.7	IFL and JKM Comparison at a Range of 12.5 Km	26
2.8	10 Degree Simple Sloping Bottom Case	29
2.9	IFL and CHSM Comparison at the Apex	31
2.10	IFL TL Contours (db) from the Source to 604 Meters	33
2.11	IFL TL Contours (db) from 600 to 1350 Meters	34
2.12	IFL TL Contours (db) from 1350 to 2100 Meters	35
2.13	IFL TL Contours (db) from 1550 to 2300 Meters	36
2.14	Normal Mode Propagation in a Wedge Shaped Ocean	39
3.1	Experimental Tank Set Up	43
3.2	Electronic Equipment Schematic	46
3.3	Pulse Length Analysis at 3.0X	49
3.4	Pulse Length Analysis at 10.4X	50
3.5	Pulse Length Analysis at 10.4X	51
4.1	Comparison of Results at 1.0X	54
4.2	Comparison of Results at 2.1X	55
4.3	Comparison of Results at 3.1X	56
4.4	Comparison of Results at 4.2X	57
4.5	Comparison of Results at 5.2X	58
4.6	Comparison of Results at 6.2X	59
4.7	Comparison of Results at 7.3X	60

4.8	Comparison of Results at 8.3X	61
4.9	Comparison of Results at 9.4X	62
4.10	Comparison of Results at 10.4X	63
4.11	Comparison of Results at 0.7X	65
4.12	Comparison of Results at 0.8X	66
4.13	Comparison of Results at 1.0X	67
4.14	Comparison of Results at 1.3X	68
4.15	Comparison of Results at 1.5X	69
4.16	Comparison of Results at 1.7X	70
4.17	Comparison of Results at 1.9X	71
4.18	Comparison of Results at 2.1X	72
4.19	Comparison of Results at 2.3X	73
D.1	Source Sensitivity Analysis at 1.0X	119
D.2	Measurements with Source Depth of 5, 7, and 9 Cm	120
D.3	Measurements with Source Depth of 11, 13, and 15 Cm	121
D.4	Measurements with Source Depth of 17, 19, and 21 Cm	122
D.5	Measurements with Source Depth of 23, 25, and 27 Cm	123
D.6	Measurements with Source Depth of 29 and 31 Cm	124

ACKNOWLEDGEMENTS

The author thanks LCDR James Nelson, USN, for his advice and assistance in developing many of the graphics plotting programs. Appreciation is also due LT Patrick LeSesne, USCG, for his many long hours spent assisting in the laboratory while compiling the experimental data. Thanks are also due to Professor Calvin Dunlap for his review of the thesis.

The author also expresses his appreciation to Dr. Alan E. Coppers for his advice and clarification given to the theoretical aspects of the thesis. Finally, the author thanks Dr. James V. Sanders for his time and effort in assisting in all aspects of research and thesis preparation. Without the knowledge, patience, and advice of the Coppers and Sanders team this thesis would not have been possible.

I. INTRODUCTION

A variety of acoustic models exist to predict transmission loss. Each of these models contain inherent strengths and weaknesses. All have shown poor results in a shallow water environment due to difficulties at the ocean bottom where there is an interface between media of different sound speeds and densities.

Since its introduction (Hardin and Tappert, 1973), the parabolic wave equation has been a widely accepted means of solution for acoustic propagation. The earliest programs used a split-step Fourier transform algorithm to solve the parabolic equation (PE). Several other solution techniques have been developed primarily to overcome difficulties that occur when the Fourier transform encounters an interface between different media (Lee and Botseas, 1982 and McDaniel and Lee, 1982).

An alternative solution technique that uses an implicit finite-difference (IFD) algorithm was developed by Lee and Papadakis (1979). This method incorporates appropriate interface conditions and allows solutions in shallow water. Starting with the IFD algorithm, Jaeger (1983) developed a computer model to predict transmission loss and acoustic pressure based on user specified bottom topography and a single sound speed profile. This computer model uses the mathematical treatment of the horizontal and sloping interfaces developed by McDaniel and Lee (1982) and Lee and McDaniel (1983). It also utilizes several design features and methods incorporated in an earlier computer program developed by Lee and Botseas (1982), and a PE computer model developed by Brock (1978).

The IFD program preserves continuity of pressure and continuity of the normal component of particle velocity at an interface between media having different sound speeds and densities. This feature makes the program unconditionally stable and better able to handle the bottom boundary condition.

Since its development, the IFD program has not been rigorously tested. This thesis analyzes the program's performance in an idealized shallow water environment. The environment includes a simple sloping sand bottom beneath an isospeed water field. The analysis begins by comparing the IFD's predictions with predictions from Jensen and Kuperman's (1980) PE model and Coppens, Humphries and Sander's (1984) image model. The analysis also includes a comparison of the model's estimated transmission loss contours with expectations based on simple physical reasoning. Finally, the thesis describes an attempt to model a shallow sloping bottom in an experimental tank. The tank contains a sand bottom sculptured with a ten degree slope. Laboratory measurements of the pressure field are taken at a frequency of 100 kHz for comparison to the predicted pressure field generated by the IFD. These comparisons of the IFD predictions with other model estimates, theory, and laboratory measurements give an indication of the IFD's performance in a shallow water environment.

II. THE IFD COMPUTER MODEL

A. BACKGROUND

An implicit finite-difference solution technique to the parabolic equation has been studied and refined by many authors. The history of this development is explained in detail by Jaeger (1983), but merits review in order to gain a perspective on an analysis of the IFD computer model. The parabolic equation is an approximation to the elliptical wave equation. The first means of solving the PE used a split-step fast Fourier transform method as developed by Tappert and Hardin (1973). This method requires periodic boundary conditions in depth because of the finite Fourier transform and handles this constraint by introducing an artificial horizontal pressure release bottom below the actual physical bottom. This method of implementing an artificial bottom was incorporated into the earliest PE models developed by Jenson and Krol (1975) and Brock (1978).

Errors in this split-step Fourier transform method were found to be proportional to the horizontal range step and the second derivative of the index of refraction. The second derivative of the index of refraction tends to be large across the ocean bottom interface. Another problem with the split-step method is that it does not consider density differences between two different media at an interface, which influences the reflection coefficient. For these reasons the split-step Fourier transform method proved to be poorly suited for a shallow water environment.

The IFD solution method was introduced by Lee and Papadakis (1979) as an alternative to the split-step method. The IFD method employs a second order central difference

formula to solve the FE in the form of a tridiagonal matrix. Although the first version of the IFD did handle discontinuities in the sound speed profile, it did not consider density discontinuities. In 1982 McDaniel and Lee introduced a method to handle a horizontal interface of different densities. In 1983, they extended their treatment to include a sloping interface. It is this version of the IFD that is used in Jaeger's computer program.

E. THE COMPUTER MODEL

The IFD computer program consists of a main program and twenty subroutines. The program utilizes a modular construction so that each of the various subroutines are called from the main program to complete a specific calculation or function when required. The IFD is run interactively from a user generated input file that contains values for frequency, one sound speed profile, a bottom profile, source/receiver depths, attenuation coefficients for both the water and the bottom, and several other input parameters that tell the the program where to obtain a solution within the field. The program initiates the calculations assuming an initial Gaussian pressure field and an artificial pressure release surface at a user specified depth.

Attenuation in both the water and the sediment is handled as complex indices of refraction. An artificial attenuation layer is established beneath the sediment to introduce attenuation above the artificial pressure release surface. The actual magnitude of this enhanced attenuation is calculated using an equation derived by Brock (1978) for use in his FE computer model.

The IFD program steps along the specified bottom profile and makes calculations down through the water/sediment column at each user specified horizontal range. The program

requires that the bottom intersect exactly at a vertical grid point. As a result, for a sloping bottom the program automatically calculates the range step to fulfill this requirement. This computer generated range step is always less than or equal to the user provided range step. As the slope of the bottom increases, the range step must decrease and more calculations and computer time are required to solve the entire pressure field. In some situations an actual modification of the user inputted bottom is required. This occurs with a very gently sloping bottom when the required range step exceeds the user specified range step. Here, the program automatically models the bottom as a series of level and sloping sections in order to ensure the user generated range step is not exceeded. This modification of the bottom is always less than or equal to one-half the vertical grid spacing and the model issues a warning to the user of the modification.

Both printed and graphical output are provided by the IFD. The printed output provides transmission loss and the real and imaginary components of the pressure field at each depth for a specified horizontal range. The graphical output is a plot of transmission loss versus range at the user specified receiver depth.

C. MODEL PROBLEMS/MODIFICATIONS

As this study of the IFD program progressed, it became necessary to modify certain aspects of the model. Most of these modifications were necessary to alter the program output into a more desirable form, but a few were implemented to correct programming deficiencies. Although this section of the thesis discusses the earliest model runs, these results are not presented in detail, but only discussed in general terms because they were obtained before

the computer model was fully modified. All modifications were made only after careful analysis of multiple model runs. It is important to realize that all results generated for comparison to other models and laboratory measurements came from a fully modified version of the IFD.

Since the ultimate objective of this study was to compare IFD model results with laboratory measurements the program was first run with input parameters which exactly modeled conditions in the tank. The experimental set up, explained later in great detail, consisted of a tank that is approximately two meters in length, one meter in depth, with a ten degree sloping sand bottom and a maximum water depth of 35 centimeters (cm). Based on a test case run by Jaeger (1983) of a simple sloping bottom, and by results shown by Jensor and Kuperman (1980) for propagation in a wedge-shaped ocean, it was expected that there would be certain recognizable patterns in the predicted propagation patterns. Specifically, since the speed of sound in the bottom exceeds that in the water (fast bottom), the simple sloping bottom supports trapped normal mode propagation (Coppens and Sanders, 1981). As the acoustic energy travels upslope toward the apex, successively lower modes are cut off and the energy contained in these modes is transmitted into the bottom. The range from the apex at which energy of the lowest mode is transmitted into the bottom is referred to as the dump distance and is a function of wavelength, wedge angle, and the ratio of sound speed in the water to the sound speed in the sediment. An empirical equation that defines this dump distance was derived by Coppens, Sanders, Ioannou, and Kawamura (1978); and was used to identify the expected ranges of these dump distances for the given scenario described above.

The initial unmodified IFD run used the parameters taken from the tank and showed no recognizable patterns in the

acoustic field. There was no observable decrease in transmission loss at the various dump distances as expected. Rather, results indicated widely fluctuating patterns in the acoustic field that appeared inconsistent with both previous studies and simple physical reasoning. Upon closer analysis, it was discovered that although the program was designed to be independent of scale there are several logic statements that are not implemented if the user provided range step is less than one meter. Because the logic statements aren't satisfied, the NEWSSEG and NEWMAT subroutines (Jaeger, 1983) are not called correctly. The NEWSSEG subroutine initializes the bottom slope and the NEWMAT subroutine computes matrix elements for the program. Obviously, errors in these two program functions seriously distort results. Because of this systematic error in the program it became necessary to scale up all tank parameters. All distances were scaled up by a factor of 1000, and frequency was scaled down by a factor of 1000. Careful analysis reveals that all input parameters are a function of either distance or frequency, so this scaling produces results that model those expected in the tank.

The second modification of the IFD was required to provide a three dimensional graphics display. As discussed earlier, the IFD provides a transmission loss plot versus range at a single depth. However, to study the model predictions in greater detail it was felt that a two dimensional analysis of the model estimates would be more meaningful. As a result a transmission loss contouring program was developed. The program (Appendix B) displays transmission loss contours for range versus depth. Use of the contour plot requires that transmission loss values generated by the IFD be sent to a data disk used by the contour routine. To facilitate this transfer a dummy variable (LIFD) was established in the PRINT2 subroutine to store the transmission loss values and then these values are written to the data disk at the end of the main program.

Another modification of the IFD output was required to change the real and imaginary components of the pressure to a single pressure amplitude magnitude. It was felt that dealing with the pressure magnitude was easier and more meaningful than with the real and imaginary components of the pressure field. This conversion was done in the PRINT2 subroutine and established a new variable (PRMAG) to represent the pressure magnitude.

A final modification in the computer program was made due to a suspected error in the computation of the attenuation in the artificial layer. Physical reasoning dictates that proper implementation of the artificial attenuation would result in a steady drop off in acoustic pressure with depth throughout the artificial layer, with pressure dropping to zero at the pressure release surface. IFD model results on the other hand showed wide fluctuations in pressure with depth in the layer and then only a minimal fall off at the pressure release surface. The equation in the IFD that actually computes the magnitude of the attenuation in the artificial layer was taken directly from Brock's (1978) PE model (Jaeger, 1983). However, Brock's equation was derived with feet as the unit of measurement while Jaeger's model is derived with meters as the unit of measurement. With this in mind, Jaeger's equation should be approximately a factor of three larger than Brock's equation to correct for the difference in units. To correct for this error the equation to calculate attenuation (ATT(I)) in the NEWMAT subroutine was increased by a factor of three. When this correction was implemented, the large fluctuations in pressure with depth were eliminated. The expected drop off in pressure with depth and the fall off of pressure to zero at the pressure release surface were noted.

A listing of the revised IFD computer program with all modifications can be seen in Appendix A.

D. MODEL VERIFICATION

1. Comparison with Jaeger Model Run

In light of the modifications to the IFD computer program just discussed, it was necessary to ensure that the changes themselves did not introduce errors into the model. So as a first step the modified IFD program was run for one of the test cases used by Jaeger in his original work. This case analyzes propagation in an environment that moves from deep to shallow water. This particular environment is depicted in Figure 2.1 and was chosen because it was very similar to the simple sloping bottom in the tank experiment. A solution is obtained for a bottom with an upslope of 8.5 degrees. An isospeed water field is used with sound speed set at 1500 m/s. Source frequency is 25 Hz. The scenario has a maximum depth of 350 meters and a range of 40 kilometers. Both the source and the receiver are set at 25 meters.

The results using the modified IFD program were the same as Jaeger's for this test case above the artificial attenuation layer. As explained earlier, the program was modified to reflect higher attenuation in the artificial layer, so as to properly reflect the effect of attenuation in this region. The modified program results did not show the large fluctuations in pressure with depth in the artificial layer but rather the gradual decline in pressure towards a value of zero at the pressure release surface. But above the artificial layer the modified program results were identical with those achieved by Jaeger with the original program. Apparently, the minor changes in the program designed to improve on the form of the program output does not hinder the model's ability to achieve a solution in the upper sediment and water column.

Figure 2.1 Jaeger's Deep-to-Shallow Water Case.

2. Comparison with Jensen and Kuperman Model Run

The second attempt at verifying the IFD model involved a comparison of model results with those achieved with a FE model designed by Jensen and Kuperman (1980). The Jensen and Kuperman Model (JKM) uses a split-step solution technique. Comparison with this particular model was chosen because it is one of the few that obtains a solution in two dimensions. Most acoustic models obtain a solution at only a single depth. In addition, Jensen and Kuperman made their model runs in a simple sloping ocean bottom environment very similar to the scenario of interest modeled in the task. This environment is depicted in Figure 2.2 and features a gently sloping bottom of 2.2 degrees. The water column has a

uniform speed of 1500 m/s. The source is placed just below the midpoint in the channel at 112 meters and is driven at a frequency of 25 Hz. The maximum depth in this scenario is 200 meters and has a maximum range of 12.5 kilometers.

The Jensen and Kuperman results were expressed as

Figure 2.2 Jensen and Kuperman Sloping Bottom Case.

transmission loss contours for range versus depth. Their study concentrated on transmission loss patterns in the sediment, but results were obtained both in the sediment and in the water. These results are compared to the IFD results at ranges of 2.5, 5.0, 7.5, 10.0, and 12.5 kilometers. These ranges were chosen for analysis because the JKM results showed the greatest variation in transmission loss with

depth and thus, makes for a more meaningful comparison with the IFD results.

The IFD and JKM results can be seen in Figure 2.3 through Figure 2.7. In all the figures, the IFD estimates are shown as a solid curve while the Jensen and Kuperman results are depicted as circular points. The first analysis is at a range of 2.5 kilometers. From Figure 2.2, it can be seen that the depth at this range is 200 meters and is in a flat bottom region. From the results shown in Figure 2.3, it is obvious that both models obtained almost identical results from the surface down to a depth of about 300 meters. There are differences between the two sets of predictions from the ocean bottom to a depth of 100 meters below this point. Below 300 meters the Jensen and Kuperman results show a very slight increase in transmission loss (TL) with depth. The IFD also shows an overall increase in transmission loss with depth but with several fluctuations in transmission loss and a marked peak at about 300 meters. So in general, the results from the two models have the same general tendencies although the IFD appears to show greater detail in results near the water/sediment boundary.

Figure 2.4 shows results at 5.0 km in range. Here, the water depth is still 300 meters and marks the very beginning of the sloping bottom section. For this range the JKM predictions are only available to a depth of 300 meters. The results are nearly identical with those obtained by the IFD. Both models show a relative minimum in transmission loss at a depth of 100 meters and then a gentle increase in TL with depth.

The model results at a range of 7.5 km are seen in Figure 2.5. At this range the bottom depth is about 150 meters and the bottom is sloping. The models show the greatest difference at this range. Both models produce nearly identical results in the water column, but beneath

Figure 2.3 IFD and JKM Comparison at a Range of 2.5 Km.

Figure 2.4 IFD and JKM Comparison at a Range of 5.0 Km.

Figure 2.5 IFD and JKM Comparison at a Range of 7.5 Km.

Figure 2.6 IFD and JKM Comparison at a Range of 10.0 Km.

Figure 2.7 IFD and JKM Comparison at a Range of 12.5 Km.

150 meters in depth the model estimates begin to show differences. The Jensen and Kuperman results show a gradual increase in TL with depth below the ocean bottom. The IFD results on the other hand, show a gradual decrease in transmission loss from 150 meters to 300 meters, with a relative minimum at 300 meters and then an increase in TL beneath this depth. Although there are some differences in model estimates near the boundary in the flat bottom sections previously discussed, the differences appear to be greater at the boundary in this sloping bottom case. In these regions away from the boundary in either the water column or in the sediment, however, both models produce similar results. The JKM's difficulty in obtaining an accurate solution at the sediment boundary is not totally unexpected. The model uses a variation of the straight split-step solution technique similar to Brock's computer model (Jensen and Kuperman, 1980), that has characteristically been unsuccessful in obtaining a reliable solution near a boundary.

The results at a range of 10.0 km are seen in Figure 2.6. At this range the water depth is approximately 100 meters and the bottom is again in a sloping region. At this range Jensen and Kuperman results are only available to about 150 meters in depth. For the data available, the models produce nearly identical results. Both models predict a minimum at about 50 meters in depth and then an almost linear increase in TL with depth. The IFD results also reflect a transmission loss maximum at about 200 meters, and then a slight decrease of TL beneath this depth. Data at these depths are not available from the Jensen and Kuperman run. In light of the results obtained at 7.5 km for a sloping bottom case one might expect that the results for the two models would show differences near the ocean bottom. However, since there is only one Jensen and Kuperman prediction available near the bottom for this range, it is

difficult to make any definite conclusions regarding differences in model performance at the boundary.

Results at the apex (range equal to 12.5 km) can be seen in Figure 2.7. Since all estimates are made in the sediment at this range, there is no boundary to contend with. At this range the two models show the best agreement. Both models show a gradual decrease in transmission loss to a depth of about 300 meters and then a gradual increase of TL with depth.

In general, there appears to be good agreement between the IFD and the JKM model results in regions not influenced by a boundary. In both the water and the deep sediment the two models produce similar results. Although this is not conclusive evidence, these similarities suggest that the IFD can successfully model acoustic propagation in these regions. Near the water/sediment boundary however, the Jensen and Kuperman and IFD predictions show marked differences. These differences appear to intensify as the bottom becomes more sophisticated. In general, the Jensen and Kuperman results do not seem to show the detail the IFD results do. Considering the different solution techniques employed by the two models, these differences in results are expected.

3. Comparison with Coppens, Humphries and Sanders Model Run

The third attempt at verifying the IFD performance was done by comparing results with an image theory model derived by Coppens, Humphries, and Sanders (1984). This Coppens, Humphries, and Sanders Model (CHSM) uses a saddle point approximation to an image model to solve for the acoustic field. Both programs were run for the scaled scenario modeled in the tank experiment. This scenario is depicted in Figure 2.8 and features a ten degree sloping

bottom. The maximum water depth for the run is 350 meters and the maximum range is two kilometers. The source generates a 100 Hz signal and is set at a depth of 175 meters.

Figure 2.8 10 Degree Simple Sloping Bottom Case.

The image model provides solutions only at the apex. The comparison of predictions at the apex (range equal to 2 kilometers), can be seen in Figure 2.9. In the Figure, the IFD values are plotted as the solid curve while the image model values are shown as circular points. The predictions are shown as values of normalized pressure amplitude for a given depth. The different models produce pressure amplitude values in different units, and thus, had to be normalized to make a comparison of values possible. This normalization was

done for each model by dividing the pressure amplitude by the maximum amplitude predicted by the model.

The results suggest that even though the models predict similar large scale trends in pressure amplitude with depth, there are several differences in detail. The plot shows that both the IFD and the image model predict an almost linear increase in amplitude with depth down to a specific maximum and then a slow decrease in amplitude beneath this maximum. The IFD however, shows the maximum at about 15.5 meters in depth while the image model maximum is deeper at 19 meters. Beneath this maximum the IFD shows a much sharper decrease in amplitude than the image model. The trend in the image model data appears smooth, while the IFD curve reflects several small scale fluctuations.

It appears that both models predict similar large scale trends in pressure amplitude for this scenario. It is difficult, if not impossible, to account for the differences in detail. As a minimum, however, this comparison indicates that the two sets of predictions are consistent with one another and can be considered reasonable in this shallow water environment.

4. Comparison with Physical Reasoning

From model comparisons it appears that the IFD at least makes a reasonable estimation of acoustic fields in a shallow water environment. IFD results are also analyzed in comparison with basic physical reasoning and theory as a fourth attempt at model verification. This analysis examines IFD transmission loss contours for the simple ten degree sloping ocean scenario seen in Figure 2.8. The scenario is two kilometers in range with a maximum depth of 350 meters. The source generates a signal at 100 Hz and is placed at 175 meters in depth.

Figure 2.9 IFI and CESM Comparison at the Apex.

Transmission loss contour plots are displayed in Figure 2.10 through Figure 2.13. All TL contours are expressed in decibels(db) and are shown as a function of depth versus range. In all figures, the ten degree sloping bottom is shown as a solid unlabeled line. The contour plots are displayed over four range subsections due to computer graphics limitations and to emphasize different significant features in the field.

The first figure shows transmission loss contours from the source to a range of 600 meters. The contours in the first 250 meters appear very symmetric, increasing outward from the source in a pattern that resembles spherical spreading. Since the IFD program assumes a Gaussian starting field this early pattern is expected.

From a range of about 300 meters to 600 meters the field in the water appears to be dominated by a surface reflection pattern. From the spotty appearance of the TL contours in this region there is an indication of an interaction of surface reflection and bottom reflection on the contours. It is possible to compare transmission loss maxima with nodes in the surface interference pattern. Based on surface interference theory these nodes should occur where (Kinsler, Frey, Coppers, and Sanders, 1982):

$$\sin(khd/r) = 0$$

or in other words;

$$khd/r = n\pi$$

where:

$$n = 0, 1, 2 \dots$$

$$r = \text{Range}$$

$$k = \text{Wavenumber}(2\pi/\lambda)$$

$$d = \text{Source Depth}$$

$$h = \text{Depth of Node}$$

$$\lambda = \text{Wavelength.}$$

Figure 2.10 IFD TL Contours (dB) from the Source to 604 Meters

Figure 2.11 IPD TL Contours(db) from 600 to 1350 Meters.

Figure 2.12 IFD TL Contours (db) from 1350 to 2100 Meters.

Figure 2.13 IFD TL Contours (db) from 1550 to 2300 Meters.

By manipulating the above equation it is possible to solve for the depth at which the nodes should be observed. For this particular scenario at a range of 604 meters, nodes should occur at integer multiples of 25 meters in depth ($25n$). Figure 2.10 shows that these transmission loss maxima do exist as expected, every 25 meters in depth at the stated range.

Based on simple physical reasoning one would expect refractive bending along the ocean bottom due to density differences between the water and the sediment. The contour plot reflects a change in basic pattern at the interface. There is a bending of the TL contours that suggest a refractive influence.

Figure 2.11 shows transmission loss contours from a range of 600 meters to 1350 meters. Again in this region the water appears to be dominated by surface reflection. Solving for the nodes in this surface interference pattern at a range of 1350 meters, it is found that these nodes should appear every 55 meters in depth. From the figure it is again seen that TL maximums do occur every 55 meters in depth as anticipated. As in the first figure there is a change in the basic TL pattern at the ocean bottom. The bending appears more accentuated than in the previous figure, but still suggests the influence of refraction at the ocean bottom.

Figure 2.12 displays transmission loss from 1350 meters to past the apex at a range of 2100 meters. In this figure, the dominance of the surface reflection mechanism is less obvious and the TL patterns become more complicated. It is in this region that the influence of trapped normal mode propagation can be seen. As discussed earlier, as acoustic energy travels up the slope toward the apex, normal modes are cut off and energy is transmitted into the bottom. According to adiabatic normal mode theory, modal separation

is range dependent (Graves, Nagel, Uberall, and Zaur, 1975 and Coppens and Sanders, 1980). The range from the apex at which the lowest mode is transmitted into the bottom, can be calculated using the following equation (Coppens, Sanders, Ioannou, and Kawamura, 1978):

$$X = \lambda/4 \sin \theta_c \tan \beta$$

where:

X = Dump Distance Of The Lowest Mode

λ = Wavelength

θ_c = Critical Angle

β = Wedge Angle.

According to adiabatic normal mode theory (Kinsler, Frey, Coppens, and Sanders, 1982), in deep water (near the source) the lower normal modes are far above cutoff and the adiabatic eigenfunctions consist of an integer number of half sine waves with zero pressure at both the top and bottom surfaces. At the cutoff of each mode, the pressure at the bottom must be maximized, resulting in an adiabatic eigenfunction that contains $1/4$, $3/4$, and $5/4$ wavelengths at the respective cutoff distances of $1X$, $3X$, and $5X$ for the three lowest modes. From Figure 2.14 it can be seen that as the normal modes travel up the sloping bottom, successive modes are forced into the bottom at distances where a particular mode reaches a depth at which it can no longer propagate. Also from the figure it is obvious that a source set at mid-depth can not excite the second mode. Since this particular geometry is present in the tank scenario it is expected that the energy associated with this second mode should not be seen. Based on this line of physical reasoning, modes should be dumped into the sediment at the first dump distance, fifth dump distance, ninth dump distance and so on. If these modes are dumped as described

Figure 2.14 Normal Mode Propagation in a Wedge Shaped Ocean.

then they should be seen in the contour plot as reductions of transmission loss in the bottom at ranges of 1952 meters, 1760 meters, 1568 meters, and so on.

From Figure 2.12 it can be seen that there is an obvious decrease in transmission loss at approximately 1950 meters (dump distance #1) and 1760 meters (dump distance #5). There is also a less clearly defined reduction in TL along the bottom at 1565 meters (dump distance #9).

The final contour plot (Figure 2.13) shows transmission loss contours from 1550 meters to 2300 meters. This figure is an extension of Figure 2.12, intended to emphasize how clearly the beam at the first dump distance is defined in the plots. The beam at the fifth dump distance is also

visible but is not nearly as well defined. From this figure, one can also see an indication of a very narrow beam in the sediment at about 1860 meters. This distance corresponds to the third dump distance (second normal mode). Based on adiabatic mode theory the second mode is not expected to be excited. However, adiabatic mode theory is only an approximation of normal mode behavior. This approximation of normal mode behavior becomes less exact as the bottom slope increases and the closer the source is to the apex. The appearance of a narrow beam at the third dump distance indicates that there is a strong possibility that the second mode is present and that the adiabatic approximation is not exact with a ten degree bottom slope.

The basic features of the IFD contour plots are consistent with both physical reasoning and theory. Basic surface reflection and bottom refraction occur where expected and behave as anticipated. In the far field, trapped normal mode propagation is observed and can be verified with simple dump distance calculations. The location of beams dumped into the bottom appear consistent with basic mode theory. In short, the transmission loss contours indicate that the IFD is making reasonable predictions of the acoustic field in a shallow water environment.

5. Verification Summary

It is difficult to say how exact the IFD predictions are for a shallow water environment based on these simple verification techniques. As a minimum it can be said that the model results are at least consistent with other model predictions and expectations based on simple physical reasoning. Model estimates are virtually the same as the Jensen and Kuperman EE model in regions not influenced by a water/sediment boundary. Close to the boundary the IFD results appear to show greater detail and variation than

this FE model. The IFD results also appear consistent with the general trends in pressure amplitude predicted by the Coppens, Humphries, and Sanders image model. Again differences were noted in the small scale structure. Finally, the IFD TI contours verify well with basic expectations based on physical reasoning and theory. Surface reflection and bottom refraction patterns are observed as anticipated. Far field normal mode propagation can be verified in the plots using simple range distance calculations. In short, all verification methods attempted, fail to uncover any inconsistency in IFD performance in a shallow water environment.

III. LABORATORY MEASUREMENTS

A. BACKGROUND

The major attempt at appraising IFD performance involved comparing model results with laboratory measurements. The shallow water environment modeled in the tank is very idealized; a ten degree sloping sand bottom with an isovelocity water column. Although this scenario appears extremely simplistic, it is one that can be reasonably modeled in the laboratory and still approximate conditions in an actual shallow water ocean environment. The methods used to model and measure the acoustic field are relatively untested. Indeed, this attempt at laboratory modeling was performed not only to verify IFD predictions, but also to see if the environment could be successfully modeled in the laboratory.

B. EXPERIMENTAL DESIGN

1. The Tank

A fiberglass coated wooden tank was used. The tank is 304 centimeters in length, 117 centimeters wide and 95 centimeters deep. Sand filling the bottom of the tank was shaped to form the ten degree sloping bottom, and measurements were taken over a range of two meters. Maximum water depth in the tank was 35 centimeters. A 100 kHz source was placed at mid-channel depth (17.5 centimeters). The layout of the tank is depicted in Figure 3.1.

Figure 3.1 Experimental Tank Set Up.

A slope of ten degrees was selected for several reasons. To begin with, even though a ten degree slope is greater than most ocean bottom slopes, it is still small enough to be considered realistic. Perhaps most important, the ten degree slope was selected because given the frequency limitations (discussed later in this chapter) and range limitations, this wedge angle allowed the source to be placed many (41.6) dump distances from the apex. A large number of dump distances was necessary to simulate a distant source.

The bottom material used in the experiment was #30 fine grade sand. The grain size ranged from 0.15 millimeters to 0.70 millimeters. The sand was treated with a technique used by Faek (1984) to remove air from the sediment. This technique used a high speed jet to agitate the sand/water mixture to remove the bubbles and then allowed the sand to settle for several days before the experiment was initiated.

Fresh(tap) water served as the medium in the tank. To remove air bubbles, the water was allowed to settle in a settling tank before being transferred to the experimental tank for use. The water in the tank was periodically treated with chlorine bleach to prevent the growth of biological material.

2. Signal Generating/Receiving Equipment

The acoustic signal used for the measurements was produced by a function generator, sent through an amplifier and then transmitted into the water by a directional transducer resonant at 100 kHz. The dimensions of the active face of the transducer were 7.0 cm in width and 2.0 cm in height. These dimensions resulted in an approximate beamwidth (angle from the acoustic axis to the first theoretical null) of 11.9 degrees in the horizontal and 46.3 degrees in the vertical. The narrow horizontal beam minimized reflections

from the sidewalls of the tank, while the wide vertical beam allowed complete ensonification of the channel in the vertical dimension.

The signal was received by an LC-5 omni-directional hydrophone, sent through an amplifier and filter, and then displayed on an oscilloscope and voltmeter. A schematic showing the electronic setup is shown in Figure 3.2.

A frequency of 100 kHz was selected for two practical reasons. First, to avoid particle scattering by the sediment, the acoustic wavelength must be at least three times larger than the grain size of the sediment (Anderson and Liebermann, 1968). The largest grain size in the sand was 0.07 cm, so that a wavelength of 1.45 cm (100 kHz) was sufficiently large enough to be immune to this effect. Second, the 100 kHz frequency and the properties of the sand provide a dump distance that is small enough to allow the source to be positioned many dump distances from the apex.

Shaping the sand bottom into a ten degree wedge with a uniform and smooth interface proved to be a long and tedious process. To facilitate this modeling, wooden supports (two-by-fours) were mounted along the length of both sides of the tank. These supports were elevated at one end of the tank to achieve the required ten degree slope. A scraping device was constructed with wooden supports and a metal scraping blade that extended across the width of the tank. This scraping device consisted of wooden supports along the top that reached across the tank and could be pulled along the elevated wooden supports on both side of the tank. This scraping device was pulled along the supports repeatedly until a smooth slope of ten degrees was sculptured from the sand.

Holes had to be drilled into the metal scraping blade because when a solid blade was used in the shallow portion of the slope, water trapped behind the blade was

- 1.) LC-5 HYDROPHONE.
- 2.) AMPLIFIER (HP 465A).
- 3.) ELECTRONIC FILTER (SK 302).
- 4.) OSCILLOSCOPE.
- 5.) VOLTMETER (HP 400D).
- 6.) FUNCTION GENERATOR WAVETEK 116.
- 7.) FREQUENCY OSCILLATOR (GR 1310).
- 8.) AMPLIFIER (HP 467A).
- 9.) FREQUENCY COUNTER (HP 5233L).
- 10.) 100 KHZ TRANSDUCER.

Figure 3.2 Electronic Equipment Schematic.

forced beneath the scraper gouging the smooth bottom. This bottom modeling technique was slow because, once a pass was made over the bottom, the water became turbid and it was then impossible to see the bottom. When the bottom was not visible, it was impossible to see where further smoothing was necessary until the water settled several hours later. In addition, as this smoothing process continued a silty residue became separated from the sand and settled out on top of the sand. This residue would be easily resuspended and eventually had to be removed using a water syphon.

C. MEASUREMENT PROCEDURES

Measurement of the pressure field within the water was done by lowering the receiver in depth at specific ranges of interest. The receiving hydrophone was attached to a pair of micrometers at right angles to one another, that was in turn bolted to a board which spanned the width of the tank. Once the board was placed close to a range of interest, one micrometer was used to give fine adjustments in range and the other in depth.

The measurements were subject to both an accuracy and a precision error. On a given day, with the water level fixed and the cross-tank support set at a particular place in the tank, it was possible to position the receiving hydrophone with an accuracy in depth and range of plus or minus 0.06 centimeters (one turn of the micrometer). To prevent the sand 'inland' of the apex from drying out when not taking measurements, enough water was added to the tank after each data run to keep the sand completely submerged. The next time measurements were taken, water had to be removed from the tank to reestablish the beach. Because of these small changes in the water level the horizontal position of the beach was subject to a precision error estimated to be within plus or minus one centimeter.

The final decision associated with the measurements centered on whether to use a triggered pulse or a continuous wave (CW) signal. With a triggered pulse it was possible to distinguish the received signal from interference caused by reflections off the side of the tank. On the other hand, by using a triggered pulse there was a possibility that the pulse length was not long enough for the acoustic energy associated with paths reflecting off the top and bottom of the water column to overlap the direct path from the source to receiver. A CW signal would avoid potential pulse length problems, but it would be impossible to distinguish between the actual signal and interference. The use of a source with a narrow horizontal beam reduced the effects of reflections from the side walls, but there was still the possibility of interference from side lobes reflected from the sides.

It was necessary to determine the best means to take measurements. This was done by taking measurements at the same location with different pulse lengths to determine the required pulse length to give consistent results and then comparing these results to those obtained with a CW signal. The third dump distance (14.4 cm from the apex) and just past the tenth dump distance (50.0 cm from the apex) were chosen, and measurements were taken for pulsed signals of 64 and 256 cycles, and a CW signal. These measurements can be seen in Figure 3.3 through Figure 3.5. In the figure depicting results at the third dump distance, the CW measurements are shown as the solid curve, the 64 trigger cycle results are displayed as circular points, and the 256 trigger cycle results are depicted as triangular points. For clarity, the measurements taken just past the tenth dump distance are shown in two figures. Figure 3.4 compares CW results (solid curve) with the 64 trigger cycle signal (circular points). Figure 3.5 compares CW measurements (solid curve) with the 256 cycle results (circular points).

Figure 3.3 Pulse Length Analysis at 3.0X.

Figure 3.4 Pulse Length Analysis at 10.4X.

Figure 3.5 Pulse length Analysis at 10.4X.

The pressure amplitude values were normalized by dividing each pressure value by the maximum pressure value in the field. The depth values were also normalized by dividing each depth by the maximum depth.

The results in Figure 3.3 show good agreement between the three curves. In particular, the results for the CW signal and the 256 cycle signal are very similar. The 64 cycle signal shows the same behavior as the CW signal, but differs in magnitude below mid-depth.

The results at 50.0 cm from the apex appear much more complicated than at the third dump distance. Figure 3.4 compares results achieved with the CW signal and the 64 cycle signal. From the figure it can be seen that the two sets of results show good agreement. The measurements depicted in Figure 3.5, which compares the 256 cycle signal and the CW signal show poorer agreement, although the general shapes of both curves remain similar.

These results indicate that interference from the side walls is sufficiently small so that it is possible to make measurements with a CW signal to at least ten dump distances from the apex. Long pulses could also be used, but the difficulty of making voltage measurements with an oscilloscope compared to reading a voltmeter, dictates that measurements should be made with a CW signal.

IV. MODEL RESULT COMPARISONS WITH LABORATORY MEASUREMENTS

A. INTRODUCTION

IFD model predictions and laboratory measurements were obtained for comparison in both a general and detailed analysis. The general analysis compared results every five centimeters from the beach to a range of 50.0 cm from the beach. The measurements were spaced to give results at approximately each of the first ten dump distances. The detailed analysis compared results every centimeter from 3.0 to 11.0 cm from the apex. These measurements were taken to observe the sensitivity of the pressure field to small changes in range.

B. THE GENERAL ANALYSIS

The general analysis compared IFD results and laboratory measurements at approximately each of the first ten dump distances (every five centimeters from the beach). These measurements were obtained by fixing the receiving hydrophone with respect to the cross board and then moving the board out in increments of five centimeters to measure the desired field as a function of depth at each range. The received CW signal was read on the voltmeter.

The comparisons are shown in Figure 4.1 through Figure 4.10. In the figures, the IFD predictions are displayed as a solid line while the experimental measurements are shown as circles connected by a dashed line. Each pressure value was normalized to unity by dividing it by the maximum pressure value for the respective curve. The depth values were also normalized by dividing each depth by the maximum depth in the water column.

Figure 4.1 Comparison of Results at 1.0X.

Figure 4.2 Comparison of Results at 2.1X.

Figure 4.3 Comparison of Results at 3.1X.

Figure 4.4 Comparison of Results at 4.2X.

Figure 4.5 Comparison of Results at 5.2X.

Figure 4.6 Comparison of Results at 6.2X.

Figure 4.7 Comparison of Results at 7.3X.

Figure 4.8 Comparison of Results at 8.3X.

Figure 4.9 Comparison of Results at 9.4X.

Figure 4.10 Comparison of Results at 10.4X.

In general, the pressure patterns predicted by the IFD and those measured, both become more complicated as the range from the beach increases. At all ranges, there is qualitative agreement in the scale of the predicted features and the scale of the measured features. Quantitative agreement is lacking. This agreement in scale but not detail, suggests that the phases among the normal modes predicted by the model do not accurately reflect the experimental situation.

C. THE DETAILED ANALYSIS

The detailed analysis compared IFD values with experimental measurements every centimeter from 3.0 cm to 11.0 cm from the beach (0.7X to 2.3X). The laboratory measurements were taken by fixing the board at one location and then using the micrometer to adjust the receiver to the desired range. The received CW signal was read from the voltmeter.

These results are depicted in Figures 4.11 through 4.19. In the figures, the IFD predictions are shown as a solid line and the experimental measurements are displayed as circles. Each pressure value was normalized to unity by dividing it by the maximum pressure for the respective curve. The depth values were normalized by dividing each depth by the maximum depth at the particular range.

There is qualitative agreement in the scale of the basic features for all ranges, but quantitative agreement is not observed. The IFD patterns change very little throughout the analysis, while the measured values change more rapidly (especially past 2.0X). The results at 2.1X and 2.3X (Figure 4.18 and Figure 4.19) show that the pressure field can change fairly significantly over a range as short as one centimeter. At those ranges where there is poor agreement between results, there is an indication of phase

Figure 4.11 Comparison of Results at 0.7X.

Figure 4.12 Comparison of Results at 0.8X.

Figure 4.13 Comparison of Results at 1.0X.

Figure 4.14 Comparison of Results at 1.3X.

Figure 4.15 Comparison of Results at 1.5X.

Figure 4.16 Comparison of Results at 1.7X.

Figure 4.17 Comparison of Results at 1.9X.

Figure 4.18 Comparison of Results at 2.1X.

Figure 4.19 Comparison of Results at 2.3X.

interference (Figure 4.18) . For distances less than $5X$, theory predicts that for a source at mid-depth, the lowest propagating mode should experience interference from only the evanescent tails of higher modes. Therefore, in the region of the detailed analysis, one expects small interference effects. The experimental results in this region however, show rather significant interference. The interference suggests unsuspected propagating modes are present, or that the evanescent tails are larger than expected. From the trend in the curves it appears as if the phase interference is not a factor from $1.5X$ inward toward the beach (Figures 4.11 through 4.15) .

V. CONCLUSIONS/RECOMMENDATIONS

A. CONCLUSIONS

1. Performance of the IFD Model

This analysis of the IFD acoustic model did not uncover any major failures of model performance in a simplified shallow water environment. But this is not to say that the model consistently and accurately performs in such an ocean scenario. Although comparison of IFD predictions with two other models and with simple physical reasoning did not uncover any inconsistencies in performance, the agreement between IFD values and laboratory measurements is insufficient to give complete confidence in the performance of either the model or the experiment.

There is reasonable agreement between the scale of the features predicted by the IFD model and the experiment. Both the model and the measurements show increasing complexity as range is increased from the beach. But despite the similarities, there is poor quantitative agreement in results. One possible cause for the differences may be that the phases of the normal modes predicted by the model are extremely sensitive to minor irregularities in the shape of the interface and the acoustic properties of the bottom. Preliminary work (LeSesne, 1984), suggests that the phase relationships between modes is strongly dependent on the distance of the source from the apex even at great ranges. Consequently, it appears that the collective influence of the normal modes is dependent upon careful geometric control of the experiment.

The detailed analysis around 2X, reveals that large phase interferences occur where only one propagating mode is

expected. This indicates the existence of an extended evanescent tail within cutoff of the higher modes. Kawamura and Iannou (1978), noted that the apparent phases of the evanescent tails are extremely sensitive to the details of the environment. The results indicate that the tail does not decay quickly, and its influence is pervasive (Figures 4.2 and 4.3). From the trend in the curves, it appears this phase interference is not a factor any closer to the beach than 1.5X (Figures 4.11 through 4.15).

2. Modeling/Measurement Procedures

The laboratory measurements represent an initial attempt at modeling an idealized shallow water environment. The experimental techniques are not without problems. All sets of measurements were repeated and showed good agreement. The reproducibility of the measurements suggest that random errors have been minimized. However, systematic errors remain which also contribute to the discrepancies between predicted and measured values. Some of these errors derive from equipment, such as the fact that the size of the hydrophone is of the same order of magnitude as the scale of variations in the pressure field. In addition, the environment in the tank may not be sufficiently close to the ideal environment assumed by the model.

E. RECOMMENDATIONS

Further study and verification of the IFD computer model is recommended. A comparison of IFD predictions with other models not analyzed in this study, may uncover the cause of the inconsistency when compared to measured values. A detailed study of the phase interaction of the normal modes, although extremely complex, may also offer insight into the IFD performance.

It appears that the modeling of an idealized shallow water environment in the laboratory is of value in acoustic model verification. Despite the experimental difficulties, there was qualitative agreement in the basic features between model predictions and laboratory measurements. But the experiment represents only an initial probe into the modeling/measurement techniques of a shallow water environment. Further refinement of these techniques and the experimental equipment may result in better quantitative agreement between model predictions and laboratory measurements.


```

**      PLR      - RANGE INCREMENT AT WHICH SOLUTION IS PASSED TO      IFD00870
**      PDZ      - CUTOUT PRINTER FILE - METERS                      IFD00880
**      PI       - DEPTH INCREMENT AT WHICH SOLUTION IS PRINTED - METERS IFD00890
**      F1       - THE VALUE OF PI                                  IFD00900
**      FFMAG    - PROPAGATION LOSS - DB                            IFD00910
**      R1       - ARRAY - MAGNITUDE OF REAL AND IMAGINARY PRESSURE IFD00920
**      R2       - RANGE AT WHICH BOTTOM DEPTH IS AVAILABLE - METERS IFD00930
**      RA1      - NEXT RANGE AT WHICH BOTTOM DEPTH IS AVAILABLE - METERS IFD00940
**      RA2      - INCREMENTED AS SOLUTION IS MARCHED CUT IN RANGE. IFD00950
**      RA1      - RANGE AT WHICH SOLUTION IS KNOWN - METERS        IFD00960
**      RA2      - RANGE AT WHICH SOLUTION IS TO BE SOLVED - METERS IFD00970
**      RHO1     - ( RA2 = RA1 + DR )                                IFD00980
**      RHO2     - DENSITY IN WATER - GM/CM**3                     IFD00990
**      RMAX     - DENSITY IN SEDIMENT - GM/CM**3                  IFD01000
**      SINE     - MAXIMUM RANGE OF SOLUTION - METERS              IFD01010
**      TEMP     - SIN ( THETA )                                    IFD01020
**      TEETHA   - TEMPERATURE VARIABLE - RADIANS                  IFD01030
**      U        - SLOPE OF BOTTOM - LEVEL INTERFACE                IFD01040
**      WLR      - THETA = 0 - - - LEVEL INTERFACE                  IFD01050
**      XK0      - THETA > 0 - - - INTERFACE SLOPES DCWN          IFD01060
**      XII      - THETA < 0 - - - INTERFACE SLOPES UP            IFD01070
**      XIAMD    - ARRAY - COMPLEX ACOUSTIC PRESSURE FIELD        IFD01080
**      XLRWS    - RANGE STEP AT WHICH SOLUTION IS WRITTEN TO CUTPUT IFD01090
**      WLR      - PLCTTER FILE                                     IFD01100
**      XK0      - REFERENCE WAVE NUMBER                           IFD01110
**      XII      - MATRIX ELEMENT, X MATRIX, LOWER DIAGONAL, ON INTERFACE IFD01120
**      XIIZ     - XII FOR SLOPING BOTTOM - METERS                 IFD01130
**      XIAMD    - REFERENCE WAVELENGTH, X MATRIX, OFF-DIAGONAL, IN WATER AND IFD01140
**      XLRWS    - MATRIX ELEMENT, X MATRIX, MAIN DIAGONAL, IN SEDIMENT IFD01150
**      XMI      - MATRIX ELEMENT, X MATRIX, MAIN DIAGONAL, ON INTERFACE IFD01160
**      XMS      - MATRIX ELEMENT, X MATRIX, MAIN DIAGONAL, IN SEDIMENT IFD01170
**      XMW      - ARRAY - MATRIX ELEMENT, X MATRIX, MAIN DIAGONAL, IN IFD01180
**      XN       - WATER                                           IFD01190
**      XN1      - REAL INDEX OF REFRACTION                        IFD01200
**      XFR      - COMPLEX INDEX OF REFRACTION SQUARED           IFD01210
**      XWR      - RANGE AT WHICH SOLUTION IS WRITTEN TO CUTPUT   IFD01220
**      XX...    - PICTTER FILE                                     IFD01230
**      XX...    - VARIABLES THAT BEGIN WITH XX HAVE NO SPECIAL PHYSICAL IFD01240
**      XX...    - SIGNIFICANCE BUT THEY CONTRIBUTE TO COMPUTATIONAL IFD01250
**      XX...    - EFFICIENCY. ALL XX VARIABLES ARE CALCULATED IN STEP IFD01260
**      XX...    - SUFFOUTLINE INITIAL, AND ALL ARE INDEPENDENT OF RANGE IFD01270
**      XX...    - AND INTERFACES. Y MATRIX, LOWER DIAGONAL, ON INTERFACE IFD01280
**      YII      - MATRIX ELEMENT, Y MATRIX, LOWER DIAGONAL, ON INTERFACE IFD01290
**      YIIV     - YII FOR LEVELING INTERFACE                     IFD01300
**      YIIZ     - YII FOR SLOPING INTERFACE                      IFD01310
**      YIIZ     - YII FOR SLOPING INTERFACE                      IFD01320
**      YIIZ     - YII FOR SLOPING INTERFACE                      IFD01330
**      YIIZ     - YII FOR SLOPING INTERFACE                      IFD01340

```



```

C          CCMCN / IN, IA, IBOT1, IFACE, IPZ, ISLOPE, ISTEP, IW2, N, NA, NBCT, NM1,
*          NSTEP, NSTEP1, NSVP, NMAX, NX1FS
C          CCMCN / REAL, ALPHA, ATT(500), BETA1, BETA2, BK(101), BZ(101), CO,
*          CSVP(101), C2, CWATER(500), DR, DRIVL, DRMAX, FZ, FRC, IDR, FCZ,
*          R1, RA1, FA2, RHCI1, FHO2, RMAX, THETA, XK0, XLAMLA, XPR, XX4, XX10,
*          XX11, XWF, WDR, ZLYR1, ZLYR2, ZR, ZS, ZSVP(101), ZAEIYR
C          DATA NIU/51, NOUT/55/
C
C      *** READ INPUT PARAMETERS
C      READ (NIU, *, END=100) FRC, ZS, ZR, CO, N
C      READ (NIU, *, END=100) RMAX, DRIVL, DRMAX, WDE, FDE, PIZ
C
C      *** READ BOTTOM PROFILE - RANGE, DEPTH
C      DC 10 I=1,101
C      READ (NIU, *, END=100) BR(I), BZ(I)
C      NBOT=I
C      *** END OF PROFILE?
C      IF (FRC(I).LT.0.0) GO TO 20
C      *** NO
C      CCNTINUE
C
C      CCNTINUE
C      *** EXTEND LAST DEPTH BEYOND MAX RANGE
C      IF (NBOT) = 1.0E+10
C      EZ(NBOT) = EZ(NBOT-1)
C
C      *** FIRST LAYER IS WATER. SECOND IS SEDIMENT.
C      *** READ MAX DEPTH, DENSITY AND ATTENUATION OF FIRST LAYER
C      READ (NIU, *, END=100) ZLYR1, RHO1, BETA1
C
C      *** READ SCUND SPEED PROFILE IN FIRST LAYER
C      DC 25 I=1,101
C      NSVP=I
C      READ (NIU, *, END=100) ZSVP(I), CSVF(I)
C      *** READ ANOTHER PROFILE PCINT?
C      IF (ZSVP(I).LT.ZLYR1) GO TO 25
C      *** NO
C      *** WAS THAT THE LAST PROFILE POINT?
C      IF (ZSVP(I).EQ.ZLYR1) GO TO 30
C      *** NO, THERE IS ERROR.
C      GO TO 101
C      CCNTINUE
C
C      *** DOES THE SCUND SPEED PROFILE START AT THE SURFACE?
C      IF (ZSVP(1).NE.0.0) GO TO 102
C      *** YES

```

C

C

C

C

C 10

C 20

C

IFD03270
IFD03280
IFD03290
IFD03300
IFD03310
IFD03320
IFD03330
IFD03340
IFD03350
IFD03360
IFD03370
IFD03380
IFD03390
IFD03400
IFD03410
IFD03420
IFD03430
IFD03440
IFD03450
IFD03460
IFD03470
IFD03480
IFD03490
IFD03500
IFD03510
IFD03520
IFD03530
IFD03540
IFD03550
IFD03560
IFD03570
IFD03580
IFD03590
IFD03600
IFD03610
IFD03620
IFD03630
IFD03640
IFD03650
IFD03660
IFD03670
IFD03680
IFD03690
IFD03700
IFD03710
IFD03720
IFD03730
IFD03740

C 25

C

C 30

C

C


```

IFD04710
IFD04720
IFD04730
IFD04740
IFD04750
IFD04760
IFD04770
IFD04780
IFD04790
IFD04800
IFD04810
IFD04820
IFD04830
IFD04840
IFD04850
IFD04860
IFD04870
IFD04880
IFD04890
IFD04900
IFD04910
IFD04920
IFD04930
IFD04940
IFD04950
IFD04960
IFD04970
IFD04980
IFD04990
IFD05000
IFD05010
IFD05020
IFD05030
IFD05040
IFD05050
IFD05060
IFD05070
IFD05080
IFD05090
IFD05100
IFD05110
IFD05120
IFD05130
IFD05140
IFD05150
IFD05160
IFD05170
IFD05180

```

```

C0 = C0/ZSVE(NSVP)
CCCONTINUE

*** INITIALIZE FANGE
RA1 = 0.0

*** INITIALIZE FOINTER THAT POINTS TO BCTTM EFCFILE PCINT
IEOT1 = 0

*** CCMPUTE REFERENCE WAVE NUMBER
XK0 = 2.0*FI*FRQ/C0

*** CCMPUTE REFERENCE WAVELENGTH
XLAMDA = CC/FRQ

*** IF DRLVL=0 SET DRLVL EQUAL TO 1/2 REFERENCE WAVELENGTH
IF ( DRLVL.EQ.0.0 ) DRLVL = 0.5 * XLAMDA

*** IF DRMAX=0 SET DRMAX EQUAL TO REFERENCE WAVELENGTH
IF ( DRMAX.EQ.0.0 ) DRMAX = XLAMDA

*** IF DRLVL GREATER THAN DRMAX SET DRLVL EQUAL TO DRMAX
IF ( DRLVL.GT.DRMAX ) DRLVL = DRMAX

*** COMPUTE ATTENUATION - SACLANT MEMO SM-121 (JENSEN + FERIA)
*** MODIFIED AS FOLLOWS:
*** IF INPUTTED BETA IS IT 0.0, ALPHA IS COMPUTED IN DB/METER
*** AND USED FOR BETA
ALPHA=FRQ*FFQ*(-0.07+(-.155*1.7)/(1.7*1.7+FRQ*FRQ*.000001))
*1.0E-09

*** INITIALIZE FOINTER THAT POINTS TO INTERFACE GFID PCINT
IIFACE = INT ( BZ(1)/DZ + 0.5 )

FETUEN
FENI

SUEROUTINE MATCCN

THIS SUBROUTINE CALCULATES VARIOUS VARIABLES NEEDED TO COMPUTE
TRIANGULAR MATRIX ELEMENTS. VARIABLES BEGINNING WITH XX HAVE
NO SPECIAL PHYSICAL SIGNIFICANCE BUT THEY CONTRIBUTE TO
COMPUTATIONAL EFFICIENCY.

CCMFLEX XN1

```



```

THIS SUBROUTINE IS CALLED AT THE START OF EACH NEW EOTICM
SEGMENT. THE SUBROUTINE DOES THE FOLLOWING TASKS FOR EACH
SECTION SEGMENT:
(1) UPDATES BOTTOM PROFILE POINTERS: IBC1 & IEOT2
(2) COMPUTES SLOPE: THETA
(3) COMPUTES NUMBER OF RANGE STEPS IN SEGMENT: NSTEP
(4) COMPUTES RANGE STEP: DR
(5) SETS SICPE FLAG: ISLOPE
    (A) ISLOPE = 1 - BOTTOM SLOPES DCWN
    (E) ISLOPE = 2 - BOTTOM LEVEL
    (C) ISLOPE = 3 - BOTTOM SLOPES UP
    (L) ISLOPE = 4 - BOTTOM SLOPES DCWN, MODIFY BC10M
    (F) ISLOPE = 5 - BOTTOM SLOPES UP, MODIFY EOTICM
(6) INITIALIZES RANGES: RA1 & RA2
(7) CHECKS THAT RANGE STEP IS LESS THAN DRMAX
(8) ISSUES ERROR OR WARNING MESSAGES AS APPROPRIATE

COMMON /IN/ IA, IBOT1, IFACE, IPZ, ISLOPE, LSTEP, IWZ, N, NA, NBCT, NM1,
* NSTEP, NSTEP1, NSVE, NWMAX, NX1FS
CCMMCN /REAL/ ALPHA, ATT(5000), BETA1, BETA2, BR(101), EZ(101), C0,
* CSVP(101), C2, CWATER(5000), DR, DR1VL, DRMAX, EZ, FRQ, FDR, PDZ,
* R1, RA1, FA2, RH, C1, RH02, RMAX, THETA, XK0, XLAMDA, APR, XX4, XX10,
* XX11, XWF, WDR, Z1VE1, ZLYR2, ZR, ZS, ZSVP(101), ZAE1YR
DATA NPOUT/55/

** UPDATE BOTICM PROFILE FCINTER
IEOT1 = IBC11 + 1
IEOT2 = IBC11 + 1
** GET STARTING AND ENDING RANGES AND DEPTHS FOR THIS SEGMENT
R1 = EF(IBC11)
Z1 = B2(IBC11)
R2 = EF(IBC12)
Z2 = B2(IBC12)
** ERROR CHECK
IF (R2 - IE.R1) GO TO 100
PUT Z1 AND Z2 CN NEAREST GRID PCINTS
ITEMP = INT ( Z1/DZ + 0.5 )
Z1 = DZ * FLOAT(ITEMP)
ITEMP = INT ( Z2/DZ + 0.5 )
Z2 = DZ * FLOAT(ITEMP)
** COMPUTE SICPE
THETA = ATAN2 (Z2-Z1, R2-R1)
DCES BC10M SLOPE DCWN, LEVEL OR UP?
IF (THETA - GT 0.0) GO TO 10
IF (THETA - LT 0.0) GO TO 20
** BOTTOM IS LEVEL
** DETIMINE NUMEER OF RANGE STEPS FOR SEGMENT

```

IFD08070
IFD08080
IFD08090
IFD08100
IFD08110
IFD08120
IFD08130
IFD08140
IFD08150
IFD08160
IFD08170
IFD08180
IFD08190
IFD08200
IFD08210
IFD08220
IFD08230
IFD08240
IFD08250
IFD08260
IFD08270
IFD08280
IFD08290
IFD08300
IFD08310
IFD08320
IFD08330
IFD08340
IFD08350
IFD08360
IFD08370
IFD08380
IFD08390
IFD08400
IFD08410
IFD08420
IFD08430
IFD08440
IFD08450
IFD08460
IFD08470
IFD08480
IFD08490
IFD08500
IFD08510
IFD08520
IFD08530
IFD08540

```

C      NSTEF = INT ( (R2-R1)/DRLVL + 0.99999 )
C      *** DETERMINE RANGE STEP
C      DR = (R2-R1) / FLOAT(NSTEP)
C      *** SET ISLOPE
C      ISLCPE = 2
C      GO TO 80
C
C      *** BOTTOM SICPES DCWN
C      *** DETERMINE NUMBER OF RANGE STEPS
C      NSTEF = INT ( (Z2-Z1+0.05)/DZ )
C      *** DETERMINE RANGE STEP
C      DR = (R2-R1)/FLOAT(NSTEP)
C      *** SET ISLOPE
C      ISLCPE = 1
C      GO TC 30
C
C      *** BOTTOM SICPES UP
C      *** DETERMINE NUMBER OF RANGE STEPS
C      NSTEF = INT ( (Z1-Z2+0.05)/DZ )
C      *** DETERMINE RANGE STEP
C      DR = (R2-R1)/FLOAT(NSTEP)
C      *** SET ISLCPE
C      ISLCPE = 3
C
C      CCNTINUE
C      *** IS RANGE STEP TOO LARGE?
C      IF ( DR.LE.LRMAX ) GC TO 80
C      *** YES, BOTTOM MUST BE MODIFIED
C      *** SET ISLCPE
C      ISLOPE = 4
C      IF ( THETA.LT.0.0 ) ISLOPE = 5
C      *** DETERMINE NUMBER OF RANGE STEPS REQUIRED TO MOVE UP
C      *** CRDCWN ONE GRID FCINT
C      NSTEP1 = INT ( DR/DRMAX + 0.99999 )
C      *** DETERMINE RANGE STEP
C      DR = DR / FLOAT(NSTEP1)
C      *** REDETERMINE NUMBER OF RANGE STEPS
C      NSTEP = NSTEP * NSTEP1
C      *** COMPUTE SLOPE OF SLOPING SECTION
C      THETA = ATAN2(DZ,DR)
C      *** COMPUTE LOCATION OF NEXT LEVEL SECTION FCILCWING A
C      SXIFS = NSTEP1/2 + 2
C      *** INDICATE TO USER THAT BOTTOM HAS BEEN MOLIFIED
C      TEMP = C.5 * LZ
C      WRITE(6,903) R1, R2, TEMP
C      WRITE(NFOUT,903) R1, R2, TEMP
C

```



```

80 C CCNTINUE
C *** INITIALIZE RA1 & RA2
  RA1 = R1
  RA2 = RA1+1F
C
C *** INDICATE TC USER HOW FAR SOLUTION FIELD HAS PROGRESSED
  WRITE(6,902) R1
C
C *** IF RANGE STEP GREATER THAN 1 (?) WAVELENGTH WRITE WARNING
  IF (DR.LE.XLAMDA) GO TO 90
  WRITE(6,901) R1, F2, DR, XLAMDA
  WRITE(NCUT,901) R1, R2, DR, XLAMDA
C
C RETURN
C
C *** ERROR EXIT
  WRITE(6,900) IBOT2, IBOT1
  WRITE(NPOT,900) IBOT2, IBOT1
C
C STOP
C
C 900 FCFORMAT(//,1X,'ERROR: THE RANGE AT BOTTOM PROFILE PCINT NUMBER ',
  * I2,' IS LESS',//,9X,' THAN THE RANGE AT BOTTOM PROFILE PCINT',
  * I2,' NUMBER ',I2,'',//,1X,' EXECUTION TERMINATED.',//)
C 901 FCFORMAT(//,1X,' WARNING: THE HORIZONTAL RANGE STEP BETWEEN RANGE R =',F8.1,
  * ' AND RANGE R =',F8.1,' (METERS) IS',F5.1,' METERS.',//,
  * ' THE REFERENCE WAVELENGTH IS',F5.1,' METERS.',//,
  * ' THE PROGRAM HAS REACHED RANGE R =',F8.1,' METERS.',//)
C 902 FCFORMAT(//,1X,' NOTE: THE BOTTOM BETWEEN RANGE',F8.1,' AND RANGE',
  * F8.1,'',//,1X,' THE BOTTOM MODIFIED BECAUSE OF ITS VERY SMALL',
  * ' SLICE.',//,8X,' THE DIFFERENCE BETWEEN THE MOLIFIED',
  * ' BOTTOM AND YOUR',//,8X,' INPUT BOTTOM IS NEVER GREATER',
  * ' THAN',F5.2,' METERS.',//)
C
C END
C
C SUBROUTINE NEWMAT
C
C THIS SUBROUTINE IS CALLED AT THE START OF EACH NEW EOTTCM
C SEGMENT. THE SUBROUTINE DOES THE FOLLOWING TASKS:
C (1) COMPUTES TRIDIAGONAL MATRIX ELEMENTS FOR MATRIX Y
C (A) MATRIX Y IS AT RANGE WHERE FIELD IS KNOWN: RANGE=RA1
C (B) Y MATRIX CN RHS OF EQUATION

```


*** SOLVE FOR SOLUTION FIELD AT RANGE RA2
CALL TRIDG (C,U,N,CR,CTWO)

*** APPLY ARTIFICIAL ATTENUATION
CALL ATTENU (U,ATT,IA,NA)

RA2P = RA2 + 0.5

*** TIME TC WRITE?

IF (RA2P.GE.XWR) CALL WRITE2

*** TIME TC PRINT?

IF (RA2P.GE.XPR) CALL PRINT2

*** UPDATE INTERFACE PCINTER
IFACE = IFACE2

RETURN
END

SUBROUTINE TRIDG (C,U,N,CR,CTWO)

*** THIS SUBROUTINE SOLVES A SET OF N - 1 (NM1) LINEAR
*** SIMULTANEOUS EQUATIONS HAVING A TRIANGONAL COEFFICIENT
*** MATRIX. MATRIX ELEMENTS IN THE LOWER DIAGONAL, MAIN DIAGONAL
*** AND UPPER TRIANGONAL ARE STORED IN C(I,1), C(I,2) AND C(I,3)
*** RESPECTIVELY. THE RHS COLUMN VECTOR IS STORED IN C(I,4).
*** THE SOLUTION FIELD IS STORED IN U(I).
{1} THE INDEX I REFERS TO ROW NUMBER.
{2} WE NEED ONLY SOLVE AN NM1 X NM1 SYSTEM (RATHER THAN
*** AN N X N SYSTEM) BECAUSE U(N) IS KNOWN: U(N)=0.0
{3} THE SUBROUTINE IS A MODIFIED VERSION OF SUBROUTINE
TRIDG FROM:

*** "APPLIED NUMERICAL ANALYSIS" (SECOND EDITION)
*** BY: CURTIS F. GERALD
*** PUBLISHED BY ADDISON-WESLEY PUBLISHING CO., 1980
(4) THE MAIN MODIFICATIONS TO THE ROUTINE IN THE TEXT
INVOLVED:

(A) INTRODUCING ARRAYS CTWO AND CR TO PRESERVE THE
ORIGINAL VALUES IN C(I,2) AND TO MAKE THE ROUTINE
MORE EFFICIENT. THIS RESULTS IN A CONSIDERABLE
SAVINGS IN EXECUTION TIME FOR THE CASE OF A
HORIZONTAL BOTTOM. (SEE SUBROUTINE TRIDI)
(B) MODIFYING THE ROUTINE TO SOLVE AN NM1 X NM1
SYSTEM.

IFD10950
IFD10960
IFD10970
IFD10980
IFD10990
IFD11000
IFD11010
IFD11020
IFD11030
IFD11040
IFD11050
IFD11060
IFD11070
IFD11080
IFD11090
IFD11100
IFD11110
IFD11120
IFD11130
IFD11140
IFD11150
IFD11160
IFD11170
IFD11180
IFD11190
IFD11200
IFD11210
IFD11220
IFD11230
IFD11240
IFD11250
IFD11260
IFD11270
IFD11280
IFD11290
IFD11300
IFD11310
IFD11320
IFD11330
IFD11340
IFD11350
IFD11360
IFD11370
IFD11380
IFD11390
IFD11400
IFD11410
IFD11420


```

CCMMCN /REAL/ ALPHA, AIT(5000), BETA1, BETA2, BR(101), BZ(101), C0
* CSVP(101), C2, CWATER(5000), DR, DRLVL, DRMAX, CZ, FRQ, EDR, EDZ,
* R1, RA1, FA2, RHC1, RHO2, RMAX, THETA, XK0, XLAME, A, XPR, XX4, XX10,
* XX11, XWF, WDR, Z1, ZR1, ZLYR2, ZR, ZS, ZSVP(101), ZAEI, YR
CCMMCN /CPLX/ A(5000), A2, C(5000,4), CR(5000), CTWC(5000),
* EYE, XLI, XLI2, XLRWS, XMI, XMS, XRI, XRI2,
* XY1, XX2, XX3, XX5, XX6, XX7, XX8, XX9, XX12, XX1M(5000), YRIZ,
* YLI, YLI2, YLRWS, YMI, YMS, YMW(5000), YRI, YRI2, YRI4, YRIZ,
* U(5000), Z25, Z26, Z27, Z28, Z29, ZZ10

** UPDATE IFACE2
IFACE2 = IFACE + 1
** UPDATE Y MATRIX, MAIN DIAGONAL, INTERFACE ELEMENT
YMI = A(IFACE)
** UPDATE Y MATRIX, MAIN DIAGONAL, WATER ELEMENT, CNE ROW
** AECVE INTEFACE
YMW(IFACE-1) = 1.0 + DR * XX1M(IFACE-1)

** UPDATE RHS
CALL RHS
** UPDATE LHS
** ** UPDATE X MATRIX ELEMENTS ONE ROW ABOVE INTERFACE
C(IFACE,1) = XLRWS
C(IFACE,2) = 1.0 - DR*XX1M(IFACE)
C(IFACE,3) = XLRWS
** ** UPDATE X MATRIX ELEMENTS ON INTERFACE
C(IFACE,1) = XII
C(IFACE,2) = A(IFACE) * ZZ7 + ZZ8
C(IFACE,3) = XRI

** SOLVE THE TRIDIAGONAL SYSTEM
CALL TRIDG(C,U,N,CR,CTWO)
** UPDATE IFACE
IFACE = IFACE2
RETURN
END

SUBROUTINE UP
THIS SUBROUTINE UPDATES THE RHS & LHS OF THE EQUATION AND
SOLVES FOR THE SOLUTION FIELD AT RA2.
(1) THE RHS COLUMN VECTOR VALUES ARE STORED IN C(I,4)
(2) THE INTERFACE AT RANGE RA1 IS AT GRIDPCINT IFACE
(3) THE INTERFACE AT RANGE RA2 IS AT GRIDPCINT IFACE2
( WHERE IFACE2 = IFACE - 1 )

```

```

IFD12390
IFD12400
IFD12410
IFD12420
IFD12430
IFD12440
IFD12450
IFD12460
IFD12470
IFD12480
IFD12490
IFD12500
IFD12510
IFD12520
IFC12530
IFD12540
IFD12550
IFD12560
IFD12570
IFD12580
IFD12590
IFD12600
IFD12610
IFD12620
IFD12630
IFD12640
IFD12650
IFD12660
IFD12670
IFD12680
IFD12690
IFD12700
IFD12710
IFD12720
IFD12730
IFD12740
IFD12750
IFD12760
IFD12770
IFD12780
IFD12790
IFD12800
IFD12810
IFD12820
IFD12830
IFD12840
IFD12850
IFD12860

```



```

*      XX1,XX2,XX3,XX5,XX6,XX7,XX8,XX9,XX12,XX1M(5000),YRI,Z,
*      YLI,YLIZ,YLWS,YMI,YMS,YMW(5000),YRI,YRIV,YRIZ,
*      U(5000),Z25,Z26,Z27,Z28,Z29,ZZ10
C      *** UDATE RHS
C      CALL RHS
C
C      *** SOLVE THE TRIDIAGONAL SYSTEM
C      CALL TRIDL (C,U,N,CR,CTWO)
C
C      RETURN
C      END
C
C      SUBROUTINE PRINT2
C
C      (1) THIS SUBROUTINE IS EFFECTIVELY THE CONTINUATION OF
C      SUBROUTINE PRINT1.
C      (2) THE FILE CREATED CORRESPONDS TO UNIT FILE NUMBER:
C      NPCUT = 55.
C      (3) THE FILENAME AND FILETYPE FOR THIS FILE ARE:
C      IFCUT PRINTER
C
C      DIMENSION FRMAG(5000), FIED(121,121)
C      COMPLEX A,A2,C,CR,CTWC,XMI,XMS,XRI,XRIZ,XX1,XX2,XX3,XX5,XX6,XX7,XX8,XX9,XX12,XX1M,YRI,YRIZ,
C      YLI,YLIZ,YLWS,YMI,YMS,YMW,Z25,Z26,Z27,Z28,Z29,ZZ10
C      U,ZZ5,ZZ6,ZZ7,ZZ8,ZZ9,ZZ10
C      COMMON /IN/IA,IBOT1,IFACE,IPZ,ISLOPE,ISTEP,IWZ,N,NA,NBOT,NM1,
C      NSTEP,NSTEP1,NSVP,NMMAX,NXIFS,IR
C      COMMON /REAL/ALPHA,ATT(5000),BETA1,BETA2,BR(101),BZ(101),CC,
C      CSVP(101),C2,CWATER(5000),DR,DRIVL,DRMAX,EZ,FRQ,FRZ,
C      R1,RA1,FA2,RHO1,RHO2,RMAX,THETA,XK0,XLAMDA,XPR,XX4,XX10,
C      XX11,XKF,XBR,ZLYR1,ZLYR2,ZR,ZS,ZSVP(101),ZAEIYA,
C      PIFD
C      COMMON /CPLX/ A(5000),A2,C(5000,4),CR(5000),CTWO(5000),
C      EVE,XLI,XLIZ,XLWS,XMI,XMS,XRI,XRIZ,XX1,XX2,XX3,XX5,XX6,XX7,XX8,XX9,XX12,XX1M(5000),
C      YLI,YLIZ,YLWS,YMI,YMS,YMW(5000),YRI,YRIV,YRIZ,
C      U(5000),Z25,Z26,Z27,Z28,Z29,ZZ10
C      DATA KPOUT/55/
C
C      *** PRINT RANGE
C      IF((RA2-LT- 1398.0)-CR-(RA2-GT. 2002.0)) GC TO 30
C      IF(RA2-GT. 2102.0) GO TO 30

```

```

C      WRITE(NPOUT,900) RA2
C      IF = IR + 1
C
C      *** COMPUTE AND PRINT PROPAGATION LOSS AT EACH IF2' TH DEPTH
C      WRITE(NPOUT,901)
C      II = 0
C      DO 20 I=IPZ,N,IPZ
C          ZI = I*PZ
C          IZL1 = INT(ZLYR1)
C          PL = CABS(U(I))
C          IF ( PL.IE.0.0 ) GO TO 10
C          PL = -20.0*ALOG10(PL) + 10.0*ALOG10(RA2)
C          GO TO 15
C          PL = 999.9
C          CCNTINUE
C          PRMAG(I) = CABS(U(I))
C          IF(ZI.GT.ZLYR1) GO TO 20
C          IF(ZI.GT.121.0) GC TO 20
C          WRITE(NFCUT,902) ZI, PL, PRMAG(I)
C          II = II + 1
C          DIFD(II,IR) = PL
C          PIFD(II,IR) = PRMAG(I)*100
C          CCNTINUE
C          CCNTINUE
C          IF(RA2.GT.199C.0) WRITE(31,36) (ATT(I),I=1,4000)
C          FCNRMAT(7F10.5)
C
C      *** DETERMINE NEXT RANGE AT WHICH TO PRINT SOLUTION
C          XPR = XPR+PIR
C
C          900 FCNRMAT(//,5X,'FANGE =',F9.3,' M',/)
C          901 FCNRMAT(15X,'DEPTH',6X,'LCSS(DB)',14X,'PRMAG(I)',/)
C          902 FCNRMAT(10X,F10.2,3X,F10.2,13X,E12.5)
C
C      RETURN
C      END
C
C      SUERCUTINE WRITE2
C
C      (1) THIS SUERCUTINE IS EFFECTIVELY THE CCNTINUATION CF
C      SUERCUTINE WRITE1
C      (2) THE SUERCUTINE WRITES RANGE, RECEIVER DEPTH AND U(I)
C      WHEN CALLED. IT THEN UPDATES THE NEXT WRITE RANGE (XWR)
C      (3) THE FILE WRITTEN INTO CORRESPONDS TO UNIT FILE NUMBER:
C          NOU = 22.

```


C
C
C

(4) THE FILENAME AND FILETYPE FOR THIS FILE ARE:
IFCUT FLCTTER

```
CCMFLX A A2 C CR CTWC EYE, XMS XRI XRIZ,  
* XLI, XLI2, XLRWS, XMI, XMS, XRI, XRIZ,  
* XX1, XX2, XX3, XX5, XX6, XX7, XX8, XX9, XX12, XX1M,  
* YLI, YLI1, YLI2, YLRWS, YMI, YMS, YMW, YRI, YRIV, YRIZ,  
* U, Z25, Z26, Z27, Z28, Z29, Z30, Z31,  
CCMCN / IN, IA, IBOT1, IFACE, IP2, ISLOPE, ISTEP, IW2, N, NA, NBCT, NM1,  
* NSTEP, NSTEP1, NSVP, NMAX, NXLS,  
CCMCN / REAL, ALPHA, ATT(5000), BETA1, BETA2, BR(101), BZ(101), C0,  
* CSVP(101), C2, CWATER(5000), DK, DRLVL, DRMAX, FZ, FRQ, FDR, EDZ,  
* R1, RA1, FA2, RHCI, RHCC2, RMAX, THETA, XKO, XIAMLA, XPR, XX4, XX10,  
* XX11, XWE, WDR, ZLYR1, ZLYR2, ZR, ZS, ZSVP(101), ZAEIYR,  
CCMCN / CPIX, XLI, XLI2, XLRWS, XMI, XMS, CR(5000), CTWC(5000),  
* EYE, XLI, XLI2, XLRWS, XMI, XMS, CR(5000), CTWC(5000),  
* XX1, XX2, XX3, XX5, XX6, XX7, XX8, XX9, XX12, XX1M(5000), YRIZ,  
* YLI, YLI1, YLI2, YLRWS, YMI, YMS, YMW(5000), YRI, YRIV, YRIZ,  
* U(5000), Z25, Z26, Z27, Z28, Z29, Z31,  
DATA NOU/52/
```

```
** WRITE RANGE, LEPTH AND U(I)  
WRITE( NOU, * ) RA2, ZR, U( IWZ )
```

```
** DETERMINE NEXT RANGE AT WHICH TO WRITE SOLUTION  
XWR = XWR + WIR
```

```
RETURN  
END
```

SUBROUTINE ATTENU(U, ATT, IA, NA)

THIS SUBROUTINE APPLIES ARTIFICIAL ATTENUATION TO THE BOTTOM-
MOST NA GRID POINTS AS PER AESD PE MODEL BY EROCK - NORDA
TECH NOTE 12 - JAN 78
(1) ATTENUATION MATRIX ATT IS CALCULATED IN SUBROUTINE
NEWMAT

```
CCMFLX U(5000)  
DIMENSION ATT(5000)
```

```
DO 10 I=1, NA  
  U( IA+I ) = U( IA+I ) * ATT( I )  
CONTINUE
```

10
C

IFD15750
IFD15760
IFD15770
IFD15780
IFD15790
IFD15800
IFD15810
IFD15820
IFD15830
IFD15840
IFD15850
IFD15860
IFD15870
IFD15880
IFD15890
IFD15900
IFD15910
IFD15920
IFD15930
IFD15940
IFD15950
IFD15960
IFD15970
IFD15980
IFD15990
IFD16000
IFD16010
IFD16020
IFD16030
IFD16040
IFD16050
IFD16060
IFD16070
IFD16080
IFD16090
IFD16100
IFD16110
IFD16120
IFD16130
IFD16140
IFD16150
IFD16160
IFD16170
IFD16180
IFD16190
IFD16200
IFD16210
IFD16220

IFD16230
IFD16240
IFD16250
IFD16260
IFD16270
IFD16280
IFD16290
IFD16300
IFD16310
IFD16320
IFD16330
IFD16340
IFD16350
IFD16360
IFD16370
IFD16380
IFD16390
IFD16400
IFD16410
IFD16420
IFD16430
IFD16440
IFD16450
IFD16460
IFD16470
IFD16480
IFD16490
IFD16500
IFD16510
IFD16520
IFD16530
IFD16540
IFD16550
IFD16560
IFD16570
IFD16580
IFD16590
IFD16600
IFD16610
IFD16620
IFD16630
IFD16640
IFD16650

```

RETURN
END

SUBROUTINE END (RA2)

THIS SUBROUTINE IS CALLED WHEN THE SOLUTION FIELD HAS REACHED
THE MAXIMUM RANGE SPECIFIED (RMAX). THE SUBROUTINE SENDS
APPROPRIATE MESSAGES TO THE TERMINAL AND STOPS EXECUTION.

WRITE (6,899) RA2
WRITE (6,900)
WRITE (6,901)

STOP

FORMAT (//, ' THE PROGRAM HAS REACHED RANGE R =', F8.1, ' METERS.' )
FCFMT (//, 'CF', RUN, //, 'PRINTER FILE:', //, 'OUTPUT HAS BEEN GENERATED.', //,
* 1CX, '(1) A FILE WITH FCFMTAD TO BE SENT TO THE PRINTER.', //,
* 15X, '(A) THIS FILE IS READY TO BE SENT TO THE PRINTER.', //,
* 15X, '(B) THE FILENAME AND FILETYPE FOR THIS FILE ARE:', //,
* 15X, '(C) IFDOUT PLOTTER FILE:', //,
* 15X, '(2) OUTPUT PLOTTER FILE:', //, 'OUTPUT HAS BEEN GENERATED.', //,
* 15X, '(A) A FILE WITH UNFORMATTED INPUT DATA FOR THE FLOT ROUTINE.', //,
* 15X, '(B) THIS FILE CONTAINS INPUT DATA FOR THE FLOT ROUTINE.', //,
* 15X, '(C) THE FILENAME AND FILETYPE FOR THIS FILE ARE:', //,
* 15X, '(D) IFDOUT PLOTTER FILE:', //, 'FOR THE PLOT ROUTINE ARE:', //,
* 15X, '(E) THE FILENAME AND FILETYPE FOR THE PLOT ROUTINE ARE:', //,
* 15X, '(F) FORTAN', //,
FCFMT (//, 'TO PLOT THE DATA, GO TO A GRAPHICS TERMINAL AND', //,
* 15X, 'ENTER COMMANDS AS FOLLOWS:', //,
* 15X, 'DEF STOR 1M', //,
* 15X, 'I CMS', //,
* 15X, 'FORTGI FLOTIFD', //,
* 15X, 'FLOTIFD', //,
* 15X, 'THEN FOLLOW PROGRAM PROMPTS.' )

RETURN
END

```

TL CONTCUR PLCT PROGRAM LISTING

113

```

20  A(I,J) = DATA(I1,J)
10  CCNTINUE
C   CCNTINUE
C   PLOTTING/GRAPHICS ROUTINE.
C   CALL TEK618
C   CALL CCMPRS
C   CALL PAGE(11,0,8.5)
C   CALL FLOWUP(1,25)
C   CALL AREA2D(8,6,0)
C   CALL FEIGHT(2)
C   CALL XNAME(1,100)
C   CALL YNAME(1,100)
C   CALL SWISSM(90,1,002,1)
C   CALL SHDCHR(35,1,002,1)
C   CALL FEIGHT(35,1,002,1)
C   CALL FASALF(1,1,002,1)
C   CALL MIXALF(1,1,002,1)
C   CALL HEADIN(1,1,002,1)
C   CALL FEIGHT(2)
C   CALL MESSAGE(1,100,3.5,10,DEGREE SIOFF$,
C   CALL MESSAGE(100,1.4,6.40)
C   CALL YAXANG(0,0)
C   CALL FEIGHT(0,0)
C   CALL RESET(SWISSM)
C   CALL GRAF(400,0,100,2000,0,121,-20,0,1,0)
C   CALL ECOMON(20,0)
C   CALL FEIGHT(0,125)
C   CALL CONMAK(A,N,1)
C   CALL CONLIN(0,0,0,1)
C   CALL CCNANG(96,56)
C   CALL FASPLN(2,56)
C   CALL LINEAR
C   CALL CCNTUR(1,1AEELS',DRAW')
C   CALL DOT
C   CALL GRID(1,1)
C   CALL ENDPL(6)
C   CALL LONEPI
C   STCF
C   ENL

```

APPENDIX C

RUNNING THE CONTOUR PLOT ON THE NPS COMPUTER

A. INTRODUCTION

This Appendix describes a procedure for running the TL contour plot on the NPS computer. Detailed instructions for running the IFD program can be found in Jaeger(1983).

B. COPYING THE FILE FOR USE

Once the TL values have been generated by the IFD program, all that is needed to produce a TL contour plot is the file PICTS FORTRAN. This file is shown in Appendix E and can be copied from a computer account maintained by the Underwater Acoustics Curriculum. To link with this account and obtain a copy of the file, the user should proceed as follows:

- (1) Log on terminal.
- (2) Enter: CP LINK 0160P 191 195 RR .
- (3) When prompted for the read password enter: UX .
- (4) Enter: ACC 195 C .
- (5) Enter: COPY PICTS FORTRAN C = = A .

At this point the PICTS FORTRAN file should reside on the user's A disk.

C. RUNNING THE PROGRAM

Before running the program the user must obtain the TL values from the IFD program. These TL values must be sent to

a data disk in order for the PLOTS program to be able to use them. This can be done by placing a WRITE statement in the IFD program that sends the values to a data disk. The modified IFD depicted in Appendix A uses this technique and can be used as a guide.

Once the data disk has been created, the user must assign temporary disk space (TDISK) to give the program sufficient room to generate the TL contours. For more information on how to assign temporary disk space, see NPS Technical Note TN-VM-C1 which is available in the computer consultant's office.

With these initial steps completed, all that remains is to compile and run the program. The program can be compiled by:

Enter: FCRTGI PLOTS .

The program must be run at the TEK618 graphics terminal under DISPLA. This can be done by:

Enter: DISPLA .

The user will then be prompted for the compiled Fortran program name and the file definitions for the data disk, before the program will run.

APPENDIX D

SOURCE DEPTH SENSITIVITY ANALYSIS

During the laboratory experiment, two supplemental sets of measurements were taken to obtain an indication of the sensitivity of the pressure amplitude to changes in the source depth. The first set of measurements was obtained by varying the source depth with the receiver fixed at 1.0X (4.8 cm from the beach), and lowered in depth to the bottom. For the second set of measurements, The receiver was fixed at the third dump distance (14.4 cm from the beach), and measurements were taken with the source fixed at 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, and and 31 cm from the surface.

The results of the first analysis are shown in Figure D.1. In the figure, pressure was normalized by dividing by the maximum pressure and depth was normalized by dividing by the depth of the water column. Although theory predicts only one propagating mode at this distance, the results show modal phase interference.

The results of the second analysis are shown in Figure D.2 through Figure D.6. In the figures, each curve represents a set of measurements taken with the source fixed at a specific depth. For convenience, more than one curve is shown on a given figure. All pressure values were normalized by dividing by the the maximum pressure in the field. The depth values were normalized by dividing by the maximum water depth.

In general, at 3.0X there appears to be modal phase interference which is influenced by source depth. Given the roughness of this analysis, the details of this interference are obscure. However, the modal interference does not appear

to be inconsistent with either the placement of the source
or with the previous experimental measurements.

Figure D.1 Source Sensitivity Analysis at 1.0X.

Figure D.2 Measurements with Source Depth of 5, 7, and 9 Cm.

Figure D.3 Measurements with Source Depth of 11, 13, and 15 cm.

Figure D.4 Measurements with Source Depth of 17, 19, and 21 Cm.

Figure D.5 Measurements with Source Depth of 23, 25, and 27 cm.

Figure D.6 Measurements with Source Depth of 29 and 31 cm.

BIBLIOGRAPHY

Anderson, C.L., and Liebermann, R.C., "Sound Velocities In Rocks And Minerals," Physical Acoustics, IV-B, Edited by W.P. Mason, Academic Press, 1968.

Paek, C.K., The Accoustic Pressure In A Wedge-Shaped Water Layer Overlying A Fast Fluid Bottom, Master's Thesis, Naval Postgraduate School, March, 1984.

Bradshaw, J.A., Laboratory Study Of Sound Propagation Into A Fast Bottom Medium, Master's Thesis, Naval Postgraduate School, June, 1981.

Frock, H.K., The AESD Parabolic Equation Model, NCEDA Technical Note 12, January, 1978.

Coppens, A.E., Humphries, and Sanders, J.V., "Propagation Of Sound Out Of A Fluid Wedge Into An Underlying Fluid Substrate Of Greater Sound Speed", Accepted for Publication by JASA, May 1984.

Coppens, A.E., and Sanders, J.V., "Propagation Of Sound From A Fluid Wedge Into A Fast Bottom," PLENUM, 1980.

Coppens, A.E., Sanders, J.V., Iannou, G.I., and Kawamura, M., Two Computer Programs For The Evaluation Of The Accoustic Pressure Amplitude And Phase At The Bottom Of A Wedge Shaped Fluid Layer Overlaying A Fast Fluid Half Space, Naval Postgraduate School Technical Report NPS 67-75-002, December, 1978.

Graves, R.F., Nagel, A., Uberall, H., and Zauer, G.I., "Range Dependent Normal Modes In Underwater Sound Propagation: Application To A Wedge Shaped Ocean," JASA, Vol. 58, December 1975.

Erdin, F.H., and Tappert, F.D., "Applications Of The Split Step Fourier Method To The Numerical Solution On Nonlinear And Variable Coefficient Wave Equations," SIAM Review, Vol. 15, 1973.

Jaeger, L.E., A Computer Program For Solving The Parabolic Equation Using An Implicit Finite-Difference Solution Method Incorporating Exact Interface Conditions, Master's Thesis, Naval Postgraduate School, September, 1983.

Jensen, F.E., and Kuperman W.A., Environmental Acoustical Modeling At SACLANTCEN, SACLANTCEN Report SR-34, November, 1979.

Jensen, F.E., and Kuperman, W.A., "Sound Propagation In A Wedge-Shaped Ocean With A Penetrable Bottom," JASA, Vol. 67, May, 1980.

Jensen, F.E., and Krol, H., The Use Of The Parabolic Equation Method In Sound Propagation Modeling, SACLANTCEN Memorandum SM-72, August, 1975.

Kawamura, M., and Iannou, I., Pressure On The Interface Between A Converging Fluid Wedge And A Fast Fluid Bottom, Master's Thesis, Naval Postgraduate School, December, 1978.

Kinsler, I.E., Frey, A.R., Correns, A.B., AND Sanders, J.V., Fundamentals Of Acoustics, Third Edition, John Wiley and Sons, 1982.

Lee, D., and Botseas, G., IFD: An Implicit Finite-Difference Computer Model For Solving The Parabolic Equation, NTSC Technical Report 6659, May, 1982.

Lee, D., Botseas, G., and Papadakis, J.S., "Finite-Difference Solution To The Parabolic Equation," JASA, Vol. 70, September, 1981.

Lee, D., and McDaniel, S.T., "A Finite-Difference Treatment Of Interface Conditions For The Parabolic Equation: The Irregular Interface," JASA, Vol. 73, May, 1983.

Lee, D., and Papadakis, J.S., Numerical Solutions Of Underwater Acoustic Wave Propagation Problems, NTSC Technical Report 5929, February, 1979.

LeSesre, F., Personal Communication, 11 May 1984.

McDaniel, S.T., and Lee, D., "A Finite-Difference Treatment Of Interface Conditions For The Parabolic Equation: The Horizontal Interface," JASA, Vol. 71, April, 1982.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2. Library, Code 0142 Naval Postgraduate School Monterey, California 93943	2
3. Commanding Officer ATTN: LT Mark E. Kosnik SWCSCCICOM SWC Dept HD Class 85, CLCVN 1 OCT 84 Newport, Rhode Island 02841	2
4. Superintendent ATTN: Dr. J.V. Sanders, Code 61 Sd Naval Postgraduate School Monterey, California 93943	8
5. Superintendent ATTN: Dr. A.B. Coppens, Code 61 Cz Naval Postgraduate School Monterey, California 93943	2
6. Superintendent ATTN: LCDR C.R. Dunlap, USN (Ret), Code 68 D1 Naval Postgraduate School Monterey, California 93943	3
7. Superintendent ATTN: Dr. C.K. Mooers, Code 68 Mr Chairman, Department of Oceanography Naval Postgraduate School Monterey, California 93943	1
8. Chief of Naval Research ATTN: Dr. Michael McKissick 800 N. Quincy St. Arlington, Va 22217	1
9. Eugene W. Brown NAVCCEANO Code 7300 Fay St. Louis, Ms 39522	1
10. Commanding Officer ATTN: Dr. James Andrews Naval Ocean Research and Development Activity NSTI Station Fay St. Louis, Ms 39522	1
11. Commanding Officer ATTN: Mr. T.N. Lawrence, Code 323 Naval Ocean Research and Development Activity NSTI Station Fay St. Louis, Ms 39522	1

12. Commanding Officer
ATTN: Dr. Robert Gardner
Naval Ocean Research and Development Activity
NSTI Station
Fay St. Louis, Ms 39522

1

13537 5

210309

Thesis

K828

Kosnik

c.1

The implicit finite-
difference (IFD) acou-
stic model in a shallow
water environment.

5 JUN 91

29 JUL 92

36105

37547

37547

210309

Thesis

K828

Kosnik

c.1

The implicit finite-
difference (IFD) acou-
stic model in a shallow
water environment.

thesK828

The implicit finite-difference (IFD) aco

3 2768 002 10503 3

DUDLEY KNOX LIBRARY