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Separation of Mixed Data Sets into Homogeneous Sets

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U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
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U.S. DEPARTMENT OF COMMERCE

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Symbols used in this report

d	difference; deviation
d.f.	degrees of freedom
f	function
ft.	feet
gdkm	geodynamic kilometer
i	subscript or superscript
j	subscript or superscript
k	kth point in a sample; number of clusters; kth cluster
km	kilometer
m	mth item; meter
mb	pressure in millibars
mi.	miles
n	nth item; number in a sample
r	sample correlation coefficient
s	sample standard deviation; second; cluster
s^2	sample variance
t	Student's "t"
x	variate
x'	variate transpose
y	variate
C	covariance matrix; Celsius
F	function
G.C.T.	Greenwich Civil Time
$\hat{P}$	probability
$\hat{P}(S X_k)$	probability of membership of X_k in the cluster s
R	correlation matrix
R	determinant of the correlation matrix R
S	type; cluster type
X	observed value of variate
$\bar{X}$	mean of variate X
X_k	vector of observations for the kth point in the sample
Y	observed value of variate
$\bar{Y}$	mean of variate Y

α	alpha; proportionality factor ($\hat{\lambda}_s$); probability level of rejection for the null hypothesis
$\hat{\lambda}_s$	lambda hat; mixing proportion for type cluster s
Λ	lambda; the diagonal matrix of eigenvectors
μ	mu; population mean
$\hat{\mu}_s$	mu hat; mean vector for cluster s
π	pi
ρ	rho; population correlation
σ	sigma; population standard deviation
σ^2	population variance
$\hat{\sigma}$	sigma hat; covariance matrix for cluster s
Σ	sigma; summation
ψ	psi
$\hat{}$	hat (caret)
$=$	equal to
$\approx$	approximately
$\overline{}$	overbar; averaging process
$\prime$	transpose

SEPARATION OF MIXED DATA SETS INTO HOMOGENEOUS SETS

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ABSTRACT. In any study, the collection, processing, and storage of data are important. Whether the data are clean, biased or contaminated is also important. Pollution or adulteration of data confuse the investigator.

Data do not necessarily fall into neatly packaged boxes or groups. Usually the data sets are mixtures of several types of phenomena. Some of these are basically deterministic in nature while others are not.

This paper illustrates the use of a clustering technique to separate mixed data sets into subsets which exhibit group characteristics. The investigator then assesses the relative importance of the subsets, the nature of the subsets, and perhaps makes an assumption as to whether a particular subset is biased, contaminated, or adulterated. That is, an assessment of the quality of the data may be made.

The techniques are applicable to any data set which is multivariate normal. Here, they are applied to weather data subsets, (1) land-sea breeze, (2) tropical stratospheric winds, (3) mid-latitude tropospheric winds, (4) mountain pass winds and temperatures, (5) surface marine weather temperatures, dew points and winds, and (6) radiosonde observation of heights, winds, temperatures, and dew points.

1. INTRODUCTION

Prehistoric man differentiated between the good and the harmful, between winter and summer, between drought and floods, and among many variable factors affecting him.

In the real world, the experienced hunter knew the differences between the bear, boar, deer, and turkey. His senses of sight, smell, and hearing aided him when he could not see the animal, but he could sense its presence. When he had some models in mind, he drew symbols or wrote words for the benefit of the inexperienced. He could even provide measurements of a sort. At this stage of the game, he began to deal more with the abstract.

Later, man attempted to record experiences, his thoughts, and his aspirations in pictographs and monuments. (Even today, artists present forms in

symbolic representation or as configurative concepts.) Undoubtedly, many numbering systems had also been developed and were then lost in antiquity, for there are some which remain indecipherable today.

The above examples may be generalized to any field. Clear-cut numerical descriptions and configurations remain important whether the field be sociology, psychology, geophysics or medicine, four of an infinite number of fields.

In this paper we illustrate some of the techniques used to differentiate groups within sets of given measurements. Hopefully, the measurements are both accurate and precise. Also, hopefully, the measurements obtained and used are those which will provide good differentiation bases.

As Friedman and Rubin (1967) quote from Bose and Roy (1938): "The problems of discrimination and classification are insistent in sciences."

Once we enter the realm of measurements and numbers in multidimensional space, assumptions and decisions are made as to the better or best characteristics. The metrics that can be used are many. Those chosen ought to provide the most useful differentiation possible.

There are many techniques and procedures used to differentiate groups. These usually involve some measures of central tendencies within the groups, some measures of differences of these central tendencies, variability within and among the groups, group shapes, and scales, etc. As more is known about the normal distribution than other distributions, it is wise, wherever possible, to transform non-normally distributed data sets to data sets which may be approximated by the normal distribution. Transformations have been of interest for many years. In fact, these are represented in the change of base in many counting systems. For example, the logarithm transformation changes a zero bounded positively skewed distribution to an unbounded distribution at both ends where the values more distant from one are scaled downward faster than those nearer one.

If the various features of a data set are each transformed to normal or near normal distributions, then the combined features may be multivariate normal. A multivariate normal distribution has normal marginal distributions. However, the fact that the marginal distributions are normal does not assure multivariate normality. If multivariate normality is assumed, then probabilistic statements can be made. Transformation will be discussed in greater detail later in the paper.

If multivariate normality is assumed and the distribution has one centroid, i.e., it is unimodal, then multivariate regression techniques can be used to produce forecast equations. If the distributions are normally distributed in the multivariate sense but the total set is multimodal, then ordinary linear regression techniques will not serve. Regression equations for clusters must be developed and the better sets of regression equations clustered and examined. The unique or best equation or set of equations can be used. If there is a best equation, it is unique.

Figure 1 illustrates a multimodal trivariate distribution. The various clusters take various ellipsoidal forms such as spheres, ellipsoids, and disks. Each one of these is trivariate normal. Each one represents a centroid or grouping of characteristics.

Some may be equal in all directions, such as in the spheres. Some may be equal in two directions but not the third, such as in the football- and disk-shaped ellipsoids. In others the distribution may be unequal in all directions. In figure 1, no scales are indicated as the illustration is for concept only. The illustration also could be considered as a higher dimensional ensemble projected onto three dimensions. When multimodal features are evident, and if one wishes to study the modes or clusters, then techniques other than regression are required to separate the data into appropriate subsets.

There are many techniques to separate distributions into parts which can be studied individually. Some of these are:

- (a) discriminant function analysis
- (b) factor analysis
- (c) principal cluster analysis
- (d) dendritic (tree) analysis
- (e) cluster analysis
- (f) clumping
- (g) numerical taxonomy
- (h) unsupervised pattern recognition
- (i) typology

Some of these are essentially the same.

Mixtures always present problems when they must be separated. Characteristics may or may not be so noticeable that classification and discrimination can be made. A mixed herd of cattle, sheep, goats, and horses may be easily separated though the sheep and goats may sometimes present a few problems. The herdsman, the separator, or the investigator must have a clear picture in his mind, i.e., a model which includes the necessary characterization(s) of the populations in which he is interested. If he doesn't have his senses of sight, smell, hearing and touch to guide him (for these provide him explicit models) and he has only some measurements of the mixture provided to him, he is in a quandry. He no longer has an explicit model. He has to start somewhere. Good (1965) discusses the philosophical problem of deciding what can be the best beginning in the problem of classification and discrimination. First of all, the investigator decides to accept the characterization of each object by a set of measurements. He believes that there should be some categories or sub-categories which will be helpful in distinguishing group characteristics. Explicitness is lost and there is no external criterion with which to define the categories. An internal criterion (Good, 1965) would be acceptable. That is, the data themselves may suggest "natural categories." The word "suggest" is necessary, for a different beginning in the treatment of the data may lead to slightly different categories. This item will be treated in more detail later.

Figure 1 A mixed set of trivariate distributions showing clusters or modes of varying sizes and shapes. These are viewed from four different points in 3-space. In each view, the three axes and a suggested line of best fit are indicated.

The next few sections take the reader through a short discussion of regression techniques and through some of the various techniques used to provide classification and discrimination within heterogeneous mixtures.

2. REGRESSION TECHNIQUES

Inevitably, due to inherent laziness or an inherent desire to get to a goal with the least expenditure of energy, physical or mental, one attempts to read relationships into sets of experiences or of data. This is a canon of science. For example, if one knows that a certain thing will happen provided that something else specific is done, then there is a clear-cut and obvious relationship between the reaction and the prior action. Though the converse may not be true, it is disregarded here.

Galton (1889) and Snedecor and Cochran (1967) noted the height of sons as related to the heights of the fathers. With the heights of the fathers as one set of data and the heights of the sons as a second set of data, Galton plotted the heights of the fathers against the heights of the sons, respectively. He noted that tall fathers did not always produce as tall or taller sons but that the height of a son seemed to be between the height of the father and the mean height of the group. That is, the height of a son regressed towards the group norm. The line of best fit for the data set was then and since then labeled the line of regression for any line relationship between data sets. Pearson and Lee (1903), Galton's associates, collected a set of data (more than a thousand) of stature, cubit, and span in family groups.

Figure 2a shows the regression of sons' statures on the fathers' statures. Please note that the heights of the sons of short fathers also tend (or regress) toward the mean. This figure is taken from Pearson's and Lee's data as illustrated, but is also shown by Snedecor and Cochran (1967). The data are scaled in the metric system here. Also, note that the data scatter uniformly along the line and do not cluster.

Figure 2b illustrates the relationship between a sister's span for a given forearm length of her brother. Both figures are adapted from Pearson and Lee with the kind permission of the Trustees of Biometrika.

Figure 3a illustrates schematically the correlation at one point. The correlation may be said to be perfect on the one hand, but also it can be said to be indeterminant.

Figure 3b illustrates the correlation between two points. The correlation may be said to be perfect on the one hand but for the space in between the points, on the line connecting the points, it may be said to be indeterminant or even zero.

Figure 3c illustrates a correlation of one with three collinear data.

Figure 3d illustrates the case of two clusters each with zero correlation, one superposed on the other.

Figure 3e illustrates the case of two clusters shown in figure 3d where the clusters are slightly separated with cluster 2 moving away on a line of 45 degrees. The correlation coefficient is something greater than zero but much less than one; i.e., $0 < r < 1$.

Figure 2a Regression of sons' statures on the fathers' stature, in. and cm., adapted from Galton (1889) and Pearson and Lee (1903).

Figure 2b Regression of sister's span, in. and cm., for given forearm of brother, adapted from Galton (1889) and Pearson and Lee (1903).

Figure 3 General schematic of points and clusters with implied correlations, r .

Figure 3f illustrates the case of two clusters shown in figure 3d where the clusters are separated still further with cluster 2 moving farther away on the 45 degree line.

Figure 3g illustrates the case of two superposed clusters. One with a correlation of plus one, the other with a correlation of minus one. The total cluster has a correlation of zero.

Figure 3h illustrates the case of two clusters one with a correlation of plus one, the other with a correlation of minus one. The second cluster is moving on a 45-degree line from the first. The distance between cluster centroids is the same as in figure 3c. The group correlation is greater than zero but less than that shown in figure 3e.

Figure 3i illustrates the case of the two clusters shown in figure 3g moving along the 45-degree line farther than shown in figure 3h. The correlation is increasing and the coefficient approaches one but not as rapidly as in figure 3f as the distance between the centroids increases.

The above discussion implies that there may be two or more clusters in any data ensemble. The scatter may be more in one cluster than in another. Also, the internal correlation (or dispersion) within each cluster may be the same or may be different from the other clusters. The total correlation attained thus may be more of the relationship among the clusters than points. As the distance between the centroids increases, the clusters are more like singleton points in the regression analyses. If there is a clustering, i.e., effectively a dearth of observations between groups either real or simply unobserved, then regression analysis whether linear or non-linear will fail. Please refer again to figure 1 which represents clusters in 3-space or clusters in n-space projected into 3-space. The representation here is multinormal. This may not be the case sometimes and some clusters or all clusters may be eccentric in shape such as eggs, starch grains, or bivalve shells.

If the variates are not all normally distributed, it is assumed that the user will transform the variates to normal variates. It is helpful to know by means of the Central Limit Theorem that, though individual data characteristics may not be normally distributed, linear functions of these tend to be normally distributed. Also, it is assumed that the user will extract the deterministic part of the variables wherever possible. Therefore, the problems of nonlinear regression are not discussed here.

The point of the entire discussion above is simply that linear regression analysis ought to be used only with a unimodal univariate or multivariate distribution. Linear predictor equations developed in linear regression models then become more useful and accurate.

Gupta and Sobel (1962) consider this problem. Aversen and McCabe (1975) present some subset selection problems for variances associated with applications to regression analysis.

3. DISCRIMINANT TECHNIQUES

3.1 Discriminant Functions

Many workers realized the problems induced by heterogeneous or mixed distributions. Among the first to attack the problem in a systematic mathematical treatment was Pearson (1894, 1901) in work on univariate distributions. Many others also have worked on this problem.

Barnard (1935) and Fisher (1936) may be considered to have first attacked the problem of classification with discrimination techniques, though the problems of classification had involved many other workers up to that time.

Let us look at a two cluster mixture from the viewpoint of separation or discrimination with subsequent rules for classification. Figure 4 shows an assemblage composed of two clusters. The clusters are shown first within a dashed circle with no axes chosen (a). Suppose that the measurements are made in terms of the x-axis drawn horizontally (b).

Projections of the cluster points are on the x-axis and in (d) on the y-axis. Visually there is separation in the (x, y) space or two space. This separation can also be shown in one space, i.e., linearly. In one space or one dimension chosen first on the x-axis, there is mixture of the projections. In (c) where the x-axis is rotated through an angle to x' , there is some separation but two points indicate some mixing. In (d) where the rotation has been carried through 90 degrees so that the x' axis is equivalent to the former y-axis, separation is complete though perhaps not the optimum. There is some angle of rotation which will produce a major separation between the clusters and a minimum variance within the clusters. The computed linear function which describes the above line after rotation is called the linear discriminant function. Brown (1947) applies these techniques to establish the discriminating procedures for azotobacter, the nitrogen-fixing bacteria. Smith (1947) provides some discrimination examples. Crutcher (1960) applies the techniques developed by Rao (1950) to the annual march of temperature and rainfall climates in the United States. Figure 5 (adapted from Crutcher, 1960) shows a two cluster (bimodal) bivariate distribution with the two dimensions shown in three-dimensional form. These three-dimensional forms are projected into two dimensions on the $\bar{x}\bar{y}$ plane. Linear discrimination is effected along the single axis pointing to the lower right ($\bar{x}\bar{y}$ plane). Figure 6 illustrates the same idea with all projections being made onto the xy plane. Tatsuoka (1971) presents similar ideas to illustrate geometrically how the discriminant function operates.

Figure 7 (Crutcher, 1960) shows three constellations for monthly average temperature galaxies where the basic variability of the constellations is different. The covariance within a galaxy is the same where the circles provide a measure of individual cluster variances and the distances between clusters is a measure of cluster variance. A point indicates one station only. Miller (1962) applies the technique to weather prediction. Applications as indicated by Fix and Hodges (1952) ran into the hundreds.

Figure 4 Schematic illustrations of discrimination for two clusters.
 (a) Two separate and distinct clusters, (b) same two clusters with projections on an arbitrary x-axis, (c) with projections on an angle from the x-axis, and (d) with projections on an axis, y-axis, $\pi/2$ radians from the x-axis.

Figure 5 Schematic illustration of discrimination in two-dimensional form with projection onto plane and onto one axis for linear discrimination. Adapted from Crutcher (1960).

Figure 6 Schematic representation of the discriminant function (dotted) line when the two bivariate populations, P_1 and P_2 , have unequal means but equal variances and covariances. If the two-dimensional function is linearized so that the data are projected onto the D axis (dotted), the discriminant criterion, C, is shown at the head of the arrow. All projections to the left of C are assigned to P_1 . Those to the right of C are assigned to P_2 . Adapted from Crutcher (1960).

Figure 7 Schematic "D" illustration of constellations of annual temperature climates for North American stations. The constellations and clusters are based on the a_0 , a_1 , and a_2 harmonic coefficients. Dispersion matrices within constellations are not significantly different but are different between constellations. Crutcher (1960).

The works of Hotelling (1931) and Mahalanobis (1936) are related closely to Fisher's (1936) discriminant criterion. Study of these papers provides excellent background. Hotelling describes these relationships in 1954. Mahalanobis' work in a sense deals with a set of multi-dimensional data for which the ensemble variance is reduced to one, i.e., the total ensemble is standardized. Euclidean measurements then are used.

There are other techniques to categorize and classify data. All of these require some transformation of data, the selection of those measurements which provide the greatest amount of information, and the elimination of those that provide the least if these may be termed unimportant. Usually, their importance is assessed in how much they contribute to the overall variability.

3.2 Factor Analysis

The basic mean structure and variance-covariance structure, i.e., the matrices of means and covariances, are the bases for factor analysis and principal component analysis. It is in the details and interpretation, some complete and some incomplete, of the internal structures that the techniques differ.

The term "factor analysis" comes from factoring procedures and techniques. There are many variants each of which is a special case of the general method of independent dimensional analysis (Tryon, 1959, Tryon and Bailey, 1970). Briefly, as with the discriminant function analysis described previously, the correlation (or covariance) matrix is the initial starting point. The methods used to extract information from this matrix are many and varied. Some are more complex than others. Some use weighting schemes either based on "a priori" knowledge and experience or physical constraints and bases. Some simply attempt to let the matrix itself determine the factors. All this leads to some confusion and some competition among the proponents of the various systems.

Multiple factor analysis with rotation to simple structure is the usual procedure in factor analysis.

The purpose of factor analysis is to explain the matrix of covariances of a multidimensional set by the least number of hypothetical factors. The correlation matrix is used. First of all, the matrix is examined to determine whether it is significantly different from zero (the identity matrix). If so, the technique then identifies, extracts, and weights proportional amounts of the correlation until the residual matrix is not significantly different from zero. Factor analysis stems mainly from the initial work of Spearman (1904, 1926). Essentially, the technique serves to study the similarities in a set of data.

Other pertinent references on factor analysis techniques are Thurstone (1947), Kendall and Smith (1950), Cattell (1952), Bartlett (1953), Fruchter (1954), Harman (1967), Sokal and Sneath (1963), Lawley and Maxwell (1963), Mulaik (1972), and Anderson and Rubin (1956).

3.3 Principal Component Analysis

Karl Pearson (1901) first proposed an empirical method for the reduction of a large body of data so that a maximum of variance could be extracted. Hotelling (1933) developed this method fully as the principle component method. This under some conditions is identical to the discriminant function [Kullback (1968), Anderson (1958), and Girshick (1936)].

For example, principal components according to Anderson (1958) are linear combinations of random or statistical variables which have specified properties in terms of variances. The first principal component is the normalized linear combination with maximum variance. In a swarm of points, each point representing an n -component vector, the distribution may be ellipsoidal. Unless the distribution is spherical, there will be an axis which is as long or longer than any other axis. This axis is called the major or principal axis. A plot of these points will reveal the ellipsoid. It is difficult to represent such an ellipsoid in more than three dimensions. Measures of the components may not directly reveal the ellipsoidal nature of the swarm. However, the variance-covariance or correlation matrix may be rotated in space such that new axes are obtained where the components, projections, or linear functions along these new axes are not correlated. The directions of these axes are known variously as characteristic vectors, latent vectors, direction cosines, or eigenvectors. If there are observations in n -dimensions, each observation as an n -dimensional vector may be projected onto each of the n -orthogonal (mutually uncorrelated) axes obtained above. The variability of these components along each axis may be determined. These variances are known respectively as characteristic roots, latent roots or eigenvalues. The sum of these is the total variance or trace of the matrix. Their product is the determinant. Each one divided by the total provides the proportional amount of variance contributed by the components along each axis. The largest axis is the major axis. The principal axes are those axes whose sum, from the largest in sequence to the next largest, accounts for all or some preselected or specified proportion, say 95 percent. It is the components along each axis that are used. Thus, it is possible to reduce a large dimensional problem to a small dimensional problem. It is also possible to select from the same data a dimensional subset ($n-1$ or less) to eliminate that (those) dimension(s) which contribute(s) little to the total overall variability.

There are tests to determine which principal components may be considered to be significantly different from the others if indeed they are. Hotelling (1933) and Bartlett (1950, 1951) provide subtests. Mulaik (1972) presents a good discussion of this problem. However, with all this, one must heed the advice of Hotelling (1957) that there may be difficulties involved in neglecting one principal axis even though it is the least in variance. Such an omission may change radically the multiple correlation used in regression. Only adequate testing will reveal whether an axis may be neglected.

In addition, the components along any one axis may be checked for evidence of heterogeneity or mixtures. If multimodality is present on any principal axis, then separation or clustering can be done for these data.

The principal component analysis is distinguished from the factor analysis

in that the principal component analysis studies the data structure from the viewpoint of differences rather than similarities.

Any degeneracy which exists in a multivariate distribution is revealed in a principal component analysis [Tatsuoka (1971)]. Therefore, principal component analysis perhaps ought to be considered to be a first stage in factor analysis, though this does not meet the views of the proponents and the opponents of the two techniques.

3.4 Multivariate Statistical Methods

As mentioned previously, the ways in which factor analyses are developed and used lead to newer systems with their respective proponents and opponents. This is so because the data sets must be discussed on the basis of each set. From set to set the bases may be different. Hotelling (1936a & b, 1957) discusses the relations between two sets of variates, simplified calculation of principal components, and then the newer multivariate statistical methods to factor analysis.

Up to the time of Hotelling's (1957) paper, factor analyses of the usual kinds were often inferior to other procedures. Unless the research worker determines and uses (an) invariant statistic(s), the results always will be difficult to assess. However, as in all investigative work of this nature, these analyses may have only heuristic or suggestive value. Hypotheses may be exposed which may be better tested by other methods.

In examination of the use of statistics in the various scientific fields, it is readily apparent that each field develops its own names and "jargon" for statistical terms within their specialized fields. Also, workers trying to reach a common goal will independently develop similar techniques. Most such techniques are beset with "nuisance" parameters induced by an arbitrarily chosen statistic. As Hotelling (1957) points out, it is the invariance of the multiple correlation coefficient as deduced by Fisher (1928) that illustrates the possibility of eliminating the "nuisance" parameters which can only becloud the issue. This puts a premium on the use of invariant statistics.

Student's (1925) "t" distribution is well known for its use to test the significance of means and difference of means and to establish confidence intervals with known probabilities. Its very usefulness in the one dimensional case led to much work towards generalizing this procedure to the multivariate case.

Hotelling (1933) in his work on principal components led to such a generalization in the " T^2 " test. The value of " T^2 " is invariant under all non-singular linear transformations among the variates. The trace or the sum of the diagonal variances of a covariance matrix is invariant. The " T^2 " distribution is a beta distribution or a variance ratio distribution.

The " D^2 " stability of Mahalanobis (1936) is closely allied to the " T^2 " stability of Hotelling (1931). This also was an attempt to arrive at the use of invariant statistics. Wilks (1932) considered generalization in multivariate analysis. Roy (1939, 1942a & b), Hsu (1938), Bartlett (1950), and Bose and Roy (1938) did considerable work in the field of multivariate analysis.

In the application of principal components in factor analysis, all components must be used. The exclusion of even the smallest component may lead to a complete change in the obtained functions to such an extent that interpretations or decisions may be quite erroneous.

Ridge regression is mentioned as an evolving technique which should receive the attention of some readers [Hoerl (1962), Hoerl and Kennard (1970a,b), and Bannerjee and Carr (1971)]. From the viewpoint of response surfaces, Davies (1956), Draper (1963), and Myers (1971) may be consulted. Though these have their impact on the problems of clustering and classification, they are not discussed further.

3.5 Multivariate Logit

Distribution of quantal responses to drugs or poisons may be better described by distributions other than the normal. The logistic curve is one of these. As Kendall and Stuart (1968) state, "The Probit and Logit transformations of percentages, respectively to normal and logistic distribution deviates, arise mainly in biological contexts and are discussed by Finney (1952)." It has not found wide application in the geophysical field. Anderson (1958) and Dempster (1973) discuss the Logit model. This would be most appropriate if the mixture is composed of a set of variables one or more of which would be considered fixed while the others are normal. This is beyond the scope of the present paper as we here utilize only those distributions which from experience are known to be approximated by the normal distribution. The use of the multivariate Logit distribution mixture separation must be deferred to later research. Hopefully, this will be within the next five years.

4. CLUSTERING

Grouping of individuals with similarities and separation of individuals with dissimilarities is a continual process. This process is evident from the smallest to the largest features in any ensemble whether it be in the universe in the development and dissolution of galaxies, in the earth in its geologic processes, or in life on earth in whatever form it may be. It is evident in the abstract world, too. Readers of Aristotle and the followers of Hippocrates and Linnaeus will recognize attempts to order chaos in the sense of clustering and differentiation, whether it be the assignment of some attribute or the recognition of some measurable quality or quantity. Much has been done and much has been written. The techniques of clustering and differentiation are as varied as those who attempt to perform those functions. The logic and the metrics may vary but the insistent theme is to group those that are alike and separate those that are unlike. Each individual is, of course, an entity in itself. A set of measurements on that individual undoubtedly constitutes a group in itself. This much is acknowledged by any worker. The job is to group those individuals together whose sets of measurements are not too far different and to put aside those whose measurements are different at some level of perception and thinking to another group or groups. Much sometimes depends on the individual whose measurements are used as the starting point. In a sense, those measurements of the first individual are in an "a priori" sense weighed most heavily in the clustering process. Coalescence begins on this individual.

Clustering is simply another term among the many in the general field of taxonomy. Taxonomy is the scientific ordering and classification of information. Taxonomic systems must be simplified representations of group characteristics and all their interrelationships. Because these are most general and therefore quite usable, perfection cannot be attained except in the case of an individual. The main function of any such system is to reduce the complex problem to a simpler problem, thus reducing the requirements of memory. References, some of which have been given previously, are Sokal and Sneath (1963), Tryon and Bailey (1970), Duda and Hart (1973), Fisher (1936), and Anderson (1958). Hartigan (1975) offers a considerable number of clustering algorithms.

Many terms are used in the grouping concept. Some are more suggestive of the process than others. These terms depend on the field of endeavor and on the basic background of the investigators. Some of the techniques are the same though the words are different. Let us look at a few of them: clusters, clumps, coalescence, condensations, aggregations, agglomerations, divisions, similarities, cohesions, linkages, hierarchies, communalities, types, and affinities. The classes obtained may be given names in the specialized fields. In the biological sciences these are the families, genera, species, subspecies, etc. In the field of the natural sciences, though not restricted to these fields, these may be clusters, universes, constellations, galaxies, etc. In the latter example, the order may be interchanged depending on the viewpoint of the problem.

In any taxonomic problem dealing with measurements, there must be some way to collect the like individuals together and to isolate, reject, and form new

groups with those which are most alike among the unlike. Some measures of likeness or similarity must be developed and accepted. As individuals are collected into small groups and as the smaller groups are collected into larger and larger ensembles, a tracing of the procedures may be kept. As diagrams or the concepts of the trace resemble some feature of the known world, these may be called tree-diagrams (branch diagrams), root diagrams (dendrodiagrams or dendritic diagrams), link diagrams, or hierarchical diagrams. These may be shown in two or three dimensions or left in abstract form.

Baker and Hubert (1975) consider procedures to measure the power of hierarchical cluster analysis. This important feature in studying the various techniques and their alternatives is not examined here in further detail.

The clustering procedures are allied closely to the previously discussed techniques of factor analysis, principal component analysis, and discriminant function analysis. Canonical analysis also may be used. The last is simply the techniques which maximize the correlations among linear functions of the data.

The specific clustering techniques used here are discussed in more detail in Wolfe's (1971b) NORMIX program, section 7.

5. TRANSFORMATIONS

The multivariate normal distribution is only one of an infinite number of useful multivariate distributions. The univariate normal distribution has been exploited more than any other because the necessary statistical tools have been developed. It is nearly so with the multivariate normal. If a distribution is multivariate normal, then any of the subspace marginal distributions are normal. Though the normality of all marginal distributions does not guarantee multivariate normality, such normality is a necessary condition for multivariate normality.

If prior experience indicates that a certain measure is usually normally distributed and tests imply non-normality, then one inference that can be made is that the data set is mixed. It is precisely the thrust of this report to separate such mixed distributions into their homogeneous parts. If any marginal distribution is mixed, then certainly any higher order is mixed or any lower order variate distribution is or may be mixed. If the set of projections onto any principal axis is mixed (multimodal) and the set of projections is a linear function of the multivariate set, then this set can be used to establish estimates of the parameters of the various homogeneous collectives.

With normality of distribution, the usual tests of significance or tests of hypothesis may be made. Otherwise, decisions based on such tests may be invalid or questionable. The robustness encountered, though, usually is such that a lack of normality does not impair the decisions very much. Far more important perhaps is the practical significance of the decision.

If it is known that an unmixed marginal distribution is not normal, then some transformation towards normality of that distribution should be sought. A marginal distribution is a subset of the higher order distribution. If the marginal distribution itself is not normal in the multivariate sense, then the still lower order marginals or smaller subsets should be checked. It may be that only one of the lowest order marginals (a one dimensional distribution) is the non-normal. If this is the case, then transformation of this distribution should be sought.

The above does not imply that it is impossible to use the non-normal distributions. It does imply that the distributions must be normally distributed or mixed normal in their distribution if the techniques discussed in this paper can be used to provide valid results.

There may be some cases where appropriate normality cannot be achieved through any transformation (Graybill, 1961, pg. 318).

For those distributions that can be normalized, the transformation process is often carried forward in graphical procedures. Boehm (1974) uses this procedure and has an electronic computer program to provide such transformation.

The methods of factor analysis, principal components, and discriminant functions are techniques to linearize a multidimensional situation to a

single univariate situation. Hopefully the final distribution or distributions in the components, factors, or clusters will allow linearization within these to the univariate problem.

There are many tests for univariate normality. Some of these (Graybill, 1961) are (1) likelihood ratio tests associated with transformation toward normality, (2) skewness and kurtosis tests, (3) omnibus tests, and (4) normal probability plots.

A useful transformation to improve normality is that proposed by Box and Tidwell (1962) for estimating a shifted power transformation $(\chi + \xi)^Y$ of a single variable. This is a generalization of many tests developed through the years [Tukey (1957) and Moore (1957)]. Simultaneous transformation of a multiple set of variables has been the subject of much research. Andrews et al. (1971) discuss the problem in more detail for the bivariate distribution. Extensions to the multivariate case are indicated.

Transformations are not discussed further here as the climatological examples used in this report do not require transformation to normality. The clustering technique used here will be applied to other climatological and geophysical problems. Many of the data distributions of the future study will require transformation to normality. The two techniques described above then will be treated in detail.

6. SEPARATION OF MIXTURES

6.1 Univariate Mixtures

Karl Pearson (1894), in his paper on "Contributions to the Mathematical Theory of Evolution," gives the procedure to mathematically dissect a mixture of two univariate normal distributions. In the general case the five parameters to be estimated are two means, two variances, and a proportionality factor. It is necessary to find a particular solution of a ninth degree polynomial equation, a nonic.

The following list is neither exhaustive nor the most important but it does provide sufficient references for the reader to pursue the subject further: Charlier (1906), Charlier and Wickseil (1924), Burrau (1934), Strömgren (1934), Essenwanger (1954), Schneider-Carius and Essenwanger (1955), Cohen (1965), and Cohen and Falls (1967). Essenwanger's work is applied to meteorology while Cohen's and Fall's work provides electronic computer routines for dissection of heterogeneous mixtures.

Hald (1952) discusses the subject of heterogeneity in the univariate case. Graphical as well as analytical techniques are discussed. The general case may be written as

$$f(x) = \sum_{i=1}^m \alpha_i f_i(x); \quad \sum \alpha_i = 1,$$

where

$$f_i(x) = (2\pi\sigma_{xi}^2)^{-1/2} \exp\{-[(x-\mu_{xi})/\sigma_{xi}]^2/2\}.$$

Even for a mixture of two distributions the solution of the nonic is a big task. Cohen and Falls (1967), using procedures developed by Charlier and Wickseil (1924) and a modern electronic computer, provide procedures to effect the requisite dissection and to obtain estimates of the parameters. Discriminant criteria can then be developed to enable classification of a new measure to one of the two groups or to classify each datum from the original data set.

The computer program, kindly lent to the authors by Cohen and Falls, has been used to dissect a mixed distribution in Section 9.

6.2 Bivariate Mixtures

Hartley (1959) provides techniques to separate two mixed bivariate normal wind distributions. Crutcher and Clutter (1962) apply these to wind distributions but encounter difficulty when the covariance matrices are singular or near singular or when the variances or covariance matrices are assumed to be different.

In the bivariate case of a unimodal distribution there are five parameters to be estimated. These are the two means, the two variances, and the correlation. When there is a mixture of two bivariate normal distributions, there

are eleven parameters to be estimated. These are the two means in each group, the two variances in each group, the correlation in each group, and the mixture parameter. If two assumptions are made, (1) the variances are all equal one to each of the other three and (2) the correlation is zero, then the simplest mixture of two circular normal distributions is obtained. If the component variances within each are equal, but unequal from one group to the other, the more complicated mixture of two circular normal distributions with different variances is obtained.

If there are "a priori" reasons to suspect that there are two groups and if the assumption of circularity provides meaningful and useful results, then it is suggested that Hartley's procedures be used. Computing time will be much less than in the clustering program discussed here.

The authors know of no other computer program available specifically to take a set of mixed bivariate normal data and dissect it even for the simplest case of two subsets with unequal means but equal variances, component and vector, and zero correlation. The cluster program used here will do the above as well as the general case. But this clustering program is far from an optimum one in terms of least time and cost. For the one or two time case, the clustering program could be used. However, for repetitive processing of many many sets of data it would be advisable to develop an optimized specific program based on Hartley's procedures. This, the authors plan to do.

The mathematics may be written for the simplest bimodal bivariate circular case as

$$f(x,y) = \sum_{i=1}^2 \alpha_i f_i(x,y); \quad \sum_{i=1}^2 \alpha_i = 1 ,$$

where

$$f_i(x,y) = (2\pi\sigma_i^2)^{-1} \exp\{ -[(X-\mu_{xi})^2/\sigma_{xi}^2 + (Y-\mu_{yi})^2/\sigma_{yi}^2] 2^{-1} \}$$

and the α_i are the proportionality factors. In the slightly more general case,

$$\begin{aligned} f_i(x,y) = [2\pi\sigma_x\sigma_y(1-\rho_{xy}^2)]^{-1/2} \exp\{ -[((X-\mu_{xi})^2/\sigma_{xi}^2) \\ -2\rho_{xy}((X-\mu_{xi})/\sigma_{xi})((Y-\mu_{yi})/\sigma_{yi}) \\ +((Y-\mu_{yi})^2/\sigma_{yi}^2)] [(1-\rho_{xy}^2)^{-1}] [2^{-1}] \} . \end{aligned}$$

6.3 Multivariate Mixtures

The general multivariate-multimodal case may be written as

$$f(x,y,\dots) = \sum_{i=1}^m \alpha_i f_i(x,y,\dots); \quad \sum \alpha_i = 1 ,$$

where

$$f_i(x,y,\dots) = [(2\pi\sigma_{xi},\sigma_{yi},\dots)|R|]^{-1/2} \exp(-\psi^2/2) ,$$

α is a proportionality factor, R is the correlation matrix, $|R|$ is the determinant of the correlation matrix, $R_{i,i}$ is the respective cofactor, and

$$\begin{aligned} \psi^2 = & |R|^{-1} [R_{xi, xi}((X-\mu_{xi})/\sigma_{xi})^2 + R_{yi, yi}((Y-\mu_{yi})/\sigma_{yi})^2 + \dots \\ & - 2R_{xi, yi}((X-\mu_{xi})/\sigma_{xi})((Y-\mu_{yi})/\sigma_{yi}) - \dots] . \end{aligned}$$

Sample estimates such as $\bar{X}$, $\bar{Y}$, ..., s_{xi} , s_{yi} , ..., r_{xy} , ..., replace the population parameters μ_{xi} , μ_{yi} , ..., σ_{xi} , σ_{yi} , ..., ρ_{xy} , ..., where the bar represents an averaging process.

7. WOLFE - NORMIX 360 COMPUTER PROGRAM

Culminating over a decade of work, Wolfe (1971b) published his NORMIX computer program. Earlier versions and other pertinent papers by Wolfe appeared in 1965, 1967 a and b, 1968, 1970, and 1971a. He provides background discussion, the necessary programs, and an example. The example chosen is the old standby example of Fisher (1936) which so many investigators use.

7.1 Maximum Likelihood Estimation

Unless "a priori" conditions indicate otherwise, the first hypothesis made is that there is a specified number of types and the probability of a datum being assigned to any one of the types is the same as to any other type. With two groups, the probability of being assigned to one of the two types is 0.50. With four groups, the probability of being assigned to one of the four types is 0.250. With no "a priori" indication, a default option of the program then provides equal mixing proportions as a first guess.

As shown by Wolfe (1970), the maximum likelihood estimates of the mixing proportion ($\hat{\lambda}_s$), mean ($\hat{\mu}_s$), and covariance ($\hat{\sigma}_s$) of the cluster (type s) in a mixture satisfy the following conditions:

$$\hat{\lambda}_s = (1/n) \sum_{k=1}^n \hat{P}(S|X_k)$$

$$\hat{\mu}_s = (1/(n\hat{\lambda}_s)) \sum_{k=1}^n X_k \hat{P}(S|X_k)$$

$$\hat{\sigma}_s = (1/(n\hat{\lambda}_s)) \sum_{k=1}^n (X_k - \hat{\mu}_s) (X_k - \hat{\mu}_s)' \hat{P}(S|X_k)$$

where the prime indicates a transpose, X_k is the vector of observations for the kth point in the sample, and $\hat{P}(S|X_k)$ is the probability of membership of X_k in clusters and is equal to $\hat{\lambda}_s$ times the ratio of the normal density of type s at X_k to the density of the mixture. In the special case where the clusters have a common covariance matrix, $\hat{\sigma}_s$ is $\hat{\sigma}$ and may be written as

$$\hat{\sigma} = (1/(n\hat{\lambda}_s)) \sum_{k=1}^n X_k X_k' - \hat{\mu}_s \hat{\mu}_s'.$$

As the unknown parameters appear on both sides of the equation, it is necessary to use an iterative process to solve the equations. For this reason, if "a priori" estimates are used the iterative processing is diminished. Computing costs therefore are much lower. Several sets of values may satisfy the equations, and the results may depend on the starting values for the iteration process. As indicated previously, a change in the order of the input data will usually change the initial estimates obtained.

Local or relative maxima or saddle points may be obtained. The procedure does not guarantee to find an absolute maximum. The initial estimates are simply modified and improved upon. Bad initial estimates or diverging iterations can cause strange estimates in the parameters such as negative variances or singular correlation matrices. When this happens, re-initialization or change of initial estimates may resolve the problem. The program also attempts to resolve the problem of singular matrices by adding a small normal random number to the diagonal terms.

7.2 Initial Estimation

It is assumed that the problem involves clustering within a multimodal multivariate data set. All measurements are standardized. That is, estimates of the individual component means and variances are computed. The squares of the deviations from the means divided by the variances produce the square of a standardized deviate. Thus, the set is reduced to a multimodal multivariate set with a zero mean and a variance of one. This is the basic Mahalanobis distance technique.

Within the above framework, if initial estimates of the cluster means and covariances are not provided, the program generates initial estimates in a KMEAN subroutine which was adapted from Ward, Hall, and Buchhorn's (1967) hierarchical grouping for minimum variance. The variance which is minimized is the sum of Mahalanobis distances between points within a cluster given by

$$d_{ij}^2 = (X_i - X_j)' C_1^{-1} (X_i - X_j) ,$$

where X_i and X_j are points in the same cluster and C_1 is the covariance matrix. The prime indicates a transpose and the superscript indicates an inverse.

The subroutine first transforms the data to principal axis factor scores,

$$Y = \Lambda^{-1/2} F'X ,$$

where F is the eigenvector matrix of C_1 and Λ is the diagonal matrix of eigenvalues. (See discussion on principal component analysis.) Then

$$d_{ij}^2 = (Y_i - Y_j)' (Y_i - Y_j)$$

which is easier to compute.

After hierarchical grouping, the within-group covariance matrix C_2 for ten clusters is determined. The number ten is arbitrary and could be changed. With this modified matrix, hierarchical grouping is performed again. A third covariance matrix C_3 within the ten clusters is obtained. New distances are obtained which are used for the third and final hierarchical grouping.

The program permits the assignment of an arbitrary number of clusters or means known as KMEANS. If the number setup is 150 or less, the procedure uses the number established. If the number of input data is greater than 150,

a leading subroutine for the KMEANS (MacQueen, 1967) precedes the hierarchical grouping. The first 150 input data points form the centroids of 150 clusters. The two closest points (clusters) are merged into a two-point cluster with a cluster mean in the location of the first of the two points read. The 151st datum is moved into the place vacated by the second of the two points. The next two closest cluster centroids are merged and the process continues until all data points have been collected into 150 clusters which then are grouped hierarchically.

7.3 Significance Tests for Number of Clusters

The user has some idea that clusters of data exist in the data set or he would not be concerned with this report or its uses in his field of endeavor. Suppositions or hypotheses concerning the number of types in the sample are listed and control is placed on each of these in the program. For example, he suspects two groups at least or perhaps four at most. Just to make sure, as far as this technique is concerned, he might place an upper limit of six. The program reaches a solution in each case which provides a relative (local) maximum to the likelihood function. The ratio of the likelihoods for any two of the hypotheses above 2, 4, or 6 provides a basis for a significance test for rejecting the null hypothesis that there is no difference between the two hypotheses, i.e., the smaller number of types against the alternative of a larger number of types.

7.4 Strategy of Use

As computing time increases with number of variates, number of input data, and null hypotheses to be checked, care should be exercised to keep time, which is transformable to cost, to a minimum. There are certain strategies for use which Wolfe (1971b) discusses.

7.5 Usage

It is appropriate that, since the program can be obtained from Wolfe (1971b), a few usage comments from his program be placed here (with his permission).

7.5.1 Storage Requirements

Storage requirements are variable, depending on the data. The arrays are dimensional at execution time. A minimal problem would require 120,000 bytes of storage (IBM 360). The first two lines of the printout give the storage required for a particular problem.

7.5.2 Restrictions

The number of types or clusters must not exceed 20. There are no fixed limits on the number of variables or sample size. However, all data and arrays must fit into core.

7.5.3 Error Messages

Bad initial estimates or diverging iterations can cause strange estimates

of the parameters, such as negative variances or singular correlation matrices which result in system diagnostics. In such a case, the user may have to specify his own initial estimates on the input form or rescale the data to eliminate dichotomous variables.

7.5.4 Input Deck

Input Deck set up cards are:

- Input Form, Cards 01-11
- Data Cards (omit if data are on tape)
- Input Form, Card 12
- Initial estimates, if any, for one hypothesis
- Input Form, Card 12
- Initial estimates, if any, for a greater number of types
- ...
- ...
- Blank Card

For example:

- Card 1 provides the user and the data used.
- Card 2 provides the title.
- Card 3 provides room for comments.
- Card 4 provides room for comments.
- Card 5 provides for the number of variables, the sample size, and input tape.
- Card 6 provides hypotheses for number of clusters.
- Card 7 provides space for assumption of same or different covariance matrices and the minimum number to be accepted into a cluster.
- Card 8 provides for the significance level for rejection of the null hypothesis.
- Card 9 provides for the entry of the number of KMEANS to be first examined and whether the iterations are to be printed.
- Card 10 provides for the establishment of a time limit and a limit on the number of iterations.
- Card 11 provides for the data format.
- Card 12 provides for the initial estimates, if any, for the number of types, the means, the standard deviations, and the correlations.

7.5.5 Validation Examples

Wolfe (1967, 1971) uses two examples to test his program. The first is the Fisher (1936) treatment of Anderson's (1935) measurements on Irises. Nearly all discrimination papers or clustering papers refer to this classic paper. The second example is an artificial set. These and another set of data on azotobacter, the nitrogen fixing bacteria [Cox and Martin (1937)], were used to validate the Wolfe program adapted for use at the National Climatic Center.

The NORMIX program has been adapted for use at the National Climatic Center. Considerable difficulty was experienced in adapting the program for use on the Univac Spectra 70/45 due to language configurations. One adaptation is an

output routine to furnish a sequential leading copy of the input data. This permits ready referencing as to a datum and its assignment to a cluster. Some modification of the output sequences were also made.

Though this point is made by Wolfe, it is given here again for emphasis. The techniques apply to only normally or near normally distributed variates. Any significant departure from normality will invalidate the probability decision levels. Therefore, if data distributions cannot be approximated by the normal distribution, transformations to normality or near normality of distribution must be made before input to the program.

A brief summary by Wolfe (1971b) is quoted below:

"The problem was to develop a computer program for cluster analysis and unsupervised pattern recognition of types with different cluster shape....

"The program will cluster a sample of thousands of objects measured on many continuous variables....

"The approach seeks maximum likelihood estimates of the parameters of multivariate distributions. The likelihood equations are solved iteratively by continually re-estimating the probability membership of each sample point in each cluster until the likelihood reaches a relative maximum. The initial estimates are derived from a minimum variance hierarchical grouping subroutine, which itself is iterative in seeking an appropriate distance function. The program prints out the means, standard deviations, and intercorrelations of the variables within clusters and the proportions of the population for each cluster. The probabilities of membership for each cluster are also printed."

Briefly some points mentioned in the foregoing section are iterated. In clustering procedures, unless there is "a priori" knowledge as to the characteristics of the clusters in the data set, the data themselves determine the clustering. This is called the unlearned procedure or learning process. The use of "a priori" estimates is termed the learned procedure. There is, then, the necessary decision as to just where to start. This is true of any discriminating procedure whether it be any of the allied techniques of analysis such as discriminant function, factor, principal component, or any other analysis.

Any change in the order of the input data will produce a difference in the output cluster characteristics (MacQueen, 1967, p. 290). Therefore, one run only provides an estimate of the clusters. A number of runs with a different ordering of the data will produce different results. Hopefully, these will not be too different and usually are not too different even though their covariance matrices may be unequal. The more distinctly different the clusters are, the more the various runs will provide the same estimates. This is not unexpected. There will be some data which will not be assigned to the same cluster each time.

8. EXAMPLES

8.1 Introduction

The techniques used here are applicable in any field as they reduce all problems to non-dimensional ones in the sense of units. Here, climatological (weather) observations are used. These were obtained from the archives of the National Climatic Center (NCC) in Asheville, N.C.

We present six data sets from among the many that could have been used. The first four present situations which are recognized as producing mixed distributions. Obviously, these are selected to demonstrate the usefulness of this procedure.

Output of this program for the six data sets used for this paper is not presented in total. The data sets are:

1. Land-sea breeze data
2. Tropical stratospheric wind data
3. Mid-latitude tropospheric wind data
4. Mountain pass wind data
5. Marine surface data
6. Radiosonde and rawinsonde data

In the first and second data sets, only selected tabular output data and computer-drawn, 0.50 probability ellipses are shown. In the third data set, a total input-output of the Wolfe (1971b) program adapted for use at the NCC is shown. Except for the input data and the modified eigenvalue-eigenvector output, the format is essentially that of the Wolfe program. Also, a considerable number of 0.50 probability ellipses are shown. The computer plot routines for these exist in programs written for the CALCOMP drum plotter and Computer Output Microfilm and the Hewlett-Packard desk-top plotter at the National Climatic Center. The computer print plot of the discriminant function for the common covariance assumption is part of the Wolfe program. The last three data sets simply illustrate selected and re-arranged output as well as the 0.25 probability ellipses.

Winds are used in all six data sets as upper winds usually are distributed in the bivariate normal sense. In the second, fifth, and sixth data sets, additional weather elements are used. Where mixtures are evident, the usual unimodal distribution cannot be used. These mixtures occur in the planetary boundary layer, in the trade-wind inversion region, in the tropopause, or in the monsoon change region. It is precisely the advisability of separating these and other mixtures that prompted the investigation of the problem and the publication of these results.

8.2 Brief Descriptions of Data and Locations

8.2.1 Land-Sea Breeze Data

San Juan, Puerto Rico, U.S.A., is a seaport located on the northern shores of the Island in its eastern portion. It is a sub-tropical station at 18°26'

north latitude and 66°00' west longitude. The data are taken from the observations made at the airport at Isle Verde. Its elevation is 20 m. This location was selected because it is a logical one for a land-sea breeze effect. The hours selected, 0600-0800 and 1200-1400, are in local standard time. The month of October was selected though any other period could have been used. One hundred observations for each hour group are used for October 1955 and the first portion of the first 3 days of October 1956. The early morning hours should show the land breeze or a balanced condition. The 1200-1400 observation hour should begin to show the effects of the sea breeze though perhaps not as strongly as later hours. The hours of 0900-1100 were not used so as to eliminate some of the overlapping of the land-sea breeze effect if any existed. This would provide a clearer separation for the technique demonstration.

8.2.2 Tropical Stratospheric Wind Data

Canton Island is in the Southern Hemisphere at latitude 2°46' and longitude 171°43' west, elevation 4 m. This location was selected because of its known, distinct quasi-biennial wind oscillation in the stratosphere. During the preparation of Technical Paper 34 (Crutcher, 1958) and U.S. NAVAER 50-1C-535 (Crutcher, 1959), the apparent biennial oscillation of the tropical tropospheric winds had been noted. U.S. NAVAER 50-1C-535 is apparently the first atlas in chart form to use the elliptical bivariate normal distribution to describe the distribution of upper winds, particularly those of the stratosphere. This followed the very important atlases of the British groups [Brooks and Carruthers (1950)] using the circular bivariate distribution and preceded their update, [Heastie and Stephenson (1960) and Tucker (1960)]. The British groups assume the circular distribution to adequately describe an upper wind distribution. This appears to be a good assumption as a first approximation if the winds are not mixed. The extension (Crutcher, 1957) of this assumption to that of the elliptical normal as a second approximation increased the representativeness of that bivariate normal from about 70 percent of the cases to about 90 percent in the troposphere. It was noted during the preparation of NAVAER 50-1C-535 that, in the tropics at the higher altitudes (lower pressure levels), the ratio of the major axis to the minor axis was seemingly extraordinarily large, near four ranging up to ten at times. An examination of the distributions generally revealed two clusters with the easterly winds being stronger and with a higher constancy than those from the west. In the meantime other investigators also were studying the problems. McCreary (1959) reported that in the stratosphere during October 1956 - July 1957 at Christmas Island, westerly winds overlay the stratospheric easterlies. This was a reversal of what was considered to be the usual pictures of easterlies over westerlies. Graystone (1959) then reported on a year to year reversal of the equatorial stratospheric winds. In the following year Reed (1960), Ebdon (1960), and Veryard (1960) provided the impetus for a great amount of research into the phenomenon now known as the quasi-biennial oscillation (QBO) of the winds in the tropical stratosphere. These same authors soon produced further work in the field [Reed et al., (1961) and Reed and Rogers (1962) in the U.S. and Veryard and Ebdon (1961a, b, and 1963) in the U.K.]. Later, an oscillation in the temperatures associated with the winds was soon reported as were oscillations in the tropopause heights, ozone, and other phenomena. Newell et al., (1974) discuss the development of research

in this field in the second volume of an important work on the general circulation of the tropical atmosphere and interactions with extratropical latitudes.

In these very important studies, the oscillation was shown to start at the higher altitude of the 10-mb level and propagate downward through the stratosphere to the 100-mb level.

Clayton (1885), Berlage (1956), and Landsberg et al., (1963) had noted this feature in surface phenomena. This feature now occupies a rather solid part in the literature of the atmospheric circulation.

Upper winds during the months of July and January for the years 1954-1964 at Canton Island at the 50-, 30-, 20-, and 10-mb pressure levels were used. For the period 1957-1967, heights and temperatures were added. Data from this station have been used many times in studies. The quasi-biennial oscillation (QBO) is barely evident at 100 mb and becomes strongest at about 30 to 25 mb. Apparently it weakens at lower pressure levels (higher altitudes), such as 10 mb, though this effect may be due to loss of observations. There is a time dependence as noted by Reed et al., (1961), Reed and Rogers (1962), Veryard and Ebdon (1961a, b), Angell and Korshover (1962), and later works.

The purpose of this paper is to provide the techniques to dissect such distributions. Dynamic or synoptic considerations are not made here except in the sense that these do effect the QBO. Newell et al., (1974) discuss the dynamic properties of the phenomenon. Dynamic considerations do create the separate distributions insofar as the calendar year dating system is involved. The results may be used to assess the extent of these effects. Only the horizontal winds and temperatures are used at the individual levels. An analytic solution or more detailed examination of the atmospheric circulation and weather is not attempted here. For example, the problem could have been made much more multidimensional by including all stratospheric levels and concomitant surface weather. The results of this section clearly demonstrate the usefulness of this separation technique.

8.2.3 Mid-Latitude Tropospheric Wind Data

A continental U.S. station was needed. Rantoul, Illinois, was selected since, in an earlier unpublished work, Crutcher and Clutter (1962) experienced some difficulty in applying the techniques of Hartley (1959). The difficulty involved singularity problems in the data when assumptions of unequal variance in the groups were made. Its latitude is $40^{\circ}18'$ north, longitude is $88^{\circ}09'$ west, and elevation is 227 m. There are some data sets which produce such near singular matrices that the techniques cannot be used in their present form. This is occasionally true of the present technique here. Ordinarily, the singularity problem is resolved in the technique by the addition of a random small amount to the matrix diagonal elements. Even then, negative variances may be encountered. Re-initialization or change in order of data entry may resolve the problem.

The period of record of data used is the month of October for the years 1950-1955. The upper wind data used are those at the 700-, 500-, and 300-mb

levels. Both input and output data are shown to better illustrate the procedures. The first two data sets described were two-dimensional. This data set example illustrates a distribution in six dimensions, two for each of three levels. For this purpose there had to be a simultaneous wind observation at each of the three levels. Therefore, each input data vector given was composed of the vector formed by the zonal and meridional component of the wind at each level, i.e., a vector composed of six components. The two-dimensional illustration for a given level then is comparable to that of another level for each datum at the given level having a matching datum at the other level.

As more than three space is difficult to illustrate except for linear reduction to three space or less, output for the six-dimensional forms is given in eigenvector and covariance (correlation matrix) form.

No attempt is made here to correlate the wind data with current or later weather. This can be done by simply extending the six-dimensional vectors to a greater number of dimensions by including the desired surface weather data.

8.2.4 Mountain Pass Wind Data

Stampede Pass, Easton, Washington, U.S.A., was selected to demonstrate the existence of two or more distinct clusters of wind, and of wind with temperature regimes, in a constricted geographic location. Its latitude is $47^{\circ}17'$ north, its longitude is $121^{\circ}20'$ west, and its elevation is 1206 m. It is almost in the lowest part of a saddle-back with a ridge running north-south upwards from the station location to a height of about 200 m above the station at a distance of about 3 km. North of the saddle, the ridges rise in an east-west fashion to heights near 3,000 m at distances ranging from 20 to 80 km.

8.2.5 Marine Surface Data

Weather observations over oceanic areas constitute an important part of the archives at the National Climatic Center. These have been the basis for the Marine Climatic Atlases program of the U.S. Navy since 1950. In some regions the weather is composed of quite distinct weather regimes over the year. Within shorter time periods there may or may not be such distinctness. There may or may not be distinct separation of weather factors between the traveling cyclones and anti-cyclones.

Observations from transient ships offer no real opportunity for time studies. For a particular spot, over time, this may or may not be important. For reasons of time continuity and a measure of pressure gradients, however, an ocean station vessel location was selected. The ocean station vessel is OSV "C" (Charlie) at $52^{\circ}45'$ north latitude and $35^{\circ}30'$ west longitude. The month of February was selected for the years of 1965-1970. The 1200Z observations are used. The elements selected are the surface winds in $\text{m}\cdot\text{s}^{-1}$, the temperatures ($^{\circ}\text{C}$), the sea level atmospheric pressures (mb), and the dew-point temperatures ($^{\circ}\text{C}$). Other elements such as wave height, wave frequency, visibility, cloud, cloud height, precipitation, and other obstructions to vision can be used. However, some of these have a lower bound of zero or are dichotomous, i.e., a yes or no condition. Transformation to near normality

of the distributions of these elements should be made before they are used with this technique.

The input data are not shown. Only the pertinent output data are shown, i.e., the means, variances or standard deviations, the correlation matrices, and the eigenvalue-eigenvector and mixture proportions.

8.2.6 Radiosonde and Rawinsonde Data

The foregoing examples clearly illustrate the usefulness of Wolfe's NORMIX and NORMAP techniques to separate mixed multivariate normal distributions. Two- or higher-dimensional vectors have been used. A radiosonde observation is composed of observations at levels in the atmosphere where significant changes are made from level to level. Also included are observations at mandatory levels so that all stations transmit data for the same levels. These can be used to produce synoptic charts of the data. Ordinarily, these charts can be studied by element and level by level or by changes within and between levels. Also, a complete observation can be studied as one vector in multidimensional space. Only the time and cost limitations of the computers restrict the problem. For example, if the winds, temperatures, dew points, and heights of selected pressure levels are used for 30 levels, an observation may be characterized by a vector of 150 components, i.e., a 150-dimension problem. Extension of this to other levels and features as well as differences level to level such as lapse rates, shears, etc., would extend the vector to still higher dimensions.

As an illustration, radiosonde and rawinsonde observations at Balboa, C.Z., Panama, are used. The 1500Z observations in the month of July for the years 1961-70 were selected. The levels selected were the surface, 950-, 850-, and 700-mb. The data used are winds in $\text{m}\cdot\text{s}^{-1}$, dry bulb and dew-point temperatures in $^{\circ}\text{C}$, and heights in gdkm. Each vector then is a 4 x 5 or 20-dimensional vector. Assumption of both equal and unequal covariance matrices is made. No input data are shown. Only the output data in terms of means, variance (standard deviations), covariance (correlation) matrices, eigenvalue and eigenvector, and mixtures are shown.

8.3 Selected Data

8.3.1 Land-Sea Breeze Data Set

8.3.1.1 Input Information.

- a. San Juan, Puerto Rico
- b. The period of record is October 1-31, 1955, and October 1-3, 1956.
- c. The data are surface winds.
- d. The number of variables is two; these are zonal and meridional wind components.
- e. The number in the sample is 200; the first 100 are from the 0600-1800 local standard time (l.s.t.) while the second 100 are from the 1200-1400 l.s.t.
- f. The minimum number to be accepted into a cluster is three.

- g. The null hypotheses are made that $(k + 1)$ clusters are not significantly different from the k clusters. The decision probability level selected is 0.01. Rejection of the hypothesis then permits the assumption of $(k + 1)$ clusters.
- h. The first 40 two-dimensional vector entries are set up as the 40 means of 40 separate and individual clusters. These are 40 points in two dimensions.
- i. Two assumptions are made. The first is the equality of covariances while the second is the non-equality of covariances.

8.3.1.2 Tables - Output Information. The program computes the necessary statistics for the three-cluster versus two-cluster including the discriminant functions and classification of entries into the clusters.

The output statistics are now shown in tables 1 and 2 for the two-cluster and the three-cluster even though the hypothesis that three types and two types were not significantly different was not rejected.

8.3.1.3 Figures and Discussion. San Juan, Puerto Rico, was selected because this is a known land-sea breeze effect location. The hour groups of 0600-0800 and 1200-1400 were selected when the land-sea breeze effect would most likely be operating. The periods of October 1-31, 1955, and October 1-3, 1956, were selected so as to provide 100 observations of the wind at each of the hour groups. The wind direction and speed are used to provide the zonal and meridional components of the wind. The first variable is the x-component or zonal (west-east) component of the wind while the second variable is the y-component or meridional (south-north) of the wind. Minus signs indicate components from the east or north.

The procedural techniques of the Wolfe NORMIX-NORMAP electronic computer routines (1971) worked well in this case.

Figure 8a shows a two-dimensional representation for the entire group of 200 observations. This may or may not be an adequate representation. No plot is made of data to visually assess the fit of the mathematical elliptical form. This form is obtained under the assumption that there is one single group. Here it is realized that such an assumption is not valid, yet it is shown for illustrative purposes. Under the assumption of two groups, the land and the sea breezes, and the equality of covariances, the breakout into two groups is shown. Elliptical error probable (e.e.p.) ellipses are shown. These are the 0.50 probability ellipses. When the axes of the ellipses are equal, the ellipses are circles and the 0.50 probability circle is called the circular error probable (c.e.p.). Figure 8b then shows a breakout into three clusters. The assumption of equality of covariances is exemplified by the fact that the ellipses all have the same shape, size, and orientation. This is not the case as shown in figures 8c and 8d where the covariances are assumed to be different. In these figures, the shape, size, and orientation may be different from ellipse to ellipse. The authors believe that assumption of unequal covariances is valid. The component means of the two-cluster configuration under the assumption of equal versus unequal covariance matrices are in the first type -0.4620 versus -0.5764 and 1.4574 versus 1.3866 and in the second type -4.7149 versus -4.7913 and -2.3420 versus -2.4596 $\text{m}\cdot\text{s}^{-1}$. The proportions assigned to the

Table 1 Surface wind statistics for San Juan, Puerto Rico, October 1-31, 1955, and October 1-3, 1956, 0600-0800 and 1200-1400 l.s.t. The sample size is 200; 100 from each hour group. The assumption is that the covariance matrices are the same in any breakdown. The variables are (1) the zonal component of the wind, positive from the west, and (2) the meridional component of the wind, positive from the south.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 2 to 1 types 32.478160 (a) 63.82 (a) 0.00000000
(b) 3 to 2 types 2.6044397 (b) 5.10 (b) 0.27672217*

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level. Minus signs indicate the wind components are from the north or east.

Characteristics of the whole sample (units are $m \cdot s^{-1}$)

	1	2	1	2
Means	-2.1114	-0.1054		
Std. Dev.	2.9022	2.3077		
Corr.	1.0000	0.4922		
	0.4922	1.0000		
Within Group	Two Clusters		Three Clusters	
Std. Dev.	2.0108	1.3527	1.8937	1.3288
Corr.	1.0000	-0.2267	1.0000	-0.1766
	-0.2267	1.0000	-0.1766	1.0000
Clusters	Two Clusters		Three Clusters	
Lambda (Proportion)	Means		Means	
(1) 0.5887	-0.4620	1.4574	(1) 0.5884	-0.4572
(2) 0.4113	-4.7149	-2.3420	(2) 0.3912	-4.9544
			(3) 0.0204	-0.1962
Discriminant Functions				
Variable (1)	0.3621			0.3785
Variable (2)	0.6432			0.6286
				-0.4352

Table 2 Surface wind statistics for San Juan, Puerto Rico, October 1-31, 1955, and October 1-3, 1956, 0600-0800 and 1200-1400 1.s.t. The sample size is 200; 100 from each hour group. The assumption is the covariance matrices are not equal. The variables are (1) the zonal component of the wind, positive from the west, and (2) the meridional component of the wind, positive from the south.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 2 to 1 types	37.528096	(a) 73.74	(a) 0.000000000
(b) 3 to 2 types	12.212320	(b) 23.94	(b) 0.00777169
(c) 4 to 3 types	0.80050798	(c) 1.56	(c) 0.99871828*

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level. Minus signs indicate the wind components are from the north or east.

Characteristics of the whole sample (units are $m \cdot s^{-1}$)

	1	2
--	---	---

Means	-2.2114	-0.1054
Std. Dev.	2.9022	2.3077
Corr.	1.0000	0.4922
	0.4922	1.0000

Characteristics of Type 1

	Proportion 0.612	Proportion 0.635
Means	-0.5764	1.3866
Std. Dev.	2.1589	1.3792
Corr.	1.0000	-0.0514
	-0.0514	1.0000

Eigen-Values	Vectors	Eigen-Values	Vectors
4.6695	0.9985	4.6571	0.9999
1.8937	-0.0552	2.1552	0.0118
			0.9999

(Table 2 continued)

<u>Characteristics of Type 2</u>		<u>Proportion 0.388</u>	<u>Proportion 0.255</u>
Means		-4.7913	-5.3239
Std. Dev.		1.8668	1.0169
Corr.		1.0000	1.0000
		-0.4905	-0.5600
		1.0000	1.0000
Eigen-			
Values			
4.0697			
1.0875			
	Eigen-		
	Values		
	2.1371		
	0.5485		
		Vectors	
		-0.5529	0.8333
		0.8333	0.5529
<u>Characteristic of Type 3</u>		<u>Proportion 0.110</u>	
Means		-4.0398	-3.4767
Std. Dev.		2.6959	0.6627
Corr.		1.0000	-0.8456
		-0.8456	1.0000
	Eigen		
	Values		
	7.5874		
	0.1198		
		Vectors	
		0.9784	0.2068
		-0.2068	0.9784

Figure 8 San Juan, Puerto Rico, surface wind distributions; period of record October 1-31, 1955, and October 1-3, 1956, hours 0600 and 0800 and 1200-1400 local standard time; $n = 200, 100$ from each period; units, $m \cdot s^{-1}$; separation shown for two and three types with covariances assumed equal and then unequal.

- (a) Total and two types, equal covariance matrices.
- (b) Three types, equal covariance matrices.
- (c) Total and two types, unequal covariance matrices.
- (d) Three types, unequal covariance matrices.

first type are 0.5887 versus 0.612 and the second type are 0.4113 versus 0.388. The comparisons indicate similarity. Under the assumption of unequal covariance matrices, the second type shown above (though losing a part to the first) then breaks down into two clusters which are proportionally 0.255 and 0.110.

Quite clearly the unequal covariance ensemble breaks down into at least three clusters, a south-southeast wind and an east-northeast wind, the early morning wind versus the noon wind, and the land breeze versus the sea breeze. The mean wind is from the east by southeast with components -2.2114 and $-0.1054 \text{ m}\cdot\text{s}^{-1}$. This is the easterly trade though it is somewhat less than the average of $3.5 \text{ m}\cdot\text{s}^{-1}$ for that region [Crutcher, Wagner, and Arnett (1966)].

The land-sea breeze situation illustrates the output of the technique in tabular form and in two-dimensional illustrations with ellipses. Two assumptions are shown. First, there is the assumption that the underlying statistics have the same covariance matrix; i.e., the underlying physical bases are operating the same. Second, there is the assumption that the underlying statistics have differing covariance matrices; i.e., there is some reason to believe that the underlying physical bases operate differently. Whether the assumptions are correct in any particular case is not known for these are not tested here. Both assumptions are made. The results are presented. The reader can make his own assessment and choose the assumption he pleases. However, statistical tests are used to reach decisions as to whether there are two groups or less, three groups or two, etc. As with all other examples, the decision level chosen to work with is the probability level of 0.01. The null hypothesis is that the distribution of $(k + 1)$ groups is not different from (k) groups; that is, rejection of the null hypothesis that there are $(k + 1)$ groups rather than (k) groups is sought.

Output or input data in computer format and the intermediate steps are not shown in this example. For these the reader is referred to Wolfe (1971b) whose electronic computer program is adapted for use here. The mid-latitude tropospheric wind data set (paragraph 8.3.3) does contain some of the intermediate steps.

8.3.2 Tropical Stratospheric Wind Data Set

8.3.2.1 Input Information.

- a. Canton Island, South Pacific, (U.S.A. and Great Britain)
- b. The periods of record are the months of July and January, 1954-1964, and for 1957-1967.
- c. The data are stratospheric winds, heights of pressure surfaces, and temperatures. The pressure levels are 50-, 30-, 20-, and 10-mb.
- d. The number of variables is two for the first period and four for the second. These are for the first period, zonal and meridional components of the wind, positive from the west and south. For the second period, these are the winds, heights, and temperatures. The units are $\text{m}\cdot\text{s}^{-1}$, m, and $^{\circ}\text{C}$.
- e. The number in the samples vary from level to level because at

times the balloons failed to reach the higher altitudes. The numbers are 263, 244, and 162, respectively, for the first three levels above. Though the 10-mb data were processed, only one cluster was determined. The results are not shown.

- f. The minimum number to be accepted into a cluster is three for the first period and five for the second.
- g. The null hypotheses are made that $(k + 1)$ clusters are not significantly different from the k clusters. The decision probability level selected is 0.01. Rejection of the hypothesis then permits the assumption of $(k + 1)$ clusters.
- h. The first 40 two-dimensional vector entries in each level are set up as the 40 means of 40 separate and individual clusters. These are 40 points in two dimensions.
- i. Two assumptions are made. The first assumption is the equality of covariances. The second assumption is the non-equality of covariances.

8.3.2.2 Tables for Wind Configurations (1954-1964). Tables 3 through 10 provide the output data in tabular form provided by the Wolfe (1971b) NORMIX-NORMAP computer routine for the first period. An asterisk indicates the rejection of the null hypothesis that $(k + 1)$ types are not significantly different from the (k) types. Thus, under the following assumption of equal then unequal covariances and at the 0.01 probability level, the greatest number of clusters (types) for the wind distributions 1954-1964 is indicated below:

	July		January	
	Equal	Unequal (Covariances)	Equal	Unequal
50-mb	2	3	2	2
30-mb	at least 4	3	1	1
20-mb	3	2	1	1
10-mb	1	1	1	1

The statistics for the single clusters are not shown.

Table 3

Upper wind statistics for Canton Island, U.S.A. and U.K. Units are $\text{m}\cdot\text{s}^{-1}$. The period of record is the months of July during 1954-1964. The pressure level is 50-mb. Sample size is 263. The assumption is that the covariance matrices are the same. The variables are (1) the zonal wind component, positive from the west, and (2) the meridional wind component, positive from the south.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 2 to 1 types	27.165587	(a) 53.61	(a) 0.00000000
(b) 3 to 2 types	61.844617	(b) 12.18	(b) 0.1605621*

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level. Minus signs indicate the wind components are from the north or east.

Characteristics of the whole sample.

	1	2	1	2
--	---	---	---	---

Means	1.1251	-0.5502		
Std. Dev.	14.0700	3.5679		
Corr.	1.0000	-0.0300		
	-0.0300	1.0000		

Within Group	Two Clusters	Three Clusters
Std. Dev.	7.9448	3.5109
Corr.	1.0000	-0.3185
	-0.3185	1.0000
Type Lambda	Means	Means
1) 0.6361	9.9084	10.0418
2) 0.3639	-14.2256	-1.6823
		-15.8612
		0.0198
		-6.7786
		-0.5528
Discriminant Functions		
Variable 1	0.1319	-0.0126
Variable 2	0.1270	0.3031

Table 4

Upper wind statistics for Canton Island, U.S.A. and U.K. Units are $\text{m}\cdot\text{s}^{-1}$. The period of record is the months of July during 1954-1964. The pressure level is 30-mb. Sample size is 244. The assumption is that the covariance matrices are the same. The variables are (1) the zonal wind component, positive from the west, and (2) the meridional wind component, positive from the south.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a)	2 to 1 types	71.531512	(a)	141.01	(a)	0.00000000
(b)	3 to 2 types	9.7739748	(b)	19.23	(b)	0.00070904
(c)	4 to 3 types	19.060173	(c)	37.42	(c)	0.00000015

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level. Minus signs indicate the wind components are from the north or east.

Characteristics of the whole sample.

	1	2	1	2	1	2
--	---	---	---	---	---	---

Means	-3.8760	-0.3253
Std. Dev.	19.5398	4.0676
Corr.	1.0000	0.0385
	0.0385	1.0000

Within Group	Two Clusters		Three Clusters		Four Clusters	
Std. Dev.	7.6585	4.0675	7.6400	3.6940	5.3554	3.6933
Corr.	1.0000	0.1121	1.0000	0.0937	1.0000	0.1113
	0.1121	1.0000	0.0937	1.0000	0.1113	1.0000
Type Lambda	Means		Means		Means	
1) 0.4827	-22.4853	-0.3004	1) 0.4828	-22.4797	1) 0.4394	-0.3111
2) 0.5173	13.4891	-0.3485	2) 0.0148	18.5728	2) 0.1584	-0.5910
			3) 0.5024	13.3385	3) 0.0147	13.5317
				-0.7669	4) 0.3875	-0.7581
Discriminant Functions						
Variable 1	-0.1314		0.1315	0.0001	0.1879	-0.0003
Variable 2	-0.0283		-0.0257	0.2707	-0.0299	0.2708

Table 5

Upper wind statistics for Canton Island, U.S.A. and U.K. Units are $\text{m}\cdot\text{s}^{-1}$. The period of record is the months of July during 1954-1964. The pressure level is 20-mb. Sample size is 162. The assumption is that the covariance matrices are the same. The variables are (1) the zonal wind component, positive from the west, and (2) the meridional wind component, positive from the south.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 2 to 1 types	71.800990	(a) 140.50	(a) 0.00000000
(b) 3 to 2 types	13.764612	(b) 26.85	(b) 0.00002132
(c) 4 to 3 types	-0.000039	(c) -0.00	(c) 1.000000*

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level. Minus signs indicate the wind components are from the north or east.

Characteristics of the whole sample.

	1	2	1	2
--	---	---	---	---

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Means	-10.0693	-0.1189
Std. Dev.	19.9881	3.3763
Corr.	1.0000	-0.0035
	-0.0035	1.0000

Within Group

	Two Clusters	Three Clusters	Four Clusters
Std. Dev.	6.6696	3.3737	5.1788
Corr.	1.0000	0.1002	1.0000
	0.1002	1.0000	0.2400

Type Lambda

1) 0.4021	Means	1) 0.1455	1) 0.1453	1) 0.1453
2) 0.5979	12.9050	2) 0.2780	2) 0.2783	2) 0.2783
	-25.5230	3) 0.5764	-26.1833	3) 0.5060

Means	1) 0.1453	1) 0.1453	1) 0.1453
	2) 0.2783	2) 0.2783	2) 0.2783
	3) 0.5060	3) 0.5060	3) 0.5060
	4) 0.0705	4) 0.0705	4) 0.0705

5.1797	3.2877	5.1797	3.2877
1.0000	0.2400	1.0000	0.2400
0.2400	1.0000	0.2400	1.0000

Discriminant Functions

Variable 1	0.1507	0.1989	0.0037	0.1988	0.0037
Variable 2	-0.0339	-0.0808	-0.0808	-0.0808	0.3027

Table 6 Upper wind statistics for Canton Island, U.S.A. and U.K. Units are $m \cdot s^{-1}$. The period of record is the months of July during 1954-1964. The pressure level is 50-mb. Sample size is 263. The assumption is that the covariance matrices are not the same. The variables are (1) the zonal wind component, positive from the west, and (2) the meridional wind, positive from the south.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a)	2 to 1 types	30.173770	(a)	59.54	(a)	0.000000000
(b)	3 to 2 types	14.343754	(b)	28.25	(b)	0.00164537
(c)	4 to 3 types	7.0614291	(c)	13.88	(c)	0.17847916*

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level.

Characteristics of the whole sample.

	1	2	1	2
Means	1.1251	-0.5502		
Std. Dev.	14.0700	3.5679		
Corr.	1.0000	-0.0300		
	-0.0300	1.0000		

Two Types With Proportions

	Type 1 (0.608)		Type 2 (0.392)		
Means	10.3910	0.2223	-13.2905	-1.4408	
Std. Dev.	7.1747	3.4534	9.2004	3.5603	
Corr.	1.0000	-0.2728	1.0000	-0.4408	
	-0.2728	1.0000	-0.4408	1.0000	
Eigen-Values	Vectors		Vectors		
52.6002	0.9865	0.1639	0.9819	0.1896	
10.8024	-0.1639	0.9865	-0.1896	0.9819	

Table 6 (continued)

Three Types With Proportions

		Type 1 (0.373)		Type 2 (0.338)	
		1	2	1	2
Means		10.7216	0.2663	4.9486	-2.2004
Std. Dev.		5.6926	3.5614	11.3340	3.9597
Corr.		1.0000	-0.6196	1.0000	0.4364
		-0.6196	1.0000	0.4364	1.0000
Eigen- Values	Vectors				
38.5133		0.8993	0.4373	0.9861	-0.1664
6.5754		-0.4373	0.8993	0.1664	0.9861

Type 3 (0.287)

Means		-15.9766	0.3409
Std. Dev.		8.1386	2.0971
Corr.		1.0000	0.0347
		0.0347	1.0000

Eigen- Values	Vectors		
66.2424		1.0000	-0.0096
4.3924		0.0096	1.0000

Four Types With Proportions

		Type 1 (0.026)		Type 2 (0.177)	
		1	2	1	2
Means		14.3882	6.6000	-7.4919	-4.1879
Std. Dev.		13.1061	2.4964	8.4065	3.5182
Corr.		1.0000	-0.9881	1.0000	-0.3777
		-0.9881	1.0000	-0.3777	1.0000
Eigen- Values	Vectors				
177.8597		0.9827	0.1851	0.9833	0.1820
0.1428		-0.1851	0.9827	-0.1820	0.9833

Table 6 (continued)

Four Types With Proportions

	Type 3 (0.553)		Type 4 (0.245)	
Means	10.8686	-0.2486	-16.0757	0.6447
Std. Dev.	6.1573	3.1438	8.4811	1.8780
Corr.	1.0000	-0.3513	1.0000	0.1235
	-0.3513	1.0000	0.1235	1.0000
Eigen- Values	Eigen- Values		Eigen- Values	
39.4747	71.9849		71.9849	
8.3209	3.4705		3.4705	
	Vectors		Vectors	
	0.9746	0.2240	0.9996	-0.0287
	-0.2240	0.9746	0.0287	0.9996

Table 7

Upper wind statistics for Canton Island, U.S.A. and U.K. Units are $\text{m}\cdot\text{s}^{-1}$. The period of record is the months of July during 1954-1964. The pressure level is 30-mb. Sample size is 244. The assumption is that the covariance matrices are not the same. The variables are (1) the zonal wind component, positive from the west, and (2) the meridional wind, positive from the south.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 2 to 1 types	85.675491	(a) 168.89	(a) 0.00000000
(b) 3 to 2 types	18.426554	(b) 36.25	(b) 0.00007627
(c) 4 to 3 types	9.2238620	(c) 18.11	(c) 0.05317713*

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level.

Characteristics of the whole sample.

	1	2	1	2
Means	-3.8760	-0.3253		
Std. Dev.	19.5398	4.0676		
Corr.	1.0000	0.0385		
	0.0385	1.0000		

Two Types With Proportions

	Type 1 (0.435)		Type 2 (0.565)	
Means	-24.1471	-0.3279	11.7512	-0.3233
Std. Dev.	4.8449	3.5602	9.8518	4.4192
Corr.	1.0000	0.1552	1.0000	0.0754
	0.1552	1.0000	0.0754	1.0000
Eigen-Values	Vectors		Vectors	
24.1008	0.9736	-0.2281	0.9991	-0.0422
12.0473	0.2281	0.9736	0.0422	0.9991

Three Types With Proportions

Type 3 (0.347)

Four Types With Proportions

Table 7 (continued)

Four Types With Proportions

	Type 3 (0.214)		Type 4 (0.376)	
Means	-5.0779	1.6368	15.3429	-0.4423
Std. Dev.	14.5772	6.1490	6.6104	2.8524
Corr.	1.0000	0.3582	1.0000	0.1930
	0.3582	1.0000	0.1930	1.0000
Eigen- Values	Eigen- Values		Eigen- Values	
218.2089	44.0661		44.0661	
32.0958	7.7677		7.7677	
	Vectors		Vectors	
	0.9845	-0.1752	0.9949	-0.1007
	0.1752	0.9845	0.1007	0.9949

Table 8

Upper wind statistics for Canton Island, U.S.A. and U.K. Units are $\text{m}\cdot\text{s}^{-1}$. The period of record is the months of July during 1954-1964. The pressure level is 20-mb. Sample size is 162. The assumption is that the covariance matrices are not the same. The variables are (1) the zonal wind component, positive from the west, and (2) the meridional wind, positive from the south.

Logarithm of the Likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a)	2 to 1 types	84.858770	(a)	166.05	(a)	0.00000000
(b)	3 to 2 types	4.5207028	(b)	8.82	(b)	0.54944347*

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level.

Characteristics of the whole sample.

	1	2	1	2
1				
2				

Means	-10.0693	-0.1189
Std. Dev.	19.9881	3.3763
Corr.	1.0000	-0.0035
	-0.0035	1.0000

Two Types With Proportions

	Type 1 (0.428)	Type 2 (0.572)
Means	11.6050	-26.3059
Std. Dev.	9.1353	4.5555
Corr.	1.0000	1.0000
	-0.1093	0.1934
		1.0000
		-0.1058
		3.1775
		0.1934
		1.0000

Eigen- Values	Vectors	Eigen- Values	Vectors
83.6403	0.9987	21.4431	0.9709
12.9522	-0.0513	9.4058	0.2395
			-0.2395
			0.9709

Table 8 (continued)

Three Types With Proportions

	Type 1 (0.290)		Type 2 (0.141)	
	1	2	1	2
Means	8.0972	0.2056	18.5108	-0.7634
Std. Dev.	19.1706	3.5413	4.2761	3.7393
Corr.	1.0000	-0.2219	1.0000	0.5415
	-0.2219	1.0000	0.5415	1.0000
Eigen- Values	Eigen- Values		Eigen- Values	
84.8178	Vectors		Vectors	
11.8221	0.9951	0.0992	0.7878	-0.6160
	-0.0992	0.9951	0.6160	0.7878

Table 9

Upper wind statistics for Canton Island, U.S.A. and U.K. Units are $\text{m}\cdot\text{s}^{-1}$. The period of record is the months of January 1954-1964. The pressure level is 50-mb. Sample size is 168. The assumption is that the covariance matrices are the same. The variables are (1) the zonal wind component, positive from the west, and (2) the meridional wind component, positive from the south.

Logarithm of the likelihood: chi-square with 4 d.f.; probability of null hypothesis.

(a)	2 to 1 types	10.845283	(a)	21.24	(a)	0.00028396
(b)	3 to 2 types	5.498598	(b)	10.74	(b)	0.02970488*

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level.

Characteristics of the whole sample.

	1	2	1	2
--	---	---	---	---

Means	-2.4529	0.6009		
Std. Dev.	12.2183	3.2754		
Corr.	1.0000	0.0280		
	0.0280	1.0000		

Within Group

Std. Dev.	6.9054	3.2747	6.8723	2.7151
Corr.	1.0000	0.0815	1.0000	0.0637
	0.0815	1.0000	0.0637	1.0000

Type Lambda

1) 0.4538	-13.5119	0.6795	1) 0.4424	-13.7874	0.5320
2) 0.5462	6.7344	0.5356	2) 0.4393	6.2786	1.9283
			3) 0.1183	7.5054	-4.0685

Discriminant Functions

Variable 1	0.1453	0.1458	0.0028
Variable 2	-0.0295	-0.0305	0.3678

Table 10

Upper wind statistics for Canton Island, U.S.A. and U.K. Units are $\text{m}\cdot\text{s}^{-1}$. The period of record is the months of January, 1954-1964. The pressure level is 50-mb. Sample size is 168. The assumption is that the covariance matrices are unequal. The variables are (1) the zonal wind component, positive from the west, and (2) the meridional wind component, positive from the south.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a)	2 to 1 types	20.949167	(a)	41.03	(a)	0.00001117
(b)	3 to 2 types	3.286572	(b)	6.42	(b)	0.77912911*

Lambda is the proportional amount assigned to each type. An asterisk indicates the rejection of the null hypothesis at the 0.01 level.

Characteristics of the whole sample.

	1	2	1	2
Means	-2.4529	0.6009		
Std Dev.	12.2183	3.2754		
Corr.	1.0000	0.0280		
	0.0280	1.0000		

Two Types With Proportions

	Type 1 (0.627)		Type 2 (0.373)	
Means	-9.5312	1.0884	9.4278	-0.2174
Std. Dev.	9.6039	2.9303	4.4498	3.6407
Corr.	1.0000	0.3176	1.0000	0.2171
	0.3176	1.0000	0.2171	1.0000
Eigen-Values	Vectors		Vectors	
93.1791	0.9945	-0.1051	0.9169	-0.3992
7.6426	0.1051	0.9945	0.3992	0.9169

Three Types With Proportions

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8.3.2.3 Figures and Discussion for Wind Configurations (1954-1964). Figures 9 through 19 show various combinations of the 0.50 probability ellipses as well as the wind plot diagrams to which these pertain. For example, figure 10a gives the 0.50 probability ellipse for the total group and a breakout into two groups. This 0.50 probability ellipse is the ellipse estimated to contain one-half of the winds from the cluster (type) to which it pertains. Figures 10b and 10c are similar illustrations for the 30- and 20-mb levels. Comparison between the results of the assumptions of equal and unequal covariance matrices are possible. Under the assumption of equal covariance matrices, the size, shape, and orientation of the cluster breakout are the same but not necessarily the same as the complete (total) distribution. That is, the flux indicated by the total may be quite different from that within the individual clusters though the individual clusters imply that the flux across the pertinent air-stream in each cluster is the same as in another. Under the assumption of unequal covariance matrices, the various clusters may show different size, scale, shape, and orientation.

In addition, other figures in the above group permit comparison from level to level of the totals and of the various types. These tabular and graphical representations quite clearly indicate that elongated elliptical distributions computed from ordinary bivariate normal statistical routines should be backed up by plot or scatter diagrams. If a plot is not available, then a wind rose of some type should be available for examination. The wind rose usually referred to is the WBAN-120 (revised) available from the National Climatic Center (1958). This is based on the work by Crutcher (1957). A decision can then be made as to whether the computed bivariate statistics are valid. These assume a unimodal bivariate distribution. In the output statistics of WBAN-120 revised format, for example, a ratio of the major to minor axes in excess of four should indicate a need for study to see whether clustering is evident. See table 9 for such a comparison. Both plots and analytic procedures such as are available in discriminant function analysis, factor analysis, or principal component analysis or other clustering routines can be used.

"A priori" considerations may also indicate that clustering techniques should be used. That is, previous research may indicate that the use of the total distribution as a unimodal bivariate (or multivariate) distribution is unwarranted and that probabilistic statements from such a model may be erroneous.

The previous example of the land-sea breeze effect at San Juan, Puerto Rico, and the present example of the stratospheric winds at Canton Island are "a priori" types.

The procedures also are helpful in establishing the quality assurance of the data. Clusters composed of only one or a few observations and far distant from the main cluster or clusters may be examined for validity. This feature will be discussed in a later section. In particular, in tables 6 and 7 and in figures 17a and 17b for the four-cluster breakout, the very low proportions in one of the clusters and the very large ratio of the major axis to minor axis indicate that these groups may be suspect for one or more reasons. The non-rejection of the null hypothesis, four clusters versus the three clusters, implies that the four-cluster breakout was not significantly different from

the three-cluster breakout. Thus, the investigator can simply stay with the three-cluster configuration while examining the isolated questionable groups indicated in the four-cluster configuration.

The equatorial stratospheric winds situation illustrates the output of the technique in tabular form and in two-dimensional depiction with 0.50 ellipses. Two assumptions are used. First, there is the assumption that the underlying distributions are the same, i.e., that the statistics have the same covariance matrices (that the underlying bases are the same). Second, there is the alternative assumption that the statistics represent different physical and dynamic situations, i.e., the covariance matrices are not the same. There is some reason to believe that the underlying physical bases operate differently. The reader can select the assumption that best fits his knowledge and experience.

Under the assumption of equal covariance matrices, one of the outputs is a plot of discriminant function assignments, such as one versus two or one versus three, etc. This is a computer tabulation type of two-dimensional plotting with each individual data point carrying the number of the cluster type to which it is assigned by the classification (discrimination) procedure. Figures 13, 14, and 15 illustrate the output for Canton Island during July at the 50-mb and 30-mb levels. Figure 13 for the July 50-mb level shows the print plot of discriminant function 1 and 2 where three types are computed. Figures 14 and 15 for the July 30-mb level show the print plots of discriminant functions 1 and 2 for the first three-type dissection and then four-type dissection. It appears here that type 2 of 3 becomes type 3 of 4 while type 3 of 2 breaks down into types 2 and 4 of 4.

Figure 9 Canton Island, U.S.A. and U.K.; upper wind distribution plots; period of record, the months of July 1954-1964; pressure levels (a) 50-, (b) 30-, (c) 20-, and (d) 10-mb; units, $\text{m}\cdot\text{s}^{-1}$.

Figure 9 (continued)

Figure 11 Canton Island, U.S.A. and U.K.; upper wind distributions; period of record July 1954-1964; pressure levels, 50-, 30-, and 20-mb; units, $\text{m}\cdot\text{s}^{-1}$; wind plot shown in figure 9; separation shown for three types, assumption of equal covariance matrices; elliptical error probable, (e.e.p.), 0.50 probability ellipses.

- (a) Three types at 50-mb, $n = 263$.
- (b) Three types at 30-mb, $n = 244$.
- (c) Three types at 20-mb, $n = 162$.

Figure 12 Canton Island, U.S.A. and U.K.; upper wind distributions; period of record July 1954-1964; pressure level, 30-mb; units, $\text{m}\cdot\text{s}^{-1}$; $n = 244$; wind plot shown in figure 9; separation shown for four types, assumption of equal covariance matrices; elliptical error probable, (e.e.p.), 0.50 probability ellipses.

SAMPLE # CANTON ISLAND
JULY 50 MB
X AND Y WIND COMPONENTS

Figure 13 Plot of Discriminant Functions 1 versus 2 for Canton Island, U.S.A. and U.K.; July winds shown in figure 9 for winds shown in figure 11, based on assumption of equal covariance matrices; period of record 1954-1964; elliptical error probable, (e.e.p.), 0.50 probability ellipses.

- (a) Three types at 50-mb.
- (b) Three types at 30-mb.
- (c) Four types at 30-mb.
- (d) Three types at 20-mb.

SAMPLE ■ CANTON ISLAND
JULY 30 MB
X AND Y WIND COMPONENTS

Figure 13 (continued)

SAMPLE # CANTON ISLAND
JULY 30 MB
X AND Y WIND COMPONENTS

DISCRIMINANT FUNCTION 1

Figure 13 (continued)

SAMPLE ■ CANTON ISLAND
JULY 20 MB
X AND Y WIND COMPONENTS

DISCRIMINANT FUNCTION 1

Figure 13 (continued)

Figure 14 Canton Island, U.S.A. and U.K.; upper wind distributions; period of record July 1954-1964; pressure levels, 50-, 30-, and 20-mb; units, $\text{m}\cdot\text{s}^{-1}$; $n = 263, 244,$ and $162,$ respectively, assumption of unequal covariance matrices; elliptical error probable, (e.e.p.), 0.50 probability ellipses.

- (a) Total for the three levels.
- (b) Type 1 for the three levels.
- (c) Type 2 for the three levels.

Figure 16 Canton Island, U.S.A. and U.K.; upper wind distributions; period of record July 1954-1964; pressure levels, 50-, 30-, and 20-mb; units, $\text{m}\cdot\text{s}^{-1}$; $n = 263, 244, \text{ and } 162$, respectively; separation shows three types, assumption of unequal covariance matrices; elliptical error probable, (e.e.p.), 0.50 probability ellipses.

- (a) Three types at 50-mb.
- (b) Three types at 30-mb.
- (c) Three types at 20-mb.

Figure 17 Canton Island, U.S.A. and U.K.; upper wind distributions; period of record July 1954-1964; pressure levels, 50- and 30-mb; units, $\text{m}\cdot\text{s}^{-1}$; $n = 263$ and 244 , respectively; separation shows four types, assumption of unequal covariance matrices; elliptical error probable, (e.e.p.), 0.50 probability ellipses.

- (a) Four types at 50-mb.
- (b) Four types at 30-mb.

Figure 18 Canton Island, U.S.A. and U.K.; upper wind distribution plots; period of record January 1954-1964; pressure levels, (a) 50-, (b) 30-, (c) 20-, and (d) 10-mb; units, $\text{m}\cdot\text{s}^{-1}$.

Figure 18 (continued)

Figure 19 Canton Island, U.S.A. and U.K.; upper wind distributions; period of record January 1954-1964; pressure level, 50-mb; wind plot shown in figure 18; units, $\text{m}\cdot\text{s}^{-1}$; separation shows for two and three types with assumption of equal then unequal covariance matrices; elliptical error probable, (e.e.p.), 0.50 probability ellipses.

- (a) Total and two types with equal covariances.
- (b) Total and two types with unequal covariances.
- (c) Three types with equal covariances.
- (d) Three types with unequal covariances.

JAN 50MB 3 TYPES

CANTON IS. SAME COVAR.

JAN 50MB. 3 TYPES

CANTON IS. DIFF COVAR.

Figure 19 (continued)

8.3.2.4 Tables and Discussions for Height, Temperature and Wind Configuration (1957-1967). The foregoing sections discussed wind configurations only for the 50-, 30-, 20-, and 10-mb levels for January and July 1954-1964. The height, temperature, and wind configurations now are discussed for the 30-mb level during the months of January and July 1957-1967.

Veryard and Ebdon (1961b) in their study of the quasi-biennial oscillation with tropical stratospheres reported that the westerly regimes were warmer than the easterlies. There was a slight lag in the temperatures behind the wind changes. Most of the data examined were for the 80- to 50-mb levels. Substantiating this feature for July but not for January are the statistics presented in tables 11 and 13 for the 30-mb level. Here, the temperature for the westerlies and easterlies was -56.8 and -56.7 during January and -51.0 and -55.0 during July. In a further breakdown, not illustrated here, one cluster in January has a temperature of -59.1 though the westerly component for the cluster is only $+4.2 \text{ m}\cdot\text{s}^{-1}$.

Tables 11 and 13 have been assembled differently from previous material. This permits a better reference for this discussion. For example, there are essentially only three clusters for the January clustering and only four clusters for the July clustering. In both months, except for a fourth cluster in July, the total sample breaks down into westerly regimes with slightly greater heights than the easterly regimes; but while there are no temperature differences in January, there are differences in July.

In January, the easterly cluster characteristics remain essentially fixed in all characteristics while the more variable westerly cluster breaks down into two clusters and then three. The last breakdown is not significant.

During July, the easterly cluster characteristics remain essentially fixed throughout subsequent breakdown of the distribution. The westerly group breaks into three significantly different groups. The difference between the most westerly group and the first easterly group is $41 \text{ m}\cdot\text{s}^{-1}$.

Examination of the standard deviations is revealing during both months. The ratios of the major axis to minor axis deviation ranges from about five for the total sample to two for the clusters.

A look at tables 12 and 14 is interesting. These tables present the correlation coefficients among the four variables, height, temperature, zonal wind component, and meridional wind component. It is difficult to assess the degrees of freedom in each case. From the QBO basis, a January or a July is expected to be consistent within itself. Therefore, a maximum of eleven points are available for the 1957-1967 period. The correlation coefficients are expected to be large, large enough to be significant. This appears to be the case for July but not for January except for the heights and the zonal winds.

The heights and the zonal winds appear to be significantly correlated for all cluster breakouts except one in each month where the correlation shifts to the meridional winds. During January there is no definite correlation between heights and temperatures at the 30-mb level. During July there is a definite correlation. This does not indicate that there is no correlation

during January or July between the heights and the temperature structure between the surface and 30 mb. This has not been examined here.

The large correlation coefficients and the large ratio of standard deviations for the total samples may be considered to be a necessary indication but not a sufficient basis for the conclusion of a two-cluster existence. Conversely, small correlation coefficients and ratio of standard deviations of one are indications of only one cluster, i.e., the total sample is a homogeneous cluster.

Table 11 January upper air statistics for Canton Island, U.S.A and U.K. Units are m, °C and m·s⁻¹. The period of record is the months of January during 1957-1967. The pressure level is 30-mb. The sample size is 244. The assumption is that the covariance matrices are not the same. The variables are (1) height, (2) temperature, (3) the zonal wind component, positive from the west, and (4) the meridional wind component, positive from the south.

Logarithm of the likelihood; chi-square with 28 d.f.; probability of null hypothesis.

		(a) 2 to 1 types	85.949209	(a) 168.02	(a) 0.00000000				
		(b) 3 to 2 types	39.701534	(b) 77.45	(b) 0.00000159				
		(c) 4 to 3 types	14.766845	(c) 28.75	(c) 0.42544651*				
		Cluster Proportion	AVG (1) m	AVG (2) °C	AVG (3) m·s ⁻¹	AVG (4) m·s ⁻¹	Std. Dev. m	Std. Dev. m·s ⁻¹	Std. Dev. m·s ⁻¹
1	1.000	23,809.5	-56.7	-5.9	+0.0	83.7	2.5	17.1	3.7
1	0.662	23,842.5	-56.8	+5.3	-0.2	74.6	2.6	7.3	3.8
2	0.338	23,744.8	-56.7	-27.9	+0.4	61.4	2.2	5.6	3.5
1	0.305	23,782.2	-57.3	+0.6	+0.0	53.7	3.2	6.7	4.6
2	0.361	23,892.9	-56.3	+9.0	-0.4	46.7	1.8	5.7	3.0
3	0.334	23,744.1	-56.7	-28.0	+0.4	61.4	2.2	5.5	3.5
1	0.181	23,777.4	-56.2	-0.6	-1.0	31.1	3.4	7.1	4.4
2	0.313	23,894.4	-55.8	+9.0	-0.9	45.6	1.5	5.9	2.8
3	0.172	23,815.2	-59.1	+4.2	+1.9	80.9	1.6	6.1	3.9
4	0.333	23,744.0	-56.7	-28.1	+0.4	61.4	2.2	5.4	3.5

Table 12

January correlation coefficients for data shown in table 11. The variables are (1) height, (2) temperature, (3) zonal wind, and (4) meridional wind at 30 mb. The degrees of freedom (d.f.) are estimated as $n/2$ where n is the number of observations in each cluster except the total sample where the degrees of freedom are estimated as the number of Januarys. The significance level is 0.05 taken from Snedecor and Cochran (1967). An underlined number indicates significance. Correlation coefficients r_{ij} , $i, j = 1, 2, 3, 4$. d.f. $r_{.05}$

Cluster

	r_{11}	r_{12}	r_{13}	r_{14}	r_{22}	r_{23}	r_{24}	r_{33}	r_{34}	r_{44}	d.f.	$r_{.05}$
1	1.000	0.242	<u>+0.612</u>	<u>+0.104</u>	1.000	-0.007	-0.199	1.000	-0.007	1.000	11	0.553
1	1.000	<u>+0.318</u>	<u>+0.304</u>	<u>+0.142</u>	1.000	-0.010	-0.264	1.000	+0.148	1.000	81	0.216
2	1.000	<u>+0.228</u>	<u>+0.351</u>	<u>+0.281</u>	1.000	<u>+0.090</u>	<u>-0.041</u>	1.000	<u>+0.193</u>	1.000	41	0.302
1	1.000	<u>+0.338</u>	-0.087	<u>+0.343</u>	1.000	-0.313	-0.286	1.000	+0.193	1.000	37	0.315
2	1.000	<u>+0.174</u>	-0.335	<u>+0.175</u>	1.000	<u>+0.100</u>	-0.214	1.000	<u>+0.276</u>	1.000	44	0.291
3	1.000	<u>+0.219</u>	<u>+0.341</u>	<u>+0.294</u>	1.000	<u>+0.062</u>	-0.027	1.000	<u>+0.223</u>	1.000	41	0.302
1	1.000	<u>+0.415</u>	-0.001	<u>+0.110</u>	1.000	-0.304	-0.374	1.000	+0.420	1.000	22	0.404
2	1.000	<u>+0.050</u>	-0.298	<u>+0.214</u>	1.000	<u>+0.038</u>	-0.071	1.000	<u>+0.417</u>	1.000	33	0.334
3	1.000	<u>+0.543</u>	<u>+0.052</u>	<u>+0.504</u>	1.000	-0.018	<u>+0.411</u>	1.000	<u>-0.333</u>	1.000	21	0.413
4	1.000	<u>+0.222</u>	<u>+0.338</u>	<u>+0.294</u>	1.000	<u>+0.072</u>	<u>-0.016</u>	1.000	<u>+0.216</u>	1.000	41	0.302

Table 13

July upper air statistics for Canton Island, U.S.A. and U.K. Units are m, °C, and $\text{m}\cdot\text{s}^{-1}$. The period of record is the months of July during 1957-1967. The pressure level is 30 mb. Sample size is 244. The assumption is that the covariance matrices are not the same. The variables are the (1) height, (2) temperature, (3) the zonal wind component, positive from the west, and (4) the meridional component, positive from the south.

Logarithm of the likelihood; chi-square with 28 d.f.; probability of null hypothesis.

		(a) 0.000000000							
		(b) 0.000000011							
		(c) 0.00146371							
		(d) 0.25821093*							
Cluster	Proportion	AVG (1) m	AVG (2) °C	AVG (3) $\text{m}\cdot\text{s}^{-1}$	AVG (4) $\text{m}\cdot\text{s}^{-1}$	Std. Dev. m	Std. Dev. °C	Std. Dev. $\text{m}\cdot\text{s}^{-1}$	Std. Dev. $\text{m}\cdot\text{s}^{-1}$
1	1.000	23,997.3	-52.7	-2.6	0.1	69.7	3.2	19.0	3.7
1	0.581	24,038.9	-51.0	11.9	-0.2	45.5	2.9	9.7	4.2
2	0.419	23,939.5	-55.0	-22.7	0.6	57.0	1.9	5.7	2.9
1	0.473	24,041.2	-51.4	15.4	0.4	45.8	2.6	6.3	3.8
2	0.108	24,028.3	-49.3	-3.7	-3.0	42.3	3.2	7.2	4.8
3	0.419	23,939.7	-55.0	-22.6	0.7	57.3	1.8	5.9	2.9
1	0.201	24,019.2	-50.4	18.2	0.4	32.5	1.9	4.6	4.2
2	0.252	24,056.7	-52.1	14.0	0.5	48.1	2.7	6.1	3.4
3	0.153	24,030.7	-50.5	-5.0	-1.6	44.3	3.7	9.2	5.4
4	0.393	29,934.9	-55.1	-23.0	0.5	54.8	1.8	5.7	2.5
1	0.202	24,019.3	-50.4	18.2	0.4	32.5	1.9	4.5	4.2
2	0.248	24,056.0	-52.1	14.0	0.6	47.8	2.7	6.2	3.4
3	0.138	24,035.8	-50.0	-2.5	-2.6	45.6	3.6	8.1	4.6
4	0.144	23,971.3	-54.9	-19.2	0.5	47.2	1.3	5.9	3.9
5	0.268	23,920.4	-55.1	-24.8	0.8	52.7	2.1	4.4	2.1

Table 14

July correlation coefficients for data shown in table 13. The variables are (1) height, (2) temperature, (3) zonal wind, and (4) meridional wind at 30-mb. The degrees of freedom are estimated as $n/2$ where n is the number of observations in each cluster except the total sample where the degrees of freedom are estimated as the number of Julys. The significance level is 0.05 taken from Snedecor and Cochran (1967). An underlined number indicates significance.

Cluster

	r_{11}	r_{12}	r_{13}	r_{14}	r_{22}	r_{23}	r_{24}	r_{33}	r_{34}	r_{44}	d.f.	$r_{.05}$
1	1.000	<u>+0.602</u>	<u>+0.696</u>	-0.009	1.000	<u>0.538</u>	-0.034	1.000	-0.078	1.000	11	0.553
1	1.000	<u>+0.279</u>	+0.158	+0.116	1.000	-0.044	+0.033	1.000	+0.075	1.000	70	0.232
2	1.000	<u>+0.359</u>	<u>+0.316</u>	+0.066	1.000	-0.020	+0.068	1.000	-0.058	1.000	52	0.269
1	1.000	<u>+0.420</u>	+0.015	+0.136	1.000	+0.349	+0.036	1.000	-0.214	1.000	57	0.257
2	1.000	<u>-0.705</u>	+0.605	-0.057	1.000	<u>-0.174</u>	+0.425	1.000	-0.314	1.000	13	0.514
3	1.000	<u>+0.351</u>	<u>+0.325</u>	+0.074	1.000	-0.047	+0.086	1.000	-0.037	1.000	52	0.269
1	1.000	<u>+0.180</u>	-0.085	+0.379	1.000	-0.277	+0.289	1.000	+0.070	1.000	25	0.381
2	1.000	<u>+0.832</u>	+0.366	<u>-0.005</u>	1.000	<u>+0.446</u>	-0.137	1.000	-0.525	1.000	30	0.349
3	1.000	<u>+0.037</u>	<u>+0.536</u>	+0.034	1.000	+0.130	+0.049	1.000	<u>+0.431</u>	1.000	19	0.433
4	1.000	<u>+0.337</u>	<u>+0.279</u>	-0.065	1.000	+0.049	+0.030	1.000	<u>-0.084</u>	1.000	48	0.276
1	1.000	<u>+0.177</u>	-0.090	+0.375	1.000	-0.275	+0.289	1.000	+0.072	1.000	25	0.381
2	1.000	<u>+0.838</u>	+0.366	<u>-0.004</u>	1.000	<u>+0.461</u>	-0.136	1.000	-0.530	1.000	30	0.349
3	1.000	<u>-0.012</u>	<u>+0.637</u>	+0.076	1.000	<u>-0.034</u>	+0.281	1.000	<u>-0.181</u>	1.000	17	0.456
4	1.000	<u>+0.257</u>	<u>-0.377</u>	+0.244	1.000	-0.553	-0.026	1.000	+0.218	1.000	17	0.456
5	1.000	<u>+0.404</u>	<u>+0.421</u>	+0.047	1.000	+0.128	+0.197	1.000	-0.293	1.000	33	0.335

8.3.3 Mid-Latitude Tropospheric Wind Data Set

8.3.3.1 Input Information.

- a. Rantoul, Illinois, U.S.A., latitude $40^{\circ}18'$ north, longitude $88^{\circ}09'$ west, elevation 227 m.
- b. The period of record is the month of October for the years 1950-1955.
- c. The data are the zonal and meridional components at the 700-, 500-, and 300-mb levels, $\text{m}\cdot\text{s}^{-1}$.
- d. The number of variables is six.
- e. The number in the sample is 503.
- f. The minimum number to be accepted into a cluster is seven, one more than the number of variates.
- g. The null hypotheses are made that $(k + 1)$ clusters are not significantly different from the k clusters. The decision probability level selected is 0.01. Rejection of the hypothesis then permits the assumption of $(k + 1)$ clusters.
- h. The first 40 six-dimensional vector entries are set up as the 40 means of 40 separate and individual clusters. These are 40 points in six dimensions.
- i. The assumption is made that the covariance matrices are not equal.

8.3.3.2 Tables. Table 15 shows the sequential input data with six variates. The variates are the zonal and meridional components of the 700-mb wind, the zonal and meridional components of the 500-mb wind, and the zonal and meridional components of the 300-mb wind in $\text{m}\cdot\text{s}^{-1}$.

Table 16 shows the computer output after the hierarchical grouping and the subsequent grouping into two clusters after 39 iterations. The two clusters with proportions of 0.171 and 0.829 comprise the total set (1.000). First, the characteristics of the total group are shown. This would be the assumption of a unimodal six-variate model. Then the characteristics of the two clusters are provided. Then, through the use of the discriminant function, assignments of each datum to the clusters may be made as shown. The probability of assignment to each cluster is printed. For example, the first datum of table 15 is assigned to cluster 2 with a probability of 0.935 versus 0.065 for cluster 1. The previous section illustrated computer plots for discriminant functions.

Table 17 provides, from the same input data, the output for three clusters rather than two. The number of iterations required in this case is 101. The probability of the null hypothesis being rejected is 0.00000349. The three-cluster configuration is judged to be better than a two-cluster configuration. Therefore, the program continues further to test a four-cluster configuration versus the three-cluster form.

Table 18 provides, again from the same input data, the output for the four-cluster configuration. The initial computing output, i.e., the first estimate of the learning process, is shown as the zero iteration. This set then becomes the initial estimation in the iteration process leading to 101 iterations before the results converged. Please note the changes from the initial estimates

to the final estimates. For example, for the zeroth iteration, the estimate for the 700-mb zonal component in the first cluster is 11.2762 and after 101 iterations the estimate is 11.1041. The respective standard deviations are 5.5488 and 7.2047. Though the means do not appear to be very different, this is almost a forty percent reduction in variance in the cluster.

The probability of rejection of the null hypothesis in the above case is 0.00980137. This is near the decision level of 0.01. Therefore, no further calculations are shown here.

Table 15 A multivariate (6) set of Rantoul, Illinois, October 1950-55, upper wind components, zonal and meridional, at the 700-, 500-, and 300-mb levels. For example, input variables 1 and 2 are the zonal and meridional components, respectively, at the 700-mb level. The units are $\text{m}\cdot\text{s}^{-1}$.

SAMPLE = RANTOUL
OCTDAER
700,500,AND 300MB
X AND Y COMPONENTS

NVBLKS= 6 NSAMPL= 503 DIFE COV MATRIX
MIN CLUSTER SIZE= 7 HYPOTHESIS TEST=0.000
INITIAL KMEANS= 40 TIME LIMIT=90 ITER LIMIT=100
NO. OF TYPES= 1 2 3 4 5 6 0 0 0 0 0 0
STORAGE REQUIREMENT= 111932 OPTIMAL BYTES
DIMENSION A(3991)

ANTICIPATED EXECUTION TIME = 21.91 MINUTES

PROGRAM NORMIX
WOLFE NORMAL MIXTURE ANALYSIS PROCEDURE(1974 REVISION)

INPUT VARIABLES						
SEQ	1	2	3	4	5	6
1	7.071	7.071	2.121	-2.121	5.563	2.248
2	4.298	10.126	1.954	4.603	10.000	0.000
3	-0.000	14.000	4.243	4.243	5.323	-2.344
4	-0.000	10.000	20.398	8.241	8.485	-8.485
5	-0.000	16.000	1.954	4.603	11.046	-4.689
6	5.470	12.887	-0.000	11.000	5.657	5.657
7	4.636	1.873	15.000	0.000	24.107	9.740
8	7.000	0.000	19.331	-8.205	20.229	-50.068
9	11.314	-11.314	21.172	-8.987	31.297	-13.285
10	12.887	-5.470	14.142	-14.142	16.108	-39.869
11	9.192	9.192	-3.126	-7.364	8.285	-3.517
12	2.828	2.828	4.636	1.873	2.828	2.828
13	3.536	3.536	2.735	6.444	7.778	7.778
14	-1.124	2.782	2.735	6.444	7.778	7.778
15	1.954	4.603	3.126	7.364	34.306	13.860
16	0.000	5.000	12.053	4.870	13.285	31.297
17	4.243	-4.243	4.243	-4.243	29.000	0.000
18	5.994	-14.835	7.778	-7.778	21.000	0.000
19	-3.126	-7.364	11.314	-11.314	17.490	-7.424
20	2.622	-6.490	19.092	-19.092	23.180	9.365
21	0.000	2.000	8.000	0.000	21.000	0.000
22	2.000	0.000	3.000	0.000	18.385	18.385
23	6.490	2.622	11.046	-4.689	30.597	12.362
24	5.523	-2.344	3.371	-8.345	11.046	-4.689
25	7.000	0.000	12.000	0.000	21.000	0.000
26	11.314	-11.314	14.728	-6.252	-36.000	0.000
27	15.762	6.368	20.000	0.000	20.251	-8.596
28	4.636	1.873	12.887	-5.470	28.536	-12.113
29	3.536	-3.536	17.490	-7.424	21.213	-21.213
30	8.285	-3.517	0.000	-14.000	9.740	-24.107
31	9.272	3.746	11.046	-4.689	4.870	-12.053
32	17.000	0.000	11.967	-5.079	6.364	-6.364
33	11.000	0.000	12.000	0.000	13.908	5.619
34	16.000	0.000	13.808	-5.861	9.000	0.000
35	18.544	7.492	15.649	-6.642	5.323	-2.344
36	9.000	0.000	11.000	0.000	7.364	-3.126
37	4.636	1.873	12.000	0.000	13.000	0.000
38	5.657	5.657	8.000	0.000	13.808	-5.861
39	6.490	2.622	8.345	3.371	18.000	0.000
40	1.954	4.603	9.272	3.746	19.000	0.000
41	12.728	12.728	10.000	0.000	20.398	8.241
42	6.364	6.364	8.485	8.485	18.544	7.492
43	12.981	5.245	17.617	7.118	12.021	12.021
44	11.000	0.000	12.053	4.870	19.471	7.867
45	16.689	6.743	14.835	5.994	22.252	8.991
46	16.000	0.000	18.000	0.000	32.451	13.111
47	13.908	5.619	15.762	6.368	26.888	10.864
48	0.000	-3.000	7.118	-17.617	18.000	0.000
49	7.071	-7.071	9.900	-9.899	9.192	-9.192
50	6.364	-6.364	10.607	-10.607	12.887	-12.887
51	2.248	-5.563	5.619	-13.908	7.118	-17.617
52	3.371	-8.345	-6.252	-14.728	0.000	-26.000
53	0.000	-9.000	0.000	-17.000	0.000	-26.000
54	-4.950	-4.950	-20.506	-20.506	-7.815	-18.410
55	0.000	-9.000	0.000	-13.000	5.994	-14.835
56	2.622	-6.490	4.495	-11.126	6.364	-6.364
57	3.371	-8.345	0.000	-8.000	0.000	-4.000
58	4.243	4.243	0.000	-5.000	0.000	-5.000
59	1.124	-2.782	1.498	-3.709	-11.126	-4.495
60	-1.854	-0.749	0.000	-2.000	-7.417	-2.997

Table 15 (continued)

SEQ	INPUT VARIABLES					
	1	2	3	4	5	6
61	-0.921	0.391	-0.921	0.391	-8.345	-3.371
62	0.000	5.000	-2.997	7.417	-0.749	1.854
63	0.000	5.000	2.344	5.523	2.735	6.444
64	0.000	3.000	0.000	2.000	1.414	1.414
65	12.000	0.000	10.000	0.000	15.649	-6.642
66	7.071	7.071	11.000	0.000	11.046	-4.689
67	6.364	6.364	6.000	0.000	5.657	-5.657
68	8.345	3.371	8.345	3.371	11.000	0.000
69	7.417	2.997	9.000	0.000	13.000	0.000
70	8.345	3.371	8.345	3.371	12.053	4.870
71	3.907	9.205	7.417	2.997	12.981	5.245
72	3.907	9.205	7.071	7.071	15.762	6.368
73	3.126	7.364	12.053	4.870	15.000	0.000
74	5.657	5.657	7.071	7.071	13.908	5.619
75	8.345	3.371	9.899	9.900	16.689	6.743
76	8.485	8.485	10.607	10.607	13.435	13.435
77	7.778	-7.778	13.000	0.000	19.331	-8.205
78	11.000	0.000	16.569	-7.033	26.000	0.000
79	12.981	5.245	18.544	7.492	25.000	0.000
80	11.314	11.314	16.649	6.743	27.816	11.238
81	10.607	10.607	13.908	5.619	14.142	14.142
82	9.192	9.192	11.314	11.314	20.398	8.241
83	8.485	8.485	16.263	16.263	30.597	12.362
84	8.485	-8.485	22.092	-9.378	31.297	-13.285
85	4.603	-1.954	17.490	-7.424	21.172	-8.987
86	11.126	4.495	12.000	0.000	22.000	0.000
87	12.981	5.245	13.908	5.619	25.961	10.489
88	8.485	8.485	11.314	11.314	19.471	7.867
89	11.000	0.000	21.000	0.000	35.000	0.000
90	17.000	0.000	25.000	0.000	49.000	0.000
91	9.205	-3.907	20.251	-8.596	38.000	0.000
92	9.000	0.000	17.000	0.000	28.743	11.613
93	-2.622	6.490	13.435	13.435	9.899	9.900
94	6.444	-2.735	4.636	1.873	24.000	0.000
95	4.000	0.000	12.887	-5.470	28.000	0.000
96	13.000	0.000	4.243	-4.243	18.000	0.000
97	16.689	6.743	13.000	0.000	10.126	-4.298
98	11.314	-11.314	18.410	-7.815	24.854	-10.550
99	9.205	-3.907	14.849	-14.849	23.933	-10.159
100	13.808	-5.861	15.649	-6.642	17.490	-7.424
101	13.808	-5.861	14.728	-6.252	13.808	-5.861
102	8.485	-8.485	21.000	0.000	27.816	11.238
103	7.071	-7.071	15.649	-6.642	11.987	-29.670
104	8.485	-8.485	9.192	-9.192	15.649	-6.642
105	6.444	-2.735	13.000	0.000	17.000	0.000
106	6.364	-6.364	7.364	-3.126	0.000	-12.000
107	2.762	-1.172	0.000	-9.000	-14.848	-34.979
108	2.828	-2.828	0.000	-6.000	-7.424	-17.490
109	0.000	-4.000	0.000	-3.000	-4.243	4.243
110	1.498	-3.709	2.762	-1.172	-3.000	0.000
111	0.000	-4.000	4.950	-4.950	10.199	4.121
112	-3.126	-7.364	0.000	-6.000	0.000	-5.000
113	-7.778	-7.778	-4.636	-11.046	-6.364	-6.364
114	-10.607	-10.607	-12.021	-12.021	4.495	-11.126
115	-3.536	-3.536	0.000	-6.000	0.000	-24.000
116	1.498	-3.709	3.746	-9.272	11.314	-11.314
117	3.536	-3.536	11.000	0.000	13.000	-5.861
118	6.444	-2.735	2.762	-1.172	21.000	0.000
119	11.046	-4.689	10.126	-4.298	20.000	0.000
120	6.444	-2.735	9.000	0.000	11.046	-4.689
121	10.126	-4.298	19.000	0.000	33.000	0.000
122	18.000	0.000	18.410	-7.815	17.490	-7.424
123	18.000	0.000	22.000	0.000	28.000	0.000
124	18.000	0.000	27.000	0.000	28.000	0.000
125	13.000	0.000	30.000	0.000	30.000	0.000
126	16.000	0.000	22.092	-9.378	37.000	0.000
127	5.657	-5.657	22.000	0.000	21.000	0.000
128	10.126	-4.298	17.490	-7.424	0.000	-22.000
129	13.808	-5.861	14.728	-6.252	0.000	-28.000
130	0.000	-10.000	5.619	-13.908	15.556	-15.556
131	4.243	-4.243	2.622	-6.490	0.000	-5.000
132	3.682	-1.563	3.000	0.000	-10.000	0.000
133	2.828	-2.828	3.000	0.000	-8.485	8.485
134	1.873	-4.636	4.603	-1.954	6.490	2.622
135	3.000	0.000	9.205	-3.907	9.272	3.746
136	3.000	0.000	3.536	-3.536	7.417	2.997
137	8.285	-3.517	8.285	-3.517	16.689	6.743
138	8.285	-3.517	11.046	-4.689	4.950	-4.950
139	11.967	-5.079	11.314	-11.314	4.495	-11.126
140	0.000	-9.000	0.000	-13.000	-16.411	-38.661
141	0.000	-8.000	0.000	-16.000	-33.941	-33.941
142	9.205	-3.907	5.245	-12.981	4.870	-12.053
143	8.485	-8.485	6.743	-16.689	14.235	-35.233
144	12.887	-5.470	7.778	-7.778	11.314	-11.314
145	12.887	-5.470	16.000	0.000	15.649	-6.642
146	7.778	-7.778	16.569	-7.033	27.615	-11.722
147	5.657	-5.657	11.314	-11.314	26.695	-11.331
148	4.243	-4.243	14.728	-6.252	23.013	-9.768
149	8.285	-3.517	10.126	-4.298	26.000	0.000
150	10.199	4.121	14.000	0.000	24.107	9.740
151	17.000	0.000	20.251	-8.596	22.000	0.000

Table 15 (continued)

SEQ	INPUT VARIABLES					
	1	2	3	4	5	6
152	17.000	0.000	14.728	-6.252	30.000	0.000
153	12.728	-12.728	12.021	-12.021	27.615	-11.722
154	15.649	-6.642	8.616	-21.325	10.864	-26.888
155	14.728	-6.252	13.435	-13.435	21.213	-21.213
156	13.808	-5.861	17.490	-7.424	21.172	-8.987
157	13.908	5.619	15.762	6.368	26.000	0.000
158	14.000	0.000	14.000	0.000	19.000	0.000
159	18.410	-7.815	17.490	-7.424	17.000	0.000
160	2.622	-6.490	6.444	-2.735	15.649	-6.642
161	-2.344	-5.523	4.950	-4.950	13.808	-5.861
162	-6.490	-2.622	3.536	-3.536	18.410	-7.815
163	-7.364	3.126	3.000	0.000	9.900	-9.899
164	-1.873	4.636	6.490	2.622	11.967	-5.079
165	1.172	2.762	3.536	-3.536	11.046	-4.689
166	1.854	0.749	3.709	1.498	12.887	-5.470
167	4.243	4.243	4.243	4.243	17.490	-7.424
168	2.344	5.523	4.603	-1.954	16.569	-7.033
169	3.536	3.536	6.490	2.622	11.967	-5.079
170	5.657	5.657	10.199	4.121	20.000	0.000
171	7.071	7.071	11.176	4.495	24.000	0.000
172	7.071	7.071	5.079	11.967	16.000	0.000
173	7.071	7.071	16.689	6.743	24.107	9.740
174	12.000	0.000	15.000	0.000	31.000	0.000
175	3.000	0.000	10.000	0.000	25.961	10.489
176	6.000	0.000	13.998	5.619	31.524	12.737
177	8.000	0.000	20.000	0.000	31.524	12.737
178	8.285	-3.517	9.205	-3.907	18.000	0.000
179	8.485	-8.485	0.707	-0.707	7.778	7.778
180	13.435	-13.435	26.000	0.000	29.456	-12.503
181	12.021	-12.021	29.456	-12.503	28.991	-28.991
182	0.000	-15.000	10.114	-25.034	20.229	-50.068
183	0.000	-14.000	11.238	-27.815	16.483	-40.796
184	5.245	-12.981	10.489	-25.961	22.627	-22.627
185	4.495	-11.126	8.485	-8.485	19.799	-19.799
186	3.746	-9.272	6.743	-16.689	10.114	-25.034
187	8.285	-3.517	12.887	-5.470	20.251	-8.596
188	7.364	-3.126	12.000	0.000	21.000	0.000
189	7.000	0.000	13.000	0.000	19.000	0.000
190	6.490	2.622	9.272	3.746	9.192	9.192
191	3.536	-3.536	11.314	-11.314	19.799	-19.799
192	13.808	-5.861	12.728	-12.728	10.114	-25.034
193	7.000	0.000	8.000	0.000	7.364	-3.126
194	5.563	2.248	10.000	0.000	9.205	-3.907
195	7.364	-3.126	11.967	-5.079	9.205	-3.907
196	4.950	-4.950	0.000	-10.000	-11.331	-26.695
197	5.000	0.000	0.000	-14.000	0.000	22.000
198	0.000	-5.000	4.870	-12.053	0.000	-14.000
199	7.364	-3.126	10.607	-10.607	9.900	-9.899
200	4.950	-4.950	11.314	-11.314	16.263	-16.263
201	13.808	-5.861	19.391	-8.205	8.991	-22.252
202	7.778	-7.778	23.395	-23.394	24.749	-24.749
203	5.619	-13.998	9.365	-23.180	8.991	-22.252
204	0.000	-17.000	17.000	0.000	13.435	-13.435
205	-5.080	-11.967	0.000	-12.000	13.808	-5.861
206	0.000	-7.000	7.364	-3.126	16.000	0.000
207	-2.344	-5.523	2.997	-7.417	13.000	0.000
208	-3.907	-9.205	2.622	-6.490	14.000	0.000
209	-9.192	-9.192	0.000	-13.000	8.285	-3.517
210	-6.364	-6.364	0.000	-7.000	7.364	-3.126
211	-5.657	-5.657	-8.485	-8.485	7.778	-7.778
212	-6.364	-6.364	-3.517	-8.285	9.205	-3.907
213	-7.071	-7.071	-3.536	3.536	6.364	-6.364
214	2.622	-6.490	1.873	-4.586	-8.205	-19.331
215	2.622	-6.490	0.000	-9.000	-8.987	-21.172
216	7.778	-7.778	-4.298	-10.126	-16.971	-16.971
217	3.746	-9.272	-2.907	-9.205	-13.435	-13.435
218	4.495	-11.126	0.000	-6.000	-12.021	-12.021
219	2.997	-7.417	-3.517	-8.285	-4.298	-10.126
220	0.000	-10.000	0.000	-5.000	-3.517	-8.285
221	-1.841	0.781	1.498	-3.709	-1.124	2.782
222	-1.414	-1.414	1.414	-1.414	2.735	6.444
223	2.121	-2.121	3.682	-1.563	3.126	7.364
224	3.536	-3.536	5.523	-2.344	5.657	5.657
225	4.603	-1.954	7.000	0.000	4.243	4.243
226	5.000	0.000	8.000	0.000	9.272	3.746
227	6.000	0.000	3.709	1.498	7.778	7.778
228	2.344	5.523	8.345	3.371	9.000	0.000
229	4.243	4.243	4.950	4.950	9.000	0.000
230	6.490	2.622	5.523	-2.344	20.000	0.000
231	7.417	2.997	0.921	-0.391	16.000	0.000
232	-0.391	-0.920	3.000	0.000	18.410	-7.815
233	0.000	3.000	-2.000	0.000	15.649	-6.642
234	2.735	6.444	0.000	3.000	12.021	-12.021
235	0.000	2.000	3.000	0.000	10.607	-10.607
236	-2.622	6.490	1.414	-1.414	4.870	-12.053
237	0.000	7.000	-1.000	0.000	-4.689	-11.046
238	-0.000	9.000	-1.873	4.636	0.000	-12.000
239	-3.746	9.272	-1.873	4.636	0.000	-10.000
240	-2.997	7.417	-0.921	0.391	10.607	-10.607
241	0.391	0.921	1.414	1.414	12.728	-12.728
242	0.000	5.000	4.495	-11.126	23.933	-10.159

Table 15 (continued)

SEQ	INPUT VARIABLES					
	1	2	3	4	5	6
243	4.950	4.950	12.000	0.000	19.331	-8.205
244	11.126	4.495	14.000	0.000	26.000	0.000
245	10.000	0.000	21.000	0.000	24.000	0.000
246	15.000	0.000	19.000	0.000	22.092	-9.378
247	10.607	-10.607	23.335	-23.334	53.389	-22.662
248	7.492	-18.544	10.489	-25.961	20.506	-20.506
249	0.000	-14.000	7.492	-18.544	16.971	-16.971
250	0.000	-12.000	7.118	-17.617	15.556	-15.556
251	3.746	-9.272	8.485	-8.485	18.410	-7.815
252	3.000	0.000	8.000	0.000	15.000	0.000
253	4.636	1.873	11.126	4.495	20.000	0.000
254	3.536	3.536	4.950	4.950	11.314	11.314
255	3.517	8.285	14.142	14.142	16.971	16.971
256	8.485	8.485	8.205	19.331	21.920	21.920
257	8.485	8.485	14.142	14.142	23.335	23.335
258	4.243	4.243	11.314	11.314	11.722	27.615
259	0.000	6.000	9.879	9.900	8.987	21.172
260	1.563	3.682	3.907	9.205	8.987	21.172
261	-3.000	0.000	2.622	-6.490	0.000	-3.000
262	5.657	-5.657	8.000	0.000	12.887	-5.470
263	9.000	0.000	9.272	3.746	10.199	4.121
264	5.657	-5.657	6.490	2.622	4.950	4.950
265	2.997	-7.417	4.603	-1.954	0.000	-7.000
266	-5.080	-11.967	-5.080	-11.967	-10.550	-24.854
267	-6.642	-15.649	-10.550	-24.854	-16.411	-38.661
268	-4.689	-11.046	-10.159	-23.933	-13.285	-31.297
269	-1.563	-3.682	-8.205	-19.331	-14.848	-34.979
270	0.000	-9.000	0.000	-14.000	0.000	-24.000
271	4.950	-4.950	4.121	-10.199	0.000	-18.000
272	2.000	0.000	4.603	-1.954	3.746	-9.272
273	8.000	0.000	8.285	-3.517	6.000	0.000
274	7.000	0.000	4.950	4.950	13.908	5.619
275	9.272	3.746	13.000	0.000	13.908	5.619
276	8.345	3.371	12.053	4.870	17.678	17.678
277	12.000	0.000	14.835	5.994	16.263	16.263
278	13.000	0.000	17.000	0.000	35.233	14.235
279	9.272	3.746	16.000	0.000	38.000	0.000
280	11.126	4.495	14.835	5.994	34.000	0.000
281	7.778	7.778	15.762	6.368	26.000	0.000
282	7.417	2.997	18.000	0.000	28.000	0.000
283	2.828	2.828	15.000	0.000	33.000	0.000
284	9.000	0.000	16.000	0.000	24.854	-10.550
285	7.000	0.000	15.000	0.000	32.218	-13.676
286	11.000	0.000	15.762	6.368	27.615	-11.722
287	14.835	5.994	12.887	-5.470	31.297	-13.285
288	16.000	0.000	15.649	-6.642	28.000	0.000
289	21.325	8.616	21.000	0.000	23.013	-9.768
290	12.887	-5.470	19.000	0.000	26.000	0.000
291	22.000	0.000	25.000	0.000	14.835	5.994
292	14.000	0.000	15.649	-6.642	21.000	0.000
293	13.908	5.619	20.251	-8.596	24.107	9.740
294	14.000	0.000	12.053	4.870	29.000	0.000
295	12.981	5.245	16.689	6.743	32.000	0.000
296	18.000	0.000	25.000	0.000	29.670	11.987
297	17.000	0.000	23.013	-9.768	43.000	0.000
298	21.000	0.000	26.000	0.000	27.000	0.000
299	10.607	-10.607	19.000	0.000	34.059	-14.457
300	7.071	-7.071	19.331	-8.205	36.820	-15.629
301	4.495	-11.126	19.331	-8.205	38.661	-16.411
302	0.000	-4.000	5.619	-13.908	22.627	-22.627
303	2.000	0.000	11.046	-4.689	18.385	-18.385
304	6.000	0.000	12.000	0.000	19.331	-8.205
305	7.417	2.997	21.000	0.000	24.000	0.000
306	18.000	0.000	17.490	-7.424	18.385	-18.385
307	15.649	-6.642	14.778	-6.252	15.556	-15.556
308	11.046	-4.689	12.887	-5.470	21.172	-8.987
309	9.272	3.746	13.000	0.000	20.251	-8.596
310	16.000	0.000	16.000	0.000	12.000	0.000
311	14.728	-6.252	11.967	-5.079	21.000	0.000
312	12.981	5.245	17.000	0.000	17.490	-7.424
313	15.762	6.368	21.000	0.000	18.410	-7.815
314	18.544	7.492	22.000	0.000	16.000	0.000
315	18.544	7.492	24.000	0.000	24.000	0.000
316	24.107	9.740	15.556	15.556	32.451	13.111
317	20.398	8.241	28.743	11.613	20.506	20.506
318	16.689	6.743	20.398	8.241	25.456	25.456
319	21.213	21.213	21.213	21.213	18.385	18.385
320	15.762	6.368	13.000	0.000	32.451	13.111
321	12.981	5.245	26.000	0.000	26.000	0.000
322	9.192	9.192	34.000	0.000	21.325	8.616
323	5.079	11.967	16.649	6.743	21.325	8.616
324	15.556	15.556	5.470	12.887	21.325	8.616
325	13.435	13.435	5.079	11.967	10.159	23.933
326	16.971	16.971	20.506	20.506	12.113	28.536
327	18.385	18.385	29.698	29.699	21.881	51.548
328	10.199	4.121	14.457	34.059	-0.000	78.000
329	9.272	3.746	13.000	0.000	-0.000	72.000
330	7.364	-3.126	5.563	2.248	-23.226	57.485
331	5.657	-5.657	3.371	-8.345	-4.689	-11.046
332	3.371	-8.345	0.000	-32.000	0.000	-41.000

Table 15 (continued)

SEQ	INPUT VARIABLES					
	1	2	3	4	5	6
333	4.870	-12.053	16.857	-41.723	20.229	-50.068
334	12.728	-12.728	24.749	-24.749	38.184	-38.184
335	19.331	-8.205	32.218	-13.676	52.469	-22.272
336	14.728	-6.252	28.536	-12.113	38.184	-38.184
337	12.887	-5.470	23.933	-10.159	49.707	-21.099
338	7.778	-7.778	19.799	-19.799	26.870	-26.870
339	7.778	-7.778	18.385	-18.385	15.734	-38.942
340	-6.364	-6.364	0.000	-15.000	7.867	-19.471
341	0.000	-8.000	0.000	-15.000	13.435	-13.435
342	-4.243	-4.243	0.000	-12.000	6.368	-15.762
343	-5.657	-5.657	-3.517	-8.285	-15.762	-6.368
344	-3.536	-3.536	-4.950	-4.950	-9.899	-9.899
345	0.000	-6.000	0.000	-9.000	-8.345	-3.271
346	0.000	-8.000	-3.126	-7.364	-7.778	-7.778
347	-6.000	0.000	-5.657	-5.657	-5.657	-5.657
348	-1.954	-4.603	-1.563	-3.682	-7.071	-7.071
349	-4.243	-4.243	-6.364	-6.364	-17.000	0.000
350	-1.954	-4.603	-7.417	-2.997	-13.908	-5.619
351	-5.000	0.000	-2.828	-2.828	-12.021	-12.021
352	1.172	2.762	-4.603	1.954	-22.252	-8.991
353	0.000	3.000	-0.375	0.927	-12.059	-4.870
354	1.172	2.762	2.344	5.523	-6.490	-2.622
355	4.243	4.243	4.689	11.046	-3.682	1.563
356	1.563	3.682	5.079	11.967	-3.371	8.345
357	2.782	1.124	3.907	9.205	-0.000	15.000
358	2.121	2.121	4.298	10.126	-0.000	19.000
359	1.563	3.682	6.364	6.364	7.815	18.410
360	15.000	0.000	9.192	-9.192	30.377	-12.894
361	10.126	-4.298	13.000	0.000	28.536	-12.113
362	11.000	0.000	22.000	0.000	23.189	9.365
363	22.000	0.000	25.000	0.000	25.961	10.489
364	18.410	-7.815	23.013	-9.768	44.000	0.000
365	14.849	-14.849	18.385	-18.385	9.740	-24.107
366	3.746	-9.272	7.071	-7.071	12.887	-5.670
367	10.126	-4.298	11.126	4.495	23.000	0.000
368	7.778	7.778	9.192	9.192	31.524	12.737
369	12.887	-5.470	14.728	-6.252	24.854	-10.550
370	21.000	0.000	30.597	12.362	54.704	22.102
371	19.000	0.000	22.252	8.991	28.743	11.613
372	12.887	-5.470	8.241	-20.398	12.728	-12.728
373	9.192	-9.192	17.678	-17.678	23.335	-23.334
374	11.046	-4.689	21.172	-8.987	33.138	-14.066
375	6.444	-6.364	13.808	-5.861	31.297	-13.285
376	6.444	-2.735	10.126	-4.298	15.762	6.368
377	8.285	-3.517	8.485	-8.485	12.887	-5.470
378	8.485	-8.485	7.071	-7.071	11.967	-5.079
379	5.657	-5.657	9.192	-9.192	11.046	-4.689
380	9.192	-9.192	12.021	-12.021	15.556	-15.556
381	14.835	5.994	26.000	0.000	31.000	0.000
382	17.000	0.000	21.000	0.000	25.000	0.000
383	12.887	-5.470	19.000	0.000	23.000	0.000
384	8.485	-8.485	18.000	0.000	25.000	0.000
385	7.778	-7.778	15.000	0.000	23.000	0.000
386	7.417	2.997	15.000	0.000	23.933	-10.159
387	9.272	3.746	17.000	0.000	24.854	-10.550
388	5.523	-2.344	18.000	0.000	26.695	-11.331
389	3.682	-1.563	10.126	-4.298	24.854	-10.550
390	3.709	1.498	14.000	0.000	23.013	-9.768
391	8.485	8.485	14.000	0.000	22.000	0.000
392	7.778	7.778	10.000	0.000	26.000	0.000
393	6.364	6.364	3.536	3.536	22.000	0.000
394	9.272	3.746	10.000	0.000	24.000	0.000
395	3.907	9.205	6.000	0.000	28.000	0.000
396	7.071	7.071	7.417	2.997	20.000	0.000
397	5.657	5.657	13.908	5.619	21.000	0.000
398	9.192	9.192	16.000	0.000	26.000	0.000
399	12.981	5.245	12.981	9.245	16.689	6.743
400	14.835	5.994	18.410	-7.815	22.000	0.000
401	13.908	5.619	19.000	0.000	16.000	0.000
402	16.000	0.000	12.981	5.245	14.000	0.000
403	17.617	7.118	19.471	7.867	21.325	8.616
404	17.000	0.000	16.971	16.971	23.335	23.335
405	22.252	8.991	15.238	35.900	16.411	38.541
406	14.849	14.849	17.182	40.502	35.355	35.355
407	17.617	7.118	21.881	51.548	22.662	53.389
408	21.000	0.000	44.000	0.000	38.184	38.184
409	11.046	-4.689	56.000	0.000	54.704	22.102
410	14.142	-14.142	27.615	-11.722	53.000	0.000
411	8.485	-8.485	31.297	-13.285	35.900	-15.238
412	7.364	-3.126	21.920	-21.920	22.627	-22.627
413	4.243	-4.243	7.071	-7.071	22.627	-22.627
414	6.000	0.000	11.967	-5.079	28.536	-12.113
415	8.000	0.000	14.728	-6.252	12.728	-12.728
416	7.000	0.000	12.887	-5.470	8.485	-8.485
417	4.000	0.000	11.046	-4.689	8.485	-8.485
418	6.490	2.622	4.602	-1.954	5.657	-5.657
419	9.000	0.000	9.000	0.000	4.636	1.873
420	7.364	-3.126	9.000	0.000	10.607	10.607
421	5.563	2.248	7.000	0.000	10.199	4.121
422	9.272	3.746	9.000	0.000	10.000	0.000

Table 15 (continued)

SEQ	INPUT VARIABLES					
	1	2	3	4	5	6
423	14.835	5.994	9.899	9.900	16.689	6.743
424	14.142	14.142	17.617	7.118	23.000	0.000
425	15.762	6.368	10.607	10.607	20.398	8.241
426	22.252	8.991	21.920	21.920	18.544	7.492
427	11.000	0.000	15.762	6.368	19.799	19.799
428	12.981	5.245	20.398	8.241	25.456	25.456
429	11.046	-4.689	18.544	7.492	15.238	35.900
430	7.364	-3.126	8.000	0.000	12.000	0.000
431	7.071	-7.071	8.000	0.000	16.000	0.000
432	4.000	0.000	10.000	0.000	14.835	5.994
433	1.124	-2.782	4.603	-1.954	12.981	5.245
434	-1.954	-4.603	2.000	0.000	1.873	-4.636
435	-0.927	-0.375	1.841	-0.781	8.485	-8.485
436	0.000	5.000	7.364	-3.126	10.126	-4.298
437	0.000	6.000	5.000	0.000	10.607	-10.607
438	1.854	0.749	6.444	-2.735	14.142	-14.142
439	5.523	-2.344	5.657	-5.657	15.359	-38.015
440	-2.828	-2.828	5.245	-12.981	31.113	-31.113
441	-5.657	-5.657	0.000	-12.000	22.627	-22.627
442	0.000	-9.000	0.000	-13.000	0.000	-14.000
443	0.000	-12.000	0.000	-13.000	-10.199	-4.121
444	0.000	-17.000	-5.861	-13.808	-37.087	-14.984
445	0.000	-13.000	-5.361	-13.808	10.489	-25.961
446	0.000	-10.000	0.000	-16.000	0.000	-22.000
447	0.000	-13.000	-7.033	-16.569	5.619	-13.908
448	3.371	-8.345	4.495	-11.126	11.046	-4.689
449	0.000	-9.000	0.000	-6.000	-4.000	0.000
450	2.997	-7.417	0.000	-12.000	0.000	-21.000
451	3.371	-8.345	4.870	-12.053	10.489	-25.961
452	4.870	-12.053	0.000	-16.000	0.000	-43.000
453	5.994	-14.835	0.000	-26.000	11.238	-27.815
454	5.619	-13.908	9.345	-23.180	11.238	-27.815
455	7.071	-7.071	11.314	-11.314	9.740	-24.107
456	7.778	-7.778	7.118	-17.617	10.489	-25.961
457	11.967	-5.079	10.607	-10.607	16.263	-16.263
458	6.444	-2.735	10.607	-10.607	16.569	-7.033
459	13.000	0.000	19.000	0.000	22.000	0.000
460	12.000	0.000	19.000	0.000	28.000	0.000
461	18.000	0.000	23.000	0.000	33.000	0.000
462	13.808	-5.861	27.000	0.000	33.000	0.000
463	6.364	-6.364	6.364	-6.364	26.695	-11.331
464	6.000	-2.735	10.000	0.000	11.046	-4.689
465	6.000	0.000	6.000	0.000	13.435	-13.435
466	7.000	0.000	10.126	-4.298	15.649	-6.642
467	9.000	0.000	13.708	-5.861	21.920	-21.920
468	13.908	5.619	11.947	-5.079	27.615	-11.722
469	12.728	12.728	18.544	7.492	32.000	0.000
470	20.398	8.241	25.034	10.114	43.000	0.000
471	25.034	10.114	41.723	16.857	46.000	0.000
472	32.451	13.111	22.627	22.627	48.214	19.480
473	25.000	0.000	45.432	18.356	60.104	60.104
474	16.263	-16.263	34.648	-34.648	41.719	-41.719
475	21.213	-21.213	27.577	-27.577	28.991	-28.991
476	15.556	-15.556	22.627	-22.627	28.284	-28.284
477	15.649	-6.642	19.799	-19.799	22.627	-22.627
478	12.728	-12.728	16.263	-16.263	24.854	-10.550
479	15.649	-6.642	20.506	-20.506	22.627	-22.627
480	16.569	-7.033	16.263	-16.263	21.213	-21.213
481	9.192	-9.192	20.251	-8.596	19.799	-19.799
482	9.192	-9.192	14.849	-14.849	14.142	-14.142
483	7.364	-3.126	10.607	-10.607	13.000	0.000
484	9.205	-3.907	11.967	-5.079	17.490	-7.424
485	5.657	5.657	15.762	6.368	20.000	0.000
486	9.899	9.900	15.556	15.556	25.961	10.489
487	14.849	14.849	17.678	17.678	21.325	8.616
488	6.364	6.364	17.617	7.118	28.743	11.613
489	5.470	12.887	8.205	19.331	26.163	26.163
490	4.298	10.126	8.987	21.172	8.596	20.251
491	7.071	7.071	12.894	30.377	28.991	28.991
492	8.596	20.251	-0.000	25.000	-0.000	58.000
493	9.192	9.192	5.861	13.808	5.861	13.808
494	14.000	0.000	9.899	9.900	5.079	11.967
495	12.053	4.870	17.617	7.118	5.470	12.887
496	11.126	4.495	8.485	8.485	5.470	12.887
497	16.689	6.743	5.657	5.657	1.414	1.414
498	10.199	4.121	6.364	-6.364	7.778	-7.778
499	12.000	0.000	9.900	-9.899	12.021	-12.021
500	14.728	-6.252	14.142	-14.142	18.385	-18.385
501	18.410	-7.815	20.506	-20.506	37.477	-37.477
502	14.728	-6.252	18.385	-18.385	26.820	-15.629
503	17.490	-7.424	17.000	0.000	35.900	-15.238

Table 16 Separation of a multivariate (6) set of Rantoul, Illinois, October 1950-55, upper wind components, zonal and meridional mixed distribution, at the 700-, 500-, and 300-mb levels into two separate distributions. The units are $\text{m}\cdot\text{s}^{-1}$.

PROGRAM NORMIX
WOLFE NORMAL MIXTURE ANALYSIS PROCEDURE (1974 REVISION)

SAMPLE = RANTOUL
OCTOBER
700, 500, AND 300MB
X AND Y COMPONENTS

SAMPLE SIZE = 503
NUMBER OF VARIABLES = 6
NUMBER OF TYPES = 2

ITERATION NUMBER 39

LIKELIHOOD OF 2 TYPES IN THIS SAMPLE = -0.73297066D 04

CHARACTERISTICS OF THE WHOLE SAMPLE

MEANS					
1	2	3	4	5	6
7.3252	-0.9486	10.6700	-2.5704	15.4049	-3.6999
STANDARD DEVIATIONS					
1	2	3	4	5	6
6.6262	6.8921	8.9594	10.3149	13.7082	16.0058
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.2215	0.7825	0.2313	0.5589	0.2311
0.2215	1.0000	0.1138	0.7581	0.1265	0.5757
0.7825	0.1138	1.0000	0.1587	0.7513	0.1977
0.2313	0.7581	0.1587	1.0000	0.1200	0.7856
0.5589	0.1265	0.7513	0.1200	1.0000	0.1266
0.2311	0.5757	0.1977	0.7856	0.1266	1.0000

CHARACTERISTICS OF TYPE 1

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.171

MEANS					
1	2	3	4	5	6
10.4545	-2.0123	16.2238	-2.6858	18.7634	-3.3254
STANDARD DEVIATIONS					
1	2	3	4	5	6
7.4317	9.0069	12.5099	17.4952	20.1227	29.7834
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.3578	0.6074	0.4401	0.5639	0.3522
0.3578	1.0000	0.0297	0.7469	0.0730	0.6333
0.6074	0.0297	1.0000	0.1637	0.7420	0.1966
0.4401	0.7469	0.1637	1.0000	0.1148	0.7884
0.5639	0.0730	0.7420	0.1148	1.0000	0.0799
0.3522	0.6333	0.1966	0.7884	0.0799	1.0000
EIGENVALUES					
1165.8027	0.1014	0.1798	0.1382	0.2729	0.4299
507.7779	0.1864	-0.0377	0.3840	-0.1749	0.7770
108.4213	0.1060	0.4467	-0.2043	0.7782	0.0692
56.7879	0.4498	-0.0490	0.7497	0.1994	-0.4398
30.0842	0.1181	0.8592	0.0471	-0.4824	-0.1135
22.0357	0.8529	-0.1618	-0.4768	-0.1293	0.0181
EIGENVECTORS					
1	2	3	4	5	6
0.8240	-0.4266	-0.3701	-0.0070	0.0091	0.0437

CHARACTERISTICS OF TYPE 2

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.829

MEANS					
1	2	3	4	5	6
6.6783	-0.7287	9.5218	-2.5466	14.7107	-3.7773
STANDARD DEVIATIONS					
1	2	3	4	5	6
6.2548	6.3457	7.5377	8.0693	11.8422	11.2131

Table 16 (continued)

CORRELATIONS						
1	2	3	4	5	6	
1.0000	0.2039	0.8038	0.1455	0.5563	0.1878	
0.2039	1.0000	0.1937	0.7943	0.1673	0.5849	
0.8038	0.1937	1.0000	0.1721	0.7626	0.2195	
0.1455	0.7943	0.1721	1.0000	0.1280	0.7914	
0.5563	0.1673	0.7626	0.1280	1.0000	0.1756	
0.1878	0.5849	0.2195	0.7914	0.1756	1.0000	
EIGENVALUES						
1	2	3	4	5	6	
236.2003	0.2483	-0.2052	0.6858	-0.0776	0.3252	0.5605
155.9890	0.2529	0.2350	0.1020	0.6736	0.5317	-0.3660
31.5369	0.3590	-0.3140	0.4636	-0.0919	-0.4232	-0.6083
28.5496	0.3591	0.4036	0.0176	0.4196	-0.6195	0.3846
8.3730	0.5713	-0.5859	-0.5400	0.0860	0.0879	0.1540
6.7008	0.5387	0.5457	-0.1120	-0.5902	0.2024	-0.1008

SAMPLE = RANDOL
OCTOBER
700,500, AND 300MS
X AND Y COMPONENTS

PROBABILITIES
OF TYPE MEMBERSHIP

1	2
1 0.065 0.935	
2 0.011 0.989	
3 0.015 0.985	
4 0.996 0.004	
5 0.022 0.978	
6 0.037 0.963	
7 0.026 0.974	
8 1.000 0.000	
9 0.119 0.881	
10 0.851 0.149	
11 0.981 0.019	
12 0.005 0.995	
13 0.007 0.993	
14 0.003 0.997	
15 0.122 0.878	
16 0.976 0.024	
17 0.034 0.966	
18 0.054 0.946	
19 0.088 0.912	
20 1.000 0.000	
21 0.003 0.997	
22 0.125 0.875	
23 0.269 0.731	
24 0.011 0.989	
25 0.005 0.995	
26 1.000 0.000	
27 0.028 0.972	
28 0.013 0.987	
29 0.294 0.706	
30 0.218 0.782	
31 0.029 0.971	
32 0.034 0.966	
33 0.011 0.989	
34 0.037 0.963	
35 0.407 0.593	
36 0.011 0.989	
37 0.008 0.992	
38 0.005 0.995	
39 0.005 0.995	
40 0.003 0.997	
41 0.323 0.677	
42 0.006 0.994	
43 0.043 0.957	
44 0.017 0.983	
45 0.025 0.975	
46 0.071 0.929	
47 0.016 0.984	
48 0.916 0.084	
49 0.010 0.990	
50 0.013 0.987	
51 0.013 0.987	
52 0.181 0.819	
53 0.020 0.980	
54 0.996 0.004	
55 0.005 0.995	
56 0.009 0.991	
57 0.012 0.988	
58 0.030 0.970	
59 0.028 0.972	
60 0.008 0.992	
61 0.008 0.992	
62 0.008 0.992	
63 0.004 0.996	
64 0.003 0.997	
65 0.017 0.983	

PROBABILITIES
OF TYPE MEMBERSHIP

1	2
66 0.010 0.990	
67 0.009 0.991	
68 0.007 0.993	
69 0.005 0.995	
70 0.007 0.993	
71 0.007 0.993	
72 0.004 0.996	
73 0.007 0.993	
74 0.005 0.995	
75 0.021 0.979	
76 0.010 0.990	
77 0.093 0.907	
78 0.024 0.976	
79 0.025 0.975	
80 0.014 0.986	
81 0.038 0.962	
82 0.010 0.990	
83 0.050 0.950	
84 0.163 0.837	
85 0.066 0.934	
86 0.008 0.992	
87 0.016 0.984	
88 0.009 0.991	
89 0.022 0.978	
90 0.114 0.886	
91 0.081 0.919	
92 0.032 0.968	
93 0.234 0.766	
94 0.048 0.952	
95 0.014 0.986	
96 0.205 0.795	
97 0.031 0.969	
98 0.058 0.942	
99 0.055 0.945	
100 0.013 0.987	
101 0.015 0.985	
102 0.403 0.597	
103 0.878 0.122	
104 0.009 0.991	
105 0.009 0.991	
106 0.045 0.955	
107 0.875 0.125	
108 0.028 0.972	
109 0.014 0.986	
110 0.010 0.990	
111 0.008 0.992	
112 0.004 0.996	
113 0.009 0.991	
114 0.005 0.995	
115 0.026 0.974	
116 0.004 0.996	
117 0.013 0.987	
118 0.037 0.963	
119 0.013 0.987	
120 0.009 0.991	
121 0.034 0.966	
122 0.023 0.977	
123 0.019 0.981	
124 0.069 0.931	
125 0.685 0.315	
126 0.106 0.894	
127 0.615 0.385	
128 0.811 0.189	
129 0.892 0.108	
130 0.006 0.994	

PROBABILITIES
OF TYPE MEMBERSHIP

	1	2
131	0.009	0.991
132	0.037	0.963
133	0.075	0.925
134	0.006	0.994
135	0.019	0.981
136	0.008	0.992
137	0.018	0.982
138	0.016	0.984
139	0.027	0.973
140	0.926	0.074
141	0.998	0.002
142	0.030	0.970
143	0.240	0.760
144	0.023	0.977
145	0.069	0.931
146	0.027	0.973
147	0.011	0.989
148	0.016	0.984
149	0.010	0.990
150	0.020	0.980
151	0.061	0.939
152	0.056	0.944
153	0.047	0.953
154	0.272	0.728
155	0.033	0.967
156	0.014	0.986
157	0.019	0.981
158	0.011	0.989
159	0.038	0.962
160	0.006	0.994
161	0.003	0.997
162	0.002	0.998
163	0.004	0.996
164	0.003	0.997
165	0.004	0.996
166	0.004	0.996
167	0.015	0.985
168	0.005	0.995
169	0.004	0.996
170	0.004	0.996
171	0.005	0.995
172	0.092	0.908
173	0.011	0.989
174	0.013	0.987
175	0.013	0.987
176	0.023	0.977
177	0.098	0.902
178	0.007	0.993
179	0.311	0.689
180	0.998	0.002
181	0.997	0.003
182	0.977	0.023
183	0.857	0.143
184	0.204	0.796
185	0.043	0.957
186	0.020	0.980
187	0.007	0.993
188	0.008	0.992
189	0.006	0.994
190	0.010	0.990
191	0.020	0.980
192	0.063	0.937
193	0.007	0.993
194	0.006	0.994
195	0.012	0.988

Table 16 (continued)

PROBABILITIES
OF TYPE MEMBERSHIP

	1	2
196	0.208	0.792
197	0.121	0.879
198	0.023	0.977
199	0.015	0.985
200	0.013	0.987
201	0.397	0.603
202	0.983	0.017
203	0.073	0.927
204	1.000	0.000
205	0.005	0.995
206	0.006	0.994
207	0.005	0.995
208	0.005	0.995
209	0.021	0.979
210	0.003	0.997
211	0.008	0.992
212	0.003	0.997
213	0.022	0.978
214	0.107	0.893
215	0.045	0.955
216	0.371	0.629
217	0.066	0.934
218	0.131	0.869
219	0.019	0.981
220	0.014	0.986
221	0.014	0.986
222	0.007	0.993
223	0.013	0.987
224	0.011	0.989
225	0.010	0.990
226	0.006	0.994
227	0.013	0.987
228	0.005	0.995
229	0.005	0.995
230	0.011	0.989
231	0.062	0.938
232	0.004	0.996
233	0.011	0.989
234	0.035	0.965
235	0.004	0.996
236	0.006	0.994
237	0.015	0.985
238	0.025	0.975
239	0.007	0.993
240	0.005	0.995
241	0.012	0.988
242	0.181	0.819
243	0.007	0.993
244	0.008	0.992
245	0.037	0.963
246	0.034	0.966
247	0.922	0.078
248	0.084	0.916
249	0.020	0.980
250	0.017	0.983
251	0.006	0.994
252	0.003	0.997
253	0.006	0.994
254	0.006	0.994
255	0.026	0.974
256	0.213	0.787
257	0.033	0.967
258	0.211	0.789
259	0.063	0.937
260	0.025	0.975
261	0.017	0.983
262	0.016	0.984
263	0.012	0.988
264	0.033	0.967
265	0.023	0.977
266	0.028	0.972
267	0.200	0.800
268	0.197	0.803
269	0.649	0.351
270	0.013	0.987
271	0.015	0.985
272	0.005	0.995
273	0.011	0.989
274	0.018	0.982
275	0.012	0.988
276	0.031	0.969
277	0.057	0.943
278	0.087	0.913
279	0.017	0.983
280	0.029	0.971
281	0.009	0.991
282	0.013	0.987
283	0.012	0.988
284	0.023	0.977
285	0.058	0.942

PROBABILITIES
OF TYPE MEMBERSHIP

	1	2
286	0.566	0.434
287	0.145	0.855
288	0.032	0.968
289	0.056	0.944
290	0.041	0.959
291	0.132	0.868
292	0.020	0.980
293	0.928	0.072
294	0.145	0.855
295	0.028	0.972
296	0.088	0.912
297	0.195	0.805
298	0.039	0.961
299	0.933	0.067
300	0.103	0.897
301	0.415	0.585
302	0.023	0.977
303	0.024	0.976
304	0.008	0.992
305	0.069	0.931
306	0.046	0.954
307	0.051	0.949
308	0.010	0.990
309	0.009	0.991
310	0.020	0.980
311	0.030	0.970
312	0.016	0.984
313	0.038	0.962
314	0.050	0.950
315	0.035	0.965
316	0.964	0.036
317	0.486	0.514
318	0.224	0.776
319	0.171	0.829
320	0.364	0.636
321	0.166	0.834
322	1.000	0.000
323	0.028	0.972
324	0.811	0.189
325	0.643	0.357
326	0.367	0.633
327	0.999	0.001
328	1.000	0.000
329	1.000	0.000
330	1.000	0.000
331	0.018	0.982
332	0.980	0.020
333	1.000	0.000
334	0.931	0.069
335	0.796	0.204
336	0.998	0.002
337	0.535	0.465
338	0.476	0.524
339	0.940	0.060
340	0.009	0.991
341	0.011	0.989
342	0.005	0.995
343	0.037	0.963
344	0.006	0.994
345	0.028	0.972
346	0.011	0.989
347	0.005	0.995
348	0.007	0.993
349	0.047	0.953
350	0.019	0.981
351	0.013	0.987
352	0.208	0.792
353	0.021	0.979
354	0.020	0.980
355	0.080	0.920
356	0.053	0.947
357	0.036	0.964
358	0.055	0.945
359	0.024	0.976
360	0.310	0.690
361	0.130	0.870
362	0.111	0.889
363	0.069	0.931
364	0.139	0.861
365	0.193	0.807
366	0.007	0.993
367	0.101	0.899
368	0.031	0.969
369	0.015	0.985
370	0.954	0.046
371	0.188	0.812
372	0.388	0.612
373	0.090	0.910
374	0.048	0.952
375	0.026	0.974

PROBABILITIES
OF TYPE MEMBERSHIP

	1	2
376	0.016	0.984
377	0.008	0.992
378	0.011	0.989
379	0.010	0.990
380	0.012	0.988
381	0.064	0.936
382	0.017	0.983
383	0.044	0.956
384	0.119	0.881
385	0.047	0.953
386	0.014	0.986
387	0.020	0.980
388	0.107	0.893
389	0.006	0.994
390	0.015	0.985
391	0.012	0.988
392	0.013	0.987
393	0.027	0.973
394	0.008	0.992
395	0.028	0.972
396	0.007	0.993
397	0.007	0.993
398	0.017	0.983
399	0.011	0.989
400	0.207	0.793
401	0.024	0.976
402	0.087	0.913
403	0.021	0.979
404	0.942	0.058
405	1.000	0.000
406	1.000	0.000
407	1.000	0.000
408	1.000	0.000
409	1.000	0.000
410	0.816	0.184
411	0.996	0.004
412	0.991	0.009
413	0.045	0.955
414	0.010	0.990
415	0.025	0.975
416	0.020	0.980
417	0.014	0.986
418	0.007	0.993
419	0.012	0.988
420	0.024	0.976
421	0.006	0.994
422	0.008	0.992
423	0.099	0.901
424	0.021	0.979
425	0.157	0.843
426	0.992	0.008
427	0.094	0.906
428	0.238	0.762
429	0.997	0.003
430	0.008	0.992
431	0.023	0.977
432	0.007	0.993
433	0.005	0.995
434	0.006	0.994
435	0.003	0.997
436	0.010	0.990
437	0.005	0.995
438	0.006	0.994
439	0.980	0.020
440	0.292	0.708
441	0.015	0.985
442	0.007	0.993
443	0.082	0.918
444	0.967	0.033
445	0.161	0.839
446	0.011	0.989
447	0.055	0.945
448	0.009	0.991
449	0.015	0.985
450	0.015	0.985
451	0.025	0.975
452	0.914	0.086
453	0.183	0.817
454	0.061	0.939
455	0.048	0.952
456	0.033	0.967
457	0.015	0.985
458	0.014	0.986
459	0.014	0.986
460	0.012	0.988
461	0.023	0.977
462	0.374	0.626
463	0.028	0.972
464	0.010	0.990
465	0.021	0.979

Table 16 (continued)

PROBABILITIES
OF TYPE MEMBERSHIP

	1	2
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466	0.005	0.995
467	0.055	0.945
468	0.071	0.929
469	0.019	0.981
470	0.265	0.735
471	0.999	0.001
472	1.000	0.000
473	1.000	0.000
474	1.000	0.000
475	0.556	0.444
476	0.241	0.759
477	0.132	0.868
478	0.030	0.970

PROBABILITIES
OF TYPE MEMBERSHIP

	1	2
--	---	---

477	0.194	0.806
480	0.037	0.963
481	0.362	0.638
482	0.032	0.968
483	0.051	0.949
484	0.007	0.993
485	0.015	0.985
486	0.029	0.971
487	0.067	0.933
488	0.015	0.985
489	0.054	0.946
490	0.087	0.913
491	0.998	0.002

PROBABILITIES
OF TYPE MEMBERSHIP

	1	2
--	---	---

492	1.000	0.000
493	0.045	0.955
494	0.312	0.688
495	0.171	0.829
496	0.041	0.959
497	0.332	0.668
498	0.035	0.965
499	0.023	0.977
500	0.023	0.977
501	0.785	0.215
502	0.149	0.851
503	0.955	0.045

LOGARITHM OF LIKELIHOOD RATIO OF 2 TO 1 TYPES = 0.189250590 03
 CHI-SQUARE WITH 54 DEGREES OF FREEDOM = 372.86
 PROBABILITY OF NULL HYPOTHESIS = 0.00000000

Table 17 Separation of a multivariate (6) set of Rantoul, Illinois, October 1950-55, upper wind components, zonal and meridional mixed distribution, at the 700-, 500-, and 300-mb levels into three distinct distributions. The units are $m \cdot s^{-1}$.

PROGRAM NORMIX
WOLFE NORMAL MIXTURE ANALYSIS PROCEDURE (1974 REVISION)

SAMPLE = RANTOUL
OCTOBER
700, 500, AND 300MB
X AND Y COMPONENTS

SAMPLE SIZE = 503
NUMBER OF VARIABLES = 6
NUMBER OF TYPES = 3

ITERATION NUMBER 101

LIKELIHOOD OF 3 TYPES IN THIS SAMPLE = -0.72717729D 04

CHARACTERISTICS OF THE WHOLE SAMPLE

MEANS					
1	2	3	4	5	6
7.3232	-0.9486	10.6700	-2.5704	15.4049	-3.6999
STANDARD DEVIATIONS					
1	2	3	4	5	6
6.6262	6.8921	8.9594	10.3149	13.7082	16.0058
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.2215	0.7525	0.2313	0.5589	0.2311
0.2215	1.0000	0.1138	0.7581	0.1265	0.5757
0.7525	0.1138	1.0000	0.1587	0.7513	0.1977
0.2313	0.7581	0.1587	1.0000	0.1200	0.7856
0.5589	0.1265	0.7513	0.1200	1.0000	0.1266
0.2311	0.5757	0.1977	0.7856	0.1266	1.0000

CHARACTERISTICS OF TYPE 1

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.182

MEANS					
1	2	3	4	5	6
10.5173	-1.7616	16.0647	-2.8166	19.4572	-3.5912
STANDARD DEVIATIONS					
1	2	3	4	5	6
7.4715	9.1043	12.1651	17.3130	19.5470	29.2311
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.2932	0.6046	0.3978	0.5377	0.3250
0.2932	1.0000	-0.0233	0.7480	0.0540	0.6252
0.6046	-0.0233	1.0000	0.1336	0.7194	0.1719
0.3978	0.7480	0.1336	1.0000	0.0917	0.7906
0.5377	0.0540	0.7194	0.0917	1.0000	0.0573
0.3250	0.6252	0.1719	0.7906	0.0573	1.0000

EIGENVALUES					
1	2	3	4	5	6
1122.2211	0.0936	0.1893	0.0998	0.3370	0.5134
481.4084	0.1910	-0.0373	0.4161	-0.1832	0.7241
107.0029	0.0873	0.4475	-0.2426	0.7334	0.0323
59.8619	0.4557	-0.0371	0.7255	0.2552	-0.4444
29.3601	0.0895	0.8629	0.0824	-0.4746	-0.1131
23.1239	0.8553	-0.1286	-0.4742	-0.1571	0.0274
EIGENVECTORS					
1	2	3	4	5	6
0.7539	0.7241	0.0323	-0.4444	-0.1131	0.0274
0.4807	0.0453	0.0505	0.0405	0.0405	0.0405

CHARACTERISTICS OF TYPE 2

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.183

MEANS					
1	2	3	4	5	6
5.9298	1.1741	11.3642	0.4841	19.9322	-4.8802
STANDARD DEVIATIONS					
1	2	3	4	5	6
4.0452	6.0597	6.8744	4.7102	8.3419	7.2647

Table 17 (continued)

CORRELATIONS					
1	2	3	4	5	6
1.0000	-0.3849	0.7454	-0.0248	0.7052	0.3055
-0.3849	1.0000	-0.5189	0.6927	-0.5375	0.4247
0.7454	-0.5189	1.0000	-0.3651	0.8079	0.0263
-0.0248	0.6927	-0.3651	1.0000	-0.3992	0.7287
0.7052	-0.5375	0.8079	-0.3992	1.0000	-0.1160
0.3055	0.4247	0.0263	0.7287	-0.1160	1.0000

EIGENVALUES					
1	2	3	4	5	6
137.9714	0.2345	0.2674	-0.1184	-0.0908	0.5475
71.7321	-0.3902	0.2424	0.7711	0.3460	-0.0810
16.9294	0.5133	0.2587	-0.0625	0.7630	0.1223
9.8691	-0.2481	0.3673	0.1214	-0.2410	0.6562
5.1576	0.6598	0.2014	0.4947	-0.4725	-0.2137
3.2296	-0.1801	0.7921	-0.3577	-0.0925	-0.4499

EIGENVECTORS					
1	2	3	4	5	6
0.2345	0.2674	-0.1184	-0.0908	0.5475	0.7426
-0.3902	0.2424	0.7711	0.3460	-0.0810	0.2609
0.5133	0.2587	-0.0625	0.7630	0.1223	-0.2621
-0.2481	0.3673	0.1214	-0.2410	0.6562	-0.5478
0.6598	0.2014	0.4947	-0.4725	-0.2137	-0.1023
-0.1801	0.7921	-0.3577	-0.0925	-0.4499	0.0351

CHARACTERISTICS OF TYPE 3

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.635

MEANS					
1	2	3	4	5	6
6.8098	-1.3264	8.9194	-3.3796	12.9361	-3.3911

STANDARD DEVIATIONS					
1	2	3	4	5	6
6.6829	6.2294	7.6649	8.4692	12.2260	11.9304

CORRELATIONS					
1	2	3	4	5	6
1.0000	0.3441	0.8404	0.1994	0.5799	0.1907
0.3441	1.0000	0.3814	0.8169	0.2775	0.6420
0.8404	0.3814	1.0000	0.2586	0.7613	0.2987
0.1994	0.8169	0.2586	1.0000	0.1608	0.8260
0.5799	0.2775	0.7613	0.1608	1.0000	0.2643
0.1907	0.6420	0.2987	0.8260	0.2643	1.0000

EIGENVALUES					
1	2	3	4	5	6
278.7235	0.2436	-0.2469	0.6157	-0.2932	0.2506
154.5198	0.2676	0.1768	0.2977	0.5868	0.6060
37.4431	0.3437	-0.3103	0.4015	-0.2494	-0.3559
22.2485	0.3694	0.4038	0.1879	0.4234	-0.6307
6.6393	0.5399	-0.6093	-0.4785	0.2521	0.0212
6.1806	0.5683	0.5266	-0.2955	-0.5145	0.2124

PROGRAM NORMIX
WOLFE NORMAL MIXTURE ANALYSIS PROCEDURE (1974 REVISION)

SAMPLE = RANTOUL
OCTOBER
700,500, AND 300MB
X AND Y COMPONENTS

PROBABILITIES OF TYPE MEMBERSHIP				PROBABILITIES OF TYPE MEMBERSHIP				PROBABILITIES OF TYPE MEMBERSHIP			
1	2	3		1	2	3		1	2	3	
1	0.249	0.001	0.750	31	0.033	0.000	0.967	61	0.009	0.001	0.991
2	0.004	0.944	0.052	32	0.028	0.000	0.972	62	0.013	0.379	0.607
3	0.001	0.993	0.005	33	0.012	0.011	0.978	63	0.006	0.081	0.913
4	1.000	0.000	0.000	34	0.039	0.000	0.961	64	0.006	0.088	0.907
5	0.001	0.998	0.001	35	0.495	0.000	0.505	65	0.023	0.014	0.963
6	0.015	0.942	0.043	36	0.011	0.006	0.983	66	0.017	0.205	0.778
7	0.032	0.009	0.959	37	0.006	0.535	0.459	67	0.019	0.038	0.943
8	1.000	0.000	0.000	38	0.005	0.675	0.321	68	0.007	0.224	0.769
9	0.027	0.960	0.013	39	0.002	0.772	0.225	69	0.006	0.183	0.811
10	0.906	0.000	0.094	40	0.004	0.618	0.377	70	0.007	0.190	0.803
11	1.000	0.000	0.000	41	0.740	0.000	0.260	71	0.007	0.716	0.277
12	0.007	0.062	0.932	42	0.006	0.536	0.458	72	0.003	0.722	0.275
13	0.008	0.196	0.796	43	0.037	0.009	0.954	73	0.004	0.864	0.133
14	0.005	0.006	0.990	44	0.014	0.379	0.607	74	0.004	0.636	0.361
15	0.241	0.000	0.759	45	0.028	0.005	0.967	75	0.019	0.504	0.477
16	0.960	0.000	0.040	46	0.065	0.001	0.934	76	0.011	0.227	0.762
17	0.041	0.000	0.959	47	0.016	0.125	0.858	77	0.018	0.955	0.027
18	0.116	0.001	0.883	48	0.907	0.000	0.093	78	0.024	0.003	0.973
19	0.198	0.001	0.802	49	0.011	0.000	0.989	79	0.050	0.206	0.743
20	1.000	0.000	0.000	50	0.014	0.000	0.986	80	0.021	0.166	0.813
21	0.008	0.052	0.941	51	0.014	0.000	0.986	81	0.053	0.067	0.881
22	0.102	0.000	0.898	52	0.164	0.000	0.836	82	0.013	0.438	0.549
23	0.219	0.000	0.781	53	0.015	0.000	0.985	83	0.203	0.000	0.796
24	0.014	0.000	0.986	54	1.000	0.000	0.000	84	0.022	0.969	0.009
25	0.003	0.614	0.383	55	0.004	0.000	0.996	85	0.031	0.844	0.125
26	1.000	0.000	0.000	56	0.008	0.000	0.992	86	0.011	0.106	0.883
27	0.034	0.000	0.966	57	0.011	0.000	0.989	87	0.017	0.138	0.845
28	0.013	0.768	0.219	58	0.089	0.000	0.911	88	0.013	0.424	0.563
29	0.087	0.900	0.012	59	0.019	0.000	0.981	89	0.017	0.754	0.229
30	0.267	0.000	0.733	60	0.008	0.000	0.991	90	0.273	0.029	0.698

Table 17 (continued)

PROBABILITIES OF TYPE MEMBERSHIP			
1	2	3	
91	0.108	0.004	0.888
92	0.032	0.008	0.960
93	0.063	0.000	0.937
94	0.063	0.536	0.419
95	0.020	0.011	0.969
96	0.340	0.000	0.659
97	0.030	0.000	0.970
98	0.060	0.811	0.129
99	0.053	0.000	0.947
100	0.020	0.000	0.980
101	0.020	0.000	0.980
102	0.773	0.038	0.189
103	0.989	0.000	0.011
104	0.010	0.001	0.989
105	0.007	0.629	0.364
106	0.048	0.000	0.952
107	0.777	0.000	0.223
108	0.021	0.000	0.979
109	0.012	0.000	0.988
110	0.008	0.000	0.991
111	0.007	0.000	0.993
112	0.003	0.000	0.997
113	0.006	0.000	0.994
114	0.003	0.000	0.997
115	0.071	0.052	0.877
116	0.005	0.006	0.988
117	0.009	0.781	0.210
118	0.052	0.107	0.841
119	0.016	0.009	0.976
120	0.009	0.340	0.652
121	0.014	0.901	0.084
122	0.019	0.000	0.981
123	0.028	0.006	0.966
124	0.149	0.018	0.833
125	0.284	0.702	0.014
126	0.082	0.000	0.918
127	0.360	0.625	0.015
128	0.855	0.000	0.145
129	0.868	0.000	0.132
130	0.009	0.001	0.991
131	0.008	0.000	0.992
132	0.025	0.000	0.975
133	0.059	0.000	0.941
134	0.006	0.002	0.992
135	0.021	0.005	0.974
136	0.010	0.001	0.989
137	0.018	0.001	0.981
138	0.016	0.000	0.984
139	0.023	0.000	0.977
140	0.835	0.000	0.165
141	0.985	0.000	0.015
142	0.030	0.000	0.970
143	0.272	0.000	0.728
144	0.024	0.000	0.976
145	0.166	0.050	0.784
146	0.008	0.943	0.049
147	0.015	0.013	0.971
148	0.007	0.870	0.122
149	0.012	0.030	0.958
150	0.021	0.018	0.962
151	0.058	0.000	0.942
152	0.054	0.000	0.946
153	0.097	0.001	0.902
154	0.179	0.000	0.821
155	0.033	0.000	0.967
156	0.025	0.001	0.973
157	0.032	0.158	0.810
158	0.013	0.010	0.977
159	0.077	0.000	0.923
160	0.008	0.448	0.544
161	0.005	0.007	0.988
162	0.011	0.000	0.988
163	0.039	0.123	0.838
164	0.006	0.723	0.271
165	0.007	0.241	0.752
166	0.002	0.774	0.224
167	0.004	0.947	0.049
168	0.004	0.831	0.164
169	0.002	0.837	0.161
170	0.002	0.838	0.160
171	0.003	0.809	0.187
172	0.106	0.672	0.222
173	0.015	0.292	0.692
174	0.013	0.399	0.588
175	0.012	0.000	0.987
176	0.036	0.000	0.963
177	0.115	0.003	0.881
178	0.008	0.025	0.967
179	0.435	0.000	0.564
180	0.938	0.062	0.000

PROBABILITIES OF TYPE MEMBERSHIP			
1	2	3	
181	1.000	0.000	0.000
182	0.999	0.000	0.001
183	0.954	0.000	0.046
184	0.149	0.000	0.851
185	0.033	0.824	0.143
186	0.023	0.000	0.977
187	0.008	0.292	0.700
188	0.005	0.672	0.323
189	0.004	0.635	0.362
190	0.010	0.062	0.929
191	0.042	0.262	0.695
192	0.049	0.000	0.951
193	0.007	0.042	0.951
194	0.007	0.271	0.722
195	0.014	0.005	0.981
196	0.100	0.000	0.900
197	0.197	0.000	0.803
198	0.024	0.000	0.976
199	0.016	0.000	0.984
200	0.021	0.025	0.954
201	0.556	0.000	0.444
202	0.996	0.000	0.004
203	0.070	0.000	0.930
204	1.000	0.000	0.000
205	0.004	0.000	0.996
206	0.010	0.003	0.987
207	0.005	0.000	0.995
208	0.004	0.000	0.996
209	0.013	0.000	0.987
210	0.003	0.000	0.997
211	0.010	0.000	0.990
212	0.002	0.000	0.998
213	0.073	0.000	0.927
214	0.078	0.000	0.922
215	0.026	0.000	0.974
216	0.255	0.000	0.745
217	0.039	0.000	0.961
218	0.084	0.000	0.916
219	0.017	0.000	0.983
220	0.012	0.000	0.988
221	0.019	0.000	0.981
222	0.007	0.000	0.993
223	0.012	0.000	0.988
224	0.010	0.000	0.989
225	0.010	0.006	0.985
226	0.006	0.051	0.943
227	0.015	0.010	0.975
228	0.003	0.708	0.289
229	0.003	0.671	0.326
230	0.019	0.070	0.911
231	0.139	0.124	0.737
232	0.009	0.485	0.506
233	0.015	0.725	0.260
234	0.009	0.965	0.026
235	0.002	0.872	0.126
236	0.003	0.942	0.055
237	0.056	0.112	0.831
238	0.032	0.839	0.129
239	0.004	0.955	0.040
240	0.002	0.977	0.022
241	0.005	0.927	0.069
242	0.563	0.001	0.436
243	0.002	0.923	0.075
244	0.010	0.206	0.784
245	0.013	0.870	0.117
246	0.067	0.003	0.930
247	0.957	0.000	0.043
248	0.092	0.000	0.907
249	0.026	0.000	0.974
250	0.019	0.000	0.981
251	0.010	0.034	0.956
252	0.003	0.336	0.660
253	0.004	0.747	0.249
254	0.008	0.027	0.964
255	0.050	0.000	0.949
256	0.340	0.000	0.660
257	0.035	0.001	0.964
258	0.149	0.000	0.851
259	0.062	0.000	0.938
260	0.023	0.000	0.977
261	0.025	0.001	0.974
262	0.012	0.653	0.335
263	0.012	0.108	0.880
264	0.039	0.015	0.946
265	0.026	0.003	0.971
266	0.017	0.000	0.983
267	0.057	0.000	0.943
268	0.115	0.000	0.885
269	0.686	0.000	0.314
270	0.011	0.000	0.989

PROBABILITIES OF TYPE MEMBERSHIP			
1	2	3	
271	0.012	0.000	0.988
272	0.007	0.122	0.871
273	0.011	0.000	0.988
274	0.015	0.417	0.568
275	0.014	0.038	0.948
276	0.029	0.006	0.965
277	0.060	0.032	0.908
278	0.068	0.000	0.932
279	0.036	0.070	0.893
280	0.055	0.464	0.481
281	0.010	0.725	0.265
282	0.008	0.784	0.208
283	0.051	0.025	0.925
284	0.012	0.879	0.108
285	0.042	0.907	0.051
286	0.885	0.070	0.045
287	0.269	0.000	0.731
288	0.029	0.000	0.971
289	0.041	0.000	0.959
290	0.016	0.888	0.096
291	0.150	0.000	0.850
292	0.020	0.000	0.980
293	0.941	0.000	0.059
294	0.105	0.646	0.249
295	0.053	0.386	0.582
296	0.107	0.008	0.885
297	0.132	0.000	0.868
298	0.059	0.000	0.941
299	0.310	0.690	0.000
300	0.034	0.945	0.021
301	0.292	0.702	0.006
302	0.065	0.028	0.907
303	0.008	0.947	0.044
304	0.003	0.902	0.095
305	0.016	0.919	0.065
306	0.037	0.000	0.963
307	0.078	0.000	0.922
308	0.015	0.058	0.927
309	0.012	0.406	0.582
310	0.019	0.000	0.981
311	0.043	0.000	0.957
312	0.022	0.001	0.977
313	0.043	0.000	0.957
314	0.036	0.000	0.964
315	0.030	0.000	0.970
316	0.980	0.000	0.020
317	0.395	0.000	0.604
318	0.175	0.001	0.823
319	0.133	0.000	0.867
320	0.432	0.000	0.568
321	0.207	0.338	0.455
322	1.000	0.000	0.000
323	0.033	0.664	0.303
324	0.977	0.003	0.020
325	0.926	0.002	0.072
326	0.227	0.000	0.773
327	0.997	0.000	0.003
328	1.000	0.000	0.000
329	1.000	0.000	0.000
330	1.000	0.000	0.000
331	0.013	0.000	0.987
332	0.968	0.000	0.032
333	1.000	0.000	0.000
334	0.993	0.000	0.007
335	0.989	0.000	0.011
336	1.000	0.000	0.000
337	0.928	0.021	0.050
338	0.773	0.000	0.227
339	0.990	0.000	0.010
340	0.015	0.000	0.985
341	0.009	0.000	0.991
342	0.009	0.001	0.991
343	0.024	0.000	0.976
344	0.006	0.000	0.994
345	0.022	0.000	0.978
346	0.008	0.000	0.992
347	0.010	0.000	0.990
348	0.005	0.000	0.995
349	0.041	0.000	0.959
350	0.014	0.000	0.986
351	0.017	0.000	0.982
352	0.142	0.000	0.858
353	0.023	0.000	0.977
354	0.021	0.004	0.975
355	0.079	0.002	0.919
356	0.050	0.001	0.949
357	0.029	0.001	0.970
358	0.041	0.000	0.959
359	0.023	0.000	0.977
360	0.380	0.000	0.620

Table 17 (continued)

PROBABILITIES OF TYPE MEMBERSHIP				PROBABILITIES OF TYPE MEMBERSHIP				PROBABILITIES OF TYPE MEMBERSHIP			
1	2	3		1	2	3		1	2	3	
361	0.032	0.937	0.031	409	1.000	0.000	0.000	457	0.015	0.000	0.985
362	0.117	0.295	0.588	410	0.981	0.000	0.019	458	0.015	0.001	0.984
363	0.084	0.000	0.916	411	0.702	0.298	0.000	459	0.018	0.197	0.785
364	0.199	0.000	0.801	412	0.997	0.000	0.003	460	0.008	0.656	0.335
365	0.430	0.000	0.570	413	0.044	0.772	0.184	461	0.038	0.032	0.930
366	0.009	0.011	0.980	414	0.010	0.719	0.271	462	0.039	0.957	0.004
367	0.020	0.926	0.053	415	0.041	0.010	0.949	463	0.039	0.304	0.658
368	0.072	0.005	0.922	416	0.026	0.005	0.968	464	0.010	0.353	0.637
369	0.027	0.032	0.940	417	0.021	0.134	0.845	465	0.023	0.583	0.394
370	0.991	0.000	0.009	418	0.011	0.008	0.981	466	0.006	0.126	0.868
371	0.314	0.120	0.565	419	0.012	0.001	0.988	467	0.177	0.029	0.794
372	0.429	0.000	0.571	420	0.024	0.002	0.975	468	0.135	0.000	0.865
373	0.198	0.000	0.802	421	0.007	0.058	0.935	469	0.075	0.063	0.861
374	0.072	0.611	0.317	422	0.010	0.012	0.978	470	0.577	0.000	0.423
375	0.011	0.926	0.063	423	0.136	0.039	0.825	471	1.000	0.000	0.000
376	0.015	0.001	0.984	424	0.053	0.004	0.943	472	1.000	0.000	0.000
377	0.009	0.000	0.991	425	0.222	0.058	0.720	473	1.000	0.000	0.000
378	0.014	0.001	0.986	426	0.993	0.000	0.007	474	1.000	0.000	0.000
379	0.010	0.000	0.990	427	0.096	0.010	0.894	475	0.967	0.000	0.033
380	0.018	0.000	0.982	428	0.183	0.002	0.816	476	0.636	0.000	0.364
381	0.100	0.102	0.798	429	0.999	0.000	0.001	477	0.122	0.000	0.878
382	0.024	0.006	0.970	430	0.009	0.158	0.833	478	0.056	0.000	0.944
383	0.031	0.785	0.184	431	0.022	0.473	0.505	479	0.183	0.000	0.817
384	0.027	0.951	0.022	432	0.008	0.047	0.945	480	0.032	0.000	0.968
385	0.019	0.905	0.076	433	0.005	0.001	0.995	481	0.704	0.206	0.090
386	0.007	0.888	0.105	434	0.008	0.013	0.980	482	0.048	0.000	0.952
387	0.025	0.684	0.291	435	0.003	0.514	0.483	483	0.052	0.000	0.948
388	0.027	0.961	0.012	436	0.012	0.652	0.336	484	0.009	0.070	0.921
389	0.005	0.786	0.209	437	0.002	0.956	0.043	485	0.015	0.766	0.220
390	0.005	0.957	0.038	438	0.003	0.878	0.119	486	0.091	0.015	0.894
391	0.014	0.514	0.472	439	0.998	0.000	0.002	487	0.129	0.300	0.871
392	0.025	0.357	0.618	440	0.891	0.022	0.086	488	0.032	0.024	0.943
393	0.022	0.801	0.177	441	0.072	0.002	0.925	489	0.123	0.000	0.877
394	0.010	0.295	0.695	442	0.005	0.000	0.995	490	0.119	0.000	0.881
395	0.149	0.138	0.712	443	0.064	0.000	0.936	491	1.000	0.000	0.000
396	0.005	0.784	0.212	444	0.896	0.000	0.104	492	1.000	0.000	0.000
397	0.005	0.813	0.182	445	0.185	0.000	0.814	493	0.061	0.089	0.850
398	0.023	0.481	0.495	446	0.008	0.000	0.992	494	0.323	0.001	0.576
399	0.012	0.048	0.939	447	0.038	0.000	0.962	495	0.136	0.001	0.864
400	0.238	0.000	0.762	448	0.007	0.000	0.993	496	0.041	0.006	0.952
401	0.024	0.001	0.975	449	0.011	0.000	0.989	497	0.419	0.000	0.581
402	0.106	0.001	0.894	450	0.012	0.000	0.988	498	0.063	0.000	0.937
403	0.019	0.001	0.980	451	0.043	0.002	0.955	499	0.023	0.000	0.977
404	0.974	0.001	0.024	452	0.894	0.000	0.106	500	0.020	0.300	0.980
405	1.000	0.000	0.000	453	0.101	0.000	0.899	501	0.847	0.000	0.153
406	1.000	0.000	0.000	454	0.062	0.000	0.938	502	0.113	0.000	0.887
407	1.000	0.000	0.000	455	0.082	0.000	0.918	503	0.989	0.008	0.003
408	1.000	0.000	0.000	456	0.027	0.000	0.973				

LOGARITHM OF LIKELIHOOD RATIO OF 3 TO 2 TYPES = 0.57933643D 02
 CHI-SQUARE WITH 54 DEGREES OF FREEDOM = 114.02
 PROBABILITY OF NULL HYPOTHESIS = 0.00000349

Table 18 Separation of a multivariate (6) set of Rantoul, Illinois, October 1950-55, upper wind components, zonal and meridional mixed distribution, at the 700-, 500-, and 300-mb levels into four distributions. The units are $\text{m}\cdot\text{s}^{-1}$.

PROGRAM NORMIX
WOLFE NORMAL MIXTURE ANALYSIS PROCEDURE (1974 REVISION)

SAMPLE = RANTOUL
OCTOBER
700, 500, AND 300MB
X AND Y COMPONENTS

SAMPLE SIZE = 503
NUMBER OF VARIABLES = 6
NUMBER OF TYPES = 4

ITERATION NUMBER 0

LIKELIHOOD OF 4 TYPES IN THIS SAMPLE = -0.727177290 04

CHARACTERISTICS OF THE WHOLE SAMPLE

MEANS					
1	2	3	4	5	6
7.3252	-0.9486	10.6700	-2.5704	15.4049	-3.6999
STANDARD DEVIATIONS					
1	2	3	4	5	6
6.6262	6.8921	8.9594	10.3149	13.7082	16.0058
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.2215	0.7925	0.2313	0.5589	0.2311
0.2215	1.0000	0.1138	0.7581	0.1265	0.5757
0.7925	0.1138	1.0000	0.1587	0.7513	0.1977
0.2313	0.7581	0.1587	1.0000	0.1200	0.7856
0.5589	0.1265	0.7513	0.1200	1.0000	0.1266
0.2311	0.5757	0.1977	0.7856	0.1266	1.0000

CHARACTERISTICS OF TYPE 1

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.406

MEANS					
1	2	3	4	5	6
11.2762	2.4484	14.1460	1.2900	20.3731	5.8172
STANDARD DEVIATIONS					
1	2	3	4	5	6
5.5488	4.8968	7.8421	8.8618	10.7092	13.4027
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.1304	0.6960	0.1793	0.4565	0.0233
0.1304	1.0000	0.0569	0.6547	-0.0423	0.4006
0.6960	0.0569	1.0000	0.1498	0.6851	0.0924
0.1793	0.6547	0.1498	1.0000	0.0492	0.7451
0.4565	-0.0423	0.6851	0.0492	1.0000	-0.0354
0.0233	0.4006	0.0924	0.7451	-0.0354	1.0000

CHARACTERISTICS OF TYPE 2

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.189

MEANS					
1	2	3	4	5	6
3.7585	4.1860	7.5600	2.5486	15.1635	-3.0406
STANDARD DEVIATIONS					
1	2	3	4	5	6
5.5488	4.8968	7.8421	8.8618	10.7092	13.4027
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.1304	0.6960	0.1793	0.4565	0.0233
0.1304	1.0000	0.0569	0.6547	-0.0423	0.4006
0.6960	0.0569	1.0000	0.1498	0.6851	0.0924
0.1793	0.6547	0.1498	1.0000	0.0492	0.7451
0.4565	-0.0423	0.6851	0.0492	1.0000	-0.0354
0.0233	0.4006	0.0924	0.7451	-0.0354	1.0000

CHARACTERISTICS OF TYPE 3

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.310

MEANS					
1	2	3	4	5	6
6.3438	-7.1940	11.4481	-9.7104	16.9286	-14.1947

Table 18 (continued)

STANDARD DEVIATIONS					
1	2	3	4	5	6
5.5408	4.8968	7.8421	8.8618	10.7092	13.4027
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.1304	0.6960	0.1793	0.4565	0.0233
0.1304	1.0000	0.0569	0.6547	-0.0423	0.4006
0.6960	0.0569	1.0000	0.1498	0.6851	0.0924
0.1793	0.6547	0.1498	1.0000	0.0492	0.7451
0.4565	-0.0423	0.6851	0.0492	1.0000	-0.0354
0.0233	0.4006	0.0924	0.7451	-0.0354	1.0000
CHARACTERISTICS OF TYPE 4					
THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.095					
MEANS					
1	2	3	4	5	6
0.7826	-5.2503	-0.4771	-5.9032	-10.1840	-11.3442
STANDARD DEVIATIONS					
1	2	3	4	5	6
5.5408	4.8968	7.8421	8.8618	10.7092	13.4027
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.1304	0.6960	0.1793	0.4565	0.0233
0.1304	1.0000	0.0569	0.6547	-0.0423	0.4006
0.6960	0.0569	1.0000	0.1498	0.6851	0.0924
0.1793	0.6547	0.1498	1.0000	0.0492	0.7451
0.4565	-0.0423	0.6851	0.0492	1.0000	-0.0354
0.0233	0.4006	0.0924	0.7451	-0.0354	1.0000
ITERATION	1	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE			
ITERATION 2	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.75273606D 04
ITERATION 3	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.74215889D 04
ITERATION 4	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.73726356D 04
ITERATION 5	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.73415493D 04
ITERATION 6	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.73171375D 04
ITERATION 7	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72971605D 04
ITERATION 8	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72839862D 04
ITERATION 9	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72759760D 04
ITERATION 10	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72705389D 04
ITERATION 11	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72664532D 04
ITERATION 12	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72632691D 04
ITERATION 13	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72607145D 04
ITERATION 14	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72585786D 04
ITERATION 15	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72567352D 04
ITERATION 16	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72551401D 04
ITERATION 17	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72537853D 04
ITERATION 18	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72526475D 04
ITERATION 19	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72516087D 04
ITERATION 20	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72507490D 04
ITERATION 21	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72499096D 04
ITERATION 22	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72491666D 04
ITERATION 23	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72485810D 04
ITERATION 24	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72481510D 04
ITERATION 25	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72478215D 04
ITERATION 26	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72475945D 04
ITERATION 27	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72474235D 04
ITERATION 28	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72473081D 04
ITERATION 29	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72472458D 04
ITERATION 30	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72472268D 04
ITERATION 31	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72472054D 04
ITERATION 32	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72471811D 04
ITERATION 33	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72471559D 04
ITERATION 34	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72471299D 04
ITERATION 35	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72471031D 04
ITERATION 36	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72470754D 04
ITERATION 37	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72470469D 04
ITERATION 38	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72470175D 04
ITERATION 39	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72469872D 04
ITERATION 40	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72469560D 04
ITERATION 41	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72469240D 04
ITERATION 42	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72468911D 04
ITERATION 43	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72468574D 04
ITERATION 44	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72468229D 04
ITERATION 45	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72467876D 04
ITERATION 46	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72467516D 04
ITERATION 47	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72467149D 04
ITERATION 48	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72466774D 04
ITERATION 49	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72466391D 04
ITERATION 50	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72466000D 04
ITERATION 51	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72465600D 04
ITERATION 52	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72465191D 04
ITERATION 53	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72464774D 04
ITERATION 54	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72464349D 04
ITERATION 55	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72463916D 04
ITERATION 56	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72463476D 04
ITERATION 57	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE				-0.72463029D 04

Table 18 (continued)

ITERATION 58	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723274170 04
ITERATION 59	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723179610 04
ITERATION 60	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723911560 04
ITERATION 61	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723298950 04
ITERATION 62	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723173960 04
ITERATION 63	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723122750 04
ITERATION 64	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723732900 04
ITERATION 65	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723256210 04
ITERATION 66	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723159650 04
ITERATION 67	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723106720 04
ITERATION 68	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723080610 04
ITERATION 69	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723416780 04
ITERATION 70	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723146300 04
ITERATION 71	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723087620 04
ITERATION 72	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723066920 04
ITERATION 73	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723058990 04
ITERATION 74	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723055380 04
ITERATION 75	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723052980 04
ITERATION 76	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723059780 04
ITERATION 77	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723053590 04
ITERATION 78	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723052490 04
ITERATION 79	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723052110 04
ITERATION 80	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723054140 04
ITERATION 81	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723052290 04
ITERATION 82	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051650 04
ITERATION 83	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051830 04
ITERATION 84	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051970 04
ITERATION 85	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051730 04
ITERATION 86	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051690 04
ITERATION 87	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051720 04
ITERATION 88	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051640 04
ITERATION 89	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051600 04
ITERATION 90	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051610 04
ITERATION 91	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051520 04
ITERATION 92	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051470 04
ITERATION 93	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051800 04
ITERATION 94	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051420 04
ITERATION 95	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051280 04
ITERATION 96	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051560 04
ITERATION 97	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723051120 04
ITERATION 98	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723050940 04
ITERATION 99	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723050950 04
ITERATION 100	LOG LIKELIHOOD OF 4 TYPES IN THIS SAMPLE	-0.723050570 04

ITERATION TIME = 5343.00 SECONDS.
TOTAL TIME USED = 11562.01 SECONDS.

ITERATION NUMBER 101

LIKELIHOOD OF 4 TYPES IN THIS SAMPLE = -0.723050570 04

CHARACTERISTICS OF THE WHOLE SAMPLE

MEANS					
1	2	3	4	5	6
7.3252	-0.9486	10.6700	-2.3704	15.4049	-3.6999
STANDARD DEVIATIONS					
1	2	3	4	5	6
6.6262	6.8921	8.9594	10.3149	13.7082	16.0058
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.2215	0.7525	0.2313	0.5589	0.2311
0.2215	1.0000	0.1138	0.7581	0.1265	0.5757
0.7525	0.1138	1.0000	0.1587	0.7513	0.1977
0.2313	0.7581	0.1587	1.0000	0.1200	0.7856
0.5589	0.1265	0.7513	0.1200	1.0000	0.1266
0.2311	0.5757	0.1977	0.7856	0.1266	1.0000

CHARACTERISTICS OF TYPE 1

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.193

MEANS					
1	2	3	4	5	6
11.1041	-0.3525	16.6560	-1.3465	22.8856	-0.6482
STANDARD DEVIATIONS					
1	2	3	4	5	6
7.2047	9.2886	11.3528	17.2976	14.8718	27.5962
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.2647	0.5390	0.3324	0.5169	0.2525
0.2647	1.0000	-0.1035	0.7443	-0.1470	0.6119
0.5390	-0.1035	1.0000	0.0730	0.7250	0.0848
0.3324	0.7443	0.0730	1.0000	-0.0241	0.7911
0.5169	-0.1470	0.7250	-0.0241	1.0000	-0.1123
0.2525	0.6119	0.0848	0.7911	-0.1123	1.0000

Table 18 (continued)

EIGENVALUES	EIGENVECTORS					
	1	2	3	4	5	6
1021.7463	0.0666	0.2519	0.1199	0.1879	0.5660	0.7497
326.0660	0.2057	-0.0519	0.4355	-0.0397	0.6673	-0.5643
104.5090	0.0270	0.5524	-0.1959	0.7483	-0.0513	-0.3054
40.0002	0.4817	0.0627	0.7053	0.1472	-0.4735	0.1439
33.4545	-0.0490	0.7903	0.0619	-0.5999	-0.0621	-0.0729
23.2217	0.8474	-0.0148	-0.5063	-0.1474	0.0608	0.0017

CHARACTERISTICS OF TYPE 2

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.293

MEANS					
1	2	3	4	5	6
6.7197	1.7879	11.3394	0.7926	20.0058	-3.5688
STANDARD DEVIATIONS					
1	2	3	4	5	6
5.2062	5.7219	6.7337	5.1816	8.3725	8.0912
CORRELATIONS					
1	2	3	4	5	6
1.0000	-0.0490	0.7322	0.2324	0.5236	0.4435
-0.0490	1.0000	-0.2795	0.6794	-0.3196	0.4835
0.7322	-0.2795	1.0000	-0.1278	0.7437	0.1412
0.2324	0.6794	-0.1278	1.0000	-0.2774	0.7870
0.5236	-0.3196	0.7437	-0.2774	1.0000	-0.0414
0.4435	0.4835	0.1412	0.7870	-0.0414	1.0000

EIGENVALUES	EIGENVECTORS					
	1	2	3	4	5	6
120.3379	0.3038	0.2911	-0.2839	0.3728	0.6914	-0.3541
102.2809	-0.2543	0.3305	0.7201	0.4626	-0.1243	-0.2796
20.2525	0.5440	0.1697	-0.2136	0.5162	-0.5472	0.2525
13.1203	-0.1758	0.4315	0.0912	0.0344	0.3317	0.8145
6.8706	0.7153	0.0421	0.5396	-0.4197	0.1338	0.0349
4.7389	-0.0663	0.7677	-0.2360	-0.4509	-0.2815	-0.2609

CHARACTERISTICS OF TYPE 3

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.421

MEANS					
1	2	3	4	5	6
6.3830	-2.6212	8.3100	-5.2661	10.5901	-4.5639
STANDARD DEVIATIONS					
1	2	3	4	5	6
6.6569	5.5893	7.5718	7.9899	11.9976	12.0984
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.3045	0.8419	0.1154	0.6065	0.1489
0.3045	1.0000	0.3415	0.7889	0.1293	0.6263
0.8419	0.3415	1.0000	0.1965	0.8310	0.3252
0.1154	0.7889	0.1965	1.0000	0.0264	0.8370
0.6065	0.1293	0.8310	0.0264	1.0000	0.3023
0.1489	0.6263	0.3252	0.8370	0.3023	1.0000

EIGENVALUES	EIGENVECTORS					
	1	2	3	4	5	6
266.8513	0.2422	0.2551	0.6124	-0.4722	-0.1661	0.5007
159.8561	0.2087	-0.1989	0.4059	0.6391	-0.5800	-0.0858
37.9134	0.3663	0.2998	0.3293	-0.1248	0.2361	-0.7721
12.6521	0.3106	-0.4448	0.2560	0.2756	0.7070	0.2558
5.5168	0.5554	0.5782	-0.4125	0.3347	0.0417	0.2707
4.2504	0.5989	-0.5228	-0.3407	-0.4076	-0.2805	-0.0841

CHARACTERISTICS OF TYPE 4

THE PROPORTION OF THE POPULATION FROM THIS TYPE = 0.094

MEANS					
1	2	3	4	5	6
5.6571	-3.2181	6.8328	-3.4952	7.2228	-6.5235
STANDARD DEVIATIONS					
1	2	3	4	5	6
6.3812	6.6990	8.7872	8.7555	18.2393	16.5218
CORRELATIONS					
1	2	3	4	5	6
1.0000	0.3470	0.9558	0.3597	0.4762	0.1332
0.3470	1.0000	0.4550	0.9402	0.6151	0.6943
0.9558	0.4550	1.0000	0.4338	0.4895	0.2214
0.3597	0.9402	0.4338	1.0000	0.5464	0.8140
0.4762	0.6151	0.4895	0.5464	1.0000	0.3738
0.1332	0.6943	0.2214	0.8140	0.3738	1.0000

Table 18 (continued)

EIGENVALUES				EIGENVECTORS			
	1	2	3	4	5	6	
529.7700	0.1370	-0.1791	0.5272	-0.1521	0.5265	0.6091	
204.9783	0.2462	0.0753	0.0656	0.6255	-0.5210	0.5164	
80.2732	0.2120	-0.2016	0.7391	-0.347	-0.4152	-0.4215	
25.7264	0.3267	0.1956	0.1260	0.6207	0.5243	-0.4232	
3.3614	0.6787	-0.6084	-0.3929	-0.1170	0.0120	-0.0310	
1.0011	0.5552	0.7164	-0.0352	-0.4105	-0.0634	0.0685	

PROBABILITIES OF TYPE MEMBERSHIP				PROBABILITIES OF TYPE MEMBERSHIP				PROBABILITIES OF TYPE MEMBERSHIP						
1	2	3	4	1	2	3	4	1	2	3	4			
1	0.622	0.051	0.327	0.000	81	0.100	0.255	0.645	0.000	161	0.011	0.228	0.761	0.000
2	0.007	0.992	0.001	0.000	82	0.027	0.872	0.012	0.088	162	0.021	0.891	0.088	0.000
3	0.008	0.991	0.002	0.000	83	0.974	0.020	0.004	0.003	163	0.009	0.984	0.007	0.000
4	1.000	0.000	0.000	0.000	84	0.064	0.933	0.003	0.000	164	0.006	0.978	0.016	0.000
5	0.004	0.996	0.000	0.000	85	0.077	0.838	0.084	0.000	165	0.015	0.993	0.383	0.010
6	0.022	0.978	0.000	0.000	86	0.020	0.786	0.193	0.000	166	0.004	0.919	0.015	0.062
7	0.036	0.031	0.933	0.001	87	0.036	0.786	0.163	0.015	167	0.011	0.879	0.000	0.109
8	1.000	0.000	0.000	0.000	88	0.028	0.803	0.019	0.150	168	0.005	0.989	0.005	0.000
9	0.060	0.935	0.005	0.000	89	0.024	0.932	0.043	0.000	169	0.004	0.919	0.027	0.050
10	0.699	0.000	0.008	0.293	90	0.375	0.406	0.016	0.203	170	0.004	0.952	0.012	0.032
11	1.000	0.000	0.000	0.000	91	0.079	0.029	0.893	0.000	171	0.006	0.965	0.003	0.027
12	0.006	0.072	0.837	0.085	92	0.029	0.019	0.942	0.010	172	0.232	0.707	0.000	0.062
13	0.018	0.469	0.321	0.191	93	0.973	0.001	0.327	0.000	173	0.037	0.619	0.332	0.012
14	0.015	0.127	0.839	0.019	94	0.044	0.671	0.001	0.284	174	0.017	0.778	0.041	0.164
15	0.002	0.000	0.000	0.998	95	0.032	0.169	0.797	1.001	175	0.022	0.006	0.960	0.013
16	0.991	0.000	0.008	0.000	96	0.946	0.012	0.042	0.000	176	0.171	0.080	0.749	0.000
17	0.017	0.014	0.004	0.965	97	0.040	0.003	0.957	0.000	177	0.087	0.010	0.902	0.000
18	0.223	0.013	0.764	0.000	98	0.064	0.893	0.043	0.000	178	0.012	0.078	0.774	0.135
19	0.366	0.046	0.589	0.000	99	0.056	0.000	0.943	0.000	179	0.796	0.005	0.188	0.010
20	1.000	0.000	0.000	0.000	100	0.018	0.026	0.845	0.111	180	0.949	0.051	0.000	0.000
21	0.011	0.841	0.147	0.001	101	0.016	0.008	0.881	0.094	181	0.997	0.003	0.000	0.000
22	0.059	0.000	0.158	0.783	102	0.847	0.047	0.106	0.000	182	1.000	0.000	0.000	0.000
23	0.307	0.000	0.692	0.001	103	0.996	0.003	0.001	0.000	183	0.988	0.000	0.012	0.000
24	0.030	0.002	0.967	0.000	104	0.011	0.005	0.884	0.100	184	0.157	0.000	0.843	0.000
25	0.006	0.758	0.195	0.041	105	0.012	0.559	0.328	0.001	185	0.054	0.934	0.012	0.000
26	0.000	0.000	0.000	1.000	106	0.016	0.009	0.689	0.286	186	0.021	0.000	0.935	0.044
27	0.132	0.124	0.743	0.001	107	0.097	0.000	0.882	0.021	187	0.012	0.533	0.390	0.065
28	0.011	0.984	0.005	0.001	108	0.004	0.000	0.957	0.039	188	0.010	0.803	0.180	0.007
29	0.155	0.844	0.001	0.000	109	0.007	0.000	0.529	0.464	189	0.007	0.688	0.284	0.022
30	0.820	0.000	0.180	0.000	110	0.004	0.002	0.833	0.161	190	0.010	0.066	0.778	0.146
31	0.025	0.003	0.972	0.000	111	0.008	0.000	0.978	0.013	191	0.072	0.668	0.258	0.002
32	0.015	0.000	0.985	0.000	112	0.003	0.001	0.995	0.002	192	0.024	0.000	0.975	0.001
33	0.013	0.061	0.894	0.032	113	0.006	0.000	0.978	0.015	193	0.006	0.158	0.697	0.139
34	0.052	0.000	0.948	0.000	114	0.023	0.000	0.403	0.574	194	0.008	0.411	0.502	0.079
35	0.729	0.000	0.271	0.000	115	0.128	0.414	0.400	0.059	195	0.013	0.024	0.815	0.148
36	0.008	0.085	0.724	0.184	116	0.009	0.035	0.899	0.057	196	0.005	0.000	0.992	0.003
37	0.011	0.497	0.478	0.014	117	0.014	0.901	0.085	0.000	197	0.153	0.000	0.847	0.000
38	0.007	0.937	0.054	0.001	118	0.083	0.447	0.011	0.458	198	0.023	0.000	0.951	0.026
39	0.005	0.881	0.024	0.091	119	0.031	0.078	0.815	0.076	199	0.016	0.001	0.981	0.002
40	0.005	0.968	0.024	0.003	120	0.010	0.568	0.352	0.070	200	0.032	0.095	0.861	0.011
41	0.944	0.044	0.012	0.000	121	0.028	0.950	0.022	0.000	201	0.026	0.000	0.011	0.963
42	0.012	0.817	0.025	0.146	122	0.019	0.000	0.981	0.000	202	1.000	0.000	0.000	0.000
43	0.038	0.153	0.580	0.229	123	0.032	0.483	0.415	0.071	203	0.139	0.000	0.855	0.006
44	0.021	0.638	0.236	0.106	124	0.093	0.681	0.142	0.084	204	1.000	0.000	0.000	0.000
45	0.056	0.720	0.224	0.000	125	0.416	0.583	0.001	0.000	205	0.006	0.000	0.994	0.000
46	0.096	0.022	0.869	0.013	126	0.073	0.001	0.926	0.000	206	0.017	0.066	0.917	0.000
47	0.032	0.757	0.186	0.025	127	0.549	0.450	0.001	0.000	207	0.006	0.000	0.992	0.002
48	0.962	0.000	0.038	0.000	128	0.031	0.000	0.002	0.967	208	0.007	0.000	0.993	0.000
49	0.009	0.001	0.876	0.113	129	0.005	0.000	0.002	0.993	209	0.018	0.000	0.982	0.000
50	0.016	0.001	0.956	0.027	130	0.014	0.012	0.975	0.000	210	0.004	0.001	0.996	0.000
51	0.012	0.000	0.980	0.007	131	0.005	0.000	0.949	0.046	211	0.006	0.001	0.017	0.976
52	0.424	0.000	0.576	0.000	132	0.005	0.000	0.544	0.450	212	0.007	0.001	0.977	0.016
53	0.007	0.000	0.943	0.050	133	0.012	0.000	0.109	0.879	213	0.510	0.420	0.070	0.000
54	1.000	0.000	0.000	0.000	134	0.006	0.007	0.958	0.030	214	0.008	0.000	0.543	0.448
55	0.004	0.000	0.863	0.132	135	0.030	0.007	0.938	0.026	215	0.002	0.000	0.734	0.263
56	0.008	0.000	0.946	0.046	136	0.014	0.005	0.936	0.024	216	0.018	0.000	0.982	0.000
57	0.008	0.000	0.780	0.212	137	0.026	0.004	0.875	0.095	217	0.006	0.000	0.982	0.012
58	0.147	0.007	0.846	0.000	138	0.009	0.005	0.653	0.333	218	0.009	0.000	0.692	0.298
59	0.004	0.000	0.451	0.545	139	0.019	0.000	0.981	0.000	219	0.011	0.000	0.973	0.016
60	0.004	0.001	0.842	0.153	140	0.003	0.000	0.015	0.983	220	0.007	0.001	0.988	0.004
61	0.003	0.003	0.859	0.135	141	0.000	0.000	0.000	1.000	221	0.023	0.001	0.924	0.052
62	0.025	0.826	0.103	0.046	142	0.024	0.000	0.976	0.000	222	0.007	0.000	0.875	0.118
63	0.009	0.131	0.721	0.119	143	0.540	0.000	0.257	0.203	223	0.010	0.000	0.699	0.291
64	0.009	0.176	0.704	0.111	144	0.030	0.001	0.969	0.000	224	0.009	0.001	0.770	0.220
65	0.045	0.660	0.271	0.024	145	0.071	0.758	0.143	0.028	225	0.007	0.010	0.786	0.196
66	0.020	0.651	0.329	0.000	146	0.017	0.964	0.018	0.000	226	0.006	0.054	0.814	0.126
67	0.027	0.399	0.574	0.000	147	0.037	0.143	0.808	0.012	227	0.028	0.042	0.863	0.067
68	0.009	0.694	0.270	0.027	148	0.013	0.931	0.055	0.000	228	0.008	0.758	0.217	0.018
69	0.009	0.424	0.555	0.012	149	0.029	0.202	0.458	0.312	229	0.006	0.851	0.077	

Table 18 (continued)

PROBABILITIES OF TYPE MEMBERSHIP				
1	2	3	4	
241	0.010	0.923	0.000	0.067
242	0.0793	0.177	0.024	0.000
243	0.005	0.982	0.009	0.004
244	0.016	0.863	0.119	0.003
245	0.028	0.901	0.071	0.000
246	0.055	0.316	0.111	0.518
247	0.972	0.000	0.028	0.000
248	0.155	0.000	0.844	0.001
249	0.036	0.000	0.964	0.000
250	0.025	0.000	0.975	0.000
251	0.017	0.124	0.858	0.001
252	0.006	0.552	0.415	0.026
253	0.009	0.962	0.028	0.001
254	0.016	0.085	0.604	0.295
255	0.236	0.017	0.747	0.000
256	0.933	0.012	0.003	0.052
257	0.104	0.012	0.548	0.337
258	0.183	0.000	0.803	0.014
259	0.096	0.000	0.904	0.000
260	0.034	0.000	0.711	0.254
261	0.032	0.002	0.958	0.008
262	0.017	0.863	0.119	0.001
263	0.011	0.314	0.551	0.123
264	0.035	0.063	0.898	0.005
265	0.017	0.033	0.945	0.005
266	0.004	0.000	0.979	0.016
267	0.004	0.000	0.674	0.323
268	0.032	0.000	0.968	0.000
269	0.143	0.000	0.857	0.000
270	0.005	0.000	0.766	0.229
271	0.004	0.000	0.966	0.029
272	0.007	0.217	0.622	0.154
273	0.010	0.005	0.982	0.003
274	0.029	0.762	0.088	0.121
275	0.018	0.095	0.883	0.005
276	0.033	0.009	0.767	0.191
277	0.060	0.061	0.699	0.180
278	0.083	0.006	0.786	0.125
279	0.043	0.798	0.013	0.146
280	0.039	0.420	0.000	0.540
281	0.019	0.950	0.006	0.026
282	0.013	0.916	0.070	0.000
283	0.043	0.914	0.043	0.000
284	0.022	0.965	0.005	0.008
285	0.091	0.909	0.000	0.001
286	0.947	0.053	0.000	0.001
287	0.961	0.035	0.004	0.000
288	0.067	0.005	0.929	0.000
289	0.272	0.001	0.727	0.000
290	0.022	0.936	0.042	0.000
291	0.412	0.015	0.525	0.049
292	0.025	0.003	0.972	0.000
293	0.989	0.000	0.011	0.000
294	0.109	0.832	0.000	0.059
295	0.049	0.523	0.001	0.427
296	0.122	0.081	0.751	0.046
297	0.124	0.001	0.875	0.000
298	0.078	0.238	0.561	0.123
299	0.806	0.194	0.000	0.000
300	0.042	0.958	0.001	0.000
301	0.125	0.875	0.000	0.000
302	0.162	0.737	0.093	0.008
303	0.018	0.980	0.003	0.000
304	0.006	0.972	0.016	0.006
305	0.052	0.907	0.040	0.000
306	0.059	0.000	0.941	0.000
307	0.074	0.009	0.854	0.263
308	0.020	0.357	0.420	0.203
309	0.014	0.908	0.036	0.042
310	0.016	0.026	0.953	0.005
311	0.083	0.032	0.878	0.007
312	0.051	0.344	0.599	0.006
313	0.126	0.080	0.792	0.002
314	0.070	0.008	0.922	0.000
315	0.075	0.066	0.859	0.000
316	0.995	0.005	0.000	0.000
317	0.299	0.037	0.070	0.593
318	0.245	0.010	0.735	0.010
319	0.887	0.017	0.096	0.000
320	0.943	0.009	0.049	0.000
321	0.180	0.761	0.057	0.001
322	1.000	0.000	0.000	0.000
323	0.073	0.861	0.085	0.000
324	0.806	0.194	0.000	0.000
325	0.934	0.063	0.001	0.000
326	0.543	0.014	0.441	0.000
327	0.984	0.000	0.000	0.016
328	1.000	0.000	0.000	0.000

PROBABILITIES OF TYPE MEMBERSHIP				
1	2	3	4	
329	1.000	0.000	0.000	0.000
330	1.000	0.000	0.000	0.000
331	0.004	0.000	0.950	0.046
332	0.919	0.000	0.081	0.000
333	1.000	0.000	0.000	0.000
334	0.999	0.000	0.001	0.000
335	0.994	0.006	0.000	0.000
336	1.000	0.000	0.000	0.000
337	0.890	0.110	0.000	0.000
338	0.925	0.004	0.071	0.000
339	0.999	0.000	0.001	0.000
340	0.027	0.009	0.964	0.000
341	0.024	0.000	0.911	0.065
342	0.014	0.014	0.966	0.007
343	0.011	0.000	0.433	0.556
344	0.002	0.000	0.820	0.178
345	0.014	0.000	0.402	0.585
346	0.002	0.000	0.674	0.324
347	0.010	0.002	0.949	0.040
348	0.002	0.001	0.883	0.114
349	0.014	0.000	0.258	0.729
350	0.004	0.000	0.882	0.114
351	0.005	0.002	0.921	0.072
352	0.005	0.000	0.995	0.000
353	0.005	0.001	0.957	0.036
354	0.009	0.027	0.854	0.110
355	0.074	0.060	0.824	0.041
356	0.046	0.012	0.941	0.001
357	0.026	0.003	0.944	0.027
358	0.039	0.001	0.924	0.036
359	0.026	0.001	0.836	0.137
360	0.994	0.002	0.004	0.000
361	0.099	0.894	0.000	0.006
362	0.243	0.253	0.500	0.004
363	0.129	0.040	0.829	0.002
364	0.186	0.004	0.805	0.005
365	0.120	0.000	0.043	0.836
366	0.011	0.034	0.951	0.003
367	0.049	0.948	0.003	0.000
368	0.037	0.068	0.000	0.895
369	0.042	0.392	0.328	0.238
370	0.999	0.000	0.001	0.000
371	0.158	0.772	0.058	0.011
372	0.661	0.000	0.339	0.000
373	0.338	0.006	0.655	0.001
374	0.076	0.870	0.054	0.000
375	0.016	0.982	0.002	0.000
376	0.017	0.002	0.913	0.068
377	0.009	0.002	0.987	0.002
378	0.014	0.007	0.829	0.150
379	0.010	0.001	0.942	0.046
380	0.018	0.005	0.870	0.107
381	0.101	0.736	0.161	0.002
382	0.025	0.424	0.469	0.082
383	0.026	0.910	0.064	0.000
384	0.067	0.921	0.012	0.000
385	0.038	0.920	0.042	0.000
386	0.013	0.971	0.005	0.011
387	0.028	0.933	0.009	0.029
388	0.066	0.934	0.000	0.000
389	0.005	0.984	0.008	0.003
390	0.010	0.988	0.002	0.000
391	0.018	0.893	0.089	0.008
392	0.022	0.970	0.009	0.000
393	0.021	0.979	0.000	0.000
394	0.017	0.932	0.047	0.004
395	0.041	0.959	0.000	0.000
396	0.006	0.990	0.003	0.000
397	0.011	0.967	0.018	0.004
398	0.024	0.922	0.054	0.000
399	0.016	0.616	0.366	0.001
400	0.403	0.001	0.596	0.000
401	0.039	0.139	0.822	0.001
402	0.116	0.283	0.294	0.007
403	0.033	0.501	0.464	0.002
404	0.986	0.011	0.003	0.000
405	1.000	0.000	0.000	0.000
406	1.000	0.000	0.000	0.000
407	1.000	0.000	0.000	0.000
408	1.000	0.000	0.000	0.000
409	1.000	0.000	0.000	0.000
410	0.944	0.004	0.052	0.000
411	0.941	0.089	0.000	0.000
412	1.000	0.000	0.000	0.000
413	0.065	0.686	0.001	0.248
414	0.011	0.967	0.005	0.017
415	0.056	0.162	0.735	0.046
416	0.031	0.054	0.863	0.052

PROBABILITIES OF TYPE MEMBERSHIP				
	1	2	3	4
417	0.026	0.234	0.713	0.026
418	0.017	0.117	0.866	0.000
419	0.308	0.017	0.934	0.041
420	0.022	0.003	0.714	0.261
421	0.010	0.097	0.864	0.028
422	0.013	0.159	0.827	0.000
423	0.098	0.891	0.011	0.000
424	0.136	0.830	0.034	0.000
425	0.151	0.846	0.003	0.000
426	1.000	0.000	0.000	0.000
427	0.100	0.013	0.797	0.089
428	0.162	0.002	0.633	0.203
429	1.000	0.000	0.000	0.000
430	0.010	0.288	0.550	0.152
431	0.037	0.730	0.230	0.002
432	0.010	0.061	0.906	0.024
433	0.007	0.007	0.934	0.053
434	0.010	0.097	0.893	0.000
435	0.005	0.886	0.075	0.035
436	0.025	0.694	0.281	0.000
437	0.004	0.990	0.005	0.002
438	0.006	0.963	0.011	0.020
439	0.432	0.000	0.000	0.568
440	0.319	0.680	0.000	0.000
441	0.041	0.958	0.001	0.001
442	0.003	0.000	0.867	0.130
443	0.009	0.000	0.016	0.975
444	0.000	0.000	0.000	1.000
445	0.049	0.001	0.001	0.949
446	0.004	0.000	0.896	0.100
447	0.158	0.000	0.515	0.327
448	0.008	0.000	0.915	0.076
449	0.009	0.000	0.767	0.224
450	0.006	0.000	0.924	0.071
451	0.048	0.021	0.306	0.624
452	0.058	0.000	0.005	0.937
453	0.180	0.000	0.820	0.000
454	0.065	0.000	0.929	0.006
455	0.056	0.005	0.491	0.449
456	0.019	0.000	0.981	0.000
457	0.022	0.001	0.975	0.001
458	0.017	0.003	0.979	0.001
459	0.015	0.599	0.308	0.079
460	0.012	0.846	0.122	0.020
461	0.038	0.657	0.215	0.090
462	0.111	0.887	0.001	0.000
463	0.034	0.448	0.004	0.514
464	0.011	0.583	0.367	0.039
465	0.024	0.650	0.008	0.319
466	0.010	0.330	0.624	0.036
467	0.228	0.288	0.022	0.461
468	0.833	0.133	0.034	0.000
469	0.151	0.842	0.002	0.005
470	0.315	0.002	0.000	0.682
471	1.000	0.000	0.000	0.000
472	1.000	0.000	0.000	0.000
473	1.000	0.000	0.000	0.000
474	1.000	0.000	0.000	0.000
475	0.998	0.000	0.002	0.000
476	0.822	0.000	0.177	0.001
477	0.134	0.000	0.866	0.000
478	0.066	0.000	0.927	0.007
479	0.231	0.000	0.769	0.000
480	0.028	0.000	0.972	0.000
481	0.413	0.582	0.005	0.000
482	0.080	0.000	0.887	0.033
483	0.093	0.000	0.906	0.001
484	0.011	0.238	0.607	0.144
485	0.032	0.942	0.025	0.000
486	0.478	0.188	0.011	0.324
487	0.859	0.076	0.014	0.022
488	0.105	0.335	0.359	0.001
489	0.115	0.000	0.001	0.884
490	0.637	0.002	0.361	0.000
491	1.000	0.000	0.000	0.000
492	1.000	0.000	0.000	0.000
493	0.097	0.723	0.180	0.000
494	0.245	0.222	0.521	0.002
495	0.063	0.017	0.139	0.791
496	0.046	0.156	0.796	0.002
497	0.601	0.012	0.387	0.000
498	0.106	0.002	0.893	0.000
499	0.024	0.000	0.976	0.000
500	0.018	0.000	0.982	0.000
501	0.993	0.000	0.005	0.000
502	0.145	0.000	0.855	0.000
503	0.984	0.015	0.000	0.000

8.3.3.3 Figures and Discussion. Figures 20(a) through (f) illustrate the tabular output of table 16. In (a), under the assumption of only one cluster at each of three levels, i.e., a unimodal bivariate distribution, the ellipse shows the relative size of the distribution at the 700-, 500-, and 300-mb levels. The 0.50 probability ellipses all have more or less the same orientations. In (b), the 700-mb unimodal assumption is illustrated along with the two clusters in the assumed bimodal bivariate distribution. The same procedure is followed in (c) and (d) for the 500- and 300-mb levels. For further comparison, (e) and (f) show the level-to-level comparison for the cluster type 1 and the cluster type 2.

Figures 21(a) through (g) illustrate the tabular output of table 17. Figure 21(a) is the same as figure 20(a). Figure 21(b) shows the 700-mb unimodal distribution of figure 21(a) with the added three-cluster breakout at the 700-mb level. The same procedure follows through figures 21(c) and 21(d). Figures 21(e), (f), and (g) show the comparison of type distributions through the three levels. For example, figure 21(e) shows roughly the same orientation but greatly increased variance with altitudes. The same is true for figures 21(f) and (g).

Figures 22(a) through (h) follow the same procedural pattern as the previous figures. Its comparable table is table 18. Here there are four clusters. Again the initial unimodal distribution is shown in figure 22(a) while (b), (c), and (d) show the four clusters at each level. The following four show the cluster types at the three levels. Though the small numbers may be difficult to read, they appear also in the tables. Of major importance here is the pictorial display of the orientations and sizes of the ellipses or clusters.

The change in orientation with altitude is noted. Some strong change in orientation between types is also noted.

Further work will be done with these distributions to determine the relationship among these clusters, their orientation and dispersion and concurrent weather.

Figure 20 Bivariate distributions of winds in $\text{m}\cdot\text{s}^{-1}$ at Rantoul, Illinois, October 1950-1955 at the 700-, 500-, and 300-mb levels. Two cluster types (1 and 2) are assumed in the total mixed observed distribution ($0.171 + 0.829 = 1.000$). (a) Total distribution, (b) 700-mb mixed, (c) 500-mb mixed, (d) 300-mb mixed, (e) Type 1, and (f) Type 2.

Figure 20 (continued)

Figure 21 Bivariate distributions of winds in $\text{m}\cdot\text{s}^{-1}$ at Rantoul, Illinois, October 1950-1955 at the 700-, 500-, and 300-mb levels. Three cluster types (1, 2, and 3) are assumed in the total mixed observed distribution ($0.182 + 0.183 + 0.635 = 1.000$). (a) Total distribution, (b) 700-mb mixed, (c) 500-mb mixed, (d) 300-mb mixed, (e) Type 1, (f) Type 2, and (g) Type 3.

Figure 21 (continued)

Figure 22 Bivariate distributions of winds in $\text{m}\cdot\text{s}^{-1}$ at Rantoul, Illinois, October 1950-1955 at the 700-, 500-, and 300-mb levels. Four cluster types (1, 2, 3, and 4) are assumed in the total mixed observed distribution ($0.094 + 0.193 + 0.293 + 0.421 \approx 1.000$). (a) Total distribution, (b) 700-mb mixed, (c) 500-mb mixed, (d) 300-mb mixed, (e) Type 1, (f) Type 2, (g) Type 3, and (h) Type 4.

Figure 22 (continued)

8.3.4 Mountain Pass Wind Data Set

8.3.4.1 Input Information.

- a. Stampede Pass, Easton, WA, U.S.A. - latitude $47^{\circ}17'$ north, longitude $121^{\circ}20'$ west, elevation 1206 m.
- b. The period of record is the month of December for the years 1966-1970. The local standard time hours are 0700 and 1300.
- c. The data are surface winds ($\text{m}\cdot\text{s}^{-1}$) and temperatures $^{\circ}\text{C}$.
- d. The number of variables is two then three. The first two are the zonal and meridional components of the wind, positive from the west and south, then third is the temperature in $^{\circ}\text{C}$.
- e. The number in the sample is 310, 155 from 0700 and 155 from 1300 l.s.t.
- f. The minimum number to be accepted into any cluster is one more than the number of variates.
- g. The null hypotheses are made that $(k + 1)$ clusters are not significantly different from the k clusters. The decision probability level selected is 0.01. Rejection of the hypothesis then permits the assumption of $(k + 1)$ clusters.
- h. The first 40 two-dimensional vector entries are set up as the 40 means of 40 separate and individual clusters. These are 40 points in two or three dimensions.
- i. The assumption is made that the covariance matrices are not equal.

8.3.4.2 Tables. Table 19 provides the output data for the two-variable wind component distributions taken in tabular form from the Wolfe (1971b) NORMIX-NORMAP computer routine. Table 20 provides the output data for the three-variable temperature and wind components.

In both tables the mixture proportions, by cluster type, the means, standard deviations, correlation matrices, and the eigenvalue-eigenvector matrices are given. An asterisk indicates the rejection of the null hypothesis that the $(k + 1)$ type is not significantly different from the (k) types.

Table 19

Surface wind statistics for Stampede Pass, Easton, WA, U.S.A. Units are $\text{m}\cdot\text{s}^{-1}$. The period of record is the month of December, 1966-1970. The sample size is 310 taken 155 from each of the local standard time hours 0700 and 1300. The assumption is that the covariance matrices are not the same. The variables are the zonal and meridional components of the wind positive from the west and south, respectively.

Logarithm of the likelihood; chi-square with 10 d.f.; probability of null hypothesis.

(a) 2 to 1 types 50.327187 (a) 99.52 (a) 0.000000000
(b) 3 to 2 types had no final solution

Lambda is the proportional amount assigned to each type. An asterisk indicates rejection of the null hypothesis at the 0.01 level. Minus signs indicate the wind components are from the north or east.

Characteristics of the whole sample.

1 2

Means -0.4915 1.1879
Std. Dev. 5.7123 1.6058
Corr. 1.0000 0.1667
0.1667 1.0000

Two Types With Proportions

Type 1 (0.516)

Means -4.8276 0.9075
Std. Dev. 3.6052 1.6646
Corr. 1.0000 -0.4170
-0.4170 1.0000

Type 2 (0.484)

4.1357 1.4865
3.4644 1.4954
1.0000 0.6036
0.6036 1.0000

Eigen-
Values
13.5690
2.1665

Vectors
0.9746 0.2239
-0.2239 0.9746

Eigen-
Values
12.9175
1.3209

Vectors
0.9597 -0.2810
0.2810 0.9597

Table 20

Surface temperature and wind statistics for Stampede Pass, Easton, WA, U.S.A. Units are $^{\circ}\text{C}$ and $\text{m}\cdot\text{s}^{-1}$. The period of record is the month of December 1966-1970. The sample size is 310 taken 155 from each of the local standard time hour 0700 and 1300. The assumption is that the covariance matrices are not the same. Variable 1 is the temperature. The wind components are positive from the west and south, respectively, as variables 2 and 3.

Logarithm of the likelihood; chi-square with 10 d.f.; probability of null hypothesis.

(a) 2 to 1 types	71.240940	(a) 140.41	(a) 0.000000000
(b) 3 to 2 types	50.982624	(b) 100.32	(b) 0.000000000
(c) 4 to 3 types	30.190515	(c) 59.31	(c) 0.00000264

Characteristics of the whole sample.

	1	2	3	1	2	3
Means	-3.7276	-0.4915	1.1880			
Std. Dev.	4.9093	5.7122	1.6057			
Corr.	1.0000	0.3966	0.1963			
	0.3966	1.0000	0.1667			
	0.1963	0.1667	1.0000			

Two Types With Proportions

	Type 1 (0.361)			Type 2 (0.639)		
Means	-5.7450	-6.3469	1.2950	-2.5879	2.8146	1.1276
Std. Dev.	6.0549	2.6497	1.6848	3.6615	4.0974	1.5561
Corr.	1.0000	0.3404	0.2216	1.0000	0.2585	0.2343
	0.3404	1.0000	-0.2746	0.2585	1.0000	0.5703
	0.2616	-0.2746	1.0000	0.2343	0.5703	1.0000
Eigen-Values		Vectors	Eigen-Values		Vectors	
37.7515	0.9883	-0.1390	-0.1172	0.5191	0.8539	-0.0367
6.6923	0.1724	0.9172	0.3591	0.8287	-0.5133	-0.2229
2.0772	0.0576	-0.3733	0.9259	0.2092	-0.0853	0.9741

Three Types With Proportions

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Table 20 (continued)

Four Types With Proportions

Type 1 (0.243)				Type 2 (0.326)			
	1	2	3		1	2	3
Means	-5.2142	-6.0801	1.4122		-3.3637	-2.5027	0.1948
Std. Dev.	3.8526	2.1293	2.0588		2.8396	3.7895	0.8571
Corr.	1.0000	-0.0634	0.5280		1.0000	0.1975	-0.1654
	-0.0634	1.0000	-0.3357		0.1975	1.0000	-0.7010
	0.5280	-0.3357	1.0000		-0.1654	-0.7010	1.0000
Eigen- Values		Vectors		Eigen- Values		Vectors	
16.3777	0.9390	0.2252	-0.2601	15.3705	0.2837	0.9589	0.0076
5.0933	-0.0827	0.8816	0.4646	7.4239	0.9463	-0.2813	0.1594
2.1440	0.3339	-0.4147	0.8464	0.3639	-0.1550	0.0380	0.9872
Type 3 (0.019)				Type 4 (0.412)			
Means	-26.2038	-10.3733	1.0727		-2.0772	4.8709	1.8429
Std. Dev.	1.3766	3.2741	0.8621		3.9258	3.1108	1.3780
Corr.	1.0000	-0.0819	-0.6418		1.0000	0.1651	0.1695
	-0.0819	1.0000	0.1371		0.1651	1.0000	0.4586
	-0.6418	0.1371	1.0000		0.1695	0.4586	1.0000
Eigen- Values		Vectors		Eigen- Values		Vectors	
10.7529	-0.0452	0.8963	0.4412	16.2040	0.9409	-0.3372	-0.0315
2.2441	0.9981	0.0591	-0.0178	9.3669	0.3222	0.9198	-0.2241
0.3610	0.0420	-0.4395	0.8972	1.4169	0.1046	0.2007	0.9740

8.3.4.3 Figures and Discussion. Figure 23a shows the single cluster distribution versus the two-cluster breakout, each cluster assumed to be bivariate normal. In essence, these assumptions are the unimodal versus the bimodal bivariate distributions. Table 19 provides the statistics for this figure. Clearly, it is seen that the total distribution is not well represented by the single unimodal elliptical bivariate normal distribution with the mean at $(-0.4915, 1.1879) \text{ m}\cdot\text{s}^{-1}$ with east-west and north-south component standard deviations of 5.7123 and 1.6058 $\text{m}\cdot\text{s}^{-1}$, respectively. The ratio is almost four to one.

The total distribution breaks out into two separate distributions, each assumed to be unimodal. The attempt to determine whether the distribution might actually be trimodal rather than bimodal met with no success. Therefore, it is assumed that the bimodal bivariate distribution is a better representation than a unimodal or trimodal bivariate representation. The two modes are east-southeast and west-southwest.

Let us now discuss the location and terrain features of Stampede Pass. Stampede Pass is located in mountainous terrain on the Main Cascade Divide at latitude $47^{\circ}17'$ north and longitude $121^{\circ}20'$ west. The elevation of the ground at the station is about 1206 m. The wind instruments are approximately 11 m higher.

East of the station the ground drops abruptly into the Yakima River Valley, about 600 m down and a little over three km distant. This land and valley fall towards the southeast. The lowest part of Stampede Pass is $5/8$ km north and 30 m lower. There is a ridge $1-1/4$ km south of the station and about 200 m higher. West of the station the land drops rapidly about 600 m over a distance of 6.4 km to the Green River Valley. This land and valley fall toward the southwest. To the north, from west northwest through east, there are ridges and peaks which rise to 1.2- and 2.7-km.

General winds from the west will be channeled from the west southwest up and around the ridge nose then turning to the east southeast and thence southeast. General winds from the east will traverse this same channel but in the opposite sense. It would seem then that a wind distribution through the pass would have to be elliptical. In addition, it would seem that traveling weather systems would create one distribution from the west southwest and one from the east southeast. This agrees with the tabular values (table 19) and the illustration (figure 23a).

Though not presented in either the table or the figure, it should be mentioned that the computer routine did attempt to converge to a solution for the trimodal assumption. The last estimates did indicate a tendency for the east southeast mode to break down into two east southeast modes located on the major axis of the mode shown but centered, one further to the east southeast and one to the west northwest.

Table 20 and figures 23b, c, and d present the situation when the temperature arguments are added to those of the wind. The singularity problem noted above is resolved and the computations indicate definitely that four cluster types are present. The null hypothesis would have been rejected at the 0.00000264

level. The selected decision level was 0.01. Therefore, there is a good probability that five or even six clusters would have been isolated if the computer had been allowed to continue. The option selected was to examine the structure through only four groups.

Figure 23b provides the two-cluster breakout with an illustration of the 0.25 error ellipses of the wind. The mixture proportion as well as the mean temperature ($^{\circ}\text{C}$) associated with each group is shown. As progress is made through the next two subfigures, note that the western group changes only slightly moving farther away as the breakdown continues. Please note the breakout of the eastern group into two groups with the easternmost group being extremely cold ($\sim -26^{\circ}\text{C}$). Note that it then remains relatively fixed while the central group of (b) breaks down into two clusters.

Note again that the easternmost group which comprises only two percent of the total is extremely cold. This leads to two conjectures:

(1) This group is an outlier and the data are bad.

(2) This group is an outlier but is a valid cluster. The output of the data permits identification of the individual datum. The two-percent cluster occurred in the same period of time and actually composed a string of data. The records were checked. The data are correct. Figure 24 is a copy of a National Meteorological Center surface analysis chart for 1200 Greenwich time on December 30. A star marks the approximate location of Stampede Pass. Note the incursion of the cold air mass from Canada. With its increasing cold and speed as seen on prior maps, it is worthwhile to read the comments by Phillips (1969) published in the Climatological Data publication of the National Oceanic and Atmospheric Administration. The temperatures mentioned are in degrees Fahrenheit.

"Washington - December 1968

"Special Weather Summary

"Until near the end of the month, weather systems from over the Pacific moved across the State at frequent intervals. In western Washington, this resulted in measurable precipitation on 23 to 28 days and on 12 to 18 days in eastern Washington.

"West of the Cascades, rather heavy precipitation was recorded on several days, falling as rain in lowlands during the first half of the month, and as snow and rain the latter half. In the mountains, most precipitation fell as snow, with near record depths for December on the ground at the end of the month.

"East of the Cascades, snow began accumulating on the ground the first of the month. After the middle of the month, most agricultural areas in southern counties were covered with 1 to 3 inches of snow. Temperatures were near or above normal for the first half of the month and slightly below normal from the 15th to the 25th.

"An outbreak of very cold arctic air accompanied by strong northerly and northeasterly winds began moving into northern valleys of eastern Washington, and other localities near the Canadian State on the 26th, spreading over most of the State on the 27th. Temperatures continued to fall for 3 days, with the lowest occurring on the 30th. In many respects, this was the most severe outbreak of cold air since the winter of 1949-50. Minimum temperatures dropped below previous records at several stations in eastern Washington and in the Cascades. The -48° recorded at Mazama and Winthrop 1 SW is a new record for the State and -43° at Chesaw 4 NNW is also below previous record of -42° at Deer Park on January 20, 1937. Other stations where minimums dropped below previously recorded low temperatures were: Anatone -32° , Chelan -18° , Colfax 1 NW -33° , Colville Airport -33° , Dayton 1 SW -25° , Holden Village -32° , Lacrosse 3 ESE -34° , Leavenworth 3 S -36° , Methow -37° , Pomeroy -27° , Pullman 2 NW -32° , Republic -38° , Rosalia -29° , Snoqualmie Pass -19° , Stampede Pass -21° , Stevens Pass -25° , Stehekin 3 NW -21° , and Waterville -33° .

"On the 30th, a warmer moist airmass from over the Pacific began moving inland over the colder air near the surface. Snow began falling during the day, becoming heavy at night and continuing through the 31st. West of the Cascades, snow depths in the lowlands ranged from 8 to 15 inches and 24 to 36 inches or more in foothills. In numerous localities, highway traffic was at a near standstill on the 31st and many offices and businesses remained closed.

"Preliminary reports from fruit producing areas indicate the low temperatures caused extensive damage to stone fruits and perhaps some damage to other fruit trees. Most of the winter wheat section was covered with 1 to 3 inches of snow, thus very little freeze damage is expected."

Note the fourth paragraph. Ludlum (1969) discusses this particular feature in Weatherwise on pages 36-37 of the February issue. Dickey and Wing (1963) also discuss the problem of arctic air flowing into the Pacific Northwest.

The above example and discussion point out the usefulness of a program such as this for editing and quality control of multivariate data; i.e., data groups other than one element at a time. Here, an outlier group was isolated but it was a valid group. The authors did not realize that this particular outlier group was embedded in the data set used. Simply, a location and a period were selected where it was thought that the program would successfully and pointedly demonstrate its capability.

Figure 23 Stampede Pass, Easton, Washington, U.S.A.; winds ($\text{m}\cdot\text{s}^{-1}$) and temperatures ($^{\circ}\text{C}$), December 1966-1970, showing breakdown of winds only into groups 2 and 3 from group 1 (a) and breakdown of wind-temperature combination into 2, 3, and 4 groups (b, c, and d).

Figure 24 Selected area of North American chart, 1200Z, Monday, December 3, 1968, NMC analysis. The star represents the approximate position of Stampede Pass, Easton, Washington.

8.3.5 Marine Surface Data Set

8.3.5.1 Input Information.

- a. QSV "C," 52°45' north latitude, 35°30' north longitude
- b. The period of record is the month of February for the years 1964 through 1972.
- c. The data are the 1200 G.C.T., pressures, temperatures, dew points, and surface winds.
- d. The number of variables is five. The wind is broken into the zonal and meridional components. The units are mb, °C, and $\text{m}\cdot\text{s}^{-1}$.
- e. The number in the sample is 251.
- f. The minimum number to be accepted into a sample is six.
- g. The null hypotheses are made that $(k + 1)$ clusters are not significantly different from the k clusters. The decision probability level selected is 0.01. Rejection of the hypothesis then permits the assumption of $(k + 1)$ clusters.
- h. The first 40 five-dimensional vector entries are set up as the 40 means of 40 separate and individual clusters. These are 40 points in five dimensions.
- i. Non-equality of covariances is assumed.

8.3.5.2 Tables - Output Information. The program computes the necessary statistics for two clusters versus one cluster, three clusters versus two clusters, and four clusters versus three clusters. These are taken from the tabular outform of the Wolfe (1971b) NORMIX computer routine.

The output statistics are now shown for the above in tables 21, 22, and 23 even though the null hypothesis is rejected for the last case.

The first variable is the surface atmospheric pressure in mb, the second variable is the dry-bulb air temperature in °C, the third variable is the dew point temperature in °C, while the fourth and fifth variables are the zonal and meridional components of the wind in $\text{m}\cdot\text{s}^{-1}$.

Table 21 1200 GCT Marine observations from Ocean Station C. for the month of February 1964 through 1972. Surface pressure (mb), temperature ($^{\circ}\text{C}$), dew point ($^{\circ}\text{C}$), and zonal and meridional components ($\text{m}\cdot\text{s}^{-1}$). Sample size is 251. The assumption is that the covariance matrices are not the same. Number of variables is 5. Number of types is 2.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 2 to 1 types 56.0886786 (a) 109.27 (a) 0.000000002

Characteristics of the whole sample.

	1	2	3	4	5
Means	1005.9072	5.4825	1.8749	2.0414	1.0315
Std. Dev.	18.1051	2.3600	3.9201	10.3810	8.2983
Corr.	1.0000	0.0494	-0.0379	-0.1026	-0.0258
	0.0494	1.0000	0.8656	-0.6136	0.3518
	-0.0379	0.8656	1.0000	-0.5179	0.3649
	-0.1026	-0.6136	-0.5179	1.0000	-0.1667
	-0.0258	0.3518	0.3649	-0.1667	1.0000

Characteristics of Type 1 Proportion 0.391

Means	1003.7997	3.5541	-1.8868	9.3676	0.6684
Std. Dev.	17.0627	1.8933	2.7773	9.4358	7.1053
Corr.	1.0000	-0.0872	-0.3678	-0.2149	-0.3972
	-0.0872	1.0000	0.6632	-0.4550	0.2966
	-0.3678	0.6632	1.0000	-0.0368	0.4421
	-0.2149	-0.4550	-0.0368	1.0000	-0.0659
	-0.3972	0.2966	0.4421	0.0659	1.0000

Eigen-Values	0.9698	0.0935	Eigen-Vectors	0.0364	-0.0103
306.6460	-0.0081	-0.1100		0.4788	0.8699
87.8493	-0.0615	-0.0649		0.8582	-0.4878
39.2844	-0.1499	0.9526		0.0639	0.0716
7.1223	-0.1819	-0.2600		-0.1698	0.0122
1.0497					

Table 21 (continued)

Characteristics of Type 2 Proportion 0.609

	1	2	3	4	5
Means	1007.2661	6.7228	4.2951	-2.6718	1.2650
Std. Dev.	18.6248	1.7062	2.3078	7.9436	8.9748
Corr.	1.0000	0.0316	-0.0360	0.0461	0.1425
	0.0316	1.0000	0.8233	-0.3347	0.5239
	-0.0360	0.8233	1.0000	-0.2660	0.6019
	0.0461	-0.3347	-0.2660	1.0000	-0.2521
	0.1425	0.5239	0.6019	-0.2521	1.0000

Eigen-
Values

349.0990
93.7420
50.8501
4.4547
0.6223

Vectors

-0.0619
0.0119
0.0583
0.8425
0.5319

0.0136
0.5748
0.7957
0.0531
-0.1827

-0.0064
0.8115
-0.5838
0.0186
0.0137

Table 22 Marine observations from Ocean Station C. February's 12Z 1964 through 1972. Surface pressure (mb), temperature ($^{\circ}\text{C}$), dew point ($^{\circ}\text{C}$), and u and v components ($\text{m}\cdot\text{s}^{-1}$). Sample size is 251. Number of variables is 5. Number of types is 3.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 3 to 2 types 34.213746 (a) 66.52 (a) 0.00529292

Characteristics of the whole sample.

	1	2	3	4	5
Means	1005.9072	5.4825	1.8749	2.0414	1.0315
Std. Dev.	18.1051	2.3600	3.9201	10.3810	8.2983
Corr.	1.0000	0.0494	-0.0379	-0.1026	-0.0258
	0.0494	1.0000	0.8656	-0.6136	0.3518
	-0.0379	0.8656	1.0000	-0.5179	0.3649
	-0.1026	-0.6136	0.5179	1.0000	0.1667
	-0.0258	0.3518	0.3649	-0.1667	1.0000

Characteristics of Type 1 Proportion 0.404

Means	1004.3747	3.5415	-1.7947	9.2681	0.5814
Std. Dev.	16.7221	1.8331	2.8160	9.2252	6.6616
Corr.	1.0000	-0.0051	-0.2979	-0.2291	-0.3637
	-0.0051	1.0000	0.6631	-0.4344	0.2463
	-0.2979	0.6631	1.0000	-0.0466	0.4377
	-0.2291	-0.4344	-0.0466	1.0000	-0.0184
	-0.3637	0.2463	0.4377	-0.0184	1.0000

Eigen-

Values

292.9837	0.9721	0.1281	0.1947	0.0230	-0.0123
81.4946	0.0014	-0.1013	0.0607	0.4627	0.8786
37.4986	-0.0517	-0.0665	0.1702	0.8606	-0.4726
7.3821	-0.1641	0.9654	0.1807	0.0647	0.0650
1.0418	-0.1594	-0.1921	0.9470	-0.2014	0.0187

Eigen-

Vectors

0.1947	0.0230	-0.0123
0.0607	0.4627	0.8786
0.1702	0.8606	-0.4726
0.1807	0.0647	0.0650
0.9470	-0.2014	0.0187

Table 22 (continued)

Characteristics of Type 2	1	2	Proportion 0.293
Means	1013.0465		7.6058
Std. Dev.	16.5701		1.7053
Corr.	1.0000		-0.0380
	-0.0380		1.0000
	0.0040		0.7820
	0.5042		-0.0466
	-0.3574		0.4095
Eigen-Values			
290.2247	0.9706		-0.0742
39.5450	-0.0056		0.0677
18.6790	-0.0008		0.0662
5.0617	0.2083		0.7341
0.6686	-0.1205		0.6683

<u>Characteristics of Type 3</u>			<u>Proportion 0.303</u>
Means	1001.0589		6.0145
Std. Dev.	19.1798		1.2040
Corr.	1.0000		-0.3396
	-0.3396		1.0000
	-0.4957		0.6806
	-0.0446		-0.3227
	-0.0783		0.2792
Eigen-			
Values			
369.3045	0.9979		0.0191
66.4312	-0.0215		-0.0524
43.9556	-0.0470		-0.0328
2.2771	-0.0225		0.9976
0.5416	-0.0313		-0.0232

Table 23 Marine observations from Ocean Station C. February's 12Z 1964 through 1972. Surface pressure (mb), temperature ($^{\circ}\text{C}$), dew point ($^{\circ}\text{C}$), and u and v components ($\text{m}\cdot\text{s}^{-1}$). Sample size is 251. The assumption is that the covariance matrices are not the same. Number of variables is 5. Number of types is 4.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 4 to 3 types 26.048976 (a) 50.54 (a) 0.12269608*

Characteristics of the whole sample.

	1	2	3	4	5
Means	1005.9072	5.4825	1.8749	2.0414	1.0315
Std. Dev.	18.1051	2.3600	3.9201	10.3810	8.2983
Corr.	1.0000	0.0494	-0.0379	-0.1026	-0.0250
	0.0494	1.0000	0.8656	-0.6136	0.3518
	-0.0379	0.8656	1.0000	-0.5179	0.3649
	-0.1026	-0.6136	-0.5179	1.0000	-0.1667
	-0.0258	0.3518	0.3649	-0.1667	1.0000

Characteristics of Type 1 Proportion 0.415

Means	1004.6928	3.6113	-1.6696	9.1991	0.6093
Std. Dev.	16.4198	1.8662	2.8892	9.1816	6.5831
Corr.	1.0000	0.0279	-0.2572	-0.2530	-0.3379
	0.0279	1.0000	0.6812	-0.4327	0.2445
	-0.2572	0.6812	1.0000	-0.0553	0.4315
	-0.2530	-0.4327	-0.0553	1.0000	-0.0167
	-0.3379	0.2445	0.4315	-0.0167	1.0000

Eigen-Values		Eigen-Vectors	
283.0719	0.9703	0.1893	-0.0123
79.4998	0.0057	0.0660	0.8794
37.6133	-0.0465	0.1842	-0.4711
7.8690	-0.1853	0.1914	0.0646
1.0262	-0.1485	0.9430	0.0198
			0.0180
			0.4599
			0.8582
			0.0672
			-0.2171

Table 23 (continued)

Characteristics of Type 2	1	2	3	4	5
Means	993.1830	6.9100	4.7324	-11.1843	9.5660
Std. Dev.	15.4344	2.2137	2.1577	5.5325	5.7345
Corr.	1.0000	-0.3363	-0.1864	-0.2038	-0.3080
	-0.3363	1.0000	0.8070	-0.3760	0.3751
	-0.1864	0.8070	1.0000	-0.3469	0.1786
	-0.2038	-0.3760	-0.3469	1.0000	0.3500
	-0.3080	0.3751	0.1786	0.3500	1.0000

Eigen- Eigen-

Values	Vectors
244.1472	0.9858
38.1681	-0.0488
23.8818	-0.0261
4.3911	-0.0857
0.6828	-0.1331
	0.1475
	-0.0649
	-0.0802
	0.7232
	0.6668
	0.0645
	0.3284
	0.2516
	-0.5913
	0.6893
	0.0430
	0.5138
	0.7522
	0.3368
	-0.2344
	0.0182
	0.7884
	-0.6032
	0.0810
	-0.0877

Characteristics of Type 3 Proportion 0.205

Means	1021.2997	7.8968	5.9650	-3.1858	7.5392
Std. Dev.	10.7461	1.1947	1.8153	4.9641	5.6589
Corr.	1.0000	-0.2843	-0.3873	0.1972	-0.3949
	-0.2843	1.0000	0.7539	-0.2296	0.6340
	-0.3873	0.7539	1.0000	-0.1704	0.4406
	0.1972	-0.2296	-0.1704	1.0000	0.1491
	-0.3949	0.6340	0.4406	0.1491	1.0000

Eigen- Eigen-

	Values	Vectors
123.5411	0.9595	0.1429
31.6815	-0.0395	0.0634
18.7018	-0.0715	0.0511
2.6280	0.0930	0.6378
0.3140	-0.2529	0.7524
		0.0214
		0.3614
		0.9056
		0.1115
		-0.1906
		-0.0199
		0.9149
		-0.3898
		0.0507
		-0.0898

Table 23 (continued)

Characteristics of Type 4 Proportion 0.278

	1	2	3	4	5
Means	1001.1050	5.9713	3.0994	0.0923	-6.2485
Std. Dev.	18.5631	1.2205	1.7767	8.2187	5.9934
Corr.	1.0000	-0.3701	-0.5146	-0.0858	-0.0108
	-0.3701	1.0000	0.6727	-0.3130	0.2402
	-0.5146	0.6727	1.0000	-0.0737	0.4216
	-0.0858	-0.3130	-0.0737	1.0000	-0.0087
	-0.0108	0.2402	0.4216	-0.0087	1.0000
Eigen- Values			Eigen- Vectors		
346.2454	0.9974	0.0434	0.0134	0.0557	-0.0046
67.1930	-0.0241	-0.0545	0.0491	0.5311	0.8438
36.5567	-0.0494	-0.0311	0.1275	0.8330	-0.5352
2.1563	-0.0464	0.9968	0.0287	0.0495	0.0302
0.5521	-0.0046	-0.0228	0.9901	-0.1358	0.0262

8.3.5.3 Figures and Discussion. Figure 25 is an attempt to illustrate a part of the output of table 21. The basic two-dimensional representation shows the decomposition of the assumed unimodal distribution into two modes or groups. The probability ellipses represent the area of the central 25 percent of the wind vector origins if the assumption which they represent is valid. As shown in table 21, the probability of the null hypothesis for two groups versus one group not being rejected is very low, i.e., 0.00000002. Therefore, the more valid assumption is that two groups better represent the data set than does one group. The single group centered at (2.0414, 1.0315) is shown for reference.

At each point, the following concurrent values are printed. the mixture proportion, the mean pressure, the mean temperature, and the mean dew point. The two groups appear to have not too different mean pressures but considerably different temperature and dew points.

Figure 26 prepared from table 22 similarly portrays the decomposition of the total group into three groups. It is quite apparent that group 1 is not much different from group 1 of figure 25. It is just as apparent that group 2 of figure 25 really breaks down into groups 2 and 3 as shown in figure 26. The pressures in groups 2 and 3 are quite different though the temperatures and dew points are not too different. The appearance here implies, as does the probability of non-rejection of the null hypothesis being small, that the trimodal representation is better than the bimodal which is in turn better than the unimodal representation.

Figure 27 prepared from table 23 depicts the further decomposition of the data set into four groups. Again, group 1 retains essentially the same mixture proportions and characteristics. The northerly group (group 4) remains almost the same as in the last decomposition. It is group 2 of figure 26 that now breaks down into groups 2 and 3 of figure 27. The pressure differences are remarkable, providing the maximum and minimum pressures for the entire four-cluster configuration. Group 3 has the highest temperatures and dew points of the entire ensemble. The 10-percent mixture proportion of group 2 leaves the impression that this is a real group though the null hypothesis is not rejected. This non-rejection probability of 0.12 implies that at our decision level of 0.01 probability, the trimodal representation is a better representation than the four-cluster configuration. However, it is interesting to conjecture that the differences shown are the difference between storm and non-storm situations with winds from the southeast quadrant.

No other representations are made though any pairing or triplets could be graphed in two and three space, respectively, and could be labeled with the fourth and fifth variable mean values.

The authors consider this to be a good representation of the clustering techniques of the Wolfe NORMIX (1971b) computer routine. It was hoped that this routine would isolate a cluster of six or more data which might be considered to be an outlier cluster which could be examined. There were 251 data. Ten percent is essentially 25 or 26 data in the clusters. The instructions provided to the computer were to collect no less than six data in a cluster. This is a default option which sets the minimum number in any cluster to be one more than the number of variables. In this case, five plus one is six.

Therefore, (a) single, doublet or triplet outlier(s) would not be isolated. In this case, if the mixture proportion ran as low as 2.5 percent, the group would be considered as an outlier group and would be examined to see whether the data were bad because of bad sensors, bad recording, bad entry of data into the archives, or simply a group of valid observations out of a very rare weather situation.

Figure 25 OSV "C" surface distribution of pressure (mb), temperature (°C), dew point (°C), and wind components ($\text{m}\cdot\text{s}^{-1}$), February, 1200 G.C.T., 1964 through 1972; $n = 251$. Covariances are assumed to be unequal. The 0.25 probability ellipses are shown for the wind distribution. The total distribution and the breakout into two clusters are shown. The mixture proportion and the averages, the pressure, the temperature, and the dew point data within each cluster are shown.

Figure 26 OSV "C" surface distribution of pressure (mb), temperature ($^{\circ}\text{C}$), dew point ($^{\circ}\text{C}$), and wind components ($\text{m}\cdot\text{s}^{-1}$), February, 1200 G.C.T., 1964 through 1972; $n = 251$. Covariances are assumed to be unequal. The 0.25 probability ellipses are shown for the wind distribution. The total distribution and the breakout into three clusters are shown. The mixture proportion and the averages, the pressure, the temperature, and the dew point data within each cluster are shown.

Figure 27 OSV "C" surface distribution of pressure (mb), temperature (°C), dew point (°C), and wind components ($\text{m}\cdot\text{s}^{-1}$), February, 1200 G.C.T., 1964 through 1972; $n = 251$. Covariances are assumed to be unequal. The 0.25 probability ellipses are shown for the wind distribution. The total distribution and the breakout into four clusters are shown. The mixture proportion and the averages, the pressure, the temperature, and the dew point data within each cluster are shown.

8.3.6 Radiosonde and Rawinsonde Data Set

8.3.6.1 Input Information.

- a. Balboa (Albrook Field), Canal Zone
- b. The period of record is the month of July 1961-1970.
- c. The data are pressures or height (mb or m) temperatures ($^{\circ}\text{C}$), dew points ($^{\circ}\text{C}$), east-west (u) wind components, and north-south (v) wind components, $\text{m}\cdot\text{s}^{-1}$ positive from the west and south.
- d. The number of variables are 20; the above elements for the surface and the 850-, 700-, and 500-mb levels.
- e. The number in the sample is 259.
- f. The minimum number to be accepted into a sample is 21.
- g. The null hypotheses are made that $(k + 1)$ clusters are not significantly different from the k clusters. The decision probability level selected is 0.01. Rejection of the hypothesis then permits the assumption of $(k + 1)$ clusters.
- h. The first 40 twenty-dimensional vector entries are set up as the 40 vector means of 40 separate and individual clusters. These are 40 points in twenty dimensions.
- i. Two assumptions are made. The first assumption is the equality of covariance matrices. The second assumption is the non-equality of the covariance matrices.

8.3.6.2 Tables and Discussion. Table 24 provides selected output data of the Wolfe (NORMIX) computer routine. The logarithm of the likelihood ratio of 2 to 1 types is 676.30485, the chi-square with 40 degrees of freedom is 1240.33, and the probability of the null hypothesis not being rejected is to seven decimals, 0.0000000. Therefore, the two-cluster configuration is not rejected. The computation failed to converge for the three-cluster versus the two-cluster configuration. Convergence under the assumption of unequal covariance matrices also failed. No figures are provided here as the number of variables is too high. A number of two-dimensional figures, however, could be made from the statistics provided.

In order to determine the capability of the NCC computer (a Univac Series 70) and in view of the previous work, a multivariate problem with 40 element vectors was chosen for Balboa, C.Z. There were eight levels of each radiosonde with five elements, pressure (or height), temperature, dew points, and the east-west and north-south components of the winds. The levels were the surface and the 950-, 900-, 850-, 800-, 700-, 600-, and 500-mb levels. The period chosen was the months of July during 1961-1970. The assumption was made that the covariance matrices were different. The computation failed to converge for a two-group separation. The number of vector elements was reduced to 20. Again the computation failed to converge.

The assumption of different covariance matrices was then replaced by the assumption of equal covariance matrices. The computation converged for a breakout of two groups but failed on three groups when the vectors were composed of 20 elements, five elements from each of four of the levels above, namely the surface, and the 850-, 700-, and 500-mb levels as these were the

only elements believed to have moisture measurements in sufficient quantities to assure enough input data.

Examination of table 24 shows that though there does not appear to be too much difference, there is some. About 95 percent of the data comprise one (Type 1) cluster while 5 percent of the data comprise the other (Type 2) cluster. Cluster 1 is cooler than cluster 2 at all levels through 500 mb, an altitude of roughly 5854 m. Cluster 1 is more moist than cluster 2 at altitudes above about 3,100 m and probably above 2000 m. Cluster 1 exhibits lower wind speeds than does cluster 2 from the surface upwards. The difference ranges from $1.2 \text{ m}\cdot\text{s}^{-1}$ at 1500 m to $1.6 \text{ m}\cdot\text{s}^{-1}$ at 5854 m. Cluster 1 winds shift from east northeast to east by southeast at the highest level while the winds in cluster 2 remain east by northeast throughout the layer. The surface pressure of cluster 1 is about 0.6 mb higher than that of cluster 2.

The types of weather that accompany these groups have not been investigated here. It would be interesting to do so. Balboa, C.Z., data were selected (1) to provide an insight into the lower level atmospheric characteristics, (2) to further understanding of the capability of this program, adopted for use on the Univac 70 at the NCC, to handle multidimensional problems, and (3) to demonstrate the utilization of such a program to consider each radio-sonde observation as a point in multidimensional space.

From the above experience it appears that the program, restricted by the present Univac 70 configuration, can handle 300 input 20-dimensional data problems. The first assumption made should be the assumption of equal covariance matrices.

Use of the program for data at other stations where greater differences may be expected may permit the use of more input data, greater dimensions, and the assumption of unequal covariance matrices.

The minimum number to be accepted into a cluster is always one more than the number of dimensions. In this case the minimum number is 21. This is about 8.1 percent. Cluster Type 2 has only about 5.4 percent. Therefore, it would be advisable to check those observations assigned to Type 2 by the discrimination function for the possibility of all or some of these being outliers. This is not done here as this is the basis for work beyond the scope of this paper.

Table 24 Means and standard deviations for the total set and clusters 1 and 2 of the data for Balboa, C.Z. These data are the pressure or height (mb or m), temperature ($^{\circ}\text{C}$), dew point ($^{\circ}\text{C}$), and the u and v components of the wind ($\text{m}\cdot\text{s}^{-1}$) at the surface, 850-, 700-, and 500-mb levels. The dimensions are 20. Equal covariance matrices are assumed for types 1 and 2.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 2 to 1 types (a) 0.054

Characteristics of the whole sample (units are $\text{m}\cdot\text{s}^{-1}$)

	Type 1			Whole Sample			Type 2		
	Proportion	Std. Dev.	Mean	Proportion	Std. Dev.	Mean	Proportion	Std. Dev.	Mean
Surface-pressure (1)	1002.20	12.18	1002.18	1002.18	12.24	1001.66	1001.66	12.18	1001.66
temperature (2)	23.91	0.92	23.92	23.92	0.92	24.07	24.07	0.92	24.07
dew point (3)	23.14	1.16	23.15	23.15	1.16	23.33	23.33	1.16	23.33
u (4)	1.22	1.20	1.23	1.23	1.20	1.27	1.27	1.20	1.27
v (5)	-1.47	1.01	-1.45	-1.45	1.02	-1.11	-1.11	1.01	-1.11
Height (1)	1500.84	9.39	1499.24	1499.24	11.53	1471.21	1471.21	9.39	1471.21
temperature (2)	17.78	0.96	17.81	17.81	0.97	18.35	18.35	0.96	18.35
dew point (3)	14.76	1.95	14.80	14.80	1.95	15.36	15.36	1.95	15.36
u (4)	-3.96	2.61	-3.99	-3.99	2.62	-4.55	-4.55	2.61	-4.55
v (5)	-2.68	2.39	-2.71	-2.71	2.40	-3.32	-3.32	2.39	-3.32
Height (1)	3141.02	11.05	3139.29	3139.29	13.21	3109.00	3109.00	11.05	3109.00
temperature (2)	9.46	1.02	9.49	9.49	1.03	10.08	10.08	1.02	10.08
dew point (3)	2.82	2.74	2.81	2.81	2.74	2.52	2.52	2.74	2.52
u (4)	-6.28	3.01	-6.34	-6.34	3.02	-7.37	-7.37	3.01	-7.37
v (5)	0.24	2.36	0.20	0.20	2.36	-0.52	-0.52	2.36	-0.52
Height (1)	5852.69	14.42	5852.75	5852.75	14.43	5853.93	5853.93	14.42	5853.93
temperature (2)	-6.67	0.90	-6.79	-6.79	0.91	-6.29	-6.29	0.90	-6.29
dew point (3)	-14.27	4.75	-14.38	-14.38	4.77	-16.36	-16.36	4.75	-16.36
u (4)	-6.16	4.00	-6.23	-6.23	4.01	-7.54	-7.54	4.00	-7.54
v (5)	1.40	3.05	1.36	1.36	3.06	0.59	0.59	3.05	0.59

9. MULTIVARIATE QUALITY ASSURANCE AND CONTROL

The capability of the NORMIX program permits a quick elementary view of principal component analysis and of multivariate quality assurance and control. The techniques of assurance and control are the same. Assurance is for incoming data or material, while control is for data or material processing. Essentially, these are filtering techniques.

Here, we look at the separation of homogeneous subsets out of mixed sets which permits the isolation of "outliers" for examination.

Figure 28 (Crutcher, 1966) is an example of a two-tailed Gaussian filter operating on a set of univariate heterogeneous data to isolate outlying data. This essentially sets up two or more groups or subsets. The progress of the sub-figures (a) through (e) schematically shows how the procedure works on a univariate distribution. The alpha level of rejection is 0.05. In this illustration the main group is isolated and the statistics estimated. The other groups have been isolated for further study as to whether they are valid data.

The NORMIX program clustering techniques essentially go through the same technique. However, the decision (alpha) level chosen for the previous examples was 0.01 probability rather than 0.05.

Use can be made of the univariate Gaussian filter or the NORMIX filter can operate on any univariate set of mixed normal distributions. Refer to the Canton Island, July, 30-mb data set. Figure 15 illustrates by means of 0.50 probability ellipses the breakdown of the total set of data into two subsets with quite different characteristics. (See section 8.3.2.3.) A univariate Gaussian filter could be used on the zonal or meridional components. To demonstrate the application of the NORMIX clustering technique as well as principal component techniques, zonal and meridional components are transformed to uncorrelated pairwise data along the major and minor axes, the two principal axes of the distribution. Prior to and after the rotation, the components are standardized to a zero mean and a variance of 1. See table 25.

Figures 29a and 29b show the frequency distribution computer printer plot of the components along the two principal axes. Figure 29a clearly delineates the two clusters. In the latter case, separation would be most difficult on a one by one basis, but at least the mean and variance can be estimated. The existence of groups with equal mean and variance would not be indicated. Clearly indicated in Figures 29a and 29b are the potential invalid outliers at +1.95 on the major axis and beyond 4.16 on the minor axis. These are simply pointed out to the reader but are not examined for validity.

Figures 30a, 30b, and 30c (Crutcher, 1966) illustrate a Gaussian filter technique in two dimensions. Figure 30a illustrates a sample drawn from a homogeneous bivariate distribution contaminated by three subsets of data. One of the subsets consisting of only a datum is first isolated, eliminated from further immediate consideration, but set aside for investigation as to its validity.

The clear separation along the major axis exhibited by the frequency diagram of figure 30a is shown by the above statistics. The separation along the minor axis is not so clearly demonstrated.

Refer to figure 29a and tables 25 and 26. Table 25 provides the statistics for the components along the major axis of the distribution of winds at Canton Island, U.S.A. and U.K., for July 1954-1964. These are standardized transformed variables and as such are dimensionless in the terms of units as in all figures in this section. The mean of the total distribution is zero and the variance is about 1.0006 to four decimals. As this is a standardized set of data, the mean should be zero and the variance one. The error in the fourth place is a numerical rounding error. The set breaks down into two clusters. The mean of the first group is -1.0420 with a standard deviation of 0.2442. The respective values for the second group are +0.7923 and 0.5127.

The output from the Cohen-Falls (1967) program provides the data in table 26. Note the output from ungrouped data as used in the NORMIX program, table 25. There are slight differences which are attributed to the use of ungrouped data and then grouped data.

The one outlier near 1.95 in figure 30a would not be set aside as a cluster or outlier by the NORMIX program. As indicated elsewhere, the minimum cluster size is always one more than the number of variates.

Table 26 also shows the output of the Cohen-Falls program for the dimensionless wind components along the minor axis as illustrated in figure 29b. It is recognized that, with the exception of the outliers, the mixed distribution has degenerated into a single standardized univariate unimodal normal distribution with a zero mean and variance of one. Note the proportion of two groups 0.998 and 0.002 with only 244 observations. Note the mean of -0.009617 in the first group with the extraordinarily large mean of 7.410603 for the second group. Remember the standard deviation of the entire set is only one. Then look at the untenable negative variance obtained which is printed as x.xxxxx. However, this negative variance indicates the difficulties in arriving at a solution and clearly implies that it is the simple case of a symmetric mixed or compound distribution with equal means and equal variances with proportions equal. Therefore, the decision will be to use the estimates for the total sample as estimates of the two groups (a mean of zero and a variance of one in each case). This is the trivial degenerate case which does provide some computing difficulty but no interpretative difficulty.

The elimination of the outliers along the minor axis and the re-standardization of the data will provide a mean of zero and a variance of one rather than the values given in table 26.

Figures 31a through 31c show the frequency distributions of the components along three principal axes of the Stampede Pass, Easton, Washington, wind and temperature standardized data. These refer to the figure 23. The existence of outliers is clearly demonstrated. Outliers lie beyond -2.56 on the first principal axis, beyond -1.68 on the second principal axis, and in both tails of the third principal axis. Undoubtedly, some of these belong to the extreme low temperature cluster isolated and discussed in section 8.3.4. The flatness

of the distribution on all three axes and the more extreme bimodality exhibited on the third axis show the existence of the groups or clusters already isolated.

Extension of the Gaussian filter to more than one dimension at a time essentially is the standardization of all components along the major axes so that the distribution may be assumed to be spherical (Crutcher, 1966). The distribution of the standardized vectors in n -dimensions then is chi-square with n degrees of freedom. The appropriate decision probability levels then are obtained from any standard text book containing chi-square tables. Alternatively, there are computer routines to compute the required values.

For example, the magnitude of the standardized vector in the one-dimensional case for the rejection of the top and bottom 0.5 percent (0.005 probability) implying non-rejection of the central 99 percent (0.99 probability) will be the square root of the 0.99 chi-square value divided by 1 (the degrees of freedom). This is 2.576 and is, of course, the t -distribution value for a large sample. In the case of a two-dimensional distribution, the magnitude of the standardized bivariate vector is the square root of $9.21/2$ or 2.146. For the three-dimensional case, the vector radius would be 1.944.

The point made here is that a datum lying on the main principal axis, the major axis, has a better chance of being valid than an outlier on only one of the original coordinate axes. The chance for an observation to be incorrect in two variables is much less than the chance for an error in the observation of one variable. An outlying datum on one axis is, of course, projected as zero on the other transformed axes so that an outlier on one axis will not be associated with an outlier on another axis. In fact, its effect is to increase the frequency count at the mean or zero of the other axes. Another way of viewing this is that the rejection ellipse has a better chance of enclosing a possible outlier than the rejection bounds on either of the two original correlated axes.

Figure 30b shows the effect of computing a new filter ellipse once the outlier shown in figure 30a has been eliminated. Now the two small contaminating subsets undetected in the first operation are isolated. Figure 30c then illustrates the last step where all data points remaining are inside the filter ellipse. All rejected data are set aside for investigation as to their validity. Before this check is made, however, these data may be processed to see whether they constitute one or more groups or clusters.

The procedure, of course, brings up the possible censoring or truncation problem. Adequate adjustments and better estimates of the parameters can be made.

There is no discussion here of the applications of higher moments to the problem of outlier detection, isolation, and removal.

Figures 32a through 32t illustrate the frequency distributions of the radio-sonde data discussed in section 8.3.6. This discussion indicated only a slight separation of clusters as contrasted with the stratospheric data of section 8.3.2. The frequency diagrams of figures 32a through 32t also show only a slight separation potential in the flat of platykurtic curves.

Figures 32a through 32t are shown more to demonstrate the outlier problem than the clustering problem. The mathematical requirements of the clustering techniques always specify that the minimum cluster size is one more than the number of variates. Thus, with twenty variables the smallest cluster size is twenty-one. A cluster of this size would really indicate a true valid cluster created by weather conditions or a continuing bias in the procedures or operating conditions. The lone outlier(s) in multi-space might go undetected. However, the frequency distribution along a principal axis or a clustering technique along each of the axes would isolate, identify, and permit removal of the questionable data. Therefore, the frequency distributions of figures 29a and b and 32a through t can be used to look at the few individual outliers. Clustering techniques would have a better chance to show this outlier as there is only one variable or a cluster size of two.

Alt et al. (1973) discuss quality assurance and control in their paper on the use of control charts for multivariable data. Crutcher and Falls (1976) discuss the testing of data sets for multivariate normality. The above two reports and this present report add to the rather meager literature on the subject of multivariate quality assurance and control.

Figure 28 Example of a two-tailed Gaussian filter operating on a set of heterogeneous data to isolate, set aside, and eliminate outlying data. The dark areas show the rejection level, 0.05 (0.025 in each tail). Crutcher (1966).

- Here are the data in histogram form.
- Here are the data with the theoretical fitting curve under the assumption of normality and independence of data. The rejection areas under each tail are shown beyond the 0.025 and 0.975 points. The observation at 75 is rejected.
- Here is the theoretical fitting curve of the data which passed the filter in (b). Again the rejection areas under each tail beyond the 0.025 and 0.975 points are shown. Data beyond 52 are rejected.
- Here is the theoretical fitting curve of the data which passed the filter in (c). Again the rejection areas under each tail beyond the 0.025 and 0.975 points are shown. Data beyond 47 are rejected.
- Here is the theoretical fitting curve for the data which passed the filter in (d). Although the rejection areas are not shown, the singleton counts below 21 and above 25 would be rejected. This rejection is an unwanted rejection but is a penalty which must be accepted at this point. The next filtering step would result in no rejection.

Figure 28 (continued)

Figure 29 Distribution of wind standardized components along the two principal axes of the Canton Island, U.S.A. and U.K., July, 30 mb.

OUTLIER

Figure 30a Schematic illustration of a sample drawn from a homogeneous bivariate distribution contaminated by a lone outlier at (100, 136) and two groups of data at approximately (-24, -12) and (4, 32). The result is a heterogeneous distribution. The ellipse shown is a theoretical 0.95 probability ellipse. The lone outlier will be rejected as not being part of the homogeneous distribution. Crutcher (1966).

Figure 30b Schematic illustration of a sample drawn from a homogeneous bivariate distribution contaminated by two small sets. The result is a heterogeneous sample. The lone outlier of figure 30a has been eliminated as it did not appear within the 0.95 probability ellipse. Here, the two contaminating sets exist outside the 0.95 probability ellipse of this figure and will be eliminated in figure 30c. Crutcher (1966).

Figure 30c Schematic illustration of a sample drawn from a homogeneous bivariate distribution. It exists as a result of the filtering action of the 0.95 probability ellipse illustrated in figures 30a and 30b. Here, the 0.95 probability ellipse contains all sample data points of the remaining group. Crutcher (1966).

STAMPEDE PASS, WASHINGTON
 SURFACE TEMPERATURE (DEGREE C), U AND V WIND COMPONENTS (MPS)
 DECEMBER 1966, 1967, 1968, 1969, 1970
 HOURS 07 AND 13 LOCAL TIME (1ST 155 INPUTS ARE HR 07)

FREQUENCY DISTRIBUTION ALONG AXIS 1	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.32000	
-4.80000	
11X	
-4.48000	
21XX	
-4.16000	
11X	
-3.84000	
11X	
-3.52000	
11X	
-3.20000	
01	
-2.88000	
01	
-2.56000	
11X	
-2.24000	
51XXXXX	
-1.92000	
51XXXXX	
-1.60000	
201XXXXXXXXXXXXXXXXXXXXX	
-1.28000	
241XXXXXXXXXXXXXXXXXXXXX	
-0.96000	
331XXXXXXXXXXXXXXXXXXXXX	
-0.64000	
241XXXXXXXXXXXXXXXXXXXXX	
-0.32000	
351XXXXXXXXXXXXXXXXXXXXX	
-0.00000	
311XXXXXXXXXXXXXXXXXXXXX	
0.32000	
341XXXXXXXXXXXXXXXXXXXXX	
0.64000	
261XXXXXXXXXXXXXXXXXXXXX	
0.96000	
241XXXXXXXXXXXXXXXXXXXXX	
1.28000	
191XXXXXXXXXXXXXXXXXXXXX	
1.60000	
91XXXXXXXXXX	
1.92000	
81XXXXXXXXXX	
2.24000	
41XXXX	
2.56000	
21XX	
2.88000	
01	
3.20000	(a)

FREQUENCY DISTRIBUTION ALONG AXIS 2	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.21000	
-3.36000	
21XX	
-3.15000	
21XX	
-2.94000	
01	
-2.73000	
11X	
-2.52000	
21XX	
-2.31000	
11X	
-2.10000	
01	
-1.89000	
21XX	
-1.68000	
01	
-1.47000	
11X	
-1.26000	
81XXXXXXXX	
-1.05000	
11XXXXXXXXXX	
-0.84000	
251XXXXXXXXXXXXXXXXXXXXX	
-0.63000	
241XXXXXXXXXXXXXXXXXXXXX	
-0.42000	
401XXXXXXXXXXXXXXXXXXXXX	
-0.21000	
341XXXXXXXXXXXXXXXXXXXXX	
0.00000	
311XXXXXXXXXXXXXXXXXXXXX	
0.21000	
321XXXXXXXXXXXXXXXXXXXXX	
0.42000	
321XXXXXXXXXXXXXXXXXXXXX	
0.63000	
161XXXXXXXXXXXXX	
0.84000	
211XXXXXXXXXXXXX	
1.05000	
161XXXXXXXXXXXXX	
1.26000	
71XXXXXX	
1.47000	
21XX	
1.68000	
01	
1.89000	(b)

Figure 31 Distribution of wind and temperature standardized components along the three principal axes of the Stampede Pass, Easton, Washington, December 1968-1970 data.

FREQUENCY DISTRIBUTION ALONG AXIS 3	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL = 0.32000	
-3.84000	
11X	
-3.52000	
01	
-3.20000	
01	
-2.88000	
11X	
-2.56000	
01	
-2.24000	
5XXXXXX	
-1.92000	
3XXXX	
-1.60000	
9XXXXXXXXXX	
-1.28000	
7XXXXXX	
-0.96000	
84XX	
-0.64000	
20XXXXXXXXXXXXXXXXXXXX	
-0.32000	
31XXXXXXXXXXXXXXXXXXXXXXXXXXXX	
-0.00000	
51XX	
0.32000	
19XXXXXXXXXXXXXXXXXXXX	
0.64000	
23XXXXXXXXXXXXXXXXXXXX	
0.96000	
25XXXXXXXXXXXXXXXXXXXX	
1.28000	
12XXXXXXXXXXXX	
1.60000	
8XXXXXXXXXX	
1.92000	
5XXXXXX	
2.24000	
4XXXX	
2.56000	
11X	
2.88000	
01	
3.20000	
01	
3.52000	
01	
3.84000	
11X	
4.16000	(c)

Figure 31 (continued)

FREQUENCY DISTRIBUTION ALONG AXIS 1	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL = 0.1000	
-4.920001	21XX
-4.510001	01
-4.100001	21XX
-3.690001	01
-3.280001	01
-2.870001	01
-2.460001	01
-2.050001	01
-1.640001	01
-1.230001	01
-0.820001	01
-0.410001	01
0.000001	01
0.410001	01
0.820001	01
1.230001	01
1.640001	01
2.050001	01
2.460001	01
2.870001	01
3.280001	01
3.690001	01
4.100001	01
4.510001	01
4.920001	01
5.330001	01

FREQUENCY DISTRIBUTION ALONG AXIS 2	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL = 0.25000	
-3.500001	21XX
-3.250001	01
-3.000001	01
-2.750001	01
-2.500001	01
-2.250001	01
-2.000001	01
-1.750001	01
-1.500001	01
-1.250001	01
-1.000001	01
-0.750001	01
-0.500001	01
-0.250001	01
0.000001	01
0.250001	01
0.500001	01
0.750001	01
1.000001	01
1.250001	01
1.500001	01
1.750001	01
2.000001	01
2.250001	01
2.500001	01
2.750001	01

FREQUENCY DISTRIBUTION ALONG AXIS 3	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL = 0.17000	
-2.550001	21XX
-2.380001	01
-2.210001	01
-2.040001	01
-1.870001	01
-1.700001	01
-1.530001	01
-1.360001	01
-1.190001	01
-1.020001	01
-0.850001	01
-0.680001	01
-0.510001	01
-0.340001	01
-0.170001	01
0.000001	01
0.170001	01
0.340001	01
0.510001	01
0.680001	01
0.850001	01
1.020001	01
1.190001	01
1.360001	01
1.530001	01
1.700001	01

FREQUENCY DISTRIBUTION ALONG AXIS 4	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL = 0.30000	
-5.440001	21XX
-5.100001	01
-4.760001	01
-4.420001	01
-4.080001	01
-3.740001	01
-3.400001	01
-3.060001	01
-2.720001	01
-2.380001	01
-2.040001	01
-1.700001	01
-1.360001	01
-1.020001	01
-0.680001	01
-0.340001	01
0.000001	01
0.340001	01
0.680001	01
1.020001	01
1.360001	01
1.700001	01
2.040001	01
2.380001	01
2.720001	01
3.060001	01

FREQUENCY DISTRIBUTION ALONG AXIS 5	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL = 0.35000	
-3.150001	21XX
-2.800001	01
-2.450001	01
-2.100001	01
-1.750001	01
-1.400001	01
-1.050001	01
-0.700001	01
-0.350001	01
0.000001	01
0.350001	01
0.700001	01
1.050001	01
1.400001	01
1.750001	01
2.100001	01
2.450001	01
2.800001	01
3.150001	01
3.500001	01
3.850001	01
4.200001	01
4.550001	01
4.900001	01
5.250001	01
5.600001	01

FREQUENCY DISTRIBUTION ALONG AXIS 6	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL = 0.36000	
-4.320001	21XX
-3.960001	01
-3.600001	01
-3.240001	01
-2.880001	01
-2.520001	01
-2.160001	01
-1.800001	01
-1.440001	01
-1.080001	01
-0.720001	01
-0.360001	01
0.000001	01
0.360001	01
0.720001	01
1.080001	01
1.440001	01
1.800001	01
2.160001	01
2.520001	01
2.880001	01
3.240001	01
3.600001	01
3.960001	01
4.320001	01
4.680001	01

Figure 32 Distribution of wind, temperature, height, and dew point standardized components along the 20 principal axes of the Balboa, C.Z., July data. Four levels are involved, surface, 850-, 700-, and 500-mb.

FREQUENCY DISTRIBUTION ALONG AXIS Y	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.34000	
-4.08000	11X
-3.74000	0
-3.40000	0
-3.06000	11X
-2.72000	11X
-2.38000	11X
-2.04000	3 XXX
-1.70000	8 XXXXXX
-1.36000	5 XXXXX
-1.02000	17 XXXXXXXXXXXXXXXX
-0.68000	20 XXXXXXXXXXXXXXXX
-0.34000	34 XXXXXXXXXXXXXXXX
0.00000	42 XXXXXXXXXXXXXXXX
0.34000	20 XXXXXXXXXXXXXXXX
0.68000	17 XXXXXXXXXXXXXXXX
1.02000	17 XXXXXXXXXXXXXXXX
1.36000	10 XXXXXXXX
1.70000	4 XXXX
2.04000	0
2.38000	11X
2.72000	0
3.06000	0
3.40000	0
3.74000	0
4.08000	0
4.42000	11X

FREQUENCY DISTRIBUTION ALONG AXIS X	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.22000	
-2.42000	11X
-2.20000	21XX
-1.98000	21XX
-1.76000	6XXXX
-1.54000	10 XXXXXXXXXXX
-1.32000	12 XXXXXXXXXXX
-1.10000	16 XXXXXXXXXXX
-0.88000	23 XXXXXXXXXXX
-0.66000	18 XXXXXXXXXXX
-0.44000	21 XXXXXXXXXXX
-0.22000	28 XXXXXXXXXXX
0.00000	28 XXXXXXXXXXX
0.22000	28 XXXXXXXXXXX
0.44000	28 XXXXXXXXXXX
0.66000	28 XXXXXXXXXXX
0.88000	11 XXXXXXXXXXX
1.10000	11 XXXXXXXXXXX
1.32000	4 XXXX
1.54000	0
1.76000	0
1.98000	0
2.20000	0
2.42000	0
2.64000	0
2.86000	0
3.08000	0

FREQUENCY DISTRIBUTION ALONG AXIS Z	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.29000	
-2.32000	11X
-2.03000	21XX
-1.74000	21XX
-1.45000	11 XXXXXXXXX
-1.16000	16 XXXXXXXXXXX
-0.87000	16 XXXXXXXXXXX
-0.58000	20 XXXXXXXXXXX
-0.29000	20 XXXXXXXXXXX
0.00000	20 XXXXXXXXXXX
0.29000	20 XXXXXXXXXXX
0.58000	20 XXXXXXXXXXX
0.87000	11 XXXXXXXXX
1.16000	11 XXXXXXXXX
1.45000	21XX
1.74000	21XX
2.03000	21XX
2.32000	0
2.61000	0
2.90000	0
3.19000	0
3.48000	0
3.77000	0
4.06000	0
4.35000	0
4.64000	0
4.93000	0

FREQUENCY DISTRIBUTION ALONG AXIS U	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.19000	
-1.71000	21XX
-1.52000	0
-1.33000	0
-1.14000	0
-0.95000	12 XXXXXXXXX
-0.76000	21 XXXXXXXXXXX
-0.57000	28 XXXXXXXXXXX
-0.38000	30 XXXXXXXXXXX
-0.19000	27 XXXXXXXXXXX
0.00000	16 XXXXXXXXX
0.19000	22 XXXXXXXXXXX
0.38000	17 XXXXXXXXXXX
0.57000	17 XXXXXXXXXXX
0.76000	8 XXXXX
0.95000	7 XXXXXXX
1.14000	0
1.33000	0
1.52000	0
1.71000	0
1.90000	0
2.09000	0
2.28000	0
2.47000	0
2.66000	0
2.85000	0
3.04000	0

FREQUENCY DISTRIBUTION ALONG AXIS V	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.27000	
-2.97000	11X
-2.70000	0
-2.43000	0
-2.16000	11X
-1.89000	11X
-1.62000	4 XXXX
-1.35000	9 XXXXXXXXX
-1.08000	20 XXXXXXXXXXX
-0.81000	15 XXXXXXXXXXX
-0.54000	30 XXXXXXXXXXX
-0.27000	45 XXXXXXXXXXX
0.00000	62 XXXXXXXXXXX
0.27000	29 XXXXXXXXXXX
0.54000	22 XXXXXXXXXXX
0.81000	14 XXXXXXXXX
1.08000	12 XXXXXXXXX
1.35000	8 XXXXXXXXX
1.62000	21XX
1.89000	0
2.16000	0
2.43000	0
2.70000	0
2.97000	0
3.24000	0
3.51000	0
3.78000	0

FREQUENCY DISTRIBUTION ALONG AXIS W	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.20000	
-3.20000	11X
-3.00000	0
-2.80000	0
-2.60000	0
-2.40000	0
-2.20000	0
-2.00000	11X
-1.80000	0
-1.60000	4 XXXXX
-1.40000	6 XXXXXXX
-1.20000	10 XXXXXXXXXXX
-1.00000	14 XXXXXXXXXXX
-0.80000	17 XXXXXXXXXXX
-0.60000	27 XXXXXXXXXXX
-0.40000	24 XXXXXXXXXXX
-0.20000	24 XXXXXXXXXXX
0.00000	29 XXXXXXXXXXX
0.20000	26 XXXXXXXXXXX
0.40000	26 XXXXXXXXXXX
0.60000	17 XXXXXXXXXXX
0.80000	17 XXXXXXXXXXX
1.00000	12 XXXXXXXXX
1.20000	4 XXXXX
1.40000	21XX
1.60000	21XX
1.80000	0

Figure 32 (continued)

FREQUENCY DISTRIBUTION ALONG AXIS13	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.27000	
-3.78000	1TX
-3.51000	2TX
-3.24000	0
-2.97000	0
-2.70000	1TX
-2.43000	0
-2.16000	5XXXXX
-1.89000	0
-1.62000	0
-1.35000	4XXXX
-1.08000	3XXXXXX
-0.81000	2XXXXXX
-0.54000	16XXXXXX
-0.27000	23XXXXXX
0.00000	24XXXXXX
0.27000	32XXXXXX
0.54000	33XXXXXX
0.81000	30XXXXXX
1.08000	22XXXXXX
1.35000	7XXXXXX
1.62000	6XXXXXX
1.89000	3XXX
2.16000	0
2.43000	0
2.70000	2TX
2.97000	0
3.24000	0

FREQUENCY DISTRIBUTION ALONG AXIS14	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.27000	
-2.70000	2TX
-2.43000	2TX
-2.16000	2TX
-1.89000	1XXXXXX
-1.62000	0
-1.35000	17XXXXXX
-1.08000	12XXXXXX
-0.81000	21XXXXXX
-0.54000	29XXXXXX
-0.27000	30XXXXXX
0.00000	24XXXXXX
0.27000	21XXXXXX
0.54000	25XXXXXX
0.81000	17XXXXXX
1.08000	1XXXXXX
1.35000	0
1.62000	0
1.89000	3XXX
2.16000	3XXX
2.43000	2XXXX
2.70000	0
2.97000	1TX
3.24000	1TX
3.51000	0
3.78000	0
4.05000	1TX

FREQUENCY DISTRIBUTION ALONG AXIS15	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.31000	
-4.03000	1TX
-3.72000	0
-3.41000	1TX
-3.10000	0
-2.79000	1TX
-2.48000	0
-2.17000	5XXXX
-1.86000	5XXXX
-1.55000	12XXXXXX
-1.24000	10XXXXXX
-0.93000	25XXXXXX
-0.62000	24XXXXXX
-0.31000	34XXXXXX
0.00000	36XXXXXX
0.31000	30XXXXXX
0.62000	23XXXXXX
0.93000	22XXXXXX
1.24000	9XXXXXX
1.55000	4XXXX
1.86000	2TX
2.17000	1XXXXXX
2.48000	0
2.79000	0
3.10000	0
3.41000	1TX
3.72000	0

FREQUENCY DISTRIBUTION ALONG AXIS16	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.18000	
-2.34000	1TX
-2.16000	0
-1.98000	1TX
-1.80000	0
-1.62000	8TX
-1.44000	0
-1.26000	8XXXXXX
-1.08000	10XXXXXX
-0.90000	10XXXXXX
-0.72000	10XXXXXX
-0.54000	22XXXXXX
-0.36000	20XXXXXX
-0.18000	25XXXXXX
0.00000	23XXXXXX
0.18000	26XXXXXX
0.36000	19XXXXXX
0.54000	19XXXXXX
0.72000	16XXXXXX
0.90000	9XXXXXX
1.08000	10XXXXXX
1.26000	10XXXXXX
1.44000	3TX
1.62000	2TX
1.80000	2TX
1.98000	1TX
2.16000	0

FREQUENCY DISTRIBUTION ALONG AXIS17	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.23000	
-3.22000	1TX
-2.99000	0
-2.76000	0
-2.53000	0
-2.30000	0
-2.07000	1TX
-1.84000	3XXX
-1.61000	0XXXXXX
-1.38000	11XXXXXX
-1.15000	8XXXXXX
-0.92000	12XXXXXX
-0.69000	30XXXXXX
-0.46000	34XXXXXX
-0.23000	22XXXXXX
0.00000	28XXXXXX
0.23000	24XXXXXX
0.46000	11XXXXXX
0.69000	18XXXXXX
0.92000	13XXXXXX
1.15000	9XXXXXX
1.38000	9XXXXXX
1.61000	4XXXX
1.84000	1TX
2.07000	2TX
2.30000	2TX
2.53000	1TX

FREQUENCY DISTRIBUTION ALONG AXIS18	
EACH X REPRESENTS 1 SAMPLE(S)	
CLASS INTERVAL= 0.24000	
-2.88000	1TX
-2.64000	1TX
-2.40000	3TX
-2.16000	5XXXX
-1.92000	3TX
-1.68000	0
-1.44000	12XXXXXX
-1.20000	10XXXXXX
-0.96000	27XXXXXX
-0.72000	27XXXXXX
-0.48000	27XXXXXX
-0.24000	27XXXXXX
0.00000	27XXXXXX
0.24000	26XXXXXX
0.48000	26XXXXXX
0.72000	12XXXXXX
0.96000	10XXXXXX
1.20000	4TX
1.44000	1TX
1.68000	0
1.92000	0
2.16000	3TX
2.40000	3TX
2.64000	0
2.88000	1TX
3.12000	0

Figure 32 (continued)

Figure 32 (continued)

Table 25 Separation of standardized transformed components along the major axis of the distribution of the Canton Island, U.S.A. and U.K., winds at the 30-mb level during the July 1954-1964. The sample size is 244. There are no dimensions in terms of units. The assumption is that the variances are not the same. NORMIX Clustering - John Wolfe Program, 1971.

Logarithm of the likelihood; chi-square with 4 d.f.; probability of null hypothesis.

(a) 2 to 1 types 81.046327 (a) 160.43. (a) 0.000000000

Characteristics of the whole sample (dimensionless).

Mean 0.0000
Std. Dev. 1.0003
Corr. 1.0000

Type 1 Proportion 0.432 Type 2 Proportion 0.568

Mean -1.0420 0.7923
Std. Dev. 0.2442 0.5127
Corr. 1.0000 1.0000

Eigen- Eigen-
Values Values
0.0597 0.2628
Vectors 1.0000
Vectors 1.0000

Table 26

Separation of standardized transformed components along the major and minor axis of the distribution of the Canton Island, U.S.A. and U.K., winds at the 30-mb level during the July 1954-1964. The sample size is 244. There are no dimensions in terms of units. The assumption is that the variances are not the same. Estimates in Mixed Normal Distribution, Cohen and Falls Program, 1967.

Major Axis

Characteristics of the whole sample (dimensionless) - grouped data.

Mean -0.000000
 Variance +1.000637
 Std. Dev. 1.000252

Type 1 Proportion 0.493 Type 2 Proportion 0.507

Mean -0.932133 0.948055
 Variance +0.111975 0.121955
 Std. Dev. +0.334614 0.349221

154

Minor Axis

Characteristics of the whole sample (dimensionless) - grouped data.

Mean -0.009180
 Variance +1.002371
 Std. Dev. +1.001184

Type 1 Proportion 0.998 Type 2 Proportion 0.002

Mean -0.009617 +7.410603
 Variance +0.875908 -4.123101
 Std. Dev. +0.93589 X.XXXXXX

10. PREDICTION

In the multimodal multivariate case, prediction first should be to the modes or clusters. Once a mixed multivariate data set has been separated into its various homogeneous parts, prediction within the cluster can be made. The characteristics of the group are known and the appropriate regression equation for each homogeneous group can be developed.

In a time series which may be composed of two or more periodic or aperiodic parts, it will be necessary to determine which components are active at the moment. Prediction is then again straightforward.

The problem still remains as to the deterministic regime operating at the moment. This may require prediction into the appropriate cluster and then prediction within the cluster.

The problem of prediction is an important one considered to be beyond the scope of the present paper. It is one to which the authors intend to return and to provide the necessary procedures to forecast to the cluster then within the cluster. Briefly, this has been touched on in section 3 on Discriminant Techniques.

SUMMARY

Clustering techniques to separate mixed distributions of meteorological data are presented. The techniques and data sets are restricted to multimodal multivariate distributions. Dichotomous or polychotomous and non-normal distributions have not been discussed.

The major element considered is wind. Three specific examples include the situations of the land and sea breeze, the quasi-biennial oscillation, and winds in a pass. Three other examples are continental tropospheric winds, a location on the ocean, and a tropical troposphere. One of the examples does include temperature; another includes temperatures, dew points, and heights of pressure surfaces; another includes pressure, temperature, and dew point; while another includes heights of the pressure surfaces and temperatures.

The electronic computer program available to make the separation of mixtures into their homogeneous parts for weather data effectively does the job.

The technique will be useful in establishing weather or climate groups which can be set aside for study and for use in guidance and forecasting.

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