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Comprehensive Tree Volume Equations for Major Species of New Mexico and Arizona:

I. Results and Methodology

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Merchantable
Volume

Stump
Volume

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Hann, David W., and B. Bruce Bare.

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Table 3.--Forked tree merchantable gross cubic foot volume data base and regression results

Species, National Forest, and regression equations	Independent variables	
	D = D.b.h. in inches	D_m = Top diameter, inside bark, in inches
	H = Total tree height in feet	P = Height to first fork divided by H
	T = 1 + (number of forks)	
	Number of observations	Relative mean squared residual (RMSQR)
White pine on Lincoln, Santa Fe, and Carson $R_{m,f} = C_0 + C_1H$ $C_0 = 0.61725167 + 0.042365845D_m$ $C_1 = 0.00598501 - 0.00073445D_m$ or $R_{m,f} = 1.04518089 - 7.88395198E-07D_m^7 + 8.39079873E-08D_m^8$	101	0.9581
Engelmann spruce-corkbark fir on Lincoln, Santa Fe, and Carson $R_{m,f} = 1.15372135 - 1.51067496E-08D_m^3$	91	
Ponderosa pine on Coconino, Tonto, Lincoln, Santa Fe, and Carson $R_{m,f} = 0.997113216 - 0.023300123D_m + C_0T^2 + C_1P$ $C_0 = 0.00364984 + 0.00079885D_m$ $C_1 = 0.002886816 + 0.02330013D_m$ or $R_{m,f} = 1.01377735 - 1.68401836E-06D_m^5$	937	.9761
Douglas-fir on Lincoln, Santa Fe, and Carson $R_{m,f} = 1.04417090 - 9.53965362E-09D_m^3$	118	
Aspen on Santa Fe and Carson $R_{m,f} = 1.07666448 - 1.18133616E-03D_m^{2.5}$	184	
White fir on Santa Fe, Carson, and Lincoln $R_{m,f} = C_0 + C_1(H/D^2)$ $C_0 = 0.673711298 + 0.055948391D_m$ or $C_1 = 1.709640750 - 0.348134538D_m$ $R_{m,f} = 0.983293776 - 2.40166266E-05D_m^4$	74	.7286

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RESEARCH SUMMARY

Comprehensive tree volume equations and the methodology used to derive them are presented for the major tree species of New Mexico and Arizona. With these equations, the following gross and net volume types can be computed for both unforked and forked trees: total stem cubic foot volume; merchantable cubic foot volume to any top diameter between 3 and 8 inches, inside bark; International 1/4-inch board foot volume to a 6-inch top, inside bark; and Scribner board foot volume to a 6-inch top, inside bark. These equations were developed in such a manner as to eliminate many of the problems of consistency, compatibility, reliability, and flexibility associated with previous sets of volume equations. This approach also allows the user to divide total stem cubic foot volume into top volume, merchantable volume, and stump volume. This last feature is useful for predicting the level of residues after harvesting to various top diameters.

INTRODUCTION

Today, as in the past, the accurate estimation of tree volume is an essential prerequisite for foresters involved with timber management planning, forest surveys, damage appraisal, timber sale preparation, trespass, and condemnation proceedings as well as growth and yield studies. To be of immediate value, volume estimates must be expressed in units of measure related directly to the products derived from the tree. The board foot and the cubic foot are traditional units of measure, although the latter is increasing in importance as utilization of the total tree becomes more common. However, it is also recognized that cubic foot volume, when combined with square feet of circumferential surface and linear feet, provides for more consistent and accurate estimates of a tree's product potential (Grosenbaugh 1954; Davis and others 1962; Bruce 1970). Thus, future needs of foresters may well require estimates in these units of measure as well as the traditional board foot. This paper is concerned solely with tree volume estimation in the Southwest. Surface area equations for selected species in the same area were provided by Hann and McKinney (1975).

All of the early volume tables and equations published for exclusive use in Arizona, New Mexico, or both were for the prediction of unforked, gross Scribner board foot volume to an 8-inch top. When volume equations were developed, the logarithmic method described by Schumacher and Hall (1933) was followed (Hornibrook 1936; Lexen and Thomson 1938a, 1938b; Peterson 1939b, 1939c, 1939d; Lexen and Peterson 1939b, 1939c, 1939d; and Krauch and Peterson 1943). When this method did not work, the alinement chart method was used (Peterson 1939a; Lexen and Peterson 1939a; and Krauch and Peterson 1943). These early equations and tables were for "blackjack" or immature ponderosa pine (Hornibrook 1936; Peterson 1939b), "yellow" or mature ponderosa pine (Hornibrook 1936; Peterson 1939c), combined ponderosa pine (Peterson 1939d), white fir (Lexen and Thomson 1938a; Lexen and Peterson 1939a), southwestern white pines (Lexen and Thomson 1938b; Peterson 1939a), Apache pine (Lexen and Peterson 1939b), Arizona pine (Lexen and Peterson 1939c), Chihuahua pine (Lexen and Peterson 1939d), and Douglas-fir (Krauch and Peterson 1934).

In addition to these, Meyer (1938) published gross volume tables for unforked ponderosa pine derived from data collected in several western States, including the Southwest. Volumes were given in three units of measure--total stem cubic foot, International 1/8-inch, and Scribner board foot.

More recently, the logarithmic method has been used to develop equations for unforked, gross Scribner board foot volume to an 8-inch top in white fir (Peterson 1958) and in Englemann spruce (Peterson 1961), and for unforked, gross cubic foot volume to both a 4- and an 8-inch top in immature and mature ponderosa pine (Gaines and Peterson 1960). Minor (1961) used weighted, least squares regression to develop an equation for unforked, gross cubic foot volume to a 4-inch top in ponderosa pine, whereas Myers (1963) used ordinary, least squares regression with segmented data to develop unforked, gross volume equations for (a) total stem cubic foot volume, (b) merchantable cubic foot volume to a 4-inch and variable top limit, and (c) International 1/4-inch and Scribner board foot volumes to a variable top limit. (Myers used the term "variable top limit"

to indicate that the merchantable top diameter was a fixed function of a tree's diameter breast height and not to indicate that the top diameter was an independent variable in the volume equation. The latter type of equation can be converted to the former by defining the functional relationship between d.b.h. and top diameter.)

The purpose of this study was to develop gross and net tree volume equations for forked and unforked trees of the major tree species found in Arizona and New Mexico. Species-National Forest (sp-NF) specific equations were developed for the following units of measure--total stem cubic foot volume, merchantable cubic foot volume for any top diameter between 3 and 8 inches, and International 1/4-inch and Scribner board foot volumes to a 6.0-inch top. The resulting equations and methodology used to derive these equations are described in this paper. Volume tables for use in the field appear in a companion piece, "Comprehensive tree volume equations for major species of New Mexico and Arizona: II. Tables for unforked trees."

The present study provides a complete set of standardized gross and net volume equations for the major species, forked or unforked, in Arizona and New Mexico. In addition, all equations are derived from a common data base. Further, modeling procedures were adopted to ensure that (a) net volumes do not exceed gross volumes, (b) reasonable board foot-cubic foot ratios prevail, and (c) Scribner board foot volumes do not exceed International 1/4-inch board foot volumes. Merchantable cubic foot volume equations for any top diameter between 3 and 8 inches also are provided, thus permitting users of the equations to adapt to changing merchantability standards.

SOURCE AND NATURE OF DATA

The data were collected from five National Forests of Arizona and New Mexico by field crews of the Division of Timber Management, Southwestern Region, Forest Service, Albuquerque, New Mexico (table 1).

The data for the Coconino, Tonto, and Lincoln National Forests are the same as used by Hann and McKinney (1975) in the development of surface area equations. On the Santa Fe and Carson National Forests, the measurement trees were selected by line sampling with a 40 basal area factor angle. The lines were located and trees designated by inventory field crews traveling to randomly selected continuous forest inventory plots. Later, a second crew felled and measured as many of the designated trees as could be finished in one working day, using procedures similar to those described by Stage and others (1968).

Tree volumes were computed using a modified version of the NETVSL computer program (Stage and others 1968). Merchantable volumes were determined to various top diameters and to a 1-foot stump on all National Forests except the Lincoln, where stump height was 1.2 feet.

Table 1.--Data sources

Species	National Forest					
	: Coconino	: Tonto	: Lincoln	: Santa Fe	: Carson	
Blackjack pine ¹ (<i>Pinus ponderosa</i> Laws.)	X	X	X	X		X
Yellow pine ¹ (<i>Pinus ponderosa</i> Laws.)	X	X	X	X		X
Douglas-fir (<i>Pseudotsuga menziesii</i> var. <i>glauca</i> [Mirb.] Franco)		X	X	X		X
White fir (<i>Abies concolor</i> [Gord. and Glend.]			X	X		X
Southwestern white pine (<i>Pinus flexilis</i> var. <i>reflexa</i> Engelm.)			X	X		X
Engelmann spruce (<i>Picea engelmannii</i> Parry)			X	X		X
Aspen (<i>Populus tremuloides</i> Michx.)				X		X
Corkbark fir (<i>Abies lasiocarpa</i> var. <i>arizonica</i> [Merriam] Lemm.)				X		X

¹In the Southwest, blackjack and yellow pines are distinguished by their bark colors; blackjack pines have very dark bark and yellow pines heavily plated, orange bark. This distinction is based on age, growth, and vigor differences rather than botanical differences, but foresters have found it useful in forest management planning.

RESULTS

Final equations for predicting the various types of tree volume follow, along with an explanation of how to apply them. Details concerning the methodology can be found in the appendixes.

A statistic that will be presented whenever possible is the relative mean-squared residuals (RMSQR) of the final equation. RMSQR is the quotient of mean-squared residuals divided by the variance of the dependent variable and, as such, is an index of fit similar to the coefficient of determination (R^2). In the case of RMSQR, however, perfect fit would result in a value of 0. A fit that would not reduce the squared residuals below the value around the mean would result in a value of 1, and a value greater than 1 would indicate that the model has increased variability. The advantage of RMSQR over R^2 is that only the former will reflect the loss in degrees of freedom that results from adding another independent variable; therefore, it serves as a better measure for comparing equations of different numbers of independent variables to see if the added variable(s) reduced variance about the regression model.

The standard error of the estimate is not reported here because the methodology used precluded the calculation of a meaningful statistic.

Gross volumes.--The appropriate equations for computing gross volume are chosen by the user on a tree-by-tree basis. If the tree is unforked, then the unforked tree volume equation is chosen based on the sp-NF combination of the tree. If the tree is forked, the unforked tree volume is first computed for the tree and then corrected for forking by the appropriate forked tree equation for the sp-NF combination of the tree. These seemingly trivial facts are brought out because the choice of which net volume equation to use is not on a tree-by-tree basis, but instead is determined for an entire data set.

A tree is considered forked if a fork of any severity occurs between breast height and the tip of the tree. For most of the species, two equations are given. The first equation utilizes tree size information only, while the second equation incorporates two independent variables that relate to the severity of forking. In all cases, the second equation has a lower RMSQR value. The position of the first fork variable, P, is defined as the height to the first fork divided by total tree height. The number of tips variable, T, can be counted directly or computed by adding 1 to the number of forks. All forks between breast height and the tip of the tree should be counted regardless of their severity.

Total Stem Gross Cubic Foot Volume--Unforked Trees

The total stem gross cubic foot volume of unforked trees is predicted from the model:

$$V_t = a_0 + a_1 D^2 H$$

where

V_t = Predicted total stem gross cubic foot volume of an unforked tree

D = Diameter at breast height (d.b.h.) in inches

H = Total tree height in feet.

The sp-NF specific, weighted least squares regression coefficients, a_0 and a_1 , are found in table 2, along with the number of trees used in their development and the resulting RMSQR. Methodological details can be found in appendix I.

Table 2.--Unforked tree total stem gross cubic foot volume data base and regression results

Species	National Forest	Number of trees	Regression coefficients		Relative mean squared residual (RMSQR)
			a_0	a_1	
White pine	Lincoln, Santa Fe, Carson	43	0.160888987	0.00203250045	0.0177
Engelmann spruce-corkbark fir	Lincoln, Santa Fe, Carson	155	.225466084	.00216969983	.0329
Yellow pine	Coconino, Tonto, Lincoln, Santa Fe, Carson	201	.237204154	.00221122919	.0292
Blackjack pine	Coconino, Tonto, Lincoln	680	.0810724804	.00198351037	.0266
Blackjack pine	Santa Fe, Carson	220	.0483082948	.00204968419	.0435
Douglas-fir	Lincoln, Tonto	108	.438373815	.00175642739	.0162
Douglas-fir	Santa Fe, Carson	81	.341133398	.00191796994	.0368
Aspen	Santa Fe, Carson	109	.0327	.00231123522	.0199
White fir	Lincoln	43	.210903832	.00183995833	.0213
White fir	Santa Fe, Carson	58	.157776859	.00200912252	.0277

Total Stem Gross Cubic Foot Volume--Forked Trees

The model for total stem gross cubic foot volume of forked trees is

$$V_{t,f} = V_t \times R_{t,f}$$

where

$V_{t,f}$ = Predicted total stem gross cubic foot volume of a forked tree

V_t = Predicted total stem gross cubic foot volume of an unforked tree

$R_{t,f}$ = Predicted ratio of actual total stem gross cubic foot volume in a forked tree divided by predicted total stem gross cubic foot volume in an unforked tree of same dimensions.

The sp-NF specific equations for $R_{t,f}$, their RMSQR, and the number of trees used are found in table 3. An examination of RMSQR values indicates that, in some cases, models are only slightly better than the mean of ratios. Therefore, the mean values for $R_{t,f}$ are also included in table 3 for those who do not feel comfortable using the equations. Methodological details can be found in appendix II.

Table 3.--Forked tree total stem cubic foot volume data base and regression results

Independent variables				
D = D.b.h. in inches				
H = Total tree height in feet				
P = Height to first fork divided by H				
T = 1 + (number of forks)				
Species, National Forest, and regression equations		: Mean : $R_{t,f}$:	: Number : of : trees :	: Relative : mean squared : residual : (RMSQR)
White pine on Santa Fe, Carson, and Lincoln				
$R_{t,f} = 8.72597123E-01 + 2.96964121E-03(H)$		1.03680	17	0.8803
or $R_{t,f} = 0.83242764 + 2.81237312E-05(H^2) + 1.67572359E-01(P)$		1.03680	17	.8137
Engelmann spruce-corkbark fir on Santa Fe, Carson, and Lincoln				
$R_{t,f} = 9.92085233E-01 + 1.87806305(D/H)^2$		1.12229	16	.7702
or $R_{t,f} = 0.49137954 + 3.73898958E-02(D) - 5.87687943E-05(H^2) + 5.08620456E-01(P)$		1.12229	16	.5139
Ponderosa pine on Lincoln, Coconino, Tonto, Santa Fe, and Carson				
$R_{t,f} = 9.86799996E-01 + 4.82642048E-01(D/H)^2$		1.02667	158	.9889
or $R_{t,f} = 0.90259661 + 1.24540324E-02(T^2) + 9.74033859E-02(P)$		1.02667	158	.9374
Douglas-fir on Santa Fe, Carson, Lincoln, and Tonto				
$R_{t,f} = 9.40292266E-01 + 3.38586950(1/H)$		1.00723	21	.9790
Aspen on Santa Fe and Carson				
$R_{t,f} = 9.49182022E-01 + 2.82537481E-05(H^2)$		1.02097	37	.9559
or $R_{t,f} = 1.36944003 + 3.87833769E-05(H^2) - 5.61174592E-02(T)$ $+ 3.69440030E-01(P^2) - 7.38880060E-01(P)$		1.02097	37	.8975
White fir on Santa Fe, Carson, and Lincoln				
$R_{t,f} = 9.10060854E-01 + 2.67134214E-01(H/D^2)$		.97591	13	.8752

Merchantable Gross Cubic Foot Volume--Unforked Trees

By definition, merchantable gross cubic foot volume in an unforked tree is equal to total stem gross cubic foot volume minus the cubic foot volume in both the top and the stump (Husch 1963). This relationship can be written as

$$V_m = V_t - V_u$$

where

V_m = Predicted merchantable gross cubic foot volume to a specified top in an unforked tree

V_t = Predicted total stem gross cubic foot volume in an unforked tree

V_u = Predicted unmerchantable gross cubic foot volume in an unforked tree (that is, the gross cubic foot volume in the top and stump of an unforked tree).

Weighted least squares regression was used to develop the following model for unmerchantable gross cubic foot volume in an unforked tree:

$$V_u = b_0 + b_1 \frac{D^3 H}{D^n} + b_2 D^2$$

where

V_u = Predicted unmerchantable gross cubic foot volume in an unforked tree

D = D.b.h. in inches

H = Total tree height in feet

D_m = Any specified top diameter between 3 and 8 inches inside bark

n = An sp-NF specific power on d.b.h.

The appropriate regression coefficients, their RMSQR, and the number of observations used can be found in table 4. The number of observations differs from the number of trees used because each tree has an unmerchantable volume for each of the several top diameters computed for the tree. Methodological details can be found in appendix III.

Table 4.--Unforked tree unmerchantable gross cubic foot volume data base and regression results

Species	National Forest	Number of observations	Power on d.b.h.	Regression coefficients			Relative mean squared residual (RMSQR)
				b_0	b_1	b_2	
White pine	Santa Fe, Carson, Lincoln	250	1.5	-0.213005397	0.00491211378	0.00606061971	0.0229
Engelmann spruce-corkbark fir	Santa Fe, Carson, Lincoln	810	1.5	-.266475206	.00612895037	.00743111599	.0503
Yellow pine	All	1,206	1.0	.0185465259	.000788175798	.00505513624	.1004
Blackjack pine	Lincoln, Coconino Tonto	4,086	1.5	-.125349396	.00360421889	.00540634204	.0591
Blackjack pine	Santa Fe, Carson	1,231	1.5	-.133967845	.00650174839	.00490223789	.0657
Douglas-fir	Lincoln, Tonto	648	1.0	-.083148657	.00121904459	.00541744598	.0547
Douglas-fir	Santa Fe, Carson	424	1.5	-.187630869	.00671872085	.00536451038	.0580
Aspen	Santa Fe, Carson	528	1.5	-.236432433	.00580208490	.00608042504	.0435
White fir	Lincoln	244	1.0	-.182699690	.00124824607	.00624477874	.3681
White fir	Santa Fe, Carson	298	1.5	-.187563245	.00632648558	.00604132385	.0293

Merchantable Gross Cubic Foot Volume--Forked Trees

The equation form for forked tree merchantable gross cubic foot volume is

$$V_{m,f} = V_m \times R_{m,f}$$

where

$V_{m,f}$ = Predicted merchantable gross cubic foot volume of a forked tree

V_m = Predicted merchantable gross cubic foot volume of an unforked tree

$R_{m,f}$ = Predicted ratio of actual merchantable gross cubic foot volume in a forked tree divided by predicted merchantable cubic foot volume in an unforked tree of same dimensions.

The sp-NF specific equations for $R_{m,f}$, their RMSQR, and the number of observations used can be found in table 5. For those who do not feel comfortable using these equations, equations that predict $R_{m,f}$ as a function of top diameter alone can also be found in table 5. Methodological details can be found in appendix IV.

Table 5.--Forked tree merchantable gross cubic foot volume data base
and regression results

Independent variables

D = D.b.h. in inches D_m = Top diameter, inside bark, in inches
H = Total tree height in feet P = Height to first fork divided by H
T = 1 + (number of forks)

Species, National Forest, and regression equations	: Number : of : observations	: Relative : mean squared : residual : (RMSQR)
White pine on Lincoln, Santa Fe, and Carson $R_{m,f} = C_0 + C_1 H$ $C_0 = 0.61725167 + 0.042365845 D_m$ $C_1 = 0.00598501 - 0.00073445 D_m$ or $R_{m,f} = 1.04518089 - 7.88395198 E-07 D_m^7 + 8.39079873 E-08 D_m^8$	101	0.9581
Engelmann spruce-corkbark fir on Lincoln, Santa Fe, and Carson $R_{m,f} = C_0 + C_1 (D/H)^2$ $C_0 = 0.88977465 + 0.01260886 D_m$ $C_1 = 2.02369991 + 0.00029092 D_m$ or $R_{m,f} = 1.09275293 + 1.66824359 E-06 D_m^7 - 2.22920473 E-07 D_m^8$	91	.9719
Ponderosa pine on Coconino, Tonto, Lincoln, Santa Fe, and Carson $R_{m,f} = C_0 + C_1 (D/H)^2$ $C_0 = 1.025717571 - 0.01509060 D_m$ $C_1 = 0.13600973 + 0.09709709 D_m$ or $R_{m,f} = 0.997113216 - 0.023300125 D_m + C_0 T^2 + C_1 P$ $C_0 = 0.00564984 + 0.00079885 D_m$ $C_1 = 0.002886816 + 0.02330013 D_m$ or $R_{m,f} = 1.01577735 - 1.68401836 E-06 D_m^5$	937	.9968
Douglas-fir on Lincoln, Santa Fe, and Carson $R_{m,f} = C_0 + C_1 H^{-1} + C_2 H^{-1} D_m$ $C_0 = 0.552116849$ $C_1 = 22.5535301$ $C_2 = 0.284962111$ or $R_{m,f} = 0.951125601 - 7.62321367 E-04 D_m^4 + 2.09712628 E-03 D_m^{3.5}$	118	.9064
Aspen on Santa Fe and Carson $R_{m,f} = C_0 + C_1 H^2$ $C_0 = 1.143728271 - 0.08851433 D_m$ or $C_1 = -0.000006413 + 0.000018827 D_m$ $R_{m,f} = 1.07666448 - 1.18133616 E-03 D_m^{2.5}$	184	.9165
White fir on Santa Fe, Carson, and Lincoln $R_{m,f} = C_0 + C_1 (H/D^2)$ $C_0 = 0.673711298 + 0.055948391 D_m$ or $C_1 = 1.709640750 - 0.348134538 D_m$ $R_{m,f} = 0.983293776 - 2.40166266 E-05 D_m^4$	74	.7286

Gross Cubic Foot Stump Volume--Unforked and Forked Trees

Unmerchantable gross cubic foot volume is composed of both stump volume and top volume. Information concerning the amount of top wood left after logging can be very useful for residue and fire assessments. Weighted least squares regression was used to fit the following gross cubic foot stump volume equation:

$$V_{st} = g_0 + g_1 D^2$$

where

V_{st} = Predicted gross cubic foot stump volume in an unforked or forked tree

D = D.b.h. in inches.

The appropriate regression coefficients, their RMSQR, and the number of trees used can be found in table 6. Stump height was 1.2 feet on the Lincoln National Forest, and 1.0 foot on the other National Forests. Methodological details can be found in appendix V. Equations for predicting d.b.h. from stump diameter have been reported previously by Hann (1976).

Table 6.--Unforked and forked tree gross cubic foot stump volume data base and regression results

Species	National Forest	Number of trees	Regression coefficients		Relative mean squared residual (RMSQR)
			g_0	g_1	
White pine	Santa Fe, Carson	18	-2.20720716E-02	6.02924330E-03	0.0553
White pine	Lincoln	42	-9.81087640E-03	6.92301900E-03	.0159
Engelmann spruce-corkbark fir	Santa Fe, Carson	161	-1.26475621E-02	6.47928905E-03	.0658
Yellow pine	Santa Fe, Carson, Coconino, Tonto	220	-7.19317815E-02	5.21911020E-03	.0417
Yellow pine	Lincoln	11	-8.93559070E-02	6.21218970E-03	.0351
Blackjack pine	Santa Fe, Carson, Coconino, Tonto	963	-8.24872240E-03	4.87880288E-03	.0505
Blackjack pine	Lincoln	57	-3.04689829E-02	6.05850250E-03	.0268
Douglas-fir	Santa Fe, Carson, Tonto	115	7.56124690E-04	5.27297220E-03	.0648
Douglas-fir	Lincoln	93	-2.21569699E-02	6.09390425E-03	.0311
Aspen	Santa Fe, Carson	145	5.55975850E-03	5.30326265E-03	.0797
White fir	Santa Fe, Carson	66	-6.57525870E-03	5.48812315E-03	.0512
White fir	Lincoln	48	-3.25893626E-02	6.52844110E-03	.0171

Gross International 1/4-Inch Board Foot Volume--Unforked Trees

Gross International 1/4-inch board foot volume to a 6-inch top in an unforked tree is predicted by

$$V_I = V_{m(6'')} \times R_{I/C}$$

where

V_I = Predicted gross International 1/4-inch board foot volume to a 6-inch top in an unforked tree

$V_{m(6'')}$ = Predicted gross merchantable cubic foot volume to a 6-inch top in an unforked tree

$R_{I/C}$ = Predicted ratio of actual gross International 1/4-inch board foot volume to a 6-inch top in an unforked tree divided by predicted gross merchantable cubic foot volume to a 6-inch top in an unforked tree.

This equation was fitted using weighted least squares regression through the origin. The resulting ratio equations, $R_{I/C}$, are of the form:

$$R_{I/C} = d_0 - d_1 D^{-1} - d_2 D^{-2} - d_3 D^{-3}$$

where

D = D.b.h. in inches.

The sp-NF specific, weighted regression coefficients for $R_{I/C}$, their RMSQR, and the number of trees used can be found in table 7. The coefficient of determination, R^2 , is also included to help bridge several gaps where comparable values of RMSQR were not computed. Methodological details can be found in appendix VI.

Table 7.--Unforked tree gross International 1/4-inch board foot volume to a 6-inch top data base and regression results

Species	National Forest	Number of trees	Regression coefficients				Relative mean squared residual (RMSQR)	Coefficient of determination (R^2)
			d_0	d_1	d_2	d_3		
White pine	Santa Fe, Carson, Lincoln	39	6.69196746	7.52011366	216.348366	0.00	0.0179	0.9842
Engelmann spruce-corkbark fir	Santa Fe, Carson, Lincoln	117	5.98736288	9.84791839	-300.8128080	2,855.342464	.1018	.9338
Yellow pine	All	201	7.10051404	7.97921881	229.556497	.00	--	.9756
Blackjack pine	Lincoln, Coconino, Tonto	613	6.84751736	7.69491322	221.377226	.00	.0753	.9265
Blackjack pine	Santa Fe, Carson	208	7.58122078	8.51941410	245.097535	.00	--	.9339
Douglas-fir	Lincoln, Tonto	102	6.58735344	.89271640	243.514909	.00	--	.9711
Douglas-fir	Santa Fe, Carson	70	6.59717456	.89404735	243.877967	.00	.0536	.9543
Aspen	Santa Fe, Carson	87	6.68808485	-1.27685063	-4.5048044	1,423.985244	.0492	.9649
White fir	Lincoln	40	6.24687520	7.01994022	201.9587285	.00	.0290	.9749
White fir	Santa Fe, Carson	47	5.73644537	1.72093361	74.57378982	.00	.0488	.9611

Gross International 1/4-Inch Board foot Volume--Forked Trees

The model for gross International 1/4-inch board foot volume to a 6-inch top for forked trees is

$$V_{I,f} = V_I \times R_{I,f}$$

where

$V_{I,f}$ = Predicted gross International 1/4-inch board foot volume to a 6-inch top in a forked tree

V_I = Predicted gross International 1/4-inch board foot volume to a 6-inch top in an unforked tree

$R_{I,f}$ = Predicted ratio of actual gross International 1/4-inch board foot volume to a 6-inch top in a forked tree divided by predicted gross International 1/4-inch board foot volume to a 6-inch top in an unforked tree.

The appropriate equations for $R_{I,f}$, their RMSQR, the number of trees used, and the mean, ratio $R_{I,f}$, can be found in table 8. Methodological details can be found in appendix VII.

Table 8.--Forked tree gross International 1/4-inch board foot volume data base and regression results

Species, National Forest, and regression equations	Independent variables		Mean : $R_{I,f}$	Number : of trees	Relative : mean squared : residual : (RMSQR)
	D = D.b.h. in inches H = Total tree height in feet	P = Height to first fork divided by H T = 1 + (number of forks)			
White pine on Santa Fe, Carson, and Lincoln $R_{I,f} = 2.21682508E-01 + 5.95430677E-05 (H^2) + 2.29900234 (H/D^2)$			1.16873	17	0.4462
Engelmann spruce-corkbark fir on Santa Fe, Carson, and Lincoln $R_{I,f} = 3.70906603E-01 + 7.71762038E-01 (1/D^2)$ or $R_{I,f} = -.46864127 + 1.74260009 (H/D^2) + 1.46864127 (P)$			1.23734	16	.4278
			1.23734	16	.4154
Yellow pine on Lincoln, Coconino, Tonto, Santa Fe, and Carson $R_{I,f} = 1.16393889 - 2.03944633 (D/H)^2$			.95773	31	.6744
Blackjack pine on Lincoln, Coconino, and Tonto $R_{I,f} = 5.49502730E-01 + 1.09541429E-01 (H/D)$ or $R_{I,f} = .61350474 + 9.60847876E-02 (H/D) - 1.52902459E-01 (T)$ $+ 7.72990514E-01 (P) - 3.86495257E-01 (P^2)$			.99776	62	.8895
			.99776	62	.8238
Blackjack pine on Santa Fe and Carson $R_{I,f} = 4.15854140E-01 + 8.37338276E+01 (1/D^2)$			1.03576	63	.7438
Douglas-fir on Santa Fe, Carson, and Lincoln $R_{I,f} = 5.38416406E+02 (H^{-1.30844077})$			2.12047	20	.7158
Aspen on Santa Fe and Carson $R_{I,f} = 5.13375186E-01 + 4.20725160E+01 (1/D^2)$			1.08110	31	.9188
White fir on Santa Fe, Carson, and Lincoln $R_{I,f} = 6.81631222E-01 + 1.35049045E-02 (D)$ or $R_{I,f} = 3.05788475E-01 + 2.34528327E-05 (H^2) + 6.94211525E-01 (P)$			.95691	12	.7677
			.95691	12	.7643

Gross Scribner Board Foot Volume--Unforked Trees

Gross Scribner board foot volume to a 6-inch top in an unforked tree is predicted by

$$V_S = V_I \times R_{S/I}$$

where

- V_S = Predicted gross Scribner board foot volume to a 6-inch top in an unforked tree
- V_I = Predicted gross International board foot volume to a 6-inch top in an unforked tree
- $R_{S/I}$ = Predicted ratio of actual gross Scribner board foot volume to a 6-inch top in an unforked tree divided by predicted gross International 1/4-inch board foot volume to a 6-inch top in an unforked tree.

Like International 1/4-inch board foot volume, this equation was fitted using weighted least squares regression through the origin. The resulting ratio equations, $R_{S/I}$, are of the form:

$$R_{S/I} = e_0 - e_1 D^{-1} - e_2 D^{-1.177748} - e_3 D^{-2}$$

where

D = D.b.h. in-inches.

The sp-NF specific, weighted regression coefficients for $R_{S/I}$, their RMSQR, and the number of trees used can be found in table 9. Methodological details can be found in appendix VIII.

Table 9.--Unforked tree gross Scribner board foot volume to a 6-inch top data base and regression results

Species	National Forest	Number of trees	Regression coefficients				Relative mean squared residual (RMSQR)
			e_0	e_1	e_2	e_3	
White pine	Santa Fe, Carson, Lincoln	32	1.00608608	2.38465985	0.00	0.00	0.0204
Engelmann spruce-corkbark fir	Santa Fe, Carson, Lincoln	71	.878453705	.00	.00	15.9984577	.1638
Yellow pine	All	200	.982101210	.926027395	.00	14.49443523	.0486
Blackjack pine	Lincoln, Coconino, Tonto	402	.96579222	.40579028	.00	16.93678414	.1035
Blackjack pine	Santa Fe, Carson	168	.993986685	1.463486622	.00	12.40584877	.1196
Douglas-fir	Lincoln, Tonto	83	1.000897473	.00	4.100072359	0.0	.0324
Douglas-fir	Santa Fe, Carson	48	.870259997	.00	.00	19.49594193	.0455
Aspen	Santa Fe, Carson	38	.887891	.00	.00	17.19374	.0831
White fir	Lincoln	36	1.0	1.88814412	.00	8.85144911	.0233
White fir	Santa Fe, Carson	31	1.01724769	1.87056853	.00	8.51445088	.0376

Gross Scribner Board Foot Volume--Forked Trees

The model for gross Scribner board foot volume to a 6-inch top for forked trees is

$$V_{S,f} = V_S \times R_{S,f}$$

where

$V_{S,f}$ = Predicted gross Scribner board foot volume to a 6-inch top in a forked tree

V_S = Predicted gross Scribner board foot volume to a 6-inch top in an unforked tree, and

$R_{S,f}$ = Predicted ratio of actual gross Scribner board foot volume to a 6-inch top in a forked tree divided by predicted gross Scribner board foot volume to a 6-inch top in an unforked tree.

The appropriate equations for $R_{S,f}$, their RMSQR, the number of trees used, and the mean ratio, $R_{S,f}$, can be found in table 10. Methodological details can be found in appendix IX.

Table 10.--Forked tree gross Scribner board foot volume to a 6-inch top data base and regression results

Species, National Forest, and regression equations	Independent variables		Mean : $R_{S,f}$:	Number : of : trees :	Relative : mean squared : residual : (RMSQR)
	D = D.b.h. in inches H = Total tree height in feet	P = Height to first fork divided by H T = 1 + (number of forks)			
White pine on Santa Fe, Carson, and Lincoln $R_{S,f} = 6.69426337E-01 + 1.41693541E-02(H) - 3.10631659E-02(D)$			1.00494	14	0.5093
Engelmann spruce-corkbark fir on Santa Fe, Carson, and Lincoln $R_{S,f} = 3.27406045E-01 + 9.24980748E-01(1/D^2)$			.98096	11	.9191
or $R_{S,f} = -0.53128818 + 2.17203301E-02(D) - 1.53128818(P^2) + 3.06257636(P)$			.98096	11	.3868
Yellow pine on Lincoln, Coconino, Tonto, Santa Fe, and Carson $R_{S,f} = 1.16596696 - 2.01070651(D/H)^2$			.96266	31	.7025
Blackjack pine on Santa Fe and Carson $R_{S,f} = 4.24761670E-01 + 1.23016529E-01(H/D)$			.86497	53	.8029
or $R_{S,f} = 6.97723772E-01 - 1.79745211E-02(D) + 8.11255589E-05(H^2) + 3.02276228E-01(P)$			.86497	53	.7426
Blackjack pine on Lincoln, Coconino, and Tonto $R_{S,f} = 1.50684356 - 1.94318670(D/H)$			1.00836	59	.8484
or $R_{S,f} = 5.46618888E-01 + 1.15209503E-01(H/D) - 1.75749455E-01(T) - 4.53381112E-01(P^2) + 9.06762224E-01(P)$			1.00836	59	.7476
Douglas-fir on Santa Fe, Carson, and Lincoln $R_{S,f} = 6.39052346E-01 + 1.18994308(H/D^2)$			.99510	17	.6622
Aspen on Santa Fe and Carson $R_{S,f} = 4.03747500E-01 + 1.17109771E-01(H/D)$			1.01472	15	.7935
White fir on Santa Fe, Carson, and Lincoln $R_{S,f} = 6.31636481E-01 + 1.50605156E-02(D)$			.93862	12	.7142

Net volumes.--The general form of the net volume equations is the same for all volume types:

$$NV = V[1.0 - (\text{PrC}) (\text{FC})]$$

where

- NV = Predicted net volume of an unforked or forked tree
- V = Predicted gross volume of an unforked or forked tree
- PrC = Predicted probability that a tree of given characteristics will have some cull (that is, the probability that the tree will be unsound)
- FC = Predicted fraction cull of a tree given that it does have some cull.

The product (PrC) (FC) estimates the average cull proportion for any tree of given characteristics. The advantage of separating the average cull rate into its components is the added flexibility it provides when sampling a stand or forest. This method not only allows the user to predict average cull proportion, but it also allows him to separate the sampled population into completely sound and unsound trees. For example, suppose a sample tree, X_i , represents n_i trees of the same characteristics. The values PrC and FC are computed for the sample tree's characteristics and then the following estimates are made:

$$\text{Number of sound trees} = n_i(1.0 - \text{PrC})$$

$$\text{Number of unsound trees} = n_i(\text{PrC})$$

$$\text{Net volume in one sound tree} = V \times (1.0)$$

$$\text{Net volume in one unsound tree} = V \times (1.0 - \text{FC})$$

$$\text{Total net volume in sound trees} = V \times n_i(1.0 - \text{PrC})$$

$$\text{Total net volume in unsound trees} = V \times n_i(\text{PrC}) (1.0 - \text{FC})$$

where

V = Gross volume of a tree of given characteristics.

It is obvious that this kind of data provides the forest manager with much more information concerning the structure of a population's net volume than does average cull proportion.

As mentioned earlier, for a given species, National Forest, and volume type, the specific equations chosen for PrC and FC are not determined on a tree-by-tree basis, but rather by the type of information collected during sampling. In addition to d.b.h. and total tree height, there are two types of additional information that a user could collect and use. One is the forking information previously discussed and the other is information about tree damaging agents. The specific equations chosen will depend upon whether the user collects all, one, or none of these additional pieces of information. Once the choice has been made, the equations will then apply to all trees, whether they be unforked, forked, undamaged, or damaged.

Probability of a Tree Being Unsound in Cubic Foot Volume--Unforked and Forked Trees

The model form for the probability of a tree being unsound in cubic foot volume to be applied to both total stem and merchantable predictions is

$$\text{PrC}_c = 1.0 / (1.0 + e^{-X_c})$$

where

PrC_c = Predicted probability of a tree being unsound in cubic foot volume

X_c = A function of the tree's measured attributes.

Weighted, nonlinear regression procedures (Hamilton 1974) were used to determine the equations for X_c . The resulting equations, along with the number of trees used, are presented in table 11. An F-statistic was used to test the significance of the models. The equation for blackjack pine on the Lincoln National Forest was significant at the 95 percent level, and all other equations were significant at the 99 percent level. Methodological details can be found in appendix X.

Table 11.--Probability of a tree being unsound in
cubic foot volume--regression results

<u>Independent variables</u>		
D = D.b.h. in inches	H = total tree height in feet	T = 1 + (number of forks)
Species, National Forest, and regression equations		:Number of : trees
White pine on Santa Fe, Carson, and Lincoln		
$X_c = -2.955536 + 0.05898932H$		60
$X_c = -3.765521 + 0.05932042H + 0.6089821T$		60
Engelmann spruce on Santa Fe, Carson, and Lincoln		
$X_c = -1.655268 + 0.001678439DH$		136
$X_c = -2.931385 + 0.1798041D + 0.3704742T$		136
Corkbark fir on Santa Fe and Carson		
$X_c = -2.994364 + 0.3277421D$		35
$X_c = -5.310682 + 0.3746394D + 1.414157T$		35
Yellow pine on Santa Fe and Carson		
$X_c = 0.9767974 + 0.03559618D$		53
Yellow pine on Lincoln, Coconino, and Tonto		
$X_c = 0.3547862 + 0.003783644D$		179
Blackjack pine on Coconino		
$X_c = -5.640547 + 0.04713862H$		306
$X_c = -6.206244 + 0.04502878H + 0.6055332T$		306
Blackjack pine on Lincoln		
$X_c = -2.672635 + 0.05792241H$		57
Blackjack pine on Tonto		
$X_c = -4.768802 + 0.2093156D$		380
Blackjack pine on Santa Fe and Carson		
$X_c = -1.418322 + 0.00006223427D^2H$		283
$X_c = -3.627838 + 0.1540565D + 0.6277967T$		283
Douglas fir on Lincoln and Tonto		
$X_c = -1.606318 + 0.0008139147DH$		114
Douglas-fir on Santa Fe and Carson		
$X_c = -2.591351 + 0.1359878D$		96
$X_c = -2.919862 + 0.001301024DH + 0.8573843T$		96
Aspen on Santa Fe and Carson		
$X_c = 1.202390 + 0.2220774D$		146
White fir on Lincoln		
$X_c = -1.730640 + 0.1221281D$		48
White fir on Santa Fe and Carson		
$X_c = -3.118569 + 0.04665243H$		66

Fraction Cull in Total Stem Cubic Foot Volume Given the Tree is Unsound--Unforked and Forked Trees

To predict the fraction of an unsound tree's total stem cubic foot volume that is in cull, the following functional relationship is used:

$$FC_t = h_t(1.0 - e^{Y_t})$$

where

FC_t = Predicted fraction cull in total stem cubic foot volume given the tree is unsound

h_t = a constant

Y_t = A function of the tree's measured attributes.

The sp-NF specific values and functions of h_t and Y_t , respectively, the number of trees used, and the resulting RMSQR values are found in table 12. An examination of the RMSQR values indicate that many of the equations might not reduce the squared deviations more than a mean value of FC_t . This is due both to the general difficulty of modeling cull rates and to the relatively small number of sample trees with cull. It is felt, however, that the equations do represent cull trends in at least the sample trees and that they also seem to behave reasonably well. It is recommended, therefore, that the equations be used but, for those who choose not to do so, mean values of FC_t have also been included in table 12. Methodological details can be found in appendix XI.

Table 12.--Fraction cull in total stem cubic foot volume, given the tree is unsound, data base and regression results

Species, National Forest, and regression results	Independent variables			Relative mean squared residual (RMSQR)
	Mean FC_t	Number of trees		
White pine on Santa Fe, Carson, and Lincoln $h_t = 0.573840391$ $Y_t = (-7.66323806E-04)D^2$	0.14358	34		0.8358
Engelmann spruce on Santa Fe, Carson, and Lincoln $h_t = 0.938462956$ $Y_t = -3.01894250E-01 + 4.30560743E-03(H) - 6.43668344E-04(D^2)$	.13981	44		.8359
Corkbark fir on Santa Fe and Carson $h_t = 0.961637598$ $Y_t = 1.36557290E-01 - 5.67626653E-02(\frac{H}{D})$	.09807	21		.8472
Yellow pine on Lincoln, Coconino, Tonto, Santa Fe, and Carson $h_t = 0.915201231$ $Y_t = (-1.63671565E-01) + (3.25503506E-05)OH$	.09303	154		.9716
Blackjack pine on Lincoln, Coconino, and Tonto $h_t = 0.970852066$ $Y_t = (-9.71749091E-03) - (6.33508782E-07)D^2H$	.01878	80		.9364
Blackjack pine on Santa Fe and Carson $h_t = 0.959154988$ $Y_t = (-2.67485517E-02) - (7.47306053E-02)H/D^2$	.04676	90		.9727
Douglas-fir on Santa Fe, Carson, Lincoln, and Tonto $h_t = 0.236595402$ $Y_t = (-2.74212212E-01) - (6.75308810E-06)D^2H$	.08489	66		.9818
Aspen on Santa Fe and Carson $h_t = 0.885502084$ $Y_t = (-2.1547562E-01) - (9.32233189E-03)D^2 + (1.31076200E-03)DH$	.28482	139		.7441
White fir on Santa Fe, Carson, and Lincoln $h_t = 0.440697488$ $Y_t = (-1.21221834E-03)D^2$	.15895	51		.7790

Fraction Cull in Merchantable Cubic Foot Volume Given the Tree Is Unsound--Unforked and Forked Trees

The following model form is used to predict the fraction of an unsound tree's merchantable cubic foot volume that is cull:

$$FC_m = h_m(1.0 - e^{-Y_m})$$

where

FC_m = Predicted fraction cull in merchantable cubic foot volume to a specified top diameter given the tree is unsound

h_m = A constant

Y_m = A function of the tree's attributes and a specified top diameter between 3 and 8 inches, inside bark.

The appropriate values of h_m and functions for Y_m , the number of observations used, and the resulting RMSQR values are found in table 13. While these models are recommended for use, equations predicting mean FC_m as a function of top diameter are also included in table 13 for those who choose not to use the FC_m equations. Methodological details can be found in appendix XII.

Table 13.--*Fraction cull in merchantable cubic foot volume, given the tree is unsound, data base, and regression results*

Independent variables

D = D.b.h. in inches

D_m = Top diameter, inside bark, in inches

H = Total tree height in feet

Species, National Forest, and regression equations	: Number : of : observations	: Relative : mean squared : residual : (RMSQR)
<hr/>		
White pine on Santa Fe, Carson, and Lincoln		
$h_m = 0.525456732$		
$Y_m = j_1 D^2$	204	0.8386
or $j_1 = -7.62194840E-04 - 1.72E-10D_m^7$		
$FC_m = 0.139967204 + 1.40417133E-07D_m^5$		
Engelmann spruce on Santa Fe, Carson, and Lincoln		
$h_m = 0.809722162$		
$Y_m = j_0 + j_1 H + j_2 D^2$	246	.8881
$j_0 = -4.79663334E-01$		
$j_1 = 5.99578762E-03$		
or $j_2 = -3.54716909E-04 - 2.30591722E-05D_m$		
$FC_m = 1.10092261E-01 + 4.70167336E-04D_m^4$		
$-8.80762813E-05D_m^5 + 4.83607897E-07D_m^7$		
Corkbark fir on Santa Fe and Carson		
$h_m = 0.945856475$		
$Y_m = j_0 + j_1 H/D$	120	.8010
$j_0 = -2.04082904E-02 + 4.51477415E-02D_m$		
or $j_1 = -1.16845115E-02 - 1.29460840E-02D_m$		
$FC_m = 7.39331940E-02 + 6.95210968E-03D_m$		
Yellow pine on Lincoln, Coconino, Tonto, Santa Fe, and Carson		
$h_m = 0.912935298$		
$Y_m = j_0 + j_1 DH$	924	.9696
$j_0 = -1.67513064E-01 - 2.39E-07D_m^5$		
or $j_1 = 3.368982527E-05 + 8.684E-11D_m^5$		
$FC_m = 9.41334189E-02 + 4.35158264E-06D_m^3$		
Blackjack pine on Lincoln, Coconino, and Tonto		
$h_m = 0.294142214$		
$Y_m = j_0 + j_1 D^2 H$	475	.9351
$j_0 = -3.39989415E-02 + 1.48516345E-04D_m^2$		
or $j_1 = -2.19796372E-06 - 8.55224950E-09D_m^2$		
$FC_m = 1.92501222E-02$		

(con.)

Table 13.--(con.)

Species, National Forest, and regression equations	: Number : of : observations	: Relative : mean squared : residual : (RMSQR)
Blackjack pine on Santa Fe and Carson		
$h_m = 0.943058463$		
$Y_m = j_0 + j_1 H/D^2$	532	0.9864
$j_0 = -1.80176132E-02 - 1.62628643E-06D_m^6$		
$+ 2.00018357E-07D_m^7$		
$j_1 = -9.68124280E-02 + 6.49858343E-06D_m^6$		
$-8.00723544E-07D_m^7$		
or		
$FC_m = 4.19699099E-02 + 1.20336014E-05D_m^5$		
$-3.54771919E-06D_m^6 + 2.55488876E-07D_m^7$		
Douglas-fir on Santa Fe, Lincoln, and Tonto		
$h_m = 0.243391121$		
$Y_m = j_0 + j_1 D^2 H$	390	.9785
$j_0 = -2.80555445E-01 - 1.2792366E-06D_m^6$		
$+ 1.24888494E-07D_m^7$		
or $j_1 = -5.87585480E-06 + 8.02594124E-11D_m^4$		
$FC_m = 7.88426507E-02 + 4.88150784E-05D_m^3$		
Aspen on Santa Fe and Carson		
$h_m = 0.881568806$		
$Y_m = j_0 + j_1 D^2 + j_2 DH$	682	.7639
$j_0 = 1.77859273 - 1.93330892D_m^{0.5}$		
$+ 4.42422923E-01D_m$		
$j_1 = -9.36182562E-03 + 2.23064483E-08D_m^6$		
$-3.90157892E-09D_m^7$		
or $j_2 = 1.38034680E-03 + 9.42523449E-11D_m^7$		
$FC_m = 2.87277280E-01 + 5.42205159E-04D_m^{2.5}$		
White fir on Santa Fe, Carson, and Lincoln		
$h_m = 0.438946854$		
$Y_m = j_1 D^2$	301	.8046
$j_1 = -1.211898445E-03 - 9.066E-09D_m^4$		
or		
$FC_m = 1.54156056E-01 + 8.17555326E-06D_m^4$		

Probability of a Tree Being Unsound in Board Foot Volume--Unforked and Forked Trees

The probability of a tree being unsound in board foot volume, to be applied to both International 1/4-inch and Scribner board foot predictions, is predicted by

$$\text{PrC}_B = 1.0 / (1.0 + e^{-X_B})$$

where

PrC_B = Predicted probability of a tree being unsound in board foot volume

X_B = A function of d.b.h., total tree height, number of tree tips, and the presence or absence of significant damage.

The equations for X_B were also fitted using weighted, nonlinear regression procedures, and the results are given in table 14, along with the number of trees used. Again, the equation for blackjack pine on the Lincoln National Forest proved to be significant at the 95 percent level, whereas all the other equations were significant at the 99 percent level.

Two damaging agents, sweep (or crook) and porcupine, proved to be useful in predicting whether a tree was sound or not. A damaging agent is recorded only if it is severe enough to (1) prevent the tree from surviving; (2) preclude the production of a marketable product; or (3) diminish the quality or quantity of that product. Damage is entered into the equations through the usage of dummy variable(s). If the tree has the particular damage, the dummy variable is set to 1.0, otherwise it is 0. Only one damaging agent is recorded per tree, and, therefore, equations with two or more dummy variables in them would have at most only one of them set to 1.0 and the rest to 0. Methodological details can be found in appendix XIII.

Table 14.--Probability of a tree being unsound in
board foot volume--regression results

Independent variables

D = D.b.h. in inches H = total tree height in feet T = 1 + (number of forks)
S = 1.0 Tree with sweep or crook Pd = 1.0 Tree with porcupine damage
0.0 Tree with no sweep or crook = 0.0 Tree with no porcupine damage

Species, National Forest, and regression equations	:Number of : trees
White pine on Santa Fe, Carson, and Lincoln $X_B = -4.421720 + 0.1001612H$	56
Engelmann spruce on Santa Fe, Carson, and Lincoln $X_B = -2.057921 + 0.1664132D$	103
or $X_B = -2.412705 + 0.1689685D + 0.2973478T$	103
Corkbark fir on Santa Fe and Carson $X_B = -2.724328 + 0.3295897D$	31
Yellow pine on Santa Fe and Carson $X_B = 1.416597 + 0.08953296D$	53
Yellow pine on Lincoln, Coconino, and Tonto $X_B = -0.3255527 + 0.09002528D$	179
Blackjack pine on Coconino $X_B = -3.790069 + 0.002540676DH$	260
Blackjack pine on Lincoln $X_B = -1.195571 + 0.03548680H$	49
Blackjack pine on Tonto $X_B = -4.866131 + 0.06649043H$	320
Blackjack pine on Santa Fe and Carson $X_B = -3.023414 + 0.05290037H$	271
or $X_B = -3.551965 + 0.1806999D + 0.6451722T$	271
or $X_B = -3.650835 + 0.06016044H + 1.278169(Pd) + 1.785158S$	271
or $X_B = -3.625993 + 0.1844583D + 0.5517561T + 1.474701S$	271
Douglas fir on Lincoln and Tonto $X_B = -4.201783 + 0.3419626D$	106
Douglas-fir on Santa Fe and Carson $X_B = -1.185559 + 0.0001857571D^2H$	84
or $X_B = -1.363273 + 0.0001709440D^2H + 2.305584S$	84
Aspen on Santa Fe and Carson $X_B = -3.889285 + 0.9000166D$	119
White fir on Lincoln $X_B = -2.260617 + 0.05019542H$	45
White fir on Santa Fe and Carson $X_B = -2.955732 + 0.2320741D$	54
$X_B = -3.023744 + 0.2225154D + 1.260367S$	54

*Fraction Cull in International 1/4-Inch Board Foot Volume Given the Tree Is Unsound--
Unforked and Forked Trees*

To predict the fraction of an unsound tree's International 1/4-inch board foot volume that is in cull, the following model is used:

$$FC_I = h_I(e^{Y_I})$$

where

FC_I = Predicted fraction cull in International 1/4-inch board foot volume to a 6-inch top given the tree is unsound

h_I = A constant

Y_I = A function of the tree's attributes.

The sp-NF specific values and functions of h_I and Y_I , respectively, the number of trees used, and the resulting RMSQR values are found in table 15. The models for Engelmann spruce and corkbark fir were too poor to be useful and, therefore, only their mean FC_I values are included. Mean values of FC_I for the other sp-NF combinations are also found in table 15. Methodological details can be found in appendix XIV.

Table 15.--*Fraction cull in International 1/4-inch board foot volume, given the tree is unsound, data base, and regression results*

Species, National Forest, and regression results	Independent variables		Number of trees	Relative mean squared residual (RMSQR)
	D = D.b.h. in inches	H = Total tree height in feet		
White pine on Santa Fe, Carson, and Lincoln				
$h_I = 0.181977197$				
$Y_I = 9.53653005E-04(D^2)$			39	0.9776
Engelmann spruce on Santa Fe, Carson, and Lincoln				
No model--Use mean FC_I			45	--
Corkbark fir on Santa Fe and Carson				
No model--Use mean FC_I			21	--
Yellow pine on Santa Fe, Carson, Lincoln, Coconino, and Tonto				
$h_I = 1.05452734$				
$Y_I = -1.89706097E-02(H)$			202	.9199
Blackjack pine on Lincoln, Coconino, and Tonto				
$h_I = 0.420800123$				
$Y_I = -1.74436591E-02(H)$			138	.9430
Blackjack pine on Santa Fe and Carson				
$h_I = 0.467888236$				
$Y_I = -1.89407426E-01(H/D)$			114	.9992
Douglas-fir on Santa Fe, Carson, Lincoln, and Tonto				
$h_I = 0.839814911$				
$Y_I = -2.92010520E-01(H/D)$			116	.9479
Aspen on Santa Fe and Carson				
$h_I = 1.39801055$				
$Y_I = -8.79518535E-03(H)$			114	.9364
White fir on Santa Fe, Carson, and Lincoln				
$h_I = 0.251235533$				
$Y_I = 5.01411097E-06(D^2H)$			62	.9972

Fraction Cull in Scribner Board Foot Volume Given the Tree Is Unsound--Unforked and Forked Trees

The following model form is used to predict the fraction of an unsound tree's Scribner board foot volume that is cull:

$$FC_S = h_S(e^{Y_S})$$

where

FC_S = Predicted fraction cull in Scribner board foot volume to a 6-inch top given the tree is unsound

h_S = A constant

Y_S = A function of the tree's attributes.

The appropriate values of h_S and functions for Y_S , respectively, the number of trees used, and the resulting RMSQR values are found in table 16. Again, the models for Engelmann spruce and corkbark fir were too poor to be useful and, therefore, only their mean FC_S values are included. Several other models proved to be no better than the mean so the user may also want to use the mean FC_S values in table 16 rather than the models. Methodological details can be found in appendix XV.

Table 16.--*Fraction cull in Scribner board foot volume, given the tree is unsound, data base, and regression results*

Independent variables
D = D.b.h. in inches H = Total tree height in feet

Species, National Forest, and regression results	:	:	:	:
	Mean	Number	Relative	
	FC_S	of	mean squared	
		trees	residual	
			(RMSQR)	
White pine on Santa Fe, Carson, and Lincoln $h_S = 0.245464500$ $Y_S = 9.55078847E-04(D^2)$	0.41683	39	1.0027	
Engelmann spruce on Santa Fe, Carson, and Lincoln No model--Use mean FC_S	.42864	45	--	
Corkbark fir on Santa Fe and Carson No model--Use mean FC_S	.57057	21	--	
Yellow pine on Santa Fe, Carson, Lincoln, Coconino, and Tonto $h_S = 1.38266375$ $Y_S = -1.94422273E-02(H)$	.36288	202	.9028	
Blackjack pine on Lincoln, Coconino, and Tonto $h_S = 0.617372520$ $Y_S = -1.96681561E-02(H)$	.20529	138	.9310	
Blackjack pine on Santa Fe and Carson $h_S = 0.646540139$ $Y_S = -2.20260760E-01(H/D)$	.27296	114	1.0062	
Douglas-fir on Santa Fe, Carson, Lincoln, and Tonto $h_S = 1.24325723$ $Y_S = -3.45196887E-01(H/D)$	.31653	116	.9126	
Aspen on Santa Fe and Carson $h_S = 1.23056500$ $Y_S = -5.46784441E-03(H)$	.93059	114	.9904	
White fir on Santa Fe, Carson, and Lincoln $h_S = 0.294076429$ $Y_S = 4.60611418E-06(D^2H)$	.39036	62	1.0026	

DISCUSSION AND CONCLUSIONS

A summary of the methodology for predicting the various volume types is given in table 17. The advantage of this approach to developing volume equations is that it avoids or eliminates many of the problems of consistency, compatibility, reliability, and flexibility found with previous sets of volume equations. The disadvantage is that the method does not lend itself well to hand computations of tree volumes. To minimize this problem, we have prepared volume tables for unforked trees ("Comprehensive tree volume equations for major species of New Mexico and Arizona: II. Tables for unforked trees"). For those with access to a computer, a FORTRAN subroutine has been written and can be obtained in versions for the CDC 6400 or the UNIVAC 1108 by writing:

Southwest Volume Subroutine
Forest and Range Evaluation Unit
Intermountain Forest and Range Experiment Station
507 - 25th Street
Ogden, Utah 84401

Table 17.--Summary of methodology for predicting various tree volume types

Predicted value	Method for prediction
Total stem gross cubic foot volume, unforked trees	V_t
Total stem gross cubic foot volume, forked trees	$V_{t,f} = V_t \times R_{t,f}$
Merchantable gross cubic foot volume, unforked trees	$V_m = V_t - V_u$
Merchantable gross cubic foot volume, forked trees	$V_{m,f} = V_m \times R_{m,f}$
Gross International 1/4-inch board foot volume, unforked trees	$V_I = V_m (6") \times R_{I/c}$
Gross International 1/4-inch board foot volume, forked trees	$V_{I,f} = V_I \times R_{I,f}$
Gross Scribner board foot volume, unforked trees	$V_S = V_I \times R_{S/I}$
Gross Scribner board foot volume, forked trees	$V_{S,f} = V_S \times R_{S,f}$
Total stem net cubic foot volume, unforked trees	$NV_t = V_t \times [1 - (PrC_c \times FC_t)]$
Total stem net cubic foot volume, forked trees	$NV_{t,f} = V_{t,f} \times [1 - (PrC_c \times FC_t)]$
Merchantable net cubic foot volume, unforked trees	$NV_m = V_m \times [1 - (PrC_c \times FC_m)]$
Merchantable net cubic foot volume, forked trees	$NV_{m,f} = V_{m,f} \times [1 - (PrC_c \times FC_m)]$
Net International 1/4-inch board-foot volume, unforked trees	$NV_I = V_I \times [1 - (PrC_B \times FC_I)]$
Net International 1/4-inch board foot volume, forked trees	$NV_{I,f} = V_{I,f} \times [1 - (PrC_B \times FC_I)]$
Net Scribner board foot volume, unforked trees	$NV_S = V_S \times [1 - (PrC_B \times FC_S)]$
Net Scribner board foot volume, forked trees	$NV_{S,f} = V_{S,f} \times [1 - (PrC_B \times FC_S)]$

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APPENDIXES

Details of Methodology

I. Total Stem Gross Cubic Foot Volume--Unforked Trees

The basic model of

$$V_t = a_0 + a_1 D^2 H \quad (1)$$

where

V_t = Predicted total stem gross cubic foot volume in an unforked tree

D = D.b.h. in inches

H = Total tree height in feet

has been used for years for predicting total stem gross cubic foot volume (Husch 1963). Weighting this model by

$$W = (D^2 H)^{-2} \quad (2)$$

was suggested by Cunia (1964) as a means for homogenizing the variance about regression. This model and weighting procedure was applied to each sp-NF combination and a plot of residuals indicated that the variance was homogenized by the weighting. However, Pearson's beta statistics computed for each equation indicated that the residuals were not normally distributed. This problem precluded the usage of this model for testing the possibility of combining species across National Forests or of combining some of the species with others, using analysis of covariance.

It was then hypothesized that the following model might eliminate the problem:

$$\ln(V_t) = a_0 + a_1 \ln(H) + a_2 \ln(D). \quad (3)$$

The plots of residuals again confirmed that variance was homogenized and Pearson's beta statistics this time indicated that the residuals were closer to being normally distributed. Analysis of covariance was then applied to see if species could be combined across forests. The results indicated that blackjack pine, Douglas-fir, and white fir on the Santa Fe and Carson National Forests could not be combined with the same species on the other National Forests. All other species were combined across Forests.

Only two "species" combinations were tested. One was whether yellow pine and blackjack pine could be combined. Results of the testing indicated that they could not be combined, a conclusion supported by the findings of Hornibrook (1936) and Myers (1963). The other combination tested was Engelmann spruce with corkbark fir. This test was made because of the small data set for corkbark fir. This test resulted in the combining of Engelmann spruce with corkbark fir.

These final data sets were fitted to equation (1) using weighted least squares regression. For aspen, this resulted in a negative intercept. Therefore, an intercept value of 0.0327 (the volume of a tree 2 inches in diameter at the root collar and 4.5 feet tall, assuming conical shape) was forced on the equation instead of forcing it through the origin.

All final equations were visually checked by plotting the predicted and actual volumes over diameter by height classes.

II. Total Stem Gross Cubic Foot Volume--Forked Trees

The ratios of actual forked volume divided by predicted unforked volume were plotted over d.b.h. (D), total tree height (H), position to fork (P), and number of tips (T) and examined for trends. From this examination, a set of 12 potential transformations on D, H, P, and T were developed and all possible combinations of three or less were screened through program REX (Grosenbaugh 1967). Selected models from this screening run were then tabled and the one with the lowest RMSQR that "behaved well" was picked as the final model. (An equation was judged to "behave well" if it did not produce unrealistic results over the expected range of usage and if it did not exhibit an undulating behavior for different combinations of tree characteristics.) Yellow pine was combined with blackjack pine, and white fir and Douglas-fir were pooled across National Forests because of the need to strengthen the data sets.

III. Merchantable Gross Cubic Foot Volume--Unforked Trees

Expressing merchantable gross cubic foot volume as the difference between total stem gross cubic foot volume and the unmerchantable gross cubic foot volume in the top and stump is a convenient technique for examining and then characterizing the effect of changing top diameter upon merchantable gross cubic foot volume. To see this advantage, consider the following:

$$\begin{aligned}\text{Unmerchantable Volume} &= (\text{Top Volume}) + (\text{Stump Volume}) \\ &= \left(\frac{k \times \pi}{4 \times 144} \right) \left(\frac{\text{Top Diameter}}{\text{in Inches}} \right)^2 \left(\frac{\text{Top length}}{\text{in Feet}} \right) \\ &\quad + \left(\frac{\pi}{4 \times 144} \right) \left(\frac{\text{Stump Diameter}}{\text{in Inches}} \right)^2 \left(\frac{\text{Stump Height}}{\text{in Feet}} \right)\end{aligned}\tag{4}$$

where

k = A form factor.

It has been shown that stump diameter and d.b.h. are highly correlated (Hann 1976) and it is also true that stump height is basically a constant; therefore, the second term of equation (4) can be expressed as

$$a_0 + a_1 D^2.\tag{5}$$

For top volume, the top diameter is specified by the user; therefore, the unknown quantities are top length and the form factor, k . To help determine the relationship of top length to d.b.h. and total tree height, the following model was fitted for each top diameter:

$$\ln(\text{TL}) = a_0 + a_1 \ln(D) + a_2 \ln(H)\tag{6}$$

where

TL = Top length in feet.

By examining the behavior of the coefficients across the various top diameter, it was found that the model

$$TL = a_1 D_m \frac{H}{D^n} \quad (7)$$

where

D_m = Top diameter inside bark, in inches

n = An sp-NF specific value of 1.0, 1.5, or 2.0

was applicable. This model assumes that the top can be approximated by a conic form (Gray 1956; Husch 1963; and Assman 1970). The power on d.b.h., n , can be thought of as determining the diameter of the top cone projected to ground level. This is because total tree height is measured as the distance from the ground level to the tip of the tree and consequently the relationship

$$\frac{TL}{D_m} = a_1 \frac{H}{D^n} \quad (8)$$

will hold for similar triangles only if D^n is directly proportional to diameter at ground level.

With this background as a basis, the model for unmerchantable volume is therefore:

$$V_u = b_0 + b_1 D_m^3 \frac{H}{D^n} + b_2 D^2 \quad (9)$$

where

V_u = Gross cubic foot volume in the top and stump for a specified top diameter.

Analysis of residuals indicated that weighting was necessary to homogenize variance. A procedure described by Hann and McKinney (1975) was therefore used to obtain the weight:

$$W = \left[\frac{D_m^3 H}{D^n} \right]^{-1} = \frac{D^n}{D_m^3 H} \quad (10)$$

Weighted least squares regression was used on each sp-NF combination to obtain the regression coefficients for n equal to 1.0, 1.5, and 2.0. The final regression equation was that which minimized RMSQR. Further refinement for a value of n was not necessary because of the small differences in the RMSQR values of the three values tested. The final merchantable gross cubic foot volume equations were visually checked by plotting the data points versus representative curves from the equations over diameter by height classes.

A word of caution concerning the interpretation of the terms in this model is appropriate. While the logic behind the two components is clear, the final equations cannot be divided into components because of multicollinearity between the two (Kmenta 1971). This can be seen by comparing the second component of the unmerchantable model to the model derived separately for stump volume. The coefficients are in the same proximity but are not equal. However, the presence of multicollinearity, so long as it is not perfect, does not bias the estimation of unmerchantable volume (Kmenta 1971).

IV. Merchantable Gross Cubic Foot Volume--Forked Trees

To maintain consistency across top diameters, the same model used in predicting total stem gross cubic foot volume was fitted to each top diameter data set. The resulting regression coefficients were examined for trends over top diameter. From this examination, the regression coefficients were modeled as linear functions of top diameter. The basic model with the coefficients as functions of top diameter was then fitted to the whole data set for a final overall slope correction on the model. Models with forking variables that proved not to be better than models without forking variables were dropped.

Because the RMSQR values of the models were high, it was decided to provide an optional model that would predict the mean ratio as a function of just top diameter. To do this, the mean ratio values for the six top diameters were weighted by their respective number of observations and then screened against 12 power transformations on top diameter. Selected models were then examined for reasonableness of behavior and the one that behaved both reasonably and minimized RMSQR was picked as the final model.

V. Gross Cubic Foot Stump Volume--Unforked and Forked Trees

The basic model:

$$V_{st} = a_0 + a_1 D^2 \quad (11)$$

where

V_{st} = Gross volume of the stump in cubic feet

was fitted to the separate sp-NF combinations. The residuals were plotted and they revealed that the basic model was appropriate but that the variance increased with the square of D^2 . Therefore, the model was refitted using a weight of

$$W = D^{-4}. \quad (12)$$

The residuals were plotted and this time they were homogeneous.

Analysis of covariance was then used to determine if species could be combined across all National Forests except the Lincoln, where stump height was 1.2 feet instead of the standard of 1.0 foot. The results indicated that they could be so combined. Analysis of covariance was also used to check to see if Engelmann spruce and corkbark fir could still be legitimately combined, and the results indicated that they could.

VI. Gross International 1/4-Inch Board Foot Volume--Unforked Trees

The approach of modeling the ratio of gross International 1/4-inch board foot volume to a 6-inch top divided by the gross merchantable cubic foot volume to a 6-inch top was used to provide the needed control on the behavior of the International 1/4-inch model. It has long been recognized that this ratio starts with a zero value at a d.b.h. near 6 inches and increases monotonically, as d.b.h. increases, to an asymptotic ratio value between 6 and 8 (Husch 1963; Avery 1967). It has also been shown that this ratio can be affected by tree height as well (Husch 1963).

With this in mind, two models were hypothesized for trial:

$$R_{I/C} = a_0 - a_1 H/D - a_2 H/(D^2) \quad (13)$$

$$R_{I/C} = b_0 - b_1 D^{-1} - a_2 D^{-2} \quad (14)$$

where

$R_{I/C}$ = Ratio of actual gross International 1/4-inch board foot volume to a 6-inch top divided by predicted gross merchantable cubic foot volume to a 6-inch top in an unforked tree.

The signs on the coefficients a_1 , a_2 , and b_1 , b_2 must be positive for the model to behave as expected.

Before these models were tried, the ratio values were formed for all trees 6.1 inches and larger and plotted for the sp-NF combinations previously delineated. An examination of these plots resulted in the decision to combine blackjack pine data sets with the yellow pine data sets and to combine both Douglas-fir data sets. The plots also showed a few outliers that were eliminated from the appropriate data sets.

The two models, (13) and (14), were then fitted to the data sets to determine which of the two minimized RMSQR. The model with tree height (13) proved to be the worst and, therefore, was eliminated. The fits-for model (14) were reasonable for only the ponderosa pine and the Douglas-fir data sets. The other data sets either had a negative sign on b_1 or b_2 or the plots of data sets versus the predicted curves indicated a poor fit. The ponderosa pine model was then "forced" onto the white pine and the white fir on the Lincoln National Forest data sets with good results. This was done by scaling the ponderosa pine model through an overall slope correction using least squares regression. Techniques similar to those described by Jensen (1964, 1973, and 1976) and Jensen and Homeyer (1970 and 1971) were used to develop better behaved (within a reasonable usage range) ratio models for Engelmann spruce-corkbark fir, aspen, and white fir on the Santa Fe and Carson National Forests data sets.

The resulting model form for white fir was the same as model (14) but, of course, these parameter estimates are not least squares estimates in this case. The form of the models for aspen and Engelmann spruce-corkbark fir was:

$$R_{I/C} = c_0 - c_1 D^{-1} - c_2 D^{-2} - c_3 D^{-3}. \quad (15)$$

For both species groups, at least one of the parameters was negative which resulted in an undesirable "peak" and "valley" in the ratio equation outside the available data range. For aspen, the undesirable points also occurred outside the reasonable usage range, but for Engelmann spruce-corkbark fir, the points did occur within the reasonable usage range. Fortunately, the magnitudes of the "peak" and "valley" were small and it was felt that they would not cause serious problems when predicting International 1/4-inch board foot volume.

The appropriate ratio models were then multiplied by their predicted merchantable cubic foot volume to a 6-inch top for each sp-NF combination and then fitted to actual International 1/4-inch board foot volume to a 6-inch top, using least squares regression through the origin to provide a slope correction. The resulting basic model form and its error structure, therefore, is the following:

$$V_I = d_1 \times (R_{I/C}) \times (V_{m(6'')}) + \epsilon \quad (16)$$

where

- V_I = Predicted gross International 1/4-inch board foot volume to a 6-inch top
 $V_{m(6'')}$ = Predicted gross merchantable cubic foot volume to a 6-inch top
 d_1 = Least squares regression coefficient
 ϵ = Residual about regression.

Next, the necessity for weighting this model was examined through the procedure described by Hann and McKinney (1975). This resulted in weights being formed of the type:

$$W = (D^e_1 H^e_2 V^e_I)^{-1}. \quad (17)$$

Unfortunately, no common set of values e_1 , e_2 , and e_3 could be found for all sp-NF combinations. The final sp-NF weights were used in weighted least squares regression through the origin to obtain the final slope correction. These slope corrections were then incorporated into the ratio models by multiplication. As a final check, the data points and regression equations were plotted over d.b.h. and total tree height to determine adequacy of fit.

When considering this approach to modeling International 1/4-inch board foot volume, the basic model form of equation (16) must be kept in mind. If a set of proposed ratio equations had been available, then the fitting of equation (16) could have been done by screening the various ratio equations times cubic foot volume to determine which was appropriate for the sp-NF combination. Unfortunately, a proposed set of ratio equations was not available; therefore, the ratio equations had to be developed using the same data set.

There is a tendency to think that what is really being fitted is the model involving the 11 to 15 independent variables that would be formed by multiplying the ratio and merchantable cubic foot volume models together. Using this approach, it was found that some independent variables proved to be insignificant while the signs and magnitude of other regression coefficients changed because of multicollinearity problems. The result was an equation with all of the undesirable properties that were to be avoided. The approach adopted here produces a model that behaves in a reasonable and consistent fashion, but sacrifices some statistical niceties.

VII. Gross International 1/4-Inch Board Foot Volume--Forked Trees

The method used to determine the ratio correction for forking upon International 1/4-inch board foot volume was the same as that used to develop the equations for forking in total stem gross cubic foot volume. All selected equations were tabulated and checked for reasonableness. From this it was discovered that the Douglas-fir equation did not behave as expected. Therefore, the following power model was fitted to Douglas-fir:

$$\ln(R_{I,f}) = a_0 + a_1 H \quad (18)$$

where

- $R_{I,f}$ = Predicted ratio of actual gross International 1/4-inch board foot volume to a 6-inch top in a forked tree divided by predicted International 1/4-inch board foot volume to a 6-inch top in an unforked tree.

This was transformed back by taking the antilog of both sides and a least squares slope correction was made on the resulting model to correct it for possible log bias. Tabulation of this model showed that it did behave reasonably.

VIII. Gross Scribner Board Foot Volume--Unforked Trees

As described by Avery (1967), the ratio of Scribner board foot volume divided by International board foot volume starts well below 1.0 at small diameters and then increases monotonically toward an asymptotic value of 1 as diameter increases. This relationship can be expressed as

$$R_{S/I} = a_0 - a_1 D^{-1} - a_2 D^{-2} \quad (19)$$

where

$R_{S/I}$ = Predicted ratio of actual gross Scribner board foot volume to a 6-inch top divided by predicted gross International 1/4-inch board foot volume to a 6-inch top.

The signs on a_1 and a_2 must be positive and a_0 must be near 1 for the model to behave reasonably.

Individual values of $R_{S/I}$ were formed and plotted across d.b.h. for all sp-NF combinations used in gross cubic foot equation development. All data below 9 inches d.b.h. and certain outliers were eliminated based on the results of the plots. This was necessary because the ratio values below 9 inches started to turn upward and exceed 1 as d.b.h. decreased. This problem was attributed to the way in which Scribner volume is estimated in program NETVSL. The elimination of trees under 9 inches should not cause problems because Scribner volume is seldom computed for trees under that limit.

Equation (19) was fitted to the corrected data set. Weights were developed using the basic model:

$$V_S = b_1 \times (R_{S/I}) \times (V_I) + \epsilon \quad (20)$$

where

V_S = Predicted gross Scribner board foot volume to a 6-inch top

b_1 = Least squares regression coefficient

ϵ = Residual about regression

and the same process as described for International 1/4-inch board foot volume. An examination of the final weighted least squares regression coefficients, after the slope correction was multiplied through the ratio model, revealed that only the models for yellow pine, both blackjack pines, and white fir on the Santa Fe and Carson National Forests were reasonable.

An abbreviated ratio model:

$$R_{S/I} = c_0 - c_1 D^{-2} \quad (21)$$

was then tried using the same weighted scheme as previously described. This model proved reasonable for Engelmann spruce-corkbark fir, aspen, and Douglas-fir on the Santa Fe and Carson National Forests.

A model of the form:

$$\ln (1.0 - R_{S/I}) = d_0 + d_1 D \quad (22)$$

was then tried on the remaining data sets to determine the power in the model:

$$R_{S/I} = e_0 - e_1 D^{-d_1} \quad (23)$$

This was also weighted and proved reasonable for Douglas-fir on the Coconino, Tonto, and Lincoln National Forests.

Both white pine and white fir on the Lincoln National Forest proved to be particularly troublesome, with all previous attempts failing to provide a reasonable model. In order to force a reasonable model on these two data sets, the following model was fitted:

$$1.0 - R_{S/I} = g_0 D^{-1} + g_1 D^{-2} \quad (24)$$

This was then transformed to

$$R_{S/I} = 1.0 - g_0 D^{-1} - g_1 D^{-2}. \quad (25)$$

Model (25) could not be corrected with a weighted slope because doing so caused the intercept to exceed 1.

Again, all final models, with the final slope correction multiplied into them, were checked by plotting. The caution given in the section on gross International 1/4-inch board foot volume for unforked trees concerning the interpretation of this approach applies even more strongly here. In this case, if all of the model components were multiplied out, there would be from 23 to 63 independent variables to fit and the result would be even more unreasonable than in the International 1/4-inch board foot volume case.

IX. Gross Scribner Board Foot Volume--Forked Trees

The techniques used were the same as for International 1/4-inch board foot volume in forked trees. In this case, however, the basic model for Douglas-fir behaved reasonably so no special effort was necessary to model it.

X. Probability of a Tree Being Unsound in Cubic Foot Volume--Unforked and Forked Trees

By definition, the probability of a tree being unsound must take on a value between 0 and 1. A form that constrains itself between these two values is the logistic function:

$$\text{PrC}_c = 1.0 / (1.0 + e^{-X_c}) \quad (26)$$

where

PrC_c = Predicted probability of a tree being unsound in total stem and in merchantable cubic foot volume

X_c = A function of the tree's measured attributes.

Hamilton (1974) developed program RISK to fit this function to a dichotomous dependent variable. The approach basically uses the first degree term of the Taylor Series expansion of the function in weighted nonlinear regression. Output, in part, consists of the regression coefficients, an F-statistic that tests the significance of the

model, t-statistics which test the significance of the regression coefficients from a value of 0, "...and a chi-square table that evaluates goodness-of-fit over the range of predictions" (Hamilton 1974). Of the three, Hamilton suggests use of the chi-square statistic for screening alternative models, but he also states that the final choice "... is left to the discretion of the user."

The dichotomous dependent variable is defined to be 1 if the tree has any cull in its total stem and 0 if not. The resulting equations for cubic foot volume, however, are to be applied to both total stem and merchantable cubic foot net volume estimation. It was felt that, in most cases, if the tree had cull in the total stem it would also have cull in the merchantable portion. In those cases where removal of the top also removed the cull portion, the fraction cull in the tree would then be 0, and this is a legitimate value in the final fraction cull model for merchantable cubic foot volume.

For each sp-NF combination, 10 models were fitted with d.b.h., total tree height, and number of tips as the tree attributes used for independent variables. Five models had one independent variable (H,D,T,DH, or D^2H) and the rest had two independent variables (H,D;H,T;D,T;DH,T; or D^2H,T). Generally, the final runs picked were the ones that minimized the chi-square statistic while maintaining the highest significance level attained by the F-statistic for the one- or two-independent variable models (that is, if some of the one- or two-variable models were significant at the 99 percent level, then the final model was picked from those that minimized the chi-square statistic). In some cases where the chi-square statistics were close, a model with a slightly higher chi-square statistic was picked if the F- and t-statistics were considerably larger.

One- and two-variable models are presented in those instances where the second variable improved the model. The added variable, number of tips in all cases, was not always statistically significant, but the resulting model behaved as expected (that is, as the number of tips increased, so did the probability of cull).

Some data sets were combined to gain data strength. Before combining, however, the preliminary models and mean values of PrC_o were examined to assure compatibility. Engelmann spruce and corkbark fir were separated because it was found that their cull structures were very different.

Damage information was available on the data sets from the Santa Fe and Carson National Forests. Therefore, in those cases where enough common damage was present to justify the effort, the previously picked models were fitted with dichotomous damage variables added. These damage variables did not prove to be significant.

All final equations were plotted and checked for reasonableness.

XI. Fraction Cull in Total Stem Cubic Foot Volume Given the Tree Is Unsound--Unforked and Forked Trees

Like the probability of a tree being unsound, fraction cull takes on a value that ranges from 0 to 1. To provide some control over the value of fraction cull, the following model was used:

$$FC_t = h_t(1.0 - e^{-Y_t}) \quad (27)$$

where

FC_t = Predicted fraction cull in total stem cubic foot volume given the tree is unsound

Y_t = A function of the tree's attributes

h_t = A final constant value.

With this model, fraction cull will have an asymptotic maximum value of h_t and, therefore, h_t should be less than or equal to 1. Unfortunately, this model does not prevent fraction cull from becoming less than 0.

The following process was used to determine the appropriate sp-NF parameters for model (27). First, the model:

$$\ln(1.0 - \frac{FC}{h}) = Y \quad (28)$$

where

FC = Actual fraction cull in total stem cubic foot volume given the tree is unsound

Y = A function of the tree's attributes

h = An intermediate constant value

was screened through linear regression using various functions of d.b.h., total tree height, number of tips, position to first fork, and damage information as independent variables in Y, and also using various dependent variables formed by varying the value of h from the maximum FC in the data set to a value of 1. For each combination group with a specified number of independent variables from the screening run, that combination of Y and h which minimized RMSQR was picked, along with several other combinations, that were almost as good. These models were next fitted, transformed back into model (27), and then tabulated and checked for reasonableness. The model with the lowest RMSQR that gave reasonable results was then fitted with least squares regression through the origin for a slope correction to adjust the model for possible log bias. This slope correction was then incorporated into the initial value of h to give the final value of h_t in model (27). Y_t is the final function of Y. As a final check, the models were again tabulated.

On the Douglas-fir data set, the final model proved to be worse than a mean value of FC_t . Therefore, the following model was fitted:

$$FC_t = b_0 - b_1 e^{Y_t} \quad (29)$$

The resulting model was better than the mean, and tabulation revealed that it behaved correctly. Model (29) was easily transformed back to model (27) by setting b_0 equal to h_t and adding $\ln(b_1/b_0)$ to Y_t .

Again, data sets were combined based on the results of the screening runs and upon the mean FC_t values of the data sets to be combined.

XII. Fraction Cull in Merchantable Cubic Foot Volume Given the Tree Is Unsound--Unforked and Forked Trees

For each sp-NF combination, the same independent variables used in the total stem cubic foot fraction cull models were fitted to each top diameter data set with a variable h in model (28). An examination of the various runs across top diameter was made and a value of h was picked that did the best overall job of minimizing the RMSQR values.

Using the value of h , the resulting regression coefficients of Y were modeled as functions of top diameter to obtain the final equations of Y_m in the model:

$$FC_m = h(1.0 - e^{Y_m}) \quad (30)$$

where

FC_m = Predicted fraction cull in merchantable cubic foot volume given the tree is unsound

Y_m = A function of the tree's attributes and top diameter

h = An intermediate constant value.

Model (30) was then fitted to the data set incorporating all top diameters with least squares regression to obtain a slope correction to adjust the model for possible log bias, and this slope correction was incorporated with h to give the final value, h_m , in the standard form:

$$FC_m = h_m(1.0 - e^{Y_m}) \quad (31)$$

where

h_m = A final constant value.

The final models were tabulated and checked for reasonableness. This procedure proved to work well on all data sets except Engelmann spruce and Douglas-fir. On these two data sets, a model of the form:

$$FC_m = b_0 - b_1 e^{Y_m} \quad (32)$$

was fitted. Tabulation of these runs indicated that they performed well. They were then converted to the standard form of model (32) using the fact that b_0 equals h_m and by adding $\ln(b_1/b_0)$ to Y_m .

Because of the high values of RMSQR, it was decided to provide an optional model to predict mean FC_m as a function of top diameter. Plots of the means over top diameter indicated that power functions of top diameter would be appropriate. The means were then weighted by their number of observations and screened with numerous power transforms on top diameter. The maximum number of independent variables in the screening run was limited to the value of 1 plus the number of maxima or minima the plots had shown. A final model was selected based on its behavior and the magnitude of its RMSQR.

XIII. Probability of Tree Being Unsound in Board Foot Volume--Unforked and Forked Trees

An examination of the data indicated that International 1/4-inch and Scribner board foot volumes were always sound or unsound together. Therefore, one model was developed that is applicable to both.

The techniques used were basically the same as those used for cubic foot volume. The damage information did prove, however, to be statistically significant at the 95 percent level on the blackjack pine and Douglas-fir data sets. The model with damage for white fir was almost significant and, because it behaved reasonably, it was also included.

Again, all curves were plotted as a final check.

XIV. Fraction Cull in International 1/4-inch Board Foot Volume Given the Tree Is Unsound--Unforked and Forked Trees

For fraction cull in International 1/4-inch board foot volume, the following model was used:

$$FC_I = h_I e^{Y_I} \quad (33)$$

where

FC_I = Predicted fraction cull in International 1/4-inch board foot volume given the tree is unsound

Y_I = A function of the tree's attributes

h_I = A constant.

This model differs from models (27) and (32) because (33) is asymptotic to 0 instead of 1 and, therefore, it will allow predicted values of FC_I to exceed 1, which is just the opposite from models (27) and (32). This change in model form was by oversight rather than design, even though it does not affect the legitimacy of the results.

To fit model (33), the following model was first fitted and screened using the same basic procedures as in the cubic foot fraction cull equations:

$$\ln(FC) = Y \quad (34)$$

where

FC = Actual fraction cull in International 1/4-inch board foot volume given the tree is unsound

Y = A function of the tree's attributes.

This model was then transformed back to the form of model (33) and h_I was found by least squares regression through the origin to correct the model for possible log bias. The models were tabulated and those that were reasonable and minimized RMSQR were picked. All reasonable models examined for Engelmann spruce and for corkbark fir proved to be much worse than the mean and, therefore, the models were dropped.

To gain data strength, yellow pine and Douglas-fir were combined across all National Forests (but not with each other).

XV. Fraction Cull in Scribner Board Foot Volume Given The Tree Is Unsound--Unforked and Forked Trees

The procedure and model forms used for fraction cull in Scribner board foot volume were virtually the same as those used for fraction cull in International 1/4-inch board foot volume.

Headquarters for the Intermountain Forest and Range Experiment Station are in Ogden, Utah. Field programs and research work units are maintained in:

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Hann, David W., and B. Bruce Bare.

1978. Comprehensive tree volume equations for major species of New Mexico and Arizona: I. Results and Methodology. USDA For. Serv. Res. Pap. INT-209, 43 p. Intermt. For. and Range Exp. Stn., Ogden, Utah 84401.

Comprehensive tree volume equations and the methods used to derive them are presented for the major tree species of New Mexico and Arizona. Selected tree volume tables for unforked trees can be found in the companion publication, "Comprehensive tree volume equations for major species of New Mexico and Arizona: II. Tables for unforked trees."

KEYWORDS: volume equations, major tree species, New Mexico, Arizona, gross volume, net volume, unforked tree, forked tree, total stem cubic foot volume, merchantable cubic foot volume, Scribner board foot volume, International 1/4-inch board foot volume.

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Table 5.--Forked tree merchantable gross cubic foot volume data base and regression results

Species, National Forest, and regression equations	Independent variables	
	D = D.b.h. in inches	D_m = Top diameter, inside bark, in inches
	H = Total tree height in feet	P = Height to first fork divided by H
	T = 1 + (number of forks)	
	Number of observations	Relative mean squared residual (RMSQR)
White pine on Lincoln, Santa Fe, and Carson		
$R_{m,f} = C_0 + C_1H$	101	0.9581
$C_0 = 0.61725167 + 0.042365845D_m$		
$C_1 = 0.00598501 - 0.00073445D_m$		
or $R_{m,f} = 1.04518089 - 7.88395198E-07D_m^7 + 8.39079873E-08D_m^8$		
Engelmann spruce-corkbark fir on Lincoln, Santa Fe, and Carson		
$R_{m,f} = 1.15372135 - 1.51067496E-08D_m^8$	91	
Ponderosa pine on Coconino, Tonto, Lincoln, Santa Fe, and Carson		
$R_{m,f} = 0.997113216 - 0.023300125D_m + C_0T^2 + C_1P$	937	.9761
$C_0 = 0.00564984 + 0.00079885D_m$		
$C_1 = 0.002886816 + 0.02330013D_m$		
or $R_{m,f} = 1.01577735 - 1.68401836E-06D_m^5$		
Douglas-fir on Lincoln, Santa Fe, and Carson		
$R_{m,f} = 1.04417090 - 9.53965362E-09D_m^3$	118	
Aspen on Santa Fe and Carson		
$R_{m,f} = 1.07666448 - 1.18135616E-05D_m^{2.5}$	184	
White fir on Santa Fe, Carson, and Lincoln		
$R_{m,f} = C_0 + C_1(H/D^2)$	74	.7286
$C_0 = 0.673711298 + 0.055948391D_m$		
or $C_1 = 1.709640750 - 0.348154538D_m$		
$R_{m,f} = 0.983293776 - 2.40166266E-05D_m^4$		