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EQUATIONS AND TABLES FOR ESTIMATING
CUBIC-FOOT VOLUME OF INTERIOR
ALASKA TREE SPECIES^{1/}

by

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INTRODUCTION

Total cubic-foot volume and merchantable cubic-foot volume inside bark for interior Alaska commercial tree species may be estimated using equations or tables given here. Diameter outside bark at breast height^{3/} (D) and total height (H) are the field measurements needed to use these equations and tables for estimating volumes of white spruce (Picea glauca (Moench) Voss), aspen (Populus tremuloides Michx.), cottonwood (P. balsamifera L. and P. trichocarpa Torr. and Gray) and paper birch (Betula papyrifera Marsh.) trees. While the equations and tables were made primarily for use in growth and yield studies, they are also suitable for other uses such as timber sales, management plans, and forest inventory. The equations will be particularly useful when electronic computer facilities are available, allowing much more rapid and accurate volume estimates than could be accomplished with the tables alone.

^{1/} Research reported here was in cooperation with the University of Alaska.

^{2/} Research foresters, project leader, silvicultural research for the Interior forests, and Station biometrician, respectively.

^{3/} Abbreviations will be used hereafter to refer to the following measures or points of measure frequently referred to in the text and tables:

- D.b.h. - diameter breast height.
- D.i.b. - diameter inside bark.
- D.o.b. - diameter outside bark.

Volumes can be estimated for:

- V_T - the entire stem including stump and top
- V_1 - the stem between a 1-foot stump and 4-inch top d.i.b.
- V_2 - the stem between a 1-foot stump and 6-inch top d.i.b.
- V_3 - the stem between a 2-foot stump and 8-inch top d.i.b.

Cubic measure, rather than board-foot measure, was selected because of the relatively small size of Interior trees. Stands of paper birch and aspen, for instance, seldom exceed 8 inches in average d.b.h. at theoretical rotation age^{4/}, and only a few stems ever exceed 13 inches. Average diameter in mature white spruce stands is usually less than 12 inches, although occasional stems may reach or even exceed 20 inches. Most mature Interior trees are in the lower limits of sawtimber size or smaller. Board feet, therefore, would be an extremely artificial and crude measure for estimating volumes for the bulk of the commercial forest resource. It might be justified on the basis of established local preference for use in sawtimber sales, but it would be a much less accurate volume measure than cubic feet.

The basic data used in constructing the equations and tables were obtained from trees distributed from the Yukon River south to the Kenai Peninsula and from the Alaska-Canada boundary west to McGrath. A total of 1,386 trees were felled and measured. Nearly all were from well- or moderately-well-stocked stands; few open grown trees were included in the basic data. The majority were from north of the Alaska Range.

METHODS

Field measurements for each sample tree included total height as well as d.o.b. and d.i.b. at 1-foot stump, 2-foot stump, b.h. (4.5 feet), and at maximum intervals of 8 feet to 8-inch, 6-inch, and 4-inch top d.i.b.

Individual tree volumes were computed graphically.^{5/} Form quotient (F_q) was calculated for each sample tree by the following formula:

$$F_q = \frac{(\text{d.i.b. at } 1/2 \text{ distance between b.h. and tip of stem})}{(\text{d.b.h., i.b.})}$$

where the numerator was read from the plotted tree curves.

^{4/} About 90 years.

^{5/} Spurr, Stephen H. Forest Inventory. 476 pp. New York: The Ronald Press Co. 1952.

Predictor equations for each volume class (V_T , V_1 , V_2 and V_3) of a species were determined by regression analysis.^{6/} Initially, the 95 and 99 percent F level stepwise results were studied for the basic equation:

$$V_i = b_0 + b_1D + b_2D^2 + b_3H + b_4D^2H + b_5F_q + b_6F_qD$$

The precision lost by eliminating F_q and F_qD did not warrant their retention. The overwhelming strength of the power variable, D^2H , was evident. Results at the two levels of F were identical.

With tree data such as from this study, it is common for the standard deviation of residuals to be proportional to volume (i.e., heteroscedastic). In this case, standard deviation would increase with increasing values of D and H and, in effect, the upper end of the regression curves at high values of V_i would strongly affect positioning of the lower end of the curves at low values of V_i . Thus, lower V_i values predicted by equation might not fit the basic data. To equalize variance and improve the fit, weighting was applied. Since volume is strongly related to D^2H , weighting the regression by $(D^2H)^{-2}$ should effectively equalize variance. However, because of expected merging^{7/}, unweighted regressions were also fitted to the data.

Interest in both the weighted and unweighted equations was twofold: (1) the stepwise results of both forms of the equation offer more opportunities to eliminate or minimize merging; and (2) the efficiency of weighted vs. unweighted equations can be determined in the manner suggested by Furnival.^{8/} The two forms of the equation fitted were:

$$\text{Unweighted: } V_i = b_0 + b_1D + b_2D^2 + b_3H + b_4D^2H$$

$$\text{Weighted: } \frac{V_i}{D^2H} = b_0' \frac{1}{D^2H} + b_1' \frac{1}{DH} + b_2' \frac{1}{H} + b_3' \frac{1}{D^2} + b_4'$$

6/ Boles, J. N. 40-series--stepwise regression system. Calif. Agr. Expt. Sta., Dept. of Agr. Econ., U. of Calif., Berkeley, 43 pp. dittoed. June 1962.

7/ For example, with certain combinations of D and H, predictor values for volume class V_3 might appear greater than corresponding V_2 values. Variation in the data and sample-size distribution would account for this.

8/ Furnival, G. M. An index for comparing equations used in constructing volume tables. Forest Sci. 7:337-341. 1961.

To express the weighted form of the regression in terms of V_i , each term was multiplied by D^2H . The standard error of estimate or root mean square of residuals ($SE_{y.x}$) of the weighted form was de-transformed as follows:

$$SE_{y.x} \text{ in units of } V_i = SE_{y.x} \text{ wt'd.} \cdot \text{antilog}_{10} (\log_{10} D^2H).$$

An approximation of this is... $SE_{y.x} \text{ wt'd.} \cdot (\overline{D^2H})$.

The relative efficiency of weighted vs. unweighted equations is...

$$\frac{SE_{y.x}}{\text{antilog}_{10} (\log_{10} D^2H) \cdot SE_{y.x} \text{ wt'd.}}.$$

Unfortunately, the multiple coefficient of determination (R^2), an index to the amount of variation accounted for by regression, could not be retrieved when weighting was used (table 1). Much of the variation associated with D^2H is removed by the weighting technique before the actual regression is performed. Thus, the part of the variation removed as a result of weighting does not appear in R^2 .

Weighting was helpful in 12 of the 16 volume equations (table 1). In every case, except for cottonwood V_3 , the equations were the final stepwise result. In cottonwood V_3 , some loss of precision was sacrificed to minimize merging.

For the dozens of equations examined, the regression coefficient for D^2H , the power variable, usually rounded to 0.002.

The equations used, their precision, and the methods of coping with merging are presented in footnote 1 of each volume table.

PRECAUTIONS IN APPLICATION

The user may occasionally need to extend volumes beyond the presented tabular limits of D and H . This can cause merging among volume classes. If it does, a correction based upon subjective interpretation is the only possible solution. The user should adjust equation-derived volumes in what he thinks to be the most logical manner. Such adjustments should rarely be necessary, however, for few trees are expected to occur beyond the range of the tables.

As mentioned, nearly all basic data were from forest-grown trees. The equations and tables may not, therefore, give accurate estimates of volume for open-grown trees. Volumes of the latter seem to vary considerably from forest-grown trees due, apparently, to differences in form. When working with open-grown trees it would be best to obtain an estimate of volumes by double sampling for the range of diameters and heights encountered, and then apply correction factors to the tabular volumes presented here.

Table 1.--Precision and efficiency data for the employed
volume equations by species and volume class

Species	: Volume : class	: Type of : regression : analysis	: SE _{y,x} :	: Efficiency, : weighted <u>vs.</u> : unweighted	: R ²
<u>Percent</u>					
White spruce	V _T	Wt'd.	4.9	2.15	Unknown ^{1/}
	V ₁	Wt'd.	5.9	1.95	Unknown
	V ₂	Wt'd.	6.4	1.70	Unknown
	V ₃	Unwt'd.	12.9	1.23	0.9807
Cottonwood	V _T	Wt'd.	5.4	2.16	Unknown
	V ₁	Wt'd.	5.8	2.16	Unknown
	V ₂	Wt'd.	7.8	1.60	Unknown
	V ₃	Unwt'd.	15.0	1.54	0.9734
Aspen	V _T	Wt'd.	7.6	>1.51	Unknown
	V ₁	Unwt'd.	11.6	1.59	0.9791
	V ₂	Unwt'd.	12.6	1.18	0.9706
	V ₃	Wt'd.	12.1	1.26	Unknown
Paper birch	V _T	Wt'd.	6.1	1.58	Unknown
	V ₁	Wt'd.	8.1	1.38	Unknown
	V ₂	Wt'd.	12.8	1.13	Unknown
	V ₃	Wt'd.	19.3	1.26	Unknown

^{1/} R² cannot be retrieved when weighting is used, as pointed out on
page 4.

Table 2.--Tree volumes inside bark (entire stem) for white spruce, interior Alaska 1/

D.B.H., inches	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	BASIS 4/
	TOTAL HEIGHT, H (FEET) 2/ Cubic feet																				Number of trees
4	.8 (1.4)	1.0 (1.7)	1.3 (2.1)	1.5 (2.4)	1.7 (2.7)	2.0 (3.0)	2.2 (3.3)	2.4 (3.6)	2.6 (3.9)	4.3	4.6										--
5																					28
6	2.1 (3.0)	(2.6) (4.0)	(3.0) (4.0)	(3.4) (4.6)	(3.8) (5.1)	(4.2) (5.7)	(4.6) (6.2)	(5.1) (6.7)	5.5 (7.3)	5.9 (7.8)	6.3 (8.4)	6.7 (8.9)	9.4	10.0	10.5						24
7	3.0 (3.9)	(3.5) (4.5)	(4.0) (5.2)	(4.6) (5.9)	(5.1) (6.6)	(5.7) (7.3)	(6.2) (7.9)	(6.7) (8.6)	(7.3) (9.3)	(7.8) (10.0)	8.4 (10.7)	8.9 (11.3)	12.0	12.7	13.4	14.1					31
8	4.9	5.7 (7.0)	6.5 (8.0)	7.4 (9.0)	(8.2) (10.0)	9.1 (11.0)	(9.9) (12.1)	(10.7) (13.1)	(11.6) (14.1)	(12.4) (15.1)	(13.3) (16.1)	(14.1) (17.1)	(14.9)	15.8	16.6	17.4					34
9																					28
10																	22.2				35
11		8.4	9.6	10.8	12.0	(13.2)	(14.4)	(15.6)	(16.8)	(18.1)	(19.3)	(20.5)	(21.7)	(22.9)	(24.1)	25.3	26.6				35
12			11.3	12.7	14.1	15.6	(17.0)	(18.4)	(19.8)	(21.3)	(22.7)	(24.1)	(25.6)	(27.0)	28.4	29.8	31.3	32.7	34.1		39
13			13.1	14.8	16.4	18.1	19.8	(21.4)	(23.1)	(24.8)	(26.4)	(28.1)	(29.7)	(31.4)	(33.1)	34.7	36.4	38.0	39.7		48
14			15.1	17.0	18.9	20.8	(22.7)	(24.7)	26.6	(28.5)	(30.4)	(32.3)	(34.2)	(36.1)	(38.1)	(40.0)	41.9	43.8	45.7	47.6	34
15			17.2	19.4	21.6	(23.7)	25.9	28.1	(30.3)	(32.5)	(34.7)	(36.9)	(39.0)	(41.2)	(43.4)	(45.6)	47.8	50.0	52.2	54.3	25
16																					
17				21.9	24.4	26.9	29.3	(31.8)	(34.3)	36.8	39.2	(41.7)	(44.2)	(46.7)	(49.1)	(51.6)	54.1	(56.6)	59.0	61.5	17
18				24.6	27.4	30.1	32.9	35.7	38.5	(41.3)	(44.1)	(46.8)	(49.6)	(52.4)	(55.2)	(58.0)	(60.8)	(63.6)	66.3	69.1	18
19				30.5	33.6	36.7	39.8	43.0	46.1	49.2	52.3	55.4	(58.0)	(61.5)	(64.9)	(68.4)	(71.9)	(75.3)	(78.8)	82.2	7
20						41.1	45.0	48.8	52.6	56.4	60.2	64.1	67.9	(71.7)	75.5	(79.4)	(83.2)	87.0	(90.8)	94.6	6
21								53.6	57.8	62.0	66.2	70.4	74.6	78.8	(83.0)	(87.2)	(91.4)	(95.6)	99.8	(104)	5
22								58.6	63.2	67.8	72.4	77.0	81.7	86.3	90.9	(95.5)	(100)	105	109	114	2
23										73.9	79.0	84.0	89.0	94.0	99.1	104	109	114	119	124	2
24										80.3	85.7	91.2	96.7	102	(108)	113	119	124	(130)	135	2
25												98.8	105	111	116	122	(128)	134	140	146	2
26																					
27												107	113	119	126	132	(139)	145	151	158	1
28												115	122	128	135	142	149	156	163	170	--
29												123	131	138	145	153	160	168	175	182	--
30												132	140	148	156	164	172	180	188	195	--
31												141	149	158	166	175	183	192	200	209	--
32												150	159	168	177	186	196	205	214	223	--
												160	170	179	189	198	(208)	218	228	237	1

1/ Table volumes obtained from weighted regression equation: $V_T = -1.1843 + 0.2050D + 0.01639H + 0.00187D^2H$. Standard error of estimate around mean volume = 1.2982 cu. ft. = 4.9%.

2/ Mid-point of class (e.g. 4 = 3.6 through 4.5 inches).

3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).

4/ Figures in parentheses indicate extent of 434 measured trees used for computing above regression equation.

Table 3.--Tree volumes inside bark (1-foot stump to 4-inch top d.i.b.) for white spruce, interior Alaska^{1/}

D.B.H., inches	TOTAL HEIGHT, H (FEET) ^{2/}																			BASIS ^{4/} Number of trees 28	
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115		120
5	(.6)	(.8)	(1.0)	(1.2)	(1.5)	(1.7)	1.9	2.2	2.4	2.6	2.8										
6	1.4 (2.7)	(1.7)	(2.0)	(2.3)	(2.7)	(3.0)	(3.3)	(3.6)	4.0	4.3	4.6	4.9									24
7	2.2 (3.2)	3.8	4.4	(3.6)	(4.0)	(4.5)	(4.9)	(5.4)	(5.8)	(6.2)	6.7	7.1	7.6	8.0	8.5						31
8				(5.0)	(5.5)	(6.1)	(6.7)	(7.3)	(7.9)	(8.4)	(9.0)	9.6	10.2	10.8	11.3	11.9					34
9	4.3	5.0	5.8	6.5	(7.2)	8.0	(8.7)	(9.4)	(10.2)	(10.9)	(11.6)	(12.4)	(13.1)	13.8	14.6	15.3					28
10		6.4	7.3	(8.2)	9.1	(10.0)	(10.9)	(11.8)	(12.7)	(13.6)	(14.5)	(15.4)	(16.3)	(17.2)	18.1	19.0	19.9				35
11																					35
12		7.8	8.9	10.0	11.1	(12.2)	(13.3)	(14.4)	(15.5)	(16.6)	(17.7)	(18.7)	(19.8)	(20.9)	(22.0)	23.1	24.2				35
13			10.6	11.9	13.3	14.6	(15.9)	(17.2)	(18.5)	(19.8)	(21.1)	(22.4)	(23.7)	(25.0)	26.3	27.6	28.9	30.2	31.5		39
14			12.5	14.1	15.6	17.1	18.6	(20.2)	(21.7)	(23.2)	(24.8)	(26.3)	(27.8)	(29.4)	(30.9)	32.4	33.5	35.5	37.0		48
15			14.5	16.3	18.1	19.9	(21.6)	(23.4)	25.2	(27.0)	(28.7)	(30.5)	(32.3)	(34.0)	(35.8)	(37.6)	39.4	41.1	42.9	44.7	34
16			16.7	18.7	20.7	(22.8)	24.8	26.9	(28.9)	(30.9)	(33.0)	(35.0)	(37.0)	(39.1)	(41.1)	(43.1)	45.2	47.2	49.3	51.3	25
17																					17
18				21.3	23.6	25.9	28.2	(30.5)	(32.8)	35.2	37.5	(39.8)	(42.1)	(44.4)	(46.7)	(49.1)	51.4	(53.7)	56.0	58.3	18
19				23.9	26.6	29.2	31.8	34.4	37.0	(39.6)	(42.2)	(44.9)	(47.5)	(50.1)	(52.7)	(55.3)	(57.9)	(60.6)	63.2	65.8	12
20					29.7	32.6	35.6	38.5	41.4	44.4	47.3	50.2	53.2	(56.1)	(59.0)	(62.0)	(64.9)	(67.8)	70.8	73.7	7
21						36.3	39.5	42.8	46.1	49.3	52.6	(55.9)	(59.2)	62.4	65.7	(69.0)	72.2	(75.5)	78.8	(82.0)	6
22						40.1	43.7	47.3	51.0	54.6	58.2	61.8	65.4	(69.1)	72.7	(76.3)	(79.9)	83.5	(87.2)	90.8	5
23								52.1	56.1	60.1	64.1	68.1	72.1	76.0	(80.0)	(84.0)	(88.0)	(92.0)	96.0	(100)	2
24								57.1	61.4	65.8	70.2	74.6	79.0	83.3	87.7	(92.1)	(96.5)	101	105	110	2
25									71.8	76.6	81.4	86.2	91.0	95.8	(100)	105	110	115	120	125	--
26									78.1	83.3	88.5	93.7	98.9	(104)	109	115	120	(125)	130	135	2
27											95.9	102	107	113	118	124	130	136	141	146	1
28											104	110	116	122	128	(134)	140	146	152	158	--
29											112	118	125	131	138	144	151	158	164	170	--
30											120	127	134	141	148	155	162	170	177	184	--
31											128	136	144	151	159	166	174	182	189	196	--
32											137	145	154	162	170	178	186	194	202	210	--
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1/ Table volumes obtained from weighted regression equation: $V_1 = -2.0555 + 0.2982D + 0.0018D^2H$. Standard error of estimate around mean volume = 1.4605 cu. ft. = 5.9%.
2/ Mid-point of class (e.g. 5 = 4.6 through 5.5 inches).
3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).
4/ Figures in parentheses indicate extent of 434 measured trees used for computing above regression equation.

Table 4.--Tree volumes inside bark (1-foot stump to 6-inch top d.i.b.) for white spruce, Interior Alaska^{1/}

D.B.H., Inches	TOTAL HEIGHT, H (FEET) ^{2/}																			BASIS ^{4/}	
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115		120
Cubic feet																					
7	1.2	(1.4)	(1.5)	(1.6)	(1.7)	(1.8)	(2.0)	(2.1)	(2.2)	(2.3)	2.4	2.6	2.7	2.8	2.9						31
8	(2.7)	3.0	3.3	(3.6)	(3.9)	(4.2)	(4.5)	(4.8)	(5.1)	(5.4)	(5.7)	6.0	6.3	6.6	7.0	7.3					34
9	3.9	4.5	5.0	5.5	(6.0)	6.5	(7.0)	(7.6)	(8.1)	(8.6)	(9.1)	(9.6)	(10.1)	10.6	11.2	11.7					28
10		5.8	6.6	(7.3)	8.1	(8.8)	(9.6)	(10.3)	(11.1)	(11.8)	(12.6)	(13.3)	(14.0)	(14.8)	15.5	16.3	17.0				35
11		7.0	8.0	9.0	10.1	(11.1)	(12.1)	(13.1)	(14.1)	(15.1)	(16.1)	(17.1)	(18.1)	(19.1)	(20.1)	21.1	22.1	28.8	30.0		35
12			9.4	10.7	12.0	13.3	(14.6)	(15.9)	(17.1)	(18.4)	(19.7)	(21.0)	(22.3)	(23.6)	24.9	26.2	27.5	34.6	36.2		39
13			10.7	12.3	13.9	15.5	17.1	(18.7)	(20.2)	(21.8)	(23.4)	(25.0)	(26.6)	(28.2)	(29.8)	31.4	33.0	40.8	42.7		48
14			11.8	13.7	15.7	17.6	(19.5)	(21.5)	23.4	(25.3)	(27.2)	(29.2)	(31.1)	(33.0)	(35.0)	(36.9)	38.8	47.1	49.3	44.6	34
15			12.9	15.1	17.4	(19.7)	22.0	24.3	(26.6)	(28.9)	(31.1)	(33.4)	(35.7)	(38.0)	(40.3)	(42.6)	44.9	47.1	49.3	51.3	25
16				16.5	19.1	21.8	24.5	(27.1)	(29.8)	32.5	35.1	(37.8)	(40.5)	(43.1)	(45.8)	(48.5)	51.1	(53.7)	56.0	58.3	17
17				17.7	20.8	23.8	26.9	30.0	33.0	(36.1)	(39.2)	(42.3)	(45.3)	(48.4)	(51.5)	(54.6)	(57.6)	(60.6)	63.2	65.8	18
18					22.3	25.8	29.3	32.8	36.3	39.8	43.3	46.9	50.4	(53.9)	(57.4)	(60.9)	(64.4)	(67.8)	70.8	73.7	12
19						27.8	31.8	35.7	39.7	43.6	47.6	(51.6)	(55.5)	59.5	63.4	(67.4)	71.3	(75.3)	78.8	(82.0)	7
20						29.7	34.2	38.6	43.0	47.5	51.9	56.4	60.8	(65.2)	69.7	(74.1)	(78.6)	83.0	(87.2)	90.8	6
21								41.5	46.5	51.4	56.3	61.3	66.2	71.2	(76.1)	(81.1)	(86.0)	(90.9)	95.9	(100)	5
22								44.4	49.9	55.4	60.8	66.3	71.8	77.3	82.7	(88.2)	(93.7)	99.1	105	110	2
23										59.4	65.4	71.5	77.5	83.5	89.5	95.6	102	108	114	120	--
24										63.5	70.1	76.7	83.3	89.9	(96.5)	103	110	116	(123)	130	2
25												82.1	89.3	96.5	104	111	(118)	125	132	140	2
26												87.6	95.4	103	111	119	(127)	135	142	150	1
27												93.2	102	110	119	127	136	144	153	161	--
28												98.9	108	117	126	136	145	154	163	172	--
29												105	115	124	134	144	154	164	174	184	--
30												111	121	132	142	153	164	174	185	195	--
31												117	128	139	151	162	173	185	196	207	--
32												123	135	147	159	171	(183)	196	208	220	1

1/ Table volumes obtained from weighted regression equation: $V_2 = -10.8448 + 2.6242D - 0.14074D^2 - 0.09628H + 0.00246D^2H$ except for volumes enclosed by solid line where: $V_1 = -2.0555 + 0.2982D + 0.00181D^2H$. Standard error of estimate around mean volume (V_2) = 1.6798 cu. ft. = 6.4%.
2/ Mid-point of class (e.g. 5 = 4.6 through 5.5 inches).
3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).
4/ Figures in parentheses indicate extent of 382 trees used for computing above regression equation V_2 .

Table 5.--Tree volumes inside bark (2-foot stump to 8-inch top d.i.b.) for white spruce, interior Alaska^{1/}

D.B.H., D ₂ / Inches	TOTAL HEIGHT, H (FEET) ^{2/}															BASIS ^{4/}				
	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	Number of trees 35
Cubic feet																				
10	4.9	5.1	(5.3)	5.4	(5.6)	(5.7)	(5.9)	(6.1)	(6.2)	(6.4)	(6.5)	(6.7)	(6.9)	7.0	7.2	7.3				
11	6.1	6.6	7.1	7.6	(8.1)	(8.6)	(9.1)	(9.6)	(10.0)	(10.5)	(11.0)	(11.5)	(12.0)	(12.5)	13.0	13.5				35
12		7.8	8.7	9.5	10.4	(11.2)	(12.1)	(12.9)	(13.8)	(14.6)	(15.5)	(16.3)	(17.2)	18.0	18.9	19.7	20.6	21.4		39
13		8.8	10.0	11.2	12.5	13.7	(15.0)	(16.2)	(17.4)	(18.7)	(19.9)	(21.2)	(22.4)	(23.6)	24.9	26.1	27.4	28.6		48
14		9.4	11.0	12.7	14.4	(16.0)	(17.7)	19.4	(21.0)	(22.7)	(24.4)	(26.0)	(27.7)	(29.3)	(31.0)	32.7	34.3	36.0	37.7	34
15		9.7	11.8	13.9	(16.1)	18.2	20.3	(22.4)	(24.5)	(26.6)	(28.8)	(30.9)	(33.0)	(35.1)	(37.2)	39.3	41.5	43.6	45.7	25
16			12.3	14.9	17.5	20.1	(22.7)	(25.3)	27.9	30.6	(33.2)	(35.8)	(38.4)	(41.0)	(43.6)	46.2	(48.8)	51.4	54.0	17
17			12.6	15.7	18.8	21.9	25.1	28.2	(31.3)	(34.4)	(37.5)	(40.6)	(43.8)	(46.9)	(50.0)	(53.1)	(56.2)	59.4	62.5	18
18				16.2	19.9	23.5	27.2	30.9	34.5	38.2	41.9	45.5	(49.2)	(52.9)	(56.5)	(60.2)	(63.9)	67.5	71.2	12
19					20.7	25.0	29.2	33.5	37.7	42.0	(46.2)	(50.5)	54.7	59.0	(63.2)	67.4	(71.7)	75.9	(80.2)	7
20					21.4	26.3	31.1	36.0	40.8	45.7	50.5	55.4	(60.2)	65.1	(70.0)	(74.8)	79.7	(84.5)	89.4	6
21							32.8	38.3	43.8	49.3	54.8	60.3	65.8	(71.3)	(76.8)	(82.3)	(87.8)	93.3	(98.8)	5
22							34.4	40.6	46.8	53.0	59.1	65.3	71.5	77.6	(83.8)	(90.0)	96.1	102	108	2
23									49.6	56.5	63.4	70.3	77.1	84.0	90.9	97.8	105	112	118	--
24									52.4	60.0	67.6	75.2	82.9	(90.5)	98.1	106	113	(121)	128	2
25											71.9	80.2	88.6	97.0	105	(114)	122	130	139	2
26											76.1	85.3	94.4	104	113	(122)	131	140	150	1
27											80.3	90.3	100	110	120	130	140	150	160	--
28											84.4	95.3	106	117	128	139	150	160	171	--
29											88.6	100	112	124	136	147	159	171	183	--
30											92.7	105	118	131	144	156	169	182	194	--
31											96.9	110	124	138	151	165	179	192	206	--
32											101	116	130	145	160	(174)	189	203	218	1

^{1/} Table volumes obtained from unweighted regression equation: $V_3 = -16.7852 + 4.6690D - 0.25937D^2 - 0.28087H + 0.00313D^2H$. Standard error of estimate around mean volume = 3.3262 cu. ft. = 12.9%; $R^2 = .9807$.

^{2/} Mid-point of class (e.g. 10 = 9.6 through 10.5 inches).

^{3/} Mid-point of class (e.g. 30 = 27.6 through 32.5 feet).

^{4/} Figures in parentheses indicate extent of 318 trees used for computing above regression equation.

Table 6.--Tree volumes inside bark (entire stem) for aspen, interior Alaska^{1/}

D.B.H., D ₂ /		TOTAL HEIGHT, H (FEET) ^{2/}												BASIS ^{4/}					
		25	30	35	40	45	50	55	Cubic feet				70	75	80	85	90	95	Number of trees
Inches																			
4		.9	1.2	1.4	1.6	1.8	2.0	2.3	2.5	2.7	2.9								--
5	(1.4)	1.7	2.0	(2.3)	(2.7)	(3.0)	(3.3)	(3.6)	4.0	(4.3)	4.6								32
6	1.9	2.3	2.8	(3.3)	(3.7)	(4.2)	(4.6)	(5.1)	(5.5)	(6.0)	6.4								31
7		3.1	3.7	4.3	4.9	(5.5)	(6.2)	(6.8)	(7.4)	(8.0)	8.6	9.2							29
8		4.0	4.8	(5.6)	(6.4)	(7.1)	(7.9)	(8.7)	(9.5)	(10.3)	11.1	11.9							30
9		5.0	6.0	(7.0)	8.0	(9.0)	(9.9)	(10.9)	(11.9)	(12.9)	(13.9)	14.9	15.9						31
10			7.4	8.6	9.8	(11.0)	12.2	(13.4)	(14.6)	(15.8)	(17.0)	(18.2)	19.4	20.6					32
11			8.9	10.3	(11.8)	13.2	14.7	(16.1)	(17.6)	(19.0)	(20.5)	(21.9)	(23.4)	(24.9)	26.3				30
12				12.2	13.9	15.7	17.4	(19.1)	(20.8)	(22.6)	(24.3)	(26.0)	(27.7)	29.5	31.2				31
13				14.3	16.3	18.3	20.3	(22.4)	(24.4)	(26.4)	(28.4)	(30.4)	32.5	(34.5)	36.5				29
14						21.2	(23.5)	(25.9)	(28.2)	(30.5)	(32.9)	(35.2)	(37.6)	(39.9)	42.2				22
15						24.3	(27.0)	29.6	(32.3)	(35.0)	(37.7)	(40.4)	(43.0)	45.7	48.4				27
16						27.6	30.6	33.7	(36.7)	(39.8)	(42.8)	(45.8)	(48.9)	51.9	55.0				13
17						31.1	34.5	38.0	41.4	44.8	(48.3)	51.7	(55.1)	(58.5)	62.0				4
18								42.5	46.4	50.2	(54.0)	57.9	(61.7)	65.6	69.4				2
19									51.6	55.9	60.2	64.4	68.7	(73.0)	77.3				1
20										61.9	66.6	71.3	(76.1)	(80.8)	85.5				2
21										68.2	73.4	78.6	83.8	89.0	94.2				--
22										74.8	80.5	86.2	91.9	97.6	103				--

1/ Table volumes obtained from weighted regression equation: $V_T = -0.01100D^2 + 0.00662H + 0.00235D^2H$. Standard error of estimate around mean volume = 1.1660 cu. ft. = 7.6%.

2/ Mid-point of class (e.g. 4 = 3.6 through 4.5 inches).

3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).

4/ Figures in parentheses indicate extent of 346 measured trees used for computing above regression equation.

Table 7.--Tree volumes inside bark (1-foot stump to 4-inch top d.i.b.) for aspen, interior Alaska^{1/}

D.B.H., D2/ Inches	TOTAL HEIGHT, H (FEET) ^{2/}															BASIS ^{4/}	
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	Number of trees	
5	(.4)	.7	1.0	(1.4)	(1.7)	(2.0)	(2.3)	(2.6)	2.9	(3.2)	3.5					32	
6	.9	1.3	1.7	(2.2)	(2.6)	(3.1)	(3.5)	(4.0)	(4.4)	(4.8)	5.3					31	
7		2.0	2.6	3.2	3.8	(4.4)	(5.0)	(5.6)	(6.2)	(6.8)	7.4	8.0				29	
8		2.7	3.5	(4.3)	(5.1)	(5.9)	(6.7)	(7.5)	(8.3)	(9.0)	9.8	10.6				30	
9		3.6	4.6	(5.6)	6.6	(7.6)	(8.6)	(9.6)	(10.6)	(11.6)	(12.6)	13.6	14.6			31	
10			5.8	7.1	8.3	(9.5)	10.8	(12.0)	(13.2)	(14.4)	(15.7)	(16.9)	18.1	19.4		32	
11			7.2	8.7	(10.2)	11.6	13.1	(14.6)	(16.1)	(17.6)	(19.1)	(20.6)	(22.1)	(23.6)	25.0	30	
12				10.4	12.2	14.0	15.7	(17.5)	(19.3)	(21.1)	(22.8)	(24.6)	(26.4)	28.1	29.9	31	
13				12.3	14.4	16.5	18.6	(20.6)	(22.7)	(24.8)	(26.9)	(29.0)	31.0	(33.1)	35.2	29	
14						19.2	(21.6)	(24.0)	(26.4)	(28.9)	(31.3)	(33.7)	(36.1)	(38.5)	40.9	22	
15						22.1	(24.9)	27.7	(30.4)	(33.2)	(36.0)	(38.7)	(41.5)	44.3	47.0	27	
16						25.3	28.4	31.6	(34.7)	(37.9)	(41.0)	(44.2)	(47.3)	50.5	53.6	13	
17						28.6	32.1	35.7	39.3	42.8	(46.4)	49.9	(53.5)	(57.0)	60.6	4	
18								40.1	44.1	48.1	(52.0)	56.0	(60.0)	64.0	68.0	2	
19									49.2	53.6	58.0	62.5	66.9	(71.4)	75.8	1	
20										59.5	64.4	69.3	(74.2)	(79.1)	84.1	2	
21										65.6	71.0	76.5	81.9	87.3	92.7	--	
22										72.1	78.0	84.0	89.9	95.9	102	--	

1/ Table volumes obtained from unweighted regression equation: $V_1 = -0.5553 - 0.02216D^2 + 0.00246D^3$. Standard error of estimate around mean volume = 2.0455 cu. ft. = 11.6%.

2/ Mid-point of class (e.g. 5 = 4.6 through 5.5 inches).

3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).

4/ Figures in parentheses indicate extent of 346 measured trees used for computing above regression equation.

Table 8.--Tree volumes inside bark (1-foot stump to 6-inch top d.i.b.) for aspen, interior Alaska^{1/}

D.B.H., D ₂ ^{4/} Inches	TOTAL HEIGHT, H (FEET) ^{2/}														BASIS ^{4/} Number of trees
	30	35	40	45	50	55	60	65	70	75	80	85	90	95	
	Cubic feet														
7	0	.4	1.0	1.5	(2.1)	(2.7)	(3.2)	(3.8)	(4.3)	4.9	5.5				29
8	.9	1.6	(2.4)	(3.1)	(3.8)	(4.6)	(5.3)	(6.0)	(6.8)	7.5	8.2				30
9	2.1	3.0	(3.9)	4.8	(5.8)	(6.7)	(7.6)	(8.6)	(9.5)	(10.4)	11.3	12.3			31
10		4.5	5.7	6.8	(7.9)	9.1	(10.2)	(11.4)	(12.5)	(13.7)	(14.8)	16.0	17.1		32
11		6.2	7.6	(9.0)	10.3	11.7	(13.1)	(14.5)	(15.9)	(17.3)	(18.7)	(20.0)	(21.4)	22.8	30
12			9.7	11.3	13.0	14.6	(16.3)	(17.9)	(19.6)	(21.2)	(22.9)	(24.5)	26.2	27.8	31
13			12.0	13.9	15.8	17.8	(19.7)	(21.7)	(23.6)	(25.5)	(27.5)	29.4	(31.3)	33.3	29
14					18.9	(21.2)	(23.4)	(25.7)	(27.9)	(30.2)	(32.4)	(34.6)	(36.9)	39.1	22
15					22.1	(24.8)	27.4	(30.0)	(32.6)	(35.1)	(37.7)	(40.3)	42.9	45.4	27
16					25.3	28.4	31.6	(34.6)	(37.5)	(40.5)	(43.4)	(46.3)	49.3	52.2	13
17					28.6	32.1	35.7	39.3	42.8	(46.1)	49.4	(52.7)	(56.1)	59.4	4
18							40.1	44.1	48.1	(52.0)	55.9	(59.6)	63.3	67.0	2
19								49.2	53.6	58.0	62.5	66.8	(70.9)	75.0	1
20									59.5	64.4	69.3	(74.2)	(78.9)	83.5	2
21									65.6	71.0	76.5	81.9	87.3	92.4	--
22									72.1	78.0	84.0	89.9	95.9	102	--

1/ Table volumes outside block enclosed by solid line obtained by unweighted regression equation: $V_2 = -3.5054 + 0.00229D^2H$.
Table volumes inside block enclosed by solid line obtained by unweighted regression equation $V_1 = -0.5553 - 0.02216D^2 + 0.00246D^2H$.
Standard error of estimate around mean volume (V_2) = 2.4248 cu. ft. = 12.6%.

2/ Mid-point of class (e.g. 7 = 6.6 through 7.5 inches).

3/ Mid-point of class (e.g. 30 = 27.6 through 32.5 feet).

4/ Figures in parentheses indicate extent of 285 measured trees used for computing above regression equation. Three trees not shown in "BASIS" column were in the 6-inch diameter class.

Table 9.--Tree volumes inside bark (2-foot stump to 8-inch top d.i.b.) for aspen, interior Alaska^{1/}

D.B.H., D2/ Inches	TOTAL HEIGHT, H (FEET)3/													BASIS4/	
	35	40	45	50	55	60	65	70	75	80	85	90	95		
	Cubic feet														
10	2.8	3.5	4.1	(4.7)	5.3	(5.9)	(6.5)	(7.1)	(7.7)	(8.3)	8.9	9.5			32
11	4.2	5.2	(6.2)	7.2	8.2	(9.2)	(10.2)	(11.2)	(12.2)	(13.2)	(14.2)	(15.2)	16.2	30	
12		6.7	8.2	9.6	11.0	(12.5)	(13.9)	(15.3)	(16.8)	(18.2)	(19.6)	21.1	22.5	31	
13		8.2	10.1	12.0	13.9	(15.8)	(17.7)	(19.6)	(21.5)	(23.4)	25.3	(27.2)	29.1	29	
14				14.4	(16.8)	(19.2)	(21.6)	(24.0)	(26.4)	(28.8)	(31.2)	(33.6)	36.0	22	
15				16.7	(19.6)	22.6	(25.5)	(28.5)	(31.4)	(34.4)	(37.3)	40.3	43.2	27	
16				19.0	22.5	26.0	(29.6)	(33.1)	(36.6)	(40.2)	(43.7)	47.2	50.8	13	
17				21.2	25.3	29.5	33.7	37.8	(42.0)	46.1	(50.3)	(54.3)	57.8	4	
18						33.0	37.8	42.6	(47.4)	52.2	(57.0)	61.5	65.2	2	
19							42.0	47.5	53.0	58.5	64.0	(68.9)	73.0	1	
20								52.5	58.8	65.0	(71.2)	(75.3)	81.3	2	
21								57.7	64.7	71.7	78.7	84.9	90.0	--	
22								62.9	70.7	78.5	86.3	93.3	99.2	--	

1/ Table volumes outside block enclosed by solid line obtained from weighted regression equation: $V_3 = -10.0346 + 2.9603D - 0.20966D^2 - 0.25373H + 0.00375D^2H$. Table volumes within block enclosed by solid line obtained by subtracting o.b. basal area at b.h. from V_2 tabular volumes. Standard error of estimate around mean volume (V_3) = 2.2803 cu. ft. = 12.1%.

2/ Mid-point of class (e.g. 10 = 9.6 through 10.5 inches).

3/ Mid-point of class (e.g. 35 = 32.6 through 37.5 feet).

4/ Figures in parentheses indicate extent of 193 measured trees used for computing the above regression equation. Thirty-two trees not shown in "BASIS" column were in the 9-inch diameter class.

Table 10.--Tree volumes inside bark (entire stem) for paper birch, interior Alaska^{1/}

D.B.H., D ₂ /Inches	TOTAL HEIGHT, H (FEET) ^{2/}											BASIS ^{4/} Number of trees			
	25	30	35	40	45	50	55	60	65	70	75		80	85	90
Cubic feet															
4	.9	1.1	1.3	1.6	1.8	2.0	2.2	2.4	2.7	2.9					34
5	1.3	1.6	(1.9)	(2.2)	(2.6)	(2.9)	(3.2)	(3.5)	(3.9)	4.2	4.5				
6	1.7	(2.2)	2.6	(3.1)	(3.5)	(4.0)	(4.4)	(4.9)	5.3	(5.8)	6.2	6.7			30
7	2.3	2.9	3.5	4.1	(4.7)	(5.3)	(5.9)	(6.5)	(7.1)	(7.7)	8.3	8.9			48
8		3.7	4.5	5.2	6.0	(6.8)	(7.5)	(8.3)	(9.1)	(9.8)	(10.6)	11.4	12.1		42
9				6.5	7.5	(8.5)	(9.4)	(10.4)	(11.3)	(12.3)	(13.3)	14.2	15.2	16.1	44
10				8.0	9.2	(10.3)	(11.5)	(12.7)	(13.9)	(15.1)	(16.2)	(17.4)	18.6	19.8	46
11				9.6	11.0	(12.4)	(13.9)	(15.3)	(16.7)	(18.1)	(19.5)	20.9	22.3	23.8	38
12					13.0	14.7	16.4	(18.1)	(19.7)	(21.4)	(23.1)	(24.8)	26.5	28.1	24
13					15.3	17.2	19.2	(21.1)	(23.1)	(25.0)	(27.0)	(29.0)	(30.9)	32.9	13
14						19.9	(22.2)	24.4	26.7	(29.0)	31.2	33.5	35.8	38.0	3
15						22.8	25.4	28.0	(30.6)	(33.2)	35.7	38.3	40.9	43.5	3
16							28.8	31.8	34.7	(37.6)	40.6	43.5	46.5	49.4	1
17								35.8	39.1	42.4	45.7	49.1	52.4	55.7	--
18								40.1	43.8	47.5	51.2	54.9	58.6	62.4	--

1/ Table volumes obtained from weighted regression equation: $V_T = -0.01408D^2 + 0.00815H + 0.00227D^2H$. Standard error of estimate around mean volume=0.6899 cu. ft. = 6.1%.

2/ Mid-point of class (e.g. 4 = 3.6 through 4.5 inches).

3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).

4/ Figures in parentheses indicate extent of 326 measured trees used for computing above regression equation.

Table 11.--Tree volumes inside bark (1-foot stump to 4-inch top d.i.b.) for paper birch, interior Alaska^{1/}

D.B.H., D2/ Inches	TOTAL HEIGHT, H (FEET)2/														BASIS4/
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	Number of trees 34
	Cubic feet														
5	.5	.7	(.9)	(1.1)	(1.3)	(1.4)	(1.6)	(1.8)	(2.0)	2.2	2.4				
6	1.1	(1.4)	1.8	(2.1)	(2.5)	(2.8)	(3.1)	(3.5)	3.8	(4.2)	4.5	4.9			30
7	1.6	2.1	2.7	3.2	(3.7)	(4.2)	(4.8)	(5.3)	(5.8)	(6.3)	6.9	7.4			48
8		2.8	3.5	4.3	5.0	(5.7)	(6.5)	(7.2)	(7.9)	(8.7)	(9.4)	10.2	10.9		42
9				5.3	6.3	(7.3)	(8.3)	(9.3)	(10.2)	(11.2)	(12.2)	13.2	14.1	15.1	44
10				6.5	7.7	(8.9)	(10.2)	(11.4)	(12.7)	(13.9)	(15.2)	(16.4)	17.7	18.9	46
11				7.6	9.1	(10.7)	(12.2)	(13.8)	(15.3)	(16.8)	(18.4)	19.9	21.5	23.0	38
12					10.6	12.5	14.3	(16.2)	(18.1)	(19.9)	(21.8)	(23.7)	25.5	27.4	24
13					12.1	14.3	16.5	(18.8)	(21.0)	(23.2)	(25.4)	(27.6)	(29.9)	32.1	13
14						16.3	(18.9)	21.5	24.1	(26.7)	29.3	31.9	34.5	37.1	3
15						18.3	21.3	24.3	(27.3)	(30.3)	33.3	36.3	39.3	42.3	3
16							23.8	27.3	30.7	(34.1)	37.6	41.0	44.5	47.9	1
17								30.3	34.3	38.2	42.1	46.0	49.9	53.8	--
18								33.6	38.0	42.4	46.8	51.2	55.6	60.0	--

1/ Table volumes obtained from weighted regression equation: $V_1 = -2.5767 + 0.9524D - 0.10446D^2 - 0.03303H + 0.00282D^2H$.
Standard error of estimate around mean volume = 0.8193 cu. ft. = 8.1%.

2/ Mid-point of class (e.g. 5 = 4.6 through 5.5 inches).

3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).

4/ Figures in parentheses indicate extent of 326 measured trees used for computing above regression equation.

Table 12.--Tree volumes inside bark (1-foot stump to 6-inch top d.i.b.) for paper birch, interior Alaska^{1/}

D.B.H., D2/ Inches	TOTAL HEIGHT, H (FEET) ^{2/}														BASIS ^{4/}
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	
Cubic feet															
7	1.0	1.2	1.4	1.6	(1.8)	(2.0)	(2.2)	(2.4)	(2.6)	(2.8)	3.0	3.2			48
8		2.3	2.7	3.1	3.4	(3.8)	(4.2)	(4.6)	(5.0)	(5.4)	(5.7)	6.1	6.5		42
9				4.7	5.3	(5.9)	(6.5)	(7.1)	(7.6)	(8.2)	(8.8)	9.4	10.0	10.6	44
10				6.5	7.4	(8.2)	(9.0)	(9.8)	(10.6)	(11.5)	(12.3)	(13.1)	13.9	14.7	46
11				7.6	9.1	(10.7)	(11.8)	(12.9)	(13.9)	(15.0)	(16.1)	17.2	18.2	19.3	38
12				10.6	12.5	14.3	(16.2)	(17.6)	(18.9)	(20.3)	(21.6)	23.0	24.3	25.8	24
13				12.1	14.3	16.5	(18.8)	(21.0)	(23.1)	(24.8)	(26.4)	28.1	29.8	31.7	13
14				16.3	18.9	21.5	(24.1)	(26.7)	(29.3)	31.7	33.7	35.6	37.5	39.4	3
15				18.3	21.3	24.3	(27.3)	(30.3)	33.3	36.3	39.3	41.9	44.5	47.9	3
16				23.8	27.3	30.7	(34.1)	37.6	41.0	44.5	47.9	51.2	54.6	58.0	1
17				30.3	34.3	38.2	42.1	46.0	49.9	53.8	57.7	61.6	65.5	69.4	--
18				33.6	38.0	42.4	46.8	51.2	55.6	60.0	64.4	68.8	73.2	77.6	--

- 1/ Table volumes outside block enclosed by solid line obtained from weighted regression equation: $V_2 = -0.07842H$
 $+ 0.00242D^2H$. Table values inside block enclosed by solid line obtained from weighted regression equation: $V_1 = -2.5767$
 $+ 0.9524D - 0.10446D^2 - 0.03303H + 0.00282D^2H$. Standard error of estimate around mean volume (V_2) = 1.2723 cu. ft. = 12.8%.
- 2/ Mid-point of class (e.g. 7 = 6.6 through 7.5 inches).
- 3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).
- 4/ Figures in parentheses indicate extent of 262 measured trees used for computing above regression equation.

Table 13.---Tree volumes inside bark (2-foot stump to 8-inch top d.i.b.) for paper birch, interior Alaska^{1/}

D.B.H., D2/ Inches	TOTAL HEIGHT, H (FEET)3/										BASIS4/	
	40	45	50	55	60	65	70	75	80	85	90	Number of trees 46
Cubic feet												
10	3.3	3.8	(4.2)	(4.6)	(5.0)	(5.4)	(5.8)	(6.3)	(6.7)	7.1	7.5	
11	5.4	6.1	(6.8)	(7.5)	(8.1)	(8.8)	(9.5)	(10.2)	10.9	11.5	12.2	38
12		8.7	9.6	10.6	(11.6)	(12.5)	(13.5)	(14.5)	(15.4)	16.4	17.4	24
13		11.2	12.8	14.0	(15.3)	(16.6)	(17.9)	(19.1)	(20.4)	(21.7)	23.0	13
14			15.2	(17.7)	19.3	21.0	(22.6)	24.2	25.8	27.4	29.0	3
15			17.1	20.1	23.1	(25.7)	(27.6)	29.6	31.6	33.5	35.5	3
16				22.4	25.9	29.3	(32.7)	35.4	37.7	40.1	42.5	1
17					28.7	32.7	36.6	40.5	44.3	47.1	49.9	--
18					31.8	36.2	40.6	45.0	49.4	53.8	57.7	--

1/ Table volumes outside block enclosed by solid line obtained from weighted regression equation:
 $V_3 = 0.16588H + 0.00249D^2H$. Table volumes within block enclosed by solid line obtained by subtracting o.b.
basal area at b.h. from V_2 tabular volumes. Standard error of estimate around mean volume (V_3) = 1.5945
cu. ft. = 19.3%.

2/ Mid-point of class (e.g. 10 = 9.6 through 10.5 inches).

3/ Mid-point of class (e.g. 40 = 37.6 through 42.5 feet).

4/ Figures in parentheses indicate extent of 128 measured trees used for computing above regression
equation. Forty-four trees not shown in the "BASIS" column were in the 9-inch diameter class.

Table 14.---Tree volumes inside bark (entire stem) for cottonwood, Interior Alaska^{1/}

D.B.H., D ₂ / Inches	TOTAL HEIGHT, H (FEET) ^{2/}																			Number of trees	BASIS ^{4/}	
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115			120

1/ Table volumes obtained from weighted regression equation: $V_T = 0.00806H + 0.00175D^2H$. Standard error of estimate around mean volume = 1.8850 cu. ft. = 5.4%.

2/ Mid-point of class (e.g. 4 = 3.6 through 4.5 inches).

3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).

4/ Figures in parentheses indicate extent of 280 measured trees used for computing above regression equation.

Table 15.--Tree volumes inside bark (1-foot stump to 4-inch top d.i.b.) for cottonwood, Interior Alaska^{1/}

D.B.H., D2/ Inches	TOTAL HEIGHT, H (FEET) ^{2/}															BASIS ^{4/} Number of trees						
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95		100	105	110	115	120	125
Cubic feet																						
5	.2	.3	(.5)	(.6)	.8	(.9)	(1.1)	(1.2)	1.4	1.5	1.7											
6	.9	1.2	1.4	1.7	2.0	2.2	(2.5)	(2.7)	(3.0)	3.2	3.5	3.8										16
7	1.6	2.0	2.4	2.8	(3.2)	3.6	(4.0)	(4.3)	(4.7)	(5.1)	5.5	5.9	6.3	6.7								17
8		2.9	3.4	3.9	(4.5)	(5.0)	(5.5)	(6.1)	(6.6)	(7.2)	7.7	8.2	8.8	9.3								15
9		3.7	4.4	5.1	5.8	(6.5)	(7.2)	(7.9)	(8.6)	(9.3)	(10.0)	(10.8)	11.5	12.2	12.9	13.6	14.3	15.0	15.7			16
10		4.5	5.4	6.3	(7.2)	8.1	9.0	9.9	(10.8)	(11.7)	(12.6)	(13.5)	14.4	15.3	16.2	17.1	18.0	18.9	19.8			14
11			6.5	7.6	(8.7)	(9.8)	10.9	(12.0)	(13.1)	(14.2)	(15.3)	(16.4)	17.5	(18.6)	19.7	20.8	(21.9)	23.0	24.1			16
12			7.5	(8.9)	10.2	11.5	(12.9)	(14.2)	(15.5)	(16.9)	(18.2)	(19.5)	(20.9)	(22.2)	23.5	(24.9)	26.2	27.5	28.9			21
13			8.6	10.2	11.8	13.4	15.0	(16.5)	(18.1)	(19.7)	(21.3)	(22.9)	(24.5)	(26.0)	27.6	(29.2)	30.8	32.4	34.0	35.5	37.1	16
14				11.6	13.5	15.3	17.2	19.0	(20.9)	(22.7)	24.6	(26.4)	(28.3)	(30.1)	(32.0)	(33.8)	(35.7)	37.5	39.4	41.2	43.1	15
15						17.3	19.4	21.6	(23.7)	25.9	28.0	(30.1)	32.3	(34.4)	(36.6)	(38.7)	(40.8)	43.0	45.1	47.3	49.4	18
16							21.8	24.3	(26.7)	29.2	(31.6)	(34.1)	(36.5)	(39.0)	(41.4)	(43.9)	(46.3)	48.8	51.2	(53.7)	56.1	25
17							24.3	(27.1)	29.9	32.7	(35.4)	38.2	(41.0)	(43.8)	(46.5)	(49.3)	(52.1)	54.9	57.6	60.4	63.2	15
18							26.9	30.0	33.2	(36.3)	(39.4)	(42.5)	45.7	(48.8)	(51.9)	(55.0)	(58.2)	(61.3)	64.4	67.5	70.7	14
19									36.6	40.1	(43.6)	(47.1)	50.6	(54.1)	(57.5)	(61.0)	(64.5)	(68.0)	(71.5)	75.0	78.5	13
20									40.2	44.0	47.9	51.8	55.7	59.6	(63.4)	(67.3)	(71.2)	(75.1)	79.0	82.8	86.7	8
21									48.2	(52.4)	(56.7)	(61.0)	65.3	69.6	(73.9)	78.2	(82.5)	(86.8)	91.0	95.3		4
22									52.4	57.2	61.9	66.6	71.3	76.0	(80.7)	85.5	90.2	(94.9)	99.6	104		2
23									56.5	62.0	67.2	72.4	77.5	(82.7)	87.9	93.0	98.2	103	108	114		1
24										67.1	(72.7)	78.4	84.0	89.6	95.3	101	106	112	118	123		2
25										72.4	(78.5)	84.6	90.7	(96.8)	103	109	(115)	121	127	134		3
26										77.8	84.4	(91.0)	(97.7)	104	111	118	124	(131)	(137)	144		6
27										83.4	90.5	97.7	105	112	119	126	134	141	148	155		--
28										89.2	(96.9)	105	(112)	120	(128)	135	(143)	151	158	166		5
29										95.1	103	112	120	128	136	145	153	161	170	178		--
30											110	(119)	128	137	146	154	163	172	(181)	190		2
31											117	126	136	146	(155)	164	174	183	193	202		1
32											124	134	144	154	165	175	185	195	205	215		--
33											132	142	153	164	174	185	196	207	217	228		--
34											139	150	162	173	185	196	208	219	230	242		--

^{1/} Table volumes obtained from weighted regression equation: $V_1 = -3.2187 + 0.8281D - 0.05908D^2 - 0.01985H + 0.00199D^2H$. Standard error of estimate around mean volume = 1.8850 cu. ft. = 5.8%.

^{2/} Mid-point of class (e.g. 5 = 4.6 through 5.5 inches).

^{3/} Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).

^{4/} Figures in parentheses indicate extent of 280 measured trees used for computing above regression equation.

Table 16.--Tree volumes inside bark (1-foot stump to 6-inch top d.i.b.) for cottonwood, interior Alaska^{1/}

D.B.H., D ₂ / Inches	TOTAL HEIGHT, H (FEET) ^{3/}																			BASIS ^{4/}										
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115		120	125								
											Cubic feet																			Number of trees
7	0	0	0	0	(.2)	.5	(.9)	(1.3)	(1.6)	(2.0)	2.4	2.7	3.1	3.5									17							
8	.4	.9	(2.3)	1.3	(1.8)	(2.3)	(2.8)	(3.3)	(3.8)	(4.2)	4.7	5.2	5.7	6.2									15							
9	1.8	2.4	(4.2)	3.0	3.6	(4.2)	(4.9)	(5.5)	(6.1)	(6.7)	(7.3)	(7.9)	8.5	9.1	9.8	10.4	11.0	11.6	12.2				16							
10	3.3	4.1	(5.6)	4.8	(5.6)	6.3	7.1	7.8	(8.6)	(9.4)	(10.1)	(10.9)	11.6	12.4	13.1	13.9	14.6	15.4	16.2				14							
11		5.8	(8.6)	6.8	(7.7)	(8.6)	9.5	(10.4)	(11.3)	(12.2)	(13.1)	(14.1)	15.0	(15.9)	16.8	17.7	(18.6)	19.5	20.5				16							
12		7.5	(11.0)	(8.8)	9.9	11.0	(12.1)	(13.1)	(14.2)	(15.3)	(16.4)	(17.5)	(18.6)	(19.7)	20.8	(21.8)	22.9	24.0	25.1				21							
13		8.6	13.5	10.2	12.2	13.5	14.8	(16.1)	(17.3)	(18.6)	(19.9)	(21.2)	(22.4)	(23.7)	25.0	(26.3)	27.5	28.8	30.1	31.4	32.7		16							
14			15.3	11.6	13.5	15.3	17.2	19.0	(20.6)	(22.1)	23.6	(25.1)	(26.6)	(28.0)	(29.5)	(31.0)	(32.5)	34.0	35.4	36.9	38.4		15							
15			17.3		17.3	19.4	21.6	(23.7)	25.8	27.5	(29.2)	30.9	(32.6)	(34.3)		(36.0)	(37.7)	39.4	41.1	42.8	44.5		18							
16			21.8			21.8	24.3	(26.7)	29.2	(31.7)	(33.6)	(35.6)	(37.5)	(39.4)	(41.4)	(43.3)	45.2	47.2	(49.1)	51.0			25							
17			24.3			24.3	(27.1)	29.9	32.7	(35.4)	38.2	(40.5)	(42.6)	(44.8)	(47.0)	(49.2)	51.4	53.6	55.7	57.9			15							
18			26.9			26.9	30.0	33.2	(36.3)	(39.4)	(42.5)	45.6	(48.1)	(50.5)	(52.9)	(55.4)	(57.8)	60.3	62.7	65.2			14							
19								36.6	40.1	(43.6)	(47.1)	50.6	(53.7)	(56.5)	(59.2)	(61.9)	(64.6)	(67.4)	70.1	72.8			13							
20								40.2	44.0	47.9	51.8	55.7	59.6	(62.7)	(65.7)	(68.8)	(71.8)	74.8	77.8	80.8			8							
21								48.2	(52.4)	(56.7)	61.0	65.3	69.3	(72.6)	(75.9)	(79.2)	82.6	85.9	89.2				4							
22								52.4	57.2	61.9	66.6	71.3	76.0	(79.7)	83.4	87.0	(90.9)	94.3	98.0				2							
23								56.9	62.0	67.2	72.4	77.5	(82.7)	87.2	91.2	95.2	99.2	103	107				1							
24								61.1	67.1	(72.7)	78.4	84.0	89.6	94.9	99.3	104	108	112	117				2							
25								65.1	72.4	(78.5)	84.6	90.7	(96.8)	103	108	(112)	117	122	127				3							
26								69.3	77.8	84.4	(91.0)	(97.7)	104	111	116	122	(127)	(132)	137				6							
27								73.5	83.4	90.5	97.7	105	112	119	126	131	136	142	148				--							
28								77.5	89.2	(96.9)	105	(112)	120	(128)	135	(141)	147	153	159				5							
29								81.5	95.1	103	112	120	128	136	145	151	157	164	170				--							
30								85.5	100	110	(119)	128	137	146	154	161	168	(175)	182				2							
31								89.5	106	117	126	136	146	(155)	164	172	179	187	194				1							
32								93.5	112	124	134	144	154	165	175	183	191	199	206				--							
33								97.5	118	132	142	153	164	174	185	195	203	211	219				--							
34								101.5	124	139	150	162	173	185	196	206	215	224	233				--							

1/ Table volumes outside block enclosed by solid line obtained from weighted regression equation: $V_2 = -7.7571 + 0.6546D + 0.00151D^2H$. Table volumes inside block enclosed by solid line obtained from weighted regression equation: $V_1 = -3.2187 + 0.8281D - 0.05908D^2 - 0.01985H + 0.00199D^2H$. Standard error of estimate around mean volume (V_2) = 2.7038 cu. ft. = 7.8%.

2/ Mid-point of class (e.g. 7 = 6.6 through 7.6 inches).

3/ Mid-point of class (e.g. 25 = 22.6 through 27.5 feet).

4/ Figures in parentheses indicate extent of 249 measured trees used for computing above regression equation V_2 .

Table 17.---Tree volumes inside bark (2-foot stump to 8-inch top d.i.b.) for cottonwood, interior Alaska^{1/}

D.B.H., D ₂ / Inches	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	BASIS ^{4/} Number of trees 14
									TOTAL HEIGHT, H (FEET) ^{2/} Cubic feet											
10	0	.4	(1.1)	1.8	2.5	3.2	(3.9)	(4.6)	(5.3)	(6.0)	6.7	7.4	8.1	8.8	9.5	10.2	10.9			
11	1.7	2.5	(3.4)	(4.2)	5.1	(5.9)	(6.8)	(7.6)	(8.5)	(9.3)	10.2	(11.0)	11.8	12.7	(13.5)	14.4	15.2			16
12	3.8	(4.8)	5.8	6.8	(7.8)	(8.8)	(9.8)	(10.8)	(11.8)	(12.9)	(13.9)	(14.9)	15.9	(16.9)	17.9	18.9	19.9			21
13	6.0	7.2	8.3	9.5	10.7	(11.9)	(13.1)	(14.3)	(15.4)	(16.6)	(17.8)	(19.0)	20.2	(21.4)	22.5	23.7	24.9	26.1	27.3	16
14		9.6	11.0	12.4	13.8	15.1	(16.5)	(17.9)	19.2	(20.6)	(22.0)	(23.4)	(24.7)	(26.1)	(27.5)	28.8	30.2	31.6	33.0	15
15				15.4	17.0	18.5	(20.1)	21.7	23.3	(24.8)	26.4	(28.0)	(29.6)	(31.1)	(32.7)	34.3	35.9	37.4	39.0	18
16					20.3	22.1	(23.9)	25.7	(27.5)	(29.3)	(31.1)	(32.9)	(34.6)	(36.4)	(38.2)	40.0	41.8	(43.6)	45.4	25
17					22.7	(25.5)	27.9	(31.9)	33.9	(36.0)	(38.0)	(40.0)	(42.0)	(44.1)	(46.1)	48.1	50.1	52.1	54.1	15
18					25.1	28.2	31.4	(34.3)	(36.6)	(38.8)	41.1	(43.4)	(45.6)	(47.9)	(50.2)	(52.4)	54.7	57.0	59.2	15
19							34.6	38.1	(41.4)	(43.9)	46.5	(49.0)	(51.5)	(54.0)	(56.6)	(59.1)	(61.6)	64.2	66.7	13
20							38.0	41.8	45.7	49.3	52.1	54.9	(57.7)	(60.5)	(63.3)	(66.1)	68.9	71.7	74.5	8
21								45.8	(50.0)	(54.3)	57.9	61.0	64.1	(67.2)	70.3	(73.4)	76.4	79.5	82.6	4
22								49.8	54.6	59.3	64.0	67.4	70.8	(74.2)	77.6	81.0	(84.3)	87.7	91.1	2
23								54.0	59.1	64.3	69.5	74.0	(77.7)	81.4	85.1	88.9	92.6	96.3	100	1
24									64.0	(69.6)	75.3	80.9	85.0	89.0	93.0	97.1	101	105	109	2
25									69.0	(75.1)	81.2	87.3	(92.4)	96.8	101	(106)	110	114	119	3
26									74.1	80.7	(87.3)	(94.0)	100	105	110	114	(119)	(124)	129	6
27									79.4	86.5	93.7	101	108	113	118	124	129	134	139	--
28									84.9	(92.6)	100	(108)	116	(122)	128	(133)	138	144	149	5
29									90.5	98.8	107	115	124	131	137	143	149	154	160	--
30										105	(114)	123	132	140	146	153	159	(165)	172	2
31										112	121	131	140	(150)	156	163	170	177	183	1
32										119	129	139	149	159	167	174	181	188	195	--
33										126	136	147	158	169	177	185	192	200	208	--
34										133	144	156	167	178	188	196	204	212	220	--

^{1/} Table volumes outside block enclosed by solid line obtained from unweighted regression equation: $V_3 = -14.9026 + 0.9689D + 0.00140D^2H$. Table volumes within block enclosed by solid line obtained by subtracting o.b. basal area at b.h. from V_2 tabular volumes. Standard error of estimate around mean volume (V_3) = 5.2204 cu. ft. = 15.0%; $R^2 = 0.9734$.

^{2/} Mid-point of class (e.g. 10 = 9.6 through 10.5 inches).

^{3/} Mid-point of class (e.g. 35 = 27.6 through 32.5 feet).

^{4/} Figures in parentheses indicate extent of 201 measured trees used for computing above regression equation V_3 . Fifteen trees not shown in the "BASIS" column were in the 9-inch diameter class.