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THE PRINCIPLE OF
LEAST ACTION IN NATURE,

ILLUSTRATED BY

ANIMAL MECHANICS.

BEING

THREE LECTURES,

DELIVERED AT THE ROYAL INSTITUTION OF GREAT BRITAIN.

BY THE

REV. SAMUEL HAUGHTON, M.D.Dubl., D.C.L.Oxon., F.R.S.,
Fellow of Trinity College, Dublin.

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THREE LECTURES

ON

THE PRINCIPLE OF LEAST ACTION IN NATURE, ILLUSTRATED BY ANIMAL MECHANICS.

LECTURE I.—Tuesday, May 23rd, 1871.

Science of Animal Mechanics defined as the application of the principles of Geometry and Mechanics to Comparative Anatomy.—General Principle of Least Action, as observed in Astronomy and Physics.—Application of this principle to Animal Mechanics.—Illustration of the Pleasures and Difficulties of the Study of Animal Mechanics, from the Lecturer's Adventures in Search of the Coefficient of Muscular Force.

I TAKE it for granted that there is no one of those whom I now address who has not read both with profit and pleasure *Gulliver's Travels*. But of the many thousands that have read that charming book, there are very few that know the real circumstances of the history of its production. It is only a fragment of a much greater work which was contemplated and which the world has lost for ever—a work which was to have been the combined result of the genius of three of the most remarkable men that our country has produced; one an Englishman, another a Scotchman, the third an Irishman. More than 150 years ago, in the good old times when Queen Anne reigned in England, and science, literature, and art were patronised by her court, three of the most remarkable men that ever lived in this city were friends and companions. The Englishman was Pope, the Scotchman was Arbuthnot, and the Irishman—I may be permitted as a fellow-countryman to say, greater than either—was our illustrious Swift. It was proposed by Pope that a novel should be written by these three men, which would have been a novelty not only in that age of literature but in our own, that the combined efforts of the genius of three such men should be brought to bear upon the production of a work of fiction. The fragments of that work of fiction remain at the end of the large editions of Pope's works under the name of *Memoirs of Martinus Scriblerus*. Pope was to have taught Martinus Scriblerus litera-

ture, logic, and metaphysics; Dr. Arbuthnot was to have taken charge of his medical education and have brought him to Holland and Germany to study medicine; while Swift was to have written his travels. *Gulliver's Travels* were produced as a separate work by Swift in consequence of the troubles at the close of the reign of Queen Anne, which scattered this remarkable triumvirate. The travels that were originally intended to illustrate the life and memoirs of Scriblerus appeared under the name of, and are known to you all as, *Gulliver's Travels*.

The accession of George I not only destroyed the prospect of the world seeing the memoirs and travels of Martinus Scriblerus, but it also lost for England the credit of producing a great work on Animal Mechanics. It can be shown from several passages in his writings, that Dr. Arbuthnot, who was himself a most skilful geometer and a most expert anatomist, had conceived the project of uniting these sister sciences of anatomy and geometry in one great science, and so creating a new field for discovery, for thought, and for research. His appointment as court-physician to George I withdrew him, I regret to say, from the ranks of science; and England lost, by his appointment to that office, the opportunity of producing a great work on Animal Mechanics. It was reserved for an Italian, Alphonso Borelli, one of the greatest men that modern times have produced, to lay the foundations of this most remarkable science. Alphonso Borelli taught mathematics in the university of Naples at the close of the seventeenth century. He was also professor of anatomy in this same university; and his book shows that the union of anatomy and geometry had the honour of being approved by the Pope, and pronounced to contain nothing dangerous to faith or morals. He produced his remarkable book on Animal Mechanics, which he entitled *De motu animalium*, in the year 1680; but he died before this book was produced, and he died also, unfortunately, without knowing what our distinguished countryman, Sir Isaac Newton, had discovered of the laws of the composition of forces. The result is that, although his book *De motu animalium* carries in every part traces of his brilliant genius, it is full of mistakes arising from the false notions on mechanics which were inevitable in the case of a person not acquainted with the composition of forces discovered by Newton. But it remains in its present state, with all its defects and errors, I say without hesitation, the only book that can be called a systematic scientific treatise on Animal Mechanics.

In later times an attempt was made by two remarkable Germans, who were brothers, Edouard and Wilhelm Weber, professors of anatomy and mathematics in the universities of Göttingen and of Leipsic. From conversing together, these two men came to the conclusion that, if mathematics or geometry and anatomy could be brought into contact, the result

would be the production of a third science, perfectly novel and of great value. They compared their observations and notes together and produced a treatise on the motions of the human body, which will always be quoted as a model of accuracy and a standard of scientific observation; but, from the necessary want of unity and uniformity produced by two minds being brought to bear upon the same subject instead of one, it cannot be regarded as so systematic or complete a treatise as that of the illustrious Borelli.

The progress of science of every description, and the extraordinary and interesting results that have been produced in recent times by the union of sciences apparently diverse from each other, have rendered it almost inevitable that now, in some quarter or other, a science of Animal Mechanics must arise which shall be worthy of being called an exact science, and worthy of taking its place amongst the most perfect of all our modern sciences.

In order to found such a science, the great want is the discovery of some general principle. I believe that I have succeeded in discovering the true principle on which this science must be founded, and have been enabled to sketch out the broad outlines of the foundation of such a science; though it must remain for abler mathematicians and more expert anatomists than I can pretend to be to fill up the details of the outline and bring them to the perfection at which I am confident they will ultimately arrive. The principle on which I propose to found the science of Animal Mechanics is a principle of almost universal application—it is called the Principle of Least Action. This is well known to mathematicians in its application to inanimate nature; and, with your permission, I will call your attention to one or two remarkable instances of the application of the principle of least action, well ascertained in physics and astronomy, before proceeding to apply it, as I hope to do in these lectures, to the study of organic nature, and particularly of muscular mechanics.

The celebrated Kepler, who died before Sir Isaac Newton's laws of gravitation were discovered, as must be well known to you all, discovered by patient observation the laws of the motions of the planets. He discovered, in particular, the important law that a planet revolves round the sun in an ellipse, the sun occupying a focus of this ellipse; and that every portion of the path of the planet is an ellipse, each portion of the path being characterised by a velocity of motion peculiar to itself, and different from that which is found in any other part. It is not necessary for me to describe to you the simple laws by which the motion of the planet varies; but if you will imagine a planet moving from point A to point B (diagram) in its path round the sun, Kepler succeeded in discovering that it moves along that path by regular laws, by a regular motion belonging to each part of that path, and never deviating from

it. He published his celebrated laws, which are known as Kepler's laws, from which, in later times, Sir Isaac Newton deduced by mathematical demonstration the celebrated law of gravitation. It is extremely interesting to us who look back upon these discoveries with the reflected light which Newton's genius has thrown upon them, to read Kepler's writings, and see in what manner this remarkable man regarded his own discoveries. Kepler was a scholar as well as a mathematician ; and he was a profound student of the *Timæus* of Plato, and was charmed with that beautiful fiction in which Plato imagines the cosmogony of the world, and describes the Demiurgus persuading Chaos to become Cosmos ; and ends by the assertion, bold for his age, considering that it was in opposition to the heathen mythology of the times, that the sun was an animal, the earth an animal, the moon an animal ; that the gods, as Venus, Diana, and Mercury, were deities who were subordinate to these great beings. Kepler was caught with the beauty of this Platonic fiction. He imagines the earth swimming in her orbit round the sun as an animal with intelligence. He discusses in his book, *Harmonices Mundi*, the possibility of the earth seeing without eyes, and hearing without ears ; and arrives at the conclusion that the earth must be an animal, for three reasons. First, the earth moves uniformly on her axis, and none but an intelligent being could know how to keep uniformly moving without going faster at one time than at another. Secondly, he says, the earth, as he has proved by his laws, describes a particular path round the sun and no other, and moves with a particular velocity at each portion of that path ; and this, he says, the earth can only do by observing the angles made by the heavenly bodies, calculating its position—going slowly when it had to go slowly, and fast when it had to go fast—by observing the planetary angles. Lastly, he says, the earth must be an intelligent animal, for the highest and best of all reasons, which he also learned from Plato—because the earth is a great geometer. The earth, he says, produces within her bosom crystals such as these which are before you, which are related to certain forms shown in the diagram—Euclidean solids capable of being inscribed in spheres. No creature, says Kepler, could know the mysterious properties of the solids inscribed in spheres, excepting an intelligent geometer. The earth produces crystals and forms closely related to these in her own bosom, and therefore by the maxim *de opifíce testatur opus* (which we may translate by “the carpenter is known by his chips”), we come to the conclusion that the earth must be a geometer, for she produces perfect geometrical forms. Kepler goes further, and proceeds to discuss the question what sort of an animal is this earth ? He says the earth is no lively animal like a lap-dog, ready to obey your nod ; she is a sluggish, lazy, intelligent creature, like an ox, or perhaps, I should say, like an elephant. Now, a very remarkable fact bearing

upon this is well known to mathematicians. The celebrated principle of least action, as it is applied in mechanics and astronomy, consists in showing that a certain integral $\int v ds$, v being the velocity at each point, and ds the element of its motion, must be the minimum. If I take the points A and B in the planets' path, s representing the sun, I only require to know those points A and B and the sun, s , to calculate for you, from the principle of least action—which I can do to the millionth part of an inch at each portion of this orbit—the path that the planet must describe, on the supposition that it is a lazy, intelligent animal, trying to swim round the sun in such a manner as to give the least trouble to itself. It is perfectly well known to astronomers that the imaginary idea of Kepler may be converted into a real fact; and that, if we grant the hypothesis that the earth is an intelligent animal swimming round the sun, we can calculate and predict its path as certainly as we could from the Newtonian law of gravitation.

Pondering this idea over in my mind, it seemed to me to afford in such a science as Animal Mechanics a possible key which would unlock the secrets of that science. It was an hypothesis invented by Kepler in the early ages of astronomy when more perfect hypotheses were impossible, but which, when carefully followed out, would lead to results as accurate and as perfect as the exact hypothesis; and the hypothesis I have to lay before you as the foundation of the science of Animal Mechanics is similar to that of Kepler, and will ultimately, I doubt not, be replaced by a higher and more perfect law, such as that with which Newton replaced Kepler's law. My hypothesis is, that in every arrangement of bones, muscles, joints, and parts of animals, the motion must be such as it would be on the hypothesis that the muscle were a living intelligent thing, trying to save itself trouble. We can calculate, as I shall show you in the subsequent lectures, with a certainty as perfect as we can calculate the path of a planet, the positions of bones and sockets as we find them in nature. If, therefore, I can prove my hypothesis, we are entitled to regard it as a key for present use, to be replaced ultimately by some higher law, but a valuable and precious key to unlock for us the secrets of animal mechanics.

Before proceeding to apply it to the muscles, I must give you one or two more illustrations of it, to show you the power of the instrument which I propose to use. When a ray of light passes from one medium into another, it always describes a bent path. A ray of light passing from the air into a glass of water will be bent at the junction of the two bodies. Taking the point A (diagram) to be the ray of light in the air, and point B the ray of light in the glass, if I imagine the fiction that the molecule of light is a living, intelligent, lazy animal, trying to go from A to B with the minimum of trouble to itself, I can predict, as certainly as by the law of refraction and reflection, the path which it would de-

scribe. This principle has been actually applied—the principle of least action—by Sir Isaac Newton to discover the path of a ray of light. It will apply, as astronomers and mathematicians are aware, to many other branches of science; it will apply also, I believe, to the unconscious actions of intelligent animals like ourselves. Some years ago, I had the opportunity of observing an unconscious application of this remarkable principle made by an extremely unintelligent class of old women. These were the oyster-women of the Mumbles Harbour, near Swansea. These poor old creatures carried their baskets down to the oyster-beds empty; they filled the baskets with oysters, and then they had to carry them to the Mumbles along a road which consisted of two parts; there was the slippery shingle of the beach where they collected the oysters, and after they left that slippery shingle there was a smooth common. Now, this placed the oyster-women in the same position as the ray of light. The velocity of the light in air and its velocity in water are different. The velocity of these poor women in the rough shingle, where they occasionally fell, and their velocity on the smooth road, were, as you may suppose, also different, and the friction different. I saw these poor women, to my great amazement, not going from point A to point B as I should have done, nor going perpendicularly so as to get from the shingle in the shortest time on to the common, but making a tack at some point which they seemed to guess at along the line of shingle, and so getting home with less trouble than they would otherwise have done. I had the curiosity to measure the angles made by their path, and made a rough calculation to determine the relative roughnesses of the two roads. I do not suppose that these poor women had any more consciousness of what they were doing than the ray of light or the planet has; they were describing a path of minimum trouble. I can hardly do them the injustice to say that they were lazy animals, because they were the most industrious, hard-working poor creatures I have ever fallen in with, but they were performing unconsciously and instinctively a great problem—the problem of doing a given amount of work with a minimum amount of effort; and I venture to say that they performed that problem as the planet describes its path, not by their own intelligence, but by the intelligence of Him who made them both.

I shall take yet another example from organic nature before proceeding to the subject of muscular action. I will take the well-known problem of the cells of bees. Every one knows that the cells of bees are constructed in hexagons, and that the ends of these cells are terminated by the faces of a rhombic dodecahedron. That was the solid so greatly admired by Plato, that he considered it was worthy of representing the earth itself. The tetrahedron represents fire; the octahedron represents air, the cube represents water, but this solid body was reserved in the Platonic system for the dignity of repre-

senting the earth itself. The bee constructs this solid; and ancient geometers such as Pappus were so struck with the beauty of these cells that they considered the bee to be a mathematician and geometer; they reckoned the bee amongst themselves as worthy of being called mathematicians, and they accepted the bee-cells as a challenge. Pappus says angrily in one of his works: "I cannot admit that the bee is so expert a geometer as we are, for we can perform a problem greater than the bee, describing polygons with the least perimeter." He proceeds to show how he could make cells more perfect than those of the bee. There is in this an unconscious supposition of the same kind as in the case of Kepler, that the bee makes the cell by some knowledge or intelligence of its own. Now, the cell of the bee possesses remarkable properties: it possesses the property of making the largest quantity of cell-space with the minimum quantity of wax; or, in other words, it performs the problem of doing a given work with the least amount of trouble to the bee. It costs the bee trouble to make wax, and, therefore, if he acted consciously or unconsciously on the principle of least action, he will make his cells in such a form as to produce a given space for the accommodation of the honey with the minimum or smallest quantity of trouble to himself and of wax as material. In all these cases, in the motion of the planet, in the motion of light, or in the case of living beings like the bee or the unconscious oyster-woman, we have the same principle at work. Nature aims at producing a given quantity of work with the least quantity of material, and this is the precise form which the principle of least action takes in muscular mechanics. Nature has to produce a certain quantity of muscle to do a certain quantity of work. The production of that quantity of muscle costs an effort which is exhausting to the animal; the muscles so produced from day to day, from hour to hour, from minute to minute must be fed by blood-vessels, must be nourished and sustained, and this causes a daily waste of labour. It is, therefore, obviously the interest of nature, whatever be the intelligence that guides her movements, it is the interest of the creatures that she makes, that they should do the work which they have to do with the minimum amount of muscle. The principle of least action is that the arrangement and mutual position of all muscular fibres, bones, and joints must be such as to produce the required effect with the minimum amount of muscular tissue. I hope to show you in my lecture on this day week by two very remarkable examples from the limbs of the tiger and the wings of the albatross a complete and perfect demonstration of the truth of this principle; and in my closing lecture I shall ask your attention to the most interesting and attractive of all the applications to which animal mechanics can be applied, namely, to the heart and other involuntary muscles of great importance.

Before proceeding to apply this principle of least action or least trouble to nature, it is necessary for us to obtain what I call the coefficient of muscular force. If you take a rope made of hemp, of silk, or of iron, engineers are well-acquainted with the importance of obtaining its coefficient, which represents the number of pounds or tons weight necessary to break it across. Now, I ask you to imagine a rope of muscle. A muscle consists of filaments or fibres very frequently parallel to each other in the form of a rope. Imagine a rope of muscle, one square inch in cross-section, hanging from the ceiling to the ground: let that muscle contract by the order of the will, what weight will it lift from the ground? This is what I define as the coefficient of muscular contraction. It has cost me twelve years of hard work to obtain the coefficients I now place before you, which are represented in pounds per square inch for human muscle. I have not succeeded in obtaining it for any other animal but man. No other animal that I have met with, not even the "hairy quadrupeds with long tails", are intelligent enough to submit to the necessary experiments. 94.7 lbs. per square inch is the weight that the arms of a young man accustomed to athletic exercise are capable of lifting. 110.4 is the corresponding coefficient for the muscles of the legs of a similar class; and 107 for the muscles covering the abdomen.

When you bear in mind what I shall now call your attention to, the extreme difficulty of obtaining these results at all, I hope you will agree with me that the differences between them fall within the necessary limits of observation and of error, and that therefore the final result of 104 per square inch may be regarded as an extremely close approximation to the real coefficient of muscular force exerted in healthy strong men. At least, it is the only coefficient that I am able to lay before you. We must undertake, and I did undertake, two extremely laborious classes of observations in order to obtain that coefficient. Without that coefficient I can make no step further in the application of geometry and mechanics to anatomy. This became an absolutely necessary preliminary step. The determining of that coefficient requires two totally distinct classes of observations. I had to make observations on the force exerted by the muscles during life; and, secondly, I had to make most careful measurements after death of those same muscles. With regard to observations on the force of the muscles exerted during life, it is not nearly so easy to make them, as many of you, who are not anatomists, would at first sight suppose. There is hardly an action of the body—the lifting my hand to my head, walking across a room—that does not involve the co-ordination and co-operation of many scores of muscles; and the moment I get many muscles into play, the difficulty of separating their action becomes immense. Although it is easy to measure the force used in rowing and

other actions with the utmost precision, when you come to distribute the action amongst the many muscles that have produced it, you find it impossible to separate and give to each muscle its part, so as to obtain the true coefficient of muscular exertion. You will only, by such processes, obtain empirical results informing you what strength animals or men could exert when performing certain actions, but you cannot work back and get the coefficient which science demands, the precise force per unit of cross-section which these muscles exert. Again, measurements of the muscles made after death presented difficulties, to which I shall presently call attention, much greater than any but an anatomist would suppose. In making my observations on the force exerted by the muscles during life, I often found (as medical men are well aware) that in several forms of disease phenomena will be presented bearing upon muscular forces that solve problems in animal mechanics that no voluntary effort on the part of the sufferer could possibly produce. Contortions of the body will be produced by the agonising spasms of cholera, of lock-jaw, or, as I have seen in cases of poisoning by strychnia, which, while they are dreadful to behold, are yet, to the intelligent observer, of most extreme importance. While you are helping a sufferer on his bed of pain there is nothing to prevent your catching the solution of your problem for the coefficient of animal mechanics. You need not be less kind or hearty in your zeal to help the sufferer because you are at the same time taking scientific note of a curious combination of muscles of which he is entirely unconscious, and that no voluntary effort on his part or on the part of any other man would enable you to obtain. Partly for this cause, and partly I hope from higher motives, I became personally and intimately acquainted with all the phenomena of cholera. In cholera, hydrophobia, lock-jaw, and for the study of the muscle of the heart in fever, it is absolutely necessary to come into contact with these diseases, to study them at the bedside, and so to become acquainted, as I did through twelve years of hard work with a most interesting class, concerning whom it will be wrong for me to proceed in my lecture without bearing my humble testimony. I am not acquainted much with the poor of England, though I doubt not their qualities are as estimable and as excellent as those of my own country; but I may be permitted to bear my humble testimony to the qualities which I have observed myself amongst our Irish poor when in sickness and in trouble. Their devotion to their friends and neighbours in time of trouble is most extraordinary. Those who have quarreled in prosperity forgive each other in times of sickness. Their impulsive nature and their heartfelt gratitude, even unto death, for a hearty word of sympathy and kindness from those who visit them, and their brave cheerfulness in facing death, cannot be described by my words. One fact stands out prominent to the observation of any person who studies

our Irish poor in time of sickness and trouble, the extreme devotion of the poor to the poor. The rich often will give money, sometimes kind words ; but the poor give all they have—their food, their money, their hearts to each other in time of sickness. I believe, and I am sure my experience will be confirmed by that of every physician who hears me, that those sufferers who have themselves drunk the bitter cup of life to its very dregs are the most ready to offer to the lips of a dying brother or sister the cup of cold water in the name of their Divine Master.

My efforts to obtain a coefficient of muscular force were not confined to observations upon the poor in hospitals ; I also had to come in contact with a more uninteresting, but perhaps not less curious, class—the criminals in our jails. It is necessary for the student of animal mechanics to become an expert in the use of the treadmill, to understand the mysteries of shot-drill, and to know how to use the crank. My object was to learn how to work upon the treadmill as an intelligent lazy burglar, trying to do my work with the least trouble to myself ; and I can assure you, after much labour, I perfectly succeeded, and can go through shot-drill, turn the crank, and work the treadmill, as the laziest burglar in London might do, working my muscles involuntarily by the principle of least action, and doing my hated task in a lazy manner, with the least trouble to myself. How did I obtain this knowledge ? How did I learn these things ? I have been taught the use of jemmies, and burglars' tools as well. I know the slate trick, which is a secret known to Irish thieves only ; I also know where to place myself to the best advantage on the treadmill ; and I learnt all this by a plan extremely simple, and which I would heartily commend to our criminal reformers as a most powerful weapon in their hands—an ounce of tobacco. An ounce of tobacco will draw the dearest secret from the heart of a burglar ; it will make the most discontented, sulky wretch in the gaol obedient and quiet for a week, the promise of an ounce of tobacco at the close. I believe it is done in some prisons, and it ought to be done in all. Instead of the extreme severity which characterises our treatment of these poor men, the occasional offer of rewards, which the men would prize, would do more to reform them than all our severity and all our stripes. I must say for them that I have found them, as a class, both English and Irish burglars and thieves, much better than I had expected.

Having made my observations upon the work done by groups of muscles in various conditions of action, I then had, as you will see, to proceed to the examination of the measurements of these muscles after death. Now, this was no easy task. The observations on the forces employed were made upon men in health—generally young men in the full vigour of life—for I was anxious to ascertain what the coefficient of muscular exertion in healthy men and in full condition was. But the examinations of muscles after death were necessarily made upon

persons who had died in their beds after they had been wasted by long illness, and in a condition of body presumably quite different from that of those same persons when in health. I saw very quickly that if I relied upon mere observations and measurements of muscles made after death, comparing them with living forces, I should get a false result, and a coefficient much too great, because the cross-section of the muscle in life is certainly much larger than it is after death from long illness. Therefore I had two courses before me : I had my choice of waiting for persons who had died suddenly a violent death, and obtaining permission to examine them, or of waiting for an opportunity of examining persons who had died violently by the hands of the law. Now, in a country like Ireland, there were unusual and extraordinary impediments to both these processes. I placed myself in communication with the hospitals of Dublin, and got early notice of every bad accident that came in. The patient in the cases, such as I was sent for generally, died, but the cause of death was so apparent that the coroner could not venture to interpose and order a legal investigation of the body, which would have given an opportunity for examination ; and the sympathy of friends in the case of accident—the determination to wake the deceased person—is so strong in Ireland that it is impossible to obtain permission from them to make an examination. A wake is a matter of great necessity in all cases in Ireland where it is possible, but in a case where the sensations and feelings have been aroused and sympathies excited by a fall from a scaffold, or an accident in a mill, or any violent death, an anatomist would be most unwise, if he attempted to make an examination in such a case. Again, when I turned my attention to executions, I had extreme difficulty, part of which is creditable to my country—social or private crime is so extremely rare in Ireland that it was almost impossible to find a case for examination. In such a case there would have been no sympathy with the convicted criminal, and the surgeon would have been permitted to conduct his investigation at his leisure ; but in the majority of cases in Ireland the crime for which executions are performed is agrarian crime, and is semipublic in its character, exciting sympathy among large masses of the misguided population who agree with the murderer ; and this calls for great care and caution on the part of any scientific man who would make a man who has shot his landlord a subject for investigation. At last, ladies and gentlemen, a brilliant idea came across my mind, which, however, I was reluctantly compelled to set aside. I thought to myself, what in the world is to hinder me from taking a farm in Westmeath, deliberately and wilfully refusing to pay my rents, and in due time shooting my landlord, and, instead of using him as a New Zealand tenant would, dissecting him at my leisure. Then I have my muscular coefficient problem and all my problems resolved. I need not say that

further consideration led me to the conclusion that there were inconveniences attached to this course, and public opinion in Ireland, I can assure you, would not sanction the shooting of a landlord for the purpose of determining the coefficient of muscular force.

In conclusion, I would wish to say a word as to the principle I have adopted. I have shown you that the planet moves in its orbit as a lazy intelligent creature would who was anxious to perform an allotted task with the least trouble to itself; that a ray of light describes its path by the same sort of apparently instinctive action; that the poor old oyster-woman, ignorant of what she is doing, instinctively walks across the varying road she has to travel in the path of the least action; that the bee constructs its wonderful cell so as to produce a given amount of storage for its honey with the least possible amount of trouble to itself; and in a case that I was not able to bring before you for want of time, of the tendons of the legs and arms of animals, I could show that these are constructed with a wonderful economy of the same kind as that with which the bee constructs its cell; and I hope to show, in a future lecture, that the limbs or muscular organs of every animal are also constructed on this strange principle; as if each one of these things was itself instinct with life and reason, as Kepler really and from his heart believed. Is it by the intelligence of the planet that it moves in its orbit? Does the light travel in its path by its intelligence? Does the poor oyster-woman calculate the road by which she goes as she instinctively walks across the strand? Who has weighed out and regulated the weight of the tendons of our arms and hands? And by what force or by what intelligence do the limbs of animals describe their proper path? Who places the socket of each joint in the exact position (which can be calculated with unerring certainty by mathematics) which enables the muscle to perform its allotted task with the least amount of trouble to itself? It is not by their intelligence, by their instinct: it is not the instinct of the planet, or of the oyster-woman, or of the bee, that guides them in their path. There is instinct; there is knowledge; there is foresight; there is calculation: it is the knowledge, the foresight, the wisdom, and the calculation, of the Great Architect and Geometrician of the Universe.

LECTURE II.—Tuesday, May 30th, 1871.

Geometrical Classification of Muscles found in Animals.—Application of the principle of Least Action to several forms of Muscle, demonstrating the possibility of “predicting” Animal Structures by Mathematical Calculations similar to those used in Astronomy and the other exact Sciences.—Special Illustrations from the Limbs of the Tiger and Wings of the Albatross.

IN bringing to a conclusion my former lecture, I acted on the principle that I laid down of the “least action.” Taking into consideration the feelings of the audience as well as of the lecturer, and judging by my own experience in hearing sermons and lectures, I thought that somewhat less than an hour’s lecture would suit your tastes. I therefore threw overboard one interesting application of the principle of least action in order to lighten the ship, and bring her safely into harbour ; but if it is your pleasure that I should now state the problem of the tendons of the fore and hind limbs of animals, which I omitted to state in my last lecture, I place myself in your hands ; but you will have yourselves to blame if we do not carry out perfectly the principle of minimum trouble and least action by this rash proposal.

I hold in my hand the flexor tendon of the hind leg of the eland. You will observe that at one extremity it branches into three distinct tendons, and at the other extremity it divaricates into two. These three tendons are the connecting ropes that join the common tendon in the foot with the three great muscles of the leg that act upon it. Three streams of force enter the common tendon through these three different lines, and they are then distributed by the intervention of this divaricating tendon into two ; these two applications of force are carried to the toes of the animal. Somewhat similar arrangements are found in the fore limb and hind limb of almost every animal. We have the muscles acting at one extremity of the tendons, which are the connecting ropes that join the muscular forces with their points of application ; and we have these both in the fore and in the hind limb. Now, in the case of many animals the fore limb and the hind limb are always used for the same purpose. In the case of the llama, the horse, and the cow, the fore limb and the hind limb have scarcely any different functions : the animals are, in the strictest sense of the word, quadrupeds ; their fore feet are used simply for the purposes of locomotion. But in the higher classes of animals, like tigers, cats, bears, dogs, monkeys, and

ourselves, we have the fore limbs more or less differentiated in function from the hind limbs, and set apart for the use of the brain as grasping organs. In man himself, as you all know, this is carried on to its very highest limit. Except when we are little children we do not run upon our fore feet ; we have lost the use of our fore limbs as feet, and we retain our hand as the highest characteristic man can possess of his great origin—the servant of his brain, a perfect instrument for carrying out the conceptions of the brain and of the intelligence with which he is endowed. We might therefore expect, and I did expect, to find, if I compared the hands and feet of higher groups of animals together, great differences in these tendons. A certain amount of friction must take place round the ankle-joint and round the wrist-joint ; and, as friction helps the weaker force, I foresaw, if my principle of least action were correct, that I should find, in the case of a hand or fore limb of an animal, that the united strength of the tendons passing from the muscles to the common tendon would be greater than the united strength of the cross tendons applied to the toes or fingers. When I grasp an object in my hand the force comes from the muscles of the arm, passes through the tendons, and is then applied to the object grasped. If the principle of least action in nature be true, the united strength of the tendons above my wrist and below my wrist will not be the same ; but advantage will be taken by nature of the necessary friction that takes place at the wrist to make the tendons in the fingers less than the tendons in the arm by exactly the extent to which they are relieved by friction. On the other hand, in using my leg as an instrument of locomotion, the force comes by reaction from the ground upward : the ground by reaction presses upon my foot ; the strength of the tendons between the foot and ankle is exactly what is necessary to prevent their rupture or injury ; and, in passing beyond my ankle into the calf of the leg, I should expect, if the principle of least action be a true principle, able to unlock the secrets of animal construction, to find the reverse of what I found in the hand ; I should expect to find the tendons which pass from the foot into the muscles having a less cross-section than those which pass from the toes into the common tendon. And this is actually the fact. I have examined upwards of eighty animals, and I find that animals might be very fairly classified, according to this peculiar arrangement of tendons in the hand or foot, as animals that possess a grasping power and use their hands as hands, and animals that are of a lower organisation, and use their fore feet not as hands but as organs of locomotion. In the hand of the tiger there is a friction of 22.7 per cent ; in the foot of 46 per cent. But those frictions, you will observe, are reversed. The tendons of the fingers in the tiger's hand are less than the tendons in the forearm ; and, *vice versâ*, the tendons of the toes of the tiger are greater than those of the leg ; so that,

although this Table represents the friction in the hand and foot of various animals, you are to remember that the friction is *plus* in one case and *minus* in the other. The strength of the tendons in the forearm always exceeds the strength of the tendons of the fingers; while the strength of the tendons of the toes exceeds the strength of the tendons of the leg. In the wolf the corresponding figures are 31.4 and 34; in the bear, 35, and 25.9; in our cousin, the negro monkey, 27.4 and 8. This animal, you see, fully justifies the title which Cuvier applied to him of quadrumanous, for he has only a friction of 8 per cent. in his foot, because he uses it very much as a hand for the purposes of climbing; this is apparent in the low coefficient of friction shown in the hind foot.

The animals whose tendons suffers least from friction, at the wrist and ankle, are the goat and the kangaroo. The wrist of the goat is so admirably constructed that no force whatever is lost. The animal climbs a hill, runs up rocks, jumps from point to point, and he does so mainly by this admirable arrangement by which no force whatever is lost by friction in the wrist. The hind foot of the kangaroo, which is the great organ of locomotion in this animal, presents no friction at the heel. The most perfect organ of locomotion that we are acquainted with amongst quadrupeds (if we may call the kangaroo a quadruped, for he only uses two feet in locomotion) is the hind leg of the boomer kangaroo. The investigation which I have carried out in between seventy and eighty distinct animals shows in a most conclusive manner that the law of least action is attended to in this department of nature down to the most minute details. Even the expense of producing a few grains more or less of this glue (for a tendon is nothing but common glue) is carefully attended to; and in the laboratory of Nature the most rigorous and parsimonious economy is observed. Not even one grain of material is ever used when less would suffice for the purpose.

This may be carried out into very minute details which time will not allow me further to develop. I will merely call your attention to a rough sketch of the hind feet of monkeys of the old and the new world, which present many remarkable differences in construction from the feet of other animals. Their feet, as I said before, are fully entitled to the name of hands. We may classify the old and the new world monkeys by the peculiar arrangement of the tendons of their feet. In both, the tendons of the feet are supplied by two great muscles—the flexor hallucis longus and the flexor digitorum longus. (Fig 1.) You can see the distribution which I have endeavoured to show here. You see that the muscle of the great toe supplies half the first toe, the whole of the second, and the whole of the fifth; the other tendon supplies half the first toe, the whole of the third, and the whole of the fourth. Therefore the most natural action for the old world monkey would be to put

the great toe opposite the third and fourth toes, or opposite the second and fifth. The first, second, and fifth, would be a combination; the first, third, and fourth, would be a combination. In the monkeys of the new world there is a totally different arrangement. It is interesting to see how profoundly different these creatures are even in the structures connected with the hand and foot. You see here (Fig. 2) one tendon supplying half the first toe and the whole of the second, third, and fourth; half the first and the whole of the fifth are supplied

Fig. 1.

Fig. 2.

by the other muscle. Therefore in the South American monkeys, best known by those beautiful little Capuchin monkeys, so called from their monk-like appearance and the devout manner of crossing their hands over their breasts, the natural action is to place the first and fifth toes together, or the first, the second, third, and fourth. In the occupation which is so common amongst them, and so useful to them, of gathering up small living creatures from their skins, you will find, if you watch the habits of these monkeys in our gardens, that they fully bear out the anatomical theory I have explained. While the monkey of Africa prefers to grasp with the first, second, and fifth toes, the monkey of South America will show equal dexterity in seizing an object with the

first and fifth alone. Fig. 3 represents the tendons of the foot of the jaguar.

I now come to the proper subject of my present lecture, which is the classification of muscles. Muscles were originally classified by Alphonso

Fig. 3.

Fig. 4.

Borelli, who divided them into various groups; but, as this is not an historical or antiquarian sketch, I shall content myself with giving you my own classification, which is based upon his, and is a considerable improvement. I divide all muscles into the following; the prismatic muscle (fig. 4), where the fibres pass parallel to each other from bone to bone; the penniform muscle (fig. 5), where the fibres radiate at equal angles from a common tendinous line, and are inserted, of course, at each extremity into the bone; the triangular muscle, where the muscular fibres proceed from a fixed line, and are inserted not into a point, but into a line, so short that we may for practical purposes regard it as a point; and the fourth class I call quadrilateral muscles, where they are drawn in lines converging from one bone to another.

If AB represent one bone, and $A'B'$ another bone (fig. 6), these bones may be curved in any way you please. In Nature, as you know, every-

thing is curved in forms that mathematicians cannot imitate. We find the muscular fibres so arranged that, if $A B$ and $A' B'$ lie in the same plane, if I produced them all, they would pass through a common point of

Fig. 5.

intersection. I ask you to take that for granted. I have proved it, but I will not trouble you with the proof. Therefore the quadrilateral muscle is really nothing but a triangular muscle with the top cut off.

If I take the triangular muscle running from a bone to a point, and cut off the top of it and place the second bone along the section so cut off, I have a quadrilateral muscle. The quadrilateral muscle is easily dealt with as long as the two bones remain in the same plane. The contraction of the fibres causes these two bones to approach in the same plane. Often, however, A B and A' B' do not always remain in the same plane ; they are not as accommodating as geometers and mathematicians would wish ; they leave their plane and form what we call in geometry skew surfaces (fig. 6). This model intended to show the manner in which

Fig. 6.

a plain quadrilateral muscle becomes what we call a skew surface. You are probably acquainted with the term from engineers using it in the construction of what are called skew bridges for railways. A skew surface is a very remarkable thing. It is made up of a number of straight lines, yet every portion of that surface, except along these particular lines, is a curve. Now I have succeeded in discovering that the particular skew surface, of which muscles are capable of assuming

the shape, is the beautiful surface known to geometers by the name of the hyperboloid of one sheet. Here you see a number of straight lines, which I shall suppose to represent muscular fibres passing from the bone A B to another bone A' B'. I have made the bones of the same length, but the results would be similar, if I made the bones of different lengths or of any curvature I pleased, and placed them in the same position. I now take this muscle and distort it out of its plane. You see I have now a curved surface in which every portion is made up of straight lines. I can curve the surface in the opposite direction, and so I make the hyperboloid of one sheet out of a plane quadrilateral muscle. This is not a mere fiction of geometers. The adductor magnus muscle in the leg of man, and the great pectoral muscle in the wing of every bird, are living examples of the reality of this curious fact, that Nature constructs not merely plane muscular structures, but that she is capable of constructing muscular surfaces belonging to the most beautiful and elegant forms that have been studied and invented by abstract geometers. A friend of mine, one of the most distinguished of living geometers, when I informed him that Nature used familiarly the hyperboloid of one sheet in making her muscles, told me that his respect for her was considerably increased. The last forms of muscle to which I shall direct your attention are the sphincter muscle, which represents a number of circular fibres surrounding an orifice, and the ellipsoidal muscle, which represents muscles of greater or less thickness surrounding a cavity called ellipsoidal, because the cavity so formed is generally egg-shaped. In the next lecture I shall direct your attention to the most important of all these forms of muscles in the human heart and the hearts of animals. I shall confine our attention to-day to the more elementary muscles represented by the prismatic, penniform, triangular, quadrilateral, and the hyperboloid muscle of one sheet.

The prismatic muscle and the penniform muscle possess the remarkable property, which can be demonstrated mathematically, that in their contraction no loss whatever takes place. Nature, therefore, according to my principle, is entitled to employ these two forms of muscles whenever she pleases. She suffers no loss or injury by using these forms of muscles, and we find, therefore, that both these classes of muscles are constantly employed. When you come to the triangular, the quadrilateral, and skew muscles, we can demonstrate by mathematics that in the use of every such muscle there is a necessary loss of force. I may, therefore, be asked—How comes it, if the principle of least action be true, that Nature ever employs muscles involving a necessary loss of force? I answer, because Nature has other problems in view than mere economy of force in a single muscle. She has to consider if she economise force simply, without regard to other circumstances, such as

beauty of form, and surface of least resistance, whether she might not lose rather than gain, taking into consideration all the conditions. I

Fig. 7.

have always maintained that beauty of form, symmetry of outline, was one of the pre-existing conditions in the mind of the contriver of the

universe, as well as economy of force. We find, therefore, that Nature never uses a triangular or quadrilateral muscle except under great necessity, of which I shall now give you a remarkable example (fig. 7).

The most wonderful triangular muscle in the world is here shown in the biceps femoris muscle, or the flexor of the thigh in the tiger. The muscular fibres radiate from O ; they are inserted from the middle of the thigh down to the heel. This enormous muscle forms one great sheet of muscle passing from the tuber ischii, and spread out over a space of three feet along the side of the leg. That muscle exists in me and in every other mammalian animal that lives. In most other animals it is arranged as a prismatic muscle. In my leg it is like a rope of parallel prismatic fibres. Now, I ask—"Why has Nature deliberately sacrificed a certain amount of force by putting a triangular muscle into the leg of the tiger to do the work which she does so effectually in my leg by a straight rope of muscle?" The answer is this, that I am a man and not a tiger ; I am not intended as a tiger is, to hide in a jungle, to jump from the jungle at a troop of horsemen going by, to take one of them and carry him off in spite of the rest, and eat him. That is not the purpose for which the Creator brought me here; but if I were brought here for such a purpose I am sure I should have a triangular muscle in my leg. The weight of muscle to give the tiger the spring which enables him to do these feats is so enormous, that if it were placed as a single rope from point to point it would not only be a great deformity in his appearance, but would seriously impede him in his progress through the jungle. The clumsy nature of this enormous rope of muscle attached to him would injure him, therefore Nature has deliberately thrown overboard the first idea that might present itself, which was to put a great rope from point to point, and to make it strong enough. "No, I cannot do that," says Nature, "I must preserve beauty of form," making the tiger (what it is) the most beautiful creature which God has created. Therefore, the tiger is given a triangular muscle with a certain amount of loss of force, but there is a gain by spreading the muscle over a great surface, a gain in the packing and shape of the leg—there is more gained than lost by the apparent sacrifice of force.

An interesting fact may here be stated, the extreme beauty of which every anatomist present will appreciate. We can demonstrate, and I ask you to take it for granted, that the resultant force of the fibres of quadrilateral and triangular muscles lies in the bisector of the vertical angle. When I take the triangular muscle in the leg of the tiger, and draw the bisector of that angle on a tiger's dead body, I find that the bisector of the angle passes rigorously through the top of the fibula, the peroneal or small bone of the tiger's leg, through the very spot in which the biceps muscle in my leg is inserted ; so that, although the tiger's muscle is triangular, it really behaves like an imaginary muscle

working along an imaginary line, passing from the same point in the tiger from which it passes in my pelvis to the very same point, the top of the fibula, to which it is attached in my leg also. Here Nature is not to be accused of departing from the principle of least work. In this case, the strict carrying out of that principle would injure her in other ways, as in the packing of the muscles; therefore she has deliberately abandoned it, making a small sacrifice in order to obtain a greater advantage. Nature always acts upon the French proverb, "*Reculer pour mieux sauter.*" I have said before that she is at perfect liberty to use, as far as I am concerned, either the penniform or the prismatic muscle; and I cannot criticise her proceedings in that respect. Still her use of the penniform muscle is very sparing, as if she did not like to use her resources except when they were absolutely necessary. We find, although there is no loss of force in a penniform muscle, it is a rare form of muscle, and only resorted to when there is a worthy object. The most remarkable example I can give you of the penniform muscle in Nature is the muscle which lifts the wing of the bird. The bird's wing is depressed by great and powerful muscles, which I shall describe at the close of the lecture: it is lifted by a small compact muscle, which is placed upon the breast of the bird, in order to keep the centre of gravity of the bird as far back as possible. It is worthy of remark, that in the case of the ostrich, which does not fly, Nature places this muscle on the neck of the bird, because it is no injury to the ostrich to have the muscle on the neck; whereas it would be destruction to any other bird to have it so. This muscle placed upon the breast works by a tendon passing through a pulley, and changes through an angle of 180 degrees in its application, so as to lift the wing of the bird. The nature of a bird's flight is this: The depressor muscles of the wing must be made enormously great, to strike the air with the utmost force; the muscles which lift the wing must be made as light and small as possible, because their only object is to bring back the wing through the air after the stroke is made; this ought to be done in the shortest possible time, because, while the wing of the bird is rising through the air, the bird is falling: therefore we find that Nature, or rather the great Author of Nature, always employs the penniform muscle to lift a bird's wing; and for this reason, since the fibres converge at an angle towards each other, by compounding their forces the velocity along the diagonal is greater than it would be in a prismatic muscle. Thus no force is lost; and the bird is enabled to repeat the downward stroke much faster than if the prismatic form of muscle had been retained.

I have selected, in illustration of triangular muscles, the triangular muscles of the fore and hind limbs of the tiger. I have selected the tiger, because he is the strongest and the handsomest animal with which we are acquainted; and strength and beauty, as long as the

world lasts, will always command attention and regard. The tiger is stronger than the lion. I should be sorry to disturb the traditions of childhood which have led any one present to regard the lion as the king of the beasts. I am sorry, however, to say that the lion is a humbug. He has a big mane, and looks grand, but he is very inferior to the tiger. He is like some human beings I am acquainted with—there is more in their appearance than you find carried out on intimate acquaintance. My reason for saying that the tiger is stronger than the lion, is a reason that will interest you. I find that the cruel Emperor Titus, A.D. 80, carried the spectacles in Rome so far as to have Bengal tigers imported from India, and compelled to fight the Numidian lions imported from Africa. In his native haunts in India, the Bengal tiger never meets the African lion. The poor Babylonian lion of Asia is a very small animal compared with the African lion; and I would back two Newfoundland dogs to fight him. The tiger sometimes meets this lion in the north of India, and it is well known that he destroys him. The Emperor Titus determined to try whether the Bengal tiger could or could not fight the large and noble African lion. The poet Martial, in his 18th epigram, *De Spectaculis*, has recorded the fact that tigers and lions fought in the amphitheatre during the reign of Titus, and that the tiger always killed the lion. There are some points of interest in the quotation, which I can verify. Martial describes the tiger as naturally a gentle animal, accustomed, he says, to lick the right hand of the keeper that trusted him; but when he came to Rome, and, as Martial observes, learned bad manners amongst the civilised Romans, he lost his native gentleness, and acquired a degree of ferocity that he never possessed in his native woods. The words he uses are:

“Lambere securi dextram consueta magistri
Tigris ab Hyrcano gloria rara jugo
Sæva ferum rabidâ laceravit dente leonem;
Res nova, non ullis cognita temporibus.
Ausa est tale nihil, sylvis dum vixit in altis,
Post quam inter nos est, plus feritatis habet.”

Accidents have happened, also, in some of our English menageries, where the barrier between the cage of the tiger and the lion has broken down, and the animals have fought. The records of all these cases, I believe, show that the tiger, if in good condition, invariably kills the lion when compelled to fight. But the best proof I can give you of the superiority of the tiger, is an experience of my own. What we learn for ourselves makes a stronger impression upon us than what we read from books. I have been for many years Secretary of the Zoological Gardens in Dublin, and have had a large number of tigers and lions under my care. Now, it occasionally happens that with such creatures as these, the claws of the fore paw, from want of natural exercise in scratching trees, like cats, grow into the foot, and cause the animal

great pain, ultimately producing death by gangrene, unless the claws be removed ; therefore it is not an unusual thing to look after a tiger's toilet, and pare the nails for him from time to time ; but it is not an operation that is at all so satisfactory in the actual performance as it appears at first sight. I have performed it repeatedly both on tigers and on lions ; and I can give you as the result of my experience, that while it requires eight men to hold down a tiger, five men will easily hold the largest lion. This greatly struck me when I ascertained it ; and having the opportunity of examining after death my two friends whose nails I cut, I found, in the cross-sections and weights of the muscles, measurements and proportions giving the tiger somewhat more than fifty per cent. of strength greater than the lion ; which quite confirmed my observation.

But the operation of cutting the tiger's claws was accompanied, in the first instance that I tried it, with some curious incidents that you might like to hear ; in fact, if they had turned out somewhat differently from what they did, I should not have had the honour of delivering this lecture before you to-day. I collected eight men to assist me ; we placed a large rope round the tiger's neck with a stop-knot ; a certain number of men held this, and the other men held the tiger's claw that was to be cut, with a rope which lifted the wrist off the ground. They were instructed carefully to keep the tiger's foot off the ground, for the tiger knew well what they were about. He watched the opportunity of putting the sore foot on the ground and slapping me with the other foot ; but as long as the sore foot could be kept off the ground and his head against the bars of the cage, he was compelled to press with his free paw against the ground to save himself from being choked, and I was quite safe. But, unfortunately, in the middle of our proceedings his companion tigress thought she would interfere, and she behaved not unlike the manner in which Jael the wife of Heber or Judith of Bethulia would have done : she came over to see what was going on, put her paw out through the bars, and struck my hat. I was proceeding to cut the tiger's claw when I felt the paw of the tigress at the back of my hat. In a moment, the body of eight men resolved itself into its component parts. Seven of them were cowards ; but one was a brave man who had led the celebrated charge of the Grenadier company of the 29th Regiment at the battle of Chillianwallah for the purpose of spiking the guns of the Sikh battery, and was promoted on the field by Lord Gough. He kept his hold of the tiger, but, to my astonishment, I saw the seven men running like drops of mercury in all directions. The tiger had thrown up the sliding door of the cage, and there was I face to face with him. I saw I was instantly to be killed, but my friend held on by the rope and puzzled the tiger. The animal then backed to the far end of the den,

and crouched upon its haunches to make a spring at me through the door. I naturally put up my hand and closed the door. The tiger rushed at the bars of the cage and broke his teeth with fury on the bars, because he was prevented from attacking me. I was not to be baffled. I collected the seven cowards again and brought them into the house. I then took the key, locked the door, and put the key in my pocket. "Now, boys", said I, "you ought to be ashamed of yourselves; you are not worthy of being called Irishmen at all. If any accident happens the tiger will eat me first, key and all, and he will sup off the rest of you at his leisure, so hold on by the ropes." We got the ropes on again, and I cut the tiger's claw; and thus I developed the greatest possible amount of muscular work that could be got out of seven cowards. After the work was over, a very remarkable scene happened. The tiger threw himself on his back and began to purr, and he made unmistakable signs that he wanted me to come over. I went over, and he put his paw with the claws in against the bars to make me rub it. He was not satisfied with that, and I patted him on the head, and put my hand to his mouth and allowed him to lick it; he also allowed me to examine the foot which I had hurt so much while operating upon him—thus carrying out to the most minute particular the character given of the gentle tiger until he came into contact with civilisation, given by Martial: "*Lambere securi dextram consueta magistri.*"

It is impossible for me to go into all the details; they must be left for future development; but I will say in brief that the principle of least action, when applied to the limbs of a tiger, require two conditions—one in the fore paw, and the other in the hind limb. In an action such as the tiger makes with the back stroke of the paw of the fore-limb, we have combined groups of muscles acting upon the arm and the fore-arm; and it is easy to prove by the principle of least action that when the resultant of the great muscles, the *latissimus dorsi* and the *teres major*, that act upon the arm, becomes perpendicular to the humerus, at the same moment of time the *triceps* muscle must be perpendicular to the line joining the olecranon process of the ulna with the centre of the elbow-joint. The most powerful stroke that the arm of a strong man can give is the back-stroke, as every swordsman knows. The back-stroke of a guardsman will cut a leg of mutton in two. The back-stroke in rackets is much more powerful than the direct stroke. It is made by bringing the muscles of the fore-arm and the humerus into co-ordination. All the efforts of the racket-player and the swordsman would not produce this effect unless the law were followed, that these two angles which have no connection with each other passed through ninety degrees together. We have, then, the very curious fact that the principle of least action in a tiger's fore-paw or arm requires that these two angles, which have no relation to each other, varying in magnitude

at every moment, must, to produce the maximum effect, pass through ninety degrees at the same moment ; and we find that Nature accepts the consequence. I know no animal in which this law is not carried out, and a corresponding law to which I shall now call attention in the hind-leg. In the hind leg the line *O...sart* (Fig. 7) joining the centre of the hip-joint with the tuber ischii, becomes at right angles to the resultant of the biceps femoris muscle, whether triangular or prismatic, at the same moment that the muscles of the calf (*G B*) become perpendicular to the line (*B Y*) which is drawn through the centre of the ankle-joint. In the case of the hind-leg the arrangement skips a joint. In the fore-arm the shoulder is related in this remarkable manner to the elbow-joint. In the hind-leg the hip-joint is not related to the knee-joint, but to the heel ; and those two angles, which have no necessary relation whatever to each other, made by one group of forces at the hip-joint, and another group of forces quite distinct from them at the heel, pass through ninety degrees together—one of the most remarkable instances on record of the skill, contrivance, and foresight, with which the frame of animals has been constructed.

You have seen on board a large steamship an engineer with a little can of oil in his hand putting his head in among moving bars of iron, poking his can of oil among little joints ; and you feel conscious that if you attempted to do it you would lose your life. That man knows to the tenth of an inch the motion of every bar—when it comes, when it retreats, when it comes forward again ; he knows that he can rely upon the motions of the bars with certainty to the hundredth part of an inch. When we see these motions regulated by the intelligence of the engineer who contrived the machine, describing their angles, and passing through each angle at the exact moment the engineer intended, no person is fool enough to believe that there is not contrivance and design. I am ashamed to say there are intelligent men who can look upon similar structures more wonderful in contrivance in the world of nature, and not recognise the hand of Him who made them.

Before parting with this subject, I may be allowed to give a word of advice to some of those who hear me. I have shown you that these two angles pass through ninety degrees together ; therefore, any arrangement of an artificial kind that interferes with the angles passing through ninety degrees together would be most injurious. I am told that it is now the habit, or has been the habit, of ladies in America—perhaps in this country—to wear high-heeled boots for the purpose of producing the Grecian bend. I am not acquainted with the subject, but the ladies present will know whether this be the fact. I would caution you against the practice. You shorten, by high-heeled boots, the distance between the points *B* and *G* ; you prevent, therefore, the beautiful play of angles and joints from coming into effect, and you sacrifice

in the movement of the limb what you gain in supposed grace of figure. If the practice continue, I should expect that our young ladies of some future period, between the bright colours of their heads and the development of the tendons in their feet, will present an appearance not unlike the flamingos that strut about the gardens in Regent's Park.

I now come to my last and most interesting application in this lecture of the principle of least action. I have shown that a quadrilateral muscle becomes occasionally a skew muscle, like the skew bridge known to engineers. Every line in it is straight, but the whole forms a curved surface, and any plane drawn across that surface would give me a conic section. (Fig. 6.) I come now to the great pectoral muscle in the wing of the bird. I have before me two diagrams that have cost me many hours of hard work. One of them represents the wing of the albatross (Fig. 8). Here is the socket, *S*, or, as anatomists call it, the glenoid cavity, of the wing; *A'A''* is the furcular clavicle; *A''B'* is the sternum; this curved line, *A'B'*, represents the origin of the great pectoral muscle; and *AB* is the insertion of the pectoral muscle into the humerus, placed so that this insertion shall occupy the same plane as the origin of the muscle. I believed that I had succeeded in carrying the principle of least action to such a point that I should be able to make a prediction. And here I would call your attention to the important fact that no science whatever is worthy of the name, no science is anything but a collection of facts, which is not able to predict consequences—when certain facts are given, to predict other facts; and in proportion as any science possesses the high prerogative of being in a condition to predict from a certain number of conditions other conditions, it deserves the name of an exact science. In other words, it has come under the control of geometry, the great queen and mistress of all sciences. I selected for the purpose of prediction the wing of the bird, and I said to myself, "I can trace accurately the origin of this great muscle, I know its insertion, and I will try and predict an unknown thing about it; viz., the position of its axis of rotation." Let *A'B'* be any curve whatever observed in Nature, as the origin of the pectoral muscle of a bird; let *AB* be any other curve observed in Nature representing the insertion of the muscle in the arm. Then draw *A'A B'B* to meet in *O*. Given these two curves, I was able to draw the bisector of the angle *A O B*. I was also able to draw a certain right line, such as *PQ* or *LM*, at right angles with that bisector, and to say, if the muscle of the bird, which is a skew quadrilateral muscle, contracts so as to produce the maximum advantage that it can produce, the axis of rotation round which the wing of the bird must turn will be a particular line that I can calculate. I shall not trouble you with the details of the calculation; they would be very uninteresting to an audience like this. You will

possibly see the importance of them when I tell you that they consist in finding a certain ellipse—the ideal ellipse as it exists in the albatross I have drawn. There is an elliptical curve which is to be calculated,

Fig. 8.

and I say that the minor axis of that ellipse is the axis of the greatest effect, or the axis of least action round which the wing of the bird revolves. The black axis *s t* represented on the diagram is the actual observed axis as I have found it in the wing of the alba-

tross ; and the red axes, such as P Q and L M, represent the axes found by me by successive approximations, each coming nearer and nearer to the real axis. I chose the albatross for the following reason. Just as I believe the tiger to be the most worthy object of study in considering the question of the arrangement of the limbs of quadrupeds, from its great strength and size and activity, so I believe the albatross to be the most wonderful of all birds with which we are acquainted, and to be worthy of study. Its habits have been described by Portuguese navigators ; and they have been described by Coleridge in the beautiful poem *The Ancient Mariner*. The albatross possesses very remarkable peculiarities. He seldom or never flaps his wing, but his soaring power in the air is prodigious. When he has once attained a certain height, he is so beautifully constructed that he is able to keep that height, or at least to lose less of it than any other known bird. The only other bird in the world to compare for a moment with the albatross in the power of soaring is the condor vulture. I have here a drawing of the wing of the condor vulture and the wing of the albatross. Any one looking at the diagram will see that, if I took a pair of shears and cut off the white feathers from the wing of the vulture, I should reduce it to the wing of the albatross. This was my main reason for choosing the wing of the albatross as a type of the perfection of flying. I studied the wings of the eagle, the hawk, the vulture, and other birds ; and I found there was a sort of type underlying them which corresponded with the wing of the albatross. I can demonstrate, but will not trouble you with the demonstration now, that the albatross wing contains all the conditions for merely soaring. It sleeps upon the water at night ; it feeds upon small floating molluscs and crustaceans which it finds in the sea, or gladly accepts from passing sailors pieces of biscuit offered from the ships. When morning comes, the albatross rises slowly and laboriously from the water. He is described by the ancient Portuguese sailors as running upon the sea, because he rises so slowly in the air that for nearly half a mile he attempts to rise from the surface, and his feet touch the waves. Slowly and painfully our poor bird rises to a height of about a thousand feet, and he seems content with this thousand feet ; he has the power of losing as little of it as any known bird. If a ship be in sight, the albatross follows the ship ; but, if no ship be in sight, he is cunning enough to look out for another albatross that sees a ship. If he sees another albatross at a distance moving in a particular direction, he knows that it sees a ship, or sees an albatross that sees an albatross that sees a ship ; and so, before ten o'clock in the morning, the ship is surrounded with flying albatrosses, soaring most gracefully in the air. Woe betide the sailor that shoots an albatross ! I was five years in obtaining this creature for dissection. Through Mr. Moore, the curator of the Museum in Liverpool, I was put in communication with a

number of sea-captains going round Cape Horn. They told me that there was great superstition among the sailors about the albatross. They all remembered the story of the Ancient Mariner, and the passage into the "silent sea", where "slimy things with legs did crawl". You remember Coleridge's words about the bird and the prejudices connected with it.

"And the good south wind still blew behind,
But no sweet bird did follow,
Nor any day for food or play
Came to the mariner's hollo !

"And I had done a hellish thing,
And it would work 'em woe :
For all averred I had killed the bird
That made the breeze to blow."

In spite of these difficulties, I obtained my albatross, and made my calculation. I was an hour dissecting the pectoral muscle, another hour making measurements upon it, and another hour transferring those measurements to paper for further measurement. After that was done, it cost me five hours of incessant labour with logarithmic tables to take out the figures and calculate the red line PQ , which represents my first approximation ; and the second approximation, LM , required ten hours of numerical work. I have applied equal labour to every one of these six birds ; viz., albatross, grebe, macaw, wood-pigeon, pheasant, and heron. My calculated red line represents what I believed would be the position of the axis of the wing corresponding with the law of least action. It comes, in every case, as you observe, uncommonly close to the black axis, ST ; it is sometimes above it, and sometimes below. It presents no suspicious nearness to the black axis, and there is a characteristic about it that I must ask your permission for two minutes to dilate upon. It is a characteristic of every real discovery that, if we make closer and closer approximations, we shall find nearer results to the true, but we shall always find certain residual phenomena left behind which our theory will not explain. Now, in the case of the vulture I have a residual phenomenon. The vulture has not only to soar like the albatross, but he has to possess a power which the albatross does not, of rising in the air in the course of an hour or two, from the level of the Pacific Ocean to the heights of Cotopaxi. He has, therefore, two problems to solve ; he has to soar, and rise to a height rapidly. There is hardly any other bird in which, if we studied their habits, we should not find that there were two or three objects to accomplish with their wings. In this calculation, I have entirely neglected those subsidiary objects ; but in the case of one or two birds like the vulture, where I have made the calculation and brought in the two conditions, I have succeeded in producing the red axis of my ellipse so as to become identical with the black

axis, *S T*. Hence, whether we take into account the other objects which wings may have to accomplish, or the necessary errors of observation, because the black line itself is only an observed line, and a line observed after death, I have reason to believe that I have succeeded in showing that we possess a power of prediction with regard to the wings of birds, and to other principles of animal mechanics, that entitles us to say that that science of animal mechanics has entered into the class or group of exact sciences.

In conclusion, I would say, with regard to prediction, you are all acquainted with the planet Neptune. In fact, the poor planet Neptune is used up ; he has been so hackneyed a subject for lecturers and audiences that I will not say anything about him. My friend Professor Tyndall has made most of you acquainted with the extraordinary prediction of conical refraction by Sir William Hamilton, whose name will be remembered by those who come after us as that of the greatest mathematician of the nineteenth century. I shall not trouble you with his theory of conical refraction, except to mention a story that possesses an interest as coming from the lips of Sir William Hamilton himself. He told me that he made the calculation late at night. He was not an experimenter, and, as you are aware, the present distinguished Provost of Trinity College, Dr. Lloyd, was the man who actually saw conical refraction first. When Sir William Hamilton took his scribbled paper to Dr. Lloyd, and asked him to make the experiment, any person, not a mathematician (Sir William Hamilton told me), and not accustomed to reading his marks, made on little scraps of paper, the backs of letters and the like, would have said, "Oh, he is taking to his friend a piece of paper on which, for fun, he has allowed a spider that he has dipped in ink to run about."

We find, then, nothing tentative in any branch of Nature. There is nothing tentative in astronomy. No planet ever seeks to move more perfectly in its orbit ; it does so from the beginning. We have no evidence that light describes its path by a succession of attempts ; it is singly, doubly, or conically refracted, according to fixed conditions, and has all the appearance of having been always so. The socket and the axis round which birds' wings revolve are placed exactly in the position best suited to produce the best effect ; and here again I find no tentative process. There is no evidence in Nature of birds with imperfect wings ; no proof of a succession of blunders before perfection was attained. All is perfect ; and all was always perfect. There have been no "tentative miracles" in nature, no failures, nor trials. The graceful limbs of the beautiful tiger and the expanded pinions of the sweet albatross of Coleridge speak to the ear of reason in language that cannot be misunderstood,

"The hand that made us is Divine."

LECTURE III.—Tuesday, June 6th, 1871.

Application of the principle of Least Action to the Heart and other involuntary Muscles.—The Mechanism of the Heart explained, and the amount of work done by it.—“Experimentum crucis” of the entire Theory, derived from the measurements of the Fibres of the Heart of Man and the Ox.—General conclusions as to the future progress of Animal Mechanics and Comparative Anatomy, when subject to the Rule of Geometry, the Queen and Mistress of all the Sciences.

I HAVE reserved for my closing lecture to day the most wonderful and remarkable of all the examples I am able to give you of the application of the principle of least action to animal mechanics. It relates to a question which deeply interests every person in this room ; it relates to the action of our hearts. It is my intention to endeavour to lay before you the work which is done by our hearts, and the manner in which those hearts do their work. The story of the heart is a most wonderful and mysterious story, and you must make allowance for the difficulty of the subject, and the defects of the lecturer, if I fail to convey perfectly to your minds all that is in my own mind respecting it. It is not easy to condense into one short hour the results of a labour of ten years. The progress of discovery is slow, and it is difficult to explain to those who have not been travelling in the same paths of research as myself, all the meaning and the bearings of the facts which I have to state ; you will therefore, I hope, excuse me if occasionally you fail to see the connecting link that joins one part of my reasoning with another, and take for granted that if I had a longer time at my disposal, or more art in the mode of laying my materials before you, I could make you perfectly understand all that I know with regard to this subject.

We have first to consider the question of the amount of work which is done by our hearts. The heart is a small muscle weighing only a few ounces, and it beats perpetually day and night, summer and winter. Frequently an old man's heart approaching a hundred years of age will be found on examination as perfect a mechanism and as complete as it was when he was a young man of twenty. In order to measure the force and power of the human heart, the most obvious method that would suggest itself is one that is impossible to adopt, because it would require the death of the person on whom the experiment was made. We have experimented on the hearts of horses, oxen, sheep, dogs, and other

animals. The first of these series of experiments was made by the celebrated Dr. Hales at the close of the last century : it consisted in measuring, by direct experiment with tubes, the amount of the hydrostatical pressure inside the cavities of the hearts of several animals. These experiments showed that the hydrostatical pressure inside the hearts of animals varies ; in the horse and ox and larger animals amounting to a pressure of nine feet perpendicular of fluid blood, and in the smaller animals to somewhat less. From these experiments we can calculate without much difficulty the total amount of work which is done by the heart of a horse, the heart of an ox, the heart of a sheep, or the heart of a dog ; but you will see it would be impossible to perform such an experiment upon our own hearts, because the experiment is necessarily accompanied with the death of the animal that is operated upon. We can calculate from these experiments also what I call the coefficient of capillary resistance. The heart pumps the blood through the large arteries of the body into the capillary vessels which permeate every tissue in our frames, and the great resistance to the action of the heart occurs in forcing the blood through these capillary vessels. I have placed before you here, as the result of direct experiment, the coefficient of capillary resistance of the sheep, $\frac{1}{18.6}$; the dog, $\frac{1}{19.6}$; the horse, $\frac{1}{39.3}$; the ox, $\frac{1}{39.8}$. You observe in the sheep and dog the coefficients are double what they are in the horse and ox. These animals group themselves naturally together into the smaller animals with a double coefficient of resistance, and the larger animals with a single coefficient. Now with which of these groups of animals are we to associate ourselves in making a calculation as to the amount of work which is performed daily by our hearts ? As I explained before, we cannot perform direct experiments upon the human subject ; but an accident placed in my power the means of making a very close approximation to this remarkable result. When the artery of a horse or of a cow is cut, we can measure with ease the height to which the blood will spout into the air ; and when the experiment is made, we are surprised at first to find that the artery does not spout to the height of nine feet. We can prove that there is an hydrostatical pressure inside the heart of the horse amounting to a nine feet column of blood, but when the artery is cut the blood will only spout to a height of two and a half feet, Nature making instinctively a spontaneous effort to shut off the pressure to save the animal from death by bleeding. If, therefore, we could by any process arrive at the precise height to which the blood would spout from our arteries if wounded, and compare them with the corresponding experiment in the sheep, the horse, and the ox, we should find which of these groups of animals man is most closely allied to with regard to the circulation of his blood.

On the 18th of March, in the year 1863, I witnessed an operation in

the theatre of the Meath Hospital in Dublin, performed upon a poor man, in whom, from various circumstances, I felt an interest. I was merely a spectator at the operation, therefore I had leisure to witness a remarkable phenomenon, and to draw inferences from it, which I could not have done if I had been actively engaged. In the course of the operation, a large artery was cut in a very unusual place, and, therefore, some delay occurred in tying it. The blood spouted in jets from the wounded artery for a minute, or two minutes, before it could be tied. When the operation was over, I examined with care the height of the table on which the man lay. The floor, which had been recently cleaned, was covered with spurts of blood which had fallen from the wounded man, and, by the application of a little geometry to the problem, I was easily able to ascertain, by taking the height of the table and the farthest positions of the spots of blood on the floor, the velocity with which the blood issued from the wounded artery. The curve described by the blood is a parabola, and, given two points on the parabola, every geometer knows that we can construct the parabola and calculate the angle of elevation and the velocity with which the fluid is projected. As soon as I had made this calculation, I found that if I had cut the artery of a man, and allowed the blood to spout directly into the air, it would spout to the height of 2.58 feet. Taking the mean of all Dr. Hales's experiments upon horses, I find that 2.53 feet is the height to which the artery of a horse will spout. We now have a most important and valuable result. We cannot compare directly the hydrostatical pressure inside the human heart with the hydrostatical pressure inside the heart of an ox, or of a cow, or of a horse: but we can, by this determination of the velocity of spouting blood, show a close relationship between the circulation in our own frames and the circulation in these animals, and, therefore, we may apply with confidence the coefficient of resistance which we find in the horse and the ox to our own cases. When this coefficient of resistance is used in the case of man, and the calculation is completed, we find the hydrostatical pressure inside the human heart to amount to 9.923 feet of blood; and, by using the number of times the heart beats—seventy times each minute—and the quantity of blood projected from the heart at each contraction, we can calculate, by very simple and elementary processes, the work done in a given time by the human heart. Now this work I shall represent for you in a form extremely easy to remember—a form which will show you the extraordinary amount of work that is done. I shall suppose that I cut out an ounce of muscle from the heart, and that I ask myself this question, What number of pounds can that contracting muscle lift in the course of a minute? I find that the contracting muscle, a single ounce in weight, of the human heart, will lift 20.576 pounds through the height of one

foot in a minute. This I believe to be a very close approximation to the power of the heart; but, inasmuch as it was not obtained by direct, but by indirect reasoning, I thought it desirable to proceed to verify it by another process; and, in the verification of this coefficient of muscular force of the heart by a second process, I made use of a very interesting phenomenon. This phenomenon was observed by the celebrated Dr. Wollaston, who wrote a paper upon it, which is published in the *Philosophical Transactions* for 1809. Dr. Wollaston was the first person who noticed that, when our muscles are contracted, they give out a deep musical note. If any of you wish to repeat the experiment for yourselves, and satisfy yourselves about it, I will inform you of the simplest modes of doing so. If you go into a room by yourself in perfect silence, place your elbows firmly on the table, and close your ears lightly with the forefingers, clenching the muscles of the forearm, you will hear immediately a deep musical hum, which never can be confounded with any other sound you heard before. Dr. Wollaston compares it most accurately to the rumbling sound produced by the noise of cabs driving over the pavement in the silence of the night. Or if you awaken at night and clench your teeth so as to call the masseter muscles into action, you will hear with the ear that lies next the pillow, which acts as a sounding board, this deep hum, which you can destroy by ceasing to clench your teeth, and renew at pleasure. You can, therefore, very soon satisfy yourself that the act of contracting the muscles is accompanied by some phenomenon that takes place in rapid succession, comparable with the motions that produce a musical note. My attention was first directed to this curious subject in a remarkable way; and it so happened that a young physician of Marseilles in a similar way was attracted to the study of this curious phenomenon. Dr. Collongues of Marseilles had charge of the cholera hospital in that town; and he observed, in studying some of the cases which had died, a remarkable fact. Dr. Collongues in Marseilles, and myself in Dublin, pursued our studies each without knowing that the other was engaged in them. A patient in cholera has his temperature much lower than the natural heat of the blood, which is 98 deg.; and it is well known to physicians that in diseases like fever, if the blood-heat rise some seven or eight degrees above that, the patient will die. It is equally well known that in diseases like cholera, if the blood-heat fall more than seven or eight degrees below 98, the patient will die. Therefore it is a strange fact that, when you examine the body of a person who has died of cholera, when you put your hand upon the body, it is warm; the temperature, which was 90 deg. before death, rises after death to 103 or 104 deg., just as if the person were still living and in the height of a violent fever. This is accompanied also with spontaneous movements occasionally of the limbs, which cause great alarm to persons

who are not acquainted with this curious fact. Happening accidentally to place my ear against the arm of a patient who had died of cholera, to my astonishment I heard the well known musical hum. I sprang to my feet, and at once placed my ear to the man's heart, thinking that he was alive, and that I might be able to save him ; but he was dead. The heart had ceased to beat, but the muscles still continued to live. The heart has been called by ancient physiologists the first to live and the last to die—*primum vivens, ultimum moriens* ; but that is not true in cases of cholera. After the heart has ceased to live, after the brain has ceased to act, when the man is dead, his muscles live ; their temperature rises, and the last traces of life remain in the body like the lingering music of the chords of a harp which the master's hand has ceased to play. I resolved to try and ascertain the precise note of this musical hum. I constructed a number of organ-pipes, and succeeded, by processes that would be too long to describe to you here, in imprisoning the musical hum in one of these pipes, where I could afterwards measure it at my leisure and determine its character. I had an organ-pipe made accurately to vibrate the note two octaves below C in the bass, which corresponds with thirty-two double vibrations in a second. The note two octaves below D in the bass corresponds with thirty-six double vibrations in the second. Now a very little trial showed me that the musical note of a muscle lay between these two notes. By fixing a second organ-pipe as exactly as I could to the musical hum of my own muscles, and then comparing the notes of the two organ-pipes by their beats, I was enabled to ascertain with, I believe, a considerable degree of precision, the exact note of the musical tone produced by the contraction of the muscles. This I made to be thirty-five and one-third vibrations per second. Shortly after I published this result, Dr. Collongues, who had moved from Marseilles to Paris, sent me a book which he had published a short time before my own, in which he had succeeded in proving, by measurements made with tuning-forks, that the vibration corresponding to the hum of the muscular contraction is thirty-six. This very remarkable result, obtained by two totally different methods of experimenting—by myself in Dublin with organ-pipes, and by Dr. Collongues in Marseilles with tuning-forks—immediately attracted attention. Dr. Collongues naturally was very uneasy about the question of priority ; and I took the opportunity of calling upon him in Paris to explain to him that I admitted his priority, and that I was more pleased to find that we had succeeded independently of each other in obtaining the same note, than if I had myself established a claim to have made the first discovery. At the close of our interview, which was very friendly, he embraced me. He had a long black beard, and I have a distinct recollection that it smelt very strongly of tobacco.

My object in determining the musical note of a muscular contraction was to calculate by a second process a coefficient which would represent the amount of work that an ounce weight of muscle would be able to perform. The manner in which I conducted the experiments was as follows. I held my arms horizontally, asking a friend to see that they neither rose above nor fell below the horizontal line. I held them in that position until I was completely tired; then I placed different weights on my arm in the same position, and I tried the experiment on many other persons. You will be astonished, if you try the experiment, how short a time you can hold out your arms perfectly horizontal. In this way, if I knew the rate at which muscular contraction takes place in the arms so held out, it was easy for a mathematician to calculate the amount of work done by each ounce of muscle engaged in holding up those arms. I had found that the musical note of the muscle vibrated thirty-five or thirty-six times in the second; and, having first got that fact, from the experiments made with holding weights in my extended arms, I was able to determine a second coefficient to be compared with the other which I had previously deduced from the hydrostatical pressure; viz., 20.576 lbs. This second coefficient came out exactly 20 lbs.; that is, the weight that can be lifted by an ounce of muscle of the heart through one foot in a single minute. This result I believe any experimenter on such subjects will admit to come in a reasonable degree close to that which I obtained from the hydrostatical pressure of the heart. I am, therefore, entitled to consider that somewhere about 20 lbs. can be lifted by every ounce of my heart in a single minute. But this conveys to your minds no adequate conception of the enormous amount of work which that represents. If I said 40 lbs. or 50 lbs. it would convey no impression to your minds; I therefore devised a plan for the purpose of showing you in this lecture how much you ought to wonder at the great work performed by the heart. I obtained from Mr. Robert Main in Oxford, and Mr. Maclaren, the celebrated trainer, the length of the Oxford and Cambridge boat-race course, and the cross-sections and plans of the Oxford eight-oared boat. The average time in which the race is rowed (it has been rowed twenty-one times in twenty-one years over the same course) is 23 minutes $3\frac{1}{2}$ seconds, and the length of the course is 4.31 miles. From these data, and from the plans and sections of the boats kindly supplied by Mr. Maclaren, I was enabled, by using Professor Rankine's well-known formulæ for the resistance of ships, to determine the amount of work done by the muscles of the young men who pull in this hardly contested race. I find that during the twenty-three minutes the race lasts, every ounce of muscle in the arms and legs of the rowers works at the rate of 20.124 lbs. lifted through one foot each minute. This comes out to be very much like the amount of the work that my heart is doing at this moment; indeed,

I am not sure that it is not doing more work than that now while I am lecturing. In the case of the young men who pull in this race for twenty-three minutes, every ounce of muscle in their arms and legs gives out a force that in a minute would lift 20 lbs. through a foot. If any of you have seen the exhausted condition of those young men when taken out of their boats after twenty-three minutes, you will, I think, agree with me that human nature could not endure such labour for forty minutes ; yet the heart of an old man close upon a hundred years of age has worked for that hundred years of his life as hard as the muscles of the young men that pull in the Oxford and Cambridge eight-oared races.

We have now discussed, and I hope satisfactorily solved, the question how much work is done by the heart ; but a question remains unanswered which no intelligent mind can avoid asking: How does the heart do that work ? I cannot pretend to tell you how ultimately it does that work, for that depends upon the problem of nerve-supply—a subject with which we are totally unacquainted. But I believe I have succeeded in making one step further in advance and getting at a slight knowledge of the arrangement of the fibres of the heart by which this enormous amount of work is possible, and have arrived at it by a strict and rigorous application of the principle of least action. I have applied the principle of least action to the construction of the heart, so as to ascertain, if possible, some law that must be fulfilled by the arrangement of the fibres which will allow of this principle being carried out. The law of muscular contraction which must be complied with is this : Let L represent the length of a muscular fibre ; an order comes from the brain or some other part of the nervous system to this fibre to contract ; it is immediately shortened to an extent that leaves it about eight-ninths of its original length. Now, when a group of fibres are so arranged, as in the example I showed you before of the triangular muscle, that each fibre in the system is not at liberty to contract to eight-ninths of its entire length, there is a necessary loss of force. Therefore, if the principle of least action applied to the heart be true, we must find such an arrangement of the fibres in the heart as will allow of every individual fibre contracting to eight-ninths of its length. The fibres of the heart have been compared by Borelli to a ball of twine ; and this has been more correctly explained by subsequent writers as two balls of twine contained in a third. There are two cavities in a heart ; we call them the right and left ventricles ; the whole heart surrounds these. Certain groups of fibres run round one cavity, certain groups run round another, and certain other groups run round both. The fibres that run round the entire heart are called common fibres, because they are fibres which are common to both cavities, while the fibres that run round each cavity separately are called proper

fibres. In these diagrams, which I have taken from Dr. Sibson's excellent work on Medical Anatomy, a general view of the arrangement of these fibres is shown. You see here represented what is called the tendinous zone or tendinous ring. The fibres start from this tendinous ring, wind round the heart in a spiral manner, and, having come to the apex of the heart, enter it and run straight back again towards the ring. In this model I endeavour to show you what I cannot show by a diagram. Supposing this to represent the heart, here is the tendinous ring which surrounds the great vessels that issue from the heart. In order to avoid confusion, I have placed only four muscular fibres on the model. Following the red one, you see it starts from this zone, twines round the heart spirally, and, as you see in the diagram, makes a complete revolution and comes back directly towards the spot from which it started. It now leaves the outside surface of the heart and enters the heart, and you may see it running almost in a straight line up the inner side of the heart. The outer fibres, therefore, wind spirally round the heart, enter it at the apex, and form, as they return to the tendinous ring or zone from which they started, the lining of the internal portion of the cavities of the heart. Imagine millions of such fibres arranged in this spiral orbicular manner, as it was called by Borelli, and you will have an idea of the complexity of the arrangements of the heart. There is a necessary space left between these outer spiral fibres and the returning fibres which you see running through the interior. This space is filled in the heart with the proper fibres. The proper fibres wind each in a spiral manner round their respective cavities and go back again, so that we have three distinct groups of spiral fibres arranged apparently in the most hopeless intricacy, but in reality according to extremely simple and beautiful geometrical laws. The law which regulates the arrangement of these fibres, so far as I have succeeded in discovering it, is this: the spiral fibre which goes round the entire of the two cavities of the heart describes a complete circumference of 180° before its return, whereas the spiral fibres that surround the right and left ventricles of the heart respectively describe an entire circumference and one-fifth over before they come back. This extra fifth of a turn I believe is for the purpose of giving a twisting motion to the cavity, just as you would wring a cloth, so that it should be completely emptied at the close of the stroke and no blood left remaining in the cavity, or even the least loss of force occasioned.

I shall take this opportunity of publicly thanking Dr. Sibson for his kindness in having placed at my disposal his unrivalled collection of dissected hearts. He allowed me to take them with me to Dublin and retain them for twelve months for study; and he placed not only his preparations but all his stores of knowledge at my disposal. As he is not present, and therefore cannot blush at what I have to say, I will

add that his great knowledge of the pathology of the heart is fully equalled by the kindness with which he places that knowledge at the disposal of the humblest searcher after truth. Jealousy is so often the characteristic of scientific men, that it is pleasant to meet a man who is entirely free from it. I suppose that this quality of jealousy which men of science possess entitles them to be considered as rising to the level of the better and the gentler sex. In the first place, each of these fibres is so arranged that it is capable of contracting to eight-ninths of its length, because I find that each of these fibres is the same length: the length of the common fibres is the same, and the length of each group of proper fibres is the same, but of course the two groups differ from each other in length. Now, since each of these fibres is so arranged spirally and is of the same length, and is capable of contracting to its full extent when ordered to do so by the brain, you will see that, as far as they are concerned, the principle of least action has been fulfilled. But there is a remarkable opportunity of applying to this case a crucial test of whether the principle of least action is or is not the great principle in muscular mechanics that I assert it to be. I have two groups of fibres, one surrounding the two cavities and another group of fibres surrounding one cavity, and by the application of a little geometrical manipulation I was able to arrive at a very remarkable result.

If I call L the length of one of these spiral fibres going round the entire heart, the volume of the whole heart will be proportional to the cube of L , which being a linear symmetrical dimension, the volume of the whole heart will be proportional to its cube; so that $L^3 - L'^3$ will be proportional to the difference in the volumes of the heart before and after contraction. But the difference in the volumes of the heart before and after contraction is the sum of the volumes of the two cavities.

I will call λ and ρ the left and right ventricles. If we take the fibres that go round a single cavity, I find that if l and l' represent their lengths before and after contraction, that in like manner $l^3 - l'^3$ will be proportional to the volume of the left ventricle.

Therefore, if the principle of least action be true, I can predict a thing that at first sight appears very strange. I can find the ratio which the volumes of the two cavities bear to each other by the measurement of the lengths of the fibres that surround them. On measuring these fibres it comes simply to this. Let L be the length of the fibres that go round the entire heart; let l be the length of the fibres that go round the left ventricle. Find those lengths and cube them. The ratio of those cubes will be proportional to the sum of the right and left ventricles divided by the left. There are theoretical grounds which I believe are almost of themselves sufficient to entitle us to believe that these two cavities are of equal volume, and therefore that this fraction will come out equal to 2. I have taken, how-

ever, a more certain mode of determining this by collecting together all the observations of direct measurement of these volumes that I can find, and I find that the mean is 2.125. From theoretical grounds I believe that more accurate experiments and observations will prove that the decimal fraction of an eighth must be struck off, and that the true proportion is represented by 2. Certainly 2 is the number given by the most accurate of the ten observers. But now to my verifications. I measured the lengths of the common fibres in the heart of a great number of oxen, and I find it to be 10.875 inches. I measured the length of the fibres that go round the left ventricles in the same hearts, and I find as the mean of many measurements 8.625. Well, I suppose there is no one present here who is a good enough arithmetician to tell me at sight what the ratio of the cubes of those numbers would be. I have cubed the numbers, and their ratio comes out 2.004. I believe that to be a remarkable result, and to entitle us to assert that the principle of least action applied to the problem of the heart is capable of solving it a step beyond what it has been solved, and bringing us within reach of the knowledge of one more of the wonderful laws of the Creator. How it would rejoice the soul of the great Kepler if he had known that the ratio of the length of the fibres in his own heart was in the proportion of cube root of 2 to 1 ! Divine Geometry ! Queen and mistress of philosophy, thy right to rule the sciences shall never be disputed !

This principle of least action applied to the heart consists, as you will see, simply in making every fibre and particle of the heart do the entire amount of work that it is capable of doing. In a somewhat analogous case, mechanical engineers have attempted to produce the same effect. If you take a fowling-piece, it is a matter of comparatively little consequence how the fibres are arranged if they be of ordinary strength. One fibre helps the other, and they all do their work ; therefore, no one thinks of inquiring how the fibres of a fowling-piece are arranged ; they are capable of resisting an explosion, because they all assist in doing it. But when you come to build up monster guns like Mr. Robert Mallet's great mortar, or Sir William Armstrong's six hundred pounder, you have to calculate with the utmost nicety what your contrivances and arrangements must be, so as to compel every fibre of steel or wrought iron in these great guns to bear its share in the work. Some few days ago, I went to the Woolwich Arsenal, to see the Armstrong six hundred pounder which exploded. It consists of eight rings ; the first, sixth, and eighth rings were burst ; the remaining five were not injured. Now, this gun, although a great attempt to solve the problem, was not a perfect gun, because a perfect gun would burst in such a manner that all the eight rings would give way together, each perishing in the effort to resist the explosion. That which human

skill is not able to effect, is solved in the arrangement of the fibres of the heart of every person in this room.

I shall now apply the principle of least action to the case of ellipsoidal muscles. We call an ellipsoidal muscle a muscle that surrounds a cavity—a muscular bag surrounding a cavity which generally contains fluids. In attempting the solution of the problem of an ellipsoidal muscle, I found myself brought into contact with a problem in architecture which has baffled architects for many years : I mean the problem of the equilibrium of an elliptical dome. Every portion of a curved ellipsoidal muscle forms a portion of a small flat dome ; and to determine the equilibrium of tensions and strains amongst the muscular fibres of such an animal structure, is the same thing as solving the problem in architecture of what are the strains in various directions in an elliptical dome. I believe I have succeeded completely in solving the problem ; and I have done so by an application of pure geometry, in which I have not used a single letter of analysis. The difficulty of constructing equilibrated domes may be illustrated when I tell you that, with the exception of the Pantheon in Paris (fortunately saved from destruction), there is not a truly equilibrated dome in existence. The dome of St. Paul's, in our own city, is braced up with double chains of iron, and other chains of timber and lead put on to cover the defects in the original structure of the dome. Even Sir Christopher Wren was not able, from his want of knowledge of the solution of this problem, to apply the principles of architecture to make the dome of St. Paul's stand by its own intrinsic strength without support. In the great dome of St. Peter's, at Rome, many hoops of iron are employed which were never intended by the great mind of Michael Angelo, who conceived it. They are confessedly failures. Brunelleschi's octagonal dome at Florence is perfectly equilibrated ; but then it is octagonal. No case exists, I believe, of a self-supporting perfectly equilibrated spherical dome but that of the Pantheon at Paris. An attempt has been made in the construction of the roof of the Albert Hall, to make an elliptical dome, but whether that construction has been successfully carried out on the principle of least action, I cannot say. The principle of least action applied to the building of a dome would require that not a single pound of material more than was sufficient was used in any part of it. The solution of the problem is so simple that I will venture to give you the result of it. Here is an ellipse that represents a section of the elliptical dome of the Hall of Albert the Good, the father of our future king. I draw a line in any direction from the centre, and I require a construction which shall give me the strain which the structure must be capable of bearing in that direction. The problem requires me to draw a line in every possible direction, radiating from the top of the dome, and to assign what amount of strength I must give the materials in that direc-

tion. In the required direction draw a radius, and draw a tangent where the radius meets the ellipse, and let fall a perpendicular on that tangent. The strain along this radius must be made proportional to the square of the perpendicular dropped on the corresponding tangent. That construction is so simple that a stonemason or a carpenter could apply it. For example, if he has an elliptical dome in which the major axis is double the minor axis, the squares of the perpendiculars will give him that the strength in the direction of the long axis must be four times the strength in the lesser axis.

I applied this principle of equilibrium to the case of the muscle which is used in placental animals for causing the birth of the young. This muscle is produced by Nature for a special purpose. As soon as it has accomplished that purpose, it is carefully removed; and therefore, if we could find in any part of Nature a test for the principle of least action, we ought to find it here. If the muscle be made by Nature too strong for the purpose intended, there is a waste of material, a waste of force; if it be made too weak, the life of the animal is risked. It is not the case of a muscle which has to overcome a resistance which it tries from day to day. If I go into a gymnasium and exercise any group of muscles in my body, there will be a gradual growth in those muscles, because there is a growing resistance day by day; but the muscle that causes the birth of the young animal never tries its strength against the resistance it is required to overcome until the moment of actual exercise arrives. By measuring the curvatures and thickness of the muscles, I ascertained that inside the ellipsoidal uterine muscle a hydrostatical pressure of 3.4 lbs. per square inch can be produced by its contraction. Dr. Matthews Duncan of Edinburgh, and Professor Tait of the same University, have made a number of valuable experiments on the strength of the membranes which this muscle has to rupture. They have tabled them; and in no case do they find that the resistance exceeds 3.1 lbs. per square inch. I am entitled to regard this as a remarkable example of the principle of least action in Nature. There is an adaptation of force to resistance—the force produced in order to overcome resistance for months not exercised against it, and found exactly of the right degree of strength, not too strong or too weak, when the time of trial comes. Here we see Nature attaining perfection at a single bound, by a process of foresight. There is no evidence whatever of the supposed necessity of an endless succession of previous blunders.

I have now to take my leave of you. I have to thank you, as I do sincerely, for the kindness with which you have listened to me; and I sincerely hope that you will make some allowance for the difficulties of my subject, as well as for its novelty, in the task which I have under-

taken of laying it before you. I come amongst you bringing you new facts on subjects which your minds are not familiar with, and I labour under the disadvantage of having to place these facts before you in the minimum of time. In fact, I have myself solved, in these lectures, the problem of least effort. I appear as a traveller from a strange country, where I have seen strange things. The pleasantest part of my life has been spent in making these researches. The pleasure of making them—the novelty of the facts which they disclose—encouraged me to come before an audience like this, in the hope of interesting others in taking up similar pursuits. I am but an humble craftsman, collecting a few stones together for the great building of Nature; but after us there must arise some great master-builder, who will perfect the task. The science of Animal Mechanics is only commencing: a vast future is before it. It would be impossible to describe all the results that must come from the careful conscientious combination of geometry and mechanics with the science of comparative anatomy. All these results must come in time. Amongst other applications, I may mention that we are even now in a position to lend valuable aid to the science of geology. You see the fossil skeletons; you see the points or processes on their bones where certain muscles were attached. We now can calculate with precision, as, I believe, within a few ounces, what the weights, the forms and sizes, of the muscles that supplied those extinct animals, must have been: therefore, if it were worth the trouble, we could reclothe with flesh the fossil megatherium, and restore the perfect form of outline which its body would have had when covered with its muscles.

In conclusion, let us suppose that this and all the other branches of science which man can study have been carried to their utmost perfection; let us suppose that man has fully explored all the secrets of Nature he is capable of obtaining, and has found a key that unlocks all her mysteries: he will still find himself only a worshipper in the temple and before the altar of an unknown God, whose true nature and moral relations to himself must be sought from other sources than those which Nature furnishes. There are truths in the system of things as real and as certain as any laws of Nature, although we cannot perceive them with our senses. My eyes cannot see them; my ears may not hear them; nor can I touch them with my hands; but they are there. I know them to be true, and that they will endure when Nature and her laws have passed away like the memory of a troubled dream. I testify what I have seen. I have many a time seen an humble earnest faith in these unseen truths cause a smile of joy to play upon the pale face distorted with pain like a sunbeam dancing on the bosom of the troubled ocean. I have seen those truths illumine with a light from heaven the dim eyes soon to be closed for ever by the

cold hand of death. These truths are more dear to me than all that Nature can teach me, because they touch my inner life and consciousness. I learned those truths as a little child upon my mother's knee ; I cherish them in my heart of hearts ; and in defence of them, if opportunity should offer and God should count me worthy, I would gladly lay down my life.