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INTERFEROMETER EXPERIMENTS IN ACOUSTICS AND GRAVITATION

PART II

By CARL BARJS, Ph. D., LL. D.

*Hazard Professor of Physics and Dean of the Graduate Department
in Brown University*

PUBLISHED BY THE CARNEGIE INSTITUTION OF WASHINGTON
WASHINGTON, MAY, 1923

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CARNEGIE INSTITUTION OF WASHINGTON

PUBLICATION No. 310

The Ford Baltimore Press
BALTIMORE, MD., U. S. A.

PREFACE.

The present report is in the main a development of the acoustic investigations with the pin-hole begun in the preceding communication (Carnegie Inst. Wash. Pub. No. 310, 1921). To this has been added an account of the progress of experiments on the value of constant of gravitation found with a vacuum apparatus—work which is feasible during a part of the summer months only. As the source of sound is necessarily of an arbitrary character, it has not seemed worth while to reduce the data to actual acoustic pressures, either positive or negative. Accordingly, the fringe deflections (s) of the mercury U-gage read by a vertical interferometer, as these are proportional to the pressures, are given throughout. Furthermore, a report of the large accumulation of numerical data obtained would have been awkward. Therefore, the graphs only have been given and are believed to suffice.

The introductory paper shows that the pin-hole probe responds effectively to nodes in organ-pipes, that it ignores the antinodes, and that an overtone may produce the fringe deflections even when the fundamental is excited. In the case of the usual air-blown diapason pipes, the presence of the air current naturally interferes with the acoustic experiment and the fringes are apt to be thrown out of the field. If, however, the pin-hole is surrounded by a minute bag of porous material like thin filter-paper (one or more ply), a consistent registry of nodes again appears, but constructed now on a base of increasing pressures, positive or negative. Even goldbeaters' parchment is sufficiently porous to function.

With a device so sensitive to nodal regions in vibrating pipes thus at hand, the construction of a pin-hole resonator suggests itself. Contrivances both of the open-pipe and closed-pipe types were tried out at length; but the latter, with but a single mouth and responding to a closed organ-pipe of the same pitch, is to be preferred in the interest of simplicity. Great difficulty was encountered in the construction of the pin-hole. Unless this is adapted to the resonator, no acoustic pressures whatever are evoked. It was eventually found that both the size and the slope of the walls of the pin-hole are critical; that a salient pin-hole generates acoustic pressure, a reëtrant pin-hole acoustic dilatation, and there is neutral behavior between the two. With a properly designed pin-hole, pressures whether positive or negative will be about equal; it follows, therefore, that by coupling them with the corresponding shanks of the mercury U-tube, an advantage in sensitiveness is secured. It does not seem feasible, however, to push the sensitiveness of the pin-hole indefinitely farther. The response is such that, below a certain threshold value of amplitude of sound-wave, the acoustic pressures evoked at the pin-hole drop off sheer. The pin-hole resonator is deaf to an organ-pipe sounding beyond a

radius of 6 meters. I have described these occurrences by an exponential expression, which fits pretty well.

It is arguable that if a single pin-hole evokes a certain acoustic pressure, two pin-holes properly adjusted in series would enhance that pressure and the project of relaying pin-holes would seem quite feasible. All experiments with this end in view remained persistently negative. With so many pin-holes in series that the excursion of the mercury in the U-tube was absolutely dead-beat, owing to the increasing air resistance introduced, the acoustic response was no better than for a single pin-hole.

The experiments have a direct bearing on the behavior of sensitive flames. If an adjusted pin-hole burner becomes turbulent by slight increase of pressure within, a sound-wave passing the burner will supply this pressure acoustically. Finally, the experiments with branched tubes and one or two (salient and reversed) pin-holes, begun at the end of the present paper, present many points of interest both on account of the low values of pitch which such structures harbor and of the eventual bearing of the data obtained on the sensitiveness of telephones.

Within its restricted field (radius 6 meters), the pin-hole resonator serves admirably for the acoustic survey of the interior of a room in which an organ-pipe is sounding. If the phenomena were visible, the room would probably have the stratified appearance of a vacuum-tube stimulated by electric discharge. For a given position of the pipe, nodal regions alternate with antinodal regions, quite irregular in distribution but none the less fixed in position.

I have cut through this acoustic topography in all directions from the position of the pipe, with the expectation of arriving at some general facts as to distribution; but except for the case of reflection from surface close at hand, I have not made much headway. It is thus very difficult to state the results otherwise than by the graphs in the text. These show a difference in the character of a survey between walls contrasted with one toward an open door; but (*a priori*) one would not be able to predict the occurrence of either type.

In the succeeding paper an account is given of the repetition, under very satisfactory instrument conditions, of certain experiments made long ago. These treat the effect (if any) of an electric current on rays of light, passing in parallel with the current through the same long channel, both in case of an exhausted tube and of an electrolytic trough. Such an effect was hardly to be expected and none was found. In the sequel some useful improvements are added to the self-adjusting interferometer, an apparatus not only of very general usefulness but almost indispensable when separate interferometer adjustments are frequently to be renewed.

The work on gravitation, which is an endeavor to ascertain to what degree the constant may be found in a self-contained apparatus under ordinary laboratory conditions, contains two groups of measurements. The first, made in a moderate vacuum of a few millimeters and with a relatively thick quartz fiber, gave a value trustworthy within a small percentage. Encouraged by this result, I installed a much thinner fiber, giving excursions 5 to 10 times larger,

the needle swinging in vacua approaching 0.001 mm. The mean elongations of the needle at night seem to be trustworthy, but the long period of the needle could not be found with anything like adequate accuracy. So dependent is this period on the thermal environment that early morning and afternoon observations may vary from 600 to 800 seconds, the individual data themselves being quite accurate. A storm passing over the laboratory dropped the period from 750 to 626 seconds, after which it rose again to 712 seconds. Of course it would be possible to determine the modulus of the fiber by a separate small body, but this would be at variance with the plan of the experiments, while the taking apart of a rigorously sealed apparatus containing a loaded quartz fiber is always a very delicate operation. The experiments will be resumed this summer under the highest attainable exhaustions, as the experiments show this to be indispensable.

CARL BARUS.

BROWN UNIVERSITY,
Providence, Rhode Island.

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CHAPTER I.

EXPERIMENTS WITH THE PIN-HOLE PROBE.

1. **Partly closed pipes.**—In the apparatus described in the last report the metal foil carrying the pin-hole was cemented normally to one end of a glass quill-tube (fig. 1, inset) about 30 cm. long, the other end communicating with the closed mercury shank of the U-tube by a short piece of rubber tubing. Changes of mercury level resulting from pressure at the pin-hole were read off by a vertical interferometer (fig. 3, inset). The length of rubber tubing used for connection at *e* here seemed to make little difference, unless this length was considerable. Deflections were reduced about one-half by lengths of tubular connector of about 1 meter between the pin-hole and the mercury surface. With quill-tube connections the decrease was often less; thus, to insert a few data for guidance:

Length of connector.....	25	65	100 cm.
Deflection, <i>s</i>	95	80	60 fringes

and the decrease would probably be still smaller if a uniform, clear-bore tube were used, instead of pieces joined together. In general, however, periodic relations occur, as investigated in §§ 5 to 7, 34, *et seq.*

The sound intensities used in the following paragraphs are merely incidental. It will therefore suffice to give the fringe deflections, *s*, in the interferometer telescope (fig. 3) in arbitrary scale-parts (collimator micrometer in .01 cm.). As a rule the fringe breadth was somewhat less than 2 scale-parts, though 3 and even 4 scale-parts per fringe were occasionally used. Taking the former estimate, the pressures Δh in centimeters of mercury ($U - U'$) to be obtained from Δs scale-parts (*s* being the reading) follow the equation

$$2\Delta h = \lambda(\Delta s/2) \text{ or } \Delta h = (\lambda/4)\Delta s = 1.5 \times 10^{-5} \Delta s \text{ cm.} = 1.5 \times 10^{-4} \Delta s \text{ mm.}$$

Pressures may thus be estimated as between 10^{-4} and 2×10^{-4} millimeter of mercury per scale-part, Δs . It was frequently convenient to shift the zero-point either to the right or left of the scale reading from $s=0$ to 100.

Using a quill tube 25 cm. long with the pin-hole at the end, the effect of partially stopping the tube was next ascertained. This could be done neatly with different lengths of the common furry tobacco-pipe cleaner. The original experiments with Helmholtz resonators excited by the telephone were very curious. Thus the g'' resonator showed strong harmonics ($s=90$ scale-parts) at d'' and g'' (the former embracing the neck connector, the latter belonging to the resonator alone) with the clear probe. Inserting the furry wire, the deflections were eventually quite as large, but of very slow growth. Indeed, the harmonic g'' , which was weak for the clear probe, was much enhanced by the stopped probe. Similar results were found with an f'' resonator, the

harmonics being d'' very strong and f'' weak for the clear probe. With the furred wire pushed in, even as far as 10 to 20 cm., both harmonics were gradually enhanced in the lapse of time, the f'' particularly, which was so faint as scarcely to be recorded in case of the clear tube. Finally, the c'' resonator with a strong harmonic at a' and a weak one at c'' retained its characteristics with the stopped pipe. Similarly, slow-growing deflections were obtained in the former report in case of slight leakage at the telephone plate or elsewhere in the closed sound chamber.

On repeating these experiments some months after with a closed c'' pipe 30 cm. long, I was unable to reproduce the former results. Furred wires 5, 10, 20 cm. long now acted nearly instantaneously, reducing the deflection to one-half or less, sometimes even to one-tenth. The deflection does not, however, diminish with the length of tube, partially stopped, if time is allowed in extreme cases (20 cm.). Longer wires were not always more harmful than shorter wires, though I obtained no definite evidence of periodicity here after many trials. Sound-waves are probably wiped out, like the waves of the sea entering a marshy shore. (*Cf.* § 5 *et seq.*)

2. Overtones in long, closed tubes.—These were of brass, about 2.5 cm. in diameter, closed at one end with the telephone (fig. 1) by which they were actuated. Lengths of 60 and 30 cm. were tried. It was not usually difficult to record with the pin-hole probe the overtones to which the pipe responded on blowing. In some of the harmonics, the pressure increments passed to pressure decrements on commutating the telephone current; but this was not the case with all the harmonics. One might conclude hastily that the commutator notes originated in the telephone, yet this is not the case as a rule. I failed to find any general result from the large number of tests made.

The closed pipe (30 cm. long with harmonics at $c' g'' e''' b'b'''$ when blown) responded to $c' e''' b'b'''$ strongly; but the g'' , for some reason, could not be evoked intensely, being in all the trials only just audible. A number of experiments were made with this pipe (fig. 1, inset, *T* telephone, *P* pipe, *ss* probe), particularly with reference to the location of the nodes in depth below the mouth. The long, rigid system *PT* was couched on a light 4-wheeled carriage, and rolled to and fro on the table over the stationary probe *ss*, to the outside of which a narrow centimeter scale had been attached. The natural overtone g'' , though not obtained on blowing, came out very strongly (fig. 1) when the pipe was excited with the telephone (1,000 ohms resistance). The nodes are quite sharp, lying at 10 cm. of depth and at the bottom. The antinode lies somewhat above 20 cm. of depth and is equally sharp. The pressure does not here fall off to zero. Probably higher overtones are in presence. No effect was produced by commutation of the telephone current. Accentuated loudness of g'' was evident to the ear.

Nothing was obtained on raising the pitch; but as the limit of the electric siren was a'' , the other overtones, $e''' b'b'''$, were unavailable.

On lowering the pitch there was no response until g' was reached. This it will be noticed is not an overtone of the closed pipe, nor would it be of the open pipe. Nevertheless, the survey in depth (fig. 2) brought out the node and antinode of the octave g'' very pronouncedly, though the telephonic resistance had to be decreased one-half (500 ohms) to obtain equivalent deflections. The pitch g' , therefore, while leaving no impression of its own, evokes the natural overtone g'' of the pipe, the node being somewhat broader and the antinode sharper. Here also commutation was ineffective.

The pitch was now lowered as far as possible on the electric siren, reaching c' , d' , e' when a response was again registered as shown in figure 3. Unfortunately, the pitch was fluctuating and not easily controlled or identified in this low region, and the three notes seemed to be about equally effective. The curve a shows the fundamental, when brought out under 500 ohms, very weak and unsteady. It was, again, very apt to be accompanied by the overtone

g'' (audibly, though intermittent), as appears from the graph b , figure 3. To be assured that there was nothing wrong with the pin-hole probe, the pitch was then raised to g' , whereupon the g'' overtone (curve c , fig. 3) came out even more strikingly than heretofore.

Finally (fig. 3, d), the pipe was removed from direct attachment (cement) to the telephone, one end of it stopped with a perforated rubber cork, and this joined at the perforation with a short rubber tube to the mouth-piece of the telephone. The pitch was thereby raised to d' . Setting the telephone for d' , the overtone near a'' nevertheless was the only one identifiable. The pressure distribution in depth, moreover, has somewhat varied compared with the cases above for similar conditions, as may be seen by inspecting the graphs b and d . It follows, therefore, that the only note which the tube will steadily maintain is g'' and that this may be evoked by the lower harmonics, either g' , c' , or even c , as well as by g'' .

Further experiments were made with the endeavor to clear up certain anomalous results with Helmholtz resonators. These were joined with a short end of rubber tubing at their necks with the mouth-piece of the telephone

into which a corresponding adjutage had been screwed. In the survey in pitch (fig. 4, *p*) the c'' appears as but a secondary maximum, the large maximum being near a' . No other maxima occur. A survey in depth below the mouth of the resonator is also shown in the same figure, both for the a' and c'' . The latter attains a maximum pressure at about 2 cm. within the mouth, and then falls off to the nearly constant values, as far as the bottom or neck. The a' increases well into the neck. It is natural, therefore, to take a' as the note of the telephone and resonator together, while c'' is the pitch of the resonator alone. To test this further, a g'' Helmholtz resonator was examined, as in figure 5, curve *p*. The note near a' is still present, though it has naturally been somewhat raised in pitch; but the proper pitch g'' , which is now remote, comes out strongly. There is still a question relative to the resonance in the U-tube. Making the telephone and the U-tube a closed region with an external pin-hole (as explained in the last report), the curves *a*, *b* of figure 6 were obtained with different resistances (500, 1,000 ohms) in the telephone circuit. In curve *c*

the telephone current is commutated. As a result some of the minima are changed to dilatations without modifying the general contours as a whole. On comparing the curves of figure 6 with those of figures 4 and 5, it is hardly probable that the note at a' in the latter is due to the note near a' of the former. Otherwise there should be evidence of the occurrence of the very loud $\#c''$ and some of the g'' of

figure 6 in both the curves of figures 4 and 5.

To contrast with the 30-cm. closed pipe, the smaller one, 14 cm. to the telephone plate and 2.5 cm. in diameter, was inserted. This, when blown, gave a sharp c'' , a shrill a''' overtone and others not available. Tested with the pin-hole probe for harmonics, the curve *p*, figure 7, was obtained at 1,000 ohms in the telephone circuit, showing besides the $\#c''$ maximum, a marked a' and a'' , both of which are abnormal. The note $\#c''$ was now examined for distribution of pressure in depth (curves $\#c''$, fig. 7) and the results found are such as would be expected for a fundamental. The graphs for 1,000 and 2,000 ohms in circuit are alike in character. The a'' was then examined for pressure distribution in depth, and though it was a little difficult to maintain this high pitch steadily, the graph of figure 7 (a'') came out clearly in all trials. The loudness is somewhat reduced, 1,000 ohms being in circuit, so that the fundamental $\#c''$ at 2,000 ohms has about the same intensity as the fundamental in the a'' curve. If, therefore, we subtract the ordinates of the former ($\#c''$, 2,000 ohms) from the latter (a'' , 1,000 ohms), we would get a graph suggesting a node or maximum pressure in the middle of the closed pipe. In other words, the closed pipe $\#c''$ would here seem to be endeavoring to vibrate like an open pipe with an antinode at the mouth and at the vibrating telephone-plate. There would

have to be but little accommodation, as an open a'' pipe is also about 14 cm. long. Another explanation, such as is given in the next paragraph, is perhaps not more exacting; but neither accounts for the rise of acoustic pressure from mouth to closed end.

In the same way as at a'' , the a' was now examined in depth with 500 ohms in circuit, the graph after many repetitions showing the character of a' curve in figure 7. This is doubly inflected, thus strongly implying the occurrence of the overtone near a''' superimposed on the fundamental. It was noticed that the tone-color changed perceptibly with slightly different speeds of the motor-break. Hence the overtones are probably intermittently present; *i. e.*, only on sharp tuning. This might account for the absence of a minimum in the a'' curve (fig. 7), even if not vouched for by evidence. The a'' being shrill, pressures are often marked even at the mouth of the pipe.

If we assume that both the a' and a'' are accentuated because they evoke the natural overtone a''' of the pipe, it must not be overlooked that this occurs in a medium of continually increasing acoustic pressure or density from the mouth of the pipe inward to the telephone-plate.

3. Pressures in wind-blown diaphon pipes. Porous envelopes.—When the pipes are blown by a bellows apparatus, the air current passing through the pipe seriously interferes with the acoustic registry. Thus, in an ordinary rectangular c'' pipe, the fringes are thrown out of the field of view for any position of the pin-hole within the pipe. Inclosing the pin-hole with a test-tube 10 cm. long and 1 cm. in diameter had no reducing effect, even when the neck was constricted so as just to admit the quill tube. A piece of pure rubber tubing closed at the outer end and loosely fitting the quill tube gave a small deflection at the middle of the organ-pipe, but not at the ends. This suggested the method of procedure; for it has been shown in the earlier work

that acoustic pressures leak (so to speak) through a porous septum and may thus be measured.

The plan, therefore, was to surround the quill tube S (fig. 8) carrying the pin-hole O with a narrow cylindrical bag EE of filter-paper, the open end of the bag being fixed to S , at aa , with cement. Pressure involving air transmission would have to pass through the pores of the paper, all other modes of ingress being sealed off. If air transmission is not essential, the flexible walls of EE are operative at once. The seal at aa , as well as at the outer end of the envelope, must be perfect; otherwise the pressures resulting from air currents are registered and the fringes leave the field throughout. Thus a bag EE merely pushed over the quill tube is unavailable. If the organ-pipe (f'') is blown intensely, a relatively large pressure will be indicated as soon as the pin-hole O enters within the plane of the mouth of the pipe. These pressures rapidly fall off to about one-fifth when the pin-hole is 1 cm. off on the outside

of the mouth, and then gradually vanish at from 1.5 to 2 cm. beyond the mouth. The graph is shown in figure 9 over the negative abscissas.

The high graph in figure 9 summarizes the results obtained with a diapason pipe 24 cm. long from open end to embouchure and giving a sharp c'' . The pipe was blown with a specially adapted Fletcher bellows, and though it was a little difficult to control the intensity, the node near the middle comes out satisfactorily. An f'' pipe, 18 cm. long, similarly blown, was much too shrill for the manometer, the fringes leaving the field, and the endeavor to reduce the intensity to manageable limits is less successful and trustworthy. Nevertheless, the character of the graph f'' is fairly well indicated. Results of the same kind were obtained with a small bag of filter-paper, E , figure 10. This was compressed, when wet, around the end of a glass rod and allowed to dry thoroughly. It was then attached to the probe S with cement and thin wire. The graphs, being essentially like figure 9, need not be given. As before, a very weak f'' note was needed to keep the fringes in the field. When the filter-paper E , figure 10, was moistened, all action ceased, to be restored, however, when the paper was again dry.

The short bag (fig. 10) easily admits of being doubled or quadrupled to reduce the sensitiveness and keep the fringes of the strong f'' in the field. Figure 11 shows the graph obtained with the envelope (inset) of two-ply filter-paper. We here definitely encounter the result which will frequently occur in the sequel, that in case of the now shrill f'' the graph is erected on a succession of increasing acoustic pressures from mouth to embouchure of the organ-pipe. The $\sharp c''$ curve, on the contrary, is erected on a succession of decreasing pressures, which dip, when the probe is near the embouchure, into marked acoustic dilatations. Nevertheless the node or maximum pressure in the f'' pipe is still in the middle. The node in the c'' pipe has been displaced apparently from 12 cm. to 8 cm. below the mouth. This is very unusual and to be ascribed to the superimposed dilatation; but the curve is otherwise smooth.

The next experiments were made by surrounding the pin-hole with a four-ply envelope of thin filter-paper (fig. 12, inset). These graphs accentuate the results of the preceding figure. The strongly blown f'' pipe, with a node in the middle, has its pressures not only built up on a base or succession of increasing pressures, but this succession actually begins with well-marked dilatations at the mouth of the pipe. The acoustic compression is zero 2 cm. within the pipe, and not again. The c'' pipe now has its node in the middle, while the compressions are zero at the mouth and at about 16 cm. within (agreeing with the preceding case). Thereafter, however, the march into dilatations becomes much more precipitous. Pressure decrements are nearly constant from 20 cm. within to the embouchure at 24 cm. It is interesting to note that at 16 cm. within, the absence of compression when the pipe is sounding passes into dilatations as the sound dies down, returning to zero when the sound ceases.

The absence of pressure increments when the bag is moistened with water has already been indicated. Similarly, if the pin-hole is surrounded by a tight cylindrical case of the thinnest mica (0.03 mm. thick), the U-tube does not react, however intensely the f'' note is sounded. With an impervious bag of very thin flexible goldbeater's skin, some response was obtained, as shown in the lower curve of figure 9. These deflections for an intense f'' were of slow growth to a maximum, as if the bag were slightly porous, or sufficiently flexible and soft to contract. One may note that this reduced curve is much like the upper curve (two-ply filter-paper) in figure 11 and erected on a base of increasing pressures. With the bags removed, the deflections were in all cases enormous, throwing the fringes far out of field.

4. Continued. Long probe tube and varied pin-holes.—Another method of reducing the effect of strength of the f'' pipe consists in elongating the quill tube of the probe as far as the less effective lengths of § 5, or of providing it with pin-holes of diameters either short of or beyond the optimum. The quill tube used was 40 cm. long, not counting the (wider) rubber connectors at the U-tube. The foil carrying the pin-hole was preferably shaped like a percus-

sion cap and cemented to the end of the quill tube, which it fitted snugly. It was then punctured with the end of the finest cambric needle. This gives a very robust apparatus not liable to spring leaks accidentally.

The graph obtained with the strongly blown f'' pipe is shown in figure 13. It is all but normal, with a node near the middle. The only suggestions of the air-current effect were obtained at the observations after 16 to 18 cm. in depth. These, as the pipe-note dies down, run as much as $s = -20$ into negative fields. In the full-sounding pipe these dilatations are overpowered by the acoustic vibrations.

Very different from this is the graph for the c'' pipe, the note of which was naturally very much weaker. Here the acoustic effect does not supervene and the graph obtained is almost identical with the corresponding graph in

figure 12. Like this, the deflection at 16 cm. of depth dies off to $s = -20$. It was therefore thought well to provide the long probe with a (one-ply) filter-paper sheath, as in figure 8. The resulting graph (the lowest in fig. 13) now lies wholly in the region of dilatations, with only the node reaching up as far as $s = 0$. Nevertheless, this is correctly placed, on a succession of increasing dilatations from mouth to embouchure.

Figure 14 (upper curve) is the graph obtained with a probe about 24 cm. long, but carrying a very fine pin-hole. The c'' pipe here exhibits the general character of figure 13, except that the node is much more accentuated and the internal $s = 0$ at about 18 cm. of depth. This also falls off to $s = -20$ as the sound dies away. At 20 to 24 cm. of depth, s even approaches -50 , in case of the full-blown pipe. The strong f'' necessarily threw the fringes out of the field of view, showing even $s = 70$ scale-parts at the mouth of the tube, $s = 20$ at 2 cm. outside, etc., as in figure 9.

It is, however, quite possible to bring the shrill f'' down to negative displacements, and this may be done by selecting a very fine pin-hole probe. The graph obtained is the lower curve in figure 14, beginning with a compression at the mouth of the pipe. The pressure distribution is peculiar, with a sharp, displaced node at 12 cm. of depth, from which the curve descends almost sheer to negative values. The record is at least a duplicate; for the shrill note in dying out increases the pressures for depths smaller than the middle, roughly speaking, and decreases them for depths beyond this. The arrows point out the trend of the weak-note curve. This seems to retain the node in its middle place, so that the shrill note is much modified by the overtones present.

5. Pin-hole probes in series.—If the pressures on the two sides of the pin-hole are different, one may argue that a series of probes should relay each other; for the inner pressure of any pin-hole would become the outer pressure of the antecedent pin-hole, in succession. A set of pin-holes was therefore prepared, consisting, as suggested in figure 15, of quill tubes s' , each 4 cm. long, provided at one end with the cap-shaped piece of copper foil, cemented in place and punctured. These were joined by short ends of rubber tube, the pin-hole ends pointing in the same direction. A similar set of quill tubes, each 4 cm. long, was also at hand, without obstruction (*i. e.*, clear bore), to be joined end to end for comparison with the pin-holes series $s's''$. The telephone-blown pipe T , sounding $\sharp c''$ (fig. 1), was used as a source of sound, the outer pin-hole being about 4 cm. within the mouth. The fringes were enlarged (3 scale-parts). The pipe s communicating with the U-gage (U , fig. 15) was about 6 cm. long.

The results obtained are shown in figure 16, where little circles indicate the fringe deflections for the clear-bore tubelets in succession (the end tube being necessarily a pin-hole), and black discs the deflections for the pin-hole series. There is practically no difference so far as intensification or increased deflection is concerned, the pin-holes, if anything, acting as a slight obstruction in the tube. Hence one must conclude (so far as I see) that the pressures on the two sides of the intermediate pin-hole are the same, there being no discernible relay effect.

What the curve, figure 16, brings out very strikingly, however, is the periodic distribution of pressure, at least in the initial parts of the tube ss'' , figure 15, connecting the end pin-hole s'' with the gage U . The first node or maximum is here probably within the tube s , so that the distance in tube-length between the first two apparent maxima is only 23 cm. The distance between the next two maxima is 27 cm., and between the minima 29 cm., which is probably an approach to the half wave-length of the pitch near $\sharp c''$. In the former report I investigated the effect of enlarging the free air space of the shank U of the gage. The results, beginning with a mere shell-like space, eventually enlarged to a volume somewhat short of a cubic decimeter, were, however, so complicated in their relations that but little interpretable information was obtained. This is not so with the behavior of the pipe ss'' , and it appears

that if successive $\lambda/2$ are added to the length, but little change of the deflection or sensitivity is to be expected. The minima, though relatively weak, do not here fall off as far as zero.

To further test this result I put pin-holes at two successive nodal points or maxima. The results were:

Pin-hole at end only.....	deflection 100
Pin-hole at second and third node.....	deflections, 100, 90, 110, etc.

for different pin-holes inserted at the second node. In like manner, pin-holes at the first and second node (remainder of the tube removed) gave the same deflection, nearly, as a single pin-hole at the end of the tube. Examined individually at the first node, about the same deflection was obtained from each pin-hole. With the pin-holes at the end of tubes each $\lambda/2$ in length and placed successively end to end, no reduction of deflection showed itself even beyond a meter.

In further experiments it appeared that the periodic effect gradually dies away and is only marked at the beginning. Thus glass quill tubes 150 cm. and 300 cm. in length with a pin-hole at the end did not further reduce the sensitivity.

The endeavor to corroborate the results for the $\#c''$ pipe with an e'' pipe was not so direct. The latter, telephone-blown, developed both an e'' (maximum deflection +60, minimum 10 scale-parts) and a $b'e''$ (minimum deflection -20 scale-parts). It was difficult to follow the periodicity with this interference beyond 25 cm. of length, though much time was given to the work. At the end of a quill tube 150 cm. long the pin-hole probe showed undiminished deflection for both notes. The same was the case with a quill tube 300 cm. long. It is probable that long tubes more easily accommodate different wave-lengths.

The outcome of this work has thus been, in general, that with a quill-tube connection between U-gage and pin-hole of more than 1 or 2 wave-lengths in length the pressures become nearly uniform. It seemed worth while, therefore, to reexamine the long (30-cm.), closed c' pipe (i. e., wind-blown) of figures 1, 2, etc., with the advantages specified, of a quill-tube connector 300 cm. long. The telephone-blown pipe gave a fundamental $b'b$, with a preference for the anomalous overtone f' . The curves obtained are given in figure 17, a being found for the position I and b for the position II of the commutator of the telephone current. In case of the curve c , a new pin-hole probe with quill tube 30 cm. long was added to the 300-cm. connecting quill pipe. When the pitch of the telephone reached f' , the note called out was the first overtone f'' of the closed $b'b$ pipe. The case presented is thus much like figures 1 and 2. But while the above graphs (for short connecting-tubes) showed no effect of reversal of the telephone current, the present curves are thereby completely reversed. Similarly, the curve c for the 330 cm. connector differs merely in pin-hole sensitivity. This result is so surprising that in figure 18 the case of a short connecting-pipe (35 cm.) has been repeated, the open circles corresponding to position II and the black disks to position I

of the switch of the telephone current. The curves (here both of low sensitivity) are practically identical. Thus the telephone is here capable of placing either a condensation or a dilatation at the nodes of the first overtone of the closed organ-pipe; but it can do this only if the length of the quill connecting-tube between U-gage and pin-hole is one or more wave-lengths (here $\lambda=90$ cm.) and not if it is but a fraction of a wave-length (short tube 35 cm.). In the latter case marked pressures only are found at the nodes and the dilatation at the antinodes is slight or wanting. The reason for the potency of the telephone is not hard to surmise; but the coöperation of the connecting-tube in the result is difficult to account for.

Figure 19 is another graph obtained a little later for a length of connecting-pipe of 180 cm. between U-gage and pin-hole. It practically tells the same story as figure 17, with the exception of a more marked tendency of the deflections to become positive, or compressions.

Finally, figure 20, the fundamental bb was now easily produced with the same quill-tube length, 180 cm. The graph was not subject to commutation, all deflections being compressions increasing from the mouth inward as far as the telephone plate. The note being relatively weak, the limiting deflection did not exceed 30 or 40 scale-parts, and irregularities in figure 20 are due to the difficulty of motor control at this low frequency.

As this experiment with pin-holes in series is of considerable importance in its bearing on the pin-hole probe, the experiments were also carried out for a closed region, as shown in figure 21. Here U is one shank of the U-gage, with its quill-tube connection, qq , leading to the telephone T . The connector qq carries a lateral branch made up of a succession of pin-holes $p'p'' \dots p$, about 4 cm. apart, held together by short ends of gum-rubber tubing. The region offered two convenient harmonics, d'' (compression) and $\sharp d''$ (dilatation), very close together, which were tested. The results at d'' were:

Number of pin-holes	1	2	3	4	5	6	7	8
Deflection	20	35	25	27	25	30	30	scale-parts

the last pin-hole being at the end of a tube 24 cm. long. The minimum at d'' was much stronger, showing:

Number of pin-holes....	1	2	3	4	5	6	7	8
Deflection	— 100	— 110	— 105	— 110	— 95	— 95	— 95	— 90 scale-parts

The pin-holes are naturally not quite equally efficient; apart from this the effect produced by 8 pin-holes in series is not very different from that of a single pin-hole, in spite of the very large resistance which must thus be introduced into $p'p$. Here it was interesting to notice that whereas with 1 pin-hole there is some vibration of the mercury columns of the gage, and with them of the fringes, the motion in the case of 5 or more pin-holes is absolutely dead-beat. With 8 pin-holes the full deflection is reached gradually and the return to zero equally so. A series of lateral pin-holes thus offers a means of damping without much loss of sensitivity, which may be advantageous; but the main result of the tests is the non-harmonic character of the action of a series of pin-holes in a lateral branch, the sound-waves proper taking the path $TqqU$, figure 21. Furthermore, the impossibility of amplifying

either the compression or the dilatation characteristic of a single pin-hole by a series of pin-holes is substantiated.

6. Evanescence of (weak) vibration in closed (tubular) regions.—The preceding methods of determining the character of the vibration in the tubes connecting the U -gage with the course of sound did not lead to results as thoroughly consistent throughout as would be desirable. I therefore, at a later date, returned to the original investigation of the case of a closed quill tube t , between the telephonic sound generator T and the gage U , figure 22 (inset). The pin-hole o , attached to a lateral branch-tube b , could be moved from point to point of t , by aid of the short rubber connectors on the sectional lengths in t . These admitted of their replacement in turn by the branch-tube b . Thus the vibratory state throughout the whole length ($t=x+y$) of t was determinable from the acoustic pressure observed at U for a given x or y location of pin-hole.

In the first experiments, t was 52 cm. long. The rather insensitive telephone, actuated by the electric siren, was kept at the pitch a' , while the probe passed from end to end of t . The results are given in the curve figure 22, and show a marked node at $x=32$ cm. from the U -tube or $y=20$ cm. from the

telephone plate. Between the position of this crest (a') along t and the U -gage ($x=32$ cm.) no acoustic pressure on vibration could be detected at any pitch; but between the node and the telephone plate ($y=20$ cm.) a succession of nodes of increasing pitch was determinable, as, for instance, a g'' maximum near 11 or 12 cm. from the plate. This made it seem probable that the air vibration evoked by the telephone plate is here largely confined in the quill tube, within the first quarter wave-length (for a' , $\lambda=76$ cm.) from the plate; and that beyond this, toward the U -gage, there is no vibration, but only an air-content relatively at rest. In other words, that within the quill tube the vibratory air motion received from the telephone is soon damped out. If this surmise should prove correct, it would throw much light on the acoustic behavior of the connector tubes t , as well as the phenomena as a whole. The answer to this question depends on the telephone used and will be treated in this and the following paragraphs.

To investigate the question fully, it is manifestly necessary to search for all the harmonics obtainable for a given position of the pin-hole branch b along the tube t . Results of this kind are given in figure 23, t length 52 cm., at positions $y=44, 39, 29, 20, 13, 8$ cm., respectively, from the telephone plate. In the first case of the lower group ($y=29$) the deflections are all very small, but a few fringes in value and largely negative. Vibration within the tube has nearly ceased. The possible low $\#c'$ maximum ($\lambda/4=30$ cm.), which it would have been interesting to search for, was beyond the range of the electric siren. Its occurrence is improbable. In a later trial, weak response was obtained, not at $\#c'$, however, but at the a below it.

At $y=20$ cm. the very marked maximum near a' , already referred to, is in presence. As the pitch rises, the vibration dies out here very slowly, the acoustic pressure eventually dipping even into negative values. The curve would probably have risen again at a'' , and a detailed investigation would have brought out more secondary sinuosities.

The next curve with the pin-hole probe about $y=13$ cm. from the plate is totally different from the preceding, and probably also contains more secondary serrations than the graph reproduces. Its chief maximum near f'' ($\lambda/4=12$ cm.) is again near a quarter wave-length from the plate. Nevertheless the former a' maximum has not vanished, though it is now nearer g' .

The group of curves in the upper part of figure 23, referring to larger and shorter distances y from the telephone plate, is in the main corroborative. At $y=39, 44$ cm., there is practically no vibration demonstrable at any pitch, excepting the small negative acoustic pressures near e'' .

The curve for $y=8$ cm. from the telephone plate, where the chief crest should lie near c''' ($\lambda/4=8$ cm., out of range), shows accentuation of the secondary crests near $\#d''$ and $\#a'$. This is to be expected from the nearness of the probe to the plate and the irregularity of tube connection there. The c''' maximum, found later, was not strong.

Elongating the quill tube t from 52 cm. to 76 cm. by the insertion of a new section, the group of curves in figure 24 was investigated. These recall the

preceding set for the same y , so that the additional elongation has had little influence. Vibration is absent practically at $y=63, 40$ cm. from the telephone plate, and the weaker maxima begin to appear at about $y=23$ cm.; but the presumptive $\lambda/4=23$ cm. crest, which should be near f' , does not appear. In its place the secondary crests are near a', e'', a'' . The curves for the pin-hole at $y=20$ and $y=13$ cm. from the telephone plate closely resemble the preceding cases (fig. 23) for like distances y and similar remarks apply.

The connector quill tube t was now further elongated from $x+y=76$ to 103 cm., and the curves of figure 25 thus obtained. As usual, there was practically no displacement above a few fringes (usually negative) at $y=36, 65$ cm., etc., from the telephone plate. The excitation is low even at $y=30$ cm. What is quite unexpected, however, is the total change of the type of curve for $y=20$ cm. The former dominant a' crest has been degraded and two new maxima near f' and f'' appear wholly out of keeping with the earlier $\lambda/4$ hypothesis. The curve has merely retained its broad response to all ranges of

pitch between f' and a'' . The curve for $y=13$ cm. is also modified. Its single high crest is still at $\lambda/4=13$ or e'' , but this occurrence must now be considered incidental.

Thus the conditions of vibration within the quill tubes near the telephone are very complicated; but from the above observations as a whole one may be inclined to conclude that the dominant excitation is present only for a distance of about a quarter wave-length from the plate. If the pin-hole probe is inserted within this interval, the U-gage, with a proper selection of pitch, registers corresponding high acoustic pressures. Beyond this distance from the telephone plate, the vibrations in the quill tube are rapidly damped out and the pin-hole probe at these points produces almost no registry of pressure. It follows, therefore, that almost any length of connector tube t (fig. 22) is permissible (a convenience which will be much used below) provided the source of sound is of the kind here treated.

These data also afford a possible clue for the interpretation of results obtained in the preceding report on the air volume of the shank of the U-tube at which the acoustic pressures are measured. This volume could be increased

from a mere crevice of about 50 c. c. to nearly 400 c. c. (the largest available) without essentially modifying the registry of the U-tube. Under the impression that the volume in question partakes of the vibration, such a result seems surprising. If the region of vibration is confined to the other end of the connector tube t , the volume conditions at the U-gage may thus be of secondary consequence.

7. The same continued. Stronger excitation.—The preceding results were obtained with a telephone of rather poor response, which needed relatively strong currents to excite it (about 200 ohms in circuit). It seemed necessary to test the case with a more sensitive instrument, requiring at least 1,000 ohms in the same circuit to keep the fringes within the field. A closer approach to the U-gage, *i. e.*, a shorter quill-tube length $t = x + y$ (fig. 22, inset), where y is the distance of the pin-hole probe from the telephone plate, also suggested itself as an essential supplementary test.

Results with this more active telephone, even when operating under 1,100 ohmic circuit resistance, are recorded in figure 26, where the distance $x = 13$ from pin-hole to U-gage is constant. The lower curves (t small), in comparison with those of the preceding paragraph, not only give evidence of high intensity but are relatively simple. Both point to a crest near a' and another near the octave a'' . The distances of

the pin-hole probe from the telephone plate, $y = 13$ and 20 cm., respectively, recalling the former e'' and g' maxima, are here ineffective. The quarter wave-length of $\#d''$, corresponding to $y = 13$, and the half wave-length of $\flat a''$ for $y = 20$ have no bearing on these results and offer no suggestion. The graphs merely report that both a' and a'' evoke a strong node within 20 cm. of the telephone plate.

The upper curves of figure 26 refer to long quill-tube connectors ($t = 122$, 147 cm.), while the pin-hole remains at the same distance $x = 13$ from the U-gage. Not only has the intensity decreased throughout, but the curves are more complicated. The a' crest is particularly weakened and for $t = 122$ it has been split into an a' and c'' maximum, while for $t = 147$ it is being obliterated. The a'' crest is broadened but retains its location. Thus at points of t far away from the telephonic source, the vibration again tends to be gradually damped out.

In the development of these results two methods were followed; in the first the distance $y = 20$ cm. of probe and telephone plate was kept constant and t successively elongated from $t = 33$ to 100 cm. The graphs so obtained

are given in order in figures 27, 28, 29. These, while evidencing much difference in detail, in their main features are not unlike. In all, the tendency to place a crest near a'' persists, though at $t=58$ it is broadened. To a less extent the

maximum near a' is the rule; at $t=58$ this is divided between ba' and c'' and at $t=100$ between ba' and b' , so that minima intervene for a' . The low pitch crests are, moreover, of much smaller intensity than the a'' crests and at $t=147$ the former seem to be vanishing. The latter fall off somewhat ($s=90$ to 60) as the tube-length t increases from 33 to 100 cm.; but these relations may be and probably are periodic, as is evident from the high value at $t=73$, and particularly at $t=147$.

One may infer in general that the high-pitch crests with the pin-hole at a given distance y

from the telephone plate are not greatly modified by the length x of the remainder of the tube t , while the low-pitch crests tend to vanish (as in fig. 26) when the quill-tube length t increases.

The second method consisted in keeping $t=100$ constant and increasing the probe-telephone distance y , successively. The graphs thus obtained are given in figures 30, 31. These, as would be expected, are far less consistent than the

preceding set, and it is hardly practicable to adequately describe them. In fact, one here explores the type of vibration at all positions y , through the whole range of pitch variation. The upper crests shift from e'' to a'' and the lower from g' to b' , and there are a variety of small maxima, some of which it was thought best to ignore.

In one respect, however, the present graphs are decisive: they completely negative the preceding results of the absence of appreciable vibration in the tube t in the part between somewhat over $\lambda/4$ from the telephone plate and the end of the U-gage. There is now no evidence of a rapid damping-out of vibrations beyond any distance from the plate within the range observed. In other words, if vibration is present at any point of the tube, then the insertion of the pin-hole probe there produces acoustic pressure, provided the point in question is the seat of a node. Thus, in figure 31, the pin-hole at $y=85$ from the telephone plate indicates a very remarkable node of pitch a' , only $x=15$ cm. from the U-gage. The whole content of t throughout its 100 cm. of length is intensely vibrating; for it is not credible that the pin-hole can do more than evoke acoustic pressure, without modifying the preëxisting node appreciably.

One feature of these graphs is perhaps intelligible, even if it does not work out quantitatively in any simple way. The small values of y , figure 30 (*i. e.*, nearness to the plate), are accompanied by the location of the dominant crests in high pitch, whereas the large values of y , figure 31, evoke a high crest (if present) in low values of pitch. In figure 30, $y=20$ would imply a half wave-length of ba'' , and $y=44$ a whole wave-length of bg'' ; if these were allowed to pass, the crest at e'' , for $y=30$, should be a half wave-length of $\#c''$, which is quite out of accord. In figure 31 the crest at a' for $y=85$ comes no nearer than a whole wave-length of g' . Finally, it is hard to understand why, at $y=59$, the complete survey from e' to a'' strikes no dominant node or crest.

8. Excitation outside the organ-pipe. Reflectors. Capsule.—The response of the pin-hole probe falls off sheer beyond the mouth of the pipe, as shown in curve a , figure 9. The endeavor to increase this by the use of reflectors has been but slightly successful. In figure 32, several cases are given. The effect (curve b) of surrounding the pin-hole with a thistle tube (mouth 3 cm. in diameter) vanishes somewhat beyond 10 cm. from the pipe. A glass bell-jar 10 cm. in diameter surrounding the pin-hole (focused) collects (curve c) the sound-rays sufficiently to produce fringe deflections within about 50 cm. between jar and pipe. A pasteboard hemisphere, however, even though 15 cm. in diameter, was scarcely superior as an amplifier to the thistle tube.

Another method of endeavoring to amplify the fringe record of the sound-wave outside of the organ-pipe is given in figure 33. Here R is the usual capsule device of the phonograph, consisting of a metal disk R with an annular ridge aa , less than a millimeter high. To this the very thin mica disk m is carefully cemented at its edges. The center of the disk R is perforated and communicates with the U-gage U by the quill tube qq , provided with a lateral branch carrying the pin-hole o . We here return to the closed region vented at o . Disk m , of thin mica, paper, and other materials, was tried out at length, but with no greater success than the moderate results obtained with concave mirrors. Well within 1 cm. of distance between the mouth-end of the organ-

pipe and the disk m , deflections up to 60 cm. were obtainable; beyond 1 cm. the effect rapidly vanished, particularly with telephone-blown pipes. At a distance of a few millimeters the harmonics could be picked out here without entering the organ-pipe. A variety of other arrangements was tried; for instance, the pin-hole was placed in qq , close to m . The results were about the same. Possibly a shank of small volume (shell-like space) at the U-tube would contribute to increased sensitivity.

9. Continued. Resonator.—Differing from the preceding results, a spherical resonator (f''), surrounding the pin-hole, gave, even with a short quill-tube connector, a record (fig. 32, curve d) more marked than the bell-jar and an evanescence with distance far more gradual.

This device, being thus the most efficient amplifier for sound-waves coming from a distance, deserves careful scrutiny, and further experiments were made with it, using the f'' pipe and bellows and a Helmholtz resonator. The quill tube between the U-gage and pin-hole was elongated to about 300 cm. and the mouth of the resonator placed toward the organ-pipe. To avoid complications from wind currents, this was directed at right angles to the axis of the resonator prolonged, or (with equal efficiency here) longitudinally away from it. Thus no air current blew towards the resonator. The results obtained are summarized in the graph e , figure 32. The curve is more or less sinuous, probably owing, as I originally supposed, to different positions of the pin-hole tube in the mouth (which modifies pitch), to reflection from objects on the table, and to other similar details; but deflection of about 5 scale-parts was obtained even at a meter. Purity of pitch seemed to be more important than loudness of note. Thus, on surrounding the resonator, except on the front side, symmetrically by a bell-jar, 6 inches in diameter, no action at all was perceptible, in spite of the reflection from the inside of the jar. No doubt there is here an interference between the bell-jar pitch and the resonator pitch; or two wave-trains (one deformed by the reflection) cross the resonator in opposite directions and obviate any cumulative effect. It was thought that sinuosities in the graph e could similarly be referred to the interaction of direct and reflected wave-trains. Curiously enough, the f'' resonator responds slightly to a c'' pitch, indicating a common overtone in c^{iv} .

The experiments were repeated with a c'' pipe and a c'' resonator, and this gave substantially the same results, although the c'' note was much less intense than the f'' note. The following is an example of the data obtained:

Distance from pipe to resonator.....	10	20	30	40	50	60	70	cm.
Deflection in scale-parts.....	35	33	30	25	25	25	20	

The organ-pipe blew across or away from the axis of the resonator. By careful adjustment of the pin-hole position in the resonator, deflections were brought up to 40 scale-parts at a distance of 30 cm. The resonator in the axis of a bell-jar reflector again refused to operate. In the present tests the f'' resonator did not respond.

More pronounced results were obtained with an open cylindrical resonator, 21 cm. long and 3 cm. in diameter, blowing f'' nearly. The pin-hole was in the center, the connector 300 cm. long. The presence of some cause producing sinuosities in the graphs was here very obvious in certain positions. Moreover, the largest deflection appeared while the f'' note was dying down, with slight lowering of pitch. Thus at 20 cm. the deflection of 45 scale-parts for the strong note increased to 60 scale-parts for the weak and flatted note, showing the relative importance of pitch. With this resonator it was easy to obtain deflection of 5 scale-parts and over, even at a distance of 2 meters.

Finally, experiments were tried with two pin-hole probes on a T-branch at the end of the 300 cm. connector quill tube. One pin-hole was surrounded with the c'' and the opposite pin-hole with the f'' spherical resonator. With a response to the c'' pipe alone of about 13 scale-parts, the f'' pipe alone gave 18 cm. This is less than half the above intensities for the same distance (20 cm.). Nevertheless, each pipe, separately, is definitely effective in presence of the other. When both pipes sounded together, the response was but 10 scale-parts, less than either alone, though there is some question with regard to the relative intensities of the blown notes when combined. The experiment shows, however, that the overtones in different parts of a region may be successively or simultaneously evoked and recorded and has a bearing on the complicated curves in the earlier report.

CHAPTER II.

ACOUSTIC TOPOGRAPHY IN A ROOM.

10. **Introductory.**—A plan of the room is given in figure 34, where W , W' denote the unbroken walls, I the interferometer and U -gauge, L the electric lantern, A other apparatus. The coördinates along which the surveys are to be made are x , y , z ; y being between walls, x toward the open door, and z above the table T . For more refined work, I , L , A , etc., should have been removed to another room; but for my present purposes this is unnecessary.

The pin-hole probe described in Chapter I has there been found useful for the location of nodes in pipes and other vessels, both telephone and wind blown. These experiments indicated the exceptional sensitivity of the probe

to nodes. It is relatively quite unresponsive to ventral segments or to wave-trains. The pressure variations in question are converted into static pressures through the intervention of the pin-hole and measured at a mercury U -gauge, read by displacement interferometry. As the pipes to be employed were to be of all kinds and intensities, the fringe displacements will again be reported, as measured on an arbitrary scale, s (0.01 cm. collimator-plate micrometer). The width of a fringe was, however, on the average about 2 scale-parts ($s=2$). Thus the corresponding pressure increments p are readily found from $p=0.00015 s$ millimeters of mercury. The apparatus can be made more sensitive by enlarging these small fringes; but these would then usually be thrown out of the field of the telescope and the screw micrometer become necessary to restore them, which is irksome, particularly with fringe U -gages.

The pin-hole probe at the end of an eighth-inch pure-rubber tube of any length (2 to 3 meters or more), at the other end of which is the U -gauge, is at once available for introduction anywhere. It fails, however, to give an

appreciable acoustic record except in the inside sounding-pipes. The plan of associating the pin-hole probe with a resonator, of either the open or closed type, thus suggests itself, and a convenient form of the open pin-hole resonator is shown in fig. 35.

11. **Pipe axis varying in altitude and azimuth.**—The work with resonators was at first very confusing because of the lack of consistency in the results obtained on different days. Thus the old cylindrical resonator *A*, at long distances, sometimes gave no fringe displacement above 1 or 2 scale-parts; at other times 15 scale-parts at 130 cm. and 10 scale-parts at 200 cm. were the rule, or even more than this (15 scale-parts at 200 cm.). Three open cylindrical f'' resonators of different diameters were therefore provided and tuned on the interferometer by comparison with the f'' pipe at 50 cm. The initial results were:

	Length.	Diameter.	Deflection (scale-parts).
	<i>cm.</i>	<i>cm.</i>	
<i>A</i>	21.4	3.1	$s=23$
<i>B</i>	21.7	4	$s=13$
<i>C</i>	23	2.1	$s=2$

This tuning by elongation must be carefully done to at least 1 mm. in length; otherwise the response dwindles rapidly in distance. The adjustment must be made, moreover, not by blowing the resonator, but by the interferometer test for resonance at a small distance (50 cm.). The final result was that *C* overtook *A* in sensitivity, whereas *B* showed little improvement. In the tests at $d=200$ cm. between f'' pipe and resonator, the axis of the latter was placed across the line of junction for greater symmetry. The position of the pin-hole was in the middle of the resonator. With distance, $d=200$ cm.

A, $s=15$ scale-parts *B*, $s=10$ scale-parts *C*, $s=20$ scale-parts

These results for s did not greatly diminish, even at $d=300$ cm., so much depending on the pitch of the f'' organ-pipe, whether awakened by a forced or by a dying air current. In the latter case, with the gradual depression of pitch, however small, the deflections usually fall off in steps, showing definite response to intermediate harmonics. It made little difference here, moreover, whether the quill-tube connection of pin-hole probe and U-gage was long (300 cm.) or short (20 cm.), so that the short connector was used for convenience, a quill-tube elbow putting the pin-hole into the middle of the resonator. Hourglass-shaped resonators (open double cone with pin-hole at the gorge), from which I expected increased sensitivity, were failures throughout; but as the resonator of small diameter (*C*), after tuning, was more efficient than either *A* or *B* of larger diameter, *C* was preferred for the following experiments. It (*RR*, fig. 35) was later provided with a central lateral tubulure, securing the pin-hole probe q_0 , without leakage.

I had frequently noticed, when the organ-pipe at a distance (200 cm.) gave a marked fringe deflection, that practically no response was obtained on moving the organ-pipe to a new position only 1 or 2 feet away on the same radius and with no change in the orientation of the pipe. The resonator was therefore now in a region which, on a much larger scale, would be called a sound shadow, though this term is here inapt and misleading. I shall call them null regions. The room (fig. 34) in which the experiments were made was an alcove about 5 meters high. The effect produced at the resonator would thus have to be an aggregate of the multiple reflections reaching it from all directions.

An early method of exhibiting the distributions in question consisted in the rotation of the f'' organ-pipe, on a long radius, through its axis in a plane either normal and symmetrical to the axis of the resonator or in a plane through the axis. The installation is shown in elevation in figure 36, where U is the closed shank of the U-tube, R the open cylindrical resonator seen end on, o the pin-hole on the quill tube, q , p the organ-pipe in its first position (90° , 180° , 270° following), a the axle and wind-supply device. The distance Ra was 120 cm., Rm' 95 cm., RE' 113 cm., RE 125 cm., Rm 143 cm. The pipe is from 17 to 18 cm. long from mouth m to embouchure E . Thus the mouth is nearly half a meter nearer R in the position 180° than in the position 0° , and yet the effect on the resonator is here much stronger for the altitude 0° than for the nearer altitude 180° , as indicated by the graph, figure 37, where little circles indicate the results for rotation in a vertical plane (fig. 36) and black disks the rotation in a horizontal plane. The mean distances in centimeters of pipe and R are marked d .

Owing to the change of pitch with the loudness of the organ-pipe, it is difficult to get a smooth curve; but the marked effect at R of the azimuth or altitude of the pipe is none the less evident. I was at first inclined to look for interferences between the wave-trains issuing at the mouth and at the embouchure of the pipe, 17.5 cm. apart. The wave-length of the f'' pipe is about 48 cm., however, and a change of path difference from the 0° to the 180° orientation, for instance, effective at the ends of the resonator, seems unlikely. These path differences are practically identical.

12. Continued. Pipe acting from different distances.—Further information as to the active and null regions in the room may be obtained by changing the distance between the fixed resonator R and organ-pipe, for a definite orientation of the latter in altitude. To do this, I attached a long board to the table, with the distance for R marked upon it, and then successively slid the pipe, held in a standard about 30 cm. above the board on the average, from position to position. These distances trend ($+x$ direction) toward the open side of the room. Beginning with the 0° (pipe-mouth away from R) for the pipe-axis and using a note of mean loudness, the curve 0° given in figure 38 was obtained. The curve is markedly sinuous and there is a null region at R when the pipe is about 160 cm. away from it. Rotation of the pipe on its own axis, except when very near R , did not in general make any difference.

In contrast with this, the survey for the pipe altitude 180° (mouth towards the resonator R) is a totally different curve. Here R is in a null region when the pipe is at a distance of about 100 cm., and the sinuosities are differently placed. Maxima and minima in the 0° and 180° graphs are almost inversions of each other. There are greater fringe deflections at the beginning, as the pipe is much nearer the resonator. Rotating the pipe on its axis here, the wind from the embouchure may produce complications. Similarly, figure 39 gives the graphs for pipe altitudes of 90° (mouth up) and 270° (mouth down). The curves are again markedly different from each other and from the curves of figure 38. In the 270° curve the pipe-mouth is relatively much nearer to the resonator than in the case of the 90° curve for the same mean distance, and hence the former shows much greater fringe deflections here.

In fact, at the mean distance of 15 cm. the fringes are thrown out of the field of view. There are no null regions at R , except when the 90° orientation is too far removed for discrimination (240 cm.); but it is curious that the fringe deflection is greater at 160 cm. than at 60 cm. In case of the 270° orientation, I tested both a weak, but full, and a strong forced pipe note, the data being distinguished in the figure by W and S . There is here relatively little difference, possibly because the mouth of the pipe is near the table, but one may notice that W and S are reversed beyond 200 cm. In the other orientations, however, the difference is often of great moment.

Accordingly, in figure 40 I repeated the experiments with slightly different altitudes (pipe-axis prolonged passing through R), for intermediate points d and for longer distances. In case of the orientation 180° the graph is practically the same as before; but for 0° and within 150 cm., a soft note being aimed at, it is quite different. The reason for this (as the sequel will show)

is to be referred chiefly to an insufficient appreciation, in this preliminary work, of the importance of small lateral differences of location for the same radial distance. In addition to this is the uncontrollable annoyance of slight but effective changes of organ-pipe pitch. There are now two null points, the old one at 160 cm. and a new one at about 250 cm.

The effect of soft (*W*) and loud (*S*) notes was specially studied in the 90° orientation (fig. 41). The *W* graph is of the same nature as the preceding curve (fig. 39). The *S* graph, however, differs from it materially, and here again there is an inversion between $d=90$ cm. and 100 cm. from *R*.

The relations met with in these cases are thus complicated by the overtones of the pipe and inadequate sharpness of the resonator. Overtones are usually evoked by an impulsive blast at the beginning. If the identical intensity of air current is gradually reached, no overtone is awakened. This phenomena was frequently noticed in the course of these experiments, a large fringe deflection persisting after an abrupt increase of the intensity of blowing, whereas a gradual growth to the same intensity produced but little deflection. Again, the change of pitch accompanying diminished intensity of blowing is immediately felt by the resonator, which often responds more strongly to the weaker note. In spite of all these misgivings, there is no doubt that the curves reproduce, though as yet crudely, because the steps in distance are too long throughout, actual conditions in the room, which itself endeavors to vibrate, possibly like a large organ-pipe, under the excitation of the small pipe within. Overtones are awakened in every special case with distinct relations to the room, depending on the position of the exciting pipe. Such distributions would necessarily be fluctuating, or very mobile in position, with small differences at the pipe, resembling the behavior of the stratified column of a cathode tube. In fact, the organ-pipe itself holds a compressed bundle of stream-lines within, as it were, very much like an electric coil grasps its magnetic lines of force. They escape at once, as seen in the graph *a* on the left of figure 9, diverging rapidly into space as soon as this confinement is relaxed. Such analogies must be used cautiously, however, since the sensitiveness of the pin-hole probe depends essentially on the proximity of nodes. It will not register a simple wave-train in free air.

13. The closed pin-hole resonator.—In the preceding paragraph the outstanding trouble encountered was ascribed to the organ-pipe, *i. e.*, to the continuity of notes lying very close together, but to only one of which the resonator responds effectively. A further difficulty is referable to two ends of the open cylindrical resonator (fig. 35), with a pin-hole in the middle within; for these ends, being 22 cm. apart, are liable to lie in regions differing acoustically. The advantage of the tube is the ease with which it may be accurately tuned by mere elongation, and hence its sensitiveness. The Helmholtz resonator, probably for this reason, was found much less responsive.

Hence the closed pin-hole resonator qR , figure 42, consisting of the cylindrical tube (1.9 cm. diam., effectively 22 cm. long) R , closed by the snugly fitting cork c , which carries the pin-hole probe qo , is preferable. The pin-hole o is at the base of the tube R , the quill tube q being connected by a length of gum-rubber tubing (2 meters) with the U-gage. This resonator has but one mouth and thus tests acoustically a single point, as it were, of the region, while the tuning may be effected with nicety by moving the cork c within R , or by elongation at the mouth of R .

The endeavor to increase the sensitivity of the closed resonator, by placing other resonators judiciously near it, met with no marked success. If the distance between the organ-pipe and the closed resonator is partly bridged by an open resonator, the deflection may sometimes be increased 20 or 30 per cent, but this would be anticipated. In all other azimuths the open resonator is liable to reduce the deflection even as much as 50 per cent. Indeed, at a distance of 10 cm. between the nearer mouths of the resonators, the reducing effect was still appreciable. Under other circumstances, difficult to state, a similar small increase of sensitivity may occur; but the expectation of relaying the closed pin-hole resonator by a sequence of auxiliary resonators adjusted with regard to it met with failures only.

I may here also advert to the difficulty of constructing a sensitive pin-hole resonator, either of the closed or open cylindrical type, simple as the device appears. Most of them give no deflection whatever, even when tested with a variety of pin-holes and when carefully tuned. There seems to be some relation between the pin-hole probe and the resonator very difficult to ascertain. Long and patient trials with all manner of devices netted me but one closed and one open resonator adequately sensitive; and these were obtained by mere accidental combinations of pin-holes and a variety of tuned cylinders. The endeavor to replace the pin-hole by a porous diaphragm (filter-paper) was in vain. The paper must be pricked and then not more than once. Elsewhere I have mentioned the disturbing effect of small objects on the table near the resonator. Even the clamps and standards should be as distant as possible. Finally, a given initially good resonator (metal foil) in the lapse of time often seems to lose its virtue. A reduction of sensitivity to one-fourth or one-fifth of the original value was met with. The whole subject will be systematically treated below, §§ 34, 35.

In a series of experiments made somewhat later, I met with better success by using conical glass tubes. Carefully grinding off the end, one eventually arrives at an optimum, the hole being somewhat larger than used in the experiments heretofore. It was further brought out that for pressures the pin-hole must be salient, *i. e.*, at the point of the tube. The same size of hole (or two or more holes) at a blunt or reëntrant end of a tube gives scarcely any deflection whatever. If the pin-hole is carried backward from the bottom of the closed resonator by a capillary tube the deflection grows rapidly less with the length of the tube.

14. **Transverse (y) acoustic survey.**—The preceding survey, made in the direction RP , figures 34, 42, toward the open end of the room, by moving the organ-pipe, may be referred to as longitudinal in the x direction. The present survey, made by moving the resonator R , figure 42, parallel to itself, is thus transverse (y direction), and it may be further desirable to test the distributions along a vertical (z) through R , all for a fixed position of the organ-pipe. This plan of attack, by moving the sensitive small resonator on a long rubber tube (2 meters long, $\frac{1}{8}$ -inch bore), is in many respects preferable, so that it may be additionally necessary to repeat the longitudinal survey by moving R for a fixed pipe and to rotate the axis of R relatively to the coördinates chosen. The work involving so many permutations is inevitably confusing, and small insets showing the position of pipe and resonator relative to each other will be found attached to each curve. The rotation for altitudes is that of figure 36, 0° referring to a trend toward the open end of the room. In all the present cases the pipe was at $x=0$, $y=0$, and at $z=40$ cm. above the table, as this was sufficiently remote to prevent direct air-current effects, even at $x=0$ and $y=0$. Negatives x and y were practically inaccessible because of apparatus,

and negative z is below the table. Guided by the above experience, it is necessary to make the observation in successive small steps of $\Delta y = 10$ cm. Accordingly, the resonator was successively displaced from $y=0$ to $y=170$ cm. in decimeters. In the graphs it was found convenient to introduce the oblique distance $d = \sqrt{y^2 + 40^2}$ between centers of organ-pipe and resonator; but the coördinate value (y) is given in small numerals in *decimeters*, on the graphs, at each point. In all these cases the pipe is oriented at 180° in altitude, *i. e.*, the mouth pointing from the door of the room. The wall is at $y=174$ cm. from the pipe.

In the first series, figure 43, the exploring resonator is in azimuth 0° . The graph is sinuous, neatly outlined, and consistent. The horizontal wave-lengths from maxima are $\Delta y = 40-0$; $80-40$; $2(100-80)$ cm., or about 40 cm.; from minima, $\Delta y = 65-30$; $100-65$, or 35 cm. The pipe λ is, however, 48 cm.; and although the two former are less than the latter (pipe 40 cm. high), they bear no immediate relation to it. For it is difficult to account for the approximate constancy of the λ found, unless there is stationary wave-production by virtue of reflection from the $+y$ wall. In such a case the organ-pipe at $y=0$, $z=40$ cm. and its image at $y=350$ cm. and $z=40$ cm. behind the wall coöperate, sending out wave-lengths of 48 cm. and nodes about 24 cm. apart in

the track of the resonator, and it will presently be argued that nodes probably coincide with the crests or maxima of the graphs. It seems advisable, however, to postpone this discussion until a larger number of relevant data have been gathered. A contribution which immediately presents itself is given by the rotation of the resonator in azimuth, at a fixed position along y . The results obtained in this way for rotations in steps of 45° are shown in figure 44, the upper curve referring to the mean position of R at $y=0$ and the lower to the position $y=50$ cm. The two graphs are opposite in phase, though the curve for $y=0$ is not as striking as would be anticipated from the nearness of R to P . At $y=50$ cm., however, the azimuth 90° corresponds to a pronounced minimum and the azimuth 270° to an equally pronounced maximum. Consequently, in making a further survey in y , these two azimuths, when the centers of P and R are coördinates, should be chosen for contrasts.

The two series of results (pipe in azimuth 180° at $y=0$ and resonator in azimuth 90° and 270° , respectively, with its center at y) are given in figure 45. The curves are pronouncedly harmonic from $y=0$ to the wall, and they are almost exact inversions of each other, crests in the one taking the place of troughs in the other throughout. Moreover, the crests seem to lie in successive levels: an initial high one ($y=10$ to 20 cm.); an intermediate lower one ($y=30$ – 80 cm.); a still lower one (100 to 140 cm.) beyond, etc. It is not feasible to go much beyond 160 cm., for with the wall at 174 cm. the resonator pitch is beginning to be modified. A curious result is the highly developed initial maximum and minimum, the position being not at the nearer $y=0$, but beyond, at $y=20$ cm. The horizontal wave-lengths Δy to be obtained roughly from these curves are successively:

<i>R</i> -azimuth, 90° :		
Crests.....	$\Delta y=35, 31, 38, 31, \text{ cm.}$	
Troughs.....	$31, 41, 30, 35, \text{ cm.}$	Mean $\Delta y=34 \text{ cm.}$
<i>R</i> -azimuth, 270° :		
Crests.....	$\Delta y=30, 33, 37, 34, \text{ cm.}$	
Troughs.....	$35, 29, 41, 35, \text{ cm.}$	Mean $\Delta y=35.5 \text{ cm.}$

The precipitous descent of the graphs between $y=70$ and 90 in one case and $y=90$ to 100 in the other makes less impression than would have been anticipated, and the mean wave-length here, $\lambda=35$ cm., does not differ from the preceding case of $\lambda=37$ cm. by more than the observations of a single curve. The graphs, figure 45, were obtained independently, one after the other.

If, instead of the y coördinate, the distance d_y between the centers of resonator and pipe be taken, the data are as follows:

	Maxima						Minima					
Resonator, $90^\circ \dots$	40	53	77	112	140	170	45	65	99	126	157	cm.
$\Delta d =$		13	24	35	28	30		20	34	27	31	cm.
Resonator, $270^\circ \dots$	..	45	64	93	126	156	..	53	75	114	146	cm.
$\Delta d =$			19	29	23	30			23	38	32	cm.

The first Δd is always small as compared with the remainder. Excluding it temporarily, the means are 29 and 26 for maxima and 30 and 33 for minima. There is no effect discernible, attributable to the position of the mouth, which

is in excess of y at 90° and the reverse for 270° . The mean interval is about 30 cm., which is nearer the semiwave length $\lambda/2=24$ than the preceding case; but it is still much too large, even if the uncertainty of measuring relatively to the bulky pipe be considered. However, it contains a suggestion that the d 's may be more closely associated with the interpretation than the y 's, seeing that one is here virtually dividing the table into half-period elements.

Now, in the line PP' , 40 cm. above the table from pipe to pipe image, the nodes should follow each other at a distance of $\lambda/2=24$ cm. apart; but as the distribution above and below is hyperbolic, the nodes at the level of the table must be farther apart. Unfortunately, the equation is cumbersome; for if r is the radius vector from the source to a node at the table (40 cm. down) and $r'=r+n\lambda/2$, the corresponding radius vector from the image to the same node, and if $z=40$ cm., $\lambda=48$ cm., $\sqrt{(r+n\lambda/2)^2-z^2}+\sqrt{r^2-z^2}=350$, the distance from source to image. Knowing r' and r for two consecutive values, $n+1$ and n , the distance apart of corresponding nodes is found by projecting the two values of r or r' on the line PP' . Without attempting to carry this out here, it is easily seen that the increased distance so obtained is inadequate; i. e., not as large as the 35 cm. and over found in the experiments, in place of $\lambda/2=24$ cm. estimated to increase to 27 cm. or 29 cm. One should expect 7 or 8 nodes in place of the 5 or 6 recognized. It seems unlikely that a frequency of a near order can be contributed by the room itself, and the reflected waves, returning from 2 meters, would seem to be too weak to compete effectively with the outgoing wave-trains. Furthermore there is another wall (fig. 34) in the direction of negative y (at $y=-130$) and one at $-x=190$ cm.; and although instruments lie in the paths here, they only introduce further complication. However, the apparent result that the room, stimulated between walls by the strong pipe-note f'' , responds with a stationary wave-train approximately in the pitch of hb' is of course out of the question.

There remains the reflection from the table, so that the pipe and its image 40 cm. below the table make the plane of the latter a region of reinforcement, or, with regard to the loss of $\lambda/2$ at the surface, a locus of nodes, though the term node would be strictly applicable only for the case of normal incidence near the line-pipe image. The next nodal locus is hyperbolic and therefore higher than 25 cm. above the table, out of reach of the resonator, which lies on it. Thus it is finally necessary to compound the phase reversed direct ray here in question with the corresponding phase reversed reflections from the walls, to get the disturbance at any point of the table. This succeeds experimentally, as I will point out presently, for walls close at hand (within a meter or two); but for walls as distant as those of the room, the reflection-wall effect is too small to account for variations as marked as those of figure 45. It is possible that a wide high wall, like W , may act obliquely by diffraction; but to speak on this subject, further inquiry will be needed. As a general fact, however, it is noticeable that a survey in y , between walls, produces a much more marked harmonic distribution of acoustic pressure along that axis than a similar survey along x toward the open door.

The striking opposition of phase which figure 45 presents for an inversion of the resonator on its middle point as an axis is perhaps more easily intelligible. As the length of the resonator is approximately $\lambda/4$, the rotation of 180° about its center will pass the mouth from a node to a loop of the stationary wave-train, or between corresponding 90° phase differences. Now, the pin-hole probe is sensitive to nodes (compressions) only and scarcely responds to wave-trains (or to the similar harmonic motion at the loops). The pin-hole resonator, however, might be thought to have the opposed quality, being stimulated by wave-trains (or loops) and probably to be insensitive to nodal phenomena or compressions. But (§ 16) this inference is not correct. We must therefore anticipate nodes at the maxima of the graph and loops at the minima, when the pin-hole resonator is used. In the same way null regions otherwise obtained are probably of the nature of ventral segments. The pin-hole alone, being far less sensitive, will usually give no record, except near the nodes within pipes. It follows from this that if half of the length of the resonator be added to the y coördinate of one of the graphs of figure 45, and half the same length be deducted from the other, *i. e.*, if the mouth of the closed resonator be taken to define the y coördinate, the two curves of figure 45 will be found to coincide at their mean position in y . Hence, if the resonator is rotated in azimuth on a vertical axis passing *through its mouth*, the data obtained should be independent of azimuth. The experiments which follow will bear this out.

15. Further longitudinal (x) surveys.—For further elucidation (figs. 46, 47), I carried out four surveys along the positive x -axis, by moving the resonator along the table, in steps of 10 cm. The f'' pipe at $x=0$ was, as before, 40 cm. above the table and kept in azimuth of 180° . The curves refer to the azimuths 0° , 90° , 180° , 270° , and to the x -coördinate of the axis through the middle point of the resonator. The conditions are here very different from the preceding case, as x points toward the open door and there is no continuous wall near at hand. So far as the curves go (within a meter), only the azimuths 0° and 180° give promise of periodicity, and this is slight. The horizontal wave-lengths, so far as discernible, do not exceed 30 cm. Thus the region of positive x is less favorable to stationary wave phenomena.

The attempt was made to complete the subject by a survey in the direction of negative x (fig. 48); for this also is shut off by a wall at $x = -180$ cm. The pipe standing as above ($z=40$ cm.), the resonator ($z=0$) was oriented at 0° . Unfortunately, the interferometer apparatus (see fig. 34) was located on the line between 0 and 50 cm., so that the pipe-note was diffracted around and the resonator moved one side of it. The graph, figure 48, shows a mere dwindling of intensity here; but it is interesting to observe that as the wall is approached there is increased indication of periodicity. A mean horizontal wave-length of about 35 cm. may be made out. The inset in figure 48 gives the usual effect of rotating the pin-hole resonator at $x = -110$ or $d = 117$ cm. for a somewhat different mounting.

Figures 49, 50, 51 are results obtained by moving the pipe ($z=40$ cm.) in the direction of positive x , for different azimuths of the pipe and resonator fixed at the origin. The steps in the two former are too large, and the curves merely indicate totally different distributions. This was remedied in figure 51 for different azimuths of the resonator ($z=0$). The graphs, nevertheless, are still largely decay curves. The case of $R\ 90^\circ$ and $P\ 180^\circ$ shows a null region at the origin, when the pipe is at $d=80$ cm. or $x=70$ cm., and the promise of

periodicity beyond. The orientations of figure 52 with the pipe ($z=40$ cm.) at 270° moving in x and the resonator ($z=0$) at the origin, at 90° and 270° in azimuth, seemed to be better adapted for a detailed survey. The case for $R\ 90^\circ$ is now a compound harmonic, which seems to dip into stationary wave-trains at a and b . The other curve is quite similar in general character, only less pronounced. Horizontal wave-lengths exceeding 35 cm. may be detected at the double inflections. I had hoped from the two positions of the resonator to get something akin to the open-pipe resonator charts (above, fig. 39, $P\ 270^\circ$), but the latter is insufficiently detailed.

To contrast with the above data for the closed resonator, I also made a survey with the open resonator, both in the $0^\circ \dots 180^\circ$ and the $90^\circ \dots 270^\circ$ positions. The graphs obtained (here omitted) were largely sinuous decay curves with few marked features, the former case ($0^\circ \dots 180^\circ$) being particularly even. This is what would have been expected, since the open resonator may be regarded as comprising two closed resonators, with a common

nodal plane in the middle. Figure 45 would therefore lead us to anticipate partial neutralizations at least at remote positions in y , since the mouths of the resonator are $\lambda/2$ apart.

16. Survey in the vertical direction (z).—This was carried out by allowing the resonator to rest horizontally on the table, in a direction normal to the f'' pipe, the latter being raised successively in steps of 10 cm., keeping it in the same azimuth of 0° .

With the resonator at the origin and the f'' pipe at $x=30$ cm., figure 53, the curves are smooth and low, showing little detail, for the two positions of the resonator tested. It is noteworthy, however, that maxima appear at $z=40$ cm. above the table rather than at the smaller z values.

The resonator was now also brought to $x=130$ cm. vertically below the pipe (figs. 54, 55). The first graph (fig. 54) shows very definitely that after $z=30$ cm. there is unmistakable periodicity, with a wave-length of not much larger than 20 cm. Here is therefore a new result, referable, no doubt, to the stationary waves produced by reflection upward from the table. For this reason it seemed desirable to repeat the work with a new installation admitting of larger displacements in z . The graphs (fig. 55) for two positions of the resonator are essentially identical and nearly so with figure 54. The z distance between crests and troughs, however, now varies between 24 and 25 cm., and thus corresponds very closely to the semiwave-length, 24 cm., of the f'' pipe. In all cases the pipe must be raised some distance (40 cm.) before the periodic distributions begin.

The behavior here in evidence is very much like Melde's experiment, though it is now made with a string of air (as it were) between the actuating organ-pipe as one attachment and the table as the other. The only adjustment possible is thus the length of the string. Since the resonator lies on the table, certainly to be regarded as a nodal surface, we look for the maximum of wave-production when the direct and return wave-trains coincide in phase at the mouth of the pipe. This will take place at intervals of $\lambda/2$, or $\Delta z=24$ cm. apart, conformably with the graphs. It would seem, however, that the maxima (in view of the loss of $\lambda/2$ at the table) should lie at $z=5\lambda/4$, $7\lambda/4$, etc., whereas in the graphs they lie at $2\lambda/2$, $3\lambda/2$, etc. The latter put a node at the mouth of the f'' pipe. This result is very important.

The x position of the vertical line of pipe and resonator is also effective. At $x=0$, for instance, the deflections were nearly uniform between $z=30$ and $z=115$ cm., varying but from 10 to 15 scale-parts, with a maximum near $z=60$ cm. For this reason a survey in z at different positions, $x=50$ and $x=80$ cm. (fig. 56), did not lead to additional results, the periodic curve being uncertain beyond $z=70$ cm.

Similarly,* with the pipe kept at the table ($z=0$) and the resonator raised vertically above it, the sinuous decay curves obtained (fig. 57) were without

* There is something about the pin-hole resonator which causes it to function with different sensitivity on different days, without any other interference whatever. The reason for this I have not been fully able to detect. It may be associated with the difficulty of constructing the resonator, to which I have already referred, but the contact breaker of the electric siren is more liable to be at fault.

novelty, except so far as they furnish strong corroborative evidence bearing on figures 54 and 55.

As the table is certainly a nodal surface, we here encounter the unexpected result of special sensitiveness to nodes on the part of the mouth of the pin-hole resonator. The opposition of phases instanced in figure 45 for a reversal of this resonator would have suggested the contrary opinion. The case is carefully tested in figure 58, where the pipe in azimuth 0° is $z=90$ cm. above the table, and the resonator vertically below the pipe is raised from the table at $z=0$. The evidence given by the curve is very satisfactory, the wave-lengths of the graph being 24 to 25 cm., or semiwave-lengths of the f'' pipe. There is complete absence of deflection at 10 to 12 cm. above the table, *i. e.*, at $\lambda/4$ for the pipe, so that the ventral segment is inactive. As the pipe at $z=90$ cm.

is approached by the resonator, the deflections naturally increase, but they do so very slowly. Obviously the present disposition with a raised pipe and with the resonator between pipe and table is the best of the methods tried for the z survey. Because of the important evidence obtained, I repeated it for an f'' pipe at $z=70$ cm. (nearly $3\lambda/2$). The results in figure 59 are of the same kind as to wave-length, inactivity for a resonator 12 cm. ($\lambda/4$) above the table, the marked effectiveness (maximum) of the distant node at the table ($z=0$) and a maximum near the pipe ($z=68$ cm.). Troughs and crests lie at positions which are multiples of $z=12$ cm. ($\lambda/4$).

The above results for normal reflection may be summarized as follows: Both the organ-pipe and the pin-hole resonator are stimulated in proportion as their mouths lie in a nodal region or surface; they remain relatively uninfluenced by a ventral segment. Consequently, an even number of half wave-

lengths lie between pipe and resonator when the response is a maximum. Although the mouths of the respective pipes are necessarily ventral segments, the anomalous features of these results disappear when it is remembered that the stimulating nodes are alternately dense and rare.

Conformably with the above, one might infer that in case of oblique reflections also, the highest crests will appear when the reflected note is returned in phase to the pipe; but this does not seem to be the case.

It remains to rotate the pipe in altitude (vertical x plane) on an axis passing through the node, $z=40$ cm., above the table on which the resonator lies (in azimuth 270°), below the pipe and normal to it. The graph is given in figure 60 and shows maxima for the horizontal positions and minima for the vertical positions of the pipe. Seeing that the mouth of the pipe is much nearer the resonator when the pipe points down, this result is striking.

17. Closed organ-pipe as a source.—This has the advantage of a single mouth, like the closed resonator. The endeavor to blow such a pipe with a telephone, as heretofore, was unsatisfactory. The note is much too fluctuating in pitch and, being richer in harmonics, is ill suited to the closed resonator. On rotating either pipe or resonator, the deflection depends on the distance between their mouths: at 10 cm. the deflection was but 22 scale-parts; at 3 cm. this increased to about 35 scale-parts, so that the pipe pointing downward gives the largest results, whereas if it points upward the deflection is scarcely 5 scale-parts.

In view of these complications, a loud, closed organ-pipe, to be blown by the bellows, was provided, constructed in plan and elevation as in figures 61, 62, where P is a thin brass pipe about 1 inch in diameter and 6 inches long, closed below with an oiled stopper S , for tuning. It is blown by the pipe-blower bwl , described in the last report (Carnegie Inst. Wash. Pub. No. 310, §§ 45, 46), where t is a $\frac{3}{8}$ -inch thin copper pipe, pinched down to a fine wedge at w . The current of wind b from a bellows produces at w a thin lamina of air, which strikes the sharp metallic lamina l , adjustable on the slide a . The embouchure O is thus adjustable. By setting $twla$ at an angle of 20° to 30° , an exceptionally loud note is producible, because of the large aperture of escape; but as the pitch decreases rapidly with the angle, the blower must be rigidly fastened. This is done by the collar cc of strip metal, tightened by the screw s . The bent ends of the strip grip the tube t as shown and are appressed by the screw s' , the edge of the wedge w reposing on the edge of the pipe. The loud pure tones are easily tuned.

The deflections obtained with the new pipe were relatively large, for rotations of the pipe at a mean distance of $z=20$ cm. apart threw the fringes completely out of the field of view, except for the azimuth 270° of the resonator, when the pipe-mouth was down (minimum distance between mouths, 10 cm.). This available azimuth of the resonator was therefore retained and the pipe vertically over the resonator rotated in altitude (in the vertical x -plane), with

the results shown in figure 63. These data (maximum for mouth down and minimum for mouth up) seem to be largely referable to the distance between mouths of pipe and resonator. Figure 64 gives the results for rotating the resonator in azimuth with the pipe horizontal. Here, also, distance is the main feature; but the data suggest the suitable adjustment for an extended probing along the y coordinate (fig. 34), measuring distances between centers of resonator and pipe.

The first survey in y , figure 65, was made by keeping the pipe horizontal (in altitude 180°) at a mean distance of $z=20$ cm. above the table on which the resonator lay. The two graphs obtained within about $\Delta y=160$ cm. (the wall being at 174 cm.) refer to the azimuths 90° and 270° of the resonator, as in figure 45. Figure 65, however, is laid off in terms of y . If figure 45 be also drawn in terms of y , the figures, as a whole, are not dissimilar. There is much less detail in figure 65, which may perhaps be attributed to a louder pipe and a less sensitive resonator. We observe the same initial high plateau terminat-

ing beyond $z=20$ cm., an intermediate plateau still quite high extending beyond $y=70$ cm. (in figure 45 beyond 80 cm.), and a final lower plateau. Figure 65 is somewhat shifted throughout compared with figure 45 and presents no approximately definite wave-length in the compound graphs, which nevertheless tend to be inversions of each other. The mean elevation, $z=20$ cm., of the pipe in figure 65 is too small, being but half that in figure 45, so that larger z values must be tried when other pipe-altitude angles will also be available.

18. The same continued. Pipe at $z=40, 60$ cm., etc.—In the preceding experiments the rays of sound strike the table at large angles of incidence, in the main almost at glancing angles. It is, therefore, desirable to raise the pipe successively, keeping it in the same orientation as before (horizontal, azimuth 180°) and retaining the two positions (azimuth 90° and 180°) normal to the pipe of the resonator on the table. Distances are still measured between centers.

The effect of rotating the resonator is given in figure 66 for the horizontal pipe at $z=40$ cm., in one case blowing up, in the other down. There is very

little difference (the down blow would naturally be a little stronger from reflection and greater nearness), so that the rotation of the pipe on its own axis through the mouth is not of consequence. It is noticeable, however, that maximum deflections occur when the resonator is in 180° of azimuth, *i. e.*, parallel to the pipe and in the same sense, with a minimum for the opposite case. In the preceding paragraph this orientation threw the fringes out of the field, so that the 90° and 270° cases are here also retained for comparison.

Figure 67 shows the results of rotation of the resonator for a vertical pipe (mean $z=40$ cm.) with the mouth downward. Here the maxima and minima occur at a resonator azimuth of 90° and 180° , respectively. Rotating the pipe in its own axis is of no consequence, as the two curves indicate. They furthermore suggest 90° and 180° azimuths as advantageous for the z survey.

Tests were also made by rotating the pipe (mean $z=40$) in a vertical plane through x , for different azimuths of the resonator, below it on the table. Figure 68 is an example of results, complicated in this case by the different distances between mouths during rotation. The maxima and minima in the upper curve lie between the quadrants, and there are large variations which contrast strongly with the small differences in the lower curve with reversed resonator.

The results of the survey with the pipe horizontal (azimuth 180°) in height $z=40$ cm. and the resonators normal to it are given in figure 69. It appears at once that the increase of height has given greater detail to the curve than was the case in figure 65 for $z=20$ cm. Moreover, the curve as a whole has shrunk in the y direction. For instance, the sheer descent now begins at 40 cm. and 60 cm., respectively, whereas in figure 65 it began at 50 and 70 cm. In figure 69, moreover, there is a definite development of maxima and minima in the same vertical. The mean wave-lengths may be estimated in one curve as 36 cm. and in the other as 35 cm., and they therefore agree closely with the case of figure 45 for the same altitude.

If the curve for R at 90° be displaced to the right 15 to 20 cm. it will nearly coincide with the curve for R at 270° , so far as its dominant features are concerned. The depth or length of the resonator does not much exceed 11 cm., but the length of the tube with stopper was 14 cm. Thus (fig. 69) in the large maximum at $y=40$, the mouth of the resonator lies at $40+7=47$ cm.; in case of the next one, at 56 cm., the mouth lies at $56-7=49$ cm., which is as near coincidence as the location of maxima permits. The same result follows for all the other cases. It seems probable, therefore, conformably with the inference above (fig. 45), if the *mouth* of the resonator be used for the specification of the coördinates x , y , z and not the middle of the pipe, that but a single graph will appear, irrespective of the azimuth of the resonator.

Figure 70 gives the results of the survey in y , with the pipe oriented as before, but $z=60$ cm. above the table. The graphs obtained are unexpectedly simple, containing but a single pronounced maximum and minimum each, and all excitation terminates beyond $y=90$ cm. The reason for this may possibly, but not probably, be referable to the temperature of the room, which

was exceptionally cold. Hence the radiators near the wall at $y=174$ cm. would interfere with the homogeneity of the air, rendering it in a measure acoustically opaque, in accordance with the effect exhaustively discussed by Tyndall. As the edge of the table is at $y=93$ cm., it might be argued that the fine shades of acoustic intensity would be wiped out by diffuse reflection from the air beyond this.

The two curves obtained independently and consecutively are nearly but not quite alike, so that they may be brought nearly but not quite into coincidence by advancing one curve in the direction y and moving the other rearward half the length of the resonator tube, i. e., what is the same thing, taking the mean abscissa for a given ordinate.

The final survey in the direction y with $z=80$ cm. for the pipe in the same orientation is given in figure 71. Here the *mouth* of the resonator and not its middle point was placed at the divisions of y , so that the curves are in fact practically coincident, as already surmised. There is also an absence of detail, but the distinguishing feature of the curve for $z=80$ cm. is the advance of the maximum to $y=60$ cm.

The deductions (figs. 72, 73) from the above results will preferably be left for discussion later.

19. The y - z location of the chief maxima. Resonator on table in y ; pipe above origin in z .—We may now proceed to get the angle of incidence of the acoustic ray from pipe to resonator when the response is an initial maximum or a minimum. Figure 71 gives both y and z at once, whereas in figures 70 and 69 the mean abscissas are to be taken; figure 65 is unavailable, because of the closeness of P and R ; the maximum would here require a negative y . The data are (including fig. 45):

Maxima $z=80$ cm.	$y=60$ cm.	Minima $z=80$ cm.	$y=30$ cm.
	60		60
	35		5
	10		
(Fig. 45)	40		10

These results are constructed in figure 72 and the graphs are astonishingly linear, considering the difficulty of measurement. The slope of the locus is thus in round numbers:

$$\tan \alpha = \frac{\Delta y}{\Delta z} = 1.25 \text{ or } \alpha = 51.3^\circ$$

The minimum line is parallel to this. Hence, if an acoustic ray strikes the table conformably with the equation $y/(z-32)=1.25$ or $y=1.25z-40$, the resonator response is unique among the maxima. With the pipe in this position a parallel line 24 cm. above it is the y - z locus of the first minima. It is noteworthy that the vertical distance apart here happens to be $\lambda/2=24$ cm.; but the case for the pipe above the resonator at the origin ($y=0$) comes out $z_0=32$ cm. or $2\lambda/3$, to which it is difficult to assign a meaning.

The tangent of the angle of incidence from P to R at this maximum would, however, be variable, being $y/z = 1.25 - 40/z$, depending upon the elevation of the pipe. It is not impossible that what is interpreted as a linear locus may really be hyperbolic; but figure 65 clearly indicates that the high maximum (pipe-level, $z=0$), here probably distorted by the wind current at the origin, tends actually to lie in negative y , conformably with the suggestion of figure 72.

Finally, if we construct the unique maxima of fringe displacement s_m in terms of the elevation z_p of the organ-pipe, a graph (fig. 73) of distinctly harmonic character results. In other words, the figures 65, 69, 70, 71 are as a whole alternately high and low, so that s_m vanishes harmonically with the

elevation of the pipe. This, when 60 cm. above the table, is more effective as a whole than when 40 cm. above. Similar harmonic relations will presently reappear in the horizontal displacement of the pipe.

20. Closed pipe. Survey in x . Pipe P and resonator R parallel.—This march was along the cleared table as far as $x=130$ cm. and toward the open door (fig. 34). The pipe was kept horizontal in azimuth 180° at a distance z above the table, while the resonator lay on it at x , also in azimuth 180° , the mouths pointing in the same direction. Under these conditions the sound was diffracted around the pipe, which is thus an obstruction, and smaller deflections might be expected. They are given in figure 74, the mouth of the resonator being at the x coördinate. The curves show little detail and are low, probably for the reason given; but one easily distinguishes the successive minima and maxima about as follows:

Min. I.	20 cm.	— cm.	Max. I.	$z=20$ cm.	$x=10$ cm. ?	Min. II.	$z=20$	$x=45$
	40	20		40	40		40	65?
	60	40		60	75		60	100
	80	60		80	90		80	40

An anterior maximum develops in $z=40$, $x=0$?, and $z=60$ cm., $x=10$. The inset (fig. 75) shows the x - z location of minima and maxima, some of them uncertain. The z - x line of first maxima has a different intercept, but is again at about the same slope as heretofore, indicating an angle to z (since

$\frac{\Delta x}{\Delta z} = \frac{60}{47} = 1.28 = \tan a$) of $a=52^\circ$. The equation with its new constant is

in round numbers $x = 1.28z - 11.5$, so that the variable slope of the angle of incidence is $x/z = 1.28 - 11.5/z$. The maximum for the pipe above the resonator at the origin ($x=0$) would occur at $z_0 = 9.0$ cm., or $3\lambda/16$, which again has no meaning and does not agree with the preceding y survey. For the upper two loci (fig. 76) the slope is similar. The maxima for some reason flatten out when x is large and the first maximum for $z=20$ is not available. This accounts for the discrepancies. The wave-lengths of the flat curves are very long. On the whole the corroboration is fairly good, in view of the nature of the evidence. It is interesting to notice that at the origin the deflection s passes through a minimum near $z=60$ cm. and a maximum near $z=80$ cm. (cf. § 25).

The second survey in x was made with the pipe horizontal and in azimuth 90° , so as to be normal to the resonator in azimuth 0° on the table. The conditions here are then the same as in the preceding paragraph, and there is no diffraction around the body of the pipe. Curiously enough, however,

figure 77, which contains the graphs for the present case, is an exhibit of wave-curves much flatter than the preceding figure, and it is now difficult to recognize the successive maxima and minima, although the usual transfer of maxima into larger x values as z increases is obvious. With the obliteration of the minimum there is often a coalescence of maxima. Some influence lurks in these phenomena which has not been ascertained and which may reside in the pin-hole resonator functioning differently in different environments, or under different excitations. If the prominent maxima

$z=80$ cm.	$x=90$ cm.
60	70
40	40?

go together, the slope of the x - y locus of corresponding maxima will be about the same as in figure 72; but the $z=40$ maximum is vague because of the obliteration of the minimum at $x=30$, $z=40$ cm. The behavior at $x=0$ for different heights z is similar to the preceding, passing through a minimum near $z=60$.

Under these circumstances, a vertical position of the pipe above the origin with the mouth downward is worth testing. I have made only one survey for $z=40$ cm., figure 78, because of the flatness of curves and their marked dependence on fractions of pitch. The curve in figure 78, however, agrees substantially with figure 74 as to maxima and minima; for the minimum at $x=20$ cm. and the maximum at 40 cm. come out pretty well, so that the deductions already made would again apply. In the same figure and for the sake of contrast, I include a survey made in y , which will be found to agree substantially with the mean curve of figure 69, for the same $z=40$ cm. Thus:

Figure 69: Max., $y=10, 50$, etc.	Figure 78: Max., $y=10, 50$, etc.
Min. 34, 68, etc.	Min. 37, 70, etc.

Hence it again appears that the survey in x toward the open door is materially different in character of results from the survey in y towards and between vertical walls. Thinking that the front part of the electric lantern of the interferometer (fig. 34) might possibly reflect or diffract, though to one side of y , I tried the difference between the front face (1 foot by 6 inches) closed and removed both for $x=50$ and $x=70$ of the resonator. There was no difference. With the same end in view, a large parabolic mirror was placed at $y=70$ cm. The results were a marked dissipation (mirror not focused) of sound; thus

Mirror in place: $s=40$ cm.; pointing downward $s=32$.
 Mirror removed: $s=47$.

indicating a large negative effect varying with the direction of the axis of the mirror. A board 4 feet by 1 foot, vertically across $x=70$ cm., gave similarly:

Board in place: $s=33$ cm.
 Board removed: $s=47$ cm.

also a dissipation (resonator at $y=20$ cm.) with the mouth toward the origin. This, however, suggested a more ample survey with reflectors as given in the next paragraph; for such results also can not but be periodic.

21. Vertical reflectors.—These experiments made in the y direction with the pipe vertical, mouth downward, at $z=40$ cm. came out with unexpected definiteness at once. They will therefore aid in disentangling the mysterious behavior of the resonator in the above paragraphs. This was placed on the table horizontally with the mouth toward the pipe (azimuth 270°). The vertical board 4 feet by 1 foot was then moved successively from $y=20$ cm. (about at the mouth of R) nearly to the wall at $y=174$ cm. The results are given in figure 79. The chief maxima are near $y=50, 100, 150$ cm., which suggests the wave-length of the f'' pipe $\lambda=48$ cm., here operating by oblique reflection. Between the maxima λ and 3λ , there is a subsidiary maximum which one might erroneously attribute to overtones, inasmuch as it is associated with weakly or strongly blown pipe-notes. Thus at 80 the largest deflection is obtained from the dying note, at 90 from a forced note; but the 2λ maximum is real. Distances between 150 cm. and the wall at 174 cm. were not available. A remarkable feature of the curve is the strong initial trough

at $y=40$ cm. If the board is placed more or less obliquely to y , the minimum tends to vanish, as shown at o' and o .

In the disposition given, two sound rays reach the resonator, one reflected from the board and the other direct, or with immediate reflection from the table. When the board is near the mouth of the resonator, these rays are reduced to one and the result is an accentuated maximum. As the board distance in y increases, the path difference steadily increases, with a simultaneous weakening of the reflected ray; nevertheless, figure 79 proves conclusively that it is not ineffective, as one might suppose.

In contrast with the preceding; the present graph, figure 80, for the same pipe position but with the resonator on the table at $y=0$ cm. (in azimuth 180°), is more regular. The curve lies in higher s . There is less repetition of sections; but nevertheless the same number of six maxima occurs, *i. e.*, each maximum in figure 79 is reproduced and often accentuated in figure 80. The former minimum of $y=40$ has become a minimum somewhat below 20 cm.

The next survey in y was made with the pipe again at $z=40$ cm. and with the resonator on the table (in azimuth 180°) at $y=40$ cm. The graph (fig. 81) shows the same number of maxima as heretofore, with the sixth soon to vanish. The feature of the curve is the enormously developed second maximum, in contrast with the first or the others.

We are no doubt trenching on the case of favorable incidence, heretofore developed. The response was always stronger to the weak notes and they were used throughout, the difference being often over 10 scale-parts.

Finally, figure 82 gives the corresponding acoustic distribution with the pipe left in its former position, but with the resonator on the table at $y=65$ cm. in azimuth 180° . The first maximum is now relatively high, while the former huge second maximum has flattened and the fifth and sixth maxima have vanished behind the wall, W . Weak notes were essential, as the strong notes gave relatively little deflection, as before. As a whole, the present work has been strikingly successful and affords clear-cut evidence in the following treatment; but it fails as yet to shed definite light on the anomalously high second or other maxima.

These maximum ordinates (s_m) for path-differences $\lambda=0, \lambda, 2\lambda, 3\lambda$ are constructed for the corresponding resonator position y_R in figure 83a. As compared with the diminishing initial maximum, the strongly harmonic distribution for the λ maximum, together with its decreasing amplitude, is well brought out. Like λ_0 , 2λ is but slightly harmonic, while the odd 3λ is again markedly so. Distances between crests and troughs are here about 40 cm., though it would be necessary for their identification to insert intermediate resonator positions, say steps not exceeding 10 cm. In figures 79 to 82, in

which the reflector only moves, the distance between crests oscillates between $\Delta y_R = 20$ to 30 cm., the mean values being 28, 29, 25, 27 in successive figures. This indicates a mean inclination of nodal planes to the table of $\sin a = (\lambda/2)\Delta y_R = (48/2) \times 27 = 9$; or $a = 63^\circ$, if the nodal surfaces trail after the reflectors at the same speed.

Figure 83*b* supplies the relations of the resonator and reflector positions along y for the λ maximum. This is probably a straight line at an angle of about 50° to the vertical, as $\Delta y_{res.}/\Delta y_{refl.} = 1.2$ nearly. The displacements of board and resonator are, in this view, proportional quantities ($y_{refl.} = 1.2y_{res.} + 40$), not very different in size.

22. Discussion of the preceding reflections.—In the graphs 79 to 82 there are three influences at work on the resonator. In the first place, the direct ray from the pipe is equivalent to a reflection from the table, a locus of reinforcement. A half wave-length has been lost. Then there are the two reflections, also with loss of $\lambda/2$, from the board normal to y and the wall at $y = -130$ cm. This may here be neglected, because it is always kept relatively very distant in the plan of the experiments. We thus have to compound the ray from the image of the source with the direct ray. The computation would be very cumbersome; but the construction in the graph itself is not so and is adequately correct. In figure 79, if we take the prominent minima, a circle is to be drawn with a center at R and a radius RP . Taking the first minimum at $y = 40$ cm., the image is I at $z = 40$, $y = 80$. Hence $IC = 27$ cm. is the path difference from the minimum at 40 cm., which is placed a little too high. In this way the following data were measured:

Minima: $y = 40$ cm., $\lambda/2 = 27$ cm.	$y = 110$ cm., $7\lambda/2 = 179$ cm.
70 $3\lambda/2 = 82$	$9\lambda/2 = 220$
93 $5\lambda/2 = 126$	
Maxima: $y = 53$ cm., $1\lambda = 50$ cm.	$y = 130$ cm., $4\lambda = 198$ cm.
83 $2\lambda = 105$	155 $5\lambda = 248$
103 $3\lambda = 148$	25 $6\lambda = 4$

These results are convincing, being quite as accurate as the location of maxima and minima will permit. The minima of figure 80 treated by the same method give:

Minima: $y = 25$ cm., $\lambda/2 = 24$ cm.	$y = 103$ cm., $7\lambda/2 = 170$ cm.
50 $3\lambda/2 = 68$	126 $9\lambda/2 = 215$
80 $5\lambda/2 = 125$	etc.

with similar data for the maxima. Among these it will be desirable to locate the high second maxima in the successive curves. They occur as follows:

	Pipe at $x=y=0$ and	Resonator at $x=z=0$ and	Reflector normal to y , for maximum at	λ	s_0	s_1
Fig. 79	$z=40$ cm.	$y=20$ cm.	$y=53$ cm.	50 cm.	67	55
Fig. 80	40	0	40	50	83	85
Fig. 81	40	40	70	50	40	75
Fig. 82	40	65	93	51	30	12

As the resonator is successively displaced along y , the second (s_1) and first (s_0) maxima are alternately in excess—harmonic conditions which are common throughout these experiments (as already indicated), but for which no adequate quantitative relations have been found.

23. Reflection from boards of different areas.—These were placed normal to x , keeping the pipe, $z=40$ cm., $x=y=0$, and the resonator at the origin below it, as in the case of figure 80. Figure 84 shows the pressures at the origin when a large board, 150 cm. high, 50 cm. broad, is moving along positive x . Comparing this with figure 80 (board over 80 by 45 cm.²), it is seen that in addition to the initial maximum ($0 \times \lambda$) the odd maxima ($\lambda, 3\lambda, 5\lambda$), have retained their strength and position, whereas the even maxima ($2\lambda, 4\lambda$) are ill defined or displaced. A feature of the curve is the marked development of the second minimum ($3\lambda/2$). Owing to the position of the collimator, this

board could not be moved in the $-y$ direction as far as needed for complete symmetry. Its weight, moreover, dislocated the fringes, so that the curve, figure 85, for the next and much smaller board (60 cm. high, 20 cm. broad) is much higher owing to the enlargement of fringes incidentally resulting; but the reduction may be made from the data for the extreme points ($y=174$ cm.) and the factor would be $55/80=0.7$, nearly. It is remarkable that this relatively narrow board produces such conspicuous alternations, at least near the origin. The maxima, however, have for some reason been shifted toward greater y . The third and fourth maxima now lack definition, but the zero'th, first, second, and fifth are firm.

Carrying the reduction in area still further, I finally selected a narrow board, 60 cm. high and 12 cm. wide, obtaining the graph figure 86 with it for the same fringes. Here the first maximum and minimum still obtrude, but the second minimum is flattening out. As a whole, these results are rather what one would expect, inasmuch as the broader boards, containing more half-period elements, retain their effectiveness as reflectors at a greater distance

from the origin. I have not been able to get much information as to the cause of the wide differences in the value of the maxima and minima. The shift of the graph, figure 84, into smaller wave-lengths is intelligible, since the large board, with its stronger reflections, forces the resonator to respond to the slightly smaller wave-lengths of the intensely sounding pipe.

Conformably with the results obtained in figures 54 to 57 for normal reflection, one might argue that in oblique reflection also the maxima of fringe deflections would be obtained when the direct ray R (fig. 85 inset) and the rays r, r' reflected from the board B meet at the pipe P again, as a node, after rising from the table at T . If this is the case, two equations must be satisfied; for the case of nodes at the table:

$$(1) \quad r + r' - R = m\lambda/2$$

m being a whole number. For the occurrence of nodes at the pipe mouth on return:

$$(2) \quad r + r' + R = n\lambda$$

n also being a whole number. Thus

$$(3) \quad R = \frac{2n - m}{4} \lambda$$

If $m=0$,	$R=\lambda/2, \quad 2\lambda/2, \quad \text{etc.}$
$m=1$,	$R=\lambda/4, \quad 3\lambda/4, \quad \text{etc.}$
$m=2$,	$R= -, \quad \lambda/2, \quad 2\lambda/2, \quad \text{etc.}$

Thus the stimulation should recur alternately for values of R made up of even and odd multiples of $\lambda/4$.

24. Reflectors displaced in $-y$.—Owing to the difficulty of managing the heavy reflectors on the table, a modification was devised (fig. 89, inset) of having the reflectors (r, r' , sheet tin-plate 70 cm. high and 50 broad) hanging from a length of gas-pipe gg , stretched between the walls. T is the table on which the resonator R lies at the origin, P being the pipe 40 cm. above it. The apparent inactivity of the wall, W'' at $y = -130$ cm., had been confusing and a survey in this direction seemed desirable. The survey in the direction of positive y with the reflector r removed gave the graph figure 87, like the preceding, but much clearer. Keeping r' at $y=40$, the reflector r was now moved in steps from $y = -130$ to 10 cm. and the graph figure 88 worked out. The curve is almost flat and not until the edge of the table at $y = -27$ cm. is passed does it ascend. Hence the $-y$ axis is here ineffective. This rather startling result, however, merely indicates that the effect of the reflector r' at close range predominates, until reflector r is also equally close. When the two, r and r' , cooperate, r is within a half wave-length in y , the paths being eventually 90 cm. and 45 cm. and the path-difference about 1 wave-length. To further interpret this curious result, I removed all the apparatus at A , figure 34, and then made another similar survey in $-y$, which now gave the very striking graph figure 89. Within the available distances, this is quite like figure 87

for $+y$ points. It seems, therefore, as if in the direction $-y$ the sound-waves were formerly broken by diffuse reflection from smaller and irregular objects, but this is not true. The new graph merely witnesses the absence of the reflector r' , r acting alone.

Great care had to be taken to keep the sheets r , r' vertical; otherwise any deflection from 0 to the limit was obtainable at the maxima. The survey in $-y$ was at best troublesome, inasmuch as the accumulation of apparatus interfered with observation. An accident at $-y=50$ changed the size of the fringes, and the higher values of figure 89 within $y=-50$ are attributable to this cause. In figure 87, excepting the initial crest, even multiples of λ correspond to low crests, odd multiples to high crests.

25. Organ-pipe displaced along y .—The question arose as to whether the removal of apparatus had materially modified the data of figure 69 above, with the pipe similarly placed ($x=y=0$, $z=40$ cm.). The new survey is given in figure 90. If the mean positions in figure 69 be taken, there is no practical difference in the y location of crests and troughs as compared with the present curve, except in case of the first maximum; but this is difficult to place in figure 69, as one of the curves terminates early at zero. The new curve has been carried beyond zero into negative y and here develops another maximum, thereafter falling off at the edge of the table, at T . There is also an apparent termination at the further table-edge T' . The size of maxima in observations made at long intervals of time apart would depend on incidental conditions (size of fringes, etc.). Then there is ever present the inevitable difficulty with slight changes of pitch in the strongly blown, as compared with the moderately blown pipe. My endeavor has been throughout to get the largest deflection, so that sometimes strong, at other times moderate, notes were used, whichever happened to fit the conditions.

With this satisfactory corroboration of the old results, it seemed further advisable to carry out a survey with pipe displaced successively in the y direction at constant z , corresponding to the z displacements at constant y above. The new results are summarized in the graphs, figures 90 to 94.

As above in the z displacement, so here the graph for $y=0$, $z=40$ (fig. 90) is most sharply featured, while the other graphs tend to show but one or two

marked maxima. Furthermore, if we regard the graphs as a whole, their maximum heights in s and other ordinates are alternately large and small. The pipe positions $y=0, 40, 80$ are thus more favorable to intense nodal phenomena than the positions $y=20$ and 60 .

In several cases I carried the resonator beyond the edges T and T' of the table. Though the fringe deflections fall off at T , there is no break of discontinuity at least at T' . Sound rays, in such a case, meet freely in the air, although there is no reflection from the table. This may be interpreted as an absence of phase reversal in all the rays, which would leave the resultant unchanged. Though I have no extensive evidence on this question, it may nevertheless be suspected that the table is not perhaps as necessary for the development of these nodal phenomena as I have hitherto supposed.

In the graphs, figures 91 to 94, it is difficult to identify the maxima corresponding to figure 90. If we plot the y positions of the pipe y_P and resonator y_R for the chief maxima, the result, figure 95, represents a complicated harmonic and is indefinite as to the maximum to be taken at $y_P=0$.

Figure 90 has another feature which deserves attention, inasmuch as the maxima (pipe at $x=y=0, z=40$) lie at $z=x=0$ and

$$y = -10, +20, +50, +80 \text{ cm.}$$

all at almost exactly $y=30$ cm. apart. This makes the oblique distance from pipe mouth to resonator mouth $\sqrt{y^2+40^2}$ or

$$d_y = 41 \text{ and } 45, 64, 89 \text{ cm.}$$

numbers which have no obvious relation to the wave-length $\lambda=48$ cm., such as the preceding paragraph indicated. In figure 92 the pipe ($y=40$) is nearest the middle ($y=33$ at m) of the table; yet the graph is nevertheless characteristically non-symmetric. If the high maxima in figures 93 and 94 be regarded as produced by reflection from the wall (the hyperbolas of the nodal loci have been sketched in), the path-differences would be 95 and 90, respec-

tively, which might be interpretable as 2λ ; but this is improbable, as it would not explain the remaining features of the curves, where the path-differences run too high in wave-lengths.

An important feature of figures 90 to 94 is the alternation of large and small amplitudes s , already adverted to, as the pipe at $z=40$ takes its successive positions $y=0, 20, 40, 60, 80$, in the xz plane. This distribution of maxima s_m of fringe deflection is shown in figure 96 and is strikingly harmonic with nearly constant amplitude. Possibly it would have been more nearly so if displacements of $\Delta y = \lambda/2 = 24$ cm. had been chosen. The pipe continually approaches the maximum in the graphs and overtakes it eventually (fig. 94). Figure 95, for a shifted pipe, is in contrast with figure 73, with its rapidly decreasing amplitude conformably with the elevation of the pipe.

26. The same continued.—Some time after the conclusion of the above work, the graph, figure 97, was investigated with the pipe at $z_p=40$ cm. and $y_p=100$, therefore, quite off the edge of the table at T' . The curve again resembles figure 93, having two large maxima, one either side of P , though the accentuated maximum at $y_R=110$ may be influenced by reflection from the lamp-case. Like figure 93, the curve is low, as its maximum value s_m in figure 96 ($y_p=100$) indicates, so that it also falls in step with the alternations recorded by that curve. Figure 97 shows no appreciable influence to be ascribed to the edge of the table at T' .

This is particularly true of the next curve, figure 98, with the pipe at $y=120$ and $z=40$ cm., in which relatively large deflections, s , again abound, extending throughout the breadth y of the table. There has not, however (as in fig. 94), been an apparent coalescence of maxima; for in figure 98 the two crests are

separated by an equally marked trough lying under the pipe. Besides the intense nodal regions occurring quite beyond the edge T' of the table, the marked nodal intensity on the table was found to extend beyond the other edge. The curve at $y_R=140$ cm. contributes a further crest, s_m , to figure 96.

Figure 99, with the pipe at $z=40$, $y=140$ cm., is again of a low order of intensity throughout, the highest crest, s_m at $y_R=110$, contributing a trough in figure 96. Otherwise, however, in spite of the displaced pipe, it is not unlike the preceding curve (98), so that a crest now lies under the pipe.

The last curve, figure 100, for which the pipe lies at $z=40$, $y=160$ (*i. e.*, a little over $\lambda/4$ from the wall), seems to be a mere decay curve, instancing the direct effect of wave-trains from the pipe on the resonator. It ceases at about $y=140$ cm., 20 cm. in horizontal displacement from the pipe, and there is no appreciable nodal intensity on the table. The semiwave-length from the wall W (λ path-differences inserted, in succession, in all the graphs) here lies beyond the pipe, so that the wall is the only nodal region attributable to reflection.

In figure 97 the path-difference λ of direct and reflected rays is ineffective, while 2λ only roughly approaches the maximum at $y=110$; 3λ would strike the table beyond the origin. In figure 98 the λ path-difference is fairly coincident with the maximum at $y=140$, but the 2λ path-difference misses the large maximum at $y=106$. In figure 100 the λ path-difference actually coincides with a trough. Hence the relations of reflected wave-fronts are again vague and untrustworthy.

If in relation to figure 96 we measure multiples of $\lambda/4=12$ cm. from the wall at $y=174$ cm., and compare them with the successive pipe positions, the relations are as follows:

$(\lambda/4) \times$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15 from wall
Dist. from															
origin.	162	150	138	126	114	102	90	78	66	54	42	30	18	6	—6 cm.
Pipe at.	160	..	140	..	..	100	..	80	..	..	40	..	20	..	.. cm.
Result	..	..	Min.	..	..	Min.	..	Max.	..	..	Max.	..	Min.	..	..

In the third row the pipe positions which correspond most closely to the $\lambda/4$ divisions are given, with the observed result in the last row. Thus there is a minimum for $6\lambda/4$ as well as for $3\lambda/4$, a maximum for $11\lambda/4$ as well as for $8\lambda/4$; so that an odd or an even number of quarter wave-lengths of pipe-distance from the wall does not seem to enter appreciably into an interpretation of the distribution of crests in figure 96, even though this has been the case in other experiments.

The conditions under diffraction from the wall W may be stated as follows, since the wave-length $\lambda=48$ cm. is relatively long: Let r be the mean radius of the largest Huyghens zone for which the wall affords available space and let there be n half-period elements within it, when the pipe is at a distance b from the wall. We may then write

$$(1) \quad r = \sqrt{nb\lambda + (n\lambda/2)^2}$$

from which

$$b = (r^2 - n^2(\lambda/2)^2) / n\lambda$$

Where the integer n passes from an even to an odd number, a curve like figure 96 should change from crest to trough, the number of half-period elements being even and odd, respectively. But the feature of the curve, so far as traced, is an almost equal distance, of about $\Delta b = 20$ cm., between crest and trough, whereas in equation (3) Δb increases rapidly as n decreases. The comparison is, however, not without interest, as when $b=0$ (in the wall), $r=n(\lambda/2)$. We may therefore assume a case in which $r=200$ cm.; $\lambda=50$ cm. (for simplicity), and $n=8$, for $b=0$. The data are then:

$n=8$	7	6	5	4	3	2	1
$b=0$	27	58	97	150	229	375	787
$\Delta b=$	27	31	39	52	79	146	412
$-db/dn=$	25	29	35	45	...	...	...
							cm.

Within the limits investigated ($y=0$ to 174 cm.), Δb increases from 27 cm. to over 52 cm., so that both the Δb is too large ($\Delta y=20$, crest to trough) and the increase is too rapid as compared with figure 96. If r , the limit of available wall-space in which the half-period areas lie, is taken twice as large ($r=400$ cm.), the Δb within the available $\Delta y=175$ of investigation would naturally be more constant, viz,

$n=16$	15	14	13	12	11	10
$\Delta b=$	26	27	31	33	36	42
$-db/dn=$	25	27	129	32	34	39
						45
						cm.

but they would, on the average, be much too large. Still, this method of viewing the phenomena, though inadequate, is better than what has preceded, seeing that there would be three or four other walls to be treated similarly. The nature of the approximation follows from

$$-\frac{db}{dn} = \frac{b}{n} + \frac{r}{n_0}$$

where $r/n_0 = \lambda/2$, a constant. Here b/n , as seen in the above data, is rapidly increasing and effective, even though $r/n_0=25$ is large and db/dn can not be less than $r/n_0 = \lambda/2$.

For comparison with the data for the f'' pipe, a survey was made with an open c'' pipe (diapason) and a corresponding c'' pin-hole resonator, of the closed type. The pipe was placed above the origin on the table, in $z=40$ cm., as usual. The note seemed relatively loud, so that larger deflections were obtained than with the other pipe; but the pitch was too fluctuating for accurate work and it was difficult to fix a steady maximum deflection. Resonance, in other words, implies sharp tuning.

The character of the survey in the y direction (the wave-length being about 63 cm.), compared with figure 90 for the f'' pipe, presented nothing very striking. The chief maximum was under the pipe. It was interesting to note that the same conical pin-hole functioned equally well for both pipes, so far as could be discerned.

27. Nodal regions in free air.—The result obtained in the last paragraph, of marked nodal activity beyond the edge of the table, though not surprising

in itself, nevertheless made it necessary to elucidate these occurrences more systematically. One is quite apt to regard the reflection from the table as an important part of the phenomena. In some instances it must be so. Incidentally the present contribution will adduce new evidence bearing on the nature of the maxima and minima observed, and the computation of wavelengths therefrom.

The first survey in $-y$ is given in figure 101, where the wall W'' of the room (fig. 34) is at $y = -130$ cm. The edge of the table is at $y = -27$ cm., so that the stretch $W''T$ is in free air. The pipe P is placed at $y = -20$ ($z = 40$, $x = 0$) vertically, very near the edge T . The resonator lies in $x = 0$, in the plane of the table (prolonged if necessary) and at y . One observes four maxima, distinguished as $-\lambda$, -2λ , -3λ , -4λ , and this corresponds to the number of nodes or hyperbolæ which can occur between W'' and P . These hyperbolæ are again roughly sketched in, the conjugate focus (not shown) being 130 cm. behind the wall W'' . It will be seen that the maxima now coincide with the hyperbolæ in turn, the first nodal surface ($\Delta y = 0 \times \lambda$) being at the wall W'' . The figure also shows the direct and reflected rays for each case, meeting at the level of the resonator in the plane of the table prolonged. The figure thus discriminates between the fourth maximum (-4λ) and the first three maxima, $-\lambda$, -2λ , -3λ . In case of the latter the rays meet in the same phase but come from opposed directions, so that there is a break of $\lambda/2$ at the node. In this event nodal condensation and rarefaction are possible in free air from the opposition of vectors. The opposition is a maximum in the level of the pipe P ; it is practically effective, however, as the experiments show, at levels much above and below it. At the maximum -4λ , the direct vector R and the reflected vector r are not in opposition relative to a vertical plane, though they would be so for a plane bisecting the angle ROr symmetrically. I suspect, therefore, that here the rays S reflected from the table provoke the node, so that here the table should be necessary.

The nodes increase in intensity from the wall W'' to the pipe P . This is not evident; for if the direct rays regularly increase, the reflected rays at the same time greatly decrease, and the intensity of interference depends essentially on the weaker ray, between P and W'' . For like reasons one is apt to ignore the wall effect at long distances, believing that, for path-differences of 3λ (150 cm.) and over, no appreciable result could be expected, even where the required amount of path-difference was demonstrable. Beyond the distance $W''P$, however, R and r coöperate, and the case of a large maximum here is more easily intelligible.

Corresponding work with the pipe at $y = -40$, $z = 40$ is given in figure 102. Here the first maximum ($-\lambda$) comes out clearly, the second (-2λ), though flattened, is correctly placed, but the third (-3λ) is apparently shifted toward the left. This may be due to the presence of the observer at the interferometer and the reflection resulting. Minima, in other words, have been obliterated.

The final graph, figure 103, with the pipe at $y = -60$, $z = 40$, has additional characteristics from the nearness (70 cm.) of the pipe to the wall W'' . The first and second maxima are correctly placed and much enhanced. The third is now impossible; but the marked height of the curve, nevertheless, persists in this region ($y = -20$ to -40). One would hesitate to ascribe it to reflection from the body of the observer. Similarly, the three curves (101, 102, 103) have a fixed maximum at $y = 0$, which, though directly possible in figure 101, would not be so in figures 102, 103. There may be reflection from the vertical interferometer (near $y = 0$) for these pipe positions. The alternation of the intensity of deflections or acoustic pressures is again evidenced by figures 101 and 102. Together with the preceding graphs, the case stands thus:

Relatively large acoustic pressure throughout: Pipe at $y = 80, 40, 0, -40$ cm.

Relatively small acoustic pressure throughout: Pipe at $y = 60, 20, -20$ cm.

The pipe at $y = -60$ cm. does not conform to this rule, possibly from the secondary disturbances mentioned. Finally, one may note the frequent occurrence of $\Delta y = 30$ cm. between well-developed maxima or minima.

In the light of these solutions, we might hope to unravel the results of the preceding paragraph, with a view to the further identification of the maxima. This does not succeed nearly as well as was confidently expected. Turning to figure 90, the first two crests on the left (hyperbolæ sketched in) may be attributed to the corresponding wall W'' , and their value would then be -4λ , -5λ (199 for 192 cm., and 238 for 240 cm.). In figure 91, the -4λ maximum lies at 198 instead of 192 cm. and the -5λ maximum is obscure. Thus even the -4λ values are bad. Possibly this may be associated with their position near the edge of the table. Moreover, the rays are opposed in direction and the observer is too near. Finally, the fit of sharp or flat notes, sometimes so distinguishable, may be a source of error in judging as to positions of maxima. There is no way at present of combating this.

In figure 90 the -5λ and $+6\lambda$ maxima happen to be nearly coincident; but the effect is not remarkable as compared with the following single $+5\lambda$

maximum. Promising as it seems that a coincidence something like this is an explanation of the occurrence of exceptionally high crests, no evidence was obtained, beyond the rough fit of -6λ and $+5\lambda$, in figure 91, showing nothing for -5λ and $+6\lambda$, moreover.

Turning to the right wall W , graph figure 90 has a maximum for a path-difference of 5λ ($y=47$), which agrees very well with experiment; but the maximum at $y=80$ would correspond to about 3.5λ and must have thus some other origin. In figure 91 the chief maximum belongs to the path-difference 5λ ($y=43$) and also fits well. Other crests are here uncertain. In the graphs, 92, 93, 94, however, the relation of the graph to the reflecting wall W is quite obscure. The hyperbolæ inserted show that these crests lie respectively at about 4.5λ , 3.5λ , 3.5λ (scant) of path-difference. As there is nothing to account for the anomalous half wave-length, some other cause for these crests must be sought. This suggests a consideration of the W' wall, normal to W and W'' ; or the wall W , which is very high compared with W'' , and may contain an unfavorable number of half-period elements, the first zone having a radius of 70 to 90 cm. It is difficult to draw quantitative inferences here.

As to the W' wall, at a distance of 190 cm. from the x - y plane of survey, as well as to the reflections across corners, these are relatively so distant that their effective participation is hardly probable.

In figure 90 the well-developed consecutive crests and troughs lie about $\Delta y=30$ cm. apart in the five cases available. There is another in figure 93. If one regards the effective impinging rays to be the axes of stationary waves produced by the totality of reflections within the room, the distance apart Δy of the crests in the graphs would conform to

$$\Delta y = \lambda/2 \sin \alpha$$

where α is the angle of incidence on the table. Thus

$$\sin \alpha = \lambda/2\Delta y = 48/60 = 0.8, \text{ or } \alpha = 53^\circ, \text{ nearly.}$$

But in figure 90 this angle would have to be constant throughout the whole survey of 120 cm. and it is difficult to understand where such parallel rays may come from. The incidence of the direct ray varies from $\alpha=0^\circ$ to $\alpha=66^\circ$, throughout the whole of which $\Delta y=30$ cm. In fact, the presence of this datum here and elsewhere is quite noteworthy.

To return again to the evidence given at the beginning of the paragraph, one may note that it implies the occurrence of nodal regions, floating, as it were, in free air, with a fixed position for each fixed position of the sounding organ-pipe. This heterogeneous air structure, which one might call flocculent, particularly characterizes the closed part of the room within the wall $WW'W''$, the ceiling, and the floor. To sustain itself, a nodal region would have to vibrate in all directions in reciprocation with the surrounding nodal regions. Thus the architecture is essentially on a large scale, so that occasional appearance of parallel regions intersected by the table, as in figure 90 and elsewhere, becomes more plausible. Nevertheless, the way of attacking these conditions quantitatively, or of accounting, for instance, for the repeated occurrence of

the angle $\alpha = 51^\circ$ to 53° , is more remote than the frequency of systematic results would lead one to expect.

28. Vertical-pressure variations at the crests and troughs.—The changes which the maxima and minima undergo along a vertical above the level of the table is investigated in figure 104. Here the graph, figure 94, with the pipe at $y=80$, $z=40$, was consulted and the resonator moved up vertically from the table, at $y=80$ and $y=20$ cm. respectively. The two graphs, figure 104, are laid off on the y position of the resonator, deflections s increasing upward, while the y coördinate of resonator and pipe increases downward. By plotting y , z , and s in this way, the pipe and resonator positions, as well as the deflections s , are all indicated. Turning first to the original maximum at the table, the deflections are seen to decrease to a minimum at $z=10$ cm. instead of $\lambda/4=12$, and then to increase to an enormous maximum at $z=30$ cm. instead of $\lambda/2=24$, the first nodal plane above the table. There is thus a tendency for the nodal plane at the table to reproduce itself at $\lambda/2$ above, which degenerates into $y=30$ cm. so far as the curve is correctly drawn. It might be supposed that the interference here is the wind from the pipe at $z=40$ in the same vertical; but the curve is too continuous throughout to supply any such evidence and the pipe blows away from the resonator. The second minimum, moreover, lies even above $5\lambda/4$, and the curves beyond the maximum largely indicate the rapidly decreasing intensity s with z .

The original minimum (fig. 104 at $y=20$ cm.) is at first too flat for the detection of characteristics. It then rises to a pronounced maximum at $z=40$ cm. above the table. Being at a long distance (60 cm.) from the pipe, there is here no draft interference. The wave-length path-difference, which lies at about $z=45$ cm., is not reached, and all intermediate and subsequent fluctuations are absent from the smooth curve observed. It does not, therefore, conform to any simple plan of node distribution upward from the table.

The graph, figure 105, is constructed in the same way and obtained by consulting figure 92, in which there is a maximum at $y=50$ cm. and a minimum at $y=90$ for the resonator reposing on the table, the pipe being at $y=40$, $z=40$. The maximum (fig. 105), with the resonator in the vertical at $y=50$, brings out the characteristics of the preceding figure. The minimum at $z=10$ is again too low, the maximum at $z=35$ is too high relatively to $\lambda/2$, and beyond this the curve merely falls off. The crests in figure 104, both when R is at $y=20$ and when at $y=70$, tend to approach the P level ($z=40$), quite so at $y=20$. The same is true in figure 105, even more nearly in both curves for R at $y=50$ and R at $y=90$.

With regard to the former minimum development (R at $y=90$), one may notice that in figure 105 it begins with a significant value, $s=18$, probably because of some alteration of adjustment from the preceding experiment. The minimum is not reached until $z=24$, which is a semiwave-length. Nevertheless, the maximum is again located at $z=40$ and near the pipe-level. The

construction and hyperbola show that the curve, when R is at $y=90$, trends as if there were a node on the table here, which is repeated when the path-difference is λ .

As in figures 104 and 105 the distance between crests and troughs is an increasing quantity upward, we may perhaps regard the distance between the first two or nearer crests to correspond roughly with the succession of nodal surfaces at the table. If the latter make an angle α with the table, this would be expressed by $\Delta z = \lambda/2 \cos \alpha$, $\Delta z/\Delta y = \tan \alpha$. Thus if $\Delta z=30$ to 35 , $\cos \alpha = \lambda/2\Delta z = 24/33 = 0.72$ or $\alpha = 44^\circ$, which is not in good agreement with the larger angle found above (51°) in the work along y . The increase of distance between crests as the resonator approaches the pipe is referable to the direct influence of the loud pipe-notes without the intervention of nodes.

29. Survey in y in the pipe-level.—The graphs of the preceding paragraph seemed to indicate that intense pressure might be expected in the z level through the mouth of the pipe P . This was therefore placed $z=40$ cm. above the origin and the resonator carried in y at the same level in the yz plane. The results are given in figure 106, in which an enormous maximum is observable when the resonator passes the pipe at $y=0$. This, of course, is to be expected. The other maxima are meager by comparison and they agree roughly with path-differences of 4λ , 5λ , reckoned from the wall W on the right at $y=174$ cm. They are quite obliterated by the proximity of the pipe (6λ , 7λ), which here blows in the y direction from $y=0$. The curve, however, is continuous and probably not much influenced by the draft from the pipe-blower, for the resonator axis was normal to the current of air and its immediate effect would have been a pressure decrement.

30. Inferences.—The above work as a whole has shown that when the reflecting plate is relatively near to the pipe and resonator, the position of maxima and minima may be predicted as a case of ordinary interference and the wave-length computed satisfactorily. The distribution of intensity among

the crests and troughs, nevertheless, remains harmonic and has not been foreseen.

If the reflecting surfaces are relatively remote, the positions of crests and troughs can not, as a rule, be found by the same method satisfactorily. In certain instances there seemed to be an approximate fit; but as a whole the attempt was unsuccessful and the distribution of nodal intensities equally puzzling. It appeared, however, that when the organ-pipe, definitely placed, is sounded, the occurrence of nodal surfaces of a fixed position in free air is demonstrable everywhere. Distributions between walls are consistent and differ from distributions between wall and door. It is not improbable, moreover, that such surfaces are regularly grouped, since they are fashioned on a large scale of architecture, and may for reasonable distances be approximately parallel. If, then, they are intersected obliquely, for instance by the plane of the table, and if α is the mean angle between the latter and the nodal surface, the distance between crests on the table should be $\Delta y = \lambda / (2 \sin \alpha)$. Hence this intercept, Δy , may have any value, depending upon the general shape of the room. If the prevailing value, $\Delta y = 30$ cm., be taken, $\sin \alpha = 24/30$ or $\alpha = 53^\circ$ roughly. This angle between surfaces is also the angle of incidence of the rays, and has been encountered more or less closely in other paragraphs (§§ 19, 20, 21, 27). The distribution of intensities would then also depend on the fixed pipe position in relation to the given walls of the room, regarded as reflectors. Such an explanation is plausible and flexible, no doubt; but it has the great disadvantage that none of the results can be reproduced by computation; and the conviction that some other explanation may be found is not removed. This is particularly so, because the pin-hole resonator is practically unresponsive, except within ranges from the pipe which are small as compared with the dimensions of the room as a whole.

The pin-hole resonator is peculiarly stimulated by a node at its mouth. If this is in a compressional phase it enters the $(\lambda/4)$ tube of the resonator and returns with change of phase to meet the ensuing rarefaction at the mouth. The mechanism to do this with wave-trains seems to function less perfectly. The organ-pipe itself, as was found, is similarly stimulated to increased activity. The evidence of harmonic distributions of acoustic pressure obtained by the pin-hole resonator is sufficient in itself to prove that direct wave-trains are relatively without importance, except very near the pipe, for these would show the same intensities everywhere, with the exceptions of the regular diminutions with distance. Hence it is not surprising that expectations as to a predominating effect attributable to moving wave-trains is usually not realized.

The structure of the acoustic model of nodal regions determinable in the ways discussed in the above paragraph is nearly always quite unsymmetrical to the organ-pipe. Hence the reason for this structure is largely to be sought outside of the pipe, the latter merely giving it the possibility of exhibition. The pipe position in a given room, moreover, determines the particular distribution of nodal regions among the infinite number available.

31. Surveys (in x and y) in the plane of the wall.—These have an advantage over the surveys across the table, because there need be no interferences (apparatus, etc.) between the pipe and the wall and no appreciable nearby reflections. Hence one should expect mere decay curves (*i. e.*, a diminution of acoustic pressure or nodal density with distance) from the normal let fall from the pipe to the wall outward, on all sides. Unsymmetric distributions are referable to causes outside of the pipe. The wall in $+y$ was inconvenient for this purpose, because of its distance from the observer and the heaters near it (hot wall). Hence the wall at $y = -130$ cm., with the pipe *behind* the observer, and 40 cm. above the plane of table (prolonged) in the xz plane, was chosen.

Figure 107 shows the distribution of acoustic pressure (deflection s) for the pipe P at $y = -60$, $P-W = 70$ cm. from the wall W , along the vertical ($x = y = 0 \pm z$) in the wall, *i. e.*, through the pipe normal. The figure also gives the $y-z$ position of the pipe and the pipe normal N . It is seen that the dis-

tribution of acoustic pressure is distinctly periodic, that the foot of the normal is in a trough, and that the pressure distribution here happens to be symmetric with regard to it, so far as the locations of maxima and minima are concerned. The nodal intensity, however, is greater for the lower points than for points above the pipe normal. The possibility that with the pipe-mouth down, sound travels with less obstruction downward than up is soon to be negated. The peculiar feature of symmetry of the rays r and r' (fig. 107) is such that they make an angle of incidence with the pipe normal of 39° at a glancing angle of 51° at the maxima.

The next survey was made with the same pipe position ($y = -60$, $P-W = 70$ cm.), but along the $\pm x$ direction through $z = 40$ cm., the pipe-level. The anticipations were (as usual) quite disappointed, for the new graph (fig. 108) shows but doubtful variations until the wall (located at $-x = 190$ cm.) is approached; at least, the marked deflections begin in the march toward the corner of the room. This might also lead one to suspect that the high maximum in figure 107 preceding may be similarly associated with deflection from the floor ($z = -100$ cm.). Neither of these surmises, however, is given any warrant in the following work.

Moving the pipe, at the same level, to $y = -80$ ($P - W = 50$ cm. from the wall), the resonator mouth (also in this level and in the wall) displaced successively in x , gave evidence of the nodal distributions shown in the curve 109. The contrast with figure 108 is astonishing; for whereas the latter varies as it approaches the corner of the room, figure 109 is nearly without displacement near that corner, but now shows very large deflections near the *free end* of the wall ($x = 42$). (Cf. fig. 34.)

These unexpectedly complicated distributions of nodal intensity in the plane of the wall, under environmental conditions apparently so simple, made it necessary to pursue other intermediate x surveys, with the pipe kept in the same level and orientation throughout ($z = 40$ cm., blowing away from wall), but with the pipe-distances $P - W = 60$ cm. and 40 cm., respectively, from the wall at $y = -130$ cm. In other words, the pipe is displaced along the same fixed pipe normal to the wall. Figure 110, which is intermediate between figures 108 and 109, shows the maxima at $y = -130$ cm., and $y = -80$ cm. of the former, dwindling, to be deleted in the latter. Again the absence of nodal intensity between $x = 0$ and 40 cm. for the pipe-distance $P - W = 70$ cm. is replaced by a high crest when $P - W = 60$ cm. and by two high crests when the pipe-distance is 50 cm. The remainder of the curves exhibit like harmonic changes less easily described. For each particular pipe-distance, *caet. par.*, there is a particular harmonic distribution of nodal density along x in the wall in the 40-cm. level above the table. These distributions are totally unlike, but pass continuously into each other, nevertheless. The same is further illustrated by the last of the x surveys, figure 111, with the pipe-distance from the wall $P - W = 40$ cm. The nodal density has now again moved from the free end of the wall toward the corner of the room, though the high maximum at $x = 0$ is retained.

It seems difficult to make out a relation for the maxima, such as was surmised in § 19, for instance; for the evidence which here abounds points to stationary locations for maxima and minima, as if the change were in intensity. Thus, at the coördinates x, z , the schedule would read

$z =$	40	50	60	70
$x =$	30 cm.,	min.	max.	max. (1)
0	max.	max. (1)	—	—
— 20	min.	min.	—	max.
— 30	max. (1)	max.	—	min.
— 50	min.	min.	—	—
— 60	max.	max.	—	max.
— 80	max.	±	max.	max.
— 110	±	±	max.	max.

The distance apart of consecutive maxima or minima for the same graph or z is usually 30 cm. For $z = 50$ cm. this is the case with all of them and the total number of such cases is 9, apart from doubtful cases. In relation to the above surmise of a shift of maxima with pipe-distance, the cases marked 1 in the schedule would be the only ones available, where $\Delta x / \Delta z = 2$ or an angle of incidence of 63.5° ; but there is no warrant here for such combinations.

In the endeavor to throw further light on the complicated subject, I carried out the three surveys along the vertical z , in the x - z plane, the pipe being

placed in its former positions, correspondingly. These additional results are given in the graphs (figs. 112, 113, 114) for pipe-distances $P-W=40, 50, 60$ cm., and they are correlative with figure 107. The pipe normal here should foot at $x=y=0, z=40$ cm. These experiments in z were made some time after those above in x and before I appreciated the fact that small differences in the lateral (x) location of the pipe would produce an effect on the graphs, very much enhanced. Consequently, the ordinate marked N in the x and z graphs may be regarded as here belonging to the pipe normal. This, however, is of little consequence, for the promise of symmetry around the pipe normal ($x=0, z=40$) has already been dissipated by the x -graphs. The z -graphs are even more unsymmetric and independent, and inferences of the kind drawn for the x -graphs are less trustworthy. None of the graphs recall the curves obtained on lifting the pipe above the table (figs. 69, 70, 71), although the experiments are essentially similar and any given survey always reproduces the same graph. Thus it seems to follow again that the cause of the distributions recorded by the graphs must be looked for in the entirety of the environment surrounding the organ-pipe, for the particular stimulation received from the latter.

32. Cylindrical survey.—The unexpectedly non-symmetric results obtained in the last paragraph along the east wall W'' (fig. 34) induced me to try a similar group of experiments relative to the center (nearly) of the table. The resonator was therefore carried in a circle of radius r_R , around the plumb-line let fall from the mouth of the pipe. The measurements were then repeated for successive levels of the pipe ($z_P=40$ cm. and $50=\lambda$ cm., nearly) and of the resonator ($z_R=0, 12=\lambda/4, 24=\lambda/2$ cm.). The relation of the different locations is suggested by the inset of figure 115, which is a reduced plan of the room, with the pipe in the vertical above P on the table. The directions $+y$ (west) and $-x$ (south) are thus toward walls, whereas $+x$ (north) and $-y$ (east) are largely toward open spaces from the table.

In the first exploration the mouth of the pipe P was $z_P=40$ cm. above the table. Carrying the mouth of the resonator on the table with its axis kept parallel to x , around the successive circles of radius $r_R=0, 10, 20, 30, 40, 50$ cm., gave the intensity distributions, s , contained in the graphs (fig. 115 and 116). These are again very definite and interrelated, but quite unsymmetrical to the pipe vertical. The oscillations in eastern and northern R displacements from the center (i. e., to open locations) are meager, while the s oscillations for southern and western directions, trending toward closed parts of the room, are very marked. As r increases from 0 to 10, 20, 30, the maximum nodal intensity lies toward the south; beyond this ($r=40, 50$ cm.) it moves to the west. All the maxima of intensity are larger than the intensity on the table, under the pipe ($r=0$) even at $r=50$ cm. (meaning a pipe-resonator distance of 64 cm.), the s value is a little in excess of $s=23$ for the normal case.

Another view of the results is given by the graph (fig. 117), which collects all the points on the line $S-N$ along x , and on the line $W-E$ along y . The

features of the latter are somewhat vague, but the S - N line presents very conspicuous crests and troughs. These are from 40 to 45 cm. apart, and situated on either side of, but without other appreciable reference to, the pipe vertical P in their contours. The inference would again be that we must look to other causes for their occurrence.

The mouth of the pipe was now raised in the same vertical to $z_P = 50$ cm. above the table, or a little over a wave-length (48 cm.). The data so obtained are constructed in the same way as before, in figures 118, 119 for the rotation of the resonator, and figure 120 for its corresponding $\pm x$ and $\pm y$ displacement. The central intensity ($r=0$) has increased, because of the λ -elevation, reproducing the results of figures 55 to 57. The curves as a whole are simpler in outline; the unique maximum is again transferred from S to W as r increases, this time a little sooner, in fact, i. e., before r reaches 30 cm. For large values of r , intermediate azimuths would have been desirable; but as the phenomena are adequately given by the linear surveys of figure 120, the extra labor was thought superfluous. The W - E line has now much more detail and

would assist in interpreting the corresponding line in figure 117. By contrast the S - N lines in both figures (117, 120) are very much alike, the maxima and minima in the latter case having been shifted into the north or positive x for the elevated pipe. This also recalls the earlier result (figs. 72 to 74). The distances apart of crests and troughs is here somewhat more difficult to define; but it is still much above 40 cm. and the pipe vertical is ignored as before.

For the fixed position of the pipe it was now desirable to raise the plane of the resonator and the graphs (figs. 121, 122, and 123) contain the nodal intensities for an elevation $z_R = 12$ cm. (a quarter wave-length) above the table. This is the level of an antinode at the pipe normal and the curves are therefore low throughout, conformably with the evidence in earlier figures (58 and 59) for normal positions. Nevertheless, as we depart from the pipe normal or center, to circumferences of radii 10, 20, 30 cm., the unique maxima in the south become prominent and beyond $r=30$ cm. they again shift to the west.

In figure 123, the S - N line has the same characteristics as in figures 117, 120 with its two marked crests. Somewhat less certainly the resemblance in W - E lines may be detected. As in all the N - S lines the central trough lies for some reason to the right of the pipe normal. The maxima are far apart apparently.

The resonator was finally transferred to a level $\lambda/2=24$ cm. above the table and moved around the pipe axis, its mouth lying on successive circumferences of radii r , with the results given in figure 124. The maxima of all the curves are curiously flattened; but there is still a gradual transfer of the chief crest from S to W , as r increases. Probably intermediate observations would have modified the curves as drawn. Since $\lambda/2$ determines a nodal plane normally below the pipe, the intensities are all relatively large, but they fall off as a whole so rapidly with r that there is no intersection and all curves may be given in a single diagram.

The graphs showing the intensity s along the S - N and W - E directions in figure 125 now exhibit a total change of form. The two crests have seemingly all but coalesced near the pipe normal. Owing to an accident which somewhat dislocated the fringes, the indentation in the middle of the S - N curve and the bulge toward the right in the other may need modification; but this is of little consequence here, since the plane at a level of $z=24$ above the table contains a single marked crest only.

It is of further interest to bring the values of intensity together when the resonator moves up from the table on the same vertical. The values of s along x_R , at the different altitudes, z_R , are from south to north,

$x_R =$	-50	-40	-30	-20	-10	± 0	10	20	30	40	50 cm.
$z_R = 0$ cm.	$s = 20$	5	25	40	40	30	20	15	15	2	1 s. p.
$= 12$ cm.	$s = 2$	0	17	12	5	3	0	4	5	0	5 s. p.
$= 24$ cm.	$s = 15$	15	22	45	47	50	40	25	12	5	1 s. p.

As constructed in figure 126, with s laid off horizontally to the right z_R vertically and with the origin of *each* tripoint s -curve at the x position of the vertical to which it belongs, these vertical distributions of nodal intensity strike the eye. The intensities s on the table for the same x positions (fig. 120) are also sketched in. Under the pipe at $z_P=50$ cm., the nodal surface at the table rapidly decreases in s intensity to the antinode at $z_R=\lambda/4$; s then rapidly increases again to the more pronounced nodal surface at $z_R=\lambda/2$, nearer the pipe. Toward the south, however, the distinction between node and antinode is not regularly sustained; at $x=-20$ cm. the antinode has gained in strength and at $x=-30$ cm. the strength of nodes and antinodes does not differ much. Beyond this ($x=-40, -50$), however, nodes and antinodes are again

sharply contrasted. Toward the north from the pipe normal ($x=0$) these differences die out more gradually, until at $x=50$ cm. the intensities are, for some further reason, reversed.

What is very surprising, however, is the observation that throughout the whole of the extent of a linear meter ($x=\pm 50$) symmetrically to the pipe normal the nodal and antinodal planes at $\varepsilon_R=0, \lambda/4, \lambda/2$ remain parallel to the table, so far as can be seen. The only distinction within this stretch is the distribution of nodal intensity as exhibited in the continuous curves sketched in, or separately in figures 117, 120, 123, 125. It would, therefore, here seem to be a mistake to associate wave-length with crests and troughs in these diagrams, however plausibly such inferences may obtrude. It is very much more probable that in the south some extraneous s distribution has been superposed which has already been encountered in the vertical survey for the y - z plane in figures 104, 105.

The west-east distributions ($\pm y$), though not so sharp in their features in figures 120, 123, and 125, show an equally pronounced character, when treated in the same way relative to vertical variations, in figure 127. At $r=0$ the second antinodal region is indicated. The tabulated values of the nodal intensity, s , for the W - E points along the successive verticals at y_R are ($\lambda=48$ cm.):

$y_R =$		-50	-40	-30	-20	-10	± 0	10	20	30	40	50 cm.
$\varepsilon_R = 0$ cm.	$s =$	35	20	35	25	20	30	32	30	15	2	7 s. p.
$= 12$ cm.	$s =$	5	12	12	0	2	3	0	0	1	0	4 s. p.
$= 24$ cm.	$s =$	4	5	22	45	52	50	45	40	20	10	1 s. p.

Each of the triplet graphs is to be referred to an origin at the x_R marked on the curve, and the (horizontal) s intervals are 20 scale-parts. Since the edges of the table are at $y = \pm 60$ cm. (nearly), the nodal plane does not extend quite so far; but it does not follow that these edges have much influence on the results, since the same kind of graphs appear in figure 126, where the edges of the table are nearly 50 cm. farther. The degradation of the nodal plane occurs first at $y=50$ cm. in the east as it did in the north. In the west it is still strong at $y=-40$, and now promises to run through another cycle of

intensity, as it did in the previous case. The antinode, very nearly $s=0$ throughout the east, begins to lose identity between $y=-20$ and -30 , to be followed by oscillations. Finally, the first nodal plane above the table degenerates here more obviously than before, between $y=-30$ and -40 .

The records in figures 126 and 127 contain an almost complete example of a free nodal region sustaining itself $z_R=24$ cm. above the table, and figure 125 gives the acoustic pressures within it. These are largest near the pipe, though not quite symmetric, and it is possible that the direct ray may here influence the resonator. In general, however, *i. e.*, at greater distances, the direct ray is relatively ineffective, and it is to the occurrence of nodal regions alone that the resonator then responds, even when the direct ray would seem to have the advantage of nearness.

33. Axial rotation of pipe. Second antinode.—As the pipe wind blows in a particular direction, it was of some importance to ascertain again whether this wind direction had any influence. The pipe was therefore rotated around its own axis in succession, producing winds blowing north, east, south, and west. In each case the fringe deflection s was 20 scale-parts, the resonator being at $r=3$ cm. (south) on the table and the pipe at $z_P=50$ cm. above the origin, as before. Thus there is no wind effect.

A further test consisted in lifting the resonator to $z_R=36$ cm. above the origin on the table. This is the second antinodal level ($3\lambda/4$), now very near the pipe at $z_P=50$ cm. The deflection obtained was about $s=30$ and is indicated in figure 127 on the $r=0$ curve. Thus, in spite of the proximity of the pipe (12 cm.), the second minimum is still distinctly present, though its intensity is naturally in excess of the $s=3$ of the first minimum. The next maximum would no doubt be enormous, as in figure 106, so that we here encounter the direct effect of wave-trains.

34. Improvements of the pin-hole resonator. Solid viscosity in mercury U-tubes.—Experiments with this end in view were made at great length, but at first little definite progress was achieved.

The plan immediately suggesting itself was the trial of an adjustable needle-valve pin-hole. The apparatus is shown in figure 128, where tt is a $\frac{1}{4}$ -inch brass tube with a conical point c , carrying the pin-hole at its end o . The tube tt is closed by a long nut m , in which a waxed screw s , terminated by the needle n , is movable. The pin-hole o could thus be completely closed or opened in any degree by advancing or withdrawing the screw s . The lateral tubulure r communicates with the interferometer U-gage. The adjustable pin-hole was inserted into the tuned resonator R , as usual. After many trials with this promising apparatus, no fringe deflection whatever was observed until the needle n had been completely withdrawn; and the simple pin-hole so obtained, being too large, functioned badly. The adjustable fixture completely failed. Tests made by inserting the conical glass pin-hole, as above, worked as well as before.

In fact, the best available pin-holes for the resonator were made from glass quill tubes drawn to a blunt cone. Occasionally this was broken off at just

the right size of hole. Usually, however, it was necessary to enlarge it by grinding, or to close the conical end by gentle fusion and regrinding when cold. In any case it is a long and tedious process, as there is no adequate guidance. Thick-walled glass tubes, of the same size externally, produced no deflection. Brass resonator tubes about 2 cm. in diameter gave the best response to f'' pipes.

To throw further light on this intricate subject, it seemed advisable to operate with pin-holes in soft sealing-wax, as these could be more easily shaped. In this way new information was obtained. A thin sheet of wax was spread on thin paper closing the end of the quill tube t , figure 129. When the wax was punctured by a needle from the inside of the tube t , producing a conical hole as exaggerated in figure 130, the fringe deflections were positive and reached a maximum with the right size of hole. But when the thin wax sheet was punctured from the outside (suggested in fig. 131), the deflection obtained was to the same degree negative. In other words, in passing from the salient pin-hole 130 to the reëntrant pin-hole 131, acoustic pressures pass to acoustic dilatations; or the pressure excess is on the reëntrant or concave side of the conical pin-hole. It follows, therefore, that a cylindrical pin-hole must be inactive, if used with a resonator, and it is possible that the interminable record of failures experienced in the present work with metal foil is attributable to this cause.

The remarkable efficiency of glass quill-tube probes, with the pin-hole at a conical end, ground off, thus finds its explanation; for these are in conformity with figure 130.

To further verify the new results, holes were bored with a fluted conical reamer in small disks of brass, and these were then soldered to the ends of quarter-inch brass tubes, as shown in figures 132 and 133. The cones ended in pin-holes, drilled from within and without, respectively, by fine needles. No. 132 gave positive deflections of about 10 scale-parts, No. 133 negative deflections of about 5 scale-parts but very definite. It was found that on enlarging No. 133 from *within* with a needle, the deflections became positive; but on further enlarging the pin-hole, now from without, they became negative again. This proves, conformably with the earlier evidence, that only a very small depth of pin-hole (probably of the thickness of a piece of paper) is effective and the remainder of the cone is without importance.

Experiments with the wax pellets at the end of quill tubes (fig. 129) were now resumed, and figure 134 shows the results obtained on puncturing a number of them side by side successively from without. The (negative) maximum here passes with two holes. In figure 135 similar results are given, obtained on puncturing successively from within the quill tube, in this case with a small as well as with a larger cambric needle. The maxima obtained in all these experiments with wax are relatively small, the glass pin-hole resonator reaching deflections of 35 scale-parts under like circumstances of position, *i. e.*, it showed 3 times greater sensitiveness. The double resonator (open tube, two mouths, pin-hole central) was usually in excess of this, but the two mouths are confusing.

Fine slits cut in wax behaved similarly to the pin-holes above mentioned, producing pressure for a salient wedge, and the reverse. The endeavor to produce the same effects with rigid slits, however, led to no results.

A pin-hole made with two pairs of knife-edges (slits) at right angles to each other, leaving a square hole between, always behaved negatively, probably because a slit underlay the hole. Its negative sensitiveness, however, was ordinary (10 scale-parts). It is quite possible that the wedge angle (figs. 130, 131) plays a part quite as important as the size of the hole, though I have been unable to get definite evidence on this question. Frequently, however, holes successively reamed with a needle will show no further increase of sensitiveness when the use of a 10° or 20° wedge-shaped-point reamer will at once double the deflection.

The case of figure 136a (salient pin-hole), when tested with a reëtrant pin-hole (fig. 133) resonator, supplied the results shown in figure 136c. The maxima and minima are throughout negative. It was supposed that if maxima were negative, minima might be positive; but this is not the case. Both salient and reëtrant pin-holes function in the same way, with an inversion of sign, but without change of sign for the same pin-hole. I have not, as a rule, been able to get more than 10 scale-parts for a reëtrant pin-hole, when a salient one gave over 30 scale-parts. The latter was usually found more sensitive.

In the preceding report (Carnegie Inst. Wash. Pub. 310, 1921, § 25) the transition of acoustic pressures into dilatations on the reversal of the conical pin-hole has already been treated in connection with the closed region. Testing this again, a glass cone giving a deflection of 40 scale-parts for pitch a' , if salient, changed this to -25 scale-parts of pitch a' when reëtrant. In pitch c'' the deflection increased (negatively) to -50 scale-parts, the salient cone showing no deflection in pitch c'' . I was not prepared, however, to find

copper foil, punctured from the outside with a fine needle, behaving similarly; but I obtained with the same adjustment -40 scale-parts of pitch a' , while the reversed foil gave $+40$ scale-parts in pitch $b'a'$.

The important question is thus adduced as to whether the pin-hole resonator with a reversed cone (fig. 137) will also produce a reversal of fringe deflections. The f'' resonator, with f'' closed organ-pipe, was therefore tested, the pin-hole cone being joined to the U-gage by a short end (20 cm.) h of rubber tubing. On trial an affirmative answer was obtained at once. Thus

Resonator.	Cone salient.	Cone reëtrant.
A.....	Deflection 60 s. p.	Deflection -20 s. p.
B.....	Deflection 40 s. p.	Deflection -25 s. p.
C.....	Deflection 35 s. p.	Deflection -25 s. p.

The negative deflections were here numerically smaller than the positive deflections, a rule which does not hold generally, however. It was further to be expected that the negative deflection would decrease with the length of connector-tube t from the bottom of the resonator R to the reëtrant pin-hole o (fig. 137), the pipe h beyond being the rubber connection to the U-gage. This was also the case, though I obtained no decided harmonic relations; for example (resonator C, t connector varied, only):

Length of t	5	10	21	46	100	150 cm.
Deflection	-37	-30	-28	-30	-12	-5

Thus the negative pin-hole resonator runs down in sensitiveness much more rapidly if treated in this way (pin-hole remote) than if the pin-hole is fixed in the bottom of the resonator and the connector-tube h of any length communicates with the U-gage. That at 50 or 100 cm. of removal from the resonator there should be such large deflections left is the surprising feature of these tests.

A final attempt to construct conical pin-hole probes was made by catching a small drop of wax at the end of a conical tube and then reaming out the wax with jeweler's broaches. It proved to be an easier method than grinding pin-holes to size, but the success was not remarkable.

In conclusion, a peculiar behavior of the U-tube, after long use, may be referred to here, and that is an exhibition of what is apparently solid viscosity. Thus, when the current of the telephone circuit is closed, the deflection appears instantaneously; but in the lapse of minutes, if the current is kept closed, the deflection gradually increases to a limit (depending on the time elapsed and on the amount of deflection), which may be 10 per cent larger than original deflection. If the current is now broken, the deflection vanishes instantly, leaving the residue of the 10 per cent in question (usually about 5 fringes) to vanish in turn gradually in the lapse of minutes. After this the zero is quite regained. We have thus a case not only of viscosity under stress, but of residual viscosity after stress ceases. Supposing there might be some electrostatic excita-

tion, I ionized the surrounding air with radium, but no effect was produced. The phenomenon can only be associated with the surface tension at the (glass-covered) free surface of the U-tube. I have not been able, thus far, to make these surfaces move quite in parallel under change of pressure. Hence capillary tensions enter in a slight extent to modify the deflections. These tensions are quite permanent in a given adjustment, as is evidenced by the permanence of the zero, even after rather rough handling. But if the gage is shaken by concussion, a new zero appears, to be permanent in its turn. It is thus in connection with the tension of the surface film of mercury (possibly enhanced by the age of the gage) that the phenomenon of solid viscosity functions. If it were also to appear in the case of rigorously pure and fresh mercury, it would have considerable theoretical interest, as the liquid surface here displays (*i. e.*, under interferometer observations) the specific properties of a solid. From a practical point of view, U-gage deflections must obviously be read off at once, so that the zero may be regained throughout.

35. Positive and negative pin-hole resonators coöperating. Paired or twinned resonators.—It was shown in the earlier report (l. c., fig. 33) that if the two shanks of the U-tube are provided with reëtrant and salient conical pin-holes, respectively, no fringe deflection is obtained when the pin-holes lie within the same closed region (T-branch with the telephone at the third end). This was again tested with the same results, the reason being, as it was thought, that pressure and dilatation are balanced in the T-branch and the U-gage remains inactive. (See § 37.)

It is obvious, however, that the positive (pin-hole cone *o* salient) and negative (pin-hole cone *o'* reëtrant) pin-hole resonators, *R* and *R'*, figure 138, must add their respective pressure effects at the gage *UU'*, if they are provided with independent connector-tubes *t* and *t'*. The plate-glass covers of the cisterns *UU'* were therefore both sealed and put in communication with two resonators (*B* and *C*) of about equal strength. The results were: *B* alone $s = +60$; *C* alone $s = -55$; together $s = 110$, the fringes passing beyond the scale limits. A large number of similar experiments all indicated that the separate deflections may be added, at least roughly. The sensitiveness may thus be nearly doubled. It is probable, moreover, that two such resonators will interfere less with each other than two positive or two negative resonators (*i. e.*, two identical resonators, generally), if placed a short distance apart (*cf.*, inset, fig. 139). The data found for the two resonators *B* and *C*, when approaching each other coaxially, mouth confronting mouth, were:

Distance between mouths...	10	6	5	4	3	2	1	0.5 cm.
Fringe deflections	$s = 110$	90	85	80	70	60	35	10

The deflection thus falls off slowly and does not quite vanish, even with a mere crevice between the mouths. If the two resonators are parallel, side by side and mouth near mouth, the conditions are even much better. Thus about $s = 100$ was obtained for this contiguous position, indicating much reduced interference. Hence if *RR'*, figure 138, are placed side by side, and

the connector tubes t, t' are of pure rubber, elongated if necessary, the twin apparatus is available for exploration.

With short connectors tt' in the interest of greater sensitiveness (the loss of deflection was here about 20 per cent per meter of t), I made a number of tests to find the maximum distance for which the twin resonator (fig. 138) would respond. The exploration proceeded in the x direction into the adjoining room and the pipe (resonators at origin) was at each stage moved about in a plane normal to x , to find the conditions of best response. The results are necessarily irregular, the only object being to find the limit. An example may be given:

$x = 100$	150	200	250	300	350	400	450	500	600 cm.
$s = 90$	100	25	33	25	10	15	4	10	1 scale parts

Thus the twinned resonators are able to hear to a distance of 6 meters, the deflection being then about a fringe. No doubt in an unbroken empty room the distance would be larger; but it is none the less suggested that the response of the pin-hole ceases abruptly, beyond a certain threshold limit of intensity.

Although a single resonator with a branched tube is ineffective for reasons already stated, the suggestion of using the bottom of the resonator R , figure 140, as a branch point for the connector tubes tt' to the gage UU' seems highly promising. One and the same resonator here functions in both (positive o and negative o') capacities. The apparatus, in fact, succeeded at once, without, as it seemed, much sacrifice of sensitivity. Figure 141a shows the fringe deflections obtained along y , for the case of short connector tubes tt' (30 cm. long), with the pipe at $x=50, y=40, z=50$, near the center of the table. In curve b the same result is extended with long connector-tubes (130 cm.). In curve c , finally, the results refer to a more distant pipe at $x=100$ cm., $y=40, z=50$. The curves fall off quite abruptly at the two edges of the table $y=-27$ and $+93$ cm. The extreme importance of tuning was noteworthy throughout, mild notes frequently giving large deflections, sometimes out of field, whereas strong notes of the same pitch to the ear dropped the fringes back to zero. A variety of experiments was made with this twin resonator, with results like the above, which need not therefore be reported.

36. Sound filtration. Resonator absorption.—Tyndall's familiar experiments with sensitive flames relatively to the transmission of sound by paper and cloth may in a measure be repeated, though merely for orientation. Subjects of this kind have been so thoroughly investigated by Professor F. R. Watson* and his collaborators at the Engineering Experiment Station of the University of Illinois, in a specially designed acoustic laboratory, that it would be idle to attempt to add to their results with the resonator in its present stage. For this reason a suitable acoustic box was dispensed with and the cloth (like a handkerchief) was laid flat on the table, covering the resonator. There is one difficulty here, inasmuch as any interference at the mouth may change the pitch and a low result be obtained for this reason. The cloth must therefore lie

* Sound Proof Partitions, by F. R. Watson, Univ. of Ill., Bull. 127, 1922.

loosely, with the edges weighted down in the case of paper, etc. A few data may be given as examples:

Full deflection	$s = 30$	Thin ($\frac{1}{4}$ -inch) loose cotton batting....	11
Loose cotton plug in mouth of resonator	0	Coarse muslin (flat).....	9
Loose cotton plug all but withdrawn..	10	Canton flannel	0.1
Handkerchief, flat	12	Filter-paper (flat).....	10
Handkerchief, loose	15	Towel paper (flat).....	5
Handkerchief, puckered	22	Foolscap (flat)	5

Thus, canton flannel is all but impervious to the ear of the resonator, under the conditions.

The marked disturbance produced by one resonator placed within a distance of $\lambda/4$ of another has often been referred to above. The actual effect may best be rated by placing the resonators coaxially with their mouths (one mouth in case of the double or open resonator) towards each other. In figure 139,

with the pipe at $z=40$ cm. above the origin and the mouth of the pin-hole resonator (R in inset) at $x=0$, $z=0$, $y=20$ (i. e., on the table), the mouth of the second resonator R' was moved from $y=20$ (coincidence) to larger y values, both resonators being coaxial in y . The normal deflection in the absence of the second resonator was $s=32$. In case of two closed resonators, the mutual interference did not cease until a distance between mouths of over 12 cm. ($\lambda/4$) had been exceeded. Moreover, the evanescence of the effect seemed to be accelerated between $y=10$ cm. and 12 cm. The open resonator behaves very differently, its interference ceasing appreciably at a distance of 6 cm. between mouths. The far, or third, mouth here dips into other nodal distributions, which render it less effective. It is not necessary that the resonators be adjusted coaxially to obtain similar results. Bags of muslin or filter-paper, tight or loose, tied about the mouth of the resonator, usually cut off all deflection. Sometimes for an intermediate size of bag a small deflection is

observed. Similarly, a pin-hole resonator coaxially within another resonator or a conical flare is at no advantage. The mouth of the pin-hole resonator sunk within a cylindrical tube is always inactive, unless the mouth of the resonator is flush with the open end of the tube or projects beyond it.

Figure 136 gives an example (curve *b*) of the reduced reflection observed when the plane reflector (12 inches broad, 18 inches high) is rotated from the original incidence, $\alpha=0^\circ$. With the plane at normal incidence, $\alpha=0^\circ$, at $y=30$ and the pipe at $z=40$ cm. above the origin, the curve *a* shows the corresponding distribution of nodal intensity found by the resonator moving in y . Thus the minimum is at $y_R=10$ cm. Leaving the resonator in this position, the board was now rotated on its vertical edge, successively, as far as 45° . The minimum gradually vanishes into the free deflection, $s=30$; but the plane is still effective as a reflector or diffractor, until the angle of rotation exceeds 30° . Much depends on the lateral size of the reflector; but I did not pursue the subject further.

37. The twinned pin-holes in branched tubes.—Thus far, all endeavors to obtain registry at the U-gage, by providing the pipes t and t' , figure 140, with salient and reëntrant pin-hole probes within, respectively, and then joining t, t' to a single pipe leading to the source of sound, proved failures. I made a final attempt by attaching the salient and reëntrant conical probes immediately at the U-gage (see inset, fig. 142) and then joining the two to a telephone by a short T-branch. This method succeeded surprisingly well, the success being referable either to the short connection (*i. e.*, to relatively great intensity of sound together with a length favorable to the harmonics) or to a more suitable selection of the aperture of the pin-hole cones.

I may premise that reversed probes, attached to the respective shanks of the U-gage and open to the air, did not respond to organ-pipe or other notes. Placed just within an adjutage of the telephone mouth-piece, both gave a strong positive registry, *i. e.*, without distinction in sign, because of the reversal. In fact, the reëntrant probe happened to be the more active. With the branched tube described (inset), the response was even more marked. The success of the experiment being thus assured, I carried it through the range in pitch given in figure 142, where the inset indicates the connections. The fringe displacement at the gage was now so ample that 5,000 ohms had to be placed in the telephone circuit of the electric siren to keep the fringes within the field. Stopping up the branch t' and opening U' to the air, or the corresponding operation for t' , gave the two lower curves (s, r) on the same diagram. Both of these are positive, since the pin-hole probes are reversed. Apart from some shift of pitch, these individual effects are nearly summed in the resultant curve. Tests with both probes, either salient or reëntrant, gave obscure differential results.

A rubber tube over a meter in length was now inserted at t'' (inset) between the telephone and the gage apparatus (T-branch). With the resistance of 5,000 ohms in circuit, no response whatever was obtainable; but on reducing

the resistance of the telephone circuit successively to 1,000 ohms, the fringe deflections rapidly increased, as shown by the curve *a*, figure 142, for the same maximum note at a pitch near *a'* here throughout. The falling off of the intensity *s* with the current is quite abrupt, so that a former inference that the pin-hole probe ceases to operate practically when a certain threshold value of intensity has been reached seems to be corroborated. A shorter tube insertion of about 30 cm. at *t''* similarly decreased *s* to about 15 scale-parts. Thus, when the telephone in figure 142 (inset) was replaced by an *f''* resonator, it was not influenced by the *f''* organ-pipe, as is the case very strikingly in the adjustments figures 138 and 140.

If the curve *a*, figure 142, be expressed by the equation $s_0 = se^{r/r_0}$ presently to be discussed, where s_0 and r_0 are constants and *s* is the fringe deflection when the impedance of the telephonic circuit is *r*, the results are, $s_0 = 155$; $r_0 = 2,000$ ohms :

<i>s</i> (observed) =	50	15	5	1	s. p.
<i>s</i> (observed) =	49.1	15.5	4.9	0.5	s. p.
<i>r</i> =	1,000	2,000	3,000	5,000	ohms

the agreement being quite as close as the observations warrant. Here s_0 is the deflection for the zero of impedance ($r=0$) and r_0 the resistance to reduce s_0 to $1/e$ of its original value. The relatively small value of r_0 which occurs here is noteworthy. Without entering into the details of the subject, some tests were also made of the effect of lengthening the quill tubes *t* and *t'*, between the pin-hole probe and the U-gage (fig. 142). The results showed not merely a shift of pitch for the high crest, but the indications of definite relation of length of tubes and maximum sensitiveness. Thus the following data were obtained (4,000 ohms in circuit) :

Length of tubes added.....	0	10	13	16	25	100 cm.
Fringe deflection, <i>s</i>	85	70	100	105	70	65

The optimum appears when each of the quill tubes *t*, *t'* is about 16 cm. long and there is a minimum within this length. The usual harmonic conditions must therefore here also be satisfied; however, the reduction for long tubes (meter) is not as marked as when tubes are inserted between pin-holes and telephone.

38. Exponential relations of acoustic pressure to the amplitude of sound-waves.—The strikingly close fit of the equation $s_0 = se^{r/r_0}$ proposed for the curve *a*, figure 142, deserves fuller consideration. Accordingly, a series of experiments was made with the short connection device (fig. 142, inset), by inserting large resistances successively in the telephone circuit and measuring the height of the unique crest near the pitch *a'*. The results are reproduced in figure 143, in which both the data for *s* and log *s* are plotted, in terms of the large resistance *r* in circuit. The curve for log *s* is again definitely linear, but with a break *B* at 8,000 ohms, owing no doubt to some incidental dislocation within the apparatus. It is thus advisable to compute the results in two series, 4,000 to 8,000 ohms and 8,000 to 18,000 ohms, separately. The former is more

accurate, since the fringe deflections are so much larger. The computations gave, $s_0 = 399$; $r_0 = 6,380$ ohms:

$r \times 10^{-3}$	4	5	6	7	8 ohms
s observed	95	65	45	32	22
s computed	94.0	65.5	45.6	31.8	22.1

true to about half a fringe. It will be seen that if the curve had been treated as an hyperbola, the inductive or other additional resistance would have to be negative. In the second case $s_0 = 190.5$, $r_0 = 8,620$ ohms:

$r \times 10^{-3}$	8	9	11	14	18 ohms
s observed	22	17	10	5	1
s computed	22.4	17.0	10.1	4.5	1.6

The constants are different here, owing to the break at B , into the cause of which it is not necessary to enter; but the present constant r_0 , which is the resistance needed to reduce the original deflection s_0 or the equivalent acoustic pressure by $1/e$, departs so much from the preceding value for long tube-connections (fig. 142, *a*) that further elucidation will be needed. A preliminary remark on the equation used may be here in place. It was inferred in the preceding report (Carnegie Inst. Wash. Pub. No. 310, 1921, § 17) that for a given telephone carrying the large external resistance r , the equation $rs = \text{const.}$ fitted the curves within a range of low frequency adequately, when an unbranched quill tube connected the telephone with the **U**-gauge then used. In the present experiments with branched tubes this hyperbolic equation (verified only for low frequencies, $n < 100$) no longer suffices, even if the form $s(r + r_0) = \text{const.}$ were substituted, for r_0 would have to be negative. The new observations were made for different lengths (x) of tubing t'' , inserted between pin-hole probes and telephone. These lengths x were additions to the 19 cm. (about) at the branch tube and 8 cm. at the telephone mouth-piece, which remain fixed, the latter being wider than the quill tubes. The results are given for s and r in figure 144 and for $\log s$ and r in figure 145. The data and constants of the equation $s_0 = se^{x/r_0}$ are recorded in table 1.

As a whole the agreement is as good as the observations warrant, when the fringes do not leave the field or more than a few fringes are in question; nevertheless, more trustworthy constants could have been obtained by judicious inspection of figure 145 than are here deduced by the computations. The trend constant is constructed in figure 146, the abscissas now denoting the added length x in centimeters, while the ordinates are s_0 , r_0 and $1/r_0$. The constants s_0 make up a sinuous line, which, however, definitely decreases as the tube-length x is longer; *i. e.*, the initial fringe deflection or acoustic pressure is largest when very little tube-length x is inserted between pin-holes and telephone. Wave-motion is gradually damped out. Similarly, r_0 decreases when the tube-length x increases, the curve again being somewhat sinuous. In other words, the resistance r_0 to be inserted to reduce s_0 by $1/e$ is much greater for short tubes, as figure 146 as a whole indicates. If we examine the curve $1/r_0$ in terms of x , it would seem that further removal of length of about 20 cm. (*i. e.*, the fixed length of **T**-tube, etc.) should reduce $1/r_0$ to zero

and s to s_0 . A number of tests were therefore made, using perforated corks, short glass T-branches, etc., to reduce x . In all of these, however (see examples a , b , fig. 144), s_0 and r_0 decreased again; or $1/r_0$ increased. The relations for short tubes are therefore as usual harmonic. In one case of the short glass T-tube, for instance (curve a), the deflection s was increased from $s=25$ to $s=42$ (curve b) at 5,000 ohms by inserting an additional 15 cm. of tubing to the telephone. Thus it is improbable that the ranges of sensitiveness contained in figures 144 and 145 can be materially increased by locating the pin-holes closer to the telephone plate.

It is, finally, necessary to consider the case of the unbranched tube, for which $rs=\text{const.}$ was temporarily accepted in the earlier report. The data

TABLE I.—Values $s_0 = se^r/r_0$ for branched tubes.

Pitch	x	$r \times 10^{-3}$	Observed s	Computed s	$\log s_0$ s_0	r_0 $(1/r_0) \times 10^4$
	cm.	ohms				ohms
f'	14	1500	>120	12.5	2.579	
		2000	88	86.5	379	3115
		3000	42	41.3		3.21
		4000	20	19.7		
ba'	25	1000	>130	150	2.508	3000
		1500	110	102	322	3.33
		2000	70	69		
		3000	32	32		
		4000	15	15		
		5000	7	7		
g'	80	7000	1	1.5		
		700	93	95	2.399	1653
		1000	60	62	251	6.05
		2000	15	15		
ba'	105	3000	4	4		
		500	80	78	2.214	1563
		1000	? 28	37	164	6.40
		2000	7	9		
		3000	2	2		
b'	0	4000	< 1	.5	2.601	6380
					399	1.57

newly investigated are given in figure 147, the adjustment being shown in the inset. The graphs a and a' contain s and $\log s$ in terms of r respectively for a salient pin-hole probe; b and b' give the corresponding values for a reëntrant probe, which happens to be somewhat less sensitive. Again, the logarithmic graphs are linear, with a break at B . In case of the salient probe, two sets of constants must therefore be computed, and they are given in table 2.

The agreement throughout is good, the error of the computation seldom exceeding a fringe in s , quite on a par with the measurements. The preliminary low-frequency equation $rs=\text{const.}$ must therefore be abandoned. The constant s_0 shows that a definite initial deflection can not be exceeded for any intensity of sound-wave obtainable from the given source. Moreover, because of the diminutions in s in rapid geometric progression, it is not possible to reach very great sensitiveness with the pin-hole probe. The

result explains absence of registry in case of the pin-hole resonator, when the sounding organ-pipe is a few meters distant, *i. e.*, the rapid decline of the deflections with distance. (§ 35.) Hence, since the amplitude a at the reso-

TABLE 2.—Values of $s_0 = se r/r_0$ for short unbranched tubes respectively salient (g') and reërrant (a').

Pitch	x	$r \times 10^{-3}$	Obs. s	Computed s	$\log s_0$	$1/r_0 \times 10^4$ r_0
g'	short	<i>ohms</i>				<i>ohms</i>
		3.57	98	98	2.564 366	1.60 6250
		4.07	83	82		
		5.07	58	57		
		6.07	38	39		
		7.07	30	30.3	2.365 232	1.25 8000
		8.07	23	22.7		
		9.07	17	17.0		
		10.02	13	13.0		
a'	short	1.12	> 130	131	2.435 272	2.83 3530
		2.12	70	68.4		
		3.12	35	35.6		
		4.12	18	18.6		
		5.12	10	9.7		
		6.12	7	5.1		
		7.12	4	2.6		

nator decreases linearly with the distance x of the pipe, the equation here takes the form $s = s_0 e^{x/r_0}$. Thus a deflection of 146 fringes at close range, falling to 54 fringes at 1 meter, would decrease further to 20 fringes at 2 meters,

7.3 at 3 meters, 2.7 at 4 meters, etc., which is about the order of values obtained in the long-range experiments of § 35. Briefly, therefore, the exponential law seems to belong to the acoustic pressures evoked by the pin-hole probe. We may write $ir = i_0 r_0$ for the effective current i in the telephone, and

regard the amplitude a of the telephone-plate and of the sound-wave produced by it proportional quantities within the limits of observation, so that $i/a = i_0/a_0 = \text{const.}$ If, finally, s is replaced by the corresponding acoustic pressure p , the equation takes the form $p_0 = p e^{a_0/a}$, where a refers to the sound-wave in the quill-tube connectors.

If finally s_0 is constructed as a function of r_0 , both increase together, rather roughly, as far as the break B in the logarithmic graphs, beyond which point s_0 decreases. With this grouping of data, the results of the former report (*l. c.*) are not in harmony. Thus, series 9, figure 18, as there obtained, would require $r_0 = 6,850$ and $s_0 = 78$ to be reproduced by the exponential equation. The old results were obtained by a metal-foil pin-hole, and either this or some other instrumental factor puts them in a different class from the newer results.

It was shown in the preceding report that a reversal of the telephone current (*i. e.*, exchange of the leads of the telephone) may reverse all the harmonics, changing pressures into dilatations, or crests into troughs. In other cases, only certain harmonics were thus reversed. In the present experiments with branched quill tubes (fig. 142, inset), reversals at the chief crest (hb') did not occur; but the intensity was about 10 to 20 per cent greater for one position of the switch than for the other. In such a case, moreover, one of the crests was very steep (hb'); the other a flat or double maximum (a' to hb').

Different common telephones, tested at random (unipolar, bipolar), showed but little difference (within 20 per cent of fringe deflection), the s -displacement depending largely upon secondary details, such as the tight clutch of the plate of the telephone, the efficiency of the break at the primary, frequency increments, etc. Moreover, a given telephone, tested from day to day, in the lapse of time behaves with marked variability, deflections changing irregularly from 40 to 60 scale-parts. This question, though pursued for some time, was not brought to an issue; but it is probable that temperature expansions of the telephone-case change the stress of the telephone-plate and that the motor break varies.

In using the branch-tube (fig. 142, inset), the relative lengths of the parts tU and $t'U'$, tT and $t'T$ are of importance, supposing the branches to be short. When the latter pair was equal, the results were strongest when $t'U'$ exceeded tU by about 4 cm. Thus the deflection for equal branch-lengths was $s = 43$ cm. The 4 cm. excess in question changed this to 63 cm. In fact the effect of a single extra centimeter of length is here quite appreciable, even though $\lambda > 60$ cm.

39. Sensitiveness of telephones. Radio-telephones.—It has just been stated that the common telephones tested evidenced about the same sensitivity; but this was by no means the case with the high-resistance telephones used in wireless telegraphy, as would be surmised. One of these, adjusted as in figure 147 (inset) threw the fringes out of scale. It was necessary to reduce the primary voltage of the small inductor one-half (from 2 to 1 storage-cell)

and insert 10,000 ohms in the secondary, to obtain fringe displacements of reasonable size at the crests.

The use of both together (T, T' , fig. 148, inset) thus produced the most sensitive apparatus so far examined. In the figure, U and U' are the two shanks of the **U**-tube, connected by quill tubes t and t' with the paired telephones T, T' . It is necessary to join t and t' , which contain the salient (p) and the reëntrant (p') pin-hole cones, by the cross-tube a , so that the whole adjustment is **H**-shaped.

With this apparatus (inductor energized by one or two storage-cells) the deflections were:

Position of switch.	r	2 cells.	Pitch.	1 cell.	Pitch.
I	43,000 ohms	$s = 105$	bb'	$s \ 27$	b'
II	43,000 ohms	95		23	

The binaural telephone is thus of a higher order of sensitiveness than the above single telephones. The apparatus, moreover, gives four times as much deflection with two cells as compared with one cell and the switching of the telephone current slightly changes the deflections.

Surveys made between d' and c''' showed the main crest to be near c'' at 13,000 ohms, near b' at 23,000 ohms, and near a' at 33,000 ohms resistance in the telephone circuit. The smaller crests, all very sharp and abrupt near e'' and a'' , behaved similarly. This may be an indication of self-induction in the resistance coils.

The lower crest (b') was particularly studied with regard to its conformity with the equation $\log s_0/s = r/r_0$, one cell actuating the small inductor. The results were as follows:

Switch I:

$r \times 10^{-3} = 14$	23	33	43 ohms	$s_0 = 152$	$\log s_0 = 2.182$
Observed $s = 83$	51	33	20	$r_0 = 49,500$ ohms;	$10^5/r_0 = 2.02$
Computed $s = 79.2$	52.1	32.7	20.6		

Switch II:

$r \times 10^{-3} = 14$	23	33	43 ohms	$s_0 = 203$	$\log s_0 = 2.307$
Observed $s = 105$	65	41	25	$r_0 = 47,400$ ohms;	$10^5/r_0 = 2.11$
Computed $s = 103$	66.4	40.8	25.1		

These data are also given in figure 148, on the left.

It was purposed to carry out the same measurements with two cells in the primary circuit, but the available resistance (except for the data already given) proved inadequate. In place of this, a weaker telephone was installed in the same **H**-branch (fig. 148, inset) and the work completed both with one and with two cells, with the following results:

1 cell:

$r \times 10^{-3} = 3.3$	5.3	7.3	9.3 ohms	$s_0 = 39.6$	$\log s_0 = 1.598$
Observed $s = 22$	17	15	11	$r_0 = 22,470$ ohms;	$10^5/r_0 = 4.45$
Computed $s = 21.4$	17.5	14.2	11.6		

2 cells:

$r \times 10^{-3} = 3.3$	5.3	7.3	9.3 ohms	$s_0 = 112.5$	$\log s_0 = 2.051$
Observed $s = 75$	66	56	52	$r_0 = 34,480$ ohms;	$10^5/r_0 = 2.90$
Computed $s = 75.3$	65.9	57.7	50.5		

These data are also given in figure 148, on the right. They are less smooth than usual, owing partly to the low fringe deflections s and partly to difficulties with the contact breaker; but they fully suffice for the present comparisons. The numerical value of the slope of the lines $\log s$ increases with the sensitivity. The relation of constants s_0 and r_0 is probably incidental and due to secondary causes not fully appreciated thus far.

To obtain a suitable datum for sensitivity in general, it will be necessary to collect for comparison the successive constants s_0 and r_0 obtained. For since $\log s = \log s_0 - r/r_0$, it follows that the deflections of $s=1$ scale-part will occur when the resistance $r' = r_0 \log s_0$ is inserted into the telephone circuit. The data are as follows:

$r_0 \times 10^{-3}$	$\log s_0$	$r' \times 10^{-4}$	Apparatus.
<i>ohms</i>		<i>ohms</i>	
2.0	2.761	0.55	Fig. 142, inset.
6.38	2.601	1.67	
8.62	2.279	1.96	
3.11	2.579	0.80	Fig. 142, inset.
3.00	2.508	0.75	
1.65	2.399	0.40	
1.50	2.214	0.34	Fig. 147, inset.
6.25	2.564	1.60	
8.00	2.365	1.90	
3.53	2.435	0.86	Fig. 148, 1 cell.
49.5	2.182	10.8	
47.4	2.307	11.0	
22.5	1.60	3.60	Fig. 148, 1 cell.
34.5	2.05	7.07	Fig. 148, 2 cells.

Since the constant r_0 is the resistance to increase the deflection s to the value s_0 (usually an excessive value, the full meaning of which on its bearing on the telephone apparatus as a whole is not easily interpreted), r_0 is of no immediate use. The quantity $r' = r_0 \log s_0$, which is the resistance to reduce the deflection to 1 scale-part or fringe, is thus a direct criterion of the sensitiveness of the telephone in a given mounting. Hence it appears that the binaural telephones, used together, are over ten times as sensitive as were the common telephones in the branch mounting used. If a definite degree is to be assigned, all telephones should of course be used singly and in the branch design of figure 142 (inset) or figure 147 (inset) for instance. These measurements were made, but are here omitted.

The data given for r' in the last binaural case shows that r' for 1 cell ($r' = 3.6$) is just about half the value for 2 cells ($r' = 7.1$) in the primary circuit, as it should be. Meanwhile the deflections s (fig. 148, right) are progressively from 3.2 to 4 times larger at the same resistances ($r = 3,300$ to 9,300 ohms), since $\log s'/s = 0.453 + 10^{-5} \times 1.55r$. In this way a rather puzzling observation is adequately cleared up, with a corroborative bearing on the exponential equation accepted.

An interesting observation with the H-branch is the rapid change of pitch with the small differences in the lengths of quill tubes Up , pT , $U'p'$, $p'T'$ and a (fig. 148). Another is the occurrence of very high crests at low notes like c' ($\lambda=127$ cm.), d' ($\lambda=113$ cm.), etc., as well as at c'' , etc. This is curious, as the parts Up , etc., were not, as a rule, larger than 10 cm. It is difficult to conjecture how such long waves are accommodated in the slender tubes. The cross-piece a is by no means indifferent. The following data are an example:

Cross-piece length..... 4 cm.;	pitch c'' ;	$s=80$
8 cm.	b'	75
38 cm.	f'	30
45 cm.	d'	20

If the first (c'') and last (d') of these data be compared, the frequency ratio is somewhat less than an octave. The induced voltage should therefore be somewhat less than 2, whereas the s ratio is 4. The case therefore resembles

the introduction of 1 and 2 cells into the primary, just discussed. In fact, if e , n , i , r , s denote secondary electromotive force, frequency, effective current, resistance, and fringe displacement, respectively, and if the frequency in pitch be regarded as in direct relation to the break-circuit speed, there are four equations available, viz,

$$e = e_0 n \quad e = ir \quad i = i_0 s \quad s_0 = s e^{r/r_0}$$

From these it follows that

$$n = \frac{i_0 r_0}{e_0} \log (s_0/s)^s$$

or, for any two frequencies and displacements, n and n' , s and s'

$$\log s_0 = \log s + ns \frac{\Delta \log s}{\Delta(n/s)} = \text{const.}$$

If the first and fifth ($n=522, 294$), second and fourth ($n=489, 348$), of the above data be combined, $\log s_0$ comes out 2.38 and 2.39, respectively, which is the order of values tabulated above. The whole subject of branched tubes is well worthy of detailed investigation; but there is no room for further results here.

CHAPTER III.

MISCELLANEOUS EXPERIMENTS.

VELOCITY OF LIGHT PASSING THROUGH PARALLEL CATHODE RAYS.

40. First method.—Many years ago I made the experiment of passing one component ray of the interferometer through a long exhausted tube, through which a current was passed, with the object of seeing whether any effect on the beam of light could be detected. The experiment has since been performed elsewhere, I believe, though I have misplaced the reference, with the same negative result which I obtained. Nevertheless, in view of the additional experience and facilities which I now possess, it seemed worth while to try it again.

In the first experiment (fig. 149), if L is a beam of white light from a collimator, $MM' NN'$ the mirrors of the interferometer, N and M half-silvers of equal thickness, and T the position of the telescope, the tube is placed at p . The ends are provided with metallic caps at the two ends and closed with glass plates. At one end is the exhaust-pipe E leading to a powerful air-pump. The induction coil C supplies the current.

The tube was 90 cm. long and about 5 cm. in diameter. Large fringes were installed. The exhaustion was carried to within 3×10^{-3} mm. Many experiments which need not be detailed here were tried, but not the slightest effect due to making and breaking the current could be detected at any pressure. A displacement of 0.1 fringe would certainly have been noticed.

41. Second method.—In the above experiment it is difficult to keep the fringes quiet while the heavy air-pumps are running. Moreover, it is obviously a more searching test if two rays are passed through the pipe p in opposite directions. Accordingly, in the new installation, figure 150 (notation the same as in fig. 149), the self-adjusting interferometer was utilized, M being the only half-silver. Fringes of any size are obtainable by rotating M' on a horizontal axis, if the vertical axis is available for coincidence of slit-images. Very large and superb fringes nearly free from quiver were therefore available and the tests were made at some length.

Again, however, not the slightest effect could be observed on making and breaking the current of the induction coil C , at any exhaustion. This was carried as far as about 4×10^{-4} mm.

The results of this very beautiful experiment are thus quite negative. Naturally, this is what one would have expected as the relative velocity of light c' , in a medium of index of refraction μ , is $c' = c/\mu \pm v(1 - 1/\mu^2)$. Unless v is very large, $c' = c$ if $\mu = 1$. To indicate the extent of the evidence given by the experiment, let c be the velocity of light and $2v$ the effect on c produced

by the twofold current in the tube. Then $2vL/c$, if L is the length of the tube, is the corresponding path-difference, so that

$$2Lv/c = \lambda$$

per fringe. Thus

$$\frac{v}{c} = \frac{5 \times 10^{-5}}{2 \times 90} = 3 \times 10^{-7}, \text{ nearly}$$

Now, as one-thirtieth fringe displacement would certainly have been detected, v/c must be smaller than 10^{-8} .

42. Collinear trains of light-waves and an electric current.—The third experiment tried out by the present apparatus bears upon the possible effect of an electric current on light-waves traveling in the same line. True, the electrolytic results of Kohrausch on the motion of the ions show but very

small velocities. But these velocities are aggregates. It is conceivable that, differentially, electronic motion at much greater speed may be taking place. As it was not difficult to adapt the interferometer for this purpose by replacing the pipe p in figure 150 by a long paraffined trough with glass end-plates carrying a solution of zinc sulphate (at maximum conduction) and the coil C by the lighting circuit of 250 volts, the experiment was duly made. The trough pp' , figure 151, was 100 cm. long and 7×7 cm.² in cross-section and nearly filled with the solution, the terminals EE' being strips of zinc. The current arriving at I through the key K passed through the ammeter A and the commutator R , thence either in one direction or the other through the trough pp' and back to the lighting circuit at I' . Between N and M pass the rays of light of the self-adjusting interferometer, here made quite coincident, but traveling in opposite directions through the trough pp' . These comprise the corresponding part of the ray parallelogram is figure 150.

There is some difficulty at first in finding the fringes, for the solution is not adequately homogeneous and the slit-images blurred. After about an hour or

more the inequalities have subsided and the fringes are now easily found and can be enlarged at pleasure in the way specified. Naturally they are somewhat sinuous, but quite fixed and remarkably sharp, so that it is possible to estimate within 0.1 fringe. When the current is turned on, the sharpness persists unaltered during the first minute or two; but thereafter the fringes usually enlarge, become more and more vague, and eventually vanish. The lapse of an hour or two nevertheless brings them back again for further trials.

The result of this experiment with fine large fringes was absolutely negative. No effect whatever could be detected (the key K was of course on a quite independent mounting from the interferometer) on closing and again opening the circuit. As the current was fully 4 amperes, the flash of spark on opening was long and care had to be taken to prevent arcing.

The degree to which the electric current and the light-ray are independent may then be found as above. Since $v/c = \lambda/2L$

$$v/c = 5 \times 10^{-5} / 200 = 2.5 / 10^{-7} \text{ per fringe.}$$

But considerably less than the displacement of 0.1 fringe would have been recognized, so that an effect is absent well within 10^{-8} . In other words, v can not have exceeded 500 cm./sec.

EXPERIMENTS WITH THE SELF-ADJUSTING INTERFEROMETER.

43. Apparatus.—This apparatus, described in an earlier report (Carnegie Inst. Wash. Pub. No. 249, part iv, 1919, chap. vii, § 54), is so convenient in many cases that it is worth further study. In figure 152, L is a beam of white light from a collimator, and M, M', N, N' the four mirrors, M being a half-silver, the others opaque. M and M' are attached to a common base, which may be slid nearer to or away from the mirrors N, N' , also attached to a common base. In this way the ray systems $abcd, a'b'c'd'$ may be brought to coincidence or separated in any reasonable degree, by sliding the system MM' correspondingly. The four mirrors have the usual plane-dot-slot adjustment, and they may be rotated on a vertical axis. Horizontal coincidence of the two component planes entering the telescope at T is thus secured and the fringes then appear at once. They may be enlarged by slight rotation of any mirror, or even by rotating a single plate compensator around a horizontal axis.

But if common plate-glass is used, while the horizontal coincidence of the rays at T is attained without difficulty in any degree, the accurate vertical coincidence of the two slit-images in the telescope is much more troublesome. It results from the fact that the half-silver M , which is twice traversed by each ray, is slightly wedge-shaped and that the deviation of the two rays is not quite the same. Usually it may be eliminated by rotation of the plate M on an axis normal to its face, until the vertical coincidence is perfect. If this fails another half-silver must be selected.

44. Billet compensator.—The Billet wedge compensator may be used at CC' , for instance, with entire success. Here C is a very sharp-angled, thick, long wedge-plate capable of being displaced vertically by a graduated micrometer screw, while C' is a similar but short and fixed wedge-plate of the same

average thickness as the other. It is usually necessary to rotate the Billet compensator on a horizontal axis, until slight differences of the deviation of the two rays are wiped out and the fringes are again sharp. I have used the two parts of a Babinet quartz compensator in the same way; but here the wedge angles are rather too large, the deviation of the rays not quite the same, and the fringes as a consequence blurred.

The adjustment is thus at once available for the graduation of the displacement of the wedge in terms of light-waves. For this purpose, T is to be a spectro-telescope and the slit of the collimator is made small enough for the appearance of the spectrum lines. If an arc lamp is used there is an abundance of sodium, and these lines are usually in presence. In this case, if the prism edge of the spectro-telescope is at right angles to the direction of the fringes, they will appear sharply for any width of slit. In general, if great accuracy as to wave-length is not necessary a wide slit under conditions stated may be used. A great advantage in relation to the intensity of spectrum is thus secured.

45. Projection.—In the case of white light from the arc lamp or the sun and bright mirrors, the fringes, if accurately produced, as just explained, are (in the absence of the telescope T) caught on a white screen 10 or 20 feet across the room, still clear, though highly magnified. The supernumerary reflection from the rear of M is lateral and may usually be screened off. The fringes are very stable and fully under the control of the compensator CC' . A lens at T may of course be introduced for projection at closer ranges. The collimator itself may be used for focusing the slit-images for small distances, but this does not necessarily focus the fringes.

If now the beam is passed through a direct-vision prism, provided with a lens if desirable, the spectrum fringes may also be projected in the focus of the lens, on a white screen. Naturally, since the slit must be wide to admit an adequate quantity of light, it is necessary that the edge of the prism be at right angles to the direction of the fringes. Since the fringes seen with white light are in the present apparatus nearly horizontal, the prism edge may then be vertical. When the spectrum fringes become inclined, as they are arcs of large circles, the prism must be rotated correspondingly on an axis normal to its edge.

46. Sliding micrometer with split mirror.—The range of the Billet compensator is too small for promiscuous work. A most serviceable micrometer is obtained by separating the mirror M' , for instance, into two parts, $M'M'_1$, with a narrow vertical crevice between them. Both must be capable of slight rotation about a vertical and horizontal (plane-dot-slot) axis. M' , moreover, is on a slide micrometer, with its screw in the direction s normal to the face of M' . The two mirrors M and $M'M'_1$ are on a common base, sliding right and left in the diagram.

This device is thus again almost self-adjusting, as follows: MM' is moved until the rays $aba'b'$ are coincident or nearly so, the spot d' falling on the mirror M' . The slit-images are then brought to coincidence horizontally and vertically, whereupon the (horizontal) fringes appear at once. They may be made of any size by rotating M' on a horizontal axis. The system MM' is then to be slid right or left until the spots dd' fall on the separated mirrors M' and M'_1 . The latter is now to be rotated around a horizontal and a vertical axis until the two slit-images coincide. Finally, the micrometer screw at s is to be given a few turns, so that M' and M'_1 may be quite coplanar. The fringes just selected will appear. The method is very rapid. Both mirrors M' and M'_1 must be firm in mounting; otherwise the quivering fringes may escape detection.

47. Adjustment for reading vertical displacements.—The scope of the self-adjusting interferometer is much enlarged by reflecting the component rays $aba'b'$ in figure 152 vertically together during a part of their path. For this purpose an obtuse-angled silvered prism P , figure 153, is convenient, together with the plane mirror below, at U . The angle ϕ of the prism need not exceed 100° ; but it must be mounted on horizontal and vertical axes, in order that the edge at ϕ may be made nearly parallel to the mirror U and normal to the rays ab .

In much of the present work, U is to be the level surface of one shank of the mercury U -tube, and U' the other, originally nearly coplanar with it. If U' is displaced, as in figure 153, over a vertical distance Δh , the component rays afb and $a'f'b'$ will now contain a path difference of

$$n\lambda = 2\Delta h \cos (\phi - 90) = 2\Delta h \sin \phi$$

where ϕ is the prism angle. If the fringes are restored by the slide micrometer $M'M'_1$ of figure 152, with a displacement of Δs ,

$$n\lambda = 2\Delta s \cos i$$

where i is the angle of incidence. Thus,

$$\Delta h = \Delta s \cos i / \sin \phi$$

i being nearly 45° and ϕ not far from 90° .

The form of prism used is given by figure 154, where F is a hollow prismatic frame of sheet brass, turned up outward at the edges, so that the silvered-glass plates pp' may be secured by steel clips cc' on both sides of the square faces of F . If p remains fixed to the frame, an excess of angle is given to p' by the thrust of the screw q near the center of the side-face. Att' is a plane-dot-slot device, operating in connection with the stiff central spring S . This prism, P , and its fixed mirror U (fig. 155) must be rigidly attached to the basal slide B of the interferometer, as, for instance, by the bent arm r , clamped at k , the distance PU being approximately chosen. P and U are first roughly adjusted with a broad beam of sunlight, sweeping by P normally to the diagram. The direct beam and the one reflected via PUP are then made to coincide on a

distant wall, by utilizing the play available at the elbow, clamps, and other stiff screws of the framework, as well as the plane-dot-slot (three axes normal to each other). The remainder of the adjustment for coincidence of slit-images is done on the interferometer, which should be in adjustment before the element figure 155 is added. With regard to the three orthogonal axes specified, one may note that rotation around y (normal to fig. 155) moves the reflected spot of sunlight up and down, whereas rotation around x (parallel to edge of prism) and around z moves the spot right and left; hence, if the initial rough adjustment is in good symmetry, the y and x axes suffice, and these are given by the plane-dot-slot machine adequately. Unfortunately, on tilting the prism system PU , the rays afb pass into eee or ccc , figure 153. The incident and emergent beams are thus no longer parallel, but inclined symmetrically to the plane of symmetry of the prism.

When the arm (fig. 155), adjusted by sunlight, is placed on the interferometer, the slit-images will usually appear; but they will not be coincident. Since the interferometer itself is in adjustment, the additional correction is again to be made with the plane-dot-slot device tt' . The fringes are then found easily by first using the spectro-telescope, and they may be changed in size by manipulating the horizontal axes of both M'_1 and M' conjointly. If the slit-images are not parallel, but at an angle with each other, the discrepancy may be corrected by rotating the whole frame 155 slightly, on a horizontal axis normal to the figure. This may be inferred from the ray systems ab , cc , ee in figure 153.

To test the apparatus, a thin silvered magnet mirror was placed on U over one of the light spots. The small mirror was slightly wedge-shaped and on calipering showed a mean thickness of 0.100 cm. $= \Delta h$. But the U plate was unfortunately not an optic flat, so that measurements in the two beams, right and left, included the surface curvature. There was also the annoyance from dust, etc., and the fact that fringes could be read much more accurately than the micrometer could be placed. For this reason I merely estimated the angles $i=45^\circ$ and $\phi=97^\circ$. Interferometer values, from $\Delta h = \Delta s \cos 45^\circ / \sin \phi$, were, for example:

TABLE 3.—Self-adjusting interferometer. Factor 0.7124 $i=45^\circ$ $\phi=97^\circ$.

Ray ab	Zero	s	Δs	Δh
	1.0390 cm.			
	(shifted)	0.8898 cm.	0.1504 cm.	0.1075 cm.
	1.0415	.8986	.1416	.1012
Ray $a'b'$				
	(shifted)	1.1894	0.1478	0.1053
	(shifted)	1.1814	.1398	.0996
	(shifted)	1.1836	.1420	.1001
	1.0417			

The difficulty here is attributable to the curvature of the U -plane, which dislocated the slit-images, modified the fringes, and required some fresh adjustment; but the example is adequate for illustration, the individual measurements being true to 0.0005 cm.

To further test the apparatus, I made a **U**-gage out of a shallow cylindrical glass dish (*A*, fig. 156) by cementing a vertical glass partition *B* across it on the sides only, leaving the bottom of *B* slightly above the bottom of *A*. A shallow charge of mercury, poured into *A*, would there establish communication between the two compartments. Light glass plates *CC'* were then floated on the mercury. This form of **U**-gage is objectionable because of the irregular distribution of capillary forces, but it is sufficient for the present purposes. With the slide micrometer already in adjustment, there was no difficulty in finding the fringes, and they were remarkably steady in spite of the mobile mirrors *U*, *U'*. The adjustment for coincidence of slit-images should be made at the prism *P*, at least very nearly.

The work done with this device, being incidental and serving merely as a test of apparatus, is of no further interest.

CHAPTER IV.

GRAVITATIONAL EXPERIMENTS.

48 Introductory.—As heretofore stated, the present research was begun with the object of reading the deflections of the gravitation needle by interferometry, a project which succeeded without great hardship. The real difficulty was found in the endeavor to interpret what had been measured, for constants several times as large as the constant of gravitation were not unusual. Even with a needle of filamentary framework, in a plenum of air, a constant nearly twice the normal value may result, and this in the summer, in a dark basement room, the mean temperature of which changes but a degree or two per day.

Naturally, the disturbing factor is effectively some kind of repulsive radiation, and the simplest working hypothesis is an assumption that the radiation in question is screened off from a corresponding end of the needle by the large attracting mass on the outside of the apparatus. Consequently, the effect of the radiation is a force always in the same sense as the gravitational attraction to be measured. It is with the object of determining to what degree the radiation discrepancy may be controlled or eliminated under ordinary laboratory surroundings that the preceding and the present experiments have been undertaken, with a full consciousness of the difficulties involved.

The large room in which the experiments were made being of importance as a typical environment, it is necessary to point out that the floor was 3 feet below the ground on the east and 10 feet on the west, the embankment sloping downward from side to side. On the east, therefore, merely the heavy basement wall screened off the sun, if present. On the south there was a large building and on the west a double wall, including the embankment specified. Solar radiation thus has the easiest access on the east, and accordingly the morning observations, whether the weather is clear or cloudy, are always largest and much too large as compared with afternoon and night observations. At the same time it is possible that such small temperature changes as occur in the room are most marked in the forenoon. Similarly, the presence of the observer near the instrument, if prolonged, produces a drift of the needle always toward larger deflections.

49. Apparatus, I.—At the end of my last experiments a form of apparatus was installed, capable of exhaustion and containing a gravitation needle made of wires as thin as possible (phosphor-bronze wire, 0.24 mm. in diameter) compatible with the lead weights, m , at the ends. The object of this was to diminish the radiation effect to an inferior limit, always remembering that the

lateral area of the shots m is necessarily in presence. The narrow case (see fig. 157) was a rectangle of brass with reëntrant sides, into which plates of thick glass could be sealed with cement free from air leakage. When the parts of the case were assembled, its inner dimensions were: Breadth 1.2 cm., height 8 cm., length 25 cm., inclosing a needle 22 cm. long. A vertical tube adapted for quartz-fiber suspension arose, in the usual way, from the middle of the top of the brass case. The torsion-head above and the bottom of the tube were sealed against air leakage by the aid of cups, into which resinous cement was poured in the molten state and allowed to solidify.

Each attracting weight M had a mounting quite independent of the case and, acting at one end of the needle only, could easily be moved, between stops, from one side of it to the other by a cranklike arrangement. If M is not too heavy, methods of multiplication are possible.

The half-silver method of reading the deflections of the needle and other details are conveniently shown by the sectional plan (fig. 157), where cc' is the rectangular brass frame holding the glass plates gg' , and mm' is the needle with a mirror at n . M is the attracting weight in one alternative position, determined by the stop on the crank mechanism. The half-silver mirror, adjustably attached to the objective of the telescope T , is shown at H . By means of it the divisions of a glass millimeter scale S , illuminated by a tungsten bulb L shining through oiled paper, are reflected upon n and thence through H into the telescope T . The total distance of S from the mirror n was found by steel tape to be $L=447.5$ cm. with allowance for the kink at H . The advantage of the arrangement lies in the halved distance of the telescope from the scale measured along the reflected beam, so that the magnification is doubled; but in view of the loss of light at the half-silver, the (distant) illumination of S must be intense. If S is long, L must be capable of being shifted along a slide. The presence of the observer at T is not to be prolonged.

The masses of the attracted lead balls at the end of the filamentary needle were $m=0.6295$ gram each, though the quartz-fiber would have been strong enough to hold a larger mass with advantage.

The attracting masses M were of different sizes in the several experiments, the smallest being $M=949$ grams, the largest $M=3,368$ grams, with an intermediate size of $M=2,950$ grams used in connection with the latter. Other masses were used later. They were turned spherical.

To determine the distance R of the center of the mass M from the center of the shot m is a difficult problem. Telescopes were used heretofore, but in the present report the best results were obtained, I believe, by calipering. Thus, in the line of centers of the two positions of M , the outside thickness of the case d , the distance of the outer end of the diameter of M and the outside of the glass face on the further side y , and finally the diameter of the ball z are carefully determined for each of the two positions of M . Then $2R=y+y'-x-z$. In this way it was found that for

$M = 949$ grams.	$R = (8.27 + 8.35 - 2.65 - 5.48)/2 = 4.25$ cm.
3368 grams.	$R = (11.17 + 11.19 - 2.71 - 8.30)/2 = 5.67$ cm.
2947 grams.	$R = (10.81 + 10.72 - 2.74 - 7.93)/2 = 5.43$ cm.

The shot is thus assumed to lie in the middle plane of the case nearly enough to come within the compensation resulting from the two positions of M . Since R enters as a square, it seems advisable not to make it too small. As R^2 increases with M , the advantage of large external masses is not great, particularly as the small masses are so much more easily and certainly controlled.

The moment of inertia of the needle, $2l=22.0$ cm. long, between centers of $m=0.6295g$ has for its main part $2ml^2=152.5$ (gcm.²). To this is to be added the moment of inertia of the stem wire ($\rho=0.0044$ gram per cm.), 3.9; of the oblique wire brace or tie, 1.0; and of the glass filament added for rigidity, 2.0; making a total of $N=159.4=2ml^2(1+s)=152.5(1+0.0452)$ gcm.²

The torsion coefficient of the wire is most conveniently found from the period T_1 of the needle vibrating in a plenum, after reduction to vacuo. This was determined with a stop-watch measuring the interval between two similar passages through the equilibrium position. As T is over 5 minutes, it is necessary that the arc of vibration be relatively large; otherwise the datum is

inaccurate to several seconds. It was found that the vibration which occurs immediately after the case is exhausted is very steady and periods were found as follows:

$$T_1 = 311.5, 310.0, 312.0, 311.5, 311.0, 311.2. \text{ Mean } 311.2 \pm 0.28 \text{ sec.}$$

Discrepancies result from radiation forces and are practically absent from the method as such. As the quantity T_1 enters in the square, this is not as good a result as I expected.

The value of the logarithmic decrement λ was obtained from observation of consecutive arcs of vibration, as shown in the example (fig. 158), where y is the scale-reading seen on the telescope. Unfortunately, the apparatus had been mounted in October of last year and the mirror in the meantime had become bluish from the sulphur sublimate coming out of the packing. It was therefore difficult to read the scale rapidly as accurately as would otherwise have been easily possible, and the results of figure 158 (series 1 and 2) are not very smooth, but they are sufficient for the present purposes. If the successive arcs are called Δy , $e^\lambda = 1.35$ in the first and 1.34 in the (better) second

series, which makes $\lambda = 0.293$ nearly. Hence the true period T of the needle is

$$T = T_1 / \sqrt{1 + \lambda^2 / 4\pi^2} = T_1 / 1.0011$$

The intervals on the stop-watch, moreover, were found too long (on comparison with a chronometer) by the factor 1.0055. This with the preceding equation gives

$$T = T_1 \times 1.0044 = 312.6 \text{ sec.}$$

With the given value of λ , the question may be answered how long it is advisable to wait for an appreciably stationary needle after the position of M is changed. The equation shows that for the n^{th} arc θ_n after the first θ_1

$$\log \theta_n = \log \theta_1 - (n-1) \log \epsilon^{\lambda}$$

The following data suffice for guidance

$\Delta y = \theta = 1$	$\frac{n=1}{2}$	$\frac{2}{0.75}$	$\frac{3}{0.56}$	$\frac{4}{0.41}$	$\frac{5}{0.31}$	$\frac{6}{0.23}$	$\frac{7}{0.17}$	$\frac{8}{0.13}$	$\frac{9}{0.10}$	$\frac{10}{0.07}$	$\frac{11}{0.05}$
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Thus at least an hour should be allowed, and observations taken $T/2$ apart. In the actual case, however, the reduction of arc is for some reason more rapid, for after half an hour the needle is appreciably stationary for any position of M . This is very convenient, though I do not understand the reason. The prolonged presence of the observer at the telescope produced a drift toward larger arcs.

50. Equations.—The approximate equation for the gravitation constant, γ , containing all quantities to be measured when the needle itself is used to find the torsion coefficient of the quartz-fiber, is

$$(1) \quad \gamma' = \frac{\pi^2 N}{L T^2 l} \frac{R^2}{M m} \Delta y$$

Δy being the ultimate double amplitude for the scale-distance L when the attracting weight M passes from side to side of m . T is the period, N the moment of inertia, and l the semi-length of the needle. Finally, R is the effective distance apart of M and m .

Since the stem is also appreciably attracted, a correction must be made for it. If t/τ is the ratio of torques for stem and mass m separately,

$$(2) \quad \frac{t}{\tau} = \frac{\rho R}{l m} (\sqrt{R^2 + l^2} - R)$$

when ρ is the mass per centimeter of the stem. Hence finally the corrected constant is

$$(3) \quad \gamma = \gamma' / (1 + t/\tau)$$

For the case of two attracting masses M' and M'' , one at each end of the needle and coöperating, we should have

$$\gamma = \kappa' \Delta y' = \kappa'' \Delta y'' = \frac{\Delta y' + \Delta y''}{1/\kappa' + 1/\kappa''}$$

If Δy is the observed deflection for both conjointly,

$$\Delta y = \Delta y' + \Delta y''$$

whence

$$(4) \quad \gamma = \frac{\Delta y}{1/\kappa' + 1/\kappa''}$$

κ' and κ'' , being the corrected coefficients, are usually not very different in value.

One may note that for measurements at one end of the needle only, while the R datum is corrected for slight asymmetry of the shot m , the $2l$ measurement is not so corrected. With two balls M' and M'' acting, respectively, on m' and m'' , the l measurement is also corrected for asymmetry. Thus, if the masses M' and M'' are carefully turned spherical, both the R and l measurements are guaranteed.

If in equation (1) we replace N by $2ml^2(1+s)$, where $s=0.0453$, a small quantity, is the ratio of the moment of inertia of the filamentary framework to that of the shot, m vanishes from the equation (except in the corrections), which now takes the simpler form

$$(5) \quad \gamma = \frac{2\pi^2 l R^2}{LM} \frac{\Delta y}{T^2} (1+s-t/\tau)$$

showing that l virtually enters in the first power only. The term t/τ is of the same order as s ($t/\tau=0.020$ to 0.024), so that the parenthesis is a correction but little over 2 per cent. Moreover, *caet. par.*, Δy increases at the same rate as T^2 , the period of the needle.

With regard to outstanding masses at the crank or at the support of M , let R' be the (oblique) distance of the center of gravity of such a mass μ , under the center of M , from the shot m , so that $R'=R/\cos \phi$, where ϕ is the (large) angle of R' to the horizontal. Thus the additional force component in the horizontal direction is

$$f = \gamma \mu m \cos \phi / R'^2 = \gamma \mu m \cos^3 \phi / R^2$$

and its relation to the gravitational pull F of M is therefore

$$(6) \quad f/F = (\mu/M) \cos^3 \phi$$

As the angle ϕ will usually exceed 45° , the weight μ will not enter with more than 30 per cent of its value. By using compensators on opposite sides of the case to the cranks, I assured myself that their presence or absence made no appreciable difference.

51. The radiant discrepancies.—It is so difficult to make a clear exposition of the relatively large thermal discrepancies which vitiate the otherwise admirable performance of the Boys quartz-fiber suspension that I have endeavored to pass in review brief estimates of the different possible activities involved. To do this it will suffice to compute the speed Δv or v of the convection current needed to produce a force equal to the gravitational attraction of a kilo-

gram ball M , for the shot $m=0.63$ gram, at the distance $R=4.3$ cm., occurring in the above observations.

We may begin with Bernoulli's equation, which for the case of constant energy and practically constant density, ρ , may be written

$$\Delta p = -(\rho/2)\Delta v^2 = -\rho v \Delta v$$

If the initial velocity is $-\Delta v/2$ and the final $+\Delta v/2$, we avoid further particularizing as to v and put

$$\Delta p = \rho/2(\Delta v)^2$$

which is numerically the same as if Δp and Δv were suddenly acquired from rest. If a is the lateral area of the shot (0.159 cm.²) and f the radiant force, $\Delta p = f/a$. If $M=10^8$ grams, $m=0.63$ gram, $R=4.3$ cm., $\gamma=10^{-8} \times 6.66$, then the velocity increment (Δv) necessary to produce a force f just equal to gravitational attraction will be ($\rho=0.0013$).

$$(\Delta v)^2 = \frac{2}{a\rho} \gamma \frac{Mm}{R^2} = 0.022 \text{ or } \Delta v = 0.15 \text{ cm./sec.}$$

If the air is exhausted to 1 cm., the speed would be $\sqrt{76}$ larger, or $\Delta v=1.3$ cm./sec. The latter velocity is out of the question under the circumstances of exhaustion at constant viscosity, and this suggests the advantage of using the needle in vacuo; but that convection currents of at least a fraction of the former velocity (1.5 mm./sec.) may be produced in a plenum can not be so easily gainsaid. The side of the shot fronting M is exceptional, as compared with the three other sides, and it is here that Δv is operative. The fatal difficulty, as it seems to me, is, however, this, that Δv may just as well be deducted from a preëxisting velocity as added to it, so that pressure should rise and the radiant forces become repulsive as well as attractive, whereas the general rule is that these forces are always attractive.

If we assume that the radiant force, equal to the gravitational attraction, is the result of a current of air traveling at speed v from shot toward M , the results are very similar, since

$$f = 10^{-6} \times 2.3 = a\rho v^2$$

Thus $v=0.10$ cm./sec. in a plenum and the question arises as to the origin of this current, persistent in relation M .

Again, the viscous drag, across a sphere, of strength f , requires a speed v given by

$$f = 10^{-6} \times 2.3 = 6\pi\eta rv$$

where $2r=0.45$ cm. is the diameter of the shot m and $\eta=170 \times 10^{-6}$ the viscosity of air. Thus

$$v = 10^{-3} \times 3.15 \text{ cm./sec.}$$

This datum is very interesting, showing that a relatively small velocity of air current would drag the ball with a force equal to the gravitation applied, and that the two preceding cases are insignificant by comparison. Moreover, f here varies as the first power of v .

But the difficulty of accounting for a current which moves persistently from m to M remains the same as before. The effect of the two gravitational fields, that of M and that of the earth, can hardly be conceived as other than static, resulting in slightly displaced isobars, no longer quite plane. One may be tempted to invent a mechanism for a slow current as follows: In consequence of the attraction of M , the air within the case near it is proportionally slightly more dense. One may imagine this air sinking as a consequence, in response to the earth's gravitational field. As the air sinks it is removed from M and consequently expands isothermally with absorption of heat from the environment. As fresh air replaces the denser air which has sunk, the general result is a circulation directed from the shot m toward M and maintained by heat energy. Nevertheless, even if such a mechanism were feasible, it would fail to clearly suggest the dependence of the radiant forces on the temperature changes of the environment.

As a final estimate, one may refer to the light or radiation pressure screened off by M , which simple mechanism has been assumed above for convenience. If p is this pressure, $f = ap = 0.16p$. In the case of full sunlight the energy received is 3 gcal./min. or $10^6 \times 2.1$ ergs./sec. As this resides in a prism 1 cm.² in section and 3×10^{10} cm. long, the energy per cm.³ is 7×10^{-5} ergs., and therefore

$$p = 7 \times 10^{-5} \text{ dynes/cm.}^2 \text{ or } f = 1.1 \times 10^{-5} \text{ dynes}$$

This is about five times as large as the gravitational pull ($10^{-6} \times 2.3$) postulated above. In other words, an amount of radiation equivalent to one-fifth of the full solar delivery would have to be screened off by M , which is itself not a cold body. It does not seem that, in a closed room, anything like this can be (differentially) available. Moreover, there is here no clear suggestion why the data should be improved by measurement in a partial vacuum.

In Nature (vol. 108, 1921, p. 40) Professor Boys discusses the same subject, with copious references to original experiments of Bennett (1792), Cavendish (1798), and Joule (1862). It is interesting to note that Cavendish, in spite of his relatively enormous apparatus, encountered similar radiation discrepancies and was fully aware of their nature. Boys and Briscoe (Phil. Mag., vol. 31, p. 59, 1891), in their remarkable development of Joule's convection radioscope, counteract extraneous radiation by a rotating envelope. Unfortunately, this device can not be applied in case of the gravitation apparatus. Boys regards the phenomena, as a whole, to result from convection currents, and no doubt in case of a plenum they largely do so. But inasmuch as the phenomena persist in large part, even at the highest exhaustions applied (within 0.003 mm.), it is difficult to believe that convection can still be operative.

In the light of the above estimates, we may state a probable case conservatively, as follows: During the daytime, M is progressively hotter than m . The result is a circulation of air within the case from m towards M near the bottom and up on the inside of the glass face nearest M . The shot m is

dragged by this current. At night, M is progressively colder than m , and the result is a reversed circulation downward on the glass face and in the direction from M to m near the bottom of the case. In a partial vacuum, while the viscosity of air remains constant, its mass or inertia diminishes, and hence the circulation is reduced. Thus the radiant forces are much smaller.

Nevertheless there are a variety of reasons which make it difficult to accept this view. It is hard to believe that two bodies, M and m , in a room of nearly constant temperature and but 4 to 5 cm. apart, can persistently differ sufficiently in temperature to evoke an air current in the case of the kind required. Again, M acts about as effectively when it is behind the case (between pier and case) as when M is in front, while the solar radiation with which M reciprocates, filtering through the walls of the room, does not come from the rear. Finally, the detailed behavior of the radiant forces does not quite conform to such a straight forward hypothesis. We may therefore regard M as a body screening off external, effectively repulsive radiation from m , at least for convenience of expression.

52. Observations. Small attracting mass $M=949$ grams.—The constants entering equation (1) for the small body were

$$\begin{array}{llll} N=159.4 \text{ gcm.}^2 & T=311.6 \text{ sec.}^* & R=4.25 \text{ cm.} & M=949 \text{ g.} \\ L=447.5 \text{ cm.} & l=11.0 \text{ cm.} & \rho=0.0044 \text{ g/cm.} & m=0.6295 \text{ g.} \end{array}$$

Hence

$$\gamma' = \kappa' \Delta y = 10^{-8} \times 9.95 \Delta y$$

In equation (2) the ratio becomes

$$\frac{t}{\tau} = \frac{0.0044 \times 4.25}{11.0 \times 0.63} (11.79 - 4.25) = 0.02035$$

Hence in equation (3)

$$\gamma = \gamma' \frac{1}{1.0204} = 10^{-8} \times 9.753 \Delta y$$

Hence the deflection to be anticipated should be

$$\Delta y_0 = \frac{\gamma}{10^{-8} \times 9.753} = 0.68 \text{ cm.}$$

The observations (M operating at one end of the needle only) made in the daytime and even at night on or after a fairly clear day were, as usual, much too large, as shown in figure 159, the needle being in a vacuum of a few millimeters. The points show the successive observations with M alternately in front and to the rear of m . Forenoon, afternoon, and night observations are distinguished by A , P , N , and little circles mark the beginning and end of each series. S denotes sunshine and C cloudy. The values of the double amplitudes Δy are attached to each series. $\Delta y_0'$ ranges as high as 0.80 cm. and the night value is not below 0.70 cm. It may thus be necessary to use the night of

* Earlier datum, left uncorrected here, as it suffices for the paragraph.

cloudy or rainy days to obtain more approximately correct values ($\Delta y_0 = 0.68$). Both elongations drift somewhat, and it is probable that in the course of a more prolonged interval of measurement both values would have been obtained, the present set being at best 3 per cent too large in their γ value. I did not, however, persevere, as I had but one of these balls ($M = 949$ grams). The l asymmetry is thus left uncompensated, which would in any case leave the interpretation in difficulty. Finally, $\Delta y = 0.7$ cm. is too small to furnish more than a corroborative value.

This relatively light ball M is interesting, as it is so easily moved and may therefore be used in a method of multiplication. If λ is the logarithmic decrement, and x the maximum deflection obtainable when the ball M is passed from side to side synchronously with and in a way to continually amplify the motion, we may write

$$(7) \quad e^\lambda = \frac{x + \Delta y/2}{x - \Delta y/2} = \frac{2x + \Delta y}{2x - \Delta y}$$

as the M effect, in this case, is merely to change the position of equilibrium and Δy is a double deflection in $\gamma = \kappa \Delta y$. Hence

$$(8) \quad \Delta y = 2x \frac{e^\lambda - 1}{e^\lambda + 1}$$

In view of the long period of over 5^m , the method is tedious; but with the above $e^\lambda = 1.34$ and $\Delta y_0 = 0.69$,

$$2x = 0.69 \frac{2.34}{0.34} = 4.8 \text{ cm.}$$

nearly. I satisfied myself as to the feasibility of this procedure by actual trial, but did not complete it, because of the eye-strain involved in reading the scale reflection in the dull mirror already specified.

53. Observations. Large attracting mass $M = 3,368$ grams.—The crank mechanism was still rigid enough to control this massive body without additional complications. Greater care had to be taken with the adjustment at the stops. The constants entering equation (1) are the same as before, so far as the needle is concerned; the new data are

$$M = 3,368 \text{ grams.} \quad R = 5.67 \text{ cm.}$$

Hence the following relations result :

$$\gamma' = 10^{-8} \times 4.960 \Delta y$$

In equation (2)

$$\frac{t}{\tau} = 0.0241$$

which in equation (3) gives

$$(9) \quad \gamma = 10^{-8} \times 4.844 \Delta y$$

so that the deflection along a horizontal scale should be

$$\Delta y_0 = 1.375 \text{ cm.}$$

Similarly the computation by equation (5) gives

$$\gamma = 10^{-8} \times 4.841 \Delta y \text{ and } \Delta y_0 = 1.376 \text{ cm., since } (1 - s + t/\tau) = 1.0212$$

The observations (mapped out consecutively, as in § 52) were begun (fig. 160) in a plenum of air on June 21. The consecutive points give the scale readings at intervals, one-half to three-quarters of an hour after turning, for alternate positions of the attracting mass M . The mean values of Δy are attached to each group of forenoon, afternoon, and night observations. Since the anticipative value of Δy_0 is but 1.37 cm., the plenum deflections ($2^{\text{h}} 62^{\text{m}}$ a. m., $2^{\text{h}} 44^{\text{m}}$ p. m.) are liable on a clear day to reach almost twice the normal value, and this in spite of the filamentary framework of the needle.

The conditions are at once improved in the observations with the needle, in a vacuum of a few millimeters, on July 22 ($\Delta y = 1.52$ a. m. and 1.45 p. m.), though clear weather still prevailed.

The improvement is enhanced on July 1, 2, 3, when the sky was heavily overcast, with much rain (R), viz,

	July 1.	July 2.	July 3.
Forenoon (A).....	$\Delta y = 1.48$	1.44	1.42
Afternoon (P).....	1.46	1.36	1.45
Night (N).....		1.35	1.34

The morning deflections almost always exceed those of the afternoon, owing, as already intimated, to the structure of the building, which has but one thick wall in the east and two with an embankment of earth in the west to shut off solar radiation. We may conclude from figure 160 that the apparatus is seeking thermal equilibrium and eventually attains or even surpasses it. In other words, it appears as if at night the radiation discrepancy was reversed and became negative, for Δy is often below the normal value. It will be seen that these low values are associated with excessive drift; thus, on the night of July 3, the first doublet is $\Delta y = 1.37$ cm., followed by a sudden displacement at the lower reading. A similar drift spoils the afternoon observation on the same day. Furthermore, as the weight is on one side only, the l asymmetry is not eliminated. For these reasons these observations, like the preceding, were continued but a few days, for corroborative evidence.

From another point of view, the set of the needle for the night observations is truly remarkable. The difference from the normal deflection is but 0.2 to 0.3 mm. on an effective radius of 8,950.0 mm.; i. e., an average angle of $\frac{2}{89,500} = 0.000028$ rad., or about 6" in the twist of the quartz-fiber is in question.

54. Observations. Two large attracting masses M' and M'' coöperating.—The pull of these masses being exerted at the two ends of the needle, respectively, the advantages of equation (4) accrue to the work. The first of the masses was the same as that used in § 53, so that in $\gamma = \kappa' \Delta y$, $\kappa' = 10^{-8} \times 4.843$. The second was a little smaller, viz, $M'' = 2,947$ grams and $R = 5.43$ cm. The latter was determined by calipering as explained, so that $R = \frac{1}{2}(10.81 + 10.72 - 2.74 - 7.93)$, 7.93 cm. being the diameter of the ball and 2.74 cm. the outside thickness of the case. Hence the approximate value of κ'' is $10^{-8} \times 5.198$. The value of t/τ is 0.0236, so that in view of the factor $1/1.0236$

$$\gamma = \kappa'' \Delta y = 10^{-8} 5.078 \Delta y$$

and the normal deflection for this ball alone should be $\Delta y_0 = 1.31$ cm. Again, since $1 + s - t/\tau = 1.0453 - 0.0236 = 1.0217$, equation (5) gives

$$\gamma = 10^{-8} \times 5.076 \Delta y \quad \text{and} \quad \Delta y_0 = 1.31 \text{ cm.}$$

If the balls are taken together and act in the same sense for the deflection Δy ,

$$(10) \quad \gamma = \frac{\Delta y}{1/\kappa' + 1/\kappa''} = \kappa \Delta y = 10^{-8} \times 2.479 \Delta y$$

which is practically the fourth part of $\kappa' + \kappa''$. The normal deflection should be

$$\Delta y_0 = \gamma/\kappa = 2.687 \text{ cm.}$$

which is the same as

$$\Delta y' + \Delta y'' = 1.375 + 1.312 = 2.687$$

The observations, begun on July 7 and continued for some time, three times daily, forenoon (*A*), afternoon (*P*), and night (*N*), with periods of from 30 minutes to 1 hour between consecutive observations, are given in figure 161.* The ultimate set of the needle at any observation was remarkably consistent, the reading being repeated at least twice, $T/2$ seconds apart. In fact, the decisive set of the needle in so delicate an instrument is profoundly impressive, the only difficulty being to decipher what is being so faithfully recorded. The scale-readings also were now clear to 0.1 or 0.2 mm. It is necessary, however, for the observer to keep as much as possible away from the instrument, even though sitting in a symmetrical position as remote as possible. On loitering near the telescope for some time (15 minutes and longer), gradual deflections

* To secure better vision the scale was slightly shifted on July 13.

of 1 mm. or more may be recognized, due no doubt to bodily heat radiation, as they gradually quite vanish again.

In figure 161, the mean value of Δy is inscribed on each group of observations. As the normal value should be $\Delta y = 2.69$ cm., it is obvious that only the night (*N*) observations are acceptable. On July 7, the apparatus, which had just been set up, was slowly approaching thermal equilibrium. On July 9, during the period of exceptional rain-storms, there was a dense fog at night, which may account in some way for the high night value. On other nights the needle showed a tendency to drift slightly, always toward larger deflections. The following is the mean record for 10 days:

June.	7	8	9	10	11	12	13	14	15	16
	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.
Forenoon (<i>A</i>)...		2.90	2.77	2.73	2.79	2.72	2.94	2.94		2.98
Afternoon (<i>P</i>)...	2.84	2.79	2.74	2.79	2.71	2.74	2.77	2.76	2.66	2.76
Night (<i>N</i>)	2.71	2.67	2.74	2.66	2.67	2.68	2.66	2.69	2.66	2.70
Weather	C	C	R	R	C	C, S	S	S	C, R	S

On July 15, a cloudy day of even temperature with a mild rain at the end, the data were nearly normal throughout the day. This is in strong contrast to the data of the clear day following, in which the radiation conditions were bad. Nevertheless, the last night doublet is $\Delta y = 2.66$ cm., so that further observations would have been normal. Similarly on July 14 the last night doublet is $\Delta y = 2.67$ cm.

If, therefore, we exclude the first night datum, as the apparatus was too fresh, and the exceptional datum during the fog of July 9, and take the last doublets specified, the mean of the eight deflections is $\Delta y = 2.666 \pm 0.0017$ cm. Therefore

$$\gamma = 10^{-8} \times 2.479 \Delta y = 10^{-8} \times 6.609$$

which is as near the normal value as one may hope to get, seeing that the small individual deflections, Δy , must be read off on a millimeter scale. If the values for the night observations (excepting June 7, 9, 16) be taken as they stand, the mean is

$$\Delta y = 2.674 \pm 0.0032 \quad \gamma = 6.629$$

Night observations made on an exhausted gravitational apparatus therefore seem to give promise of trustworthy results, particularly if a season of uniform temperature is chosen and a relatively finer quartz-fiber is inserted. While the exhaustion here in question has not been carried further than a few millimeters, the experiments are all aiming at the final test under complete exhaustion.

55. Apparatus II. New thin quartz-fiber.—The encouraging results obtained in the last paragraph indicate that for further development a much more sensitive quartz-fiber suspension will have to be installed. Search was therefore made for a finer quartz filament and one was eventually found giving a deflection Δy of over 13 cm. with the same needle and attracting bodies similarly placed. In compensation for this advantage, however, the period T of the needle was over 12 minutes and the logarithmic decrement very large (about $\lambda=0.9$), so that the determination of T was to be the marked difficulty encountered in the work.

The case of the apparatus was new, but consisted of a recessed brass rectangle like the preceding, closed with thick glass plates, sealed in place. Resinous cement, in the molten state, was poured into the crevice between glass plate and brass rim, and subsequently remelted several times locally, with a hot, flat metallic strip, to obviate leakage. Yet with all the care taken, it was not easy to get the case quite tight.

The needle being the same as before, the mass of each shot was 0.6295 gram and the length of needle (determined by a distant telescope and new scale) $2l=22.05$ cm., the moment of inertia $N=159.4$, about as above.

The masses and distances between centers for the two attracting bodies were

$$\begin{aligned} M' &= 3.368 \text{ grams.} & R' &= \frac{1}{2}(11.63 + 11.58 - 8.29 - 3.12) = 5.89 \text{ cm.} \\ M'' &= 2.947 \text{ grams.} & R'' &= \frac{1}{2}(11.20 + 11.12 - 7.92 - 3.13) = 5.63 \end{aligned}$$

the latter determined by calipering, as above explained.

The scale-distance from mirror, found by steel tape, was $L=445.8$ cm. The same half-silver device for reading was employed, a movable frosted tungsten lamp, sliding behind the distant glass scale, to illuminate the particular part of it needed in the reading.

As the deflection Δy is large, it was sufficient to pass the weight M from one side to the other, to obtain arcs large enough for the measurement of the logarithmic decrement. An example of two such experiments is given in figure 162, the needle being in a vacuum of about 0.5 cm. The needle comes to rest after about eight semiperiods, practically therefore within an hour, so that this time will have to be allowed between the readings for Δy .

The logarithmic decrement is unfortunately no longer constant. If computed from observations 3 elongations apart, the values come out for the two graphs

$$\begin{array}{cc} e^{\lambda}=2.41 & e^{\lambda}=2.44 \\ 2.44 & 2.45 \\ 2.48 & 2.53 \end{array}$$

As the period T is to be determined from the first two arcs on passing the weight M from side by side, the corresponding values of $e^{\lambda}=2.47=2.49$ were computed. These make $\lambda=0.909$ on the average, and hence the period is

$$T=T_1/\sqrt{1+\lambda^2/4\pi^2}=T_1/1.010$$

T_1 being the damped period. This is to be increased by the correction, here 1.0056, for the stop-watch, so that $T=T_1/1.0047$.

The value of T_1 , like the value of λ , is variable in a vacuum of a few millimeters to an unexpected extent and therefore very difficult to ascertain, possibly even with a complete and rigorous exhaustion. For this I was not quite ready. The method here, as above, consisted in finding with a stop-watch the interval of time elapsed between two similar passages through the known position of equilibrium. The reading itself was certainly accurate to the fraction of a second, but the periods found were liable to vary from about 12 to 13 minutes, and this apparently quite irrationally. Sometimes two consecutive periods would be nearly the same; at other times quite different. In this dilemma the only recourse seemed to be the very tedious process of determining T_1 for each exchange of the position of the weight M . After this had been done for a time, it was found that the periods depended to a surprising extent on the accurate selection of the position of equilibrium, the error being of an order of 5 seconds per millimeter of departure of the assumed equilibrium from the true position of equilibrium on the scale. Moreover, if y is the true zero and y_T the convenient value assumed in measuring T , then falling from the high elongations, if $y-y_T$ is positive, T is low, rising from the low elongations, if $y-y_T$ is positive, T is high, and vice versa. Fortunately the y 's are nearly known, being the limits of the Δy sought. Hence if $y-y_T$ has the same sign and value at both elongations, the mean T , for the two elongations, is the correct value. After prolonged trial this method also proved to be far from satisfactory. Some independent radiation discrepancy often intervenes, because of which two consecutive observations are respectively too large and too small, so that the mean value of the pair is probably the closest approach to T obtainable, always supposing that $y-y_T$ is very small (1 or 2 mm.). The following example may be cited:

	July 18.				July 19.				July 20.			
High elongation..	12 ^m 29 ^s	12 ^m 35 ^s	12 ^m 3 ^s	12 ^m 22 ^s	12 ^m 12 ^s	12 ^m 4 ^s	12 ^m 11 ^s	12 ^m 18 ^s				
Low elongation..	12 15	12 17	12 11	12 19	12 31	12 48	12 28	12 30				
Mean	12 22	12 26	12 17	12 20	12 20	12 26	12 20	12 24				

Naturally, the final mean, $T=742$ sec., is still quite deficient. Thus, in addition to the high value of λ , variations in the radiating forces during a period are fruitful sources of discrepancy. One may therefore anticipate better

results from the measurement of semiperiods. The motion of the needle is throughout faster, and small departures of the fiducial mark from the true position of equilibrium will represent smaller time-errors. In fact, consecutive values of T were here also liable to be too large and too small, respectively, so that their mean is the closest approach to T obtainable. The value of $y - y_T$, if small, did not seem to show more than a qualitative basis for discrimination. Since there is drift, the actual value of y is not sharply determinable. The following is an example of the mean values of T , found from semiperiods:

TABLE 4.—Doubled semiperiods, $T/2$.

Date.	P	Mean.	N	Mean.
	<i>sec.</i>	<i>sec.</i>	<i>sec.</i>	<i>sec.</i>
July 21.....	746	756	752	753
	766		754	
	736	749	...	...
	762		...	
July 22.....	756	758	742	759
	760		776	
	750	760	774	762
	770		750	
July 23.....	758	753	754	756
	748		758	
	764	755	766	...
	746		...	
July 24.....	750	750	754	766
	750		778	
	758	762	762	772
	766		782	
July 25.....	...	...	761	761
	...		762	
	...	...	770	766
	...		762	
Mean		755		758

On July 21 there was too much drift, so that the results are unsatisfactory. One may note that the high and low individual values often change places. Moreover, even widely different individual T values, if consecutive, are symmetrical to the mean, approximately; for instance (semi-periods),

$$\left. \begin{matrix} 7^m & 4^s \\ 5^m & 53^s \end{matrix} \right\} 6^m \ 28^s \qquad \left. \begin{matrix} 7^m & 19^s \\ 5^m & 30^s \end{matrix} \right\} 6^m \ 24^s$$

These are usually observed in the forenoon and are worthless, both in T and Δy . Obtained in this way, the mean of eight afternoon doublets was $T=755$ seconds and of eight night doublets $T=758$ seconds, both derived from semiperiods. This is much in excess of the datum (742 seconds) obtained from the observation of whole periods above. In this dilemma, the only way out seems to be to adopt temporarily the mean value $T_1=750$ seconds for the damped period, until the apparatus can be rigorously exhausted for the measurement of T .

If the value of $T_1=750$ seconds is thus provisionally accepted, the period is $T=T_1/1.0047=746.4$ seconds.

Finally, the correction for the stem attraction $t/\tau = (R\rho/lm)(\sqrt{R^2 + l^2} - R)$, where $\rho = 0.0044$ g/cm. and $m = 0.63$ gram, comes out $t/\tau = 0.0247$ for the larger M' and 0.0241 for the smaller M'' . The factor $(1 + s - t/\tau)$ in equation (5) thus becomes

$$(1.0453 - 0.0247) = 1.0206 \text{ and } (1.0453 - 0.0241) = 1.0212$$

in the two cases. Further, if $\gamma = \kappa \Delta y$

$$\kappa' = 10^{-9} \times 9.781 \text{ for the larger } M' \text{ and } \Delta y$$

$$\kappa'' = 10^{-9} \times 10.218 \text{ for the smaller } M'' \text{ and } \Delta y''$$

and finally,

$$\gamma = \frac{\Delta y}{1/\kappa' + 1/\kappa''} = 10^{-9} \times 4.997 \Delta y$$

so that the normal deflection is $\Delta y_0 = 13.33$ cm., with the understanding that κ and Δy_0 are merely provisional, like the T_1^2 which they embody.

56. Observations Thin quartz-fiber. Large M.—The new observations are given in figures 163, 164 and were all obtained in a vacuum of a few millimeters (2 to 5). Throughout a single group, forenoon, afternoon, or night (A, P, N), the scale is left untouched, but in different groups slight changes are often advantageous. On July 17, soon after the apparatus was set up, and the following day there is considerable drift, while the apparatus seeks thermal equilibrium with its environment. So the values of Δy (marked on each group) are high, even at night. On July 19, 20, the night observations have apparently become normal ($\Delta y = 13.43, 13.40$). On the following day, July 21, there was a fall of atmospheric temperature and a peculiar behavior of the apparatus, particularly at night. There is excessive drift at the lower elongations. If all observations are taken, $\Delta y = 13.14$ cm., a datum below the normal value and suggesting a reversal of radiant forces. Their effect is here for the first time definitely repulsive, whereas, as an almost uniform rule, they act as an increment to gravitational attraction. If the exceptionally low observation (9.55) is excluded, the mean would be $\Delta y = 13.38$ cm., or nearly normal. If the smaller observation (9.00) is excluded, the mean is but $\Delta y = 12.88$ cm., excessively low. Although there is nothing else against the exceptional reading (9.55), it is an isolated case thus far, though it occurs again below, for it would mean that the usual radiation from without inward turns with quick fall of atmospheric temperature into a radiation from within outward, reversing the radiant forces. Very high deflections are in evidence during the forenoon of July 23 and the radiating forces dissipate slowly. On July 26 (fig. 164), it was thought desirable to restore the position of the scale, on account of the continuous seasonal drift which the needle had experienced. This drift is without consequence, as it is probably due to the slow yielding of the cement by which the hooks on the needle and torsion-head are fastened to the hooks at the ends of the quartz-fiber, to obviate all sudden accidental dis-

placements at these contacts. It was similarly present in early experiments and finally reached a limit. It is always marked in a fresh apparatus, in the beginning. The isolated low value of Δy found on July 21 is again repeated on July 31, after the violent thunderstorm had suddenly cooled the atmosphere. As a result, even the afternoon observations are nearly normal; but the first

doublet at night is as low as $\Delta y = 12.66$, the mean value being 13.24 for all observations. There can be no doubt, therefore, that the radiation forces are here repulsive, a condition which is short-lived and soon vanishes, as in the preceding case. If, therefore, we look at the cause of the usual discrepancy as an effectively repulsive radiation from without, screened off by the mass M (attraction incremented), we here meet with a radiation contributed by the mass M (attraction decremented), because of the deficiency from without, a

sudden fall of atmospheric temperature being in question. Thus the phenomenon is of rare occurrence.

On August 1 and 2 there is actually a nearly symmetrical drift, and the night data found under these circumstances would seem to be the more trustworthy. They make a strong contrast with the irregular drift on August 3, when atmospheric temperatures at night were low and their effect exaggerated by an accidentally open window in the laboratory. The night mean falls as low as $\Delta y = 13.01$ cm., with the least doublet but 12.42 cm. On August 5 the scale was shifted; but low values continue, here as well as on August 6, with

a minimum doublet of 13.29 cm. Low night values are still the rule for the remainder of the curve. These are, in my opinion, due to the relatively cool nights during this season; but on August 4 a new air-pump was installed, giving vacua below 1 mm., and this may have contributed to reduce the data for Δy , though I do not think so.

If we regard the observations on August 1 and 2, under good conditions, again, it is probable that the full-drift curves for an interval of 24 hours would be bow-shaped and concave upward at the upper points, concave downward at the lower points of elongation.

It is curious to note that the diurnal drift (scale reset July 26 because of it) is always larger on a given day at the lower points (where the eastern

mass M' is in front, or on the south side of the case) than at the upper (M' north side of m). The drift below is in the opposite direction to that in which M' pulls, and it is chiefly here that the radiation discrepancy vanishes. This very much resembles a viscous effect, since any given pull of M acts on the viscous deformation produced, left by the preceding opposite pull. The upper points show the reciprocal effect, only less pronounced. The seasonal drift vanishes for old needles (as in the experiments, February 5), but the diurnal drift does not, and it is always more pronounced at the lower points.

When the eastern mass M' is behind the case there is less protection from the outside southeastern (solar) radiation. Since the diurnal drift would respond to such a radiation, we infer that it is rather this mechanism than viscosity which is operative. There is, however, an uncertainty in superposing the diurnal on the seasonal drift. If we suppose the latter to be upgoing for both upper and lower points and superpose on this a diurnal drift upgoing below and downgoing above, we will most nearly reproduce the actual occurrences in figures 163 and 164 or figure 161.

57. Mean results.—Returning to the laboratory in September, I made a few supplementary night observations on the now well-seasoned apparatus. They are given in full in table 5.

TABLE 5.—Example of the values of successive scale readings y and deflections Δy , observed at night.

Date.	Time.	y	Δy	Date.	Time.	y	Δy
		cm.	cm.			cm.	cm.
Sept. 8.....	5h 45m	28.78	13.58	Sept. 12.....	6h 40m	28.30	13.56
	7 15	42.30			7 50	41.88	
	8 15	28.75			8 45	21.35	
	10 00	42.38			9 45	41.89	
Sept. 9.....	7 00	41.58	13.55	Sept. 13.....	6 20	41.70	13.25
	8 00	27.95			7 10	28.46	
	9 00	41.58			8 00	41.72	
	10 00	28.02		Sept. 14.....	7 40	42.58	13.52
Sept. 10.....	6 45	28.25	13.47		9 00	29.06	
	7 45	41.73		Sept. 15.....	6 15	27.30	13.41
	8 45	28.26			7 35	40.84	
	9 45	41.72			8 15	27.55	
Sept. 11.....	7 00	41.82	13.50				
	8 00	28.47					
	9 00	41.82					
	10 00	28.37					

Obtained during a season of more equable temperature with an old apparatus, the earlier of these data for Δy are probably the most trustworthy of the lot, but later divergencies appeared. Whenever the Δy is derived from triplets, etc., it is included in table 6 to find the mean value.

In figure 165 I have constructed most of the data for Δy , investigated both for the morning and afternoon observations (a. m. observations are incomplete and erratic), as well as the atmospheric temperature in degrees Fahrenheit

for the successive days in July, August, and September. Though there are a few divergencies (July 30 for instance), the graphs for afternoon and night run closely alike, the former being naturally above the latter throughout. Correlative peaks and depressions may be picked out in turn. The correspondence with the temperature graph θ is not so striking, though some relation is perceptible. A comparison with the daily change of temperature was also made, but the resemblance is less close. Hence the warm days contribute the larger radiant forces, and they vanish with the day to about the same limit, both in warmer and colder summer weather.

The current deflections Δy obtained in the course of 24 days' examination, together with 8 observations of a later date, are given in table 6.

The forenoon observations, being excessive, particularly on sunny days, were eventually discontinued. The afternoon data are still much too large. The

TABLE 6.—Deflections Δy on successive days. Case exhausted to a few millimeters.

Date.	A	P	N	Date.	A	P	N
	cm.	cm.	cm.		cm.	cm.	cm.
July 17			13.86	August 1	14.50	13.97	13.56
18	14.93	14.18	13.94	2		13.75	13.55
19		13.99	13.43	3	14.10	13.76	13.01
20	13.74	14.03	13.40	* 4		13.46	13.03
21	13.80	13.82	13.38	5		13.65	13.28
22	14.58	13.79	13.26	6			13.48
23	15.15	13.89	13.32	7		13.47	13.32
24		13.93	13.50	8		14.01	13.12
25		13.85	13.33	9		14.33	13.70
26		14.03	13.54	Sept. 8			13.58
27		14.01	13.64	9			13.55
28	14.83	13.91	13.49	10			13.47
29	14.45	13.89	13.57	11			13.50
30	14.20	14.22	13.50	12			13.56
31	14.08	13.65	13.24	13			13.25
				14			13.52
				15			13.41

* New air-pump.

night observations, after the first two days in which a steady state is approached, seem to oscillate about a normal value. The mean values of the 30 night observations after July 18 are, in decades, $\Delta y = 13.429, 13.354, 13.466$ cm., which when averaged* give

$$\Delta y = 13.416 \pm 0.033$$

* Taken individually, the mean error would be 0.0312.

whence

$$\gamma = 10^{-8} \times 4.997 \Delta y = 10^{-8} \times 6.704$$

This result is obtained by introducing the provisional period $T=750$ sec. of § 55, which thus appears to be too small, as the γ obtained is too large. If the mean whole-period observations $T=742$ sec. had been accepted, the result would be larger still. With the mean afternoon half-period observations, $T=755$ sec., the result comes out $\gamma=10^{-8} \times 6.615$, which is about as much too small. For the mean night half-period observations $T=758$ sec., the result, $\gamma=10^{-8} \times 6.563$, is thus much too small. It will therefore be the chief burden of the succeeding paragraphs to endeavor to find a more trustworthy value of T , if possible, and this should apparently closely approach $T=752$ sec.

One may notice in passing that the introduction of the finer quartz-fiber, in spite of the larger deflection Δy , has been of no material benefit in the accuracy of the data for either Δy or T .

58. Higher exhaustion.—The endeavor was now made to reach more perfect vacua, with the object of reducing the logarithmic decrement λ of the needle. The latter was put in vibration by exchanging the positions of the large weights M .

At the stage of exhaustion for 0.1 or 0.2 mm., the logarithmic decrement, $\lambda \log e$, fell for the needle from the original value of 0.385 to $\lambda \log e=0.331$, 0.344, etc., in experiments made on different days. The needle still stuck to the glass face (east shot forward, west shot rearward) for some time after the exhaustion had ceased. It usually had to be gently tapped off and then gradually swung freely. The radiant forces are thus much in excess when the shot are near the glass plates, and the needle, taking always the same oblique position during exhaustion, must be not quite symmetric to the case.

Another curious result was the occasional occurrence of large differences between logarithmic decrements, according as the first swing was from large or from small scale-readings, y ; *i. e.*, according as the eastern M is in front (F , here associated with small values of λ) or in the rear (R , associated with large values of λ). The value of $\lambda \log e$ for the latter case being as above, of the order of 0.34, the former values (readings falling from low y) were even as small as $\lambda \log e=0.28$ in two successive tests.

This shows that the values of λ are in some way closely associated with the occurrence of radiant forces. The successive arcs of vibration sometimes give evidence of asymmetry of field of the same kind. Thus at higher exhaustions and in two consecutive cases (M positions exchanged),

Δy	$\lambda \log e$	Δy	$\lambda \log e$
10.50	0.2638	19.60	0.3454
5.72	.3560	8.85	.2841
2.52	.1634	4.60	.3661
1.73	.2838	1.98	.2714
.90		1.06	
Mean... ..	.267		.317

the needle falling from low and high values of y , respectively. It is obvious, therefore, that the vibration is associated with a persistent drift of the needle, so that arcs would be described with mean velocities, $v \pm v'$, v' being impressed by the drift. The latter is a residuum of the agencies which during exhaustion fix the bodies m to the glass plates of the cases.

The vacuum was now increased to a few thousandths of a millimeter as tested by the McLeod gage (0.001 to 0.005 mm.), and observations made in the course of time to test the leakage of the case of the needle. The mean values of $\lambda \log e$ conveniently found from four arcs $y_1 \dots y_4$ in the form $\log (y_1 - y_2) - \log (y_3 - y_4) = 2\lambda \log e$ were

Time.....	0	1	4.5	8	29	hours
$\lambda \log e$	0.293	0.320	0.352	0.365	0.434	

In another of the many experiments made,

Time.....	0	1	31	hours
$\lambda \log e$	0.267	0.317	0.455	

These results, constructed in curve *a*, figure 167, are discouraging, for neither is the case and appurtenances * adequately tight for exhaustions of this severity nor is the logarithmic decrement small enough to be fully available for computing T . If the pump is kept running with the case open, the needle shows a tendency to stick to the case, and there are other interferences. Experiments of this kind were carried on for several weeks, introducing a great variety of improvements, but not with any appreciably greater success.

59. Telescope at a distance.—In determining the gravitational deflections, there is no objection to the nearness of the telescope to the needle, as the reading requires but the momentary presence of the observer after the position of the needle has been fixed. For this reason the advantage gained by anchoring the telescope on the same pier which carries the case is available. But in measuring the period T and the logarithmic decrement λ , even if both are roughly known, a longer stay at the telescope is necessary. Thus the radiation from the body of the observer becomes appreciable and some of the drift in the above work is due to this cause.

Hence the telescope is to be removed to a considerable distance from the case (8 to 10 feet), and if a stable support can there be located, the appropriate modification of the above method is feasible in the way shown in figure 166. Here S is the scale at the distance $L=447$ cm. from the mirror m in the case C , H being the half-silver. Near it is placed the lens L of long focus ($F=185$ cm.), and at further long distance ($L'=261$ cm.) the telescope T , the line of vision Tm being at right angles to the direction of the impinging light SH . The latter is screened off as far as possible. Without the lens L the millimeter divisions for a telescope of moderate magnifying power (focal length=22 cm.) would be so small as to be useless; but with the lens the definition is admirable and there is abundance of light if S is illuminated by a tungsten burner, as above.

* The leak was eventually located in the rubber-tube connections.

With this arrangement measurements of λ were carried out from five successive elongations y or four arcs Δy . Nevertheless the zigzag march of the consecutive values of λ had not been eliminated. Two series, for instance, gave

$\lambda \log e =$	0.351	0.360
	.328	.316
	.338	.337
	<hr/>	<hr/>
Mean...	.336	.332

but the mean values (found as above) agree more closely than heretofore.

The case was now freshly exhausted to 2.6×10^{-3} millimeters of pressure. Measurements of λ were made from five elongations throughout consecutive days. The mean results of each quintuplet were as follows:

Time.....	0	3	23	37	hours
$\lambda \log e$	0.280	0.276	0.322	0.345	
		.276		.341	

Though the individual values still zigzagged, the mean results are satisfactory and trustworthy. At the outset (0 hour), λ is naturally uncertain

and happens to be larger than the 3-hour value. These data are summarized by the graph *b*, figure 167, and resemble the other curve (167, *a*). The increase of λ is, again, due to the slow infiltration of air through the rubber tubes, which still made up the connections. The graph suggests a method of finding the viscosity of air at extremely high exhaustions, presently to be further developed.

Together with the trustworthy values of λ thus obtained, it was hoped that definite values for T , the period of the needle, would be determinable, but all such attempts, though made with the greatest care, failed utterly of yielding adequate data. The results were no better than those recorded in § 55, table 4, under much smaller vacua, and are therefore not worth reproducing here. The reason for this seems to be that the needle is not vibrating under simple harmonic conditions, for besides the torque of the fiber there are the variable field of radiant forces and probably a not quite uniform gravitational field, by all of which the moving needle is influenced. Moreover, when the torque is so small as in the present case, the two other agencies are more nearly of the same order and the data correspondingly irregular. In the above experiments, § 54, with a thicker fiber, the larger torques and smaller deflec-

tions both produced a more nearly normal environment for the vibration of the needle.

60. Removal of the gravitational field.—This is done by placing the masses MM' in the *neutral* position during observation. The plan is thus to deflect the needle from rest to the maximum elongation by placing MM' in either of the extreme positions and with the needle so deflected for an instant to quickly put the weights in the neutral position again, *i. e.*, in the plane of the case prolonged. It is thus possible to determine both the period T of the needle and the logarithmic decrement, the former by timing successive passages through the equilibrium position with the stop-watch and the latter by reading the elongations. After a group of observations for M in front and M in the rear respectively have been completed, the approximate data for all subsequent series are given, a convenience which greatly facilitates the work. The zero-point was nearly constant. For example (equilibrium at $y=36.00$ cm.),

y	Δy	$\lambda \log e$	y	Δy	$\lambda \log e$
25.55 cm.	14.19 cm.	0.341	46.90 cm.	14.94 cm.	0.349
39.74	5.80		31.96	6.08	
33.94	2.80		38.04	2.86	
36.74	1.27		35.18	1.27	
35.57	...		36.45	...	

The arcs are smaller than in the above work, but the needle comes to rest sooner and the radiation is taken symmetrically from both sides. The periods are found from observations between the first and second and the second and third elongations. So far as the method goes, the values of T and λ are quite accurately found in this way, the difficulty being in the interpretation, as the needle does not swing under simple harmonic conditions. Table 7 is a list of the data obtained, the work being continuous with that of § 59.

The table also contains a second series of data made in October during the few available days in which the steam heat was shut off. In the warmer months it had made little difference whether the needle moved from the front (F , southern exposure) or from the rear (R) of the case in its first and longest arc of vibration. In the second group of experiments, however, there is considerable difference, as is indicated by the letters F and R . In table 7 these give the temporary position of the eastern weight M , which starts the vibrations as explained, or of the first elongation of the eastern end of the needle. Apart from the cold rainstorm at the beginning, the F period is always much larger than the R period immediately following. Since the needle has two ends, radiation forces necessarily act differentially; but in some way the effective radiation urges the needle into increased vibration in the F case and retards it, or acts like an elastic buffer, in the R case. The enormous differences observed are quite astonishing. They are therefore omitted from the discussions below.

The observations for λ , together with those of the last paragraph antedating them, are given in figure 168 (later data crossed) in terms of the time elapsed

after an exhaustion to 2.6×10^{-3} mm. As a whole, they seem to be trustworthy, though strongly modified by the radiant forces. As the air slowly leaks in through the rubber tubes, λ increases gradually from day to day, the vacuum remaining within 0.1 mm. even at the end. Unfortunately, at the highest vacuum applied, i. e., immediately after exhaustion, the logarithmic decrement is still too large for the purposes of precision. The curve suggests that for vacua within 2×10^{-3} mm. λ will decrease rapidly; but it also accentuates the need of the highest vacua attainable. So far as determinable in a heated room, the plenum value of $\lambda \log e$ was estimated as 1.

TABLE 7.—Successive values of the semiperiod $T/2$ and the logarithmic decrement λ , in the morning (*A*), afternoon (*P*) and at night (*N*). Original exhaustion to 2×10^{-3} mm., very slowly decreasing in the lapse of time.

Time.	$\lambda \log e$	$T/2$	Time.	$\lambda \log e$	$T/2$
<i>hours.</i>			<i>hours.</i>		
<i>N</i> 44		6m 43 ^s	<i>P</i> 262	.383	6m 19 ^s
45		3	<i>N</i> 265	.400	5 50
<i>A</i> 61	.341	5 43	<i>P</i> 1.5	.262	6 16 <i>F</i>
62	.349	45	† <i>P</i> 2.5	.279	5 13 <i>R</i>
<i>P</i> 63	.346	5 55	* <i>P</i> 4.0	.272	5 56 <i>R</i>
65	.355	6 15	<i>P</i> 75	.359	6 15 <i>F</i>
67	.354	6 4	<i>P</i> 76		5 34 <i>R</i>
<i>N</i> 69	.355	6 6	<i>N</i> 79	.371	5 29 <i>R</i>
<i>A</i> 85	.370	5 5	<i>N</i> 80	.365	6 31 <i>F</i>
<i>P</i> 88	.363	5 50	* <i>A</i> 92	.381	6 2 <i>F</i>
91	.361	6 21	<i>P</i> 171	.387	5 53 <i>R</i>
<i>N</i> 94	.362	6 1	172	.408	6 31 <i>F</i>
<i>A</i> 108	.381	5 1	<i>A</i> 189	.412	4 38 <i>R</i>
109	.392	5 21	<i>P</i> 190	.408	5 28 <i>F</i>
<i>P</i> 113		6 22	196	.412	6 5 <i>R</i>
* <i>P</i> 115		6 37	<i>N</i> 197	.428	6 25 <i>F</i>
<i>P</i> 162	.380	6 34	<i>P</i> 268	.447	5 12 <i>R</i>
<i>N</i> 166	.390	6 1	269	.457	6 17 <i>F</i>
<i>N</i> 167	.368	6 36	<i>N</i> 271	.443	5 28 <i>R</i>
<i>A</i> 256	.399	4 56	272	.443	6 28 <i>F</i>
<i>P</i> 258	.383	5 20			

* Steam heat on thereafter; later again turned off. All observations made during the prolonged absence of steam heat.

† Cold storm.

Unlike λ , the values for the period T (fig. 168) show no marked improvement as the vacuum increases. They rather vary with the time of day, being smallest in the morning and largest toward the end of the day, and they drop off at night (*A* denoting forenoon, *P* afternoon, *N* night). The latitude of values is enormous, ranging from less than 10 minutes to over 13 minutes. The former cases (low T) are usually encountered in the morning, and probably indicate repulsive radiant forces acting from the plates toward the middle plane of the case, like an elastic buffer, whereby the period is shortened. The large values of T found in the late afternoon might in the same way be supposed to result from radiant attractions, which by a reversed procedure lengthen the period. But this is improbable. In all cases the screening masses M are quite out of the way, and, curiously enough, the zero-point is

relatively steady, changing scarcely 1 or 2 mm., while the static gravitational deflection alone is over 130 mm.

Finally, there seems to be little promise, in case of T , of modifying the unfavorable conditions by higher exhaustions. It is not, therefore, the large logarithmic decrement λ (adequately determinable, but without relation to T), but the radiant discrepancy, which vitiates the meaning of the otherwise accurately measureable values of T . In other words, the effect of the changes of λ on T is nearly negligible in comparison with the enormous fluctuations of T resulting from modified environmental radiation.

Furthermore, whereas the deflections Δy are largest in the morning and least at night, the T values pursue a nearly opposite course, increasing from morning to afternoon, except that the night periods again drop off (fig. 168). Hence the value of γ if computed in the morning with $\Delta y/T$ would be enormously too large, both factors conspiring in the same direction. As the day progresses, both approach normal values, so that with $\Delta y/T$ the γ rapidly decreases. It is rather a pity that there is no compensation in the march of these occurrences. There is, however, one very promising inference to be

drawn from the behavior of T , in its passage through a maximum in the afternoon, as shown in figure 168; for it seems obvious that, both when the environmental temperature is gradually increasing (morning) and when it is gradually decreasing (night), small values of T , the period of the needle, occur. This was also shown strikingly during a violent rainstorm, in which the atmospheric temperature dropped about 20° F. rather suddenly. Periods were observed at that time, as follows:

Exhaustion, 3×10^{-3} mm.	Before storm..... $3^h 15^m$ p. m.	$T = 752$ sec.
	During storm..... 4 15	626
	After storm..... 6 00	712

Under the high exhaustion in question, the T discrepancy can scarcely be identified with anything else than some form of radiation pressure. Since M is out of the way, there is an exchange of radiation between the inner walls of the case and the needle. If the case is warmer, the needle meets repulsive agencies toward the center from the excess of radiation inward. If the masses m at the end of the needle are the warmer, the needle radiates toward the case outward and therefore experiences a repulsive reaction which functions just like the preceding instance of a hotter case. Now, if this field of radiant repulsive force were symmetrical and uniform, no effect would be produced; but if it increases toward the walls of the case, the effect will be virtually

an increase of the torsion coefficient of the quartz-fiber, so that T comes out smaller. That the radiant field is far from uniform is evidenced by the tendency of the needle to stick to the glass plates of the case whenever the latter is freshly exhausted and even when the exhaustions are high throughout. The needle usually has to be tapped free. It is further evidenced by the good γ values obtained when the excursions of the needle end are relatively small, as in §§ 53, 54 ($\Delta y = 1.34$ cm. and 2.66 cm. as compared with $\Delta y = 13.5$ cm.; shot excursions, $\Delta x = 0.016$ cm., 0.032 cm., and 0.16 cm.).

It would then follow that the large deflections are obtained when the radiant field is nearly uniform and that the larger values of the period T are the more nearly correct. Excessive values of T would then correspond to a radiant field increasing from the plates of the case to the needle, and this, as one would naturally suppose, is a rare occurrence, as in figure 168. The inference is also in keeping with the γ values thus obtained.

Finally, the deflections Δy are modified and complicated by the screening effect of the presence of the masses M , which in the T experiments are absent.

61. Estimate of radiation temperature differences.—With the data of the preceding paragraph, it will now be possible to get some idea as to what the temperatures of case and needle between which radiation takes place would have to be. Although the glass plates are impervious to dark radiation from without, the heat will nevertheless accumulate in them, to be radiated in turn from the inside towards the needle. Thus the plates are merely a temporary obstacle to the passage of radiation, eventually acting much like a relay.

Let us suppose that the radiant repulsion from case to needle, the elastic buffer effect explained in § 60, can be expressed as an increment a_r of the modulus of torsion a of the needle. Then, if the two periods with and without radiation forces are T' and T , the equations will be ($N = 2ml^2$ being the moment of inertia of the needle):

$$(1) \quad T' = 2\pi \sqrt{\frac{N}{a + a_r}} \quad T = 2\pi \sqrt{\frac{N}{a}}$$

from which

$$(2) \quad a_r = 8\pi^2 ml^2 \Delta \frac{1}{T^2}$$

m being the mass of the shot and Δ a symbol of finite differences. Furthermore, if f_r is the radiant force at each end and $\Delta y_r/4L$ half the double deflection in radians (Δy_r distance between elongations; L distance of scale from mirror):

$$a_r = 2f_r l / \left(\frac{\Delta y_r}{4L} \right) = 8f_r l L / \Delta y_r$$

If we equate these values of a_r

$$(3) \quad f_r = \pi^2 \left(\frac{ml}{L} \Delta \frac{1}{T^2} \right) \Delta y_r$$

where the quantity in parenthesis is the virtual modulus of the radiant forces, f_r , and Δy_r the double deflection referable to it. To find $\Delta I/T^2$, the least and maximum T found in vacuo in the last paragraph, *i. e.*, $T'=5$ minutes and $T=6.5$ minutes, may be inserted, so that $\Delta(I/T^2)=4.5 \times 10^{-6}$, nearly. As $m=0.63g$, $l=11$ cm., $L=450$ cm., the modulus is 70×10^{-8} , nearly.

To determine the radiation forces in vacuo, it will be necessary to postulate the case of black body, or full radiation, *i. e.*, to assume Stefan's law, so that if θ and θ_0 are the absolute temperatures between which radiation takes place and E the energy radiated (ergs per cm.² per sec.)

$$(4) \quad E = K(\theta^4 - \theta_0^4) = 4K\theta^3 \Delta\theta$$

as $\theta - \theta_0$ is relatively small here. The value of θ may be taken as 300° and $K=5.3 \times 10^{-5}$ is the datum found at the Reichsanstalt. Thus,

$$E = 4 \times 5.3 \times 10^{-5} \times (300)^3 \Delta\theta = 10^3 \times 5.7 \Delta\theta \text{ ergs./sec.}$$

This radiation may at the outset be supposed to reach the needle from a single cm.² of each plate situated on the normal to needle and plate, as shown in figure 169. There are two coöperating hemispheres centered at s and s' , so that $2E$ is contained within a sphere whose radius is the velocity of light. Let ρ be the energy density at a distance r from s or s' , implying

$$(5) \quad \rho = \rho_0/r^2$$

and

$$2E = \int_0^c \rho_0/r^2 \cdot 4\pi r^2 dr = 4\pi \rho_0 c$$

whence

$$(6) \quad \rho_0 = E/2\pi c$$

This energy density ρ_0 is thus the light pressure at a distance of 1 cm. from the element s or s' . If we suppose (as is nearly the case) the shot m of frontal area A to be situated there, the force f_r upon it will be

$$(7) \quad f_r = \rho_0 A = EA/2\pi c = \frac{10^3 \times 5.7 A \Delta\theta}{2\pi c}$$

where $A=0.16$ cm.² Hence

$$f_r = 10^{-9} \times 4.8 \Delta\theta$$

for the square centimeter of plate taken. There is an inconsistency in equation (1) as compared with equation (5), which does not condition simple harmonic motion. It must be supposed, therefore, that for small excursions the equations would not be materially altered.

If now we equate the values of f_r in (3) and (7),

$$(8) \quad 70 \times 10^{-8} \Delta y_r = 10^{-9} \times 4.8 \Delta\theta$$

or

$$\Delta\theta = 145\Delta y_r$$

if but a single square centimeter on each glass face radiates toward m . As there are a number of such square centimeters contributing radiation, a second integration is implied, by which the coefficient of (8) would be decreased 10 or 20 times; but this detail is beyond the purpose of the present estimate. One may conclude, I think, that $\Delta\theta$ and Δy_r are numerically not of the same order, or that an increment of deflection of $\Delta y_r = 1$ cm. should be equivalent to a temperature difference of the order of 10° C. between plates and shot m . This is too large a value to persist in a room of practically constant temperature in the summer. It is thus more than probable that at a vacuum of 10^{-3} mm. or 1.4 dynes per cm.², or a force $1.4 \times 0.16 = 0.22$ dyne on each shot, so small a part as 10 or 20 times the $10^{-8} \times 70 \times \Delta y_r$ instanced in equation (3) may remain fluctuating in value, with small changes in the thermal environment. In fact, from this point of view, a vacuum of 10^{-3} mm. is still an enormous agency as compared with the small radiation discrepancies involved.

The conclusion would, then, be that it is not light pressure but a radiometer effect which militates against the experiments, and that its complete elimination, even at the highest exhaustions attainable, would not be feasible.

62. Measurement of γ in terms of the viscosity of air.—This method was tested in the last report (Carnegie Inst. Wash. Pub. No. 310, § 94) with the rather unpromising case of a straw-shafted needle, but the results obtained were surprisingly good. Uniform motion at a distance from the turning-points is the usual occurrence. For the present needle with a filamentary suspension, the equations for the spheres m and m' are known and the results therefore more trustworthy.

The experiment is made in a plenum of air, therefore, of known viscosity. A maximum deflection of the m balls is produced by temporarily placing the M balls in position. When this is attained, the latter are reversed and the velocity v with which the needle passes through the equilibrium position is measured with a stop-watch. The M balls are then again reversed and the measurement of v repeated, etc. In a plenum, the motion of the needle through the zero-point is practically uniform and therefore the frictional force is equal to the gravitational pull. Moreover, if two points of the scale near to and equidistant from the zero-point are selected for releasing and arresting the stop-watch, the torsion of the fiber acts to the same degree to produce an acceleration and a retardation and is thus eliminated.

The frictional resistance encountered by either round mass m of radius r is by Stokes's equation $6\pi\eta rv$, where η is the viscosity of air. The velocity v of the balls m is

$$v = \Delta x / \Delta t = (l/2L)\Delta y / \Delta t$$

l being the semi-length of the needle and L the scale-distance from the mirror. Δy is the telescopic excursion in the time Δt , symmetrically to the position of

equilibrium, and the latter must first be found by putting M and M' in the neutral plane. We obtain

$$\gamma \frac{mM}{R^2} = 6\pi\eta r \frac{l}{2L} \frac{\Delta y}{\Delta t}$$

whence

$$\gamma = \frac{R^2}{Mm} \frac{3\pi\eta r l}{L} \frac{\Delta y}{\Delta t}$$

If we insert the values for the first weight M

$$\begin{array}{ll} M = 3,368 \text{ grams.} & R = 5.67 \text{ cm.} \\ m = .6295 \text{ grams.} & \eta = .00019. \\ L = 447.5 \text{ cm.} & r = .23 \text{ cm.} \\ l = 11.0 \text{ cm.} & \end{array}$$

$$\gamma = 10^{-7} \times 1.536 \Delta y' / \Delta t$$

For the second weight,

$$M = 2947 \text{ grams.} \quad R = 5.43 \text{ cm.}$$

whence

$$\gamma = 10^{-7} \times 1.609 \Delta y'' / \Delta t$$

Hence, when both coöperate

$$\gamma = 10^{-7} \times 3.145 \Delta y / \Delta t$$

Unfortunately, the steam heat had been turned on permanently in the building and further measurement of γ was therefore out of the question. The position of equilibrium of the needle continually shifted. Nevertheless, I was anxious to test the equation, to see how far the results would diverge from a normal value, even under these extremely unfavorable conditions. The time taken by the needle to pass over $\Delta y = 2$ cm. of scale near the zero-point were

Rightward ... 12	13	15.8	12.2	sec.
Leftward 16	16	15.5	16.0	sec.
Mean $\Delta y / \Delta t = 0.15$	0.14	0.128	0.145	cm./sec.
$10^8 \times \gamma = 4.6$	4.4	4.1	4.6	

These values are of course small, which may in part be due to the rough values of η and r inserted or to an absence of corrections for the framework or the equations. It is chiefly due to the presence of relatively enormous and varying radiant forces. By no other method would it have been possible to get a value of γ under these circumstances anything like as near as this, and the reason is that all radiant forces of fixed direction are eliminated. Those effective are the ones which shift with the weights M .