FOREST
VALUATION

HERMAN H. CHAPMAN

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FOREST VALUATION

BY

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FIRST EDITION

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To
HENRY SOLON GRAVES
FORESTER

IN RECOGNITION OF THE SERVICES WHICH HE IS RENDERING TO THE
NATION AND TO FORESTRY BY MAINTAINING A HIGH STAND-
ARD OF EFFICIENCY AND INTEGRITY IN PUBLIC OFFICE,
AND BY UPHOLDING BOTH THE SPIRIT AND THE
LETTER OF NATIONAL FOREST POLICY

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PREFACE

THE term Forest Valuation, applied to this volume, covers only a portion of the subject of Forest Finance, which is usually separated into two parts, termed, respectively, Valuation and Statics. Valuation deals with the determination of the value of standing timber, both mature and immature, and of forest soil, and the forest as a whole. Forest Statics deals with the comparison of financial results obtained in forestry with those yielded by other enterprises, and compares the relative merits of different methods of treating the forest. This subject is discussed in Chapter VIII. Since Forest Valuation includes the greater portion of the text, and this term is more familiar to American foresters, it has been adopted as the title, in place of the more comprehensive term, Forest Finance.

The literature of Forest Finance is largely of German origin, and has been developed in the latter half of the nineteenth century, by numerous German and French authorities. The standard English text is found in "Forest Management," by Sir William Schlich, Vol. III, Part II, Forest Valuation.* This author follows the continental treatment of the subject. In 1909, a pamphlet on "Forest Finance" was published by C. A. Schenck, Ph.D., Forester to the Biltmore estate,† which gives a synopsis or outline of a series of lectures on the subject. Dr. Schenck's treatment is thoroughly American and contains valuable suggestions for the advanced student of Forest Finance.

Unfortunately, neither of these publications treats the subject in a sufficiently elementary manner to serve as a textbook for students of forestry or for persons desiring information who are not already thoroughly familiar with the economic and mathe-

* Fourth Edition, Revised. Bradbury, Agnew & Co., Ltd., 10 Bouverie St., London, 1911.

† *Inland Press*, Asheville, N. C., 1909.

mathematical principles on which the theory of Forest Finance is based. Forest Valuation deals with periods of time far longer than are considered in most financial calculations. The theory and application of compound interest as a mathematical means of determining value needs a more complete exposition than is usually given even in courses in economics, before the significance of the methods taught in valuation can be grasped. Forest Valuation, while disguised as a mathematical science, is in reality the application of broad economic principles, equally serviceable in the determination of all values.

In order that the reader who is not thoroughly grounded in economics may readily grasp these principles as applied in actual Forest Valuation, Chapters I to IV have been devoted to a summary of economic subjects and tenets, with especial emphasis on interest. The ideas set forth follow the mathematical methods used by many economists and expressed by Prof. Irving Fisher of Yale University.* These chapters will serve the additional purpose of stating the economic basis upon which the discussion of disputed questions in the text is made to rest, and will enable students who disagree with any of these fundamental definitions or principles to substitute those of other authorities more to their liking.

To the general reader and even to the student, the use of formulæ as a means of expressing economic relations is apt to convey the impression that the subject is a difficult one to grasp. The effort to attach the proper significance to symbols must precede that required to reach the mathematical solution. The first difficulty is lessened by the consistent use of symbols to which a definite meaning is attached. An entire chapter (V) has been devoted to an explanation of the derivation of the formulæ of compound interest. A further difficulty has arisen in the past, because the methods of treatment of the subject of Forest Valuation bore no relation to, and were apparently wholly at variance with, the customs and principles of business accounting. An effort has been made in the present volume to indicate

* "Elementary Principles of Economics." The Macmillan Company, New York, July, 1912.

these relations, and enable accountants and business men to harmonize the methods used in computing cost and value for forest property, with the forms and ideas with which they are familiar.

Lastly, the emphasis has been shifted from the determination of expectation value of forest soil, so conspicuous a feature of European Forest Finance, to the consideration of stumpage values, sale values, damages, and other subjects of greater practical interest in this country.

The volume is intended primarily for use as a textbook. But in recognition of the frequent need for information on such subjects as damages, stumpage values and costs of forest production, a consistent effort has been made to treat the entire subject in a manner that may be readily grasped by the average reader, without previous preparation or study.

No problems have been included in the text. These should be supplied by instructors to suit the local economic conditions. The table of logarithms has been included for convenience, to enable the student to familiarize himself with its use in solving problems of valuation.

Acknowledgment is made to Prof. Fred R. Fairchild of Yale University for review and criticism of Chapter X on Taxation, to Mr. J. E. Glass, accountant for John Wiley & Sons, Inc., for suggestions regarding forms for forestry accounts shown in Chapter VIII, and to my colleagues on the faculty of the Yale Forest School for valuable suggestions and assistance.

HERMAN H. CHAPMAN.

NEW HAVEN, CONN.,
October 22, 1914.

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ERRATA.

Page 51. TABLE III.

For "profit" read "loss".

For "loss" read "profit".

Page 134. Lines 12 and 13.

For "this value is $(C' + E') - (C + E)$ in which $(C' + E')$ represents" read "the original value of S_v is $C' - C$, in which C' represents".

Last line.

For " $(C' + E')$ " read "the item".

Footnote.

First line, for "the" read "an".

Second line, for "endures" read "occurs".

Third line, for the entire expression

$$\begin{aligned} & \text{"(E' - E) (1.0p^b - 1) instead of E' - E" read} \\ & \text{"(E' - E) (1.0p^b - 1)"} \\ & \qquad \qquad \qquad \frac{\text{"(E' - E) (1.0p^b - 1)"} }{1.0p^b} \end{aligned}$$

Page 135.

First line,

for " $(C + E)$ " read " $C' - C$ ".

Formula O₂. Change to read

$$L = \frac{Y + S_v + E}{1.0p^{n-a}} - (S_v + E) + (C' - C).$$

Page 157. Formula IIIa.

Instead of " $(E \cdot 1.0p^{20} - E + E') \cdot 1.0p^{10} - E'' + E'''$ etc." read

" $(E \cdot 1.0p^{20} - E + E') \cdot 1.0p^{10} - E' + E''$ etc."

Page 247. For formula O₂ read

$$L = \frac{Y + S_v + E}{1.0p^{n-a}} - (S_v + E) + (C' - C).$$

FOREST VALUATION

CHAPTER I

VALUES

1. Wealth and Property. — Wealth is defined as material objects owned by human beings. Forest wealth consists of forest land and its resources, principally trees, but including all other materials such as forage and game.

Property is the right represented by ownership, and not the material object itself. The benefits derived from wealth make its ownership desirable. The right to enjoy these benefits is indicated by the term property. The satisfactions to be derived from ownership of forest property, aside from the pleasures of sport and recreation, take the form of money income from the sale of forest products.

2. The Standard of Value. — Value is an expression of the relative desirability of different forms of wealth or property in terms of a common standard. The form of wealth accepted as this standard is termed money, and is exchangeable for all other kinds of wealth or property. The present standard in this country is gold. The value of money itself can only be measured by its purchasing power, which means its relative desirability compared with other goods. Changes in the quantity and rapidity of circulation of money cause its purchasing power or value to fluctuate, and a decline in value of money means higher prices for all other property. Prices in the immediate past have risen. If continued, this tendency will be a conservative factor in the valuation of forests; while in contrast, such property as long term bonds will tend to fall in value, because it is redeemable in money.

3. Prices. — A price, when expressed in terms of the common standard of value, is the amount of money accepted in exchange for a unit of goods. Until the actual exchange is consummated the price is not definitely established, but exists merely as an appraisal or opinion in the mind of the prospective buyer or the present owner. Prices resulting from exchanges are agreements between two persons as to the value of a unit of property. Market prices originate in centers of trade and form the basis of numerous transactions. Prices are therefore of human and mental origin and are established by economic causes which influence the *opinions* of the majority of purchasers and sellers. While in the main, prices move in response to supply and demand, yet the fluctuations in price of certain forms of property, such as real estate and stocks, demonstrate that it is possible to create market prices either higher or lower than circumstances justify, by influencing general or public opinion through optimism, misrepresentation, or concealment of facts.

4. Values. — Value may be defined as the price of a given unit of wealth, multiplied by the quantity or number of units. Thus the value of a given quantity of cordwood is the price per cord multiplied by the number of cords on hand. This definition holds good if prices are accepted as the final standard of value. For finished products, especially staple commodities, actual prices established by exchanges may be taken as determining value. But for forms of goods in an unfinished state, or for productive wealth, such as land or growing timber, prices furnish only contributory evidence of value, which in many instances is entirely misleading. The very fact that prices for finished products are the basis of value indicates that the value of productive property must be derived from that of its products. Ignorance of the true productiveness of property, and of the mathematical relation between this income and the value of the property, is the principal cause of divergence between current prices and true value for such forms of wealth as forest lands and young timber.

5. Demand. — The influences which determine prices are twofold, corresponding roughly to the two persons concerned

in the transfer. Demand, or the need of the purchaser, is weighed against outlay or effort on the part of the owner. The necessities and desires of the human mind and body, such as food, clothing and shelter, create the demand for any form of material wealth. Civilized society differs from primitive conditions by the greater extent and complexity of its needs. Values are thus created which have no existence among savages. This fact is strikingly illustrated by the history of the timberlands belonging to Indian tribes. To the American Indians the pine timber was of no value whatever, and their use of the forest was confined to its game and fish, the destruction of which they deeply resented. Title to much of this forest land was given to various tribes by the government. This timber soon came to have a high value to the white population, and the Indians were either defrauded of values which they had failed to appreciate, or found themselves enriched by these same values which they had no part in creating.

6. Effort or Outlay. — Human demand or needs would not alone create values. Materials indispensable to life remain without value when they can be obtained without effort. Property which is worth owning has cost the original owner some effort either for its acquisition or improvement. This influences his idea of its value. A prospective purchaser gauges his price by the possibility of securing a similar article through his own efforts. The supply of finished products is greatly influenced by cost of production.

7. Ownership. — The whole structure of values rests on the institution of private property, for without the right of ownership there could be no measurable value. In a primitive state of society each individual is able, after a fashion, to supply his own needs. But with specialization and division of labor, producers of most forms of wealth need very little of it themselves and must sell it, while the purchasers would be unable to produce for themselves the small quantities they need at anything like the price at which it is sold. Purchasers therefore seek the lowest price, while owners, in order to dispose of their surplus or to increase their output, often underbid one another. With unlimited

competition among producers, prices tend frequently to fall below the cost of production. But where the supply of an article of wealth is limited, and no satisfactory substitutes are available, the owner may demand and receive exorbitant prices, profiting by the needs of the consumers. In such cases there is no relation between prices and cost of production. Famine prices for food are obtained in times of flood or temporary scarcity. Elements which tend to increase the advantage of the owner and the margin between costs and value are: first, absence or monopoly of transportation facilities; second, the elimination of competition, by monopoly of the sources of supply or manufacture; and third, monopoly of markets. In some lines of production this control, through the power incidental to ownership, has curtailed production and has had a marked effect upon prices. Combinations or trade agreements may bring about this result. This condition is manifested in the lumber trade chiefly by the maintenance of local retail prices regardless of the fluctuations of the general lumber market. Rail competition will prevent the possibility of combination on the part of manufacturers of lumber for an indefinite period.

8. Prices of Finished Products. — Since the immediate needs of persons and their satisfaction from material sources create value, it follows (§ 4) that the prices established for goods ready for immediate consumption or use are the basis or source of value for all forms of wealth. Both needs and prices are a matter of to-day. The element of cost, in so far as it affects prices, is completed in the final product. Purchasers and venders are separated into two classes with sharply divided interests, the one obliged to buy, the other with no recourse but eventually to sell. Under such circumstances prices absolutely determine value. The price of lumber, ties, cordwood and other forest products depends to as great an extent upon the demand and markets as upon the cost of production, and in periods of depression, lumber frequently sells for less than this cost. But whatever the conditions, these market prices determine the value of forest products.

9. Prices of Raw Materials and Natural Resources. — Raw materials, such as pig iron or wool, are valuable to owner or purchaser only for further manufacture. A lowering of the cost of production makes it possible to sell these products more cheaply, but scarcity and increased demand may completely offset this tendency and cause higher prices. The value of raw products is derived directly from the price of finished goods. Logs and timber stumpage derive their value from retail and wholesale lumber prices. Property used for production is valuable only because of its products. The value of agricultural land depends upon its adaptability for certain kinds of farm crops, and upon the prices these crops will bring. The market price for lumber is the source of value for forest land.

Prices for land should therefore depend entirely upon future income derived from its products or use. Costs already incurred for clearing or improvements are not considered, for if land is worth either more or less than the cost of clearing, its value and not its cost will fix its price. If two grades of land, one good and one poor, cost the same amount for clearing, they will still sell for entirely different prices. Both the owner and the purchaser of land desire it merely for its future use and income. Sale is seldom forced, since the owner can probably make a living by developing and using the property. The price paid for such property thus depends upon expected income whose value is determined by the prices of the products which the land yields.

10. Future Costs. — While costs already incurred do not determine the value of productive wealth, future costs which intervene between the present moment and the final attainment of income must be subtracted from the value of this income. If it can be shown that the future cost of obtaining income exceeds the future value of the income, the property itself will be worthless. Thus the value of standing timber is the margin left after subtracting from the sale value of the lumber the estimated costs of logging, manufacturing and transporting this lumber to market. At present prices, nearly all standing timber has a stumpage value, but in the past only the most accessible

trees had any value. Wild land or stump land may be worthless under present conditions, in spite of a fertile soil, if it is evident that the labor of clearing exceeds the discounted value of the income from the crops which may be raised. Most land is cleared at an economic loss, and there is a tendency to hold cut-over or stump lands at prices which greatly exceed their real value as reduced by the future costs of clearing.

11. The Element of Time in Values. — As prices and values center upon the satisfaction of human needs in the present moment, the tendency is to avoid delay and reduce as far as possible the period which elapses between effort and satisfaction. Yet this element of time intervenes in every process of production. Hunting and fishing supply food within a very short period, which explains the dependence placed upon this source of living by primitive people. In agriculture, the period which must elapse between effort and satisfaction is extended to cover the crop season, and enough surplus must be produced to last a year. Modern engineering, illustrated by railroad construction, requires still greater delay before any use or income can be derived from the property. If the laborers who construct such works were also the owners, and dependent for their living upon the receipts from freight and passenger service, they would starve long before the completion of the road. The development of civilization is marked by the lengthening of the time intervening between the inception of undertakings and the final realization of income. This time element must always be considered in the determination of present values.

12. Capital. — Men must live while engaged in production, and they are enabled to undertake enterprises requiring time, only when they have enough goods stored up to last them until their completion. The wealth with which to supply them consists of food, clothing and dwellings, back of which are factories, raw products and land. The amount of energy that can be diverted from immediate production of food to the task of clearing unproductive land or constructing other improvements depends upon the surplus wealth available for these purposes. A poor man works a lifetime to clear up a wooded farm and at

first must earn wages by outside labor in order to live. But a person possessed of a few thousand dollars or its equivalent can hire this work done and accomplish it in a short time.

The term capital is used to signify a stock of wealth existing at a given instant of time, and therefore available for future use. All wealth is capital. Efforts to separate wealth into classes, such as that which is intended for consumption, and that used for the production of other wealth, lead to confusion. It is immaterial whether a man's capital is in the form of food, buildings, land or money. He will endeavor to secure by exchange a proper balance between the different kinds of capital he possesses. The exchange of wheat for money, its reservation for seed or its conversion into flour does not alter its status as capital.

13. The Capitalist. — With sufficient capital a man is not only free from immediate personal wants but by employment and the payment of wages, can provide for the daily wants of others. On this basis all modern business enterprises are conducted. The time needed in constructive works may be lessened by increasing the number of persons employed and the capital expended, but only to a limited extent. For crop production, and in the growing of trees, the time required cannot be appreciably reduced. Since it is only by the use of capital that any enterprise requiring time can be undertaken, capitalists make possible the entire fabric of modern civilization, and without this accumulated surplus we would revert to primitive conditions. In forest production the element of time is so important that the trend is toward state and national forestry, for governments represent the accumulated capital of the entire community and can best afford to await the maturing of timber crops by the slow process of growth.

14. Interest. — The present moment is the basis for computing and comparing all values. Although use and income, on which value depends, continue into the future indefinitely, yet no one will provide for future wants and leave present demands wholly unsatisfied. To every one the enjoyment of wealth in the present is valued more highly than at any future

period, and the less capital a man has the greater is his preference for its immediate use. This preference can be satisfied by borrowing. The borrower obtains the benefits he seeks immediately, but the lender is forced to wait until the capital is returned before enjoying it. For this gain in "time value," the borrower pays interest. Upon money loans interest is repaid in money, the amount paid bearing a definite relation to the amount borrowed and to the length of time elapsing before repayment. Since interest and principal are in the same commodity, money, the relative amount and value of the interest can be expressed as a per cent of the principal. This per cent gives the rate of interest paid or earned. The period for which this rate is customarily quoted is one year. If the period intended is less than a year the fact must be clearly stated. High rates of interest may be based on the month, in order to make the rate sound less exorbitant.

Interest, therefore, is the price of time differences or the standard by which the value of immediate possession and enjoyment of income can be determined, when compared with the value of the same income if its receipt is postponed to a future period.

15. Discount. — All future values can be standardized by reducing them to their equivalent present value by means of a given rate of interest. This process of reduction is termed discount. The discounting of money loans or demand notes consists of receiving in advance the value of a sum receivable in the future, less the difference in value due to the time elapsing before this payment is due. In the same way the present value of all forms of property is derived by consciously or unconsciously discounting its future value which takes the form of future income. The process of discounting is therefore the most important element in the valuation of forest property.

16. Compound Interest and Discount. — Interest upon money loans is due and payable either annually or at shorter intervals. When received it may be added to the principal at the discretion of the owner. It may then be loaned and will in turn earn interest. At the end of each period of payment, provided no capital has been withdrawn, and interest has been promptly

reinvested, the total investment will have increased by an amount representing the interest on the total sum on hand at the beginning of the period. A sum earning interest in this manner increases at a compound or geometric rate.

Should the annual interest earned be withdrawn annually, neither the principal nor the interest earned would increase in amount. Should this interest be saved but not reinvested, the total value of principal and accumulated interest would increase by the same amount annually. This is termed simple interest. In neither of these cases is compound interest obtained. Compound interest is paid by savings banks on deposits which remain untouched for periods of six months or more.

Since the earning of interest on money, without expenditure or withdrawal of the earnings, is equivalent to depriving the owner of this capital of all use of his property for the period covered by the loan, the difference in value of the capital at the beginning and end of the period measures the money value of the period of waiting. When money is loaned for periods exceeding one year, this difference in value must be computed by compound interest. Any lesser sum would fall short of the demonstrated earning power of money capital at a given rate of interest. Since compound interest is thus required for money loans it must be accepted also as the standard of time differences in value for all other forms of income.

By means of compound interest at a given rate the exact ratio is obtained by which present values may be converted into future values, or, by discount, future values may be reduced to their present equivalents. When both the present and future values are known, a rate of compound interest may be found which will equalize the difference in time and convert the one value into the other.

17. Sale Value. — The sale value of property is indicated by recent prices for similar property in the same locality. In the absence of actual sales it is difficult to determine present sale value.

Sale values are not uniform for the same kind and quality of goods, even in the same locality, unless there is a complete

general knowledge of current prices and the desire to buy is approximately equal to the pressure for sales. Should the owner stand in need of money he will tend to lower prices, and vice versa. Forced sales for cash or to liquidate assets are made at prices much below the market and establish what are termed wrecking values. The difference in value due to forced sales is greatest with property used in production or for special purposes, of which real estate is a good example. A forced sale of forest lands, especially of young timber with no market value, will seldom secure more than a fraction of the true value of the property.

18. Appraised Value. — Values must frequently be established in the absence of sales, or as a basis for such transactions when previous sales are either lacking or unreliable as a standard of value. Instances are found in condemnation proceedings, in settlements for damages, and in inventories or the valuation of assets for purposes of determining the financial status of a business. The principle underlying appraisals is that the value to the owner, for continued ownership and use, must be determined rather than its possible value for sale, since the owner must not be forced to accept a lower value than that represented by his use or enjoyment of the property. Appraised values, for this reason, frequently differ sharply from sale values, especially in the case of slow-moving assets.

CHAPTER II

OUTLAY AND INCOME

19. Proprietary Accounts versus Specific or Valuation Accounts. — Modern accounting distinguishes two classes of accounts, proprietary accounts and specific or valuation accounts. Proprietary accounts deal with the proprietor or owner's interest, and show what his outlay has been, the resulting income already received and the balance which represents net capital. These accounts show the owner as creditor of a business venture, the credit representing his investments. Expenses are debited and income or profits credited. Such accounts normally show a credit balance. The credit items are the plus items of the account. Dealing as it does entirely with outlay and income, it records only actual transactions or past events.

Specific or valuation accounts deal with the assets or goods for which these expenses are incurred, and from which the income is expected. In contrast to economic accounts, they determine the value of these assets, and may thus introduce elements derived from the future as well as the past (§ 9).

20. Capital Accounts. — Outlay or expenditure of capital is undertaken for one of two purposes. Either an exchange is effected by which the proprietor receives goods or property of equal value, or the outlay is in return for services and the equivalent in value must be sought in the effect or results of such services.

In accounting, these two classes of outlay are rigidly separated wherever it is possible to do so. The cost of tangible assets, especially of those which have a more or less durable character and permanent value, is entered in a capital account. This account is corrected by entering deductions from cost for forms of capital which depreciate in value and may finally become worthless. This factor of depreciation is common to all forms

of property constructed by human labor, as, for example, machinery and buildings. The charging off of depreciation, unless accompanied by an appropriation of income to replace the capital originally expended, corresponds to a loss of capital. If replaced from income, it serves to diminish the net income available for dividends. In either case it improves the accuracy of the capital account.

A capital account ought to show at any time the total amount of capital invested in permanent or durable assets, or the actual cost of these assets, whether or not there has been any subsequent increase in value. The correction for depreciation, which is justified by the fact of diminishing value, is in itself purely a matter of accounting. The amount of depreciation written off on the books may not, and usually does not, coincide exactly with the actual loss in value at the time. Should no account be taken of depreciation, the capital account would merely show that the owner had neglected to reduce the "value" or appropriate any of his income as replacement of capital cost. This is an unwise and dangerous practice, but it is optional with the owner except where provision for depreciation is prescribed by law for the protection of stockholders in order to bring the capital account into closer agreement with the actual value of the assets.

21. Outlay and Income Accounts. — Outlay and income accounts, more often termed profit and loss accounts, deal first with the expenditures for which services are received and which are not therefore chargeable to capital account or regarded as investments. A second class of expenses usually included in this account is the cost of raw materials to be manufactured, goods to be resold, supplies to be expended in service, and fuel. These items represent capital and their value will be included in an inventory. But in most forms of business this investment is transitory in character, the raw materials are turned over as quickly as possible, and the resultant income is expected to more than cancel these outlays during the current year.

Outlay and income accounts are for this reason kept as open accounts, to be balanced, usually, at annual intervals. Outlay

is debited under such heads as wages, transportation, fuel, insurance, light, rent, repairs, interest, taxes, supplies and cost of raw materials or goods. Income is credited, and in mercantile business consists almost wholly of receipts from sale of goods, to which is added incidental receipts from all other sources, such as rent and interest. The capital account, kept separate from this mass of fluctuating items of expense and income, is brought up to date each year by entering against it the net increase or decrease in actual capital resulting from the transactions of the year. Should any of the assets whose cost is included in the capital account be sold, the income thus received must be credited in the capital account to replace the cost of this asset. In the same manner, loss in value of capital is replaced through the depreciation account. Excess of sale value over cost of assets would be credited as profits.

22. Investments versus Expenses. — The distinctions, or classifications in accounts, adopted by experience as suitable for modern business conducted on the basis of annual returns, should not be allowed to conceal the economic similarity of all forms of outlay and income. Outlay is undertaken with the sole purpose of thereby securing income. Those forms of outlay customarily entered in the capital account, for which tangible assets are received, are, it is true, regarded in the light of investments rather than expense by the owner. But their acquisition is merely the first step in securing income, and the culmination of the process is certain to require services or labor. The income will be the result of both classes of expenditure. That these are really identical is seen by the fact that improved property, purchased as capital, is largely the result of the services of former owners. The further expenditure for services by the new owner may have a sale value which a future purchaser would enter not as expense but as capital. Expenses for all kinds of service and supplies, instead of being an economic loss as must of necessity be assumed in the practice of accounting, is the most important form of investment, without which there could be practically no income, and the capital would then be without value. The controlling purpose of the pro-

prietor is to obtain as great a ratio of income as possible for each unit of capital employed, whether this outlay takes the form of investment or subsequent expenses.

23. The Relation in Time between Outlay and Income. — The element of time enters into every process of production (§ 11). Effort normally precedes satisfaction. Money income is to business what the satisfaction of personal wants is to an individual. Since outlay and effort are synonymous, it follows that in all forms of business, investment must precede the receipt of income.

This process is easily recognized in the period of installation, especially in such forms of business as require the completion of material structures. But it is equally true of the current business of a going concern. The receipt of income is reckoned from the moment a trade is concluded. Actual payment may be delayed, but this is a matter of accounting. Upon the delivery of the goods, all expense connected with the securing of this income terminates, except where contracts stipulate future maintenance in good order for a stated period. Consequently, the wages and all other charges incurred in the securing of this income normally precede the final sale. The postponement of payment of these expenses is also a mere matter of accounting. The liability or indebtedness for expenses is incurred previous to receipt of income in practically every case.

24. The Total Investment. — The total capital invested in a business is always greater than that represented by the cost of the assets listed under the capital account. Were income and the items charged under expense to occur simultaneously, the one should cancel the other, leaving the surplus or profit to apply as interest or dividends on the capital originally invested. After a business is established, this actually takes place and unless the scope of operations is enlarged, the capital required should not subsequently increase. But this apparent coincidence of income and outlay does not alter the time relation shown to exist between them. The expenditures of to-day are for the purpose of securing future income, while the income of to-day is the result of past outlays.

These outlays, whether in form of services or purchases, must be paid for by the proprietor at the time they are incurred, either in money or credit. This evidently requires an addition to the total capital invested, sufficient to meet all expenses covering the period of installation or formation of the business, or the time elapsing between outlay and income. If the income is regular and continuous, this required capital ceases to increase as soon as current income exceeds current outlay. But the sum invested up to this point remains in the business, for whatever portion may be paid back out of income an equal sum is at once required to meet the expenses for which the corresponding future income has not yet materialized.

The capital required to meet these current expenses and carry them continuously, thus bridging the gap between outlay and income, is termed working or floating capital, in contrast to that represented by more durable assets shown in the capital account.

For business in which income is realized only at long intervals, expenses continue during the period, and the amount of working capital tied up in wages and supplies increases rapidly to large proportions, and is as suddenly decreased when the income is received. Such a condition is represented by the business of driving logs down streams. Contractors engaged in this work receive their settlement when the logs reach their destination, which is once a year, while frequently a drive is hung up and lays over a season. It was formerly the custom in some states to transfer this indebtedness or outlay to the shoulders of the laborers, who were paid in time checks made payable in cash at a date subsequent to the probable delivery of the drive. The men, being without capital, cashed these checks at discounts which allowed the bankers or speculators who furnished the funds to make very large profits. These profits were in some cases split with the contractor who issued the checks. Legislation has in most instances required cash payment of wages, thus requiring the contractor to assume this obligation and raise the necessary working capital. Where, as in the case of forest plantations, the period elapsing between outlay and

income becomes abnormally long, the amount of capital required to carry the project to completion is proportionally increased.

25. Cost Accounts. — A cost account differs from a general profit and loss account in that it analyzes the cost of producing a given unit of output, instead of accounting for the current outlay and income of the business as a whole. Such an account traces the history of the product from the time of its purchase or inception until it is sold. The costs incurred fall into two groups: specific costs, which include that for raw materials and labor directly applied in production; and overhead charges, which include taxes, insurance, light, fuel, superintendence, rent and other items, which must be apportioned on an equitable basis over the entire output. Cost accounts in forest production sum up the cost of growing a crop of timber from origin to maturity. Owing to the great length of the period involved, the factor of compound interest on invested funds, when introduced as a cost in such accounts, becomes one of the largest items of expense.

26. Income. — In § 8 and § 9 it appears that prices for finished products are the source of value for all forms of property. Value, as distinguished from cost, depends upon net income and looks to the future. The income from finished products, which gives them their value, corresponds with the services they are capable of rendering by being used or consumed.

The sources of income from productive property consist either of finished products or materials in a raw or unfinished state, while the income derived from the business of operating such properties is the money received from the sale of the output.

27. The Economic Opportunity. — To derive or produce an income, some human want must be supplied. The greater the range and diversity of these wants and the greater the purchasing power or capacity of the average individual to gratify them, the more numerous and promising will be the economic opportunities for business. Inequalities in the distribution of wealth, and the existence of a pauper class, greatly diminish the total business of a nation and divert much of it into wasteful and injurious channels.

28. The Business Venture. — Income can be produced only by risking capital and expending effort on the chance that this outlay will be returned “with interest.” In a properly conducted business this risk is assumed by the proprietor, who makes himself personally responsible for the management. Ownership is represented in modern business by capital stock. By means of this same device, extensive frauds are possible by which the real managers of a business shift both risk and loss to the stockholders or owners. Legitimate risk, borne by those responsible for the results, is unavoidable and is one of the chief characteristics of the business venture. Aside from the risk of accidental losses by fire, theft and other factors, the final ever-present risk is that income will be insufficient to offset the required outlay, and insolvency result.

29. Exchange. — Income from business (§ 26) is derived almost entirely from sales of merchandise or manufactured products, which constitute an exchange of goods for money. The transaction is in theory completed with the delivery of the goods, but payment is often deferred and constitutes a debt on the part of the purchaser, which is a debit or asset to the business. The losses which occur through ultimate failure of debtors to pay are finally debited as an expense.

30. Disposition of Income. — With the receipt of income the cycle of effort is completed and the purposes of the business are accomplished. An accounting must now be rendered to the owner. In case the proprietor’s account and ownership is terminated by a sale of the entire business, the settlement would consist of:

Payment of outstanding bills, wages and other outside obligations.

Return of outstanding and borrowed capital.

Distribution of surplus as profit to owners of the capital invested.

In the annual accounting of a going concern the profit and loss account takes care of the expenses. The surplus or deficit in annual income, shown as profit or loss, is carried into the annual balance sheet for future disposal.

31. Returns on Capital. — Out of the net profits after cancelling the expenses for the year, provision must first be made for the replacement of capital assets lost by depreciation or otherwise. The remainder is available as returns on capital.

Here a sharp distinction must be made between the total amount of capital required to finance a venture and the persons or sources from which this capital is derived. From the standpoint of the business itself, the source or ownership of this capital has no effect upon the total net profits but affects merely the apportioning of these profits.

A proprietor who furnishes his entire capital, both fixed and working, takes all the profits and assumes the risk of loss. But it is an almost universal custom to borrow part of the needed capital on the security of the business or of the fixed assets. In this case profits are earned on the total invested capital as before, but the division is on a different basis. The borrowed capital receives a fixed rate, agreed upon at the time of securing the loan, and guaranteed by the proprietor. He receives in turn the entire surplus remaining after this obligation is met.

32. Interest versus Dividends. — The sum guaranteed to the lender of borrowed capital is interest. This is paid out of net income and is as much a part of the net earnings of the business as the remainder. The residue is either left in the business as additional capital, or paid to the proprietors as dividends.

Interest, at the rate demanded on borrowed funds, will require a given sum on the entire capital. Should the business earn a net income exceeding this sum, the proprietor receives the excess, which raises the rate earned by his own capital, or his dividends, above the rate of interest paid to creditors. The greater the proportion of capital he borrows, the greater will be the rate of dividends on the lessened capital of the proprietor. But on failure of the business to earn the rate paid on borrowed funds, the owner bears the entire loss, and the smaller the proportion which his capital bears to the whole investment, the greater is his relative loss.

In an insolvent concern, the owner's capital, as well as divi-

dends, is sacrificed before any loss of either interest or capital is permitted to fall upon the holder of mortgages or liens. For this reason bankrupts may fraudulently conceal the assets and thus defraud creditors.

33. Profits. — According to Marshall,* “When a man is engaged in business his profits for the year are the excess of his receipts from his business during the year over his outlay for his business. The difference between the value of his stock and plant at the end and at the beginning of the year is taken as part of his receipts, or as part of his outlay, according as there has been an increase or decrease of value. What remains of his profits after deducting interest on his capital at the current rate may be called his earnings of undertaking or management.” According to this definition, interest paid on borrowed money is excluded from profits, but interest on the owner’s capital is considered as constituting part of the profits. This is the distinguishing point between a purely economic or impersonal consideration of a business and a personal account with the proprietor. In an economic consideration of the general question of profits on an industry as a whole, such as is needed, for instance, in the process of appraising stumpage values for standing timber (Chapter XI), the distribution of the required capital between proprietors and creditors has no significance. The entire net revenue available as returns on capital for the current year must be regarded as profits.

34. Wages versus Profits. — Wages are an outlay, or an expense, which serves to diminish profits. The wage or salary, ultimately met from income, is the share of the wage earner in the returns of the enterprise.

In most undertakings the owners themselves give at least part of their time to the management of the business. In very small enterprises the owner may even do most of his own work, thus saving the expense of employing labor. The larger forms of business, in which the ownership is scattered in the form of capital stock, are managed by proxy, through boards of direc-

* “Principles of Economics,” by Alfred Marshall, 5th Ed., Vol. I, p. 142. Macmillan & Co., London, 1907.

tors who devote but a small portion of their time to these duties. There is an almost complete separation between wages or salaries, and income in the form of dividends. Even in such cases, capable managers are frequently given an interest in the business and their returns thus include both wages and dividends, although the two items are separately accounted for. In small ventures managed by the owner, this distinction is often neglected, and the owner's profits are the sole source of compensation for his time. In such cases a sum representing the wages of the manager or overseer should be charged to expense. Should the business pay less than this sum it is unprofitable.

CHAPTER III

INTEREST

35. The Problem of Interest. — In proprietary accounts, all items, with the exception of interest, are easily classified as costs or as income. Interest may be regarded as either cost or income, according to the point of view. Since these two conceptions are not interchangeable, a clear understanding of this problem is necessary to avoid errors in dealing with questions of valuation accounting.

36. Interest as a Cost. — Interest when regarded as a cost must be considered in its relation, first to the owners of capital, and second, to the capital itself. In accounting, the interest paid upon borrowed capital is regarded by the proprietors as a cost. But it must be noted that the net profits of the business are first computed, and from these profits, rather than from gross income, the cost of interest is met (§ 32). This item of cost is entirely a personal matter between the owner and his creditors. Should there be no borrowed capital, this cost item would disappear from the account, and dividends or additions to surplus, to an equal amount, would be substituted.

But since borrowed capital is entitled to receive interest at a standard rate, and this "cost" must be met, the owner considers that his own capital is entitled to receive interest at a like rate. He guarantees payment on loans, but must depend on his own exertions to obtain this interest on both the borrowed funds and on his own investment. Should he prefer the role of money lender, his capital will earn this rate of interest without the risks attendant on the business venture. The risks of lending money upon good security are much less than those accompanying other forms of business. Shall he therefore charge interest upon his entire capital as a cost which must be met? Those who take this view will include interest, not only on

borrowed funds but on the total invested capital, in the same category as other expenses, that is, as a part of the so-called carrying charges of the business.

The interest to which the owner is entitled cannot be entered in proprietary accounts as an actual charge or cost similar to interest on borrowed funds. His share of interest is never actually expended, and the idea of cost exists merely by implication. But in a cost account, which is intended to present definitely the relation between total cost of production and resulting income, interest may be calculated and entered as a part of the total cost.

37. Interest as Income. — Whether or not interest is regarded as a cost, it always represents income on the capital investment, and in this respect differs from other costs. All other items of expense, such as taxes, wages and cost of supplies, while met eventually from income, go to persons or enterprises other than the capitalists who finance the venture. And just as the creditors of the business, during the formative period, are paid in advance by the capitalist, who receives returns finally from income, so the capitalist who only loans money and does not assume the risks of ownership may receive his interest in advance of income, provided it falls due and is paid by the proprietor. But this cost to the latter is ultimately returned to him from income, which must also pay him returns on his own investment. In case the income is continuous and is accounted for annually, both borrowed and invested capital receive income directly from the net returns.

38. The "Rate of Interest." — Interest has been defined as the price of time differences in the enjoyment of income (§ 14). Since the use or borrowing of capital is the means of anticipating income (§ 12), and the value of both capital and interest is expressed in money, it follows that interest is the price of the use of money.

This price for money loans is termed the "rate of interest." As money is more desirable than other forms of capital because of its universal acceptability as a medium of exchange, and as the risk in money lending is normally less (§ 36) than in business,

this rate of interest, which represents the possible income on capital in the form of money, becomes the standard by which the desirability of any other form of investment is gauged.

39. The Profits of the Undertaking.—Unless there is a chance of earning a larger rate of income upon the capital required in an undertaking than could be earned by lending this capital to others, there would be no incentive to assume the risks of the business. In order to make a business really profitable, the net income should exceed this “rate of interest.” This surplus is the goal of the whole operation and is termed the “profits of the undertaking,” as distinct from the term “profits,” which includes interest (§ 33). It is the personal reward of the individual who has borne the responsibility of the business. There is no good English word to indicate this person. Unfortunately we have appropriated the word “undertaker” for other uses. The French term “entrepreneur” is commonly accepted by economists. The more recent English substitute “enterpriser” is rather clumsy, but will be used in this text.

The term “profits of the undertaking” would therefore exclude interest as a necessary “cost” and regard the margin remaining as the real profit.

40. Annual Profits.—On this basis, a business paying annual dividends is considered successful if the average dividends are larger than the rate of interest which the firm has to pay on borrowed capital. If the dividends are less than this, the business is considered unprofitable to the owner, although it may be relatively more profitable to continue it than to abandon or sell it, with resultant loss or impairment of capital. As a matter of fact, only the more capable and experienced managers succeed in earning an “enterpriser’s profit,” and one per cent of independent concerns annually become insolvent, thus failing to earn a net income sufficient even to protect the capital investment.*

41. Deferred Profits.—In an enterprise in which outlay precedes income by an interval greater than one year, the method of accounting for profits depends upon whether this condition

* The Percentage of Failures, *Dun’s Review*, May 31, 1913. R. S. Dun & Co., New York.

of affairs is normal or accidental, expected or unforeseen. If profits should normally be earned annually, failure to do so in any given year would be regarded as a loss by the proprietor, but this would be mentally "written off" against the current year and not carried as a debit to be recovered from the income of a subsequent year.

If the enterprise is planned to extend several years before realizing profits, as in the case of the voyages of whaling ships, or the production of timber from plantations, the annual interest which remains unpaid is not considered lost, since it is the expectation of the owners that this will be more than made up when the "ship comes in." But the profits of the undertaking may still be computed separately from the standard interest "cost" or income. This must be done by calculating compound interest on all cash expenses as long as they remain unpaid, which gives the sum that could be earned by the capital as loans in money form for an equal period.

The deferred income may greatly exceed the total cash outlay and the enterprise yet fail to earn compound interest at a standard rate. In this case the deferred "profits of the undertaking" disappear, and the status of the venture is the same as that of the business whose annual dividends fall below the standard rate of interest.

The production of timber as a business is based largely on the theory of deferred profits, and both the determination of these profits and the methods of accounting employed should be made to conform to the essential differences between such a business and one conducted on the basis of annual profits.

42. Income versus Profits.—In determining the amount or rate of profit which should be earned in order to properly compensate the enterpriser, the status of interest must be held clearly in mind. It is a "cost" merely for purposes of separating that portion of the income which his capital could earn from the additional income earned by his personal efforts. In impersonal consideration of the legitimate profits of a business, the distinction between borrowed capital and proprietary capital is rejected, and the interest upon the entire capitalization is

included in the income earned and not in costs. Should the fallacy be introduced of considering interest as a cost in computing profits, the apparent profit required would be increased abnormally. For instance, it is stated that a certain business should earn a profit of 10 per cent, which, with interest at 6 per cent, indicates an enterpriser's profit of 4 per cent. By considering this interest as an actual cost, an additional profit of 10 per cent might be expected. But, in reality, this 10 per cent would be the enterpriser's profit. The total net income must then be 16 per cent, and the enterpriser's gain is $2\frac{1}{2}$ times as great as in the original case. If an enterpriser's profit of 10 per cent is necessary, the "profits" of the business must be 16 per cent and not 10 per cent.

The larger the amount of capital invested in a business, the smaller, as a rule, will be this margin between income and interest, which represents the enterpriser's gain. Both income and "gain," while larger in amount, average less in proportion than on small investments. This shrinkage does not come entirely on the margin of gain, since funds can probably be borrowed at lower rates in such undertakings. The longer the time elapsing before income is received, the less will be this margin of profit over compound interest at standard rates. For very long investments, the enterpriser's gain tends to disappear, and is entirely dependent on the rate of interest chosen as representing the legitimate income on the capital itself.

In both the above cases, in spite of the shrinking of enterpriser's profits, the income or profit over expenses remains in the form of interest returns, and, especially on long time investments, the final results may still be considered by the investor as highly satisfactory.

This attitude is partly due to ignorance. The owner of unproductive property, such as unimproved real estate, seldom keeps a cost account which shows him the total outlay represented by the purchase price plus taxes and improvements, and still more rarely does he add the "cost" of the unearned interest on this total from year to year. When he sells this property after the lapse of several years, for more than double what he

paid for it, he does not realize that a savings bank at 4 per cent might have paid him more for the uninterrupted use of the same capital than his profit amounts to.

Yet this attitude of investors towards deferred profits is not explained wholly by ignorance. If an investor is in a position to get along without income for a long period, at the end of that time the "loss" of annual income has but little significance and he sees only the accumulated total of his profit. He compares this with his original outlay and judges results on that basis. This tends to make him satisfied with a return which in reality represents low rates of interest on such investments.

43. Influences Determining the "Rate of Interest." — The current rate of interest is a price, determined by the demand and supply of money for borrowing purposes. This price or rate has no greater stability than the average prices of other commodities. It fluctuates during the year, and over longer periods, in response to changing economic conditions. Nor is this rate the same for all classes of loans. It varies with the security offered, the length of the period covered by the loan, and the personality of the individuals negotiating it. In spite of these variations, an average rate of interest may be roughly approximated for a given time and place; but this is determined far more closely if the circumstances affecting the particular loan are also known.

44. The Influence of Personality on the Rate of Interest. — Just as prices are the resultant of human opinions interpreting desires or demand and weighing them against efforts or supply, so the rate of interest depends even more completely upon the human character and mind.

Impatience for income, and preference for present over future income, tends to raise the price the individual will pay for this privilege. Persons who lack foresight, are improvident and are lacking in self-control will borrow at high rates. The same is true of persons who lack capital and are in need of present income. On the other hand, individuals who desire to provide for the future, who possess the power of self-denial and who are already possessed of considerable capital will borrow, if at all,

only at reasonable rates. As investors, the former group desire to "get rich quick" and by seeking abnormal profits usually lose what capital they may possess. The latter group are apt to favor long time investments which give a certain return even at a low rate.

45. Risk and Expense. — As most loans of money are made on the basis of a security pledged as a guarantee for repayment, and worth more than the amount of the loan, the risk of loss of principal or interest is correspondingly reduced. In case of unreliable parties, notes are guaranteed by some one who is able to pay. Risky loans are made at higher rates than safe loans, the extra rate being intended to cover average losses. The exorbitant rates charged by loan sharks are not justified on this basis, although loans of this character require a somewhat higher rate than business loans. Such rates are due to personal elements, exaggerated by defective laws and bad economic conditions, which permit the exploitation of a class of persons unable to protect themselves.

Expense of placing loans and of collecting interest tend to increase the rate of interest charged. This expense is greater for short loans, and the funds are apt to be idle part of the time.

It follows that the lowest rates of interest are received on absolutely safe securities, which run for a long period and upon which the interest payments are made without expense to the owner. Government securities and bonds of high grade fulfil these conditions.

46. Fluctuating Value of Money. — Should money fall in value or in purchasing power, the rate of interest tends to rise to a degree corresponding to the average annual loss in the value of the capital. Only by this means can a lender of capital escape actual loss. If the fall in value amounts to 1 per cent per year, securities upon which the normal rate of interest is 4 per cent will be worth in purchasing power only 99 per cent of their original value at the close of the year. A rate of 5 per cent enables the investor to replace this difference by adding it to the capital.

Such losses usually escape the notice of the owner, who imag-

ines himself as wealthy as before even if he spends the entire income from his investment.

47. The Rate of Interest for Business Investments. — The rate of interest for a business is the rate at which money will seek investment in the enterprise. It is a question whether or not this basic rate should include an enterpriser's profit. The rate which the lender of capital will accept on the security of the business, or what the proprietor can obtain as the basis for a loan, is a conception tallying rather more closely with actual conditions. Such a rate may be said to attract capital, but it may not attract the enterpriser, who hopes to do better. The "rate" adopted as the standard for an investment becomes the basis for computing both the cost of production (§ 25) and the value of the assets (Chapter IV).

48. The Effect of Deferred Income upon the Rate of Interest. — A mathematical comparison of simple and compound interest leads to the conclusion that, since the latter, as applied to deferred returns, is the exact equivalent of the former when returns are annual, a given basic rate should be the same whether the investment pays annual or deferred income.

But under the definition that the "rate of interest" means the rate which will attract money to a business, the above conclusion must be tested by the "price making" factors which influence the minds of investors. Should these factors be found to differ for investments of the two classes, the basic rate of interest applicable to each case will differ to a corresponding degree.

49. Comparison of Results of Simple and Compound Interest. — The economic results obtained in actual practice from an undertaking which produces simple or annual interest are not the same as those obtained by compound interest, however similar may be their mathematical relation.

To make the returns from a business which starts with a definite amount of capital, and earns annual net profits at x per cent, equal to those from the investment of a similar sum which will produce compound interest after a term of years at the same rate, x per cent, requires several assumptions. The

entire earnings of the business would have to be retained as capital and not a cent withdrawn as dividends. This capital must then be utilized in such a manner that it will produce net earnings equal in rate to the earnings on the original capital. Each year this process must be repeated, resulting in an expansion of the business at a geometric rate. If this result is obtained, the total capital at the end of the period will not be worth one cent more than the funds accumulated in the second investment, where all income is deferred until the end of the period.

Should the proprietor withdraw any portion of the earnings during the entire period, or should the business fail to expand annually at the required per cent, the resultant rate of compound interest earned by annual profits will be lowered below that demanded in the comparison.

50. Limitation of Opportunity. — Such compound expansion of a business is possible only where the economic opportunity (§ 27) is practically unlimited, or increases at a rate which keeps pace with the growth of the business. This occurs only as a temporary condition, both in business and in all other forms of activity.

Animal and plant life have the capacity for expansion at geometric ratios comparable to enormous rates of compound interest; such events seldom occur, but when they do, the result is calamitous. English sparrows imported into America found an environment not used by any existing species of bird. Until this niche had been completely occupied, the sparrow increased at a compound rate. The species is now merely holding its own.

In manufacturing, the production of new articles and the growth of a demand for them, as in the case of automobiles, creates an opportunity which permits of a trade expansion at geometric or compound rates for a while. This expansion took place when bicycles were first made popular. It cannot continue in any line at the same rate over an extended period. A lessening, not of the annual demand, but merely of its rate of increase, has the effect of preventing further expansion at a like rate, and with the establishment of equilibrium, not only

will compound interest fall to a lower rate, but annual profits will be reduced as well. This explains the fact, accepted by economists, that the larger the aggregation of capital employed in an undertaking, the smaller will be the annual rate of profits which justifies the investment.

The conclusion is irresistible, that the mathematical equality between simple and compound returns does not hold good. The latter tends to fall below the former, and, what is more important, this discrepancy is accepted as equitable by investors. The recognition of this fact will go far towards clearing up the false impressions which at present obscure the question of rates of interest applicable to forest investments.

51. Effect of the Rate of Interest upon Accumulations of Compound Interest.—The cumulative effect of compound interest bears an intimate relation to the rate of interest. Low rates cause a very slow increase and may be applied with some reason to investments which run for a very long period. High rates cause a very rapid increase which soon passes the bounds of possible attainment in practice. The following diagram reveals this relation. (See Diagram I.)

TABLE I

COMPARISON OF THE RESULTS OF SIMPLE AND COMPOUND INTEREST
ON \$100 AT THE END OF A PERIOD OF 50 YEARS

Rate of interest earned	Value of capital and interest at end of period	Rate of simple in- terest to be earned and saved to give equal final results
Per Cent		Per Cent
1	\$164	1.28
2	269	3.38
3	438	6.76
4	710	12.20
5	1,146	20.92
6	1,842	34.84
7	2,946	56.92
8	4,690	91.80
10	11,739	232.78

A comparison of the economic results of simple and compound interest is further brought out by the above table which gives

DIAGRAM I

PERIODS REQUIRED FOR \$1.00 TO MULTIPLY, AT DIFFERENT RATES OF COMPOUND INTEREST*

* A rate of 10 per cent, compounded, multiplies 20 times in 31 years. A rate of 1 per cent compounded, doubles in 70 years. Even 6 per cent would, in 70 years, multiply the principal 59 times.

the total amount to which the sum of \$100 will increase in 50 years, provided the investor complies with the conditions outlined in Article 49.

The final column shows the rate which the original investment would have to earn annually to get the same results, provided the entire income were placed in a deposit vault and saved until the end of the period without reinvestment. Should the proprietor spend the annual income, he would have merely the original capital of \$100 left at 50 years.

52. The Rate of Interest in Forest Investments. — To determine the basic rate of interest applicable to investments in forestry, two factors must be analysed, namely, the relative security of the investment, and the financial nature of the enterprise. It is claimed by advocates of a high rate on forest investments that this is justified by the ever-present risks from fire and other destructive agencies, and also by the length of the period elapsing between outlay and income, which makes the investment less desirable unless it can be shown to be more profitable.

The first of these claims must be admitted. Risk should be provided against by demanding higher interest. This subject is discussed in Chapter XIII. Risks must be judged on the basis of comparison with those assumed in other lines of investment, and due weight must be given to the present development of measures of protection on the part of states and associations.

The second claim is fallacious. To demand a higher rate of interest the longer the returns are deferred is a subversion of the economic laws applicable to all forms of investment. The desirability of the investment as affected by the difference between annual and deferred returns will not modify the rate of interest which should apply, but rather, will determine the class of persons who are apt to choose such an investment. And since those persons who are the most apt to favor long-term investments with deferred income are those possessed of foresight, making provision for their children, and with sufficient capital for their personal needs (§ 44), such persons will accept a lower rather than a higher rate of interest, and in many cases will not

even compute the probable rate, being content with the general prospect of a future value greatly in excess of present investment.

53. Comparison of Interest Rates in Forestry with Other Investments. — Proper rates for forest investments can only be judged on the basis of comparison with other forms of enterprise. If the scope of the comparison is confined to forest production and the account covers the period requisite for growth, a comparison between forestry and business producing annual income is impossible. The goal of forest management is the forest which will yield annual returns, but each crop represents the accumulated outlay throughout its life and must be so judged in comparing interest rates.

The only enterprises familiar to the public, which are reckoned on a basis of compound interest, are savings banks and life insurance companies. The former have paid an average of 3 per cent, but of late years 4 per cent is common. Most banks do not permit the accumulation of compound interest on accounts to run more than 20 years without some sign of active interest on the part of the owner, manifested either by withdrawals or additional deposits. If the owner is alive and can refrain from depleting his account for fifty years and the bank remains solvent, he can obtain compound interest for that period. Such cases practically never occur.

The average period covered by the risk of a life insurance policy is not over $13\frac{1}{2}$ years,* and most companies calculate that they can earn compound interest at about 4 per cent on the money invested in policies for this average period.

For periods longer than fifty years there is no basis for comparison by which the rates reasonably applicable to forest investments can be judged. Based upon the economic laws outlined in Articles 47 to 50, it must follow that those rates will be less than 4 per cent provided the investment offers equal security with life insurance and savings banks. Should the security be considered less safe, the increased rate demanded would offset the reduction in rate called for by the length of the period of investment, and the investment should not require

* The Brown Book of Life Insurance Economics. Edition of 1911-12, p. 13.

a rate appreciably higher than 4 per cent for periods of over 50 years.

If these arguments are admitted, it is possible to show that the income which may actually be earned by forest investments covering long periods will be equivalent to the rates demanded by the character of the investment, even though the earnings may fall as low as 3 per cent for periods of over 50 years, and $2\frac{1}{2}$ per cent to 2 per cent for periods exceeding 100 years.

CHAPTER IV

VALUATION OF ASSETS

54. Valuation Accounts. — A valuation account is a specific statement (§ 19) of the value of assets, whether material or consisting of intangible rights. It informs the owner of the present condition of the property, and enables him to compare this condition and value with the total net cost or investment (§ 22), and thus determine whether the business as a whole is gaining or losing (§ 28). To accomplish this purpose, the account should show what the business or property is actually worth at the present moment, not merely what it was worth when the assets were first acquired.

55. The Inventory. — In order to determine value we must know the quantity of goods on hand and what each unit is worth. This process of taking stock is termed an inventory. It is not complete with the mere listing of goods, but includes their valuation. In forest management the process of estimating standing timber and other forest resources, often termed reconnaissance, is the means of obtaining this data.

56. Cost as a Basis of Value. — Conservative accounting seeks to avoid inflation of the value of assets, with its corresponding indication of false profits in the balance sheet. If an asset has been purchased in good faith, the purchase price is taken to represent its true value. All items of investment representing assets of such a character that they would be included in the capital account will be accepted at cost, less depreciation, in the valuation of assets. This does not mean that the cost of an article represents its present value, but merely that cost is definitely known and, when corrected for depreciation, is accepted as a conservative basis for value.

57. Expenses and Interest versus Value. — While accountants approve of the acceptance of capital expenditures or original

cost of purchase of tangible assets as a basis of value, they condemn the assumption that subsequent expenses, or items customarily entered under outlay and income, or profit and loss, add anything to the value of assets or furnish a basis for valuing these assets. The theory is that such expenditures are made for the direct purpose of producing income sufficient to offset them, and that if this result is not secured, the loss must be shown as a deficit, and not concealed by adding an equivalent of this loss to the value of the assets.

This theory is justified for business undertakings supposed to produce annual income, for if the income which should cancel expenses fails to do so, the deficit cannot be construed to represent anything but absolute loss, leading to insolvency.

This reasoning is then extended to apply to undertakings in their formative period (§ 23), and the practice of retaining investments in the inventory at their original cost is highly commended, in spite of the fact that practically all expenses during this period require additions to capital (§ 24), and this treatment of assets is thus bound to result in an apparent deficit or state of insolvency.

Yet no one would undertake such long time investments unless he expects an increase in the value of his assets in a measure corresponding to or paralleling the accumulation of cost and interest. That this fact is assumed has a remarkable confirmation in the revolutionary practice of accounting authorized in case of railroad properties during the course of construction and before it is possible to earn an income. In Germany and England it is permitted by law to include as an addition to the value of the assets, not only the revenue expenditures during construction, but the interest on bonds and dividends to stockholders.* This is the very process so vigorously and justly condemned in case of business undertakings which should pay annual dividends, nor would it be tolerated in the case of railroads except during the period of construction.

A case closely parallel to the above is found in forest planta-

* "Modern Accounting," by Henry Rand Hatfield, pp. 76, 77. D. Appleton & Co., 1909.

tions. Shall the value of such property be based on the purchase price, plus expenses, plus unpaid or deferred interest, representing the total "cost" or "cost value" of the property? As in the case of the railroad this might be justified. But such "cost" is only an indication of value in either of these instances. Unless the property is of such a character and its promise of future income is such that this increase in value is sure to occur and is evidently taking place, no one would be deceived into thinking that such a cost calculation corresponded in the least with value. Even then the value might be more, or less, and there is no reason for assuming that it is accurately gauged by cost.

58. Essential Difference between Cost and Value. — The reader must keep clearly in mind that the valuation of assets is a process essentially different from the account of cost and expenses (Chapter I, Articles 2 to 11 inclusive). If costs are taken as the basis of value, as is done in some instances (§ 56), they are merely the source of the information used, but this information might as readily be obtained from entirely different sources, and frequently must be so acquired. Value is determined for the purpose of comparison with costs to indicate profit or loss. If it were identical with cost, no comparison would be possible and the owner could not determine his probable profits.

59. Sale Value as a Basis of Value. — In case a business changes hands by sale, the sale value so determined is accepted as the value of the business. In this transaction the price paid for the assets must be entered as their value on the books. Such sales thus establish a new recorded or book value for these assets. The fact of the sale is the most convincing evidence of value, although even this is not final proof, for one or the other of the parties may have been deceived or handicapped in the transaction. Nor will a past sale be accepted as absolutely determining a future sale value even for the same property; in fact it is usually the reason for desiring a different value. Sales of property or business have a profound effect upon proprietary accounts, in that the former owner receives at once all his income and capital and can balance his books and determine

his total net profit, while the purchaser is saddled with a cost or investment which forms the opening entry in a similar account, and no future acts will serve to reduce this initial cost.

60. Appraisal of Value (§ 18). — In determining the true value of assets, either as a guide to the owners in negotiating a sale, or for compensation for damages or in condemnation proceedings, the appraiser bases his opinion upon the most reliable sources of information at hand. If a basis for sales is sought, the average sale value of similar property in the immediate past is given most weight. But where the owner does not desire to sell, the appraiser must seek to establish the value to the owner if the property remains in his possession, and this rests wholly on its usefulness to him, not on what someone else would pay for it.

61. Future Income as the Basis of Value. — The only true basis of value recognized by economists is income (§§ 4, 26). When a sale of property occurs, the "income" from the sale should coincide with the value of the property. The value which the owner or purchaser places on the property is determined by the value of all the future income which he expects to derive from it.

In explaining values on this basis the true nature of income must be kept in mind. "Final" income takes the form of actual enjoyment of the uses or services of possessions. Real estate is valuable to an owner, not merely for its money income, but for the personal enjoyment which he gets from the rights of possession. In this must be included the associations and neighborhood of which it makes him a part. The sentiment attached to the retention of an inherited homestead, the prejudice against leaving surroundings grown familiar by long residence will, by contributing to the peace of mind of the owner, enhance the value of his land in his opinion in a manner fully as concrete as money income. Only poverty or actual want will counterbalance these very real values. This element of social income must never be lost sight of in the valuation of property. But the appraisal of income must ordinarily be based upon money returns which can reasonably be expected in the

future, and the present value of the property rests upon the mathematical relation between the future income and its discounted value.

62. Capital Value or Expectation Value. — Capital value, or expectation value, is the net value of property based upon future income. It is the capitalized value of future income minus future expenses. The terms "capitalized" and "discounted" are synonymous. Therefore capital value is the discounted net value of future income. Finally, since all future income is included in present value, capital value is the sum of the discounted values of all items of future net income. This definition holds good whether the income is received at regular daily, monthly or annual intervals, or at intervals of two or more, even of 100 years, or is irregular in amount and paid at irregular intervals.

An example of regular annual income is the interest on a money loan. This interest, capitalized, corresponds to the value of the loan of capital. This relation established between capital and income through the rate of interest (§ 38) is the means of determining the capital value of all other forms of property.

For money alone the value of the capital and of its use determines the value of the income or rate of interest, and the value of money is neither raised nor lowered by fluctuations in the rate of interest.

For other property the relation is:

1. Capital or property earns income.
2. The value of this income is appraised.
3. This value is then discounted to obtain the value of the capital.
4. The "rate of interest" is the measure of discount.

The term "capital value" has so far found no place in the literature of forest valuation. Instead, such authorities as Schlich have translated continental terms into the English "expectation value." This term is descriptive of the fact that such values are based on future expectations, not fully determined; but it is open to objections, since it conveys the

impression that such values are different or obtained in a different manner from the capitalized values computed for all other forms of property. On the contrary, the process is identical and it would be a distinct advantage to recognize this fact in terminology. A third term, also identical in meaning with the two above mentioned, is "capitalized rental value," sometimes abbreviated to "rental value." Two objections to the term "rental value" are that it may be confused with rent, which is only one form of income, and that it may be taken to mean the value of one of the current or periodical payments of income instead of its true meaning, which is the sum of the discounted values of all items of future income. The term "capitalized rental value" seeks to emphasize this latter meaning, but the use of the word "rental" might be taken to mean that such values are determined only for property producing income at regular intervals. Since it makes no difference what the character or interval of payment is, the term "capital value" is the most comprehensive. The one great objection to this term is that the word capital is commonly used to denote the amount of an investment, or the total funds furnished by capitalists (§§ 12-13), and is in this sense allied with cost rather than value of assets (§ 73). Were all assets in the form of money, this would make no difference (§ 62), but as cost and value for other assets diverge widely, some term should be used which is clearly understood as referring to value rather than cost. Another possibility is the term "income value," or "capitalized income value." "Income value" is open to the same objection urged against "rental value," — that it might be understood to mean the value of one year's income, or of one payment. "Capitalized income value" comes nearest to fully describing the value indicated and the means of obtaining it.

The valuation of forest property differs from that of property earning annual income only from the fact that the income may occur at intervals separated by long periods. Hence the tendency, expressed by the term "expectation value," to emphasize this "expectation." As will be shown in Chapter V, the formulæ employed in the capitalization of annual and of intermittent

income, such as is received from forest crops, are identical in character, and it is more important to recognize this essential similarity than to seek to differentiate forest valuation from other forms of appraisal. In the text, the synonymous terms "capital value" and "expectation value" will be used interchangeably, rather than the more cumbrous term "capitalized income value."

63. Future Prices and Values. — The determination of capital value rests upon the ability of the appraiser: first, to ascertain the amount and character of the income, its duration and the total for the entire life of the enterprise; and second, to appraise the probable value of this income for the time when it is received, by a conjecture as to prices for the future products (Chapter XII). But the future is shrouded in uncertainty, which becomes more impenetrable the longer the period over which such predictions must extend. The past is an open book. The knowledge thus obtained by experience is the guide in the process of appraisal. It is comparatively easy to determine the probable productiveness of property for any business which presents the results of past experience. In new and untried ventures this is uncertain. But future prices are far more of a problem, since they are the resultant of the complex of all the economic forces of society. Attempts may be made to predict changes in prices, due either to changing price levels based on fluctuation of the value of money, or changes in the supply and demand for the specific product. Such predictions can only be based on present tendencies interpreted in the light of past, especially recent, experience.

When reliable authority is lacking for prophesying the probable nature and amount of changes in prices, although they are certain to occur, the appraiser is forced to accept present prices as the basis for future income. This principle finds its expression in appraisals for damages, when speculative increase in future income is not admitted in valuations.

64. Future Expenses. — The net present value of income is equal to the margin over future expenses (§ 10). These future probable expenses must be appraised as carefully as income.

Most elements of expense can be judged by past costs for similar undertakings. Future expenses may and will vary from present standards, for part of this expense is determined by prices, part by wages and part by legislation and taxation. Uncertainties regarding the trend of any of these elements of cost interfere with the accuracy of the appraisal of value. A fruitful source of uncertainty is future taxation. Legislation tending to substitute a definite arrangement for this fluctuating item is a great aid in appraising the value of forest property.

65. The Time Element in Capital Value. — When once the future income and future expenses have been appraised, the determination of the capital value of the property is a mathematical calculation, dependent on the rate of interest chosen as the basis of comparison between the present and the future values involved. Each item of income is discounted for the period of time intervening between the present moment and the receipt of the income payment (§ 11). The sum of these values equals the amount of money which is equivalent in present value to the expected future income. The property is therefore considered as having this value. For this "time difference" in value the rate of interest is the measure. In the same way, future expenses are discounted to get net value.

The greater the time interval between the present moment and a future transaction, the less will be the importance of this transaction when discounted to determine its present money value. The net capital value of property is practically determined by events occurring within the next fifty years, not merely because these events can be more accurately prophesied, but through the operation of the factor of discount, based as it is on the laws of compound interest. Not only are profits that are deferred for fifty years of little significance to the investor (§ 41), but the future values themselves are correspondingly small when viewed from the present standpoint. The immediate future occupies the minds of men and its relative importance is mathematically expressed in discounting to obtain capital values.

66. Effect of Rate of Interest on Capital Value. — The rate of interest is a barometer which measures the fluctuations that

occur between the relative values of future and present income. An increase in this rate, or measure, increases the difference in values due to this time element. The greater this difference, the smaller will be all capital values. A permanent doubling of the rates of interest would diminish the value of productive capital to one-half its former worth. This premise is based on the economic facts discussed in Articles 60 to 64, which refer value directly to future income. High rates of interest indicate risk, unsettled conditions and instability of character, and are thus directly related to influences tending to depress property values. Low rates of interest correspondingly raise the capital value of property, by diminishing the loss or discount necessary in arriving at the present value of the future income.

67. Uncertainty of Capital Value. — The premise that value is based wholly on future net income is sometimes questioned, because of the uncertainty which surrounds every element of future occurrence. The determination of value on this basis, by mathematical deductions, is a guess, to the extent that these future elements are subject to change. Yet every sale of property is the result of a guessing contest as to these future elements, rather than a calculation of past costs. As two minds are better than one and as the guess is made the basis of an actual exchange of property, a sale as soon as it is effected becomes a new indicator or basis for the next guess. The uncertainty of future values is identical in nature with other risks incurred in business and adds the element of chance which becomes the basis for practically all forms of speculation. The difference between "legitimate" and "speculative" value lies merely in the degree of uncertainty surrounding the future conditions upon which the guess as to present value is based. The value of bonds fluctuates but slightly and then only because of changes in the "rate of interest," for the income is fixed and secure and the price or value of the bond is easily calculated mathematically by use of the desired rate of interest in discounting this income. By contrast, the uncertainty and fluctuation in the value of industrial stocks is due to a corresponding uncertainty regarding future dividends.

68. Independence of Capital Value and Past Outlay or Cost.

—It is essential to thoroughly establish the idea of this independence of capital or expectation value from all elements of past cost. It cannot be said that past costs are without influence on value. In most cases they have been the direct means of creating this value. This leads to the erroneous conclusion that such costs are identical with value or are value. The fallacy of this idea is easily shown. Cost is an individual outlay. Value is the product of social conditions. Costs are undertaken in order to obtain value and, when contrasted, are negative where value is positive.

To illustrate: Two parcels of land are acquired and cleared at an expense in each instance of \$25 per acre. One tract has such poor soil that it will raise nothing of value and cannot be sold for \$5 per acre. The other is suitable for truck gardening and eventually sells for \$500 per acre. Such variations in value, entirely independent of cost, may be caused by differences in location or by the development of new crops or new markets. All such fluctuations are due directly to the prospects for future income.

The true relation between cost and value is that between a means and an end. The cost is assumed for the purpose of obtaining income and the value is derived from the prospect of this income. If the outlay has been effectual in insuring the future income the rise in value will follow. If the outlay is ineffectual for any cause, chiefly through errors of judgment or through incompetence, there will be no corresponding increase in value. On the other hand, value will increase and decrease from social and economic causes not even remotely associated with the owner's outlay. For these reasons it must be accepted that past cost or outlay, while it may furnish useful information for the calculation of value, does not determine value.

69. Effect of Capital Value on Sale Value.—Sale value depends directly upon the demonstrated ability of property to earn income. Whenever this earning capacity is uncertain, especially if it appears probable that it may increase in the future, sale values move in response to the guesses of those

who exchange property, as to the probable future income. Inflated ideas of future income, if commonly accepted, result in inflated sale values, as instanced in real estate booms, high values for western fruit lands and flurries in mining stocks. Could sound, reliable information regarding the possible future income from property be a matter of common knowledge, inflation of values would become very difficult. Occasionally, by rapid development of a region, values which at first seemed high materialize by the assurance of income resulting from this increase in population. More often the boom is overdone, and sooner or later, values collapse to a level corresponding to the measure of income possible. In every case the sale value is a crude attempt at approximating capital value, and the disagreement between them is due to ignorance of the investing public as to the true present value of the possible income.

70. Effect of Capital Value upon Appraised Value. — The appraiser must therefore take into account the actual sources of value in appraising property in order to check the sale value and detect any glaring discrepancies between the reasonable capital value of the net income and the price asked for the property. If the owners of wood lots are selling their timber at one-half to one-third of its real value, because they are ignorant of both the quantity of standing timber they possess and the current prices for stumpage, or if these current prices manifestly are so low that the purchaser is able to make a very large profit, the appraiser can go back of sale values and determine the value of such stumpage directly from income (§ 61). Where no sale values exist, the determination of capital value is his only recourse and is fully satisfactory.

71. Inflation of Capital Values. — Value thus depends upon future income which may never materialize. On the chances of obtaining this income, investors are willing to purchase property at values computed by discounting these chances. The *rate* of income earned on the past cost or invested capital of the owner does not fix the price of the property. The amount and value of this income are the determining factors. The price is determined by discounting this value, using a rate of interest

which is satisfactory to both the purchaser and owner. Should a business be sold which represents an investment of \$50,000, and earns a net income of \$25,000 per year, or 50 per cent on cost, the purchaser would probably be satisfied with 20 per cent per year. The price will then be $\frac{25,000}{.20}$, or \$125,000, and the former owner has exchanged his income of \$25,000 for capital on which he in turn must earn 20 per cent in order to be as well off as before.

In the large operations of modern finance, especially in the formation of trusts and transportation monopolies, it was considered that unlimited opportunities for future income would be created by the control of prices and railroad rates. This belief was then capitalized on a generous scale. The final step was to sell these securities to the public, thus realizing, for the original owners, the capitalized income which had not been earned. This inflated income must now materialize or the value of these investments will correspondingly diminish, as has been the case with many such ventures in the recent past.

72. The Regulation of Capitalization. — It is possible to prevent the fraud and burdens of over-capitalization by giving to states or the national government the power of supervision over such semi-public operations. Since income is the key to capitalization, the exaction of extortionate rates or prices should be checked. Capitalization may also be regulated to conform to the reasonable requirements and to the true earning power of the business. The effect of such regulation will cause disproportionate differences between the capital value placed upon a business and its actual cost, to shrink to reasonable proportions, and the balance sheet to show a margin of profit more in keeping with the best interests of the public from whom must come the income upon which these values rest.

73. The Problem of the Balance Sheet. — Accounting practice is based on an equation in which

$$\text{goods or assets} = \text{liabilities} + \text{proprietorship}.$$

The left-hand member of this equation represents the tangible and intangible assets, at book value, or whatever value is

adopted as correct according to the conditions and principles employed in determining such values. Accounts receivable and debts owing to the business are part of the assets.

The right-hand member represents the capital paid in plus the borrowed capital. Debts owed by the business and accounts payable plus borrowed funds constitute the liabilities.

In balancing the equation, the "value" of assets is first determined. The liabilities are rigid, therefore the balance represents the proprietorship. When this is originally expressed as capital stock, an excess of value is shown as surplus, while a deficit is carried to the left member to balance.

Changes in the status of the business, which are constantly occurring, take one of two forms; a change in value of capital resulting from net income or loss, or a change in said value independent of such net income or loss. The cash for the purchase of assets must come either from the proprietors as additional capital, or from surplus derived from income. On the other hand, sales are mere exchanges, bringing in an equivalent asset in cash.

It is evident, then, that the assets are increased only by

Additional capital invested by proprietors.

Net income, from

Excess of sale value over cost of assets.

Revenue from use of assets.

Appreciation in value of assets.

The last item is a potential income, which may or may not be eventually realized. In the same way, assets are decreased by

Withdrawal of capital by proprietors.

Losses or destruction of assets not insured.

Dividends.

Expenses.

All items of current income or expense effect a corresponding change in the net value of the capital, but as the balance sheet is not usually made up oftener than once a year, these items are carried in the profit and loss account, and only the surplus or deficit entered in the balance sheet.

The commercial balance sheet attempts to show, first, the

status of the assets, and second, the relation existing, as a result of this status, between assets and proprietors.

The problem of the balance sheet lies in the treatment of potential income, and its effect on the value of the assets. As long as the assets are entered at cost, and income appears only after it has been actually received, no potential or unearned profits appear in the balance. On this basis, the excess of annual costs over annual income may be added to capital as *cost* of assets, as is permitted in railroad construction (§ 57), and by the government in computing the income tax (§ 162). Unearned interest on such costs would be excluded in a balance sheet.

But if the assets are revalued, at "actual" value, based perhaps on present increased sale value, but due entirely to increased prospects of income capitalized as expectation value, the alteration of the value of the assets, thus justified, indicates an apparent profit to the owners exactly equalling this increase, not yet realized nor available for the payment of dividends.

Not only does the balance sheet fail to distinguish between actual and potential profits, but it entirely ignores the element of time in indicating the relative value of the profits earned, compared with the capital invested.

The fundamental relations exhibited in the balance sheet, and its connection with the profit and loss account, may be set forth in the following framework.

BALANCE SHEET				
Assets		Liabilities, Proprietorship		
+	-	-		+
Cash.			=	Capital paid in.
Capital assets in form of goods or property.				Borrowed capi- tal or liabili- ties.
Deficit.				Surplus.

The diagram corresponds to an equation in which the opposite sides may show both plus and minus items. A minus item, by transfer from one side to the other, becomes plus. The items included in each subdivision represent a separate equation.

CHANGES IN STATUS DURING YEAR

I. CAPITAL ACCOUNT

a. Deductions from Capital

1. Depreciation

+	-		-	+
	Losses not covered by insurance. Depreciation in value not offset by income or depreciation fund.	=	Corresponding reduction in value of proprietorship constituting a loss of capital or of surplus.	

2. Withdrawal of Capital

	Withdrawal of cash or other assets by transfer to proprietors.	=	Cancellation, or reduction in value of capital stock.	
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These withdrawals may be induced by the sale of certain of the capital assets, or by their depreciation in value. Instead of creating a depreciation fund, the investment in a business may be steadily reduced. This would be the course pursued in lumbering, should it be found impossible to reinvest the income in new timber stumpage or otherwise continue the operation. The cancellation of bonds issued against standing timber must keep pace with the cutting of this timber.

b. Additions to Capital

1. Additional Investment

+	-		-	+
Purchase of additional assets. Cash.		=		Additional capital paid in.

2. Income

Depreciation fund. Increase of surplus.		=		Net income (gross income minus annual expenses, interest and dividends) transferred from profit and loss account.
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3. Capitalized Expenses

Increase in book value, <i>i.e.</i> , in cost of capital assets already owned.		=		Additional capital to meet deficit caused by excess of expenses over income.
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This last transaction consists in marking up the capital assets on the basis not of value, but of cost, to coincide with the additional capital advanced to meet annual expenses in absence of revenue. Normally the deficit would appear as such in the balance sheet and not in the capital account.

The appreciation in value of capital, or unrealized profits, requires no corresponding investment, and should be distinguished from increase in cost by a separate account.

4. Appreciation of Assets

+	-	-	+
Appreciation or marking up of value of capital assets.		=	Corresponding increase in proprietorship representing potential profits.

II. PROFIT AND LOSS ACCOUNT

This account records the cash outlay and income during the year, and its balance, indicating a net income or deficit, is then transferred to the capital account, and appears in the balance sheet.

+	-		-	+
4. Depreciation fund.		=	1. Cost of capital assets. 2. Annual expenses. 3. Interest on borrowed capital. 5. Dividends.	Gross income, including sale of capital assets.
6. Additions to surplus.				

Item 1 represents cost of assets sold, and this amount is credited to the capital asset account. Item 2 includes the current expenses of operation including taxes. Item 3, interest on borrowed capital is met before further disposition of income is permitted. In item 4, the depreciation fund thus

appropriated from income offsets the loss from this source in the capital account. Item 5, dividends are then appropriated, and the remainder of the annual income (item 6) represents profits remaining in the business.

A deficit, instead of a net income, occurs if the gross income is insufficient to cancel the first three items. When a regular annual provision is made for depreciation this may also cause a deficit. In case this deficit occurs, it results either in diminishing the surplus, or the value of the proprietor's equity, or else it is considered as a natural and legitimate cost to be added to capital expenditure. This difference in the point of view coincides with the difference between enterprises yielding annual income and investments on which income is naturally deferred for longer periods.

The failure of the commercial balance sheet to fully inform the investor as to the true economic status of his business, for the reasons mentioned (page 48), suggests a supplementary statement, in which the factor of time, and the increase in value of assets due to future or potential income, may be expressed. Such a statement is merely a prospectus or forecast, but will be of great value if its true character is clearly explained. Its principal use is found in connection with a business in which the actual returns are deferred rather than annual.

III. PROSPECTUS OR FORECAST

+ Debit		- Credit	
+	-	-	+
Expectation value of future income, capi- talized as val- ue of assets.	Future costs to be subtracted from income previous to capitalization.	= Past income. Interest cred- ited on past income.	Total cash in- vestment for all purposes. Interest defer- red on cash investment. Potential loss.
Potential profit.			

To obtain this last balance, the cost of assets must be shifted to the "credit" column, thus comparing cost with capitalized or expectation value and revealing the potential profit by which the true economic status of the business may be gauged.

The sources of confusion in the treatment of assets by accountants are now revealed. Costs and value are diametri-

cally opposite in character, yet custom and the practical necessities of commercial accounting compel a consolidation or compromise between these elements, as a result of which the valuation of assets is sometimes based on cost, with or without "expenses," sometimes on capital value, and often on both cost and value. Sprague* both sums up and explains this practice by the statement: "The aspect of assets as the present worth of future services (capitalized income)† is entirely based on opinion, while the aspect which regards them (assets) as the resultant of services (cost) is based upon facts."

It is evident that the proprietor is at liberty to compute his potential profits by capitalizing future net income on any basis which he sees fit, and may charge against this value a cost on which he adds the unearned interest on his capital. He may in this manner satisfy himself as to his economic status, and determine whether to sell, buy, or continue to operate. But in public accounts a limit must be placed both upon inclusion of costs not actually incurred, and of profits not yet realized. Hence the full and perfect expression of economic status is seldom permitted or secured by the commercial balance sheet, and must be left to supplementary calculations in the nature of forecasts.

In this economic comparison, the proprietary or right-hand member of the ordinary balance sheet does not appear, and the account deals entirely with the material assets. The balance is struck between past costs, with interest, less past income, and future value, discounted, less future expenses. Instead of being upon the same side of the equation, necessitating an arbitrary choice between absolutely opposite elements, the equation lies between cost and value, and the balancing quantity is the potential or unearned profit or loss, expressed as capitalized or expectation value.

The incorporation of a portion of this potential profit into the regular balance sheet, previous to actual sale of the business, is an action dictated entirely by motives of expediency.

* "The Philosophy of Accounts," by Charles E. Sprague, 54 West 32d St., New York, p. 41, Art. 111. Published by author (at above address). 1908.

† Material in parentheses inserted by author.

CHAPTER V

FORMULÆ OF COMPOUND INTEREST

74. The Discount Factor. — The mathematical relation between present and future values is expressed by formulæ. By substituting the desired rate of interest in the formula the ratio is found by which the one value may be converted into the other. This ratio may be termed the “discount factor.”

75. Rentals. — The term “rental” is used to denote a definite sum recurring as income at regular intervals. The interval may be of any length, even one hundred years, provided the payment is assumed to recur periodically. Since income to one person means outlay to another, the amount of an expense occurring at regular periods is obtained by treating it as a rental. The sum of the future values of a series of payments constituting a rental must frequently be determined, as well as the discounted or present value of this sum.

The formulæ needed in valuation are those which will determine:

The future value of single sums.

The present value of future sums.

The future value of the sum of rentals.

The present value of the sum of future rentals.

The ratio between present and future values.

76. Future Value or Cost of Single Sums. —

V = value of capital or sum.

V_0 = value at beginning of period.

V_n = value at end of period.

V_1, V_2 = value at end of years indicated by numeral.

n = period intervening before final reckoning.

p = rate of interest adopted.

$\frac{p}{100} = 0.0p$, interest expressed decimally.

The expression $0.0p$ is an arithmetical term, in which the letter p is substituted for the numeral representing the rate of interest. For a given rate, as 4 per cent, the expression reads 0.04 ; for 10 per cent, it reads 0.10 .

$0.0p$ is the ratio between capital and interest for one year. Therefore $1 + 0.0p$, or $1.0p$, is the ratio between the capital at the beginning and end of the year, assuming that the interest is added to the principal.

$$\begin{aligned} V_1 : V_0 &:: 1.0p : 1. \\ V_1 &= V_0 \times 1.0p. \end{aligned}$$

At the end of the second year

$$\begin{aligned} V_2 : V_1 &:: 1.0p : 1. \\ V_2 &= V_1 \times 1.0p \\ &= (V_0 \times 1.0p) 1.0p \\ &= V_0 \times 1.0p^2, \\ V_3 &= (V_0 \times 1.0p^2) 1.0p \\ &= V_0 \times 1.0p^3 \end{aligned}$$

and

$$\begin{aligned} V_n &= (V_0 \times 1.0p^{n-1}) 1.0p \\ &= V_0 \times 1.0p^n. \end{aligned} \tag{I}$$

This formula is used to determine the accumulation of cost on an investment which does not produce income until the close of a stated period. In such a cost account (§§ 25 and 36) all investments are charged with interest. Sums representing expenditures which occur only once and cannot be treated as rentals, as, for instance, the cost of planting trees, are totalled for n years by this means.

All formulæ dealing with future values of present sums show also the present cost of past expenditures. They reckon forward from beginning to end of an investment.

77. Present, Expectation, or Capital Value of Future Sums. — In computing future sums, the value at a future period has no special significance until it is discounted to the present.

$$\begin{aligned} \text{If} \quad & V_n = V_0 \times 1.0p^n \\ \text{then} \quad & V_0 = \frac{V_n}{1.0p^n}. \end{aligned} \tag{II}$$

The arithmetical equivalent of $\frac{1}{1.0p^n}$ may be expressed as a decimal.

Discount formulæ are used in valuation of assets. The discounting of future "profits" or net income is identical with the determination of capital value. There can be no value in excess of this discounted net income (§ 62).

To obtain this net present value, future costs may be first discounted and then subtracted from discounted gross income. Or this subtraction may be made at any future period and the *net* income discounted. If the values are properly computed to the same year, before striking the balance, the discounted results are identical. For instance, an expense c occurs in the year d , which lies between the years 0 and n .

The present value or equivalent of c will be $\frac{c}{1.0p^d}$.

The future equivalent of c at the end of the year n will be $c \times 1.0p^{n-d}$, since $n - d$ is the period over which interest on c would accumulate.

The present equivalent of $c \times 1.0p^{n-d}$ will be

$$\frac{c \times 1.0p^{n-d}}{1.0p^n} = \frac{c \times 1.0p^n}{1.0p^{n+d}} = \frac{c}{1.0p^d}.$$

Hence, to determine the *net* present value of future income, future expenses may be computed *forward* to the year in which the income is received, and subtracted from this income, and the resultant net income discounted to the present. This accomplishes the same purpose as discounting each item of expense to the present before subtracting it from gross income. The choice of these methods is merely a matter of convenience.

78. Use of Logarithms and Tables. — The arithmetical equivalents of these formulæ are calculated by the use of logarithms. The numeral representing the desired rate of interest is substituted for p , and the number of years in the period for n . For 4 per cent and 50 years, $1.0p^n$ becomes 1.04^{50} . The logarithm of 1.04 is multiplied by 50. The result is the logarithm of 1.04^{50} , from which the ratio is obtained. In the case cited, the ratio is 7.1067, which indicates that \$1.00 will increase in

value to \$7.1067 if placed at compound interest at 4 per cent for fifty years.

The value of all other formulæ are derived from the expression $1.0p^n$. The values for discount, $\frac{1}{1.0p^n}$, are obtained by reducing the fraction to a decimal. In the above case, $\frac{1}{7.1067}$ gives a discount factor of 0.1407, which means that \$1.00 payable in fifty years has a present value, at 4 per cent, of \$0.1407.

These values may be expressed in tables, which make such calculations unnecessary. (Table VI, appendix.)

79. Future Value of Temporary Annual Rentals. — The future value, or cost, of a rental continuing for a temporary period represented by n annual payments, equals the sum of all these items plus interest upon each from the date of payment to the end of the year n .

The payments are regarded as occurring in each instance at the *end* of the year, although they might be made during the year. The final payment thus incurs no interest charge.

Let r = annual rental.

Then, final rental for year n = r .

Rental for year $n - 1$, in year n = $r \times 1.0p$.

Rental for $n - 2$, in year n = $r \times 1.0p^2$.

Rental for first year, in year n = $r \times 1.0p^{n-1}$.

These terms form an increasing series showing a geometrical progression. The ratio of increase or multiple is the factor $1.0p$. The sum of these terms gives the total value or expense of the rental at the end of the period. The calculation of these separate values is unnecessary. The sum may be obtained by formula.

Let V_r = total value of rental.

(1) $V_r = r + r(1.0p) + r(1.0p^2) + r(1.0p^3) + \dots + r(1.0p^{n-1})$.

Multiply the equation by the ratio $1.0p$.

(2) $V_r(1.0p) = r(1.0p) + r(1.0p^2) + r(1.0p^3) + \dots + r(1.0p^n)$.

Subtract (1) from (2).

$$V_r(1.0p) - V_r = r(1.0p^n) - r,$$

$$V_r(1.0p - 1) = r(1.0p^n - 1),$$

$$V_r = r \frac{(1.0p^n - 1)}{1.0p - 1}.$$

$$\begin{aligned}
 \text{Since } 1.0p - 1 &= 0.0p, \\
 V_r &= \frac{r(1.0p^n - 1)}{0.0p} \\
 &= \frac{r}{0.0p}(1.0p^n - 1).
 \end{aligned}
 \tag{III}$$

Outlay for fire protection or sums paid annually for administration serve as examples of annual rentals. As income we may have receipts from grazing or hunting privileges. The above formula holds good only during the period that such rentals remain equal in amount and value as is the case with actual rent. Should they fluctuate annually, each item must be separately computed.

If rentals remain the same for a period and then rise or fall to a different level for a second period, the above formula is used to compute the total to the end of the first period. This value then bears interest for the remaining period as a definite sum (I), and the value of the new rental r_1 for this second period is computed in the same manner as before and added to the total. For two periods termed a and b this reads:

$$V_r = \left\{ \frac{r(1.0p^a - 1)}{0.0p} \right\} 1.0p^b + \frac{r_1(1.0p^b - 1)}{0.0p}. \tag{IIIa}$$

80. Formulæ for Geometric Series.—In the series whose sum is obtained by Formula III, the ratio $1.0p$ is greater than 1, which gives an increasing series.

Let g = first term of series.
 q = ratio or multiple (such as $1.0p$).
 Q = sum of series.

Then, by demonstration above (III),

$$Q = \frac{g(q^n - 1)}{q - 1}. \tag{Q_1}$$

The sum of future values of rentals forms such a series.

Should the ratio be less than 1, each succeeding term will be diminished in value, which gives a decreasing series.

The sum of this series is obtained by the same formula, but a more convenient form of expression, obtained by multiplying both numerator and denominator by -1 , is

$$Q = \frac{g(1 - q^n)}{1 - q}. \tag{Q_2}$$

The sum of present or discounted values of rentals forms such a descending series.

These formulæ apply either to temporary rentals, with a limited number of terms, or to perpetual rentals, with an infinite number of terms.

For an infinite increasing series

$$Q = \frac{g(q^x - 1)}{q - 1} = \infty \text{ (infinity)}. \quad (Q_3)$$

For an infinite decreasing series, since the value of q^n diminishes in direct proportion to the exponent used,

$$q^x = 0.$$

Therefore

$$Q = \frac{g(1 - q^x)}{1 - q} = \frac{g}{1 - q}. \quad (Q_4)$$

81. Present, Expectation, or Capital Value of Temporary Annual Rentals. — Let R = present, expectation, or capital value of the sum of a rental.

Should each rental be discounted to the present by Formula I, the sum of the resulting terms is

$$R = \frac{r}{1.0p} + \frac{r}{1.0p^2} + \frac{r}{1.0p^3} + \dots + \frac{r}{1.0p^n}.$$

Applying Formula Q_2 for the summation of decreasing temporary rentals,

$$\begin{aligned} g &= \frac{r}{1.0p}, \\ q &= \frac{1}{1.0p}, \\ R &= \frac{\frac{r}{1.0p} \left(1 - \frac{1}{1.0p^n} \right)}{1 - \frac{1}{1.0p}}. \end{aligned}$$

Multiplying numerator and denominator by $1.0p \times 1.0p^n$,

$$\begin{aligned} R &= \frac{r(1.0p^n - 1)}{(1.0p - 1) 1.0p^n} \\ &= \frac{r(1.0p^n - 1)}{0.0p \times 1.0p^n}. \end{aligned} \quad (IV)$$

The same result may be obtained by discounting the final future value of the sum of these rentals to the present (II).

$$\text{This future value } V_r = \frac{r(1.0p^n - 1)}{0.0p}.$$

$$\text{Hence } \frac{r(1.0p^n - 1)}{0.0p} \times \frac{1}{1.0p^n} = \frac{r(1.0p^n - 1)}{0.0p \times 1.0p^n}.$$

Since capital value is the net value of income discounted to the present, the application of this formula is evident. It may be used to discount either annual income or annual expenses. Should the two occur simultaneously, only the net annual profit or loss is thus discounted.

82. Future Value of Temporary Intermittent Rentals Due First at n Years, and t Times at Intervals of n Years Thereafter. — A rental which occurs at intervals greater than one year is termed intermittent, although if the interval is regular it does not differ in character but only in degree from annual or monthly rentals.

Let n = period between rentals.

t = number of payments expected.

nt = total period covered by the investment.

r' = intermittent rentals due at intervals of n years.

V_r = total value of intermittent rentals at end of period.

The future value of the sum of these rentals, if computed in the year of payment of the final rental, is

$$\begin{aligned} \text{Final payment} &= r', \\ \text{Payment, } n \text{ years preceding} &= r'(1.0p^n), \\ \text{Payment, } 2n \text{ years preceding} &= r'(1.0p^{2n}), \\ \text{First payment} &= r'(1.0p^{(t-1)n}). \end{aligned}$$

By use of Formula Q_1 , the sum of these terms is found by substituting the ratio $1.0p^n$ for q and the factor t for n .

The formula then reads:

$$V_r = \frac{r'(1.0p^{nt} - 1)}{1.0p^n - 1}. \quad (V)$$

Either income or expenses which terminate at a definite date are far more apt to occur annually than at longer intervals

(III and IV), and if intermittent and terminating, such as a succession of thinnings from a crop of timber previous to final cutting, they are also apt to be of different values, requiring separate valuation. If assumed to be equal in value, they may usually be assumed to recur perpetually (IX).

83. Present, Expectation, or Capital Value of Temporary Intermittent Rentals, Due First at n Years, and t Times at Intervals of n Years Thereafter. — The present, expectation, or capital value of intermittent rentals limited in number is found as in the case of annual rentals.

The series decreases.

For rentals due first after n years and t times at intervals of n years thereafter, this series is

$$R = \frac{r'}{1.0p^n} + \frac{r'}{1.0p^{2n}} + \frac{r'}{1.0p^{3n}} + \dots + \frac{r'}{1.0p^{tn}}.$$

Then (Q₂),

$$\begin{aligned} g &= \frac{r'}{1.0p^n} \\ q &= \frac{1}{1.0p^n} \\ Q &= \frac{\frac{r'}{1.0p^n} \left(1 - \frac{1}{1.0p^{nt}} \right)}{1 - \frac{1}{1.0p^n}}. \end{aligned}$$

Multiplying numerator and denominator by $1.0p^n \times 1.0p^{nt}$,

$$R = \frac{r' (1.0p^{nt} - 1)}{(1.0p^n - 1) 1.0p^{nt}}. \quad (\text{VI})$$

The same result is obtained by discounting the future value of the series (V) to the present (II) using the factor $\frac{1}{1.0p^{nt}}$ as the substitute for $\frac{1}{1.0p^n}$.

$$R = \frac{r' (1.0p^{nt} - 1)}{1.0p^n - 1} \times \frac{1}{1.0p^{nt}} = \frac{r' (1.0p^{nt} - 1)}{(1.0p^n - 1) 1.0p^{nt}}.$$

84. Future Value of Temporary Intermittent Rentals, Due First in a Years, and t Times at Intervals of n Years Thereafter. — When the value of the sum of temporary rentals is

determined for the date of the final payment, falling in the year nt , the interval elapsing subsequent to this payment is zero. When this payment falls at some previous date, a , the total value at that date accumulates interest during the remaining period. This period is equal to $n - a$ years.

The series forming the sum of the rentals is

$$V_r = r' (1.0p^{n-a}) + r' (1.0p^{2n-a}) \cdot \cdot \cdot + r' (1.0p^{tn-a}).$$

By use of Formula Q₁, substituting the ratio $1.0p^n$ for q , and the factor t for n , this becomes

$$V_r = \frac{r' (1.0p^{nt} - 1) 1.0p^{n-a}}{1.0p^n - 1}. \quad (\text{VII})$$

By multiplying Formula V by $1.0p^{n-a}$ (see Formula I), the same result is obtained.

85. Present, Expectation, or Capital Value of Temporary Intermittent Rentals, Due First in a Years, and t Times at Intervals of n Years Thereafter. — The present value of the first payment, due in a years, is $\frac{r'}{1.0p^a}$ (II). The period a may be of any length and may exceed n . The present value of subsequent payments diminishes by the constant ratio $\frac{1}{1.0p^n}$, forming the series

$$R = \frac{r'}{1.0p^a} + \frac{r'}{1.0p^{a+n}} + \frac{r'}{1.0p^{a+2n}} \cdot \cdot \cdot + \frac{r'}{1.0p^{a+(t-1)n}}.$$

To obtain the sum of present values (Q₂),

$$g = \frac{r'}{1.0p^a},$$

$$q = \frac{1}{1.0p^n},$$

$$Q = \frac{\frac{r'}{1.0p^a} \left(1 - \frac{1}{1.0p^{nt}} \right)}{1 - \frac{1}{1.0p^n}}.$$

$$\frac{r'}{1.0p^a} = r' (1.0p^{-a}).$$

Multiplying numerator and denominator by $1.0p^n \times 1.0p^{nt}$,

$$R = \frac{r' (1.0p^{nt} - 1) 1.0p^{n-a}}{(1.0p^n - 1) 1.0p^{nt}}. \quad (\text{VIII})$$

This formula is also derived from Formula VII by discounting the final value to the present (II), substituting $\frac{1}{1.0p^{nt}}$ for $\frac{1}{1.0p^n}$.

Formulae V to VIII are seldom used in forest valuation (§ 82).

86. Present, Expectation, or Capital Value of Perpetual Rentals. — Under the assumption that a rental is perpetual (§ 80), the intermittent rental r , payable every n years, forever will have a future value equal to infinity (∞).

Its present value (Q_4) is

$$\begin{aligned} g &= \frac{r'}{1.0p^n}, \\ q &= \frac{1}{1.0p^n}, \\ Q &= \frac{\frac{r'}{1.0p^n}}{1 - \frac{1}{1.0p^n}}, \\ R &= \frac{r'}{1.0p^n - 1}. \end{aligned} \quad (\text{IX})$$

If the first rental is paid in the year a instead of at n years, as in case of revenue from thinnings, the present value of the sum of a perpetual series due every n years thereafter (Q_4) is

$$\begin{aligned} g &= \frac{r'}{1.0p^a}, \\ q &= \frac{1}{1.0p^n}, \\ Q &= \frac{\frac{r'}{1.0p^a}}{1 - \frac{1}{1.0p^n}}, \\ R &= \frac{r' (1.0p^{n-a})}{1.0p^n - 1}. \end{aligned} \quad (\text{X})$$

But should the period of discount lie between the year a and the final year n , when the crop is harvested, as it does when the value of growing timber is computed at a years, this formula (Q_4) is

$$\begin{aligned} g &= \frac{r'}{1.0p^{n-a}}, \\ q &= \frac{1}{1.0p^n}, \\ Q &= \frac{\frac{r'}{1.0p^{n-a}}}{1 - \frac{1}{1.0p^n}}, \\ R &= \frac{r'(1.0p^a)}{1.0p^n - 1}. \end{aligned} \quad (\text{Xa})$$

When the first rental is due in the present year, the series becomes

$$\begin{aligned} R &= r' + \frac{r'}{1.0p^n} + \frac{r'}{1.0p^{2n}} + \dots + \frac{r'}{1.0p^\infty}, \\ Q &= \frac{r'}{1 - \frac{1}{1.0p^n}}, \\ R &= \frac{r'(1.0p^n)}{1.0p^n - 1}. \end{aligned} \quad (\text{XI})$$

By separating the first payment from the series, the remaining terms equal $\frac{r'}{1.0p^n - 1}$ (see IX), and the sum is

$$R = r' + \frac{r'}{1.0p^n - 1}. \quad (\text{XIa})$$

An annual rental, payable perpetually, has a present, expectation, or capital value (Q_4) of

$$\begin{aligned} g &= \frac{r}{1.0p}, \\ q &= \frac{1}{1.0p}, \end{aligned}$$

$$Q = \frac{\frac{r}{1.0p}}{1 - \frac{1}{1.0p}},$$

$$R = \frac{r}{1.0p - 1},$$

$$= \frac{r}{0.0p}. \quad (\text{XII})$$

Formulae IX to XII are of great importance in forest valuation. The conception that a rental may be perpetual is difficult to grasp. But this is merely another way of stating that the property producing the rental does not depreciate in value. Granting this premise, the receipt of income for any given period leaves the property capable of yielding another return of equal value. Since this assumption can be extended indefinitely, the expected income may be regarded as perpetual.

Formula XII is customarily used to determine the money capital which will produce an annual income of r . If we assume that money remains of the same value, and nothing occurs to impair the integrity of the capital, it can be imagined that this income is perpetual. In the case of land whose fertility is maintained by proper methods, so that the net income does not represent a drain of capital, or soil depreciation, the same conditions are true, and the capital value of the soil may be found by use of Formula XII.

This premise will hold for forest lands producing crops at long intervals. The trees maintain the productiveness of the soil. In this case, the net income, being intermittent, is capitalized by Formula IX.

For practically all other forms of investment, revenue is apt to be accompanied by depreciation of capital, which causes the income to diminish and finally terminate, thus requiring, instead of the simple formulæ just quoted, the use of the formulæ IV, VI or VIII for the present value of temporary rentals.

These values are *less* than those obtained by $\frac{r'}{1.0p^n - 1}$ or $\frac{r}{0.0p}$

and the erroneous use of these latter formulæ leads to over-valuation of assets.

87. Relation Between Future Value of Temporary Annual Rentals and Present, or Capital, Value of Same.—In the formula for future value of temporary annual rentals, expressed as $\frac{r}{0.0p} (1.0p^n - 1)$ (§ 79), the portion $\frac{r}{0.0p}$ represents the capital value of a perpetual annual rental (XII), or the capital which will produce an annual income of r . This formula might be expressed $\frac{r}{0.0p} (1.0p^n) - \frac{r}{0.0p}$.

Substituting R for $\frac{r}{0.0p}$,

R = capital value of annual rental r .

$$\begin{aligned} \text{Then} \quad \frac{r}{0.0p} (1.0p^n) - \frac{r}{0.0p} &= R \times 1.0p^n - R \\ &= R (1.0p^n - 1). \end{aligned} \quad (\text{IIIb})$$

This expression gives the sum of the compound interest, or deferred income, earned by the capital R in n years, and is obtained by computing the value of R at n years at p per cent, and then subtracting the original capital.

The numeral 1 substituted for R indicates that at any future period the original capital remains intact. Since the sum of the future values of an annual rental is thus shown to equal the compound interest or income on its capitalized value, the formula $R (1.0p^n - 1)$ may be used to compute the total cost not only of annual rentals, but of interest on invested capital whenever the capital so invested remains intact at the end of the period, as is the case with land.

88. Conversion of Intermittent into Annual Rentals.—To convert an intermittent rental occurring every n years into its equivalent annual rental, it is incorrect to divide the amount of the intermittent rental by the factor n . The intermittent rental is not received until n years from daté, while the annual rental is received in installments over the entire period, which makes the average period of deferred payment less than half as long.

The capital value of an intermittent rental is $\frac{r'}{1.0p^n - 1}$ and of an annual rental $\frac{r}{0.0p}$. Since the annual income for each is required to be equal, these expressions must have the same value.

$$\text{If} \quad \frac{r'}{1.0p^n - 1} = \frac{r}{0.0p},$$

$$\text{then} \quad r = \frac{r'}{1.0p^n - 1} \times 0.0p. \quad (\text{XIII})$$

The annual rental equivalent to an intermittent rental falling due first at an interval shorter or longer than n is found in the same way by multiplying the capital values given in formulæ XI and XII respectively by $0.0p$. The results show that notwithstanding the fact that the intermittent rentals are equal in amount and paid at equal intervals, the capital value, and its equivalent in permanent annual income, constantly rises until one of the intermittent payments is received, which at once diminishes this capital value by the exact amount of the payment. If exchanged for an annuity at this time, the permanent income thus secured is the lowest for any portion of the period, while a similar exchange effected just previous to receipt of intermittent income secures the maximum annual revenue.

89. The Ratio of Income or Earnings. — The “rate of interest,” p per cent, is accepted as a fixed standard in computing cost (I); or in discounting for value (II). But when both cost, or investment, and income are known, the ratio of this income to cost represents the earnings of the capital invested, or dividends. A rate of interest can be found by which the discounted value of income is made to exactly equal cost, or the accumulated costs to just balance income. This rate is the per cent earned by the investment.

To distinguish this rate from the “rate of interest” p , the term x will be used.

x = per cent representing income earned.

Then

$$\begin{aligned} 1.0x^n &= \frac{V_n}{V_0}, \\ 1.0x &= 1 + 0.0x, \\ 1 + 0.0x &= \sqrt[n]{\frac{V_n}{V_0}}, \\ 0.0x &= \sqrt[n]{\frac{V_n}{V_0}} - 1, \\ x &= 100 \left(\sqrt[n]{\frac{V_n}{V_0}} - 1 \right). \end{aligned} \tag{XIV}$$

The rate x enables the owner to compare the earning power of his investment with enterprises which produce annual dividends (Chapter VIII).

Instead of solving the above equation, the value of x may be found by inspection of Table VI. The value of $1.0x^n$ is calculated, and the nearest equivalent figures must be found opposite n years in the table. By interpolation, x may then be found to within one-tenth of 1 per cent.

CHAPTER VI

INVESTMENTS AND COSTS IN FOREST PRODUCTION

90. The Business of Forest Production versus Lumbering. — Lumbering is a term applied to that portion of the business of forest utilization which begins with the removal of the forest products from the soil and is completed when the wood is placed upon the market as lumber, ties, shingles or other products. The stage dealing with the felling and transportation of the raw product to place of manufacture is termed "logging," while the manufacture itself is spoken of as "milling."

Forest production is a separate undertaking, which precedes lumbering, and deals entirely with the growing of forest crops, principally wood products. Until recently, the raw material on which the enormous investments in logging and milling are dependent, was drawn entirely from virgin stands grown without human aid or expense. Ultimately, forest production must rank in importance with lumbering, for under modern conditions, forests will seldom renew themselves unaided. The only large and complete organization of forest production as a business in this country at present is found in the administration of the national forests. The operations of certain states are gradually taking on a systematic form. But, as a whole, the productive side in the management of private timberlands has not yet been organized into a systematic effort, and takes the form of tentative or experimental measures, completely overshadowed by the operations of lumbering.

Logging and milling are indispensable adjuncts to production, but without it they are self-destructive. These three operations complete the chain of effort by which the income from forestry is finally secured. Owners of large tracts of timber usually undertake to log them in order to produce income from their holdings, and professional lumbermen prefer to own their stump-

age. When stumpage is owned, logged and manufactured by the same person or corporation, both costs and net profits are based on the total operation. Frequently an owner of stumpage will contract the logging and do his own milling. Both logging and milling may be contracted, the owner receiving the final net revenue from sale of lumber. Or, the owner may sell his stumpage, permitting the logger and mill man to make what profit they can. On the books of a firm operating with their own stumpage, a technical separation of costs is made between stumpage, logging and manufacture. The total profits as well should theoretically be distributed between these three departments, but for practical purposes this is unnecessary. Where desired, the "value" of stumpage is charged as a cost against logging. By increasing this value, logging costs are advanced in the same proportion, and the *profits* disappear into stumpage values, without in any way altering their amount or character. But should stumpage be owned separately, and its sale constitute a transaction taking place between different firms, the costs and profits arising from sale of stumpage are actual.

Forest production as a separate business, or even as a separate department of the general business of lumbering, is completed with the sale of the timber or products on the stump. This separation from lumbering is illustrated by the operations of the United States Forest Service, which performs no part of the logging or manufacturing.

When an owner of forest property finds it necessary to engage in logging and manufacturing, he should separate the costs and profits of these enterprises from the profits on growing or holding his timber. Often the owner is in position to log his own timber. This may result in securing a combined or total profit far in excess of the receipts obtainable from sale of stumpage. In general, lumbering, to pay as an enterprise, requires large tracts and a supply of timber which will last for a reasonable period, and the owners of small tracts must either sell their timber on the stump or contract for its manufacture.

91. Proprietary Accounts in Forestry.—Proprietary accounts, showing the amount of capital invested, and the income and

outlay, or profit and loss in the business of forest production, will deal with the operations of the owner as a unit, and not with the stand. If such accounts are kept according to the customary forms, only actual cash outlay or income can be recorded. The expenditures might be separated into capital account for such items as land, and profit and loss for wages, taxes and other current expenses.

Unfortunately, both for the accountant and the proprietor, such a business may fail to pay dividends for an extended period. Whether it does so or not depends upon the condition of the forest, for if the timber is already produced or grown, the income is within reach. With a young forest, or plantation, additional capital is required to cover every item of annual outlay in excess of income. Since no dividends are produced, interest on borrowed capital increases at a compound rate, actually when this interest is also borrowed and paid, or by analogy, when its payment is merely deferred.

Unless this condition is foreseen, and the enterprise floated without borrowing, or the loan is negotiated on the basis of deferred instead of annual interest, which is not customary, the proprietor must pay this interest or bankruptcy ensues. No existing custom of book-keeping will permit the charging of unearned interest as a cost or actual expenditure, *unless* it is paid out as above indicated. Hence, in the *absence* of borrowed funds, the accumulating "costs" represented by deferred interest (§ 36) are not shown on the books.

For a forest investment, representing, as it usually does, timber in all stages of growth, some capable of yielding income at once, while other portions require outlay for many years, it will be necessary to keep a proprietary account in the usual form, omitting the item of deferred interest on the owner's capital. Such an account will eventually show net profits, regarding interest as income (§ 37), but it will not reveal actual "cost," nor the profits of the undertaking (§ 39) or enterpriser's gain.

92. Cost Accounts in Forestry. — A serious defect in such proprietary accounts is that the value of the element of time is

wholly neglected (§ 11) and profits received after many years are shown in amount only, as though of equal value with profits of the year. In contrast, cost accounts (§ 25) seek to eliminate such inconsistencies and express costs in their true economic relation to income. This is accomplished by the adoption of a rate of interest (§§ 52 and 53) by which all expenditures are calculated forward to the year when the income, for which the cost was incurred, is finally received (§ 23). In practice, a separate cost account could be kept for each stand, lot or subdivision. The summary or total to a given date (the present) of these accounts would reveal to the proprietor his exact investment, including the "unearned" interest. This summary should differ from the corresponding items in the proprietary account by just the amount of this interest.

93. Investments, or Permanent Outlay.—Expenditures for property of a durable character in forestry are confined to land and to permanent roads, houses, telephones or other improvements. The original cost of these may be considered a capital expenditure.

94. Expenses, or Temporary Outlay in Forestry.—Temporary or current expenses would ordinarily include raw materials as well as items like wages (§ 21). But when wages are expended on planting trees and when the raw material consists of standing timber, which will remain as property for a period of years, this treatment is open to question. *After* the forest property has passed through its initial stage of construction, and is producing annual revenue, many items, such as reforestation, which at first call for an initial investment, can be regarded as current outlay. In this class will fall all expenses for upkeep of improvements.

95. Land.—Land is the one investment which can be credited as capital not ordinarily subject to depreciation, and indispensable to forestry. It remains after the timber is harvested. The purchase price of land is the only land value which can legitimately be entered in the cost account, and this valuation can remain the same as long as the owner retains the land for forest production. A sale of the property, terminating the

ownership and cost account, calls for a revaluation of land (§§ 61 and 62).

Bare land suitable only for forest crops does not command a high price (§ 96).

96. Standing Timber. — By definition, real estate comprises land and all that stands thereon. A purchaser of land acquires all permanent improvements. These improvements may be sold separately, but always with the stipulation that they are to be removed within a given time.

Standing timber is part of the real estate, and a purchase of land includes the timber, unless excepted by the owner subject to removal by him. Timber stumpage is sold by the owner on contract which allows the purchaser a definite time for removal, failing in which, the purchaser, in certain states,* loses his right to the timber. These timber rights sometimes run for periods of twenty years and while in force constitute a lien on the property.

By the time a crop of timber is large enough to cut, its value greatly exceeds that of the land. Virgin timber, containing material of high quality, shows a still greater excess of value (§ 184). In the past, lumbermen were quite indifferent to land values, regarding the title as desirable only because it made them independent of restrictions as to time and rate of cutting. What they purchased was the timber. Yet timber, whether young or old, is a "vanishing" asset. It must be converted eventually into income, and is subject to losses. Owing to the difficulty, in some cases, of distinguishing between the value of the land and of the timber, and the great preponderance of the latter value, the cost of standing timber is usually regarded as a capital expenditure. Bonds may be issued, with this standing timber as security, provision being made to retire these bonds at a rate to more than keep pace with the cutting of the timber. Such bonds are in this sense paid out of "capital," as this "capital" is converted into income. Bonds could not be floated on timber which was not about to be, or in process of being, cut.

* This subject has caused extensive litigation, and has resulted in conflicting practice. State supreme court decisions must be ascertained in each locality.

97. Permanent Improvements and Equipment.—On any property managed exclusively for forest production, and of sufficient size to demand continuous care and administration, buildings must be provided for the permanent force, both officers and laborers, especially since these tracts are not often conveniently near towns. Special telephone construction is often required. These are legitimately classed as capital expenditures. Stables, draft animals and tools come in this category, but proper allowance for depreciation must be made.

In cost accounts for individual stands, it is proper to distribute these overhead charges over the entire territory which they cover, pro-rating them on a per acre basis. The initial cost for these items may be included with the cost of land and timber. The annual charge for upkeep, or its equivalent in depreciation, must be entered as a current annual expense on the same acreage basis.

98. Roads and Transportation Systems.—Lack of permanent roads or other means of transportation is characteristic of American forest conditions. To yield a steady income, a forest must produce wood annually for market, which requires the transportation system to be in continual repair.

In inaccessible regions, timber must exist in sufficiently large bodies to justify construction of a system of transportation for its removal. The cost of transportation contains two elements: first, cost of construction; second, cost of transport. Construction cost, no matter how cheapened, cannot be reduced below a certain limit. Cost of transport is lessened by improving the means of transportation. When railroads, roads or flumes are built for merely temporary use, the entire cost of construction, as well as transport, falls on the timber removed at the time, and is pro-rated for each unit of product as, for instance, per thousand feet of lumber.

Since transportation is a feature belonging strictly to logging, it affects forest production through its influence on the value of stumpage. It is the universal practice in America to allow the cost of construction, as well as transport, to fall upon the logger, who takes it out of the price of stumpage.

In permanent forest property, this construction cost may be assumed by the forest owner, whether or not he intends to do the logging. This is necessary in order to permit of the radical change in logging from the removal of practically the entire crop at one time to the system of continuous removal of smaller quantities over the entire area. Under the latter system, no logging contractor could afford to construct his own transportation facilities.

Such permanent systems are a capital expenditure similar in character to land, and the overhead costs thus incurred should be pro-rated by area and charged as an addition to the cost of acquiring the property. The maintenance of transportation systems is an annual expense to be pro-rated in the same way. The owner's benefit from thus assuming costs belonging to logging is obtained by appropriating a larger share of the general profits through increased value of stumpage.

99. Silvicultural Operations. — All labor expended directly upon the forest crop constitutes a charge against the particular area or stand which is benefited. Such costs include planting, with the cost of the plants; weeding or removal of injurious "weed" trees or brush when the plantation is young; protection from insects by measures requiring expenditure of labor on the area; and thinning, provided this is done at a cost exceeding the value of the product obtained. Costs incurred in natural reproduction, by leaving a portion of the stand to serve as seed trees, correspond to the cost of planting and should be charged against the new crop. Practically all silvicultural expenses occur as single or initial costs, and not as annual expenses, when reckoned in cost accounts for the stand. In a forest under perfect management, costs of most of these operations constitute a nearly constant annual expense for the forest as a whole, and would so appear in the proprietary account.

100. Protection Expenses. — Initial expenses for protection from fire take the form of measures for disposal of slash left from logging. This expense is chargeable to the cost of the new crop and may be so treated in a cost account. But in practice, where the owner is not doing his own logging, as in the United

States Forest Service operations, this cost is transferred wholly or in part to the logger, and taken out of the price of stumpage. By this means, the true nature of the expense is concealed. The loss in stumpage value is a present cost or investment in the new crop as surely as if the owner received full value for the timber and re-expended the amount required for brush burning. This initial cost, then, accumulates interest until the new crop is cut (§ 107). For this reason, it has been advocated as an alternative, that a sum representing merely the annual interest on this initial expense could be expended annually on the area to secure better *protection* from fire, without increasing the total outlay.

Fire lines will frequently be necessary, particularly where a bad hazard occurs and where an investment has been made in plantations. This constitutes a capital expenditure, especially where the initial cost of the lines greatly exceeds their subsequent cost of maintenance. The charge must be pro-rated over the area benefited, dated from the year it was incurred. Subsequent maintenance of these lines, a necessity often lost sight of, is an annual charge per acre.

Lookout towers and installation of telephones used in fire protection are of the same class of expenditure as fire lines.

The labor employed in fire protection for patrol constitutes an annual charge.

Extra costs of extinguishing fire, if borne by the owners, constitute losses, entered against the area burned, and recoverable in some cases as damages.

101. Administrative Expenses. — With a large area and well-developed system of forest production, the administration of the tract involves not only all business matters dealing with sale of products, taxes, protection and silviculture, but includes as well the proper technical direction of the work to assure the success of the measures for forest production which are the basis of the business. The salary of a technical forester is, therefore, a necessity, and this must be pro-rated on an area basis. Should the tract be too small to justify such expense, the owner must seek occasional technical advice and trust the

management to a foreman who can follow directions. Such advice is best obtained, for very small areas, through the State forester. For larger areas it is practicable for several owners to co-operate and employ a forester.

102. Taxes and Insurance. — Taxes are separately discussed in Chapter X. Under present systems of taxation they tend to increase and this, coupled with the uncertainty as to the amount and time of these increases, makes it difficult to predict the total cost of this item or prescribe the exact method of calculating it. In this chapter, taxes will be assumed as a fixed annual expense, to be added to that for administration and protection.

Insurance is not as yet available in this country on growing or standing timber. The owner bears his own risk and losses.

103. Interest Charges. — These do not appear in proprietary accounts except when money is borrowed (§ 91). In cost accounts, past interest may at any time be calculated at whatever rate is considered equitable. Since the periods intervening between outlay and income may extend to over 100 years, the interest charges will often be found to far exceed the sum of all actual expenditures.

104. Classification of Costs in Forest Production. — Following the suggestions in the above analysis of costs, the investments in forest production may be grouped, for computation, into three classes:

Group A.

Capital expenditures for

Land.

Permanent improvements and equipment.

Permanent roads and firelines.

Group B.

Crop expenses for

Standing timber, which is often included with cost of land.

Planting or cost of regeneration.

Brush disposal or initial fire protection.

Weeding.

Protective operations.

Thinning.

Group C.

Annual or current expenses for

Protection.

Administration.

Taxes.

Maintenance of permanent improvements.

105. Calculation of Total or Final Costs. — To compute the total final cost, with interest included, at the end of a given period, the items under each of these three classes may be totalled.

The "cost" of each is the amount which should be charged against the resulting income, which in forestry is the stumpage value of the timber plus the revenue from other sources.

In Group]A, the original capital, if maintained properly, remains intact at the end, and only the interest can be charged as an expense to the crop. Let S_c = cost of soil and permanent improvements. Then total cost is

$$S_c (1.0p^n) - S_c. \quad (\S 87, \text{IIIb})$$

In Group B, only those crop expenses which occur in the same year may be computed as one total. These expenses must eventually be met entirely from income and do not constitute capital expenditures. With the cutting of the timber, no portion of the remaining assets are available from which to return these costs.

The total cost of all such items is found by Formula I in which

C = initial investment,

$C (1.0p^n)$ = cost at end of period.

Expenses incurred in year d will cost

$$C_d (1.0p^{n-d}).$$

Let C_o, C_d, C_e, C_f = Initial expenses incurred in the beginning and in the years d, e and f . The sum of these costs is

$$C_o (1.0p^n) + C_d (1.0p^{n-d}) + C_e (1.0p^{n-e}) + C_f (1.0p^{n-f}).$$

In Group C, all annual expenses are totalled. The sum of this expense with interest is found by Formula IIIb (§ 87).

Substitute e for r to represent annual expense.

Then $\frac{e}{0.0p} = R = E$, capitalized annual expense.

The sum of these expenses, provided they remain at a constant annual amount, is

$$E(1.0p^n) - E = E(1.0p^n - 1).$$

The analogy of this calculation to that used in Group A is evident. The difference is that in A, S_c represents existing capital which remains as an asset, while in C, E represents a fictitious capital which may have no existence and is never expended. It represents the capital which would be tied up, in order to produce an income to just balance the expense e . This capital might, if the owner desired, be actually deposited at interest, when the expense e would be permanently taken care of and the capital remain intact at the end as in Group A. But the proprietor usually meets this expense out of income from other sources. The capital represented by E is, therefore, merely a mathematical device, and the expression $\frac{e}{0.0p}$ better expresses the status of the item, as it emphasizes the annual expense.

Should expenses increase, the modified Formula IIIa is used, by substituting E' for $\frac{r_1}{0.0p}$ (§ 79).

The total actual cost of production for timber grown under management covers the period from its origin to its removal. Adding the summaries for the three groups, we have

$$\begin{aligned} G_c &= C(1.0p^n) + S_c(1.0p^n - 1) + E(1.0p^n - 1) \\ &= (C + S_c + E)1.0p^n - (S_c + E). \end{aligned} \quad (A)$$

This formula suffices when all initial crop expenses occur in the first year.

Income during this period serves to cancel an equal value of accumulated costs in the year in which it is received. Annual income usually serves merely to reduce the net annual cost, and, therefore, does not appear in the formula. But income received from the sale of products obtained from advance cutting, or thinnings, bears no constant relation to the total expenses for

the year. All income is equivalent to negative costs, and in this case the item is best carried with interest to the final year, when its total may be subtracted (§ 77). Otherwise a separate balance would be required for the year of receipt of this income, which would serve no useful purpose.

Let T_p, T_q, T_r = value of thinnings or other incidental revenue received at p, q and r years. The future value, at end of period, will be

$$T_p (1.0p^{n-p}) + T_q (1.0p^{n-q}) + T_r (1.0p^{n-r}).$$

Subtracting this income from total cost, including C_d, C_e and C_f , we have the net cost of the growing stock, G_e .

$$\begin{aligned} G_e = & (C + S_e + E) 1.0p^n + C_d (1.0p^{n-d}) \\ & + C_e (1.0p^{n-e}) + C_f (1.0p^{n-f}) - T_p (1.0p^{n-p}) \\ & - T_q (1.0p^{n-q}) - T_r (1.0p^{n-r}) - (S_e + E). \end{aligned} \quad (A_1)$$

There is no way of avoiding these separate calculations for single sums occurring in odd years. As each item is independent of the others, the apparent complexity of these formulæ need not be a source of confusion.

To obtain the cost of a forest, including both soil and timber, the cost of soil is not subtracted.

Let F_e = cost of forest.

Then $F_e = (C + S_e + E) 1.0p^n - E$,

and for the year a ,

$$^aF_e = (C + S_e + E) 1.0p^a - E. \quad (A_2)$$

106. Total Cost of Investments in Standing Timber. — Only a few instances are found in America where forest crops have been brought from seed to maturity under management, and it is still less frequently that actual total costs of production have been recorded. Practically all past transactions in land, for forest purposes, dealt with property which already contained standing timber.

Cost accounts for such property begin with the purchase price and carry all subsequent *net* expenses to the present, to final sale of the property, or to the year when the timber is sold. Since such property is purchased in a partially or fully developed

condition, with respect to the timber crop, and may include certain permanent improvements, the cost to the purchaser may not coincide with the cost of production, for the sale is usually the means of realizing profit on the part of the original owner (§ 90). This initial cost is usually a lump sum, which includes the value not only of all separate stands of timber, but does not often separate the value of the land from that of the timber crop. An analysis of this cost must precede any attempt to institute a cost-keeping system separately for different parcels or stands.

Separation of the cost of land and timber, while usually postponed in such instances until actual separation occurs through cutting, is necessary if the true cost of holding the timber is to be computed, and the land permanently devoted to forestry. But if the purchase is a timber speculation, and the land, after the timber is stripped from it, is to be sold or abandoned, there will be no need of this distinction.

In the former case,

$C + S_c$ = cost of timber and land, or purchase price of property.

The purchase price of the property is substituted for cost of land plus silvicultural operations, in Formula A. The net cost of production for timber depends now upon the value agreed upon for the land *at time of purchase*, for this must be subtracted as representing a capital investment. The formula is in all respects identical with A, and interest accumulates for the period elapsing between date of purchase and date of final sale or income, which is assumed as n in the problem.

When interest is thus computed as a cost on the investment in stumpage, as well as on taxes and other expenses, a rate of 6 per cent will cause this "cost" to double in twelve years. The stumpage value must double in a shorter period or there will be no income to replace current expenses. This raises the important question as to the length of time which it is possible for private owners to hold stumpage without cutting it (§ 181). For periods of forty years or over, it might easily be shown that the costs of raising a new crop of timber leave a higher margin of profit than those incurred in carrying a mature crop with a high initial value, for the same length of time.

107. Costs of Forestry Compared with Destructive Lumbering.—Owners of timberlands have the choice of two lines of policy. They may desire to retain the lands only until the timber is removed, when every effort will be made to sell or dispose of them. Or they may plan to hold these lands indefinitely, with the intention of producing or securing a new growth of trees. The former policy is usually accompanied by destructive lumbering, so termed because the purpose of the operation is to remove every tree of merchantable value, at the same time undertaking no measures to reproduce or perpetuate the forest.

The financial distinction between these two policies lies in the character of the investment required for forestry. Destructive lumbering squeezes the last cent of capital from the forest, wrecking it as a productive enterprise, and converting it into money profits. In this process, the cost of transportation is reduced to a minimum by use of temporary structures and complete removal of the timber, by elimination of all care in felling and skidding needed in preventing injury to young timber, and by neglect of the fire trap caused by the slash, and abandonment of the land to promiscuous broadcast burning. Under such conditions, stumpage brings its highest prices and the profits to logger and owner are greatest in amount. But there they cease.

In any other form of business, similar results may be obtained should the owner close out the enterprise by the sale, not only of the net product, but of his reserve stock, fixtures and capital assets. The profits obtained on stumpage under destructive lumbering are higher than the business of forest production can earn or is entitled to, and the entire structure of lumbering as it is carried on to-day will crumble away with the exhaustion of the raw materials, having converted the chief asset, productiveness of forest lands, into money.

The investment which forestry may require, and which destructive lumbering avoids, consists of

Additional cost of transportation in logging.

Additional felling and skidding costs.

Brush disposal.

Cost of seed trees.

With the entire cost of building roads charged against the logging, and pro-rated over the amount of timber logged, any reduction in this quantity, especially if the area is not correspondingly reduced, increases the unit cost of logging by nearly the same ratio. It is not often possible to cheapen the means of transportation to conform to the lessened cut (§ 98).

Felling and skidding will cost more in proportion to the care needed in avoiding damage to young timber.

Brush disposal (§ 100) is almost always needed. It frequently happens that as a result of this measure, the costs of skidding are correspondingly reduced.

A question may well be raised as to whether the sum of these extra expenses constitutes a loss of profits on the crop being cut or a diversion of profits into initial investments on the next crop. Should the total extra cost, thus incurred, be charged as initial cost or silvicultural expenses, to accumulate at compound interest, it would constitute a heavy burden on the succeeding crop. Viewed from the standpoint of destructive lumbering, this cost, taken from profits, is a deliberate reduction of ordinary profits and *must* be charged against future income. But from the standpoint of forest production, this excess profit is not a true profit at all, but the wrecking value of the "plant" added to legitimate profit. Normal profit is that which will leave the productive capital intact. From this standpoint these "extra" costs of logging would be considered as "ordinary" costs, falling on the logging and cancelled by income from sale of logs, and would *not* be regarded as an additional investment in the new crop. These two opposing tendencies are best illustrated by the expense of burning brush (§ 100).

The investment represented by the stand of timber left after logging is more easily classified. Should this consist merely of seed trees, it can be regarded as a loss only in case these trees will not live until the harvesting of the next crop. If they survive, they represent an *investment* from which the returns will be:

The future value of the trees themselves;

The value of the new crop.

This investment feature is even more conspicuous in the case of stands in which a large per cent remains after logging.

In either case, the *sale* of the greater portion of the merchantable stand gives the basis for an *appraisal* of the remaining timber. A balance may be struck between past costs and income and the remaining crop credited with its present value, which is then charged as an initial investment for the ensuing period. This would show profits on the entire crop, and a reinvestment of the portion left standing. If this appraisal is not used, the profits are confined to the timber actually cut, and the investment would not be charged against the new crop or period.

The former method of accounting is preferable. The use of profits to cancel investment may be pushed to illegitimate lengths, as when income from the sale of timber is allowed to "wipe out" the cost of the land, which is then carried as having no "cost" or "value" for a succeeding period. In this case, income from sales should be permitted to offset only the cost of the timber sold, while the land (§ 96) must be separately valued, and will be charged with interest in the future reckoning.

108. Cost of Many-aged Forests. — Proprietary accounts of investment for large forest properties are kept very simply (§ 91). The same is true of such an account when dealing with single stands composed of trees of many ages. But the question arises as to whether income from such a stand should cancel original capital investments, or be regarded purely as interest or dividends on this capital.

If the forest is established and is renewed, so that, whether the cutting be annual or at intermittent periods, it may be maintained in amount and value indefinitely, the cost of the stand can be considered as a unit, regardless of the separate age classes (§ 161). This cost may be the price of purchase or the cost of production (§ 109). In a stand of all ages, producing annual income, this cost does not increase and the net income is merely the annual dividends on the investment. None of this income is needed for the replacement of capital.

In a forest yielding an intermittent rental, the cost or total investment continues to increase (§ 92) until a payment of

income is received. This payment must be regarded as cancelling the additional capital invested since the last payment.

Where yields are irregular in time and value, and yet there always remains a considerable stand of timber on the ground, the relation between cost and income becomes a matter of judgment or adjustment. The object should be to insure the return of the capital invested in the timber which is removed, and carry as a cost or further investment only a reasonable balance of original cost against future income. The cost of a forest is the sum of the net costs of the separate stands or parcels.

109. Cost of Producing a Normal Forest. — A normal forest is one from which an annual income may be obtained without diminishing the amount and value of the capital or assets. This requires the presence of equal productive areas in each age class.

Granting that the cost of each parcel of land is charged to this forest in the year in which the crop upon it is started, and that it takes n years to establish the forest, an area equal to $\frac{1}{n}$ of the whole must be reproduced each year. The total annual expense e applies to the entire area of the forest. C is required to plant the parcel. The capital tied up in the investment for the year will then be $C + S_e + E$ (§ 105).

Since an equal area and capital investment is added each year, on which interest accumulates till the year n , the sum of costs forms a series, with $(C + S_e + E)$ as the first term and $1.0p$ as the ratio of increase. See Formula III.

The sum of these costs is

$$\frac{(C + S_e + E)(1.0p^n - 1)}{0.0p}.$$

But the capital E is fictitious (§ 104) and this has been introduced n times. In the same way, S_e may be deducted as a capital asset, totalling nS_e .

This makes the net cost of production for the timber alone as

$$G_e = \frac{(C + S_e + E)(1.0p^n - 1)}{0.0p} - n(S_e + E). \quad (A_3)$$

CHAPTER VII

THE VALUATION OF FORESTS

110. Valuation Accounts in Forestry.—Just as the proprietary and cost accounts in forestry (Chapter VI) deal with the personal investments of the owner and past elements of cost and income, so the valuation accounts in forestry deal entirely with material assets (§ 54) and determine their value. The basis of this determination is the forest inventory (§ 55) which shows the area of land of different qualities, the amount and kind of timber and of other resources, and any other property included in the forest.

111. Income as the Basis of Value of Forests.—This physical property must be valued or appraised (§§ 18, 60). Following the conservative systems of accountants, the values entered for land, timber, buildings or other forms of property cannot exceed what was actually paid for them (§ 56). Such an assertion does not mean that the actual value of the property remains at cost, but that growth in value is not considered equivalent to realized income. An increase in value of assets may and does occur, and, with growing timber, is the rule; but the accountant in all such cases desires to await the actual *sale* of the product, and credits this increase in value only when received in the form of money returns.

While this custom protects the proprietor or stockholder, no pretense is made that the book values, based on cost, represent the actual value of the property. In Chapter IV the basis of value is seen to be future income (§§ 61 and 62) and the true value of all assets is derived from this income, discounted to the present, minus future costs.

In order to grasp the principles employed in appraising forest values, the relations between physical assets, income and value must again be emphasized. First, the forest land, with

its growth or stand of timber, is the property to be valued. Second, this land, by producing timber, and the timber, by growing, yields saleable products whose final sale produces income. Third, the income, expressed in money, has an appraised value. As the final step, the value of the income is discounted to determine the value of the physical capital which will produce it.

This process cannot be short-circuited. The sale value of forest land or timber does not represent any quality inherent in the land or timber *except* the capacity to produce income, and the value of the property *must first* be based on the valuation of the income.

The income from forests is principally the value of timber sold on the stump, or the proportion of profits from logging which is due to the owner of stumpage. Secondary income is obtained from thinnings or advance cuttings, and from pasturage, or hunting and fishing rights. Other sources of income are by-products, such as naval stores, or tree seeds, nuts, bark, sugar, shrubs, herbs and litter.

The present value of the forest is the sum of the present or discounted values of all items of income expected in the future, less the present value of all items of anticipated expense (§ 62).

Sale value bears the same relation to capital or expectation value for forests as for any other form of property (§ 59). But since sale values are the expression of average general opinion, the sale value of forest land and young timber will be apt to depart widely from the capital or expectation value of such property in regions where the business of forest production is not recognized, and lands are abandoned after logging or devoted to agriculture and grazing.

112. Value of Forest Property for Destructive Lumbering. — When the owner intends to remove the timber and devote the land to other uses, his income will consist of:

Yield of timber;

Value of land for other purposes after timber is removed.

As long as the timber remains, the land cannot be utilized fully for anything but the timber crop. Grazing is only a secondary source of revenue. When cleared, its value is considered

as depending solely on the net future income from other uses, such as agriculture or grazing. Since these are enterprises not directly connected with forestry, it may be technically assumed that the forest owner sells the land, and the sale value received completes his income and terminates the enterprise.

The total present value of forest property based on the income from destructive lumbering is determined as follows:

F_s = sale value of forest property, including both land and timber:

Y = yield or net stumpage value of timber in final cutting.

T_p, T_q, T_r = yield or value of thinnings or timber removed before final cutting.

S_s = sale value of land when finally cleared of standing timber.

e = annual expenses.

E = capital value of annual expenses.

a = present year, or the year a dating from the origin of the stand.

n = year in which final cutting is made, dating from the origin of the stand.

The period of discount will then be $n - a$ years.

In the year of sale, and previous to its consummation, a and n coincide. The value of the property for immediate sale is then

$$F_s = Y + S_s. \quad (B)$$

When the year of valuation a precedes n , the value is as follows:

Value of Y for year a

$$= \frac{Y}{1.0p^{n-a}} \text{ (see II).}$$

Value of T_p, T_q, T_r for year a

$$= \frac{T_p}{1.0p^{p-a}}, \frac{T_q}{1.0p^{q-a}}, \frac{T_r}{1.0p^{r-a}}.$$

Value of S_s for year a

$$= \frac{S_s}{1.0p^{n-a}}.$$

Cost of e for $n - a$ years = $\frac{E(1.0p^{n-a} - 1)}{1.0p^{n-a}}$ (see IIIb, II), which becomes $E - \frac{E}{1.0p^{n-a}}$.

Omitting the item of thinnings,

$$\begin{aligned} {}^aF_s &= \frac{Y + S_s - E(1.0p^{n-a} - 1)}{1.0p^{n-a}} \\ &= \frac{Y + S_s + E}{1.0p^{n-a}} - E. \end{aligned} \quad (B_1)$$

Including thinnings whose income is expected in the future, or between the years a and n ,

$$\begin{aligned} {}^aF_s &= \frac{Y + S_s + E}{1.0p^{n-a}} - E + \frac{T_p}{1.0p^{p-a}} + \frac{T_q}{1.0p^{q-a}} + \frac{T_r}{1.0p^{r-a}} \\ &= \frac{Y + S_s + E + T_p(1.0p^{n-p}) + T_q(1.0p^{n-q}) + T_r(1.0p^{n-r})}{1.0p^{n-a}} - E. \end{aligned} \quad (B_2)$$

113. Value of Forest Property for Continuous Forest Production. — The substitution of sale value of land for its value determined from income was a makeshift justified by the change in its use. True forest land, devoted permanently to producing timber, must be valued on the basis of the sum of all future income which it is capable of producing. This will include the net value, not merely of the present stand, but of a succession of timber crops, extending to infinity (§ 86).

The relative present value of the first crop compared with others to follow is very large, but merely because the interval between payments is correspondingly long. The income from sale of a crop of timber does not differ in character from the monthly payment of rent, and the value of the forest is as truly the sum of *all* present values from future crops as that of the house is the capitalized net rental or income. In the same way that the value of the house represents, not the gross rent, but the rent minus all outlay such as taxes, repairs and depreciation, so from all future income from timber, deductions for expenses must be made to get total net value.

114. Value of a Forest of Even Age Just Previous to Cutting.—

A forest or stand of even age may be cut clear at one time and replanted. During the period immediately following the cutting, the soil is bare of trees, while at all other times it is occupied by timber of constantly increasing size.

Since every item of either income or expense must be assumed to recur for each future timber crop, the total present or discounted value for each item is found by Formulæ IX to XII.

For the year n , and just previous to receipt of income Y , the value of income Y , being the sum of all future payments discounted to the present year, is

$$Y + \frac{Y}{1.0p^n} + \frac{Y}{1.0p^{2n}} + \dots + \frac{Y}{1.0p^\infty} = Y + \frac{Y}{1.0p^n - 1} \quad (\S 85, \text{XIa}).$$

The cost of planting or reproduction, which in destructive lumbering does not occur, falls here in the same year as Y and recurs every n years.

Value of expense C , totalled in the same manner, is

$$C + \frac{C}{1.0p^n} + \frac{C}{1.0p^{2n}} + \dots + \frac{C}{1.0p^\infty} = C + \frac{C}{1.0p^n - 1}.$$

The annual expenses e recur at intervals of 1 year, by definition, and are computed from the *end* of each year.

Value of the sum of annual expenses e :

$$\frac{e}{1.0p} + \frac{e}{1.0p^2} + \frac{e}{1.0p^3} + \dots + \frac{e}{1.0p^\infty} = \frac{e}{1.0p - 1} = \frac{e}{0.0p} = E.$$

Let F_v = expectation or capital value of forest property for continuous production.

Then

$$\begin{aligned} F_v &= Y + \frac{Y}{1.0p^n - 1} - \left(C + \frac{C}{1.0p^n - 1} \right) - E \\ &= Y + \frac{Y - C}{1.0p^n - 1} - (C + E), \end{aligned} \quad (\text{C})$$

which is the value of forest property just previous to removal of the timber.

In this formula, no account has been taken of other items of

expense corresponding to C but falling at different years, nor of income from thinnings corresponding to Y but received previous to this final cut.

The present value of these items is found by Formula X, since each single payment, while not repeated during the rotation, is assumed to recur in the next rotation, and the total of all future payments forms the series.

Let T_p = income from thinning in year p , recurring every n years forever.

Then $\frac{T_p}{1.0p^p}$ is the value of T_p in the first year, n .

The series is

$$\frac{T_p}{1.0p^p} + \frac{T_p}{1.0p^{p+n}} + \frac{T_p}{1.0p^{p+2n}} + \dots + \frac{T_p}{1.0p^\infty}.$$

The sum is (Formula Q₄)

$$\frac{\frac{T_p}{1.0p^p}}{1 - \frac{1}{1.0p^n}} = \frac{T_p (1.0p^{n-p})}{1.0p^n - 1}.$$

By substituting q , or r , for p , the present capital value of other items of income or of expense, for any year, is obtained.

The present value of forest property just previous to removal of a final cut of timber is summed up as

$$F_v = Y - (C + E) + \left\{ \begin{array}{l} Y + T_p (1.0p^{n-p}) + T_q (1.0p^{n-q}) \\ + T_r (1.0p^{n-r}) - C - C_d (1.0p^{n-d}) \\ - C_e (1.0p^{n-e}) - C_f (1.0p^{n-f}) \end{array} \right\} \frac{1}{1.0p^n - 1} \quad (C_1)$$

115. Value of an Even-aged Forest for Any Year.—The value of the forest for any other year, or during its growth, is obtained by discounting the above value for the period previous to this final cut. The forest has its maximum value just before the principal yield is removed. The further into the future this income is postponed, the less becomes the present value of the property.

Should the value be sought for the year a , and this year be but one year previous to the cut, the value of the forest is discounted $n - a$, or one, year. As a is dated earlier and earlier in the life of the stand, the discount period $n - a$ increases, until when $a = 0$, just after the felling of the old crop and previous to the starting of the new one, we get the smallest value to which the property shrinks in the entire rotation.

With the advance of the date of calculation to the year a , both the future yield Y and the cost of planting C occur first in $n - a$ years. The annual expenses, since they are calculated for the same interval, and extend to infinity in both cases, have the same capital value E .

By Formula Xa, the value of the forest is

$$\begin{aligned} {}^aF_v &= \frac{Y (1.0p^a)}{1.0p^n - 1} - \frac{C (1.0p^a)}{1.0p^n - 1} - E, \\ &= \frac{(Y - C) 1.0p^a}{1.0p^n - 1} - E. \end{aligned} \quad (D)$$

Items of either income or expense occurring at other than n years will fall either previous to or subsequent to year a . Those preceding a are not considered in the value of the standing crop (§ 68), but their occurrence in the subsequent rotations affects the value of the forest.

A thinning occurring in the year r subsequent to a has a value at present (year a) of $\frac{T_r}{1.0p^{r-a}}$.

The sum of future values (Q_4) is

$$\frac{\frac{T_r}{1.0p^{r-a}}}{1 - \frac{1}{1.0p^n}} = \frac{T_r(1.0p^{n+a-r})}{1.0p^n - 1}.$$

A thinning occurring in the year p previous to a_2 has a present value (year a) of $\frac{T_p}{1.0p^{p+n-a}}$.

The sum of future values (Q_4) is

$$\frac{\frac{T_p}{1.0p^{p+n-a}}}{1 - \frac{1}{1.0p^n}} = \frac{T_p (1.0p^{a-p})}{1.0p^n - 1}.$$

These expressions are equally applicable to costs occurring before or after the year a . The present value, in year a , of an even-aged forest or stand, is summed up as

$${}^aF_v = \left\{ \begin{array}{l} Y(1.0p^a) + T_p(1.0p^{a-p}) + T_q(1.0p^{a-q}) \\ + T_r(1.0p^{n+a-r}) - C(1.0p^a) - C_d(1.0p^{a-d}) \\ - C_e(1.0p^{a-e}) - C_f(1.0p^{a-f}) \end{array} \right\} \frac{1}{1.0p^n - 1} - E.$$

This formula may be expressed in another form.

$${}^aF_v = \frac{\left(Y + \frac{T_p}{1.0p^p} + \frac{T_q}{1.0p^q} + \frac{T_r(1.0p^n)}{1.0p^r} - C - \frac{C_d}{1.0p^d} - \frac{C_e}{1.0p^e} - \frac{C_f}{1.0p^f} \right) 1.0p^a}{1.0p^n - 1} - E. \quad (D_1)$$

The chronological relations of these items are set forth in Diagram II. The present year is the basis of valuation. Past occurrences are ignored. But in the terminology, the numerical value of a is reckoned from the past year 0, or date of origin of the crop, which is also the basis for the other dates.

The year $2a$ is just n years in the future. The Formula D_1 expresses the value of the items computed at compound interest to the year $2a$, when the balance is struck which gives net value. This net income recurs at intervals of n years from date, and its present value is $\frac{r'}{1.0p^n - 1}$, hence the above result.

116. Value of Forest Soil.—The value given in Formula C was for a forest just previous to removal of the final cut. This value steadily diminishes as the date of this cut is assumed to be further into the future until the lowest ebb is reached, just before the discount period overlaps the date of the previous cut. A similar fluctuation in value occurs in the case of an account

DIAGRAM II.

Method by which the items of future income and cost for timber crops are balanced n years from date, and discounted to the present year as the first term of a series of future payments due at intervals of n years, and constituting the sum of a perpetual rental and the expectation value of the property.

in a savings bank from which the accumulated interest is withdrawn, say every 10 years. For land which is bare of trees but in condition to be planted at once, the period intervening to reduce the value of this prospective income coincides with that for growth of the crop. But should the land first require draining, or other preparation as in the case of peat lands or shifting sands, a still further shrinkage of prospective value occurs, because of the lengthened time and extra expense intervening.

This discounted value of future income, computed for a date just previous to the origin of an even-aged stand of timber, gives the value of forest land bare of trees.

This value is obtained from Formula C, by omitting the value of the timber Y . Let S_v = expectation value of forest land.

$$\text{Then} \quad S_v = \frac{Y - C}{1.0p^n - 1} - (C + E), \quad (E)$$

or

$$S_v = \frac{\{Y + T_p(1.0p^{n-p}) + T_q(1.0p^{n-q}) + T_r(1.0p^{n-r}) - C - C_d(1.0p^{n-d}) - C_e(1.0p^{n-e}) - C_f(1.0p^{n-f})\}}{1.0p^n - 1} - (C + E). \quad (E_1)$$

S_v , the expectation or capital value of forest land bare of trees, may be likened to the value of bare land suitable for farm crops or orchards. In the case of farm land, it may happen that the sale value of one annual crop will equal or exceed the value of the land, and farms are usually sold subject to crop removal or in the winter. With orchards, the value of the bearing trees greatly enhances that of the bare soil. Even a young orchard increases land values. In the latter case, it is of small moment whether value is separated into the value of the land and that of the fruit trees, for both are sold together. With forest trees, the relation between land values and ultimate income is exactly the same, but the period covered by the growth of the stand is so long that one is apt to lose sight of the fact that this so-called value of land is merely the discounted value of the future crops of timber.

This soil value S_v represents the value of all future income dating from a point just subsequent to the complete removal of

a crop of timber. It is the value of all future crops exclusive of the one occupying the ground.

But S_v represents this value correctly, only for the year of cutting, or just previous to planting. Dismissing the idea that S_v represents the actual "intrinsic" value of the soil itself, and regarding it solely as the discounted net value of the future receipts from timber, it is seen that this value S_v holds good only for the one year. Just previous to the cut, the property is worth $Y + S_v$. Just subsequent to cutting, the value is S_v . But when this net income is discounted to any previous year a , its value

ceases to be S_v and becomes $\frac{S_v}{1.0p^{n-a}}$. The value of the entire property, soil and timber, as determined from the net revenue (see § 112, Formulæ B₁ and B₂), is the sum of the net income from the standing crop plus the discounted net income from future sources, represented by $\frac{S_v}{1.0p^{n-a}}$. The value of the stand-

ing timber is $\frac{Y}{1.0p^{n-a}} - \left(E - \frac{E}{1.0p^{n-a}}\right)$.

The sum is, therefore,

$${}_aF_v = \frac{Y + S_v + E}{1.0p^{n-a}} - E. \quad (D_2)$$

But since S_v reappears every n years as the net value of bare soil, it may be assumed that this represents the value of the soil during the intervening period, and that the excess of value for the property represents the value of the trees or growing stock. Under this assumption, S_v might be regarded as the fixed capital, corresponding to a sum of money, and the value of the timber as the accumulating interest earned by this capital.

To determine what annual interest this soil capital is earning, it is necessary merely to multiply S_v by the rate of interest p used in calculating it.

$$S_v \times 0.0p = \text{soil rent (§ 162)}. \quad (E_2)$$

Expectation value of soil is merely an expression of net income. Instead of being permanent or fixed, it is a prediction or forecast, and changes constantly (as do all values) not only with every

change in economic conditions affecting the price of products, but with the opinions of property owners as to the amount and present value of the anticipated income. Since it is the furthest removed from actual income, soil value is the most unreliable of all forest values, and is subject to greater proportional fluctuations than values of standing timber.

117. Value of Many-aged Forests Producing Regular Income. — The true character of this capital value of bare forest soil is further emphasized in the valuation of stands of timber composed of trees of many different ages.

Two cases may be presupposed. First, the existing age classes are so distributed that an amount of timber can be cut at equal intervals of time, and of approximately equal quantity and value. Second, the age classes are irregular in quantity and unevenly distributed as to age. This second example is the prevalent type of many-aged forest.

In neither case is the land ever clear of timber, unless the young age classes are destroyed by fire or logging. It follows that for a forest of this character the value of bare land cannot be determined, except by making an artificial assumption of an even-aged stand of equal total yield with that of the many-aged forest, and in which the trees take an equal time to develop. But the trees in the all-aged stand take longer to grow than they would if even-aged, on account of suppression or shading. The correct determination of soil value, apart from the stand, for such forests is therefore quite unsatisfactory. With a slow-growing species the values indicated for bare soil may be negative and are usually very low.

But the forest itself, or total property, both soil and timber, even after a cutting has just been made, has a positive and often a high value.

Whether the interval between cuttings m is long or short, the value of the forest just after the cut is

$$F_v = \frac{Y - C}{1.0p^m - 1} - (C + E). \quad (F)$$

But in this case, the formula, although similar to that (E) for

bare soil, differs in the period m and gives, *not* the value of soil, but of the soil and timber remaining after the cut. It is the value of the real estate or forest.

The term F_v is thus used to indicate the value of the forest just following a cut.

For any year previous to a cut, the value of the property may now be found by discounting the net value of this yield to the present, and adding the discounted value of F_v .

This gives

$$\begin{aligned} {}^aF_v &= \frac{Y + E}{1.0p^{m-a}} - E + \frac{F_v}{1.0p^{m-a}} \\ &= \frac{Y + F_v + E}{1.0p^{m-a}} - E. \end{aligned} \quad (F_1)$$

The value of a forest producing a regular annual income is found by capitalizing this net income, using Formula XII, $\frac{r}{0.0p}$.

The annual income will be $Y + T_p + T_q + T_r$.

Annual expenses are $C + n \times e$, since e is the annual expense for but one area or age class and there are n age classes in the forest.

Other expenses, as C_d , C_e and C_f , also occur annually.

$$F_v = \frac{Y + T_p + T_q + T_r - (C + C_d + C_e + C_f + n \times e)}{0.0p}. \quad (F_2)$$

The net income, which is thus capitalized to obtain value, is termed the forest rent (§ 162).

118. Value of Many-aged Forests Producing Irregular Income. — Should the value of irregular many-aged forests be sought, the period must first be determined, which is required by the average trees to become merchantable. The stand must then be roughly grouped into its age classes, and the approximate per cent of total area agreed upon which each age class occupies. Each of these yields must then be discounted to the present, with a value determined by its area. To get the *net* value of each crop, the future expenses must be pro-rated according to the same distribution of area. Finally, the *total value*

is obtained if each yield is treated as a recurring rental (Formula X, Diagram II). An example of such a valuation would be:

Crops cut

in year a , $\frac{1}{x}$ total area,

in year b , $\frac{1}{y}$ total area,

in year c , $\frac{1}{z}$ total area.

If total value of all crops is Y and total expense e is capitalized as E , and there is no expense for reproduction, the value of the forest is:

$${}^aF_v = \frac{\frac{Y}{x} (1.0p^{n-a}) + \frac{Y}{y} (1.0p^{n-b}) + \frac{Y}{z} (1.0p^{n-c})}{1.0p^n - 1} - E.$$

Should an expense for reproduction, C , occur at the time of cutting, or any other initial expense be incurred at that time, this expense is subtracted from Y in that year.

As

$${}^aF_v = \frac{\frac{Y-C}{x} (1.0p^{n-a}) + \frac{Y-C}{y} (1.0p^{n-b}) + \frac{Y-C}{z} (1.0p^{n-c})}{1.0p^n - 1} - E. \quad (F_3)$$

Formulae are merely convenient methods of calculating results after the facts are obtained. Any large area of forest requires careful analysis and separation into its component parts before any of the methods of computing value can be applied.

119. Value of Timber Separate from Land.—When for any reason the value of the timber must be separated from the value of bare land, it may be done by subtracting this latter value S_v from the formulae giving the total value of the property.

Formula D_2 then becomes

$${}^aG_v = \frac{Y + S_v + E}{1.0p^{n-a}} - (S_v + E) \quad (G)$$

and Formula F_1 reads

$${}^aG_v = \frac{Y + F_v + E}{1.0p^{m-a}} - (S_v + E), \quad (G_1)$$

provided in this case that the value of the soil can be determined or agreed upon apart from that of the poles, saplings and seedlings which increase the value of F_v over that of S_v .

For a normal forest (§ 109) producing annual income, the expectation value of the standing timber, separate from the land, may be found by subtracting $n \times S_v$ from Formula F_2 which then reads

$$G_v = \frac{Y + T_p + T_q + T_r - \{C + C_d + C_e + C_f + n(e)\}}{.0p} - n(S_v). \quad (G_2)$$

CHAPTER VIII

FOREST STATICS — THE BALANCE SHEET — PROFITS

120. Determining the Profits of a Forest Investment.—Forest statics is defined by Schlich * as “the science which weighs and considers the comparative merits of the different methods of treatment to which forests may be subjected. The financial results, or income-yielding power of an undertaking, are expressed by the proportion existing between the yield and the capital which produces it.”

Two distinct methods are possible in preparing a financial statement in forestry (§ 73). The first may be termed the specific balance sheet and conforms to the principles of accounting, by recording only the actual cash investments as credits, and valuing the assets preferably at actual cost. By means of the supplementary profit and loss account, the net income, or loss, is annually transferred to the balance sheet.

This specific balance sheet does not reveal profits until they are actually realized. It is therefore extremely conservative. For a business based on deferred income as in forest production, the annual net deficit of expenses over income may continue for many years, requiring continuous increase in invested capital, which would be further increased by borrowing, and adding to expense the actual cost of interest on these borrowed funds. The balance sheet indicates an increasing deficit, up to the time the timber is cut.

With the receipt of this income, the total assets are suddenly increased to several times the total investment, showing an apparent profit of hundreds of per cent in case of even-aged stands harvested after from 40 to 60 years.

* “Manual of Forestry,” Volume III, Forest Management, by Sir William Schlich, 4th Ed., Chapter VI, p. 149. Bradbury, Agnew Co., Ltd., 10 Bouverie St., London. 1911.

Such results evidently do not fully accomplish one important purpose of a balance sheet, which is to reveal to the owner the true financial status of his investment. Two elements of economic importance have been neglected, which entirely alter this status. The expectation value, not the cost of the assets, is the true measure of the debit entries, while the profits finally credited are not revealed in their true relation to costs unless the time required to earn these profits is given proper weight in the balance by exhibiting the interest at p per cent which should have been earned on the invested capital during the period.

A statement which includes these two factors, expectation value and interest on costs, may be termed the "economic forecast." Revaluation of assets to coincide with present value rather than cost is practiced in commercial accounting when circumstances seem to justify it (§ 73). But the addition of unearned interest as a cost is universally excluded in specific accounts. The economic forecast would include such interest on costs, for in no other way can the owner inform himself of the relative value of the profits derived from income, or whether there is any enterpriser's gain. The apparently enormous final profits shown by the specific balance sheet may, when tested by this economic comparison, be revealed as equivalent to less than p per cent, which is no profit at all as measured by the standard of enterpriser's gain (§ 42).

In this economic statement the proprietary side ordinarily shown as credit is excluded altogether, and the costs are shifted to the right-hand column in order to compare them with actual value (§ 73), and reveal potential profit or loss. The statement formulated in this manner is thus purely impersonal, and does not distinguish between borrowed and personal capital. It is similar to that which would be formed by comparing a cost account for manufactured articles, as credit, with the sale value of the article as debit. But in the case of productive property, this comparison must be made far in advance of the receipt of final or total income and is a balancing of cost against expectation value for the entire enterprise.

That the economic statement will show entirely different

results from the specific balance sheet is evident. When the accountant insists on retaining land and timber in the balance sheet at their original cost, the credits will exceed the debits by the amount of annual expenses not entered as a capital expenditure. This deficit is further increased by interest paid on borrowed capital if such expense has been incurred. By contrast, the economic statement shows an even greater "cost," since interest is added as a cost item to all capital invested, whether borrowed or not. The assets, on the other hand, are treated without any regard for actual cost, their value being determined entirely on the basis of future income (Chapter VII).

The value of forest property, which is used in the economic statement, is its expectation or capital value, until it reaches maturity, when the capital value of the standing timber is merged in the sale value from which it is derived. The comparison for timber property at any year previous to maturity of the timber will show.

Debits.	Credits.
Expectation value of property, consisting of Young timber, Forest soil. Indicated or potential loss.	Cost of property, consisting of Cost of soil } including Cost of crop } Interest on all items of cost. Indicated or potential profits.

The economic forecast in forestry thus consists of balancing past costs, with compound interest to date, against future income (§ 62) discounted to date, as represented by expectation value of assets. By thus comparing the relative values of past net outlay and future net income, for any year a , the exact economic status is revealed for that date.

This method of valuation for forest assets may be compared to the appraisal of the property and business of a lumber company for purpose of sale. Cost would not finally determine the value placed upon any item of such property. The mill and logging road, machinery and stock, would be valued on the basis of their future usefulness to the purchaser and present owner. The timber would be appraised at going prices for equally

good stumpage. Should the sale be consummated these appraised values are realized, and the profit shown in this tentative or economic statement becomes an actual profit, entered specifically on the books.

To what extent the value of assets in a specific balance sheet should be increased above cost of purchase, is a question to be determined by the character of the business. Unless these increased values are eventually realized, the profit shown by reason of such increase exists merely in anticipation, and until it is secured, this profit cannot be distributed as dividends.

There is considerable difference between the financial status of a lumber company conducting an extensive operation with stumpage held merely as a timber mine to furnish wood until exhausted, and that of a forest owner engaged in growing timber as a business. The former concern is obliged to adhere rigidly to the accounting principles commonly accepted in business circles. Its timber will be entered at cost, unless a bond issue is floated, when it is customary to appraise the timber at its present stumpage value and to make a far more complete estimate of its amount. Bonds are secured by value, not by cost of property bonded. The interest on these bonds must be met annually or foreclosure follows. Taxes, protection and interest are, therefore, formidable items of expense and timber must be cut each year to produce sufficient income to cancel these costs for the entire remaining body of stumpage.

But because of these conditions, the actual status of the lumberman's investment tends to follow a similar course to that of an investment in growing timber. The first years will tax all the resources of the owners to keep ahead of their obligations. Need for income is urgent. The tendency is to greatly increase the size and capacity of mills and operations. This universal tendency causes overproduction of lumber and defeats its own ends by lowering prices.

But once a concern has weathered this initial period, and reduced its indebtedness and carrying charges, the final years of the operation become more and more profitable. It follows that those operators who have the greatest amount of capital

behind them, and who do the least borrowing and maintain operations in a region after others have ceased, reap large profits, while the small investors, or firms attempting too much with limited capital, are often bankrupted in the first stage of the business.

The investor must, therefore, distinguish sharply between the commercial or specific trial balance, and the economic forecast which accounts for time, includes interest on the investment as a cost, and bases the value of assets not on cost but on income, by the process of discount. Such a "balance" informs the owner of his true economic status, thus enabling him to determine his future policy — whether to sell, purchase or continue to operate. Economic, and not specific, profits are discussed in this chapter.

121. The Rate of Interest in its Relation to Profits. — In securing an accurate statement or balance between past costs and future income or capital value, the effect of the element of time upon such relative values must be eliminated or equalized (§ 11). This is done by employing the same rate of interest (§ 38), p per cent, on both sides of the equation.

The total amount of compound interest is increased by employing a higher rate, while the same increase in the rate has the opposite effect of lowering the present or discounted value of assets. This effect is due in each case to increasing the difference in value caused by the element of time. It follows that an increase in the standard rate of interest, p per cent, tends to obliterate the margin of profit on the credit side of the balance, and replace it with a debit deficit or loss. In other words, if the investor is satisfied with 3 per cent, and that rate is used in computing values for the balance sheet, the statement might indicate a profit, but if he uses 6 per cent as the necessary basis of comparison with other investments, the resulting balance may indicate a loss, although the cash income and expenses remain the same in either case. The change is wrought wholly in discount or expectation value and in the accumulated "cost" of compound interest.

A rate of interest may be found for any given total of expense, on the one hand, and of income, on the other, which will just bal-

ance the cost or credit side with the asset or debit side of the balance sheet. This rate would in most instances include a fraction or decimal, such as 4.27 per cent. The rate which thus balances cost and income is the rate of interest *earned* by the investment. And since money is never borrowed at odd rates, and the *earning* power of an investment does not determine the rate of interest used in calculating its value (§ 62), these items of cost and value seldom balance exactly, but always show a profit or loss.

Yet the fact that this rate exists indicates a second method of determining enterprisers' profits. The *rate earned*, expressed in terms of compound interest, eliminates the element of time as effectually as does discount, and enables the investor to compare the income rather than the expectation value of two investments. The forest investment must earn p per cent before it becomes as desirable as the money it represents. If x per cent is earned, then when x per cent $> p$ per cent the excess represents the ratio of profit or enterpriser's gain, per dollar of investment, in excess of the standard rate. If x per cent $< p$ per cent, the rate x per cent still indicates net income, but compared with p per cent, there is a loss, or rather a failure to realize the amount desired.

122. Profits in Destructive Lumbering. — It is assumed that owners who do not intend to continue in forest production will sell the land or devote it to some other purpose. Its sale or appraised value for this purpose is credited as income.

Let P = profit.

$$\begin{aligned} P &= Y + S_s - (C + S_c) 1.0p^n - E (1.0p^n - 1) \\ &= Y + S_s + E - (C + S_c + E) 1.0p^n. \end{aligned} \quad (H)$$

This formula is expanded, by the addition of other items of both cost and income, to the form,

$$\begin{aligned} P &= Y + S_s + E + T_p (1.0p^{n-p}) + T_q (1.0p^{n-q}) \\ &\quad + T_r (1.0p^{n-r}) - (C + S_c + E) 1.0p^n \\ &\quad - C_d (1.0p^{n-d}) - C_e (1.0p^{n-e}) - C_f (1.0p^{n-f}). \end{aligned} \quad (H_1)$$

This formula gives the profit on a single stand of timber, for the year of sale, assuming that both timber and land are

sold, or that the timber is immediately removed and the land sold or appraised.

Actual profits on the investment in land and timber exclusive of lumbering, for a large operation, must be based on actual sale of land rather than appraisal at time of sale of timber, and if this sale is delayed, as it may be, for many years, the transaction is not closed. A new account may be opened for the denuded land, crediting it to the original account preferably at the cost, not the present sale value of land, and debiting the land account with this investment. If a "cost" account is kept as before, the land will be charged with interest until sold.

123. Profits on a Stand of Timber. — Whether or not the owner intends his land for future crops of timber, the profit on the existing stand may be separately stated, by charging interest on the cost of land as an expense against the timber, and crediting the land at its original cost, when the timber is sold.

$$\begin{aligned} P &= Y + S_c - (C + S_c) 1.0p^n - E(1.0p^n - 1) \\ &= Y + S_c + E - (C + S_c + E) 1.0p^n. \end{aligned} \quad (K)$$

This formula differs from H only by the introduction of S_c as an asset in place of S_s , and may be expanded in the same manner as H_1 .

124. Anticipated Profits on Young Timber. — Assuming that S_c is credited at the time the timber is cut, the profits of that year may be anticipated for the year a in one of two ways. The net profit (K) may be discounted to the present (II), or the balance may be struck for the given year by direct computation of value and cost for that year. In the latter case,

$$\begin{aligned} \text{Value} &= \frac{Y + S_c + E}{1.0p^{n-a}} - (S_c + E). \\ \text{Cost} &= (C + S_c + E) 1.0p^a - (S_c + E). \\ P &= \frac{Y + S_c + E}{1.0p^{n-a}} - (C + S_c + E) 1.0p^a. \end{aligned} \quad (K_1)$$

This formula may be expanded by the addition of income and cost items, regardless of whether these costs occur after the year of valuation, or income occurs previous to this year.

The expression $\frac{T_p}{1.0p^{p-a}}$ gives the present equivalent of T_p in the year a whether $a > p$ or the reverse. Therefore,

$$P = \frac{Y + S_c + E}{1.0p^{n-a}} + \frac{T_p}{1.0p^{p-a}} + \frac{T_q}{1.0p^{q-a}} + \frac{T_r}{1.0p^{r-a}} \\ - (C + S_c + E) 1.0p^a - C_d (1.0p^{a-d}) \\ - C_e (1.0p^{a-e}) - C_f (1.0p^{a-f}). \quad (K_2)$$

125. Profits from Continuous Forest Production. — If instead of abandoning the business, forest production is to be continued, the sale value S_s credited for land does not express the true conditions. The value S_v , representing the expectation value of the soil or the value of all future crops, must be substituted for S_s .

Formula H then becomes

$$P = Y + S_v + E - (C + S_c + E) 1.0p^n. \quad (L)$$

The net profits from future crops, expressed as

$$\frac{Y - C}{1.0p^n - 1} - (C + E),$$

may be substituted for the expression S_v in the formula, which does not alter the result. Both the net profit on the present crop, and on the future crops, S_v , should include all items of cost and income which appear in Formula H₁, but the repetition of these terms in Formula L has been omitted.

126. Anticipation of Continuous Profits. — To anticipate the total net profit, for the year a , including net income from all future crops, the value S_v , once determined, is substituted for S_c as an asset in Formula K₁ which then reads

$$P = \frac{Y + S_v + E}{1.0p^{n-a}} - (C + S_c + E) 1.0p^a. \quad (L_1)$$

This formula is modified by the same additions for extra items of cost or income, as Formula K₂. But Formula K₁ gives the profit on the present stand of timber, exclusive of future crops or of profit on the sale of the land, while L₁ includes these

future profits. Should Formula L_1 be derived in the same way as K_1 the result stands

$$P = \frac{Y + S_v + E}{1.0p^{n-a}} - (C + S_c + E) 1.0p^a - (S_v - S_c),$$

which indicates a means of separating the profits on the standing timber from that on the future crops, or "soil," parallel to the method employed for sale values in § 123.

The true derivation of Formula L_1 is from value (§ 116, D_2) and cost (§ 105, A_2) of the entire property, timber and soil.

Then

$$P = {}^aF_v - {}^aF_c. \quad (L_2)$$

127. Profits Expressed in Soil Values. — In the year in which the enterprise of planting or reproduction is undertaken, and previous to the actual planting, the net value of all future income is S_v , or, expressing this value as discounted from future crops, it equals

$$\frac{Y - C}{1.0p^n - 1} - (C + E).$$

The cost includes merely the price paid for the soil S_c . Then,

$$P = S_v - S_c. \quad (L_3)$$

Since this profit is shown for the year previous to undertaking forestry, it is taken as an indication of the advisability of such an enterprise. This relation shows clearly the nature of S_v , which is merely a calculation of discounted net income. S_v does not indicate what a purchaser should pay for land. It indicates the highest price which he can afford to pay and still expect to earn the desired rate, p per cent, on his undertaking. If more than this is paid, a prospective loss is incurred, *i.e.* he cannot expect to earn as much as p per cent. If the land costs less than S_v he has good prospects of earning a profit at p per cent, or a rate of income, x , exceeding p per cent on his investment. If a purchaser is so foolish as to deliberately convert his expected profits ($S_v - S_c$) into costs, by making $S_c = S_v$, he is merely handing over these profits to the one who sells him the property.

S_p increases in value with every change in prospective income or expenses which alters the net value of this income, and is much larger for soils of good than poor quality. But there is evidently no profit in devoting to forestry lands which will sell for other purposes for a sum larger than S_p .

In countries where forestry is extensively practiced on lands owned by states, or on private estates which have practically never changed hands, there is no way of determining the cost of land. The value of the soil, S_p , is calculated for the purposes of taxation and as a necessary step in valuing growing timber. This value may be recalculated at any time. It is evident that the soil value may always be placed at a figure which will permit the investment to earn exactly the rate of interest agreed upon as the standard for forest investments. And in practice, if during the crop period it becomes evident through price changes or other influences that the investment, with its former soil value, will earn more than p per cent, this value is promptly raised, and the rate of earnings is thus arbitrarily reduced to p per cent once more. The real status of the investment is thus concealed in the capitalized value of the soil. Applied practically to a private operation, this process would consist of entering the land in the capital account as an asset having a value in excess of its cost. If this value were made great enough, the investment never could earn more than p per cent upon said value. But figured on the original cost, a much greater rate of profit may be shown. This practice of capitalizing income as soil value is universal in European forest finance, and the profits of forestry, standardized to p per cent, may be discovered only by comparing the values of land computed for different sites and species, and by a knowledge of past changes in these soil values.

If S_p equals S_c , it may be substituted for S_c in Formulæ A, A_1 and A_2 . This is done under the conditions just discussed, where S_c is not determinable. The effect of this substitution is to make the computed cost of timber in any year of the rotation just equal to its expectation value in that year, and to reduce profit, outside of soil value, to zero.

In the same way, it can be shown that for definite costs and income, the profit determined for any year may be converted into that for any other year by the use of Formulæ I and II.

Substituting S_v for S_c in Formula L_1 , thus reducing profit to zero, this formula becomes

$$\frac{Y + S_v + E}{1.0p^n} - (C + S_v + E) = 0.$$

Then

$$\frac{Y + S_v + E}{1.0p^n} - (C + E) = S_v.$$

Resuming the value of S_c as cost of land, the profit is

$$\begin{aligned} P &= \frac{Y + S_v + E}{1.0p^n} - (C + E) - S_c \\ &= S_v - S_c. \quad (\text{See } L_3.) \end{aligned}$$

Since Formulæ L_1 and L_3 are thus derived from Formula L by discount (II), the reverse is true, and for any year a

$$P = (S_v - S_c) 1.0p^a. \quad (L_4)$$

128. Profits Expressed as a Ratio of Income to Capital. — The economic forecast shows profits as a net sum or balance between assets and cost, which sum cannot be realized in money without a sale of ownership. But the net future income which is accountable for this balance of profit shown may be also expressed in terms of interest earned on the cost instead of as capitalized value (§ 121). The rate or ratio of income to capital depends directly upon the period of time elapsing between expenditure and income (§ 23), a fact often lost sight of. In quick profits, earned largely by the sale of property at an advance in capitalized value rather than by awaiting the slow realization of income, the element of time is lost sight of; yet every profit should be standardized by reducing it to the rate of interest which it represents on the investment for the period of waiting. Should a property be sold at double its cost, the profit is 100 per cent on cost. But with the element of time introduced, the *rate* earned would be as follows (Formula XIV):

Sale at end of	Rate earned, per cent.
1 year	100
5 years	14.9
10 years	7.2
20 years	3.6
50 years	1.4
100 years	.7

129. Earning Power of Capital Invested in Forest Production. — In no form of business does the per cent earned by the total amount of capital invested equal the per cent earned by the proprietor's investment unless he furnishes the entire capital without borrowing (§ 32).

In business receiving annual income, the capital needed to carry the current expenses is usually borrowed, and therefore does not share in the profits but receives p per cent only.

These arguments are used as a justification for separating the capital invested in a forest crop into two portions, one of which shall be computed at p per cent, and not share in the profits, while the remainder is the basis upon which the rate of income is computed. By thus reducing the amount of profit-sharing capital, the *rate* of profit is increased or reduced on this capital as the earnings exceed or fall below p per cent on the *total* investment.

For instance, it may be arbitrarily assumed that the owner acquires the land, but borrows the funds needed for all expenses of crop production (Group A, § 104), as silvicultural expenses, and maintenance or annual charges. It has been shown that such a transaction would seldom occur. But if it does, these expenses must first be returned from income, with compound interest at p per cent. The remaining sum is net income earned by the owner's capital, S_c .

$$\begin{aligned}\text{Net earnings} &= Y + S_v - \{C(1.0p^n) + E(1.0p^n - 1)\} \\ &= Y + S_v + E - (C + E)1.0p^n.\end{aligned}$$

The substitution of S_c or of S_s for S_v will adjust this formula to any desired condition (§§ 122, 123 and 125).

In this formula, $E = \frac{e}{0.0p}$, and as p is determined, this term has a definite value (but see Formula N₂).

Capital invested by owner = S_c ,
 Rate earned = x per cent,
 Period of time = n years.
 By use of Formula XIV (§ 89),

$$S_c (1.0x^n) = Y + S_v + E - (C + E) 1.0p^n,$$

$$1.0x^n = \frac{(Y + S_v + E) - (C + E) 1.0p^n}{S_c} \quad (N)$$

or
$$x = 100 \left\{ \sqrt[n]{\frac{Y + S_v + E - (C + E) 1.0p^n}{S_c}} - 1 \right\}.$$

This division of capital is purely arbitrary. An equally justifiable assumption is that the owner also invests the capital needed for silvicultural operations and all initial expenses (Group B, § 104). In this case, the borrowed capital E is first repaid with p per cent interest from the gross income and x per cent determined for the remainder.

$$\text{Net earnings} = Y + S_v - E (1.0p^n - 1),$$

$$\text{Capital invested by owner} = S_c + C.$$

$$\text{Then} \quad (S_c + C) 1.0x^n = Y + S_v + E - E (1.0p^n),$$

$$1.0x^n = \frac{Y + S_v + E - E (1.0p^n)}{S_c + C}, \quad (N_1)$$

or
$$x = 100 \left\{ \sqrt[n]{\frac{Y + S_v + E - E (1.0p^n)}{S_c + C}} - 1 \right\}.$$

Both of these solutions (N and N₁) give a value for x , greater or less than that actually earned by the entire capital investment. The difference is greatest when only the investment in soil is used as a basis. Method N₁ has been advocated for use in this country.*

The true "economic" per cent (§ 121) earned by an investment is the rate paid on *all* capital invested for periods of one year or

* Forest Management of Loblolly Pine, Bulletin 11, U. S. Department of Agriculture, W. D. Sterrett, Jan. 23, 1914, p. 21.

over, regardless of ownership. This investment must include the annual expenses which are not cancelled by annual income. In this case,

$$\text{Net earnings} = Y + S_v,$$

$$\text{Capital invested by owner} = S_c + C + n(e) \text{ (see § 105).}$$

$$\text{Then net earnings} = Y + S_v - \{(S_c + C + E) 1.0x^n - E\},$$

$$1.0x^n = \frac{Y + S_v + E}{S_c + C + E}, \quad (\text{N}_2)$$

$$\text{or} \quad x = 100 \left\{ \sqrt[n]{\frac{Y + S_v + E}{S_c + C + E}} - 1 \right\}.$$

But in this formula, $E = \frac{e}{0.0x}$, and as x appears on both sides of the equation, a solution is not reached. The value of x may be obtained by trial. A trial per cent x' is first substituted for x in the expression $\frac{e}{0.0x}$ and the value E' determined. The solution then gives a value x'' . This new value is then substituted for x' and $\frac{e}{0.0x''}$ gives E'' . The second solution of the equation will give a value for x usually within one-tenth of one per cent of true value.

This result is satisfactory, for the rate, x , is a prediction or estimate in most instances; or where results are at hand, it is merely a calculation intended to gauge the relative merits of the investment compared with others. For such purposes, even $\frac{1}{2}$ per cent is sufficiently close.

In the solution of the value of x , the use of logarithms is not usually necessary. The value may be found in Table VI, opposite n years, and interpolated to one-tenth of one per cent.

This method of gauging the profits of forest production is far more suitable to new conditions characteristic of the industry in America than the method of capitalizing income in the form of soil value or computing the "forest per cent." In almost every instance, except for lands owned by the nation, the cost of acquiring the property is known, or is a determining factor to be decided previous to undertaking forestry. It is upon this

investment that profits must be computed whether or not the capital invested in future expenses is also entitled to share in the profit. The rate of interest earned offers an intelligible standard of comparison with the common forms of business enterprise.

The profit (§ 39) or enterpriser's gain is then found as a ratio instead of a lump sum, by subtracting p per cent from x per cent.

European methods of computing the rate x earned by capital invested in forestry differ from those advocated above. The capital investment is regarded as the cost of the forest soil, in accord with Method N. But instead of computing the rate of compound interest which is earned on S_c , the first step is to calculate the annual "rent" earned on S_v , which is equivalent to $S_v \times 0.0p$. This, divided by S_c , gives $0.0x$, and x is termed the mean annual forest per cent. As expressed by Schlich,

$$x = \frac{S_v}{S_c} \times p. \quad (\text{N}_3)$$

The per cent x in this formula is more sensitive than in Formula N. When $x > p$, a higher rate is shown for x than that given by method N. This is due to the effect of using different rates of interest, x versus p , in discounting by the two methods.

Formulæ N to N₃ express profits only for the beginning of the enterprise, as forecasts or as summaries. When the potential annual or current rate of interest is desired, termed the current annual forest per cent, this may be found, after the timber becomes of merchantable value, by first determining the annual increase in this stumpage value (Chapter XII, § 182).

Let Y_n = stumpage value in year n .
 Y_{n+1} = stumpage value in year $n + 1$.

From the increase thus shown must be deducted the annual expenses e . The investment consists of the cost of the forest for the year n , consisting of soil and timber, and represented (Formula A₂) by aF_c .

The formula for current forest per cent is

$$x = \left(\frac{Y_{n+1} - Y_n - e}{{}_nF_c} \right) 100 \quad (\text{N}_4)$$

for which may be substituted

$$x = \left(\frac{Y_{n+1} - Y_n - e}{S_c + Y_n} \right) 100. \quad (N_5)$$

For a normal forest (§ 109) the forest per cent is obtained by methods identical with those applied in Formulæ N_3 and N_4 . The value of such a forest (F_2) divided by its cost, which is either the cost of purchase or of production (§ 109), gives the basis for the following formula

$$x = \frac{F_v}{F_c} \times p. \quad (N_6)$$

130. The Relative Importance of Profits in Private versus Public Forestry. — In Chapter III it was emphasized that for very long periods the operation of the laws of compound interest requires the lowering of the rate of interest, “ p per cent,” used to gauge the relative value of investments. It is easily shown that enterprises running for periods of over 50 years cannot earn to exceed 4 per cent except under the most favorable circumstances, although the specific net profits above cash outlay may exceed this outlay by several hundred per cent (§§ 53 and 128).

Private industry cannot be expected to undertake the business of forest production on a scale commensurate with our future needs, for the following reasons:

1. A higher rate of interest is required on private than on public undertakings.
2. The average business life of an individual is shorter than the period required to produce a forest crop.
3. The private enterprise depends for its existence solely on profits, while the public undertaking includes the benefit to the community as the principal item of income, and may therefore be conducted at an actual money loss and still be justified.
4. In many instances, lands are already the property of the state or nation, and no cost or interest on land need be charged. The sole cost of public forestry in such instances is the sum of actual outlays for protection and silvicultural measures.
5. Public forestry can and should be conducted on a very

large scale as it is on the national forests, with resulting economy in all measures of protection, technical administration and improved efficiency which will increase future revenue.

6. Private ventures are based on the assurance of an enterpriser's gain or net profit above p per cent, which is a matter of indifference in public enterprises. The necessary lowering of the interest rate on long investments, the postponement of results and the increasing importance of the governing rate of interest as affecting the size of the indicated profits lessen the individual's interest in forest production as a business venture, and strengthen the reasons for state or national enterprise.

After the forest capital has been destroyed, its restoration by planting or seeding can be of interest to private owners only for species, regions and sites which promise reasonably quick returns and high values, such as are shown for white pine in New England or Loblolly pine in Maryland. But before destructive lumbering has made such restoration necessary, especially for forests which show a succession of age classes, such as spruce in the northeast, the private owner may often continue to operate indefinitely, with profit, by removing only the more mature timber. This difference is due to the fact that such a forest produces income annually with which to cancel expenses. Under such circumstances the specific balance sheet shows an annual profit; there is no accumulation of unpaid interest, and by foregoing the "profits" he might obtain at any time by withdrawing the entire capital, the owner substitutes a permanent annual income.

In considering questions of public policy, no sound conclusions can be reached if the financial questions involved are neglected. The difficulties of inducing private owners to voluntarily enter forest production as a definite business are evident. If this course is made mandatory by legislation, as is seriously contemplated in some states, it must first be shown that the expenses thus required are in reasonable relation to the income that can be expected from forestry. Otherwise it would appear that the state is attempting to shift onto private shoulders a responsibility which it should publicly assume.

This public sentiment for regulation is the expression of an instinctive protest against destructive lumbering as a policy, for it is far more expensive and wasteful to tear down a business and then rebuild it than to continue it as a going concern. Whether or not forest owners can afford any other policy than destructive lumbering, they should in any case use every effort to do as little harm as possible to reproduction, protect cut-over lands from fire, and induce the formation of a new crop of natural seedlings on absolute forest soils. In proportion to the effectiveness of such measures for perpetuating the productiveness of forest lands in private hands, the desire on the part of the public for drastic interference with private rights will lose its force.

131. Forms for Specific Accounts in Forestry. — The accounts kept by the proprietors of a large business, involving the ownership and management of many separate parcels of forest land, and a more or less continuous annual income and outlay, must show specifically the cash transactions and actual cost of the property.

The capital accounts should record cost of permanent assets. But to this cost may be added the annual excess of actual cash expenses over annual income, incurred for the care and maintenance of the property (§ 57). The only alternative is to exhibit a corresponding deficit in the balance sheet. The choice between these methods will be based on the character of the assets. For property increasing in value by natural growth of timber there can be no serious objection to the process of adding actual costs to the capital account.

A depreciation fund is seldom required for such an enterprise. The equipment subject to depreciation is of such small total value that it will usually appear as expense.

BALANCE SHEET

<i>Assets</i>	<i>Liabilities and Proprietorship</i>
Capital assets:	Capital paid in.
Land.	Surplus.
Timber.	
Other property	Loans (liabilities).
less depreciation.	
Current assets:	Current liabilities:
Cash.	Accounts payable.
Accounts receivable.	

TRADING, AND PROFIT AND LOSS ACCOUNT

A. — LAND

<i>Debit</i>	<i>Credit</i>
Cost, to date of sale.	Sales.
Balance, gross profit, to general outlay and income account.	Balance, loss, to general outlay and income account.

B. — STUMPAGE

<i>Debit</i>	<i>Credit</i>
Cost, to date of sale.	Sales.
Balance, gross profit, to general outlay and income account.	Balance, loss, to general outlay and income account.

C. — GENERAL OUTLAY AND INCOME ACCOUNT

<i>Debit</i>	<i>Credit</i>
Silvicultural operations.	Leases.
Protection.	Grazing.
Maintenance.	Secondary forest products.
Administration.	Thinnings.
Taxes.	Profit, from lands.
Loss, from lands.	Profit, from timber stumpage.
Loss, from timber stumpage.	Balance (loss).
Balance (profit).	

Balance (loss).	Balance (profit).
Interest on loans.	
Depreciation.	
Surplus (profit for period).	Deficit (loss for period).

CAPITAL ACCOUNTS

D. — LAND.

<i>Debit</i>	<i>Credit</i>
Cost, by parcels.	By cost to date of sale.
Additions to cost by reason of annual cash expenses.	Balance.

E. — STUMPAGE.

<i>Debit</i>	<i>Credit</i>
Cost, by parcels.	By cost to date of sale.
Additions to cost by reason of annual cash expenses.	Balance.

F. — DISTRIBUTION OF ANNUAL LOSS, FROM PROFIT AND LOSS ACCOUNT

<i>Debit</i>	<i>Credit</i>
Deficit.	By increase of capital assets.

G. — DISTRIBUTION OF SURPLUS

<i>Debit</i>	<i>Credit</i>
To dividends.	Surplus, profit and loss account.

H. — CHANGES IN VALUE OF CAPITAL ASSETS

<i>Debit</i>	<i>Credit</i>
Appreciation or marking up of value of:	To appreciation of assets.
Stumpage.	
Land.	
<hr/>	
Depreciation of assets.	To losses of timber by reason of fire, etc., not insured.

132. Forms of Economic Accounts in Forestry. — During the formative or constructive period of a forest investment, previous to the receipt of the principal income, the economic status of the entire investment may be shown by a cost account, in which accrued interest is charged against all outstanding capital investments.

OUTLAY AND INCOME ACCOUNT

<i>Debit</i>	<i>Credit</i>
Outlay for all sources.	Income from all sources.
Accrued interest on outlay.	Accrued interest on income.
	Balance, representing net investment or cost.

Since such an account is intended merely for the owners' information, the "interest" cost is a legitimate one. In the same way the "economic" balance sheet (§ 73) can be made up by discounting the value of future income and entering the assets on the basis of their capital or expectation value.

"ECONOMIC" BALANCE SHEET

<i>Debit</i>	<i>Credit</i>
Expectation value of land and young timber.	Total net cost of land and timber from outlay and income account.
Value of improvements.	Net cost of improvements (original cost, less sums returned for depreciation).
Potential deficit.	Potential surplus.

It is this balance which is the object of nearly all the calculations discussed under forest finance.

CHAPTER IX

THE APPRAISAL OF DAMAGES

133. Principles Underlying Appraisal of Damages. — In appraisals to determine damages, the principles underlying the valuation of forest property reach their fullest development and receive their most complete application. When the economic basis of forest valuation is understood (Chapter IV) it will be seen that the guiding principles of legal practice indicate a consistent effort to apply this basis in determining compensation for damages.

The uncertainty and difficulty of authoritative determination of future values is here, as elsewhere, the most serious drawback to the unqualified adoption of expectation values as the basis for measuring losses or damages. Whatever substitute methods are adopted, as, for instance, the determination of cost, are dictated by the greater relative certainty of the figures thus obtained, which consideration may outweigh in importance the fact that these figures do not accurately represent actual value or loss.

These legal principles may be summarized as follows:

a. Damages are payable in money. Compensation, not physical restoration, is required.

b. The difference in value of the property before and after the damage is the measure of damages. The value of the portion destroyed is not in itself the measure of damages, but is a means of ascertaining this difference in total value. Soil and timber are real estate. The value of this real estate as a whole, before and after the injury, is the measure of damages.

c. Damages must be appraised on the basis of the most profitable use to which a property is adapted, as indicated by the use of similar contiguous property. Sale value may or may not be a correct index of real value.

d. Values must ordinarily have a commercial or utilitarian basis, but æsthetic values, or the value of "legitimate gratification" (§ 4), must be recognized whenever based on elements generally accepted by the public at large. Sentimental value peculiar to the owner cannot be admitted.

e. Loss of income may be made the basis of damages; but this loss should be discounted to the present, and will then equal the loss in capital value.

f. Cost of restoration, while frequently ruled out, is admitted when shown to be less than value, or when value is difficult to determine.

g. Ordinary "profits" or income is the basis of damages. Excessive profits, which do not allow for normal losses, and speculative or contingent profits, which depend upon uncertain future factors, such as increased prices, are not admitted. The element of time, intervening between the damage and the realization of profit, does not bar the consideration of these profits provided they are reasonably certain to occur and are properly discounted.

h. Damages must be actual, present, imminent or reasonably certain to occur. The damage itself may be indirect, but it must be proximate, or traceable directly to the offending cause, as, for instance, the destruction of crops, due to cattle, through leaving a gate open. The physical destruction of property by fire is an actual present damage. The cause may be a spark from a defective locomotive igniting the débris upon an improperly cleared right of way, several miles from the property destroyed. The determination of the money value of the damage is the only element of uncertainty in the process.*

* The entire subject of damages is the source of wide differences of legal opinion and practice in different states. Precedents and rules adopted in one state are frequently at variance with the decisions in other states. The measurement of damages is largely a determination of facts rather than of legal principles; the ascertaining of the amount and character of the injury done to the property and consequent income of the plaintiff. The above rules appear to agree with the principles laid down in Sedgwick on Damages, 9th Ed., Vol. 3, §§ 931 to 933. The contention of foresters that expectation value, where it can be determined, is the basis of damages, finds its clearest legal expression in rule *b* above, in which form it is receiving increasing recognition in state courts. See reference above for citations of cases.

134. Elements of Damage to Forest Property. — Damage may occur to forest property through injury or destruction of timber, soil or permanent and temporary improvements. For permanent improvements, the cost of replacement or repair is a fair basis of valuation. For temporary improvements, the present value would be found by subtracting depreciation from cost, or estimating the future usefulness of the improvements destroyed. The problems discussed in this chapter are those which deal with damage to timber crops and forest soil.

The separate elements which may constitute damages are itemized as follows:

Mature timber.

Young or immature timber.

Forest soil.

Watershed protection.

Reduction in total value of property not otherwise damaged.

Æsthetic values.

135. Physical Separation of Timber from Soil. — Damage to forest property may take many forms, but only those sources of injury traceable to human agencies financially capable of making good the loss, are of interest in appraisal of damages. All losses involving actual destruction of property are irreparable in the sense that the entire value destroyed is lost to some person. Losses from insects, wind or disease must be appraised for the information of the owner. But when the loss is to be shifted to another, and collected by legal processes, its appraisal must rest upon facts that can be accepted by a jury.

Timber trespass takes the form of removing forest products from the soil without the consent of the owner and, incidentally, of injury thereby caused to unmerchantable trees. The main element of damage is the value of the timber destroyed or taken.

Damage by fire results in the destruction, or partial destruction, of the tree growth, or merely in injury and retardation of growth. Except in the most unusual conflagrations, the timber even if completely killed is not consumed or removed in the sense that it is in lumbering. Much of this fire-killed timber, if large

enough to be merchantable, can be salvaged. After the great fires of 1910 in the Pacific northwest about 90 per cent of fire-killed timber owned by private parties was marketed. If not merchantable, the inflammable dead material, rotting and falling, greatly increases the fire hazard (§ 200). In practically no case does fire bring about a complete separation of the timber from the soil.

136. Separation of Value of Timber from Value of Soil. — As a step in the appraisal of damages it may be necessary to determine separately the value of timber and of soil.

By Formula G the separate value of timber is

$$\frac{Y + S_v + E}{1.0p^{n-a}} - (S_v + E),$$

which gives the value in the year a for the timber by subtracting the value S_v from the total value of the property. The residual value, land, then equals S_v , and if the damage just equals the value of the timber, it may be appraised by this means. But this assumes that the physical separation actually takes place, and that the soil or property S_v is left, as a result, in the same condition which it would be in after lumbering and brush disposal. This premise becomes evident by analysis of the above formula which equals

$$\begin{aligned} {}^aG_v &= \frac{Y}{1.0p^{n-a}} - \left(E - \frac{E}{1.0p^{n-a}} \right) - \left(S_v - \frac{S_v}{1.0p^{n-a}} \right) \\ &= \left(\frac{Y + S_v + E}{1.0p^{n-a}} - E \right) - S_v. \end{aligned}$$

In this form the separate values of land and timber are clearly shown. But the expression $S_v - \frac{S_v}{1.0p^{n-a}}$ is equivalent to the difference in value of S_v if freed for use now, and its present value if freed for use only after the lapse of $n - a$ years. This difference is equal to the present, discounted or capital value of the interest on S_v for $n - a$ years, or

$$\frac{S_v (1.0p^{n-a} - 1)}{1.0p^{n-a}}.$$

It is important to understand the significance of this treatment of soil value in valuing timber. One way of describing this "expense" is that the owner of the *timber* is charged with the interest on the soil value, or "soil rent," as a *cost* of bringing the timber crop to maturity, dating from the present year. This is the assumption made by Schlich.* This would class soil rent as an actual future cash expense similar to taxes. But soil rent cannot be an *actual future* cost or outlay, unless a different person owns the soil and rents it to the one who grows the trees. The capital value of the property, timber and soil, takes no account of this cost (Formula D₂), for $\frac{S_v}{1.0p^{n-a}}$ is the discounted net income, which constitutes soil value.

A separation of the present net value of the standing timber from the value of subsequent crops (soil S_v) gives

$$\frac{Y + E}{1.0p^{n-a}} - E = \text{value of timber,}$$

$$\frac{S_v}{1.0p^{n-a}} = \text{present value of soil if released in } n - a \text{ years.}$$

Comparing this with

$$\frac{Y + S_v + E}{1.0p^{n-a}} - (S_v + E) = \text{value of timber,}$$

$$S_v = \text{value of soil,}$$

the sum of values for soil and timber, or the value of the property, is identical, but the proportion of this total value assigned to the timber is greater by the first method than by the second, by just the amount of the discounted interest on S_v .

In the second method, instead of assuming that the timber owner is charged with interest on S_v , a much clearer conception is that the land owner, who in fact is the same person, *benefits* by the *release* of his land value from its necessary use for $n - a$ years to produce the revenue Y . He therefore cannot demand the full value $\frac{Y + E}{1.0p^{n-a}} - E$ for the crop, but accepts a value smaller by the exact amount of the excess in value of

* Schlich's "Manual of Forestry," 4th Ed., Vol. III, Part II, Chapter III, p. 133.

S_v over $\frac{S_v}{1.0p^{n-a}}$. The trespasser, in settling the damages, thus "purchases" the timber separately and is given the benefit of subtracting the discounted future expense of "soil rent" in reduced damages. It is now possible for the owner to obtain this soil rent on another crop of timber that can be planted immediately. He is not justified in collecting it in the form of increased net damages.

The subtraction of soil rent from present value of standing timber in appraisal of damages is therefore a mere matter of book-keeping between the owner and the trespasser, by which the *owner* is the loser in damages, but is compensated by the release of the land.

It is evident that unless these conditions are actually secured, and the land not only released from the crop but left in the condition which would normally result from logging, such an assumption absolutely fails to appraise the true damages, which are in excess of the amount indicated.

In European practice, the expectation value for the soil S_v is first computed. This value is then introduced not only in determining expectation value of standing timber, but is also substituted for the *cost* of the soil.

The effect of using S_v as the so-called "cost" of the soil is to eliminate "profit" or enterpriser's gain, and to balance costs and income at p per cent (§ 127). Not only will the costs balance the income at the final year, but, by formula K_1 , substituting S_v for S_c ,

$$\frac{Y + S_v + E}{1.0p^{n-a}} - (S_v + E) = (C + S_v + E) 1.0p^a - (S_v + E),$$

which indicates that this so-called "cost" is exactly equal to expectation value of timber for any year during the growth of the crop. This means that if actual cost of soil is not known, or does not enter into the problem, as in the case of government lands, and the returns from forestry are computed on the basis of a fixed rate of interest, p per cent, throwing all "profit" into terms of soil value (§ 127), standing timber of a given age

will have the same value whether computed as a cost or as a capital value. To compute this value under actual conditions in this country, either the standard rate of interest for many sections must be very low or a negative soil value may have to be used, which is perfectly practical. The method is so foreign to our ideas of finance that it should be discarded in favor of the more intelligible plans of utilizing actual costs and sale value, or of discarding altogether the subtraction of interest on soil as a deduction from damages.

137. A Basis of Damages: Cost of Replacement. — The objections to the use of cost of replacement as a measure of damages are that it does not represent the true value of the property nor measure the loss (§§ 133*b*, 68). Its merits lie in the certainty of the figures, derived as they are from past experience.

Very young timber or reproduction may be completely destroyed by fire. It has no sale value, except, rarely, for Christmas trees. The expectation value of such young stands is difficult to compute. But the cost of planting and the annual charge for protection and administration can be easily determined.

Assuming that the trees have been completely killed and that no further expense is needed to prepare the ground for a new crop, the cost, including interest on soil, the use of which is lost, will be, for a years,

$$(C + S_c + E) 1.0p^a - (S_c + E). \quad (A)$$

By substituting the capital value of soil for S_c , this cost becomes equal to the expectation value of the young timber (§ 127). But this value S_v is even more unreliable than the expectation value of the young stand (§ 116). If S_c is charged as a cost, it will be determined by the price paid for the soil, or the sale value of similar bare land in the vicinity.

Where, as in government forests in the west, land has no purchase price, the practice is to omit this item from cost. The cost of the crop then becomes

$$(C + E) 1.0p^a - E.$$

The effect of this omission is to decrease the damages demanded.

The cost C is included except when natural reproduction is sure to occur. In this case, C may be omitted, and the damages would then amount merely to the annual expenses

$$E(1.0p^a - 1).$$

In ordinary cases, the entire annual charges, including administration, taxes and upkeep, would constitute the cost. But in national forests, administration expenses are frequently offset by revenues from grazing, while taxes are not paid. The item e may then include only the cost of fire protection. The rate of interest adopted in government forestry is seldom over 3 per cent. For these reasons, damages *based on cost* for private forests usually exceed those claimed for government timber of the same quality.

Excessive costs of replacement are sometimes demanded on the basis that the trees burned should be physically replaced with others of like size. An owner who has a young grove destroyed feels that the small value represented by the cost of growing the trees does not compensate him for the loss of the grove. His trees are gone and he will have to wait an equal period for a new plantation to attain like size. If repeatedly burned, he would never secure his grove. The excess value indicated in such a case actually exists, both as sentiment on the part of the owner and as an addition to the value of the entire property (§ 133*b*). Neither of these elements are best valued by attempting to estimate the cost of immediate physical restoration of the property, but are discussed under other headings (§§ 138 and 147).

With increasing age of the timber, the relative merits of cost as a basis, compared with value, become less and less. Wherever there is a marked divergence in the two, whether cost exceeds or is less than value, the collection of damages based on cost cannot be justified by any principle of law. Its use is clearly a make-shift and will be confined to young timber whose age does not exceed one-half the period required to bring the trees to maturity.

138. A Basis of Damages: Sale Value. — Sale value is a far more reliable indication of damage than cost. In forest property, it is always a question as to whether the timber destroyed or removed can be valued separately, and by this means the loss in value to the property as a whole be determined.

The ultimate sale value of the timber is a reliable basis for damages, as it is the basis of capital value not only for the timber but for the soil as well (Chapter VII). Whenever timber is destroyed which has a present sale value, this value must be ascertained, and will be accepted as the measure of damages for the timber destroyed, unless it can be proved that other damage has been suffered. If the timber is young and growing rapidly, or if it is at present inaccessible but certain to be developed in the near future, its present stumpage value or sale value may not represent its true value to the owner, who intends to hold it. By proving with reasonable certainty the existence of this higher value, sale value even for merchantable timber can be set aside. But the burden of proof is on the owner, and courts will accept sale value for timber unless a strong case for higher values can be shown.

The loss in sale value of the entire property is not so easily gauged. It is a great advantage to be able to value the income, in the form of timber, rather than to judge of the effect of this loss of timber upon the selling value of the property. In theory, the latter loss more accurately gauges the damage (§ 133*b*). There are cases when the value of the materials destroyed is wholly inadequate as a measure of the loss resulting to the owner.

The most direct case is where a portion of a large stand is destroyed, appreciably reducing the total amount that may be logged. This raises the cost per thousand feet, or unit, for logging the remainder (§§ 98 and 174). The increased cost is subtracted from stumpage value of the remaining timber. The total loss is equal to the sale value of the timber burned plus the loss in sale value for the entire remaining body of timber.

A woodlot frequently adds materially to the sale value of a farm. Its destruction would represent a loss many times greater

than the value of the wood, and, under definition *b*, this difference in value for the entire property must be the basis of damage.

For such losses, affecting the value of the entire property, the appraised loss in sale value is the safest guide, as it eliminates sentiment or personal considerations (§ 133*d*).

Were the general public fully alive to prospective values of young timber and of forest lands, and agreed upon the rate of interest which should apply to forest investments, and if a sufficient movement in forest real estate existed so that these opinions could be crystallized by sales, there is no doubt that sale value for immature timber would approximate its real value so closely that it might be accepted as a basis for damages as readily as the sale value of mature or saleable trees. This would be an immense advantage, as there is never any objection to accepting sale values whenever they are evidently just to the owner. But with the growing of trees as a business still in its infancy, or perhaps not practiced at all, the sale value of such young timber is likely to be far below its capital or expectation value. Under principle *c* (§ 133) a great injustice would be done the owner by accepting such a basis of settlement.

In regions where immature timber is commonly regarded as having little if any value, the claimant must prove the injustice of such valuation by demonstrating his intention and ability to bring the young trees to maturity and to realize the ultimate revenue. It would be difficult for a lumberman who customarily abandons his lands to fire, to do this. But the United States Forest Service has no trouble in proving the value which it places upon such young growth. In regions where forestry is extensively practiced, and upon lands which the owner has had under management, such as plantations, not only will sale value more nearly approximate real value, but it will be easier to secure the true expectation value independent of the standard set by sales.

139. A Basis of Damages: Expectation or Capital Value. — Not only does sale value fail to establish the value of property to the owner for his own use, when this value is unfamiliar to the public at large, but it frequently happens that there are no sales

of similar property on which to base such values. There is a further difficulty in judging of the loss in sale value resulting from the damage. For these reasons, and in order to base the damage fairly upon loss in income or use, the capital value or expectation value of the property, as well as of the portion destroyed or income lost, must be calculated, in spite of its uncertainties. These uncertainties (§ 67) lie in the determination of:

Future yield of merchantable material.

Date of maturity or final cutting.

Future price of products.

Rate of interest adopted.

Authoritative studies of the yields per acre for important commercial species are becoming available which indicate not only the amount of merchantable material to be expected, but the proper age at which to cut the stand. In the absence of such data, experience of farmers and woodlot owners in regions of second growth can be cited.

It might seem that the certainty of the upward tendency of prices for wood products would justify the adoption of higher stumpage values as a basis of damages for destruction of young timber (Chapter XII). This argument is ingeniously used by the Forest Service to justify valuation of very young timber on the basis of cost and thus secure higher values than would be shown by capitalizing income on the basis of present stumpage prices. It is probably true that owners forced to accept settlement on terms of expectation value based on present prices, for timber which will not mature for several decades, are losing a profit which in all reason they may expect. But the determination of the *amount* of this increase introduces a speculative element into a calculation already sufficiently complicated, and runs directly counter to principle *g* (§ 133). Damages are intended to fully compensate the owner for reasonable profits, and no objection can be raised to admitting the effects of the growth of the timber or its increase in *quality*, but the fluctuation of stumpage prices will not ordinarily be admitted by a court and it is the part of wisdom to base the evidence upon prevailing prices. In such calculations as an owner may make for his own

information, he is at liberty to speculate on the increase in prices as he sees fit.

The rate of interest will remain a matter of dispute until forest production takes its place as a recognized business. The higher the rate used, the smaller and more conservative will be the value placed upon the damage. This is directly opposite to the effect of a high rate used in calculating cost of replacement. In such a case, the higher the interest rate, the greater will be the cost, and the smaller the value of the crop (§ 121).

140. Damages to Merchantable Timber.—Timber killed by fire is saleable if accessibly located and a market exists capable of absorbing it before it deteriorates from rot and insects. If sales can be made, the damage is the difference in value of the timber before and after the fire. To this loss must be added the loss in stumpage value of timber not burned but whose logging costs are increased as a result of the diminished quantity of stumpage (§ 138).

Let Y' = value of damaged timber,
 L = total damage.

The loss is expressed as

$$L = Y - Y'. \quad (O)$$

Timber cut illegally may be settled for with or without suit, on the basis of the stumpage value.* But it is poor satisfaction to an owner to collect merely the present value for small timber which he desired to preserve. Additional damages can be obtained by the owner, if he can prove:

a. That the expectation value of the timber is greater than its present stumpage value. Unless increase in prices is permitted, this is not always easy to show.

b. That the sale value of the entire property has been injured by the cutting.

* An important principle of damages widely recognized is that in case of *wilful* trespass, the value of the products wherever found, is the measure of the damages. In such cases the value of the timber after felling, transportation, or manufacture may be the basis of damages. Sedgwick on Damages, 9th Ed., Vol. III, § 934.

c. That its value for other uses, as park purposes, has been injured.

He may also instigate a criminal action against the perpetrator provided the trespass is wilful. In case of a logging contract under which the contractor is required to leave trees of certain sizes, constant inspection by the owner, cancellation of the contract, or perhaps a clause requiring payment of double value for forbidden timber is more effectual than damage suits.

Merchantable timber injured by fire but not killed must be appraised on the basis of its present saleability, and the extent and character of the damage to its sale value. If it can be sold at once, the loss will be measured by deterioration in grade and quantity due to the fire. If it must remain for some years, a further deterioration may be expected as the result of fungi and insects. If this can be measured, it should be allowed. Timber whose sale is necessarily postponed, even if of merchantable size, should be valued by the same method as young or immature timber. Its expectation value, as reduced by rot and insects, and not its present sale value, should be the basis of damage.

141. Damage to Immature Timber: Partial Loss. — For reasons given in § 138, sale value for immature timber destroyed does not compensate the owner, and it rarely happens that the sale value of the property, either before the damage or after it occurs, is a safe index of the damage. Cost of replacement may be used, as suggested in § 137. But for timber more than half grown or approaching maturity, capital or expectation value must be the means of approximating the loss in value to the property, and this method should always be used as a check on cost, even for very young stands.

Where a stand is injured but not killed, the trees will continue to occupy the soil until they reach merchantable size and are cut. The damage to the crop is measured by the loss in value of the timber when cut, traceable to the fire.

Y = original expected value of timber when cut,

Y' = expected value as reduced by fire damage.

The future net value of each of these crops in the year n is, respectively, when S represents either S_c , S_v or S_s ,

$$Y - (S + E) (1.0p^{n-a} - 1),$$

$$Y' - (S + E) (1.0p^{n-a} - 1),$$

as the expenses continue undiminished. The difference in net value, representing damages, is $Y - Y'$. Discounting this loss to the present year a gives $\frac{Y - Y'}{1.0p^{n-a}}$. (O₁)

Under normal conditions, the expected value of the crop cannot exceed

$$Y - (S + E) (1.0p^{n-a} - 1).$$

But if $Y' < (S + E) (1.0p^{n-a} - 1)$,
then $Y - Y' > Y - (S + E) (1.0p^{n-a} - 1)$,

and damages will exceed the expectation value of the timber. That this result coincides with actual conditions can be easily shown. A damaged crop of timber which will just pay expenses can be left till mature. But what is to be done with a stand damaged so severely that it will not meet these future expenses? Its presence prevents the use of the land for a new crop, yet the cost of removing it is a net loss. Assuming that the timber will be left standing, $\frac{Y - Y'}{1.0p^{n-a}}$ should be used whenever Y' can be determined. The total loss to the value of the *property* rather than to the timber is thus measured (§ 133b).

142. Damage to Immature Timber: Total Loss. — When immature timber is totally destroyed or so badly injured that it must be removed if any further profitable use is to be made of the soil, two factors influence the method of appraisal. The soil is released by this removal, and its value advanced from $\frac{S_v}{1.0p^{n-a}}$ to S_v (§ 136). But the value S_v is materially altered to the injury of the owner. As S_v is merely the value of the discounted net income, all future expenses directly diminish this value, as is seen in the formula for S_v or

$$S_v = \frac{Y - C}{1.0p^n - 1} - (C + E).$$

The initial expense C includes brush disposal, soil preparation, and the cost of securing reproduction. The removal of burned and worthless timber increases C abnormally. The expense E representing $\frac{e}{0.0p}$ includes fire protection. A fire trap, caused by débris, must either be removed as an initial cost or an additional annual expense e' be incurred, proportionally increasing E to E' for at least a few years.

If the value of the burned timber is appraised as

$$\frac{Y + S_v + E}{1.0p^{n-a}} - (S_v + E),$$

the owner must be left in possession of a value for land, S_v , undiminished by such expenses (§ 136). The expense of restoring this value is $(C' + E') - (C + E)$, in which $(C' + E')$ represent the costs of securing and protecting the new crop after the fire.* For instance, should natural reproduction, by sprouts or seed, be the normal method, but as a result of the fire, planting becomes necessary, the cost of planting constitutes a damage. If the owner would have had to plant anyway, the damage includes only the cost of removing the débris or protecting the plantation from extra fire hazard.

The damage where young timber is totally destroyed may be summarized as:

$$\text{Timber destroyed} = \frac{Y + S_v + E}{1.0p^{n-a}} - (S_v + E),$$

$$\text{Further damage to property} = S_v - S'_v,$$

$$\text{Total damage} = \frac{Y + S_v + E}{1.0p^{n-a}} - (S'_v + E).$$

In other words, the value of the soil resulting from the fire can be substituted for its value previous to the fire, thus covering the total loss in value of the entire property. But as $(C' + E') -$

* This formula is technically incorrect, since the additional expense $e' - e$ endures probably for but a limited period b . Hence the increased cost of this item is $(E' - E)(1.0pb - 1)$ instead of $(E' - E)$. This alteration may be made in formula O₂.

$(C + E)$ as an expense is the equivalent of $S_v - S'_v$ in future value, the practical application of this problem consists in estimating the costs of restoring the soil to its original condition, S_v , and adding these future costs to the value of the timber destroyed. It must be emphasized that this is not an awarding of double damages but that the two items are absolutely distinct and their sum represents the loss in value of the property. The formula should, therefore, be expressed,

$$L = \frac{Y + S_v + E}{1.0p^{n-a}} + C' + E' - (C + E). \quad (O_2)$$

The cost of removing dead timber and débris and any extraordinary cost of securing reproduction is thus seen to be a legitimate damage wholly apart from and in addition to the value of the timber destroyed.

143. Damage to Forest Soil. — Damage to the soil results frequently from fire, and sometimes this is so serious that the soil is entirely destroyed. This damage takes two forms, actual physical injury by destruction of litter, humus and beneficial bacteria with resulting impoverishment of the soil, and loss of profits or net income from either present or future crops of timber, without necessarily injuring the soil itself. Both results usually occur and there is no accurate way of measuring the injury to the soil, in terms of money, *except* through loss in probable income from timber crops.

Damage to soil resulting in destruction of litter and humus, but which does not kill the standing or growing trees, results in retarding their growth. This decreases the volume and value of the crop. The damage is measured by the reduced value of the standing timber (§ 141, O_1).

Damage to soil accompanied by complete destruction of timber can be measured only by the probable effect of this injury on the value of a *new crop* of trees. When the soil is largely vegetable, as in swamps or on high mountain slopes, its destruction, especially on slopes, may prevent all further tree growth. The damage then equals S_v in addition to the value of the timber burned. When the soil is partially destroyed,

it will be difficult to estimate accurately the probable loss on a new crop, unless the effect of similar burns can be cited.

The cost of restoring soil value by removal of débris (§ 142, O₂) is wholly apart from the appraisal of physical damage to the soil itself. Except in unusual cases, this physical damage is exceedingly difficult to measure in terms of value, and, where timber is destroyed, destruction of soil represents but a small proportion of the total loss. This item of damage is, therefore, neglected in most instances.

144. Damage to Single Trees.—When the expectation value of young timber is made the basis of damages, and the present value of future crops is calculated, the future expenses deducted from value are based on the acreage protected and utilized by the crop. The yield of the timber is also computed on the basis of area, and in this way the normal loss in numbers which occurs in all stands with increasing age is allowed for.

In scattered stands of timber, the individual tree becomes more important, and this importance increases in stands composed of trees of many ages. Damage or loss in such forests cannot as easily be appraised as for crops of even-aged trees. The mature timber can be estimated according to its merchantable contents. If these trees are still small, though merchantable, it may be possible to show that their future increase in volume and quality, with consequent increase in value, is such that this discounted value exceeds the present value of the trees.

If it be assumed that all trees now merchantable will remain standing, and future expenses are neglected, the expectation value so obtained will be too high. Such figures must be reduced:

By allowing for loss of numbers;

By pro-rating the crop expenses among the reduced stand, per tree, according to present volume.

The immature portion of such stands cannot, with any degree of accuracy, be valued on the basis of individual trees. The loss in numbers previous to maturity increases with decreasing age, and the proportional expense to be charged to each tree becomes wholly problematical.

145. Damage to Many-aged Stands. — The value of damage to a many-aged stand is determined by the character and value of the stand. Should the timber be entirely destroyed, its value corresponds to that given in § 117 or § 118, according to the age classes represented. The keynote in valuing damage to stands containing both immature and mature trees of different ages on the same area is to attempt to separate the stand into its age classes according to the *area* which each really occupies. This permits of the distribution of future costs and avoids double valuation.

For instance, suppressed and stunted reproduction growing under old timber may be considered as occupying little if any area. But if this reproduction would have been the means of forming a second crop after logging, it must be valued. Its age should be taken not as the actual age resulting from suppression, but as the age of trees of similar size grown in open land. Means of determining accurately the proportional areas occupied by age classes in such forests are not yet perfected. But an approximation of the area and average "economic" age of the crop, or age of an even-aged stand of equal volume and sizes, is the only satisfactory basis for arriving at damages.

The first step is to decide upon the area which should be assigned to immature timber as a whole. The remaining area is that occupied by mature trees. The appraiser then decides whether to divide this immature timber into several age classes, or to assume an average age for the entire crop and treat it as an even-aged stand of that age. If more than one age class is required, the area and average age for each is determined. In this manner the even-aged stand and area become the basis of appraisal of damages for many-aged forests (§ 140 to 142).

146. Damage to Watersheds. — The protective value of forest cover affects, first, the soil itself and its ability to produce timber crops, and second, the property and interests affected by stream flow.

Soils subject to erosion may be completely ruined, even for tree growth, by the effect of removing the timber cover through fire or logging. This loss would be measured as in § 143.

But the damage inflicted on other property caused by the increasing irregularity of stream flow, floods followed by almost complete cessation of flow, the silting up of streams and reservoirs, and the consequent damage to navigation, waterpowers, irrigation and agriculture are additional to the loss of the timber and soil on the property itself. Deliberate ruin or removal of necessary timber cover, and destruction of grass and brush by over-grazing, probably renders the owner of such lands liable for the resulting damages to others. But the connection between cause and effect, while proved beyond a doubt, is very difficult to express in money value. Large areas of watershed contribute to the result, and the damage extends over many years and varies according to rainfall and other circumstances. To determine the proportion of this ultimate probable damage for which the denudation of one acre is responsible, is a mere guess. Such protective areas should be acquired by national or State governments and the damage prevented, rather than to attempt to force private owners to preserve the forest for the benefit of others, or endeavor to collect damages from such owners for the injuries caused by their abuse of the forest cover.

In the absence of commercial value, as where forests are inaccessible and cannot be cut, the cost of re-establishing the protective cover, either by natural or artificial means, and of protecting this area from fire until it reaches the same state of effectiveness as that which was removed, can properly be considered as a measure of damages. It is also possible in the case of forest users, such as lumbermen, to set an arbitrary value per acre for the protective influences of the cover, and bind the user by private contract to pay this sum in case of fire occurring through the carelessness of his employees.

As knowledge of the effect of forests on flow of streams is obtained, it will become possible to arrive at a value per acre of the protective influence of the forest cover on different watersheds. This value may then be used in damage suits.

147. Æsthetic Values. — The fundamental character of the value represented by an appeal to the æsthetic sense must be recognized in appraising damages (§ 4). The gratifications for

which mankind will pay money are by no means confined to the material demands of the body. Simply because it is difficult to place a money value upon such gratification is no reason for denying its existence. Scenery and beauty are capitalized by those who provide hotels and camps for visitors, and the common rights in scenery, especially in the enjoyment of woods and forests, clash repeatedly with the private property rights of those who own and may wish to cut the timber, usually to the loss of the public, without recourse to damage suits. Such rights, just as in the case of watersheds, are best protected by public ownership of lands and forests which have peculiar or exceptional value.

But the private owner who has acquired property of scenic and æsthetic value is not obliged to accept merely the commercial value of the income from wood products as full settlement for damages caused by fire or theft. The additional value is as real to him as similar values are to the public at large. It is just this common or universal acceptance of this form of value which permits of its appraisal as an element of damages. Purchasers can usually be found for estates possessing such advantages, and at prices which would probably compensate the owner for any reasonable cost he has incurred to secure the results. For this reason, the æsthetic values destroyed may frequently be gauged roughly by the loss in sale value of the entire property. Where landscape features are affected the damage can best be appraised by experts familiar with landscape gardening rather than forestry.

148. Punitive Damages. — Damages in excess of loss in value are sometimes provided for by law in case of destruction of property belonging to the state. This is not justified by any principle applicable to private owners, but is rather an attempt to enforce punitive measures as a matter of public education. It lies halfway between civil judgments and criminal penalties, in which the offender is punished as a public example. The state of New York has such a law, providing a penalty of \$10.00 per tree destroyed on the Forest Preserve. The advantage of combining punitive measures with civil suits for damages is

extremely doubtful, and in suits instituted by New York, this provision, while held as a club over the heads of the offending parties, has not been used as the basis of determining the damages. Even in private contracts it has been found advisable to secure compliance with contract terms by other means than the enforcement of punitive damages for unauthorized acts.

CHAPTER X

FOREST TAXATION

149. Sources of Revenue from which to pay Taxes. — Taxes are levied to raise funds with which to meet the expenses of government. They therefore constitute an annual charge against the wealth of the community. In both theory and practice taxes are levied against persons instead of property, and are gauged according to ability to pay, and not on the basis of benefits received. But the effect of taxation upon the value of productive property may best be studied by regarding the enterprise as the source of the revenue required to pay the taxes. The expense of taxes must be met eventually from income, as are all other annual expenses. Taxes must be paid whether or not this income from the taxed property is forthcoming, and in the absence of such revenue, or previous to its receipt, must be met by the investment of additional capital (§ 131).

Such investments of capital add nothing to the value of the property, and must eventually be recovered from income. Should the taxes reach a proportion where the entire net income is absorbed, the property becomes worthless to the owner (§ 61) and will be abandoned, as was the case with much cut-over timberland in the Lake States in the early '90's when owners could see no prospect of income sufficient to pay the annual taxes.

150. Tax on Income. — A tax on income is the most logical method of preventing excessive taxation. The relation between the income and the tax is definite and the collection of the tax follows the receipt of income, thus relieving the owner of the necessity of borrowing or making additional unprofitable investment of capital. The determination of net income raises the question as to the character of expenses to be permitted as deduc-

tions from gross income. A tax levied on gross income must of necessity be at a lower rate than one levied on net income, and is more easily computed. But net income is a more scientific basis of taxation since the net, after subtracting expenses incidental to securing it, is the only portion of gross income which becomes the property of the owner available to meet taxes.

151. Tax on Value of Property. — Value of property is derived from net income (§ 64) and may therefore be taxed directly, without injustice, provided the assessed value of the property does not exceed its expectation or capital value, and the rate of taxation on this capital value bears the proper ratio to the legitimate tax on income.

This ratio is dependent on the rate of interest used as the standard for determining the capital value (p per cent). The annual equivalent of intermittent income may always be found (XIII). The relation between this annual income and value is

$$\begin{aligned}\text{Value} &= \frac{\text{Income}}{.0p} \\ &= \text{Income} \times \frac{100}{p}.\end{aligned}$$

The expression $\frac{100}{p}$ is the ratio between a tax on income and on capital value. It follows that a tax equal to a per cent of value must take $\frac{100}{p} \times a$ per cent of income. A tax rate of 10 mills, or 1 per cent, when the rate of interest is 5 per cent, will require $\frac{100}{5} \times 1$ per cent, or 20 per cent of the income, while a 20-mill tax absorbs 40 per cent of income. With a rate of interest of 10 per cent the proportion of taxes to income would in this case be respectively 10 per cent and 20 per cent.

The amount of the annual tax is the product of the assessed value and the tax rate. The annual tax, rather than either the rate or the assessed value alone, determines the relative per cent of income demanded. A low valuation permits a proportionally higher tax rate, while high tax rates may be lowered with

no corresponding reduction of taxes by increasing the assessed valuation.

In practice, sale value is made the basis of valuations for assessment purposes, and since sale value may not coincide with expectation value (§§ 17, 59, 69), this may cause both assessed value and taxes to depart widely from the desired ratio.

A tax on sale value secures the desired ratio of net income most accurately when property is producing this income annually. When the income is intermittent and deferred, great discrepancies and inequalities are almost certain to exist in the ratio between taxes and final income.

152. Taxable Value of Property. — The taxable value of property is the value determined from future net income, or its capital value (§ 62) calculated by disregarding the future expenses represented by the proposed taxes, but deducting the discounted value of all other future expenses. The net income which represents this net value is then divided between the owner and the taxing power, and the owner's final net income is what is left after paying his taxes.

153. Effect of Taxes on Property Values. — This net taxable value is merely the sum of values belonging jointly to the owner and the taxing power or public. Nominally the complete ownership is vested in the proprietor. Practically the power to take income carries with it and is a manifestation of the sovereign right of ownership, and on failure of the owner to pay taxes this right is exercised by confiscating the property. But the payment of the tax results no less in an actual transfer of value from owner to public. The residual value after subtracting the taxes from net income is the real value of the property to the owner, who is justified in considering it as worthless when the total net *income* is absorbed by taxation.

It might be assumed that in absence of taxation, property would be worth the additional value represented by the capitalized taxes. This is literally true when property is exempted from taxation and at the same time receives all the benefits of governmental activities. Just as business expenses are expected to result in income greater than the expense, so the benefits of

a government conducted by and for the people, and respecting the rights of property, should, on the whole, give a full equivalent in service for the taxes collected. Whatever the effect may be upon the individual property owner, the entire level of values is raised and the public or common share in this increase should absorb less than the total gain.

Perfect equality of taxation on all forms of income and capital value would have no visible effect upon *relative* values of different forms of property. But if the burden of taxes is unequally distributed, either by deliberate intent as is proposed by the advocates of single tax on land, or by exemptions of certain forms of private property from taxation, or merely by reason of the imperfect working of the machinery for assessment and collection of taxes, this balance is disturbed. The value of property over-burdened with taxation is then confiscated in actual reality and to just the extent of the excess of taxes above the average level. This serves to depress the sale value of the property and lower its apparent taxable value, resulting either in diminished taxes or an increased tax rate which still further depresses values. The end of the vicious process is physical confiscation of the property, destruction of the private enterprise dependent on it, and the cessation of public revenue from taxes formerly received. Meanwhile, property bearing less than its share of taxes is automatically increased in value and made prosperous at the expense of the over-burdened forms of enterprise, but only to a certain extent, for the effect of the destruction of any form of industry is a general lowering of the value of all forms of property or of the total wealth of the community. Universal equality in taxation on the basis of ability to pay (§ 149) is, therefore, the goal of all genuine efforts at tax reform.

154. The General Property Tax. — The general property tax is a direct tax levied on the assessed value of all forms of tangible and intangible property on the theory that equality in taxation is thereby secured. In an agricultural age and region results were fairly equitable. With increasing complication of modern industrial development, the method has become less and less effectual, since it fails to secure equal proportions

of net income from different forms of property and industry. As a means of taxing standing timber, it is a conspicuous failure in this respect, as the tax rates and the total taxes paid on timber bear no fixed relation whatever to the ultimate value of the income.

155. The Problem of Taxation for Timberlands. — The failure of the general property tax when applied to timberlands is due to several causes. Imperfect and unequal valuations are more prevalent with this class of property than with other forms, owing to the inherent difficulty of estimating and appraising standing timber and the unfamiliarity of tax assessors with this work. This is not a defect of the system, but merely in its application.

There are three fundamental difficulties in securing equitable taxation of timber under this system, namely:

The difficulty of distinguishing capital from income for purposes of taxation.

The practice of collecting taxes in advance of the receipt of income.

The element of time and the resulting problem of interest in its effect on the relation between taxes, income and taxable value.

156. Distinction Between Capital and Income in Timber Property. — Regarded as a crop, artificially produced, the entire value of timber represents the income earned by the capital invested in the soil. In the form of an even-aged stand, the increasing value of the crop is merely the accumulating income which, when finally cut, exposes the soil capital once more (§ 116). In determining the value of this soil capital, all future expenses are deducted at compound interest. The annual income equivalent to this final income (Formula XIII) is termed soil rent, and the point of view which regards soil alone as capital, timber as income, and requires all values to be based on compound interest, is termed the theory of soil rent.

In contrast to this conception, land and timber together may be regarded as capital. A forest which has been brought to a condition of complete normality (§ 109) capable of yielding a

continuous annual income without diminishing the value of the capital, has an actual capital or property value which includes the entire value of all standing timber. The annual net cash income from this forest represents interest on this entire forest value, and is termed forest rent. As this income cancels annual expenses, compound interest on the forest as a whole has no chance to accumulate. The point of view which regards the forest, including timber, as capital, and does not deduct compound interest on costs but treats the entire forest as a business capable of producing annual net returns, is termed the theory of forest rent.

The latter conception more nearly approximates the attitude of investors in American forest property. Yet both the conditions and the thoughts of investors occupy a position midway between the extremes of soil rent and forest rent. Timber is almost universally acquired in the form of native or virgin stands, already grown, and is purchased as capital. The entire business, rather than the stand, or parcel, is the basis of annual income, if such income is secured. The *source* of this income is largely the increase in value of the timber, rather by price increment than through growth, and in the realization of this income the capital itself is liquidated (§ 132). The operation thus resembles a speculation similar to the holding of unimproved real estate, rather than a productive business as contemplated by the idea of forest rent. Under such conditions, the distinction between capital and income cannot be co-ordinated with the physical distinction between wood and soil, or between wood capital and wood increment. It is a mere matter of book-keeping based on cash investments and cash returns.

157. The Problem of Interest in Forest Taxation. — But the problem of interest appears in such investments in the form presented by the distinction between the theories of soil rent and forest rent (§ 35). When the investment as a whole produces no annual income for a period of years, unearned interest on the annual expenses accumulates and this time element affects both final "cost" and capital value of the property.

The capital value of a forest which will produce a sustained

income which is realized at intervals greater than one year, may be less or greater than that of a forest producing income of the same total value, but in annual installments. The difference in value is due to the same factors which cause the difference between soil value and forest value, *i.e.*, the income is discounted by compound interest, and the discounted total of future or intervening annual expenses is deducted from value.

As touching taxation the question assumes this form: Shall taxes be levied on actual present value of property, or shall deductions from this value be permitted, including interest on past costs, in arriving at the net taxable value? And shall taxes paid in advance of income be computed as if drawing compound interest in calculating the per cent of final income which such advance payments of taxes absorb?

158. Effect of Present Condition of Forest upon Choice of Methods of Taxation. — These problems are best presented by a concrete example in which the effects of the two opposing theories are illustrated.

A stand of Loblolly Pine in Maryland* yields products worth on the stump:

At 20 years of age	\$17.81 per acre.
At 30 years of age	61.14 per acre.
At 40 years of age	108.91 per acre.
At 50 years of age	168.49 per acre.

Land costs \$5.00 per acre. Annual expenses, exclusive of taxes, are 3 cents per acre. The cost of securing reproduction is placed at \$5.00 per acre.

An interest rate of 5 per cent will be assumed for the problem.

A tax rate of 20 mills on full value will be assumed. This is nearly twice as heavy as the average rate, yet it is exceeded in many forest regions, especially in new and poorly developed localities.

The area of the forest will be taken as 50 acres, and the total yield in 50 years will be $50 \times \$168.49$ or \$8424.50. Three cases are assumed. Case A is for an even-aged stand cover-

* Bulletin 11, U. S. Dept. of Agriculture, Jan. 23, 1914, Table 15, page 19.

ing the entire area, and cut at the end of 50 years, thus giving the conditions described under soil rent. Case *C* is for a stand of all ages. There are here 50 age classes, each occupying one acre. The yield on one acre is cut each year perpetually. This crop may either occupy a separate area as an even-aged stand of one acre, or be scattered as single trees throughout the forest. The results will be identical. Case *B* is for a forest giving an intermittent yield every 10 years, thus presenting five age classes of 10 acres each.

Except for this difference in the arrangement and present condition of the age classes of timber, the financial results for these forests are assumed to be identical. The capital value (expectation value) of each forest can be computed as a basis for taxation.

Table II presents the relation between a tax of 2 per cent on capital value and an equivalent tax on income from sale of timber in each case.

TABLE II

COMPARISON OF RESULTS FROM TAXATION OF CAPITAL VALUE OF FOREST SOIL AND OF SOIL AND TIMBER ON A FOREST OF 50 ACRES

	Basis of taxation.	Character of forest.	Cutting area.	Period between cuts.	Value of yield for period.		Cash expenses for period.	
					Total for period.	Per acre.	Total.	Per acre.*
			Acres.	Years.				
A.....	Forest soil.	Even-aged.	50	50	\$8424.50	\$168.49	\$325.00	\$6.50
B.....	Soil and timber.	5 age classes.	10	10	1684.90	168.49	65.00	6.50
C.....	Soil and timber.	All-aged.	1	1	168.49	168.49	6.50	6.50

* The cash expenses, \$5.00 per acre for planting, and \$1.50 per year, or $\$.03 \times 50$ for protection of the entire tract, are charged against the area cut over within the period.

	Receipts less expenses.		Cash expenses compounded to year of cutting at 5 per cent.		Net value of stumpage.		Capital value of property after removal of yield.		Annual taxes at 20 mills on capital value.	
	Total.	Per acre.	Total.	Per acre.	Total.	Per acre.	Total.	Per acre.	Total.	Per acre.
A.....	\$8099.50	\$161.99	\$3180.87	\$63.61	\$5243.63	\$104.86	\$ 500.95	\$10.02	\$10.02	\$.20
B.....	1619.90	161.99	100.31	10.03	1584.59	158.46	2519.62	50.39	50.39	1.00
C.....	161.99	161.99	6.50	6.50	161.99	161.99	3239.80	64.79	64.79	1.29

TABLE II. — (Continued)

	Total cash taxes for 50 years.		Taxes compounded to year of cutting at 5 per cent.		Compound interest on taxes.		Difference in taxes and interest paid on forests A and B, compared with C.*	
	On total area.	Per acre.	Total.	Per acre.	Total.	Per acre.	Total.	Per acre.
A.....	\$ 500.95	\$10.02	\$2097.46	\$41.95	\$1596.51	\$31.93	\$1142.34	\$22.84
B.....	2519.62	50.39	3169.00	63.38	649.38	12.99	70.50	1.41
C.....	3239.80	64.79	3239.80	64.79				
A'.....	2178.50	43.57	4326.50	86.53	2148.00	42.96		

(A'. Taxes levied on sale value, revalued at 20 years and every 10 years thereafter.)

* This saving is largely due to the fact that the capital values of forests A and B are computed for the year in which these values are lowest, and remain at this valuation for fifty years. See §161.

Per Cent of Values Taken by Taxation, Computing Taxes with Compound Interest to Year of Final Yield

	Full stumpage value.	Net cash profit on stumpage.	Net value of stumpage.
	Per cent	Per cent	Per cent
A.....	24.5	25.9	40
B.....	37.6	39.0	40
C.....	38.4	40.0	40
A'.....	51.3	53.4	82.5

In the problem discussed, the tax rate of 2 per cent on full value is nearly twice the average rate of taxation on present values of forest property. But the crop of timber chosen as the basis of the problem is more profitable than that produced by most species and localities. The coincidence between total cash taxes for 50 years at 2 per cent and capital value would hold good only for the same period and rate, since $2 \text{ per cent} \times 50 = 100 \text{ per cent of value}$.

159. Scientific Taxation: Forest Property Tax. — The value given for the forest in case C is found by deducting the annual cash expenses, \$5.00 for planting and 3 cents $\times$ 50 acres or \$1.50 for protection. This gives \$161.99 as the "net" value of the annual crop, which, capitalized at 5 per cent, equals \$3239.50. (Formula XII.)

The tax, 2 per cent, yields \$64.79 annually. This is just 40 per cent of "net" stumpage value and a slightly smaller per cent of sale value. The annual profit or net income of

the forest coincides with net value of the yield, and the tax, therefore, takes 40 per cent of this profit.

Since no one would dispute the desirability of subtracting these current cash expenses previous to taxing stumpage value, it is evident that for forests producing an annual income, the tax on forest capital and on forest products are equal in value and interchangeable.

160. Scientific Taxation: Forest Land Tax. — The value given for forest soil in case *A* is found by discounting the value of the final crop for 50 years, treating it as a recurring income, and subtracting the discounted cost of planting and annual expenses in the same manner (§ 116). This gives \$500.95 as the capital value of the soil just after removal of the crop.

The tax of 2 per cent on this value for 50 years amounts, in cash, to but \$10.02 per acre, as against a cash total of \$64.79 paid by forest *C*. A study of the table explains this discrepancy. The sum of \$31.93 represents compound interest on these taxes during the 50-year period. The further sum of \$22.84 represents a saving due to the fact that the capital value adopted as a permanent basis for taxation is determined just subsequent to the removal of the periodic cut of timber. This requires that the future costs for planting and protection, amounting with interest to \$63.61, be deducted from stumpage value previous to discounting (§ 105) to obtain capital value.

As a result of this latter deduction or allowance, the total taxes with interest, paid on forest *A*, amount to but 25.9 per cent of the stumpage value minus cash expenses, or 24.5 per cent of sale value. The ratio of 40 per cent applies only to the reduced value or net profit on the investment in land.

Forest *B* presents conditions midway between these two extremes. The capital value is found by discounting the decennial yield and deducting the discounted 10-year accumulation of expenses, giving a value of \$2519.62 (§ 117, F_1). Upon this value a 2 per cent tax gives a total cash payment of \$50.39 per acre during the 50-year period, which is increased by compound interest to \$63.38, leaving a small margin of \$1.41 as a saving due to the factors explained for forest *A*.

In computing the comparative per cents of stumpage value taken by a 2 per cent tax on capital value, compound interest has in each case been added to the taxes, although this item does not constitute a cash outlay. It represents, rather, the equivalent to the community of receiving taxes in advance of income, or may be considered as the discounted value of a 40 per cent income tax paid in annual installments throughout the growth of the crop instead of at the close of the period.

161. Comparison of Taxes on Forest Rent versus Soil Rent. —

The point at issue between these two systems, both based on the taxation of capital value, is evidently the double deduction from income, first, of the future costs of production with interest, exclusive of soil, and second, of the interest on taxes paid previous to receipt of income. A forest created by planting or human effort passes through stage *A* before it can be brought to condition *C*. Shall it, at the latter date, be taxed as for *C* or continue to pay the reduced taxes computed for *A*?

This problem depends for its answer upon whether future conditions, or past conditions, are permitted to determine the burden of taxation. Both cases, *A* and *C*, as well as case *B*, represent actual present value, based on future conditions. When forest *A* is brought into condition *C*, its present value is no longer *A* but *C* and, based on value, it would pay the taxes demanded of *C*.

Taxes are based upon value and not upon cost. Future costs diminish value (§ 64) but past costs are disregarded in computing value and do not directly affect it (§ 68). By this reasoning, the increasing tax which would accompany increased capital value is apparently justified (§ 165). But just as property may have a definite value and still represent a net loss to the owner (§ 68), it may be shown that an owner of timberland cannot afford to pay taxes continuously for 50 years and then have these back taxes completely ignored and be taxed on the full value of his crop. The public, basing the tax on present value, may ignore past taxes, but the owner, looking ahead, is not so apt to overlook this future possibility. This situation can only be met by a contract or tax agreement by which the public consents

to forego taxing the full final value *C* and confines its tax to present value *A*, in return for which concession the owner agreed to undertake the growing of the crop. Past errors in taxation will ordinarily have to stand.

162. Scientific Taxation: Income or Products Tax.—The tax on income, correctly applied, is the mathematical equivalent of the tax on capital, provided the controlling rate of interest can be agreed upon. But the same difficulty is encountered in computing this tax as was detected in comparing taxes on forests *A* and *C*. Table II shows income taxes purporting to represent the equivalent of 2 per cent on capital value, yet varying from 24.5 per cent to 38.4 per cent of the gross income.

Net cash income from either form of forest, *A*, *B* or *C*, is the same, provided *interest* on expenses is neglected, and totals \$161.99 per acre. The "net" income corresponding to the capital value is diminished in cases *A* and *B* by deductions for interest, which accounts for these discrepancies. The question is, will such deductions be permitted in levying an income tax on present incomes?

This question is open to the same solution as that proposed for the taxation of capital values. If the income tax is to be substituted for an annual capital tax on standing timber, the full rate can be levied without injustice. But if the income is not available until a distant future period on crops still immature, the lesser rate should be promised and contracted for as a future system of taxation binding on both parties. *Future expenses must* be considered in a scheme of taxation. Past accumulations of interest "cost" may be successfully ignored.

This view is confirmed by the attitude of the U. S. Collector of Internal Revenue in interpreting the income tax on corporations dealing in timberlands.* The ruling reads: "In cases where lands are purchased for cash, the cost being the proceeds of the sale of the capital stock of the company, the earning value of the money invested, that is, the interest which might have been received on this money had it been otherwise employed, cannot be added to the initial cost of the land to represent its

* Letter, Jan. 6, 1914, to P. S. Ridsdale, Sec. American Forestry Association.

aggregate cost at time of sale. The compensation for such investment is the anticipated dividends which will be paid to the stockholders out of the profits when the lands are sold. The addition of the earning power of the money invested to the original cost of the lands would operate as a deduction of the dividend from gross income. Dividends are not deductible, either directly or indirectly. Interest purported to accrue on investments of this character, made by stockholders, cannot be added to the initial cost of the assets for the purpose of fixing a cost price as the basis for determining the profits accruing to the corporation when such assets are sold."

Since these regulations state previously that "the cost of assets (land and timber) for the purpose of determining the amount of income resulting from a sale is held to mean the purchase price of the lands plus the taxes and other carrying charges paid thereon prior to the sale," it is clearly the position of the government that interest on such carrying charges will not be permitted as a deduction from income. The national income tax on timberlands would, therefore, be based upon the theory of forest rent, ignoring interest, in spite of the fact that this income is deferred until some future year as in case *A*, and not earned annually as in case *C*.

The tax would thus take the same per cent of income, whether this income were received in six months or sixty years. For a speculative business, where profits are not dependent on a definite period of time, this method of taxing income may not act as a deterrent. But with the production of forest crops, the announcement in advance that "no deductions from the income tax will be permitted for interest on future costs incurred in advance of income," would have a direct influence on future profits and act to prevent the undertaking.

The controversy as to whether interest is a cost or an income as affecting taxation either of income or capital values cannot be settled, but may be dismissed, with this summary; past interest accrued on the capital invested by the proprietor or owner is not an actual cost, but merely a method of gauging profits and will usually be neglected in taxation. But future

interest, in the form of discount, *cannot* be neglected for it vitally affects present values, which are the basis of present taxation. Whether or not an income tax to be collected at a future period shall be reduced on account of deductions from income of interest on costs, depends upon the desirability of encouraging industry by granting this absolutely just concession in advance.

163. Scientific Taxation: Combined Capital Tax and Income Tax. — The two systems of taxation, capital tax and income tax, are thus seen to be interchangeable for property producing annual income (C), but to give widely divergent results for property whose income is intermittent (A). In the latter case, either system is open to grave objections. The capital tax can properly be computed only for the year following the yield, when it is most difficult to determine it exactly (§ 116). The income tax wholly fails to satisfy the need for current revenue for which taxes are levied. The third difficulty is that during the growth of the timber and in absence of definite provision to the contrary, the constant tendency will be to levy the capital tax upon the existing value of capital including the timber as if this temporary capital value were permanent and capable of producing permanent annual income (§ 156).

In order to secure regular annual revenue from taxation in the absence of regular income from lumbering or cutting, and at the same time convince the public that owners of timber property are paying their full share of taxes, it is theoretically possible to assess the capital value of the property at a portion of its value and postpone the collection of the remainder as an income tax.

In case A, the soil has a capital value of \$10.02, but can be purchased at \$5.00 per acre.* A tax on \$5.00 at 2 per cent will amount in 50 years to \$20.93, leaving \$21.04 available as a tax on stumpage at time of cutting. This amounts in each case to 12.5 per cent of the "gross" value, or 20 per cent of the "net" value of the yield.

* This discrepancy in values is possible in regions where the value of land for forest production is not fully appreciated.

The true equivalents between capital tax and income tax, and the ratio of division between them, depend first upon the proportion of capital value which is taxed, second, upon the rate of taxation applied to it, and third, upon whether the community is willing in the future to abide by its agreement and permit the owner to continue to pay taxes on the basis of capital values which no longer represent the present value of the property. If the rate is full, the only margin left for an income tax is due to a low valuation of the land. If the land is valued at its full capital value, only a reduced rate will leave a margin for income tax on stumpage. And in case the full rate is collected for the full period on its original capital value, the public is entitled to an additional income tax only by repudiating the owner's claim to deduction of interest on costs from gross stumpage value. In practice, capital values of land tend to advance and the stumpage tax might well be levied on the increased value of timber which is the cause of this advance in land value.

In European systems of taxation both capital tax and income tax are usually levied, but the scientific relation between them is not determined. Each tax is assessed independently, but the combined taxes are sufficiently low so that the property is not over-taxed.

164. Taxes under the General Property Tax. — Under the general property tax, the principle of taxing capital value, theoretically the basis of the tax, is actually entirely superseded by the plan of taxing sale value regardless of income. While standing timber may be legitimately taxed as capital when, as in case *C*, only the annual growth is removed, the effect of annually taxing the sale value of even-aged crops is entirely different. The distinction between capital and income cannot be drawn with any accuracy. Case *A* presupposes that the entire timber value represents income only and not capital.

The injustice in the system consists in taxing annually, on its full capital value, a property which is not producing annual income, and which is being held and increased in value by a

process similar to that of savings.* It is as if the owner of property capable of yielding a definite annual income of \$168.49 (Table II) allows this income to accumulate, although by so doing the earning power of the property is not increased. The tax assessor promptly adds this increment to the assessed value of the property, whereupon the owner, if he can, as promptly withdraws the surplus to escape this excessive tax. By inspection of Table II it will be noted that the value of the yield just previous to cutting, for an even-aged forest, is \$8424.50, which would be the "full" assessed value of the property at that time. After cutting, the assessed value should not exceed \$500.95, and in practice will be nearer \$250.00 if land sells for \$5.00 per acre. The average value for the whole period would correspond closely to \$3239.80.

The tax assessor, under the present plan, bases his increases upon market or sale value, and as the value of young timber is seldom recognized, this sale value usually equals the value of the soil for other purposes, plus the market value of the timber. In this way the property escapes the full effect of repeated taxation of capital value.

A comparison of values for the crop used in Table II gives the following results.

TABLE III

COMPARISON OF SALE VALUE WITH EXPECTATION VALUE OF AN EVEN-AGED STAND, INCLUDING BOTH LAND AND TIMBER

Age, years.	Sale value with land at \$5.00.	Expectation value with land at \$10.02.
10	\$5.00	\$24.83
20	22.81	40.83
30	66.14	66.90
40	113.91	109.36
50	173.49	178.51

The expectation value is based in each instance on the value of the crop at 50 years (Formula G_1), and on an interest rate of

* The Economic Problem of Forest Taxation, by Fred Rogers Fairchild, *Yale Review*, February, 1909. See also "The Nature of Capital and Income," by Irving Fisher, pp. 249-255.

5 per cent. Basing the taxes on sale value and revaluing every 10 years, the total cash taxes paid during the 50-year period amount to \$43.57 per acre, which is slightly less than the amount paid on forests *B* or *C* for the same period and over four times as large as the sum paid by forest *A*, which is taxed solely on the value of soil.

Adding compound interest to these annual taxes (Formula IIIa), the total tax, with interest, amounts in 50 years to \$86.53, equalling 51.3 per cent of gross stumpage value, or 53.4 per cent of net cash profit on stumpage. But basing the net value of the stumpage on the margin over costs with interest or \$104.86, we find taxes and interest absorbing 82.5 per cent of this margin.

These taxes are computed as follows:

Let e = annual tax on original assessed value.
 e' = annual tax on revaluation at 20 years.
 e'' = annual tax on revaluation at 30 years.
 e''' = annual tax on revaluation at 40 years.

Then $E, E', E'', E''' = \frac{e}{0.0p}, \frac{e'}{0.0p}, \frac{e''}{0.0p}, \frac{e'''}{0.0p}.$

Sum of taxes (Formula IIIa),

$[\{ (E \cdot 1.0p^{20} - E + E') 1.0p^{10} - E'' + E''' \} 1.0p^{10} - E'' + E'''] 1.0p^{10} - E'''.$

$\$5.00 \times 0.02 = \0.10 , annual taxes for 20 years.

$$\frac{10}{0.05} = \$2.00 (E).$$

$\$2.00 \times 2.6533 - \$2.00 = \$3.30$, taxes at 20th year.

$\$22.81 \times 0.02 = \0.45 , annual taxes for next 10 years.

$\frac{45}{0.05} = \$9.12 (E')$ (discrepancy due to decimal values, omitted in text).

$\$(3.30 + 9.12) 1.6289 - \$9.12 = \$11.12$, taxes at 30th year.

$\$66.14 \times 0.02 = \1.32 , annual taxes for next 10 years.

$$\frac{1.32}{0.05} = \$26.45 (E'').$$

$\$(11.12 + 26.45) 1.6289 - \$26.45 = \$34.76$, taxes at 40th year.

$\$113.91 \times 0.02 = \2.37 , annual taxes for last 10 years.

$$\frac{2.37}{0.05} = \$47.56 (E''').$$

$\$(34.76 + 47.56) 1.6289 - \$47.56 = \$86.53$, taxes at 50th year.

165. Effect of the General Property Tax on Forest Production.

— The effect upon the owner's profits of the system of taxing sale value annually depends upon the length of time the present owner has held the property, the purchase price, the amount of back taxes already paid and the period which must elapse previous to cutting. The relation of taxes to the value of the property, regardless of ownership, is independent of the purchase price or of back taxes, and deals only with the future. A rapid rise in prices for timber products so enhances the value of standing timber as well as of forest soil that taxes previously paid, on very low values, do not absorb an unfair percentage of present value. Whatever over-taxation would have occurred with stationary values has been offset by reason of this increase in capital value of the property. A continuous increase in value of forest property at the same rate as in the past would continue to absorb a large portion of the over-taxation due to reassessments based on sale value. But it does *not* absorb the compound interest item which accumulates on past taxes in absence of income from sales.

Should the owner have paid practically no taxes up to the present year, he can pay annually on full value for a definite period before the amount of his tax with interest becomes equivalent to the taxes paid on forest *A*. Should he cut his timber sooner, he pays less than this tax. For every year which the timber stands beyond this period, the tax becomes proportionally greater than its just equivalent.

Assuming that the tax of \$41.97, the total paid in 50 years on forest *A*, is just, how long can an owner permit his timber to stand before incurring over-taxation. The sum of annual taxes with interest must equal \$41.97.

$$E(1.0p^x - 1) = 41.97.$$

Omitting all consideration of back taxes, the results are as follows:

TABLE IV

PERIODS WITHIN WHICH PROPERTY TAX ON TIMBER BECOMES EQUIVALENT
TO TAX ON SOIL VALUE

Age of timber at present.	Present value.	Annual tax 2 per cent of present value.	Tax capitalized (E).	$41.97 \div E$, or $1.0p^x - 1$.	Period indi- cated.
Years	Dollars	Dollars	Dollars		Years
50	173.49	3.47	69.40	0.6048	9 to 10
40	113.91	2.28	45.56	0.9212	13 to 14
30	66.50	1.33	26.60	1.576	19 to 20
20	22.81	0.46	9.12	4.600	35 to 36

The indicated period is found by looking up the factor $1.0p^x$ in the table of values for 5 per cent (Table VI). The value corresponding to the factor is found opposite the year indicating length of period.

In the owner's mind these periods will be lengthened if he bases his standard of value on the capital value of the forest rather than on the soil, and would be shortened in proportion to the back taxes which he has paid. Increasing prices for forest products would lengthen the period, and future increases in the assessed value of the property would shorten it. But at some future date, not very far distant for mature timber when taxed at full value, the timber must be cut or the process of confiscation begins. After the year in which the accumulating cost of annual taxes equals the equitable proportion of value available for taxation, the tax steadily confiscates an increasing per cent of the capital value.

With the tax of 2 per cent on soil value, the per cent of "net" income appropriated was shown to be 40 per cent. This in effect appropriates 40 per cent of S_v , so that for forest land worth \$10.00 the owner's equity amounts to but \$6.00, and the state has taken \$4.00 in value for taxing purposes.

But with assessed value increased at 10-year periods the per cent of the net future income taken for taxes is 82.5 per cent. Since this net income is the basis of capital value of soil, the state thus appropriates \$8.25 in land value, leaving the owner a residual value of \$1.75 exclusive of the sum required to reimburse his costs and taxes with interest at 5 per cent. But if he

has paid \$5.00 for this land he cannot afford to give the state in taxes two-thirds of this cost price as well as his entire net profit over 5 per cent. Such a method of taxation therefore tends to prevent the private owner from undertaking the business of forest production.

Upon mature timber the effect is to stimulate cutting in order to save taxes, whether or not over-taxation has already occurred. Few owners stop to compute how much taxes timber should bear. They know that the way to escape paying any more taxes is to cut the timber. If this is the purpose of the tax it accomplishes its object thoroughly, and is a great stimulus to over-production of timber. The only incentive to hold mature stumpage under this system is the hope that increased prices will offset the taxes. The injurious effects of a bad tax system are not reflected fully by depression of values (§ 153), since the resulting losses are pocketed by the owners as past costs, and values for stumpage continue to reflect only future elements of cost and profit. But upon the business of lumbering, in a region of large surplus supplies of stumpage, the effect is disastrous, forcing the weaker operators into bankruptcy and stimulating rapid and wasteful cutting on the part of even the strongest holders.

166. Tax Reform for Forest Property. — From the analysis of the financial relation of taxes to values, the most serious flaws in the system of taxing forests on their sale value appear to be:

1. Actual over-taxation through taxing annually the value of the entire property including timber, during periods when no income is produced, resulting ultimately in confiscation of an inequitably large proportion of ultimate or total income.

2. Uncertainty as to the extent of this over-taxation through the arbitrary power vested in local assessors to raise valuations at any time.

Tax reform must remove these two obstacles by substituting some system of scientific taxation which will secure equality between forest taxes and those imposed on other property when gauged by the per cent of income taken, and thus remove the element of uncertainty which is even worse than the over-taxation itself.

But in effecting this reform the annual local revenues from taxation must be maintained. Reduced revenue at present, even if made up in later years, causes heavier taxation of other forms of property (§ 153). This is the explanation of the failure of many attempts to encourage forestry by granting tax rebates and exemptions, a form of privilege quickly resented and rendered ineffectual by assessors through the simple device of increasing the assessed valuation of other property belonging to the beneficiary. From the facts discussed in Article 161 it appears that tax reform must of necessity look to the future rather than attempt to correct past errors. On this basis, the problem divides itself into:

The taxation of forest land not containing merchantable trees.

The taxation of land bearing merchantable timber.

The efforts to secure an equitable tax on forest land devoid of merchantable timber are best directed, at present, along the lines discussed in Article 163. A combined land and products tax best meets the needs of the situation and is most easily understood. It must not be forgotten that the products tax is justified only when land is not taxed at its full value, and to the extent of the margin of accumulated value remaining untaxed.

Efforts to secure this result have recently been made by several states in the northeast. Connecticut permits no increase in assessed value of land for fifty years and limits the tax rate to 10 mills. An additional income tax of 10 per cent is then assessed on the stumpage value of wood products when harvested. Massachusetts permits the revaluation of land exclusive of timber at any time so that it is probable that the assessed value will be raised to correspond closely with expectation value of the bare soil. Taxes are paid annually at local rates. An income tax is then levied on forest products at 6 per cent of the stumpage value. This tax is equitable only in case the assessed value of land will average lower than expectation value, to an extent sufficient to make up the difference represented by the tax on stumpage.

Pennsylvania arbitrarily fixes the value of land at \$1.00 per

acre. The state then pays local taxes at 2 per cent of this valuation, thus assuming the burden of annual taxation for the joint benefit of the owner and community. These taxes are never refunded by the owner. The owner pays a 10 per cent stumpage tax when timber is cut.

In all these laws only land voluntarily offered by the owner and approved by the state forester, or in Massachusetts by the local assessors, for classification, comes within the operation of the law. This will result in a very gradual application of the system.

No valid objection can be urged to the adoption of similar measures in any region, for they do not reduce the present taxes and will result in greatly increased revenue in the future by bringing into productive use forest lands which may otherwise remain waste.

The future taxation of lands bearing merchantable timber is not so easily provided for. The immediate substitution of an income tax which takes the same proportion of income as the existing capital tax does of value, is entirely impractical, owing to the condition of the capital (§ 156). Not only would the annual taxes from a given forest vary from the former amounts paid annually, being either greater or less according to rapidity of logging, but this variation would be general; and regions where logging is being actively conducted would be in receipt of largely increased temporary revenue, while districts as yet untouched would be entirely without income from timber values. Under the general property tax the local revenue is secured only as long as the timber remains standing, after which the land alone may be taxed. The loss of current revenue after cutting is inevitable under any system.

A plan proposed* for securing equality of revenue between counties, for conditions prevalent in regions of heavy virgin stands and large operations, contemplates the postponement of the annual taxes on timber until the timber is cut, when they will be paid in one sum but without compound interest. To

* Report of Forestry Committee, 5th Conservation Congress, November, 1913, page 128.

equalize local revenues the state shall act as banker, collecting the tax, advancing funds to counties as yet undeveloped, and recouping itself from counties where active logging is in process.

In eastern states a different plan has been adopted. Logging is largely in second growth and scattered, so that the problem of equalizing the revenues from income is not urgent. The plan is to continue the existing valuation of property containing timber of taxable value, but to permit no further increase in assessed valuation of this timber. When the timber is cut, the property is revalued and thereafter taxed only on the value of the land. In Connecticut this protection from increased valuation is offset by a slight tax on yield, increasing 1 per cent for each decade, for 50 years, so that the advantage to the owner of standing timber is not very great.

Massachusetts has adopted a plan which is worthy of special mention. The land containing merchantable timber may be revalued at time of classification and pays a land tax similar to non-forested land. This tax is subtracted from the total tax paid on the entire property previous to classification. The balance is the additional amount of tax which the property must continue to pay if local revenues are to be maintained. This is termed the commutation tax. The amount of this tax cannot be subsequently increased, by reason of increased value. The principle is that the timber is exempt from annual taxation, yet the property pays this commutation tax to tide over the gap between the present year and the receipt of the income tax. On cutting, the timber pays a 10 per cent income tax, which extinguishes the commutation tax. A particularly valuable provision is that as soon as timber having a total value equal to that of the stumpage originally assessed, has paid a products tax, the timber remaining or the increase resulting from growth is freed from annual taxation. The community has in the meantime suffered no loss of local revenue and is in receipt of an income tax in lieu of increased taxes on advancing values.

An additional advantage of this plan is that it permits of the consolidation, for taxing purposes, of areas of any size and condition of timber, with land bare of merchantable timber. The

amount of the commutation tax is made a lien on the entire consolidated tract and extinguished by cutting on any portion of it.

Some such plan of taxation is recommended as possessing greater merits than an immediate substitution of income tax for capital tax in timbered regions.

No law providing for proper taxation of forest lands should give arbitrary powers to a state official to prescribe or regulate the scientific methods to be followed by the owner in growing trees. The one condition to be insisted on must be that the land, especially that without merchantable timber, shall be brought under forest cover composed of useful species, and maintained as forest land. Cancellation of classification should follow failure of owners to comply with these basic conditions.

To prevent owners from taking advantage of such laws in order to escape taxation on property which they may ultimately intend for some other purpose, two precautions are used: the limitation imposed by classification of lands suitable for forest purposes, and provision for payment of a penalty upon cancellation of this classification. Connecticut and Vermont prescribe that lands above a certain value per acre, not including timber values, cannot be classified. Massachusetts, in permitting any lands to be classified regardless of value, protects the public by permitting revaluations, which is perhaps the better plan.

On cancellation of classification the penalty imposed should not exceed the difference saved on taxes by the classification. Connecticut determines whether the property at time of cancellation has increased in value and assesses this increase at one-half the average rate of 10 mills, or 5 mills per year, multiplied by the period of classification. The owner thus pays his back taxes (without interest), on the theory that the increase has been uniform throughout the period. Massachusetts releases the owner by the payment of the products tax on all timber standing at the time of cancellation.

The apparent difficulties of securing the adoption of rational tax systems for forest property, not the least of which are the

constitutional provisions of many states requiring all property to be taxed "equally" under the general property tax, should not discourage efforts at genuine reform. The future practice of forestry by private owners must be accompanied by the adoption of a reasonable and adequate system of taxation for forest property.

CHAPTER XI

STUMPAGE VALUES

167. Definition of Stumpage Value. — Stumpage value is the sale value of merchantable standing timber — its value on the stump previous to felling and removal. The term “merchantable” is not so easily defined. Timber is merchantable when it is of sufficient size to furnish wood products suitable for definite commercial purposes. There can be no fixed standard of sizes to coincide with the general definition of “merchantable” timber. Trees merchantable for cordwood or paper pulp are not large enough to yield saw timber. It follows that stumpage value is the sale value of timber intended for certain specific uses. The industry which can pay the most for the timber determines its stumpage value. In the form of stumpage, timber is a portion of the real estate (§§ 134 to 136). When purchased with land it may be held indefinitely, but when bought or sold separately it must usually be removed within a stipulated period. The value of stumpage is ordinarily separated from that of the land, even when both are purchased, though in the past the land was often considered of no value except for the timber.

168. Sale Value of Stumpage. — Small bodies of timber, such as woodlots, are frequently sold for a lump sum. The value of the timber is thus agreed upon without determining its price per unit, which is known only to the purchaser after he has cut the timber. Areas of considerable size may be sold on the basis of a price per acre.

This crude method is giving place, for large tracts or timber aggregating considerable value, to the determination of quantity and agreement upon a price per unit, thus arriving at total value. But neither quantity nor price are easily ascertained

for standing timber. Products already harvested and measured, like wheat or coal, resemble logs or lumber, but the standing trees are like the grain before the harvest, or the coal yet in the mine; yet not quite the same, for the trees can all be seen, and their contents, quality and value ascertained with some definiteness. This requires agreement upon a unit of measurement for the product, as, for instance, board feet by a given log rule, cords or posts. The methods of determining the quantity of stumpage are discussed in Chapter XIV.

169. Stumpage Prices. — Prices for stumpage are determined by actual sales, whenever a price per unit of product is quoted. But the price obtained for one body of stumpage is seldom a safe basis for appraising the value of another stand. For no other product is there such need of checking sale values by a careful appraisals as for timber stumpage. Practically every body of timber should have a separate price, determined by the factors which are peculiar to the stand in question. Standard or uniform prices are often established in a region, for small bodies of stumpage bought by a single company, but this uniformity is usually explained by the fact that the purchaser is in position to dictate the price, and could, if he wished, pay more for the better lots.

170. Factors Determining Stumpage Prices. — The value of stumpage is derived directly from that of the finished products into which it is finally manufactured (§ 9). The stages in this process, in reverse order, are:

$$\begin{array}{l}
 \text{Income from} \\
 \text{retail lumber} \quad \left. \vphantom{\begin{array}{l} \text{Income from} \\ \text{retail lumber} \end{array}} \right\} = \\
 \left\{ \begin{array}{l} \text{Cost of retail business} \\ \text{Retailers profits} \\ \text{Freight charges} \\ \text{Cost of lumber} \\ \text{f.o.b. mills} \end{array} \right\} = \text{Income from manufacture or milling} = \\
 \left\{ \begin{array}{l} \text{Cost of manufacture} \\ \text{Milling profits} \\ \text{Per unit of product} \\ \text{Overrun} \\ \text{Cost for logs at mill} \end{array} \right\} = \text{Income from logging} =
 \end{array}$$

$$\begin{array}{l}
 \left\{ \begin{array}{l} \text{Cost of transportation} \\ \text{Cost of felling and skidding} \\ \text{Logging profits} \\ \text{Cost of stumpage} \end{array} \right\} = \text{Income from timberlands, or} \\
 \hspace{15em} \text{Stumpage value} = \\
 \left\{ \begin{array}{l} \text{Cost of production, or carrying charges} \\ \text{Profits} \end{array} \right.
 \end{array}$$

It is evident that the gross income at any stage in this economic series just equals the total costs plus profits incident to the entire chain of processes underlying this income and preceding it in point of time, and that the profits, or losses, are the factors which balance the successive equations. Stumpage value is equivalent to cost of purchase, and tends to increase by higher prices for lumber, and to decrease by higher costs or by larger profits in retailing, manufacture or logging.

There may be a great difference between what is paid for stumpage and what should be paid. This difference is added to the profit of the logger or lumberman (§ 168).

But actual values for stumpage, as established by sales of merchantable timber, do not fluctuate with the annual changes in lumber prices. Increased prices for lumber are not immediately followed by similar increases in stumpage prices, nor do the depressions in price, which are so characteristic of the lumber trade, make themselves felt at once in a lowered price for stumpage. Stumpage prices are sluggish rather than sensitive.

Three factors serve to explain this apparent failure of stumpage to behave as indicated by the economic relations set forth above. The first is the human element in price making (§§ 5-7). At best, it takes an expert thoroughly familiar with the costs and reasonable profits of such technical and diversified industries as logging and milling to determine the proper relations between prices f.o.b. at mill for lumber and prices for stumpage. The purchaser possesses this knowledge but the stumpage owner may not have it. Since it is to the purchaser's interest to keep the owner in ignorance, prices are allowed to remain as long as possible at the level set by previous sales. This condition is disturbed only when competition between two purchasers brings

prices up to their proper level to the benefit of the stumpage owners. The same condition is secured when timber is sold by a lumber company which is fully acquainted with its value.

The second factor is the controlling influence of transportation and logging costs. Logging alone frequently requires a large investment in railroads, flumes and equipment. It is often impossible for the owner of a small body of timber to market it, except by means of the facilities owned by a lumber company. The owner feels entitled to the stumpage value which results from these increased transportation facilities. The lumberman endeavors to secure the stumpage at the value which it had before his improvements were constructed. When he is the only purchaser, his price, rather than that desired by the owners, is usually determined upon as the value of the stumpage.

The third factor is the element of time. Present prices of finished lumber can under no circumstances determine present stumpage prices absolutely. In the first place they do not apply to present but to past stumpage values, and the value of present stumpage will depend upon the price obtained when its product, in turn, reaches the market (§ 11).

But the period required to bring the timber from the stump to the retailer, or even to the cars, is not the most important element of time involved, and neither the discount factor nor the speculative probability of a change in value for this short period would have much influence on stumpage prices. A more important fact is that large bodies of stumpage cannot be logged at one time. The operation must extend over several years. Therefore the prices finally received for this lumber will not be this year's or next year's prices, but an average of many years. Averages tend to eliminate annual fluctuations. Stumpage values do not fall when lumber prices decline because the owners of stumpage expect that before they have finished cutting, these lumber prices will have risen again.

171. Appraisals of Stumpage Value. — An appraisal of stumpage value requires:

(a) Acceptance of a definite class of products or stage of manufacture as furnishing a basis of prices from which to undertake the appraisal. The further removed this basis is from stumpage value, the greater will be the difficulty of correctly appraising the intervening elements of cost and profit. But the more stable and widely accepted are the prices which furnish this basis, the more reliable will be the result.

(b) Appraisal of expenses of the intervening operations as logging, or logging and milling, including depreciation of capital assets employed in these enterprises.

(c) Determination of legitimate profits for logging and milling, and of the proper basis and method of computing what such profits should be.

(d) Residual or stumpage value after subtracting elements *b* and *c* from *a*, and correcting for the error caused by overrun, in order to express value of stumpage in terms of log scale.

172. The Price Basis in Appraisals. — The acceptance of the retailer's price as the starting point for appraising stumpage values would interpose retail costs and profits as an element in the appraisal. This would result in *appraising* the value of lumber f.o.b. cars at mill, instead of accepting the current prices already determined for this lumber by sales; such a course is obviously undesirable.

Were the prices of logs standardized, and determined in open competitive market as are wholesale lumber prices, there would be a similar argument for accepting log prices instead of substituting an appraisal of the value of logs. This method has been used on Puget Sound where such a log market exists. Whenever logging is conducted as a separate business, and not merely by contract for the manufacturer, the price of logs *does* determine what may be paid for stumpage, and must be taken as the basis of appraisals.

It is claimed that in markets established for logs, prices for the logs are not as high as an equitable appraisal would indicate owing to the ability of the mill owners to keep these prices under control. Where the log market is the only outlet for the

logs, this fact would not justify an appraisal of stumpage based on higher log prices, unobtainable in practice, although it might justify holding rather than selling the timber.

But in the majority of cases, the business of logging is either conducted by the same firms which manufacture the timber, or the manufacturer purchases the stumpage, contracts the logging and thus bases the price which he can pay for the stumpage on the price of lumber at the mill. Stumpage appraisals must thus conform strictly to the prevalent economic conditions, and must be based on the value of the product put forth by the operator who purchases the timber, provided he conforms to general customs in the locality.

Mill prices for lumber fluctuate not only with the season and condition of the market, but are affected by freight charges. In certain localities, prices at large central markets for lumber may be taken as standard, and the proper discounts made for local freight rates.

The main source of variation in mill prices lies in the variety of products, and of grades of the same product, into which the timber is manufactured. This difference exists originally in the quality of the trees or logs, and is increased by the methods of handling the raw material or logs by the millman. One mill may, from the same material, produce a much greater per cent of the more valuable grades than another, whose principal output may be dimension lumber.

These differences in the quality of the grades produced vitally affect stumpage value and give to old timber a value at present prices which second growth timber may never obtain in the future because of inferior quality.

Two plans may be used in obtaining a price basis for the appraisal of a given body of stumpage. The first, advocated by the U. S. Forest Service in Washington and Oregon, is based on grade prices at a central market, discounted for freight rates to the required locality.

The first step is to estimate in the field the per cent of each of certain standard grades which will probably be yielded by the body of timber to be appraised. The average price of each

grade is obtained and a per cent of this price taken equal to the per cent of the total stand represented by the given grade. These fractional prices, when totalled, give an average price weighted both by quantity and value for the grades expected.*

The second plan depends upon "mill run" prices. In the absence of stable central markets, average prices for the total output of mills, termed mill run prices, are advocated. This method is defective for several reasons. The higher the average prices for lumber, the more profitable it becomes to manufacture low grades. In periods of depression these grades may be left to waste in the woods. An increased output of cheap grades, which always follows higher prices, may actually lower rather than raise the average mill run prices for lumber. Again, when prices are based on average mill run, the per cent of grades sawed may not coincide with those which can be produced from the timber to be appraised, either because the mill may have been cutting in timber better or poorer than the average, or, because the timber to be appraised may be better or poorer than the average. Mill run prices for timber known to be of

* The following figures, used in determining the average price of lumber in connection with a recent timber sale in Washington, were furnished by Asst. District Forester, F. E. Ames, U. S. Forest Service, and serve to illustrate this method.

DOUGLAS FIR GRADES, PRICES AND AVERAGE VALUE

No. 1 V. G. Flooring.....	3% @	\$26.00	\$0.78
No. 2 V. G. Flooring.....	5% @	23.00	1.15
No. 2 and Bet. F. G. Flooring.....	3% @	15.00	.45
No. 2 Clear and Bet. Finish.....	5% @	26.00	1.30
No. 2 Clear and Bet. Siding and Rustic..	4% @	17.00	.68
No. 3 Clear Flooring. Siding and Rustic	8% @	13.50	1.08
No. 1 Shop.....	4% @	16.00	.64
Car Sills.....	6% @	16.50	.99
Timbers.....	15% @	10.30	1.55
Dimension.....	28% @	9.10	2.55
Boards and Shiplap.....	12% @	8.75	1.05
Dunnage.....	7% @	6.00	.42
			<hr/> \$12.64

The grade prices represent the average of the quarterly averages from July 1, 1911 to July 1, 1914. Underweights not included.

similar quality to that appraised removes this objection. Average mill run prices for several mills in a region are serviceable only when the timber is remarkably uniform in quality and grade.

173. Depreciation. — Depreciation plays so much more important a part in stumpage appraisals than in forest production that it should be given special consideration in this chapter.

Depreciation is a sum written off or subtracted from the cost of capital assets, to correspond roughly or approximately with a shrinkage or disappearance of value through diminishing usefulness. An investment in a fixed or durable asset is not an "expense" at the time of purchase, but an exchange of cash for another form of property. The real expense is incidental to the use or service which this property yields, by reason of which it diminishes in value. Depreciation on property for which there exists a continuous need corresponds to actual deterioration and final discarding of the article as worn out. But where the period and amount of use is limited and the property will outlast its usefulness, the residual value will coincide with the value of the material of which it is composed, for removal or use in some other line. This shrinkage in value not accompanied by physical deterioration may be termed "amortization."

When depreciation is added to other expenses, the sum gives the total actual outlay or cost of the business, in which total the investment in capital assets subject to depreciation is excluded, as not constituting a cost.

A depreciation *fund* is a fund created by the setting aside annually, from net income, amounts intended to eventually equal the total depreciation (§ 73). When the period is known with reasonable certainty, the amounts annually needed to accomplish this purpose can be computed in advance.

Let A = original capital investment.
 W = final or wrecking value.
 D = total depreciation or $A - W$.
 n = years in period.

Four methods may be used in determining depreciation.

Method A. The total depreciation may be refunded by depositing each year in a savings bank, at p per cent, or investing and reinvesting, annual sums in such a way as to earn compound interest, so that the total of principal and interest at the end of the period equals the depreciation. This fund is not returned to the owners until the expiration of the period. Their capital investment remains at its original figure, and the annual dividends are declared on this basis.

Let X = annual sum for depreciation.

Then $\frac{X}{0.0p}(1.0p^n - 1) = D$, (See III.)

$$X = \frac{D(0.0p)}{1.0p^n - 1}. \quad (P)$$

This method is seldom, if ever, employed in connection with annual business, although it is practical as the basis of a sinking fund for the payment of bonds.

The other three methods neglect the factor of compound interest on the annual payments, and undertake to repay, through a series of years, sums whose cash total will just equal the total depreciation.

Method B. This method proposes to pay the total depreciation in equal annual instalments covering the requisite period. The formula is

$$X = \frac{D}{n} \quad \text{or} \quad \frac{A - W}{n}. \quad (P_1)$$

The method is used more widely than any other and is easily understood and applied. Actual return to investors of this annual appropriation for depreciation would have the result of diminishing the capital investment annually.

Method C. Instead of depreciating the capital by equal amounts annually, the depreciation may be made to equal a fixed per cent annually of the residual value of the capital; its value less previous depreciation. By this means, depre-

ciation expense is heaviest in the earlier years, and steadily diminishes.

Let z = per cent of depreciation, and
 $0.0z$ = per cent expressed decimally.

Then $A(1 - 0.0z)^n = W$,

$$0.0z = 1 - \sqrt[n]{\frac{W}{A}},$$

$$z = 100 \left(1 - \sqrt[n]{\frac{W}{A}} \right). \quad (P_2)$$

This method is seldom employed as it is inconvenient to write off large sums for depreciation in the earlier stages of the business, especially in lumbering.

Method D. This method proposes to increase the amount charged to depreciation annually, and combine it with an annual interest return on the diminished or residual capital, in such amounts that the sum of interest and depreciation will require equal annual payments. Depreciation, by reducing the invested capital, correspondingly lowers the interest returns required by a given rate of interest. This reduction in interest may be added annually to depreciation. It only remains to determine what annual sum in excess of interest on the original investment is required for the first payment, to secure this balance between the two elements of interest and depreciation, so that the sum of the increasing payments on account of depreciation will at the close of the period just equal the total amount to be written off.

In this case, either of two solutions is possible to determine the annual payment.

Let Z = annual payment for interest and depreciation.

Let $d = \frac{1.0p^n - 1}{0.0p}$, when p = rate of interest to be earned on capital A .

Then
$$Z = \frac{A (1.0p^n) - W}{d},^*$$
 (P₃)

or
$$Z = A (0.0p) + \frac{D}{d}.\dagger$$
 (P₄)

The first formula utilizes the wrecking value, while the second introduces total depreciation, and is somewhat simpler. The factor d is easily computed from Table VI, or, for values of p not given, by logarithms, in which the logarithm of $1.0p$ is multiplied by n to obtain the logarithm of $1.0p^n$.‡

* "Modern Accounting," by Henry Rand Hatfield, page 132.

$$\{ (A \cdot 1.0p - X) 1.0p - X \} 1.0p \dots \text{to } n \text{ terms} = W.$$

$$A (1.0p^n) - X (1 + 1.0p + 1.0p^2 \dots 1.0p^{n-1}) = W.$$

$$A (1.0p^n) - X \frac{1.0p^n - 1}{1.0p - 1} = W \text{ (by Formula Q}_1\text{).}$$

$$\begin{aligned} X &= \frac{A (1.0p^n) - W}{\frac{1.0p^n - 1}{0.0p}} \\ &= \frac{A (1.0p^n) - W}{d}. \end{aligned}$$

† Proposed by W. B. Hunter, formerly special examiner in U. S. Bureau of Corporations.

Let
$$X = A (0.0p) + e.$$

Then, in second year,

$$(A \cdot 0.0p - e) 0.0p = \text{interest on capital.}$$

$$e (0.0p) = \text{increase in depreciation.}$$

$$e + e (0.0p) = e (1.0p) = \text{depreciation.}$$

In third year,

$$(A \cdot 0.0p - e \cdot 1.0p) 0.0p = \text{interest.}$$

$$e (1.0p) \times 0.0p = \text{increase in depreciation.}$$

$$e (1.0p) - e (1.0p) \times 0.0p = e (1.0p^2), \text{ depreciation.}$$

$$D = e + e (1.0p) + e (1.0p^2) \dots 1.0p^{n-1}$$

$$= \frac{e (1.0p^n - 1)}{0.0p}$$

$$\frac{1.0p^n - 1}{0.0p} = d.$$

$$e = X - A (0.0p).$$

$$D = (X - A \cdot 0.0p) d.$$

$$X = A (0.0p) + \frac{D}{d}.$$

‡ The objections urged against this method are: first, that it allows interest on invested capital as a cost (Hatfield); second, that a plant depreciates more rapidly during the first few years than thereafter, and loss during that time would

174. Milling Costs and Profits. — The cost of milling or manufacture may be standardized for operations of a given size. The current expenses, or operating costs, do not vary greatly. For operations involving a definite quantity of stumpage, the duration of the operation is largely determined by the capacity of the mill.

The factor which indicates the most profitable size for a mill is the relation between the depreciation of fixed costs or capital expenditures for the plant and machinery, and the period and quantity of output over which this cost must be distributed. The value of the mill when there is no more timber to cut is practically nothing, and that of second-hand machinery, very small. The difference between these final values and initial cost, which represents depreciation, is one of the principal elements of expense (§ 173).

The size of this item of depreciation and the relation which it bears to the total output, and to the total cost of producing each unit of product, evidently depends more upon the total length of time over which the plant can continue to operate than upon its daily capacity. With mills of equal efficiency, the one with a twenty-year cut may be considered as having about half the depreciation expense per unit of output that one with but ten years' cut must write off.

Provided the supply of logs is assured, and the period over which to distribute the cost of depreciation can be approximated, the requisite *profits* in manufacturing may be gauged with reasonable uniformity. Uncertainty as to future timber supply makes it necessary to guard against a shortage or complete exhaustion of raw material by a correspondingly large appropriation from gross income for depreciation. Should this shortage not materialize, the concern would later find its

find the proprietors without adequate provision for meeting this loss. The first objection does not apply when the purpose of the computation is to determine *profits* in advance, since these are not treated as costs. The second objection holds good if it is considered necessary to make the amounts written off for depreciation agree closely with actual loss in value. The results obtained by this method agree more closely with those of Method A than with Method B or C.

financial position and average profits correspondingly, and justly, increased by these earlier sacrifices.

The costs and profits of manufacture are both included in contract prices for sawing or manufacturing products, which is the basis of operations of many portable saw mills.

175. Logging Costs and Profits. — It is in the logging, rather than the milling end of lumbering, that the greatest variations in costs occur. The cost of felling and bucking the trees into logs, ties or poles may be standardized. But the cost of transportation of the product from the site to the mill is determined by a number of exceedingly variable factors, and is the principal part of logging.

These factors are: accessibility and quantity of stumpage, and methods of transportation adapted to climate, to the topography and to the size of the timber. The direct relation of total quantity or volume upon the cost per unit of the combined expense of constructing the transportation routes and moving the product needs to be emphasized (§ 98), and explains the great differences in value of timber of equal quality but differing merely in quantity available.

Accessibility is a relative condition depending upon the total cost per unit of product to bring the timber to the mill, and the margin of profit remaining for distribution between the owner of stumpage and the logger. When the logger sees so small a margin that he demands it all, the timber is for the present inaccessible. Accessibility depends, then, upon the total volume available as well as upon the location of the site and the difficulties and costs of transportation intervening between site and mill.

Methods of transportation exercise a profound influence upon logging costs. The factor of climate exerts its chief influence upon the bottom, making ice roads possible in some regions, while in others rain greatly increases the difficulties by softening the soil. The driving of streams, supplemented by the sleigh haul, may be superseded by rail transportation involving an investment of an entirely different character. For short distances, in regions of second growth, wagon haul is the chief

dependence of the operator. The overhead cable to supplement the railroad may prove cheapest, as in cypress swamps or over broken gorges, while elsewhere, skidding by ground cables or with animals is most efficient.

In appraisals, average logging costs for the specific body of timber, based on the methods accepted by experience in that region as best and cheapest, will be the basis of costs. Excessive costs due to deliberate use of methods shown to be antiquated or expensive, when the operator is in position to use better methods, or when others can be found who will do so, cannot be permitted to influence or increase the factor of logging costs. An additional cost is incurred in logging under forestry principles (§§ 107 and 176).

The profits in logging are less easily gauged than in milling. The great variation in conditions, which makes each logging job or "chance" a separate venture, prevents the standardization of methods and increases the uncertainty of the total estimated cost, and consequently of the risk of loss. Added to this is the risk from natural agencies, as floods, fire, drought, lack of cold weather, too much snow or any other variation from normal which either directly damages the timber or the means of transportation, or unduly increases the labor and cost of logging.

Logging work is less closely supervised from the main office than is milling. Hence woods bosses are more largely thrown on their own resources. The main attributes of woods foremen even now is brute force and practical experience. This type of boss lacks ability to apply methods of scientific efficiency. The average risk in logging is probably double that incurred in manufacture.

Depreciation as an item of expense in logging chiefly concerns the means of transportation. The entire initial cost for temporary railroads, flumes or roads, which will be abandoned on completion of the operation, is depreciated or charged off as a cost of logging. The costs for railroads, or roads which will afterwards be used for permanent traffic, will be maintenance and the difference in value of the road at the beginning and end of the operation. A logging company which subsequently sells

the road to a railroad company for continuous operation may secure a price equal to the cost of the road, in which case no depreciation is incurred. Such a result cannot be assumed, and depreciation must always be charged against the transportation system.

The cost of all logging equipment, such as cables, engines, horses, and other supplies, should be reckoned not in the form of a capital expenditure but as depreciation.

176. Stumpage as a Capital Investment and as a Cost of Raw Materials. — Methods of appraising the value of stumpage on the basis of the present net residue, after subtracting logging and manufacturing costs and profits, are based on the supposition that this timber will be removed within a short time, not exceeding three to five years. Such values are applicable to short-term contracts or to small bodies of timber, and to purchases in which land is not included. But the purchase for cash of large bodies of stumpage introduces an additional element of cost which must be considered by the purchaser. The time which must elapse previous to the actual completion of cutting necessitates the holding of this stumpage, with accompanying annual expenses (§ 170). This would apparently lower the price the purchaser can pay, in the same manner that the absence of a market at present, even if it is sure to develop later, depresses present values. When transportation facilities, mills and markets are ready, the price will be correspondingly higher than in regions where development must follow purchase.

It is not possible, however, to arrive at a proper appraised value for large areas of standing timber by the method of computing the value for immediate cutting, and then discounting this value for the average period which the timber must stand.

In spite of the impossibility of immediate cutting, the timber will usually sell for the value shown *without* such discount. This is explained by the element of increasing prices, to which may be added growth of timber and improvement in quality. It would be unfair to discount a future stumpage value based on present sales, when everyone, purchaser and owner alike,

expects stumpage to increase in value (§ 188). Were it definitely known that no such increase would occur, the expectation value of the revenue from cutting the timber would be less than the present sale value for immediate cutting. As it is, the expectation value of the stumpage is based on this probable increase and its present sale value is not depressed by the postponement of cutting. Again, the great advantages accompanying the ownership of large bodies of timber and the reduction of depreciation costs (§ 173) and transportation charges (§ 174) in themselves more than offset such possible discount in value.

The value of stumpage for immediate purchase as a capital investment may therefore be appraised on the basis of what it is worth on the completion of the necessary transportation and milling facilities, or, if these are assured, it may safely be based on present value.

For stumpage which is paid for in small instalments just previous to or following the cutting of the timber, the value is manifestly that existing at the time of payment. On long-term contracts, as a result of such postponed payments, a readjustment of stumpage values is required at reasonable intervals, on account of the expected increase in future prices of lumber. This method of payment, by which the operator is relieved not only of the investment with its attendant annual expense, but also of the risk, and of any losses which may result from fire or any other agency, puts stumpage in the same class as that of raw materials, rather than capital. The attendant advantages to the purchaser, corresponding to the prospective loss and risk of holding timber, are recognized as having a value great enough to form a material inducement in sales of timber on National Forests. It is usually sufficient to offset the additional cost of brush burning and other silvicultural measures required by the government.

177. The Determination of Legitimate Profits. Overturn Methods. — Costs must be gauged in relation to the quantity of product resulting from a given expenditure, and a cost account seeks to determine the cost and the resulting profit on each unit (§ 25). All factors of cost, including depreciation, may be

expressed on the basis of a thousand board feet, cord or other unit of product.

The surplus after subtracting costs from the average price of the product is the net profit, from which is paid the returns on the invested capital and the enterpriser's gain for services (§ 39). If the cost of stumpage is fixed, this, added to other costs, determines the logger's profit. In appraisals of stumpage, on the other hand, the net surplus after paying all other expenses must be divided between stumpage and profit, and the determination of legitimate profit must precede that of stumpage value.

On small ventures, where the enterpriser depends almost entirely on his individual efforts and the amount of capital invested is insignificant, most of the annual costs are met from income, depreciation is a small item, and profit is usually gauged by the margin of income over costs for each unit. Logging contractors are satisfied if they realize, say, \$1 profit per thousand board feet log scale above their computed expenses. Profits may always be expressed in this manner, whether or not the calculation stops at this point. This has been termed the overturn method of expressing profits, since the working capital used for wages and other current expenses is turned over or used once for each unit of output.

The contractor bases his profit on costs of logging, exclusive of stumpage.

Let Lc = logging costs.
 Mc = milling costs.
 D = depreciation costs $\begin{cases} LD \text{ for logging,} \\ MD \text{ for milling.} \end{cases}$
 S = sale value of lumber or of logs.
 P = profit.
 Y_m = stumpage value of 1000 board feet of
 manufactured lumber.
 $q\%$ = legitimate profit.

Then $P = q\% (Lc + D).$
 $Y_m = S - (Lc + D) - q\% (Lc + D).$

That is, if the price for logs is taken as the basis of prices, and the logger computes his requisite profit roughly as a per cent of average logging costs, stumpage value is the residue after subtracting costs and profit from sale prices.

In the same way, milling costs and profit may be subtracted, and (Method A)

$$Y_m = S - (Lc + Mc + D) - q\% (Lc + Mc + D). \quad (R)$$

Should milling require a different profit from logging, these costs and profits may be separated, using r per cent for profit in milling.

$$Y_m = S - (Mc + MD + Lc + LD) - r\% (Mc + MD) - q\% (Lc + LD). \quad (R_1)$$

The computation of profits as a per cent of operating costs, if based on the results of individual operations, would place a premium on extravagance. But when gauged by average costs of well-conducted operations of similar size, and applied in determining appraisals in advance of cutting, no such objection can be urged.

Method B assumes that stumpage is a part of the total operating costs, and that the money invested in it is entitled to share in the profit. By thus adding stumpage to the sum of the other expenses, the total of costs plus profit equals gross income, or sale value. Profit thus becomes a fixed per cent of this sale value, and is independent of operating costs. Any increase in operating expense of logging is subtracted from cost of stumpage, and vice versa.

The formula is:

$$Y_m = S - (Lc + Mc + D) - q\% (Lc + Mc + D + Y_m),$$

which reduces to

$$Y_m = \frac{S}{1 + q\%} - (Lc + Mc + D). \quad (R_2)$$

This formula was formerly advocated by the Forest Service for use in small sales where but little capital is invested, but has been set aside in favor of Method A. The profit $q\%$ should be exclusive of a salary of \$1200 to \$1500 for the operator (§ 34).

Method C is to share the profit or margin between the owner of stumpage and the operator. Such a plan requires complete confidence in the integrity of the operator or a right to inspect his books, and is not generally practiced, though one of the most equitable of methods.

It is the only one of the three methods proposed above, which can be based on the actual costs of a given operation, with safety to both parties. Letting q per cent equal the per cent of net income taken for stumpage value, the formula is

$$Y_m = q\% \{S - (Lc + Mc + D)\}. \quad (R_3)$$

In the following diagram the basis and results of these three methods is shown, and the effect of an increase of operating costs upon profits and stumpage values.

DIAGRAM III

COMPARISON OF COSTS, PROFITS AND STUMPAGE VALUE BY OVERTURN METHODS

Sale Value 17.00			
Method A	Costs 11.00	Profit 2.20	Stumpage 3.80
	Costs 13.00	Profit 2.60	Stumpage 1.40
	Profit 20% of Costs		
Method B	Costs 11.00	Stumpage 3.45	Profit 2.55
	Costs 13.00	Stumpage 1.45	Profit 2.55
	Profit 15% of Sale Value		
Method C	Costs 11.00	Profit 3.00	Stumpage 3.00
	Costs 13.00	Profit 2.00	Stumpage 2.00
	Profit 50% of Net Income		

Operating costs, profits and stumpage value based on 1 M board feet of lumber, scaled in the log.

178. The Investment Basis for Profits. — Although profit may be expressed in terms of per cent of cost, or of sale value for a unit of product, it can be correctly gauged only as a return on invested capital in which is included the enterpriser's gain (§§ 32, 33). The knowledge that a profit of \$1 per thousand feet has been earned does not suffice as a basis for dividends. The reward for personal services, which is so large an element in small undertakings, disappears with large enterprises, and net income

or profits must be converted into annual dividends earned on invested capital.

The capital upon which this income is earned is the entire amount of fixed and working capital employed in the business, including that which is borrowed. It is of no significance that working capital is turned over several times in a season, as is done in railroad logging, except as it increases the total net income which is the product of the profit per thousand feet multiplied by the year's output. This income is not the product solely of the working capital, but of the total investment.

Interest on borrowed capital is paid out of this net income, and it is immaterial whether the capital entitled to share in the income is taken as the entire investment, or as the portion owned by the proprietors, excepting that the latter are entitled to a larger rate of profit in proportion to the lessening of their per cent of total capital and corresponding increase of risk (§ 32). In stumpage appraisals it is advisable to compute the profit on the total investment, thus eliminating the purely personal element as to what proportion of the capital is borrowed.

It must be self-evident that interest is not a "cost" in stumpage appraisals, but it may be so regarded without harm, provided it is not added to operating costs in Methods A, B and C (§ 177), and is subtracted from income allowed by the investment method. To permit profits to be earned on income represented by interest on owner's capital, results in serious inflation of profits and resultant heavy losses in stumpage values (§ 42).

The main difficulty in computing profits in the form of a return on the average annual investment arises from the element of depreciation, and its effect in reducing the capital invested year by year.

It is possible to earn annually about the same net surplus, available for depreciation and profits. To base the amount of these profits upon the maximum investment, which occurs normally in the first or earlier years, results in a steadily increased *rate* of profit on the capital remaining invested in subsequent years, and an average rate considerably greater than intended, with corresponding loss in stumpage value.

By the use of either of the formulæ given in Article 175, Method D (P_3 , P_4), a fixed rate of profit may be found, which will apply to the residual capital actually invested annually. This is accomplished through the increasing payments for depreciation. The sum of interest and depreciation, or Z , calls for equal income annually. This conforms with the financial conditions which can normally be secured in lumbering. To the owners, it makes very little actual difference whether their annual income is termed dividends or depreciation. They expect to allow the larger portion of their capital to remain invested throughout the life of the enterprise, and this result is accomplished by increasing depreciation from small beginnings. Apparently, there is no valid objection to the adoption of this method of determining the average per cent of profit to be allowed on invested capital.

Should the method of allowing equal annual amounts for depreciation be adopted (§ 175, Method B) the average amount of capital invested during the prospective life of the enterprise must be determined. The investment for each year is taken at the beginning of the year. The average for the period is then equal to the residual or final investment (W) plus one-half the total depreciation, plus one-half the annual depreciation. For an investment of \$20,000, with a residual value of \$10,000, over a period of five years, the values at the beginning of each year are

First year.....	\$20,000
Second year.....	18,000
Third year.....	16,000
Fourth year.....	14,000
Fifth year.....	12,000
Average investment.....	\$16,000

which equals

Residual investment.....	\$10,000
One-half total depreciation.....	5,000
One-half annual depreciation.....	<u>1,000</u>
Average investment.....	\$16,000

This amount would be taken as the basis for computing the per cent of annual profit to be allowed.

179. Overrun. — When the amount and per cent of profit have been determined, the remainder, or stumpage value, is a definite proportion or per cent of the sale value of the manufactured products, and equals a stated price per thousand board feet. Should stumpage appraisal indicate a value equaling 20 per cent of the wholesale price or mill price of lumber worth \$20 per thousand, the price of stumpage will be \$4 per thousand feet of sawed lumber.

But this does not indicate that the true price of stumpage is \$4. It is probably considerably more. The contents of logs, and not the measured products, is the basis of payment, not only when the timber is bought on the stump, but when it is paid for after being felled and measured.

Only in case the logs are measured by a rule which will give the actual sawed contents, will the appraised stumpage value per thousand feet be the same when measured by log scale and by sawed contents. All log scales in extensive commercial use give less than the contents ordinarily sawed from logs, and this discrepancy increases as utilization becomes closer. The excess of manufactured lumber over the contents measured by the log scale is termed overrun.

With certain log rules, notably the Doyle and Doyle-Scribner, this excess amounts to 25 per cent and over, and increases with small timber.

Should the overrun be 25 per cent, it indicates that for timber worth \$4 per thousand, each thousand feet log scale produces 1.25 thousand feet of lumber. The value of the stumpage is therefore $1.25 \times \$4$, or \$5 per thousand.

If

Y_m = stumpage value per thousand feet of sawed lumber,

s = per cent of overrun per thousand feet of log scale,

Y = resulting stumpage value per thousand feet log scale,

then

$$Y = Y_m \times 1.0s.$$

180. Stumpage Values for Different Species in Mixed Stands.

— In a mixed stand containing two or more species, the sale value of the products of these species usually differs considerably. Often only one valuable species will be removed from a mixed stand. As values increase, other species become profitable, and finally, all may be taken.

At a point just before the inferior species become sufficiently valuable to be logged, the operator is under the necessity of constructing his transportation system, and incurring his entire cost for equipment and overhead expenses, in order to log a portion of the stand, thus moving a smaller volume of lumber, at relatively greater expense per unit.

Assuming that the total costs, including depreciation, amount to \$12 per thousand board feet, and the most valuable species is worth \$20, the margin for profit and stumpage is \$8. A species worth but \$12 on the market could apparently not be marketed at all. But there is very little additional investment involved in logging this second species. By doing so, the total average cost of logging may be reduced, by lessening the transportation expense per unit, the average of overhead charges, and the depreciation, to \$10 per thousand feet for the entire operation. This adds a profit of \$2 to the more valuable species, and leaves a margin of \$2 for the inferior one, for profit and stumpage.

But such a cost distribution is unfair to the inferior species, since the operation does not depend on its presence and the capital expenditures must be incurred without its inclusion. It has been suggested that the more valuable species or group be made to bear both the entire cost of depreciation, which represents the capital expenditures, and the overhead charges. The inferior group is left to bear merely its proportion of operating costs. This plan is rather crude where there are several species, all of different values.

The following method is quoted verbatim from the Manual of Stumpage Appraisals, U. S. Forest Service, November, 1914. This plan offers an equitable solution of the problem.

“The most satisfactory method is to pro-rate the gross annual

depreciation and profit over the difference between operating cost and selling price, for each species, in the quantities entering into the annual cut.

To illustrate:

A yearly cut is made up of 4 million feet of sugar pine, 3 million feet of yellow pine, and 2 million feet of white fir (giving a total of 9 million feet).^{*} The margins between selling prices and costs of production, exclusive of depreciation and profit, are:

Species.	Selling price. *	Operating cost.	Margin.
Sugar pine.....	\$20.00	\$10.00	\$10.00
Yellow pine.....	18.00	10.00	8.00
White fir.....	(16.00)	(10.00)	6.00

The total net value, or sum of the margins over which depreciation and profit may be pro-rated, is thus:

Sugar pine.....	\$10.00 × 4000 M =	\$40,000
Yellow pine.....	8.00 × 3000 M =	24,000
White fir.....	6.00 × 2000 M =	12,000
Total.....		\$76,000

The annual sum of depreciation and profit (investment method) which must be paid out of this total has been computed as \$34,200. Hence $\frac{\$34,200}{76,000} = \0.45 . That is, every dollar of margin between operating costs and selling price must pay 45 cents toward profit and depreciation. Put in quantitative terms, by species, we have the following charges per *M* feet for depreciation and profit:

Sugar pine.....	10 × \$0.45 =	\$4.50
Yellow pine.....	8 × 0.45 =	3.60
White fir.....	6 × 0.45 =	2.70

By this method inferior species which yield no margin between operating costs and selling price, or a negative margin, but which must be included in the sale for silvicultural reasons,

* The bracketed material does not appear in the original quotation and has been inserted as additional explanation.

are automatically relieved of profit and depreciation, and the charge upon the other timbers for these items proportionately increased.

The same result is readily obtained on a thousand foot basis, using the per cents of the different species in the cut. That is, to obtain the *average* margin (of sale value above operating costs), multiply the margin for each species by the per cent of this species in total cost:

45 per cent sugar pine at \$10.00.....	\$4.50
33 per cent yellow pine at \$8.00.....	2.64
22 per cent white fir at \$6.00.....	1.32
Total.....	\$8.46

Depreciation and profit per *M* feet $\left(\frac{\$34,200}{9000} \right)$ \$3.80.

$\frac{\$3.80}{8.46} = \0.45 , to be taken from each dollar of margin for these items.

This method of adjusting the prices of the more and less valuable species is believed to accord with customary business practice. Volume of money handled, rather than quantity of this or that product, is the usual basis for figuring carrying charges, depreciation and returns. In logging, improvements are frequently constructed primarily to take out certain valuable species. Inferior timbers may be cut or left as the market warrants. In such cases operators will usually cut inferior species if a profit can be netted over bare operating costs, figuring that the cost of improvements is borne wholly by the better stuff. The foregoing is believed to be a logical and rational application of this principle."

For a mixed stand, the method proposed is, therefore, to compute the operating costs on the basis of the total amount to be logged, including inferior species. The profit and depreciation are then determined as a lump sum, sufficient to pay the required per cent on the total investments. Finally, this sum is charged as an additional "cost" against the sale value prorated as outlined above between the different species. The

remaining margin between the sale value of a species and these total charges is its appraised stumpage value.

181. Carrying Charges versus Profits, on Stumpage. — In Chapter X (§ 158), it was demonstrated that a forest capable of producing annual income was also capable of paying increased taxes, for the reason that compound interest did not accumulate on such a forest. The owner may, for his own information, and to gauge the advisability of reinvesting income, compute the interest on his investment for the interval elapsing between cost and the income directly resulting from such cost. But the total cash investment in the business does not increase by reason of this interest. Neither the unearned interest, nor the actual cash expenses for taxes and protection serve to increase the capital cost of stumpage, as long as sufficient income is secured each year from some portion of the forest or business to cancel these costs.

There is the same difference between tracts of mature stumpage held for speculation without cutting, and tracts on which an operation is conducted, as there is between an even-aged and an all-aged forest, Cases *A* and *C* (§ 158). The first case is paralleled by the young forest; the second, by the forest already capable of revenue. For timber, or that portion of the timber holdings which cannot be cut for from 20 to 40 years, a separate cost account based on economic factors (§ 106) would show identical costs, whether or not the specific stand is a part of a large holding which produces annual revenue, or is part of a speculative holding producing no income. But there is an enormous specific difference in the financial status of the owners of the respective tracts, which has an important bearing on the problem of carrying charges and increased stumpage values.

The non-productive speculative holding requires an increase in stumpage values sufficient to ultimately offset all annual expenses, and earn compound interest on the entire investment, up to the year when cutting begins, in order to show a profit sufficiently large to include an enterpriser's gain (§ 33).

The productive holding, which is being operated, and is producing annual income at the present time, will under certain

conditions continue to pay a profit as great as it pays now, if stumpage does not increase in value, or, rather, if the margin of prices over cost of operation does not increase in per cent (§ 187). Were the forest itself composed of stands of all ages, which are cut upon reaching maturity, this condition would actually occur, and advancing prices for stumpage would simply increase the profits.

The conditions confronting the holder of stumpage, who is also an operator, partake of both of these extremes or lie between them. The total investment in stumpage is not maintained, except through extending the area by purchase, but diminishes constantly by cutting, with a corresponding lightening of the annual carrying charges, and a reduction of the capital investment (§ 175).

It is hardly to be expected that a company planning extensive operations over a long period, will inaugurate a scheme of cutting so comprehensive that they will be enabled to earn their carrying charges for the first few years, and at the same time pay dividends on the total capital investment. The actual annual outlay may absorb every cent of annual income for a while. When interest on bonds is added to other carrying charges the burden in the earlier years is still heavier, and it may require additions to capital to meet these charges. Some companies owning extensive timber properties have lost this timber by foreclosure, because of non-payment of interest on bonds through inability to obtain this additional capital.

With reduction in interest on bonds, in taxes and in other expenses, a company comes sooner or later to a condition permitting of the payment of dividends. Up to that time, it might be assumed that the value of stumpage must increase, in order that the dividends when received shall be large enough to compensate for their absence in earlier years. From then on, with both the annual expenses and dividends paid from the annual net income, an increase in stumpage value or in net income means a corresponding increase in surplus and dividend rates. When a company is able, by operating, to secure a sufficient net income to meet all expenses and pay dividends from the

start, stumpage value does not need to increase beyond the amount needed to meet any increased costs, in order to maintain the dividend rate. Furthermore, this rate is certain to increase, on account of reduced carrying charges, even if the value of stumpage remains stationary.

The claim that the value of stumpage *must* increase at a rate sufficient to return compound interest on all costs, or the result will be financial loss or bankruptcy, must therefore be flatly rejected except for purely speculative holdings, or for operators attempting to carry far larger holdings than they can take care of with their own capital. An owner of stumpage must either have sufficient capital to carry all charges for an indefinite period, or he must operate and obtain revenue annually. In either case, if he expands his operations by borrowing on the security of the timber, he must conduct such a transaction with considerably greater caution and conservatism than is necessary in many other forms of business, or he may lose his holdings. For a concern which is earning annual dividends, increased costs, especially taxation, is the only factor which will necessitate the realization of higher values in order to maintain profits. Very strong combinations of capital holding timberlands and operating, may, by a policy of cutting calculated to cancel carrying charges, put themselves annually in a better financial condition, until, after the period of pressure and over-production has passed, they will reap relatively large profits, representing more than average dividends on outstanding capital investments.

This distinction between the separation of a business into parcels, and the treatment of the business as a unit, is recognized by the U. S. Commissioner of Internal Revenue in interpreting the income tax law as applied to corporations dealing in timberlands. A cost account is to be kept for each parcel of land as the basis for determining net income (§ 25), and on sale of this parcel the value received can be reduced by the amount of the original cost of the parcel, but cannot be absorbed by applying it to cancel capital cost for any other parcel. On the other hand, net *income* is treated as applying, not to the parcel,

but to the entire business of the concern. If actual cash expenses in any year exceed actual income, such expenses can be added to capital cost, and pro-rated among the parcels. The net income, after subtracting capital cost of the parcel sold, may be used, as far as needed, to cancel actual outlay on the entire tract for the year, the surplus income remaining after this total expense is met being the only portion subject to the federal income tax.

CHAPTER XII

FUTURE VALUE OF FOREST PRODUCTS

182. Factors Affecting Future Value. — Since the value of forest property is derived from income, an appraisal of the future value of forest products must precede that of the property itself and forms the basis for this appraisal. The examiner must base his conclusions, first, on the character and present condition of the property and its probable productiveness, and second, upon the factors which will affect the value of the products secured. The elements demanding consideration are:

The Stand.

- Growth in volume.
- Improvement in quality of products.
- Deterioration of standing timber.
- Closeness of utilization.

Price Levels.

- General price changes.
- Changes in prices for forest products.
- Effects of substitutes for wood.

Future Operating Costs.

Local Factors.

- Future transportation facilities.
- Industries and markets.
- Future supply of timber.

183. The Stand; Growth in Volume. — The future growth in volume of a stand of timber may be predicted for short periods by measurements of past growth on average trees. For periods exceeding a decade this method becomes unreliable by reason of the progressive loss in numbers and altered rate of growth of single trees which takes place in all stands as they increase

in age. Yield tables attempt to state the actual results in volume per acre, produced by stands at different ages. When applied to young timber growing upon sites of the same quality, and of approximately equal density or degree of stocking, the growth and future volume of such stands is predicted with sufficient accuracy. For scattered timber or poorly stocked stands, a reducing factor is required to prevent over-estimates. Methods of studying and predicting yields per acre form part of the subject of forest mensuration. Tables of growth and yield are published by the U. S. Forest Service and in various state bulletins, but are still very deficient in number and reliability.

184. The Stand; Improvement in Quality of Products. — Volume alone does not measure the increase in value of a stand of timber. As the trees become larger, the quality of the products which may be cut increases, for two reasons. The lower branches on young trees form knots as long as they remain attached, either alive or dead. After the branch falls and the scar heals, the wood laid on is free from knots, or clear, and produces the most valuable grades. The slabs contain the greatest per cent of clear lumber of any portion of the tree, and old trees contain enormously greater per cents of high-grade products than young timber. The second factor of increase in quality is due to the larger sizes which can be cut from older trees. Both in length and breadth, higher prices are paid for increased dimensions.

Both volume and quality increase the value per acre entirely independent of changes in prices due to economic conditions outside the stand.

185. Deterioration of Standing Timber. — After timber reaches a certain age and condition, the stand as a whole ceases to increase in volume, and at a much later period it begins to diminish in quality as well. The trees continue to grow as long as they live, but the total growth of the stand is offset by the loss of trees through windfall, insects and disease.

The remaining trees are putting on clear lumber, but in old stands, decay caused by fungi becomes increasingly prevalent, and finally offsets this increase in quality.

In Washington it was shown that Douglas fir stands yielding 65 thousand board feet per acre at their maximum, deteriorate to 25 thousand feet in 100 years, an average annual loss of 400 board feet in volume alone. Similar losses are occurring in all over-mature stands regardless of species or locality.

186. Closeness of Utilization. — Present volume of standing timber is expressed in terms of the quantities actually utilized rather than the cubic contents of the stand. A future increase in the closeness of utilization thus has the effect of increasing the apparent volume of the stand. Its effect on future income is of similar importance to that of actual growth in volume and quality. In regions where markets for the poorer grades are uncertain, these grades are seldom manufactured or included in an estimate (§ 172). The future increase in merchantable volume from closer utilization alone is a large factor.

In addition to the gain in volume per tree the total merchantable yield of the stand is increased by removal of smaller trees and of species formerly unprofitable.

187. Price Levels; General Price Changes. — General changes in prices, caused by fluctuations in the value of money (§ 3) would raise the cost of logging and milling in equal ratio to the value of the product. When the ratio of profit to costs remains the same, stumpage values will increase at the same rate or per cent as will prices for lumber. For instance, should lumber, now worth \$15, increase to \$20, or $33\frac{1}{3}$ per cent, stumpage value worth at present \$3 should then be worth \$4. In this case costs might be assumed originally as \$9.60 and profit \$2.40, or 25 per cent of cost. An advance of $33\frac{1}{3}$ per cent in prices of supplies and labor would bring costs up to \$12.80, and 25 per cent profit would equal \$3.20, or a total of \$16, thus leaving the required margin of \$4 for stumpage.

Prof. Irving Fisher predicts an average annual increase in general prices of 2 per cent per year for the next fifteen years.*

188. Price Levels; Changes in Prices of Forest Products. — The prices for a given product will depart from the average

* *American Economic Review*, Sept., 1912, Vol. II, No. 3, p. 553. Managing Editor, Prof. Davis R. Dewey, 491 Boylston St., Boston.

price level when the demand for the specific goods increases relatively faster than the supply. Both demand and supply may be increasing, or both decreasing, or demand may increase while supply diminishes. In any case, the effect on prices is gauged by the difference rather than by the direction of these forces.

When population is increasing, while the area occupied by timber tends to decrease, the maintenance of the same per capita rate of consumption for timber causes demand to progressively outrun supply; and this factor has resulted in the past two decades in an increase in lumber prices approximately twice as rapid as the movement of prices in general. This increase has had a pronounced effect on stumpage values. Instead of increasing by the same per cent as the price of lumber, the value of stumpage tends to absorb the entire surplus not taken by increased costs (§ 170). The profits of the logger and millman tend to maintain a fairly constant ratio to cost of operation, except where, by retarding the advance in stumpage values, they are able to appropriate a larger share of this increase. In the above illustration, the price of lumber can be assumed to have increased from \$15 to \$25, or $66\frac{2}{3}$ per cent, while general prices and the cost of logging advance but $33\frac{1}{3}$ per cent. If a profit of 25 per cent on operating costs is still considered satisfactory, these costs advance only to \$16, and the margin of value for stumpage becomes \$9, an increase of 200 per cent.

In regions where timber was formerly barely accessible (§ 174) this increase in value sometimes amounts to several hundred per cent on cost. Stumpage in general doubled and trebled in value in most regions of large supplies in the decade of 1898-1908.

189. Effects of Substitutes on Wood Prices. — Increasing prices for lumber and other wood products act in two ways to diminish consumption of wood. Less wood is used merely because it costs more and the enterprises depending on wood are curtailed; and substitutes appear, made possible by this same increase in price.

The extent to which substitutes will succeed in permanently displacing wood as a raw material depends upon three factors:

The relative adaptability of wood and its substitutes for the services demanded.

The relative cost, measured in first cost of raw material, labor of construction and durability or cost of depreciation.

The habits and customs of consumers as influenced by advertising and imitation.

When no substitutes can be found which will give the kind of service demanded, the use of wood will be continued, at constantly advancing prices, until the supply is completely exhausted, when the need for which this wood was used will remain unsatisfied. But ordinarily, high prices will force the use of some substitute which, while not so satisfactory, is so much cheaper that it is accepted.

Formerly wood was used for many purposes because it was cheaper than substitutes. Board fences and sidewalks have been replaced by wire and cement, since the combined cost of labor, material and depreciation is less for the latter materials and they give better satisfaction.

Wood for buildings has until recently been much cheaper than brick or stone, there being a saving in labor and material. Old, well-constructed wooden houses depreciated slowly. At present it is still cheaper to build wooden houses, but there is less difference in the cost of materials, while the poor quality of much of the lumber used increases depreciation in favor of brick, cement or stone. A further element of cost is the insurance of wooden buildings demanded by the extra fire risk.

The use of wood for boxes and crates has been considerably lessened by substitutes in the form of fibre and cardboard, which are cheaper and fully as satisfactory for many purposes. Shingles have been extensively replaced by patent roofing which is cheaper in both labor and material, although possibly not as durable.

The effect of advertising and habit is especially noticeable in the last two instances. Manufacturers of patent roofing and of cheap boxes have expended large sums to capture the trade and educate consumers into using their products. Such changes

are made with reluctance and suspicion, but once undertaken, the imitative faculty, which plays so strong a part in human affairs, may cause a very rapid extension of the substitution and force producers of lumber to fight for the retention of trade formerly theirs by right of original possession.

In spite of the inroads which substitutes have made on consumption of wood as the result of high prices and trade developments, there remain many uses for which wood is so pre-eminently adapted, that the total consumption is almost certain to increase in the future rather than to diminish. For railroad ties, the resilience or elasticity of wood, which absorbs the shock of traffic, combined with the small labor cost of track repairing, and increased safety of travel, when compared with the substitutes so far employed, make it certain that the price of railroad ties will be much higher than at present before the railroads discontinue the use of the wooden tie.

A still more striking case is the use of wood for paper pulp. Formerly confined largely to spruce and poplar, pulp is now made from many hardwoods and from southern longleaf pine. Paper can be made from many other materials, such as rags or corn stalks, but the cost of assembling sufficient quantities at the mill is greatly in favor of wood.

The general effect of substitutes is to diminish the use of cheap grades of wood, and to increase the supply available for more important uses. This tendency is illustrated in the case of fuel. Wood for domestic fuel is still used in enormous quantities, but only at points near the source of supply. Its bulk and weight prevent its transportation by freight for long distances to large centers of consumption. It is probably worth as much, if not more, for this purpose in most localities than it ever was. It would have been impossible for our forests to supply enough fuel for the entire demands of modern industry. The substitution of coal has relegated wood to the position of a luxury for the fireplace in most large cities; has ruined the markets for cordwood formerly existing in connection with certain manufacturing enterprises, and in some regions near such factories has caused the value of woodlands to decline. Yet the

general effect of this substitution, where the market has been spoiled for cordwood, is to encourage the growth of timber of larger dimensions, suitable for ties, telegraph poles and lumber, from which the owner may realize eventually as large a profit as from his cordwood. Substitutes thus tend to bring about longer rotations, greater diversity of wood products, higher values per acre, and increased usefulness of each acre of forest, and do not, in general, interfere seriously or permanently with the industry of wood production. Radical changes in the management of forests may become necessary by these economic changes in utilization and markets, and the rate of interest earned in private forestry may be diminished by the necessity of longer rotations and more specialized products. New uses for wood spring up to take the place of former demands now obsolete.

The use of substitutes will undoubtedly prevent wood prices from rising as rapidly as in the decade 1898-1908 and probably explains in some measure the slower increase which has occurred since that date. But substitutes will never do away with the use and demand for wood, nor cause the average value of wood products and timber stumpage to be lowered to any appreciable extent. Prices are still far below the level established in Europe, and it is reasonable to expect that these prices will continue to increase slightly faster than those for other goods, for some time to come.

190. Future Operating Costs. — Future costs of logging and milling will increase approximately at the same rate as general prices. Added to this are the special factors which may cause operating costs to vary from this general trend.

The cost of labor is the largest item in logging. Throughout northern regions the effectiveness of woods labor is rapidly diminishing. Old-time American lumberjacks are very rare, and a large part of the work is performed by inexperienced foreigners, or shifting labor from the city slums, resulting in an increased cost considerably greater than can be accounted for by increased wages. This tendency is offset by improved efficiency, as operators learn from experience the use of the best technique.

The recent development of logging engineering as a profession on the Pacific Coast is in response to a recognition by operators of the possibility of greater efficiency and economy in transportation.

Extraordinary increases in cost, if general, must ultimately be taken from the value of stumpage. In the same way, a general reduction of costs, even if due solely to improved methods and greater efficiency, will increase the value of stumpage.

191. Local Factors: Future Transportation Facilities. — With increasing population, transportation facilities are certain to improve. Tracts of timber of considerable extent and volume will be certainly reached in time by the construction of railroads or other special means of transportation. A far more serious question arises as to the permanence of such facilities. If there is not sufficient use for them to justify their continuous maintenance after the present merchantable stand is removed, they will be abandoned. The rails and ties are removed, the flumes, bridges, trestles and corduroy rot away, the grades become gullied and the roads grow up to brush. To remove any timber in the future these structures must be replaced. There must be a sufficient stand of second growth not only to furnish the material for reconstruction but to warrant the expense (§ 98). The original stand of virgin timber may have been several hundred years old, and far more valuable per thousand feet than this second growth (§ 184). There are many districts where a second operation will be financially impossible, and timber grown after logging will remain inaccessible unless a very considerable increase occurs in the value of stumpage in general. This condition applies to mountainous or non-agricultural regions. Elsewhere, increasing population assures a future market for all timber.

A similar increase in average transportation costs per thousand board feet occurs with the exhaustion of the more accessible bodies of virgin timber. This tendency was for a long period overcome by a corresponding expansion of the size of operations, the amount of capital invested, and the quantity of timber tributary to a single transportation system. The old days of

hand logging, short drives and heavy timber were succeeded by large operations, which tapped reservoirs of timber of great extent, and distributed the costs over enough stumpage to bring them even below those of the simpler undertakings. When this increasing cost is no longer offset by increased volume to be transported, it at once depresses stumpage values. This means that average quoted prices for stumpage may not rise as rapidly as lumber prices, because the later quotations are for stumpage differently situated. The rise in values occurs, but is evident only for the same or similar timber.

An important consideration in forest production is the possibility of substituting the cost of growing trees in accessible situations, for the cost of transportation from regions much farther from markets. As accessible timber disappears, the difference in stumpage value between timber easy and difficult to obtain will become larger and larger, and this margin of value can be expended upon forest production on accessible sites.

192. Local Factors: Industries and Markets. — Transportation charges are subtracted from income in arriving at stumpage value. It follows that the farther from the mill the timber lies, the less will be its value. In the manufacture of lumber, portable mills brought to the timber greatly reduce the cost of hauling this lumber to the market by leaving the waste from sawing in the woods. But the greatest value for many woods lies in their use for special products, such as spools, excelsior, boxes, match stock and other specialties. These factories pay larger prices for stumpage since they utilize the tree far more completely and obtain much greater returns on a given amount of wood. Local industries of this character must operate continuously in order to be profitable. They remain in a region as long as they can obtain a sufficient supply of raw material. The reduction of the supply to a point where the mills can only run part time, or must suspend operations for several years, results in driving out these industries. The remaining stumpage must then be marketed at more distant points, perhaps for a less intensive use, and there is a very considerable loss in value.

193. Local Factors: Future Supply of Timber. — From the factors discussed in Articles 191 and 192 it is apparent that the local value of timber stumpage in the future depends largely upon the total quantity of timber produced in that locality, and the continuity of the supply. If virgin timber is being appraised, its value must often be based on the premise that it will be logged at the time the remaining stumpage is cut, since it is worth more now than it may ever be again. If the timber is of second growth, or is in a region of second growth, the extent, average age, vigor and utility of this second growth is of the greatest importance in appraising the value of any separate parcel or tract. Where most of the cut-over areas are ravaged by fire, or otherwise rendered worthless, an isolated patch of reproduction or plantation not only costs more to protect, but its stumpage value is permanently less in such a region than if it were a part of an abundant second growth. It may pay better to plant trees in regions where natural reforestation is abundant than under the reverse conditions. By following out this argument it will be seen that extensive operations on the part of states or the national government to protect and reproduce large areas of forest will have the effect of raising stumpage prices for those private owners who grow trees in such localities.

194. The Rate of Increase in Stumpage Value. — By carefully weighing both general tendencies and the equally important local factors, the examiner may appraise the probable stumpage value of timber with considerable accuracy for a period of 10 to 15 years in the future. Beyond this period, so many unforeseen events may occur to affect prices that only a semblance of accuracy can attach to such appraisals. One of two suppositions can be made. The value may be assumed to remain constant beyond a certain year. The tendencies of prices and the history of wood consumption justify so conservative a conclusion.

The second assumption is that prices will continue to advance at the same per cent as in the past. The validity of this conclusion depends upon the meaning attached to the phrase;

whether simple or compound interest is meant, that is, whether the original value or the increasing value is the basis of the annual percentage. Stumpage as a residual value is subject to more extensive proportional changes than the price of timber products which causes these changes (§ 188). Then, while prices and stumpage value may rise at a rate equal to simple interest on the value at a given date, the rate of interest, simple or compound which this rise represents, when based on increased prices or values, becomes less and less.

Table V illustrates these relations. Should stumpage rise from an original value of \$5 per thousand, by the sum of \$5 every 10 years, or 50 cents per year, the rate of increase would be equivalent to 10 per cent simple interest annually on the original value.

TABLE V
RATE OF ANNUAL INCREASE IN VALUE OF STUMPAGE

Period, years.	Value of stumpage, dollars.	Rate of simple interest on original value, per cent.	Rate of com- pound interest on original value, per cent.	Rate of simple interest on value for each decade, per cent.	Rate of com- pound interest on value for each decade, per cent.
0	5.00				
10	10.00	10	7.2	10	7.2
20	15.00	10	5.6	5	4.1
30	20.00	10	4.7	3.3	2.9
40	25.00	10	4.1	2.5	2.3
50	30.00	10	3.65	2	1.85

Values of stumpage are more apt to follow the law of increase set forth in this table than to increase continuously at a fixed rate of compound interest. It follows that as prices and values advance, the *rate* of increase must fall. By first predicting the amount of the increase, or the rate expressed in terms of simple interest on original value, the rate of compound interest for these values may be computed if desired (§ 89, Formula XIV; see also § 49).

The true rate of increase in stumpage value depends upon the sum of increased values due to growth in volume, improved quality, closer utilization and price increment. These sources

may be expressed separately, as if each represented a rate of interest, as

$$V_0 (1.0a) (1.0b) (1.0c) = V_1,$$

which is slightly greater than

$$V_0 \cdot 1.0 (a + b + c) = V_1.$$

In ascertaining the values of the factors a , b , etc., for a period of some years, these values must either be taken as representing simple interest, or, if compounded, the rate quoted for the entire period must be much lower than the probable annual increase in the earlier years.

195. Increasing Future Values as a Basis of Appraisals. — As set forth in Article 139, the elements of increase in value inherent in the character and condition of the property itself must be accepted as reliable and occurring in the natural course of events, and will be recognized in appraisals for public purposes, such as damage suits and condemnation proceedings. These include growth and quality increment. But the factors of closer utilization and rising prices depend upon outside influences shown to be affected by a complex of causes and therefore not capable of reliable diagnosis.

The acceptance of such increased prices as a basis of appraisals must be determined by the purpose of the appraisal, and by the evidence on which the conclusions rest. Reasonable conservatism is required in appraisal of damages. A more liberal construction is permitted in condemnation proceedings. In either case, evidences to prove the probability of rising prices must take into account both the general and local factors discussed in this chapter, and it will usually be found advisable to make very moderate claims on this score.

As a basis of valuation to determine a tentative sale value, it is equally necessary to treat the element of future prices conservatively. The purchaser is entitled to this source of future profits, and should base his price for the property upon present rather than future prices for products. As tending merely to show the probable amount and present value of these

profits, for the information of the owner, no objection can be urged to any assumption which he can reasonably justify.

196. Revision of Stumpage Values in Long-term Contracts. — In timber sale contracts, where timber is paid for at time of purchase, the owner of the stumpage, in return for the risks incidental to carrying this investment, is entitled to share in the increased values which may materialize (§ 176). For periods of less than five years this factor is immaterial. If the contract is for longer periods, at a fixed price, the purchaser is usually willing to operate at a loss during the earlier portion of the term, in expectation of realizing increasing profits later on. In this way the increase may be shared with the owner even on the basis of a fixed contract price. Contractors desire certainty in the element of cost of raw materials, even at some sacrifice of possible profits.

The ideal arrangement is some form of revision at stated intervals, by which the value of stumpage is readjusted to changing conditions. Such a revision may be based on a reappraisal. This would take into account the element of closer utilization, as well as advancing prices. The factor of growth is accounted for by the scale, while changes in quality are usually neglected, but can be considered in reappraisal. This method is advocated in the Forest Service for regions where close competition exists for government timber.

A second plan for revision confines the changes to the single factor of prices for lumber, neglecting those of utilization and quality. The point to be determined is the per cent of increase in lumber prices which should be appropriated by stumpage. On the basis that timber prices have advanced in the past more rapidly than costs, the Forest Service has claimed as much as 75 per cent of this increase for stumpage. Their present basis is to appropriate 50 per cent of increased prices as stumpage value. Revision, without reappraisal, is undertaken at intervals of five years. The prices of lumber accepted as a basis (§ 172) are averaged for three years previous to revision.

CHAPTER XIII

RISKS

197. Risks versus the Rate of Interest in Forestry. — The risks to which forest property is exposed tend to discourage the investment of funds in forest production, in the same manner that similar risks restrain investors in other enterprises. This tendency is expressed by an increase in the basic rate of interest at which capital will seek investment in forestry (p per cent). A high rate of interest is the expression of a fear entertained for the safety of the principal. The use of such a rate gives very low expectation values for probable future income.

When the per cent which can be earned by forest enterprises is lower than the rate of interest demanded, capital is presumably unwilling to invest in forestry because the average risk of loss exceeds the probable income. Should individuals still be willing to undertake forestry, they do so because they place a lower estimate upon this risk than that commonly prevailing, or expect to combat it by effectual preventive measures; hence their individual "rate of interest" is gauged proportionally lower and indicates possible profit.

198. The Character of Risks in Forestry. — Risks may be classed as physical, moral and financial. Physical risks are those incurred through the operation of the forces of nature. In case of fire, the physical risk is greatly increased by the addition of a moral risk. Moral risks are those arising through possible human actions, either deliberate or accidental. Financial risks include both physical and moral elements, since all risks are measured by the probable financial losses that may be inflicted. But the risk of loss of profits exists quite independent of these other factors, and is influenced by changing economic conditions, such as demand, prices, future taxation and any other

element which serves to diminish profits below the expected amount.

199. The Gauging of Risks. — There are two elements to a risk: the hazard, or degree of probability that damage will occur, as indicated by past history and present conditions; and the liability, or value of the property subject to damage. The liability, in forest investments, is measured by expectation value for partially mature stands; but for plantations, cost is the basis considered, and extra efforts will be made to protect such plantations because of the investment which they represent. The hazard must be gauged, during the growth of the crop, by the increasing or decreasing susceptibility or resistance of the trees to fire, wind, insects and fungi, and by the probability of these events occurring, through prevalence of the causes.

When the liability is large, and the hazard is extreme, active preventive measures involving considerable expense are justified.

With a small present liability and extreme hazard, the question arises whether protective expenses will pay for themselves.

When liability is large, but the hazard negligible, protective measures may involve a needless expense, and diminish profits and value.

With small present liability, and negligible hazard, the expenditure justified for forest protection is still further reduced.

The financial character of the risk thus indicates the expense which should be incurred for protection. The largest expenditures are justified when property of considerable value is in great danger of being destroyed.

200. Physical Risks: Fire. — Fire is by far the greatest risk in forest production. The resistance of trees to fire damage depends on the character of the bark, which differs with the species, and on its thickness, which increases with age for all species. Sandy soils in all regions are far more subject to fire than soils which retain moisture. In many localities fires burn annually on such sites. As a consequence, the tree species which inhabit these situations have developed a fire-resistant bark which often renders them immune to the effects of ordinary

fires. This immunity is attained at an early age, but seldom covers the period of reproduction, or the seedling stage of growth.

On soils and in localities where fires burn only at long intervals, most tree species are very susceptible to injury, and, as the fires which do occur on these sites are unusually severe, practically the entire stand may be killed. The younger the stand, the greater the per cent of mortality and consequent proportion of value destroyed.

The fire hazard is largely due to the inflammable character of the forest floor. The entire mass of accumulated litter and humus will burn when dry enough. Most of the undergrowth is also extremely inflammable. Shade thrown by a dense cover reduces the risk by preventing extreme desiccation, and keeping out brush and grass. Of the materials forming the forest floor and undergrowth, grass, when dry, is the most inflammable. Pine needles come next, while hardwood leaves do not cause a serious blaze. Humus or duff, when dry enough to burn, causes a slow and very destructive ground fire, difficult to extinguish.

The hazard is reduced by atmospheric moisture. Except on the sandiest soils, belts of heavy rainfall will give rise to rank vegetation. These are regions of slight hazard. Over most of the forest areas in this country droughts occur either annually or at greater intervals, of sufficient severity to create an excessive fire hazard.

This risk is enormously increased by logging, because of the great increase in litter and slash incidental to the operations. As the liability is reduced in like proportion, the temptation to ignore the hazard and abandon the attempt to protect the residual forest values is very great.

201. Wind. — Wind resistance is greatest with small timber, and becomes less with increasing bulk, weight and age. The form, distribution and penetration of the roots of different species determine their resistance to wind-throw. Shallow, rocky soils, or those underlaid by hardpan or water, induce windfall. Regions subject to frequent tornadoes present a wind hazard which must be reckoned with in planning the sequence of logging operations. This risk is excessive only for

mature and over-mature timber, and is a serious factor in operations where a portion of the stand is left for a second crop or as seed trees. If all parts of a tract are accessible, wind-thrown timber can be marketed without loss, provided the quantity thrown does not exceed the capacity of the operation as gauged by the length of time the down trees can be left without deterioration.

202. Insects. — Imported insects as, for example, the gypsy and brown-tail moths are far more dangerous to forests than native insects, as they are practically without any natural parasites. The introduction of these enemies now promises to keep the gypsy moth in check.

Native insects occasionally develop possibilities of injury, and may destroy great areas of forest. They are finally checked by disease or parasites. The worst of these insects are the bark-boring beetles. Defoliating insects, like the larch sawfly or spruce bud worm, may kill timber over entire regions. The destruction of native birds explains the increase of certain forms of injurious insects. Woodpeckers are very useful in keeping boring larvæ under control.

203. Fungous Diseases. — The only conspicuous instance of widespread destruction of an American species due to fungous attacks is found in the ravages of the imported disease of the chestnut, *Diaporthe parasitica*, which threatens to exterminate what is considered the most important commercial hardwood over wide regions east of the Appalachians. Great damage is done to the heartwood of all species of trees by fungi which gain entrance through wounds or knots. These fungi do not kill the trees as they have no effect on the living tissue. This damage is greatest in old timber and is not a serious factor in second growth.

204. Climatic Injuries. — Excessive drought in rare instances kills many trees of the more susceptible species in exposed regions or localities. Trees unused to swampy conditions are killed by prolonged flooding of the roots.

Minor injuries are caused by breakage in sleet storms or heavy snows, permitting fungi to enter. Reproduction is especially

sensitive to frost, drought and excessive moisture, but the young trees soon become hardy.

205. Injurious Fumes. — The physical injury caused by fumes is due to human activities. The fumes from copper smelters will sometimes kill timber seventy-five miles from the location of the smelter.*

206. Moral Risks: Trespass. — Timber products are frequently stolen, but the theft is easily discovered, and it is not difficult to identify the trespasser. Losses from this source are the result of neglect of the property, and are easily eliminated by proper marking and posting of boundaries, and the employment of a custodian. The expense of such protection is serious for small tracts whose owners reside at considerable distances, but is immaterial for larger holdings. The character of the population and ease with which small quantities of timber can be transported determine the degree of hazard.

Trespass is frequent in connection with logging operations, where only a portion of the stand is to be cut, or where the logger attempts to enlarge his operation at the expense of adjoining owners. Reputable firms incur considerable expense in relocating boundaries not only to prevent others from trespassing, but to avoid accidental encroachment on adjoining lands by their own employees.

207. Moral Risks: Fire. — Some fires are started by lightning, but the great majority are due to human agencies. Most of these fires are caused by careless or unintentional acts, but in some regions the habit of burning is firmly fixed, and occasionally fires are set through malice. The moral risk thus incurred is so great in some southern states that it presents an almost insuperable obstacle to forest production. In other localities this risk is chiefly due to avoidable factors, such as railroad operation, and yields to preventive measures and education.

208. Financial Risks. — The risk of excessive future taxation is discussed in Chapter X. Unless this risk is removed, it will prevent the practice of private forestry in the regions affected.

* Deerlodge National Forest, Anaconda, Montana, 1912.

The financial risk is in reality a summary of the factors enumerated above plus the economic risk of failure to secure remunerative prices (Chapter XII). The economic situation, except in certain inaccessible localities, is so favorable that the only serious risks are those discussed in this chapter.

209. Control of Risks: Insurance. — Insurance against losses follows rather than precedes other measures for the control of risks. When the risk is large, and the probable amount of damage uncertain, the premiums will be prohibitively high. At the present time, no insurance against fire is in force on standing timber in the United States, and no companies will accept such risks. Lloyds of London have insured a large tract of timber in Canada.*

The unreliability of appraised values in case of damage, and the great variation in value of stumpage, adds to the difficulty of such insurance projects. Standing timber in European states is widely insured at low rates. Tornado insurance on timber is available in the United States, but has not been taken advantage of by owners of timber stumpage.

210. Public Measures. — Within the last decade it has been demonstrated that the fire risk can be largely eliminated in northern and western states by the enforcement of efficient fire laws and the organization of state and local forces of fire wardens. These systems are as yet not completely successful, but promise to become so, especially in states where politics has been eliminated from the organization. Education of the public, punishment of offenders, and prompt detection and suppression of incipient fires form the groundwork of these systems. Through their operation, risks will eventually be reduced to the point where it will become possible to insure standing timber from fire.

These state forces are aided materially by semi-public co-operative associations of land owners, which expend large sums in fire patrol and suppression. The operations of these associa-

* "Logging," by Ralph Clement Bryant, p. 37. John Wiley & Sons, Inc., New York, 1913.

tions are conspicuous in the Pacific Northwest, and have extended to Michigan and New England.

The importation of injurious fungi and insects is now guarded against by national statutes and inspection. One serious fungous disease, the white pine blister rust, has been checked by this means. State inspection and control of injurious insects and diseases, supplemented by the activities of the national government, furnish a means of preventing future outbreaks similar to that of the chestnut bark disease. Such aid should be chiefly preventive in character, and can be invoked but seldom for the control of native insects.

211. Private Measures. — Private owners may be forced by state legislation to take measures to eliminate unusual fire risks caused by their own operations. Laws making brush or slash burning compulsory, especially along rights of way, are already adopted in some states. Top-logging is intended for the same purpose.

In addition to these compulsory private measures, owners of forest property must expect to take extra precautions against fire, similar to those adopted by factories and stores to supplement the work of the fire department. Firebreaks should be constructed and maintained for the protection of plantations, and to guard against excessive hazards. The owner and his tenants and employees should be prepared for fire fighting and equipped with the most suitable tools for the purpose.

212. Effect of Risks on the Business of Forest Production. — Mature timber, while in danger of being killed by fire, insects or disease, or blown down, can usually be salvaged with only partial loss. To the greatest danger of all, fire, it is moderately resistant. Since systematic fire protection has been instituted in the North and West, losses have averaged less than two-tenths of one per cent annually, on the value protected. The risk has never deterred investors from making purchases and speculating on values of stumpage.

With young, immature timber and reproduction, the same cannot be said. A comparison of risks shows that the hazard is less on small or young trees, from practically all sources excepting

fire. But the risk of destruction from this source has in the past been so great as to constitute a prohibitive hazard. Destruction of young timber is a total loss, for there is no salvage. The dead trees are quite worthless, and even constitute an additional expense (§ 136). The period over which the investment is exposed to risk is correspondingly longer for young timber than for mature trees, which can be removed at the owner's convenience.

As long as the public and the land owners themselves remained completely indifferent to young growth, and regarded fires on cut-over lands as of no consequence, the risk incurred in the business of forest production was prohibitive. One could as well expect to succeed in a mercantile business when thieves, incendiaries and dishonest debtors abounded, and there existed no effective law or public sentiment to check their operations. The first step in establishing the social conditions which are now making forest production possible was the creation of a new public conscience and understanding on the subject of forest property and forest fires. The elimination of theft of national and state timber, the tremendous progress in fire protection in northern states, and the large sums spent in eastern states for suppression of forest insects and diseases indicate that this constructive work is succeeding. Under proper social conditions, such as have been secured in France and Germany, the element of risk in forest production becomes less than that incurred in almost any other form of enterprise. The employment of a high rate of interest in forestry is merely the expression of this undeveloped state of civilization as touching forest property, and should not be confused with the returns or earning power of forest investments.

CHAPTER XIV

FIELD APPRAISALS OF TIMBER STUMPAGE

213. The Scope of Field Appraisals. — The value of timber stumpage, while based on the species, quantity and quality of the timber, is determined only when the value of the products f.o.b. at mill and the intervening costs of logging and milling have been appraised (Chapter XI).

The field appraisal thus deals first, with the reconnaissance of the timber (§ 110), and second, with the appraisal of logging costs. The cost of milling, depending as it does on the size of the mill and duration of the operation (§ 173), is affected by the total volume of timber accessible to the mill, and its ownership, and this cannot always be determined when examining a given tract. Appraisals, except when they cover a large area, are therefore most definite when made in connection with mills already operating.

The choice of general means of transportation must occasionally be left to the field examiner, but more often this is already indicated by local custom, and it remains only to determine on the ground whether any modifications of the plan are necessary because of the nature of the topography and the transportation problems which arise. The scope of the actual field appraisal of costs is usually confined to ascertaining the local costs of logging. These costs should be appraised separately for each so-called "logging chance," or separate body of timber forming a logging unit. In flat regions, the division of a larger area into units is not important and depends on the convenience of locations and routes for main spurs. But in a rough or mountainous country the "chance" is a distinct topographic unit, such as a gulch or small basin, containing enough timber to furnish employment for a camp of reasonable size for one or

more seasons. The timber on such a unit normally comes out over the same route — a branch railroad spur or road.

Of first importance in the examination is the necessity of sizing up this local transportation problem, deciding upon the best method of removing the timber, and estimating the difficulties of construction and cost of overcoming them. The cost is gauged, not on the basis of the total expense of construction, but with relation to the amount of timber reached (§ 174). In high and precipitous mountains it is necessary to decide how far up the slope the operator can go without exceeding the permissible cost of getting out his logs.

This transportation cost is thus directly dependent upon the estimate of the standing timber. The field examination necessary to determine it is either a part of the work of timber reconnaissance, or some additional work is done following the completion of the cruise, to make sure of the salient points.

214. Timber Reconnaissance: The Map. — In the timber examination the tract is ordinarily crossed and recrossed at intervals of 40 rods or less, and the intimate and systematic knowledge of the topography and surface thus gained permits the formulation of the plan for logging. The main drainage and its tributaries are located and the character of the slopes and surface noted.

This knowledge, when necessary, may be systematized in the form of a topographic map which shows all features that affect logging. If the topography is so level that transportation presents no difficulties, such a map serves no useful purpose. If the map does not indicate conditions with sufficient accuracy and detail to enable the manager to plan the general location of his transportation system, it fails of its purpose. Actual lines of railroad or road must be located on the ground, but the entire system can nearly always be roughly planned from a good map. Since such a map can be made as a part of the work of timber estimating, the expense is minimized and the effort justified. Otherwise the information so vitally necessary to the planning of transportation routes exists only in the mind of the field examiner or in the form of incomplete notes.

215. Estimation of Standing Timber. — The estimation of standing timber presents one of the most difficult problems in the business of lumbering. This might not be true were there no limits to the cost which can be incurred in making such an estimate. But it is quite possible for a careful detailed measurement of the timber on a tract to cost over \$1.00 per acre. It is a recognized precept in business that the cost of stock taking must be kept as low as possible, for the work adds nothing to the value or the profits of the business, and is necessary merely as a basis upon which to plan actual operations.

Methods of timber estimating are determined by the relation between the cost of doing the work and the value of the timber. The amount of care and expense justified increases with rising stumpage values. Timber is always estimated in terms of the product which has the highest market value, or is most readily saleable. When pulpwood is used, the cord is the customary unit. But should the stand be situated in a region where cordwood is used only for fuel, the number of ties or poles and the contents in lumber of suitable trees would be first estimated and only the residue expressed in cords. By far the greater quantity of timber is estimated in terms of board feet, since it is used in this form.

216. Log Rules. — The measurement of standing trees for board foot contents is beset by the fundamental difficulty that no commonly accepted standard of measurement has been or is likely to be adopted in this country which will properly express this contents. Innumerable log rules are in use, of which a few are now widely recognized. But these rules are very defective, seldom giving a sufficiently large contents for logs (§ 179). In spite of this fact, it will seldom be possible to adopt the actual sawed output as a basis for timber estimating, as long as logging contracts and other woods operations, such as felling, are based on the old standards as expressed by the log rules in common local use. Any great increase in the measured volumes of the logs would require reduction in the scale of payment or result in loss of profits by the millman. The only safeguard is a knowledge of the amount of overrun usually obtained with the

given rule in the class of timber estimated (§ 179). Quantities estimated will therefore be expressed as log contents measured by the local rule, but in computing values the overrun on each thousand feet of logs will be allowed for. Log rules have been devised which give very closely the actual sawed contents of logs. The use of such a rule insures a proper standard of measurement for the contents of standing trees.*

217. Closeness of Utilization as Affecting Timber Estimates.

— Another factor which influences timber estimating is the closeness with which the timber is utilized (§ 186). Cheap lumber necessitates the wasting of the poorer grades of boards, as the cost of manufacture exceeds the sale value. Under this condition, limby tops, defective logs and small trees are left in the woods. In such forests as exist in Washington State, the difference between utilizing only the highest grade product and taking common and low grade lumber may make 50 per cent difference in the total quantity removed (§ 186). In general, an estimate is made on the basis of present market conditions, and does not attempt to anticipate future values, though this will have weight with a far-sighted purchaser. Estimates made in past periods were low compared with present estimates and must be revised as utilization becomes more complete.

218. Field Methods of Timber Estimating. — With these points decided, the problem remains of how to cover the area with the least possible cost. It is desired to obtain the total quantity of timber, by species, with a knowledge of sizes and quality. Cruisers of considerable experience in a restricted locality can size up timber by walking through it, and from their general knowledge of what similar stands have cut out, will make a guess at the total stand. This method is comparatively useless for large tracts, or with men of lesser experience or unfamiliar with a region, and is always subject to large errors.

On the other hand, the cruiser may go to the trouble of measuring the contents of each tree separately. To justify this course the trees must be very valuable and the area small. Woodsmen

* "Forest Mensuration," by Henry Solon Graves, Chap. III, pp. 27-35. John Wiley & Sons, Inc., N. Y., 1906.

almost invariably guess at the contents of the tree, either as a whole or by their knowledge of the contents of logs of certain sizes. A better method is to use tables giving volumes in board feet, or in other units, for average trees of given diameters and heights, when such tables are available and are known to be reliable.

219. Strip Methods. — By far the greater amount of timber estimating is done by methods involving the measurement of only a portion of the stand, in an effort to get an average which will apply to the whole area. Even if all the trees on a tract are observed, they may be merely counted, and an average volume obtained by guess or by tallying the sizes of a certain proportion of the stand.

The most widely used methods consist of running parallel compass courses through the area at definite intervals, and measuring the trees standing on a strip of a definite width with the course as a center. In dense brush, or with small timber, this strip may be but 4 rods wide. Where tree callipers are used to get the diameter of each tree and the men employed to do the work have but little experience in timber cruising, this width of 4 rods is seldom exceeded. More experienced woodsmen, who estimate the timber by eye, usually cover a strip 8 or 10 rods wide. The number and contents of the trees on these strips form the basis of the estimate. Should the strips so measured be accepted without question as giving an average stand for the whole area, the total estimate is found by using the multiple determined by the per cent of the area measured.

220. Errors in These Methods. — But this is seldom the case. Timber is never uniformly distributed even over small areas, but varies enormously from acre to acre in size and density. Topographic and site factors exercise a marked influence on the stand. The timber tends to run in belts along stream bottoms and slopes. Chances of obtaining a better average are improved by arranging the strips to cross these belts, which object is most frequently attained by running up and down slopes rather than along them.

For a given per cent of the total area estimated, the chances

for accuracy are improved in direct ratio to the size of the area units separately measured. Should the figures be desired by separate forty-acre tracts, an estimate covering 10 per cent of the total would leave opportunities for large errors, while for a section of 640 acres, 10 per cent might give quite close results. Large and scattered timber requires a more complete estimate than small trees densely stocked. Very irregular or bunchy stands may require greatly increased work to arrive at a fair result. One method of dealing with this fundamental difficulty is to merely accept the result as within the allowable error for such work. Astonishing errors creep into timber estimates, even with experienced men.

221. Types. — A second plan is to subdivide an area into types by means of a sketch map. In timber estimating, these type divisions are for the purpose of improving the possibility of getting a fair average stand. They are really stand classes. Blank or open areas are mapped and excluded in computing the total estimate. Only the area of strips run within a given type is used in determining the stand on that type. The amount of work which this method entails is somewhat greater than when the timber is calculated on the basis of the whole area, but this is often justified and the plan is used in a rough form by nearly all woodsmen.

222. Correction Factor. — Another method of improving the final estimate without measuring every tree is the correction of the computed total, whenever the observations and judgment of the cruisers indicate its necessity. This correction must be based on actual inspection of the entire body of timber. When running strips, short side trips are made to view the timber which lies outside the strip. Should this timber run heavier or lighter than that measured, the total will be reduced or increased by an estimated per cent. This plan must not be confused with the method of guessing at the total stand, for it is based on a careful estimate, tree by tree, of a portion of the area. Should it be impossible for any reason to actually inspect the remaining timber, there can be no basis for correcting the result, which must stand for what it is worth.

223. Quality of Timber. — To judge of the quality of the timber, attempts have been made to grade logs, according to the relative value of their contents, into perhaps four classes. This is actually done in Puget Sound where a well-established log market exists and logs are purchased on the basis of the log grade or class. The difficulty with such a basis ordinarily is that logs vary in value from several sources; knots or sap in young timber, rot and wind shake in old trees. A dividing line or standard is not sufficiently definite, nor, except under the conditions cited above, are logs ever sold on this basis. They are paid for at a flat rate, according to the average value of the contents. The only recourse of the cruiser is to attempt to size up the quality of the trees by their size, apparent age, clearness or freedom from knots, and soundness or freedom from interior defects. With a knowledge of the average per cent of grades produced in mills cutting similar timber, the cruiser estimates the per cent of each grade that the stand will cut. Should the timber be older, larger or better than the average stands, he increases his estimate of the higher grades to correspond.

224. Reports. — The information which should be given in a satisfactory report on a tract of timber is shown under the following heads.

I. *Area.*

1. Description or location of specific tract.
2. Area.
3. Subdivisions of area.
 - a. Blanks.
Water.
Barren.
Burns.
 - b. Reproduction or immature timber.
 - c. Forest types.
 - d. Age classes, where distinguishable.

II. *Volume.*

1. Species: Separate estimates for conifers and hardwoods, and for each commercial species.
2. Volume in merchantable units.
 - a. Board feet.
 - b. Cords.
 - c. Pieces (poles, ties, etc.).
3. Deductions from gross volume.
 - a. Dead timber (probable durability).
 - b. Cull, due to defects (rot, wind shake).

III. *Damage.*

1. Rot: Extent, character and amount of damage.
2. Fire: Standing dead timber, damage to live timber.
3. Insects: Location, prevalence, character of damage.
4. Windfall: Location, amount and condition.

IV. *Quality.*

1. Size (if necessary, for two or more size classes).
 - a. Average diameters, at $4\frac{1}{2}$ feet (or at stump height).
 - b. Average heights.
Number of merchantable 16-ft. logs.
Total height of tree in feet.
2. Form of bole.
 - a. Straight or crooked.
 - b. Taper, expressed in inches per 16-ft. log.
 - c. Clear length. Limbs, character of knots.
3. Grades.
Per cent of probable output in each of several standard grades for species and locality.

V. *Young timber.*

1. Species.
2. Area, in acres or in per cent of total area.

3. Average age. Range of ages.
4. Sizes. Average and range of diameters and heights.
5. Condition. Thrift, damage, with causes.
6. Chances of survival. Competition with older timber, and with worthless species, or brush.
7. Probable commercial value at maturity.

VI. *Soil. Quality. Agricultural value.*

VII. *Logging conditions; summary for "logging chance" rather than for each subdivision of estimate.*

1. Topography. By map or description.
Character of slopes. Obstacles to logging.
2. Surface.
 - a. Underbrush.
 - b. Rock.
 - c. Character of bottom.
3. Transportation.
 - a. Streams, character for driving, cost of improvements.
 - b. Routes for roads, or railroad spurs.
 - c. Methods of skidding and hauling.

VIII. *Modifications of logging required by silvicultural demands. Summary for logging chance.*

1. Amount and character of merchantable timber to be left standing.
2. Methods of brush disposal, and precautions required for protection of young timber (if not summed up for larger areas as a whole).

IX. *General conditions affecting appraisal.*

1. Market value of lumber, or price basis.
2. Size of mill, and cost of milling, with profits.
Investment required.
3. Cost of main transportation system.

X. *Specific conditions affecting appraisal.*

1. Specific costs of logging the "chance."
2. Specific appraisal of value of standing timber per unit of log scale, based on total cost and total quantity, or distributed according to value of species (§ 180).

XI. *Appraised value of standing timber, by species, for specific units of product, modified for overrun to apply to standing timber.*

CHAPTER XV

COMPARISON OF FOREST VALUES WITH AGRICULTURAL VALUES

225. Agricultural Soils. — Agricultural soils are soils which will produce agricultural crops continuously in paying quantities, under proper cultivation and treatment. Many classes of soils will produce scant crops for short periods, after which they must be abandoned. Such soils should seldom be used for crops at all but kept under forest cover. Permanent agricultural fitness of soil is determined by six factors: quality of the soil itself, topography or slope, climate, water supply, the personal or human factor, and the economic factors.

226. Quality of Soil. — Soils to be agricultural must be free from excessive quantities of rock, and not too sandy, since loose-textured soils will not retain sufficient moisture or soluble plant food to permit satisfactory growth. They should be underlaid by a permeable subsoil which permits sufficient drainage. Deep sands, and stiff clay underlaid by hard-pan, are familiar types of non-agricultural lands.

227. Slope. — The slope is important, since crops must be cultivated, and upon steep slopes, if not terraced, the loosened surface washes away, deep gullies are formed and the land is permanently ruined for agriculture. Slopes varying from 10 to 20 per cent, according to rainfall and quality of soil, will prevent permanent cultivation.

228. Climate. — Crops will grow in all but the coldest climates if given sufficient water. But in mountainous regions, lands at high elevations resemble regions near the arctic circle. Frosts occur every month in the year, and the growing of all agricultural crops, except a poor quality of hay, is prevented. Such lands are non-agricultural, regardless of the quality of soil.

229. Water. — In regions of sufficient rainfall, low-lying lands are often swampy. Drainage transforms such soils into permanent agricultural lands of high value. In dry regions, irrigation brings otherwise desert soils under cultivation. Trees ordinarily require fully as much soil moisture as crops. Desert lands are therefore of no value for forestry, but undrained swamp land will produce valuable though slow-growing species.

230. Personal Factors. — The personal element is a large factor in successful agriculture. Experience is the foremost need, but industry, economy and good business capacity are equally helpful. The advancement of agricultural knowledge by experiment stations, and the general trying out of new methods, often make success possible on soils and in climates which were once thought too hostile to permit of agricultural use.

231. Economic Factors. — The development of transportation, and increasing values for farm products, have a stimulating effect on agriculture, and farms can be made to pay in regions near large markets, which would be too poor to cultivate in less accessible localities. This last factor, coupled with the advance in knowledge of farming methods, tends to bring more and more land into agricultural use, and the standard of classification between agriculture and forestry, for this reason, can never be a hard and fast one.

But the differences between good and poor soil will remain as great as ever and some soil will always be too poor to farm. In densely populated eastern states sandy and rocky land in abundance is found, upon which the inhabitants eke out a miserable existence and which no amount of skillful management will make into profitable farms.

232. Comparison of Agriculture with Forestry as a Source of Livelihood. — The use of land for the production of agricultural crops differs in many important particulars from its use for the production of timber crops. Agriculture, the direct source of all food supplies, yields immediate livelihood to the farmer, whose returns are based on an annual cycle. A large element of labor enters into crop production, both for preparing and clearing the soil, and in actual cultivation and harvesting. Added to this is

the investment in buildings, fences, machinery and live stock. Agriculture requires large continuous outlays per acre, and in return must produce adequate income to compensate the farmer. If the soil is unsuited to agriculture, this outlay, which is as necessary on poor soil as on good, is inadequately rewarded, and the farmer impoverished.

Trees grow best on good agricultural soils and produce their greatest values in such situations. But timber crops do not provide as stable a source of livelihood as agriculture. The revenue from sales of timber is realized only after the lapse of the long period of growth. Mature stands may be sold or held at will and the income varied largely at the discretion of the owner as long as there are trees of suitable size for cutting.

Over long periods, even at best, the net annual revenue from forest property is much lower than from agricultural crops. Relatively, forestry may be far the more profitable business. The expenses per acre are correspondingly lower than in agriculture. While not giving a solid basis for continuous livelihood except in the business of lumbering, which requires large areas of land, forestry on the other hand does not require constant time and attention. Timber if given protection from fire will grow without cultivation. It therefore forms an ideal investment for surplus income, but a poor basis for an individual living.

The economic advantage of forestry over agriculture becomes evident only on poor agricultural soils. Trees of fair quality will still grow on soils utterly unfit for farming, and owing to the small expense attendant on this crop, will yield profitable returns on such soils.

233. Non-agricultural Soils. — Since food and livelihood and permanent homes are more important to mankind than fuel and timber, soils which will repay the farmer should be devoted to farm crops. But the attempt to cultivate soils essentially non-agricultural leads to very serious results both for the individual and for society. Soils are non-agricultural when they cannot continue year after year, with proper care, to produce paying crops. Non-agricultural soils, when cultivated, either

wash away by erosion, blow into sand dunes, lose their small store of fertility by too active drainage, or for other reasons fail to respond to the efforts of the farmer.

234. Exploitation of Land Purchasers. — Intelligent farmers refuse to buy or work such lands. Uninformed purchasers, either foreigners or city emigrants unused to farming, buy these lands because they are cheap. They are even less able to make a living upon them than are the more experienced farmers who have passed them by. The effort results in financial and often in moral bankruptcy. Lands of this character thus change hands often, and since the real estate business is based on land transfers, it thrives on such conditions.

Whatever hurts the individual hurts the whole community. The exploitation of well-meaning land-hungry investors by professional land locators and real estate sharks is a business closely akin to more pronounced forms of swindling. Local sentiment in such communities will be largely molded by these selfish predatory interests and will resist attempts to properly classify such land as non-agricultural. Thus a condition is created containing limitless possibilities of evil to the public at large.

235. Land Classification. — The solution of this problem lies in the scientific classification of lands, to determine whether or not agriculture is possible. If unsuitable for farms, then the lands fall naturally into the class of forest soils, and should be removed from the market as agricultural land, and devoted to forest production. The best way to accomplish this removal is for the states to purchase such soils for forest reserves or for the national government to acquire them for stream protection.

236. Basis of Comparison of Agricultural and Forest Values of Land. — To compare the value of land for agriculture with its value for forestry, each value must if possible be expressed in money. In each case the basis of value is future net income capitalized (§ 62). If we include for both uses all items of future income, minus future costs, and can appraise this income and its present value accurately, the comparison will reveal the relative value of the property and determine whether the soil is best suited

to farming or forestry. Unfortunately, the uncertainties affecting the determination of these future items, in both cases, prevent a satisfactory mathematical comparison of such values. Yet whatever the basis on which the comparison is made, it is founded on the economic relations which such calculations set forth.

237. Expectation Value of Agricultural Land. — The basis of agricultural value is the net revenue from crops. The actual cost of labor and use of machinery is deducted from the sale value of the crops and the average net return is capitalized. Owing to the fact that on small farms the proprietor puts in his own labor and raises food for his family, this net income is not often accurately computed. Cash rent paid for farm land separates the costs from the income, since the tenant presumably gets only his just share, including profit, and the rent represents the net income earned by the soil. Since this is annual, it is capitalized by the formula $\frac{r}{0.0p}$ (XII), on the basis that the farm will produce this rental without deteriorating. A farm earning \$5 per acre rent will, at 5 per cent, be worth \$100 per acre.

238. Sale Value of Agricultural Land. — Improved farm land is transferred with sufficient frequency to establish sale values, in any region where farming is a well-developed industry. These sale values will, on the whole, coincide quite closely with the capital or appraised values of farms based on net income. But traditions, demand, and increasing or declining population affect farm values and prices to a great extent. Values of farm property are sluggish at times and again, in boom periods, outstrip conservative appraisals of income. In this the human element in price making is seen to play a large part (§§ 17, 59, 69.)

239. Timber as an "Agricultural" Value. — Improved farm land is not the true basis of comparison with forest land. Land in its wild state ranges from heavily timbered tracts through all degrees of stocking and of young timber, to natural prairies needing only to be broken for a crop. Land which

has been stripped of timber, but is covered with stumps which interfere with the cultivation of the surface and prevent the use of machinery, is the condition in which the prospective farmer in most cases has to accept property in timber regions to-day. In former times, timber itself had no value, hence the cost of removing it, including the burning of the logs, gave such lands a low initial value. It is still necessary to cut and remove standing timber from agricultural soils, but the timber now has a stumpage value and its removal will cost the owner nothing, while its sale supplies him with capital with which he can undertake the work of clearing the land of stumps. The presence of merchantable timber upon such soils thus aids, apparently, in establishing agricultural use.

In heavily timbered regions, this seldom works out. Stumpage cannot be marketed at will and in many regions must await the slow development of transportation and the gradual cutting of the more accessible timber. The owner cannot afford to sacrifice it for agriculture, except on a small portion of the area and, consequently, sells this land with its timber to some lumber company. In time, the land is stripped of timber and the stump land is offered to purchasers for agricultural use.

Timber values on agricultural soils cannot be regarded in any sense as agricultural values, but are the equivalent of so much money capital to the owner of the land. Granting that purchasers who secure timber obtain more valuable property than those who get only stump land, it is still evident that the agricultural value of forest land must be based solely on the land bare of all timber.

240. Stump Land versus Cleared Land. — The value of stump land is less than that of open meadow or prairie land in its wild state, by the amount of the extra cost of clearing and preparing it for cultivation. The crops are the only source of value, and since improved land is now bearing crops its value rests on a firm basis. Between this value and that of stump land is interposed the future costs of clearing and breaking, and the time necessary to do the work. Fencing, development of water, and buildings must be added, before agriculture can be suc-

cessful. These future costs depress the real value of stump land, and cause a wide divergence between the true values of wild land and improved farms.

241. Cost of Clearing Stump Lands. — The appraisal of the cost of clearing land is not difficult, but if based on the actual cost of wages for the labor required in clearing up ground at one operation, it is apt to be too high. Land is seldom cleared in that way, except by land companies supplied with large capital. The settler clears gradually, using his own labor at slack seasons, cultivating between the stumps, grazing the land, and allowing the stumps to rot. The roots of hardwood and sap pine stumps decay in from three to ten years and the stumps are easily gotten out. The stumps of heart pine and certain other conifers do not rot readily but will pull much more easily after a few years than at first. The loss in time is great, since the full utility of the soil for crops is not attained for years. The settler may even be investing an amount of labor and capital in the clearing for which, judged commercially, he will never be repaid. But clearing will be attempted successfully even on lands on which its cost apparently exceeds the final value of the land. Therefore the cost of clearing, provided the soil is good, should not be regarded as indicating a negative expectation value for stump land. Should the cost exceed the value of cleared land, a very low value for stump land is indicated, but this value will still be positive in most cases.

242. Sale Value of Stump Lands. — Sale values for stump lands are often falsely or inaccurately determined and are set at far too high a figure. This is due to the lack of general knowledge of the costs and delays of clearing. The values for highly productive improved lands, perhaps bearing orchards, are compared with stump lands, to the enhancement of value for the latter in the minds of purchasers, many of whom know nothing of the problems involved. It is safe to say that without the necessary additional capital to hire the clearing done, such lands can never be developed by this class of investors out of the income from the property, while persons of small means cannot buy stump land at high initial prices and ever

expect to lift the mortgages which they must usually accept. In contrast to boom sale values upon such lands, the appraised value of stump land should be reasonably low compared with improved tracts in the same neighborhood.

243. Summary of Elements of Value for Forest Land. — The elements of value which must be considered in arriving at the value of forest lands are:

Value of mature or merchantable timber.

Expectation value of young timber.

Value of land based on future timber crops.

To these values must be added:

Value of improvements as roads, fire lines.

Value of protective influence of the forest.

The determination of value for merchantable timber is discussed in Chapter X. Actual prices received for similar timber are commonly accepted as indicating this value.

The value of young timber and of land is found by the methods described in Chapter VI, when an appraisal is necessary.

Roads and other improvements do not influence value on the basis of what they cost, but by their effect in cheapening transportation, lowering fire risks and increasing the net income or lowering the rate of interest (§§ 211 and 68). In proof of this statement it is recognized that in agricultural appraisals the cost of dwellings and other improvements on farms is not the basis on which the farm is valued, although they add to its value by providing the immediate facilities for deriving the income by cultivation.

The value of the protective influence has been discussed in Chapter VII (§ 146). This value benefits adjoining property rather than the land itself, but is an important factor when weighing the relative merits of agricultural versus forest use. Fortunately, slopes valuable for their protective forest cover are usually too steep to be tilled without inducing erosion, and the demonstration of this fact would at once prevent the classification of such soils as agricultural by an intelligent investigator.

Of these five items of value only the revenue from sale of mature timber is immediately available. This is comparable with the revenue from the current crop on agricultural land. When ready to harvest, it greatly exceeds the latter as it represents a long accumulation.

244. Value of Young Timber as a Part of the Value of Forest Soil. — When comparing the value of land for forestry with agricultural values, we assume that the land has some value for agriculture. Absolute forest soils, whose agricultural use is impossible, need not be valued in order to determine the class to which they belong.

Where agriculture is possible, but its profitableness is questionable, and the cost of clearing is high, the value of the land for forestry may greatly exceed any value for farming. Here a calculation of forest values will be of great assistance in determining the best disposition of the land.

Agricultural soil is always regarded as bare land and the value as that of the soil itself, not including crops which may be growing. This is merely because of the short crop period. In reality the standing crop increases the value of the property while present and is the source of this value, and a purchaser of the land must either permit the former owner to harvest the crop or pay him for it. Forest land to be compared with agricultural land on the same basis must be bare of timber. But it is the timber crop that gives the land whatever forest value it has (§ 116). In actual practice, forest land is seldom in this bare condition. As seen in Chapter VII, the actual value of forest property depends upon the closeness of the date of realizing the income from the timber. The value of the timber may be separated from land if necessary, but this separation is artificial (§ 135). The real value of forest property is based on the actual condition of the stand, its age and size. Forest values differ from agricultural values in that they fluctuate over long periods with the growth and removal of the crops instead of maintaining a reasonably constant relation from year to year. The lowest ebb is represented by bare land, and in a system of clear cutting this condition is attained but once in the period of

growth of the crop, just previous to planting. In any system of natural seeding, and especially with selection cutting, the value of forest "land" never reaches this low point, but is maintained at a higher level by the presence of partly grown timber which will bring in revenue in a shorter period than a full crop rotation.

If young timber is present on forest soils, it is not possible to consider this timber as having no value without rejecting the basis of all values, which is future income. We cannot consider merely the forest value of the land itself, for this is derived wholly from timber. The expectation value of the young timber is more definite than the same value calculated for the soil itself (§ 116), as it is nearer and more certain than that from non-existent future crops, or "soil." If land bearing young timber is cleared for agriculture, the forest value sacrificed is greater than that of forest land by just the difference that the age of this timber makes in the present value of the income expected from it. To ignore this value would be the same as to consider a half grown crop of wheat as adding nothing to the sale value of agricultural land.

Nor can we ignore the future income from young timber as an element of forest value on the grounds that this is not a present but a future value, perhaps not attainable for several decades. If agricultural values were based merely on the revenue from the next year or decade, the capital value of farm land would be much lower than it is. All future revenue is included in such agricultural values as surely as it must be included in finding the present value of forest property (§ 85). The mere fact that forest income is irregular makes it no less necessary to value the entire future income in finding the value of the property for forestry.

245. Sale Value of Forest Property. — As in the case of agricultural soils, sales are the generally recognized basis of value for forest property, soil and standing crops included. But the sale value of forest property may suffer from ignorance or lack of recognition of the true worth of the property (§ 111). If young timber has been allowed to burn up in the past, or if

insufficient time has elapsed since logging began on the virgin forest, the fact that young timber will grow into a valuable merchantable stand, or second growth, is not generally appreciated. The forester, a specialist on appraisal of growing timber, can estimate this value on its true basis. But the public will place a much lower estimate on the property, or may ignore altogether the element represented by the young timber and value the land itself on an arbitrary basis not related to its use for forest production, but based on grazing or other uses.

Purchasers of land follow a sound policy in acquiring it at as low a value as possible, and if bought for forestry they gain by this condition of undeveloped sale values. But as a basis for *classification*, the acceptance of existing sale values of land and young timber, merely because the public are not familiar with true values, is wholly inexcusable. When this condition is further aggravated by accepting an inflated agricultural value for unimproved stump lands, the comparison is unfavorable to the securing of a true and lasting classification.

246. Discrimination against Forest Values. — Where economic pressure is strong for the listing and opening of lands for new settlers, the tendency is to deliberately ignore the expectation value of young timber as well as of the land, and to consider only the mature timber as indicating the sum of forest values. Or, perhaps, the forest value of the land is recognized, but not that of young trees. This is even more inconsistent than the first plan. The argument is advanced that the value of young timber will not be realized for many years while that of agricultural crops applies to the present moment. But if these timber values are based on present stumpage prices (§ 195) and properly discounted to the present, it must be evident from the discussion in Chapter VI that expectation value is a true present value, strictly comparable with that for agriculture (§ 244). Rejection of such appraisal can be justified only in two ways, — when forest values are actually less than agricultural values for stump land, or on the basis of political expediency.

247. Discrimination in Favor of Forest Values. — The recognition of increasing future prices brings an element of specu-

lation into such appraisals, and when this is done, the present values thus found are no longer based on present but on future conditions. It is this element, and not the discounting of future income to the present, which would unfairly alter the basis of appraisal in favor of forest values. A parallel case would be found were it estimated that prices for farm crops would increase and that the value of land at present should be based on an appraisal of this future increase in annual income.

Future growth in volume is a justifiable assumption, as is also an improvement in quality of timber and of unit price based on this quality, to conform to the higher prices now received for similar grades (§§ 183, 184). But the discounting of future increases in the general price level for timber is as little justified in the case of valuation for comparison with agriculture as it is in valuing damages to young timber. In either case, present values must be based on present prices, and future increases may be used only by the owner in computing possible or speculative profits.

248. Results, when Values of Bare Land are Compared. — From the above discussion it is assumed that a true comparison of forest with agricultural values is based on a computation or appraisal of the expectation or present capital value of the property for each use, the forest values present in the form of timber and land being compared with the value of stump land for agriculture. The highest value for both purposes will always be found on the richest agricultural soils. But it will seldom if ever be true that the forest value of bare soil will exceed its agricultural value if the soil is fairly adapted to crop production. Only upon the poorer or non-agricultural soils will land itself, even at a low forest value, be worth more for wood crops than for agriculture.

249. Results, when Full Value of Property is used in Comparison. — With the growth of the timber, however, a corresponding increase takes place in the forest values of the property or "land," until at some point during this growth the actual present value of the timber and land exceeds the agricultural values. At any time previous to this, it is justifiable

to destroy the young timber and clear the land, although in doing so a greater value is sacrificed or wasted than if stump land only were so used. The destruction of merchantable timber in former days was justified on this basis, since the value of the land for agriculture exceeded the total timber values present.

But pioneer conditions have passed, and the destruction of merchantable timber values is no longer thought of except where these pioneer conditions are reproduced by lack of transportation and markets. Even as the former settlers found that with increasing values the uncleared portions of the farm became a source of unexpected wealth, so in the future it will pay the settler to protect young growth and keep a portion of his farm in timber.

The mature timber upon agricultural land adds a forest value to the value of the stump land. The land is worth just as much more to the settler as the price he can get for his standing timber. The only drawback is that until this is sold and removed he cannot begin his clearing. But upon most claims there is enough poorly timbered soil to occupy the time and energies of the settlers until such sale is effected. Should there be no such openings, and the entire claim be heavily timbered, agricultural use is inevitably postponed until logging commences, for the relative value of the timber is so great as to preclude any other course.

250. National Forest Policy in Land Classification. — In classifying national forest land in pursuance of the policy of granting homesteads, land should not be listed for settlement while the total forest values exceed the agricultural value of stump land. This necessitates the sale and removal of mature timber from agricultural soils, previous to listing. But owing to the pressure for land throughout the west the tendency is at present to ignore the appraised value of immature timber and even the capital value of forest land, and to permit the listing of lands containing a certain amount of merchantable timber. This reservation is beneficial to the settler for it gives him a subsidy or value in addition to his land.

Since this procedure clearly sacrifices all forest values except

the present saleable timber, it will inevitably result in listing rapidly all lands of agricultural value whether or not they are covered with young trees, provided only that the merchantable stand is not too dense or valuable. If this policy is demanded by dearth of farm land in these regions it should be carried out, but the basis upon which the classification is made must be frankly recognized as expediency rather than sound appraisals of value.

251. Reciprocal Values in Forestry and Agriculture. — Forest and agricultural values cannot be considered as wholly separate. On every agricultural unit there is need of a certain amount of forest land to furnish wood for fuel, fence posts and even lumber, and as shelter for stock and windbreaks. In treeless prairies the planted grove greatly enhances the desirability of the whole property as a residence and adds in this way far more to the sale value of the farm than is accounted for by the possible revenue which this grove could produce. In wooded regions the labor of clearing is so great that two or even three decades must elapse before the entire farm will be brought under cultivation by the efforts of the farmer, unless aided by investing the necessary capital in clearing on a large scale. Meanwhile the woodlots will grow at an appreciable rate and young timber at first but a few feet tall will become large enough for use. In many regions this wooded portion may also be grazed without undue injury to the trees, if the latter have sufficient height.

Every effort should be made by owners of such tracts to spare and encourage areas of reproduction and of young timber, more especially when such growth occurs on the poorer portions, steeper slopes and more inaccessible parts of the farm. In this way each part of the farm unit is put to its best use, the maximum returns are obtained, and the sale value of the entire property will usually exceed the sum of the forest and agricultural values when separately considered.

APPENDIX

Formulæ.

Summary of Formulæ of Compound Interest.

Definitions of Symbols.

Summary of Formulæ of Forest Valuations.

Compound Interest Tables (Table VI).

Tables of Logarithms (Table VII).

SUMMARY OF FORMULÆ OF COMPOUND INTEREST

Character of payments.	Values at the end, future values, cost.	Values at the beginning, present values, capitalized income, capital value, expectation value.
Single sums.....	$V_0 \times 1.0p^n$ (I)	$\frac{V_n}{1.0p^n}$ (II)
Temporary rentals, annual	$\frac{r(1.0p^n - 1)}{0.0p}$ (III)	$\frac{r(1.0p^n - 1)}{0.0p(1.0p^n)}$ (IV)
intermittent	$\frac{r'(1.0p^{nt} - 1)}{1.0p^n - 1}$ (V)	$\frac{r'(1.0p^{nt} - 1)}{(1.0p^n - 1)1.0p^{nt}}$ (VI)
	$\frac{r'(1.0p^{nt} - 1)1.0p^{n-a}}{1.0p^n - 1}$ (VII)	$\frac{r'(1.0p^{nt} - 1)1.0p^{n-a}}{(1.0p^n - 1)1.0p^{nt}}$ (VIII)
Perpetual rentals, intermittent	∞	$\frac{r'}{1.0p^n - 1}$ (IX)
		$\frac{r'(1.0p^{n-a})}{1.0p^n - 1}$ (X)
		$\frac{r'(1.0p^n)}{1.0p^n - 1}$ (XI)
annual	∞	$\frac{r}{0.0p}$ (XII)
Conversion of intermittent into annual rentals, $\frac{r'}{1.0p^n - 1} \times 0.0p$ (XIII)		
Ratio of income to capital investment, $x = 100 \left(\sqrt[n]{\frac{V_n}{V_0}} - 1 \right)$ (XIV)		

SUPPLEMENTARY FORMULÆ

$$\left\{ \frac{r}{0.0p} (1.0p^a - 1) \right\} 1.0p^b + \frac{r_1}{0.0p} (1.0p^b - 1). \quad (\text{IIIa})$$

$$R (1.0p^n - 1). \quad (\text{IIIb})$$

$$\frac{r' (1.0p^a)}{1.0p^n - 1}. \quad (\text{Xa})$$

$$r' + \frac{r'}{1.0p^n - 1}. \quad (\text{XIa})$$

DEFINITIONS OF SYMBOLS

- a* — Year, previous to *n*, for which value or cost of property is sought.
- d* — $\frac{1.0p^n - 1}{0.0p}$, ratio used in Formulæ P₃ and P₄ in determining the amount (*Z*) to be written off annually for profit and depreciation.
- e* — Annual expenses, constituting one year's net outlay.
- g* — First term of a geometric series.
- m* — Interval between intermittent yields in a many-aged forest.
- n* — Period required to produce a crop of timber. Rotation.
Interval between yields produced by even-aged stands of timber.
- p* — Standard rate of interest in forestry.
- q* — Ratio, or multiple, for a geometric series.
- q%* — Per cent of legitimate profit to be allowed in logging and milling, or in logging alone, in stumpage appraisals.
- r* — Annual rental, regarded either as income or outlay.
- r'* — Intermittent rental, due at regular intervals greater than one year.
- r%* — Per cent of legitimate profit to be allowed in milling alone, in stumpage appraisals.
- s* — Per cent of overrun per thousand board feet, log scale.
- t* — Number of payments expected in a temporary intermittent rental.

- x — Rate of dividends or interest earned by an investment.
 z — Per cent of value to be written off annually for depreciation in Formula P_2 .
 A — Original capital investment in assets subject to depreciation.
 C — Cost of initial silvicultural and protective measures, incurred in first year.
 C_d, C_e, C_f — Cost of silvicultural and protective measures, incurred in subsequent years, d, e and f .
 D — Total amount to be written off for depreciation during the life of an enterprise.
 E — Expectation or capital value of annual expenses, e , by Formula XII, $\frac{e}{0.0p}$.
 aF_c — Total cost of forest property, including both land and growing stock or timber, for the year a . For year n , substitute nF_c or F_c .
 F_s — Sale value of forest property, both land and timber, in the year n or when timber is mature.
 aF_s — Expectation value of this sale value for the year a , previous to the year n or year of sale.
 F_v — Expectation value (capital value) of forest property, including both land and timber. Applicable to both even-aged and many-aged stands.
 aF_v — Expectation value of forest property for year a previous to year n .
 G_c — Cost of growing stock or standing timber.
 aG_v — Expectation value of growing stock, or standing timber, separate from land, for the year a previous to year n .
 L — Total amount of loss or damages to forest property.
 LC — Logging costs.
 LD — Depreciation of logging investments.
 MC — Milling costs.
 MD — Depreciation of milling investments.
 P — Profit or enterpriser's gain, remaining after paying interest at p per cent on capital investment.
 Q — Sum of terms in a geometric series.

- R — Expectation or capital value of the sum of payments constituting a rental.
- S — Sale value of lumber or of logs. Value of soil.
- S_c — Cost of soil and of permanent improvements.
- S_s — Sale value of soil.
- S_v — Expectation or capital value of forest soil.
- T_p, T_q, T_r — Net income from sale of thinnings in years p, q and r .
- V — Value of a single sum.
- V_0 — Value of sum at beginning of period.
- V_a — Future value in year a of the sum of items of income and expense on a crop of timber.
- V_n — Value of a single sum at end of a period of n years.
- V_r — Future value of the sum of payments constituting a rental.
- W — Final, wrecking or residual value of assets in logging or milling, after writing off total depreciation.
- X — Annual sum required to be written off for depreciation.
- Y — Sale value, or appraised value, of timber stumpage.
- Y' — Sale value, or appraised value, of timber stumpage, resulting from fire or other damage.
- Y_m — Stumpage value of 1000 board feet of manufactured lumber, used as basis to determine Y .
- Z — Annual sum which combines income or profit on invested capital with depreciation, the sum constituting one of a series of equal annual payments.

SUMMARY OF FORMULÆ IN FOREST VALUATION

COSTS

Cost of a single even-aged stand of timber.

	INDEX LETTER	ART.
$G_c = (C + S_c + E) 1.0p^n - (S_c + E)$	A	105
$G_c = (C + S_c + E) 1.0p^n + C_d(1.0p^{n-d}) + C_e(1.0p^{n-e})$ $+ C_f(1.0p^{n-f}) - T_p(1.0p^{n-p}) - T_q(1.0p^{n-q})$ $- T_r(1.0p^{n-r}) - (S_c + E)$	A ₁	105

Cost of a forest, including timber and soil, for year a .

${}^aF_c = (C + S_c + E) 1.0p^a - E$	A ₂	105
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Cost of a normal many-aged forest.

$$G_c = \frac{(C + S_c + E)(1.0p^n - 1)}{0.0p} - n(S_c + E). \quad A_3 \quad 109$$

VALUES

Sale value of a single even-aged stand in year n , including value of soil.

$$F_s = Y + S_s. \quad B \quad 112$$

Expectation or capital value of above for year a , previous to year n .

$${}^aF_s = \frac{Y + S_s + E}{1.0p^{n-a}} - E. \quad B_1 \quad 112$$

$${}^aF_s = \frac{Y + S_s + E + T_p(1.0p^{n-p}) + T_q(1.0p^{n-q}) + T_r(1.0p^{n-r})}{1.0p^{n-a}} - E. \quad B_2 \quad 112$$

Expectation or capital value of forest property in the year n , previous to removal of crop.

$$F_v = Y + \frac{Y - C}{1.0p^n - 1} - (C + E). \quad C \quad 114$$

$$F_v = Y - (C + E) + \frac{\begin{cases} Y + T_p(1.0p^{n-p}) + T_q(1.0p^{n-q}) \\ + T_r(1.0p^{n-r}) - C - C_d(1.0p^{n-d}) \\ - C_e(1.0p^{n-e}) - C_f(1.0p^{n-f}) \end{cases}}{1.0p^n - 1} \quad C_1 \quad 114$$

Expectation or capital value of forest property in the year a , previous to year n .

$${}^aF_v = \frac{(Y - C) 1.0p^a}{1.0p^n - 1} - E. \quad D \quad 115$$

$${}^aF_v = \frac{\begin{cases} Y + \frac{T_p}{1.0p^p} + \frac{T_q}{1.0p^q} + \frac{T_r(1.0p^n)}{1.0p^r} - C \\ - \frac{C_d(1.0p^n)}{1.0p^d} - \frac{C_e(1.0p^n)}{1.0p^e} - \frac{C_f(1.0p^n)}{1.0p^f} \end{cases} 1.0p^a}{1.0p^n - 1} - E. \quad D_1 \quad 115$$

$${}^aF_v = \frac{Y + S_v + E}{1.0p^{n-a}} - E. \quad D_2 \quad 116$$

Expectation or capital value of forest soil.

$$S_v = \frac{Y - C}{1.0p^n - 1} - (C + E). \quad \dots \quad E \quad 116$$

$$S_v = \frac{\begin{cases} Y + T_p(1.0p^{n-p}) + T_q(1.0p^{n-q}) + T_r(1.0p^{n-r}) \\ - C - C_d(1.0p^{n-d}) - C_e(1.0p^{n-e}) - C_f(1.0p^{n-f}) \end{cases}}{1.0p^n - 1} - (C + E)$$

E₁ 116

$$\text{Soil rent} = S_v \times 0.0p. \quad \dots \quad E_2 \quad 116$$

Expectation or capital value of a many-aged forest
in year m , just subsequent to removal of crop.

$$F_v = \frac{Y - C}{1.0p^m - 1} - (C + E). \quad \dots \quad F \quad 117$$

Expectation or capital value of a many-aged forest
in year a , previous to year m .

$${}^aF_v = \frac{Y + F_v + E}{1.0p^{m-a}} - E. \quad \dots \quad F_1 \quad 117$$

Expectation or capital value of a many-aged forest
producing annual income.

$$F_v = \frac{Y + T_p + T_q + T_r - \{C + C_d + C_e + C_f + n(e)\}}{0.0p}, F_2 \quad 117$$

Expectation or capital value of a many-aged forest
for irregular yields and periods.

$$F_v = \frac{\frac{Y-C}{x}(1.0p^{n-a}) + \frac{Y-C}{y}(1.0p^{n-b}) + \frac{Y-C}{z}(1.0p^{n-c})}{1.0p^n - 1} - E$$

F₃ 118

Expectation or capital value of growing stock or
timber separate from soil, in year a , previous to
year n , for a single even-aged stand.

$${}^aG_v = \frac{Y + S_v + E}{1.0p^{n-a}} - (S_v + E). \quad \dots \quad G \quad 119$$

Expectation or capital value of growing stock or timber separate from soil, in year a , previous to year m , for a many-aged stand.

$${}^aG_v = \frac{Y + F_v + E}{1.0p^{m-a}} - (S_v + E). \quad . \quad . \quad . \quad . \quad . \quad G_1 \quad 119$$

PROFITS

Profits in destructive lumbering, on timber and land, for year n or at time of sale of timber.

$$P = Y + S_s + E - (C + S_c + E) 1.0p^n \quad . \quad . \quad . \quad H \quad 122$$

$$P = Y + S_s + E + T_p (1.0p^{n-p}) + T_q (1.0p^{n-q}) + T_r (1.0p^{n-r}) - (C + S_c + E) 1.0p^n - C_d (1.0p^{n-d}) - C_e (1.0p^{n-e}) - C_f (1.0p^{n-f}). \quad H_1 \quad 122$$

Profits in destructive lumbering, on timber alone, for year n .

$$P = Y + S_c + E - (C + S_c + E) 1.0p^n. \quad . \quad . \quad . \quad K \quad 123$$

Profits in destructive lumbering, on timber alone, for year a , previous to year n .

$$P = \frac{Y + S_c + E}{1.0p^{n-a}} - (C + S_c + E) 1.0p^a. \quad . \quad . \quad . \quad K_1 \quad 124$$

$$P = \frac{Y + S_c + E}{1.0p^{n-a}} + \frac{T_p}{1.0p^{p-a}} + \frac{T_q}{1.0p^{q-a}} + \frac{T_r}{1.0p^{r-a}} - (C + S_c + E) 1.0p^a - C_d (1.0p^{a-d}) - C_e (1.0p^{a-e}) - C_f (1.0p^{a-f}). \quad . \quad . \quad . \quad K_2 \quad 124$$

Profits in continuous forest production, for year n .

$$P = Y + S_v + E - (C + S_c + E) 1.0p^n. \quad . \quad . \quad . \quad L \quad 125$$

Profits in continuous forest production, for year a , previous to year n .

$$P = \frac{Y + S_v + E}{1.0p^{n-a}} - (C + S_c + E) 1.0p^a \quad . \quad . \quad . \quad L_1 \quad 126$$

$$P = {}^aF_v - {}^aF_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad L_2 \quad 126$$

$$P = (S_c - S_v) 1.0p^a. \quad . \quad . \quad . \quad . \quad . \quad . \quad L_4 \quad 126$$

Profits in continuous forest production for year o , previous to origin of crop.

$$P = S_v - S_c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad L_3 \quad 127$$

INTEREST EARNED

Rate of interest earned on soil capital.

$$1.0x^n = \frac{Y + S_v + E - (C + E) 1.0p^n}{S_c} \quad . \quad . \quad . \quad N \quad 129$$

Rate of interest earned on soil and initial investments.

$$1.0x^n = \frac{Y + S_v + E - E(1.0p^n)}{S_c + C} \quad . \quad . \quad . \quad . \quad N_1 \quad 129$$

Rate of interest earned on entire investment.

$$1.0x^n = \frac{Y + S_v + E}{S_c + C + E} \quad . \quad . \quad . \quad . \quad N_2 \quad 129$$

Mean annual forest per cent.

$$x = \frac{S_v}{S_c} \times p. \quad . \quad . \quad . \quad . \quad N_3 \quad 129$$

Current annual forest per cent.

$$x = \left(\frac{Y_{n+1} - Y_n - e}{F_c} \right) 100 \quad . \quad . \quad . \quad . \quad N_4 \quad 129$$

$$\text{or } x = \left(\frac{Y_{n+1} - Y_n - e}{S_c + Y_n} \right) 100. \quad . \quad . \quad . \quad . \quad N_5 \quad 129$$

Current annual forest per cent, for normal forest.

$$x = \frac{F_v}{F_c} \times p. \quad . \quad . \quad . \quad . \quad N_6 \quad 129$$

DAMAGES

Damages to mature timber.

$$L = Y - Y'. \quad . \quad . \quad . \quad . \quad O \quad 140$$

Damages to young timber for year a , previous to year n .

$$L = \frac{Y - Y'}{1.0p^{n-a}} \quad . \quad . \quad . \quad . \quad O_1 \quad 141$$

Damages to timber and soil, for year a .

$$L = \frac{Y + S_v + E}{1.0p^{n-a}} + C' + E' - (C + E). \quad . \quad . \quad . \quad O_2 \quad 142$$

DEPRECIATION

Depreciation, Method A.

$$X = \frac{D (0.0p)}{1.0p^n - 1} P \quad 175$$

Depreciation, Method B.

$$X = \frac{D}{n} = \frac{A - W}{n} P_1 \quad 175$$

Depreciation, Method C.

$$z = 100 \left(1 - \sqrt[n]{\frac{W}{A}} \right) P_2 \quad 175$$

Depreciation, Method D.

$$Z = \frac{A (1.0p^n) - W}{d} P_3 \quad 175$$

$$Z = A (0.0p) + \frac{D}{d} (\text{Hunter's formula}). P_4 \quad 175$$

GEOMETRIC SERIES

Sum of rentals, increasing temporary series.

$$Q = \frac{g (q^n - 1)}{q - 1} Q_1 \quad 80$$

Sum of rentals, decreasing temporary series.

$$Q = \frac{g (1 - q^n)}{1 - q} Q_2 \quad 80$$

Sum of rentals, increasing perpetual series.

$$Q = \infty Q_3 \quad 80$$

Sum of rentals, decreasing perpetual series.

$$Q = \frac{g}{1 - q} Q_4 \quad 80$$

STUMPAGE VALUES

Stumpage appraisal on basis of manufactured lumber;
profits, a per cent of operating costs.

$$Y_m = S - (LC + MC + D) - q\% (LC + MC + D) \quad R \quad 177$$

$$Y_m = S - (MC + MD + LC + LD) - r\% (MC + MD) - q\% (LC + LD). \quad R_1 \quad 177$$

Stumpage appraisal on basis of manufactured lumber;
profits, a per cent of sale value.

$$Y_m = \frac{S}{1 + q\%} - (LC + MC + D) \quad R_2 \quad 177$$

Stumpage appraisal on basis of manufactured lumber;
profits, a per cent of margin between operating costs
and sale value.

$$Y_m = q\% \{S - (LC + MC + D)\} \quad R_3 \quad 177$$

Stumpage appraisal on basis of manufactured lumber;
reduction to log scale.

$$Y = Y_m (1.05) \quad R_4 \quad 179$$

TABLE VI

COMPOUND INTEREST TABLES

The period, n years, in left-hand column.

Column I, Formula I. — Future value of \$1 at end of n years.

Column II, Formula II. — Present, expectation or capital value of \$1 due in n years.

Column III, Formula III. — Future value at end of n years of an annual rental of \$1 payable n times.

Column IV, Formula IV. — Present, expectation or capital value of an annual rental of \$1 payable n times.

$\frac{1}{2}$ PER CENT

<i>n</i> years.	I.	II.	III.	IV.
1	1.0050	.9950	1.0000	.9950
2	1.0100	.9901	2.0050	1.9851
3	1.0151	.9851	3.0150	2.9702
4	1.0202	.9802	4.0301	3.9505
5	1.0253	.9754	5.0503	4.9259
6	1.0304	.9705	6.0755	5.8904
7	1.0355	.9657	7.1059	6.8621
8	1.0407	.9609	8.1414	7.8230
9	1.0459	.9561	9.1821	8.7791
10	1.0511	.9513	10.2280	9.7304
11	1.0564	.9466	11.2792	10.6770
12	1.0617	.9419	12.3356	11.6189
13	1.0670	.9372	13.3972	12.5562
14	1.0723	.9326	14.4642	13.4887
15	1.0777	.9279	15.5365	14.4166
16	1.0831	.9233	16.6142	15.3399
17	1.0885	.9187	17.6973	16.2586
18	1.0939	.9141	18.7858	17.1728
19	1.0994	.9096	19.8797	18.0824
20	1.1049	.9051	20.9791	18.9874
21	1.1104	.9006	22.0840	19.8880
22	1.1160	.8961	23.1944	20.7841
23	1.1216	.8916	24.3104	21.6756
24	1.1272	.8872	25.4320	22.5629
25	1.1328	.8828	26.5591	23.4456
26	1.1385	.8784	27.6919	24.3240
27	1.1442	.8740	28.8304	25.1980
28	1.1499	.8697	29.9745	26.0677
29	1.1556	.8653	31.1244	26.9330
30	1.1614	.8610	32.2800	27.7941
31	1.1672	.8567	33.4414	28.6508
32	1.1730	.8525	34.6086	29.5033
33	1.1789	.8482	35.7817	30.3515
34	1.1848	.8440	36.9606	31.1955
35	1.1907	.8398	38.1454	32.0354
36	1.1967	.8356	39.3361	32.8710
37	1.2027	.8315	40.5328	33.7025
38	1.2087	.8274	41.7354	34.5299
39	1.2147	.8232	42.9441	35.3531
40	1.2208	.8191	44.1588	36.1722
41	1.2269	.8151	45.3796	36.9873
42	1.2330	.8110	46.6065	37.7983
43	1.2392	.8070	47.8306	38.6053
44	1.2454	.8030	49.0788	39.4082
45	1.2516	.7990	50.3242	40.2072
46	1.2579	.7950	51.5758	41.0022
47	1.2642	.7910	52.8337	41.7932
48	1.2705	.7871	54.0978	42.5803
49	1.2768	.7832	55.3683	43.3635
50	1.2832	.7793	56.6452	44.1428
55	1.3156	.7601	63.1200	47.9782
60	1.3488	.7414	69.7600	51.7020
65	1.3829	.7231	76.5800	55.2764
70	1.4178	.7053	83.5600	58.9304
75	1.4536	.6879	90.7200	62.4745
80	1.4903	.6710	98.0600	65.7988
85	1.5280	.6545	105.6000	69.1099
90	1.5665	.6383	113.3000	72.3268
95	1.6061	.6226	121.2200	75.4747
100	1.6467	.6073	129.3400	78.5449
105	1.6882	.5923	137.6400	81.5306
110	1.7309	.5777	146.1800	84.4531
115	1.7746	.5635	154.9200	87.2985
120	1.8194	.5496	163.8800	90.0736

1 PER CENT

n years.	I.	II.	III.	IV.
1	1.0100	.9901	1.0000	.9901
2	1.0201	.9803	2.0100	1.9704
3	1.0303	.9706	3.0301	2.9410
4	1.0406	.9610	4.0604	3.9020
5	1.0510	.9515	5.1010	4.8534
6	1.0615	.9420	6.1520	5.7955
7	1.0721	.9327	7.2135	6.7282
8	1.0829	.9235	8.2857	7.6517
9	1.0937	.9143	9.3685	8.5660
10	1.1046	.9053	10.4622	9.4713
11	1.1157	.8963	11.5668	10.3676
12	1.1268	.8874	12.6825	11.2551
13	1.1381	.8787	13.8093	12.1337
14	1.1495	.8700	14.9474	13.0037
15	1.1610	.8613	16.0969	13.8651
16	1.1726	.8528	17.2579	14.7179
17	1.1843	.8444	18.4304	15.5622
18	1.1961	.8360	19.6147	16.3983
19	1.2081	.8277	20.8109	17.2260
20	1.2202	.8195	22.0190	18.0456
21	1.2324	.8114	23.2392	18.8570
22	1.2447	.8034	24.4716	19.6604
23	1.2572	.7954	25.7163	20.4558
24	1.2697	.7876	26.9735	21.2434
25	1.2824	.7798	28.2432	22.0232
26	1.2953	.7720	29.5256	22.7952
27	1.3082	.7644	30.8209	23.5596
28	1.3213	.7568	32.1291	24.3164
29	1.3345	.7493	33.4504	25.0658
30	1.3478	.7419	34.7849	25.8077
31	1.3613	.7346	36.1327	26.5423
32	1.3749	.7273	37.4941	27.2696
33	1.3887	.7201	38.8690	27.9897
34	1.4026	.7130	40.2577	28.7027
35	1.4166	.7059	41.6603	29.4086
36	1.4308	.6989	43.0769	30.1075
37	1.4451	.6920	44.5076	30.7995
38	1.4595	.6852	45.9527	31.4847
39	1.4741	.6784	47.4123	32.1630
40	1.4889	.6717	48.8864	32.8347
41	1.5038	.6650	50.3752	33.4997
42	1.5188	.6584	51.8790	34.1581
43	1.5340	.6519	53.3978	34.8100
44	1.5493	.6454	54.9318	35.4554
45	1.5648	.6391	56.4811	36.0945
46	1.5805	.6327	58.0459	36.7272
47	1.5963	.6265	59.6263	37.3537
48	1.6122	.6203	61.2226	37.9740
49	1.6283	.6141	62.8348	38.5881
50	1.6446	.6080	64.4632	39.1961
55	1.7284	.5786	72.8400	42.1430
60	1.8166	.5505	81.6600	44.9521
65	1.9093	.5238	90.9300	47.6247
70	2.0066	.4983	100.6600	50.1644
75	2.1090	.4742	110.9000	52.5841
80	2.2166	.4500	121.6600	54.8858
85	2.3296	.4292	132.9600	57.0742
90	2.4485	.4084	144.8500	59.1750
95	2.5733	.3886	157.3300	61.1394
100	2.7046	.3697	170.4600	63.0259
105	2.8425	.3518	184.2500	64.8197
110	2.9875	.3347	198.7500	66.5272
115	3.1399	.3185	213.9900	68.1518
120	3.3001	.3030	230.0100	69.6979

$1\frac{1}{2}$ PER CENT

<i>n</i> years.	I.	II.	III.	IV.
1	1.0150	.9852	1.0000	.9852
2	1.0302	.9707	2.0150	1.9559
3	1.0457	.9563	3.0452	2.9122
4	1.0614	.9422	4.0909	3.8544
5	1.0773	.9283	5.1523	4.7826
6	1.0934	.9145	6.2290	5.6972
7	1.1098	.9010	7.3230	6.5982
8	1.1265	.8877	8.4328	7.4859
9	1.1434	.8746	9.5593	8.3605
10	1.1605	.8617	10.9027	9.2222
11	1.1779	.8489	11.8633	10.0711
12	1.1959	.8364	13.0412	10.9075
13	1.2130	.8240	14.2368	11.7315
14	1.2318	.8118	15.4504	12.5434
15	1.2502	.7999	16.6821	13.3432
16	1.2690	.7880	17.9324	14.1313
17	1.2880	.7764	19.2014	14.9076
18	1.3073	.7649	20.4894	15.6726
19	1.3270	.7536	21.7907	16.4262
20	1.3469	.7425	23.1237	17.1686
21	1.3671	.7315	24.4705	17.9001
22	1.3876	.7207	25.8376	18.6208
23	1.4084	.7100	27.2251	19.3309
24	1.4295	.6995	28.6335	20.0304
25	1.4509	.6892	30.0630	20.7196
26	1.4727	.6790	31.5140	21.3986
27	1.4948	.6690	32.9867	22.0676
28	1.5172	.6591	34.4815	22.7267
29	1.5400	.6494	35.9987	23.3761
30	1.5631	.6398	37.5387	24.0158
31	1.5865	.6303	39.1018	24.6461
32	1.6103	.6210	40.6883	25.2671
33	1.6345	.6118	42.2986	25.8790
34	1.6590	.6028	43.9331	26.4817
35	1.6839	.5939	45.5921	27.0756
36	1.7091	.5851	47.2760	27.6607
37	1.7348	.5764	48.9851	28.2371
38	1.7608	.5679	50.7109	28.8051
39	1.7872	.5595	52.4607	29.3646
40	1.8140	.5513	54.2679	29.9158
41	1.8412	.5431	56.0819	30.4590
42	1.8688	.5351	57.9231	30.9940
43	1.8969	.5272	59.7920	31.5212
44	1.9253	.5194	61.6889	32.0406
45	1.9542	.5117	63.6142	32.5523
46	1.9835	.5042	65.5684	33.0565
47	2.0133	.4967	67.5519	33.5532
48	2.0435	.4894	69.5652	34.0426
49	2.0741	.4821	71.6087	34.5247
50	2.1052	.4750	73.6828	34.9997
55	2.2679	.4409	84.5296	37.2715
60	2.4432	.4093	96.2147	39.3803
65	2.6320	.3799	108.8000	41.3373
70	2.8355	.3527	122.3640	43.1549
75	3.0546	.3274	136.9670	44.8409
80	3.2907	.3039	152.7110	46.4073
85	3.5450	.2821	169.6600	47.8603
90	3.8189	.2618	187.9300	49.2090
95	4.1141	.2431	207.6000	50.4618
100	4.4320	.2256	228.8030	51.6247
105	4.7746	.2094	251.6330	52.7036
110	5.1436	.1944	276.2380	53.7055
115	5.5411	.1805	302.7330	54.6351
120	5.9693	.1675	331.2880	55.4985

2 PER CENT

n years.	I.	II.	III.	IV.
1	1.0200	.9804	1.0000	.9804
2	1.0404	.9612	2.0200	1.9416
3	1.0612	.9423	3.0604	2.8839
4	1.0824	.9238	4.1216	3.8077
5	1.1041	.9056	5.2040	4.7135
6	1.1262	.8880	6.3081	5.6014
7	1.1487	.8706	7.4343	6.4720
8	1.1717	.8535	8.5830	7.3255
9	1.1951	.8368	9.7546	8.1622
10	1.2190	.8203	10.9497	8.9826
11	1.2434	.8043	12.1687	9.7868
12	1.2682	.7885	13.4121	10.5753
13	1.2936	.7730	14.6803	11.3484
14	1.3195	.7579	15.9739	12.1062
15	1.3459	.7430	17.2934	12.8493
16	1.3728	.7284	18.6393	13.5777
17	1.4002	.7142	20.0121	14.2919
18	1.4282	.7002	21.4123	14.9920
19	1.4568	.6864	22.8406	15.6785
20	1.4859	.6730	24.2974	16.3514
21	1.5157	.6598	25.7833	17.0112
22	1.5460	.6468	27.2990	17.6580
23	1.5769	.6342	28.8450	18.2922
24	1.6084	.6217	30.4219	18.9139
25	1.6406	.6095	32.0303	19.5235
26	1.6734	.5976	33.6709	20.1210
27	1.7069	.5859	35.3443	20.7069
28	1.7410	.5744	37.0512	21.2813
29	1.7758	.5631	38.7922	21.8444
30	1.8114	.5521	40.5681	22.3965
31	1.8476	.5412	42.3794	22.9377
32	1.8845	.5306	44.2270	23.4683
33	1.9222	.5202	46.1116	23.9886
34	1.9607	.5100	48.0338	24.4986
35	1.9999	.5000	49.9945	24.9986
36	2.0399	.4902	51.9944	25.4888
37	2.0807	.4806	54.0343	25.9695
38	2.1223	.4712	56.1149	26.4406
39	2.1647	.4619	58.2372	26.9026
40	2.2080	.4529	60.4020	27.3555
41	2.2522	.4440	62.6100	27.7995
42	2.2972	.4353	64.8622	28.2348
43	2.3432	.4268	67.1595	28.6616
44	2.3901	.4184	69.5027	29.0800
45	2.4379	.4102	71.8927	29.4902
46	2.4866	.4022	74.3306	29.8923
47	2.5363	.3943	76.8172	30.2866
48	2.5871	.3865	79.3535	30.6731
49	2.6388	.3790	81.9406	31.0521
50	2.6916	.3715	84.5794	31.4236
55	2.9717	.3365	98.5865	33.1748
60	3.2810	.3048	114.0520	34.7609
65	3.6225	.2760	131.1250	36.1973
70	3.9995	.2500	149.9780	37.4986
75	4.4158	.2265	170.7900	38.6763
80	4.8754	.2051	193.7720	39.7445
85	5.3828	.1858	219.1400	40.7111
90	5.9431	.1683	247.1570	41.5869
95	6.5617	.1524	278.0850	42.3800
100	7.2446	.1380	312.2320	43.0984
105	7.9987	.1250	349.9300	43.7489
110	8.8312	.1132	391.5590	44.3382
115	9.7503	.1026	437.5150	44.8719
120	10.7652	.0929	488.2580	45.3554

$2\frac{1}{2}$ PER CENT

<i>n</i> years.	I.	II.	III.	IV.
1	1.0250	.9756	1.0000	.9756
2	1.0506	.9518	2.0250	1.9274
3	1.0769	.9286	3.0756	2.8560
4	1.1038	.9060	4.1525	3.7620
5	1.1314	.8839	5.2563	4.6458
6	1.1597	.8623	6.3877	5.5081
7	1.1887	.8413	7.5474	6.3494
8	1.2184	.8207	8.7361	7.1701
9	1.2489	.8007	9.9545	7.9799
10	1.2801	.7812	11.2034	8.7521
11	1.3121	.7621	12.4835	9.5142
12	1.3449	.7436	13.7956	10.2578
13	1.3785	.7254	15.1404	10.9832
14	1.4130	.7077	16.5190	11.6909
15	1.4483	.6905	17.9319	12.3814
16	1.4845	.6736	19.3802	13.0550
17	1.5216	.6572	20.8647	13.7122
18	1.5597	.6412	22.3863	14.3534
19	1.5987	.6255	23.9460	14.9789
20	1.6386	.6103	25.5447	15.5892
21	1.6796	.5954	27.1833	16.1845
22	1.7216	.5809	28.8629	16.7654
23	1.7646	.5667	30.5844	17.3321
24	1.8087	.5529	32.3490	17.8850
25	1.8539	.5394	34.1578	18.4244
26	1.9003	.5262	36.0117	18.9506
27	1.9478	.5134	37.9120	19.4640
28	1.9965	.5009	39.8598	19.9649
29	2.0464	.4887	41.8563	20.4535
30	2.0976	.4767	43.9027	20.9303
31	2.1500	.4651	46.0003	21.3954
32	2.2038	.4538	48.1503	21.8492
33	2.2589	.4427	50.3540	22.2919
34	2.3153	.4319	52.6129	22.7238
35	2.3732	.4214	54.9282	23.1452
36	2.4325	.4111	57.3014	23.5563
37	2.4933	.4011	59.7339	23.9573
38	2.5557	.3913	62.2273	24.3486
39	2.6196	.3817	64.7830	24.7303
40	2.6851	.3724	67.4026	25.1028
41	2.7522	.3633	70.0876	25.4661
42	2.8210	.3545	72.8398	25.8206
43	2.8915	.3458	75.6608	26.1664
44	2.9638	.3374	78.5523	26.5038
45	3.0379	.3292	81.5161	26.8330
46	3.1139	.3211	84.5540	27.1542
47	3.1917	.3133	87.6679	27.4675
48	3.2715	.3057	90.8596	27.7732
49	3.3533	.2982	94.1311	28.0714
50	3.4371	.2909	97.4843	28.3623
55	3.8888	.2571	115.551	29.7140
60	4.3998	.2273	135.992	30.9087
65	4.9780	.2009	159.120	31.963
70	5.6321	.1775	185.284	32.898
75	6.3722	.1569	214.888	33.645
80	7.2096	.1387	248.383	34.452
85	8.1570	.1226	286.280	35.096
90	9.2289	.1084	329.154	35.666
95	10.4416	.0958	377.664	36.171
100	11.8137	.0846	432.549	36.614
105	13.3661	.0748	494.644	37.007
110	15.1226	.0661	564.902	37.355
115	17.1098	.0584	644.392	37.664
120	19.3581	.0517	734.326	37.934

3 PER CENT

n years.	I.	II.	III.	IV.
1	1.0300	.9709	1.0000	.9709
2	1.0609	.9426	2.0300	1.9135
3	1.0927	.9151	3.0909	2.8286
4	1.1255	.8885	4.1836	3.7171
5	1.1593	.8626	5.3091	4.5797
6	1.1941	.8375	6.4684	5.4172
7	1.2299	.8131	7.6625	6.2303
8	1.2668	.7894	8.8923	7.0197
9	1.3048	.7664	10.1591	7.7861
10	1.3439	.7441	11.4639	8.5302
11	1.3842	.7224	12.8075	9.2526
12	1.4258	.7014	14.1920	9.9540
13	1.4685	.6810	15.6178	10.6350
14	1.5126	.6611	17.0863	11.2961
15	1.5580	.6419	18.5989	11.9379
16	1.6047	.6232	20.1569	12.5611
17	1.6528	.6050	21.7616	13.1661
18	1.7024	.5874	23.4144	13.7535
19	1.7535	.5703	25.1169	14.3238
20	1.8061	.5537	26.8704	14.8775
21	1.8603	.5375	28.6765	15.4150
22	1.9161	.5219	30.5368	15.9369
23	1.9736	.5067	32.4529	16.4436
24	2.0328	.4919	34.4265	16.9325
25	2.0938	.4776	36.4593	17.4131
26	2.1566	.4637	38.5530	17.8768
27	2.2213	.4502	40.7096	18.3270
28	2.2879	.4371	42.9300	18.7641
29	2.3566	.4243	45.2189	19.1885
30	2.4273	.4120	47.5754	19.6004
31	2.5001	.4000	50.0027	20.0004
32	2.5751	.3883	52.5028	20.3888
33	2.6523	.3770	55.0778	20.7658
34	2.7319	.3660	57.7302	21.1318
35	2.8139	.3554	60.4621	21.4872
36	2.8983	.3450	63.2759	21.8323
37	2.9852	.3350	66.1742	22.1672
38	3.0748	.3252	69.1594	22.4925
39	3.1670	.3158	72.2342	22.8082
40	3.2620	.3066	75.4013	23.1148
41	3.3599	.2976	78.6633	23.4124
42	3.4607	.2890	82.0232	23.7014
43	3.5645	.2805	85.4839	23.9819
44	3.6715	.2724	89.0484	24.2543
45	3.7816	.2644	92.7199	24.5187
46	3.8950	.2567	96.5015	24.7754
47	4.0119	.2493	100.3965	25.0247
48	4.1323	.2420	104.4084	25.2667
49	4.2562	.2350	108.5406	25.5017
50	4.3839	.2281	112.7969	25.7298
55	5.0821	.1968	136.072	26.7744
60	5.8916	.1697	163.053	27.6756
65	6.8300	.1464	194.333	28.452
70	7.9178	.1263	230.594	29.123
75	9.1789	.1089	272.630	29.702
80	10.6409	.0940	321.363	30.201
85	12.3357	.0811	377.857	30.701
90	14.3005	.0699	443.349	31.002
95	16.5782	.0603	519.273	31.323
100	19.2186	.0520	607.288	31.599
105	22.2797	.0449	709.323	31.838
110	25.8282	.0387	827.608	32.043
115	29.9420	.0334	964.733	32.220
120	34.7110	.0288	1123.70	32.373

$3\frac{1}{2}$ PER CENT

<i>n</i> years.	I.	II.	III.	IV.
1	1.0350	.9662	1.0000	.9662
2	1.0712	.9335	2.0350	1.8997
3	1.1087	.9019	3.1062	2.8016
4	1.1475	.8714	4.2149	3.6731
5	1.1877	.8420	5.3625	4.5151
6	1.2293	.8135	6.5502	5.3286
7	1.2723	.7860	7.7794	6.1145
8	1.3168	.7594	9.0517	6.8740
9	1.3629	.7337	10.3685	7.6077
10	1.4106	.7089	11.7314	8.3166
11	1.4600	.6849	13.1420	9.0016
12	1.5111	.6618	14.6020	9.6633
13	1.5640	.6394	16.1130	10.3027
14	1.6187	.6178	17.6770	10.9205
15	1.6753	.5969	19.2957	11.5174
16	1.7340	.5767	20.9710	12.0941
17	1.7947	.5572	22.7050	12.6513
18	1.8575	.5384	24.4997	13.1897
19	1.9225	.5202	26.3572	13.7098
20	1.9898	.5026	28.2797	14.2124
21	2.0594	.4856	30.2695	14.6980
22	2.1315	.4692	32.3289	15.1671
23	2.2061	.4533	34.4604	15.6204
24	2.2833	.4380	36.6665	16.0574
25	2.3632	.4231	38.9499	16.4815
26	2.4460	.4088	41.3131	16.8904
27	2.5316	.3950	43.7591	17.2854
28	2.6202	.3817	46.2906	17.6670
29	2.7119	.3687	48.9108	18.0358
30	2.8068	.3563	51.6227	18.3920
31	2.9050	.3442	54.4295	18.7363
32	3.0067	.3326	57.3345	19.0689
33	3.1119	.3213	60.3412	19.3902
34	3.2209	.3105	63.4532	19.7007
35	3.3336	.3000	66.6740	20.0007
36	3.4503	.2898	70.0076	20.2905
37	3.5710	.2800	73.4579	20.5705
38	3.6960	.2706	77.0289	20.8411
39	3.8254	.2614	80.7249	21.1025
40	3.9593	.2526	84.5503	21.3551
41	4.0978	.2440	88.5095	21.5991
42	4.2413	.2358	92.6074	21.8349
43	4.3897	.2278	96.8487	22.0627
44	4.5433	.2201	101.2383	22.2828
45	4.7024	.2127	105.7817	22.4955
46	4.8669	.2055	110.4840	22.7009
47	5.0373	.1985	115.3510	22.8994
48	5.2136	.1918	120.3883	23.0912
49	5.3961	.1853	125.6018	23.2766
50	5.5849	.1791	130.9979	23.4556
55	6.6331	.1508	160.947	24.2641
60	7.8781	.1260	196.517	24.9447
65	9.3567	.1060	238.763	25.5168
70	11.1128	.0900	288.938	26.0004
75	13.1986	.0758	348.531	26.4067
80	15.6757	.0638	419.307	26.7488
85	18.6179	.0537	503.368	27.0368
90	22.1122	.0452	603.205	27.2793
95	26.2623	.0381	721.780	27.4798
100	31.1914	.0321	862.612	27.6554
105	37.0456	.0270	1029.874	27.8002
110	43.9986	.0227	1228.53	27.9221
115	52.2565	.0191	1464.471	28.0247
120	62.0643	.0161	1744.69	28.1111

4 PER CENT

n years.	I.	II.	III.	IV.
1	1.0400	.9615	1.0000	.9615
2	1.0861	.9246	2.0400	1.8861
3	1.1249	.8890	3.1216	2.7751
4	1.1609	.8548	4.2465	3.6299
5	1.2167	.8219	5.4103	4.4518
6	1.2653	.7903	6.6330	5.2421
7	1.3159	.7599	7.8983	6.0021
8	1.3686	.7307	9.2142	6.7327
9	1.4233	.7026	10.5828	7.4353
10	1.4802	.6756	12.0061	8.1109
11	1.5395	.6496	13.4864	8.7605
12	1.6010	.6246	15.0258	9.3851
13	1.6651	.6006	16.6268	9.9856
14	1.7315	.5775	18.2919	10.5631
15	1.8009	.5553	20.0236	11.1184
16	1.8730	.5339	21.8245	11.6523
17	1.9479	.5134	23.6975	12.1657
18	2.0258	.4936	25.6454	12.6593
19	2.1068	.4746	27.6712	13.1339
20	2.1911	.4564	29.7781	13.5903
21	2.2788	.4388	31.9602	14.0292
22	2.3699	.4220	34.2480	14.4511
23	2.4647	.4057	36.6179	14.8568
24	2.5633	.3901	39.0826	15.2470
25	2.6658	.3751	41.6459	15.6221
26	2.7725	.3607	44.3117	15.9828
27	2.8834	.3468	47.0842	16.3296
28	2.9987	.3335	49.9676	16.6631
29	3.1187	.3207	52.9663	16.9837
30	3.2434	.3083	56.0849	17.2920
31	3.3731	.2965	59.3283	17.5885
32	3.5081	.2851	62.7015	17.8736
33	3.6484	.2741	66.2095	18.1476
34	3.7943	.2636	69.8579	18.4112
35	3.9461	.2534	73.6522	18.6646
36	4.1039	.2437	77.5983	18.9083
37	4.2681	.2343	81.7022	19.1426
38	4.4388	.2253	85.7993	19.3679
39	4.6164	.2166	90.4091	19.5845
40	4.8010	.2083	95.0255	19.7928
41	4.9931	.2003	99.8265	19.9931
42	5.1928	.1926	104.8200	20.1856
43	5.4005	.1852	110.0124	20.3708
44	5.6165	.1780	115.4129	20.5488
45	5.8412	.1712	121.0294	20.7200
46	6.0748	.1646	126.8706	20.8847
47	6.3178	.1583	132.9454	21.0429
48	6.5705	.1522	139.2632	21.1951
49	6.8333	.1463	145.8337	21.3415
50	7.1067	.1407	152.6671	21.4822
55	8.6464	.1157	191.159	22.1086
60	10.5196	.0951	237.991	22.6235
65	12.7987	.0781	294.967	23.0466
70	15.5716	.0642	364.290	23.3945
75	18.9453	.0528	448.642	23.6281
80	23.0498	.0434	551.245	23.9154
85	28.0436	.0357	676.090	24.1085
90	34.1193	.0293	827.983	24.2673
95	41.5114	.0241	1012.785	24.3977
100	50.5049	.0198	1237.622	24.5050
105	61.4470	.0163	1511.175	24.5931
110	74.7597	.0134	1843.992	24.6656
115	90.9566	.0110	2248.915	24.7251
120	110.663	.0090	2741.558	24.7741

4½ PER CENT

<i>n</i> years.	I.	II.	III.	IV.
1	1.0450	.9569	1.0000	.9569
2	1.0920	.9157	2.0450	1.8727
3	1.1412	.8765	3.1370	2.7490
4	1.1925	.8386	4.2782	3.5875
5	1.2462	.8022	5.4707	4.3900
6	1.3023	.7679	6.7169	5.1579
7	1.3609	.7348	8.0192	5.8927
8	1.4221	.7032	9.3800	6.5959
9	1.4861	.6729	10.8021	7.2688
10	1.5530	.6439	12.2882	7.9125
11	1.6229	.6162	13.8412	8.5289
12	1.6959	.4897	15.4640	9.1186
13	1.7722	.5643	17.1599	9.6829
14	1.8519	.5400	18.9321	10.2229
15	1.9353	.5167	20.7841	10.7395
16	2.0224	.4945	22.7193	11.2340
17	2.1134	.4732	24.7417	11.7072
18	2.2085	.4528	26.8551	12.1660
19	2.3079	.4333	29.0634	12.5933
20	2.4117	.4146	31.3716	13.0079
21	2.5202	.3968	33.7831	13.4047
22	2.6337	.3797	36.3034	13.7844
23	2.7522	.3634	38.9370	14.1478
24	2.8760	.3477	41.6892	14.4955
25	3.0054	.3327	44.5652	14.8282
26	3.1407	.3184	47.5706	15.1466
27	3.2820	.3047	50.7113	15.4513
28	3.4279	.2916	53.9933	15.7429
29	3.5840	.2790	57.4230	16.0219
30	3.7453	.2670	61.0071	16.2889
31	3.9139	.2555	64.7524	16.5444
32	4.0900	.2445	68.6662	16.7889
33	4.2740	.2340	72.7562	17.0229
34	4.4664	.2239	77.0303	17.2468
35	4.6673	.2143	81.4966	17.4610
36	4.8774	.2050	86.1640	17.6660
37	5.0969	.1962	91.0413	17.8622
38	5.3262	.1878	96.1382	18.0500
39	5.5659	.1797	101.4644	18.2297
40	5.8164	.1719	107.0303	18.4016
41	6.0781	.1645	112.8467	18.5661
42	6.3516	.1574	118.9248	18.7235
43	6.6374	.1507	125.2764	18.8742
44	6.9361	.1442	131.9138	19.0184
45	7.2482	.1380	138.8500	19.1563
46	7.5744	.1320	146.0982	19.2884
47	7.9153	.1263	153.6726	19.4147
48	8.2715	.1209	161.5879	19.5356
49	8.6437	.1157	169.8594	19.6513
50	9.0326	.1107	178.5030	19.7620
55	11.2563	.0888	227.9180	20.2480
60	14.0274	.0713	289.4980	20.6380
65	17.4807	.0572	366.2380	20.9509
70	21.7841	.0459	461.8700	21.2021
75	27.1470	.0368	581.2670	21.4118
80	33.8301	.0296	729.5580	21.5653
85	42.1585	.0237	914.6330	21.6951
90	52.5371	.0190	1145.2700	21.7992
95	65.4708	.0153	1432.6840	21.8828
100	81.5885	.0123	1790.8600	21.9499
105	101.674	.0098	2237.2000	22.0036
110	126.704	.0079	2703.4300	22.0468
115	157.897	.0063	3486.6000	22.0815
120	196.768	.0051	4350.4000	22.1093

5 PER CENT

<i>n</i> years.	I.	II.	III.	IV.
1	1.0500	.9524	1.0000	.9524
2	1.1025	.9070	2.0500	1.8504
3	1.1576	.8638	3.1525	2.7232
4	1.2155	.8227	4.3101	3.5460
5	1.2763	.7835	5.5256	4.3295
6	1.3401	.7462	6.8019	5.0757
7	1.4071	.7107	8.1420	5.7864
8	1.4775	.6768	9.5491	6.4632
9	1.5513	.6446	11.0266	7.1078
10	1.6289	.6139	12.5779	7.7217
11	1.7103	.5847	14.2068	8.3064
12	1.7959	.5568	15.9171	8.8623
13	1.8856	.5303	17.7130	9.3936
14	1.9799	.5051	19.5986	9.8986
15	2.0789	.4810	21.5786	10.3797
16	2.1829	.4581	23.6575	10.8378
17	2.2920	.4363	25.8404	11.2741
18	2.4066	.4155	28.1324	11.6896
19	2.5270	.3957	30.5390	12.0853
20	2.6533	.3769	33.0660	12.4622
21	2.7860	.3589	35.7193	12.8212
22	2.9253	.3418	38.5052	13.1630
23	3.0715	.3256	41.4305	13.4886
24	3.2251	.3101	44.5020	13.7986
25	3.3864	.2953	47.7271	14.0939
26	3.5557	.2812	51.1135	14.3752
27	3.7335	.2678	54.6691	14.6430
28	3.9201	.2551	58.4026	14.8981
29	4.1161	.2429	62.3227	15.1411
30	4.3219	.2314	66.4388	15.3725
31	4.5380	.2204	70.7608	15.5928
32	4.7649	.2099	75.2988	15.8027
33	5.0032	.1999	80.0638	16.0025
34	5.2533	.1904	85.0670	16.1929
35	5.5160	.1813	90.3203	16.3742
36	5.7918	.1727	95.8363	16.5469
37	6.0814	.1644	101.6281	16.7113
38	6.3855	.1566	107.7095	16.8679
39	6.7048	.1491	114.0950	17.0170
40	7.0400	.1420	120.7998	17.1591
41	7.3920	.1353	127.8398	17.2944
42	7.7616	.1288	135.2318	17.4232
43	8.1497	.1227	142.9933	17.5459
44	8.5572	.1169	151.1430	17.6628
45	8.9850	.1113	139.7002	17.7741
46	9.4343	.1060	168.6852	17.8801
47	9.9060	.1009	178.1194	17.9810
48	10.4013	.0961	188.0254	18.0772
49	10.9213	.0916	198.4267	18.1687
50	11.4674	.0872	209.3480	18.2559
55	14.6356	.0683	272.7130	18.6335
60	18.6792	.0535	353.5840	18.9293
65	23.8399	.0419	456.7980	19.1191
70	30.4264	.0329	588.5290	19.3427
75	38.8327	.0257	756.6540	19.4849
80	49.5614	.0202	971.2290	19.5965
85	63.2544	.0158	1245.0880	19.6838
90	80.7304	.0124	1594.6100	19.7523
95	103.035	.0097	2040.7000	19.8058
100	131.501	.0076	2610.0300	19.8479
105	167.833	.0060	3336.6600	19.8808
110	214.202	.0047	4264.0300	19.9066
115	273.382	.0037	5447.6400	19.9268
120	348.912	.0029	6958.2400	19.9427

$5\frac{1}{2}$ PER CENT

<i>n</i> years.	I.	II.	III.	IV.
1	1.0550	.9479	1.0000	.9479
2	1.1130	.8985	2.0550	1.8463
3	1.1742	.8516	3.1680	2.9979
4	1.2388	.8072	4.3423	3.5052
5	1.3070	.7651	5.5811	4.2703
6	1.3788	.7252	6.8881	4.9955
7	1.4547	.6854	8.2669	5.6830
8	1.5347	.6516	9.7216	6.3346
9	1.6191	.6176	11.2563	6.9522
10	1.7081	.5854	12.8754	7.5376
11	1.8021	.5549	14.5835	8.0925
12	1.9012	.5260	16.3856	8.6185
13	2.0058	.4986	18.2868	9.1171
14	2.1161	.4726	20.2926	9.5896
15	2.2325	.4479	22.4087	10.0376
16	2.3553	.4246	24.6411	10.4622
17	2.4848	.4024	26.9964	10.8646
18	2.6215	.3815	29.4812	11.2461
19	2.7656	.3616	32.1027	11.6077
20	2.9178	.3427	34.8683	11.9504
21	3.0782	.3249	37.7861	12.2752
22	3.2275	.3079	40.8643	12.5832
23	3.4262	.2919	44.1118	12.8750
24	3.6146	.2767	47.5380	13.1517
25	3.8134	.2622	51.1526	13.4139
26	4.0231	.2486	54.9660	13.6625
27	4.2444	.2356	58.9891	13.8981
28	4.4778	.2233	63.2335	14.1214
29	4.7241	.2117	67.7114	14.3331
30	4.9840	.2006	71.4355	14.5337
31	5.2581	.1902	77.4194	14.7239
32	5.5473	.1803	82.6775	14.9042
33	5.8524	.1709	88.2248	15.0751
34	6.1742	.1620	94.0771	15.2370
35	6.5138	.1535	100.2514	15.3906
36	6.8721	.1455	106.7652	15.5361
37	7.2501	.1379	113.6373	15.6740
38	7.6488	.1307	120.8873	15.8047
39	8.0695	.1239	128.5361	15.9287
40	8.5133	.1175	136.6056	16.0461
41	8.9815	.1113	145.1180	16.1575
42	9.4755	.1055	154.1005	16.2630
43	9.9907	.1000	163.5760	16.3630
44	10.5405	.0948	173.5727	16.4579
45	11.1266	.0899	184.1192	16.5477
46	11.7385	.0852	195.2457	16.6329
47	12.3841	.0807	206.9842	16.7137
48	13.0653	.0765	219.3684	16.7902
49	13.7838	.0725	232.4336	16.8628
50	14.5420	.0688	246.2175	16.9315
55	19.0046	.0526	327.3563	17.2251
60	24.8381	.0403	433.4200	17.4498
65	32.4623	.0308	572.0364	17.6216
70	43.4150	.0230	771.1818	17.7630
75	56.7414	.0176	1,013.4800	17.8614
80	72.4703	.0138	1,299.4600	17.9309
85	94.7152	.0106	1,703.9127	17.9898
90	123.7883	.0081	2,232.5145	18.0349
95	161.7855	.0062	2,923.3727	18.0694
100	211.4463	.0047	3,826.2963	18.0958
105	276.3503	.0036	5,006.3691	18.1160
110	361.2768	.0028	6,550.4873	18.1315
115	472.0413	.0021	8,564.3873	18.1433
120	616.9357	.0016	11,198.8309	18.1523

6 PER CENT

n years.	I.	II.	III.	IV.
1	1.0600	.9434	1.0000	.9434
2	1.1236	.8900	2.0600	1.8334
3	1.1910	.8396	3.1836	2.6730
4	1.2625	.7921	4.3746	3.4651
5	1.3382	.7473	5.6371	4.2124
6	1.4185	.7050	6.9753	4.9173
7	1.5036	.6651	8.3938	5.5824
8	1.5938	.6274	9.8975	6.2098
9	1.6895	.5919	11.4913	6.8017
10	1.7908	.5584	13.1808	7.3601
11	1.8983	.5268	14.9716	7.8860
12	2.0122	.4970	16.8699	8.3838
13	2.1329	.4688	18.8821	8.8527
14	2.2609	.4423	21.0151	9.2950
15	2.3966	.4173	23.2760	9.7122
16	2.5404	.3936	25.6725	10.1059
17	2.6928	.3714	28.2129	10.4772
18	2.8543	.3503	30.9057	10.8276
19	3.0256	.3305	33.7600	11.1581
20	3.2071	.3118	36.7856	11.4699
21	3.3996	.2942	39.9927	11.7641
22	3.6035	.2775	43.3923	12.0416
23	3.8197	.2618	46.9958	12.3034
24	4.0489	.2470	50.8156	12.5504
25	4.2919	.2330	54.8645	12.7834
26	4.5494	.2198	59.1564	13.0032
27	4.8223	.2074	63.7058	13.2105
28	5.1117	.1956	68.5281	13.4062
29	5.4184	.1846	73.6398	13.5907
30	5.7435	.1741	79.0582	13.7648
31	6.0881	.1643	84.8017	13.9291
32	6.4534	.1550	90.8898	14.0840
33	6.8406	.1462	97.3432	14.2302
34	7.2510	.1379	104.1838	14.3681
35	7.6861	.1301	111.4348	14.4982
36	8.1473	.1227	119.1209	14.6210
37	8.6361	.1158	127.2681	14.7368
38	9.1543	.1092	135.9042	14.8460
39	9.7035	.1031	145.0585	14.9491
40	10.2857	.0972	154.7620	15.0463
41	10.9029	.0917	165.0477	15.1380
42	11.5570	.0865	175.9505	15.2245
43	12.2505	.0816	187.5076	15.3062
44	12.9855	.0770	199.7580	15.3832
45	13.7646	.0727	212.7435	15.4558
46	14.5905	.0685	226.5081	15.5244
47	15.4659	.0647	241.0985	15.5890
48	16.3939	.0610	256.5645	15.6500
49	17.3775	.0575	272.9584	15.7076
50	18.4202	.0543	290.3359	15.7619
55	24.6507	.0406	394.1783	15.9905
60	32.9883	.0303	533.1383	16.1611
65	44.1458	.0226	719.0966	16.2891
70	59.0772	.0169	967.9533	16.3845
75	79.0587	.0126	1,300.9783	16.4558
80	105.7985	.0095	1,746.6416	16.5091
85	141.5287	.0071	2,343.0450	16.5489
90	189.4698	.0053	3,141.1633	16.5787
95	253.5538	.0039	4,209.2300	16.6009
100	339.3125	.0029	5,638.5416	16.6175
105	454.0770	.0022	7,551.2833	16.6299
110	607.6591	.0016	10,110.9850	16.6392
115	813.1867	.0012	13,536.4450	16.6461
120	1088.2280	.0009	18,120.4667	16.6513

7 PER CENT

n years.	I.	II.	III.	IV.
1	1.0700	.9346	1.0000	.9328
2	1.1449	.8736	2.0700	1.8043
3	1.2250	.8163	3.2142	2.6228
4	1.3108	.7629	4.4400	3.3857
5	1.4026	.7130	5.7514	4.0986
6	1.5007	.6663	7.3529	4.7657
7	1.6058	.6227	8.6542	5.3886
8	1.7182	.5820	10.2600	5.9700
9	1.8385	.5439	11.9786	6.5143
10	1.9671	.5083	13.8159	7.0228
11	2.1049	.4751	15.7843	7.4971
12	2.2522	.4440	17.8886	7.9414
13	2.4098	.4150	20.1400	8.3557
14	2.5785	.3878	22.5500	8.7442
15	2.7590	.3624	25.1286	9.1071
16	2.9522	.3387	27.8886	9.4457
17	3.1588	.3161	30.8400	9.7686
18	3.3800	.2959	34.0000	10.0571
19	3.6165	.2765	37.3786	10.3343
20	3.8697	.2584	40.9528	10.5928
21	4.1406	.2415	44.8657	10.8343
22	4.4304	.2257	49.0057	11.0600
23	4.7405	.2109	53.4343	11.2714
24	5.0724	.1971	58.3200	11.4685
25	5.4275	.1842	63.2500	11.6528
26	5.8075	.1722	68.6786	11.8242
27	6.2140	.1609	74.4857	11.9857
28	6.6490	.1504	80.7000	12.1357
29	7.1144	.1406	87.3346	12.2757
30	7.6124	.1314	94.4628	12.4071
31	8.1452	.1228	102.0742	12.5300
32	8.7154	.1147	110.2700	12.6457
33	9.3255	.1072	118.9500	12.7528
34	9.9783	.1002	128.2618	12.8528
35	10.6768	.0937	138.2400	12.9457
36	11.4241	.0875	148.9157	13.0343
37	12.2239	.0818	160.3414	13.1157
38	13.0795	.0765	172.5642	13.1914
39	13.9950	.0715	185.6428	13.2628
40	14.9747	.0668	199.6386	13.3300
41	16.0230	.0624	214.6143	13.3928
42	17.1446	.0583	230.6371	13.4514
43	18.3448	.0545	247.7828	13.5057
44	19.6290	.0509	266.1428	13.5571
45	21.0030	.0476	285.7571	13.6043
46	22.4332	.0445	306.1886	13.6485
47	24.0463	.0416	329.2328	13.6900
48	25.7888	.0387	354.1257	13.7314
49	27.5306	.0363	379.0086	13.7657
50	29.4577	.0339	406.5386	13.8000
55	41.3162	.0242	575.9458	13.9385
60	57.9482	.0173	813.5458	14.0371
65	81.2755	.0123	1,146.7928	14.1085
70	113.9929	.0088	1,614.1844	14.1585
75	159.8823	.0062	2,141.1757	14.1959
80	224.2440	.0045	2,269.7471	14.2200
85	314.5138	.0032	3,160.6285	14.2385
90	441.1230	.0023	4,478.7682	14.2514
95	618.7000	.0016	6,287.4714	14.2614
100	867.7600	.0011	8,824.2857	14.2685
105	1217.0812	.0008	12,382.2855	14.2728
110	1707.0235	.0006	17,372.5886	14.2757
115	2394.1978	.0004	24,371.7642	14.2785
120	3357.9923	.0003	34,188.5400	14.2800

8 PER CENT

n years.	I.	II.	III.	IV.
1	1.0800	.9259	1.0000	.9250
2	1.1664	.8573	2.0800	1.7825
3	1.2597	.7938	3.2463	2.5762
4	1.3605	.7350	4.5062	3.3112
5	1.4693	.6806	5.8366	3.9912
6	1.5869	.6302	7.3362	4.6212
7	1.7138	.5840	8.9225	5.1987
8	1.8509	.5403	10.6303	5.7450
9	1.9990	.5002	12.4875	6.2462
10	2.1589	.4632	14.4862	6.7087
11	2.3317	.4289	16.6463	7.1375
12	2.5182	.3971	18.9775	7.5350
13	2.7196	.3676	21.4950	7.9037
14	2.9372	.3405	24.2150	8.2425
15	3.1722	.3152	27.1525	8.5587
16	3.4260	.2919	30.3250	8.8513
17	3.7000	.2703	33.7500	9.1200
18	3.9960	.2502	37.4500	9.3712
19	4.3157	.2317	41.4463	9.6025
20	4.6610	.2145	45.7625	9.8175
21	5.0339	.1987	50.4237	10.0150
22	5.4366	.1839	55.4575	10.2000
23	5.8716	.1703	60.8950	10.3700
24	6.3413	.1577	66.7663	10.5275
25	6.8486	.1460	73.1075	10.6737
26	7.3964	.1352	79.9800	10.8087
27	7.9882	.1252	87.3525	10.9337
28	8.6272	.1159	95.3400	11.0500
29	9.3174	.1073	103.9675	11.1575
30	10.0629	.0994	113.2862	11.2562
31	10.8678	.0920	123.3475	11.3487
32	11.7371	.0852	134.2138	11.4337
33	12.6763	.0789	145.9537	11.5125
34	13.6904	.0730	158.6300	11.5862
35	14.7853	.0676	172.3163	11.6537
36	15.9684	.0626	187.1050	11.7162
37	17.2460	.0580	203.0750	11.7737
38	18.6249	.0537	220.3113	11.8275
39	20.1159	.0497	238.9488	11.8775
40	21.7250	.0460	259.0625	11.9237
41	23.4630	.0426	280.7875	11.9662
42	25.3400	.0395	304.2500	12.0050
43	27.3672	.0365	329.5900	12.0425
44	29.5567	.0338	356.9588	12.0762
45	31.9213	.0313	386.5163	12.1075
46	34.4750	.0290	418.4375	12.1362
47	37.2330	.0269	452.9125	12.1625
48	40.2117	.0249	490.1463	12.1875
49	43.4207	.0230	530.2588	12.2112
50	46.9029	.0213	573.7863	12.2325
55	68.9160	.0145	848.9500	12.3175
60	101.2605	.0099	1,253.2563	12.3750
65	148.7849	.0067	1,847.3113	12.4150
70	218.6150	.0046	2,720.1875	12.4412
75	321.2177	.0031	4,002.7213	12.4600
80	471.9761	.0021	5,887.2013	12.4725
85	693.4888	.0014	8,656.1100	12.4812
90	1,018.9649	.0010	12,724.5613	12.4862
95	1,497.1993	.0007	18,702.4913	12.4900
100	2,199.8838	.0005	27,486.0475	12.4925
105	3,232.3656	.0003	40,392.0700	12.4950
110	4,749.4130	.0002	59,355.1625	12.4962
115	6,978.4677	.0001	87,218.3463	12.4975
120	10,253.6792	.0001	128,158.4900	12.4989

10 PER CENT

n years.	I.	II.	III.	IV.
1	1.1000	.9091	1.0000	0.908
2	1.2100	.8264	2.1000	1.735
3	1.3310	.7513	3.3100	2.486
4	1.4641	.6830	4.6410	3.169
5	1.6105	.6209	6.1050	3.790
6	1.7716	.5645	7.7160	4.354
7	1.9487	.5132	9.4870	4.867
8	2.1436	.4665	11.4360	5.334
9	2.3580	.4241	13.5800	5.758
10	2.5938	.3855	15.9380	6.144
11	2.8531	.3505	18.5310	6.494
12	3.1385	.3186	21.3850	6.813
13	3.4523	.2897	24.5230	7.102
14	3.7976	.2633	27.9760	7.366
15	4.1773	.2394	31.7730	7.605
16	4.5950	.2176	35.9500	7.823
17	5.0545	.1978	40.5450	8.021
18	5.5600	.1799	45.6000	8.200
19	6.1160	.1635	51.1600	8.364
20	6.7276	.1486	57.2760	8.513
21	7.4004	.1351	64.0040	8.648
22	8.1404	.1228	71.4040	8.771
23	8.9545	.1117	79.5450	8.882
24	9.8500	.1015	88.5000	8.984
25	10.8349	.0923	98.3490	9.076
26	11.9184	.0839	109.1840	9.160
27	13.1103	.0763	121.1030	9.236
28	14.4213	.0693	134.2130	9.306
29	15.8634	.0630	148.6340	9.369
30	17.4498	.0573	164.4980	9.426
31	19.1948	.0521	181.9480	9.478
32	21.1143	.0474	201.1430	9.525
33	23.2257	.0431	222.2570	9.568
34	25.5483	.0391	245.4830	9.608
35	28.1032	.0356	271.0320	9.643
36	30.9135	.0324	299.1350	9.675
37	34.0049	.0294	330.0490	9.705
38	37.4054	.0273	364.0540	9.726
39	41.1460	.0243	401.4600	9.756
40	45.2605	.0221	442.6050	9.778
41	49.7866	.0201	487.8660	9.798
42	54.7655	.0183	537.6550	9.816
43	60.2420	.0166	592.4200	9.833
44	66.2662	.0151	652.6620	9.848
45	72.8928	.0137	718.9280	9.862
46	80.1822	.0125	791.8220	9.874
47	88.2004	.0113	872.0040	9.886
48	97.0207	.0103	960.2070	9.896
49	106.7228	.0094	1,057.2280	9.905
50	117.3926	.0085	1,163.9260	9.914
55	189.0668	.0053	1,880.6680	9.946
60	304.4944	.0033	3,034.9440	9.966
65	490.3932	.0020	4,893.9320	9.979
70	789.7876	.0013	7,887.8760	9.986
75	1,271.9648	.0008	12,709.6480	9.991
80	2,048.5188	.0005	20,475.1880	9.994
85	3,299.1742	.0003	32,981.7420	9.996
90	5,313.3659	.0002	53,123.6590	9.997
95	8,557.2549	.0001	85,562.5490	9.998
100	13,781.6139	.00007	137,806.1390	9.9992
105	22,195.5102	.00005	221,945.1020	9.9994
110	35,746.1983	.00003	357,451.9830	9.9996
115	57,569.8666	.00002	575,688.6660	9.9997
120	92,717.0213	.00001	927,160.2130	9.9998

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	*0300	*0724	*1147	*1570	*1993	*2415	424
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
104	7033	7451	7868	8284	8700	9116	9532	9947	*0361	*0775	416
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	*0195	*0600	*1004	*1408	*1812	*2216	*2619	*3021	404
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	*0207	*0602	*0998	397
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	434	43	87	130	174	217	260	304	347	391	434
	433	43	87	130	173	217	260	303	346	390	433
	432	43	86	130	173	216	259	302	346	389	432
	431	43	86	129	172	216	259	302	345	388	431
	430	43	86	129	172	215	258	301	344	387	430
	429	43	86	129	172	215	257	300	343	386	429
	428	43	86	128	171	214	257	300	342	385	428
	427	43	85	128	171	214	256	299	342	384	427
	426	43	85	128	170	213	256	298	341	383	426
	425	43	85	128	170	213	255	298	340	383	425
	424	42	85	127	170	212	254	297	339	382	424
	423	42	85	127	169	212	254	296	338	381	423
	422	42	84	127	169	211	253	295	338	380	422
	421	42	84	126	168	211	253	295	337	379	421
	420	42	84	126	168	210	252	294	336	378	420
	419	42	84	126	168	210	251	293	335	377	419
	418	42	84	125	167	209	251	293	334	376	418
	417	42	83	125	167	209	250	292	334	375	417
	416	42	83	125	166	208	250	291	333	374	416
	415	42	83	125	166	208	249	291	332	374	415
	414	41	83	124	166	207	248	290	331	373	414
	413	41	83	124	165	207	248	289	330	372	413
	412	41	82	124	165	206	247	288	329	371	412
	411	41	82	123	164	206	247	288	329	370	411
	410	41	82	123	164	205	246	287	328	369	410
	409	41	82	123	164	205	245	286	327	368	409
	408	41	82	122	163	204	245	286	326	367	408
	407	41	81	122	163	204	244	285	326	366	407
	406	41	81	122	162	203	244	284	325	365	406
	405	41	81	122	162	203	243	284	324	365	405
	404	40	81	121	162	202	242	283	323	364	404
	403	40	81	121	161	202	242	282	322	363	403
	402	40	80	121	161	201	241	281	322	362	402
	401	40	80	120	160	201	241	281	321	361	401
	400	40	80	120	160	200	240	280	320	360	400
	399	40	80	120	160	200	239	279	319	359	399
	398	40	80	119	159	199	239	279	318	358	398
	397	40	79	119	159	199	238	278	318	357	397
	396	40	79	119	158	198	238	277	317	356	396
	395	40	79	119	158	198	237	277	316	356	395
	394	39	79	118	158	197	236	276	315	355	394
	393	39	79	118	157	197	236	275	314	354	393
	392	39	78	118	157	196	235	274	314	353	392
	391	39	78	117	156	196	235	274	313	352	391
	390	39	78	117	156	195	234	273	312	351	390
	389	39	78	117	156	195	233	272	311	350	389
	388	39	78	116	155	194	233	272	310	349	388
Diff.		1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
112	9218	9606	9993	*0380	*0766	*1153	*1538	*1924	*2309	*2694	386
113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	*0320	379
115	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
117	8186	8557	8928	9298	9668	*0038	*0407	*0776	*1145	*1514	370
118	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	9543	9904	*0266	*0626	*0987	*1347	*1707	*2067	*2426	360
121	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	*0258	*0611	*0963	*1315	*1667	*2018	*2370	*2721	*3071	352
124	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	387	39	77	116	155	194	232	271	310	348	387
	386	39	77	116	154	193	232	270	309	347	386
	385	39	77	116	154	193	231	270	308	347	385
	384	38	77	115	154	192	230	269	307	346	384
	383	38	77	115	153	192	230	268	306	345	383
	382	38	76	115	153	191	229	267	306	344	382
	381	38	76	114	152	191	229	267	305	343	381
	380	38	76	114	152	190	228	266	304	342	380
	379	38	76	114	152	190	227	265	303	341	379
	378	38	76	113	151	189	227	265	302	340	378
	377	38	75	113	151	189	226	264	302	339	377
	376	38	75	113	150	188	226	263	301	338	376
	375	38	75	113	150	188	225	263	300	338	375
	374	37	75	112	150	187	224	262	299	337	374
	373	37	75	112	149	187	224	261	298	336	373
	372	37	74	112	149	186	223	260	298	335	372
	371	37	74	111	148	186	223	260	297	334	371
	370	37	74	111	148	185	222	259	296	333	370
	369	37	74	111	148	185	221	258	295	332	369
	368	37	74	110	147	184	221	258	294	331	368
	367	37	73	110	147	184	220	257	294	330	367
	366	37	73	110	146	183	220	256	293	329	366
	365	37	73	110	146	183	219	256	292	329	365
	364	36	73	109	146	182	218	255	291	328	364
	363	36	73	109	145	182	218	254	290	327	363
	362	36	72	109	145	181	217	253	290	326	362
	361	36	72	108	144	181	217	253	289	325	361
	360	36	72	108	144	180	216	252	288	324	360
	359	36	72	108	144	180	215	251	287	323	359
	358	36	72	107	143	179	215	251	286	322	358
	357	36	71	107	143	179	214	250	286	321	357
	356	36	71	107	142	178	214	249	285	320	356
	355	36	71	107	142	178	213	249	284	320	355
	354	35	71	106	142	177	212	248	283	319	354
	353	35	71	106	141	177	212	247	282	318	353
	352	35	70	106	141	176	211	246	282	317	352
	351	35	70	105	140	176	211	246	281	316	351
	350	35	70	105	140	175	210	245	280	315	350
	349	35	70	105	140	175	209	244	279	314	349
	348	35	70	104	139	174	209	244	278	313	348
	347	35	69	104	139	174	208	243	278	312	347
Diff.	1	2	3	4	5	6	7	8	9	Diff.	

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
125	096910	7257	7604	7951	8298	8644	8990	9335	9681	*0026	346
126	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	*0253	338
129	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	113943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	*0245	330
132	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	*0012	323
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
138	9879	*0194	*0508	*0822	*1136	*1450	*1763	*2076	*2389	*2702	314
139	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	347	35	69	104	139	174	208	243	278	312	347
	346	35	69	104	138	173	208	242	277	311	346
	345	35	69	104	138	173	207	242	276	311	345
	344	34	69	103	138	172	206	241	275	310	344
	343	34	69	103	137	172	206	240	274	309	343
	342	34	68	103	137	171	205	239	274	308	342
	341	34	68	102	136	171	205	239	273	307	341
	340	34	68	102	136	170	204	238	272	306	340
	339	34	68	102	136	170	203	237	271	305	339
	338	34	68	101	135	169	203	237	270	304	338
	337	34	67	101	135	169	202	236	270	303	337
	336	34	67	101	134	168	202	235	269	302	336
	335	34	67	101	134	168	201	235	268	302	335
	334	33	67	100	134	167	200	234	267	301	334
	333	33	67	100	133	167	200	233	266	300	333
	332	33	66	100	133	166	199	232	266	299	332
	331	33	66	99	132	166	199	232	265	298	331
	330	33	66	99	132	165	198	231	264	297	330
	329	33	66	99	132	165	197	230	263	296	329
	328	33	66	98	131	164	197	230	262	295	328
	327	33	65	98	131	164	196	229	262	294	327
	326	33	65	98	130	163	196	228	261	293	326
	325	33	65	98	130	163	195	228	260	293	325
	324	32	65	97	130	162	194	227	259	292	324
	323	32	65	97	129	162	194	226	258	291	323
	322	32	64	97	129	161	193	225	258	290	322
	321	32	64	96	128	161	193	225	257	289	321
	320	32	64	96	128	160	192	224	256	288	320
	319	32	64	96	128	160	191	223	255	287	319
	318	32	64	95	127	159	191	223	254	286	318
	317	32	63	95	127	159	190	222	254	285	317
	316	32	63	95	126	158	190	221	253	284	316
	315	32	63	95	126	158	189	221	252	284	315
	314	31	63	94	126	157	188	220	251	283	314
	313	31	63	94	125	157	188	219	250	282	313
	312	31	62	94	125	156	187	218	250	281	312
	311	31	62	93	124	156	187	218	249	280	311
	310	31	62	93	124	155	186	217	248	279	310
	309	31	62	93	124	155	185	216	247	278	309
	308	31	62	92	123	154	185	216	246	277	308
	307	31	61	92	123	154	184	215	246	276	307
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
140	146128	6438	6748	7055	7367	7676	7985	8294	8603	8911	309
141	9219	9527	9835	*0142	*0449	*0756	*1063	*1370	*1676	*1982	307
142	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	*0168	*0469	*0769	*1068	301
145	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	6351	6670	6959	7248	7536	7825	8113	8401	8689	289
151	8977	9264	9552	9839	*0126	*0413	*0699	*0986	*1272	*1558	287
152	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	*0051	281
155	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755	*0029	*0303	*0577	*0850	*1124	274
159	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	306	31	61	92	122	153	184	214	245	275	306
	305	31	61	92	122	153	183	214	244	275	305
	304	30	61	91	122	152	182	213	243	274	304
	303	30	61	91	121	152	182	212	242	273	303
	302	30	60	91	121	151	181	211	242	272	302
	301	30	60	90	120	151	181	211	241	271	301
	300	30	60	90	120	150	180	210	240	270	300
	299	30	60	90	120	150	179	209	239	269	299
	298	30	60	89	119	149	179	209	238	268	298
	297	30	59	89	119	149	178	208	238	267	297
	296	30	59	89	118	148	178	207	237	266	296
	295	30	59	89	118	148	177	207	236	266	295
	294	29	59	88	118	147	176	206	235	265	294
	293	29	59	88	117	147	176	205	234	264	293
	292	29	58	88	117	146	175	204	234	263	292
	291	29	58	87	116	146	175	204	233	262	291
	290	29	58	87	116	145	174	203	232	261	290
	289	29	58	87	116	145	173	202	231	260	289
	288	29	58	86	115	144	173	202	230	259	288
	287	29	57	86	115	144	172	201	230	258	287
	286	29	57	86	114	143	172	200	229	257	286
	285	29	57	86	114	143	171	200	228	257	285
	284	28	57	85	114	142	170	199	227	256	284
	283	28	57	85	113	142	170	198	226	255	283
	282	28	56	85	113	141	169	197	226	254	282
	281	28	56	84	112	141	169	197	225	253	281
	280	28	56	84	112	140	168	196	224	252	280
	279	28	56	84	112	140	167	195	223	251	279
	278	28	56	83	111	139	167	195	222	250	278
	277	28	55	83	111	139	166	194	222	249	277
	276	28	55	83	110	138	166	193	221	248	276
	275	28	55	83	110	138	165	193	220	248	275
	274	27	55	82	110	137	164	192	219	247	274
	273	27	55	82	109	137	164	191	218	246	273
	272	27	54	82	109	136	163	190	218	245	272
	271	27	54	81	108	136	163	190	217	244	271
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
160	204120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	*0051	*0319	*0586	*0853	*1121	*1388	*1654	*1921	267
163	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	217484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	*0193	256
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
171	2966	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	8046	8297	8548	8799	9049	9299	9550	9800	*0050	*0300	250
174	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	243038	3266	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	*0176	245
178	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	272	27	54	82	109	136	163	190	218	245	272
	271	27	54	81	108	136	163	190	217	244	271
	270	27	54	81	108	135	162	189	216	243	270
	269	27	54	81	108	135	161	188	215	242	269
	268	27	54	80	107	134	161	188	214	241	268
	267	27	53	80	107	134	160	187	214	240	267
	266	27	53	80	106	133	160	186	213	239	266
	265	27	53	80	106	133	159	186	212	239	265
	264	26	53	79	106	132	158	185	211	238	264
	263	26	53	79	105	132	158	184	210	237	263
	262	26	52	79	105	131	157	183	210	236	262
	261	26	52	78	104	131	157	183	209	235	261
	260	26	52	78	104	130	156	182	208	234	260
	259	26	52	78	104	130	155	181	207	233	259
	258	26	52	77	103	129	155	181	206	232	258
	257	26	51	77	103	129	154	180	206	231	257
	256	26	51	77	102	128	154	179	205	230	256
	255	26	51	77	102	128	153	179	204	230	255
	254	25	51	76	102	127	152	178	203	229	254
	253	25	51	76	101	127	152	177	202	228	253
	252	25	50	76	101	126	151	176	202	227	252
	251	25	50	75	100	126	151	176	201	226	251
	250	25	50	75	100	125	150	175	200	225	250
	249	25	50	75	100	125	149	174	199	224	249
248	25	50	74	99	124	149	174	198	223	248	
247	25	49	74	99	124	148	173	198	222	247	
246	25	49	74	98	123	148	172	197	221	246	
245	25	49	74	98	123	147	172	196	221	245	
244	24	49	73	98	122	146	171	195	220	244	
243	24	49	73	97	122	146	170	194	219	243	
242	24	48	73	97	121	145	169	194	218	242	
241	24	48	72	96	121	145	169	193	217	241	
240	24	48	72	96	120	144	168	192	216	240	
Diff.		1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
180	255273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	267172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	9513	9746	9980	*0213	*0446	*0679	*0912	*1144	*1377	*1609	233
187	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	278754	8982	9211	9439	9667	9895	*0123	*0351	*0578	*0806	228
191	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	8853	9071	9289	9507	9725	9943	*0161	*0378	*0595	*0813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	9630	9843	*0056	*0268	*0481	*0693	*0906	*1118	*1330	*1542	212
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	239	24	48	72	96	120	143	167	191	215	239
	238	24	48	71	95	119	143	167	190	214	238
	237	24	47	71	95	119	142	166	190	213	237
	236	24	47	71	94	118	142	165	189	212	236
	235	24	47	71	94	118	141	165	188	212	235
	234	23	47	70	94	117	140	164	187	211	234
	233	23	47	70	93	117	140	163	186	210	233
	232	23	46	70	93	116	139	162	186	209	232
	231	23	46	69	92	116	139	162	185	208	231
	230	23	46	69	92	115	138	161	184	207	230
	229	23	46	69	92	115	137	160	183	206	229
	228	23	46	68	91	114	137	160	182	205	228
	227	23	45	68	91	114	136	159	182	204	227
	226	23	45	68	90	113	136	158	181	203	226
	225	23	45	68	90	113	135	158	180	203	225
	224	22	45	67	90	112	134	157	179	202	224
	223	22	45	67	89	112	134	156	178	201	223
	222	22	44	67	89	111	133	155	178	200	222
	221	22	44	66	88	111	133	155	177	199	221
	220	22	44	66	88	110	132	154	176	198	220
	219	22	44	66	88	110	131	153	175	197	219
	218	22	44	65	87	109	131	153	174	196	218
	217	22	43	65	87	109	130	152	174	195	217
	216	22	43	65	86	108	130	151	173	194	216
	215	22	43	65	86	108	129	151	172	194	215
	214	21	43	64	86	107	128	150	171	193	214
	213	21	43	64	85	107	128	149	170	192	213
	212	21	42	64	85	106	127	148	170	191	212
Diff.		1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
205	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	322219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	8380	8583	8787	8991	9194	9398	9601	9805	*0008	*0211	203
214	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202
215	332138	2640	2842	3044	3246	3447	3649	3850	4051	4253	201
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	8456	8656	8855	9054	9253	9451	9650	9849	*0047	*0246	199
219	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	342423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	8305	8500	8694	8889	9083	9278	9472	9666	9860	*0054	194
224	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
225	352183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	9935	*0025	*0215	*0404	*0593	*0783	*0972	*1161	*1350	*1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	*0143	*0328	*0513	*0698	*0883	185

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	212	21	42	64	85	106	127	148	170	191	212
	211	21	42	63	84	106	127	148	169	190	211
	210	21	42	63	84	105	126	147	168	189	210
	209	21	42	63	84	105	125	146	167	188	209
	208	21	42	62	83	104	125	146	166	187	208
	207	21	41	62	83	104	124	145	166	186	207
	206	21	41	62	82	103	124	144	165	185	206
	205	21	41	62	82	103	123	144	164	185	205
	204	20	41	61	82	102	122	143	163	184	204
	203	20	41	61	81	102	122	142	162	183	203
	202	20	40	61	81	101	121	141	162	182	202
	201	20	40	60	80	101	121	141	161	181	201
	200	20	40	60	80	100	120	140	160	180	200
	199	20	40	60	80	100	119	139	159	179	199
	198	20	40	59	79	99	119	139	158	178	198
	197	20	39	59	79	99	118	138	158	177	197
	196	20	39	59	78	98	118	137	157	176	196
	195	20	39	59	78	98	117	137	156	176	195
	194	19	39	58	78	97	116	136	155	175	194
	193	19	39	58	77	97	116	135	154	174	193
	192	19	38	58	77	96	115	134	154	173	192
	191	19	38	57	76	96	115	134	153	172	191
	190	19	38	57	76	95	114	133	152	171	190
	189	19	38	57	76	95	113	132	151	170	189
	188	19	38	56	75	94	113	132	150	169	188
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
235	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	*0030	181
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	389166	9343	9520	9698	9875	*0051	*0228	*0405	*0582	*0759	177
246	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	9674	9847	*0020	*0192	*0365	*0538	*0711	*0883	*1056	*1228	173
252	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	406540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	9933	*0102	*0271	*0440	*0609	*0777	*0946	*1114	*1283	*1451	169
258	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	414973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	9956	*0121	*0286	*0451	*0616	*0781	*0945	*1110	*1275	*1439	165
264	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	187	19	37	56	75	94	112	131	150	168	187
	186	19	37	56	74	93	112	130	149	167	186
	185	19	37	56	74	93	111	130	148	167	185
	184	18	37	55	74	92	110	129	147	166	184
	183	18	37	55	73	92	110	128	146	165	183
	182	18	36	55	73	91	109	127	146	164	182
	181	18	36	54	72	91	109	127	145	163	181
	180	18	36	54	72	90	108	126	144	162	180
	179	18	36	54	72	90	107	125	143	161	179
	178	18	36	53	71	89	107	125	142	160	178
	177	18	35	53	71	89	106	124	142	159	177
	176	18	35	53	70	88	106	123	141	158	176
	175	18	35	53	70	88	105	123	140	158	175
	174	17	35	52	70	87	104	122	139	157	174
	173	17	35	52	69	87	104	121	138	156	173
	172	17	34	52	69	86	103	120	138	155	172
	171	17	34	51	68	86	103	120	137	154	171
	170	17	34	51	68	85	102	119	136	153	170
	169	17	34	51	68	85	101	118	135	152	169
	168	17	34	50	67	84	101	118	134	151	168
	167	17	33	50	67	84	100	117	134	150	167
	166	17	33	50	66	83	100	116	133	149	166
	165	17	33	50	66	83	99	116	132	149	165
	164	16	33	49	66	82	98	115	131	148	164
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
265	423246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	9752	9914	*0075	*0236	*0398	*0559	*0720	*0881	*1042	*1203	161
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
271	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	439333	9491	9648	9806	9964	*0122	*0279	*0437	*0594	*0752	158
276	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
279	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	447158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
281	8706	8861	9015	9170	9324	9478	9633	9787	9941	*0095	154
282	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
283	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	454845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
286	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
287	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
288	9392	9543	9694	9845	9995	*0146	*0296	*0447	*0597	*0748	151
289	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	462398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
294	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
295	469822	9969	*0116	*0263	*0410	*0557	*0704	*0851	*0998	*1145	147
296	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	164	16	33	49	66	82	98	115	131	148	164
	163	16	33	49	65	82	98	114	130	147	163
	162	16	32	49	65	81	97	113	130	146	162
	161	16	32	48	64	81	97	113	129	145	161
	160	16	32	48	64	80	96	112	128	144	160
	159	16	32	48	64	80	95	111	127	143	159
	158	16	32	47	63	79	95	111	126	142	158
	157	16	31	47	63	79	94	110	126	141	157
	156	16	31	47	62	78	94	109	125	140	156
	155	16	31	47	62	78	93	109	124	140	155
	154	15	31	46	62	77	92	108	123	139	154
	153	15	31	46	61	77	92	107	122	138	153
	152	15	30	46	61	76	91	106	122	137	152
	151	15	30	45	60	76	91	106	121	136	151
	150	15	30	45	60	75	90	105	120	135	150
	149	15	30	45	60	75	89	104	119	134	149
	148	15	30	44	59	74	89	104	118	133	148
	147	15	29	44	59	74	88	103	118	132	147
	146	15	29	44	58	73	88	102	117	131	146
	145	15	29	44	58	73	87	102	116	131	145
	144	14	29	43	58	72	86	101	115	130	144
	143	14	29	43	57	72	86	100	114	129	143
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
301	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
302	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
305	484300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
308	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	9958	*0099	*0239	*0380	*0520	*0661	*0801	*0941	*1081	*1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	498311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
316	9687	9824	9962	*0099	*0236	*0374	*0511	*0648	*0785	*0922	137
317	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
321	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
322	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
323	9203	9337	9471	9606	9740	9874	*0009	*0143	*0277	*0411	134
324	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
325	511883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	518514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
331	9828	9959	*0090	*0221	*0353	*0484	*0615	*0745	*0876	*1007	131
332	521138	1209	1400	1530	1661	1792	1922	2053	2183	2314	131
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	525045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
336	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
338	8917	9045	9174	9302	9430	9559	9687	9815	9943	*0072	128
339	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	142	14	28	43	57	71	85	99	114	128	142
	141	14	28	42	56	71	85	99	113	127	141
	140	14	28	42	56	70	84	98	112	126	140
	139	14	28	42	56	70	83	97	111	125	139
	138	14	28	41	55	69	83	97	110	124	138
	137	14	27	41	55	69	82	96	110	123	137
	136	14	27	41	54	68	82	95	109	122	136
	135	14	27	41	54	68	81	95	108	122	135
	134	13	27	40	54	67	80	94	107	121	134
	133	13	27	40	53	67	80	93	106	120	133
	132	13	26	40	53	66	79	92	106	119	132
	131	13	26	39	52	66	79	92	105	118	131
	130	13	26	39	52	65	78	91	104	117	130
	129	13	26	39	52	65	77	90	103	116	129
	128	13	26	38	51	64	77	90	102	115	128
	127	13	25	38	51	64	76	89	102	114	127
Diff.	1	2	3	4	5	6	7	8	9	Diff.	

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
341	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
343	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	537819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	9076	9202	9327	9452	9578	9703	9829	9954	*0079	*0204	125
347	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	*0106	123
355	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	6423	6544	6664	6785	6905	7025	7146	7267	7387	120
361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	9907	*0026	*0146	*0265	*0385	*0504	*0624	*0743	*0863	*0982	119
364	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	562293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	9374	9491	9608	9725	9842	9959	*0076	*0193	*0309	*0426	117
372	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	574031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	128	13	26	38	51	64	77	90	102	115	128
	127	13	25	38	51	64	76	89	102	114	127
	126	13	25	38	50	63	76	88	101	113	126
	125	13	25	38	50	63	75	88	100	113	125
	124	12	25	37	50	62	74	87	99	112	124
	123	12	25	37	49	62	74	86	98	111	123
	122	12	24	37	49	61	73	85	98	110	122
	121	12	24	36	48	61	73	85	97	109	121
	120	12	24	36	48	60	72	84	96	108	120
	119	12	24	36	48	60	71	83	95	107	119
	118	12	24	35	47	59	71	83	94	106	118
	117	12	23	35	47	59	70	82	94	105	117
	116	12	23	35	46	58	70	81	93	104	116
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
380	579784	9898	*0012	*0126	*0241	*0355	*0469	*0583	*0697	*0811	114
381	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	114
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	585461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	9950	*0061	*0173	*0284	*0396	*0507	*0619	*0730	*0842	*0953	112
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	596597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	9883	9992	*0101	*0210	*0319	*0428	*0537	*0646	*0755	*0864	109
399	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	109
400	602060	2169	2277	2386	2494	2603	2711	2819	2928	3036	108
401	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	607455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
407	9594	9701	9808	9914	*0021	*0128	*0234	*0341	*0447	*0554	107
408	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	106
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	618048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	9093	9198	9302	9406	9511	9615	9719	9824	9928	*0032	104
417	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	115	12	23	35	46	58	69	81	92	104	115
	114	11	23	34	46	57	68	80	91	103	114
	113	11	23	34	45	57	68	79	90	102	113
	112	11	22	34	45	56	67	78	90	101	112
	111	11	22	33	44	56	67	78	89	100	111
	110	11	22	33	44	55	66	77	88	99	110
	109	11	22	33	44	55	65	76	87	98	109
	108	11	22	32	43	54	65	76	86	97	108
	107	11	21	32	43	54	64	75	86	96	107
	106	11	21	32	42	53	64	74	85	95	106
	105	11	21	32	42	53	63	74	84	95	105
	104	10	21	31	42	52	62	73	83	94	104
	103	10	21	31	41	52	62	72	82	93	103
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
420	623249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
425	628389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
426	9410	9512	9613	9715	9817	9919	*0021	*0123	*0224	*0326	102
427	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
431	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	101
432	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
434	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
435	638489	8589	8689	8789	8888	8988	9088	9188	9287	9387	100
436	9486	9586	9686	9785	9885	9984	*0084	*0183	*0283	*0382	99
437	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375	99
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
439	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	3551	3650	3749	3847	3946	4044	4143	4242	4340	98
441	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	648360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	9335	9432	9530	9627	9724	9821	9919	*0016	*0113	*0210	97
447	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	97
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	3309	3405	3502	3598	3695	3791	3888	3984	4080	96
451	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	658011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
457	9916	*0011	*0106	*0201	*0296	*0391	*0486	*0581	*0676	*0771	95
458	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
459	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
460	662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	94
461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	104	10	21	31	42	52	62	73	83	94	104
	103	10	21	31	41	52	62	72	82	93	103
	102	10	20	31	41	51	61	71	82	92	102
	101	10	20	30	40	51	61	71	81	91	101
	100	10	20	30	40	50	60	70	80	90	100
	99	10	20	30	40	50	59	69	79	89	99
	98	10	20	29	39	49	59	69	78	88	98
	97	10	19	29	39	49	58	68	78	87	97
	96	10	19	29	38	48	58	67	77	86	96
	95	10	19	29	38	48	57	67	76	86	95
Diff.	1	2	3	4	5	6	7	8	9	Diff.	

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
465	667453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967	*0060	*0153	93
468	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	676694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	9428	9519	9610	9700	9791	9882	9973	*0063	*0154	*0245	91
479	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	91
480	681241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	685742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930	*0019	*0107	89
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	694605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	9057	9144	9231	9317	9404	9491	9578	9664	9751	87
501	9838	9924	*0011	*0098	*0184	*0271	*0358	*0444	*0531	*0617	87
502	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	703291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	94	9	19	28	38	47	56	66	75	85	94
	93	9	19	28	37	47	56	65	74	84	93
	92	9	18	28	37	46	55	64	74	83	92
	91	9	18	27	36	46	55	64	73	82	91
	90	9	18	27	36	45	54	63	72	81	90
	89	9	18	27	36	45	53	62	71	80	89
	88	9	18	26	35	44	53	62	70	79	88
	87	9	17	26	35	44	52	61	70	78	87
	86	9	17	26	34	43	52	60	69	77	86
	85	9	17	26	34	43	51	60	68	77	85
Diff.	1	2	3	4	5	6	7	8	9	Diff.	

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
510	707570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	9270	9355	9440	9524	9609	9694	9779	9863	9948	*0033	85
513	710117	0202	0287	0371	0456	0540	0625	0710	0794	0879	85
514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	711807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
519	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
520	716003	6087	6170	6254	6337	6421	6504	6588	6671	6754	83
521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	*0077	83
525	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	728354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	*0055	*0136	*0217	*0298	*0378	*0459	*0540	*0621	*0702	81
538	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
543	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	736397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	9572	9651	9731	9810	9889	9968	*0047	*0126	*0205	*0284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	86	9	17	26	34	43	52	60	69	77	86
	85	9	17	26	34	43	51	60	68	77	85
	84	8	17	25	34	42	50	59	67	76	84
	83	8	17	25	33	42	50	58	66	75	83
	82	8	16	25	33	41	49	57	66	74	82
	81	8	16	24	32	41	49	57	65	73	81
	80	8	16	24	32	40	48	56	64	72	80
	79	8	16	24	32	40	47	55	63	71	79
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
555	744293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	*0045	*0123	*0200	*0277	*0354	*0431	77
563	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	752048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	759668	9743	9819	9894	9970	*0045	*0121	*0196	*0272	*0347	75
576	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	75
577	1176	1251	1326	1402	1477	1552	1627	1702	1777	1853	75
578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
579	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
580	763428	3503	3578	3653	3727	3802	3877	3952	4027	4101	75
581	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	767186	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	*0042	74
589	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	74
590	770852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	774517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PRO. PARTS	78	8	16	23	31	39	47	55	62	70	78
	77	8	15	23	31	39	46	54	62	69	77
	76	8	15	23	30	38	46	53	61	68	76
	75	8	15	23	30	38	45	53	60	68	75
	74	7	15	22	30	37	44	52	59	67	74
	73	7	15	22	29	37	44	51	58	66	73
	72	7	14	22	29	36	43	50	58	65	72
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
600	778151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72
601	8374	8947	9019	9091	9163	9235	9308	9380	9452	9524	72
602	9596	9669	9741	9813	9885	9957	*0029	*0101	*0173	*0245	72
603	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	781755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	788875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	9581	9651	9722	9792	9863	9933	*0004	*0074	*0144	*0215	70
617	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
619	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	795880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	69
632	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	68
635	802774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	68
641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	809560	9627	9694	9762	9829	9896	9964	*0031	*0098	*0165	67
646	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
P. 20. 2. 15 S	73	7	15	22	29	37	44	51	58	66	73
	72	7	14	22	29	36	43	50	58	65	72
	71	7	14	21	28	36	43	50	57	64	71
	70	7	14	21	28	35	42	49	56	63	70
	69	7	14	21	28	35	41	48	55	62	69
68		7	14	20	27	34	41	48	54	61	68
Diff.		1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
650	812913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	816241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	9610	9676	9741	9807	9873	9939	*0004	*0070	*0136	66
661	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	822822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	829304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	9047	*0011	*0075	*0139	*0204	*0268	*0332	*0396	*0460	*0525	64
677	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832509	2573	2637	2700	2764	2828	2892	2956	3020	3083	64
681	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	835091	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	9478	9541	9604	9667	9729	9792	9855	9918	9981	*0043	63
692	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	841985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PRO. P.TS	67	7	13	20	27	34	40	47	54	60	67
	66	7	13	20	26	33	40	46	53	59	66
	65	7	13	20	26	33	39	46	52	58	65
	64	6	13	19	26	32	38	45	51	57	64
	63	6	13	19	25	32	38	44	50	56	63
	62	6	12	19	25	31	37	43	50	56	62
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
700	845098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	848189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	850033	0095	0156	0217	0279	0340	0401	0462	0524	0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	1320	1381	1442	1503	1564	1625	1686	1747	1809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	854306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	9739	9799	9859	9918	9978	*0038	*0098	*0158	*0218	*0278	60
725	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	3382	3442	3501	3561	3620	3680	3739	3799	3858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	866287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	9290	9349	9408	9466	9525	9584	9642	9701	9760	59
741	9818	9877	9935	9994	*0053	*0111	*0170	*0228	*0287	*0345	59
742	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	872156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PRO. PTS	62	6	12	19	25	31	37	43	50	56	62
	61	6	12	18	24	31	37	43	49	55	61
	60	6	12	18	24	30	36	42	48	54	60
	59	6	12	18	24	30	35	41	47	53	59
	58	6	12	17	23	29	35	41	46	52	58
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
750	875061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	877947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	9669	9726	9784	9841	9898	9956	*0013	*0070	*0127	*0185	57
759	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	57
760	880814	0871	0928	0985	1042	1099	1156	1213	1271	1328	57
761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	883661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	886491	6547	6604	6660	6716	6773	6829	6885	6942	6998	56
771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	889302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
776	9862	9918	9974	*0030	*0086	*0141	*0197	*0253	*0309	*0365	56
777	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	894870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	9821	9875	9930	9985	*0039	*0094	*0149	*0203	*0258	*0312	55
795	900367	0422	0476	0531	0586	0640	0695	0749	0804	0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
P. R. P. T. S.	57	6	11	17	23	29	34	40	46	51	57
	56	6	11	17	22	28	34	39	45	50	56
	55	6	11	17	22	28	33	39	44	50	55
	54	5	11	16	22	27	32	38	43	49	54
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
800	903090	3144	3199	3253	3307	3361	3416	3470	3524	3578	54
801	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	905796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
806	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908485	8539	8592	8646	8699	8753	8807	8860	8914	8967	54
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	9556	9610	9663	9716	9770	9823	9877	9930	9984	*0037	53
813	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	911158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	913814	3867	3920	3973	4026	4079	4132	4184	4237	4290	53
821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	916454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	9601	9653	9706	9758	9810	9862	9914	9967	*0019	*0071	52
832	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	921686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	926857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
P. R. P. T. S.	55	6	11	17	22	28	33	39	44	50	55
	54	5	11	16	22	27	32	38	43	49	54
	53	5	11	16	21	27	32	37	42	48	53
	52	5	10	16	21	26	31	36	42	47	52
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
850	929419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
851	9930	9981	*0032	*0083	*0134	*0185	*0236	*0287	*0338	*0389	51
852	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	931966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	937016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	942008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
880	944483	4532	4581	4631	4680	4729	4779	4828	4877	4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	946943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	9878	9926	9975	*0024	*0073	*0121	*0170	*0219	*0267	*0316	49
892	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	951823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
P. R. P. T'S.	51	5	10	15	20	26	31	36	41	46	51
	50	5	10	15	20	25	30	35	40	45	50
	49	5	10	15	20	25	29	34	39	44	49
	48	5	10	14	19	24	29	34	38	43	48
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE VII.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	956649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	*0042	*0090	*0138	*0185	*0233	*0280	*0328	*0376	*0423	48
913	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	48
915	961421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	966142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	8530	8576	8623	8670	8716	8763	8810	8856	8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	9882	9928	9975	*0021	*0068	*0114	*0161	*0207	*0254	*0300	47
934	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
935	970812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
940	973128	3174	3220	3266	3313	3359	3405	3451	3497	3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	975432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
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TABLE VII.—LOGARITHMS OF NUMBERS.

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951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	984527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	989005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9995	9939	9983	*0028	*0072	*0117	*0161	*0206	*0250	*0294	44
978	990339	0383	0428	0472	0516	0561	0605	0649	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	993436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	997823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
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TABLE VII.—LOGARITHMS OF NUMBERS.

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1002	0868	0911	0954	0998	1041	1084	1128	1171	1214	1258	43
1003	1301	1344	1388	1431	1474	1517	1561	1604	1647	1690	43
1004	1734	1777	1820	1863	1907	1950	1993	2036	2080	2123	43
1005	002166	2209	2252	2296	2339	2382	2425	2468	2512	2555	43
1006	2598	2641	2684	2727	2771	2814	2857	2900	2943	2986	43
1007	3029	3073	3116	3159	3202	3245	3288	3331	3374	3417	43
1008	3461	3504	3547	3590	3633	3676	3719	3762	3805	3848	43
1009	3891	3934	3977	4020	4063	4106	4149	4192	4235	4278	43
1010	004321	4364	4407	4450	4493	4536	4579	4622	4665	4708	43
1011	4751	4794	4837	4880	4923	4966	5009	5052	5095	5138	43
1012	5181	5223	5266	5309	5352	5395	5438	5481	5524	5567	43
1013	5609	5652	5695	5738	5781	5824	5867	5909	5952	5995	43
1014	6038	6081	6124	6166	6209	6252	6295	6338	6380	6423	43
1015	006466	6509	6552	6594	6637	6680	6723	6765	6808	6851	43
1016	6894	6936	6979	7022	7065	7107	7150	7193	7236	7278	43
1017	7321	7364	7406	7449	7492	7534	7577	7620	7662	7705	43
1018	7748	7790	7833	7876	7918	7961	8004	8046	8089	8132	43
1019	8174	8217	8259	8302	8345	8387	8430	8472	8515	8558	43
1020	008600	8643	8685	8728	8770	8813	8856	8898	8941	8983	43
1021	9026	9068	9111	9153	9196	9238	9281	9323	9366	9408	42
1022	9451	9493	9536	9578	9621	9663	9706	9748	9791	9833	42
1023	9876	9918	9961	*0003	*0045	*0088	*0130	*0173	*0215	*0258	42
1024	010300	0342	0385	0427	0470	0512	0554	0597	0639	0681	42
1025	010724	0766	0809	0851	0893	0936	0978	1020	1063	1105	42
1026	1147	1190	1232	1274	1317	1359	1401	1444	1486	1528	42
1027	1570	1613	1655	1697	1740	1782	1824	1866	1909	1951	42
1028	1993	2035	2078	2120	2162	2204	2247	2289	2331	2373	42
1029	2415	2458	2500	2542	2584	2626	2669	2711	2753	2795	42
1030	012837	2879	2922	2964	3006	3048	3090	3132	3174	3217	42
1031	3259	3301	3343	3385	3427	3469	3511	3553	3595	3638	42
1032	3680	3722	3764	3806	3848	3890	3932	3974	4016	4058	42
1033	4100	4142	4184	4226	4268	4310	4353	4395	4437	4479	42
1034	4521	4563	4605	4647	4689	4730	4772	4814	4856	4898	42
1035	014940	4982	5024	5066	5108	5150	5192	5234	5276	5318	42
1036	5360	5402	5444	5485	5527	5569	5611	5653	5695	5737	42
1037	5779	5821	5863	5904	5946	5988	6030	6072	6114	6156	42
1038	6197	6239	6281	6323	6365	6407	6448	6490	6532	6574	42
1039	6616	6657	6699	6741	6783	6824	6866	6908	6950	6992	42
1040	017033	7075	7117	7159	7200	7242	7284	7326	7367	7409	42
1041	7451	7492	7534	7576	7618	7659	7701	7743	7784	7826	42
1042	7868	7909	7951	7993	8034	8076	8118	8159	8201	8243	42
1043	8284	8326	8368	8409	8451	8492	8534	8576	8617	8659	42
1044	8700	8742	8784	8825	8867	8908	8950	8992	9033	9075	42
1045	019116	9158	9199	9241	9282	9324	9366	9407	9449	9490	42
1046	9532	9573	9615	9656	9698	9739	9781	9822	9864	9905	41
1047	9947	9988	*0030	*0071	*0113	*0154	*0195	*0237	*0278	*0320	41
1048	020361	0403	0444	0486	0527	0568	0610	0651	0693	0734	41
1049	0775	0817	0858	0900	0941	0982	1024	1065	1107	1148	41
1050	021189	1231	1272	1313	1355	1396	1437	1479	1520	1561	41
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	41	4	8	12	16	21	25	29	33	37	41
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

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