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T A B L E S
D' I N T É G R A L E S D É F I N I E S.

T A B L E S
D'INTÉGRALES DÉFINIES

PAR

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P R É F A C E.

Dans la construction de ces *Tables d'Intégrales définies* j'avais en vue un objet quadruple.

En premier lieu je voulais réunir les uns auprès des autres les différents résultats, épars par-ci et par-là, que l'on avait obtenus au sujet de ces fonctions, par beaucoup de méthodes intrinsèquement différentes, et pour la plupart plus ou moins indirectes. Il résultait de cette dispersion des formules obtenues, que l'on ne pouvait en tirer tout le profit possible, tant pour la pratique, — c'est-à-dire pour les cas, où l'on pourrait avoir besoin des valeurs d'une certaine intégrale définie, — que pour la théorie elle-même, — c'est-à-dire pour l'emploi de ces formules dans la déduction d'autres intégrales définies, et pour la vérification de nouvelles formules de ce genre, à l'égard desquelles on pouvait entretenir des doutes, par rapport à la priorité ou à l'originalité. Une collection d'intégrales définies bien ordonnée peut certainement obvier à tous ces inconvénients.

De ce point de vue suivait naturellement une autre considération non moins importante. Après avoir réuni les diverses formules, il importait beaucoup de connaître leur méthode de déduction: et cela d'autant plus, que plusieurs méthodes employées en d'autres temps avec une confiance absolue, ne sont maintenant plus à l'abri d'objections, en quelques cas très-fondées. Dès-lors, pour être sûr d'un résultat quelconque, il fallait absolument que l'on fût à même de juger de la validité et de l'exactitude de la méthode employée. Or il ne pouvait entrer dans le plan de rédaction de ces Tables, déjà assez volumineuses, d'ajouter à côté de chaque intégrale la méthode à l'aide de laquelle on l'avait déduite, quand même il eût été possible de l'indiquer d'une manière courte, précise et certaine: de plus il y a beaucoup de formules, qui peuvent être trouvées de plus d'une manière. J'ai tâché de subvenir à cette difficulté d'une autre manière, qui, à ce que j'espère, ne manquera pas d'approbation. A côté de chaque formule in-

tégrale se trouve une notice bibliographique indiquant, où l'on peut en trouver la déduction, ou même plusieurs déductions diverses, s'il y en a. De cette façon chacun est mis en état de juger par lui-même de la validité des résultats indiqués, et de répéter lui-même les calculs nécessaires, s'il pourrait le juger convenable.

Mais par ces notices bibliographiques elles-mêmes il était en même temps possible de remplir un troisième but, celui de donner un tableau historique et bibliographique de cette branche de l'analyse. Pour que ce tableau fût complet, il aurait fallu que j'eusse pu parcourir tous les ouvrages, où pourraient se trouver des intégrales définies. C'était une entreprise à peu près impossible, puisque d'une part je n'aurais jamais pu m'assurer de n'avoir omis aucun livre, et que je ne me trouvais pas dans une condition de pouvoir les avoir tous sous les yeux : tandis que d'autre part le travail serait devenu d'une telle longueur que je n'aurais pas osé l'entreprendre. Néanmoins je dois confesser que de ce côté-là mes desirs, peut-être trop ardents par l'intérêt personnel que je portais naturellement au succès de mon entreprise, n'ont pas été remplis comme je l'avais désiré, ni comme je l'avais espéré. J'avais demandé par la voie de quelques journaux scientifiques l'envoi des notes ou des mémoires monographiques, qui pourraient exister sur la théorie des intégrales définies : et j'avoue avoir assez compté sur l'intérêt que les formules, dont je me proposais la récolte, doivent inspirer aux Analystes, pour attendre quelques fruits de cette démarche : mais personne n'a répondu à l'appel. Tout dépendait donc de moi-même et c'est par le sommaire des livres, des journaux et des mémoires consultés (pages 20 et 21) que l'on pourra juger jusqu'à quel point les Tables peuvent être censées complètes. Si toutefois, comme je n'en doute guère, il y a des écrits, qui portent sur cette matière et qui pourtant ne se trouvent pas sur cette liste, qu'on veuille bien trouver l'excuse de leur omission dans ce que je viens de dire ; l'espèce de reproche, qui s'y trouve, n'a son origine que dans mon désir de rendre mon excuse plus fondée. Quant à ce sommaire, il donne lieu à quelques observations. Les journaux mathématiques Anglais et Américains y manquent complètement, puisque je n'ai pas eu moyen de m'en procurer l'étude : la même observation se répète pour les livres et les monographies de ces pays, que l'on trouve peu chez nous. J'ai été bien fâché que tel ait été le cas, puisque bien certainement le fruit de leurs études m'eût fourni bien des données intéressantes : toutefois le Journal de Mathématiques pures et appliquées, rédigé par M. J. LIOUVILLE, nous donne quelques-unes de ces recherches, et j'ai dû me contenter de celles-là. Quant aux Mémoires des diverses Académies et Institutions scientifiques, il y avait de ce côté-là une occasion magnifique et unique pour notre patrie dans la Bibliothèque de l'Académie Royale des Sciences à Amsterdam ; et comme l'entrée m'y était ouverte primitivement par les soins bienveillants

de M. W. VROLIK, Secrétaire de cette Académie, et plus tard par les droits acquis par mon titre de membre, j'ai tâché de ne rien laisser désirer à cet égard. Si l'on prend la peine de feuilleter les Tables, on voudra bien admettre, j'espère, que si tout le matériel ne s'y trouve pas, c'est bien au moins le plus grand nombre des formules trouvées, que l'on y rencontre. Je dois encore ajouter ici qu'en général je n'ai admis dans les Tables que toutes les intégrales définies, que j'ai pu trouver évaluées avant la fin de l'année 1853, lorsque j'ai commencé la rédaction en tables. Voici un sommaire — par ordre alphabétique des noms d'auteurs — des mémoires principaux, contenus dans les diverses Collections Académiques et dans les journaux scientifiques, que j'ai consultés: on y rencontre bien des noms éminents.

Abel, Cr. 2. 22.

Abria, L. 4. 248.

Arndt, Gr. 4. 436. — Gr. 6. 187. — Gr. 6. 434. — Gr. 10. 225. — Gr. 10. 233. — Gr. 10. 240. — Gr. 10. 247. — Gr. 10. 250. — Gr. 10. 253. — Gr. 10. 455. — Gr. 11. 70.

Bertrand, L. 8. 110.

Besge, L. 14. 31.

Bidone, Mém. Turin. 1812. 231.

Biersens de Haan, Verh. Kon. Akad. Amsterdam. Dl. 2. bl. 19. — Gr. 13. 193.

Binet, C. R. 9. 39. — C. R. 12. 958. — P. 27. 123.

Björling, Gr. 21. 26.

Boncompagni, Cr. 25. 74.

Ossian Bonnet, L. 6. 238. — L. 14. 249. — L. 17. 265.

Boole, Phil. Trans. 1844. 225. — L. 13. 111.

Catalan, L. 4. 323. — L. 5. 110. — L. 6. 340. — L. 6. 419. — L. 8. 239.

Cauchy, Mém. Paris. 1823. 603. — Sav. Etr. I. 1827. p. 3. Notes. — Sav. Etr. I. 1827. p. 599. — C. R. 11. 1008. — C. R. 16. 122. — P. 19. 511. — P. 28. 147.

Cayley, L. 12. 231. — L. 13. 245. — L. 13. 264.

Cellérier, L. 8. 255.

Chev. Cisa de Grésy, Mém. Turin. T. 20. 1821. 209.

Clausen, Cr. 7. 309. — Gr. 3. 336.

Clausius, Cr. 34. 123.

Dedekind, Cr. 45. 370.

Delaunay, L. 2. 355.

Dienger, Cr. 34. 75. — Cr. 37. 363. — Cr. 38. 266. — Cr. 38. 331. — Cr. 41. 137. — Cr. 42. 285. — Gr. 8. 450. — Gr. 10. 107. — Gr. 10. 341. — Gr. 11. 88. — Gr. 11. 94. — Gr. 12. 81. — Gr. 12. 97. — Gr. 12. 210. — Gr. 12. 409. — Gr. 12. 416. — Gr. 13. 280. — Gr. 13. 424. — Gr. 14. 223. — Gr. 15. 119.

Dirksen, Ber. Abh. Berlin. 1848. 120.

Euler, N. C. Petr. 6. 115. — N. C. Petr. 14. 129. — N. C. Petr. 16. 91. — N. C. Petr. 19. 3. — N. C. Petr. 19. 30. — N. C. Petr. 19. 60. — N. C. Petr. 20. 59. — Act. Petr. T. 1. 1777. P. 2. p. 3. — Act. Petr. T. 1. 1777. P. 2. p. 29. — Mém. Pétersbourg. T. 6. 1814.

Fuss, Mém. Pétersbourg. T. 11. 1820.

Grunert, Cr. 8. 146. — Gr. 2. 266. — Gr. 4. 113. — Gr. 6. 448. — Gr. 17. 313.

Hill, Cr. 3. 104. — Cr. 3. 132. — Cr. 7. 102.

Hoppe, Cr. 40. 139. — Cr. 40. 142.

Jacobi, Cr. 10. 101. — Cr. 11. 307. — Cr. 15. 1. — Cr. 32. 8. — L. 10. 229.

Jürgensen, Cr. 23. 143.

Kausler, Mém. Pétersbourg. T. 3. 1811.

Kummer, Cr. 14. 148. — Cr. 17. 210. — Cr. 17. 228. — Cr. 20. 1. — Cr. 25. 1.

Lamé, L. 2. 147.

Laplace, Mém. Acad. Paris. 1778. 227. — Mém. Acad. Paris. 1782. 1. — Mém. Inst. 1809. 353. — P. 15. 229.

Lebesgue, L. 15. 215.

Lefort, L. 11. 142.

Legendre, Mém. Inst. 1809. 416.

Lejeune-Dirichlet, C. R. 8. 157. — Abh. Berlin. 1835. — Cr. 4. 94. — Cr. 15. 258. — Cr. 17. 57.

Libri, Cr. 7. 224.

Lindmann, K. Danske Handl. 1850. — Gr. 16. 94. — Gr. 17. 455.

Liouville, Cr. 11. 1. — Cr. 13. 219. — L. 2. 135. — L. 4. 317. — L. 5. 311. — L. 11. 464. — L. 17. 448.

Lobatto, Cr. 9. 260. — Cr. 11. 171. — L. 5. 115.

Lobatschewsky, Mém. Kasan. 1835. 1. — Mém. Kasan. 1835. 211. — Mém. Kasan. 1836. 1. — Cr. 24. 162.

- Malmsten*, K. Stockh. Handl. 1841. — Cr. 35. 55. — Cr. 38. 1.
- Mösta*, Gr. 10. 449. — Gr. 10. 455.
- Oettinger*, Cr. 35. 13. — Cr. 38. 162. — Cr. 38. 216.
- Pioch*, Mém. Cour. Bruxelles. T. 25. 1843.
- Plana*, Mém. Turin. 23. 1818. 7. — Mém. Turin. 25. 1820. — Mém. Brux. 10. 1837. — Cr. 17. 1. — Cr. 17. 163. — Cr. 17. 345.
- Poisson*, Mém. Inst. 1811. 163. — Mém. Paris. 1816. 71. — Mém. Paris. 1823. — P. 16. 215. — P. 17. 612. — P. 18. 295. — P. 19. 404. — P. 20. 222. — P. 19. 60. — L. 2. 184. — L. 2. 224.
- Raabe*, Cr. 15. 355. — Cr. 16. 219. — Cr. 23. 105. — Cr. 25. 146. — Cr. 25. 160. — Cr. 25. 169. — Cr. 28. 10. — Cr. 37. 356. — Cr. 42. 348.
- Ramus*, Dauske Afh. 7. 265. — Overs. Danske Forh. 1844. — Cr. 24. 257.
- William Roberts*, L. 10. 453. — L. 11. 157. — L. 11. 201. — L. 11. 471. — L. 12. 449. — L. 15. 238. — L. 16. 1.
- Schaar*, N. Mém. Bruxelles. T. 23. 1850. 117. — Mém. Cour. Brux. T. 22. 1848.
- Schaeffer*, Cr. 30. 277. — Cr. 37. 127.
- Schellbach*, Cr. 48. 207.
- Scherk*, Cr. 10. 97.
- Schlömilch*, Cr. 33. 268. — Cr. 33. 316. — Cr. 33. 325. — Cr. 33. 353. — Cr. 36. 268. — Cr. 36. 271. — Cr. 42. 125. — Gr. 1. 263. — Gr. 1. 360. — Gr. 1. 417. — Gr. 3. 9. — Gr. 3. 278. — Gr. 4. 23. — Gr. 4. 71. — Gr. 4. 167. — Gr. 4. 316. — Gr. 4. 364. — Gr. 5. 90. — Gr. 5. 152. — Gr. 5. 204. — Gr. 6. 200. — Gr. 6. 213. — Gr. 7. 38. — Gr. 7. 100. — Gr. 7. 270. — Gr. 9. 5. — Gr. 9. 307. — Gr. 9. 379. — Gr. 10. 340. — Gr. 10. 424. — Gr. 10. 440. — Gr. 11. 63. — Gr. 11. 174. — Gr. 11. 189. — Gr. 12. 198. — Gr. 12. 208. — Gr. 18. 391.
- von Schmidten*, Cr. 5. 392.
- Serret*, L. 8. 1. — L. 8. 489. — L. 9. 193. — L. 9. 436.
- Smaasen*, Cr. 42. 222.
- Stegmann*, r. 21. 377.
- Stern*, Gr. 7. 108.
- Svanberg*, L. 11. 197.
- Tchebicheff*, L. 8. 235.

Thomson, L. 10. 137.

Tortolini, Cr. 34. 101.

Winckler, Cr. 45. 102.

Enfin je m'étais encore proposé un quatrième but: savoir la critique des intégrales définies, que je trouvais. Mais de ce côté-là surgirent bien des obstacles. L'on ne pourrait exiger, que j'eusse fait la révision de tous les calculs, en général assez longs, dont les résultats seulement remplissent tant de volumes: mais la seule et la moindre difficulté ne se trouvait pas là; une autre était d'une importance bien plus grande pour la rédaction de ces Tables. Ce n'est pas seulement pourtant contre les anciennes méthodes, que se sont élevées des objections dont j'ai parlé précédemment, mais plusieurs des méthodes nouvelles ou récemment appliquées ont subi le même sort. En un mot telle, méthode employée, et par suite admise par tel Analyste, est rejetée comme fausse par un autre: donc les résultats obtenus par l'un ne sont pas admis par celui d'une opinion contraire. Fallait-il que je me fusse posé en juge? Je me suis souvent fait cette question: mais je n'osais le faire, je ne croyais pas les fondemens de cette partie de l'Analyse toujours basés sur des principes d'une telle stabilité, que l'on aurait le droit absolu de juger sans merci les pensées, les recherches d'un autre. C'est pourquoi j'ai aussi admis dans les Tables les résultats obtenus par des méthodes, que pour moi-même je ne saurais regarder comme valides. Je m'y suis résolu d'autant plus volontiers que l'annexion des noms des auteurs, qui ont déduit ces résultats au moins douteux, donne pour ainsi dire des poids, qui en indiquent et en mesurent la certitude et la validité. Dans cette catégorie tombent par exemple toutes les intégrales définies, qui se trouvent Partie I, Section IV, Tables 96, 97, 98, 102, en tant que les fonctions circulaires directes aient la première puissance de la variable pour argument. Ces intégrales de fonctions circulaires directes de x seulement, prises entre les limites zéro et l'infini, — dont les valeurs, entre autres d'après M. RAABE, ont obtenu leurs places aux Tables indiquées — sont évidemment indéterminées selon ma manière de voir: néanmoins, d'après les principes exposés j'ai cru devoir les admettre dans les Tables. Quiconque parcourt ces Tables peut s'assurer lui-même, par l'inspection des citations bibliographiques de l'exactitude et de la validité de la méthode de M. RAABE. Il y a encore une autre classe d'intégrales définies, que je m'occupe d'étudier, savoir celles qui contiennent un élément k , que l'on fait diverger vers l'infini: jusques à présent je ne suis pas toujours du même avis que plusieurs auteurs, qui en ont fait usage: mais puisque en premier lieu mes recherches n'ont pas encore atteint le but proposé, et que d'un autre côté les Tables ont déjà été rédigées avant la fin de l'année 1854, j'ai jugé à propos de ne pas admettre mes résultats dans

les Tables : pourtant je dois avertir que les valeurs des intégrales mentionnées sont d'une grande influence sur plusieurs autres intégrales définies, dont on acquiert la valeur au moyen d'elles *). Néanmoins des observations critiques de ma part ne se font pas désirer : on les trouve là, où les résultats divergents de différents Mathématiciens demandaient un jugement : là aussi, où une faute de calcul, qui me frappait, rendait le résultat vicieux.

Voilà mes considérations, quant au but que je me suis proposé et à la manière dont j'ai cherché à l'atteindre — en recueillant les formules déduites par d'autres auteurs. Je ne manquais pas de reconnaître bientôt, que je me trouvais dans une position favorable pour en déduire des résultats nouveaux, et je vais m'expliquer dans quelle voie je me suis engagé à cet égard. En premier lieu il était aisé quelquefois de déduire d'autres intégrales par voie d'addition ou de soustraction des résultats déjà obtenus. D'une autre part plusieurs des sections me semblaient présenter des lacunes en quelques points, et n'être pas assez complètes ; j'ai tâché d'y remédier en transformant les intégrales définies d'une autre catégorie, par la méthode connue de transformation de la variable, en d'autres formules telles, qu'elles pussent prendre place là, où ces lacunes se trouvaient à remplir. Il va sans dire que je n'ai pas eu l'intention d'épuiser toutes les substitutions possibles ; j'ajouterai seulement que je ne les ai appliquées que là, où cette application était directement permise, c'est-à-dire où dans la détermination des limites elles n'offraient pas des maxima ou des minima, qui pouvaient donner lieu à des incertitudes ou au moins à des objections. De plus j'ai toujours donné la préférence à de telles intégrales résultantes, qui pouvaient se prêter à la méthode suivante, qui m'a fourni en grande partie les résultats nouveaux, que l'on trouvera dans les Tables.

Cette méthode, exposée et appliquée plus amplement dans une „Note sur une méthode pour la réduction d'intégrales définies et sur son application à quelques formules spéciales”, que l'Académie Royale des Sciences m'a fait l'honneur de faire imprimer dans le deuxième Volume de ses Mémoires, revient à celle d'intégration partielle. Elle est contenue dans la formule

$$\left. \int_a^b f(x) d.F(x) - f(x).F(x) \right\}_a^b + \int_a^b F(x) d.f(x) = 0.$$

On peut appliquer cette formule de transformation à une intégrale définie, aussitôt que la fonction intégrée se laisse diviser en deux facteurs dont l'un peut être considéré comme la différentielle de quelque fonction connue ; car dès-lors cette intégrale définie rentre sous la forme du troisième

*) Depuis cette Note a été présentée et se trouvera dans le Volume VII de ces Mémoires.

terme de l'équation précédente, et la formule donnera lieu à la détermination du premier terme, — qui est une nouvelle intégrale définie, en général d'une forme tout-à-fait différente, — à moins que le deuxième terme $f(x) \cdot F(x) \Big|_a^b$, c'est-à-dire pris de la limite a jusques à l'autre b , n'offre de difficultés ou d'obstacles, qu'il reste continu entre ces limites, et que sa valeur soit finie et assignable pour ces limites elles-mêmes. L'on observera sans peine dans les Tables les fruits, que cette méthode a portés.

Quant aux résultats — leur nombre est près de trois mille deux-cent — que j'obtenais ainsi par une des méthodes mentionnées, et qui n'étaient pas encore trouvés par d'autres, ils se distinguent par un manque de bibliographie; on y trouve seulement un renvoi vers une autre intégrale définie, dont elle a été déduite en général à l'aide d'une des trois méthodes précédentes. Je n'ai pas ajouté de quelle méthode j'ai fait usage dans chaque cas spécial, puisqu'en général on peut aisément s'en assurer par l'observation et par la comparaison de la formule employée et du résultat obtenu. De la sorte chacun peut lui-même reprendre les calculs nécessaires pour s'assurer de l'exactitude d'une telle intégrale définie, faculté qu'il m'importait beaucoup d'offrir, et qui était rigoureusement nécessaire afin que ces résultats nouveaux pussent être d'un même poids que les autres, que j'avais recueillis et munis de leurs passe-ports de bibliographie.

Ensuite j'ai encore à ajouter quelques observations explicatives, et justifiantes au besoin, sur la rédaction et la classification de ces Tables, qui n'ont pas laissé de me causer quelquefois maint embarras. Il me paraissait nécessaire en premier lieu que la division fût naturelle, d'une autre part que la recherche d'une intégrale définie put toujours se faire aisément. Mais quiconque veut se souvenir de la variété des formes, qui se fait observer parmi les intégrales définies, reconnaîtra qu'une division bonne, naturelle et simple n'est pas chose aussi facile, que cela peut paraître au premier abord. L'incertitude sur le nombre de formules à enregistrer, dont dépendait naturellement le nombre des Tables, rendait cette division encore plus difficile au commencement, et j'ai été obligé de temps en temps à modifier les règles qui me servaient à la classification. C'est pourquoi l'exposition des principes que j'ai suivis pourra montrer de quelle manière j'ai cherché à atteindre ce but, aussi près qu'il m'était possible.

La première division (voir Page 3) en trois Parties est fondée sur le nombre de fonctions, qui se trouvent sous le signe d'intégration définie, suivant que ce nombre est d'une seule, de deux ou de plus de deux.

La deuxième division en trente-cinq Sections ne donnera guère lieu à plus de difficultés. J'ai pris en considération les cinq fonctions diverses: Algébriques, Exponentielles, Logarithmes, Circulaires Directes (autrement dites goniométriques), Circulaires Inverses: et chaque Section I à V contient les intégrales définies, dont l'argument ou la fonction intégrée appartient exclusivement à une seule de ces fonctions. La Section VI contient les autres fonctions, telles que fonctions Elliptiques, le Logarithme Intégral, l'Exponentielle Intégrale, la Sinus Intégrale, la Cosinus Intégrale, les fonctions $B'(x)$ et $B''(x)$ de M. RAABE. Les fonctions Hyperboliques, qui peuvent être représentées par des fonctions Exponentielles, ne sont pas admises comme distinctes, et l'on trouvera toujours leurs valeurs exprimées à l'aide de ces dernières. Dans la deuxième Partie, Sections VII à XX, qui contient les intégrales définies, dont les arguments sont composés de deux sortes de fonctions différentes, les six sortes de fonctions mentionnées précédemment se trouvent combinées deux à deux en respectant l'ordre donné à ces fonctions dans la Partie première. Enfin dans la Partie troisième, le même ordre est observé dans les combinaisons respectives. Elle contient Sections XXI à XXXIV les intégrales définies d'un argument, qui est composé de trois sortes différentes de fonctions, et dans la Section XXXV celles, qui en contiennent plus de trois. Les diverses combinaisons y sont à peu près toutes représentées; car, dans la Partie deuxième manque seulement la combinaison: Fonctions Circulaires Inverses et Autres; et dans la Partie troisième, — si l'on excepte la catégorie de „Autres Fonctions”, qui ne s'y trouve que cinq fois — seulement la combinaison: Fonctions Exponentielles et Logarithmes et Circulaires Inverses. Il faut toutefois faire remarquer, que plusieurs de ces Sections ne sont représentées que par un petit nombre d'intégrales définies.

Il fallait subdiviser ces Sections en Tables. En premier lieu la considération des limites, entre lesquelles l'intégration définie doit avoir lieu, s'offrait comme argument principal: ces limites diffèrent généralement auprès des différentes fonctions et donc dans chaque Section. Ici ce sont les limites 0, ± 1 et $\pm \infty$, qui sont les plus naturelles, comme pour les fonctions Algébriques, Exponentielles, Logarithmiques, Circulaires Inverses: là ce sont au contraire les multiples et les parties aliquotes de π , comme pour les fonctions Circulaires Directes. Dans les Parties deuxième et troisième ce sont tantôt les limites de la première catégorie, tantôt celles de la seconde, qui s'offrent le plus, sans ordre apparent. Le choix des limites a donc dépendu en général du nombre des formules, qui venaient s'y soumettre; les limites, qui ne valaient que pour un petit nombre d'intégrales définies, se trouvant toutes réunies sous le nom de „Limites diverses.” J'insère ici un extrait du sommaire des Tables pour offrir un coup d'oeil sur la division

B

à cet égard: cet aperçu servira bien mieux à donner une idée de cette classification que plusieurs réflexions ou observations ne pourraient le faire.

			Limites.				Limites.
P. I. S.	I. T.	1—16.	0, 1.	P. I. S.	IV. T.	105—107.	λ, μ .
		T. 17.	— 1, 1.	S.	V. T.	108.	0, 1.
		T. 18—28.	0, ∞ .			T. 109.	0, ∞ .
		T. 29, 30.	— ∞ , ∞ .			T. 110.	diverses.
		T. 31, 32.	1, ∞ .	S.	VI. T.	111.	diverses.
		T. 33, 34.	0, p .	P. II. S.	VII. T.	112.	0, 1.
		T. 35.	diverses.			T. 113—141.	0, ∞ .
S.	II. T.	36—39.	0, ∞ .			T. 142—148.	— ∞ , ∞ .
		T. 40.	— ∞ , ∞ .			T. 149.	0, p .
		T. 41.	diverses.			T. 150.	$p, \pm \infty$.
S.	III. T.	42—44.	0, 1.	S. VIII.	T.	151—178.	0, 1.
		T. 45.	diverses.			T. 179—185.	0, ∞ .
S.	IV. T.	46—52.	$0, \frac{1}{4}\pi$.			T. 187.	1, ∞ .
		T. 53—77.	$0, \frac{1}{2}\pi$.			T. 188.	0, p .
		T. 78—86.	0, π .			T. 189.	p, q .
		T. 87—91.	0, 2π .			T. 190. (Log. de Log.)	0, 1.
		T. 92.	$-\frac{1}{4}\pi, \frac{1}{4}\pi$.			T. 191.	p, ∞ .
		T. 93.	$-\frac{1}{2}\pi, \frac{1}{2}\pi$.	S.	IX. T.	192.	0, 1.
		T. 94.	$p\pi, q\pi$.			T. 193—231.	0, ∞ .
		T. 95.	0, 1.			T. 232—234.	— ∞ , ∞ .
		T. 96—99.	0, ∞ .			T. 235, 236.	1, ∞ .
		T. 100, 101.	— ∞ , ∞ .			T. 237.	$0, \frac{1}{4}\pi$.
		T. 102.	$\frac{\pi}{2}, \infty$.			T. 238—243.	$0, \frac{1}{2}\pi$.
		T. 103, 104.	0, p .			T. 244—249.	0, π .
						T. 250.	0, 2π .
						T. 251, 252.	0, p .

			Limites.				Limites.
P. II. S.	IX. T. 253.		$\lambda, \mu.$	P. II. S.	XVI. T. 362.		$0, \lambda.$
	T. 254.		$p, \infty.$		T. 363.		$\lambda, \frac{1}{2} \pi.$
	T. 255.		diverses.		T. 364.		$\lambda, \mu.$
S.	X. T. 256—262.		$0, 1.$		T. 365.		diverses.
	T. 263—269.		$0, \infty.$	S. XVII. T. 366.			$0, 1.$
	T. 270.		$1, \infty.$	S. XVIII. T. 367.			diverses.
	T. 271.		diverses.	S. XIX. T. 368, 369.			$0, \frac{1}{2} \pi.$
S.	XI. T. 272.		diverses.		T. 370—372.		$0, \pi.$
S.	XII. T. 273—275.		$0, \infty.$		T. 373.		$0, 2 \pi.$
	T. 276.		$-\infty, \infty.$		T. 374.		diverses.
	T. 277.		diverses.	S. XX. T. 375.			diverses.
S.	XIII. T. 278—285.		$0, \infty.$	P. III. S.	XXI. T. 376.		$0, 1.$
	T. 286.		$-\infty, \infty.$		T. 377—381.		$0, \infty.$
	T. 287—295.		$0, \frac{1}{2} \pi.$		T. 382.		$-\infty, \infty.$
	T. 296.		$0, \pi.$		T. 383.		diverses.
	T. 297.		$-\frac{1}{2} \pi, \frac{1}{2} \pi.$	S. XXII. T. 384.			$0, \frac{1}{4} \pi.$
	T. 298.		diverses.		T. 385—399.		$0, \infty.$
S.	XIV. T. 299.		$0, \infty.$		T. 400.		diverses.
S.	XV. T. 300.		$0, \infty.$	S. XXIII. T. 401.			$0, \infty.$
S.	XVI. T. 301.		$0, 1.$	S. XXIV. T. 402.			$0, \infty.$
	T. 302.		$0, \infty.$	S. XXV. T. 403—409.			$0, 1.$
	T. 303—329.		$0, \frac{1}{4} \pi.$		T. 410.		$0, \frac{1}{4} \pi.$
	T. 330—352.		$0, \frac{1}{2} \pi.$		T. 411, 412.		$0, \frac{1}{2} \pi.$
	T. 353—355.		$0, \pi.$		T. 413.		$0, \pi.$
	T. 356.		$0, 2 \pi.$		T. 414—419.		$0, \infty.$
	T. 357—360.		$\frac{1}{4} \pi, \frac{1}{2} \pi.$		T. 420.		$-\infty, \infty.$
	T. 361.		$0, p \pi.$		T. 421.		diverses.

	Limites.		Limites.
P. III. S. XXVI. T. 422—424.	0, 1.	P. III. S. XXIX. T. 435.	0, ∞ .
T. 425.	0, ∞ .	S. XXX. T. 436—438.	0, $\frac{1}{2}\pi$.
T. 426.	1, ∞ .	T. 439.	0, π .
T. 427.	diverses.	T. 440.	diverses.
S. XXVII. T. 428.	diverses.	S. XXXI. T. 441.	0, ∞ .
S. XXVIII. T. 429.	0, $\frac{1}{2}\pi$.	S. XXXII. T. 442.	diverses.
T. 430.	0, π .	S. XXXIII. T. 443.	diverses.
T. 431—433.	0, ∞ .	S. XXXIV. T. 444.	diverses.
T. 434.	diverses.	S. XXXV. T. 445—447.	diverses.

Mais à présent les intégrales entre les mêmes limites, appartenant à une même Section, devaient entrer dans des cadres assez nuancés pour ainsi dire, pour pouvoir facilement faire saisir les distinctions établies entre elles. Il me semblait qu'il devait y avoir de l'inconvénient dans des tables trop étendues, puisqu'alors il serait nécessairement plus difficile de trouver une intégrale définie quelconque, que l'on chercherait. D'un autre côté il ne fallait pas rendre les tables trop petites et augmenter ainsi outre mesure le nombre des distinctions devenues par là nécessairement minutieuses. Là donc, où il était besoin d'une telle restriction, je me suis borné au nombre d'environ vingt-cinq formules pour chaque Table; j'ai dû régler la classification d'après cette limite arbitraire, et pour cela admettre des distinctions trop minutieuses pour être universellement admissibles. Toutefois ces divisions moins naturelles n'ont été nécessaires que dans un petit nombre de cas: quelquefois même je n'ai pas subdivisé des Tables d'une étendue plus grande (voir, p. a. T. 1, 40, 49, 68, 85, 127, 135, 195, 202, etc.).

En général je me suis demandé pour les fonctions Algébriques :

- 1°. si elles étaient rationnelles ou irrationnelles: — c'est-à-dire quant à la forme: p. ex. x^p , quoique p fût fractionnaire, est considéré comme rationnel, $x^{p-\frac{1}{2}}$ au contraire est considéré comme une fonction irrationnelle.
- 2°. si elles étaient entières ou fractionnaires: — de même quant à la forme; x^{p-1} est considéré entier, même dans le cas que p était assujetti à la condition de ne pas surpasser l'unité, mais x^{-p} est regardé comme une fraction.
- 3°. si elles étaient monômes ou polynômes. Les formes $(a+x)^b$, quoique proprement des monômes, ont été rangés parmi les polynômes, et bien comme des puissances de binômes.

Quelquefois la subdivision se règle d'après puissances, et alors aussi d'après puissances numériques (pour l'exposant a spécial) et puissances algébriques (pour cet exposant a général).

Après des fonctions Exponentielles et Logarithmiques la même distinction de formes rationnelles ou irrationnelles, de formes entières ou fractionnaires, de formes monômes ou polynômes est retenue: cette distinction offrant là aussi beaucoup de facilité pour la classification.

Quant aux fonctions Circulaires Directes, j'ai toujours considéré la Sinus, la Cosinus et la Tangente comme des fonctions entières; pour la Cotangente, la Sécante et la Cosécante j'ai pris en général leurs valeurs fractionnaires exprimées en Sinus et en Cosinus; néanmoins j'ai pensé devoir quelquefois m'abstenir de cette distinction, quand pour la symétrie des résultats il importait de les réunir dans un même cadre.

Les fonctions Circulaires Inverses offraient peu de difficultés: quelquefois seulement j'ai été obligé de faire une distinction entre celles, qui avaient pour argument un simple x , et celles dont l'argument était une fonction quelconque de x .

C'est d'après les principes exposés que les intégrales définies sont rangées dans les Tables respectives: le sommaire (voir Pages 5 à 19) en fait voir le résultat: j'ose espérer que leur emploi prouvera que l'arrangement est convenable.

Quelques mots suffiront pour faire comprendre la construction des Tables elles-mêmes. En tête de chaque Table on trouve au milieu son numero, à gauche la description des fonctions intégrées, à droite les limites de l'intégration: ce sont les mêmes trois arguments principaux, qui figurent dans le sommaire des Tables. Alors viennent les intégrales définies elles-mêmes, numérotées, afin de pouvoir facilement les citer: les intégrales plus générales suivent celles qui sont spéciales ou les cas spéciaux des premières. Or ces cas spéciaux des formules générales ne sauraient toujours être omis comme sous-entendus dans celles-ci, puisque d'une part les valeurs deviennent pour la plupart beaucoup plus simples, et que d'un autre côté ces valeurs spéciales de quelque constante sont bien loin d'être toujours permises. Après de chaque formule sont notées, s'il le faut, les équations de limite auxquelles quelque constante peut être soumise: dans le cas contraire les premières lettres de l'alphabet $a, b, c, \dots$ désignent en général des quantités entières, les lettres $p, q, r, \dots$ au contraire des quantités quelconques, entières ou fractionnaires, rationnelles ou irrationnelles. Toutefois toutes ces quantités sont regardées comme positives, à moins que le contraire ne soit expressément énoncé; x est toujours réservé pour indiquer la variable de l'intégration

Dans les valeurs des intégrales définies l'on observe diverses fonctions, outre celles dont il a été question déjà à l'occasion de la division des Tables: on les trouve Page 22, 23, avec les notations respectives, ainsi que je les ai employées. Ce sont: les quatre fonctions Hyperboliques, — les coefficients du binôme, — les factorielles $c^{a/b}$, laquelle notation exprime le produit $c(c+b)(c+2b)\dots(c+[a-6]b)$, — les coefficients Bernoulliens B_{2a-1} , tandis que les fonctions correspondantes B_{2a} désignent les coefficients de la série pour la sécante, — les trois séries hypergéométriques de M. KUMMER, — la fonction $L(a)$ de M. LOBATSCHESKY. De plus la lettre α désigne souvent une quantité arbitraire ou indéterminée, et k une quantité qui devient infinie: i est la racine carrée de l'unité négative, la quantité ainsi dite imaginaire la plus simple, — A la constante du Logarithme Intégral, évaluée à 18 décimales (voir GRUNERT, Archiv der Mathematik und Physik, Th. XI. Seite 323), — e la base des Logarithmes naturels, évaluée à 105 décimales (voir GRUNERT, Archiv der Mathematik und Physik, Th. III. Seite 28), — π le rapport de la circonférence du cercle à son diamètre, évalué à 530 décimales.

Quelquefois on rencontre des sommations, c'est-à-dire des séries, soit finies, soit infinies; elles sont désignées par le signe $\sum_b^a$, où a et b sont les limites entre lesquelles on doit donner à l'argument, qui est représenté par la lettre n , toutes les valeurs entières possibles. Lorsqu'il y a des sommations doubles, la première se fait ordinairement suivant l'argument n , la seconde suivant l'argument m : la forme des sommations elle-même en décide toujours aisément.

Encore une observation quant à la notation des fonctions Circulaires Directes. Il me semblait plus clair de prendre le signe $\text{Sin.}^2 x$ pour la seconde puissance de $\text{Sin. } x$, tandis que la Sinus d'une Sinus de x est désigné par $\text{Sin.}(\text{Sin. } x)$: on sait que dans les derniers temps on a proposé le premier signe pour la seconde fonction. Encore $\text{Sin. } x^2$ ou plutôt $\text{Sin.}(x^2)$ est ici la Sinus de x^2 . De même j'ai donné la préférence aux signes $\text{Arcsin. } x$, $\text{Arcos. } x$ etc. sur les autres signes $\frac{1}{\text{Sin.}} x$, $\frac{1}{\text{Cos.}} x$, etc. et cela seulement pour l'exactitude de l'impression, car je craignais que dans les formules, où des fonctions Circulaires Directes se trouvaient mêlées à des fonctions Circulaires Inverses, l'on ne confondît entre les deux fonctions absolument diverses $\frac{1}{\text{Sin.}} x$ et $\frac{1}{\text{Sin. } x}$. J'insiste sur ces raisons pour le choix de ces signes, puisque d'un point de vue purement théorique les autres notations pourraient bien être préférables.

Comme la publication de ces Tables est la première entreprise de ce genre, je ne doute pas qu'elles ne soient sujettes à des défauts: je n'ai qu'à prier ceux, qui en feront une étude particulière, de vouloir bien me faire part de leurs observations critiques, que je recevrai avec reconnaissance.

Il me reste enfin à faire observer, que je dois l'impression de ces Tables, dont la précision et l'élégance, faisant honneur à la typographie de M. KRÖBER, m'ont beaucoup facilité la correction, à la munificence de l'Académie Royale des Sciences, qui a bien voulu les insérer dans sa collection de Mémoires, et en a ainsi rendu la publication possible.

D. BIERENS DE HAAN.

Deventer, Décembre 1855.

Pendant que ces tables d'Intégrales Définies étaient livrées à l'impression je me suis occupé de la théorie de ces fonctions et de la critique des diverses méthodes d'évaluation. Lorsque ce travail était assez avancé, j'ai pu confronter mes résultats avec ceux, que j'avais accueillis dans mes Tables, sans toutefois en avoir alors revisé les calculs, comme je viens de dire plus haut. Le résultat de cette confrontation n'était pas toujours favorable à mes Tables; quelquefois une faute s'y était glissée par suite d'un signe ou d'une notation mal copiés, — et une transcription totale, quatre fois répétée durant la rédaction, n'en avait pas diminué le danger; tantôt j'avais admis un résultat lui-même fautif. Et, ce qui ne valait guère mieux, ces fautes s'étaient nécessairement répétées dans les nouvelles intégrales, que j'en avais déduites. Par la nature des fonctions en question, les formules juxtaposées ne donnent lieu en général à aucune comparaison mutuelle et sont tout-à-fait indépendantes les unes des autres, et même elles se déduisent souvent de formules qui se trouvent éparses dans toute l'étendue de cet ouvrage; de sorte que la correction des épreuves, — ouvrage naturellement assez épineux et dont je ne pouvais partager les difficultés avec un autre, — ne mettait pas ces fautes en évidence. Comme l'exactitude pourtant est de première nécessité pour ces Tables, j'ai pris le parti de faire moi-même le calcul de chaque formule, et ce travail, souvent assez pénible, m'a fourni la liste suivante de corrections et d'observations criti-

ques: elle faillit plusieurs fois me décourager de mon ouvrage, qui contenait encore tant de fautes, notamment dans les quarante-trois feuilles déjà imprimées avant cette révision: toutefois je puis alléguer en ma faveur que près de six-cents formules fautives sont telles par suite d'une faute qui se trouvait dans un résultat acquis par un autre.

Mais en même temps j'étais contraint à présent de me déclarer à l'égard de la validité de certains résultats, déjà indiqués ci-devant: je l'ai fait en les pourvoyant d'un signe d'interrogation ou en les notant comme fautives dans la liste mentionnée. Parmi les résultats que je n'ai pas révisés, se trouvent ceux qui dépendent des facultés à un nombre fractionnaire ou négatif de

termes de Mr. OETTINGER, savoir $a^{\frac{b}{c}/d}$, $a^{-b/d}$, ainsi que les résultats de M. LOBATSCHESKY, consignés dans un Mémoire en langue Russe.

Deventer, Mars 1858.

B. D. II.

OBSERVATIONS ET CORRECTIONS,

EN PARTIE CRITIQUES.

Ad Page 22. Après les chiffres de la Constante du Logarithme Intégral ajoutez encore (après avoir ôté le dernier 1): 06065124, où les deux dernières figures ne sont pas certaines: voyez Grunerts Archiv, Th. 29. S. 240.

Quant aux 530 décimales de π , je les donne ici, non puisque on en fera usage dans le calcul, mais comme un exemple intéressant de la perfection des méthodes dans l'Analyse, qui nous permettent une telle exactitude sans exiger pour cela des travaux extraordinaires. J'insère un tableau des diverses recherches relatives à cette constante, qui offre des données curieuses sur les progrès de ces méthodes dans le cours de vingt-et-un siècles: encore faut-il observer que les quelques décimales des siècles passés ont exigé des calculs bien autrement longs et pénibles, que les derniers résultats.

<i>Années.</i>	<i>Calculateurs.</i>	<i>Déc. calc. Déc. exactes.</i>		<i>Littérature.</i>
250 (a J.C.)	Archimède	2		<i>Archimedis</i> , de dimensione circuli.
1461	Regiomontanus	3		<i>Regiomontanus</i> , de quadratura circuli adversus Nic. de Cusa.
1580	Joh. Rheticus	8		<i>Rheticus</i> , Canon Doctrinae Triangulorum.
1585	Adr. Metius	8		<i>Metius</i> , Manuale Geometriae Practicae L. B.
1579	Fr. Vieta	11		<i>Vieta</i> , Canon Mathem. s. ad Triangula. Par 1579.
1597	Adr. Romanus	16		<i>Romanus</i> , In Archimedis Circ. Dimens. Exp. et Anal. Würzb.
1619	Lud. van Ceulen	32		<i>L. à Ceulen</i> , De circulo et adscriptis. Ed. Snellius L. B. 1619.
1621	Will. Snellius	34		<i>Snellius</i> , Cyclometricus de circ. dimens. L. B. 1621.
1717	Abr. Sharp	75	72	<i>A. S (harp) Philomath</i> , Geometry improv'd. Lond. 1717.
....	Machin		100	<i>Jones's</i> , Synopsis Palmariorum.
1719	de Lagny	128	114	Mém. de Paris 1719, p. 155.
1790	de Vega	141	136	N. Act. Petr. T. 9. Hist., p. 41.
1812	Rutherford	205	152	Phil. Trans. 1841. P. 2, p. 283.
....	(Anonyme)		154	Manuscrit de la Bibliothèque Radcliffe à Oxford.
1844	Dahse	205	200	Journ. v. Crelle, Bd. 27. S. 198.
	Clausen		256	Astron. Nachr., N. 184.
1853	Shanks		318	Proceed. Royal Society, Jan. 20. 1853, p. 273.
1853	Richter	353	330	Grunert's Archiv, Th. 21. S. 119.
1854	Richter		400	Grunert's Arch., Th. 22. S. 173 — corrigé. ib., Th. 23. S. 476.
1853	Rutherford		440	Proceed. Royal Society, Jan. 20. 1853, p. 273.
1855	Richter		500	Grunert's Arch., Th. 25. S. 472.
1853	Shanks		530	Proceed. Royal Society, Jan. 20. 1853, p. 273.

Ad. Page 23, 1^{re} Ligne, au lieu de: $q + 2$ lisez: $q + 1$

<i>T.</i>	<i>N.</i>	<i>au lieu de :</i>	<i>lisez :</i>	<i>T.</i>	<i>N.</i>	<i>au lieu de :</i>	<i>lisez :</i>
1.	15.	$b + c$	$b + c - 1$	9.	S.	$a + b/2$	$a + b/1$
	16.	$b - c - 1$	$b - c + 1$	10.	4		$2a + b$
	19.	$=$	$= \frac{1}{2}$	13.	$2b + 1$		$2b + 3$
2.	2, 5.	fautives?		10, 11.	hg		$\frac{h}{g}l$
3.	2.	0	-1	26.	$=$		$= (-1)^c$
	11.	$\frac{1^{n/1}}{(p+2)}$	$\frac{1^{n-1/1}}{(p+1)}$	11.	5 à 7.	fautives?	
	20.	A	$-A$	12.	4.	$(1-x)^p$	$(1-x)^{2p}$
	27.	$\frac{i}{2a}$	$\frac{i}{4a}$	16.	$+ \vee 3$		$+ 3 \vee 3$
4.	1.	$\left(\frac{1}{2}\right)^n$	$\left(-\frac{1}{2}\right)^n$	19 à 22.	fautives?		
	10.	fautive?		13.	2.	$(1-x^3)$	$(1-x^3)^2$
	11.	$(1+p)^a$	$(1+p)^{1-a}$		3.	$(1-x^3)^2$	$(1-x^3)$
	13.	$a^p(1+a)^p$	$a^p(1+a)^q$		9.	$\vee 3$	1
	17.	$\frac{\sum}{0} \binom{a}{n}$	$\frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)} \sum_0 \left(-\frac{a}{n}\right)$	14.	3.	$b - 1$	$\frac{a+b}{2} - 1$
	23.	$x^p + x^{-p}$	$x^{pi} + x^{-pi}$		5.	$(1-x^q)$	$(1-x^q)^p$ V. Schubert.
5.	2.	$1+x$	$1-x$		6.	$(1-x^a)$ (partout)	$(1-x^a)-$
	27.	$\frac{a}{b}$	$a + 1$		7, 8, 10, 11, 18 à 21.	fautives?	
			$b + 1$	15.	13.	fautive?	
6.	1.	$+\frac{1}{4}$	$-\frac{1}{4}$		18.	$\frac{\pi}{2b}$	$\frac{\pi}{b}$
	3.	$a - 1$	$a + 1$	16.	5.	$x^2 -$	$x^2 +$
	4.	$(1+a)^{q-1}$	$(1+a)^{-q}$		12.	$\vee ()$ (partout)	$\vee ()^3$
		a^q	a^{1-q}	17.	1, 2.	fautives: la valeur est 0.	
	5.	$p^q(1+p)^r$	$p^r(1+p)^q$	18.	9.	ne vaut qu'entre a et $\pm \infty$.	
	8.	a (partout)	r		10.	qr	q
	18.	1	$(-1)^n$		11.	$(1+x)^a$	$1+x^a$
7.	5.	$=$	$= -$		19.	$(1+x)^{2a+1}, \pi p^2$	$(1+x)^{2a}, \pi p$
	6.	$\frac{1}{2} \lambda$	λ		24.	$c - b$	$c + b + 1$
	16.	dx	xdx		25.	$c - b + 2$	$c - b + 1$
8.	3.	b	a		26.	$b - c - 1, p\pi$	$b - c + 1, \pi$
	7.	$q^{p+1} \sin. q \pi$	$q^p \sin. p \pi$	19.	3, 4', 4''.	fautives; la valeur est 0.	
9.	1.	$1 - x$	$1 - x^a$		11.	x^2	x
	2.	a^2	$a+1, 2$		12.	fautive; elle est:	$\frac{\pi}{3 \vee 3}$
	3.	$(2p-1)^{a-2}$	$(2p-1)^{a-1/2} 1^{a-b+1/2}$		18.	$\pm q$	$\pm qi$
				20.	9.	$\frac{\pi}{2a}, a > b$	$\frac{\pi}{2ac}, 2a+1 > 2b$
					10.	$a < b$	$2a+1 < 2b$
					12.	$-x$	$+x$, Gr. 16. 94.

<i>P.</i>	<i>N.</i>	<i>au lieu de :</i>	<i>lisez :</i>	<i>T.</i>	<i>N.</i>	<i>au lieu de :</i>	<i>lisez :</i>
20.	16.	$a + 1$	$a + 2$	31.	9.	$1^{cl}, b-c-1$	$p^{cl}, b-c+1$
	17.	$-x^2, 2b$	$+x^2, 4b$. Gr. 16. 94.		19.	$1 + x^2 + x^4$	$1 - x^2 + x^4$
21.	3.	$a + 2$	$a + 1$		21.	$(x-1)^p$	$(x-1)^{-p}$
	8.	$\frac{1}{2a}$	$\frac{\pi}{2^{a+1}}$		22.	$(x-1)^{p-1}$	$(x-1)^p$
					8, 23.	fautives?	
22.	2, 11 à 13.	fautives?				$-2p)^{1/2}$	$(1-2p)^{1/2}$
	4.	$=$	$= 2$	32.	5.	$(2p-1)^{a'-2}$	$(2p-1)^{a-1'-2} 1^{a-b+1/1}$
	10.	$b-1, -x$	$-1, +x$		12.	$b - \frac{1}{2}$	$b - 1$
25.	9.	ax	x				
		πi	$-\pi i$	34.	19.	$=$	$= -$
	10.	b					
	12.	q^p	q^{2-p}	35.	2.	elle est $\frac{1^{2q11}}{(1q1)^2} \cdot \frac{\pi}{p \cdot 2^{2q+1}}$	
26.	8.	$p \pi, Cos.$	$p \pi, Cosec.$		6.	$p \leq 1$ est la condition de N ^o 5.	
	12.	$\frac{2\pi}{a}$	$\frac{2\pi}{q}$		7.	p^2	p
					19.	fautive.	
27.	1.	$= 2$	$=$		22.	$-x^2$	$-x$
	6.	$(2p-1)^{a+1-2}$	$(1-2p)^{a-1'-2} 1^{b-a+1/1}$		23.	$-2b)^{a-1}$	$-2b)^{a+1}$
	12.	$b - \frac{1}{2}$	$b - 1$	36.	7.	ajoutez: Schellbach, Cr. 48. 207.	
					8.	" Raabe, Cr. 48. 137.	
	15.	$=$	$= \frac{1}{3}$		11.	... 737 ...	... 777 ...
	16.	$=$	$= \frac{2}{3}$		11.	$\frac{1}{p}$	$\frac{1}{b}$
	19.	$\frac{b}{2} + 1$	$\frac{1}{2} + b$	37.	5, 6, 8.	fautives.	
	24, 25.	ne valent qu'entre 0 et 1.			10.	$=$	$= (-1)^q$
28.	5.	$p -$	$p +$		13.	$2q$	q
	6.	fautive?		38.	1, 5.	$-px, -qx$	px, qx
	10.	$\frac{l}{q}$	$\frac{l}{p}$		17.	ajoutez: Poisson, P. 20. 222.	
	11.	$p^2 x$	px^2		21, 22.	$=$	$dx =$
	22, 23.	x^b	x^6	39.	2.	ajoutez: Schellbach, Cr. 18. 207, où fautive.	
29.	5.	fautive, elle est 0.			10.	$e^{px} + 1, \pi$	$e^{2px} + 1, \pi + 2.$
	7.	fautive, elle est π .			11.	$=$	$= -$
30.	4, 8, 9, 12.	fautives?			12.	$e^{-x} dx$	$e^x dx$
	18.	$\frac{1}{2} b^2$	$b \text{ Cos. } \lambda$		16.	$+ e^{q-p} x$	$+ e^{p-q} x$
31.	6.	$(x-1)_{1+p}$	$(x-1)^{p-1}$		19.	ne vaut qu'entre $-\infty$ et $+\infty$.	
	7.	$b+c+1$	$b+c+11$		20.	seulement le $+$ des doubles signes.	
					21.	$q \text{ Sin. } \lambda$	$q^2 \text{ Sin. } \lambda$
					16.	8. $\frac{\pi i}{1}$	$\frac{\pi i}{4}$
					25, 26.	fautives.	

T.	N.	au lieu de:	lisez:	T.	N.	au lieu de:	lisez:
41.	5.	pour $p < 1$: pour $p > 1$ elle est ∞ .		50.	14.	<i>Sec.</i>	<i>Cosec.</i>
	17.	$(q^{eri})^a$	$(q^{eri})^{-a}$	51.	1.	$= \frac{1}{2}$	$=$
43.	16.	$-\frac{1}{q}$	$\frac{1}{q} +$		9, 10.	fautives?	
	17.	$=$	$= -$		11.	$=$	$= \pi$
44.	2.	$a - 1$	a		18.	$2 \vee 3$	$3 \vee 3$
	5.	$= -$	$=$	52.	18.	$\frac{\text{Sin. } 4q x. \text{Cos. } q x}{\{(Cos. x - Sin. x)\}}$	$\frac{2 \text{Sin. } q+1 x. \text{Cos. } q-1 x}{\{(Cos.^2 x - Sin.^2 x)\}}$
	6.	fautive?			19.	$=$	$= \frac{1}{p^3}$
	8.	$- 2 l 2$	$+ 2 l 2$	53.	18.	$\text{Sin. } p x$	$\text{Sin. } p-1 x$
45.	6.	fautive: α est 1.			22.	$\left(\frac{1}{2} p\right)$	$\frac{1}{2} (p$
	13.	ne vaut qu'entre 0 et 1.		54.	2.	$q - 1$	$q - 2$
46.	16.	$= \frac{1}{2}$	$=$		7.	$(2a - 1)^2 p^2$	$(2a - 1)^2 - p^2$
	17.	<i>Tang.</i>	<i>Cot.</i>		15.	$2p + 1$	$2p+1$
	19.	$Tg.p x + Cot.p x, \pm \frac{1}{4} p \pi$	$Tg.p i x + Cot.p i x, \pm \frac{1}{2} p \pi$	55.	12, 16 à 18.	Partout $a > b$.	
47.	2, 3.	$2a + b +$	$2a + 2b +$	56.	4.	$1^{a/2}$	$2a^{1/2}$
	4.	dx	$\frac{dx}{\text{Cos.}^4 x}$		8, 9.	$b + 2$	$b + 1$
	21.	$\text{Cos.}^2 x$	$\text{Cos.}^2 x. \text{Tang.}^p x$		10 à 12.	$\text{Cos. } x$	$\text{Cos.}^{2(} x$
48.	6.	$\frac{\text{Sin.}^2 x}{1 + 3 \text{Sin.}^2 x. \text{Cos.}^2 x}$	$\frac{1}{1 - 3 \text{Sin.}^2 x. \text{Cos.}^2 x}$		16.	$\frac{a+b}{2} - 1$	$\frac{a-b}{2} + 1$
	p. 92.	(titre) monôme.	binôme.	57.	1.	$2a - b$	$b - 2a$
	8.	$- l$	$+ l$		5.	$2a - 2b$	$2b - 2a$
	19.	dx	$\text{Tang. } x dx$		8, 12.	$\frac{q \pi}{2}$	$\frac{1}{2} p \pi$
49.	3.	b	6		14.	1^{b-1}	$1-b$
	4.	$\pm \frac{2-a}{16}$	$\mp \frac{2-a}{16}$	58.	2.	ne vaut que pour $b = -1$.	
	10.	$=$	$= -$		3 à 7.	ne valent que pour $c = 0$.	
	12.	$\frac{1}{2} \text{Sin. } \lambda$	$\text{Sin. } \lambda$	59.	7, 12 à 15.	fautives?	
	16.	$- \text{Sin. } x$	$+ \text{Sin. } x$	60.	2, 3.	a (partout)	$a \pi$
	19, 21.	4	8		4.	$2n + 1$	$2n - 1$
	24.	$\text{Cos. } 2x$	$\text{Sin. } 2x$		9.	fautive.	
	25.	$1 + p$	$1 - p$ V. T. 31. N°. 25.	61.	14.	$=$	$= -$
	28.	$q \Gamma (2q)$	$4 \Gamma (2q)$	62.	4, 5, 7, 8.	fautives?	
	29.	$\frac{p \pi}{2q}$	$\frac{p \pi}{q}$	63.	1.	$q x$	$4 q x$
			$\frac{dx}{\text{Cos. } x}$		2.	$= \frac{1}{a}$	$= - a$
50.	8, 9.	dx			4.	$p \pi$	$\frac{1}{2} p \pi$

T.	N.	au lieu de:	lisez:	T.	N.	au lieu de:	lisez:
63.	8.	$p - 1$	$p - q$	74.	3.	$\frac{2}{b \vee (a + b)}$	$\frac{2a}{b \vee (a + b)}$; c'est le co-
	11.	$\text{Cos. } 2x$	$\text{Cos.}^2 2x$			$a F'$	efficient de tout le reste.
64.	8.	$\text{Cos. } 2x$	$\text{Cos. } 2x. \text{Cos.}^2 x$			$E' (\quad , F ($	$E \left(\frac{\pi}{4}, F \left(\frac{\pi}{4},$
	14.	$\text{Cos. } p+q-1 x$	$\text{Sin. } p+q-1 x$				$1 - p^2$
65.	10.	$\bar{q}$	$\bar{p}$	75.	20.	$3 - p^2$	$1 - p^2$
	15.	$\bar{2}$	3		24.	$+ \text{Sin.}^2 \lambda$	$- \text{Sin.}^2 \lambda$
66.	4, 5.	$=$	$= \frac{1}{2}$		26.	$a - b + \frac{1}{2}$	$a - b + 1$
	10.	$\frac{\text{Cos.}^2 x}{1 -}$	$\frac{1}{1 - 3}$	76.	4.	$\frac{1}{p^3}, \frac{1}{(1 - p^2)p}$	$\frac{1}{p(1 - p^2)}, \frac{1}{1 - p^2} - \frac{1}{p}$
	21.	$1 + a^2$	$-1 + a^2$		5.	$\frac{1}{p^2}$	$\frac{1}{1 - p^2}$
	22.	$1 + a^2$	$1 - a^2$ V. T. 23. N°. 2.		11.	$p - \frac{1}{2}$	$p + \frac{1}{2}$
67.	2, 3.	$1 - p$	$1 + p$ V. T. 31. N°. 22.		13.	$(p + q - 1), \frac{1}{2}(q + 1)$	$(p - q - 1), 1 - \frac{1}{2}(p + q)$
	4.	-2λ	$-2\lambda - \text{Sin. } \lambda$		18.	$\text{Sin.}^2 \mu$	$\text{Sin. } \mu$
	13.	$7p^4 q^2 + 7$	$3p^4 q^2 + 3$		20.	4	2
	14, 15.	$6p^2 q^2$	$2p^2 q^2$	77.	1, 2.	pour $k = \infty$	pour $k = 0$
68.	2.	$=$	$= 2$	78.	3.	2^{2a}	2^{2a+1}
	3.	2	$\bar{3}$		4.	-1	$+1$
	4.	$1 - 3$	$1 -$		14.	fautive: il y manque	le facteur: x .
	26.	$\text{Tang } p x$	$\text{Tang. } p+2 x$		15.	$(-1)^a$	$(-1)^{a-1}$
	27.	$1 - a,) + a + 1$	$1 - q,) + a$	79.	8.	$a^{b/1}$	$a^{b-1/1}$
	30.	4	2	82.	19.	$\}^3 a + 1$	$\}^2. 2^a$
69.	9.	$\frac{1}{2} \lambda \text{Sin. } \lambda$	$\lambda \text{Sin. } \lambda - 1$	83.	1.	$= -$	$=$
70.	3, 4, 11 à 14.	fautives?			12, 13.	ne valent qu'entre 0 et $\frac{1}{2} \pi$.	
	20.	$- \text{Cos.}^2 x$	$. \text{Cos.}^2 x$		14.	Sin.	Sin.^2
71.	3, 4, 7 à 10, 12 à 14, 17.	fautives?		84.	15, 20.	p^a	p^{a-1}
	16.	$2n + 1$	$2n - 1$		25.	$\text{Cos. } 2ex$	$\text{Sin. } 2ex$
	18, 19.	$=$	$= \frac{1}{2}$	85.	10, 11.	$=$	$= \pi$
72.	1, 11.	$n^{n/2}$	$2^{n/2}$		21.	$\}^{2a}$	$\}^1$
	6.	4	3		30.	2	$2p q$
	7.	fautive.			32.	fautive, sa valeur est	∞ .
	13 à 15.	$\text{Cos. } x$	$\text{Cos.}^2 x$	86.	5.	$= 2$	$= 1$
	14.	$2b + 1$	1				
	21, 23.	$2 \vee 3$	$3 \vee 3$				
74.	1.	$F' ($	$F \left(\frac{\pi}{4},$				

<i>T.</i>	<i>N.</i>	<i>au lieu de :</i>	<i>lisez :</i>	<i>T.</i>	<i>N.</i>	<i>au lieu de :</i>	<i>lisez :</i>
86.	13.	E	F	106.	5.	$\text{Sin.}^{2a+1} \lambda$	$\text{Sin.}^{2a-1} \lambda$
	14, 15.	$p^2 + q, p^2 - q$	$p^2 + q^2, p^2 - q^2$	107.	17.	$\frac{1 + \text{Sin.}^2 \mu}{2 \text{Cos.} \lambda} \text{Sin.} \mu$	$\frac{\text{Sin.} \mu}{\text{Cos.} \lambda} \Gamma \left(\frac{\text{Sin.} \mu}{\text{Sin.} \mu} \right) - \frac{\text{Sin.} \mu \text{Sin.}^2 \lambda}{2 \text{Cos.} \lambda}$
	11.	$p^2 q$	$p^2 - q^2$				
	15.	$q \vee$	$q^2 \vee$				
87.	15.	=	= π				
88.	8.	fautive, sa valeur est 0.		21.		$\frac{1}{\text{Sin.} \mu}$	$\text{Sin.} \mu$
89.	10, 12.	$a - 1$	$a + 1$				
	15.	$2 \pi a$	2π	108.	5.	$-\frac{1}{2} p, -p$	$-p, -4p$
	16.	=	$dx =$		6.	$\text{Sin.} \frac{1}{2} p$	$\text{Sin.}^4 p$
90.	21.	$\frac{\pi}{2}$	π		10.	$= -\frac{\pi}{16}$	$= \frac{\pi^2}{16}$
	23.	$\frac{\pi}{2}$	$a \pi$	109.	2, 3, 7 à 10.	fautives: elles sont ∞ .	
92.	12.	=	= —	110.	2.	fautive: elle est ∞ .	
93.	1.	$b - 2$	$b - 1$		3.	π^2	$-\pi^2$
	2.	$\left(\frac{a+b}{2} - 1 \right) \Gamma \left(\frac{b-a}{2} - 1 \right)$	$\left(\frac{a+b+1}{2} \right) \Gamma \left(\frac{b-a+1}{2} \right)$		4.	$=, \left(\frac{\pi}{2} \right), \frac{4}{p+2m}$	$=, \left(\frac{\pi}{2} \right), \frac{2}{p+2m-1}$
94.	2.	ne vaut que pour $a = \infty$.		112.	1.	3	2
	4.	fautive.			6.	ne vaut qu'entre 0 et ∞ .	
	14.	=	= a		7, 12.	fautives.	
95.	5.	$a x$	πx		11.	$\frac{d}{da}$	$\frac{d}{da} \cdot l$
	6.	1	π	113.	8.	a^x	x^x
	9.	=	= 2		9.	e^x	e^{-x}
96.	1 à 6, 12, 14.	fautives.			15.	$a + bi$	$a - bi$
97.	1 à 8.	fautives.			17.	ajoutez: Raabe, Cr.	48. 160,
98.	6.	=	= —	111.	4.	=	= $2-a$
	7 à 9.	fautives.		115.	10.	ôtez e^b	
99.	1 à 4.	$\vee \text{Sin.}, \vee \text{Cos. Sin.} \vee, \text{Cos.} \vee$		116.	1.	fautive,	
	5, 6.	fautives.			5.	$\frac{1}{3}$	$-\frac{1}{3}$
101.	1, 2.	q^2	q		8.	$\sum_0^{\infty} (-1)^n$	$\sum_0^a$
102.	1, 3, 5 à 13.	fautives.			9.	a	p
104.	4.	$2 \text{Cos.}^2 \mu$	$2 \text{Cos.}^3 \mu$	117.	3.	256	252
	7 à 9.	$(\text{Sin.} \mu, q)$	$(\text{Sin.} \nu, q)$		4.	1680	240
	10 à 12.	$\text{Sin.}^2 \lambda$	$\text{Cos.}^2 \lambda$		11, 12, 14.	0	1
	14.	$\text{Sin.} \mu$	$\text{Sin.} \nu$				
105.	3.	$\frac{1}{\text{Sin.}^3 \mu} \frac{1}{4n^2}$	$\frac{3n/2}{4n+1 \cdot 2}$				

T.	N.	au lieu de :	lisez :	T.	N.	au lieu de :	lisez :
117.	19.	x^2	x^3	127.	29.	$\sum_1$	$\sum_0$
	23.	$2a + 1$	$2a - 1$		33.	$q^2 l q$	$q^3 l q$
119.	1.	x^2	$\frac{1}{2} x^2$	128.	6.	$(-1)^c$	$(-1)^{c+1}$ V. T. 168. N°. 20.
	2.	$e^x - 2$	e^x V. T. 117. N°. 5.	129.	5.	x	xi
	3, 4.	$\Gamma(q)$	$\Gamma(q+1)$ V. T. 117. N°. 16, 17.		10.	fautive.	
	8.	e^{-x}	$(-e^x)$		11.	q	$-q$
120.	1.	$= -$	$=$		12.	$(-pq)^{n-1}$	$(pq)^{n-1}$
	8.	16	12	130.	11, 13.	fautives.	
121.	5.	e^{qx}	$-e^{qx}$	131.	8.	fautive.	
	6.	$-e^{-qx}$	$+e^{-qx}$	132.	9, 10.	changez le dénominateur avec N°. 12, 13.	
	8.	$3 Sec.^3$	$2 Sec.^3$		10, 13.	$=$	$= -$
	9.	$3 \pi^3$	$2 \pi^3$		10.	$e^{-pq} Ei. (pq)$	$e^{pq} Ei. (-pq)$
	12.	e^{-x}	1		14.	$+ e^{pq}$	$-e^{pq}$
	19.	$Sin.$	$Cos.$				$\Gamma\left(\frac{q+1}{2}\right) \Gamma\left(\frac{1}{2}\right)$
	19, 21.	$e^{bx}, e^{-bx}, \left(\frac{2\pi}{b}\right), b\pi, e^{px}, e^{-qx}, (2b\pi), q\pi$		134.	2.	fautive: sa valeur est l	$\Gamma\left(\frac{q}{2}\right)$
		, où $q^2 = a^2 b^2 < 1$, a arbitraire.			3.	$2p$	2
122.	1.	$-e^{-x}$	$+e^{-x}$		6.	$(1 - e^{-px})e^{-(q+1)x}$	$(e^{-qx} - 1)e^{-p+1}e$
	6.	π	π^2				$\frac{b-1}{2}$
	7, 8.	changez les coefficients $p+q$ et $p-q$: dans les numérateurs liez les puissances de e par le signe $+$.		14.	ôtez: $\left(\frac{c+1}{2}\right)^2$		
	10.	$e^{\pi x} - , e^{\pi x} +$	$e^{\pi x} + , e^{\pi x} -$		16.	$e^{-px} +$	$e^{-px} -$
123.	5.	$2a - 1$	$2a - 1$	135.	10, 15.	ajoutez: Féaux, Funct. Transc.	
			$1a$		12.	$+$	$-$
	8.	$2a + 1$	$2a - 1$		18.	$dx \left\{ \frac{e^{-x}}{x} - \right.$	$dx \left\{ \frac{1}{x} \frac{1}{p-1-e^{-x}} - \right.$
124.	9.	$-2 Cos. \lambda, 3 Sin. \lambda$	$+2 Cos. \lambda, Sin. \lambda$		19.	$-e^{-x}$	$+e^{-x}$
125.	8.	$2 +$	$2 -$		24, 25.	$e^{1-a} x$	$e^{1-p} x$
	12.	3	2		28.	$pq +$	$pq -$
126.	3.	$= -$	$=$		30.	$q - \frac{1}{2}$	$\left(q - \frac{1}{2}\right)$
	4 à 7, 17.	fautives?			31.	e^{px}	$e^{-\frac{1}{2}px}$
	8.	e^x	e^{-x}	136.	3.	$=$	$= -$
	10, 11.	ne valent que pour a négatif.			1.	$q\pi$	$q\pi$
	12, 14.	ne valent que pour b négatif.			2.	p	$4p$
127.	13.	$b - a$	$b - c$		5.	$e^{qx} - e^{-qx}$	$(e^x - e^{-x})^2$
	17.	$=$	$= -$		10.	fautive.	
	27.	dx	$2dx$				

T.	N.	au lieu de:	lisez:	T.	N.	au lieu de:	lisez:
136.	12.	$\frac{x}{-}$	$\frac{dx}{x}$	145.	4, 8.	$=$	$= -$
	18.	dx	$\frac{dx}{x^p}$; liez les fractions par +.		6.	$4 \ell q$	ℓq
137.	1, 2.	fautives.			7.	$\frac{1a/1}{2a/2}$	$\frac{1a-1,1}{a^{a1}}$
	3.	dx	$(1 - e^{-x})^2 dx$		9.	6	3
	6.	$e^x +$, Sec. ²	$e^x -$, Sec.		12.	24	4
	12.	$(p - n)$	$(p + n)$		13.	360	15
138.	4, 7.	ajoutez: Poisson, P. 20. 222, $p \leq \pi$.			14.	720	6
	9.	$\frac{1}{4}q$	$\frac{1}{4}q$		15.	$q + 1$	$q - 1$
	12.	dx	$2 dx$		19.	$=$	$= 2$
139.	8.	$c - n$	$c - n + 1$		21, 22.	$e^{bx}, e^{-bx}, \left(\frac{2\pi}{b}\right), a\pi, e^{qx}, e^{-qx}, (2b\pi), q\pi$	
	10.	$(-q)$	q			où $q^2 = \alpha^2 b^2 < 1$, α arbitraire.	
140.	11.	q^n	$(-q)^n$		27.	$(2\pi)^{a+1}$	$(2\pi)^{2a+1}$
	13, 14.	$=$	$= -$	146.	3 à 5.	$4 p q$	$4 \vee p q$
	15.	$x p \vee$	$p \vee x$		4, 5.	$a^{2n/-1}$	$(a + n - 1)^{2n/-1}$
	17, 18.	$+ x \}$, $- x \}$	$+ p \}$, $- p \}$	147.	12, 13.	changez les conditions.	
141.	16, 17.	fautives.		148.	1, 5.	fautives?	
	22.	π	2π	149.	5.	$(-1)^a \sum 1$	$\sum (-1)^n$
	23.	$- \pi$	2π		8.	$e^{-x}, (e^{\frac{1}{2}x} - e^{-\frac{1}{2}x})^4, 2(\ell 2)^2$	$2e^{-x}, (e^x + 2e^{-x} - 2)^2, (\ell 2)^2$
	24.	e^{-x} (partout)	e^x		9.	e^{-x}	$2e^{-x}$
		$a^{-p}, (a + b)^{-p}$	$a^{1-p}, (a + b)^{1-p}$		11.	$(p^2 - 1)(e^x + e^{-x}) - 2(p^2 + 1)$	$(e^{-\frac{1}{2}x} - e^{\frac{1}{2}x})^4 \vee \{(p^2 - 1)(e^x + e^{-x}) + 2(p^2 + 1)\}$
142.	3.	π	2π		12.	$\frac{e^x - 1}{(p^2 - 1)(e^{2x} + 1) + 2(p^2 + 1)e^x}$	$\frac{1}{e^{2x} - 1}$
143.	4, 5.	e^{2ax}	e^{-2ax}		13, 14.	$=$	$= -$
	5.	e^x , Tang. $\frac{\pi}{2a}$	e^{-x} , Tang. $\frac{\pi}{a}$		150.	3.	fautive.
	8, 9.	$=$	$= -$		11.	(limite) q	$2 \ell q$
	15.	$\frac{a\pi}{b}, \frac{a+2}{b}$	$\frac{a\pi}{2b}, \frac{a+2}{2b}$		13.	(") ℓp	$- \ell p$
144.	4.	$\frac{a}{2p}$	$\frac{a}{2b}$	151.	1, 14.	ℓ	$- \ell$
	7.	$\frac{1a/1}{2a/1}$	$\frac{1a-1/1}{a^{a1}}$		7.	$\angle^a$	$\frac{1}{q} p^{b+1}$
	8.	$=$	$= -$		11.	$(-1)^a \sum 1$	$\sum (-1)^n$
	9.	e^x	e^{-x} V. T. 182. N ^o . 7.		13.	$p + 1, p + 3$	$p + 2, p + 4$
	10.	$q + 1$	$q - 1$		15.	{	{ -
	12.	$\text{Sin. } p -$	$\text{Sin. } p\pi +$	152.	1.	fautive: sa valeur est ∞ .	
145.	3.	Sa valeur est $\frac{1}{2}\pi^2$.		153.	5, 6, 20.	ℓx	$\ell \frac{1}{x}$

<i>T.</i>	<i>N.</i>	<i>au lieu de:</i>	<i>lisez:</i>
153.	12.	$x^p - q$	$-x^{p-q}$
	13.	$-x^{p+q}$	$+x^{p+q}$
	14.	$= \frac{1}{2}$	$=$
154.	8.	$=$	$= -$
155.	9.	1680	240
157.	4, 10.	$\sum$	$\sum_1$
	8, 9.	$a/1$	$a-1, 1$
	11, 12.	$b/1$	$b-1, 1$
158.	3, 9.	$x^b, x^{-b}, \left(\frac{2\pi}{b}\right), b\pi$	$x^q, x^{-q}, (2b\pi), q\pi$
		, où $q^2 = a^2 b^2 < 1$, a arbitraire.	
	5.	$2a -$	$2a - 1$
	6.	$= (-1)^a$	$= -$
	12.	x^{-1}	x^{b-1}
160.	4.	$1+x^2$	$(1+x^2)^2$
	8.	$\frac{p}{1+p^2}, \frac{1}{4}p^2$	$\frac{-2}{1+p^2}, \frac{1}{2}\frac{p}{1+p^2}$
	9, 13, 14.	l	$-l$
	17.	$1+x^2$	$1-x^2$
	18.	$= 2$	$=$
161.	8.	2560	1260
	12.	2^{a+2}	$2a+2$
	13.	$n+1$	n
	14, 15.	$\frac{1^{a+1,1}}{a}$	$1^{a-1,1}$
	16.	$q+1$	$q+2$
	17.	$\frac{1^{a+1,1}}{2^{a+1}}$	$-\frac{1^{a-1,1}}{2^a}$
162.	2.	$\frac{3}{4}$	$\frac{4}{3}$
163.	8, 9.	fantives.	
165.	1, 2, 5, 11.	l	$-l$
	4.	$l(r$	$l(-r; \text{elle ne vaut qu'entre } -1+1.$
	6.	q^2	q
	8.	$-q^2$	$+q^2$
	13.	$F(p, \lambda)$	$\{F(p, \lambda)\}^2$
	14.	$Sin.$	$Sin.^2$

<i>T.</i>	<i>N.</i>	<i>au lieu de:</i>	<i>lisez:</i>
165.	19.	$-x$	$-x^2$
	21.	$F(p) +$	$F'(p) -$
	25.	$1^{n/2}$	$3^{n/2}$
	27.	$F(p, \lambda)$	$F\{1/(1-p^2), \lambda\}$
	28.	$2 \sqrt{\quad}$	$2 \sqrt{\quad}$
166.	3.	$Cos.^2 \mu, Cot.$	$Sin.^2 \mu, Tang.$
	9.	$1/(Cos.^2$	$\sqrt{(Cos.^2$
	15, 16.	$=$	$= -$
	21.	$+\frac{2 Sin.^2 \lambda}{(1-x^2 Sin.^2 \lambda)^2}$	$-\frac{2 Sin. \lambda}{x(1-x^2 Sin.^2 \lambda)}$
168.	9.	$\sum_0$	$\sum_1$
	11.	dx	$2 dx$
	15.	$(pn+1)l$	$(pn+1)^2 l$
	20.	r^{-b}	r^b
169.	1.	$= -$	$=$
	2.	$=$	$= -$
	3, 4.	$q^2 + (lx)^2$	$\frac{1}{4}q^2 + (lx)^2$
	3.	$\sum_1$	$\frac{2}{q} \sum_0$
	4.	$\sum_1$	$\frac{1}{q^3} \sum_0$
	6.	$2n-2$	$2n+2$
	14.	$epq Ei.(pq)$	$epq Ei.(-pq)$
	15.	$l, 1+pq$	$lx, 1-pq$
	16.	$1-pq$	$1+pq$
170.	3.	$1 + \frac{1}{x lx}$	$-1 - \frac{1}{x lx}$
171.	2.	Z' (partout)	$l \Gamma$
	3.	$(1-x^p) x^q$	$(1-x^q) x^p$
	5.	$l \left\{ \left(\frac{a}{a+1} \right)^{n-1} \right.$	$l \{$
	13.	$\frac{1}{lx} + dx, \Gamma(a+b+p)$	$\frac{p}{x} - \frac{1}{(1-x)x} - \frac{dx}{lx},$ $\Gamma(a-b+p)$
	19.	$l \frac{(1+p+s)(1+q+r)}{(1+p+r)(1+q+s)}$	$l \frac{\Gamma(1+p+s)\Gamma(1+q+r)}{\Gamma(1+p+r)\Gamma(1+q+s)}$

<i>T.</i>	<i>N.</i>	<i>au lieu de:</i>	<i>lisez:</i>	<i>T.</i>	<i>N.</i>	<i>au lieu de:</i>	<i>lisez:</i>
171.	26.	$\frac{1}{1-x} \left. \begin{array}{l} dx \\ l x \end{array} \right\}$	$\frac{x}{1-x} \left. \begin{array}{l} dx \\ x l x \end{array} \right\}$	177.	15.	$x q - \frac{1}{2}$	$x q - \frac{1}{2}$
172.	2, 7.	fautives.			19.	$\frac{1}{x}$	$\frac{1}{2}$
	9.	$\frac{q \pi}{2 p}$	$\frac{q \pi}{4 p}$		20.	$\frac{2}{1-x^2}$	$\frac{x}{1-x^2}$
173.	1.	dx	$x dx$		21.	x^{q-1}	x^{p-1}
	2, 12.	dx	$-l x dx$	178.	4.	$\sum_1$	$\sum_0$
	2.	$\frac{q}{4 \pi}$	$\frac{\pi}{q}$		6, 7.	$l x$	$-l x$
	4.	π^{2n}	$(2 \pi)^{2n}$		8.	$q > p > 0$	$q > p > 0$
	8.	$\frac{1}{4} \pi$	$\frac{1}{4 \pi}$	179.	3.	$x^{1-p}, p+1$	$x^{2-p}, p-1$
	9.	$2 q + 3, 2 q + 1$	$2 q + 3 \pi, 2 q + \pi$	180.	5.	B'	B''
	10.	$1 + x^2$	$1 - x^2$	181.	16.	Col.	Tang.
	11.	$\frac{1}{2 \pi} - l 2$	$\frac{1}{2} \left(\frac{1}{2} - l 2 \right)$		17.	n'est pas à sa place ici.	
	14.	$q^2 \pi^2$	$-q^2 \pi^2$	182.	2.	$(l x)^2$	$l x$
	17.	$\frac{dx}{1-x^2} =$	$\frac{dx}{1+x^2} = 2, \text{ V. T.}$		5.	$(l x)^2$	$l (x^2)$
	18.	$1 \angle \pi$	$\pi 1 \angle 2 \text{ V. T. } 138. \text{ N}^\circ 18.$		6.	$\frac{a}{2 p}$	$\frac{a}{2 b}$
174.	2, 3.	fautives,			15.	$l q -$	$l q +$
	5.	dx	$(1-x^2) dx$		19.	$= -$	$=$
	13.	$1 + x^2$	$1 - x^2$		22.	$2 q$	q
175.	9.	$-x^{2p} +$	$-2 x^{2p} +$	183.	8.	$2 q^{2a+1}$	$(2 q)^{2a+1}$
	12.	$1 - x^2$	$1 - x q \text{ V. T. } 135. \text{ N}^\circ 27.$		10.	$dx = p$	$\frac{dx}{x} = l q$
176.	1.	$\frac{1-x}{x}$	$\frac{1-x}{2}$		14.	$(l q^2)$	$(l q)^2$
	4.	$1 - x^2$	$1 + x^2$		17.	$p - q$	$p \pi$
	5.	$\sum_0$	$\sum_1$	184.	5, 6.	$x^b, x^{-b}, q \pi, a \pi$	$x^q, x^{-q}, b \pi, q \pi$
	4, 5.	π^{p-1}	1			, où $q^2 = a^2 b^2 < 1.$	
	6.	$-\pi$	$-\frac{1}{\pi}$		15.	$l q$	$\frac{1}{2} (l q)^2$
	8.	$(l x^2)$	$(l x)^2$		16.	$\frac{1}{1-q}$	$\frac{-1}{1+q}$
	11.	$-\sum$	$-\pi \sum$		18.	$)^p$ (partout)	$)^{1-p}$
177.	6.	$l x$	$4 l x \text{ V. T. } 138. \text{ N}^\circ 18.$	185.	1.	$l x$	$l(1+x)$
	9.	$\frac{1}{2} \pi$	$\frac{1}{2 \pi}$		2.	$\frac{1}{2} \pi$	2π
	13, 14.	$2 n p \pi$	$n p \pi$		10.	$\frac{1}{2} + a$	$\frac{1}{2} - a$
					14.	$2 dx$	$x dx$

T.	N.	au lieu de:	lisez:	T.	N.	au lieu de:	lisez:
157.	2.	Σ	$\Sigma (-1)^{n-1} \cdot$	195.	15 à 17.	Dans les conditions changez p en q et q en p .	
	4, 13.	$l x$	$- l x$	197.	20. 125	115	
	8.	$\frac{1}{6} \pi^2 -$	$\frac{1}{12} \pi^2 +$ V. T. 160. N°.	199.	5.	où $s = -p$.	
	12.	$d x$	$q d x$	200.	6.	$a - 1$	$a + 1$
	15.	$-\frac{1}{2} q$	$+\frac{1}{2}$	201.	5, 13, 14.	fautives.	
	16.	$\frac{(l x)^b}{x^2} = \frac{b}{a}$	$\frac{(l x)^{b-1}}{x} = \frac{\pi}{a}$ V. T. 20. N° 1.	202.	14.	$r - 1$	$r + 1$
	17.	$\surd (2n + 1)$	$\surd (2n + 1)^3$		15.	$=$	$= 2r$
188.	1, 14.	$=$	$= -$		25.	$- 2n)^a -$	$- 2n)^a +$
	2.	$= -$	$=$		32.	$a - 1$	$a + 1$
	4.	$\int_0$	$\int_1$		36.	$\frac{b}{2}$	$\frac{b}{2} + 1$
	8.	$\left(l \frac{-1 + \surd 5}{2} \right)^2$	$\left(l \frac{-1 + \surd 5}{2} \right)$		10, 41.	$2p + q$	$2p + a$
	12.	$l \frac{3 - \surd 5}{2} +$	$l \frac{3 - \surd 5}{2} -$	203.	10. 1201	1201	
	15.	$=$	$= \frac{1}{2}$		11, 12.	α y doit être l'unité.	
189.	2.	$\frac{d x}{x}$	$\frac{d x}{x^2}$	204.	9, 10.	fautives: elles sont infinies.	
	10.	$- l$	$+ l$		15.	ajoutez: Liouville, P. 21. 71.	
	11, 12.	fautives.		205.	3, 4, 12 à 18, 26, 27.	fautives: elles sont infinies.	
	13.	$q - p x^2$	$q^2 - p x^2$	206.	3 à 8, 11, 12, 15 à 19.	fautives: elles sont infinies.	
	18.	$(l x)^2$	$l(x^2)$		13, 14.	$a^2 - x^2$	$x^2 - a^2$
190.	4.	$\surd (2n + 1)$	$\surd (2n + 1)$ V. T. 381. N° 14.		20.	$=$	$= -$
	5.	$\Sigma \{ - \}$	$\Sigma (-1)^n \{ + \}$	207.	1 à 3.	fautives: elles sont infinies.	
	6.	$+ A$	$- A$		17.	$\text{Cos.} \left(\frac{n\pi}{b} - a \right)$	$\text{Cos.} \left(\frac{2n\pi}{b} - a \right)$
	8.	$\surd (2n + 1)$	$\surd n$		18.	$\text{Cos.} \left(\frac{2n+1}{2b} \pi - a \right)$	$\text{Cos.} \left(\frac{2n+1}{b} \pi - a \right)$
192.	4, 13.	n'appartiennent pas ici: dans 4) il doit être q^{x-1} .			26.	$\frac{\pi}{2}$	$2 \surd 2$
	9.	Σ	$\Sigma (-1)^n$		27.	q^{2p-1}	q^{2p-1}
	10.	$\frac{1}{2} \pi$	$\frac{1}{4} \pi$	209.	10.	$3 a q$	$2 a q$
193.	1 à 11, 21 à 23.	fautives?		210.	1 à 4.	ne valent qu'entre les limites $-x$ et x .	
					1, 2.	$g + 2 h x$	$g^2 + 2 h x$
					5.	fautive.	
					11 à 13.	fautives: elles sont infinies.	
				212.	5, 8 à 11.	fautives: elles sont infinies.	
					14, 15.	$e^{-p} q$	$(e^{-p} - 1)$
					15.	$\text{Cos.} p x$	$\text{Cos.} l x - 1$
					16.	1	2
				213.	13, 14.	$=$	$= \frac{1}{2}$
					19.	$(b + x i)^{-a}$ partout	$(b + x i)^{-a-1}$

T. N.	au lieu de:	lisez:	T. N.	au lieu de:	lisez:
211 à 218.	ne valent pas.		229. 10.	$\frac{1}{4 \sqrt{q}}$	$\frac{1}{2 \sqrt{2q}}$
219. 6.	$=$	$= -$	230. 1, 7.	$\sqrt{2\pi}$	$1 \sqrt{2a\pi}$
220. 5, 6.	$e^{2ac} + e^{-2ac}$	$e^{2aq} + e^{-2ac}$	2 à 6, 8 à 12. fautives, quoique Cauchy ordonne la différentiation.		
6.	$q + x^2$	$q^2 + x^2$	231. 7.	Cos.^2	Sin.^2
7.	$\text{Cos. } 2ax dx$	$(p + \text{Cos. } 2ax) dx$	232. 4, 7.	$r \pm qi$	$q \pm ri$
8, 9.	fautives.		14.	$= ($	$= (-$
10.	dx	xdx	19, 20.	3	2
11.	$+ pe^{-a}$	$+ pe^{-2a}$	233. 3, 7.	$r \pm qi$	$q \pm ri$
13.	p^2 (numérat.)	p	20.	$\text{Sin. } rt -$	$\text{Sin. } rt +$
221. 1 à 1, 18 à 20.	fautives.		22.	$b - c$	$b + 1$
7.	$1 - pe^u$	$1 + pe^l$	23.	$- 2px$	$+ 2px$
10.	$2q$	$2p$	24.	$a + 1$	$2a + 1$
15.	$\text{Cos. } rx$	$\text{Cos. } 2rx$	234. 2 à 4.	fautives.	
222. 2, 13, 14.	fautives,		235. 2 à 7, 10 à 13.	fautives, quoique Cauchy ordonne	
3.	π	$\frac{\pi}{2q^2}$	236. 3 à 8, 11 à 14.	la différentiation.	
2q			21.	pour a	pour $\frac{1}{a}$
7, 11.	dx	$2dx$	237. 8.	{ 1	{ 2 V. T. 238. N°. 19.
8, 10.	$- e^{-2ac}$	$+ e^{-2ac}$	9.	dx	adx
223. fautive.			10.	$\text{Cos.}^2 x$	$\text{Tang. } x$
224. 16 à 23.	fautives.		11.	$=$	$= -$
225. 3 à 10, 22, 24, 26 à 33.	fautives.		16.	$+ l$	$- l$
16.	$\frac{2}{3}$	$\frac{\pi}{3}$	238. 2.	$\frac{1}{2} \pi l 2$	$\frac{1}{4} \pi$ V. T. 267. N°. 2.
23.	n'est pas fautive.		19.	{ 1 -	{ 2 -
226. 5, 6.	fautives.		2.	$+ \pi$	$- \pi$
227. 4.	$+ 1 \sqrt{\frac{\pi}{2b}}$	$- 1 \sqrt{\frac{\pi}{2b}}$	3.	$=$	$= -$
9, 10.	sans restrictions pour a .		4.	$(-1)^{2n-1}$	$(-1)^{n-1}$
228. 2 à 6, 8 à 12.	fautives, quoique Cauchy ordonne la différentiation.		5.	2^{n-2}	2^{2n-2}
229. 1.	$= -$	$=$	7, 17.	fautives; elles sont infinies.	
5.	$\sum_1^\infty$	$\sqrt{2} \sum_1^\infty \text{Sin.} \left(\frac{2n-1}{4} \pi \right)$	15.	$+ (-1)^n 2a, 2n-1$	$\} + \sum_1^\infty (-1)^n 2a, 2n$
6.	$\frac{1}{b} \sum_1^\infty$	$\frac{1}{a} \sqrt{2} \sum_1^\infty \text{Cos.} \left(\frac{2n-1}{4} \pi \right)$	20.	$2^{p-2} \left\{ \Gamma \left(\frac{1}{2} p \right) \right\}^2$	$\frac{2^{p-1}}{p} \left\{ \Gamma \left(\frac{p+1}{2} \right) \right\}^2$
8.	$\frac{1}{2}$	$\frac{1}{1 \sqrt{2}}$	22.	$\frac{\pi}{p}$	$\frac{\pi}{4p}$
9.	$\frac{1}{4q} \left(1 + \frac{1}{q} \right)$	$\frac{1}{2q \sqrt{2q}} \left(b + \frac{1}{2q} \right)$			

T.	N.	au lieu de :	lisez :	T.	N.	au lieu de :	lisez :
240.	4.	{1	{2	257.	6.	$\frac{\pi}{2}$	π
	5.	$\text{Cos. } n \lambda$	$(-1)^{n-1} \text{Cos. } n \lambda$				$2p$
	7.	$S. (2n)^{2m+2}$	$(2n+1)^{2m+2}$		14.	$\frac{1}{2^{2\mu+2}} \left[1 \right]$	$\frac{1}{2^{2p+1}} \left[2 \right]$
		, où $c = q - \sqrt{q^2 - 1}$.		258.	28.	$x - \frac{1}{x}$	$x^2 - 1$
	14.	$\sqrt{1+q}$	$\sqrt{1+q^2}$	259.	1.	$-\pi$	$+\pi$
241.	1.	ne vaut qu'entre les limites 0 et 2π .			2.	$(-1)^n$	$(-1)^{n-1}$
	12.	$= -$	$=$		5.	$(2n)^{2m+2}$	$(2n+1)^{2m+2}$
	14.	dx	$q dx$	260.	15.	$4p$	4
	16.	$=$	$= \pi -$	265.	28.	$x dx$	dx
242.	7.	9	3	267.	24.	$2 \text{Sin.}^2 \lambda$	$2 \text{Cos.}^2 \lambda$
	8.	$3 \sqrt{3}$	$\sqrt{3}$	269.	1.	$1+x$	$1+x^2$
243.	6.	$\frac{\pi}{2} - \frac{1}{2}$	$-\frac{\pi}{1 \sqrt{2}} +$	271.	9.	$= -$	$=$
	18.	Sin.^2	Cos.^2	272.	2.	$=$	$= -$
	19.	$\text{Sin. } 2x, = \frac{2}{p^2}$	$\text{Cos. } x + p^2 \text{Sin. } x \text{Cos. } x, =$	277.	8.	$2p$	p
244.	12.	$\frac{1}{q^2}$	$\frac{1}{q^3}$	279.	1.	$2a+2$	$2a-2$
	13.	a^3	a^2		8.	$= 2$	$= r$
	15.	9Σ	$12 \Sigma (-1)^n$	281.	11.	$\sum_0^{\infty} \frac{n}{n^2 + (a-1)^2}$	$\sum_0^{a-2} \frac{p}{p^2 + (n+1)^2}$
245.	13.	πq	$\pi q \lambda$	282.	4.	4	2
246.	4.	$2n-1$	$2n+1$	283.	2.	$(-1)^n$	$(-1)^{n-1}$
	8.	fautive, elle ne vaut que pour $\text{Cos } hp, \lambda$ et $\text{Sin } hp, \lambda$ au lieu de $\text{Cos. } \lambda$ et de $\text{Sin. } \lambda$.		286.	13, 14.	$-s^2, \frac{\pi}{p^2 + s^2}$	$+s^2, \frac{\pi}{1 \sqrt{(p^2 + s^2)}}$
	9.	$= 2$	$=$	291.	4.	$\frac{1}{2}$	$\frac{1}{2^a}$
	21.	$p^2 - 1 +$	$1 - p^2 +$		6.	ôtez $\sqrt{\pi}$.	
247.	5.	où $a < 1$.		293.	17.	$=$	$= -$
	24.	$p^2 + pq$	$2p^2 + pq$	296.	22.	$=$	$= \frac{1}{2} \pi$
249.	4, 6.	gh	qh		23.	$= \frac{1}{2}$	$=$
	6.	$= -2$	$= -$	297.	6.	-1	1
	22.	ne vaut qu'entre 0 et ∞ .			7.	$e^{(q+1)x+1}$	$e^{-(q+1)x-1}$
	23.	fautive par suite de N°. 22.		301.	15.	$=$	$= -$
250.	3, 4.	a^n	p^n		23.	$q \sqrt{\quad}, = 4$	$2q \sqrt{\quad}, = 2$
251.	5 à 8.	ne valent qu'entre 0 et ∞ .					
	9, 10.	ne valent que pour $r = 1$.					
253.	2.	$\text{Sin.}^2 \lambda, \text{Sin.}^2 \mu$	$\text{Sin.}^2 \lambda, \text{Sin. } \mu$				
255.	3.	$2 \text{Sin.}^2 \mu (\text{Cos. } \lambda$	$2 \text{Sin.}^3 \mu (\text{Cos. } \lambda$				

T. N.	au lieu de :	lisez :	T. N.	au lieu de :	lisez :
303. 13.	$-x$	$+x$	333. 8.	$(q-1)x$	$(q+1)x$
306. 7.	$= -$	$= - \frac{1}{4}$	336. 5.	$2p-1, 4p$	$p-1, 2p$
8.	$\sum_0^{\infty} \frac{n^{2a+1}}{1}$	$\sum_1^{\infty} \frac{1}{n^{2a+1}}$	338. 6.	$\frac{1}{2}$	$\frac{1}{4}$
9.	$2x)^a$	$2x)^{a-1}$	339. 14, 16.	$\frac{1}{2}$	$\frac{1}{2}\pi$
307. 1.	0	1	340. 15.	4	2
309. 7.	$4a$	$4a^2$	344. 8.	$\text{Sin. } x$	$\text{Sin.}^2 x$
10.	$\frac{1}{2}$	$\frac{1}{3}$	14.	$\frac{2}{q}$	$\frac{2}{q^2}$
12.	p	$p-1$	345. 14.	π	π^2
310. 12.	$x+$	$x-$	350. 5.	$\text{Sin. } x +$	$\text{Sin. } x -$
311. 5.	$l \text{ Tang.}$	$l \text{ Cot.}$	352. 2.	$l \text{ Cos. } x, dx$	$l \text{ Sin. } x, dx$
312. 10.	$2a+2$	$2a+1$	7.	n	n^2
11.	$2a-1$	$2a+1$	353. 16.	$\text{Cot.}, =$	$\text{Cos.}, = 2$
313. 5.	π	π^2	362. 9.	$-\text{Sin. } x)$	$-\text{Sin.}^2 x)$
17.	$-(p+q)$	$+(p+q)$	363. 8.	$\pi +$	$\pi + \pi$
314. 2.	16	8	369. 7.	$\frac{1}{2}\pi$	$\frac{1}{8}\pi$
316. 9.	π	1	372. 2.	$\sqrt{(1+a^2-2a)\text{F}}, -\frac{2}{b}$	$\sqrt{(1+b^2-2b)\text{F}}, -\frac{2}{b}$
11.	l	$-l$			$+(1+b)(b-a^2)\text{F}(b-\frac{2}{b})$
318. 10.	$\text{Cot.}^p x, \sum$	$-\text{Cot.}^p x, \sum_0$	7.	c	p
319. 9.	Sin.	Sin.^3	374. 2.	fautive.	
320. 3, 10.	$2, 2^a$	$(2\pi)^{2a}$	3, 5.	Cos.^{a+1}	Cos.^{a-1}
18.	dx	$\text{Sec. } 2x dx$	375. 14.	où $p = \text{Sin. } \lambda$.	
20.	4	2	379. 2.	$pq+a-$	$pq+a-1$
321. 16.	$l \text{ Tang.} \left(\frac{\pi}{4} \pm, = \pm \text{ Tang.} \left(\frac{\pi}{4} - \right) \right), = -$		388. 20.	$(-1)^{n-1}$	$(-1)^n$ V. T. 439. N°. 13.
322. 22.	fautive.		22.	ax	$2ax$ V. T. 439. N°. 11.
325. 1.	8	4	390. 15, 16.	ne valent pas, puisque T. 116. N°. 4 ne vaut plus pour $q > 2$.	
13.	$=$	$= -$	396. 22, 29.	$\frac{e^q - e^{-q}}{4}$	$\frac{e^q - e^{-q}}{2}$
327. 2.	dx	$dx l \text{ Tang. } x$	24, 25.	$\frac{e^q - e^{-q}}{2}$	$(e^q - e^{-q})$
329. 15.	$l \text{ Tang. } x$	$l \text{ Cot. } x$	399. 2 à 8, 10 à 16.	fautives?	
22.	$\text{Cos. } 2x$	Cos.			
331. 10.	0, $\frac{1}{2}$	1, $\frac{1}{n}$			
12.	$\text{Sin. } p$	$\text{Sin. } x$			

<i>T.</i>	<i>N.</i>	<i>au lieu de:</i>	<i>lisez:</i>
408.	5, 9.	$p^2 <$	$q^2 <$
409.	5, 6, 12, 13.	$p^2 <$	$q^2 <$
	9.	$+ (e^{p\pi})$	$- (e^{p\pi})$
	11.	$+ (e^{p\pi})$	$+ (- e^{p\pi})$
	13.	$=$	$= 1$
410.	6.	$=$	$= -$
411.	10.	$\frac{\pi}{1-p^2}$	$\frac{\pi}{1/(1-p^2)}$
418.	9.	$1-$	$1+$

<i>T.</i>	<i>N.</i>	<i>au lieu de:</i>	<i>lisez:</i>
422.	8.	$\frac{dx}{x}$	$\frac{dx}{x^2}$
	11.	$+$	$-$
		$\frac{2}{1-x^2}$	$\frac{2}{1-x^2}$
	31.	$=$	$= \frac{1}{2}$
423.	24.	changez $1/(1-x^2)$ et $(1-x^2)$.	
	30.	$x^2 \cos.^2 \lambda$	$x^2 \sin.^2 \lambda$
439.	15.	$p \pi$ (partout)	$p x$

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D'INTÉGRALES DÉFINIES

PAR

D. BIERENS DE HAAN.

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1.	F. Alg. rat. ent.	Lim. 0 et 1
2.	" " " fract. à dén. monôme	" " " "
3.	" " " " " " $a + bx^c$	" " " "
4.	" " " " " " $(a + bx^c)^d$	" " " "
5.	" " " " " " $(a + bx^c)^d x^e$	" " " "
6.	" " " " " " $(a + bx^c)^d (a' + b' x^{c'})^{d'} x^{e'}$	" " " "
7.	" " " " " " trinôme	" " " "
8.	" " " " " " " composé	" " " "
9.	" " irrat. ent. à fact. $(1 - x)^a$ et $(1 - x^2)^a$	" " " "
10.	" " " " " " $(1 - x^a)^b$	" " " "
11.	" " " fract. à dén. monôme	" " " "
12.	" " " " " " $(1 \pm x)^a$ et $(1 \pm x^2)^a$	" " " "
13.	" " " " " " $(1 - x^a)^b$ pour a spécial	" " " "
14.	" " " " " " " " général	" " " "
15.	" " " " " " composé avec fact. monôme	" " " "
16.	" " " " " " " sans " "	" " " "
17.	" "	Lim. — 1 et + 1
18.	" " rat. fract. à dén. x^a et $(1 \pm x)^a$	Lim. 0 et ∞
19.	" " " " " " $1 + x^a$ pour a spécial	" " " "
20.	" " " " " " " " général	" " " "
21.	" " " " " " $(1 \pm x^a)^b$	" " " "
22.	" " " " " " à fact. monôme et binômes	" " " "
23.	" " " " " " " binômes $(1 \pm x)^a$	" " " "
24.	" " " " " " " $(1 \pm x^a)^b$	" " " "
25.	" " " " " " trinôme	" " " "
26.	" " " " " " autre dén. polynôme	" " " "

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27.	F. Alg. irrat. fract. à dén. binôme	Lim. 0 et ∞
28.	" " " " autre dén.	" " " "
29.	" " rat. " " dén. $1 \pm x^a$	Lim. $-\infty$ et ∞
30.	" " " " " autre dén.	" " " "
31.	" " " "	Lim. 1 et ∞
32.	" " irrat. "	" " " "
33.	" " ent.	Lim. 0 et p
34.	" " fract.	" " " "
35.	" "	Lim. diverses

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36.	F. Expon. Forme ex^a	Lim. 0 et ∞
37.	" " . Autre forme ent.	" " " "
38.	" " . Forme fract. à dén. binôme	" " " "
39.	" " . " " " " polynôme	" " " "
40.	" "	Lim. $-\infty$ et ∞
41.	" "	Lim. diverses

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42.	F. Logar. Forme rat. ent.	Lim. 0 et 1
43.	" " . " " fract.	" " " "
44.	" " . " " irrat.	" " " "
45.	" "	Lim. diverses

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46.	F. Circ. Dir. rat. ent.	Lim. 0 et $\frac{\pi}{4}$
47.	" " " " fract. à dén. monôme	" " " "
48.	" " " " " " " binôme	" " " "
49.	" " " " " " " composé	" " " "
50.	" " " " irrat. " " " d'un fact. monôme	" " " "
51.	" " " " " " " de deux fact. monômes	" " " "
52.	" " " " " " " à fact. binômes	" " " "
53.	" " " " rat. ent. à un fact.	Lim. 0 et $\frac{\pi}{2}$
54.	" " " " " " Fact. Sin. ax et un autre	" " " "

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55.	F. Circ.	Dir.	rat.	ent.	Fact.	$\text{Cos.}^a x$	et un autre	Lim. 0 et $\frac{\pi}{2}$
56.	"	"	"	"	"	Produit de deux puissances.		" " " "
57.	"	"	"	"	"	Trois fact.	Sin. ou Cos.	" " " "
58.	"	"	"	"	"	Fact.	$\text{tg}^a x$ et autres	" " " "
59.	"	"	"	"	"	comp. à arg.	$\text{tg} x$	" " " "
60.	"	"	"	"	"	"	autre arg. monôme	" " " "
61.	"	"	"	"	"	"	arg. binôme	" " " "
62.	"	"	"	"	"	fract. à num.	monôme et dén. $\text{Sin.}^a x$, $\text{Cos.}^a x$	" " " "
63.	"	"	"	"	"	"	autre dén. monôme	" " " "
64.	"	"	"	"	"	"	binôme et dén. monôme	" " " "
65.	"	"	"	"	"	dén.	de 1 ^{er} degré	" " " "
66.	"	"	"	"	"	"	de plus haut degré	" " " "
67.	"	"	"	"	"	"	puissance de binômes	" " " "
68.	"	"	"	"	"	"	produit de monôme et binômes	" " " "
69.	"	"	"	"	"	"	trinôme	" " " "
70.	"	"	"	"	"	compos. à arg.	$\text{tg} x$	" " " "
71.	"	"	"	"	"	"	autre arg.	" " " "
72.	"	"	"	"	"	irrat. ent.		" " " "
73.	"	"	"	"	"	fract. à dén.	monôme	" " " "
74.	"	"	"	"	"	"	binôme du 1 ^{er} degré	" " " "
75.	"	"	"	"	"	"	du 2 ^d	" " " "
76.	"	"	"	"	"	"	produit de monôme et binômes	" " " "
77.	"	"	"	"	"	composée		" " " "
78.	"	"	"	"	"	rat. ent.	monôme	Lim. 0 et π
79.	"	"	"	"	"	"	trinôme	" " " "
80.	"	"	"	"	"	"	composée	" " " "
81.	"	"	"	"	"	fract. à dén.	monôme	" " " "
82.	"	"	"	"	"	"	binôme de 1 ^{er} degré	" " " "
83.	"	"	"	"	"	"	2 ^e	" " " "
84.	"	"	"	"	"	"	à un fact. trinôme	" " " "
85.	"	"	"	"	"	"	et autres	" " " "
86.	"	"	"	"	"	irrat. "		" " " "
87.	"	"	"	"	"	rat. ent.		Lim. 0 et 2π
88.	"	"	"	"	"	fract. à dén.	monôme et binôme	" " " "
89.	"	"	"	"	"	"	trinôme à Cos.	" " " "

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90.	F. Circ. Dir. rat. fract. à dén. trinôme à <i>Sin.</i> et <i>Cos.</i>	Lim. 0 et 2π
91.	" " " irrat. "	" " " "
92.	" " " fract.	Lim. $\frac{\pi}{4}$ et $\frac{\pi}{2}$
93.	" " "	Lim. $-\frac{\pi}{2}$ et $\frac{\pi}{2}$
94.	" " "	Lim. $p\pi$ et $q\pi$
95.	" " "	Lim. 0 et 1
96.	" " " rat. ent. à un fact.	Lim. 0 et ∞
97.	" " " " " " plusieurs fact.	" " " "
98.	" " " " de forme fract.	" " " "
99.	" " " irrat.	" " " "
100.	" " " rat. ent. à un fact.	Lim. $-\infty$ et ∞
101.	" " " " " " deux fact.	" " " "
102.	" " " "	Lim. $\frac{\pi}{2}$ et ∞
103.	" " "	Lim. 0 et p
104.	" " " irrat. fract.	Lim. 0 et λ
105.	" " " " ent.	Lim. λ et μ
106.	" " " " fract. à dén. rat.	" " " "
107.	" " " " " " " irrat.	" " " "

V. FONCTIONS CIRCULAIRES INVERSES. T. 108 à 110.

108.	F. Circ. Inv.	Lim. 0 et 1
109.	" " "	Lim. 0 et ∞
110.	" " "	Lim. diverses

VI. DIVERSES FONCTIONS. T. 111.

111.	F. diverses	Lim. diverses
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PARTIE DEUXIÈME.

VII. FONCTIONS ALGÈBRIQUES ET EXPONENTIELLES. T. 112 à 150.

112.	F. Alg.	et Expon.	Lim. 0 et 1
113.	" " rat. ent.	" " monôme e^{ax}	Lim. 0 et ∞
114.	" " " "	" " " e^{ax^b} pour b spécial	" " " "
115.	" " " "	" " " " " général	" " " "

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116.	F. Alg. rat. ent.	et Expon. monôme d'autre forme	Lim. 0 et ∞
117.	" " " " monôme	" " bin. $e^{ax} \pm 1$ en dén. Num. alg.	" " " "
118.	" " " " "	" " " " " " " " et exp.	" " " "
119.	" " " " "	" " " $(e^{ax} \pm 1)^2$ en dén.	" " " "
120.	" " " " "	" " " $e^{ax} \pm e^{-ax}$ " " Num. alg.	" " " "
121.	" " " " "	" " " " " " " " et exp.	" " " "
122.	" " " " "	" " " $(e^{ax} \pm e^{-ax})^2$	" " " "
123.	" " " " binôme	" " " en dén.	" " " "
124.	" " " " "	" " polyn. en dén. Num. alg.	" " " "
125.	" " " " "	" " " " " " " et exp.	" " " "
126.	" " " fract. à dén. monôme	" " monôme en num.	" " " "
127.	" " " " " " x^a pour a spéc.	" " polyn. " "	" " " "
128.	" " " " " " " " " génér.	" " " " " "	" " " "
129.	" " " " " " " $x \pm q$	" " monôme	" " " "
130.	" " " " " " " $x^2 \pm q^2$	" " "	" " " "
131.	" " " " " " " $(x^a \pm q^a)^b$	" " "	" " " "
132.	" " " " " " " autre dén.	" " "	" " " "
133.	" " " " " " " dén. prod. de polyn.	" " "	" " " "
134.	" " " " " " " x	" " bin. $e^{ax} \pm 1$ en dén. à un terme	" " " "
135.	" " " " " " " monôme	" " " " " " " plus. termes	" " " "
136.	" " " " " " " "	" " " $e^{ax} \pm e^{-ax}$ en dén.	" " " "
137.	" " " " " " " "	" " trinôme en dén.	" " " "
138.	" " " " " " " binôme	" " binôme " "	" " " "
139.	" " " irrat. ent.	" "	" " " "
140.	" " " " fract.	" "	" " " "
141.	" " " " rat. ent.	" " sous forme irrat.	" " " "
142.	" " " " "	" " monôme	Lim. — ∞ et ∞
143.	" " " " " x	" " binôme en dén.	" " " "
144.	" " " " " "	" " polynôme en dén.	" " " "
145.	" " " " " x^a	" " " " "	" " " "
146.	" " " " fract.	" " dén. à fact. x^a	" " " "
147.	" " " " "	" " " sans fact. x^a	" " " "
148.	" " " " irrat.	" "	" " " "
149.	" " " " "	" "	Lim. div. 0 et ∞
150.	" " " " "	" "	Lim. div. pet $\pm \infty$

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VIII. FONCTIONS ALGÈBRIQUES ET LOGARITHMES. T. 151 à 191.

151.	F. Alg. rat. ent.	et Log. en num.	Lim. 0 et 1
152.	" " " fract. à dén. mon. ou bin.	" " " lx	" " " "
153.	" " " " " autre dén.	" " " "	" " " "
154.	" " " " " dén. binôme	" " " $(lx)^2, (lx)^3$	" " " "
155.	" " " " " " "	" " " $(lx)^4, (lx)^5, (lx)^6, (lx)^7$	" " " "
156.	" " " " " trinôme	" " " $(lx)^a$ pour a spécial	" " " "
157.	" " " " " bin. $x \pm b$	" " " " " " général	" " " "
158.	" " " " " autre dén. binôme	" " " " " " "	" " " "
159.	" " " " " dén. trinôme	" " " " " " "	" " " "
160.	" " " " "	" " " " de forme div. à un fact.	" " " "
161.	" " " " "	" " " " " " deux "	" " " "
162.	" " irrat. ent.	" " " "	" " " "
163.	" " " fract.	" " " lx	" " " "
164.	" " " " "	" " " $(lx)^a$	" " " "
165.	" " " " "	" " " de fonct. ent.	" " " "
166.	" " " " "	" " " " fract.	" " " "
167.	" " rat. ent.	" " " dén. lx	" " " "
168.	" " " " "	" " " $(lx)^a$	" " " "
169.	" " " " "	" " " de forme $a \pm (lx)^b$	" " " "
170.	" " " fract. à dén. monôme	" " " "	" " " "
171.	" " " " " " $1 \pm x$	" " " lx	" " " "
172.	" " " " " " $1 \pm x^a$	" " " $(lx)^b$	" " " "
173.	" " " " " " "	" " " de forme $1 \pm (lx)^b$	" " " "
174.	" " " " " " trinôme	" " " "	" " " "
175.	" " " " " " prod. de fact.	" " " lx	" " " "
176.	" " " " " " " " "	" " " d'autre forme.	" " " "
177.	" " irrat. "	" " " "	" " " "
178.	" " rat.	" " " sous forme irrat.	" " " "
179.	" " " fract. à dén. x^p	" " "	Lim. 0 et ∞
180.	" " " " " " binôme	" " $(lx)^a$	" " " "
181.	" " " " " " "	" " d'autre forme	" " " "
182.	" " " " " " $(a \pm x^b)^c$	" " "	" " " "
183.	" " " " " " autre dén.	" " lx	" " " "
184.	" " " " " " " "	" " d'autre forme	" " " "
185.	" " irrat. "	" " "	" " " "

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187. " "	" "	Lim. 1 et ∞
188. " "	" "	Lim. div. 0 et p
189. " "	" "	" " p et q
190. " "	" " de Log.	Lim. 0 et 1
191. " "	" " " "	Lim. 0 ou 1 et ∞

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192.	F. Alg.									et Circ. Dir.		Lim. 0 et 1
193.	" "	rat. ent.								" "	" " " " " " " " " " " "	Lim. 0 et ∞
194.	" "	" fract. à dén. x								" "	" en num.d'un fact. monôme . . .	" " " " "
195.	" "	" " " " "								" "	" de fact. diff. monômes. .	" " " " "
196.	" "	" " " " "								" "	" polynôme	" " " " "
197.	" "	" " " " "								" "	" x^a pour a spécial " " " " " " " " " " " "	" " " " "
198.	" "	" " " " "								" "	" " de fact. diff. . .	" " " " "
199.	" "	" " " " "								" "	" polynôme	" " " " "
200.	" "	" " " " "								" "	" " génér. " " " " " " " " " " " "	" " " " "
201.	" "	" " " " "								" "	" " " " " " " " " " " "	" " " " "
202.	" "	" " " " "								" "	" " " " " " " " " " " "	" " " " "
203.	" "	" " " " "								" "	" $a \pm x$ " " " " " " " " " " " "	" " " " "
204.	" "	" " " " "								" "	" $1 \pm x^2$ " " " " " " " " " " " "	" " " " "
205.	" "	" " " " "								" "	" $a^2 + x^2$ " " " " " " " " " " " "	" " " " "
206.	" "	" " " " "								" "	" $a^2 - x^2$ " " " " " " " " " " " "	" " " " "
207.	" "	" " " " "								" "	" $a \pm x^b$ " " " " " " " " " " " "	" " " " "
208.	" "	" " " " "								" "	" $(a \pm x^b)^c$ " " " " " " " " " " " "	" " " " "
209.	" "	" " " " "								" "	" binôme " " " " " " " " " " " "	" " " " "
210.	" "	" " " " "								" "	" trinôme " " " " " " " " " " " "	" " " " "
211.	" "	" " " " "								" "	" quadrinôme " " " " " " " " " " " "	" " " " "
212.	" "	" " " " "								" "	" prod.d.mon.et bin. " " " " " " " " " " " "	" " " " "
213.	" "	" " " " "								" "	" " polynômes " " " " " " " " " " " "	" " " " "
214.	" "	" " " " "								" "	" x " " " " " " " " " " " "	" " " " "
215.	" "	" " " " "								" "	" " " " " " " " " " " "	" " " " "
216.	" "	" " " " "								" "	" $1 + x^2$ " " " " " " " " " " " "	" " " " "
217.	" "	" " " " "								" "	" $a^b + x^b$ " " " " " " " " " " " "	" " " " "
218.	" "	" " " " "								" "	" $(a^2 - x^2)x$ " " " " " " " " " " " "	" " " " "
219.	" "	" " " " "								" "	" x " " " " " " " " " " " "	" " " " "
220.	" "	" " " " "								" "	" binôme " " " " " " " " " " " "	" " " " "

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221. F. Alg.	rat. fract.	à dén.	binôme	et Circ.	Dir.	en num.	trinôme	$a - b \cos. x + c$.	Lim. 0 et ∞
222. " "	" "	" "	polynôme	" "	" "	" "	" "	" "	" "
223. " "	" "	irrat.	ent.	" "	" "	" "	" "	" "	" "
224. " "	" "	"	fract. à dén.	$1/x$	" "	"	en num.	monôme à un fact.	circ. de x
225. " "	" "	" "	" "	" "	" "	" "	" "	" "	" "
226. " "	" "	" "	" "	" "	" "	" "	" "	deux "	" "
227. " "	" "	" "	" "	" "	" "	" "	" "	binôme	" "
228. " "	" "	" "	" "	" "	" "	" "	circ. de $x - \frac{1}{x}$	" "	" "
229. " "	" "	" "	"	autre dén.	" "	" "	" "	" "	" "
230. " "	" "	" "	"	dén.	binôme	"	circ. de $x - \frac{1}{x}$	" "	" "
231. " "	" "	" "	" "	" "	" "	dén.	" "	" "	" "
232. " "	rat.	" "	" "	" "	" "	num.	$\sin. x$	Lim. $-\infty$ et ∞	" "
233. " "	" "	" "	" "	" "	" "	"	$\cos. x$	" "	" "
234. " "	" "	" "	" "	" "	" "	"	d'autre forme	" "	" "
235. " "	fract.	" "	" "	" "	" "	$\sin. x$	" "	Lim. 1 et ∞	" "
236. " "	" "	" "	" "	" "	" "	$\cos. x$	" "	" "	" "
237. " "	rat.	ent.	" "	" "	" "	en dén.	" "	Lim. 0 et $\frac{\pi}{4}$	" "
238. " "	" "	" "	" "	" "	" "	ent.	" "	Lim. 0 et $\frac{\pi}{2}$	" "
239. " "	" "	" "	" "	" "	" "	en dén.	monôme	" "	" "
240. " "	" "	" "	" "	" "	" "	" "	binôme	" "	" "
241. " "	" "	" "	" "	" "	" "	" "	d'autre forme	" "	" "
242. " "	" "	" "	" "	" "	" "	" "	sous forme irrat.	à dén.	monôme
243. " "	" "	" "	" "	" "	" "	" "	" "	" "	polynôme
244. " "	" "	" "	" "	" "	" "	ent.	" "	Lim. 0 et π	" "
245. " "	" "	" "	" "	" "	" "	en dén.	binôme $a + b$	" "	" "
246. " "	" "	" "	" "	" "	" "	" "	" $a - b$	" "	" "
247. " "	" "	" "	" "	" "	" "	puiss.	de binôme	" "	" "
248. " "	" "	" "	" "	" "	" "	trinôme	$1 - a \cos. x + b$	" "	" "
249. " "	" "	" "	" "	" "	" "	" "	d'autre forme	" "	" "
250. " "	" "	" "	" "	" "	" "	" "	" "	Lim. 0 et 2π	" "
251. " "	" "	" "	" "	" "	" "	" "	" "	Lim. 0 et γ	" "
252. " "	" "	" "	" "	" "	" "	" "	sous forme irrat.	Lim. 0 et λ	" "
253. " "	" "	" "	" "	" "	" "	" "	" "	Lim. λ et μ	" "
254. " "	" "	" "	fract.	" "	" "	" "	" "	Lim. div. p et $+\infty$	" "
255. " "	" "	" "	" "	" "	" "	" "	" "	Lim. diverses	" "

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256. F. Alg. rat. ent.	et Circ. Inv. de x	Lim. 0 et 1
257. " " " fract. à dén. monôme	" " " " "	" " "
258. " " " " " polynôme	" " " " " à un fact.	" " "
259. " " " " " " "	" " " " " plus. "	" " "
260. " " " " " " prod. de fact.	" " " " "	" " "
261. " " irrat. "	" " " " "	" " "
262. " " fract.	" " " d'autre forme	" " "
263. " " rat. ent.	" " " de x	Lim. 0 et ∞
264. " " fract. à dén. monôme	" " " " "	" " "
265. " " " " " binôme	" " " " "	" " "
266. " " " " " $x(q^2 + x^2)$	" " " " "	" " "
267. " " " " " prod. de binômes	" " " " "	" " "
268. " " irrat.	" " " " "	" " "
269. " " fract.	" " " d'autre forme	" " "
270. " " " "	" " "	Lim. 1 et ∞
271. " " " "	" " "	Lim. diverses

XI. FONCTIONS ALGÈBRIQUES ET AUTRES FONCTIONS. T. 272.

272. F. Alg.	et autres fonctions	Lim. diverses
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XII. FONCTIONS EXPONENTIELLES ET LOGARITHMES. T. 273 à 277.

273. F. Expon.	et Log., Fonct. ent.	Lim. 0 et ∞
274. " " polynôme en dén.	" " en num. le	" " "
275. " " " " "	" " " " $l(p^2 \pm x^2)$	" " "
276. " " " " "	" "	Lim. $-\infty$ et ∞
277. " " " " "	" "	Lim. diverses

XIII. FONCTIONS EXPONENTIELLES ET CIRCULAIRES DIRECTES. T. 278 à 298.

278. F. Expon. $e^{\pm ax}$	et Circ. Dir. ent. à un fact.	Lim. 0 et ∞
279. " " " "	" " " " "	" " "
280. " " e^{-ax^2}	" " " " "	" " "
281. " " en dén. binôme à exp. $e^{\pm ax}$	" " " en num.	" " "
282. " " " " " " et en num.	" " " " "	" " "
283. " " num. e^{-x^2}	" " " " dén. trinôme	" " "
284. " " $e^{\pm ax}$ ou $e^{\pm ax^2}$	" " " d'autre forme	" " "

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285. F. Expon. d'autre forme	et Circ. Dir.	Lim. 0 et ∞
286. " "	" " " ent.	Lim.— ∞ et ∞
287. " " e^{+ax}	" " " "	Lim. 0 et $\frac{\pi}{2}$
288. " " à exp. circ. dir.	" " " "	" " " "
289. " " " " " "	" " " en dén. <i>Sin.</i> $2x$	" " " "
290. " " " " " "	" " " " " à une autre fonct. monôme	" " " "
291. " " " " " "	" " " " " plus. " "	" " " "
292. " " en dén. polynôme	" " " " num.	" " " "
293. " " " " " "	" " " " " déu.	" " " "
294. " " " num.	" " " " " trinôme	" " " "
295. " " " "	" " " de forme irrat.	" " " "
296. " " " "	" " " "	Lim. 0 et π
297. " " " "	" " " "	Lim. — $\frac{\pi}{2}$ et $\frac{\pi}{2}$
298. " " " "	" " " "	Lim. diverses

XIV. FONCTIONS EXPONENTIELLES ET CIRCULAIRES INVERSES. T. 299.

299. F. Expon.	et Circ. Inv.	Lim. 0 et ∞
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XV. FONCTIONS EXPONENTIELLES ET AUTRES FONCTIONS. T. 500.

300. F. Expon.	et autres Fonctions	Lim. 0 et ∞
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XVI. FONCTIONS LOGARITHMES ET CIRCULAIRES DIRECTES. T. 501 à 565.

301. F. Log.	et Circ. Dir.	Lim. 0 et 1
302. " "	" " " "	Lim. 0 et ∞
303. " " de Circ. Dir. en num. $(l \text{ Sin. } ax)^b$	" " " ent.	Lim. 0 et $\frac{\pi}{4}$
304. " " " " " " $(l \text{ Cos. } ax)^b$	" " " " " " " "	" " " "
305. " " " " " " $(l \text{ tg. } ax)^b$	" " " " " " " "	" " " "
306. " " " " " " "	" " " " " d'autre forme	" " " "
307. " " " " " " $l \text{ Sin. } ax, l \text{ Cos. } ax$	" " " rat. en dén. monôme	" " " "
308. " " " " " " $(l \text{ Sin. } ax)^b, (l \text{ Cos. } ax)^b$	" " " " " " " "	" " " "
309. " " " " " " $l \text{ tg. } ax$	" " " " " " " "	" " " "
310. " " " " " " $(l \text{ tg. } ax)^b$	" " " " " " " "	" " " "
311. " " " " " " $l \text{ tg. } ax$	" " " " " " binôme	" " " "
312. " " " " " " $(l \text{ tg. } ax)^b$	" " " " " " " "	" " " "
313. " " " " " " $l \text{ tg. } ax$	" " " " " " à fact. mon. et bin.	" " " "
314. " " " " " " $(l \text{ tg. } ax)^b$	" " " " " " " " " " " "	" " " "

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315.	F. Log. de Circ. Dir. en num.	$lty\left(\frac{\pi}{4} + x\right)$	et Circ. Dir. rat. en dén.		Lim. 0 et $\frac{\pi}{4}$
316.	" " " " " " "	d'autre forme à un fact.	" " " " " "		" " " "
317.	" " " " " " "	à deux fact.	" " " " " "		" " " "
318.	" " " " " " "	Log. de Log.	" " " " " "		" " " "
319.	" " " " " " "	$(lty.x)^a$	" " " en dén. irrat.		" " " "
320.	" " " " " " "	d'autre forme	" " " " " "		" " " "
321.	" " " " " " "	dén. fonet. monôme	" " " ent.		" " " "
322.	" " " " " " "	" "	" " " en dén. monôme		" " " "
323.	" " " " " " "	" "	" " " d'autre forme		" " " "
324.	" " " " " " "	binôme	" " " ent.		" " " "
325.	" " " " " " "	" "	" " " en dén. rat. monôme		" " " "
326.	" " " " " " "	" "	" " " irrat.	"	" " " "
327.	" " " " " " "	" "	" " " " composée		" " " "
328.	" " " " " " "	sous forme irrat.	" " "		" " " "
329.	" " " " " " "	de Circ. Dir.	" " " d'autre forme		" " " "
330.	" " " " " " "	en num. $lSin.x$	" " " ent.		Lim. 0 et $\frac{\pi}{2}$
331.	" " " " " " "	$lCos.x$	" " " "		" " " "
332.	" " " " " " "	. Pr. de $lSin.x$ et $lCos.x$	" " "		" " " "
333.	" " " " " " "	$(lty.x)^a$	" " " "		" " " "
334.	" " " " " " "	et Circ. Dir. Log. de Circ. Dir.	d'autre forme sans fact. circ.		" " " "
335.	" " " " " " "	" " " " " "	" " avec " "		" " " "
336.	" " " " " " "	en num. $(lSin.x)^a$	et Circ. Dir. rat. en dén. monome		" " " "
337.	" " " " " " "	$(lCos.x)^a$	" " " " " "	"	" " " "
338.	" " " " " " "	$(lty.x)^a$	" " " " " "	"	" " " "
339.	" " " " " " "	d'autres fonet. ent.	" " " " " "	"	" " " "
340.	" " " " " " "	de fonet. fract.	" " " " " "	"	" " " "
341.	" " " " " " "	. Produits	" " " " " "	"	" " " "
342.	" " " " " " "	de circ. monôme	" " " " " "	binôme	" " " "
343.	" " " " " " "	binôme	" " " " " "	"	" " " "
344.	" " " " " " "	" "	" " " " " "	puiss. de binôme.	" " " "
345.	" " " " " " "	" "	" " " " " "	à fact. bin. et autre.	" " " "
346.	" " " " " " "	" "	" " " " " "	trinôme	" " " "
347.	" " " " " " "	de circ. monôme	" " " de forme irrat.		" " " "
348.	" " " " " " "	polynôme	" " " " " "	"	" " " "
349.	" " " " " " "	sous forme irrat.	" " "		" " " "
350.	" " " " " " "	en dén. monôme	" " "		" " " "

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351. F. Log. en dén. binôme $q \pm (l \sin. x)^2$	et Circ. Dir.	Lim. 0 et $\frac{\pi}{2}$
352. " " " d'autre forme binôme	" " "	" " " "
353. " " et Circ. Dir. Log. de Circ. Dir. sans fact. circ.		Lim. 0 et π
354. " " " " " " " avec " "		" " " "
355. " de " "	et Circ. Dir. fract.	" " " "
356. " " " "		Lim. 0 et 2π
357. " $(ltg.x)$	" " "	Lim. $\frac{\pi}{4}$ et $\frac{\pi}{2}$
358. " $(ltg.x)^a$ pour a spécial	" " "	" " " "
359. " " " " général	" " "	" " " "
360. " en dén.	" " "	" " " "
361. " " " "		Lim. 0 et $p\pi$
362. " " " "		Lim. 0 et λ
363. " " " "		Lim. λ et $\frac{\pi}{2}$
364. " " " "		Lim. λ et μ
365. " " " "		Lim. diverses

XVII. FONCTIONS LOGARITHMES ET CIRCULAIRES INVERSES. T. 566.

366. F. Log.	et Circ. Inv.	Lim. 0 et 1
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XVIII. FONCTIONS LOGARITHMES ET AUTRES FONCTIONS. T. 567.

367. F. Log.	et autres Fonctions	Lim. diverses
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XIX. FONCTIONS CIRCULAIRES DIRECTES ET CIRCULAIRES INVERSES. T. 568 à 574.

368. F. Circ. Dir. ent.	et Circ. Inv.	Lim. 0 et $\frac{\pi}{2}$
369. " " " fract.	" " "	" " " "
370. " " " ent.	" " "	Lim. 0 et π
371. " " " fract. à dén. monôme	" " "	" " " "
372. " " " " " polynôme	" " "	" " " "
373. " " " " " " "		Lim. 0 et 2π
374. " " " " " " "		Lim. diverses

XX. FONCTIONS CIRCULAIRES DIRECTES ET AUTRES FONCTIONS. T. 575.

375. F. Circ. Dir.	et autres Fonctions	Lim. diverses
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XXI. FONCTIONS ALGÈBRIQUES ET EXPONENTIELLES ET LOGARITHMES. T. 376 à 383.

376. F. Alg. ent.	et Expon. monôme	et Log. Lim. 0 et 1
377. " " "	" " "	" " Lim. 0 et ∞
378. " " fract. à dén. monôme et binôme	" " "	" " " " " "
379. " " " " " puiss. de binôme	" " "	" " " " " "
380. " " rat.	" " en dén. polynôme	" " " " " "
381. " " irrat.	" " "	" " " " " "
382. " " "	" " "	" " Lim.— ∞ et ∞
383. " " "	" " "	" " Lim. diverses.

XXII. FONCTIONS ALGÈBRIQUES ET EXPONENTIELLES ET CIRCULAIRES DIRECTES. T. 384 à 400.

384. F. Alg. rat. ent.	et Expon.	et Circ. Dir.	Lim. 0 et $\frac{\pi}{4}$
385. " " " " x^a pour a spécial	" " $e^{\pm px}$	" " " monôme . . .	Lim. 0 et ∞
386. " " " " " " " général	" " "	" " " " " . . .	" " " "
387. " " " " "	" " "	" " " polynôme. . .	" " " "
388. " " " " "	" " e^{-x^2}	" " "	" " " "
389. " " " " "	" " e^{-ax^2}	" " "	" " " "
390. " " " " "	" " d'autre forme	" " "	" " " "
391. " " " " "	" " en dén. binôme	" " "	" " " "
392. " " " fract. à dén. x	" " monôme	" " " monôme . . .	" " " "
393. " " " " " " "	" " "	" " " (F. polyn. en num.)	" " " "
394. " " " " " " x^a	" " en num.	" " "	" " " "
395. " " " " " " $x^2 + a^2$	" " " "	" " "	" " " "
396. " " " " "	" " en dén. polyn.	" " "	" " " "
397. " " irrat. ent.	" " "	" " "	" " " "
398. " " " fract. à dén. $\sqrt{x}$	" " "	" " "	" " " "
399. " " " " " à autre dén.	" " "	" " "	" " " "
400. " " " " "	" " "	" " "	Lim. diverses.

XXIII. FONCTIONS ALGÈBRIQUES ET EXPONENTIELLES ET CIRCULAIRES INVERSES. T. 401.

401. F. Alg.	et Expon.	et Circ. Inv.	Lim. 0 et ∞
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XXIV. FONCTIONS ALGÈBRIQUES ET EXPONENTIELLES ET AUTRES. T. 402.

402. F. Alg.	et Expon.	et autres Fonct.	Lim. 0 et ∞
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XXV. FONCTIONS ALGÈBRIQUES ET LOGARITHMES ET CIRCULAIRES DIRECTES. T. 405—421.

403. F. Alg. rat. ent.	et Log.	et Circ. Dir. de Log.	Lim. 0 et 1
404. " " " fract. à den. binôme	" "	" " " " " "	" " " "
405. " " " " " autre dén.	" "	" " " " " "	" " " "
406. " " " " " en den. lx	" "	" " " " " "	" " " "
407. " " " " " $\sqrt[l]{lx}$	" "	" " " " " "	" " " "
408. " " " fract.	" " " " $q^2 + (lx)^2$	" " " " " "	" " " "
409. " " irrat. "	" " " " " " "	" " " " " "	" " " "
410. " " rat. ent.	" " de	" " " " " "	Lim. 0 et $\frac{\pi}{4}$
411. " " " " "	" " "	" " " " " "	Lim. 0 et $\frac{\pi}{2}$
412. " " " fract.	" " "	" " Den. $x^2 + (l \text{ Cos. } x)^2$	" " " "
413. " " " ent.	" " "	" " " " " "	Lim. 0 et π
414. " " " fract. à den. x^a	" " "	et " " " " " "	Lim. 0 et ∞
415. " " " " " " $b^2 \pm x^2$	" " de	" " monôme	" " " "
416. " " " " " " " "	" " "	" " polynôme	" " " "
417. " " " " " " " "	" " lax	et " " " " " "	" " " "
418. " " " " " " " $b^n \pm x^n$	" " "	" " " " " "	" " " "
419. " " " " " " autre dén.	" " "	" " " " " "	" " " "
420. " " " " " "	" " "	" " " " " "	Lim. — ∞ et ∞
421. " " " " " "	" " "	" " " " " "	Lim. diverses.

XXVI. FONCTIONS ALGÈBRIQUES ET LOGARITHMES ET CIRCULAIRES INVERSES. T. 422—427.

422. F. Alg. rat.	et Log. en num.	et Circ. Inv.	Lim. 0 et 1
423. " " irrat.	" " " "	" " " "	" " " "
424. " " " " " dén.	" " " " " "	" " " " " "	" " " "
425. " " " " " "	" " " " " "	" " " " " "	Lim. 0 et ∞
426. " " " " " "	" " " " " "	" " " " " "	Lim. 1 et ∞
427. " " " " " "	" " " " " "	" " " " " "	Lim. diverses.

XXVII. FONCTIONS ALGÈBRIQUES ET LOGARITHMES ET AUTRES. T. 428.

428. F. Alg.	et Log.	et autres Fonct.	Lim. diverses.
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XXVIII. FONCTIONS ALGÈBRIQUES ET CIRCULAIRES DIRECTES ET CIRCULAIRES INVERSES.

T. 429—454.

429. F. Alg.	et Circ. Dir.	et Circ. Inv.	Lim. 0 et $\frac{\pi}{2}$.
430. " " " " " "	" " " " " "	" " " " " "	Lim. 0 et π

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431. F. Alg. rat. fract.	et Circ. Dir.	et Circ. Inv.	Lim. 0 et ∞
432. " " irrat. " à dén. binôme	" " "	" " "	" " " "
433. " " " " " autre dén.	" " "	" " "	" " " "
434. " " " " "	" " "	" " "	Lim. diverses.

XXIX. FONCTIONS ALGÈBRIQUES ET CIRCULAIRES DIRECTES ET AUTRES. T. 455.

435. F. Alg.	et Circ. Dir.	et autres Fonct.	Lim. 0 et ∞
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XXX. FONCTIONS EXPONENTIELLES ET LOGARITHMES ET CIRCULAIRES DIRECTES. T. 456—440.

436. F. Expon. monôme	et Log.	et Circ. Dir. ent.	Lim. 0 et $\frac{\pi}{2}$
437. " " "	" "	" " " fract.	" " " "
438. " " en dén. binôme	" "	" " " "	" " " "
439. " " "	" "	" " "	Lim. 0 et ∞
440. " " "	" "	" " "	Lim. diverses.

XXXI. FONCTIONS EXPONENTIELLES ET CIRCULAIRES DIRECTES ET CIRCULAIRES INVERSES. T. 441.

441. F. Expon.	et Circ. Dir.	et Circ. Inv.	Lim. 0 et ∞
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XXXII. FONCTIONS EXPONENTIELLES ET CIRCULAIRES DIRECTES ET AUTRES. T. 442.

442. F. Expon.	et Circ. Dir.	et autres Fonct.	Lim. diverses.
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XXXIII. FONCTIONS LOGARITHMES ET CIRCULAIRES DIRECTES ET CIRCULAIRES INVERSES. T. 445.

443. F. Log.	et Circ. Dir.	et Circ. Inv.	Lim. diverses.
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XXXIV. FONCTIONS LOGARITHMES ET CIRCULAIRES DIRECTES ET AUTRES T. 444.

444. F. Log.	et Circ. Dir. .	et autres Fonct.	Lim. diverses.
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XXXV. FONCTIONS ALGÈBRIQUES ET PLUSIEURS FONCTIONS. T. 445—447.

445. F. Alg. rat. ent.	et plusieurs Fonct.	Lim. diverses.
446. " " fract.	" " "	" "
447. " " irrat "	" " "	" "

ABBREVIATIONS ET NOTATIONS.

Mém. Inst.	Mémoires de l'Institut. — Classe des Sciences physiques et mathém. Paris.
Mém. Acad.	Mémoires de l'Académie Royale des Sciences. Paris.
Sav. Etr.	Mémoires présentés à l'Acad. Royale des Sc. par divers Savans. Paris.
C. R.	Comptes Rendus des Séances hebdomadaires de l'Acad. des Sc. Paris.
Comm. Petr.	Commentaria Petropolitana.
N. C. Petr.	Nova Commentaria Petropolitana.
A. Petr.	Acta Petropolitana.
N. A. Petr.	Nova Acta Petropolitana.
Mém. Petr.	Mémoires de l'Académie de St. Pétersbourg.
Mém. Turin.	Mémoires de l'Académie de Turin.
Mém. Brux.	Nouv. Mém. de l'Acad. Roy. des Sc. et Belles Lettres de Bruxelles.
Mém. Kasan.	Mémoires de l'Académie de Kasan.
Abh. Berlin.	Abhandlungen der K. Akademie der Wissenschaften zu Berlin.
Phil. Trans.	Philosophical Transactions.
Verh. K. Ak. Wet.	Verhandelingen der K. Akademie van Wetenschappen. Amsterdam.
Handl. Stockh.	Kongl. Vetenskaps Academiens Handlingar. Stockholm.
Danske Handl.	Danske Videnskab Akademiens Handlingar.
Overs. Handl.	Overs. over det Kongl. Danske Videnskab. Selskabs Forhandl.
Gött. Stud.	Göttinger Studien.
P.	Journal de l'École Polytechnique.
Bull. Phil.	Bulletin de la Société Philomatique.
C.	Crelle, Journal für reine und angewandte Mathematik.
Gr.	Grunert, Archiv der Mathematik und Physik.
L.	Liouville, Journal de Mathématiques pures et appliquées.
Lim. Imag.	A. L. Cauchy, Mém. sur les intégrales définies prises entre des limites imaginaires. Paris. Debure. 1825. 4°. 69 Pages.
Rés. Leç.	A. L. Cauchy, Résumé des Leçons données à l'Ec. Polyt. sur le Calcul Infinitésimal. T. I (et seul). Paris. Debure. 1823. 4°. XII et 172 Pages.
Exerc.	Cauchy, Exercices Mathématiques. Paris. 4°.
Eul. Int.	R. Dedekind, Ueber die Elemente der Theorie der Euler'schen Integrale. Göttingen. Huth. 1852. 4°. 23 S.
Calc. Int.	Euler, Institutiones Calculi Integralis. IV Vol. Petrop. 1792—1794. 4°.
Funct. Transc.	B. J. Féaux, De functione transcendente, quae littera Γ () obsignatur: sive de integrali Euleriano secundae speciei. Monast. Copenh-rath 1844. 4°. 43 Pages.
Chal.	Fourier, Théorie Analytique de la Chaleur. Paris. Firmin Didot. 1822. 4°. XXII et 639 Pages.
Transf.	Ph. P. Helmling, Transformation und Ausmittlung bestimmter Integrale. Dorpat. Laakman. 1851. 4°. 35 S.
Transf. II.	Ph. P. Helmling, Transformation und Ausmittlung bestimmter Integrale mit besonderer Rücksicht auf grössere Werthe der Grenzen und implicirten Constanten. Mitau und Leipzig. Reyher. 1854. 4°. IV und 146 S.

ABBREVIATIONS ET NOTATIONS.

Réfr.	Kramp, Analyse des réfractions astronomiques et terrestres. Leipzig. Schwickert. 1799. 4°. XX et 210 S.
Probab.	Laplace, Théorie analytique des Probabilités. Paris. Courcier. 1812. 4°. 465 Pages et quatre Suppléments.
Exerc.	A. M. Legendre, Exercices de Calcul Intégral sur divers ordres de transcendantes et sur les Quadratures. 3 Vol. Paris. Courcier. 1811—1818. 4°.
Int.	R. Lobatto, Lessen over de Integraal-Rekening. I. 's Gravenh. Van Cleef. VI en 466 Bladz.
Adn.	L. Mascheroni, Adnotationes ad Calculum Integrale Euleri. Ticini. Galeatis. 1790. 4°. 72 Pag.
Int. Déf.	A. Meyer, Exposé élément. de la Théorie des Intégrales définies. Bruxelles. Muquardt. 1851. 8°. 513 Pages.
Int.	Moigno, Leçons de Calcul Intégral. I. Paris. Bachelier. 1844. 8. XLVIII et 783 Pages.
Def. Int.	H. Mosely, Definite Integrals (Encycl. Metropol. Re-issue). London. Griffin. 1849. 4°. 51 Pages.
Ausw.	M. Ohm, Die Auswerthungsmethoden bestimmter Integrale, so wie die Theorie der Reihen und der Integrale des Fourier. Nürnberg. Korn. 1852. 8°. XII und 437 S.
Chal.	S. D. Poisson, Théorie mathématique de la Chaleur. Paris. Bachelier. 1835. 4°. 532 Pag. et Supplement. Paris. Bachelier. 1837. 4°. 72 Pag.
Int.	J. L. Raabe, Die Integralrechnung. III Th. Zürich. Orell. 1839, 1843, 1847. 8°.
J. B. Funct. Mat.	J. L. Raabe, Die Jacob-Bernoullische Function. Zürich. Orell. 1848. 4°. 51 S. Rogner, Materialien aus der höheren Analysis. Gratz. Hesse. 1853. 8°. XIV und 463 S.
Beitr.	O. Schlömilch, Beiträge zur Theorie bestimmter Integrale. Jena. Frommann. 1843. 4°. 103 S.
An. Stud.	O. Schlömilch, Analytische Studien. II Th. Leipzig. Engelmann. 1848. 8°. 209 und 197 S.
Int	O. Schlömilch, Handbuch der Integralrechnung. Greifswald. Otte. 1847. 8°. 214 S.
Höh. An.	O. Schlömilch, Compendium der höhern Analysis. Braunschweig. Vieweg. 1853. 8°. XVI und 550 S.
Samml.	J. A. Schubert, Sammlung von Differential- und Integral-Formeln. Dresden. Arnold. 1845. 8°. XIV und 173 S.
Samml.	L. A. Sohnke, Sammlung von Aufgaben aus der Differential- und Integral-Rechnung. Halle. Schmidt. 1850. 8°. VI und 338 S.
Transf.	A. F. Svanberg, Observations sur la transformation des Intégrales multiples. Ups. Leffler 1845. 4°. 13 Pag.
Anal.	J. Vieille, Cours complémentaire d'Analyse et de Mécanique rationnelle. Paris. Bachelier. 1851. 8°. VII en 400 Pages.

ABBREVIATIONS ET NOTATIONS.

$A = 0$, 577215 664901 532861.... Constante du Logarithme intégral.
 $e = 2$, 718281 828459 045235 360287 471352 662497 757247 093699 959574
 966967 627724 076630 353547 594571 382178 525166 427427 466....
 Base des Logarithmes naturels.

$\pi = 3$, 141592 653589 793238 462643 383279 502884 197169 399375 105820
 971914 592307 816406 286208 998628 034825 312117 067982 148086
 513282 306647 093844 609550 582231 725359 408128 481117 450284
 102701 938521 105559 644622 948954 930381 964428 810975 665933
 446128 475648 233786 783165 271201 909145 648566 923460 348610
 454326 648213 393607 260249 141273 724587 006606 315588 174881
 520920 962829 254091 715364 367892 590360 011330 530548 820466
 521384 146951 941511 609433 057270 365759 591953 092186 117381
 932611 793105 118548 074462 379962 749567 351885 752724 891227
 938183 011949 129833 673362 440656 643086 021394 88.... Circonfé-
 rence du cercle dont le diamètre est l'unité.

$i = \sqrt{-1}$

$\text{Sinhp. } a = \frac{e^a - e^{-a}}{2}$	, Sinus hyperbolique	Ces notations ne sont employées, qu'au- tant qu'elles portent sur des constantes : elles ne sont donc pas admises comme argument dans les tables, mais dans les formules, où elles se trouvent, on y a substitué les va- leurs équivalentes en exponentielles.
$\text{Coshp. } a = \frac{e^a + e^{-a}}{2}$	, Cosinus "	
$\text{Tghp. } a = \frac{e^a - e^{-a}}{e^a + e^{-a}}$	, Tangente "	
$\text{Cothp. } a = \frac{e^a + e^{-a}}{e^a - e^{-a}}$	, Cotangente "	

$\ln = \int_1^a \frac{dx}{x}$, le Logarithme naturel

$li(a) = \int_0^a \frac{dx}{lx}$, le Logarithme intégral

$Ei(a) = \int_{-a}^{\infty} \frac{e^{-x} dx}{x}$, l'Exponentielle intégrale

$Si(a) = \int_0^a \frac{\text{Sin. } x dx}{x}$, le Sinus intégral

$Ci(a) = \int_{\infty}^a \frac{\text{Cos. } x dx}{x}$, le Cosinus intégral

$\Psi'(a) = \frac{d \cdot \ln \Gamma(a)}{da}$

Ces fonctions sont comprises sous
la dénomination d' "autres fonctions."

$\binom{a}{b}$ le coefficient $b^{\text{ième}}$ de la puissance $a^{\text{ième}}$ du binôme.

$c^{a|b}$ factorielle (Notation de Kramp).

B_{2p-1} coefficient ou nombre Bernoullien.

ABBREVIATIONS ET NOTATIONS.

$$\left. \begin{aligned} \varphi(p, q, r) &= 1 + \frac{p}{q} \frac{r}{1} + \frac{p \cdot p + 1}{q \cdot q + 2} \frac{r^2}{1 \cdot 2} + \dots \\ \psi(p, q) &= 1 + \frac{q}{1 \cdot p} + \frac{q^2}{1 \cdot 2 \cdot p \cdot p + 1} + \dots \\ \chi(p, q, r) &= 1 - \frac{p}{1} \frac{q}{r} + \frac{p \cdot p + 1}{1 \cdot 2} \frac{q \cdot q + 1}{r^2} - \dots \end{aligned} \right\} \begin{array}{l} \text{Notations employées par} \\ \text{Kummer, Cr. 17. 228.} \end{array}$$

$$\left. \begin{aligned} B'(x) &= \frac{x^{2a+2}}{2a+2} - \frac{1}{2} x^{2a+1} + \frac{1}{2} \binom{2a+1}{1} B_1 x^{2a} - \frac{1}{4} \binom{2a+1}{3} B_3 x^{2a-2} + \dots \\ &\quad + \frac{(-1)^{a-1}}{2a} \binom{2a+1}{2a-1} B_{2a-1} x \\ B''(x) &= \frac{x^{2a+1}}{2a+1} - \frac{1}{2} x^{2a} + \frac{1}{2} \binom{2a}{1} B_1 x^{2a-1} - \frac{1}{4} \binom{2a}{3} B_3 x^{2a-3} + \dots \\ &\quad + \frac{(-1)^{a-1}}{2a} \binom{2a}{2a-1} B_{2a-1} x \end{aligned} \right\} \begin{array}{l} \text{Notations em-} \\ \text{ployées par Raabe} \\ \text{Cr. 42. 348 et} \\ \text{comprises parmi} \\ \text{les "autres fonc-} \\ \text{tions."} \end{array}$$

$$1. (a) = \int_a^0 dx l \cos. x = al 2 - \frac{1}{2} \sum_1^{\infty} (-1)^n \frac{\sin. 2na}{n^2}. \text{Lobatschewsky, Mém. Kasan. 1836. 1.}$$

$$Y(p, q) = \int_0^q \frac{E(p, \varphi)}{1 - p^2 \sin.^2 \varphi} d\varphi.$$

ABBREVIATIONS DANS LE SOMMAIRE DES TABLES.

F.	Fonction.
Alg.	Algébrique.
Exp.	Exponentielle.
Log.	Logarithme.
Circ. Dir.	Circulaire Directe.
Circ. Inv.	Circulaire Inverse.
rat.	rationnelle.
irrat.	irrationnelle.
ent.	entière.
fract.	fractionnaire.
mon.	monôme.
bin.	binôme.
trin.	trinôme.
polyn.	polynôme.
num.	numérateur.
dén.	dénominateur.
fact.	facteur.
prod.	produit.
puiss.	puissance.
comp.	composée.
arg.	argument.

PARTIE PREMIÈRE.

TABLES

D'INTÉGRALES DÉFINIES

PARTIE PREMIÈRE.

F. Alg. rat. ent.	TABLE 1.	Lim. 0 et 1.
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- 1) $\int x^{p-1} dx = \frac{1}{p}$ Cauchy, Cours Lec. 32. — Plana, Cr. 17. 1. Il observe que Cavalleri a trouvé 2) et Wallis 3).
- 2) $\int x^a dx = \frac{1}{a+1}$
- 3) $\int x^b dx = \frac{b}{a+b}$
- 4) $\int (1-x^2)^a dx = \frac{2^{2a}}{2a+1} \frac{(1a/1)^2}{1^{2a/1}}$ Plana, Cr. 17. 1.
- 5) $\int (1-x) x^{p-1} dx = \frac{1}{p(1+p)}$ Cisa de Grésy, Mém. Turin 1821. 209. I. N°. 5.
- 6) $\int (1-x)^{a-1} x^{a-1} dx = 2 \frac{1^{a-1/1}}{(a+1)^{a/1}}$ Euler, Calc. Int. 4. S. 3. 13.
- 7) $\int (1-x)^p x^{1-p} dx = p \frac{1-p}{2} \frac{\pi}{\sin p \pi} = \int (1-x)^{1-p} x^p dx$ Oettinger, Cr. 35. 162.
- 8) $\int (1-x)^{p-1} x^{q-1} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$ Poisson, P. 19. 404. N°. 72. — Jacobi, Cr. 11. 307. — Plana, Cr. 17. 1. — Grunert, Gr. 2. 266.
- 9) $= \frac{1^{p+1}}{q^{p+1}} \cdot \frac{1}{p}$ pour p et q entiers; Legendre, Exerc. 3. 34. — Schlomilch, Gr. 4. 23. — Cisa de Grésy, Mém. Turin 1821. 209. I. N°. 2.
- 10) $= \frac{1^{p-1/1} 1^{q-1/1}}{1^{p+q-1/1}}$ pour tout p et q ; Oettinger, Cr. 35. 13. — Lobatschewsky, Mém. Kasan 1835. 211.

$$11) \int (1-x)^{p-1} x^{q-1} dx = \frac{p+q}{pq} \cdot \frac{1 \cdot p+q+1}{p+1 \cdot q+1} \cdot \frac{2 \cdot p+q+2}{p+2 \cdot q+2} \dots \text{Cisa de Grésy, Mém. Turin 1821. 209. 1. N°. 4.}$$

C'est l'intégrale Eulérienne de première espèce $B(p, q)$ ou $\left[\frac{p}{q} \right]$. Binet en traite P. 27. 123. — Lejeune Dirichlet, Cr. 15. 258. — Schaar, Mém. Cour. Brux. T. 22.

$$12) \int (1-x)^{q-p-1} x^{p+a-1} dx = \frac{p^{a/1} \Gamma(p) \Gamma(q-p)}{q^{a/1} \Gamma(q)} \text{ Schlömilch, Stud. I. 24.}$$

$$13) \int (1-x)^{c \pm b-1} x^{a \pm b-1} dx = \frac{1^{c \pm b-1/1} 1^{a \pm b-1/1}}{1^{a+c \pm 2b-1/1}} = \int (1-x)^{a \pm b-1} x^{c \pm b-1} dx \quad \left. \begin{array}{l} \text{Oettinger, Cr.} \\ 35. 13. \end{array} \right\}$$

$$14) \int (1-x)^{b-a-1} x^{a-1} dx = \frac{1^{a-1/1} 1^{b-a-1/1}}{1^{b-1/1}} = \int (1-x)^{a-1} x^{b-a-1} dx$$

$$15) \int (1-x)^{b-p} x^{p+c} dx = \frac{(1+p)^{c/1} (1-p)^{b/1}}{1^{b+c/1}} \frac{p \pi}{\text{Sin. } p \pi} = \int (1-x)^{p+c} x^{b-p} dx$$

$$16) \int (1-x)^{b-p} x^{p-c} dx = \frac{(1-p)^{b/1}}{p^{c/1} 1^{b-c-1/1}} \frac{p \pi}{\text{Sin. } p \pi} = \int (1-x)^{p-c} x^{b-p} dx$$

Oettinger, Cr. 38 162.

$$17) \int (1-x^2)^q x^{2a-1} dx = \frac{1^{a/1} 1^{q/1}}{2^{a/1} 1^{a+q/1}}$$

$$18) \int (1-x^2)^q x^{2a} dx = \frac{2^{q/2}}{(2a+1)^{q+1/2}}$$

$$19) \int (1-x^2)^b x^{2a+1} dx = \frac{2^{a/2} 2^{b/2}}{4^{a+b/2}} \text{ Olm, Ausw 49.}$$

$$20) \int (1-x^b)^p x^{q-1} dx = b^p \frac{1^{p/1}}{q^{p+1/b}} \text{ pour } p \text{ et } q \text{ entiers; Euler, Calc. Int. 4. S. 3. 1. — Id., N. C. Petr. 16. 91. — Kramp, Réfr. 3. 76. — Plana, Cr. 17. 1. — Oettinger, Cr. 35. 13.}$$

$$21) = \frac{\Gamma(\frac{b}{q}) \Gamma(p+1)}{q \Gamma(\frac{b}{q} + p + 1)} = \frac{p}{q + b p} \frac{\Gamma(\frac{b}{q}) \Gamma(p)}{q \Gamma(\frac{b}{q} + p)} \text{ pour } q \text{ aussi des fractions; Plana, Cr. 17. 1.}$$

$$22) = \frac{1^{p/1}}{b (\frac{q}{b})^{p+1/1}} = \frac{1^{\frac{q}{b}/1} 1^{p/1}}{q \cdot 1^{\frac{q}{b} + p/1}} = \frac{1^{\frac{q}{b}/1}}{q (p+1)^{\frac{q}{b}/1}} \text{ pour } p \text{ et } q \text{ entiers: Oettinger, Cr. 35. 13.}$$

$$23) \int (1-x^b)^{a-1} x^{a b-1} dx = \frac{2}{b} \cdot \frac{1^{a-1/1}}{(a+1)^{a/1}} \text{ Euler, Calc. Int. 4. S. 3. 14.}$$

$$24) \int (1-x^a)^{b-1} x^{a-1} dx = \frac{1}{a b} \text{ Euler, Calc. Int. 4. S. 3. 17.}$$

$$25) \int (x^a-1) (1-x)^b dx = 1^{b/1} \left\{ \frac{1^{a/1}}{2^{a+b/1}} - \frac{1}{2^{b/1}} \right\} \text{ Lobatschewsky, Mém. Kasan. 1835. 211.}$$

$$\begin{aligned}
26) \int (x^b - x^{b+c})^g x^{a-1} dx &= \frac{1^{\frac{a+b}{c}g/1} \cdot 1^{g/1}}{(a+bg) \cdot 1^{\frac{a+bg+cg}{g}/1}} = {}^{cg}C_{a+bg, a+bg+c, a+bg+2c, \dots, a+(b+c)g} \cdot \frac{1 \cdot 2 \dots g-1}{1} \quad \text{Oettinger, Cr. 35. 13.} \\
27) \int (1-x)^{a-1} (1+qx^b)^c x^{p-1} dx &= 1^{a-1/1} \sum_0^\infty \binom{c}{n} \frac{q^n}{(p+nb)^{a/1}}, q^2 < 1 \\
28) \int (1-x)^{a-1} (1+x^b)^c x^{p-1} dx &= 1^{a-1/1} \sum_0^\infty \binom{c}{n} \frac{1}{(p+nb)^{a/1}} \\
29) \int (1-x)^{a-1} (1-x^b)^c x^{p-1} dx &= 1^{a-1/1} \sum_0^\infty \binom{c}{n} (-1)^n \frac{1}{(p+nb)^{a/1}}, a+c > 0 \\
30) \int \left\{ (1+x)^{p-1} (1-x)^{q-1} + (1+x)^{q-1} (1-x)^{p-1} \right\} dx &= 2^{p+q-1} B(p, q) \quad \text{Binet, P. 27. 123. N°. 3.}
\end{aligned}
\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{Schlömilch, Stud. I. 13.}$$

$$\begin{aligned}
1) \int \frac{dx}{x^p} &= \infty \quad \text{Cauchy, Cours Lec. 32.} \\
2) \int (1-\frac{1}{x^b})^c x^{a-1} dx &= \frac{1}{a} b^c \frac{1^{c/1}}{(b-a)^{c/b}} = \frac{1^{c/1}}{a(1-\frac{a}{b})^{c/1}} \\
3) \int (\frac{1}{x}-1)^p dx &= \frac{p \pi}{\sin. p \pi} \\
4) \int \frac{(1-x)^p}{x^{p+1}} dx &= \frac{-\pi}{\sin. p \pi} \\
5) \int \frac{(1-x)^{p+b}}{x^{p+a}} dx &= \frac{(1+p)^{a/1}}{2^{a-b+1/1} p^{b/1}} \cdot \frac{p \pi}{\sin. p \pi} \\
6) \int \frac{(1-x)^{p+1}}{x^p} dx &= \frac{1+p}{2} \frac{p \pi}{\sin. p \pi}
\end{aligned}
\left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \text{Oettinger, Cr. 35. 13.}$$

$$\begin{aligned}
1) \int \frac{x^{p-1} dx}{1+x} &= \frac{1}{2} Z' \left(\frac{p+1}{2} \right) - \frac{1}{2} Z' \left(\frac{p}{2} \right) \quad \text{Legendre, Exerc. 5. 4.} \\
2) &= \sum_0^\infty \frac{(-1)^{n-1}}{p+n+1} \quad \text{pour } p \text{ entier; } \quad \text{Arndt, Gr. 6. 434.}
\end{aligned}$$

$$3) \int \frac{dx}{p-x} = l \frac{p}{1-p}, \quad p < 1; \quad \text{Cauchy, Exerc. 1827. p. 125.}$$

$$4) \int \frac{1-x^{p-1}}{1-x} dx = A + Z'(p) \quad \begin{array}{l} \text{Legendre, Exerc. 4. 50. — Cauchy, P. 28. 147. I. § 6. — Cisa} \\ \text{de Gréssy, Mém. Turin 1821. 209. I. N°. 28. — Lobatschewsky,} \\ \text{Mém. Kasan 1835, 211. — Schlömilch, Gr. 4. 167. — Id., Gr.} \\ \text{9. 5. — Id., Stud. I 6.} \end{array}$$

$$5) \quad = \sum_{n=0}^{p-2} \frac{1}{n+1} \quad \text{où } p \text{ entier; } \quad \text{Cauchy, Cours Leç. 32. — Dienger, Gr. 8. 451.}$$

$$6) \int \frac{1-x^k}{1-x} dx = A + lk, \quad \text{pour } k = \infty \quad \text{Legendre, Exerc. 5. 12.}$$

$$7) \int \frac{1-x^p}{1-x} x^{q-1} dx = Z'(p+q) - Z'(q) \quad \text{Legendre, Exerc. 4. 50.}$$

$$8) \int \frac{x^q - x^p}{1-x} dx = Z'(1+p) - Z'(1+q) \quad \text{Legendre, Exerc. 4. 50. — Schlömilch, Gr. 4. 167.}$$

$$9) \int \frac{dx}{p+qx} = \frac{1}{q} l \frac{p+q}{p} \quad \text{Meyer, Int. Déf. 95.}$$

$$10) \int \frac{(1-x)^{b-a-1}}{1-px} x^{a-1} dx = \frac{\Gamma(a)\Gamma(b-a)}{\Gamma(b)} \xi(a, b, p), \quad b > a > 0 \quad \text{Schaeffer, Cr. 37. 127.}$$

$$11) \int \frac{1-a^x}{1-ax} (1-x)^p dx = \sum_{i=1}^c \frac{a^{n-1} \cdot 1^{n/1}}{(p+2)^{n-1/1}} \quad \text{Lindmann, Stockh. Handl. 1850.}$$

$$12) \int \frac{dx}{1+x^2} = \frac{1}{2} \pi \quad \text{Ohm, Ausw. 2. — Raabe, Int. 136.}$$

$$13) \int \frac{x^p dx}{1+x^2} = \frac{1}{4} Z' \left(\frac{p+3}{4} \right) - \frac{1}{4} Z' \left(\frac{p+1}{4} \right) \quad \begin{array}{l} \text{Legendre, Exerc. 5. 16. — Lindmann Stockh.} \\ \text{Handl. 1850.} \end{array}$$

$$14) \int \frac{x^p - x^q}{1+x^2} dx = \frac{1}{2} Z' \left(\frac{q+1}{2} \right) - \frac{1}{2} Z' \left(\frac{p+1}{2} \right) \quad \text{Malmsten, Cr. 38. 1.}$$

$$\left. \begin{array}{l} 15) \int \frac{1-x^3}{1-x^4} dx = \frac{\pi}{8} + \frac{3}{8} l 2 \\ 16) \int \frac{1-x}{1-x^4} x^2 dx = -\frac{\pi}{8} + \frac{3}{8} l 2 \end{array} \right\} \quad \text{Poisson, Mém. Inst. 1811, 163.}$$

$$17) \int \frac{x^{b-1} + x^{a-b-1}}{1+x^a} dx = \frac{\pi}{a} \operatorname{Cosec.} \frac{b\pi}{a} \quad \text{Euler, Calc. Int. T. 4, S. 3. 70. — Id., N. C. P. 19. 3.}$$

$$18) \int \frac{x^{a-1} dx}{1+x^b} = \frac{1}{2b} Z' \left(\frac{a+b}{2b} \right) - \frac{1}{2b} Z' \left(\frac{a}{2b} \right) \quad \text{Legendre, Exerc. 5. 4.}$$

$$19) \int \frac{x^{b-1} - x^{a-b-1}}{1-x^a} dx = \frac{\pi}{a} \text{Cot.} \frac{b\pi}{a} \quad \text{Euler, Calc. Int. T. 4, S. 3. 70. — Id., N. C. P. 19. 3. — Legendre, Mém. Inst. 1809. 416. N° 45. — Id., Exerc. 2. 44. — Id., ib. 5. 13.}$$

$$20) \int \frac{x^{a-1} - x^{b-1}}{1-x^b} dx = \frac{1}{b} \left\{ \Lambda - Z' \left(\frac{a}{b} \right) \right\} \quad \text{Raabe, Cr. 25. 160.}$$

$$21) \int \frac{x^{a-1} - x^{ap+b-1}}{1-x^a} dx = \frac{1}{a} \left\{ \Lambda + Z' \left(p + \frac{b}{a} \right) \right\} \quad \text{Schlömilch, Stud. I. 7.}$$

$$22) \int \frac{x^{a-1} dx}{1-x^b} = -\frac{1}{b} \sum_1^b \text{Cos.} \frac{2qn\pi}{b} l 2 \text{Sin.} \frac{n\pi}{b} + \frac{1}{b} \sum_1^b \text{Sin.} \frac{2qn\pi}{b} \left(\frac{\pi}{2} - \frac{n\pi}{b} \right) \quad \left. \begin{array}{l} \text{Lebesgue, L.} \\ 15. 215. \end{array} \right\}$$

$$23) \quad = -\frac{1}{b} \sum_1^b \text{Cos.} \frac{2qn\pi}{b} l \text{Sin.} \frac{n\pi}{b} - \frac{\pi}{b^2} \sum_1^b n \text{Sin.} \frac{2qn\pi}{b}$$

$$24) \int \frac{x^{q-1} + x^{p-1}}{x^{p+q} + 1} dx = \frac{\pi}{p+q} \text{Sec.} \frac{q-p}{q+p} \frac{\pi}{2} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{Raabe, Int. 146. — Ohm, Ausw. 14.}$$

$$25) \int \frac{x^{q-1} - x^{p-1}}{x^{p+q} - 1} dx = \frac{\pi}{p+q} \text{Tang.} \frac{q-p}{q+p} \frac{\pi}{2}$$

$$26) \int \frac{e^{p^i} dx}{1 + e^{2p^i} x^a} = -\frac{1}{a} \sum_0^{a-1} \left\{ \text{Cos.} \frac{2n+1}{a} \pi l (\text{Cos.} p - \text{Cos.} \frac{2n+1}{a} \pi) \right\} + \frac{\pi}{2a \text{Sin.} \frac{\pi}{a}} +$$

$$+ \frac{i}{2a} \sum_0^{a-1} \left\{ \text{Sin.} \frac{2n+1}{a} \pi l \frac{1 - \text{Cos.} \left(\frac{2n+1}{a} \pi + p \right)}{1 - \text{Cos.} \left(\frac{2n+1}{a} \pi - p \right)} \right\} \quad \text{Dienger, Cr. 38. 331.}$$

$$27) \int \frac{e^{b p^i} x^{b-1} dx}{1 + e^{2a p^i} x^{2a}} = -\frac{1}{2a} \sum_0^{a-1} \left\{ \text{Cos.} \frac{2n+1}{2a} b \pi l \left(\text{Cos.} p - \text{Cos.} \frac{2n+1}{2a} \pi \right) \right\} + \frac{\pi}{4a} \text{Cosec.} \frac{b\pi}{2a} +$$

$$+ \frac{i}{2a} \sum_0^{a-1} \left\{ \text{Sin.} \frac{2n+1}{2a} b \pi l \frac{1 - \text{Cos.} \left(\frac{2n+1}{2a} \pi + p \right)}{1 - \text{Cos.} \left(\frac{2n+1}{2a} \pi - p \right)} \right\} \quad \text{Dienger, Cr. 38. 331}$$

$$1) \int \frac{x^{a-1} dx}{(1+x)^b} = \left(\frac{1}{2} \right)^a \sum_0^{\infty} \binom{b-a-1}{n} \left(\frac{1}{2} \right)^n \quad \text{Legendre, Exerc. 5. N° 6.}$$

$$2) \int \frac{x^{a-1} dx}{(1+x)^{2a+b}} = \frac{1^{a-1/2} \Gamma \left(\frac{1}{2} \right)}{2^{2a+b} \Gamma \left(a + \frac{1}{2} \right)} \sum_0^{\infty} \binom{b}{2n} \frac{1^{n/2}}{(2a+1)^{n/2}} + \frac{1^{a-1/2}}{a \cdot 2^{2a+b}} \sum_0^{\infty} \binom{b}{2n+1} \frac{1^{n/2}}{(a+1)^{n/2}} \quad \left. \begin{array}{l} \text{Legendre} \\ \text{Exerc. 5.} \\ \text{N° 8.} \end{array} \right\}$$

- 3) $\int \frac{x^{p-1} dx}{(1+x)^{2p}} = \frac{1}{2^{2p}} \frac{\Gamma(p) \Gamma(\frac{1}{2})}{\Gamma(p+\frac{1}{2})}$ Legendre, Exerc. 5. N°. 7.
- 4) $\int \frac{x^{q-1} + x^{p-q-1}}{(1+x)^p} dx = \frac{\Gamma(q) \Gamma(p-q)}{\Gamma(p)}$ Legendre, Exerc. 4. N°. 101.
- 5) $\int \frac{x^{q-1} + x^{p-1}}{(1+x)^{p+q}} dx = B(q, p)$ Binet, P. 27. 123.
- 6) $\int \frac{x^{p-1} dx}{(1-x)^p} = \frac{\pi}{\sin. p \pi}, p^2 < 1$; Cisa de Grézy, Mém. Turin 1821. 209. I. § 7. — Oettinger, Cr. 35. 13.
- 7) $\int \frac{x^p dx}{(1-x)^p} = \frac{p \pi}{\sin. p \pi} \left\{ \begin{array}{l} p^2 < 1 \\ \text{Oettinger, Cr. 35. 13.} \end{array} \right.$
- 8) $\int \frac{x^p dx}{(1-x)^{p+1}} = -\frac{\pi}{\sin. p \pi}$
- 9) $\int \frac{x^{p+1} dx}{(1-x)^p} = \frac{1+p}{2} \frac{p \pi}{\sin. p \pi} \left\{ \begin{array}{l} p^2 < 1 \\ \text{Oettinger, Cr. 38. 162.} \end{array} \right.$
- 10) $\int \frac{x^{p+a} dx}{(1-x)^{p+b}} = \frac{(1+p)^{a/1}}{1^{a-b+1/1} p^{b/1}} \cdot \frac{p \pi}{\sin. p \pi}$
- 11) $\int \frac{x^{a-2} dx}{(1+px)^a} = \frac{(1+p)^a}{a-1}$ Legendre, Exerc. 4. 118.
- 12) $\int \frac{x^{a-1} dx}{(1+px)^b} = \frac{1}{(a-1)(1+p)^b} \xi\left(b, a, \frac{p}{1+p}\right), a > 1, p \geq -\frac{1}{2}$; Schaeffer, Cr. 37. 127.
- 13) $\int \frac{x^{q-1} (1-x)^{p-1}}{(x+a)^{p+q}} dx = \frac{\Gamma(q) \Gamma(p)}{\Gamma(p+q)} \frac{1}{a^q (1+a)^p}$ Abel, Cr. 2. 22.
- 14) $\int \frac{x^{p-1} (1-x)^{q-1} dx}{(r+sx)^{p+q}} = \frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)} \frac{1}{(r+s)^p r^q}$ Schlömilch, Höh. Anal. 85.
- 15) $\int \frac{x^{p-1} (1-x)^{q-1} dx}{(1+ax)^{p+q}} = \frac{1}{(1+a)^p} B(q, p)$
- 16) $\int \frac{x^{p-1} (1-x)^{q-1} dx}{(1+ax)^r} = B(p, q) \left\{ 1 - \binom{r}{1} \frac{p}{p+q} a + \binom{r}{2} \frac{p \cdot p+1}{p+q \cdot p+q+1} a^2 + \dots \right\}$ Bonecompagni, Cr. 25. 74.
- 17) $\int \frac{x^{b-1} (1-x)^{c-b-1}}{(1-qx)^a} dx = \sum_0^{\infty} \binom{a}{n} \frac{b^{n/1}}{c^{n/1}} q^n$ Schlömilch, Stud. I. 24.
- 18) $\int \frac{x^q dx}{(1+x^2)^2} = \frac{1-q}{8} \left\{ Z' \left(\frac{q+3}{4} \right) - Z' \left(\frac{q+1}{4} \right) \right\} + \frac{1}{4}$ Legendre, Exerc. 5. 17.

- 19) $\int \frac{x^{2p-2} dx}{(1-x^2)^p} = \frac{\Gamma(2p-1) \Gamma(1-p)}{2^{2p-1} \Gamma(p)}$ Legendre, Exerc. 4. 117.
- 20) $\int \frac{x^{a-1} dx}{(1-x^b)^c} = \infty$ Oettinger, Cr. 35. 13.
- 21) $\int \left(\frac{d^{2a}}{dp^{2a}} \cdot \frac{1}{x-p} \right) dx = \frac{d^{2a}}{dp^{2a}} \cdot \frac{1-p}{p}$
- 22) $\int \left(\frac{d^{2a-1}}{dp^{2a-1}} \cdot \frac{1}{x-p} \right) dx = \infty$ } Cauchy, P. 19. 511.
- 23) $\int \frac{x^p + x^{-p}}{(1+x^q)^2} x^{q-1} dx = \frac{\pi}{q^2} \frac{p}{e^{\frac{p\pi}{q}} - e^{-\frac{p\pi}{q}}}$ Euler, N. A. Petr. 3. 3.

- 1) $\int \frac{x^{p-1} + x^{-p}}{1+x} dx = \pi \operatorname{Cosec}. p \pi$ Legendre, Exerc. 4. 96
- 2) $\int \frac{x^p - x^{1-p}}{1+x} \frac{dx}{x} = \pi \operatorname{Cot}. p \pi$ Legendre, Exerc. 4. 54.
- 3) $\int \frac{x^{-p} - 1}{1-x} dx = -A - Z'(1-p)$
- 4) $\int \frac{x^{-p} - x^{-q}}{1-x} dx = Z'(1-q) - Z'(1-p)$ } $p < 1$; Legendre, Exerc. 5. 3.
- 5) $\int \frac{x^{-p} - x^p}{1-x} dx = \frac{1}{p} - \pi \operatorname{Cot}. p \pi$ }
- 6) $\int \frac{x^{p-1} - x^{-p}}{1-x} dx = \pi \operatorname{Cot}. p \pi$ Legendre, Exerc. 4. 98. — Serret, L. S. 1.
- 7) $\int \left(\frac{1-x}{x} \right)^p \frac{dx}{1-x} = \pi \operatorname{Cosec}. p \pi, p < 1$; Oettinger, Cr. 35. 13.
- 8) $\int \frac{x^q - x^p}{1-x} \frac{dx}{x} = Z'(p) - Z'(q)$ Legendre, Exerc. 4. 50. — Id., ib. 5. 3. — Stern, Cr. 21. 377. — Schlömilch, Beitr. III. 9.
- 9) $\int \frac{(x^p - x^{-p})(x^q - x^{-q})}{1+x^2} dx = 2\pi \frac{\operatorname{Sin}. \frac{1}{2} p \pi \cdot \operatorname{Sin}. \frac{1}{2} q \pi}{\operatorname{Cos}. p \pi + \operatorname{Cos}. q \pi}, p < 1, q < 1$; V. T. 38. N°. 15.
- 10) $\int \frac{(x^p + x^{-p})(x^q + x^{-q})}{1+x^2} dx = 2\pi \frac{\operatorname{Cos}. \frac{1}{2} p \pi \cdot \operatorname{Cos}. \frac{1}{2} q \pi}{\operatorname{Cos}. p \pi + \operatorname{Cos}. q \pi}, p < 1, q < 1$; V. T. 38. N°. 14.

- 11) $\int \frac{x^p + x^{-p}}{1 + x^2} dx = \frac{\pi}{2} \operatorname{Sec.} \frac{p\pi}{2}$ V. T. 38. N°. 16.
- 12) $\int \frac{x^{p-1} - x^{1-p}}{1 - x^2} dx = \frac{\pi}{2} \operatorname{Cot.} \frac{p\pi}{2}$ Legendre, Exere. 4. 98. — Cauchy, P. 19. 511.
- 13) $\int \frac{x^p - x^{-p}}{1 - x^2} dx = -\frac{\pi}{2} \operatorname{tg.} \frac{p\pi}{2}$ Legendre, Exere. 4. 98.
- 14) $\int \frac{x^p - x^{-p}}{1 - x^2} x dx = -\frac{1}{p} + \frac{\pi}{2} \operatorname{Cot.} \frac{1}{2} p\pi$ V. T. 38. N°. 13.
- 15) $\int \frac{(x^p - x^{-p})(x^q + x^{-q})}{1 - x^2} dx = \frac{-\pi \operatorname{Sin.} p\pi}{\operatorname{Cos.} p\pi + \operatorname{Cos.} q\pi}, p < 1; \text{ V. T. 38. N°. 13.}$
- 16) $\int \frac{x^p + x^{q-p}}{1 + x^q} \frac{dx}{x} = \frac{\pi}{q} \operatorname{Cosec.} \frac{p\pi}{2q}$ Euler, Calc. Int. 4. S. 5. N°. 155.
- 17) $\int \frac{x^{p-q} + x^{p+q}}{1 + x^{2p}} \frac{dx}{x} = \frac{\pi}{2p} \operatorname{Sec.} \frac{q\pi}{2p}$ Euler, Calc. Int. 4. S. 3. N°. 71. — Id., N. C. P. 19. 30.
- 18) $\int \frac{x^{p+q} - x^{p-q}}{1 - x^{2p}} \frac{dx}{x} = -\frac{\pi}{2p} \operatorname{Tang.} \frac{q\pi}{2p}$
- 19) $\int \frac{x^{p(q+r)} + x^{p(q-r)}}{1 + x^{2pq}} \frac{dx}{x} = \frac{\pi}{2pq} \operatorname{Sec.} \frac{r\pi}{2q}$ Euler, Calc. Int. 4. S. 3. N°. 72
- 20) $\int \frac{x^{p(q+r)} - x^{p(q-r)}}{1 - x^{2pq}} \frac{dx}{x} = -\frac{\pi}{2pq} \operatorname{Tang.} \frac{r\pi}{2q}$
- 21) $\int \frac{x^q - x^{-q}}{x^p - x^{-p}} \frac{dx}{x} = \frac{\pi}{2p} \operatorname{Tang.} \frac{q\pi}{2p}$ Euler, N. A. Petr. 3. 3. — Poisson, P. 18. 293. N°. 22. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.
- 22) $\int \frac{x^q + x^{-q}}{x^p + x^{-p}} \frac{dx}{x} = \frac{\pi}{2p} \operatorname{Sec.} \frac{q\pi}{2p}$
- 23) $\int \frac{dx}{x^{1-p} + x^{1+p}} = \frac{\pi}{4p}$ V. T. 38. N°. 8.
- 24) $\int \frac{1}{(x + x^{-1})^{2p}} \frac{dx}{x} = \frac{\{\Gamma(p)\}^2}{4\Gamma(2p)}$ Schlömilch, Gr. 6. 213.
- 25) $\int \frac{x^{q-p} + x^{p-q}}{(x + x^{-1})^{p+q}} \frac{dx}{x} = \frac{1}{2} B(p, q)$ V. T. 39. N°. 16.
- 26) $\int \frac{x^{2p} + x^{-2p}}{(x + x^{-1})^{2q}} \frac{dx}{x} = \frac{\Gamma(q+p)\Gamma(q-p)}{2\Gamma(2q)}$ V. T. 39. N°. 18.
- 27) $\int \frac{x^{ak} - x^{bk}}{x - 1} \frac{dx}{x} = l \frac{a}{b}, \text{ pour } k = \infty; \text{ Euler, N. C. Petr. 20. 59.}$

- 1) $\int \frac{x}{1+x^2} \frac{dx}{1+px} = -\frac{1}{1+p^2} \left\{ l \frac{1+p}{1^2} + \frac{1}{4} p \pi \right\}$ Bertrand, L. S. 110.
- 2) $\int \frac{x^{\eta-1}}{(1-x)^\eta} \frac{dx}{1+px} = \frac{\pi}{(1+p)^\eta} \operatorname{Cosec.} q \pi$ Legendre, Exerc. 4. 118. — Boncompagni, Cr. 25. 74.
- 3) $\int \frac{1-x^a}{(1+x)^{a+1}} \frac{dx}{1-x} = \frac{1}{2^{a-1}} \sum_{n=1}^a \frac{2^n}{n}$ Serret, L. S. 1.
- 4) $\int \frac{x^{\eta-1}}{(1-x)^\eta} \frac{dx}{x+a} = \frac{(1+a)^{\eta-1}}{a^\eta} \pi \operatorname{Cosec.} q \pi$
- 5) $\int \frac{x^{\eta-1}}{(1-x)^{1-r}} \frac{dx}{(x+p)^{\eta+r}} = \frac{\Gamma(q) \Gamma(r)}{\Gamma(q+r)} \frac{1}{p^\eta (1+p)^r}$
- 6) $\int \frac{x^{r-1} dx}{(1-x)^r (1+px)^a} = \frac{\pi}{\operatorname{Sin.} r \pi} \frac{1}{(1+p)^r} \sum_{n=0}^{\infty} (-1)^n \binom{a-1}{n} \binom{r}{n} \left(\frac{p}{1+p} \right)^n$ Legendre, Exerc. 4. 119.
- 7) $\int \frac{x^{r+p-2}}{(1-x)^p} \frac{dx}{(1+qx)^r} = (1+q)^{1-r-p} \frac{\Gamma(r+p-1) \Gamma(1-p)}{\Gamma(r)}$ Legendre, Exerc. 4. 115. où $r+p > 1, p < 1, q+1 > 0$.
- 8) $\int \frac{x^{r-1}}{(1-x)^r} \frac{dx}{(1+px)(1+qx)} = \frac{\pi}{(p-q) \operatorname{Sin.} r \pi} \left\{ \frac{p}{(1+p)^a} - \frac{q}{(1+q)^a} \right\}$ Legendre, Exerc. 4. 120.
- 9) $\int \frac{1}{(1-x)^{1-p} x^p} \frac{dx}{a-bx} = \frac{\pi}{(a-b)^{1-p} a^p \operatorname{Sin.} p \pi}$
- 10) $\int \frac{1}{(1-x)^{1-p} x^p} \frac{dx}{(a-bx)^{c+1}} = \frac{p^{c+1}}{1^{c+1}} \frac{\pi \operatorname{Cosec.} p \pi}{a^p (a-b)^{c-1-p}} \sum_{c=0}^{\infty} \frac{1-p, 2-p, \dots, c-p-n}{c+p-1, c+p-2, \dots, p+n} \binom{c}{n} \left(\frac{a-b}{a} \right)^n$ Legendre, Exerc. 4. 120. où $1 > p > 0$; $a \neq 0$, $a \neq b$; Dienger, Cr. 42. 283.
- 11) $\int \left(\frac{x^{\eta-1}}{1+px} + \frac{x^{-\eta}}{p+x} \right) dx = \frac{\pi}{p^\eta} \operatorname{Cosec.} q \pi$ Legendre, Exerc. 4. 137.
- 12) $\int \left(\frac{x^{rp-1}}{1-x} - \frac{r x^{r^2 p-1}}{1-x^r} \right) dx = l r$ Stern, Cr. 21. 377.
- 13) $\int \left(\frac{x^{p-1}}{1-x} - \frac{q x^{p\eta-1}}{1-x^\eta} \right) dx = l q$ Legendre, Exerc. 4. 56. — Stern, Cr. 21. 377. — Arndt, Gr. 10. 253.
- 14) $\int \left(\frac{b x^{b-1}}{1-x^b} - \frac{x^{ab-1}}{1-x} \right) dx = A + \frac{1}{b} \sum_1^b Z' \left(a + \frac{b-n}{b} \right)$
- 15) $\int \left(\frac{b x^{b-1}}{1-x^b} - \frac{1}{1-x} \right) dx = -\frac{1}{b} Z'(ab) + \frac{1}{b} \sum_1^b Z' \left(a + \frac{b-n}{b} \right)$
- 16) $\int \left(\frac{1}{1-x} - \frac{p x^{p-1}}{1-x^p} \right) dx = l p$ Legendre, Exerc. 5. 12. — Schlömilch, Stud. I. 7. — Arndt, Gr. 10. 253.

$$17) \int \left(\frac{kx^{k-1}}{1-x^k} - \frac{x^k}{1-x} \right) dx = \Lambda, \text{ pour } k = \infty; \text{ Legendre, Exerc. 5. 12.}$$

$$18) \int \left(\frac{e^{pi}}{1 + e^{api} x^a} + \frac{e^{-pi}}{1 + e^{-api} x^a} \right) dx = 2 \sum_0^{\infty} \frac{1}{na+1} \cos. (na+1)p \left\{ \begin{array}{l} a^2 p^2 < \pi^2; \\ \text{Dienger, Cr. 38. 331.} \end{array} \right.$$

$$19) \int \left(\frac{e^{pi}}{1 + e^{api} x^a} - \frac{e^{-pi}}{1 + e^{-api} x^a} \right) dx = 2 \sum_0^{\infty} \frac{(-1)^n}{na+1} \sin. (na+1)p \left\{ \right.$$

$$1) \int \frac{dx}{1-x+x^2} = \frac{2\pi}{3\sqrt{3}} \left\{ \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. § 105.} \end{array} \right.$$

$$2) \int \frac{dx}{1+x+x^2} = \frac{\pi}{3\sqrt{3}}$$

$$3) \int \frac{dx}{1-2x \cos. \lambda + x^2} = \frac{1}{\sin. \lambda} \operatorname{Arctg.} \frac{\sin. \lambda}{1 - \cos. \lambda} \text{ Euler, Calc. Int. 4. S. 5. 46.}$$

$$4) \int \frac{dx}{1+2x \cos. \lambda + x^2} = \frac{\lambda}{2 \sin. \lambda} \text{ Legendre, Exerc. 4. 105.}$$

$$5) \int \frac{\cos. \lambda - x}{1-2x \cos. \lambda + x^2} dx = l(2 \sin. \frac{1}{2} \lambda) \text{ Euler, Calc. Int. 4. S. 5. N. 55.}$$

$$6) \int \frac{1-x^2}{1+2x \cos. \lambda + x^2} dx = \cos. \lambda l \{ 2(1+\cos. \lambda) \} + \frac{1}{2} \lambda \sin. \lambda - 1 \text{ Poisson, Mém. Inst. 1811. 163. N°. 27.}$$

$$7) \int \frac{x^p + x^{-p}}{1+2x \cos. \lambda + x^2} dx = \frac{\pi}{\sin. p \pi} \frac{\sin. p \lambda}{\sin. \lambda}, p < 1; \text{ Legendre, Exerc. 4. 103.}$$

$$8) \int \frac{x^a dx}{1+x+x^2} = \frac{1}{3} \left\{ Z' \left(\frac{a+2}{3} \right) - Z' \left(\frac{a+1}{3} \right) \right\}$$

$$9) \int \frac{x^a dx}{1-x+x^2} = \frac{1}{6} \left\{ Z' \left(\frac{a+5}{6} \right) - Z' \left(\frac{a+2}{6} \right) + Z' \left(\frac{a+4}{6} \right) - Z' \left(\frac{a+1}{6} \right) \right\} \left\{ \begin{array}{l} \text{Legendre, Exerc.} \\ 5. 16. \end{array} \right.$$

$$10) \int \frac{x^a dx}{1 \pm x \sqrt{2} + x^2} = \frac{1}{8} \left\{ Z' \left(\frac{a+7}{8} \right) - Z' \left(\frac{a+1}{8} \right) + Z' \left(\frac{a+5}{8} \right) - Z' \left(\frac{a+3}{8} \right) \right\} \\ \mp \frac{1}{8\sqrt{2}} \left\{ Z' \left(\frac{a+6}{8} \right) - Z' \left(\frac{a+2}{8} \right) \right\}$$

$$11) \int \frac{xdx}{1-2px+x^2} = \frac{1}{2} l \{ 2(1-p) \} + \frac{p}{2 \sin. \{ \operatorname{Arccos}(-p) \}} \operatorname{Arccos}(-p), p < 1; \text{ Cauchy, Sav. Etr. 1827. 599. Suppl. 1.}$$

$$12) \int \frac{x dx}{1-2px+x^2} = \frac{1}{2} l\{2(p-1)\} - \frac{p}{2\sqrt{p^2-1}} l\{p+\sqrt{p^2-1}\}, p > 1; \text{Cauchy, Sav. Etr. 1827. 599. Suppl. 1.}$$

$$13) \int \frac{x^c dx}{1+2x \cos. \frac{a\pi}{b} + x^2} = \frac{1}{\sin. \frac{a\pi}{b}} \sum_0^{c-1} (-1)^{n-1} \sin. \frac{na\pi}{b} \left\{ Z' \left(\frac{b+c+n}{2b} \right) - Z' \left(\frac{c+n}{2b} \right) \right\} \left. \begin{array}{l} \text{pour} \\ a+b \\ \text{impair.} \end{array} \right\}$$

$$14) = \frac{1}{\sin. \frac{a\pi}{b}} \sum_0^{\frac{1}{2}(c-1)} (-1)^{n-1} \sin. \frac{na\pi}{b} \left\{ Z' \left(\frac{b+c-n}{b} \right) - Z' \left(\frac{c+n}{b} \right) \right\} \left. \begin{array}{l} \text{pour} \\ a+b \\ \text{pair.} \end{array} \right\}$$

Ces deux formules chez Malmsten, Cr. 38. 1.

$$15) \int \frac{1-x \cos. \lambda - x^{a+1} \cos. (a+1)\lambda + x^{a+2} \cos. a\lambda}{1-2x \cos. \lambda + x^2} dx = \sum_0^a \frac{\cos. n\lambda}{n+1} \left. \begin{array}{l} \\ \text{Dienger, Gr. 8. 450.} \end{array} \right\}$$

$$16) \int \frac{\sin. \lambda - x^a \sin. (a+1)\lambda + x^{a+1} \sin. a\lambda}{1-2x \cos. \lambda + x^2} dx = \sum_0^a \frac{\sin. n\lambda}{n+1}$$

$$17) \int \frac{\sin. \lambda - q^a x^a \sin. (a+1)\lambda + q^{a+1} x^{a+1} \sin. a\lambda}{1-2qx \cos. \lambda + q^2 x^2} (1-x)^p dx = \frac{\Gamma(p+1)}{q} \sum_1^a \frac{q^n \Gamma(n) \sin. n\lambda}{\Gamma(n+p+1)} \left. \begin{array}{l} \\ \text{Lindmann,} \\ \text{Stockh. Handl.} \\ \text{1850. I.} \end{array} \right\}$$

$$18) \int \frac{\cos. \lambda - qx - q^a x^a \cos. (a+1)\lambda + q^{a+1} x^{a+1} \cos. a\lambda}{1-2qx \cos. \lambda + q^2 x^2} (1-x)^p dx = \frac{\Gamma(p+1)}{q} \sum_1^a \frac{q^n \Gamma(n) \cos. n\lambda}{\Gamma(n+p+1)}$$

$$19) \int \frac{1+x^2}{1-x^2+x^4} dx = \frac{1}{2} \pi \quad \text{Raabe, Cr. 37. 356.}$$

$$20) \int \frac{x^{b-1} + x^{2a-b-1}}{1+2x^a \cos. \lambda + x^{2a}} dx = \frac{\pi \sin. \frac{a-b}{a} \lambda}{a \sin. \lambda \sin. \frac{b\pi}{a}} \quad \text{Euler, Calc. Int. 4. S. 5. N°. 185.}$$

$$21) \int \frac{x^{a-b-1} + x^{a+b-1}}{1+2x^a \cos. \lambda + x^{2a}} dx = \frac{\pi \sin. \frac{b}{a} \lambda}{a \sin. \lambda \sin. \frac{b\pi}{a}} \quad \text{Euler, Calc. Int. 4. S. 5. N°. 191.}$$

$$22) \int \frac{1-x}{1-2x \cos. \lambda + x^2} dx = \frac{1}{\sin. \lambda} \sum_1^\infty \frac{\sin. n\lambda}{n(n+1)} \quad \text{Schlömilch, Gr. 4. 23.}$$

- 1) $\int \frac{x^{1+p} + x^{1-p}}{(1 + 2x \cos. \lambda + x^2)^2} dx = \frac{\pi}{2 \sin. p\pi} \frac{p \sin. \lambda. \cos. p\lambda - \cos. \lambda. \sin. p\lambda}{\sin^3. \lambda}$ Legendre, Exerc. 4. 108.
- 2) $\int \frac{x^a dx}{(1 + x + x^2)^2} = 2 \frac{1-a}{9} Z' \left(\frac{a+2}{3} \right) - \frac{2-a}{9} Z' \left(\frac{a+1}{3} \right) + \frac{a}{9} Z' \frac{a}{3} + \frac{1}{3}$
- 3) $\int \frac{x^a dx}{(1 - x + x^2)^2} = \frac{1-a}{9} \left\{ Z' \left(\frac{a+5}{6} \right) - Z' \left(\frac{a+2}{6} \right) \right\} - \frac{2-a}{18} \left\{ Z' \left(\frac{a+4}{6} \right) - Z' \left(\frac{a+1}{6} \right) \right\} + \frac{a}{18} \left\{ Z' \left(\frac{a+3}{6} \right) - Z' \left(\frac{a}{6} \right) \right\} + \frac{1}{3}$ Legendre, Exerc. 5. 17.
- 4) $\int \frac{x^a dx}{(1 \pm x + 2 + x^2)^2} = \frac{1-a}{8} \left\{ Z' \left(\frac{a+7}{8} \right) - Z' \left(\frac{a+3}{8} \right) \right\} - \frac{1}{8} \left\{ Z' \left(\frac{a+5}{8} \right) - Z' \left(\frac{a+1}{8} \right) \right\} \mp \frac{2-a}{16} \left\{ Z' \left(\frac{a+6}{8} \right) - Z' \left(\frac{a+2}{8} \right) \right\} \mp \frac{a+2}{16} \left\{ Z' \left(\frac{a+4}{8} \right) - Z' \left(\frac{a}{8} \right) \right\} + \frac{1}{2}$
- 5) $\int \frac{x^{q-1} dx}{1 + p x \cos. \lambda + p^2 x^2 (1-x)^q} = \frac{\pi}{p^q \sin. q \pi. \sin. \lambda} \sin. \left\{ \lambda - q \operatorname{Arctg.} \frac{p \sin. \lambda}{1 + p \cos. \lambda} \right\}$ Legendre, Exerc. 4. 120.
- 6) $\int \left(\frac{x^p}{1 + 2 q x \cos. \lambda + q^2 x^2} + \frac{x^{-p}}{x^2 + 2 q x \cos. \lambda + q^2} \right) dx = \frac{\pi \sin. p \lambda}{q^{p+1} \sin. p \pi. \sin. \lambda}$ Legendre, Exerc. 4. 138.
- 7) $\int \left\{ \frac{1 + q x \cos. \lambda}{1 + 2 q x \cos. \lambda + q^2 x^2} x^{p-1} + \frac{x + q \cos. \lambda}{x^2 + 2 q x \cos. \lambda + q^2} \frac{1}{x^p} \right\} dx = \frac{\pi}{q^{p+1} \sin. q \pi} \cos. p \lambda$
- 8) $\int \left(\frac{\sin. \lambda}{1 + 2 x \cos. \lambda + x^2} - \frac{\lambda}{(1+x)^2} \right) dx = 0$ Malmsten, Cr. 38. 1.
- 9) $\int \frac{x^p - 2 \cos. \lambda + x^{-p}}{x^q - 2 \cos. \mu + x^{-q}} \frac{dx}{x} = \frac{\pi \sin. \left(\frac{\pi - \mu}{q} p \right)}{q \sin. \mu. \sin. \frac{p \pi}{q}} - \frac{\pi - \mu}{q \sin. \mu} \cos. \lambda$
- 10) $\int \frac{x^p + x^{-p}}{x^q - 2 \cos. \lambda + x^{-q}} \frac{dx}{x} = \frac{\pi \sin. \left(p \frac{\pi - \lambda}{q} \right)}{q \sin. \lambda. \sin. \frac{p \pi}{q}}$ Euler, N. A. Petr. 3. 3.
- 11) $\int \frac{x^p + x^{-p}}{x^q + \left(a + \frac{1}{a} \right) + x^{-q}} \frac{dx}{x} = \frac{\frac{p}{a q} - a \frac{p}{q}}{a - \frac{1}{a}} \frac{\pi}{q \sin. \frac{p \pi}{q}}$
- 12) $\int \frac{x^q + x^{-q}}{x^p + 2 \cos. \lambda + x^{-p}} \frac{dx}{x} = \frac{\pi \sin. \frac{q \lambda}{p}}{p \sin. \lambda. \sin. \frac{q \pi}{p}}$ Poisson, P. 18. 295. N°. 28.

- 1) $\int x^{a-b-1} (1-x)^{\frac{b-a}{a}} dx = \frac{\pi}{a} \operatorname{Cosec.} \frac{b\pi}{a}$ Euler, N. A. P. I. 2. p. 3.
- 2) $\int x^{a+p+\frac{1}{2}} (1-x)^{b-p-\frac{1}{2}} dx = \frac{(1+2p)^{a^2} (1-2p)^{b^2}}{1^{a+b+1/2}} \frac{\pi \operatorname{Sec.} p\pi}{2^{a+b+1}} = \int x^{b-p-\frac{1}{2}} (1-x)^{a+p+\frac{1}{2}} dx$
- 3) $\int x^{-a+p+\frac{1}{2}} (1-x)^{b-p-\frac{1}{2}} dx = \frac{(1-2p)^{b/2}}{(2p-1)^{a-2}} \frac{2^{a-b-1} \pi}{\operatorname{Cos.} p\pi} = \int x^{b-p-\frac{1}{2}} (1-x)^{-a+p+\frac{1}{2}} dx$
- 4) $\int dx \sqrt{1-x^2} = \frac{1}{2} \pi$ Euler, Calc. Int. I. P. 1. S. 1. 8. 340. — Plana, Cr. 17. 1.
- 5) $\int (1-x^2)^{a-\frac{1}{2}} dx = \frac{(a+1)^{a/2}}{1^{a/2}} \frac{\pi}{2^{2a+1}}$ Laplace, Prob. I. 34.
- 6) $\int x^{2a-1} dx \sqrt{1-x^2} = \frac{2^{a-1/2}}{3^{a/2}}$
- 7) $\int x^{2a} dx \sqrt{1-x^2} = \frac{3^{a-1/2}}{4^{a/2}} \frac{\pi}{4}$
- 8) $\int x^{2a} (1-x^2)^{b-\frac{1}{2}} dx = \frac{1^{a/2} 1^{b/2}}{1^{a+b/2}} \frac{\pi}{2^{a+b+1}}$ Euler, Calc. Int. I. P. I. S. 1. 8. 340. — Oettinger, Cr. 35. 13.
- 9) $= \frac{1^{a/2} 1^{b/2}}{2^{a/2} (2+2a)^{b/2}} \frac{\pi}{2}$ Kramp, Réfr. 3. 79.
- 10) $\int x^{2a} (1-x^2)^{b+\frac{1}{2}} dx = \frac{3^{b/2} 3^{a-1/2}}{3^{a+b/2}} \frac{\pi}{4}$ Oettinger, Cr. 35. 13.
- 11) $\int x^{2a-1} (1-x^2)^{b-\frac{1}{2}} dx = \frac{2^{a-1/2} 1^{b/2}}{1^{a+1/2}}$ Euler, Calc. Int. I. P. 1. S. 1. 8. 340. — Oettinger, Cr. 35. 13.
- 12) $= \frac{2^{a-1/2} 1^{b/2}}{1^{a/2} (1+2a)^{b/2}}$ Kramp, Réfr. 3. 79.
- 13) $\int x^{2a-1} (1-x^2)^{b+\frac{1}{2}} dx = \frac{2^{a-1/2}}{(2b+1)^{a/2}}$ Oettinger, Cr. 35. 13.
- 14) $\int (1-x^2)^{1-\frac{1}{2}a} (1-p^2 x^2)^{1-\frac{1}{2}a} x^a dx = \frac{\Gamma\left(\frac{a+1}{2}\right) \Gamma\left(2-\frac{a}{2}\right)}{1^{\left\{\pi, a-1, a-3, a-5\right\}}} \frac{1}{p^3}$
 $\left\{ \frac{1+(a-3)p+p^2}{(1+p)^{a-3}} - \frac{1-(a-3)p+p^2}{(1-p)^{a-3}} \right\}$ Ramus, Overs. Danske Forh. 1844.

$$1) \int x^{a-1} (1-x^b)^{\frac{c-b}{b}} dx = \frac{b}{ac} \cdot \frac{2b \cdot a+c}{a+b \cdot c+b} \cdot \frac{3b \cdot a+c+b}{a+2b \cdot c+2b} \cdots \quad \text{Euler, Calc. Int. I. P. 1. S. 1. 8. 364.}$$

$$2) \quad = \frac{1^{\frac{a}{b}} 1^{\frac{c-b}{b}}}{a \cdot 1^{\frac{a+c}{b}-1}} \quad \text{Oettinger, Cr. 35. 13.}$$

$$3) \quad = \frac{\Gamma\left(\frac{a}{b}\right) \Gamma\left(\frac{c}{b}\right)}{b \Gamma\left(\frac{a+c}{b}\right)}, \text{ où } b \text{ peut être fractionnaire; } \quad \text{Legendre, Mém. Inst. 1809. 416. N°. 55. — Id., Exerc. 2. 56. — Plana, Cr. 17. 1.}$$

$$4) \int x^{a-1} (1-x^a)^{\frac{b-a}{a}} dx = \frac{1}{b} \quad \text{Euler, N. A. P. I. 2. p. 3. — Id., Calc. Int. 4. S. 3. 129.}$$

$$5) \int (1-x^a)^{\frac{b-a}{a}} x^{a-b-1} dx = \frac{\pi}{a} \operatorname{Cosec.} \frac{b\pi}{a} \quad \text{Euler, Calc. Int. 4. S. 3. 131.}$$

$$6) \int x^{a+b-1} (1-x^b)^c dx = \frac{1^{\frac{a}{b}+1/1} 1^{\frac{c}{d/1}}}{(a+b) 1^{\frac{a}{b}+\frac{c}{d}+1/1}}$$

$$7) \int x^{a+bd-1} (1-x^b)^{\pm \frac{h}{g}+c} dx = \frac{1}{a+bd} \frac{(\pm h+g)^{c/g} (a+b)^{d/b} 1^{\pm \frac{h}{g}/1} 1^{\frac{a}{b}}}{(ag \pm bh + bg)^{c+d/bg} 1^{\frac{a}{b} \pm \frac{h}{g}/1}} bc g^d$$

$$8) \int x^{a-bd-1} (1-x^b)^{\frac{h}{g}-c} dx = \frac{\left(\frac{a}{b} + \frac{h}{g}\right)^{c+dl-1}}{\left(\frac{h}{g}\right)^{c/g-1} \left(\frac{a}{b}\right)^{d/l-1}} \frac{1}{a-bd} \frac{1^{\frac{a}{b}} 1^{\frac{h}{g}/1}}{1^{\frac{a}{b}+\frac{h}{g}/1}}$$

$$9) \int x^{\pm a+bd-1} (1-x^b)^{\frac{h}{g}-c} dx = \frac{1}{a+bd} \frac{\left(1 \pm \frac{a}{b}\right)^{d/1}}{\left(\frac{h}{g}\right)^{c/g-1} \left(1 \pm \frac{a}{b} + \frac{h}{g}\right)^{d-c/1}} \frac{1^{\pm \frac{a}{b}/1} 1^{\frac{h}{g}/1}}{1^{\pm \frac{a}{b}+\frac{h}{g}/1}}$$

$$10) \int x^{a-bd-1} (1-x^b)^{\pm \frac{h}{g}+c} dx = \frac{1}{a-bd} \frac{\left(1 \pm \frac{h}{g}\right)^{c/1}}{\left(\frac{a}{b}\right)^{d-1} \left(1 + \frac{a}{b} \pm \frac{h}{g}\right)^{c-d/1}} \frac{1^{\frac{a}{b}/1} 1^{\pm \frac{h}{g}/1}}{1^{\frac{a}{b} \pm \frac{h}{g}/1}}$$

$$11) \int x^{-a+bd-1} (1-x^b)^{\frac{h}{g}+c} dx = \frac{1}{b d - a} \frac{(h+g)^{c/g} (b-a)^{d/b}}{(bh+bg-a)^{c+d/bg}} \cdot \frac{1^{h/g} 1^{\frac{a}{b}}}{1^{h/g} 1^{\frac{a}{b}}} bc g^d$$

$$12) \int x^{a-1} (1-x^b)^{\pm \frac{h}{g}+c} dx = \frac{(g \pm h)^{c/g}}{(ag + bg + bh)^{c/bg}} \frac{1^{\frac{a}{b}/1} 1^{\pm \frac{h}{g}/1}}{1^{\frac{a}{b} \pm \frac{h}{g}/1}} \frac{bc}{a}$$

$$13) \int x^{a-1} (1-x^b)^{\frac{h}{g}-c} dx = \frac{(ag+bh)^{c-bg}}{h^{c/g}} \frac{1^{\frac{a}{b}/1} 1^{\frac{h}{g}/1}}{1^{\frac{a}{b}+\frac{h}{g}/1}} \frac{1}{ab^c}$$

$$14) \int x^{a-1} (1-x^b)^{-\frac{a}{b}+c} dx = \frac{(b-a)^{c/b}}{b^{c/b}} \frac{1^{\frac{a}{b}/1}}{1^{\frac{a}{b}/1}} \frac{1}{a}$$

$$15) \int x^{a+bc-1} (1-x^b)^{\frac{h}{g}} dx = \frac{(a+b)^{c/b}}{(ag+bh+bg)^{c/bg}} \frac{1}{a+b} \frac{1^{\frac{a}{b}/1} 1^{\frac{h}{g}/1}}{1^{\frac{a}{b}+\frac{h}{g}/1}} g^c$$

$$16) \int x^{a-bc-1} (1-x^b)^{\frac{h}{g}} dx = \frac{(ag+bh)^{c/-bg}}{a^{c/-b}} \frac{1}{a-b} \frac{1^{\frac{a}{b}/1} 1^{\frac{h}{g}/1}}{1^{\frac{a}{b}+\frac{h}{g}/1}} \frac{1}{g^c}$$

$$17) \int x^{ab-1} (1-x^b)^{\frac{h}{g}} dx = \frac{g^{a/g}}{ab(h+g)^{a/g}}$$

$$18) \int x^{ab} (1-x^b)^{\frac{h}{g}} dx = \frac{(b+1)^{a/b}}{(g+bh+bg)^{a/bg}} \frac{1^{\frac{h}{g}/1} 1^{\frac{1}{a}/1}}{1^{\frac{h}{g}+\frac{1}{a}/1}} \frac{g^a}{1+ab}$$

$$19) \int x^{ab-1} (1-x^b)^{\pm \frac{h}{g}+c} dx = \frac{(g+h)^{c/g} g^{a/g}}{ab(h+g)^{c+a/g}}$$

$$20) \int x^{ab} (1-x^b)^{\pm \frac{h}{g}+c} dx = \frac{(g+h)^{c/g} (1+b)^{a/b}}{(g+bh+bg)^{a+clbg}} \frac{1^{\frac{h}{g}/1} 1^{\frac{1}{a}/1}}{1^{\frac{h}{g}+\frac{1}{a}/1}} \frac{bc g^a}{1+ab}$$

$$21) \int x^{ab-1} (1-x^b)^{\frac{h}{g}-c} dx = \frac{g^{a/g}}{ab \cdot h^{c/-g} (h+g)^{a-c/g}}$$

$$22) \int x^{ab} (1-x^b)^{\frac{h}{g}-c} dx = \frac{(1+b)^{a/b}}{h^{c/-g} (g+bh+bg)^{a-clbg}} \frac{1^{\frac{h}{g}/1} 1^{\frac{1}{a}/1}}{1^{\frac{h}{g}+\frac{1}{a}/1}} \frac{g^a}{1+ab}$$

Les Intégrales 6 à 22 se trouvent toutes: Oettinger, Cr. 35. 13.

$$23) \int x^{b-a-1} (1-x^b)^{\frac{a}{b}-1} dx = \frac{\pi}{b} \operatorname{Cosec.} \frac{a\pi}{b} \quad \text{Euler, Calc. Int. T. I. S. 1.8. 352. — Oettinger, Cr. 35. 13.}$$

$$24) \int x^{a+bc-1} (1-x^b)^{-\frac{a}{b}+g} dx = \frac{(b-a)^{g/b} (b+a)^{c/b}}{b^{c+g/b}} \frac{a}{a+b} \frac{\pi}{b \operatorname{Sin.} \frac{a\pi}{b}}$$

$$25) \int x^{a+bc-1} (1-x^b)^{-\frac{a}{b}+c} dx = \left(1 - \frac{a^2}{b^2}\right) \left(1 - \frac{a^2}{b^2}\right) \dots \left(c^2 - \frac{a^2}{b^2}\right) \frac{a}{1^{2c/1}} \frac{1}{a+b} \frac{\pi}{b \operatorname{Sin.} \frac{a\pi}{b}}$$

$$26) \int x^{a-bc-1} (1-x^b)^{-\frac{a}{b}+g} dx = \frac{(b-a)g/b}{(b-a)c/b} \frac{b^{c-g}}{1g^{-c/1}} \frac{\pi}{b \operatorname{Sin.} \frac{a\pi}{b}}$$

$$27) \int x^{a-bc-1} (1-x^b)^{-\frac{a}{b}+c} dx = (-1)^c \frac{\pi}{b} \operatorname{Cosec.} \frac{a\pi}{b}$$

Des formules 24—27 voyez Oettinger, Cr. 38. 162.

$$1) \int \frac{(1-x)^{p+\frac{1}{2}}}{x^{p+\frac{1}{2}}} dx = \frac{2p+1}{2} \pi \operatorname{Sec.} p\pi$$

$$2) \int \frac{(1-x)^{p-\frac{1}{2}}}{x^{p+\frac{1}{2}}} dx = \pi \operatorname{Sec.} p\pi$$

$$3) \int \frac{(1-x)^{p-\frac{1}{2}}}{x^{p-\frac{1}{2}}} dx = \frac{1-2p}{2} \pi \operatorname{Sec.} p\pi$$

$$4) \int \frac{(1-x)^{p+a+\frac{1}{2}}}{x^{p+b+\frac{1}{2}}} dx = \frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2} 1^{a-b+1/1}} (-1)^b 2^{b-a} \pi \operatorname{Sec.} p\pi$$

$$5) \int \frac{(1-x^b)^{\frac{h}{g}-c}}{x^{a+bd+1}} dx = \frac{-1}{a+bd} \frac{\left(-\frac{a}{b} + \frac{h}{g}\right)^{c+d/-1}}{\left(\frac{h}{g}\right)^{c/-1} \left(\frac{a}{-b}\right)^{d/-1}} \cdot \frac{1^{-a/1} 1^{\frac{h}{g}1}}{1g^{-a/1} 1^{\frac{h}{g}1}}$$

$$6) \int \frac{(1-x^b)^{\frac{h}{g}+c}}{x^{a+bd+1}} dx = \frac{-1}{a+bd} \frac{\left(1 \pm \frac{h}{g}\right)^{c/1}}{\left(-\frac{a}{b}\right)^{d/-1} \left(1 - \frac{a}{b} \pm \frac{h}{g}\right)^{c-d/1}} \frac{1^{-a/1} 1^{\pm \frac{h}{g}1}}{1^{\pm \frac{h}{g}1} 1^{\frac{a}{b}1}}$$

$$7) \int \frac{(1-x^b)^{\frac{a}{b}-c}}{x^{a+bg+1}} dx = 0$$

Oettinger, Cr. 38. 162.

Oettinger, Cr. 35.
13.

$$1) \int \frac{x^{a-\frac{1}{2}} dx}{\sqrt{1-x}} = \frac{1^{a/2}}{2^{a/2}} \pi \quad \text{Schlömlich, Stud. I. § 2.}$$

$$2) \int \frac{x^a dx}{\sqrt{1-x}} = \frac{2^{a/2}}{3^{a/2}} 2 \quad \text{Schlömlich, Gr. 5. 90.}$$

$$3) \int \frac{x_b^a dx}{(1-x)_b^a} = \frac{a}{b} \pi \operatorname{Cosec.} \frac{a\pi}{b} \quad \text{Euler, N. C. P. 6. 115.}$$

- 4) $\int \frac{x^{p-\frac{1}{2}} dx}{(1-x)^p} = \frac{2^{1-2p}}{1-2p} \frac{\Gamma(p+\frac{1}{2}) \Gamma(1-p)}{\sqrt{\pi}}, p < \frac{1}{2};$ Legendre, Exerc. 4. 126.
- 5) $\int \frac{x^{a+p+\frac{1}{2}} dx}{(1-x)^{p+b+\frac{1}{2}}} = \frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2} 1^{a-b+1/1}} (-1)^b 2^{b-a} \pi \text{ Sec. } p \pi$
- 6) $\int \frac{x^{p+\frac{1}{2}} dx}{(1-x)^{p+\frac{1}{2}}} = \frac{2p+1}{2} \pi \text{ Sec. } p \pi$
- 7) $\int \frac{x^{p-\frac{1}{2}} dx}{(1-x)^{p+\frac{1}{2}}} = \pi \text{ Sec. } p \pi$
- 8) $\int \frac{x^{p-\frac{1}{2}} dx}{(1-x)^{p-\frac{1}{2}}} = \frac{1-2p}{2} \pi \text{ Sec. } p \pi$
- 9) $\int dx \sqrt{\frac{1-x^2}{1+x^2}} = \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2}\pi} - \frac{\pi\sqrt{2}\pi}{\{\Gamma(\frac{1}{4})\}^2}$ Catalan, L. 4. 323.
- 10) $\int \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2}\pi$ Euler, Calc. Int. T. 1. P. 1. S. 1. S. § 330, 356. — Oettinger, Cr. 35. 13.
- 11) $\int \frac{x dx}{\sqrt{(1-x^2)}} = 1$ Euler, Calc. Int. T. 1. P. 1. S. 1. S. 330.
- 12) $\int \frac{x^{2a} dx}{\sqrt{(1-x^2)}} = \frac{3^{a-1/2}}{2^{a,2}} \frac{\pi}{2}$ Euler, Calc. Int. T. 1. P. 1. S. 1. S. 330. — Oettinger, Cr. 35. 13. — Schlömilch, Stud. I. 2.
- 13) $\int \frac{x^{2a-1} dx}{\sqrt{(1-x^2)}} = \frac{2^{a-1/2}}{1^{a/2}}$ Euler, Calc. Int. T. 1. P. 1. S. 1. S. 330. — Plana, Cr. 17. 1. — Oettinger, Cr. 35. 13. — Dienger, Cr. 38. 266. — Schlömilch, Stud. I. 2.
- 14) $\int dx \sqrt{\frac{1-p^2 x^2}{1-x^2}} = 1 - \sum_1 \left\{ \frac{1^{n-1/2}}{2^{n/2}} \right\}^2 (2n-1) p^{2n}$ Ohm, Ausw. 26.
- 15) $\int \frac{dx \sqrt{x}}{\sqrt{(1-x^2)}} = \frac{1-\sqrt{3}}{\sqrt{3}} F'\left(\cos. \frac{\pi}{12}\right) + \frac{2\sqrt{3}}{\sqrt{3}} E'\left(\cos. \frac{\pi}{12}\right)$ Legendre, Exerc. 1. 39.
- 16) $\int \frac{dx \sqrt{x^2}}{\sqrt{(1-x^2)}} = \frac{3\sqrt{3}}{\sqrt{3}} E'\left(\sin. \frac{\pi}{12}\right) - \frac{3+\sqrt{3}}{2\sqrt{3}} F'\left(\sin. \frac{\pi}{12}\right)$ Legendre, Exerc. 1. 40.
- 17) $\int \frac{x^{2p} dx}{(1-x^2)^{p+\frac{1}{2}}} = \frac{\pi}{2} \text{ Sec. } p \pi$ Euler, N. C. P. 6. 115.
- 18) $\int \frac{x^{p-\frac{1}{2}} dx}{(1-x^2)^p} = \frac{2^{1-p}}{2p-1} \frac{\Gamma(p+\frac{1}{2}) \Gamma(1-p)}{\sqrt{\pi}} \text{ Sin. } \left(\frac{2p-1}{4} \pi\right), p < 1;$ Legendre, Exerc. 4. 124.
- 19) $\int \frac{x^{2a-1} dx}{(1-x^2)^{b-\frac{1}{2}}} = (-1)^{b-1} \frac{2^{a/2}}{2^a \cdot 1^{b-1/2} 3^{a-b/2}}$ Oettinger, Cr. 35. 13.

$$20) \int \frac{x^{2a-1} dx}{(1-x^2)^{b-\frac{1}{2}}} = (-1)^{b-1} \frac{2^{a-1/2} (2a-2b+1)^{b/2}}{1^{a/2} 1^{b/2}} \quad \text{Kramp, Refr. 3. 81.}$$

$$21) \int \frac{x^{2a} dx}{(1-x^2)^{b-\frac{1}{2}}} = (-1)^{b-1} \frac{3^{a-1/2}}{1^{b-1/2} 4^{a-b/2}} \frac{\pi}{4} \quad \text{Oettinger, Cr. 35. 13.}$$

$$22) \quad = (-1)^{b-1} \frac{1^{a/2} (2a-2b+2)^{b/2}}{2^{a/2} 1^{b/2}} \frac{\pi}{2} \quad \text{Kramp, Refr. 3. 81.}$$

$$\left. \begin{aligned} 23) \int \frac{x^{p-1} + x^{q-1}}{(1-x^2)^{\frac{p+q}{2}}} dx &= \frac{\text{Cos.} \left(\frac{q-p}{4} \pi \right)}{2 \text{Cos.} \left(\frac{q+p}{4} \pi \right)} \text{B} \left(\frac{p}{2}, \frac{q}{2} \right) \\ 24) \int \frac{x^{p-1} - x^{q-1}}{(1-x^2)^{\frac{p+q}{2}}} dx &= \frac{\text{Sin.} \left(\frac{q-p}{4} \pi \right)}{2 \text{Sin.} \left(\frac{q+p}{4} \pi \right)} \text{B} \left(\frac{p}{2}, \frac{q}{2} \right) \end{aligned} \right\} \quad \text{Serret, L. 8. 1.}$$

$$25) \int \frac{dx}{\sqrt{1+px^2}} = \frac{1}{\sqrt{p}} \ln \{ \sqrt{p} + \sqrt{1+p} \} \quad \text{Schlömlich, Beitr. III. 7.}$$

$$1) \int \frac{dx}{\sqrt{1-x^3}} = \frac{2}{\sqrt[3]{27}} \text{F}' \left(\text{Sin.} \frac{\pi}{12} \right) \quad \text{Legendre, Exerc. 1. 145.}$$

$$2) \int \frac{dx}{\sqrt[3]{1-x^3}} = \frac{4}{3 \sqrt[3]{4} \sqrt[3]{3}} \text{F}' \left(\text{Cos.} \frac{\pi}{12} \right) \quad \text{Legendre, Exerc. 1. 41.}$$

$$3) \int \frac{x dx}{\sqrt[3]{1-x^3}^2} = \frac{\sqrt[3]{4}}{2 \sqrt[3]{3}} \frac{\pi}{\text{F}' \left(\text{Cos.} \frac{\pi}{12} \right)} \quad \text{Legendre, Exerc. 1. 118.}$$

$$\left. \begin{aligned} 4) \int \frac{x^{3a} dx}{\sqrt[3]{1-x^3}} &= \frac{1^{a/3} 2 \pi}{3^{a/3} 3 \sqrt[3]{3}} \\ 5) \int \frac{x^{3a-1} dx}{\sqrt[3]{1-x^3}} &= \frac{3^{a-1/3}}{2^{a/3}} \end{aligned} \right\} \quad \text{Kausler, Mém. Pétersb. 1811. T. 3.}$$

$$6) \int \frac{dx}{\sqrt{1-x^4}} = \frac{1}{2} \sqrt[4]{2} \cdot \text{F}' \left(\text{Sin.} \frac{\pi}{4} \right) \quad \text{Legendre, Exerc. 1. 146.}$$

$$7) \int \frac{x^3 dx}{\sqrt{1-x^4}} = \frac{1}{2} \quad \text{Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 337.}$$

$$8) \int dx \sqrt{\frac{1-p^2x^4}{1-x^4}} = \frac{cF'(c)+bF'(b)}{(b+c)^2} + \frac{b-c}{(b+c)^2} \{E'(b)-E'(c)\}, \text{ où } b^2 = \frac{(1+\sqrt{p})^2}{2(1+p)}, c^2 = \frac{(1-\sqrt{p})^2}{2(1+p)}; \text{ Legendre, Exerc. 6. 308.}$$

$$9) \int \frac{dx}{\sqrt{1-x^6}} = \frac{1}{\sqrt[3]{27}} F' \left(\text{Sin.} \frac{5\pi}{12} \right) \\ 9') = \frac{\sqrt[3]{3}}{\sqrt[3]{27}} E' \left(\text{Sin.} \frac{\pi}{12} \right) \quad \left. \vphantom{\int \frac{dx}{\sqrt{1-x^6}}} \right\} \text{Legendre, Exerc. 1. 147.}$$

$$10) \int \frac{dx}{\sqrt{1-x^8}} = \frac{1}{\sqrt[4]{2}} F' \left(\text{Tg.} \frac{\pi}{8} \right) \quad \text{Legendre, Exerc. 1. 148.}$$

$$11) \int \frac{dx}{\sqrt{1-x^{12}}} = \frac{1}{2\sqrt[3]{3}} F' \left(\text{Sin.} \frac{\pi}{4} \right) + \text{Sin.} \frac{\pi}{12} F' \left(\frac{\sqrt[3]{2}-\sqrt[3]{3}}{1+\sqrt[3]{3}} \right) \quad \text{Legendre, Exerc. 1. 150.}$$

$$1) \int \frac{x^{a-b-1} dx}{(1-x^a)^{\frac{a-b}{a}}} = \frac{\pi}{a} \text{Cosec.} \frac{b\pi}{a} \quad \text{Euler, Calc. Int. T. 1. P. 1. S. 1. S. 352.}$$

$$2) \int \frac{x^{b-1} dx}{(1-x^a)^{\frac{b}{a}}} = \frac{\pi}{a} \text{Cosec.} \frac{b\pi}{a} \quad \text{Euler, Calc. Int. T. 1. P. 1. S. 1. S. 352, 366. — Id., N. C. P. 6. 115. — Oettinger, Cr. 35. 13.}$$

$$3) \int \frac{x^{b-1} dx}{(1-x^a)^{\frac{a+b}{2a}}} = \frac{\pi}{a} \text{Sec.} \frac{b\pi}{2a} \quad \text{Euler, Calc. Int. T. 1. P. 1. S. 1. S. 366.}$$

$$4) \int \frac{dx}{\sqrt[q]{1-x^q}} = \frac{\pi}{q} \text{Cosec.} \frac{\pi}{q} \quad \text{Ohm, Ausw. 14. — Raabe, J. 147.}$$

$$5) \int \frac{x^{p-1} dx}{\sqrt[q]{1-x^q}} = \frac{\pi}{q} \text{Cosec.} \frac{p\pi}{q}, p > q-1; \quad \text{Minding, Samml. 115.}$$

$$6) \int \frac{x^{p-1} dx}{(1-x^a)^{\frac{q-a}{a}}} = \frac{\Gamma \left(\frac{q}{a} \right) \Gamma \left(\frac{p}{a} \right)}{a \Gamma \left(\frac{p+q}{a} \right)} = \int \frac{x^{q-1} dx}{(1-x^a)^{\frac{p-a}{a}}} \quad \text{Raabe, J. 217.}$$

$$7) \int \frac{x^{a-bd-1} dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{\left(\frac{a}{b} - \frac{h}{g} \right)^{c+d/-1}}{\left(-\frac{h}{g} \right)^{c/-1} \left(\frac{a}{b} \right)^{d/-1}} \frac{1}{a-bd} \frac{1^{\frac{a}{b}/1} 1^{-\frac{h}{g}/1}}{1^{\frac{a}{b}-\frac{h}{g}/1}} \\ 8) \int \frac{x^{a-bd-1} dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{1}{a+bd} \frac{\left(1 + \frac{a}{b} \right)^{d/1}}{\left(-\frac{h}{g} \right)^{c/-1} \left(1 + \frac{a}{b} - \frac{h}{g} \right)^{d-c/1}} \cdot \frac{1^{\frac{a}{b}/1} 1^{-\frac{h}{g}/1}}{1^{\frac{a}{b}-\frac{h}{g}/1}} \quad \left. \vphantom{\int \frac{x^{a-bd-1} dx}{(1-x^b)^{\frac{h}{g}+c}}} \right\} \text{Oettinger, Cr. 35. 13.}$$

- $$9) \int \frac{x^{a-1} dx}{(1-x^b)^{\frac{a}{b}}} = \frac{1}{a} \frac{b^2}{b^2-a^2} \cdot \frac{4b^2}{4b^2-a^2} \cdots$$
- $$10) \int \frac{x^{a-1} dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{(bh-ag)^{c/bg}}{ab^c h^{c/g}} \frac{1^{\frac{a}{b}} 1^{-\frac{h}{g}}}{1^{\frac{a}{b}-\frac{h}{g}}}$$
- $$11) \int \frac{x^{a-1} dx}{(1-x^b)^{\frac{a}{b}+c}} = 0$$
- $$12) \int \frac{x^{a-bc-1} dx}{(1-x^b)^{\frac{a}{b}}} = 0$$
- $$13) \int \frac{x^{a+bc-1}}{(1-x^b)^{\frac{h}{g}}} dx = \frac{1}{a+bc} \frac{(a+b)^{c/b}}{(ag-bh+bg)^{c/bg}} \frac{1^{\frac{a}{b}} 1^{-\frac{h}{g}}}{1^{\frac{a}{b}-\frac{h}{g}}} g^c$$
- $$14) \int \frac{x^{a-bc-1}}{(1-x^b)^{\frac{h}{g}}} dx = \frac{1}{a-bc} \frac{(bh-ag)^{c/bg}}{ac^{c/b}} \frac{1^{\frac{a}{b}} 1^{-\frac{h}{g}}}{1^{\frac{a}{b}-\frac{h}{g}}} \frac{1}{g^c}$$
- $$15) \int \frac{x^{h+ab-1}}{(1-x^b)^{\frac{h}{b}}} dx = \frac{1}{ab+h} \frac{(b+h)^{a/b}}{ba^{a/b}} \frac{1^{\frac{h}{b}} 1^{-\frac{h}{b}}}{1^{\frac{h}{b}-\frac{h}{b}}}$$
- $$16) \int \frac{x^{ab-1} dx}{(1-x^b)^{\frac{h}{g}}} = \frac{g^{a/g}}{ab(g-h)^{a/g}}$$
- $$17) \int \frac{x^{ab} dx}{(1-x^b)^{\frac{h}{g}}} = \frac{1}{1+ab} \frac{(b+1)^{a/b}}{(g-bh+bg)^{a/bg}} \frac{1^{\frac{1}{a}} 1^{-\frac{h}{g}}}{1^{\frac{1}{a}-\frac{h}{g}}} g^a$$
- $$18) \int \frac{x^{ab-1} dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{1}{ab} \frac{g^{a/g}}{h^{c/g} (g-h)^{a-c/g}}$$
- $$19) \int \frac{x^{ab} dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{1}{1+ab} \frac{(1+b)^{a/b}}{h^{c/g} (bh-g-bg)^{a-c/bg}} \frac{1^{\frac{1}{a}} 1^{-\frac{h}{g}}}{1^{\frac{1}{a}-\frac{h}{g}}} g^a$$
- $$20) \int \frac{x^{h-cg-1} dx}{(1-x^b)^{\frac{h}{g}+c}} = 0$$

Oettinger, Cr. 35. 13.

$$\left. \begin{aligned} 21) \int \frac{x^{a+bc-1} dx}{(1-x^b)^{\frac{a}{b}+g}} &= (-1)^g \frac{(a+b)^{clb}}{a^{glb} 1^{c-g/l}} \frac{a b^{g-c}}{a+bc} \frac{\pi}{b \sin. \frac{a\pi}{b}} \\ 22) \int \frac{x^{a+bc-1} dx}{(1-x^b)^{\frac{a}{b}+c}} &= (-1)^c \frac{\pi}{b} \operatorname{Cosec.} \frac{a\pi}{b} \\ 23) \int \frac{dx}{(1-\sqrt[b]{x})^b} &= b\pi \operatorname{Cosec.} b\pi \\ 24) \int \frac{x^{q-1} dx}{(1-\sqrt[b]{x})^{bq}} &= b\pi \operatorname{Cosec.} bq\pi \end{aligned} \right\} \text{Oettinger, Cr. 38. 162.}$$

$$\left. \begin{aligned} 23) \int \frac{dx}{(1-\sqrt[b]{x})^b} &= b\pi \operatorname{Cosec.} b\pi \\ 24) \int \frac{x^{q-1} dx}{(1-\sqrt[b]{x})^{bq}} &= b\pi \operatorname{Cosec.} bq\pi \end{aligned} \right\} \text{Krampe, Réfr. 3. 83.}$$

$$\begin{aligned} 1) \int \frac{(1-x)^{-\frac{1}{2}} - 1}{x} dx &= 2 \, l \, 2 \quad \text{Arndt, Gr. 6. 187.} \\ 2) \int \frac{\sqrt{x} + \frac{1}{x}}{1+x^2} dx &= \frac{1}{2} \pi \sqrt{2} \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 2. — Plana, Mém. Brux. T. 10. — Mascheroni, Adm. p. 53 la trouvait fautivement} = \pi \sqrt{2}. \\ 3) \int \frac{x^a + x^{a+\frac{1}{2}} - 2x^{2a}}{1-x} \frac{dx}{x} &= 2 \, l \, 2 \\ 4) \int \frac{x^a + x^{a+\frac{1}{2}} + x^{a+\frac{1}{2}} - 3x^{3a}}{1-x} \frac{dx}{x} &= 3 \, l \, 3 \end{aligned} \left\{ \text{Legendre, Exerc. 4. 55.} \right.$$

$$5) \int \frac{\sqrt{\frac{1}{x}} - 1}{1-x} dx = l \, 4 \quad \text{Poisson, Mém. Inst. 1811. 163. N°. 54.}$$

$$6) \int \frac{dx}{\sqrt{x-x^2}} = \pi \quad \text{Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 335. — Dienger, Cr. 42. 283.}$$

$$\left. \begin{aligned} 7) \int \frac{(1-x)^a}{\sqrt{x(1-x)}} dx &= \frac{1^{a/2}}{2^{a/2}} \pi \\ 8) \int \frac{(1-x)^a x^b dx}{\sqrt{x(1-x)}} &= \frac{1^{a/2} 1^{b/2}}{2^{a+b/2}} \pi \end{aligned} \right\} \text{Ohm, Ausw. N°. 46.}$$

$$9) \int \frac{x^a dx}{\sqrt{x(1-x)}} = \frac{1^{a/2}}{2^{a/2}} \pi \quad \text{Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 335. — Oettinger, Cr. 35. 13. — Ohm, Ausw. 14 trouve } \frac{\pi}{2} \text{ au lieu de } \pi \text{ faut.}$$

$$10) \int \frac{dx}{\sqrt{x(1+px)}} = \frac{2}{\sqrt{p}} l \left\{ \sqrt{p} + \sqrt{1+p} \right\} \quad \text{Schlömilch, Beitr. III. 7.}$$

- 11) $\int \frac{dx}{x^{\frac{2}{3}} \sqrt[3]{1-x^2}} = \frac{3}{\sqrt[3]{3}} F' \left(\sin. \frac{\pi}{12} \right)$ Legendre Exerc. 1. 39.
- 12) $\int \frac{dx}{x^{\frac{1}{3}} \sqrt[3]{1-x^2}} = \frac{1}{\sqrt[3]{3}} F' \left(\cos. \frac{\pi}{12} \right)$ Legendre, Exerc. 1. 40.
- 13) $\int \frac{dx}{x^{a+bd+1} (1-x^b)^{\frac{h}{g}+c}} = \frac{\left(-\frac{a}{b} - \frac{h}{g}\right)^{c+d/-1}}{\left(-\frac{h}{g}\right)^{c/-1} \left(-\frac{a}{b}\right)^{d/-1}} \frac{1-\frac{a}{b} \sqrt[1]{1} \ 1-\frac{h}{g} \sqrt[1]{1}}{1-\left(\frac{a}{b} + \frac{h}{g}\right) \sqrt[1]{1}} \frac{1}{a+b \ d}$ Oettinger, Cr. 35. 13.
- 14) $\int \frac{x^{ab-1} dx}{\sqrt{x^{2a}-x^{3a}}} = \frac{2b-1/2}{(b-1)a \cdot 1^{b-1/2}}$
- 15) $\int \frac{1}{a-bx} \frac{dx}{\sqrt{x(1-x)}} = \frac{\pi}{\sqrt{a(a-b)}}$ pour $b \text{ ni} = a$,
ni = 0;
- 16) $\int \frac{1}{(a-bx)^{c+1}} \frac{dx}{\sqrt{x(1-x)}} = \frac{1^{c/2}}{2^{c/2} (a-b)^c} \frac{\pi}{\sqrt{(a^2-ab)}} \sum_0^{\infty} \binom{c}{n} \frac{1^{n/2}}{(2c-1)^{n-2}} \left(\frac{a-b}{a}\right)^n$ Dienger, Cr. 42. 483.
- 17) $\int \left(\frac{x}{1-x}\right)^p \frac{dx}{\sqrt{x(1-x)}} = \pi \text{ Sec. } p \pi, \quad p^2 < \frac{1}{4};$ Ohm, Ausw. N°. 46.
- 18) $\int \frac{x^{\frac{a}{2b}-x^{\frac{a}{2b}}}}{1-x} \frac{dx}{\sqrt{x}} = \frac{\pi}{2b} \text{ Tang. } \frac{a\pi}{2b}$ Malmsten, Cr. 38. 1.

- 1) $\int \frac{x^{p-1/2} dx}{(1-x)^p (1+qx)^p} = \frac{2\Gamma(p+\frac{1}{2})\Gamma(1-p)}{\sqrt{\pi}} \cos. 2p(\text{Arctg. } \sqrt{q}) \cdot \frac{\sin. \{(2p-1)\text{Arctg. } \sqrt{q}\}}{(2p-1) \sin. (\text{Arctg. } \sqrt{q})} \left\{ \begin{array}{l} p < 1, \\ 1 > q > 0; \end{array} \right.$
- 2) $\int \frac{x^{p-1/2} dx}{(1-x)^p (1-qx)^p} = \frac{2\Gamma(p+\frac{1}{2})\Gamma(1-p)}{\sqrt{\pi}} \frac{(1-\sqrt{q})^{1-2p} - (1+\sqrt{q})^{1-2p}}{(2p-1) 2 \sqrt{q}} \left\{ \begin{array}{l} \text{Legendre,} \\ \text{Exerc. 4.} \\ 124, 145. \end{array} \right.$
- 3) $\int \frac{x^2}{1+x^4} \frac{dx}{\sqrt{1-x^4}} = \frac{\pi}{8}$
- 4) $\int \frac{1+px^2}{1+x^4} \frac{dx}{\sqrt{1-x^4}} = \frac{1+p}{8} \pi + \frac{1}{4} \sqrt{2} \cdot F' \left(\sin. \frac{\pi}{4} \right)$
- 5) $\int \frac{1+p+2x^2-(1-p)x^4}{1+x^4} \frac{dx}{\sqrt{1-x^4}} = \frac{1+p}{4} \pi + \frac{1}{2} \sqrt{2} \cdot F' \left(\sin. \frac{\pi}{4} \right)$
- 6) $\int \frac{x dx}{\sqrt{(p^2+x^2)(1-x^2)}} = \frac{\pi}{2} - \text{Arctg. } p$ Poisson, P. 16. 215. N°. 11.

$$7) \int \frac{x^{\frac{q}{2}} dx}{\{(1-x)(1-p^2 x)\}^{\frac{q+1}{2}}} = \frac{(1-p)^{-q}-(1+p)^{-q}}{2pq} \text{ B } \left(\frac{q+2}{2}, \frac{1-q}{2} \right)$$

$$8) \int \frac{x^{\frac{q}{2}} dx}{\{(1-x)(1-x \text{ Tang.}^2 \lambda)\}^{\frac{q+1}{2}}} = \frac{\text{Sin. } q \lambda \text{ Cos. } q \lambda}{q \text{ Tang. } \lambda} \text{ B } \left(\frac{q+2}{2}, \frac{1-q}{2} \right)$$

$$9) \int \left(\frac{x^{a-1}}{1-\sqrt[p]{x}} - \frac{px^{p^a-1}}{1-x} \right) dx = p \, l \, p$$

$$10) \int \left(\frac{a}{1-x} - \frac{x^{p-1}}{1-\sqrt[a]{x}} \right) dx = a \, l \, p + \sum_1^a \text{Z}' \left(p + \frac{b-n}{a} \right)$$

$$11) \int \left(\frac{k}{1-x} - \frac{\sqrt[k]{x}}{1-\sqrt[k]{x}} \right) dx = k \, \Lambda, \text{ pour } k = \infty; \text{ Legendre, Exerc. 5. 12.}$$

$$12) \int \frac{1-x^2}{\sqrt{1+\alpha^2 x^2}} \cdot \frac{1-\alpha^2 b^2 x^2}{\sqrt{1+b^2 x^2}} x^2 dx = 0 \quad \text{Ramus, Overs. Danske Forh. 1844.}$$

$$13) \int \left(\frac{px^{pq-1}}{1-x} - \frac{x^{q-1}}{1-\sqrt[p]{x}} \right) dx = \frac{d}{dq} \cdot l \frac{\Gamma(q) \Gamma\left(q+\frac{1}{p}\right) \Gamma\left(q+\frac{2}{p}\right) \dots \Gamma\left(q+\frac{p-1}{p}\right)}{\Gamma(qp)}$$

Boncompagni, Cr. 25.
74.

Arndt, Gr. 10. 253.

Schaar, Mém.
Cour. Brux.
T. 22.

$$1) \int \frac{dx}{x} = l \, \alpha, \text{ où } \alpha \text{ arbitraire; Cauchy, Cours. Lec. 24.}$$

$$2) \qquad \qquad = - (2k+1) \pi i, \text{ où } k \text{ arbitraire;}$$

$$3) \int \frac{dx}{x^2} = -2$$

$$4) \int \frac{dx}{x^a} = \frac{(-1)^a}{a-1} [\text{Cos.} \{ (a-1)(2k+1)\pi \} - 1], \text{ où } k \text{ arbitraire;}$$

$$5) \qquad \qquad = 0 \text{ pour } a \text{ impair}$$

$$6) \qquad \qquad = \frac{2}{1-a} \text{ pour } a \text{ pair}$$

$$7) \int \frac{dx}{\sqrt{1-2px+p^2}} = 2, \text{ pour } p < 1; \text{ Poisson, Chal. 113.}$$

Poisson, P. 18. 295. N°. 33.

Poisson, P. 18. 295. N°. 34.

$$\begin{aligned}
& \left. \begin{aligned} 8) \int \frac{dx}{1 \sqrt{(1-2px+p^2)}} &= \frac{2}{p}, \text{ pour } p > 1; \\ 9) \int \frac{p-x}{1 \sqrt{(1-2px+p^2)^3}} dx &= 0, \text{ pour } p < 1; \\ 10) &= \frac{2}{p^2}, \text{ pour } p > 1; \end{aligned} \right\} \text{Poisson, Chal. 113.} \\
& \left. \begin{aligned} 11) \int \frac{px-q}{1 \sqrt{(p^2-pqx+q^2)^3}} dx &= 0, \text{ pour } p > q; \\ 12) &= -\frac{2}{q^2}, \text{ pour } p < q; \end{aligned} \right\} \text{Plana, Mém. Turin. 1820. 389. N°. 9.} \\
13) \int \frac{dx}{1 \sqrt{\left\{ (1-2pqx+p^2q^2) \left(1-\frac{2p}{q}x+\frac{p^2}{q^2} \right) \right\}}} &= \frac{1}{p} l \frac{1+p}{1-p} \quad \text{Legendre, Exerc. 5. 124. — Liouville, L. 2. 135.} \\
14) \int \frac{dx}{1 \sqrt{\left\{ (1-2px+p^2) (1-2qx+q^2) \right\}}} &= \frac{1}{1 \sqrt{pq}} l \frac{1+1 \sqrt{pq}}{1-1 \sqrt{pq}}, \text{ pour } p^2 < 1, q^2 < 1; \\
15) &= \frac{1}{1 \sqrt{pq}} l \frac{1 \sqrt{p+1 \sqrt{q}}}{1 \sqrt{p-1 \sqrt{q}}}, \text{ pour } q^2 < 1 < p^2; \\
16) &= \frac{1}{1 \sqrt{pq}} l \frac{1 \sqrt{q+1 \sqrt{p}}}{1 \sqrt{q-1 \sqrt{p}}}, \text{ pour } p^2 < 1 < q^2; \\
17) &= \frac{1}{1 \sqrt{pq}} l \frac{1 \sqrt{pq+1}}{1 \sqrt{pq-1}}, \text{ pour } p^2 > 1, q^2 > 1; \end{aligned} \right\} \begin{array}{l} \text{Poisson,} \\ \text{P. 19.} \\ \text{404. N°.} \\ \text{82.} \end{array}
\end{aligned}$$

$$\begin{aligned}
1) \int \frac{dx}{x} &= \infty \quad \text{Cauchy, Cours. Lec. 24. — Meijer, Int. déf. 98.} \\
2) \int \frac{x^{p-1} dx}{1+x} &= \frac{\pi}{\text{Sin. } p \pi} \quad \left\{ \begin{array}{l} \text{où } 0 < p < 1; \text{ Legendre, Exerc. 4. 95. — Poisson, P. 19. 404.} \\ \text{N°. 72. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Id., Lim.} \\ \text{Imag. Add. 12. — Id., P. 28. 147. I. N°. 2. — Bonnet, L. 6.} \\ \text{238. — Lejeune-Dirichlet, Cr. 15. 258. — Jürgensen, Cr. 23.} \\ \text{142. — Oettinger, Cr. 35. 13. — Winckler, Cr. 45. 102. — Grunert, Gr. 2. 266. — Schlömilch, Gr. 3. 278. — Serret, L. 8. 1.} \end{array} \right. \\
3) &= \Gamma(p) \Gamma(1-p) \\
4) \int \frac{x^{1-p} dx}{1+x} &= -\pi \text{Cosec. } p\pi \quad \text{Oettinger, Cr. 35. 13.} \\
5) \int \frac{x^{p-1} dx}{x+a} &= \pi a^{p-1} \text{Cosec. } p\pi, \quad 0 < p < 1; \quad \text{Schlömilch, Gr. 12. 198. — Dedekind, Cr. 45. 370.}
\end{aligned}$$

- 6) $\int \frac{x^{p-1} dx}{x + e^{qi}} = \frac{\pi}{\sin. p \pi} e^{(p-1)qi}, 0 < p < 1, q^2 < \pi^2$; Minding, Taf. II.
- 7) $\int \frac{x^{p-1} dx}{qx + 1} = \frac{\pi}{q^p} \operatorname{Cosec}. p \pi, p < 1$; Dedekind, Cr. 45. 370.
- 8) $\int \frac{x^{p-1} dx}{1-x} = \pi \operatorname{Cot}. p \pi, 0 < p < 1$; Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Id., Lim. Imag. Add. 13. — Grunert, Gr. 2. 266. — Schlömilch, Gr. 3. 278. — Meijer, Int. Déf. 155 a faut. $\frac{1}{2} \pi \operatorname{Cot}. p \pi$.
- 9) $\int \frac{(x-a)^{p-1} dx}{b-x} = \pm \frac{(-1)^p \pi}{(b-a)^{1-p} \sin. p \pi}, \pm \text{ selon que } \begin{matrix} a > b \\ a < b \end{matrix}$; Jürgensen, Cr. 23. 142.
- 10) $\int \frac{x^{p-r-1} (1+x)^{1-p}}{1-q^r+x} dx = \frac{\Gamma(r) \Gamma(p-r)}{\Gamma(p)} \xi(r, p, q), p > r$; Schaeffer, Cr. 37. 127.
- 11) $\int \frac{x^{b-1} dx}{(1+x)^a} = \frac{\pi}{a} \operatorname{Cosec}. \frac{b \pi}{a}$ Euler, Calc. Int. T. 4. S. 5. 124.
- 12) $\int \frac{x^{q-1} dx}{(1+x)^p} = \frac{\Gamma(q) \Gamma(p-q)}{\Gamma(p)}, q < p$; Legendre, Exerc. 4. 99. — Poisson, P. 19. 404. N°. 72. — Cauchy, P. 28. 147. I. N°. 2.
- 13) $\int \frac{x^{p-1} dx}{(1+x)^{p+q}} = \frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)}$ Lobatschewsky, Mém. Kasan. 1835. 1. — Grunert, Gr. 2. 266.
- 14) $= B(p, q) = \int \frac{x^{q-1} dx}{(1+x)^{p+q}}$ Binet, P. 27. 123. — Schaar, Mém. Cour. Brux. T. 22.
- 15) $= \left(\frac{p}{q} \right)$ Lejeune-Dirichlet, Cr. 15. 258.

C'est l'Intégrale Eulérienne de première espèce.

- 16) $\int \frac{x^{q-1} + x^{p-1}}{(1+x)^{p+q}} dx = 2 B(q, p)$ Binet, P. 27. 123.
- 17) $\int \frac{x^{p-1} dx}{(b+x)^{a+1}} = \frac{(-1)^a (p-1)^{a'-1}}{1^{a'}} \frac{\pi}{\sin. p \pi} b^{p-a-1}$
- 18) $\int \frac{x^{p-1} dx}{(1+x)^{a+1}} = \frac{(p-1)^{a'-1} (-1)^a \pi}{1^{a'}} \sin. p \pi$
- 19) $\int \frac{x^{a+p} dx}{(1+x)^{2a+1}} = \frac{(-1)^a \pi}{\sin. p \pi} \cdot \frac{p^2 \cdot p^2 - 1^2 \cdot p^2 - 2^2 \cdot \dots \cdot p^2 - a^2}{1^{2a+1/1}}$
- 20) $\int \frac{x^a dx}{(1+x)^{a+p+1}} = \Delta^a \left(\frac{1}{p} \right)$ Cauchy, P. 28. 147. I. § 2.

$$\begin{aligned}
21) \int \frac{x^{1-p} dx}{(1+x)^3} &= \frac{1-p}{2} p \pi \operatorname{Cosec}. p \pi \\
22) \int \frac{x^p dx}{(1+x)^3} &= \frac{1-p}{2} p \pi \operatorname{Cosec}. p \pi \\
23) \int \frac{x^{p+1} dx}{(1+x)^3} &= \frac{1+p}{2} p \pi \operatorname{Cosec}. p \pi \\
24) \int \frac{x^{p+c} dx}{(1+x)^{b+c+2}} &= \frac{(1+p)^{c/1} (1-p)^{b/1}}{1^{c-b/1}} p \pi \operatorname{Cosec}. p \pi = \int \frac{x^{b-p} dx}{(1+x)^{b+c+2}} \\
25) \int \frac{x^{p+c} dx}{(1+x)^{b-c+2}} &= \frac{(1+p)^{c/1}}{p^{b/1} 1^{c-b+2/1}} p \pi \operatorname{Cosec}. p \pi \\
26) \int \frac{x^{p-c} dx}{(1+x)^{b-c+2}} &= \frac{(1-p)^{b/1}}{(1-p)^{c-1/1} 1^{b-c-1/1}} p \pi \operatorname{Cosec}. p \pi = \int \frac{x^{b-p} dx}{(1+x)^{b-c+2}} \\
27) \int \frac{x^{b-a-1} dx}{(1+x)^b} &= \frac{1^{a-1/1} 1^{b-a-1/1}}{1^{b-1/1}} = \int \frac{x^{a-1} dx}{(1+x)^b} \\
28) \int \frac{x^{a \pm b-1} dx}{(1+x)^{a+c \pm 2b}} &= \frac{1^{a \pm b-1/1} 1^{c \pm b-1/1}}{1^{a+c \pm 2b-1/1}} = \int \frac{x^{c \pm b-1} dx}{(1+x)^{a+c \pm 2b}} \\
29) \int \left(x^{q-p} - \frac{x^q}{(1+x)^p} \right) dx &= \frac{q}{q-p+1} \frac{\Gamma(q) \Gamma(p-q)}{\Gamma(p)} \quad \text{Legendre, Exerc. 4. 109.}
\end{aligned}$$

Oettinger, Cr. 38. 162

Oettinger, Cr. 35. 13.

$$\begin{aligned}
1) \int \frac{dx}{1+x^2} &= \frac{1}{2} \pi \quad \text{Euler, Calc. Int. T. 1. P. 1. S. 1. S. 353. — Poisson, P. 18. 295. N°. 26.} \\
2) \int \frac{dx}{p^2 + x^2} &= \frac{\pi}{2p}, p > 0: \quad \text{Liouville, Cr. 13. 219. — Schlömilch, Gr. 4. 71. — Id., Gr. 10. 410. — Cisa de Grézy, Mém. Turin. 1821. 209. II. 58.} \\
3) \int \frac{dx}{1-x^2} &= \frac{1}{2} l(-1) \quad \text{Poisson, P. 18. 295. N°. 26. — Plana, Cr. 17. 1.} \\
3') &= 0 \quad \text{Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Schlömilch, Gr. 3. 278.} \\
4) \int \frac{dx}{b^2 - x^2} &= 0. \quad \text{Legendre, Exerc. 5. 13. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 31. — Plana, Mém. Turin. 1818. 7 Art. 1. 3, 4. — Cisa de Grézy, Mém. Turin. 1821. 209. II. 56. — Schlömilch, Gr. 7. 270.} \\
4') &= \frac{1}{2b} l(-1) \quad \text{Poisson, P. 18. 295. N°. 38. — Cisa de Grézy, Mém. Turin. 1821. 209. II. 58.} \\
4'') &= \alpha, \text{ où } \alpha \text{ arbitraire; Arndt, Gr. 10. 240. C'est la vraie valeur.}
\end{aligned}$$

- 5) $\int \frac{x^{2p-1}}{1+x^2} dx = \frac{\pi}{2} \operatorname{Cosec}. p\pi, 1 > p > 0$; Schlömilch, Beitr. III. 4.
- 6) $\int \frac{x^{p-1}}{1+x^2} dx = \frac{\pi}{2} \operatorname{Cosec}. \frac{p\pi}{2}, 2 \geq p \geq 0$; Cauchy, Cours. Leç. 34. — Id., P. 19. 511. — Id., Exerc. 1826. p. 95.
- 7) $= \frac{\pi}{i^{p-1} + (-i)^{p-1}}, 2 > p > 0$; Meijer, Int. Déf. 154.
- 8) $\int \frac{x^p dx}{1+x^2} = \frac{\pi}{2} \operatorname{Sec}. \frac{p\pi}{2}, 1 \geq p \geq 0$; Schlömilch, Gr. 3. 278.
- 9) $\int \frac{x^{p-1} dx}{1-x^2} = \frac{\pi}{2} \operatorname{Cot}. \frac{p\pi}{2}, 1 > p$; Cauchy, Cours. Leç. 34. — Meijer, Int. Déf. 155.
- 10) $\int \frac{dx}{1+x^3} = \frac{2\pi}{9} \vee 3$
- 11) $\int \frac{x^2 dx}{1+x^3} = \frac{2\pi}{9} \vee 3$ Euler, N. C. P. 6. 115. — Id., Calc. Int. T. 1. P. 1. S. 8. 353.
- 12) $\int \frac{dx}{1-x^3} = \frac{1}{9}\pi - \frac{1}{3}l(-1)$ Plana, Mém. Turin. 1820.
- 13) $\int \frac{dx}{1+x^4} = \frac{1}{4}\pi \vee 2$ Euler, N. C. P. 6. 115. — Id., Calc. Int. T. 1. P. 1. S. 1. 8. 353. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.
- 14) $\int \frac{x^2 dx}{1+x^4} = \frac{1}{4}\pi \vee 2$ Euler, N. C. P. 6. 115. — Id., Calc. Int. T. 1. P. 1. S. 1. 8. 353.
- 15) $\int \frac{dx}{1+x^6} = \frac{1}{3}\pi$
- 16) $\int \frac{x^2 dx}{1+x^6} = \frac{1}{6}\pi$
- 17) $\int \frac{x^4 dx}{1+x^6} = \frac{1}{3}\pi$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 353. — Poisson, L. 2. 224.
- 18) $\int \frac{dx}{(\pm p + qi)^2 + x^2} = \frac{\pi}{2(p \pm q)}$ Ohm, Ausw. 2.

- 1) $\int \frac{x^{q-1} dx}{1+x^p} = \frac{\pi}{p} \operatorname{Cosec}. \frac{q\pi}{p}, p > q > 0$; Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 351. — Poisson, P. 16. 215. N°. 6. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Mascheroni, Adnot. p. 63. — Oettinger, Cr. 35. 13. — Schlömilch, Gr. 3. 278.
- 2) $= \infty, q > p$; Poisson, P. 16. 215. N°. 6.

- 3) $\int \frac{x^{p-q-1} dx}{1+x^p} = \frac{\pi}{p} \operatorname{Cosec.} \frac{q\pi}{p}, p > q > 0$; Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 351. — Oettinger, Cr. 35. 13.
- 4) $\int \frac{x^q dx}{p^2 + x^{2q}} = \frac{\pi 1^{\frac{q}{p}}}{2 p q} \operatorname{Sec.} \frac{\pi}{2 q} \left\{ \begin{array}{l} q \geq 1; \\ q \leq 1; \end{array} \right. \text{Raabe, Int. 416.}$
- 5) $\int \frac{dx}{p^2 + x^{2q}} = \frac{\pi 1^{\frac{q}{p}}}{2 p^2 q} \operatorname{Cosec.} \frac{\pi}{2 q} \left\{ \begin{array}{l} q \geq 1; \\ q \leq 1; \end{array} \right. \text{Raabe, Int. 416.}$
- 6) $\int \frac{x^{2b} dx}{1+x^{2a}} = \frac{\pi}{2a} \operatorname{Cosec.} \left(\frac{2b+1}{2a} \pi \right), 2a+1 > 2b$; Poisson, P. 16. 215. N°. 6. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Serret, L. 8. 1. — Grunert, Gr. 2. 266.
- 7) $= \infty, a < b$; Poisson, P. 16. 215. N°. 6.
- 8) $\int \frac{x^{q-1} + x^{p-1}}{x^p + q + 1} dx = \frac{2\pi}{p+q} \operatorname{Sec.} \frac{q-p}{q+p} \frac{\pi}{2}$ Ohm, Ausw. 14. — Raabe, Int. 147.
- 9) $\int \frac{x^{2bc+c-1} dx}{1+x^{2ac}} = \frac{\pi}{2a} \operatorname{Cosec.} \left(\frac{2b+1}{2a} \pi \right), a > b \left\{ \begin{array}{l} a > b \\ a < b \end{array} \right. \text{Poisson, P. 16. 215. N°. 6.}$
- 10) $= \infty, a < b$
- 11) $\int \frac{x^{a-1} dx}{e^{qi} + x^b} = \frac{\pi}{b} e^{\left(\frac{a}{b}-1\right)qi} \operatorname{Cosec.} \frac{a\pi}{b}, a < b, q^2 < \pi^2$; Minding, Taf. 11.
- 12) $\int \frac{1+x^2}{1-x^{2(2b+1)}} x^{a-1} dx = \frac{\pi}{4b+2} \left\{ \operatorname{Cosec.} \frac{a\pi}{4b+2} + \operatorname{Cosec.} \left(\frac{a+2}{4b+2} \pi \right) \right\}, 4b > a > 0$; Lindmann, Gr. 14. 94.
- 13) $\int \frac{x^{q-1} dx}{1-x^p} = \frac{\pi}{p} \operatorname{Cot.} \frac{q\pi}{p}, p > q$; Mascheroni, Adn. p. 64. — Legendre, Exerc. 5. 13. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Schlömilch, Gr. 3. 278.
- 14) $\int \frac{x^{2b} dx}{1-x^{2a}} = \frac{\pi}{2a} \operatorname{Cot.} \left(\frac{2b+1}{2a} \pi \right), 2a+1 > 2b$; Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Grunert, Gr. 2. 266.
- 15) $\int \frac{x^{q-1} - x^{p-1}}{x^p + q - 1} dx = \frac{2\pi}{p+q} \operatorname{Tang.} \left(\frac{q-p}{q+p} \frac{\pi}{2} \right)$ Ohm, Ausw. 14. — Raabe, Int. 147.
- 16) $\int \frac{1-x^2}{1-x^{2b}} x^{a-1} dx = \frac{\pi}{2b} \operatorname{Sin.} \frac{\pi}{b} \operatorname{Cosec.} \frac{a\pi}{2b} \operatorname{Cosec.} \left(\frac{a+1}{2b} \pi \right), b-1 > \frac{1}{2} a > 0 \left\{ \begin{array}{l} b-1 > \frac{1}{2} a > 0 \\ b-1 < \frac{1}{2} a > 0 \end{array} \right. \text{Lindmann, Gr. 14. 94.}$
- 17) $\int \frac{1-x^2}{1-x^{4b}} x^{a-1} dx = \frac{\pi}{2b} \left\{ \operatorname{Cot.} \frac{a\pi}{4b} + \operatorname{Cot.} \left(\frac{a+1}{4b} \pi \right) \right\}, 2b-1 > \frac{1}{2} a > 0 \left\{ \begin{array}{l} b-1 > \frac{1}{2} a > 0 \\ b-1 < \frac{1}{2} a > 0 \end{array} \right. \text{Lindmann, Gr. 14. 94.}$

$$\begin{aligned}
& \left. \begin{aligned} 1) \int \frac{dx}{(1+x^2)^a} &= \frac{1^{a-1/2} \pi}{2^{a-1/2} 2} \\ 2) \int \frac{dx}{(p+x^2)^{a+1}} &= \pm \frac{\pi}{2} \frac{d^a}{dp^a} \cdot \frac{1}{1 \swarrow p} \\ 3) &= \frac{1^{a/2}}{1^{a/1}} \frac{\pi}{2^{a+2} p^a 1 \swarrow p} \end{aligned} \right\} \text{Cauchy, Cours. Lec. 33.} \\
& \left. \begin{aligned} 4) \int \frac{dx}{(1+x^2)^3} &= \frac{3}{16} \pi \\ 5) \int \frac{x^2 dx}{(1+x^2)^3} &= \frac{1}{16} \pi \\ 6) \int \frac{x^4 dx}{(1+x^2)^3} &= \frac{3}{16} \pi \end{aligned} \right\} \text{Poisson, L. 2. 224.} \\
& 7) \int \frac{dx}{(q^2-x^2)^2} = 0 \quad \text{Plana, Mém. Turin. 1818. 7. Art. 1. N°. 3.} \\
& \left. \begin{aligned} 8) \int \frac{dx}{(q^2+x^2)^{a+1}} &= \frac{1}{2^a} \frac{1^{a/2}}{q^{2a+1} 1^{a/1}} \\ 9) \int \frac{x^{p-1} dx}{(1+x^2)^a} &= \frac{\Gamma(\frac{1}{2}p) \Gamma(a-\frac{1}{2}p)}{2 \Gamma(a)}, 1 > p > 0; \end{aligned} \right\} \text{Schlömlich, Beitr. III. 12, 13.} \\
& 10) \int \frac{x^{b-1} dx}{(1+x^a)^c} = \left(1-\frac{b}{a}\right) \left(1-\frac{b}{2a}\right) \dots \left(1-\frac{b}{(c-1)a}\right) \frac{\pi}{a} \operatorname{Cosec.} \frac{b\pi}{a} \quad \text{Euler, Calc. Int. T. 1. S. 5. 159.} \\
& \left. \begin{aligned} 11) \int \frac{x^{-a+c(b+1)-1} dx}{(1+x^b)^{g+c-1}} &= \frac{(b-a)^{c/b} (b+a)^{g-1/b}}{bg+c/b} \frac{a\pi}{b} \operatorname{Cosec.} \frac{a\pi}{b} \\ 12) \int \frac{x^{-a+c(b+1)-1} dx}{(1+x^b)^{2c-1}} &= \left(1-\frac{a^2}{b^2}\right) \left(1-\frac{a^2}{b^2}\right) \dots \left(c^2-\frac{a^2}{b^2}\right) \frac{a}{1^{2c/1}} \frac{1}{a+bc} \frac{\pi}{b} \operatorname{Cosec.} \frac{a\pi}{b} \\ 13) \int \frac{x^{-a+b(g+1)-1} dx}{(1+x^b)^{g-c+1}} &= (-1)^c \frac{(b-a)^{g/b}}{(b-a)^{c/b} 1^{g-c/1} b^{g-c+1}} \frac{\pi}{b} \operatorname{Cosec.} \frac{a\pi}{b} \end{aligned} \right\} \text{Oettinger, Cr. 38. 162.} \\
& \left. \begin{aligned} 14) \int \frac{x^{p+q-1} dx}{(1+x^q)^2} &= \frac{p\pi}{q^2} \operatorname{Cosec.} \frac{p\pi}{q}, p < q; \\ 15) \int \frac{x^{p-1} dx}{(1+x^q)^2} &= \frac{p-q}{q^2} \pi \operatorname{Cosec.} \left(\frac{p-q}{q}\right), 2q > p > q; \end{aligned} \right\} \text{Ohm, Ausw. N° 20.}
\end{aligned}$$

- 1) $\int \frac{dx}{(1+x)x^p} = \pi \operatorname{Cosec}. p\pi, 1 > p > 0; \text{ Ramus, Cr. 24. 257. — Oettinger, Cr. 35. 13.}$
- 2) $\int \frac{dx}{(1+x)^{c-b+2} x^{b+p}} = \frac{(1+p)^{c-1}}{1^{c-b+2/1} p^{b+1}} p\pi \operatorname{Cosec}. p\pi \text{ Oettinger, Cr. 38. 162.}$
- 3) $\int \frac{(1+x)^q - 1}{(1+x)^{p+q}} \frac{dx}{x} = Z'(p+q) - Z'(q) \text{ Lindmann, Stockh. Handl. 1850. II.}$
- 4) $\int \frac{x^{p-1} + x^{-p}}{1+x} dx = \pi \operatorname{Cosec}. p\pi, p < 1; \text{ Dedekind, Cr. 45. 370.}$
- 5) $\int \frac{x^{q-1} - x^{-p}}{1-x} dx = \pi \operatorname{Sin}. \{(q+p)\pi\} \cdot \operatorname{Cosec}. q\pi \cdot \operatorname{Cosec}. p\pi \text{ Svanberg, Transf. § 5.}$
- 6) $\int \frac{x^{p-1} - a^{p-q} x^q}{x} \frac{dx}{x-a} = \pi a^{p-1} (\operatorname{Cot}. q\pi - \operatorname{Cot}. p\pi), \begin{matrix} 1 > p > 0 \\ 1 > q > 0 \end{matrix}; \text{ Minding, Taf. II.}$
- 7) $\int \frac{x^p - x^{-p}}{1-x^2} dx = -\frac{\pi}{2} \operatorname{Tang}. \frac{p\pi}{2}, 1 > p > 0; \text{ Schlömilch, Gr. 3. 278.}$
- 8) $\int \frac{dx}{(1+x^2)x^p} = \frac{1+p}{2} p\pi \operatorname{Cosec}. p\pi \text{ Oettinger, Cr. 38. 162.}$
- 9) $\int \frac{x^b}{1+x^a} \frac{dx}{x} = \frac{\pi}{a} \operatorname{Cosec}. \frac{b\pi}{a} \text{ Euler, Calc. Int. 4. S. 5. 155.}$
- 10) $\int \frac{x^{bc+b-1}}{1-x^b} \frac{dx}{x^a} = (-1)^c \frac{\pi}{b} \operatorname{Cosec}. \frac{a\pi}{b}$
- 11) $\int \frac{dx}{x^{a+(b+1)g+1} (1+x^b)^{c-g+1}} = \frac{(-1)^g}{a+bc} \frac{(b-a)^{c/b}}{(b-a)^{g/b} 1^{c-g/1}} \frac{\pi}{b^{c-g+1}} \operatorname{Cosec}. \frac{a\pi}{b}$
- 12) $\int \frac{dx}{x^{a+(b+1)c+1} (1+x^b)} = (-1)^c \frac{\pi}{b} \operatorname{Cosec}. \frac{a\pi}{b}$
- 13) $\int \frac{x^{-p}}{x^q - x^{-q}} \frac{dx}{x} = -\frac{\pi}{2q} \operatorname{Tang}. \frac{p\pi}{2q} \text{ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.}$
- 14) $\int \frac{x^p + x^{-p}}{x^q + x^{-q}} \frac{dx}{x} = \frac{\pi}{q} \operatorname{Sec}. \frac{p\pi}{2q} \text{ V. T. 40. N°. 29.}$
- 15) $\int \frac{x^p - x^{-p}}{x^q - x^{-q}} \frac{dx}{x} = \frac{\pi}{q} \operatorname{Tang}. \frac{p\pi}{2q} \text{ Malmsten, Cr. 38. 1, où faut. } \frac{\pi}{2q}.$
- 16) $\int \left\{ \frac{1}{x^p} - \frac{1}{(1+x)^p} \right\} x^q dx = \frac{q}{q-p+1} \frac{\Gamma(q)\Gamma(p-q)}{\Gamma(p)} \text{ Legendre, Exerc. 4. 109.}$

$$17) \int \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^p} \right\} \frac{dx}{x} = \Lambda + Z'(p) \quad \text{Schlömilch, Stud. I. § 6. — Id., Gr. 9. 5.}$$

$$18) \int \left\{ \frac{1}{(1+x)^p} - \frac{1}{(1+x)^q} \right\} \frac{dx}{x} = Z'(q) - Z'(p) \quad \text{Schlömilch, Beitr. III. 9.}$$

$$19) \int \frac{x^p - a^p}{x-1} \frac{x^{-p} - 1}{x-a} dx = \frac{1}{a-1} \{ 2\pi(a^p-1) \text{Cot. } p\pi - (a^p+1)la \}, p^2 < 1; \quad \text{Minding, Taf. II.}$$

$$20) \int \left\{ \frac{q^p x^{p-1}}{(1+qx)^p} - \frac{(1+qx)^{p-1}}{q^{p-1} x^p} \right\} dx = \pi \text{Cot. } p\pi \quad \text{Legendre, Exerc. 4. 143.}$$

$$1) \int \frac{dx}{(1+x)(2+x)} = l2 \quad \text{Schlömilch, Gr. 9. 5.}$$

$$\left. \begin{aligned} 2) \int \frac{dx}{(ax+b)(cx+d)} &= \frac{1}{ad-bc} l \frac{ad}{bc}, \text{ où } a < b, c < d; \\ 3) \int \frac{dx}{(x+1)(x+a)} &= \frac{1}{a-1} la \\ 4) \int \frac{dx}{(ax+1)(a+x)} &= \frac{1}{a^2-1} la^2 \end{aligned} \right\} \quad \text{Dedekind, Cr. 45. 370.}$$

$$5) \int \frac{x^{p-1}}{1+x} \frac{dx}{1+ax} = \frac{1-a^{1-p}}{1-a} \pi \text{Cosec. } p\pi, p < 1; \quad \text{Svanberg, Transf. § 5.}$$

$$6) \int \frac{x^p}{1+x} \frac{dx}{x+a} = \frac{a^p-1}{a-1} \pi \text{Cosec. } p\pi, p^2 < 1; \quad \text{Minding, Taf. II.}$$

$$7) \int \frac{x-1}{qx+1} \frac{x^{p-1} dx}{x+q} = \frac{q^{a-1}-q^{-a}}{q-1} \pi \text{Cosec. } p\pi, p < 1; \quad \text{Dedekind, Cr. 45. 370.}$$

$$8) \int \left(\frac{x^q}{(1+x)^{1+q}} - \frac{x^p}{(1+x)^{1+p}} \right) dx = Z'(1+p) - Z'(1+q) \quad \text{Legendre, Exerc. 4. 110}$$

$$9) \int \frac{(a-xi)^{-p} + (a+xi)^{-p}}{2} \frac{(b-xi)^{-q} + (b+xi)^{-q}}{2} dx = \frac{\pi}{2} (a+b)^{1-p-q} \frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)} \left\{ \begin{array}{l} \text{Cauchy,} \\ \text{Lim.} \end{array} \right.$$

$$10) \int \frac{(a-xi)^{-p} - (a+xi)^{-p}}{2} \frac{(b-xi)^{-q} - (b+xi)^{-q}}{2} dx = -\frac{\pi}{2} (a+b)^{1-p-q} \frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)} \left\{ \begin{array}{l} \text{Imag.} \\ \text{108.} \end{array} \right.$$

$$11) \int \frac{(a-xi)^{-b} + (a+xi)^{-b}}{2} x^{2c} dx = 0, \quad b > 2c+1; \quad \text{Cauchy, P. 28. 147. 1. N°. 3.}$$

- 12) $\int \frac{(a-xi)^{-b} - (a+xi)^{-b}}{2} x^{2c-1} dx = 0$, $b > 2c$; Cauchy, P. 28. 147. 1. N°. 3.
- 13) $\int \frac{(1+px)^{-b} + (1+qx)^{-b}}{2} x^{a-1} dx = (pq)^{\frac{1}{2}a} \frac{\Gamma(a)\Gamma(b-a)}{\Gamma(b)} \text{Cos.} \left(a \text{Arccos.} \frac{p+q}{2\sqrt{pq}} \right)$
- 14) $\int \frac{(1+px)^{-b} - (1+qx)^{-b}}{2} x^{a-1} dx = -(pq)^{\frac{1}{2}a} \frac{\Gamma(a)\Gamma(b-a)}{i\Gamma(b)} \text{Sin.} \left(a \text{Arcsin.} \frac{p-q}{2i\sqrt{pq}} \right)$
- 15) $\int \frac{x^p - x^q}{x-1} \frac{dx}{x+a} = \frac{\pi}{1+a} \left\{ \frac{a^p - \text{Cos. } p\pi}{\text{Sin. } p\pi} - \frac{a^q - \text{Cos. } q\pi}{\text{Sin. } q\pi} \right\}$, $p^2 < 1$, $q^2 < 1$;
- 16) $\int \frac{x^p - 1}{x-1} \frac{dx}{x+a} = \frac{\pi}{1+a} \left\{ \frac{a^p - \text{Cos. } p\pi}{\text{Sin. } p\pi} - \frac{1}{\pi} l a \right\}$, $p^2 < 1$;
- 17) $\int \frac{x^p - x^{p-q}}{x-1} \frac{x^q - a^q}{x-a} dx = \frac{\pi}{a-1} \frac{\text{Sin. } q\pi}{\text{Sin. } p\pi} \left\{ \frac{a^{p+q} - 1}{\text{Sin. } \{(p+q)\pi\}} + \frac{a^l - a^p}{\text{Sin. } \{(p-q)\pi\}} \right\}$, $(p+q)^2 < 1$, $(p-q)^2 < 1$;
- 18) $\int \frac{x^p - a^p}{x-a} \frac{x^p - 1}{x-1} dx = \frac{\pi}{a-1} \left\{ \frac{a^{2p} - 1}{\text{Sin. } 2p\pi} - \frac{a^p}{\pi} l a \right\}$, $4p^2 < 1$;

Minding,
Taf. II.

- 1) $\int \frac{x}{a^2 + x^2} \frac{dx}{1+x} = \frac{1}{1+a^2} \left(\frac{a\pi}{2} - l a \right)$
- 2) $\int \frac{x}{a^2 + x^2} \frac{dx}{1-x} = \frac{-1}{1+a^2} \left(\frac{a\pi}{2} + l a \right)$
- 3) $\int \frac{1}{a^2 + x^2} \frac{dx}{1+x} = \frac{1}{1+a^2} \left(\frac{\pi}{2a} + l a \right)$
- 4) $\int \frac{1}{a^2 + x^2} \frac{dx}{1-x} = \frac{1}{1+a^2} \left(\frac{\pi}{2a} - l a \right)$
- 5) $\int \frac{1-x}{1+x} \frac{dx}{1+x^2} = 0$ Arndt, Gr. 10. 225.
- 6) $\int \frac{ax-1}{x+a} \frac{dx}{1+x^2} = \frac{1}{2} l a^2$ Arndt, Gr. 11. 70.
- 7) $\int \frac{dx}{(1+x^2)(p^2+x^2)} = \frac{\pi}{2p(p+1)}$ Schlömilch, Gr. 4. 71. — Id., Gr. 10. 440.
- 8) $\int \frac{x^2}{p^2+x^2} \frac{dx}{q^2+x^2} = \frac{\pi}{2(p+q)}$ Cisa de Grésy, Mém. Turin. 1821. 209. II. 62.

$$\left. \begin{aligned} 9) \int \frac{g + h x^2}{b^2 + x^2} \frac{dx}{c^2 - x^2} &= \frac{g - h b^2}{b^2 + c^2} \frac{\pi}{2 b} \\ 10) \int \frac{x^{p-1}}{b^2 + x^2} \frac{dx}{c^2 - x^2} &= \frac{b^{p-2} + c^{p-2} \operatorname{Cos.} \frac{p \pi}{2}}{b^2 + c^2} \frac{\pi}{2} \operatorname{Cosec.} \frac{p \pi}{2} \end{aligned} \right\} \text{Legendre, Exerc. 5. 13.}$$

$$11) \int \frac{x^2}{q^2 + x^2} \frac{dx}{x^2 - p^2} = \frac{q}{2} \frac{\pi}{p^2 + q^2} \quad \text{Cisa de Grésy, Mém. Turin. 1821. 209 H. 61.}$$

$$12) \int \frac{x}{q^2 + x^2} \frac{dx}{1 - x^2} = -\frac{1}{1 + q^2} \log q \quad \text{V. T. 24. N°. 1, 2.}$$

$$13) \int \frac{1}{q^2 + x^2} \frac{dx}{1 - x^2} = \frac{1}{1 + q^2} \frac{\pi}{2 q} \quad \text{V. T. 24. N°. 3, 4.}$$

$$14) \int \frac{x^2}{q^2 + x^2} \frac{dx}{1 - x^2} = -\frac{1}{1 + q^2} \frac{q \pi}{2} \quad \text{V. T. 24. N°. 1, 2.}$$

$$15) \int \frac{dx}{(1 + x^4)(1 + x^6)} = \frac{\pi}{4} \left(\sqrt{2} - \frac{1}{3} \right)$$

$$16) \int \frac{x^{2b}}{1 + x^{2a} 1 + x^{3a}} dx = \frac{\pi}{4a} \left\{ \operatorname{Cosec.} \left(\frac{2b+1}{2a} \pi \right) + \operatorname{Sec.} \left(\frac{2b+1}{2a} \pi \right) \right\} + \frac{\pi}{6a} \frac{1 + 4 \operatorname{Cos.} \left(\frac{2b+1}{3a} 2\pi \right) + 4 \operatorname{Cos.} \left(\frac{2b+1}{3a} 2\pi - \frac{2b+1}{a} \pi \right)}{\operatorname{Sin.} \left(\frac{2b+1}{a} \pi \right)}$$

$$17) \int \frac{x^{p-1}}{1 + x^a 1 + x^b} dx = \frac{\pi}{2a \operatorname{Sin} p\pi} \left\{ \frac{\operatorname{Cos.} \left(\frac{1-a}{a} p\pi \right) + \operatorname{Cos.} \left(\frac{(1-a)(p-b)}{a} \pi \right)}{1 + \operatorname{Cos.} \left(\frac{b}{a} \pi \right)} + \frac{\operatorname{Cos.} \left(\frac{3-a}{a} p\pi \right) + \operatorname{Cos.} \left(\frac{(3-a)(p-b)}{a} \pi \right)}{1 + \operatorname{Cos.} \left(\frac{b}{a} 3\pi \right)} + \dots \right\} \\ + \frac{\pi}{2b \operatorname{Sin} p\pi} \left\{ \frac{\operatorname{Cos.} \left(\frac{1-b}{b} p\pi \right) + \operatorname{Cos.} \left(\frac{(1-b)(p-a)}{b} \pi \right)}{1 + \operatorname{Cos.} \left(\frac{a}{b} \pi \right)} + \frac{\operatorname{Cos.} \left(\frac{3-b}{b} p\pi \right) + \operatorname{Cos.} \left(\frac{(3-b)(p-a)}{b} \pi \right)}{1 + \operatorname{Cos.} \left(\frac{a}{b} 3\pi \right)} + \dots \right\}$$

Dans 16), 17) a et b sont des entiers quelconques; $1 + x^a$ et $1 + x^b$ n'ont pas de racine commune; p est > 0 et quelconque, rationnel ou irrationnel, mais $< (a+b)$; les dénominateurs des derniers termes sont pour les deux séries respectivement: $1 + \operatorname{Cos.} \left(\frac{2a-1}{a} b\pi \right)$ et $1 + \operatorname{Cos.} \left(\frac{2b-1}{b} a\pi \right)$. — Des formules (15) à (17) voyez Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

$$18) \int \left(\frac{x}{p^2 r^2 + x^2} - \frac{x}{q^2 r^2 + x^2} \right) dx = \frac{1}{2} \log \frac{q}{p} \quad \text{Schlömlich Gr. 5. 152.}$$

- 1) $\int \frac{dx}{1+x+x^2} = \frac{2\pi}{3\sqrt{3}}$ Dienger, Gr. 10. 107.
- 2) $\int \frac{dx}{1-2x\cos\lambda+x^2} = \frac{\pi-\lambda}{\sin\lambda}, \frac{\pi}{2} > \lambda > 0$; Schlömilch, Gr. 10. 424. — Ohm, Ausw. 3. la trouve fautive ment égale à $-\lambda: \sin\lambda$.
- 3) $\int \frac{dx}{1+2x\cos\lambda+x^2} = \frac{\lambda}{\sin\lambda}, \frac{\pi}{2} > \lambda > 0$; Schlömilch, Gr. 10. 424.
- 4) $\int \frac{x^{p-1}dx}{1+x+x^2} = \frac{2\pi}{\sqrt{3}} \sin\left(\frac{1-p}{3}\pi\right) \operatorname{Cosec} p\pi, 0 < p < 2$; Dienger, Gr. 10. 107.
- 5) $\int \frac{x^p dx}{1+2x\cos\lambda+x^2} = \frac{\pi}{\sin p\pi} \frac{\sin p\lambda}{\sin\lambda}, p^2 < 1, \lambda^2 < \pi^2$; Legendre, Exerc. 4. 102. — Schlömilch, Gr. 12. 198.
- 6) $\int \frac{x^p dx}{1+2q\cos\lambda+q^2x^2} = \frac{\pi}{q^{p+1}\sin p\pi} \frac{\sin p\lambda}{\sin\lambda}, p^2 < 1, \lambda^2 < \pi^2$; Plana, Mém. Brux. T. 10. — Legendre, Exerc. 4. 102, trouve fautive ment qp au lieu de $qp+1$.
- 7) $\int \frac{1+ax}{1+2ax+(a^2+b^2)x^2} x^{p-1} dx = \frac{\pi}{\sqrt{(a^2+b^2)^p}} \operatorname{Cosec} p\pi \operatorname{Cos} \left(p \operatorname{Arctg} \frac{b}{a}\right)$
- 8) $\int \frac{bx}{1+2ax+(a^2+b^2)x^2} x^{p-1} dx = \frac{\pi}{\sqrt{(a^2+b^2)^p}} \operatorname{Cosec} p\pi \operatorname{Sin} \left(p \operatorname{Arctg} \frac{b}{a}\right)$
- 9) $\int \frac{x^p dx}{1+x^2+2ax\cos\left(\operatorname{Arctg} \frac{b}{a}\right)} = \pi \operatorname{Cosec} p\pi \operatorname{Sin} \left(p \operatorname{Arctg} \frac{b}{a}\right) \operatorname{Cosec} \left(\operatorname{Arctg} \frac{b}{a}\right)$
- 10) $\int \frac{x^{1-p} dx}{b^2+(x+a)^2} = \frac{\pi i}{b \sin p\pi} \frac{(a-bi)^{1-p} - (a+bi)^{1-p}}{2}$
- 11) $= \frac{\pi}{\sqrt{(a^2+b^2)^p}} \frac{\operatorname{Sin} \left\{ (1-p) \operatorname{Arctg} \frac{b}{a} \right\}}{\operatorname{Sin} p\pi \operatorname{Sin} \left(\operatorname{Arctg} \frac{b}{a}\right)}$
- 12) $\int \frac{x^{1-p} dx}{1+2q\cos\lambda+q^2x^2} = \frac{\pi}{q^p} \frac{\operatorname{Sin} \{(1-p)\lambda\}}{\operatorname{Sin} \lambda \operatorname{Sin} \{(1-p)\pi\}}, p < 1, q < 1$
- 13) $\int \frac{x^p dx}{q^2+2q\cos\lambda+x^2} = \frac{\pi q^{p-1}}{\sin p\pi} \frac{\sin p\lambda}{\sin\lambda}, p^2 < 1, \lambda^2 < \pi^2$; Schlömilch, Gr. 12. 198.
- 14) $\int \frac{dx}{1-2x\cos\frac{2a\pi}{b}+x^2} = \frac{1}{2b \sin \frac{2a\pi}{b}} \sum_{i=1}^{b-1} \operatorname{Sin} \frac{2na\pi}{b} \operatorname{Cot} \frac{n\pi}{b}$ Schlömilch, Gr. 10. 424.

- 15) $\int \frac{x^{p-1} dx}{1+x^2+x^4} = \frac{\pi}{\sqrt{3}} \operatorname{Cosec} \frac{p\pi}{2} \cdot \operatorname{Sin} \left(\frac{\pi}{3} - \frac{p\pi}{6} \right), 0 < p < 4; \text{ Dienger, Gr. 10. 341.}$
- 16) $\int \frac{1+x^2}{1-x^2+x^4} dx = \pi \quad \text{Raabe, Cr. 37. 356.}$
- 17) $\int \frac{dx}{a+bx^2+cx^4} = \frac{\pi}{2\sqrt{\{ab+2a\sqrt{ac}\}}}$
- 18) $\int \frac{x^2 dx}{a+bx^2+cx^4} = \frac{\pi}{2\sqrt{\{bc+2c\sqrt{ac}\}}}$
- 19) $\int \frac{dx}{x^2+(a+bx^2)^2} = \frac{\pi}{2a\sqrt{1+4ab}}$
- 20) $\int \frac{x^2 dx}{x^2+(a+bx^2)^2} = \frac{\pi}{2b\sqrt{1+4ab}}$
- 21) $\int \frac{p^2-q^2+x^2}{(p^2+q^2)^2+2(p^2-q^2)x^2+x^4} dx = \frac{1}{2} \frac{p\pi}{p^2+q^2}$
- 22) $\int \frac{dx}{(p^2+q^2)^2+2(p^2-q^2)x^2+x^4} = \frac{1}{4p} \frac{\pi}{p^2+q^2}$
- 23) $\int \frac{x^{b-1} dx}{1-2x^a \operatorname{Cos} \lambda + x^{2a}} = \frac{\pi}{a} \operatorname{Cosec} \lambda \cdot \operatorname{Cosec} \frac{b\pi}{a} \cdot \operatorname{Sin} \left\{ \frac{b(\pi-\lambda)+a\lambda}{a} \right\} \quad \text{Euler, Calc. Int. T. 4. S. 5. 179.}$
- 24) $\int \frac{x^{b-1} dx}{1+2x^a \operatorname{Cos} \lambda + x^{2a}} = \frac{\pi}{a} \operatorname{Cosec} \lambda \cdot \operatorname{Cosec} \frac{b\pi}{a} \cdot \operatorname{Sin} \left(\frac{a-b}{a} \lambda \right) \quad \text{Euler, Calc. Int. T. 4. S. 5. 183.}$
- 25) $\int \frac{x^{a+b-1} dx}{1+2x^a \operatorname{Cos} \lambda + x^{2a}} = \frac{\pi}{a} \operatorname{Cosec} \lambda \cdot \operatorname{Cosec} \frac{b\pi}{a} \cdot \operatorname{Sin} \frac{b\lambda}{a} \quad \text{Euler, Calc. Int. T. 4. S. 5. 191.}$

Plana, Mém. Turin. 1820.

Ohm, Ausw. N°. 2.

- 1) $\int \frac{x^{p+1} dx}{(1+2x \operatorname{Cos} \lambda + x^2)^2} = \frac{\pi}{2 \operatorname{Sin} p\pi} \frac{p \operatorname{Sin} \lambda \cdot \operatorname{Cos} p\lambda - \operatorname{Cos} \lambda \cdot \operatorname{Sin} p\lambda}{\operatorname{Sin}^3 \lambda} \quad \text{Legendre, Exerc. 4. 108.}$
- 2) $\int \frac{1}{1+2x \operatorname{Cos} \lambda + x^2} \frac{dx}{x^p} = \frac{\pi}{\operatorname{Sin} p\pi} \frac{\operatorname{Sin} p\lambda}{\operatorname{Sin} \lambda}, p^2 < 1; \text{ Legendre, Exerc. 4. 102.}$
- 3) $\int \frac{x+a}{b^2+(x+a)^2} \frac{dx}{x^p} = \frac{\pi}{\sqrt{(a^2+b^2)^p}} \frac{\operatorname{Cos} (p \operatorname{Arctg} \frac{b}{a})}{\operatorname{Sin} p\pi} \quad \text{Cauchy, P. 28. 147. I. § 2.}$
- 4) $= \frac{\pi}{\operatorname{Sin} p\pi} \frac{(a-bi)^{-p} + (a+bi)^{-p}}{2} \quad \text{Plana, Mém. Brux. 1837.}$

$$\begin{aligned}
 5) \int \frac{1}{b^2 + (x+a)^2} \frac{dx}{x^p} &= \frac{\pi}{\sqrt{(a^2 + b^2)^p}} \frac{\text{Sin.} \left(p \text{ Arctg.} \frac{b}{a} \right)}{\text{Sin.} p \pi} \quad \text{Cauchy, P. 28. 147. I. § 2.} \\
 6) &= \frac{\pi i}{\text{Sin.} p \pi} \frac{(a - bi)^{-p} - (a + bi)^{-p}}{2} \\
 7) &= \frac{\pi}{\sqrt{(a^2 + b^2)^{p+1}}} \frac{\text{Sin.} \left(p \text{ Arctg.} \frac{b}{a} \right)}{\text{Sin.} p \pi \cdot \text{Sin.} \left(\text{Arctg.} \frac{b}{a} \right)} \\
 8) \int \frac{1}{1 + 2 q x \text{Cos.} \lambda + q^2 x^2} \frac{dx}{x^p} &= \frac{\pi}{q^{p-1}} \text{Sin.} p \lambda \cdot \text{Cosec.} p \pi \cdot \text{Cos.} \lambda, p < 1, q < 1; \\
 9) \int \frac{x^p - 2 \text{Cos.} \mu + x^{-p}}{x^q - 2 \text{Cos.} \lambda + x^{-q}} \frac{dx}{x} &= \frac{2 \pi \text{Sin.} \left(p \frac{\pi - \lambda}{q} \right)}{q \text{Sin.} \lambda \cdot \text{Sin.} \frac{p \pi}{q}} - \frac{2 (\pi - \lambda) \text{Cos.} \mu}{q \text{Sin.} \lambda} \\
 10) \int \frac{x^p + x^{-p}}{x^q - 2 \text{Cos.} \lambda + x^{-q}} \frac{dx}{x} &= \frac{2 \pi \text{Sin.} \left(p \frac{\pi - \lambda}{q} \right)}{q \text{Sin.} \lambda \cdot \text{Sin.} \frac{p \pi}{q}} \\
 11) \int \frac{x^{\pm p}}{x^q - 2 \text{Cos.} \lambda + x^{-q}} \frac{dx}{x} &= \frac{\pi \text{Sin.} \left(p \frac{\pi - \lambda}{q} \right)}{q \text{Sin.} \lambda \cdot \text{Sin.} \frac{p \pi}{q}} \\
 12) \int \frac{x^p + x^{-p}}{x^q + \left(a + \frac{1}{a} \right) + x^{-q}} \frac{dx}{x} &= \frac{\frac{p}{a^q} - a^{-\frac{p}{q}}}{a - \frac{1}{a}} \frac{2 \pi}{a} \text{Cosec.} \frac{p \pi}{q} \\
 13) \int \frac{dx}{x^6 + a x^4 + b x^2 + c} &= \frac{p \pi}{p(p^2 - a) \sqrt{c - 2c}} \\
 14) \int \frac{x^2 dx}{x^6 + a x^4 + b x^2 + c} &= \frac{\pi \sqrt{c}}{p(p^2 - a) \sqrt{c - 2c}} \\
 15) \int \frac{x^4 dx}{x^6 + a x^4 + b x^2 + c} &= \frac{\frac{1}{2} \pi \sqrt{c}}{p(p^2 - a) \sqrt{c - 2c}}
 \end{aligned}$$

Plana. Mém. Brux. 1837

Euler. N. A. Petr. III. 3.

Poisson L. 2. 224.

, où p la plus grande racine de $(Z^2 - a)^2 - 8Z\sqrt{c} - 4b = 0$.

$$1) \int \frac{x^{\frac{1}{2}p-1} dx}{(1+x)^a} = 2 \frac{\Gamma(\frac{1}{2}p)\Gamma(a-\frac{1}{2}p)}{\Gamma(a)}, 1 > p > 0; \text{ Schlömilch, Beitr. III. 13.}$$

$$2) \int \frac{x^{p-\frac{1}{2}} dx}{1+x} = \pi \operatorname{Sec.} p \pi$$

$$3) \int \frac{x^{p-\frac{1}{2}} dx}{(1+x)^2} = \frac{1-2p}{2} \pi \operatorname{Sec.} p \pi$$

$$4) \int \frac{x^{p+\frac{1}{2}} dx}{(1+x)^2} = \frac{2p+1}{2} \pi \operatorname{Sec.} p \pi$$

$$5) \int \frac{x^{p+a+\frac{1}{2}} dx}{(1+x)^{a+b+2}} = \frac{(1+2p)^{a+1/2} (1-2p)^{b/2}}{1^{a+b+1/2} 2^{a+b+1}} \pi \operatorname{Sec.} p \pi = \int \frac{x^{b-p-\frac{1}{2}} dx}{(1+x)^{a+b+2}} \quad \left. \begin{array}{l} \text{Oettinger, Cr. 38.} \\ 162. \end{array} \right\}$$

$$6) \int \frac{x^{p-a+\frac{1}{2}} dx}{(1+x)^{-a+b+2}} = \frac{(1-2p)^{b/2}}{(2p-1)^{a+1/2}} 2^{a-b-1} \pi \operatorname{Sec.} p \pi = \int \frac{x^{b-p-\frac{1}{2}} dx}{(1+x)^{-a+b+2}}$$

$$7) \int \frac{x^{p+a+\frac{1}{2}} dx}{(1+x)^{a-b+2}} = (-1)^b \frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2} 1^{a-b+1/2}} 2^{b-a} \pi \operatorname{Sec.} p \pi$$

$$8) \int \frac{x^{\frac{1}{2}(p-q-1)} dx}{(1+x)^{p+\frac{1}{2}}} = \frac{\Gamma\left(\frac{p-q+1}{2}\right) \Gamma\left(\frac{p+q}{2}\right)}{\Gamma(p+\frac{1}{2})}$$

$$9) \int \frac{x^{\frac{1}{2}(p-q-1)} dx}{(1+x)^{p-\frac{1}{2}}} = \frac{\Gamma\left(\frac{p-q+1}{2}\right) \Gamma\left(\frac{p+q}{2}-1\right)}{\Gamma(p-\frac{1}{2})}$$

$$10) \int \frac{x^{p-q} dx}{(1+x^2)^{p+\frac{1}{2}}} = \frac{\Gamma\left(\frac{p-q+1}{2}\right) \Gamma\left(\frac{p+q}{2}\right)}{2 \Gamma(p+\frac{1}{2})}$$

$$11) \int \frac{x^{p-q} dx}{(1+x^2)^{p-\frac{1}{2}}} = \frac{\Gamma\left(\frac{p-q+1}{2}\right) \Gamma\left(\frac{p+q}{2}-1\right)}{2 \Gamma(p-\frac{1}{2})}$$

Meijer, Int. Déf. 329.

$$12) \int \frac{x^{2a} dx}{(2+x^2)^{b+\frac{1}{2}}} = 2^{a-b-\frac{1}{2}} \frac{\Gamma(a+\frac{1}{2})\Gamma(b-a)}{\Gamma(b+\frac{1}{2})} \quad \text{Schlömilch, Gr. 6. 213.}$$

$$13) \int \frac{dx}{1 \vee (1+x^4)} = \Gamma'\left(\sin. \frac{\pi}{4}\right) \quad \text{Legendre, Exerc. 1. 146.}$$

$$14) \int \frac{1-x}{1 \vee (1-x^4)^3} dx = 0 \quad \text{Euler, N. C. Petr. 6. 115.}$$

- 15) $\int \frac{dx}{\sqrt[3]{1+x^6}} = \frac{2}{\sqrt[3]{3}} E' \left(\sin, \frac{5\pi}{12} \right)$ Legendre, Exerc. 1. 147.
- 16) $= \frac{2}{\sqrt[3]{3}} F' \left(\sin, \frac{\pi}{12} \right)$
- 17) $\int \frac{dx}{\sqrt[3]{1+x^3}} = \sec. \frac{\pi}{8} \sqrt[3]{\frac{1}{2}} F' \left(Tg, \frac{\pi}{8} \right)$ Legendre, Exerc. 1. 148.
- 18) $\int \frac{dx}{\sqrt[3]{1+x^{12}}} = \frac{1}{2\sqrt[3]{3}} \sec. \frac{\pi}{12} F' \left(\sin, \frac{\pi}{4} \right) + Tg. \frac{\pi}{12} F' \left(\sqrt[3]{\frac{2-\sqrt[3]{3}}{1+\sqrt[3]{3}}} \right)$ Legendre, Exerc. 1. 149.
- 19) $\int \frac{x^{2a} dx}{\left(\frac{1}{x^2} + x^2 \right)^{b+\frac{1}{2}}} = \frac{\Gamma \left(\frac{b+a+1}{2} \right) \Gamma \left(\frac{b-a}{2} \right)}{4 \Gamma \left(\frac{b}{2} + 1 \right)}$ Schlömilch, Gr. 6. 213.
- 20) $\int \frac{x^{\frac{1}{2}b \pm a-1} dx}{1+x^b} = \frac{\pi}{b} \sec. \frac{a\pi}{b}$ Euler, N. C. P. 6. 115.
- 21) $\int \frac{x^{p-1} dx}{\sqrt[3]{1+x^q}} = 2^{\frac{2p}{q}} B(q-2p, p), q > 2p;$
- 22) $\int \frac{x^{q-p-1} dx}{\sqrt[3]{1+x^q}} = 2^{2-\frac{2p}{q}} B(2p-q, q-p), q < 2p;$ } Plana, Cr. 17. 163.
- 23) $\int \frac{x^{b(a+1)-1} dx}{(1+x^b)^{a+\frac{c}{b}+1}} = \frac{1}{c} \frac{1^{\frac{c}{b}} 1^{a+1}}{1^{\frac{c}{b}+a+1}}$ Oettinger, Cr. 35. 13.
- 24) $\int \frac{x^{a-1} dx}{(1-x^b)^a} = \frac{\pi}{b} \operatorname{Cosec}. \frac{a\pi}{b}$
- 25) $\int \frac{x^{a+b-1} dx}{(1-x^b)^a} = \frac{a\pi}{b^2} \operatorname{Cosec}. \frac{a\pi}{b}$ } Euler, N. C. Petr. 6. 115.

- 1) $\int \frac{1}{1+x} \frac{dx}{\sqrt{x}} = \pi$ Dedekind, Cr. 45. 370.
- 2) $\int \frac{1}{1-x} \frac{dx}{\sqrt{x}} = 0$ Schlömilch, Beitr. III. § 6.
- 3) $\int \frac{dx}{(1+x)^2 x^{p+\frac{1}{2}}} = \frac{2p+1}{2} \pi \sec. p\pi$ Oettinger, Cr. 38. 162.

- 4) $\int \frac{dx}{(1+x)^2 x^{p-\frac{1}{2}}} = \frac{1-2p}{2} \pi \operatorname{Sec.} p \pi$
- 5) $\int \frac{dx}{(1+x) x^{p-\frac{1}{2}}} = \pi \operatorname{Sec.} p \pi$
- 6) $\int \frac{dx}{x^{p+b+\frac{1}{2}} (1+x)^{a-b+2}} = (-1)^b \frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2} 1^{a-b+1/2}} 2^{b-a} \pi \operatorname{Sec.} p \pi$
- 7) $\int \left(\frac{x^{\frac{1}{2}p} - x^{-\frac{1}{2}p}}{x-1} \right)^2 dx = 2(1-p \pi \operatorname{Cot.} p \pi), p^2 < 1; \text{ Minding, Taf. II.}$
- 8) $\int \frac{1}{\left(x + \frac{1}{x}\right)^{2(p-a)}} \frac{dx}{x} = \frac{\{\Gamma(p-a)\}^2}{2\Gamma(2p-2a)} \text{ Schlömilch, Gr. 6. 213.}$
- 9) $\int \frac{dx}{(q^2+x^2) \sqrt{p^2+x^2}} = \frac{1}{q \sqrt{p^2-q^2}} \operatorname{Arctg.} \frac{\sqrt{p^2-q^2}}{q}, q < p;$
- 10) $= \frac{1}{q \sqrt{q^2-p^2}} \operatorname{Arctg.} \frac{q + \sqrt{q^2-p^2}}{q}, q > p;$ Lobatto, Int. § 53.
- 11) $\int \frac{dx}{(1+px^2) \sqrt{1+9p^2x}} = \frac{x}{1+\sqrt{p}} \text{ Legendre, Exerc. 1. 140.}$
- 12) $\int \left\{ 1 - \frac{x^2+1}{(x^4+1)} \right\} \frac{dx}{x} = -l 2$
- 13) $\int \left\{ 1 - \frac{ax^2+c}{\sqrt{a^2x^4+2(ac-b^2)x^2+c^2}} \right\} \frac{dx}{x} = l \frac{ac-b^2}{ac}$ Winckler, Cr. 15. 102.
- 14) $\int \frac{dx}{1-(1+p^2x)(1+q^2x)(1+r^2x)} = \frac{2}{\sqrt{p^2-r^2}} \operatorname{F} \left(\frac{1}{2}, 1, \frac{p^2-q^2}{p^2-r^2} \right)$
- 15) $\int \frac{dx}{1-x(x+p^2)(x+q^2)(x+r^2)} = \frac{2}{\sqrt{p^2-r^2}} \operatorname{F} \left(\frac{1}{2}, 1, \frac{p-q^2}{p^2-r^2} \right)$
- 16) $\int \frac{dx}{1-(p^2-l^2x)(q^2+m^2x)(r^2+n^2x)} = \frac{\pi}{2pmn \operatorname{Sin.} \varphi} \operatorname{K} \left(\frac{n}{m}, \frac{p^2m^2-q^2l^2}{p^2n^2-r^2l^2} \right)$ où $\operatorname{Cos.} \varphi = \frac{r}{p}$,
et $\operatorname{Cos.} \varphi = \frac{r l}{p n}$; Jacob, Cr. 10. 101.
- 17) $\int \frac{x^{p-q} dx}{(a+bx+cx^2)^{p+\frac{1}{2}}} = \frac{1}{\Gamma(p+\frac{1}{2})} \left(\frac{c}{a} \right)^{\frac{q}{2}} \frac{\pi}{c} \sum_{n=0}^{\infty} \frac{(q-n)^{2n/4}}{2^n 2(21-ac)^n} \frac{\Gamma(p-n)}{(b+21-ac)^{p-n}}$ Schlömilch, Cr. 33. 268. — Id., Stud. I. 17.
- 18) $\int \frac{x^{p+q} dx}{(a+bx+cx^2)^{p+\frac{1}{2}}} = \frac{1}{\Gamma(p+\frac{1}{2})} \left(\frac{a}{c} \right)^{\frac{q}{2}} \frac{\pi}{c} \sum_{n=0}^{\infty} \frac{(q-n+1)^{2n/4}}{2^n 2(21-ac)^n} \frac{\Gamma(p-n)}{(b+21-ac)^{p-n}}$

$$\begin{aligned}
 19) \int \frac{p + \sqrt{\frac{1}{2}x}}{x + p \sqrt{2x + p^2} \sqrt{q^2 - x^2}} dx &= \frac{\pi}{2 \sqrt{2} q \cdot (q + p \sqrt{2} q + p^2)} \quad \text{Poisson, Chaleur. N°. 159.} \\
 20) \int \frac{q + \frac{1}{p} \sqrt{2x}}{q^2 + \frac{q}{p} \sqrt{2x + \frac{x}{p^2}}} \frac{dx \sqrt{\frac{1}{2}x}}{1 + r^2 x^2} &= \frac{\pi}{2 \sqrt{r} 1 + p q \sqrt{r}} \left. \begin{aligned} & \\ & \end{aligned} \right\} \text{Poisson, Chaleur. Suppl. Note B.} \\
 21) \int \frac{q + \frac{1}{p} \sqrt{2x}}{q^2 + \frac{q}{p} \sqrt{2x + \frac{x}{p^2}}} \frac{dx}{1 + r^2 x^2} &= \frac{\pi}{2 r 1 + p q \sqrt{r}} \\
 22) \int \frac{x^b}{\sqrt{\{1 + (2 - 4p^2)x^2 + x^4\}} (1 + x^2)^3} dx &= \frac{3}{8p^2} \{E'(p) - F'(p)\} + \frac{1}{2} F'(p) \left. \begin{aligned} & \\ & \end{aligned} \right\} \text{Schlömlich, Beitr. III. 16.} \\
 23) \int \frac{x^b}{\sqrt{\{1 + (2 - 4p^2)x^2 + x^4\}} (1 + x^2)^5} dx &= \frac{2p^2 + 1}{8p^2} E'(p) - \frac{1 - p^2}{8p^2} F'(p) \\
 24) \int \frac{x^b - 1}{1 - x} \frac{dx}{x} &= \pi \text{ Cot. } b \pi \quad \text{Dedekind, Eul. Int. S. 22.}
 \end{aligned}$$

$$\begin{aligned}
 1) \int \frac{dx}{x \pm q} &= 0 \quad \text{Grunert, Gr. 2. 266.} \\
 2) \int \frac{dx}{1 + x^2} &= \pi \quad \text{Ohm, Ausw. N° 2.} \\
 3) \int \frac{dx}{x^2 + p^2} &= \frac{1}{p} \pi \\
 4) \int \frac{x dx}{x^2 + p^2} &= 0 \\
 5) &= l a, \text{ pour } -\infty = -a(\infty); \\
 6) \int \frac{dx}{x^2 - p^2} &= 0 \quad \text{Poisson, P. 18. 295. N°. 41.} \\
 7) \int \frac{(-xi)^{p-1}}{1 + x^2} dx &= \pi \{(-i)^{p-1} + (i)^{p-1}\}, p < 2; \quad \text{Cauchy, Cours. Lec. 34.} \\
 8) &= \pi \\
 9) \int \frac{(-xi)^{p-1}}{1 - x^2} dx &= \pi \text{ Cos. } \frac{p\pi}{2} \left. \begin{aligned} & \\ & \end{aligned} \right\} \text{Meijer, Int. Déf. 154.}
 \end{aligned}$$

$$10) \int \frac{(-x i)^{p-1}}{1-x^2} dx = \frac{\pi}{2} \{(-i)^p + (i)^p\}, p < 2; \text{ Cauchy, Cours. Lec. 34.}$$

$$11) \int \frac{x^{2b} dx}{1+x^{2a}} = \frac{\pi}{a} \operatorname{Cosec.} \left\{ \frac{2b+1}{2a} \pi \right\}, 2b < 2a-1; \text{ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. (pour } a \text{ et } b \text{ quelconques). — Serret, L. 8. 1. — Grunert, Gr. 2. 266.}$$

$$12) \int \frac{x^{2b-1} dx}{1+x^{2a}} = 0, 2b < 2a-1;$$

$$13) \int \frac{x^{2b} dx}{1+x^{2a-1}} = \frac{\pi}{2a-1} \operatorname{Cot.} \left\{ \frac{2b+1}{2a-1} \frac{\pi}{2} \right\} \left. \vphantom{\int} \right\} \text{ Grunert, Gr. 2. 266.}$$

$$14) \int \frac{x^{2b-1} dx}{1+x^{2a-1}} = \frac{\pi}{2a-1} \operatorname{Tang.} \frac{b\pi}{2a-1}$$

$$15) \int \frac{x^{2b} dx}{1-x^{2a}} = \frac{\pi}{a} \operatorname{Cot.} \left\{ \frac{2b+1}{2a} \pi \right\}, 2b < 2a-1; \text{ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. (pour } a \text{ et } b \text{ quelconques). — Grunert, Gr. 2. 266.}$$

$$16) \int \frac{x^{2b-1} dx}{1-x^{2a}} = 0, 2b < 2a-1;$$

$$17) \int \frac{x^{2b} dx}{1-x^{2a-1}} = \frac{\pi}{2a-1} \operatorname{Cot.} \left\{ \frac{2b+1}{2a-1} \frac{\pi}{2} \right\} \left. \vphantom{\int} \right\} \text{ Grunert, Gr. 2. 266.}$$

$$18) \int \frac{x^{2b-1} dx}{1-x^{2a-1}} = -\frac{\pi}{2a-1} \operatorname{Tang.} \frac{b\pi}{2a-1}$$

$$1) \int \frac{dx}{(r+xi)^p (s-xi)^q} = 2\pi (r+s)^{1-p-q} \frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)}, \left. \vphantom{\int} \right\} \begin{array}{l} r > 0, s > 0, \\ 1 > p > 0, 1 > q > 0; \end{array}$$

$$2) \int \frac{dx}{(r+xi)^p (s+xi)^q} = 0 \left. \vphantom{\int} \right\} \text{ Cauchy, Lim. Imag. N. 102-106}$$

$$3) \int \frac{dx}{(r-xi)^p (s-xi)^q} = 0$$

$$4) \int \left(\frac{p-qi}{x-r-si} + \frac{p+qi}{x-r+si} \right) dx = 2\pi q + 2p\alpha, \text{ pour } -\infty = -\alpha(\infty). \text{ Cauchy, Cours. Lec. 32.}$$

$$5) \int \left(\frac{p-qi}{x-r-si} - \frac{p+qi}{x-r+si} \right) dx = 2\pi q \text{ Cauchy, Cours. Lec. 32. — Grunert, Gr. 2. 266.}$$

$$6) \int \left(\frac{1}{x-r-si} + \frac{1}{x-r+si} \right) dx = 0 \text{ Grunert, Gr. 2. 266}$$

- 7) $\int \frac{1}{1 - qxi)^p} \frac{dx}{1 + x^2} = \frac{\pi}{(1 + q)^p}$ V. T. 113. N°. 17. et T. 147. N°. 8.
- 8) $\int \frac{(p + xi)^{a-1}}{(1 - p - xi)^{a+b}} dx = 2 \operatorname{Sin.} a \pi \frac{\Gamma(a) \Gamma(b)}{\Gamma(a+b)}, -\infty < a < 1;$
- 9) $\int \frac{(p + xi)^{q-1}}{(1 - p - xi)^{1+r}} dx = \frac{2 \operatorname{Sin.} r \pi \operatorname{Sin.} q \pi}{\operatorname{Sin.} \{(q+r) \pi\}} \frac{\Gamma(q) \Gamma(r)}{\Gamma(q+r)}, -\infty < (q+r) < 1;$
- 10) $\int \frac{dx}{1 + x + x^2} = \frac{2\pi}{1 \sqrt{3}}$ Ohm, Ausw. 8.
- 11) $\int \frac{dx}{1 - x + x^2} = \frac{2\pi}{1 \sqrt{3}}$ Ohm, Ausw. 9
- 12) $\int \frac{x - a}{(x - a)^2 + b^2} dx = \alpha$, pour $-\infty = -\alpha(\infty)$; Cauchy, Cours. Leç. 32.
- 13) $= 0$
- 14) $\int \frac{dx}{(x - a)^2 + b^2} = \frac{1}{b} \pi$ } Cauchy, Cours. Leç. 32. — Grunert, Gr. 2. 265.
- 15) $\int \frac{dx}{1 \pm x \sqrt{3} + x^2} = 2\pi$ }
- 16) $\int \frac{1 + x^2}{1 - x^2 + x^4} dx = 2\pi$ } Raabe, Cr 37. 356.
- 17) $\int \frac{dx}{1 - 2x \operatorname{Cos.} \lambda + x^2} = \pi \operatorname{Cosec.} \lambda$ Schlömilch, Int. 117.
- 18) $\int \frac{a + bx}{x^2 + 2cx \operatorname{Cos.} \lambda + c^2} dx = \frac{\pi}{c \operatorname{Sin.} \lambda} (a - \frac{1}{2} b^2)$ Plana, Mém. Turin. 1818. 7. Art. 1. N°. 12.

- 1) $\int \frac{(x-1)^p}{x} dx = -\pi \operatorname{Cosec.} p \pi$
- 2) $\int \frac{(x-1)^{1-p}}{x} dx = -\pi \operatorname{Cosec.} p \pi$ } Oettinger, Cr. 35. 13.
- 3) $\int \frac{dx}{x^p} = \frac{1}{p-1}, p > 1;$
- 4) $= \infty, p \leq 1;$ } Raabe, Int. 120.

- 5) $\int \frac{(x-1)^{1-p} dx}{x^3} = \frac{1-p}{2} p \pi \operatorname{Cosec.} p \pi$
- 6) $\int \frac{(x-1)^{1+p} dx}{x^3} = \frac{1+p}{2} p \pi \operatorname{Cosec.} p \pi$
- 7) $\int \frac{(x-1)^{p+c} dx}{x^{b+c+2}} = \frac{(1+p)^{c+1} (1-p)^{b/2}}{1^{b+c+1}} p \pi \operatorname{Cosec.} p \pi = \int \frac{(x-1)^{b-p} dx}{x^{b+c+2}}$
- 8) $\int \frac{(x-1)^{p+c} dx}{x^{c-b+2}} = \frac{(1+p)^{c+1}}{(1+p)^{b-1/2} 1^{c-b+2/2}} \pi \operatorname{Cosec.} p \pi$
- 9) $\int \frac{(x-1)^{p-c} dx}{x^{b-c+2}} = \frac{(1-p)^{b/2}}{1^{c-1/2} 1^{b-c-1/2}} p \pi \operatorname{Cosec.} p \pi = \int \frac{(x-1)^{b-p} dx}{x^{b-c+2}}$
- 10) $\int \frac{(x-1)^{a-1} dx}{x^b} = \frac{1^{a-1/2} 1^{b-a-1/2}}{1^{b-1/2}} = \int \frac{(x-1)^{b-a-1} dx}{x^b}$
- 11) $\int \frac{(x-1)^{a \pm b-1} dx}{x^{a \pm 2b+c}} = \frac{1^{a \pm b-1/2} 1^{c \pm b-1/2}}{1^{a \pm 2b+c-1/2}} = \int \frac{(x-1)^{c \pm b-1} dx}{x^{a \pm 2b+c}}$
- 12) $\int \frac{(x^b-1)^a dx}{x^{ab+c-1}} = \frac{1}{c} \frac{1^{c/2} 1^{a/2}}{1^{b+a/2}}$
- 13) $\int \frac{dx}{1+x} = \infty$
- 14) $\int \frac{x^p dx}{1+x} = \infty$
- 15) $\int \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx = B(p, q)$ Binet, P. 27. 123.
- 16) $\int \frac{dx}{1+x^2} = \frac{\pi}{2}$ Raabe, Int. 136. — Ohm, Ausw. 3.
- 17) $\int \frac{x^{q-1} + x^{p-1}}{x^{p+q} + 1} dx = \frac{\pi}{p+q} \operatorname{Sec.} \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\}$
- 18) $\int \frac{x^{q-1} - x^{p-1}}{x^{p+q} - 1} dx = \frac{\pi}{p+q} \operatorname{Tang.} \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\}$
- 19) $\int \frac{1+x^2}{1+x^2+x^4} dx = \frac{1}{2} \pi$ Raabe, Cr. 37. 356.
- 20) $\int \frac{dx}{x(x-1)^p} = \pi \operatorname{Cosec.} p \pi$ Oettinger, Cr. 35. 13.

$$\begin{aligned}
 21) \int \frac{dx}{x^3 (x-1)^p} &= \frac{1-p}{2} p \pi \operatorname{Cosec}. p \pi \\
 22) \int \frac{dx}{x^3 (x-1)^{p-1}} &= \frac{1+p}{2} p \pi \operatorname{Cosec}. p \pi \\
 23) \int \frac{dx}{x^{b+c+2} (x-1)^{b+p}} &= \frac{(1+p)^{c/1}}{(1+p)^{b-1/1} 1^{c-b+1/1}} \pi \operatorname{Cosec}. p \pi \\
 24) \int \frac{x^{p-1} - x^{-p-1}}{x^q - x^{-q}} dx &= \frac{\pi}{2q} \operatorname{Sec}. \frac{p\pi}{2q} \quad \text{Malmsten, Cr. 38. 1.}
 \end{aligned}
 \left. \vphantom{\int} \right\} \text{Oettinger, Cr. 38. 162.}$$

$$\begin{aligned}
 1) \int \frac{(x-1)^{p-\frac{1}{2}}}{x} dx &= \pi \operatorname{Sec}. p \pi \\
 2) \int \frac{(x-1)^{p-\frac{1}{2}}}{x^2} dx &= \frac{1-2p}{2} \pi \operatorname{Sec}. p \pi \\
 3) \int \frac{(x-1)^{p+\frac{1}{2}}}{x^2} dx &= \frac{2p+1}{2} \pi \operatorname{Sec}. p \pi \\
 4) \int \frac{(x-1)^{p+a+\frac{1}{2}}}{x^{a+b+2}} dx &= \frac{(1+2p)^{a+1/2} (1-2p)^{b/2}}{1^{a+b+1/1} 2^{a+b+1}} \pi \operatorname{Sec}. p \pi = \int \frac{(x-1)^{p+b-\frac{1}{2}}}{x^{a+b+2}} dx \\
 5) \int \frac{(x-1)^{p-a+\frac{1}{2}}}{x^{a+b+2}} dx &= \frac{-2p)^{b/2}}{(2p-1)^{a/2-2}} 2^{a-b-1} \pi \operatorname{Sec}. p \pi = \int \frac{(x-1)^{p+b-\frac{1}{2}}}{x^{b-a+2}} dx \\
 6) \int \frac{(x-1)^{p+a+\frac{1}{2}}}{x^{a-b+2}} dx &= (-1)^b \frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2} 1^{a-b+1/1}} 2^{b-a} \pi \operatorname{Sec}. p \pi = \int \frac{(x-1)^{p-b-\frac{1}{2}}}{x^{a-b+2}} dx \\
 7) \int \frac{(x^b-1)^{-1+\frac{a}{b}}}{x} dx &= \frac{\pi}{b} \operatorname{Cosec}. \frac{a\pi}{b} \quad \text{Oettinger, Cr. 35. 13.} \\
 8) \int \frac{(x^b-1)^{\frac{a}{b}+c}}{x^{(c+g)b+1}} dx &= \frac{(b-a)^{c/b} (b+a)^{g/b}}{b^{c+g/b}} \frac{a}{a+bc} \frac{\pi}{b} \operatorname{Cosec}. \frac{a\pi}{b} \\
 9) \int \frac{(x^b-1)^{\frac{a}{b}+c}}{x^{2bc+1}} dx &= \left(1 - \frac{a^2}{b^2}\right) \left(4 - \frac{a^2}{b^2}\right) \dots \left(c^2 - \frac{a^2}{b^2}\right) \frac{a}{1^{2c/1}} \frac{1}{a+bc} \frac{\pi}{b} \operatorname{Cosec}. \frac{a\pi}{b} \\
 10) \int \frac{(x^b-1)^{\frac{a}{b}+g}}{x^{b(g-c)+1}} dx &= (-1)^c \frac{(b-a)^{c/b} b^{g-c-1}}{(b-a)^{g/b} 1^{c-g/1}} \pi \operatorname{Cosec}. \frac{a\pi}{b}
 \end{aligned}
 \left. \vphantom{\int} \right\} \text{Oettinger, Cr. 38. 162.}$$

$$11) \int \frac{(x^b - 1)^{-\frac{a}{b} + c}}{x} dx = (-1)^c \frac{\pi}{b} \operatorname{Cosec}. \frac{a\pi}{b} \quad \text{Oettinger, Cr. 38. 162.}$$

$$12) \int \frac{\left(1 + \frac{1}{x^2}\right) \left(x - \frac{1}{x}\right)^{2a}}{\left(x^2 + \frac{1}{x^2}\right)^{b+\frac{1}{2}}} dx = 2^{a-b-\frac{1}{2}} \frac{\Gamma(a + \frac{1}{2}) \Gamma(b - a)}{\Gamma(b + \frac{1}{2})} \quad \text{Schlömlich, Gr. 6. 213.}$$

$$\left. \begin{aligned} 13) \int \frac{dx}{x \sqrt{x^2 - 1}} &= \frac{1}{2} \pi \operatorname{Cosec}. \frac{\pi}{2} \\ 14) \int \frac{dx}{x \sqrt{x^p - 1}} &= \frac{\pi}{p} \operatorname{Cosec}. \frac{\pi}{p} \end{aligned} \right\} \quad \text{Raabe, Int. 147. — Ohm, Ausw. 14.}$$

$$\left. \begin{aligned} 15) \int \frac{dx}{x(x-1)^{p-\frac{1}{2}}} &= \pi \operatorname{Sec}. p \pi \\ 16) \int \frac{dx}{x^2(x-1)^{p-\frac{1}{2}}} &= \frac{1-2p}{2} \pi \operatorname{Sec}. p \pi \\ 17) \int \frac{dx}{x^2(x-1)^{p+\frac{1}{2}}} &= \frac{2p+1}{2} \pi \operatorname{Sec}. p \pi \\ 18) \int \frac{dx}{x(x^b-1)^{\frac{a}{b}}} &= \frac{\pi}{b} \operatorname{Cosec}. \frac{a\pi}{b} \\ 19) \int \frac{dx}{x(x^b-1)^{\frac{a}{b}+c}} &= (-1)^c \frac{\pi}{b} \operatorname{Cosec}. \frac{a\pi}{b} \\ 20) \int \frac{dx}{x^{(c-g)b+1} (x^b-1)^{a+g}} &= (-1)^g \frac{(b+a)^{c/b} a^{bg-c-1}}{a^{g/b} 1^{c-g/1} a+b^c} \pi \operatorname{Cosec}. \frac{a\pi}{b} \end{aligned} \right\} \quad \text{Oettinger, Cr. 38. 162.}$$

$$\left. \begin{aligned} 1) \int dx \sqrt{p^2 - x^2} &= \frac{1}{4} p^2 \pi \\ 2) \int x dx \sqrt{p^2 - x^2} &= \frac{1}{3} p^3 \\ 3) \int x^2 dx \sqrt{p^2 - x^2} &= \frac{1}{16} p^4 \pi \end{aligned} \right\} \quad \text{Sohnke, Samml.}$$

$$\begin{aligned}
 4) \int x^{2b} dx \sqrt{p^2 - x^2} &= \frac{1^{b/2}}{2^{b+1/2}} p^{2b+2} \frac{\pi}{2} \\
 5) \int x^{2b+1} dx \sqrt{p^2 - x^2} &= \frac{2^{b/2}}{3^{b+1/2}} p^{2b+3} \\
 6) \int dx \sqrt{p^2 - x^2}^{2b} &= \frac{2^{b/2}}{3^{b/2}} p^{2b+1} \\
 7) \int dx \sqrt{p^2 - x^2}^{2b-1} &= \frac{1^{b/2}}{2^{b/2}} p^{2b} \frac{\pi}{2} \\
 8) \int x dx \sqrt{p^2 - x^2}^b &= \frac{1}{b+2} p^{b+2} \\
 9) \int x^{2b+1} dx \sqrt{p^2 - x^2}^c &= \frac{2^{b/2}}{(c+2)^{b+1/2}} a^{2b+c+2} \\
 10) \int x^{2b} dx \sqrt{p^2 - x^2}^{2c-1} &= \frac{1^{b/2} 1^{c/2}}{2^{b+c/2}} a^{2(b+c)} \frac{\pi}{2} \\
 11) \int x^{2b} dx \sqrt{p^2 - x^2}^{2c} &= \frac{1^{b/2} 2^{c/2}}{3^{b+c/2}} a^{2(b+c)+1}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Sohnke, Samml.} \\ \\ \\ \\ \text{Dienger, Cr. 38. 266.} \end{array}$$

$$\begin{aligned}
 1) \int \frac{dx}{1+x^2} &= \text{Arctg. } p \quad \text{Raabe, Int. 136.} \\
 2) \int \frac{k dx}{k^2 + x^2} &= \frac{1}{2} \pi, \text{ pour } k = 0; \quad \text{Schlömlich, Gr. 11. 63.} \\
 3) \int \frac{dx}{x^2 - p^2} &= -\infty \left\{ \begin{array}{l} \text{Bidone, Mém. Turin, 1812. 231. Art. 1. N. 31. — Plana, Mém. Turin,} \\ \text{1818. 7. Art. 1. N. 4.} \end{array} \right. \\
 4) &= \frac{1}{2p} \ln \left| \frac{0}{2p} \right| \\
 5) \int \frac{dx}{\sqrt{1-x^2}} &= \text{Arcsin. } p, p^2 < 1; \quad \text{Raabe, Int. 135.} \\
 6) \int \frac{dx}{\sqrt{p^2 - x^2}} &= \frac{1}{2} \pi \quad \text{Cauchy, Cours. Leç. 32.} \\
 7) \int \frac{x dx}{\sqrt{p^2 - x^2}} &= p \quad \text{Sohnke, Samml.}
 \end{aligned}$$

$$\begin{aligned}
 & \left. \begin{aligned}
 8) \int \frac{x^2 dx}{\sqrt{(p^2 - x^2)}} &= \frac{1}{4} p^2 \pi \\
 9) \int \frac{x^3 dx}{\sqrt{(p^2 - x^2)}} &= \frac{2}{3} p^3 \\
 10) \int \frac{x^{2b} dx}{\sqrt{(p^2 - x^2)}} &= \frac{1^{1/2}}{2^{b/2}} p^{2b} \frac{\pi}{2} \\
 11) \int \frac{x^{2b+1} dx}{\sqrt{(p^2 - x^2)}} &= \frac{2^{b/2}}{3^{b/2}} p^{2b+1}
 \end{aligned} \right\} \text{Sohinke, Samml.} \\
 & \left. \begin{aligned}
 12) \int \frac{dx}{\sqrt{(p^2 - x^2)(b^2 - x^2)}} &= \frac{1}{b} F' \left(\frac{p}{b} \right) \\
 13) \int \frac{x^2 dx}{\sqrt{(p^2 - x^2)(b^2 - x^2)}} &= b \left\{ F' \left(\frac{p}{b} \right) - E' \left(\frac{p}{b} \right) \right\} \\
 14) \int \frac{x^4 dx}{\sqrt{(p^2 - x^2)(b^2 - x^2)}} &= \frac{1}{3} b^3 \left\{ \left(2 + \frac{p^2}{b^2} \right) F' \left(\frac{p}{b} \right) - 2 \left(1 + \frac{p^2}{b^2} \right) E' \left(\frac{p}{b} \right) \right\} \\
 15) \int \frac{x^b dx}{\sqrt{(p^2 - x^2)}} &= \frac{1^{b/2}}{2^{b/2}} p^b \pi
 \end{aligned} \right\} \text{Poisson, L. 2. 184.} \\
 & \left. \begin{aligned}
 16) \int \frac{dx}{x \sqrt{\frac{p^2 - c^2 x^2}{p^2 - x^2}}} &= \frac{p \pi}{2} \left[1 - \sum_{n=1}^{\infty} \frac{1}{2n-1} \left\{ \frac{1^{n/2}}{2^{n/2}} \left(\frac{c}{p} \right)^n \right\}^{n+1} \right] \\
 17) \int \frac{dx}{\sqrt{(p-x)(2bx-x^2)}} &= \frac{\pi}{\sqrt{2} b} \sum_{n=0}^{\infty} \left(\frac{1^{n/2}}{2^{n/2}} \right)^{n-1} \left(\frac{p}{2b} \right)^n
 \end{aligned} \right\} \text{Rogner, Mater.} \\
 & 18) \int \frac{dx}{(q^2+x)\sqrt{(1-x)}} = \frac{1}{\sqrt{(1+q^2)}} \left\{ l \frac{\sqrt{(1+q^2)} - \sqrt{(1-p)}}{\sqrt{(1+q^2)} + \sqrt{(1-p)}} + l \frac{\sqrt{(1+q^2)} + 1}{\sqrt{(1+q^2)} - 1} \right\}, p \leq 1; \text{Raabe, Int. 421.} \\
 & 19) \int \frac{2x^2 - b^2 - p^2}{\sqrt{\{(b^2 + p^2 - x^2)(b^2 p^2 - (b^2 + p^2)x^2 + x^4)\}}} dx = \frac{1}{2} \pi, b \geq p; \text{Dienger, Gr. 10. 341.} \\
 & 20) \int \frac{x^{b-1} (p-x)^{c-1} dx}{\{(g-h)x + hp+k\}^{b+1}} = \frac{\Gamma(b)\Gamma(c)}{\Gamma(b+c)} \frac{p^{b+c-1}}{(gp+k)^b (hp+k)^c} \text{Winckler, Cr. 45. 102.}
 \end{aligned}$$

$$\begin{aligned}
 & 1) \int_{-1}^0 \frac{dx}{x} = -\infty \quad \text{Cauchy, Cours. Lec. 24.} \\
 & 2) \int_0^1 (1 - p^2 x^2)^{q-1} dx = \frac{3q^{1/2}}{2q\Gamma(q+1)} \frac{\pi}{2p} \quad \text{Lindmann, Stockh. Handl. 1850. III.}
 \end{aligned}$$

$$3) \int_0^1 \frac{x^q dx}{1+p^2 x^2} = \frac{1}{4p^{q+1}} \left\{ Z' \left(\frac{q+3}{4} \right) - Z' \left(\frac{q+1}{4} \right) \right\} \quad \text{Lindmann, Stockh. Handl. 1850. III.}$$

$$4) \int_0^{V^{(1-p^2)}} \frac{(x^2+p^2)^{\frac{2b+1}{2}} dx}{(x^2+p^2)^{\frac{2b+1}{2}}} = \frac{1}{2b+2} \left\{ \frac{1-p^{2b+2}}{(1-p^2)^{b+1}} + \frac{(-1)^b p^{b+\frac{1}{2}} p^{2b+2}}{1^{b+1}} \frac{d^b}{dt^b} \left\{ \frac{1}{2} \frac{1}{t^3} \frac{t^{b+1} (1-p^2) q}{t-1 (1-p^2) q} \right\} \right\},$$

après la différentiation on doit mettre q au lieu de η ;

$$5) \int_0^{V^{(1-p^2)}} \frac{(x^2+p^2)^{2a} dx}{(x^2+p^2)^{2a}} = \sum_0^{\infty} p^{2 \cdot 2a-n} (1-p^2)^{n+\frac{1}{2}} \binom{2a}{n} \frac{1}{2n+1}$$

Sur les intégrales 4), 5) voyez Cauchy, Sav. Etr. 1827. 124. Note 16.

$$6) \int_1^p \frac{dx}{\sqrt{(x^2-1)}} = l \{ p + 1 \sqrt{p^2-1} \}, p \leq 1; \quad \text{Ohm, Ausw. 10.}$$

$$7) \int_1^p \frac{dx}{\sqrt{(q^2+x) \sqrt{1-x}}} = \frac{-2i}{\sqrt{(1+q^2)}} \text{Arctg.} \frac{1 \sqrt{p^2-1}}{(1+q^2)} \quad \text{Raabe, Int. 421.}$$

$$8) \int_a^b \frac{dx}{\sqrt{(x^2-a^2)(b^2-x^2)}} = \frac{1}{b} F' \left\{ \frac{1}{b} \sqrt{b^2-a^2} \right\}$$

$$9) \int_a^b \frac{x^2 dx}{\sqrt{(x^2-a^2)(b^2-x^2)}} = \frac{1}{b} E' \left\{ \frac{1}{b} \sqrt{b^2-a^2} \right\}$$

Poisson,
l. 2.
184.

$$10) \int_a^b \frac{x^4 dx}{\sqrt{(x^2-a^2)(b^2-x^2)}} = \frac{b^3}{3} \left[2 \left(1 + \frac{a^2}{b^2} \right) E' \left\{ \frac{1}{b} \sqrt{b^2-a^2} \right\} - \frac{a^2}{b^2} F' \left\{ \frac{1}{b} \sqrt{b^2-a^2} \right\} \right]$$

$$11) \int_a^{\infty} \frac{dx}{x^2-a^2} = \infty$$

Bidone, Mém. Turin. 1812. 231. Art. 2. N. 31. — Plana, Mém. Turin. 1818. 7. Art. 1. N. 4.

$$12) = -\frac{1}{2a} l \frac{0}{2a}$$

$$13) \int_p^{\infty} \frac{dx}{x(x+1)} = l \frac{1+p}{p} \quad \text{Schlömilch, Gr. 4. 71. — Arndt, Gr. 10. 225.}$$

$$14) \int_{\text{Sec. } \lambda}^{\infty} \frac{dx}{x^2-1} = \frac{1}{2} l \frac{1+\text{Cos. } \lambda}{1-\text{Cos. } \lambda} \quad \text{Legendre, Exerc. 5. 76.}$$

$$15) \int_{-p}^p \frac{dx}{x} = 0 \quad \text{Cauchy, Sav. Etr. 1827. 599. P. 2. § 3. — Matzka, Gr. 20. 1.}$$

- 16) $\int_a^\infty \frac{dx}{(b-x)(x-a)^p} = \frac{\pi}{(-1)^{1-p}(b-a)^p} \operatorname{Cosec.} p \pi, b < a;$
- 17) $\int_{-\infty}^a \frac{dx}{(b-x)(x-a)^p} = \frac{\pi}{(-1)^p(b-a)^p} \operatorname{Cosec.} p \pi, b > a;$
- 18) $\int_{-\infty}^a \frac{(a+x)^{b-1}(a-x)^{c-1} dx}{\{g(a+x)+h(a-x)+2k\}^{b+c}} = \frac{\Gamma(b)\Gamma(c)}{2\Gamma(b+c)} \frac{a^{b+c-1}}{(ga+k)^b(ha+k)^c}$ Winckler, Cr. 45. 102.
- 19) $\int_p^q \frac{dx}{x} = \frac{1}{2} l \left(\frac{q}{p} \right)^2 - l \alpha$ Arndt, Gr. 10. 240.
- 20) $= l \frac{q}{p}$ Ohm, Ausw. 1, 2.
- 21) $= \frac{1}{2} l \left(\frac{p}{q} \right)^2$ Matzka, Gr. 20. 1.
- 22) $\int_p^q \frac{dx}{(r^2+x)\sqrt{(1-x^2)}} = \frac{1}{1\sqrt{(1-r^2)}} l \left\{ \frac{(1+r^2)+1}{1\sqrt{(1+r^2)+1}} \frac{(1-p)}{(1-q)} - \frac{(1+r^2)-1}{1\sqrt{(1+r^2)-1}} \frac{(1-q)}{(1-p)} \right\}$ Raabe, Int. 421.
- 23) $\int_0^{a-2b} x(a-2b-x)^{a-1} dx = \frac{(a-2b)^{a-1}}{a(a+1)}$ Hoppe, Cr. 40. 142.

- 1) $\int e^{-x} dx = 1$ Cauchy, Cours. Lec. 32. — Grunert, Gr. 2. 266.
- 2) $\int e^{-px} dx = \frac{1}{p}$ Cauchy, Cours. Lec. 32. — Cisa de Grézy, Mém. Turin. 1821. 209. II. N°. 45. — Lionville, L. 4. 317. — Oettinger, Cr. 35. 13. — Grunert, Gr. 2. 266.
- 3) $\int p^x dx = -\frac{1}{l p}, p < 1;$ Poisson, P. 19. 404. N°. 76.
- 4) $\int e^{px} dx = \infty$ Cisa de Grézy, Mém. Turin. 1821. 209. II. N. 45. — Cauchy, Cours. Lec. 24.
- 5) $\int e^{-pxi} dx = \frac{1}{p i}$
- 6) $\int e^{-(p+q^2)x} dx = \frac{p+qi}{p^2+q^2}$
- Meyer, Int. Déf. 99.

$$7) \int e^{-x^2} dx = \frac{1}{2} \sqrt{\pi}$$

Sur cette intégrale on peut voir: Laplace, Mém. de l'Ac. 1778. 227. § 23. — Id., ib. 1782. I. § 4. — Id., Mém. Inst. 1809. 353. § 3. — Id., Probabilités. L. 1. N. 24. — Legendre, Exerc. 2. 81. — Fourier, Chaleur. 360. — Kramp, Réfr. 3. N. 66. — Bidone, Mém. Turin. 1812. 231. Art. 1. N. 20. — Binet, P. 27. 123. — Oettinger, Cr. 35. 13. — Roberts, L. 16. 1. — Grunert, Gr. 2. 266.

$$8) \int e^{-p^2 x^2} dx = \frac{1}{2p} \sqrt{\pi} \quad \text{Bidone, Mém. Turin. 1812. 231. Tableau. — Cisa de Grésy, Mém. Turin. 1821. 209. II. N°. 35. — Boncompagni, Cr. 25. 74. — Winckler, Cr. 45. 102. — Schlömilch, Gr. 5. 90. — Id., Stud. I. 12.}$$

$$9) \int e^{px^2 i} dx = \frac{1}{2} e^{\frac{\pi i}{4}} \sqrt{\frac{\pi}{p}} \quad \text{Schlömilch, Stud. I. 13. — Schaar, Mém. Brux. T. 24.}$$

$$10) \int e^{\frac{2\pi x^2 i}{p}} dx = \frac{1+i}{4} \sqrt{p} \quad \text{Schaar, Mém. Brux. T. 24.}$$

$$11) \int e^{-x^4} dx = \frac{1}{2} \sqrt{(\pi_1 \sqrt{2\pi})}, \text{ où } \pi_1 = 1, 31102\ 87371\ 46059\ 87: \\ \text{Laplace, Mém. Ac. 1782. § 5.}$$

$$12) \int e^{-x^p} dx = \frac{1}{p} \Gamma\left(\frac{1}{p}\right) \quad \text{Legendre, Mém. Inst. 1809. 416. N°. 81. — Id., Exerc. 2. 81.}$$

$$13) \quad = \frac{1}{p^{1/p}} \quad \text{Oettinger, Cr. 35. 13.}$$

$$14) \int e^{-pe^{bx}} dx = -\frac{1}{p} \operatorname{li}(e^{-p}) \quad \text{Winckler, Cr. 45. 102.}$$

$$15) \int e^{-x^{\frac{2}{1+2a}}} dx = \frac{1a+1/2}{2a+1} \sqrt{\pi} \quad \text{Kramp, Réfr. 3. 67.}$$

$$16) \int e^{-\frac{1}{x^2}} dx = \sqrt{\pi} \quad \text{Kramp, Réfr. 3. 74.}$$

$$17) \int e^{-\frac{1}{x^a}} dx = \frac{\sqrt[a]{a}}{(a-1)^{1/a}} \quad \text{Kramp, Réfr. 3. 73.}$$

$$1) \int e^{-x^2+p-\frac{p^2}{4x^2}} dx = \frac{1}{2} \sqrt{\pi} \quad \text{Cauchy, Sav. Etr. 1827. 124. Notes. N°. 2.}$$

$$2) \int e^{-x^2 - \frac{p^2}{4x^2}} dx = \frac{1}{2} e^{-p} \sqrt{\pi}$$

Sur cette intégrale voyez : Laplace, Nouv. Bull. de la Soc. Philom. N. 43. — Id., Probab. L. 1. N°. 26. — Poisson, Nouv. Bull. de la Soc. Philom. N°. 50. — Id., P. 16. 215. N°. 8. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 23. — Cisa de Grézy, Mém. Turin. 1821. 209. II. § 37. — Cauchy, Sav. Etr. 1827. 124. Notes. N°. 2. — Von Schmidten, Cr. 5. 388. — Kummer, Cr. 17. 228. — Boole, L. 13. 111. — Bonnet, L. 14. 249. — Helmling, Transf. 27.

$$3) \int e^{-p^2 x^2 - \frac{q^2}{x^2}} dx = \frac{1}{2p} e^{-2pq} \sqrt{\pi} \quad \text{Schlömilch, Gr. 9. 379. — Helmling, Transf. 27.}$$

$$4) \int e^{-(x - \frac{p}{x})^{2b}} dx = \frac{1}{2b} \Gamma\left(\frac{1}{2b}\right) \quad \text{Boole, L. 13. 111.}$$

$$5) \int e^{-x^2 + px} dx = \frac{1}{2} e^{\frac{1}{4}p^2} \sqrt{\pi} \quad \text{Cisa de Grézy, Mém. Turin. 1821. 209. II. N°. 39.}$$

$$6) \int e^{-(q^2 x^2 + px)} dx = e^{-\frac{p^2}{4q^2}} \sqrt{\frac{\pi}{2q}} \quad \text{Meyer, Int. Déf. 118; fautive selon Helmling, Transf. 6.}$$

$$7) \int e^{\left(\frac{x^2 + q^2}{p^2 + x^2}\right)^{ai}} dx = \frac{1}{2} p e^{\frac{2qai}{p} + \frac{\pi i}{4}} \sqrt{\frac{\pi}{a}} \quad \text{Schlömilch, Stud. II. 24.}$$

$$8) \int e^{-x + 2p\sqrt{x}} dx = p e^{p^2} \sqrt{\pi} - 1 \quad \text{V. T. 37. N°. 5.}$$

$$9) \int (e^{px} + e^{-px}) e^{-x^2 - \frac{1}{4}p^2} dx = \sqrt{\pi} \quad \text{Cauchy, Sav. Etr. 1827. 124. Note 2.}$$

$$10) \int (e^{-x} - 1)^q e^{-px} dx = \frac{\Gamma(q+1) \Gamma(p)}{\Gamma(p+q+1)} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{Cauchy, P. 28. 147. P. 3. N°. 1.}$$

$$11) \quad = \Delta^q \cdot \frac{1}{p} \text{ pour } q \text{ entier}$$

$$12) \int (e^{2px} + e^{-2px}) e^{-x^2} dx = e^{p^2} \sqrt{\pi} \quad \text{Cauchy, Cours. Leç. 40.}$$

$$13) \int (e^{2px} + e^{-2px}) e^{-q^2 x^2} dx = \frac{\sqrt{\pi}}{2q} e^{\frac{p^2}{q^2}} \quad \text{Schlömilch, Stud. I. 12. — Helmling, Transf. 7.}$$

$$14) \int (e^{p\sqrt{x}} - e^{-p\sqrt{x}}) e^{-x} dx = p e^{\frac{p^2}{4}} \sqrt{\pi} \quad \text{Helmling, Transform. 11.}$$

- 1) $\int \frac{dx}{e^{2x} + 1} = \frac{1}{2} l 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.
- 2) $\int \frac{e^{px} - e^{-px}}{e^{\pi x} - 1} dx = \frac{1}{p} - \text{Cot. } p$ Malmsten, Cr. 35. 55.
- 3) $\int \frac{e^{-qx} - e^{-px}}{1 - e^{-x}} dx = Z'(p) - Z'(q)$ V. T. 5. N°. 8.
- 4) $\int \frac{1 - e^{-px}}{1 - e^{-x}} dx = -A - Z'(1 - p)$ V. T. 5. N°. 3.
- 5) $\int \frac{e^{-px} - e^{-qx}}{1 - e^{-x}} dx = Z'(1 - p) - Z'(1 - q)$ V. T. 5. N°. 4.
- 6) $\int \frac{dx}{1 + e^{2px}} = \frac{1}{2p} l 2$ Lobatschewsky, Mém. Kasan. 1836. 1. II. N°. 20.
- 7) $\int \frac{e^{(q-p)x} + e^{-(q+p)x}}{1 + e^{-2px}} dx = \frac{\pi}{2p} \text{Sec. } \frac{q}{2p} \pi$ V. T. 5. N°. 17.
- 8) $\int \frac{dx}{e^{px} + e^{-px}} = \frac{\pi}{4p}$ Poisson, P. 19. 404. N°. 77. — Raabe, Cr. 42. 348.
- 9) $\int \frac{e^{px} + e^{-px}}{e^{qx} + e^{-qx}} dx = \frac{\pi}{2q} \text{Sec. } \frac{p}{2q} \pi$
- 10) $\int \frac{e^{px} - e^{-px}}{e^{qx} - e^{-qx}} dx = \frac{\pi}{2q} \text{Tang. } \frac{p}{2q} \pi$
- 11) $\int \frac{p^{rx} + p^{-rx}}{q^{rx} + q^{-rx}} dx = \frac{\pi}{2rlq} \text{Sec. } \left(\frac{\pi}{2} lq \right)$
- 12) $\int \frac{p^{rx} - p^{-rx}}{q^{rx} - q^{-rx}} dx = \frac{\pi}{2rlq} \text{Tang. } \left(\frac{\pi}{2} lq \right)$
- 13) $\int \frac{e^{px} - e^{-px}}{e^{2\pi x} - 1} dx = \frac{1}{p} - \frac{1}{2} \text{Cot. } \frac{1}{2} p$ Malmsten, Cr. 35. 55.
- 14) $\int \frac{(e^{2px} + e^{-2px})(e^{2qx} + e^{-2qx})}{e^{\pi x} + e^{-\pi x}} dx = 2 \frac{\text{Cos. } p \cdot \text{Cos. } q}{\text{Cos. } 2p + \text{Cos. } 2q}$, $p < \frac{\pi}{2}, q < \frac{\pi}{2}$;
- 15) $\int \frac{(e^{2px} - e^{-2px})(e^{2qx} - e^{-2qx})}{e^{\pi x} + e^{-\pi x}} dx = 2 \frac{\text{Sin. } p \cdot \text{Sin. } q}{\text{Cos. } 2p + \text{Cos. } 2q}$ Poisson, P. 17. 612. N°. 21.
- 16) $\int \frac{e^{2px} + e^{-2px}}{e^{\pi x} + e^{-\pi x}} dx = \frac{1}{2} \text{Sec. } p$ Poisson, P. 18. 295. N°. 22. — Legendre, Exerc. 5. 45. — Malmsten, Cr. 38. 1 la trouve fautive $\frac{1}{2} \text{Sec. } \frac{1}{2} p$.

- 17) $\int \frac{e^{2px} - e^{-2px}}{e^{\pi x} - e^{-\pi x}} dx = \frac{1}{2} \text{Tang. } p$ Poisson, P. 18. 295. N°. 22. — Legendre, Exerc. 5. 45. — Malmsten, Cr. 38. 1 la trouve fautive *Tang. p.*
- 18) $\int \frac{(e^{px} - e^{-px})(e^{qx} + e^{-qx})}{e^{\pi x} - e^{-\pi x}} dx = \frac{\text{Sin. } p}{\text{Cos. } p + \text{Cos. } q}, p < \pi;$ Poisson, Mém. Inst. 1811. 163. N°. 26. — Id., P. 18. 295. N°. 21. — Plana, Mém. Turin. 1818. 7. Art. 4. N°. 20.
- 19) $\int \frac{(e^{px} + e^{-px})(e^{qx} + e^{-qx})}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} dx = 4 \frac{\text{Cos. } p \cdot \text{Cos. } q}{\text{Cos. } 2p + \text{Cos. } 2q}, q < \pi, p < \pi;$
- 20) $\int \frac{(e^{px} - e^{-px})(e^{qx} - e^{-qx})}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} dx = 4 \frac{\text{Sin. } p \cdot \text{Sin. } q}{\text{Cos. } 2p + \text{Cos. } 2q}$ Poisson, P. 17. 612. N°. 21.
- 21) $\int \frac{e^{px} + e^{-px}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} = \text{Sec. } p$, $p^2 < \frac{1}{4} \pi^2;$
- 22) $\int \frac{e^{px} - e^{-px}}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} = \text{Tang. } p$ Raabe, Cr. 42. 348.
- 23) $\int [(1 - e^{-x})^{-\frac{1}{2}} - 1] dx = 2 \log 2$ V. T. 15. N°. 1.

- 1) $\int \frac{e^x dx}{e^{2x} - 2 \cos. \lambda + 1} = \frac{\pi - \lambda}{2 \text{Sin. } \lambda}$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.
- 2) $\int \frac{e^{px} + e^{-px}}{e^x + e^{-x} + 2 \cos. \lambda} dx = \frac{\pi \text{Sin. } p \lambda}{\text{Sin. } \lambda \cdot \text{Sin. } p\pi}, p < 1;$ Poisson, P. 18. 295. N°. 28.
- 3) $\int \frac{e^{(p+q)x} + e^{-(p+q)x} - e^{(p-q)x} - e^{-(p-q)x}}{e^{2px} - 2 + e^{-2px}} dx = \frac{\pi}{2p} \text{Tang. } \frac{q\pi}{2p}, p > q;$
- 4) $\int \frac{e^{(p+q)x} + e^{-(p+q)x} + e^{(p-q)x} + e^{-(p-q)x}}{e^{2px} + 2 + e^{-2px}} dx = \frac{\pi}{2p} \text{Sec. } \frac{q\pi}{2p}$ Raabe, Int. 143.
- 5) $\int \frac{e^{px} + e^{-px}}{e^{qx} - 2 \cos. \lambda + e^{-qx}} dx = \frac{\pi \text{Sin. } \left(p \frac{\pi - \lambda}{q}\right)}{q \text{Sin. } \lambda \cdot \text{Sin. } \frac{p\pi}{q}}$ V. T. 8. N°. 10.
- 6) $\int \frac{e^{px} + e^{-px}}{e^{qx} + 2 \cos. \lambda + e^{-qx}} dx = \frac{\pi \text{Sin. } \frac{p\lambda}{q}}{q \text{Sin. } \lambda \cdot \text{Sin. } \frac{p\pi}{q}}$ V. T. 8. N°. 12.

- 7) $\int \frac{e^{px} - 2 \cos. \lambda + e^{-px}}{e^{qx} + 2 \cos. \mu + e^{-qx}} dx = \frac{\pi \sin. \left(p \frac{\pi - \mu}{q} \right)}{q \sin. \mu \cdot \sin. \frac{p \pi}{q}} - \frac{\pi - \mu}{q \sin. \mu} \cos. \lambda$ V. T. 8. N°. 9.
- 8) $\int \frac{dx}{(e^{px} + e^{-px})^2} = \frac{1}{2 p^2} \log 2$ V. T. 38. N°. 6.
- 9) $\int \frac{dx}{(e^x + e^{-x})^{2p}} = \frac{\{\Gamma(p)\}^2}{4 \Gamma(2p)}$ V. T. 5. N°. 24.
- 10) $\int \frac{e^{px}}{(e^{px} + 1)^2} dx = \frac{\pi}{8p}$ V. T. 38. N°. 6.
- 11) $\int \frac{e^{px} - 1}{(e^{px} + 1)^2} dx = \frac{1}{p} (1 - \log 2)$ V. T. 38. N°. 6.
- 12) $\int \frac{e^{-x} dx}{(e^x + e^{-x})^{2p+1}} = \frac{1}{p 2^{2p+2}} + \frac{\{\Gamma(p)\}^2}{8 \Gamma(2p)}$ V. T. 39. N°. 9.
- 13) $\int \frac{(e^{2px} - e^{-2px})(e^{\pi x} - e^{-\pi x})}{(e^{\pi x} + e^{-\pi x})^2} dx = \frac{p}{\pi} \sec. p, p < \frac{\pi}{2};$ V. T. 38. N°. 16.
- 14) $\int \frac{(e^{2px} - e^{-2px})(e^{qx} - e^{-qx})}{(e^{qx} + e^{-qx})^2} dx = \frac{p\pi}{2q^2} \sec. \frac{p\pi}{2q}, q > p;$ V. T. 38. N°. 9.
- 15) $\int \frac{(p-\pi)\{e^{(\pi+p)x} - e^{-(\pi+p)x}\} + (p+\pi)\{e^{(\pi-p)x} - e^{-(\pi-p)x}\}}{(e^{\pi x} - e^{-\pi x})^2} (e^{qx} - e^{-qx}) dx = \frac{q \sin. p}{\cos. p + \cos. q}, p+q < \pi;$ V. T. 38. N°. 18.
- 16) $\int \frac{e^{(q-p)x} + e^{(q+p)x}}{(e^x + e^{-x})^{p+q}} dx = \frac{1}{2} B(p, q)$ Binet, P. 27. 123.
- 17) $\int \frac{e^{2px} + e^{-2px}}{(e^x + e^{-x})^{2q}} dx = \frac{1}{2} B(q+p, q-p)$
- 18) $= \frac{\Gamma(q+p)\Gamma(q-p)}{2 \Gamma(2q)}$ Meyer, Int. Déf. 312.
- 19) $\int \left\{ e^{px} - \frac{1}{(1 + e^{-x})^p} \right\} e^{-(q+1)x} dx = \frac{q}{q-p+1} \frac{\Gamma(q)\Gamma(p-q)}{\Gamma(p)}$ V. T. 22. N°. 16.
- 20) $\int \frac{e^x \pm \cos. \lambda}{(e^x + e^{-x} - 2 \cos. \lambda)^2} dx = \frac{\pi - \lambda}{4 \sin. \lambda} \mp \frac{1}{4} \frac{1}{1 - \cos. \lambda}$ V. T. 39. N°. 1.
- 21) $\int \frac{(e^{px} - e^{-px})(e^{qx} - e^{-qx})}{(e^{qx} + e^{-qx} + 2 \cos. \lambda)^2} dx = \frac{-p \pi \sin. \frac{p \lambda}{q}}{q \sin. \lambda \cdot \sin. \frac{p \pi}{q}}, p < 1;$ V. T. 39. N°. 5.

- 1) $\int e^x dx = \infty$ Cauchy, Cours. Lecq. 24.
 - 2) $\int e^{pxi} dx = 0$ Poisson, P. 19. 404. N°. 69.
 - 3) $\int e^{-x^2} dx = \sqrt{\pi}$ Poisson, Chal. 74. — Grunert, Gr. 2. 266.
 - 4) $\int e^{-px^2} dx = \sqrt{\frac{\pi}{p}}$ Ohm, Ausw. 20.
 - 5) $\int e^{x^2i} dx = \frac{1+i}{2} \sqrt{2\pi}$ Cauchy, Lim. Imag. § 189. — Id., P. 19. 511.
 - 6) $\int e^{px^2i} dx = e^{\frac{1}{4}\pi i} \sqrt{\frac{\pi}{p}}$ Schaar, Mém. Brux. T. 25.
 - 7) $\int e^{-x^2i} dx = \frac{1-i}{2} \sqrt{2\pi}$ Cauchy, P. 19. 511.
 - 8) $\int e^{-px^2i} dx = e^{\frac{\pi i}{4}} \sqrt{\frac{\pi}{p}}$ Schlömilch, Stud. I. 13.
 - 9) $\int e^{-x^2+2px} dx = e^{p^2} \sqrt{\pi}$ Poisson, Chaleur. 74. — Cisa de Grésy, Mém. Turin. 1821. 209. I. 3^a.
 - 10) $\int e^{-x^2-2px} dx = e^{p^2} \sqrt{\pi}$ Cauchy, P. 19. 511.
 - 11) $\int e^{-px^2-qx} dx = e^{\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}}$ Cauchy, Exerc. 1827. p. 233. — Id., P. 19. 511.
 - 12) $\int e^{-px^2+qx} dx = e^{\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}}$ Ohm, Ausw. 20.
 - 13) $\int e^{-(x^2+2px)i} dx = \frac{1+i}{\sqrt{2}} e^{-p^2i} \sqrt{\pi}$ Cauchy, Lim. Imag. 190.
 - 14) $\int e^{(px^2+qx)i} dx = e^{\left(\frac{\pi}{4}-\frac{q^2}{4p}\right)i} \sqrt{\frac{\pi}{p}}$ Schlömilch, Stud. I. 13.
 - 15)
$$= (1+i) e^{\frac{-q^2i}{4p}} \sqrt{\frac{\pi}{2p}} \left\{ \right.$$
 - 16)
$$= (1-i) e^{\frac{q^2i}{4p}} \sqrt{\frac{\pi}{2p}} \left\{ \right.$$
- Cauchy, P. 19. 511.

$$17) \int e^{(px^2 - qx)i} dx = (1+i) e^{-\frac{q^2 i}{p}} \sqrt{\frac{\pi}{2p}} \quad \text{Lejeune-Dirichlet, C. R. S. 157.}$$

$$18) \int e^{-(x^2 + p + 2qx)} dx = e^{-p+q^2} \sqrt{\pi} \quad \text{Fourier, Chaleur. 364.}$$

$$19) \int e^{-(px^2 + qx + r)} dx = e^{-r + \frac{q^2}{4p}} \sqrt{\frac{\pi}{p}}, \quad \text{si la partie réelle de } p \text{ est } > 0; \left. \vphantom{\int} \right\} \begin{array}{l} \text{Cauchy,} \\ \text{Exerc.} \\ 1827. \text{ p.} \\ 233. \end{array} \quad \text{ou } p, q \text{ et } r \text{ sont imaginaires;}$$

$$20) \quad = \infty, \quad \text{si la partie réelle de } p \text{ est } < 0;$$

$$21) \int e^{-px^2 - \frac{q}{x^2}} dx = e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{p}}$$

$$22) \int e^{-x^2 - \frac{b^2}{4x^2}} dx = e^{-b} \sqrt{\pi}$$

$$23) \int e^{(px^2 + \frac{q}{x^2})i} dx = (1+i) e^{2i\sqrt{pq}} \sqrt{\frac{\pi}{2p}}$$

$$24) \int e^{-(px^2 + \frac{q}{x^2})i} dx = (1-i) e^{-2i\sqrt{pq}} \sqrt{\frac{\pi}{2p}}$$

$$25) \int e^{-(p+qi)^2 \left\{ x + \frac{r+si}{p+qi} \right\}^2} dx = \frac{\sqrt{\pi}}{p+qi} : p > q;$$

$$26) \int e^{-qi-p)^2 \left\{ x - \frac{r+si}{p-qi} \right\}^2} dx = \frac{\sqrt{\pi}}{qi-p}$$

$$27) \int e^{-p \left\{ x \pm \sqrt{\frac{r+si}{p}} \right\}^2} dx = \sqrt{\frac{\pi}{p}}$$

$$28) \int \frac{e^{-bx}}{1+e^{-ax}} dx = \frac{\pi}{a} \operatorname{Cosec.} \frac{b\pi}{a} \quad \text{V. T. 22. N°. 9.}$$

$$29) \int \frac{e^{px} + e^{-px}}{e^{qx} + e^{-qx}} dx = \frac{\pi}{q} \operatorname{Sec.} \frac{p\pi}{2q}$$

$$30) \int \frac{e^{px} - e^{-px}}{e^{qx} - e^{-qx}} dx = \frac{\pi}{q} \operatorname{Tang.} \frac{p\pi}{2q} \quad \text{Ohm, Ausw. 14.}$$

$$31) \int \frac{e^{p-q} x}{(e^x + e^{-x})^{p+q}} dx = \frac{1}{2} B(p, q) \quad \text{Binet, P. 27. 123.}$$

$$32) \int \frac{(1+e^{-x})^q - 1}{(1+e^{-x})^{p+q}} dx = Z'(p+q) - Z'(q) \quad \text{V. T. 22. N°. 3.}$$

F. Exponent.	TABLE 40 suite.	Lim. — ∞ et ∞ .
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$$33) \int \left(\frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^p} \right) dx = A + Z'(p) \quad \text{V. T. 22. N}^\circ. 17.$$

$$34) \int \left(\frac{1}{(1+e^{-x})^q} - \frac{1}{(1+e^{-x})^p} \right) dx = Z'(p) - Z'(q) \quad \text{V. T. 22. N}^\circ. 18.$$

F. Exponent.	TABLE 41.	Limites diverses.
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$$1) \int_0^\infty \frac{dx}{e^x - p} = -\frac{1}{p} l(1 - p e^{-q}) \quad \text{Hoppe, Cr. 40. 139.}$$

$$2) \int_0^1 e^{-x^q} dx = \frac{q \cdot 2q \cdot 3q \dots}{q + 1 \cdot 2q + 1 \cdot 3q + 1 \dots} \quad \text{Oettinger, Cr. 33. 13.}$$

$$3) \int_0^1 x^{-p^x} dx = \sum_1^\infty \frac{p^{n-1}}{n^n} \quad \text{Kummer, Cr. 17. 210.}$$

$$4) \int_0^1 e^{2a\sqrt{x}} dx = \frac{1}{a} \left(e^{2a} - \frac{1}{2a} e^{2a} + \frac{1}{2a} \right) \quad \text{Dienger, Cr. 46. 119.}$$

$$5) \int_0^1 p^{q^x} dx = \frac{p^q - 1}{q l p} \quad \text{Meyer, Int. Déf. 98.}$$

$$\left. \begin{aligned} 6) \int_0^1 e^x dx &= e - 1 \\ 7) \int_0^{2\pi} \frac{dx}{1 - p e^{xi}} &= 2\pi, p < 1; \end{aligned} \right\} \quad \text{Moigno, Int. 33, 138.}$$

$$\left. \begin{aligned} 8) \int_0^{2\pi} \frac{p e^{xi} dx}{p e^{xi} \pm q e^{ri}} &= 0, p < q; \\ 9) &= 2\pi, p > q; \end{aligned} \right\} \quad \text{Ohm, Ausw. 18.}$$

$$\left. \begin{aligned} 10) \int_0^q p^x dx &= \frac{p^q - 1}{l p} \\ 11) \int_{-1}^{+1} p^x dx &= \frac{p^x - 1}{p l p} \end{aligned} \right\} \quad \text{Meyer, Int. Déf. 93.}$$

$$12) \int_{-\pi}^{\pi} e^{pxi} dx = 0 \quad \text{Poisson, P. 19. 404. N}^{\circ}. 78.$$

$$13) \int_{-\pi}^{\pi} (q + p e^{xi})^a dx = 2 \pi q^a$$

$$14) \int_{-\pi}^{\pi} (p e^{xi})^a dx = 0$$

$$15) \int_{-\pi}^{\pi} \frac{dx}{(p e^{xi})^a} = 0$$

$$16) \int_{-\pi}^{\pi} (p e^{xi})^q dx = \frac{2}{q} p^q \sin. q \pi, q < 1;$$

$$17) \int_{-\pi}^{\pi} \frac{dx}{(q e^{xi} + p e^{xi})^a} = 2 \pi (q e^{xi})^a, p < q;$$

$$18) \quad = 0 \quad , p > q;$$

$$19) \int_{-\pi}^{\pi} e^{-axi} e^{pxi} dx = \frac{2\pi}{1^{a/1}} p^a \quad \text{Cauchy, Exerc. 1826. p. 205.}$$

$$20) \int_{-\pi}^{\pi} \frac{(e^{xi})^{a+1} dx}{\sqrt{(1 - 2 e^{xi} \cos. \lambda + e^{2xi})}} = 0 \quad \text{Dirksen, Ber. der Berlin. Acad. 1848. 120.}$$

$$21) \int_{-\infty}^0 e^x dx = 1 \quad \text{Cauchy, Cours, Lecq. 24.}$$

Ohm, Ausw. 18.

$$1) \int \left(l \frac{1}{x} \right)^p dx = 1^{p/1}, \text{ pour } p \text{ entier; Euler, Calc. Int. 4. S. 3. 7.}$$

$$2) \quad = \Gamma(p+1), \quad \infty > p > -1; \text{ c'est l'Intégrale Eulérienne de seconde espèce.}$$

Voyez: Legendre, Exerc. 2. 54. — Id., Mém. Inst. 1809. 416. N^o. 53. — Binet, P. 27. 123. — Cisa de Grésy, Mém. Turin. 1821. 209. I. N^o. 16. — Schaar, Mém. Cour. Brux. T. 23. — Lejeune-Dirichlet, Cr. 15. 258.

$$3) \int \left(l \frac{1}{x} \right)^2 dx = 2 \quad \text{Plana, Cr. 17. 1.}$$

$$4) \int \{1 + l(1 + px)\} dx = \frac{1+p}{p} l(1+p) \quad \text{Dienger, Cr. 38. 331.}$$

$$5) \int l x \cdot l(1-x) dx = 2 - \frac{1}{6} \pi^2 \quad \text{V. T. 152. N°. 9, T. 160. N°. 9 et T. 42. N°. 2.}$$

$$6) \int l x dx = -\Lambda \quad \text{Mascheroni, Adn. p. 18.}$$

$$7) \int \{l((x))\}^p dx = (-1)^p \Gamma(p+1), \quad -\infty > p > -1; \quad \text{Ohm, Ausw. 14.}$$

$$8) \int \left(l \frac{1}{x}\right)^{p-1} dx \, l \frac{1}{x} = Z'(p) \Gamma(p) \quad \text{V. T. 377. N°. 1.}$$

$$9) \int l(x+q) dx = (1+q) \{l(1+q) - 1\} - q \{l(q) - 1\} \quad \text{Raabe, Cr. 23. 146.}$$

$$1) \int \frac{dx}{lx} = \Lambda + lo \quad \text{Cisa de Grézy, Mém. Turin. 1821. 209. Art. 1. N°. 25, 27.}$$

$$2) \quad = -\infty \quad \text{Legendre, Exerc. 3. 57.}$$

$$3) \int \frac{dx}{\left(l \frac{1}{x}\right)^p} = \frac{\pi}{\Gamma(p)} \text{Cosec. } p \pi \quad \text{V. T. 126. N°. 8.}$$

$$4) \int \frac{dx}{l l x} = 0 \quad \text{Mascheroni, Adn. p. 18.}$$

$$5) \int \frac{dx}{l p + l x} = \frac{1}{p} li. p \quad \text{Schlömilch, Gr. 5. 204.}$$

$$6) \int \frac{dx}{q - l x} = -e q li. (e^{-q}) \quad \text{V. T. 129. N°. 4.}$$

$$7) \int \frac{dx}{q + l x} = e^{-q} li. (e^q) \quad \text{V. T. 129. N°. 9.}$$

$$8) \int \frac{dx}{q^2 + (l x)^2} = \frac{1}{q} \left\{ Ci. (q) Sin. q - Si. (q) Cos. q + \frac{\pi}{2} Cos. q \right\} \quad \text{V. T. 130. N°. 4.}$$

$$9) \int \frac{l x}{q^2 + (l x)^2} dx = Ci. (q) Cos. q + Si. (q) Sin. q - \frac{\pi}{2} Sin. q \quad \text{V. T. 130. N°. 5.}$$

$$10) \int \frac{dx}{q^2 - (lx)^2} = \frac{1}{2q} \{e^{-q} Ei.(q) - e^q Ei.(-q)\} \quad \text{V. T. 130. N}^\circ. 10.$$

$$11) \int \frac{lx}{q^2 - (lx)^2} dx = -\frac{1}{2} \{e^{-q} Ei.(q) + e^q Ei.(-q)\} \quad \text{V. T. 130. N}^\circ. 12.$$

$$12) \int \frac{dx}{q^4 - (lx)^4} = \frac{1}{4q^3} \{-e^{-q} Ei.(q) + e^q Ei.(-q) - 2Ci.(q)Sin.q + 2Si.(q)Cos.q - \pi Cos.q\} \quad \text{V. T. 132. N}^\circ. 1.$$

$$13) \int \frac{lx}{q^4 - (lx)^4} dx = \frac{1}{4q^2} \{-e^{-q} Ei.(q) + e^q Ei.(-q) - 2Ci.(q)Cos.q - 2Si.(q)Sin.q + \pi Sin.q\} \quad \text{V. T. 132. N}^\circ. 2.$$

$$14) \int \frac{(lx)^2}{q^4 - (lx)^4} dx = \frac{1}{4q} \{-e^{-q} Ei.(q) + e^q Ei.(-q) + 2Ci.(q)Sin.q - 2Si.(q)Cos.q + \pi Cos.q\} \quad \text{V. T. 132. N}^\circ. 3.$$

$$15) \int \frac{(lx)^3}{q^4 - (lx)^4} dx = \frac{1}{4} \{e^{-q} Ei.(q) + e^q Ei.(-q) + 2Ci.(q)Cos.q + 2Si.(q)Sin.q - \pi Sin.q\} \quad \text{V. T. 132. N}^\circ. 4.$$

$$16) \int \frac{dx}{(q-lx)^2} = -\frac{1}{q} - e^q Ei.(-q) \quad \text{V. T. 43. N}^\circ. 6.$$

$$17) \int \frac{dx}{(q+lx)^2} = \frac{1}{q} + e^{-q} Ei.(q) \quad \text{V. T. 43. N}^\circ. 7.$$

$$18) \int \frac{dx}{(lp+lx)^2} = -\frac{1}{lp} + \frac{1}{p} li.(p) \quad \text{V. T. 43. N}^\circ. 5.$$

$$1) \int dx \sqrt[l]{\frac{1}{x}} = \frac{1}{2} \sqrt[l]{\pi} \quad \text{Euler, Calc. Int. 4. S. 3. 16. — Id., N. C. P. 16. 91. — Plana, Cr. 17. 1.}$$

$$2) \int dx \left(l \frac{1}{x}\right)^{\frac{2a+1}{2}} = \frac{3a^2}{2a-1} \sqrt[l]{\pi} \quad \text{Euler, Calc. Int. 4. S. 3. 29. — Id. N. C. P. 16. 91.}$$

$$3) \int dx ll \sqrt[l]{\frac{1}{x}} = -A - lq \quad \text{V. T. 273. N}^\circ. 2.$$

$$4) \int \frac{dx}{\sqrt[l]{l \frac{1}{x}}} = \sqrt[l]{\pi} \quad \text{Euler, Calc. Int. 4. S. 5. 211. — Id., C. P. 5. 41. — Id., N. C. P. 16. 91. — Legendre, Exerc. 2. 81. — Plana, Cr. 17. 1.}$$

$$5) \int \frac{dx}{\sqrt[l]{\left(l \frac{1}{x}\right)^3}} = -2 \sqrt[l]{\pi} \quad \text{V. T. 126. N}^\circ. 3.$$

$$6) \int \frac{dx}{\left(l \frac{1}{x}\right)^{a+1/2}} = \frac{(-2)^a \sqrt{\pi}}{1^{a/2}} \quad \text{V. T. 126. N}^\circ. 7.$$

$$7) \int \frac{dx \, ll \frac{1}{x}}{\sqrt[l]{l \frac{1}{x}}} = - (A + 2 l 2) \sqrt{\pi} \quad \text{V. T. 273. N}^\circ. 3.$$

$$8) \int \frac{dx}{\sqrt[l]{l \left(\sqrt[l]{l} \frac{1}{x}\right)}} \, ll \left(\sqrt[l]{l} \frac{1}{x}\right) = - (A + l q - 2 l 2) \sqrt{\pi q} \quad \text{V. T. 273. N}^\circ. 4.$$

$$1) \int_0^\infty l x \, l \frac{b^2 + x^2}{a^2 + x^2} dx = \pi (a - b) + \pi l \frac{b^b}{a^a} \quad \text{Schl\"omilch, Gr. 4. 306.}$$

$$2) \int_0^\infty dx \left(l \frac{1}{x}\right)^{\frac{2a-1}{2}} = \frac{1^{a,2}}{2^{a-1}} \sqrt{\pi} \quad \text{V. T. 142. N}^\circ. 8.$$

$$3) \int_0^\infty dx \left(l \frac{1}{x}\right)^a = 0 \quad \text{V. T. 142. N}^\circ. 9.$$

$$4) \int_0^\infty dx \sqrt[p]{l \left(\sqrt[p]{l} \frac{1}{x}\right)} = \frac{1}{p} \sqrt[p]{\frac{\pi}{p}} \quad \text{V. T. 142. N}^\circ. 7.$$

$$5) \int_0^p \frac{dx}{lx} = li.(p) \quad \text{Schl\"omilch, Gr. 5. 204. — Mascheroni, Adn. 4 propose de l'appeler hyperlogarithme. — Voyez sur le Logarithme Integral: Soldner, Th\'eorie et Tables d'une nouvelle fonction transcendante. Munich. 1809.}$$

$$6) \int_0^{e^p} \frac{dx}{lx} = \frac{1}{2} l \alpha^2 + li.(p), p > 1: \quad \text{Arndt, Gr. 10. 247.}$$

$$\left. \begin{aligned} 7) \int_1^e \frac{dx}{l \frac{1}{x}} &= -\infty \\ 8) \int_e^\infty \frac{dx}{l \frac{1}{x}} &= -\infty \end{aligned} \right\} \quad \text{Cisa de Gr\'esy, M\'em. Turin. 1821-209. Art. 1. N}^\circ. 27.$$

$$9) \int_1^{\infty} \frac{dx}{l \frac{1}{x}} = -\infty \quad \text{Cisa de Grésy, Mém. Turin. 1821. 209. Art. 1. N° 27.}$$

$$10) \int_1^{\frac{1}{e}} \frac{dx}{lx} = \infty \quad \text{V. T. 112. N° 4.}$$

$$11) \int_0^{e^{-q}} \frac{dx}{lx + q} = -\infty \quad \text{V. T. 150. N° 6.}$$

$$12) \int_{e^{-q}}^1 \frac{dx}{lx + q} = +\infty \quad \text{V. T. 149. N° 15.}$$

$$13) \int_{\frac{1}{e}}^1 dx \vee l \frac{1}{x} = \frac{1}{2} \vee \pi \quad \text{V. T. 112. N° 6.}$$

$$14) \int_1^e \frac{dx \, lx}{(1+lx)^2} = \frac{1}{2}e - 1 \quad \text{V. T. 112. N° 5.}$$

$$15) \int_0^{e^{-q}} \frac{dx}{lx} = A + lq + \sum_1^{\infty} (-1)^n \frac{q^n}{n \Gamma(n+1)} = Ei.(-q) \quad \text{V. T. 150. N° 4.}$$

$$16) \int_0^{\vee^{\frac{1}{e}}} \frac{dx}{\vee - (1+q \, lx)} = \frac{1}{\vee^q e} \vee \frac{\pi}{p} \quad \text{V. T. 150. N° 1.}$$

$$1) \int Tang. x \, dx = \frac{1}{2} l 2 \quad \text{Meyer, Int. Déf. 121.}$$

$$\left. \begin{aligned} 2) \int Tang.^a x \, dx &= \sum_0^{\infty} \frac{(-1)^n}{a + 2n + 1} \\ 3) \int Tang.^{2a} x \, dx &= (-1)^a \frac{\pi}{4} + \sum_0^{a-1} \frac{(-1)^n}{2a - 2n - 1} \end{aligned} \right\} \quad \text{Arndt, Gr. 6. 434.}$$

$$\left. \begin{aligned} 4) &= (-1)^a \frac{\pi}{4} + (-1)^a \sum_1^a \frac{(-1)^n}{2n - 1} \\ 5) \int Tang.^{2a+1} x \, dx &= (-1)^a \frac{1}{2} l 2 + (-1)^a \sum_1^a \frac{(-1)^n}{2n} \end{aligned} \right\} \quad \text{Cauchy, Cours. Lec. 32.}$$

- 6) $\int \text{Tang.}^{2a+1} x \, dx = (-1)^a \frac{1}{2} l \, 2 + \sum_0^{a-1} \frac{(-1)^n}{2a-2n} \quad \text{Arndt, Gr. 6. 434.}$
- 7) $\int \text{Tang.}^p x \, dx = \frac{1}{4} \left\{ Z' \left(\frac{p+3}{4} \right) - Z' \left(\frac{p+1}{4} \right) \right\} \quad \text{V. T. 3. N}^\circ. 13.$
- 8) $\int \text{Tang.}^p x. \text{Sin.}^2 x \, dx = \frac{1+p}{8} \left\{ Z' \left(\frac{p+3}{4} \right) - Z' \left(\frac{p+1}{4} \right) \right\} - \frac{1}{4} \quad \text{V. T. 46. N}^\circ. 7, 9.$
- 9) $\int \text{Tang.}^p x. \text{Cos.}^2 x \, dx = \frac{1-p}{8} \left\{ Z' \left(\frac{p+3}{4} \right) - Z' \left(\frac{p+1}{4} \right) \right\} + \frac{1}{4} \quad \text{V. T. 4. N}^\circ. 18.$
- 10) $\int \text{Tang.}^p x. \text{Cos.} \, 2x \, dx = \frac{1}{2} - \frac{p}{4} \left\{ Z' \left(\frac{p+3}{4} \right) - Z' \left(\frac{p+1}{4} \right) \right\} \quad \text{V. T. 46. N}^\circ. 7, 9.$
- 11) $\int \text{Cos.}^{p-1} 2x. \text{Tang.} \, x \, dx = -\frac{1}{4} \left\{ Z' \left(\frac{p}{2} \right) - Z' \left(\frac{p+1}{2} \right) \right\} \quad \text{V. T. 3. N}^\circ. 1.$
- 12) $\int (\text{Cos.}^{p-1} 2x - \text{Sec.}^p 2x) \text{Cot.} \, x \, dx = \pi \text{Cot.} \, p \pi \quad \text{V. T. 5. N}^\circ. 6.$
- 13) $\int (\text{Cos.}^{p-1} 2x + \text{Sec.}^p 2x) \text{Tang.} \, x \, dx = \frac{1}{2} \pi \text{Cosec.} \, p \pi \quad \text{V. T. 5. N}^\circ. 1.$
- 14) $\int (\text{Sin.}^{a+1} 2x - 1) \text{Tang.} \left(\frac{\pi}{4} + x \right) \, dx = -\frac{1}{2} \sum_0^a \frac{1}{n+1} \quad \text{V. T. 3. N}^\circ. 4.$
- 15) $\int (\text{Sin.}^q 2x - \text{Sin.}^p 2x) \text{Tang.} \left(\frac{\pi}{4} + x \right) \, dx = \frac{1}{2} \left\{ Z' (p+1) - Z' (q+1) \right\} \quad \text{V. T. 3. N}^\circ. 8.$
- 16) $\int (\text{Sin.}^p 2x - \text{Sin.}^{1-p} 2x) \text{Tang.} \left(\frac{\pi}{4} + x \right) \, dx = \frac{1}{2} \pi \text{Cot.} \, p \pi \quad \text{V. T. 5. N}^\circ. 2.$
- 17) $\int (\text{Sin.}^{p-1} 2x + \text{Cosec.}^p 2x) \text{Tang.} \left(\frac{\pi}{4} + x \right) \, dx = \frac{1}{2} \pi \text{Cosec.} \, p \pi \quad \text{V. T. 5. N}^\circ. 1.$
- 18) $\int (\text{Tang.}^p x + \text{Cot.}^p x) \, dx = \frac{1}{2} \pi \text{Sec.} \, \frac{1}{2} p \pi \quad \text{V. T. 5. N}^\circ. 11.$
- 19) $\int (\text{Tang.}^p x + \text{Cot.}^p x) \text{Sin.} \, 2x \, dx = \frac{1}{2} \frac{p \pi}{e^{4p\pi} - e^{-4p\pi}} \quad \text{V. T. 4. N}^\circ. 23.$

- 1) $\int \frac{\cos^q 2x}{\cos^{q+2} x} dx = \frac{2^{2q}}{2q+1} \frac{\{\Gamma(q+1)\}^2}{\Gamma(2q+1)}$ V. T. 1. N°. 4.
- 2) $\int \frac{\cos^b 2x \cdot \sin^{2a-1} x}{\cos^{2a+b+1} x} dx = \frac{1}{2a} \frac{1^{a/1} 1^{b/1}}{1^{a+b/1}}$ V. T. 1. N°. 17.
- 3) $\int \frac{\cos^b 2x \cdot \sin^{2a} x}{\cos^{2a+b+2} x} dx = \frac{2^{b/2}}{(2a+1)^{b+1/2}}$ V. T. 1. N°. 18.
- 4) $\int \frac{\sin^{2p-2} x}{\cos^p 2x} dx = \frac{\Gamma(2p-1)\Gamma(1-p)}{2^{2p-1}\Gamma(p)}$ V. T. 4. N°. 19.
- 5) $\int \frac{1 - \text{Tang. } x}{\cos 2x} \sin^2 x dx = \frac{3}{4} l 2 - \frac{\pi}{8}$ V. T. 3. N°. 16.
- 6) $\int \frac{1 - \text{Tang. }^3 x}{\cos 2x} \cos^2 x dx = \frac{3}{4} l 2 + \frac{\pi}{8}$ V. T. 3. N°. 15.
- 7) $\int \frac{\text{Tang.}^{p-1} x - \text{Cot.}^{p-1} x}{\cos 2x} dx = \frac{\pi}{2} \text{Cot.} \frac{p\pi}{2}$ V. T. 5. N°. 12.
- 8) $\int \frac{\cos^{a+1} 2x - 1}{\text{Tang. } x} dx = -\frac{1}{2} \sum_0^a \frac{1}{n+1}$ V. T. 3. N°. 4.
- 9) $\int \frac{\cos^q 2x - \cos^p 2x}{\text{Tang. } x} dx = \frac{1}{2} \{Z'(p+1) - Z'(q+1)\}$ V. T. 3. N°. 8.
- 10) $\int \frac{\cos^p 2x - \cos^{1-p} 2x}{\text{Tang. } x} \frac{dx}{\cos 2x} = \frac{1}{2} \pi \text{Cot. } p\pi$ V. T. 5. N°. 2.
- 11) $\int \frac{1 - \sec^p 2x}{\text{Tang. } x} dx = \frac{1}{2} \{\Lambda + Z'(1-p)\}$ V. T. 5. N°. 3.
- 12) $\int \frac{\cos^p 2x - \sec^p 2x}{\text{Tang. } x} dx = -\frac{1}{2p} + \frac{\pi}{2} \text{Cot. } p\pi$ V. T. 5. N°. 5.
- 13) $\int \frac{\sin^p 2x - 1}{\sin^p 2x} \text{Tang.} \left(\frac{\pi}{4} + x \right) dx = \frac{1}{2} \{\Lambda + Z'(1-p)\}$ V. T. 5. N°. 3.
- 14) $\int \frac{\sin^{2p} 2x - 1}{\sin^p 2x} \text{Tang.} \left(\frac{\pi}{4} + x \right) dx = -\frac{1}{2p} + \frac{\pi}{2} \text{Cot. } p\pi$ V. T. 5. N°. 5.
- 15) $\int \frac{(\text{Tang.}^p x + \text{Cot.}^p x)}{\cos 2x} \text{Tang. } x dx = -\frac{1}{p} + \frac{\pi}{2} \text{Cot.} \frac{1}{2} p\pi$ V. T. 5. N°. 14.

$$16) \int (Tang.^p x + Tang.^{-p} x) (Tang.^q x + Tang.^{-q} x) dx = 2\pi \frac{Cos.^{\frac{1}{2}p} \pi \cdot Cos.^{\frac{1}{2}q} \pi}{Cos.^p \pi + Cos.^q \pi}, p < 1, q < 1; \quad \text{V. T. 5. N}^\circ 10.$$

$$17) \int Tang.^p x - Tang.^{-p} x (Tang.^q x - Tang.^{-q} x) dx = 2\pi \frac{Sin.^{\frac{1}{2}p} \pi \cdot Sin.^{\frac{1}{2}q} \pi}{Cos.^p \pi + Cos.^q \pi}, p < 1, q < 1; \quad \text{V. T. 5. N}^\circ 9.$$

$$18) \int \frac{(Cos. x - Sin. x)^p}{Sin.^p x Sin. 2x} dx = -\frac{1}{2} \pi Cosec. p \pi \quad \text{V. T. 31. N}^\circ 1.$$

$$19) \int \frac{(Cos. x - Sin. x)^{1-p} Sin.^p x}{Cos.^3 x} dx = \frac{1-p}{2} p \pi Cosec. p \pi \quad \text{V. T. 31. N}^\circ 5.$$

$$20) \int \frac{(Tang.^p x - Cot.^p x) (Tang.^q x + Cot.^q x)}{Cos. 2x} dx = \frac{-\pi Sin. p \pi}{Cos.^p \pi + Cos.^q \pi}, p < 1, q < 1; \quad \text{V. T. 5. N}^\circ 15.$$

$$21) \int \left(\frac{Cos. x - Sin. x}{Cos. x} \right)^{p-1} \frac{dx}{Cos.^2 x} = \pi Cosec. p \pi \quad \text{V. T. 5. N}^\circ 7.$$

$$22) \int Sin. (p Tang. x) \frac{dx}{Sin. 2x} = \frac{1}{2} Si. (p) \quad \text{V. T. 192. N}^\circ 5.$$

$$23) \int Cos. (p Cot. x) \frac{dx}{Sin. 2x} = -\frac{1}{2} Ci. (p) \quad \text{V. T. 254. N}^\circ 1.$$

$$24) \int \frac{Cos. (q Tang. x) - Cos. (q Cot. x)}{Cos. 2x} dx = \frac{1}{2} \pi Sin. q \quad \text{V. T. 192. N}^\circ 11.$$

$$1) \int \frac{Tang. x}{1+p Tang. x} dx = -\frac{1}{1+p^2} \left\{ l \frac{1+p}{1 \vee 2} - \frac{1}{4} p \pi \right\} \quad \text{V. T. 6. N}^\circ 1.$$

$$2) \int \frac{dx}{1-Sin. x \cdot Cos. x} = \frac{2\pi}{3 \vee 3} \quad \text{V. T. 7. N}^\circ 1.$$

$$3) \int \frac{dx}{1+Sin. x \cdot Cos. x} = \frac{\pi}{3 \vee 3} \quad \text{V. T. 7. N}^\circ 2.$$

$$4) \int \frac{dx}{1-Sin.^2 x \cdot Cos.^2 x} = \frac{\pi}{2 \vee 3} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{V. T. 48. N}^\circ 2, 3.$$

$$5) \int \frac{Sin. 2x}{1-Sin.^2 x \cdot Cos.^2 x} dx = \frac{\pi}{3 \vee 3} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$6) \int \frac{Sin.^2 x}{1+3 Sin.^2 x \cdot Cos.^2 x} dx = \frac{1}{2} \pi \quad \text{V. T. 31. N}^\circ 19.$$

- 7) $\int \frac{dx}{1 - 3 \sin^2 x \cos^2 x} = \frac{1}{2} \pi$ V. T. 7. N°. 19.
- 8) $\int \frac{\text{Tang. } x}{1 - \cos. \lambda. \sin. 2x} dx = \frac{\pi - \lambda}{2 \text{Tang. } \lambda} - l \left(2 \sin. \frac{1}{2} \lambda \right)$ V. T. 7. N°. 3, 5.
- 9) $\int \frac{\text{Tang.}^a x}{1 \pm \sin. 2x \sqrt{\frac{1}{2}}} dx = \frac{1}{8} \left\{ Z' \left(\frac{a+7}{8} \right) - Z' \left(\frac{a+1}{8} \right) + Z' \left(\frac{a+5}{8} \right) - Z' \left(\frac{a+3}{8} \right) \right\} \mp$
 $\mp \frac{1}{8} \sqrt{2} \left\{ Z' \left(\frac{a+6}{8} \right) - Z' \left(\frac{a+2}{8} \right) \right\}$ V. T. 7. N°. 10.
- 10) $\int \frac{\text{Tang. } x}{1 - p \sin. 2x} dx = \frac{1}{2} l \{ 2(1-p) \} + \frac{p}{2} \frac{\text{Arccos. } (-p)}{\sin. \{ \text{Arccos. } (-p) \}}, \text{ pour } p < 1;$
11) $= \frac{1}{2} l \{ 2(p-1) \} - \frac{p}{2 \sqrt{(p^2-1)}} l \{ p + \sqrt{(p^2-1)} \}, \text{ pour } p > 1;$ V. T. 7.
N°. 11, 12.
- 12) $\int \frac{\text{Tang.}^c x dx}{1 + \sin. 2x \cos. \frac{a\pi}{b}} = \text{Cosec.} \frac{a\pi}{b} \sum_0^{c-1} (-1)^{n-1} \sin. \frac{na\pi}{b} \left\{ Z' \left(\frac{c+b+n}{2b} \right) - Z' \left(\frac{c+n}{2b} \right) \right\}, \text{ pour } a+b \text{ impair};$
13) $= \text{Cosec.} \frac{a\pi}{b} \sum_0^{\frac{1}{2}(c-1)} (-1)^{n-1} \sin. \frac{na\pi}{b} \left\{ Z' \left(\frac{b+c-n}{b} \right) - Z' \left(\frac{c+n}{b} \right) \right\}, \text{ pour } a+b \text{ pair};$ V. T. 7.
N°. 13, 14.
- 14) $\int \frac{\text{Tang.}^a x}{1 + \sin. x \cos. x} dx = \frac{1}{3} \left\{ Z' \left(\frac{a+2}{3} \right) - Z' \left(\frac{a+1}{3} \right) \right\}$ V. T. 7. N°. 8.
- 15) $\int \frac{\text{Tang.}^a x}{1 - \sin. x \cos. x} dx = \frac{1}{6} \left\{ Z' \left(\frac{a+5}{6} \right) - Z' \left(\frac{a+2}{6} \right) + Z' \left(\frac{a+4}{6} \right) - Z' \left(\frac{a+1}{6} \right) \right\}$ V. T. 7. N°. 9.
- 16) $\int \frac{\text{Tang.}^p x + \text{Cot.}^p x}{1 + \sin. 2x \cos. \lambda} dx = \frac{\pi \sin. p \lambda}{\sin. p \pi \sin. \lambda}$ V. T. 7. N°. 7.
- 17) $\int \frac{1 - \text{Tang. } x}{1 - \sin. 2x \cos. \lambda} dx = \frac{1}{\sin. \lambda} \sum_1^{\infty} \frac{\sin. n \lambda}{n(n+1)}$ V. T. 7. N°. 22.
- 18) $\int \frac{1 - \text{Tang. } x \cos. \lambda - \text{Tang.}^{a+1} x \cos. \{(a+1)\lambda\} + \text{Tang.}^{a+2} x \cos. a \lambda}{1 - \cos. \lambda \sin. 2x} dx = \sum_1^a \frac{\cos. n \lambda}{n+1}$ V. T. 7. N°. 15.
- 19) $\int \frac{\sin. \lambda - \text{Tang.}^a x \sin. \{(a+1)\lambda\} + \text{Tang.}^{a+1} x \sin. a \lambda}{1 - \cos. \lambda \sin. 2x} dx = \sum_1^a \frac{\sin. n \lambda}{n+1}$ V. T. 7. N°. 16.

- 1) $\int \frac{Tang^p x + Cot^p x}{(1 + Sin. 2x. Cos. \lambda)^2} Sin. 2x dx = \frac{\pi}{Sin. p \pi. Sin.^3 \lambda} (p Sin. \lambda. Cos. p \lambda - Cos. \lambda. Sin. p \lambda) \quad \begin{matrix} \text{V. T. 8.} \\ \text{N}^\circ. 1. \end{matrix}$
- 2) $\int \frac{Tang.^a x. Cos.^2 x}{(1 + Sin. x. Cos. x)^2} dx = 2 \frac{1-a}{9} Z' \left(\frac{a+2}{3} \right) - \frac{2-a}{9} Z' \left(\frac{a+1}{3} \right) + \frac{a}{9} Z' \left(\frac{a}{3} \right) + \frac{1}{3} \quad \begin{matrix} \text{V. T. 8.} \\ \text{N}^\circ. 2. \end{matrix}$
- 3) $\int \frac{Tang.^a x. Cos.^2 x}{(1 - Sin x. Cos. x)^2} dx = \frac{1-a}{9} \left\{ Z' \left(\frac{a+5}{6} \right) - Z' \left(\frac{a+2}{6} \right) \right\} - \frac{2-a}{18} \left\{ Z' \left(\frac{a+4}{6} \right) - Z' \left(\frac{a+1}{6} \right) \right\} +$
 $+ \frac{a}{18} \left\{ Z' \left(\frac{a+3}{6} \right) - Z' \left(\frac{a}{6} \right) \right\} + \frac{1}{3} \quad \text{V. T. 8. N}^\circ. 3.$
- 4) $\int \frac{Tang.^a x. Cos.^2 x}{(1 \pm Sin. x. Cos. x \sqrt{2})^2} dx = \frac{1-a}{8} \left\{ Z' \left(\frac{a+7}{8} \right) - Z' \left(\frac{a+3}{8} \right) \right\} - \frac{1}{8} \left\{ Z' \left(\frac{a+5}{8} \right) - Z' \left(\frac{a+1}{8} \right) \right\}$
 $\pm \frac{2-a}{16} \sqrt{2} \left\{ Z' \left(\frac{a+6}{8} \right) - Z' \left(\frac{a+2}{8} \right) \right\} \mp \frac{a\sqrt{2}}{16} \left\{ Z' \left(\frac{a+4}{8} \right) - Z' \left(\frac{a}{8} \right) \right\} + \frac{1}{2} \quad \begin{matrix} \text{V. T. 8.} \\ \text{N}^\circ. 4. \end{matrix}$
- 5) $\int \frac{Sin^{p-1} 2x}{(Cos. x + Sin. x)^{2p}} dx = \frac{1}{2^{p+1}} \frac{\Gamma(p)\Gamma(\frac{1}{2})}{\Gamma(p+\frac{1}{2})} \quad \text{V. T. 4. N}^\circ. 3.$
- 6) $\int \frac{Tang.^q x - Tang.^p x}{Cos. x - Sin. x} \frac{dx}{Cos. x} = Z'(1+p) - Z'(1+q) \quad \text{V. T. 3. N}^\circ. 8.$
- 7) $\int \frac{Cot.^q x - Cot.^p x}{Cos. x - Sin. x} \frac{dx}{Cos. x} = Z'(1-p) - Z'(1-q) \quad \text{V. T. 5. N}^\circ. 4.$
- 8) $\int \frac{Tang.^{p-1} x + Cot.^p x}{Cos. x + Sin. x} \frac{dx}{Cos. x} = \pi Cosec. p \pi \quad \text{V. T. 5. N}^\circ. 1.$
- 9) $\int \frac{Tang.^{p-1} x - Cot.^p x}{Cos. x - Sin. x} \frac{dx}{Cos. x} = \pi Cot. p \pi \quad \text{V. T. 5. N}^\circ. 6.$
- 10) $\int \frac{Cot.^p x - 1}{Cos. x - Sin. x} \frac{dx}{Cos. x} = \Lambda - Z'(1-p) \quad \text{V. T. 5. N}^\circ. 3.$
- 11) $\int \frac{Tang.^p x - Cot.^p x}{Cos. x - Sin. x} \frac{dx}{Cos. x} = \pi Cot. p \pi - \frac{1}{p} \quad \text{V. T. 5. N}^\circ. 5.$
- 12) $\int \frac{Cos. 2x}{1 + Sin. 2x. Cos. \lambda} \frac{dx}{Cos.^2 x} = Cos. \lambda l \{2(1 + Cos. \lambda)\} + \frac{1}{2} Sin. \lambda - 1 \quad \text{V. T. 7. N}^\circ. 6.$
- 13) $\int \frac{Tang.^p x + Tang.^q x}{Tang.^{p+q} x + 1} \frac{dx}{Sin. 2x} = \frac{1}{2} \frac{\pi}{p+q} Sec. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\} \quad \text{V. T. 31. N}^\circ. 17.$
- 14) $\int \frac{Tang.^q x - Tang.^p x}{Tang.^{p+q} x - 1} \frac{dx}{Sin. 2x} = \frac{1}{2} \frac{\pi}{p+q} Tang. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\} \quad \text{V. T. 31. N}^\circ. 18.$

- $$15) \int \frac{Tang.^q x - Tang.^p x}{Cos. x - Sin. x} \frac{dx}{Sin. x} = Z'(p) - Z'(q) \quad \text{V. T. 5. N}^\circ. 8.$$
- $$16) \int \frac{Tang.^p x - Tang.^{1-p} x}{Cos. x - Sin. x} \frac{dx}{Sin. x} = \pi Cot. p \pi \quad \text{V. T. 5. N}^\circ. 2.$$
- $$17) \int \frac{Tang.^q x - Cot.^q x}{Tang.^p x - Cot.^p x} \frac{dx}{Sin. 2 x} = \frac{\pi}{4 p} Tang. \frac{q \pi}{2 p} \quad \text{V. T. 5. N}^\circ. 21.$$
- $$18) \int \frac{Tang.^q x + Cot.^q x}{Tang.^p x + Cot.^p x} \frac{dx}{Sin. 2 x} = \frac{\pi}{4 p} Sec. \frac{q \pi}{2 p} \quad \text{V. T. 31. N}^\circ. 24.$$
- $$19) \int \frac{1}{Tang.^p x + Cot.^p x} \frac{dx}{Sin. 2 x} = \frac{\pi}{4 p} \quad \text{V. T. 5. N}^\circ. 23.$$
- $$20) \int \left\{ \frac{Sin. \lambda}{1 + Sin. 2 x Cos. \lambda} - \frac{\lambda}{(Sin. x + Cos. x)^2} \right\} dx = 0 \quad \text{V. T. 8. N}^\circ. 8.$$
- $$21) \int \frac{dx}{(Tang. x + Cot. x)^{2p} Sin. 2 x} = \frac{\{\Gamma(p)\}^2}{4 \Gamma(2 p)} \quad \text{V. T. 5. N}^\circ. 24.$$
- $$22) \int \frac{Sin.^p x}{(Cos. x - Sin. x)^{p+1}} \frac{dx}{Cos. x} = -\pi Cosec. p \pi \quad \text{V. T. 4. N}^\circ. 8.$$
- $$23) \int \frac{Sin.^p x}{(Cos. x - Sin. x)^p} \frac{dx}{Cos.^2 x} = p \pi Cosec. p \pi \quad \text{V. T. 4. N}^\circ. 7.$$
- $$24) \int \frac{Sin.^p x}{(Cos. x - Sin. x)^p} \frac{dx}{Cos. 2 x} = \frac{1}{2} \pi Cosec. p \pi \quad \text{V. T. 31. N}^\circ. 20.$$
- $$25) \int \frac{Sin.^p x}{(Cos. x - Sin. x)^{p-1}} \frac{dx}{Cos.^3 x} = \frac{1+p}{2} p \pi Cosec. p \pi \quad \text{V. T. 31. N}^\circ. 22.$$
- $$26) \int \frac{Sin.^p x}{(Cos. x - Sin. x)^p} \frac{dx}{Sin. 2 x} = \frac{1}{2} \pi Cosec. p \pi \quad \text{V. T. 4. N}^\circ. 6.$$
- $$27) \int \frac{Tang.^{p-q} x + Cot.^{p-q} x}{(Tang. x + Cot. x)^{p+q}} \frac{dx}{Sin. 2 x} = \frac{1}{4} B(p, q) \quad \text{V. T. 5. N}^\circ. 25.$$
- $$28) \int \frac{Tang.^{2p} x + Cot.^{2p} x}{(Tang. x + Cot. x)^{2q} Sin. 2 x} dx = \frac{\Gamma(p+q) \Gamma(q-p)}{q \Gamma(2 q)} \quad \text{V. T. 5. N}^\circ. 26.$$
- $$29) \int \frac{Tang.^p x + Cot.^p x}{Tang.^q x + Cot.^q x + 2 Cos. \lambda} \frac{dx}{Sin. 2 x} = \frac{\pi Sin. \frac{p \lambda}{q}}{2 q Sin. \lambda. Sin. \frac{p \pi}{2 q}} \quad \text{V. T. 8. N}^\circ. 12.$$
- $$30) \int \frac{Tang.^p x + Cot.^p x - 2 Cos. \lambda}{Tang.^q x + Cot.^q x - 2 Cos. \mu} \frac{dx}{Sin. 2 x} = \frac{\pi Sin. \left(\frac{\pi - \mu}{q} p \right)}{2 q Sin. \mu. Sin. \frac{p \pi}{q}} + \frac{\mu - \pi}{2 q} \frac{Cos. \lambda}{Sin. \mu} \quad \text{V. T. 8. N}^\circ. 9.$$

- 1) $\int dx \sqrt{1 - \text{Tang.}^4 x} = \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2}\pi} - \frac{\pi\sqrt{2}\pi}{\{\Gamma(\frac{1}{4})\}^2} \quad \text{V. T. 12. N}^\circ. 9.$
- 2) $\int \frac{\text{Tang.}^3 x}{\sqrt{\text{Cos.} 2x}} dx = \frac{1}{2} \quad \text{V. T. 13. N}^\circ. 7.$
- 3) $\int \frac{dx \sqrt{\text{Cos.} 2x}}{\text{Cos.}^2 x} = \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2}\pi} - \frac{\pi\sqrt{2}\pi}{\{\Gamma(\frac{1}{4})\}^2} \quad \text{V. T. 12. N}^\circ. 9.$
- 4) $\int \frac{dx \sqrt{\text{Cos.} 2x}}{\text{Cos.}^3 x} = \frac{1}{4} \pi \quad \text{V. T. 9. N}^\circ. 4.$
- 5) $\int \frac{\text{Cos.}^{a-\frac{1}{2}} 2x}{\text{Cos.}^{2a+1} x} dx = \frac{(a+1)^{a/1}}{1^{a/1}} \frac{\pi}{2^{2a+1}} \quad \text{V. T. 9. N}^\circ. 5.$
- 6) $\int \frac{\text{Sin.}^{2a-1} x}{\text{Cos.}^{2a+2} x} dx \sqrt{\text{Cos.} 2x} = \frac{2^{a-1/2}}{3^{a/2}} \quad \text{V. T. 9. N}^\circ. 6.$
- 7) $\int \frac{\text{Sin.}^{2a} x}{\text{Cos.}^{2a+3} x} dx \sqrt{\text{Cos.} 2x} = \frac{3^{a-1/2}}{4^{a/2}} \frac{\pi}{4} \quad \text{V. T. 9. N}^\circ. 7.$
- 8) $\int \frac{\text{Sin.}^{2a-1} x}{\text{Cos.}^{2a+2b-1} x} \text{Cos.}^{b-\frac{1}{2}} 2x dx = \frac{2^{a-1/2} 1^{b/2}}{1^{a+b/2}} \quad \text{V. T. 9. N}^\circ. 10.$
- 9) $\int \frac{\text{Sin.}^{2a} x}{\text{Cos.}^{2a+2b} x} \text{Cos.}^{b-\frac{1}{2}} 2x dx = \frac{1^{a/2} 1^{b/2}}{1^{a+b/2}} \frac{\pi}{2^{a+b+1}} \quad \text{V. T. 9. N}^\circ. 8.$
- 10) $\int \frac{\text{Sin.}^{2p} x}{\text{Cos.}^{p+\frac{1}{2}} 2x. \text{Cos.} x} dx = \frac{1}{2} \pi \text{Sec.} p \pi \quad \text{V. T. 12. N}^\circ. 17.$
- 11) $\int \frac{(\text{Cot.} x - 1)^{p-\frac{1}{2}}}{\text{Cos.}^2 x} dx = \frac{2p+1}{2} \pi \text{Sec.} p \pi \quad \text{V. T. 32. N}^\circ. 3$
- 12) $\int \frac{(\text{Cot.} x - 1)^{p-\frac{1}{2}}}{\text{Sin.} 2x} dx = \frac{1}{2} \pi \text{Sec.} p \pi \quad \text{V. T. 32. N}^\circ. 1.$
- 13) $\int \frac{\text{Tang.}^{p-1} x + \text{Tang.}^{q-1} x}{\text{Cos.}^{\frac{p+q}{2}} 2x} \text{Cos.}^{p+q-2} x dx = \frac{1}{2} \text{Cos.} \left(\frac{q-p}{4} \pi \right) \cdot \text{Sec.} \left(\frac{q+p}{4} \pi \right) \text{B} \left(\frac{1}{2} p, \frac{1}{2} q \right) \quad \text{V. T. 12. N}^\circ. 23.$
- 14) $\int \frac{\text{Tang.}^{p-1} x - \text{Tang.}^{q-1} x}{\text{Cos.}^{\frac{p+q}{2}} 2x} \text{Cos.}^{p+q-2} x dx = \frac{1}{2} \text{Sin.} \left(\frac{q-p}{4} \pi \right) \cdot \text{Sec.} \left(\frac{q+p}{4} \pi \right) \text{B} \left(\frac{1}{2} p, \frac{1}{2} q \right) \quad \text{V. T. 12. N}^\circ. 24.$
- 15) $\int (\sqrt{\text{Tang.} x} + \sqrt{\text{Cot.} x}) dx = \frac{1}{2} \pi \sqrt{2} \quad \text{V. T. 15. N}^\circ. 2.$
- 16) $\int (\text{Cos.} 2x)^{a-\frac{1}{2}} \cdot \text{Cos.} (p \text{Tang.} x) \frac{dx}{\text{Cos.}^{2a+1} x} = \frac{1^{a/2}}{2^{a+2} 1^{a/1}} \left\{ 1 + \sum_1 (-1)^n \frac{(\frac{1}{2} p)^{2n}}{1^{n/1} (a+1)^{n/1}} \right\} \quad \text{V. T. 192. N}^\circ. 6.$

$$1) \int \frac{\text{Cos.}^{-\frac{1}{2}} 2x - 1}{\text{Tang. } x} dx = \frac{1}{2} l 2 \quad \text{V. T. 15. N}^{\circ}. 5.$$

$$2) \int \frac{dx}{\text{Cos. } x \vee \text{Cos. } 2x} = \frac{1}{2} \pi \quad \text{V. T. 12. N}^{\circ}. 10.$$

$$3) \int \frac{\text{Sin. } x}{\text{Cos.}^2 x \vee \text{Cos. } 2x} dx = 1 \quad \text{V. T. 12. N}^{\circ}. 11.$$

$$4) \int \frac{\text{Sin.}^{2a-1} x}{\text{Cos.}^{2a} x \vee \text{Cos. } 2x} dx = \frac{2^{a-1/2}}{1^{a/2}} \quad \text{V. T. 12. N}^{\circ}. 13.$$

$$5) \int \frac{\text{Sin.}^{2a} x}{\text{Cos.}^{2a+1} x \vee \text{Cos. } 2x} dx = \frac{3^{a-1/2} \pi}{2^{a/2}} \quad \text{V. T. 12. N}^{\circ}. 12.$$

$$6) \int \frac{dx}{\text{Cos.}^2 x} \vee \frac{\text{Cos.}^2 x - p. \text{Sin.}^2 x}{\text{Cos. } 2x} = 1 - \sum_1 \left\{ \frac{1^{n-1/2}}{2^{n/2}} \right\}^2 (2n-1) p^{2n} \quad \text{V. T. 12. N}^{\circ}. 14.$$

$$7) \int \frac{dx}{\text{Cos.}^2 x} \vee \frac{\text{Cos.}^4 x - p^2 \text{Sin.}^4 x}{\text{Cos. } 2x} = \frac{c \text{F}'(c) + b \text{F}'(b)}{(b+c)^2} + \frac{b-c}{(b+c)^2} \{ \text{E}'(b) - \text{E}'(c) \},$$

$$\text{où } 2c^2 = \frac{(1-\vee p)^2}{1+p}, \quad 2b^2 = \frac{(1+\vee p)^2}{1+p}; \quad \text{V. T. 13. N}^{\circ}. 3.$$

$$8) \int \frac{\text{Sin.}^{p-\frac{1}{2}} 2x}{\text{Cos.}^p 2x. \text{Cos. } x} dx = \frac{2}{2p-1} \frac{\Gamma(p+\frac{1}{2}) \Gamma(1-p)}{\vee \pi} \text{Sin.} \left\{ \frac{2p-1}{4} \pi \right\} \quad \text{V. T. 12. N}^{\circ}. 13.$$

$$9) \int \frac{\text{Sin.}^{2a-1} x}{\text{Cos.}^{2a-2b+2} x. \text{Cos.}^{b-\frac{1}{2}} 2x} dx = (-1)^{b-1} \frac{2^{a/2}}{1^{b-1/2} 3^{a-b/2}} \frac{1}{2a} \quad \text{V. T. 12. N}^{\circ}. 19.$$

$$10) \int \frac{\text{Sin.}^{2a} x}{\text{Cos.}^{2a-2b+3} x. \text{Cos.}^{b-\frac{1}{2}} 2x} dx = (-1)^{b-1} \frac{3^{a-1/2}}{1^{b-1/2} 4^{a-b/2}} \frac{\pi}{4} \quad \text{V. T. 12. N}^{\circ}. 21.$$

$$11) \int \frac{(\text{Cos. } x - \text{Sin. } x)^{a-\frac{1}{2}}}{\text{Cos.}^{a+1} x} \frac{\text{Tang.}^b x}{\vee \text{Sin. } x} dx = \frac{1^{a/2} 1^{b/2}}{2^{a+b/2}} \quad \text{V. T. 15. N}^{\circ}. 8.$$

$$12) \int \frac{(\text{Cos. } x - \text{Sin. } x)^{a-\frac{1}{2}}}{\text{Cos.}^{a+1} x \vee \text{Sin. } x} dx = \frac{1^{a/2}}{2^{a/2}} \pi \quad \text{V. T. 15. N}^{\circ}. 7.$$

$$13) \int \frac{(\text{Sin. } x - \text{Cos. } x)^{p+\frac{1}{2}}}{\text{Sin.}^{p+\frac{1}{2}} x. \text{Cos.}^2 x} dx = \frac{2p+1}{2} \pi \text{Sec. } p \pi \quad \text{V. T. 11. N}^{\circ}. 1.$$

$$14) \int \frac{(\text{Sin. } x - \text{Cos. } x)^{p-\frac{1}{2}}}{\text{Sin.}^{p+\frac{1}{2}} x. \text{Cos. } x} dx = \pi \text{Sec. } p \pi \quad \text{V. T. 11. N}^{\circ}. 2.$$

$$15) \int \frac{1}{\vee \text{Sin.}^2 x. \text{Cos. } x} \frac{dx}{\vee \text{Cos. } 2x} = \frac{3}{\vee 3} \text{F}' \left(\text{Sin.} \frac{\pi}{12} \right) \quad \text{V. T. 15. N}^{\circ}. 11.$$

- 16) $\int \frac{1}{\sqrt{\sin x \cos^2 x}} \frac{dx}{\sqrt{\cos 2x}} = \frac{1}{\sqrt{3}} F' \left(\cos, \frac{\pi}{12} \right)$ V. T. 15. N°. 12.
- 17) $\int \frac{\sqrt{\tan x}}{\sqrt{\cos 2x} \cos x} \frac{dx}{\cos x} = \frac{1-\sqrt{3}}{\sqrt{3}} F' \left(\cos, \frac{\pi}{12} \right) + \frac{2\sqrt{3}}{\sqrt{3}} E' \left(\cos, \frac{\pi}{12} \right)$ V. T. 12. N°. 15.
- 18) $\int \frac{\sqrt{\tan^2 x}}{\sqrt{\cos 2x} \cos x} \frac{dx}{\cos x} = \frac{3\sqrt{3}}{\sqrt{3}} E' \left(\sin, \frac{\pi}{12} \right) - \frac{3+2\sqrt{3}}{2\sqrt{3}} F' \left(\sin, \frac{\pi}{12} \right)$ V. T. 12. N°. 16.

- 1) $\int \frac{dx}{\cos x \sqrt{\sin x (\cos x - \sin x)}} = \pi$ V. T. 15. N°. 6.
- 2) $\int \frac{dx}{\cos x \sqrt{\sin x (\cos x + p \sin x)}} = \frac{2}{\sqrt{p}} l \{ \sqrt{p} + \sqrt{1+p} \}$ V. T. 15. N°. 10.
- 3) $\int \frac{1}{a \cos x - b \sin x} \frac{dx}{\sqrt{\sin x (\cos x - \sin x)}} = \frac{\pi}{\sqrt{a(a-b)}}, b \text{ ni} = a, \text{ ni} = 0;$ V. T. 15. N°. 15.
- 4) $\int \frac{\sin^a x}{\cos^{a+1} x} \frac{dx}{\sqrt{\cos x (\cos x - \sin x)}} = \frac{2^{a/2}}{3^{a/2}} 2$ V. T. 12. N°. 2.
- 5) $\int \frac{\sin^a x}{\cos^{a+1} x} \frac{dx}{\sqrt{\sin x (\cos x - \sin x)}} = \frac{1^{a/2}}{2^{a/2}} \pi$ V. T. 15. N°. 9.
- 6) $\int \frac{dx}{\cos x \sqrt{(\cos^2 x + p \sin^2 x)}} = \frac{1}{\sqrt{p}} l \{ \sqrt{p} + \sqrt{1+p} \}$ V. T. 12. N°. 25.
- 7) $\int \frac{\tan x}{\sqrt{(p \cos^2 x + \sin^2 x)}} \frac{dx}{\sqrt{\cos 2x}} = \frac{\pi}{2} - \text{Arctg. } p$ V. T. 16. N°. 6.
- 8) $\int \frac{\sqrt{\cot x} - 1}{\cos x - \sin x} \frac{dx}{\cos x} = l 4$ V. T. 15. N°. 5.
- 9) $\int \frac{(1 - \tan x)^{-\frac{1}{2}} - 1}{\sin 2x} dx = l 2$ V. T. 15. N°. 1.
- 10) $\int \frac{1}{\tan^2 x + \cot^2 x} \frac{dx}{\sqrt{\cos 2x}} = \frac{\pi}{8}$ V. T. 16. N°. 3.
- 11) $\int \frac{\cot^2 x}{\tan^2 x + \cot^2 x} \frac{dx}{\sqrt{\cos 2x}} = \frac{\pi}{8} + \frac{1}{4} \sqrt{2} F' \left(\sin, \frac{\pi}{4} \right)$ V. T. 16. N°. 4.
- 12) $\int \frac{\sin^{p-\frac{1}{2}} x}{(\cos x - \sin x)^{p+\frac{1}{2}}} \frac{dx}{\cos x} = \pi \text{ Sec. } p \pi$ V. T. 12. N°. 6.

$$13) \int \frac{\sin p^{-\frac{1}{2}} x}{(\cos x - \sin x)^{p-\frac{1}{2}}} \frac{dx}{\cos^2 x} = \frac{2p-1}{2} \pi \sec p \pi \quad \text{V. T. 12. N}^\circ. 7.$$

$$14) \int \frac{1}{(\cot x - 1)^{p+\frac{1}{2}}} \frac{dx}{\cos^2 x} = \frac{2p+1}{2} \pi \sec p \pi \quad \text{V. T. 32. N}^\circ. 17.$$

$$15) \int \frac{1}{(\cot x - 1)^{p+\frac{1}{2}}} \frac{dx}{\sin^2 x} = \frac{1}{2} \pi \sec p \pi \quad \text{V. T. 32. N}^\circ. 15.$$

$$16) \int \frac{\sin p^{-\frac{1}{2}} 2x}{(\cos x - \sin x)^{2p}} \frac{dx}{\cos x} = \frac{2\frac{1}{2}-p}{1-2p} \frac{\Gamma(p+\frac{1}{2}) \Gamma(1-p)}{\sqrt{\pi}}, \quad p < \frac{1}{2}; \quad \text{V. T. 12. N}^\circ. 4.$$

$$17) \int \left(\frac{\sin x}{\cos x - \sin x} \right)^p \frac{dx}{\cos x \sqrt{\sin x (\cos x - \sin x)}} = \pi \sec p \pi, \quad p < \frac{1}{2}; \quad \text{V. T. 15. N}^\circ. 17.$$

$$18) \int \frac{\sin^{\frac{1}{2}q} x \cos^q x dx}{\left\{ (\cos x - \sin x) \cos(x+\lambda) \cos(x-\lambda) \right\}^{\frac{q+1}{2}}} = \frac{1}{q} \sin q \lambda \operatorname{Cosec} \lambda B \left(\frac{q+2}{2}, \frac{1-q}{2} \right) \quad \text{V. T. 16. N}^\circ. 8.$$

$$19) \int \frac{\sin^a x \cos^{1-\frac{1}{2}a} 2x}{(\cos^2 x - p^2 \sin^2 x)^{\frac{1}{2}a-1}} \frac{dx}{\cos^4 x} = \frac{\Gamma\left(\frac{a+1}{2}\right) \Gamma\left(2-\frac{a}{2}\right)}{\sqrt{\pi(a-1)(a-3)(a-5)}} \left\{ \frac{1+(a-3)p+p^2}{(1+p)^{a-3}} - \frac{1-(a-3)p+p^2}{(1-p)^{a-3}} \right\} \quad \text{V. T. 16. N}^\circ. 14.$$

$$1) \int \sin bx dx = 0, \quad \text{pour } b = 4a; \quad \left. \begin{array}{l} 2) \\ 3) \\ 4) \end{array} \right\}$$

$$= \frac{1}{4a+1}, \quad \text{pour } b = 4a+1;$$

$$= \frac{1}{2a+1}, \quad \text{pour } b = 4a+2;$$

$$= \frac{1}{4a+3}, \quad \text{pour } b = 4a+3;$$

$$5) \int \cos bx dx = 0, \quad \text{pour } b = 4a; \quad \left. \begin{array}{l} 6) \\ 7) \\ 8) \end{array} \right\}$$

$$= \frac{1}{4a+1}, \quad \text{pour } b = 4a+1;$$

$$= 0, \quad \text{pour } b = 4a+2;$$

$$= \frac{-1}{4a+3}, \quad \text{pour } b = 4a+3;$$

Meyer, Int. Déf. 97.

- 9) $\int \sin. x \, dx = 1$
- 10) $\int \cos. x \, dx = 1$
- 11) $\int \cos.^2 x \, dx = \frac{1}{4} \pi$ Liouville, Cr. 13. 219.
- 12) $\int \sin.^{2a} x \, dx = \frac{1^{a/2}}{2^{a/2}} \frac{\pi}{2}$ Cauchy, Cours. Leç. 32. — Poisson, Chal. 78. — Dienger, Cr. 38. 331.
- 13) $\int \sin.^{2a+1} x \, dx = \frac{2^{a/2}}{3^{a/2}}$ Cauchy, Cours. Leç. 32. — Dienger, 38. 266. — Oettinger, Cr. 38. 162.
- 14) $= \frac{(1^{a/1})^2}{1^{2a+1/1}} 2^{2a}$
- 15) $= \frac{1^{a/1}}{1^{a+1/1}} \frac{1}{2} \sqrt{\pi}$
- 16) $\int \cos.^{2a} x \, dx = \frac{1^{a/2}}{2^{a/2}} \frac{\pi}{2}$ Cauchy, Cours. Leç. 32.
- 17) $\int \cos.^{2a+1} x \, dx = \frac{2^{a/2}}{3^{a/2}}$ Cauchy, Cours. Leç. 32. — Oettinger, Cr. 38. 162.
- 18) $\int \sin.^p x \, dx = 2^{p-2} \frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma(p)}$ Lobatschewsky, Mém. Kasan. 1835. 1.
- 19) $= \frac{1 \sqrt{\pi}}{2(\frac{1}{2}p)^{1/2-1}}$ Lobatschewsky, Mém. Kasan. 1835. 211.
- 20) $\int \sin.^{p-1} x \, dx = \frac{1}{2} \sqrt{\pi} \frac{\Gamma(\frac{1}{2}p)}{\Gamma(\frac{1+p}{2})}$ Raabe, Int. 222.
- 21) $\int \cos.^p x \, dx = \frac{\pi}{2^{p+1}} \frac{\Gamma(p+1)}{\{\Gamma(\frac{1}{2}p+1)\}^2}$ Lobatschewsky, Mém. Kasan. 1835. 211.
- 22) $= 2^{p-1} \frac{\{\Gamma(\frac{1}{2}p+1)\}^2}{\Gamma(p+1)}$ Serret, L. 8. 1.
- 23) $\int \cos.^{p-1} x \, dx = \frac{1}{2} \sqrt{\pi} \frac{\Gamma(\frac{1}{2}p)}{\Gamma(\frac{1+p}{2})}$ Raabe, Int. 222.

$$24) \int \sin^{2a+1} x dx = \frac{1^{a/1} \cdot 1^{a/1}}{1^{2a+1/1}} 2^{2a} \quad \text{Oettinger, Cr. 38. 162.}$$

$$25) \int \tan^{2p-1} x dx = \frac{1}{2} \pi \operatorname{Cosec}. p \pi, \quad 1 > p > 0; \quad \text{Bonnet, L. 6. 233. — Oettinger, Cr. 38. 162.}$$

$$26) \int \tan^p x dx = \frac{1}{2} \pi \operatorname{Sec}. \frac{1}{2} p \pi, \quad 1 > p > 0; \quad \text{Cauchy, Exerc. 1826. p. 205. — Schlömilch, Gr. 6. 200.}$$

$$1) \int \sin^q x \cdot \sin. \{(q+2)x\} dx = \frac{1}{q+1} \operatorname{Cos}. \frac{q\pi}{2} \quad \text{Serret, L. 8. 1.}$$

$$2) \int \sin^{q-1} x \cdot \sin. q x dx = \frac{1}{1-q} \operatorname{Cos}. \frac{q\pi}{2} \quad \text{Serret, L. 8. 489.}$$

$$3) \int \sin^{2a+1} x \cdot \sin. \{(2b+1)x\} dx = (-1)^b \frac{\pi}{2^{2a+2}} \frac{(a+b+2)^{a-b/1}}{1^{a-b/1}}, \quad a > b; \quad \text{Jacobi, Cr. 15. 1.}$$

$$4) \quad = 0, \quad a < b; \quad \text{Ohm, Ausw. 13.}$$

$$5) \int \sin^{2a} x \cdot \sin. \{(2b+1)x\} dx = \frac{1^{2a/1}}{2^2 - (2b+1)^2 \cdot 4^2 - (2b+1)^2 \dots (2a)^2 - (2b+1)^2} \frac{1}{2b+1} \quad \text{Ohm, Ausw. 13.}$$

$$6) \int \sin^{2a} x \cdot \sin. p x dx = \frac{1}{p} \frac{1^{2a/1}}{2^2 - p^2 \cdot 4^2 - p^2 \dots (2a)^2 - p^2} \left\{ 1 - \operatorname{Cos}. \frac{p\pi}{2} \left(1 - \frac{p^2}{1 \cdot 2} - \frac{p^2 \cdot 2^2 - p^2}{1 \cdot 2 \cdot 3 \cdot 4} - \dots - \frac{p^2 \cdot 2^2 - p^2 \dots (2a-2)^2 - p^2}{1^{2a/1}} \right) \right\}$$

$$7) \int \sin^{2a+1} x \cdot \sin. p x dx = \frac{1}{p} \operatorname{Cos}. \frac{p\pi}{2} \frac{1^{2a+1/1}}{1^2 - p^2 \cdot 3^2 - p^2 \dots (2a+1)^2 - p^2} \left\{ \frac{p^2}{1} + \frac{p^2 \cdot 1^2 - p^2}{2 \cdot 3} + \dots + \frac{p^2 \cdot 1 - p^2 \dots (2a-1)^2 p^2}{1^{2a+1/1}} \right\}$$

Ces deux formules se trouvent chez Raabe, Int. 153.

$$8) \int \sin^q x \cdot \operatorname{Cos}. \{(q+2)x\} dx = \frac{-1}{q+1} \sin. \frac{q\pi}{2} \quad \text{Serret, L. 8. 1.}$$

$$9) \int \sin^{q-2} x \cdot \operatorname{Cos}. q x dx = \frac{1}{q-1} \sin. \frac{q\pi}{2} \quad \text{Serret, L. 8. 489.}$$

$$10) \int \sin^{2a} x \cdot \operatorname{Cos}. 2b x dx = (-1)^b \frac{\pi}{2^{2a+1}} \frac{(a+b+1)^{a-b/1}}{1^{a-b/1}}, \quad a > b; \quad \text{Jacobi, Cr. 15. 1.}$$

$$11) \quad = 0, \quad a < b;$$

$$12) \int \sin^{2a+1} x \cdot \operatorname{Cos}. 2b x dx = \frac{1^{2a+1/1}}{1^2 - (2b)^2 \cdot 3^2 - (2b)^2 \dots (2a+1)^2 - (2b)^2} \quad \left. \vphantom{\int} \right\} \text{Ohm, Ausw. 13.}$$

$$13) \int \text{Sin}^{2a} x. \text{Cos} p x dx = \frac{1}{p} \text{Sin} \frac{p\pi}{2} \frac{1^{2a/1}}{2^2 - p^2, 4^2 - p^2, \dots, (2a)^2 - p^2} \left\{ 1 - \frac{p^2}{1.2} - \frac{p^2.2^2 - p^2}{1.2.3.4} - \dots - \frac{p^2.2^2 - p^2 \dots (2a-2)^2 - p^2}{1^{2a-1}} \right\}$$

$$14) \int \text{Sin}^{2a+1} x. \text{Cos} p x dx = \frac{1^{2a+1/1}}{1^2 - p^2, 3^2 - p^2, \dots, (2a+1)^2 - p^2} \left\{ 1 - \frac{1}{p} \text{Sin} \frac{p\pi}{2} \left(\frac{p^2}{1} + \frac{p^2.1^2 - p^2}{1.2.3} + \dots + \frac{p^2.1^2 - p^2 \dots (2a-1)^2 - p^2}{1^{2a+1-1}} \right) \right\}$$

Sur ces deux formules voyez Raabe, Int. 153.

$$15) \int \text{Sin}^p x. \text{Cos} \left\{ p \left(\frac{\pi}{2} - x \right) \right\} dx = \frac{\pi}{2p+1} \quad \text{Cauchy, Exere. 1826. p. 255.}$$

$$1) \int \text{Cos}^{q-1} x. \text{Sin} \{ (q+1)x \} dx = \frac{1}{q} \quad \text{Serret, L. S. 1. — Id., L. S. 483. — Kummer, Cr. 20. 1.}$$

$$2) \int \text{Cos}^a x. \text{Sin} a x dx = \frac{1}{2^{a+1}} \sum_{i=1}^a \frac{2^a}{a} \quad \text{Serret, L. S. 1.}$$

$$3) \int \text{Cos}^q x. \text{Sin} \{ (q+2b)x \} dx = (q+2b) \sum_{i=1}^b (-1)^{n-1} \frac{2^{2n-2} (q+b+1)^{n-1/1} (b-n+1)^{n-1/1}}{(q+1)^{2n/1}} \quad \text{Serret, L. S. 1.}$$

$$4) \int \text{Cos}^{2a} x. \text{Sin} p x dx = \frac{1}{p} \frac{1^{2a/1}}{2^2 - p^2, 4^2 - p^2, \dots, 2a^2 - p^2} \left\{ 1 - \text{Cos} \frac{p\pi}{2} - \frac{p^2}{1.2} - \frac{p^2.2^2 - p^2}{1.2.3.4} - \dots - \frac{p^2.2^2 - p^2 \dots (2a-2)^2 - p^2}{1^{2a/1}} \right\}$$

$$5) \int \text{Cos}^{2a+1} x. \text{Sin} p x dx = \frac{1}{p} \frac{1^{2a+1/1}}{1^2 - p^2, 3^2 - p^2, \dots, (2a+1)^2 - p^2} \left\{ p \text{Sin} \frac{p\pi}{2} - \frac{p^2}{1} - \frac{p^2.1^2 - p^2}{1.2.3} - \dots - \frac{p^2.1^2 - p^2 \dots (2a-1)^2 - p^2}{1^{2a+1-1}} \right\}$$

Raabe, Int. 153. déduit ces deux formules.

$$6) \int \text{Cos}^p x. \text{Cos} q x dx = \frac{\pi}{2^{p+1}} \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+q}{2}+1\right) \Gamma\left(\frac{p-q}{2}+1\right)} \quad \begin{array}{l} \text{Cauchy, Lim. Imag. 124. — Ca-} \\ \text{talan, L. 5. 110. — Serret, L. S.} \\ \text{1. — Id., L. S. 489. — Kummer,} \\ \text{Cr. 17. 210. — Id., Cr. 20. 1. —} \\ \text{Lobatschewsky, Mém. Kasan. 1835.} \\ \text{211. — Schlämle, Stud. 1. 24.} \end{array}$$

$$7) \int \text{Cos}^p x. \text{Cos} q x dx = \frac{\pi}{(p+1) 2^{p+1} \text{B}\left(\frac{p+q}{2}+1, \frac{p-q}{2}+1\right)} \quad \begin{array}{l} \text{Serret, L. S. 1. — Binet, P.} \\ \text{27. 123.} \end{array}$$

$$8) \int \text{Cos}^q x. \text{Cos} \{ (q+2b)x \} dx = 0 \quad \begin{array}{l} \text{Poisson, P. 19. 404. N°. 76. — Id., Conn. des Temps. 1836. p. 1. —} \\ \text{Serret, L. S. 1.} \end{array}$$

$$9) \int \text{Cos}^q x. \{ \text{Cos} (q-2b)x \} dx = \frac{\pi}{2^{q+1}} \frac{(q-b+1)^{b/1}}{1^{b/1}}, \quad q > b-1; \quad \begin{array}{l} \text{Poisson, P. 19. 404. N°. 76. —} \\ \text{Binet, P. 27. 123. — Serret, L.} \\ \text{S. 1. — Jacobi, Cr. 15. 1.} \end{array}$$

- 10) $\int \text{Cos.}^a x. \text{Cos.} \{(a+2p)x\} dx = \frac{\text{Sin. } p \pi}{2^{a+1}} \frac{1^{a/1} \Gamma(p)}{\Gamma(a+p+1)}$ Kummer, Cr. 20. 1.
- 11) $\int \text{Cos.}^p x. \text{Cos.} \{(2b-p)x\} dx = \frac{\pi}{2^{p+1}} \frac{\Gamma(p+1)}{1^{b/1} \Gamma(1+p-b)}$ Cauchy, Bull. d. Sc. Math. de Férussac. 1825. N°. 250. — Hill, Cr. 7. 102.
- 12) $\int \text{Cos.}^a x. \text{Cos. } 2bx dx = \frac{\pi}{2^{a+1}} \frac{1^{a/1}}{\Gamma(\frac{1}{2}a+b+1) \Gamma(\frac{1}{2}a-b+1)}$ Lobatschewsky, Mém. Kasan. 1835. 211.
- 13) $\int \text{Cos.}^q x. \text{Cos. } qx dx = \frac{\pi}{2^{q+1}}$ Serret, L. 8. 1 — Id., L. 8. 489. — Lobatschewsky, Mém. Kasan. 1835. 211. — Poisson, P. 19. 404. N°. 76. (la trouve faut.) — Id., Conn. des Temps. 1836. p. 1.
- 14) $\int \text{Cos.}^{q-1} x. \text{Cos.} \{(q+1)x\} dx = 0$ Cauchy, Exerc. 1826. p. 205. — Serret, L. 8. 1. — Id., L. 8. 489. — Kummer, Cr. 20. 1. — Lindmann, Stockh. Handl. 1850. II.
- 15) $\int \text{Cos.}^{a+b} x. \text{Cos.} \{(a-b)x\} dx = \frac{\pi}{2^{a+b+1}} \frac{1^{a+b/1}}{1^{a/1} 1^{b/1}}$ Cauchy, Lim. Imag. 123. — Oettinger, Cr. 38. 216.
- 16) $\int \text{Cos.}^{2a} x. \text{Cos. } 2bx dx = \frac{\pi}{2^{2a+1}} \frac{1^{2a/1}}{1^{a+b/1} 1^{a-b/1}}$ Cauchy, Exerc. de Math. 2. 368. — Oettinger, Cr. 38. 216.
- 17)
$$= \frac{\pi}{2^{2a+1}} \frac{(a+b+1)^{a-b/1}}{1^{a-b/1}} \left. \vphantom{\frac{\pi}{2^{2a+1}}} \right\} \text{Jacobi, Cr. 15. 1.}$$
- 18) $\int \text{Cos.}^{2a+1} x. \text{Cos.} \{(2b+1)x\} dx = \frac{\pi}{2^{2a+2}} \frac{(a+b+2)^{a-b/1}}{1^{a-b/1}} \left. \vphantom{\frac{\pi}{2^{2a+2}}} \right\}$
- 19) $\int \text{Cos.}^{2a} x. \text{Cos. } px dx = \frac{1^{2a/1}}{2^2 - p^2 \cdot 4^2 - p^2 \dots (2a)^2 - p^2} \frac{1}{p} \text{Sin. } \frac{1}{2} p \pi$ $\left. \vphantom{\frac{1^{2a/1}}{2^2 - p^2 \cdot 4^2 - p^2 \dots (2a)^2 - p^2}} \right\}$ Raabe, Int. 153.
- 20) $\int \text{Cos.}^{2a+1} x. \text{Cos. } px dx = \frac{1^{2a+1/1}}{1^2 - p^2 \cdot 3^2 - p^2 \dots (2a+1)^2 - p^2} \text{Cos. } \frac{1}{2} p \pi$ $\left. \vphantom{\frac{1^{2a+1/1}}{1^2 - p^2 \cdot 3^2 - p^2 \dots (2a+1)^2 - p^2}} \right\}$
- 21) $\int \text{Cos.}^{p-2b} x. \text{Cos. } px dx = \frac{-\Gamma(b+p)}{1^{b/1} \Gamma(p)} \frac{\pi}{2^{p+2b-1}} \sum_0^{\infty} \frac{b^{n/-1}}{(p+b-1)^{n/-1}} \binom{b}{n}$ Schlömilch, Cr. 33. 353.

- 1) $\int \text{Sin.}^{2a} x. \text{Cos.}^{2b} x dx = \frac{1^{a/2} 1^{b/2}}{2^{a+b/2}} \frac{\pi}{2}$ Jacobi, Cr. 15. 1.
- 2)
$$= \frac{\Gamma(a+\frac{1}{2}) \Gamma(b+\frac{1}{2})}{2 \Gamma(a+b+1)}$$
 Schlömilch, Gr. 4. 316.

$$3) \int \sin^{2a} x \cdot \cos^{2b+1} x \, dx = \frac{1}{2a+1} \frac{1^{a/2} 2^{b/2}}{3^{a+b/2}} \quad \text{Oettinger, Cr. 38. 162.}$$

$$4) \int \sin^{2a+1} x \cdot \cos^{2b} x \, dx = \frac{1^{a/2} 1^{b/2}}{3^{a+b/2}} \quad \text{Ohm, Ausw. 49.}$$

$$5) \int \sin^{2a+1} x \cdot \cos^{2b+1} x \, dx = \frac{1^{a+1/2} 1^{b/2}}{1^{a+b+1/2}} \frac{1}{2(a+1)} \quad \begin{array}{l} \text{Oettinger, Cr. 38. 162. — Lobatschewsky.} \\ \text{Mém. Kasan. 1835. 211.} \end{array}$$

$$6) \int \sin^{2p-1} x \cdot \cos^{1-2p} x \, dx = \frac{1}{2} \pi \operatorname{Cosec}. p \pi \quad \begin{array}{l} \text{Bonnet., L. 6. 238. — Oettinger, Cr. 38. 162. —} \\ \text{Id., Cr. 38. 216.} \end{array}$$

$$7) \int \sin^{p-1} x \cdot \cos^{q-1} x \, dx = \frac{\Gamma(\frac{1}{2}p) \Gamma(\frac{1}{2}q)}{2 \Gamma(\frac{p+q}{2})} \quad \text{Raabe, Int. 222.}$$

$$\left. \begin{aligned} 8) \int \sin^{a-1} x \cdot \cos^{2b+1} x \, dx &= \frac{2^{b/2}}{a^{b+2/2}} \\ 9) \int \cos^{a-1} x \cdot \sin^{2b+1} x \, dx &= \frac{2^{b/2}}{a^{b+2/2}} \end{aligned} \right\} \quad \text{Oettinger, Cr. 38. 162.}$$

$$\left. \begin{aligned} 10) \int \cos^{a+b} x \cdot \operatorname{Tang}^{2a+1} x \, dx &= \frac{1^{a/2} 1^{b/2}}{1^{a+b/2}} \frac{1}{2b} \\ 11) \int \cos^{a+1} x \cdot \operatorname{Tang}^{2a+1} x \, dx &= \frac{1}{2(a+1)} \\ 12) \int \cos^{2a+1} x \cdot \operatorname{Tang}^{2a+1} x \, dx &= \frac{1^{a/2} 1^{a/2}}{2 \cdot 1^{2a+1/2}} \\ 13) \int \cos^{2a+2b-2} x \cdot \operatorname{Tang}^{2a-1} x \, dx &= \frac{1^{a-1/2} 1^{b-1/2}}{2 \cdot 1^{a+b-1/2}} \end{aligned} \right\} \quad \text{Oettinger, Cr. 38. 162.}$$

$$14) \int \cos^{2a-2} x \cdot \operatorname{Tang}^{p-1} x \, dx = \frac{\Gamma(\frac{1}{2}p) \Gamma(a - \frac{1}{2}p)}{2 \Gamma(a)} \quad \text{V. T. 21. N°. 9.}$$

$$15) \int \cos^{2p-1} x \cdot \operatorname{Tang}^{p-q} x \, dx = \frac{\Gamma(\frac{p-q+1}{2}) \Gamma(\frac{p+q}{2})}{2 \Gamma(p + \frac{1}{2})} \quad \text{V. T. 27. N°. 10.}$$

$$16) \int \sin^{a+b+1} x \cdot \cos^{a-b+1} x \, dx = \frac{\Gamma(\frac{a+b}{2} + 1) \Gamma(\frac{a+b}{2} - 1)}{2 \cdot 1^{a+1/2}} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211.}$$

- 1) $\int \text{Sin.}^{2a-1} x. \text{Cos.}^{2b-1} x. \text{Sin.} b x dx = 0, b > 2a;$
- 2) $\int \text{Sin.}^{2a} x. \text{Cos.}^{b-2a-2} x. \text{Cos.} b x dx = 0, b > 2a+1;$
- 3) $\int \text{Sin.}^{2a-1} x. \text{Cos.}^{2b-2a-1} x. \text{Cos.} 2bx dx = \frac{\Gamma(\frac{1}{2}) \Gamma(b-a+\frac{1}{2}) \Gamma(a) \Gamma(b-a)}{2 \Gamma(b) \Gamma(b+\frac{1}{2}) \Gamma(\frac{1}{2}-a)}$ Kummer, Cr. 17. 210. — Schlömilch, Stud. I. 24.
- 4) $= (-1)^a \frac{1^{2a-1/1} 1^{2b-2a-1,1}}{1^{2b-1/1}}$
- 5) $\int \text{Sin.}^{2a-2b-1} x. \text{Cos.}^{2a-1} x. \text{Cos.} 2bx dx = (-1)^{b-a} \frac{1^{2a-1/1} 1^{2b-2a-1/1}}{1^{2b-1,1}}$ Oettinger, Cr. 38. 216.
- 6) $\int \text{Sin.}^{p-q-1} x. \text{Cos.}^{q-1} x. \text{Sin.} p x dx = \frac{1^{q-1/1} 1^{p-q-1/1}}{1^{p-1/1}} \text{Sin.} \left\{ \frac{p-q}{2} \pi \right\}$
- 7) $\int \text{Sin.}^{p-q-1} x. \text{Cos.}^{q-1} x. \text{Cos.} p x dx = \frac{1^{q-1/1} 1^{p-q-1/1}}{1^{p-1/1}} \text{Cos.} \left\{ \frac{p-q}{2} \pi \right\}$ }, où p et q des fractions seulement; Oettinger, Cr. 38. 216.
- 8) $\int \text{Sin.}^{p-1} x. \text{Cos.}^{q-1} x. \text{Cos.} \{(p+q)x\} dx = B(p, q) \text{Cos.} \frac{q\pi}{2}$ Serret, L. 8. 1.
- 9) $= \frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)} \text{Cos.} \frac{p\pi}{2}$ }, $2 > p > 0;$
- 10) $\int \text{Sin.}^{p-1} x. \text{Cos.}^{q-1} x. \text{Sin.} \{(p+q)x\} dx = \frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)} \text{Sin.} \frac{p\pi}{2}$ Serret, L. 8. 489. — Kummer, Cr. 17. 210. — Id., Cr. 20. 1. — Schlömilch, Stud. I. 24.
- 11) $= \frac{\Gamma(q)}{\Gamma(p+q) \Gamma(1-p)} \frac{\pi}{2 \text{Cos.} \frac{1}{2} p\pi}$
- 12) $= B(p, q) \text{Sin.} \frac{q\pi}{2}$ } Serret, L. 8. 1.
- 13) $\int \text{Sin.}^{2a-1} x. \text{Cos.}^{2b-1} x. \text{Cos.} \{2(a+b)x\} dx = (-1)^a \frac{1^{2a-1/1} 1^{2b-1/1}}{1^{2a+2b-1/1}}$ }, où p et q des fractions seulement;
- 14) $\int \text{Sin.}^{b-1} x. \text{Cos.}^{a-b-1} x. \text{Cos.} a x dx = \frac{1^{a-b-1/1}}{1^{a-1/1} 1^{b-1/1}} \frac{\pi}{2} \text{Cos.} \frac{p\pi}{2}$
- 15) $\int \text{Sin.}^{p-1} x. \text{Cos.}^{q-p-1} x. \text{Cos.} q x dx = \frac{1^{p-1/1} 1^{q-p-1/1}}{1^{q-1/1}} \text{Cos.} \frac{p\pi}{2}$ } Oettinger, Cr. 38. 216.
- 16) $\int \text{Sin.}^{p-1} x. \text{Cos.}^{q-1} x. \text{Cos.} \{(p+q)x\} dx = \frac{1^{p-1/1} 1^{q-1/1}}{1^{p+q-1/1}} \text{Cos.} \frac{p\pi}{2}$

$$17) \int \sin^{2p-1} x \cdot \cos^{2q-2p-1} x \cdot \cos 2qx dx = \frac{(2-2p)^{q-1/2} 1^{q-p/2} 1^{p/2} \pi}{1^{2q-1/1}} - \frac{\pi}{2} \cot p\pi \left. \vphantom{\int} \right\} \text{où } p \text{ et } q \text{ des fractions seulement;}$$

$$18) \int \sin^{2p-1} x \cdot \cos^{2q-1} x \cdot \cos \{2(p+q)x\} dx = \frac{(2-2p)^{p+q-1/2} 1^{p/2} 1^{q/2} \pi}{1^{2p+2q-1/1}} - \frac{\pi}{2} \cot p\pi \left. \vphantom{\int} \right\} \text{Oettinger, Cr. 38. 216.}$$

$$19) \int \cos^a x \cdot \sin b x \cdot \sin x dx = \frac{b \pi 1^{a/1}}{2^{a+2} \Gamma\left(\frac{a+b+3}{2}\right) \Gamma\left(\frac{a-b+3}{2}\right)} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211.}$$

$$20) \int \cos^a x \cdot \sin a x \cdot \sin 2bx dx = \frac{\pi}{2^{a+2}} \frac{a^{b/1-1}}{1^{b/1}} \left. \vphantom{\int} \right\} \text{Poisson, P. 19. 40 f. N°. 76. — Id., Conn. des Temps. 1836. 1.}$$

$$21) \int \cos^a x \cdot \cos a x \cdot \cos 2bx dx = \frac{\pi}{2^{a+2}} \frac{a^{b/1-1}}{1^{b/1}} \left. \vphantom{\int} \right\}$$

$$22) \int (2 \cos x)^{a-1} \cdot \cos \{(a+1)x\} \cdot \cos 2bx dx = \frac{\pi}{4} \frac{(a-b+1)^{b-1/1}}{1^{b-1/1}} \quad \text{Kummer, Cr. 17. 210.}$$

$$1) \int \cos^{p+2b} x \cdot \sin p x \cdot \text{Tang.}^a x dx = \frac{\Gamma(b+p)}{1^{b/1} \Gamma(p)} \cdot \frac{\pi}{2^{p+2b-1}} \sum_0^a \binom{b}{n} \frac{b^{n/1-1}}{(p+b-1)^{n/1-1}} \quad \text{Schlömleeh, Cr. 33. 353.}$$

$$2) \int \cos^{p-2} x \cdot \text{Tang.}^b x \cdot \sin p x dx = 0 \quad \text{V. T. 58. N°. 4, 7.}$$

$$3) \int \cos^{p-2} x \cdot \text{Tang.}^b x \cdot \cos p x dx = 0 \quad \text{V. T. 58. N°. 5, 6.}$$

$$4) \int \cos^{p+b-1} x \cdot \text{Tang.}^{c-1} x \cdot \cos p x \cdot \sin \{(b+1)x\} dx = (-1)^{\frac{c}{2}} \frac{\pi}{2^{p+b-1}} \frac{\Gamma(b+p)}{1^{b/1} \Gamma(p)} \sum_0^a (-2)^n \binom{c-1}{n} \frac{b^{n/1-1}}{(p+b-1)^{n/1-1}}$$

$$5) \int \cos^{p+b-1} x \cdot \text{Tang.}^c x \cdot \cos p x \cdot \cos \{(b+1)x\} dx = (-1)^{\frac{c}{2}} \frac{\pi}{2^{p+b-1}} \frac{\Gamma(b+p)}{1^{b/1} \Gamma(p)} \sum_0^a (-2)^n \binom{c}{n} \frac{b^{n/1-1}}{(p+b-1)^{n/1-1}}$$

$$6) \int \cos^{p+b-1} x \cdot \text{Tang.}^c x \cdot \sin p x \cdot \sin \{(b+1)x\} dx = (-1)^{\frac{c}{2}} \frac{\pi}{2^{p+b-1}} \frac{\Gamma(b+p)}{1^{b/1} \Gamma(p)} \sum_0^a (-2)^n \binom{c}{n} \frac{b^{n/1-1}}{(p+b-1)^{n/1-1}}$$

$$7) \int \cos^{p+b-1} x \cdot \text{Tang.}^{c-1} x \cdot \sin p x \cdot \cos \{(b+1)x\} dx = (-1)^{\frac{c}{2}+1} \frac{\pi}{2^{p+b-1}} \frac{\Gamma(b+p)}{1^{b/1} \Gamma(p)} \sum_0^a (-2)^n \binom{c-1}{n} \frac{b^{n/1-1}}{(p+b-1)^{n/1-1}}$$

Sur ces 4 formules voyez: Schlömleeh, Cr. 33. 353.

$$1) \int \cos. (p \operatorname{Tang.} x) dx = \frac{1}{2} \pi e^{-p} \quad \text{Serret, L. 8. 489. — Dienger, Gr. 10. 341.}$$

$$2) \int \sin. (p \operatorname{Tang.} x) dx = \frac{1}{2} \{e^{-p} Ei. (p) - e^p Ei. (-p)\} \quad \text{V. T. 204. N°. 7.}$$

$$3) \int \cos.^2 (p \operatorname{Tang.} x) dx = \frac{1}{4} \pi (1 + e^{-2p}) \quad \text{V. T. 205. N°. 22.}$$

$$4) \int \sin.^2 (p \operatorname{Tang.} x) dx = \frac{1}{4} \pi (1 - e^{-2p}) \quad \text{V. T. 205. N°. 21.}$$

$$5) \int \sin. (p \operatorname{Tang.} x). \operatorname{Tang.} x dx = \frac{1}{2} \pi e^{-p} \quad \text{V. T. 204. N°. 3.}$$

$$6) \int \cos. (p \operatorname{Tang.} x). \operatorname{Tang.} x dx = -\frac{1}{2} \{e^{-p} Ei. (p) + e^p Ei. (-p)\} \quad \text{V. T. 204. N°. 8.}$$

$$7) \int \operatorname{Tang.} (p \operatorname{Tang.} x). \operatorname{Tang.} x dx = \frac{\pi}{e^{2p} + 1} \quad \text{V. T. 204. N°. 9.}$$

$$8) \int \sin. (p \operatorname{Tang.} x). \sin. 2x dx = \frac{1}{2} p \pi e^{-p} \quad \text{V. T. 208. N°. 3.}$$

$$9) \int \cos. (p \operatorname{Tang.} x). \cos.^2 x dx = \frac{1+p}{4} \pi e^{-p} \quad \text{V. T. 208. N°. 7.}$$

$$10) \int \cos. (p \operatorname{Tang.} x). \sin.^2 x dx = \frac{1-p}{4} \pi e^{-p} \quad \text{V. T. 208. N°. 8.}$$

$$11) \int \cos. (p \operatorname{Tang.} x). \cos. 2x dx = \frac{1}{2} p \pi e^{-p} \quad \text{V. T. 59. N°. 9, 10.}$$

$$12) \int \sin. (p \operatorname{Tang.} x). \operatorname{Tang.}^{2a+1} x dx = (-1)^a \frac{\pi}{2} e^{-p} \quad \text{V. T. 205. N°. 27.}$$

$$13) \int \cos. (p \operatorname{Tang.} x). \operatorname{Tang.}^{2a} x dx = (-1)^a \frac{\pi}{2} e^{-p} \quad \text{V. T. 205. N°. 26.}$$

$$14) \int \cos. \left(p \operatorname{Tang.} \frac{x}{q} \right). \operatorname{Tang.}^2 x dx = -\frac{p}{2} e^{-pq} \quad \text{V. T. 205. N°. 12.}$$

$$15) \int \cot. \left(p \operatorname{Tang.} \frac{x}{q} \right). \operatorname{Tang.} x dx = \frac{\pi q}{e^{2pq} - 1} \quad \text{V. T. 205. N°. 16.}$$

$$16) \int \cos.^{2a-1} x. \cos. (2 \operatorname{Tang.} x \setminus c) dx = \frac{\sqrt{\pi}}{2\Gamma(a+\frac{1}{2})} \{ \Gamma(a) \psi(1-a, c) + \Gamma(-a) e^a \psi(1+a, c) \} \quad \text{Kummer, Cr. 17. 225.}$$

$$\left. \begin{aligned} 17) \int \cos^{p-1} x. \sin. \{(p+1)x\} \sin. (c \operatorname{Tang.} x) dx &= \frac{\pi}{2 \Gamma(p+1)} c^p e^{-c} \\ 18) \int \cos^{p-1} x. \cos. \{(p+1)x\} \cos. (c \operatorname{Tang.} x) dx &= \frac{\pi}{2 \Gamma(p+1)} c^p e^{-c} \end{aligned} \right\} \text{Kummer, Cr. 17. 228.}$$

$$19) \int \{ \cos. (q \operatorname{Tang.} x) + \operatorname{Tang.} x. \sin. (q \operatorname{Tang.} x) \} dx = \pi e^{-q} \quad \text{V. T. 204. N}^\circ. 18.$$

$$\begin{aligned} 1) \int \cos. 2q x. \cos. (2p \cos. x) dx &= \frac{1}{2q} \sin. q \pi \left\{ 1 + \sum_1^{\infty} (-1)^n \frac{p^{2n}}{1^2 - q^2. 2^2 - q^2 \dots n^2 - q^2} \right\} \quad \text{Kummer, Cr. 17. 210.} \\ 2) \int \sin. (a \sin. x). \sin. 2x dx &= -\frac{2}{a} \quad \text{V. T. 192. N}^\circ. 1. \\ 3) \int \sin. (a \cos. x). \sin. 2x dx &= -\frac{2}{a} \quad \text{V. T. 192. N}^\circ. 1. \\ 4) \int \sin. (p \cos. x). \operatorname{Tang.} x dx &= \sum_1^{\infty} \frac{1}{2n+1} \frac{p^{2n-1}}{1^{2n-1/1}} \quad \text{V. T. 192. N}^\circ. 5. \\ 5) \int \sin. (p \cot. x). \operatorname{Tang.} x dx &= \frac{\pi}{2} (1 - e^{-q}) \quad \text{V. T. 212. N}^\circ. 4. \\ 6) \int \operatorname{Tang.} (p \cot. x). \operatorname{Tang.} x dx &= \frac{\pi}{2} \frac{e^p - e^{-p}}{e^p + e^{-p}} \quad \text{V. T. 212. N}^\circ. 5. \\ 7) \int \sin. (p \cot. q x). \operatorname{Tang.}^{2a-1} x dx &= (-1)^a \frac{\pi}{2} e^{-p} \quad \text{V. T. 212. N}^\circ. 14. \\ 8) \int \cos. (p \cot. q x). \operatorname{Tang.}^{2a} x dx &= (-1)^a \frac{\pi}{2} e^{-p} \quad \text{V. T. 212. N}^\circ. 15. \\ 9) \int \sin. (a \cos. x). \cos. q x. \operatorname{Tang.} x dx &= \frac{1-q}{(lq)^2 + a^2} \frac{a}{q} \quad \text{V. T. 192. N}^\circ. 4. \\ 10) \int \sin. (\tfrac{1}{2} p \pi - q \operatorname{Tang.} x). \operatorname{Tang.}^{p-1} x dx &= \frac{1}{2} \pi e^{-q} \quad \text{V. T. 204. N}^\circ. 14. \\ 11) \int \cos. (\tfrac{1}{2} p \pi - q \operatorname{Tang.} x). \operatorname{Tang.}^p x dx &= \frac{1}{2} \pi e^{-q} \quad \text{V. T. 204. N}^\circ. 15. \end{aligned}$$

$$12) \int \{1 - \cos. (p \cot. x)\} \operatorname{Tang}^2 x dx = \frac{\pi}{2} (e^{-p} + p - 1) \quad \text{V. T. 212. N}^\circ. 13.$$

$$13) \int \{ \cos. (a \cot. x) - \cos. (b \cot. x) \} \operatorname{Tang}^2 x dx = \frac{\pi}{2} (e^{-b} - e^{-a}) + \frac{b-a}{2} \pi \quad \text{V. T. 212. N}^\circ. 7.$$

$$1) \int \cos^{p-2} x. \cos. (p \operatorname{Tang} x - p x) dx = \frac{\pi}{\Gamma(p+1)} \left(\frac{p}{e}\right)^p \quad \text{Legendre, Exerc. 3. 40.}$$

$$\left. \begin{aligned} 2) \int \cos^{p-1} x. \cos. \{c \operatorname{Tang} x - (p+1)x\} dx &= \frac{\pi}{\Gamma(p+1)} c^p e^{-c} \\ 3) \int \cos^{p-1} x. \cos. \{c \operatorname{Tang} x + (p+1)x\} dx &= 0 \\ 4) \int \cos^{p-1} x. \cos. \{c \operatorname{Tang} x + (p-1)x\} dx &= \frac{\pi}{2^p} e^{-c} \end{aligned} \right\} \quad \text{Kummer, Cr. 17. 228.}$$

$$5) \int \cos^{p-2} x. \cos. \{p x - c \operatorname{Tang} x\} dx = \frac{\pi}{\Gamma(p)} e^{-c} c^{p-1} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211.}$$

$$6) \int \cos^{p-1} x. \cos. \{c \operatorname{Tang} x + b x\} dx = \frac{\pi e^{-c} \Gamma(p)}{2^p \Gamma\left(\frac{p-b+1}{2}\right) \Gamma\left(\frac{p+b+1}{2}\right)} \varphi\left(\frac{b-p+1}{2}, 1-p, 2c\right) -$$

$$- \frac{\pi c^p e^{-c} \cos. \left\{\frac{p-b}{2} \pi\right\}}{\Gamma(p+1) \sin. p \pi} \varphi\left(\frac{b+p+1}{2}, 1+p, 2c\right)$$

$$7) \int \cos^{p-b-1} x. \cos. \{c \operatorname{Tang} x + (p+b-1)x\} dx = \frac{\pi}{\Gamma(1-p)} e^{-c} 2^{p-b} c^{-p} \sum_0^{\infty} (-1)^n \frac{p^{n/1} b^{n/1}}{1^{n/1} (2c)^n}$$

De ces deux intégrales voyez: Kummer, Cr. 17. 228.

$$8) \int \cos. (2x - 2 \operatorname{Tang} x) dx = \frac{2\pi}{e^2} \quad \text{V. T. 209. N}^\circ. 19.$$

$$9) \int \sin. \left(\frac{1}{2} p \pi - p \operatorname{Tang} x\right). \operatorname{Tang}^{a-1} x dx = \frac{1}{2} \pi e^{-p} \quad \text{V. T. 205. N}^\circ. 24.$$

$$10) \int \cos. \left(\frac{1}{2} p \pi - p \operatorname{Tang} x\right). \operatorname{Tang}^a x dx = \frac{1}{2} \pi e^{-p} \quad \text{V. T. 205. N}^\circ. 25.$$

$$11) \int \sin^{p-1} x \cdot \cos^{q-1} x \cdot \cos. \{c \operatorname{Tang}. x + (p+q)x\} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \cos. \frac{p\pi}{2} \varphi(p, 1-q, c) + \\ + c^q \cos. \frac{p\pi}{2} \Gamma(-q) \varphi(p+q, 1+q, c)$$

$$12) \int \sin^{p-1} x \cdot \cos^{q-1} x \cdot \sin. \{c \operatorname{Tang}. x + (p+q)x\} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \sin. \frac{p\pi}{2} \varphi(p, 1-q, c) + \\ + c^q \sin. \frac{p\pi}{2} \Gamma(-q) \varphi(p+q, 1+q, c)$$

$$13) \int \sin^{p-1} x \cdot \cos^{q-1} x \cdot \sin. \left\{c \operatorname{Tang}. x + (p+q)x - \frac{p\pi}{2}\right\} dx = 0$$

$$14) \int \sin^{p-1} x \cdot \cos^{q-1} x \cdot \sin. \{c \operatorname{Tang}. x - (p+q)x\} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \sin. \frac{p\pi}{2} \varphi(p, 1-q, -c) - \\ - c^q \sin. \left\{\left(\frac{p}{2} + q\right)\pi\right\} \Gamma(-q) \varphi(p+q, 1-q, -c)$$

$$15) \int \sin^{p-1} x \cdot \cos^{q-1} x \cdot \cos. \{c \operatorname{Tang}. x - (p+q)x\} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \cos. \frac{p\pi}{2} \varphi(p, 1-q, -c) + \\ + c^q \cos. \left\{\left(\frac{p}{2} + q\right)\pi\right\} \Gamma(-q) \varphi(p+q, 1-q, -c)$$

$$16) \int \sin^{p-1} x \cdot \cos^{q-1} x \cdot \sin. \{c \operatorname{Tang}. x - (p+q)x + (\frac{1}{2}p+q)\pi\} dx = \frac{\pi \Gamma(p)}{\Gamma(1-q)\Gamma(p+q)} \varphi(p, 1-q, c)$$

$$17) \int \sin^{p-1} x \cdot \cos^{q-1} x \cdot \sin. \{c \operatorname{Tang}. x - (p+q)x + \frac{1}{2}p\pi\} dx = \frac{\pi}{\Gamma(q+1)} c^q \varphi(p+q, 1+q, -c)$$

Kummer, Cr. 17. 228, a déduit les formules 11 à 17.

$$1) \int \frac{\sin. p x \cdot \cos^{p-1} x}{\sin. x} dx = \frac{1}{2} \pi \quad \begin{array}{l} \text{Cauchy, Exerc. 1826. p. 205. -- Serret, L. 8. 1. -- Id., L. 8.} \\ \text{489. -- Liouville, Cr. 13. 219. -- Kummer, Cr. 17. 228. --} \\ \text{Schlömlich, Cr. 33. 353. -- Id., Gr. 6. 200. -- Id., Stud. I. 15.} \end{array}$$

$$2) \int \frac{\sin. (2k+1)x}{\sin. x} dx = \frac{1}{2} \pi, \text{ pour } k = \infty; \quad \text{Schlömlich, Beitr. 1. § 4.}$$

$$3) \int \frac{\cos. 2p-1 x}{\sin. 2p-1 x} dx = \frac{1}{2} \pi \operatorname{Cosec}. p\pi, 1 > p > 0; \quad \text{Oettinger, Cr. 38. 162.}$$

$$4) \int \frac{\text{Cos.}^{2a-1} x}{\text{Sin.}^{2b} x} dx = \frac{(-1)^b}{2a} \frac{2^{a/2}}{1^{b/2} 1^{a-b/2}} \quad \text{V. T. 12. N}^\circ. 19.$$

$$5) \int \frac{\text{Cos.}^{2a} x}{\text{Sin.}^{2b} x} dx = (-1)^b \frac{\pi}{4} \frac{1^{a/2}}{1^{b/2} 2^{a-b/2}} \quad \text{V. T. 12. N}^\circ. 21.$$

$$6) \int \frac{\text{Cos.}^{2p-2} x}{\text{Sin.}^{2p-1} x} dx = \frac{\Gamma(2p-1) \Gamma(1-p)}{2^{2p-1} \Gamma(p)} \quad \text{V. T. 4. N}^\circ. 19.$$

$$7) \int \frac{\text{Sin.}^{2a-1} x}{\text{Cos.}^{2b} x} dx = \frac{(-1)^b}{2a} \frac{2^{a/2}}{1^{b/2} 1^{a-b/2}} \quad \text{V. T. 12. N}^\circ. 19.$$

$$8) \int \frac{\text{Sin.}^{2a} x}{\text{Cos.}^{2b} x} dx = (-1)^b \frac{\pi}{4} \frac{1^{a/2}}{1^{b/2} 2^{a-b/2}} \quad \text{V. T. 12. N}^\circ. 21.$$

$$9) \int \frac{\text{Sin.}^{2p-2} x}{\text{Cos.}^{2p-1} x} dx = \frac{\Gamma(2p-1) \Gamma(1-p)}{2^{2p-1} \Gamma(p)} \quad \text{V. T. 4. N}^\circ. 19.$$

$$\left. \begin{aligned} 10) \int \frac{\text{Sin.} \{(2-p)x\}}{\text{Sin.}^p x} dx &= \frac{1}{1-p} \text{Cos.} \frac{1}{2} p \pi, 2 > p > 0; \\ 11) \int \frac{\text{Cos.} \{(2-p)x\}}{\text{Sin.}^p x} dx &= \frac{1}{1-p} \text{Sin.} \frac{1}{2} p \pi, 1 > p > 0; \end{aligned} \right\} \quad \text{Schlömilch, Gr. 6. 200.}$$

$$1) \int \frac{\text{Sin.} q x}{\text{Tang.} x} dx = 2 \pi q^2 \quad \text{V. T. 354. N}^\circ. 3.$$

$$2) \int \frac{\text{Sin.}^{4a} x}{\text{Tang.} x} dx = \frac{2 \pi}{a k} \sum_1^{k-1} \text{Cos.} \frac{2 a n \pi}{k} l \text{Sin.} \frac{n \pi}{2 k}, \text{ où } k = \infty; \quad \text{V. T. 356. N}^\circ. 4.$$

$$3) \int \frac{\text{Cos.}^{a-1} x \cdot \text{Sin.} \{(a+1)x\}}{\text{Tang.} x} dx = \frac{1}{2} \pi \quad \text{Lindmann, Stockh. Handl. 1850.}$$

$$4) \int \frac{dx}{\text{Tang.}^p x} = \frac{1}{2} \pi \text{Sec.} p \pi, 1 > p > 0; \quad \text{Schlömilch, Gr. 6. 200.}$$

$$\left. \begin{aligned} 5) \int \frac{\text{Sin.} 2x}{\text{Tang.}^p x} dx &= \frac{1}{2} p \pi \text{Cosec.} \frac{p \pi}{2}, 2 > p > 0; \\ 6) \int \frac{\text{Cos.} 2x}{\text{Tang.}^p x} dx &= \frac{1}{2} p \pi \text{Sec.} \frac{p \pi}{2}, 1 > p > 0; \end{aligned} \right\} \quad \text{Schlömilch, Gr. 6. 200. — Id., Stud. I. 15.}$$

$$7) \int \frac{\text{Sin.}^{2a-2} x}{\text{Tang.}^{p-1} x} dx = \frac{\Gamma(\frac{1}{2} p) \Gamma(a - \frac{1}{2} p)}{2 \Gamma(a)} \quad \text{V. T. 21. N}^\circ. 9.$$

$$8) \int \frac{\sin^{2p-1} x}{\tan^{p-1} x} dx = \frac{\Gamma\left(\frac{p-q+1}{2}\right) \Gamma\left(\frac{p+q}{2}\right)}{2 \Gamma(p + \frac{1}{2})} \quad \text{V. T. 27. N}^\circ. 10.$$

$$9) \int \frac{\sin^p x \cos^{p-2} x}{\tan^q x} dx = \frac{\Gamma(p+q-1)}{2 \Gamma(p) \Gamma(q)} \pi \operatorname{Cosec} \frac{1}{2} q \pi, 2 > q > 0; \left\{ \begin{array}{l} \text{Schlömlich, Gr. 6.} \\ 200. - \text{Id., Cr. 33.} \\ 333. - \text{Id., Stud. I.} \\ 15. \end{array} \right.$$

$$10) \int \frac{\cos^p x \cos^{p-2} x}{\tan^q x} dx = \frac{\Gamma(p+q-1)}{2 \Gamma(p) \Gamma(q)} \pi \operatorname{Sec} \frac{1}{2} q \pi, 1 > q > 0;$$

$$11) \int \frac{\cos^2 x}{\cos 2x} dx = 0 \quad \text{V. T. 21. N}^\circ. 7.$$

$$12) \int \frac{\sin^2 x}{\cos 2x} dx = -\frac{1}{4} \pi \quad \text{V. T. 24. N}^\circ. 14.$$

$$13) \int \frac{\cos^2 x}{\cos 2x} dx = \frac{1}{4} \pi \quad \text{V. T. 24. N}^\circ. 13.$$

$$14) \int \frac{\tan^{p-1} x}{\cos 2x} dx = \frac{1}{2} \pi \operatorname{Cot} \frac{1}{2} p \pi \quad \text{V. T. 19. N}^\circ. 9.$$

$$15) \int \frac{dx}{\cos 2x \tan^{p-1} x} = -\frac{1}{2} \pi \operatorname{Cot} \frac{1}{2} p \pi \quad \text{V. T. 19. N}^\circ. 9.$$

$$1) \int \frac{\sin^{p-1} x - \sin^{1-p} x}{\cos x} dx = \frac{1}{2} \pi \operatorname{Cot} \frac{1}{2} p \pi \quad \text{V. T. 5. N}^\circ. 12.$$

$$2) \int \frac{\sin^p x - \operatorname{Cosec}^p x}{\cos x} dx = -\frac{1}{2} \pi \tan \frac{1}{2} p \pi \quad \text{V. T. 5. N}^\circ. 13.$$

$$3) \int \frac{\sin^p x - \sin^q x}{\cos x} dx = \frac{1}{2} \left\{ Z' \left(\frac{q+1}{2} \right) - Z' \left(\frac{p+1}{2} \right) \right\} \quad \text{V. T. 3. N}^\circ. 14.$$

$$4) \int \frac{\cos^{p-1} x - \sec^{p-1} x}{\sin x} dx = \frac{1}{2} \pi \operatorname{Cot} \frac{1}{2} p \pi \quad \text{V. T. 5. N}^\circ. 12.$$

$$5) \int (\sec x - 1)^p \tan x dx = -\pi \operatorname{Cosec} p \pi \quad \text{V. T. 31. N}^\circ. 1$$

$$6) \int (\sec x - 1)^{1-p} \sin 2x dx = (1-p) p \pi \operatorname{Cosec} p \pi \quad \text{V. T. 31. N}^\circ. 5.$$

- 7) $\int (\operatorname{Cosec}. x - 1)^{1-p} \operatorname{Sin}. 2x dx = (1-p)p\pi \operatorname{Cosec}. p\pi$ V. T. 31. N°. 5.
- 8) $\int \frac{a + b \operatorname{Tang}^2 x}{\operatorname{Cos}. 2x} dx = \frac{a-b}{4} \pi$ V. T. 24. N°. 9.
- 9) $\int \frac{(\operatorname{Cosec}. x - 1)^p}{\operatorname{Tang}. x} dx = -\pi \operatorname{Cosec}. p\pi$ V. T. 31. N°. 1.
- 10) $\int \frac{\operatorname{Tang}^{p-1} x - \operatorname{Tang}^{1-p} x}{\operatorname{Cos}. 2x} dx = \pi \operatorname{Cot}. \frac{1}{2} p\pi$ V. T. 47. N°. 7 et T. 92. N°. 1.
- 11) $\int \frac{\operatorname{Sin}^{p-1} x + \operatorname{Sin}^{q-1} x}{\operatorname{Cos}^{p+q-1} x} dx = \frac{1}{2} \operatorname{Cos}. \left(\frac{q-p}{4} \pi \right) \cdot \operatorname{Sec}. \left(\frac{q+p}{4} \pi \right) \operatorname{B} \left(\frac{1}{2} p, \frac{1}{2} q \right)$ V. T. 12. N°. 23.
- 12) $\int \frac{\operatorname{Sin}^{p-1} x - \operatorname{Sin}^{q-1} x}{\operatorname{Cos}^{p+q-1} x} dx = \frac{1}{2} \operatorname{Sin}. \left(\frac{q-p}{4} \pi \right) \cdot \operatorname{Cosec}. \left(\frac{q+p}{4} \pi \right) \operatorname{B} \left(\frac{1}{2} p, \frac{1}{2} q \right)$ V. T. 12. N°. 24.
- 13) $\int \frac{\operatorname{Cos}^{p-1} x + \operatorname{Cos}^{q-1} x}{\operatorname{Sin}^{p+q-1} x} dx = \frac{1}{2} \operatorname{Cos}. \left(\frac{q-p}{4} \pi \right) \cdot \operatorname{Sec}. \left(\frac{q+p}{4} \pi \right) \operatorname{B} \left(\frac{1}{2} p, \frac{1}{2} q \right)$ V. T. 12. N°. 23.
- 14) $\int \frac{\operatorname{Cos}^{p-1} x - \operatorname{Cos}^{q-1} x}{\operatorname{Cos}^{p+q-1} x} dx = \frac{1}{2} \operatorname{Sin}. \left(\frac{q-p}{4} \pi \right) \cdot \operatorname{Cosec}. \left(\frac{q+p}{4} \pi \right) \operatorname{B} \left(\frac{1}{2} p, \frac{1}{2} q \right)$ V. T. 12. N°. 24.
- 15) $\int (\operatorname{Tang}^p x - \operatorname{Cot}^p x) (\operatorname{Tang}^q x + \operatorname{Cot}^q x) \frac{dx}{\operatorname{Cos}. 2x} = \frac{-2\pi \operatorname{Sin}. p\pi}{\operatorname{Cos}. p\pi + \operatorname{Cos}. q\pi}, p < 1; \begin{matrix} \text{V. T. 47. N}^\circ. \\ 20 \text{ et T. 92.} \\ \text{N}^\circ. 4. \end{matrix}$
- 16) $\int (\operatorname{Tang}^p x + \operatorname{Cot}^p x) (\operatorname{Tang}^q x + \operatorname{Cot}^q x) dx = \frac{4\pi \operatorname{Cos}. \frac{1}{2} p\pi \cdot \operatorname{Cos}. \frac{1}{2} q\pi}{\operatorname{Cos}. p\pi + \operatorname{Cos}. q\pi}$ V. T. 47. N°. 16 et T. 92. N°. 2.
- 17) $\int (\operatorname{Tang}^p x - \operatorname{Cot}^p x) (\operatorname{Tang}^q x - \operatorname{Cot}^q x) dx = \frac{4\pi \operatorname{Sin}. \frac{1}{2} p\pi \cdot \operatorname{Sin}. \frac{1}{2} q\pi}{\operatorname{Cos}. p\pi + \operatorname{Cos}. q\pi}$ V. T. 47. N°. 17 et T. 92. N°. 3.

- 1) $\int \frac{dx}{2 - \operatorname{Sin}. x} = \frac{2\pi}{3\sqrt{3}}$ V. T. 7. N°. 1.
- 2) $\int \frac{dx}{2 + \operatorname{Sin}. x} = \frac{\pi}{3\sqrt{3}}$ V. T. 7. N°. 2.
- 3) $\int \frac{dx}{1 - \operatorname{Sin}. x \cdot \operatorname{Cos}. x} = \frac{4\pi}{3\sqrt{3}}$ V. T. 48. N°. 2 et T. 92. N°. 5.
- 1) $\int \frac{dx}{1 + \operatorname{Sin}. x \cdot \operatorname{Cos}. x} = \frac{2\pi}{3\sqrt{3}}$ V. T. 25. N°. 1.

- 5) $\int \frac{dx}{1 - \sin. x. \cos. \lambda} = (\pi - \lambda) \operatorname{Cosec.} \lambda \quad \text{V. T. 7. N}^\circ. 3.$
- 6) $\int \frac{dx}{1 + \sin. x. \cos. \lambda} = \lambda \operatorname{Cosec.} \lambda \quad \text{V. T. 7. N}^\circ. 4.$
- 7) $\int \frac{\sin. x}{\sin. x \pm q \cos. x} dx = \frac{q}{1 + q^2} \left\{ \frac{\pi}{2q} \pm lq \right\} \quad \text{V. T. 24. N}^\circ. 3, 4.$
- 8) $\int \frac{\cos. x}{\sin. x \pm q \cos. x} dx = \frac{1}{1 + q^2} \left\{ \pm \frac{q\pi}{2} - lq \right\} \quad \text{V. T. 24. N}^\circ. 1, 2.$
- 9) $\int \frac{dx}{p + q \cos. x} = \frac{1}{\sqrt{p^2 - q^2}} \operatorname{Arccos.} \frac{q}{p} \quad , \text{ pour } q < p; \quad \text{Lobatto, Int. 53.}$
- 10) $= \frac{1}{\sqrt{q^2 - p^2}} l \frac{q + \sqrt{q^2 - p^2}}{q} \quad , \text{ pour } q > p;$
- 11) $= \frac{1}{p} \quad , \text{ pour } q = p;$
- 12) $\int \frac{dx}{-p + q \cos. x} = -\frac{1}{\sqrt{q^2 - p^2}} l \frac{\sqrt{q^2 - p^2} - q}{p} \quad , \text{ pour } p < q; \text{ (val. princ.)}$
- 13) $= -\infty \quad , \text{ pour } p = q;$
- 14) $\int \frac{\operatorname{Tang.}^p x}{1 + \sin. 2x. \cos. \lambda} dx = \frac{\pi}{\sin. p \pi} \frac{\sin. p \lambda}{\sin. \lambda}, \lambda^2 < \pi^2, p^2 < 1; \quad \text{V. T. 25. N}^\circ. 5.$
- 15) $\int \frac{\operatorname{Tang.}^{p-1} x}{1 + \sin. x \cos. x} dx = \frac{2\pi}{\sqrt{3}} \operatorname{Cosec.} p \pi. \sin. \left\{ \frac{1-p}{2} \pi \right\}, 1 > p > 0; \quad \text{V. T. 25. N}^\circ. 4.$
- 16) $\int \frac{\cos. x - \sin. x}{\cos. x + \sin. x} dx = 0 \quad \text{V. T. 24. N}^\circ. 5.$
- 17) $\int \frac{q \sin. x - \cos. x}{\sin. x + q \cos. x} dx = lq \quad \text{V. T. 24. N}^\circ. 6.$

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$$1) \int \frac{\sin.^4 x}{1 + p \sin.^2 x} dx = \frac{\pi}{2p^2 \sqrt{1+p}} + \frac{p-2}{4p^2} \pi \quad \text{Ramus, Danske Aft. 6. 265.}$$

$$2) \int \frac{dx}{1 + p^2 \operatorname{Tang.}^2 x} = \frac{\pi}{2} \frac{1}{1+p} \quad \text{Mösta, Gr. 10. 449.}$$

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- 3) $\int \frac{Tang.^2 x}{1 + p^2 Tang.^2 x} dx = \frac{\pi}{2p(p+1)}$ V. T. 24. N°. 7.
- 4) $\int \frac{dx}{4 - Sin.^2 x} = \frac{\pi}{2\sqrt{3}}$ V. T. 65. N°. 1, 2.
- 5) $\int \frac{Sin. x}{4 - Sin.^2 x} dx = \frac{\pi}{3\sqrt{3}}$ V. T. 65. N°. 1, 2.
- 6) $\int \frac{dx}{1 - Sin.^2 x. Cos.^2 x} = \frac{\pi}{\sqrt{3}}$ V. T. 65. N°. 3, 4.
- 7) $\int \frac{Sin.^2 x}{1 - Sin.^2 x. Cos.^2 x} dx = \frac{2\pi}{3\sqrt{3}}$ V. T. 65. N°. 3, 4.
- 8) $\int \frac{dx}{1 - Sin.^2 x. Cos.^2 \lambda} = \frac{1}{2} \pi Cosec. \lambda$ V. T. 65. N°. 5, 6.
- 9) $\int \frac{Sin. x}{1 - Sin.^2 x. Cos.^2 \lambda} dx = \frac{1}{2} (\pi - 2\lambda) Cosec. \lambda$ V. T. 65. N°. 5, 6.
- 10) $\int \frac{Cos.^2 x}{1 - Sin.^2 x. Cos.^2 x} dx = \pi$ V. T. 25. N°. 16.
- 11) $\int \frac{dx}{p^2 + Tang.^2 x} = \frac{\pi}{2p(p+1)}$ V. T. 24. N°. 7.
- 12) $\int \frac{Cos.p x + Cos.q x}{Cos.p+q x + 1} Tang. x dx = \frac{\pi}{p+q} Sec. \left\{ \frac{q-p}{p+q} \frac{\pi}{2} \right\}$ V. T. 31. N°. 17.
- 13) $\int \frac{Cos.p x - Cos.q x}{Cos.p+q x - 1} Tang. x dx = \frac{\pi}{p+q} Tang. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\}$ V. T. 31. N°. 18.
- 14) $\int \frac{Tang. x}{Cos.p x + Sec.p x} dx = \frac{1}{4p} \pi$ V. T. 5. N°. 23.
- 15) $\int \frac{Cos.^2 x. Tang.p-1 x}{1 - 3 Sin.^2 x. Cos.^2 x} dx = \frac{\pi}{\sqrt{3}} Cosec. \frac{1}{2} p\pi. Sin. \left\{ \frac{2-p}{6} \pi \right\}, 4 > p > 0 :$ V. T. 25. N°. 15.
- 16) $\int \frac{Cos.^2 x}{Sin.^2 \lambda + Cos.^2 \lambda. Cos.^2 x} dx = \frac{\pi}{2} \frac{1 - Sin. \lambda}{Cos.^2 \lambda}$
- 17) $\int \frac{Cos. x}{Sin.^2 \lambda + Cos.^2 \lambda. Cos.^2 x} dx = - Sec. \lambda l Tang. \frac{1}{2} \lambda$
- 18) $\int \frac{dx}{p^2 Cos.^2 x + q^2 Sin.^2 x} = \frac{\pi}{2pq}$ Catalan, L. 6. 349. — Tortolini, Cr. 34. 101.

$$19) \int \frac{Tang.^r x}{p^2 \cos.^2 x + q^2 \sin.^2 x} dx = \frac{1}{2} \pi p^{r-1} q^{-r-1} Sec. \frac{1}{2} r \pi \quad \text{Schlömlich, Höh. An. 85.}$$

$$20) \int \frac{\cos.^a x \cdot \cos. a x}{\cos.^2 x + b^2 \sin.^2 x} dx = \frac{\pi}{2} \frac{b^{a-1}}{(b+1)^a} \quad \text{Serret, L. 8. 489.}$$

$$21) \int \frac{\sin. 2 x}{a^2 \sin.^2 x + \cos.^2 x} dx = \frac{2}{1+a^2} l a \quad \text{V. T. 24. N°. 12.}$$

$$22) \int \frac{\sin. 2 x}{\sin.^2 x + a^2 \cos.^2 x} dx = \frac{-2}{1+a^2} l a \quad \text{V. T. 24. N°. 12.}$$

$$1) \int \frac{Tang. x}{(Sec. x - 1)^p} dx = \pi Cosec. p \pi \quad \text{V. T. 31. N°. 20.}$$

$$2) \int \frac{\sin. 2 x}{(Sec. x - 1)^p} dx = (1-p) p \pi Cosec. p \pi \quad \text{V. T. 31. N°. 21.}$$

$$3) \int \frac{\sin. 2 x}{(Cosec. x - 1)^p} dx = (1-p) p \pi Cosec. p \pi \quad \text{V. T. 31. N°. 21.}$$

$$4) \int \frac{\sin. 2 x \cdot \cos. x}{(1 - \cos.^2 \lambda \cdot \sin.^2 x)^2} dx = \frac{\pi - 2 \lambda}{\sin. 2 \lambda \cdot \cos. \lambda} \quad \text{V. T. 66. N°. 9.}$$

$$5) \int \frac{\cos.^2 x \cdot Tang. p+1 x}{(1 + \sin. 2 x \cdot \cos. \lambda)^2} dx = \frac{\pi}{2 \sin. p \pi} \frac{p \sin. \lambda \cdot \cos. p \lambda - \cos. \lambda \cdot \sin. p \lambda}{\sin.^3 \lambda} \quad \text{V. T. 26. N°. 1.}$$

$$6) \int \frac{\sin.^{1-p} x \cdot \cos. p x}{(\cos. x + \sin. x)^3} dx = \frac{1-p}{2} p \pi Cosec. p \pi \quad \text{V. T. 18. N°. 22.}$$

$$7) \int \frac{dx}{(p^2 \cos.^2 x + q^2 \sin.^2 x)^2} = \frac{\pi}{4} \frac{p^2 + q^2}{p^3 q^2} \quad \text{Tortolini, Cr. 34. 101.}$$

$$8) \int \frac{\sin.^2 x}{(p^2 \cos.^2 x + q^2 \sin.^2 x)^2} dx = \frac{\pi}{4} \frac{1}{p q^3} \quad \text{Grunert, Cr. 8. 146. — Tortolini, Cr. 34. 101.}$$

$$9) \int \frac{\cos.^2 x}{(p^2 \cos.^2 x + q^2 \sin.^2 x)^2} dx = \frac{\pi}{4} \frac{1}{p^3 q}$$

$$10) \int \frac{dx}{(p^2 \cos.^2 x + q^2 \sin.^2 x)^3} = \frac{\pi}{16} \frac{3 p^4 + 2 p^2 q^2 + 3 q^4}{p^5 q^5} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Tortolini, Cr. 34. 101.}$$

$$11) \int \frac{\sin.^2 x}{(p^2 \cos.^2 x + q^2 \sin.^2 x)^3} dx = \frac{\pi}{16} \frac{3 p^2 + q^2}{p^3 q^5}$$

- $$\begin{aligned}
 12) \int \frac{\text{Cos.}^2 x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^3} dx &= \frac{\pi}{16} \frac{p^2 + 3q^2}{p^5 q^3} \\
 13) \int \frac{dx}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^4} &= \frac{\pi}{32} \frac{5p^6 + 7p^4 q^2 + 7p^2 q^4 + 5q^6}{p^7 q^7} \\
 14) \int \frac{\text{Sin.}^2 x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^4} dx &= \frac{\pi}{32} \frac{5p^4 + 6p^2 q^2 + q^4}{p^5 q^7} \\
 15) \int \frac{\text{Cos.}^2 x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^4} dx &= \frac{\pi}{32} \frac{p^4 + 6p^2 q^2 + 5q^4}{p^7 q^5} \\
 16) \int \frac{\text{Sin.}^4 x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^4} dx &= \frac{\pi}{32} \frac{5p^2 + q^2}{p^3 q^7} \\
 17) \int \frac{\text{Cos.}^4 x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^4} dx &= \frac{\pi}{32} \frac{p^2 + 5q^2}{p^7 q^3} \\
 18) \int \frac{\text{Sin.}^2 x \cdot \text{Cos.}^2 x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^4} dx &= \frac{\pi}{32} \frac{p^2 + q^2}{p^5 q^5} \\
 19) \int \frac{\text{Cos.} 2x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^2} dx &= \frac{\pi}{4} \frac{q^2 - p^2}{p^3 q^3} \quad \text{V. T. 67. N}^\circ 8, 9. \\
 20) \int \frac{\text{Cos.} 2x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^3} dx &= \frac{3\pi}{16} \frac{q^4 - p^4}{p^5 q^5} \quad \text{V. T. 67. N}^\circ 11, 12. \\
 21) \int \frac{\text{Cos.} 2x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^4} dx &= \frac{\pi}{32} \frac{5q^6 - p^2 q^4 + p^4 q^2 - 5p^6}{p^7 q^7} \quad \text{V. T. 67. N}^\circ 14, 15. \\
 22) \int \frac{\text{Cos.}^{2r-1} x \cdot \text{Sin.}^{2s-1} x}{(p^2 \text{Cos.}^2 x + q^2 \text{Sin.}^2 x)^{r+s}} dx &= \frac{\Gamma(r) \Gamma(s)}{\Gamma(r+s)} \frac{1}{2 p^{2r} q^{2s}} \quad \text{Schlömlich, Höh. Anal. 85.} \\
 23) \int \frac{\text{Sin.}^{p-1} 2x}{(\text{Sin.} x + \text{Cos.} x)^{2p}} dx &= \frac{1}{2^p} \frac{\Gamma(p) \Gamma(\frac{1}{2})}{\Gamma(p + \frac{1}{2})} \quad \text{V. T. 49. N}^\circ 5. \text{ et T. 92. N}^\circ 9. \\
 24) \int \frac{dx}{(\text{Tang.} x + \text{Cot.} x)^2} &= \frac{1}{16} \pi \quad \text{V. T. 21. N}^\circ 5.
 \end{aligned}$$
- Tortolini, Cr. 34. 101.

- $$\begin{aligned}
 1) \int \frac{\text{Tang.}^p x}{\text{Sin.} x + \text{Cos.} x} \frac{dx}{\text{Sin.} x} &= \pi \text{Cosec.} p \pi \quad \text{V. T. 22. N}^\circ 1. \\
 2) \int \frac{\text{Tang.}^{p-1} x + \text{Cot.}^p x}{\text{Sin.} x + \text{Cos.} x} \frac{dx}{\text{Cos.} x} &= \pi \text{Cosec.} p \pi \quad \text{V. T. 22. N}^\circ 4.
 \end{aligned}$$

- 3) $\int \frac{1}{1 + \sin. x. \cos. x} \frac{dx}{\text{Tang.}^{p-1} x} = \frac{2\pi}{1 \vee 3} \text{Cosec. } p \pi. \sin. \left\{ \frac{1-p}{2} \pi \right\}, 1 > p > 0; \text{ V. T. 25. N}^\circ. 4.$
- 4) $\int \frac{\sin.^2 x}{1 - 3 \sin.^2 x. \cos.^2 x} \frac{dx}{\text{Tang.}^{p-1} x} = \frac{\pi}{1 \vee 3} \text{Cosec. } \frac{1}{2} p \pi. \sin. \left\{ \frac{2-p}{6} \pi \right\}, 4 > p > 0; \text{ V. T. 25. N}^\circ. 15.$
- 5) $\int \frac{\sin.^2 x}{p^2 \cos.^2 x + \sin.^2 x \cos. 2x} \frac{dx}{\cos. 2x} = -\frac{1}{2} \frac{p \pi}{1 + p^2} \text{ V. T. 24. N}^\circ. 14.$
- 6) $\int \frac{\cos.^2 x}{p^2 \cos.^2 x + \sin.^2 x \cos. 2x} \frac{dx}{\cos. 2x} = \frac{1}{2p} \frac{\pi}{1 + p^2} \text{ V. T. 24. N}^\circ. 13.$
- 7) $\int \frac{\sin.^2 x}{\cos.^2 x + p^2 \sin.^2 x \cos. 2x} \frac{dx}{\cos. 2x} = -\frac{1}{2p} \frac{\pi}{1 + p^2} \text{ V. T. 24. N}^\circ. 13.$
- 8) $\int \frac{\cos.^2 x}{\cos.^2 x + p^2 \sin.^2 x \cos. 2x} \frac{dx}{\cos. 2x} = \frac{1}{2} \frac{p \pi}{1 + p^2} \text{ V. T. 24. N}^\circ. 14.$
- 9) $\int \frac{\text{Cot.}^p x}{1 - (2q - q^2) \cos.^2 x} dx = \frac{1}{(1-q)^{p+1}} \frac{\pi}{2} \text{Sec. } \frac{1}{2} p \pi, p^2 < 1; \left. \begin{array}{l} 10) \int \frac{\text{Cot.}^{p-1} x}{1 - (2q - q^2) \cos.^2 x} dx = \frac{1}{(1-q)^p} \frac{\pi}{2} \text{Cosec. } \frac{1}{2} p \pi, 2 > p > 0; \\ 11) \int \frac{\text{Cot.}^p x}{1 - q \cos.^2 x} dx = \frac{1}{1 \vee (1-q)^{p+1}} \frac{\pi}{2} \text{Sec. } \frac{1}{2} p \pi, p^2 < 1; \end{array} \right\} \text{ Schlömilch, Stud. I. 15.}$
- 12) $\int \frac{1}{1 + \sin. 2x. \cos. \lambda} \frac{dx}{\text{Tang.}^p x} = \frac{\pi}{\sin. p \pi} \frac{\sin. p \lambda}{\sin. \lambda}, \lambda^2 < \pi^2, p^2 < 1; \text{ V. T. 25. N}^\circ. 5.$
- 13) $\int \frac{\text{Tang.}^p x + \text{Cot.}^p x}{1 + \cos. \lambda. \sin. 2x} dx = \frac{2\pi}{\sin. p \pi} \frac{\sin. p \lambda}{\sin. \lambda}, p^2 < 1; \text{ V. T. 48. N}^\circ. 16 \text{ et T. 92. N}^\circ. 7.$
- 14) $\int \frac{1}{\sin.^p x + \text{Cosec.}^p x} \frac{dx}{\text{Tang. } x} = \frac{\pi}{4p} \text{ V. T. 5. N}^\circ. 23.$
- 15) $\int \frac{\sin.^p x + \sin.^q x}{\sin.^{p+q} x + 1} \frac{dx}{\text{Tang. } x} = \frac{\pi}{p+q} \text{Sec. } \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\} \text{ V. T. 31. N}^\circ. 17.$
- 16) $\int \frac{\sin.^p x - \sin.^q x}{1 - \sin.^{p+q} x} \frac{dx}{\text{Tang. } x} = \frac{\pi}{p+q} \text{Tang. } \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\} \text{ V. T. 31. N}^\circ. 18.$
- 17) $\int \frac{\cos.^p x + \text{Sec.}^p x}{\cos.^q x + \text{Sec.}^q x} \text{Tang. } x dx = \frac{\pi}{2q} \text{Sec. } \frac{p \pi}{2q} \text{ V. T. 31. N}^\circ. 24.$
- 18) $\int \frac{\text{Cot.}^p x}{\text{Tang.}^q x - \text{Cot.}^q x \sin. 2x} \frac{dx}{\sin. 2x} = -\frac{\pi}{4q} \text{Tang. } \frac{p \pi}{2q} \text{ V. T. 22. N}^\circ. 13.$

- 19) $\int \frac{1}{Tang.^p x + Cot.^p x \sin. 2x} \frac{dx}{dx} = \frac{\pi}{4p}$ V. T. 49. N°. 19 et T. 92. N°. 10.
- 20) $\int \frac{Tang.^q x + Tang.^p x}{Tang.^{p+q} x + 1} \frac{dx}{\sin. 2x} = \frac{\pi}{p+q} Sec. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\}$ V. T. 49. N°. 13 et T. 92. N°. 11.
- 21) $\int \frac{Tang.^q x - Tang.^p x}{Tang.^{p+q} x - 1} \frac{dx}{\sin. 2x} = \frac{\pi}{p+q} Tang. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\}$ V. T. 49. N°. 14 et T. 92. N°. 12.
- 22) $\int \frac{Tang.^q x + Cot.^q x}{Tang.^p x + Cot.^p x} \frac{dx}{\sin. 2x} = \frac{\pi}{2p} Sec. \frac{q}{2} \frac{\pi}{p}$ V. T. 22. N°. 14.
- 23) $\int \frac{Tang.^q x - Cot.^q x}{Tang.^p x - Cot.^p x} \frac{dx}{\sin. 2x} = \frac{\pi}{2p} Tang. \frac{q}{2} \frac{\pi}{p}$ V. T. 22. N°. 15.
- 24) $\int \frac{\sin.^2 x}{(1 + \sin. 2x \cos. \lambda)^2} \frac{dx}{Tang.^{p+1} x} = \frac{\pi}{2 \sin. p \pi} \frac{p \sin. \lambda \cos. p \lambda - \cos. \lambda \sin. p \lambda}{\sin.^3 \lambda}$ V. T. 26. N°. 1.
- 25) $\int \left(\frac{Tang.^p x - Cot.^p x}{\cos. x - \sin. x} \right)^q dx = 2(1 - 2p \pi \cot. 2p \pi)$ V. T. 28. N°. 7.
- 26) $\int \frac{Tang.^p x}{(1 + Tang. x)^3} \frac{dx}{\sin. 2x} = \frac{1+p}{4} p \pi \operatorname{Cosec}. p \pi$ V. T. 18. N°. 23.
- 27) $\int \frac{\cos.^{2a} x}{(1 - q \cos.^2 x)^{a+1}} \frac{dx}{Tang.^p x} = \frac{(p+1)^{a/2}}{2^{a/2}} \frac{\pi \operatorname{Sec}. \frac{1}{2} p \pi}{2(1-a)^{\frac{1}{2}(p+1)+a+1}}, p^2 < 1, q^2 < 1;$ Schlömilch, Stud. 1. 15.
- 28) $\int \frac{1}{(\operatorname{Cosec}. x - 1)^p} \frac{dx}{Tang. x} = \pi \operatorname{Cosec}. p \pi$ V. T. 31. N°. 20.
- 29) $\int \frac{(1 + Tang. x)^q - 1}{(1 + Tang. x)^{p+q} \sin. 2x} \frac{dx}{dx} = \frac{1}{2} \{Z'(p+q) - Z'(q)\}$ V. T. 22. N°. 3.
- 30) $\int \{(1 + Tang. x)^{-q} - (1 + Tang. x)^{-p}\} \frac{dx}{\sin. 2x} = \frac{1}{4} \{Z'(p) - Z'(q)\}$ V. T. 22. N°. 18.
- 31) $\int \left\{ Cot.^p x - \frac{1}{(1 + Tang. x)^p} \right\} \frac{Tang.^{q+2} x}{\sin.^2 x} dx = \frac{q}{q-p+1} \frac{\Gamma(q) \Gamma(p-q)}{\Gamma(p)}$ V. T. 22. N°. 16.

- 1) $\int \frac{1 - q \cos. 2x}{1 - 2q \cos. 2x + q^2} Tang.^p x dx = \frac{1}{4} \pi \operatorname{Sec}. \frac{1}{2} p \pi \left\{ 1 + \left(\frac{1-q}{1+q} \right)^p \right\}$, pour $q^2 < 1$; Cauchy, Lim. Imag. 116.
- 2) $= \frac{1}{4} \pi \operatorname{Sec}. \frac{1}{2} p \pi \left\{ 1 - \left(\frac{q-1}{q+1} \right)^p \right\}$, pour $q^2 > 1$; Cauchy, Lim. Imag. 122.

- 3) $\int \frac{q \sin. 2x}{1 - 2q \cos. 2x + q^2} \text{Tang. } p x dx = \frac{1}{4} \pi \text{Cosec. } \frac{1}{2} p \pi \left\{ 1 - \left(\frac{1-q}{1+q} \right)^p \right\}$, pour $q^2 < 1$; Cauchy, Lim. Imag. 117.
- 4) $= \frac{1}{4} \pi \text{Cosec. } \frac{1}{2} p \pi \left\{ 1 + \left(\frac{q-1}{q+1} \right)^p \right\}$, pour $q^2 > 1$; Cauchy, Lim. Imag. 122.
- 5) $\int \frac{\cos. a x \cos. a x}{1 - 2q \cos. 2x + q^2} dx = \frac{\pi}{2(1-q^2)} \left(\frac{1+q}{2} \right)^a$, $a > 0$, $p^2 < 1$;
- 6) $\int \frac{\cos. a x \sin. a x \sin. 2x}{1 - 2q \cos. 2x + q^2} dx = \frac{\pi}{4q} \left\{ \left(\frac{1+q}{2} \right)^a - \frac{1}{2^a} \right\}$ Poisson, P. 19. 404. N°. 76.
- 7) $\int \frac{dx}{1 - c^2 (a^2 \sin.^2 x + b^2 \cos.^2 x)} = \frac{\pi}{2 \sqrt{(1 - a^2 c^2)(1 - b^2 c^2)}}$ Plana, Cr. 17. 345.
- 8) $\int \frac{dx}{a + b \sin.^2 x + c \cos.^2 x} = \frac{\pi}{2 \sqrt{(a+b)(a+c)}}$ Lobatto, Int. 53.
- 9) $\int \frac{\cos.^3 x}{1 + 2 \cos. \lambda \sin. x + \sin.^2 x} dx = \cos. \lambda l \{ 2(1 + \cos. \lambda) \} + \frac{1}{2} \lambda \sin. \lambda$ V. T. 7. N°. 6.
- 10) $\int \frac{\text{Tang. } p x}{\text{Tang. } q x + \text{Cot. } q x - 2 \cos. \lambda} \frac{dx}{\sin. 2x} = \frac{\pi \sin. \left\{ p \frac{\pi - \lambda}{q} \right\}}{2q \sin. \lambda \sin. \frac{p\pi}{q}}$ V. T. 26. N°. 10.
- 11) $\int \frac{\text{Tang. } p x + \text{Cot. } p x - 2 \cos. \mu}{\text{Tang. } q x + \text{Cot. } q x - 2 \cos. \lambda} \frac{dx}{\sin. 2x} = \frac{\pi \sin. \left\{ p \frac{\pi - \lambda}{q} \right\}}{q \sin. \lambda \sin. \frac{p\pi}{q}} + \frac{\lambda - \pi}{q} \frac{\cos. \mu}{\sin. \lambda}$ V. T. 26. N°. 9.
- 12) $\int \frac{\sin. p x - 2 \cos. \lambda + \text{Cosec. } p x}{\sin. q x - 2 \cos. \mu + \text{Cosec. } q x} \frac{dx}{\text{Tang. } x} = \frac{\pi \sin. \left\{ \frac{\pi - \mu}{q} p \right\}}{q \sin. \mu \sin. \frac{p\pi}{q}} + \frac{\mu - \pi}{q} \frac{\cos. \lambda}{\sin. \mu}$ V. T. S. N°. 9.
- 13) $\int \frac{\sin. p x + \text{Cosec. } p x}{\sin. q x + 2 \cos. \lambda + \text{Cosec. } q x} \frac{dx}{\text{Tang. } x} = \frac{\pi \sin. \frac{p\lambda}{q}}{q \sin. \lambda \sin. \frac{p\pi}{q}}$ V. T. S. N°. 12.
- 14) $\int \frac{\sin. 2x \sin. \{(2k+1)x\}}{1 - p \cos. 2x + p^2 \cos. x} dx = 0$, $p^2 < 1$, pour $k = \infty$;
Schlömilch, Beitr. II. 1.

$$15) \int \frac{\sin. 2x \cos. \{(2k+1)x\}}{1-2p \cos. 2x + p^2} \frac{dx}{\sin. x} = 0 \quad , p^2 < 1 \text{ , pour } k = \infty ;$$

Schlömilch, Beitr. II. 1.

$$16) \int \frac{dx}{\{1+q^2(1-p^2 \sin.^2 x)\}(1-p^2 \sin.^2 x)} = \frac{\pi}{2\sqrt{1-p^2}} - \frac{\pi q^2}{2\sqrt{\{(1+q^2)(1-p^2 q^2 + q^2)\}}} \quad \text{Roberts, L. 11. 157.}$$

$$1) \int \frac{\sin. (a \text{ Tang. } x)}{\sin. x \cos. x} dx = \frac{1}{2} \pi \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211. la trouve faut. = } \pi.$$

$$2) \int \sin. (q \text{ Tang. } x) \frac{dx}{\text{Tang. } x} = \frac{1}{2} \pi (1 - e^{-q}) \quad \text{V. T. 212. N° 4.}$$

$$3) \int \text{Tang. } (q \text{ Tang. } x) \frac{dx}{\text{Tang. } x} = \frac{1}{2} \pi \frac{e^q - e^{-q}}{e^q + e^{-q}} \quad \text{V. T. 212. N° 5.}$$

$$4) \int \sin. (q \text{ Tang. } x) \frac{dx}{\cos. 2x \text{ Tang. } x} = \frac{1}{2} \pi (1 - \cos. q) \quad \text{V. T. 212. N° 11.}$$

$$5) \int \sin. (q \text{ Tang. } x) \frac{\text{Tang. } x}{\cos. 2x} dx = -\frac{1}{2} \pi \cos. p \quad \text{V. T. 204. N° 22.}$$

$$6) \int \cos. (q \text{ Tang. } x) \frac{dx}{\cos. 2x} = \frac{1}{2} \pi \sin. p \quad \text{V. T. 204. N° 21.}$$

$$7) \int \sin. (q \text{ Tang. } x) \frac{dx}{\cos. 2x} = \text{Ci. } (q) \sin. q - \text{Si. } (q) \cos. q \quad \text{V. T. 206. N° 9.}$$

$$8) \int \cos. (q \text{ Tang. } x) \frac{\text{Tang. } x}{\cos. 2x} dx = \text{Ci. } (q) \cos. q + \text{Si. } (q) \sin. q \quad \text{V. T. 206. N° 10.}$$

$$9) \int \sin. (q \text{ Tang. } x) \cdot \sin. (\pi \text{ Tang. } x) \frac{dx}{\cos. 2x} = \frac{\pi}{2} \sin. q, \text{ pour } 0 < q < \pi; \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{V. T. 204. N° 23, 24.}$$

$$10) \quad \quad \quad = 0 \quad , \text{ pour } q > \pi;$$

$$11) \int \text{Tang. } (p \text{ Tang. } x) \frac{\text{Tang. } x}{\cos. 2x} dx = -\frac{1}{2} \pi \quad \text{V. T. 206. N° 15.}$$

$$12) \int \frac{\text{Tang. } x}{\text{Tang. } (q \text{ Tang. } x)} dx = \frac{\pi}{e^{2q} - 1} \quad \text{V. T. 204. N° 10.}$$

$$13) \int \frac{\text{Tang. } x}{\text{Tang. } (p \text{ Tang. } x)} \frac{dx}{\cos. 2x} = \frac{1}{2} \pi \quad \text{V. T. 206. N° 19.}$$

$$14) \int \frac{Tang. x}{Sin. (p Tang. x)} \frac{dx}{Cos. 2x} = 0 \quad \text{V. T. 206 N}^{\circ} 20.$$

$$15) \int Cos.^2 (p Tang. x) \frac{dx}{Cos. 2x} = \frac{1}{4} \pi Sin. 2p \quad \text{V. T. 206. N}^{\circ} 21.$$

$$16) \int Sin. (p Tang. x) \frac{dx}{Tang.^{2a-1} x} = (-1)^a \frac{\pi}{2} e^{-p} \quad \text{V. T. 212. N}^{\circ} 14.$$

$$17) \int Cos. (p Tang. x) \frac{dx}{Tang.^{2a} x} = (-1)^a \frac{\pi}{2} e^{-p} \quad \text{V. T. 212. N}^{\circ} 15.$$

$$18) \int Cos. (p Tang. x) \left(\frac{Cos. x}{Cos. 2x} \right)^2 dx = \frac{\pi}{4} (Sin. p - p Cos. p) \quad \text{V. T. 208. N}^{\circ} 17.$$

$$19) \int \frac{1 - Cos. (p Tang. x)}{Tang.^2 x} dx = \frac{\pi}{2} (e^{-p} + p - 1) \quad \text{V. T. 212. N}^{\circ} 13.$$

$$20) \int \frac{Cos. (Tang. x) - Cos.^2 x - 1}{Tang. x} dx = -\Lambda \quad \text{V. T. 212. N}^{\circ} 6.$$

$$21) \int \frac{Cos. (a Tang. x) - Cos. (b Tang. x)}{Tang.^2 x} dx = \frac{\pi}{2} (e^{-b} - e^{-a}) + \frac{b-a}{2} \pi \quad \text{V. T. 212. N}^{\circ} 7.$$

$$22) \int \frac{Cos.^{p-1} x}{Sin. x} Sin. (a Tang. x + p x) dx = \frac{1}{2} \pi \quad \text{Kummer, Cr. 17. 228. — Id., Cr. 20. 1.}$$

$$23) \int \frac{Sin. p x - Sin. (p x - a Tang. x)}{Sin. x} Cos.^{p-1} x dx = \frac{1}{2} \pi \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211. trouve fautivement: \pi.}$$

$$1) \int Sin. (q Cot. x) \frac{dx}{Tang. x} = \frac{1}{2} \pi e^{-q} \quad \text{V. T. 204. N}^{\circ} 3.$$

$$2) \int Cos. (q Cot. x) \frac{dx}{Tang. x} = -\frac{1}{2} \{e^{-q} Ei.(q) + e^q Ei.(-q)\} \quad \text{V. T. 204. N}^{\circ} 8.$$

$$3) \int Tang. (q Cot. x) \frac{dx}{Tang. x} = \frac{\pi}{e^{2q} + 1} \quad \text{V. T. 204. N}^{\circ} 9.$$

$$4) \int Cot. (q Cot. x) \frac{dx}{Tang. x} = \frac{\pi}{e^{2q} - 1} \quad \text{V. T. 204. N}^{\circ} 10.$$

$$5) \int \sin. (q \cot. x) \frac{\text{Tang. } x \, dx}{\cos. 2x} = \frac{1}{2} \pi (\cos. q - 1) \quad \text{V. T. 212. N}^{\circ}. 17.$$

$$6) \int \sin. (q \cot. x) \frac{dx}{\cos. 2x \cdot \text{Tang. } x} = \frac{1}{2} \pi \cos. q \quad \text{V. T. 204. N}^{\circ}. 22.$$

$$7) \int \cos. (q \cot. x) \frac{dx}{\cos. 2x \cdot \text{Tang. } x} = -\text{Ci.}(q) \cdot \cos. q - \text{Si.}(q) \cdot \sin. q \quad \text{V. T. 206. N}^{\circ}. 10.$$

$$8) \int \sin. (q \cot. x) \frac{dx}{\text{Tang.}^{2b+1} x} = (-1)^b \frac{1}{2} \pi e^{-q} \quad \text{V. T. 205. N}^{\circ}. 27.$$

$$9) \int \cos. (q \cot. x) \frac{dx}{\text{Tang.}^{2b} x} = (-1)^b \frac{1}{2} \pi e^{-q} \quad \text{V. T. 205. N}^{\circ}. 26.$$

$$10) \int \cos. (q \cot. x) \frac{dx}{\text{Tang.}^2 x} = -\frac{1}{2} q e^{-q} \quad \text{V. T. 205. N}^{\circ}. 12.$$

$$11) \int \cos. (q \cot. x) \left(\frac{\sin. x}{\cos. 2x} \right)^2 dx = \frac{\pi}{4} (\sin. p - p \cos. p) \quad \text{V. T. 208. N}^{\circ}. 17.$$

$$12) \int \cot. (q \cot. x) \frac{dx}{\text{Tang. } x} = \frac{\pi}{e^q - 1} \quad \text{V. T. 205. N}^{\circ}. 18.$$

$$13) \int \cot. (q \cot. x) \frac{dx}{\text{Tang. } x \cdot \cos. 2x} = -\frac{1}{2} \pi \quad \text{V. T. 206. N}^{\circ}. 19.$$

$$14) \int \text{Cosec. } (q \cot. x) \frac{dx}{\text{Tang. } x \cdot \cos. 2x} = 0 \quad \text{V. T. 206. N}^{\circ}. 20.$$

$$15) \int \cos.^2 (q \cot. x) \frac{dx}{\cos. 2x} = -\frac{1}{4} \pi \sin. 2p \quad \text{V. T. 206. N}^{\circ}. 21.$$

$$16) \int \sin. (p \sin. x) \frac{dx}{\text{Tang. } x} = \sum_1^{\infty} \frac{p^{2n-1}}{1^{2n-1/1}} \frac{1}{2n+1} \quad \text{V. T. 192. N}^{\circ}. 5.$$

$$17) \int \sin.^q x \cdot \sin. (a \sin. x) \frac{dx}{\text{Tang. } x} = \frac{1-q}{(lq)^2 + a^2} \frac{a}{q} \quad \text{V. T. 192. N}^{\circ}. 4.$$

$$18) \int \sin. (p \text{Cosec. } x) \cdot \sin. (p \cot. x) \frac{dx}{\cos. x} = \pi \sin. p \quad \text{V. T. 192. N}^{\circ}. 10.$$

$$19) \int \sin. (p \text{Sec. } x) \cdot \sin. (p \text{Tang. } x) \frac{dx}{\sin. x} = \pi \sin. p \quad \text{V. T. 192. N}^{\circ}. 10.$$

$$20) \int \sin. (\frac{1}{2} p \pi - q \cot. x) \frac{dx}{\text{Tang.}^{p-1} x} = \frac{1}{2} \pi e^{-q} \quad \text{V. T. 205. N}^{\circ}. 24.$$

$$21) \int \cos. (\tfrac{1}{2} p \pi - q \cot. x) \frac{dx}{\text{Tang.}^p x} = \frac{1}{2} \pi e^{-q} \quad \text{V. T. 205. N}^\circ 25.$$

$$22) \int \frac{\cos. (q \sin. x) - \cos. (q \operatorname{cosec}. x)}{\cos. x} dx = \frac{1}{2} \pi \sin. q \quad \text{V. T. 192. N}^\circ 11.$$

$$23) \int \frac{\cos. (q \cos. x) - \cos. (q \sec. x)}{\sin. x} dx = \frac{1}{2} \pi \sin. q \quad \text{V. T. 192. N}^\circ 11.$$

$$24) \int \{ \sin. (q \cot. x) + \text{Tang.} x \cos. (q \cot. x) \} \frac{dx}{\text{Tang.} x} = \pi e^{-q} \quad \text{V. T. 204. N}^\circ 15.$$

$$1) \int dx \sqrt{1 - q^2 \sin.^2 x} = 1 - \sum_1^\infty \left\{ \frac{1^{n-1/2}}{n^{n/2}} \right\}^2 (2n-1) q^{2n}, q < 1; \quad \text{V. T. 12. N}^\circ 14.$$

$$2) \quad = E'(q), \text{ la Fonction elliptique complète de seconde espèce.} \\ \text{Legendre, Exerc. 1. 138.}$$

$$3) \int \sin. x dx \sqrt{1 - q^2 \sin.^2 x} = \frac{1}{2} \left\{ 1 + \frac{1-q^2}{2q} l \frac{1+q}{1-q} \right\}, q < 1; \quad \begin{array}{l} \text{Catalan, L. 4. 323. —} \\ \text{Lobatto, L. 5. 113. —} \\ \text{Dienger, Cr. 46. 119. —} \\ \text{Grunert, Gr. 4. 113.} \end{array}$$

$$4) \int \sin.^2 x dx \sqrt{1 - q^2 \sin.^2 x} = \frac{2q^2-1}{3q^2} E'(q) + \frac{1-q^2}{3q^2} F'(q), q < 1; \\ 5) \int \cos.^2 x dx \sqrt{1 - q^2 \sin.^2 x} = \frac{1+q^2}{3q^2} E'(q) - \frac{1-q^2}{3q^2} F'(q), q < 1; \quad \left. \begin{array}{l} \text{Legendre, Exerc.} \\ \text{1. 138.} \end{array} \right\}$$

$$6) \int \sin.^3 x dx \sqrt{1 - q^2 \sin.^2 x} = \frac{3q^2-1}{8q^2} + \frac{1-q^2}{16} \frac{1+3q^2}{q^3} l \frac{1+q}{1-q}, q < 1;$$

$$\text{Lobatto, L. 5. 113. — Grunert, Gr. 4. 113. — Catalan, L. 4. 323. la trouve faut.} = \frac{3q-2}{q^2} + \dots$$

$$7) \int dx \sqrt{1 - \sin.^2 \lambda \sin.^2 x} = \cos.^2 \lambda \operatorname{cosec}. \lambda E'(\sin. \lambda) \quad \text{Lobatschewsky, Mém. Kasan. 1835. 1.}$$

$$8) \int \sin. x \cos.^2 x dx \sqrt{1 - q^2 \sin.^2 x} = \frac{1}{4} + \frac{1-q^2}{8q^2} + \frac{(1-q^2)^2}{16q^3} l \frac{1-q}{1+q}, q < 1; \quad \begin{array}{l} \text{Dienger,} \\ \text{Cr. 46. 119.} \end{array}$$

$$9) \int \text{Tang.}^2 x dx \sqrt{1 - q^2 \sin.^2 x} = \infty, q < 1; \quad \text{Legendre, Exerc. 1. 138.}$$

$$10) \int dx \sqrt{1 - q^2 \sin.^2 x}^3 = \frac{4-2q^2}{3} E'(q) - \frac{1-q^2}{3} F'(q), q < 1; \quad \begin{array}{l} \text{Legendre, Exerc. 1. 138. —} \\ \text{Poisson, L. 2. 181.} \end{array}$$

$$11) \int dx \sqrt{1 - p^2 \cos^2 x} = 1 - \sum_1 \left\{ \frac{1^{n-1/2}}{n^{n/2}} \right\}^2 (2n-1) p^{2n} \quad \text{V. T. 12. N}^\circ. 14.$$

$$12) \int \sin^p x \cos^{3-p} x (1 - q^2 \sin^2 x)^{1-p} dx = \frac{\Gamma\left(\frac{p+1}{2}\right) \Gamma\left(2 - \frac{p}{2}\right)}{q^3 \sqrt{\{\pi(p-1)(p-3)(p-5)\}}} \\ \left\{ \frac{1 + (p-3)q + q^2}{(1+q)^{p-3}} - \frac{1 - (p-3)q + q^2}{(1-q)^{p-3}} \right\} \quad \text{V. T. 9. N}^\circ. 14.$$

$$13) \int \text{Tang.}^{2a+1} x \cos^{a+\frac{1}{2}b} x dx = \frac{2^{a/2}}{b^{a+1/2}} \\ 14) \int \text{Tang.}^{2a+1} x \cos^{a+b+\frac{1}{2}} x dx = \frac{1^{b+1/2} 1^{a/1}}{(2b+1)^{a+b+1/2}} 2^a \\ 15) \int \text{Tang.}^2 x dx \sqrt{\cos^3 x} = \frac{1}{3} \quad \left. \vphantom{\int \text{Tang.}^{2a+1} x \cos^{a+\frac{1}{2}b} x dx} \right\} \text{Oettinger, Cr. 38. 162.}$$

$$16) \int \sin^{\frac{2a}{b}-1} x \cos^{2c-\frac{2a}{b}-1} x \cos 2cx dx = \frac{\pi}{2 b^{2c-1}} \cos cc. \frac{a\pi}{b} \frac{(b-2a)^{2c-1} b}{1^{2c-1/1}}$$

$$17) \int \sin^{\frac{2a}{b}-1} x \cos^{\frac{2c}{d}-\frac{2a}{b}-1} x \cos \frac{2cx}{d} dx = \pi \text{Secc.} \frac{a\pi}{b} \frac{d.2d.3d.....}{2c.2c+d.2c+2d...} \frac{2bd. 2bd+bc...}{2bd-2ad.2bd-2ad+bc...}$$

Voyez de ces deux formules: Oettinger, Cr. 38. 216.

$$18) \int \sin^{a-1} x dx \sqrt{1 + \cos 2x} = \frac{1}{a} \sqrt{2} \\ 19) \int \sin^{a-1} x (1 + \cos 2x)^{b+\frac{1}{2}} dx = \frac{1^{\frac{1}{2}a/1} 1^{b/1}}{1^{\frac{1}{2}a+b/1}} \frac{2^b}{a} \sqrt{2} \quad \left. \vphantom{\int \sin^{a-1} x dx \sqrt{1 + \cos 2x}} \right\} \text{Oettinger, Cr. 38. 162.}$$

$$20) \int dx \sqrt[3]{\sin x} = \frac{1-\sqrt[3]{3}}{\sqrt[3]{3}} F' \left(\cos, \frac{\pi}{12} \right) + \frac{2\sqrt[3]{3}}{\sqrt[3]{3}} E' \left(\cos, \frac{\pi}{12} \right) \quad \text{V. T. 12. N}^\circ. 15.$$

$$21) \int dx \sqrt[3]{\sin^2 x} = \frac{3\sqrt[3]{3}}{\sqrt[3]{3}} E' \left(\sin, \frac{\pi}{12} \right) - \frac{3+2\sqrt[3]{3}}{2\sqrt[3]{3}} F' \left(\sin, \frac{\pi}{12} \right) \quad \text{V. T. 12. N}^\circ. 16.$$

$$22) \int dx \sqrt[3]{\cos x} = \frac{1-\sqrt[3]{3}}{\sqrt[3]{3}} F' \left(\cos, \frac{\pi}{12} \right) + \frac{2\sqrt[3]{3}}{\sqrt[3]{3}} E' \left(\cos, \frac{\pi}{12} \right) \quad \text{V. T. 12. N}^\circ. 15.$$

$$23) \int dx \sqrt[3]{\cos^2 x} = \frac{3\sqrt[3]{3}}{\sqrt[3]{3}} E' \left(\sin, \frac{\pi}{12} \right) - \frac{3+2\sqrt[3]{3}}{2\sqrt[3]{3}} F' \left(\sin, \frac{\pi}{12} \right) \quad \text{V. T. 12. N}^\circ. 16.$$

- 1) $\int \frac{\text{Sin.}^{p-\frac{1}{2}} x}{\text{Cos.}^{2p-1} x} dx = \frac{2^{\frac{1}{2}-p}}{2p-1} \frac{\Gamma(p+\frac{1}{2})\Gamma(1-p)}{\sqrt{\pi}} \text{Sin.} \left\{ \frac{2p-1}{4} \pi \right\}, p < 1; \text{ V. T. 12. N}^\circ. 18.$
- 2) $\int \frac{\text{Cos.}^{p-\frac{1}{2}} x}{\text{Sin.}^{2p-1} x} dx = \frac{2^{\frac{1}{2}-p}}{2p-1} \frac{\Gamma(p+\frac{1}{2})\Gamma(1-p)}{\sqrt{\pi}} \text{Sin.} \left\{ \frac{2p-1}{4} \pi \right\}, p < 1; \text{ V. T. 12. N}^\circ. 18.$
- 3) $\int \frac{dx}{\sqrt{\text{Cos.} x}} = \sqrt{2} \cdot \text{F}' \left(\text{Sin.} \frac{\pi}{4} \right) \text{ V. T. 13. N}^\circ. 6.$
- 4) $\int (\sqrt{\text{Tang.} x} + \sqrt{\text{Cot.} x}) dx = \pi \sqrt{2} \text{ V. T. 50. N}^\circ. 15 \text{ et T. 92. N}^\circ. 13.$
- 5) $\int \frac{\text{Cos.} x - \text{Sin.} x}{\sqrt{\text{Cos.}^3 2x}} dx = 0 \text{ V. T. 27. N}^\circ. 14.$
- 6) $\int \frac{\text{Cos.} x + \text{Sin.} x}{\text{Cos.} 2x \sqrt{\text{Sin.} 2x}} dx = 0 \text{ V. T. 28. N}^\circ. 2.$
- 7) $\int \frac{dx}{\sqrt{\text{Sin.} x}} = \frac{1}{\sqrt{3}} \text{F}' \left(\text{Cos.} \frac{\pi}{12} \right) \text{ V. T. 15. N}^\circ. 12.$
- 8) $\int \frac{dx}{\sqrt{\text{Sin.}^2 x}} = \frac{3}{\sqrt{3}} \text{F}' \left(\text{Sin.} \frac{\pi}{12} \right) \text{ V. T. 15. N}^\circ. 11.$
- 9) $\int \frac{dx}{\sqrt{\text{Cos.} x}} = \frac{1}{\sqrt{3}} \text{F}' \left(\text{Cos.} \frac{\pi}{12} \right) \text{ V. T. 15. N}^\circ. 12.$
- 10) $\int \frac{dx}{\sqrt{\text{Cos.}^2 x}} = \frac{3}{\sqrt{3}} \text{F}' \left(\text{Sin.} \frac{\pi}{12} \right) \text{ V. T. 15. N}^\circ. 11.$
- 11) $\int \frac{dx}{\text{Tang.} x} \sqrt{\text{Tang.}^2 x} = \frac{1}{2} \pi \text{Cosec.} \frac{\pi}{q} \text{ V. T. 32. N}^\circ. 13.$
- 12) $\int \frac{\text{Tang.} x}{\sqrt{\text{Tang.}^2 x}} dx = \frac{1}{2} \pi \text{Cosec.} \frac{\pi}{q} \text{ V. T. 32. N}^\circ. 13.$
- 13) $\int dx \sqrt{\frac{1-p^2 \text{Sin.}^2 x}{\text{Sin.} x}} = \frac{2c \text{F}'(c) + 2b \text{F}'(b)}{(b+c)^2} + 2 \frac{b-c}{(b+c)^2} \left\{ \text{E}'(b) - \text{E}'(c) \right\} \left. \vphantom{\int dx \sqrt{\frac{1-p^2 \text{Sin.}^2 x}{\text{Sin.} x}}} \right\} \begin{array}{l} \text{Legendre,} \\ \text{Exerc. 6. 308} \end{array}$
, où $2c^2 = \frac{(1-\sqrt{p})^2}{1+p}$ et $2b^2 = \frac{(1+\sqrt{p})^2}{1+p}$;
- 14) $\int \frac{dx}{\text{Cos.}^2 x} \sqrt{1-p^2 \text{Sin.} x} = \infty \text{ Legendre, Exerc. 1. 138.}$
- 15) $\int (\text{Sec.} x - 1)^{p+\frac{1}{2}} \text{Sin.} x dx = \frac{2p+1}{2} \pi \text{Sec.} p \pi \text{ V. T. 32. N}^\circ. 3.$

$$16) \int (\sec x - 1)^{p-\frac{1}{2}} \operatorname{Tang} x dx = \pi \sec p \pi \quad \text{V. T. 32. N}^\circ. 1.$$

$$17) \int (\operatorname{Cosec} x - 1)^{p-\frac{1}{2}} \cos x dx = \frac{2p+1}{2} \pi \sec p \pi \quad \text{V. T. 32. N}^\circ. 3.$$

$$18) \int (\operatorname{Cosec} x - 1)^{p-\frac{1}{2}} \frac{dx}{\operatorname{Tang} x} = \pi \sec p \pi \quad \text{V. T. 32. N}^\circ. 1.$$

$$\left. \begin{aligned} 1) \int \frac{dx}{\sqrt{(a+b \cos x)}} &= \frac{2}{\sqrt{a+b}} F' \left(\sqrt{\frac{2b}{a+b}} \right) \\ 2) \int \frac{dx}{\sqrt{(a-b \cos x)}} &= \frac{2}{\sqrt{a+b}} \left\{ F' \left(\sqrt{\frac{2b}{a+b}} \right) - F \left(\frac{1}{4} \pi, \sqrt{\frac{2b}{a+b}} \right) \right\} \\ 3) \int \frac{\cos x}{\sqrt{(a-b \cos x)}} dx &= \frac{2}{b \sqrt{a+b}} \left\{ a F' \left(\sqrt{\frac{2b}{a+b}} \right) - F \left(\frac{1}{4} \pi, \sqrt{\frac{2b}{a+b}} \right) \right\} \\ &\quad - (a+b) \left\{ E' \left(\sqrt{\frac{2b}{a+b}} \right) - E \left(\frac{1}{4} \pi, \sqrt{\frac{2b}{a+b}} \right) \right\} \\ 4) \int \frac{\cos x}{\sqrt{(a+b \cos x)}} dx &= \frac{2}{b \sqrt{a+b}} \left\{ (a+b) E' \left(\sqrt{\frac{2b}{a+b}} \right) - a F' \left(\sqrt{\frac{2b}{a+b}} \right) \right\} \\ 5) \int \frac{dx}{\sqrt{(3 \pm \cos 2x)}} &= \frac{1}{2} F' \left(\sin \frac{\pi}{4} \right) \quad \text{V. T. 13. N}^\circ. 6. \\ 6) \int \frac{\sin^2 x}{\sqrt{(3 + \cos 2x)}} dx &= \frac{\{\Gamma(\frac{1}{4})\}^2}{8 \sqrt{\pi}} - \frac{\pi \sqrt{\pi}}{\{\Gamma(\frac{1}{4})\}^2} \quad \text{V. T. 13. N}^\circ. 9. \\ 7) \int \frac{\sin^3 x}{\sqrt{(3 - \cos 2x)}} dx &= \frac{1}{4} \sqrt{2} \quad \text{V. T. 13. N}^\circ. 7. \\ 8) \int \frac{\operatorname{Tang}^{p+\frac{1}{2}} x}{(\cos x + \sin x)^2} dx &= \frac{2p+1}{2} \pi \sec p \pi \quad \text{V. T. 27. N}^\circ. 4. \\ 9) \int \frac{\sin x}{(\sec x - 1)^{p+\frac{1}{2}}} dx &= \frac{2p+1}{2} \pi \sec p \pi \quad \text{V. T. 32. N}^\circ. 17. \\ 10) \int \frac{\operatorname{Tang} x}{(\sec x - 1)^{p-\frac{1}{2}}} dx &= \pi \sec p \pi \quad \text{V. T. 32. N}^\circ. 15. \end{aligned} \right\} \begin{array}{l} , a > b > 0 : \\ \text{Dienger, Gr.} \\ 13. 424. \end{array}$$

$$11) \int \frac{\cos. x}{(\operatorname{Cosec}. x - 1)^{p+\frac{1}{2}}} dx = \frac{2p+1}{2} \pi \operatorname{Sec}. p \pi \quad \text{V. T. 32. N}^\circ. 17.$$

$$12) \int \frac{\sin. x^{p+\frac{1}{2}}}{(1 - \sin. x)^{p-\frac{1}{2}} \operatorname{Tang}. x} dx = \frac{1-2p}{2} \pi \operatorname{Sec}. p \pi \quad \text{V. T. 12. N}^\circ. 8.$$

$$1) \int \frac{dx}{\sqrt{1 + \sin.^2 x}} = \frac{1}{2} \sqrt{2} \cdot F' \left(\sin. \frac{\pi}{4} \right) \quad \text{V. T. 13. N}^\circ. 6.$$

$$2) \int \frac{\cos.^2 x}{\sqrt{1 + \sin.^2 x}} dx = \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2}\pi} - \frac{\pi\sqrt{2}\pi}{\{\Gamma(\frac{1}{4})\}^2} \quad \text{V. T. 12. N}^\circ. 9.$$

$$3) \int \frac{\sin.^2 x}{\sqrt{1 + \sin.^2 x}} dx = \frac{1}{2} \sqrt{2} \cdot F' \left(\sin. \frac{\pi}{4} \right) + \frac{\pi\sqrt{2}\pi}{\{\Gamma(\frac{1}{4})\}^2} - \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2}\pi} \quad \text{V. T. 75. N}^\circ. 1, 2.$$

$$4) \int \frac{\sin.^3 x}{\sqrt{1 + \sin.^2 x}} dx = \frac{1}{2} \quad \text{V. T. 13. N}^\circ. 7.$$

$$5) \int \frac{\sin. x}{\sqrt{p^2 + \sin.^2 x}} dx = \frac{\pi}{2} - \operatorname{Arctang}. p \quad \text{V. T. 16. N}^\circ. 6.$$

$$6) \int \frac{\sin. x}{\sqrt{1 + p^2 \sin.^2 x}} dx = \frac{1}{p} \operatorname{Arctang}. p \quad \left. \vphantom{\int} \right\} , p^2 < 1;$$

$$7) \int \frac{\sin.^3 x}{\sqrt{1 + p^2 \sin.^2 x}^3} dx = \frac{1}{p^3} \left\{ -\frac{p}{1+p^2} + \operatorname{Arctang}. p \right\} \quad \left. \vphantom{\int} \right\} \text{Dienger, Cr. 46. 119.}$$

$$8) \int \frac{\cos. x}{\sqrt{p^2 + \cos.^2 x}} dx = \frac{\pi}{2} - \operatorname{Arctang}. p \quad \text{V. T. 16. N}^\circ. 6.$$

$$9) \int \frac{dx}{\sqrt{1 - p^2 \sin.^2 x}} = F'(p), \text{ la Fonction elliptique complète de première espèce; } \quad \text{Legendre, Exerc. 1. 138.}$$

$$10) \int \frac{\sin. x}{\sqrt{1 - p^2 \sin.^2 x}} dx = \frac{1}{2p} \ell \frac{1+p}{1-p}, p^2 < 1; \quad \text{Dienger, Cr. 46. 119.}$$

$$11) \int \frac{\sin.^2 x}{\sqrt{1 - p^2 \sin.^2 x}} dx = \frac{1}{p^2} \{F'(p) - E'(p)\} \quad \left. \vphantom{\int} \right\} p^2 < 1;$$

$$12) \int \frac{\cos.^2 x}{\sqrt{1 - p^2 \sin.^2 x}} dx = F'(p) - \frac{1}{p^2} \{F'(p) - E'(p)\} \quad \left. \vphantom{\int} \right\} \text{Legendre, Exerc. 1. 138. — Dienger, Cr. 46. 119.}$$

$$13) \int \frac{Tang.^2 x}{\sqrt{1-p^2 \sin.^2 x}} dx = \infty \quad \text{Legendre, Exerc. 1. 138.}$$

$$14) \int \frac{Cos.^2 x}{\sqrt{1-p^2 \sin.^2 x}} dx = F'(p) - \frac{2}{p^2} \{F'(p) - E'(p)\} \quad \text{V. T. 75. N°. 11, 12.}$$

$$15) \int \frac{\sin.^4 x}{\sqrt{1-p^2 \sin.^2 x}} dx = \frac{2}{3} \frac{1+p^2}{p^4} \{F'(p) - E'(p)\} - \frac{1}{3p^2} F'(p) \quad \begin{array}{l} \text{Dienger, Cr. 46. 119.} \\ \text{Poisson, L. 2. 184.} \end{array}$$

$$16) \int \frac{\sin. x. Cos.^2 x}{\sqrt{1-p^2 \sin.^2 x}} dx = \frac{1}{2p^2} + \frac{1-p^2}{4p^3} l \frac{1-p}{1+p} \quad \text{Dienger, Cr. 46. 119.}$$

$$17) \int \frac{Tang.^2 \frac{1}{2} x}{\sqrt{1-p^2 \sin.^2 x}} dx = 2 \sqrt{1-p^2} + F'(p) - 2 E'(p) \left\{ \begin{array}{l} , p < 1; \\ \text{Legendre, Exerc. 1. 138.} \end{array} \right.$$

$$18) \int \frac{dx}{\sqrt{1-p^2 \sin.^2 x}^3} = \frac{1}{1-p^2} E'(p)$$

$$19) \int \frac{\sin. x}{\sqrt{1-p^2 \sin.^2 x}^3} dx = \frac{1}{1-p^2} \quad \text{Plana, Cr. 17. 345.}$$

$$20) \int \frac{\sin.^2 x}{\sqrt{1-p^2 \sin.^2 x}^3} dx = \frac{1}{p^2 (3-p^2)} E'(p) - \frac{1}{p^2} F'(p) \quad \text{Roberts, L. 11. 157.}$$

$$21) \int \frac{2 \sin.^2 x - 1}{\sqrt{1-p \sin.^2 x}} dx = \left(\frac{2}{p^2} - 1 \right) F'(p) - \frac{2}{p^2} E'(p) \quad \text{Ramus, Danske Afh. 6. 265.}$$

$$22) \int \frac{\sin.^6 x}{\sqrt{1-p^2 \sin.^2 x}} dx = \frac{3}{8p^2} \{E'(p) - F'(p)\} + \frac{1}{2} F'(p) \quad \text{V. T. 28. N°. 22.}$$

$$23) \int \frac{\sin.^6 x. Cos.^4 x}{\sqrt{1-p^2 \sin.^2 x}} dx = \frac{2p^2+1}{8p^2} E'(p) - \frac{1-p^2}{8p^2} F'(p) \quad \text{V. T. 28. N°. 23.}$$

$$24) \int \frac{\sin. x. Cos. x}{\sqrt{Cos.^2 \mu - \sin.^2 \lambda. \sin.^2 x}} dx = \frac{Cos. \mu}{\sin.^2 \lambda} - \frac{1}{\sin.^2 \lambda} \sqrt{Cos.^2 \mu + \sin.^2 \lambda} \quad \text{Poisson, Chaleur. § 216.}$$

$$25) \int dx \sqrt{\frac{1-p^2 \sin.^4 x}{1+\sin.^2 x}} = \frac{cF'(c)+bF'(b)}{(b+c)^2} + \frac{b-c}{(b+c)^2} \{E'(b)-E'(c)\}, \text{ où } 2c^2 = \frac{(1-\sqrt{p})^2}{1+p}, 2b^2 = \frac{(1+\sqrt{p})^2}{1+p}, \text{ V. T. 13. N°. 8.}$$

$$26) \int \frac{Tang.^{2a} x. Cos.^2 x}{(1+Sec.^2 x)^{b+\frac{1}{2}}} dx = 2^{a-b+\frac{1}{2}} \frac{\Gamma(a+\frac{1}{2}) \Gamma(b-a)}{\Gamma(b+\frac{1}{2})} \quad \text{V. T. 27. N°. 12.}$$

$$\begin{aligned}
 1) & \int \frac{dx}{\cos^2 x \sqrt{(1-p^2 \sin^2 x)}} = \infty \\
 2) & \int \frac{dx}{\cos^2 \frac{1}{2} x \sqrt{(1-p^2 \sin^2 x)}} = 2 \sqrt{(1-p^2)} + 2 F'(p) - 2 E'(p) \\
 3) & \int \frac{dx}{\sin^2 x \sqrt{(1-p^2 \sin^2 x)}} = F'(p) - E'(p) - \sqrt{(1-p^2)} \\
 4) & \int \frac{\sin x}{1-p \sin x} \frac{dx}{\sqrt{(1-p^2 \sin^2 x)}} = \frac{1}{p^3} E'(p) + \frac{1}{(1-p^2)p} F'(p) \\
 5) & \int \frac{1}{1-p \sin x} \frac{dx}{\sqrt{(1-p^2 \sin^2 x)}} = \frac{1}{p^2} E'(p) + \frac{p}{1-p^2} \\
 6) & \int \frac{dx}{\sqrt{\{ \cos x (\cos x + p^2 \sin x) (\cos x + q^2 \sin x) (\cos x + r^2 \sin x) \}}} = \frac{2}{\sqrt{(p^2-r^2)}} F\left(q, \sqrt{\frac{p^2-q^2}{p^2-r^2}}\right) \\
 7) & \int \frac{dx}{\sqrt{\{ \cos x (p^2 \cos x + \sin x) (q^2 \cos x + \sin x) (r^2 \cos x + \sin x) \}}} = \frac{2}{\sqrt{(p^2-r^2)}} F\left(q, \sqrt{\frac{p^2-q^2}{p^2-r^2}}\right)
 \end{aligned}$$

, $p < 1$;
Legendre, Exerc. 1.
138.

Roberts, L. 11. 157.

Dienger, Gr.
11. 88.

V. T. 28.
N°. 14.

V. T. 28.
N°. 15.

Dans les formules 4, 5, on a $\cos. q = \frac{r}{p}$, où $p > q > r$;

$$\begin{aligned}
 8) & \int \frac{dx}{\sqrt{\{ \cos x (p^2 \cos x + l^2 \sin x) (q^2 \cos x + m^2 \sin x) (r^2 \cos x + n^2 \sin x) \}}} = \frac{\pi}{2 p m n \sin. q} \\
 & F\left(q, \frac{n}{m} \sqrt{\frac{p^2 m^2 - q^2 l^2}{p^2 n^2 - r^2 l^2}}\right), \text{ où } \cos^2 q = \frac{r l}{p n}, p m > q l, p n > r l; \text{ V. T. 28. N°. 16.} \\
 9) & \int \frac{dx}{\sqrt{\{ \sin x (p^2 \cos x + l^2 \sin x) (q^2 \cos x + m^2 \sin x) (r^2 \cos x + n^2 \sin x) \}}} = \frac{\pi}{2 l q r \sin. q} \\
 & F\left(q, \frac{r}{p} \sqrt{\frac{p^2 m^2 - q^2 l^2}{p^2 n^2 - r^2 l^2}}\right), \text{ où } \cos^2 q = \frac{p n}{r l}, q l > p m, r l > p n; \text{ V. T. 28. N°. 16.} \\
 10) & \int \frac{\sin^{p-\frac{1}{2}} x}{\cos x + \sin x} \frac{dx}{\cos^{p+\frac{1}{2}} x} = \pi \sec. p \pi \text{ V. T. 27. N°. 2.} \\
 11) & \int \frac{1}{\cos x + \sin x} \frac{dx}{\cos x \cdot \text{Tang.}^{p-\frac{1}{2}} x} = \pi \sec. p \pi \text{ V. T. 28. N°. 5.} \\
 12) & \int \frac{1}{(\cos x + \sin x)^2} \frac{dx}{\text{Tang.}^{p+\frac{1}{2}} x} = \frac{2p+1}{2} \pi \sec. p \pi \text{ V. T. 27. N°. 4.} \\
 13) & \int \frac{\sin^{\frac{1}{2}(p+q-1)} x}{(\cos x + \sin x)^{p+\frac{1}{2}}} \frac{dx}{\cos^{\frac{1}{2}(q+1)} x} = \frac{\Gamma\left(\frac{p+q}{2}\right) \Gamma\left(\frac{p-q+1}{2}\right)}{\Gamma(p+\frac{1}{2})} \text{ V. T. 27. N°. 8.}
 \end{aligned}$$

- 14) $\int \frac{1}{(\operatorname{Cosec}. x - 1)^{\mu-1}} \frac{dx}{\operatorname{Tang}. x} = \pi \operatorname{Sec}. p \pi \quad \text{V. T. 32. N}^\circ 15.$
- 15) $\int \frac{\operatorname{Sin}^{q+1} x}{(1 - p^2 \operatorname{Sin}^2 x)^{\frac{q+1}{2}}} \frac{dx}{\operatorname{Cos}. q x} = \frac{(1-p)^{-q} - (1+p)^{-q}}{4 p q} \operatorname{B} \left(\frac{q+2}{2}, \frac{1-q}{2} \right) \quad \text{V. T. 16. N}^\circ 7.$
- 16) $\int \frac{\operatorname{Sin}^{q+1} x}{(\operatorname{Cos}^2 \lambda - \operatorname{Sin}^2 x \operatorname{Sin}^2 \lambda)^{\frac{q+1}{2}}} \frac{dx}{\operatorname{Cos}. q x} = \frac{\operatorname{Sin}. q \lambda}{2 q \operatorname{Sin}. \lambda} \operatorname{B} \left(\frac{2+q}{2}, \frac{1-q}{2} \right) \quad \text{V. T. 16. N}^\circ 8.$
- 17) $\int \frac{1}{\operatorname{Sin}. x + \operatorname{Cos}. x} \frac{dx}{\sqrt{\operatorname{Sin}. 2 x}} = \frac{\pi}{2} \sqrt{2} \quad \text{V. T. 23. N}^\circ 1.$
- 18) $\int \frac{\operatorname{Sin}. x \operatorname{Cos}. x}{1 - \operatorname{Sin}^2 \lambda \operatorname{Sin}^2 x} \frac{dx}{1 - (\operatorname{Cos}^2 \mu - \operatorname{Sin}^2 \lambda \operatorname{Sin}^2 x)} = \frac{1}{\operatorname{Sin}^2 \lambda \operatorname{Sin}^2 \mu} \left\{ \frac{\pi}{2} - \mu - \operatorname{Arccos}. \left(\frac{\operatorname{Sin}. \mu}{\operatorname{Cos}. \lambda} \right) \right\}$
- 19) $\int \frac{\operatorname{Cos}^2 x}{1 - \operatorname{Sin}^2 \lambda \operatorname{Sin}^2 x} \frac{dx}{\sqrt{(\operatorname{Cos}^2 \mu - \operatorname{Sin}^2 \lambda \operatorname{Sin}^2 x)}} = \operatorname{Sec}. \mu \operatorname{F}' \left(\frac{\operatorname{Sin}. \lambda}{\operatorname{Cos}. \mu} \right) -$
 $-\frac{\operatorname{Cos}. \lambda}{\operatorname{Sin}^2 \lambda \operatorname{Sin}. \mu} \left\{ \operatorname{F}' \left(\frac{\operatorname{Sin}. \lambda}{\operatorname{Cos}. \mu} \right) \operatorname{E} \left(\frac{\operatorname{Sin}. \lambda}{\operatorname{Cos}. \mu}, \frac{\pi}{2} - \mu \right) - \operatorname{E}' \left(\frac{\operatorname{Sin}. \lambda}{\operatorname{Cos}. \mu} \right) \operatorname{F} \left(\frac{\operatorname{Sin}. \lambda}{\operatorname{Cos}. \mu}, \frac{\pi}{2} - \mu \right) \right\}$
- 20) $\int \frac{\operatorname{Cos}^2 x}{1 + p^2 \operatorname{Sin}^2 x} \frac{dx}{\sqrt{(1 - q^2 \operatorname{Sin}^2 x)}} = \frac{\operatorname{Tang}. \mu}{2 \sqrt{(\operatorname{Cos}^2 \mu + q^2 \operatorname{Sin}^2 \mu)}} \left\{ \begin{array}{l} \operatorname{Catalan}, \\ \text{L. G.} \\ 419. \end{array} \right.$
 $[\pi + 2 \operatorname{F}'(q) \operatorname{F} \{ \sqrt{(1 - q^2)}, \mu \} - 2 \operatorname{F}'(q) \operatorname{E} \{ \sqrt{(1 - q^2)}, \mu \} - 4 \operatorname{E}'(q) \operatorname{F} \{ \sqrt{(1 - q^2)}, \mu \}], \text{ où } p = \operatorname{Cot}. \mu ;$
- 21) $\int \frac{\operatorname{Sin}. x}{\sqrt{\left(\frac{\operatorname{Cos}^2 x}{r^2} + \frac{\operatorname{Sin}^2 x}{q^2} \right)}} \frac{dx}{\sqrt{\left(\frac{\operatorname{Cos}^2 x}{r^2} + \frac{\operatorname{Sin}^2 x}{p^2} \right)}} = \frac{p q r}{\sqrt{(r^2 - p^2)}} \operatorname{F} \left\{ \operatorname{Arccos}. \frac{p}{r}, \sqrt{\frac{r^2 - q^2}{r^2 - p^2}} \right\}, r > q > p; \quad \text{Jacobi, Cr. 10. 101.}$

- 1) $\int \operatorname{Sin}. \left(\frac{1}{k} \operatorname{Sec}. x \right) \frac{dx}{\sqrt{\operatorname{Cos}^3 x}} = \left(\operatorname{Cos}. \frac{1}{k} + \operatorname{Sin}. \frac{1}{k} \right) \sqrt{\frac{k \pi}{4}}, \text{ pour } k = \infty;$
- 2) $\int \operatorname{Cos}. \left(\frac{1}{k} \operatorname{Sec}. x \right) \frac{dx}{\sqrt{\operatorname{Cos}^3 x}} = \left(\operatorname{Cos}. \frac{1}{k} - \operatorname{Sin}. \frac{1}{k} \right) \sqrt{\frac{k \pi}{4}} \quad \text{Poisson, Mém. Ac. 1816. 71. N}^\circ 40.$
- 3) $\int \operatorname{Sin}. (p \operatorname{Tang}. x) \frac{dx}{\operatorname{Cos}. x \sqrt{\operatorname{Sin}. 2 x}} = \frac{1}{2} \sqrt{\frac{\pi}{p}} \quad \text{V. T. 224. N}^\circ 4.$
- 4) $\int \operatorname{Cos}. (p \operatorname{Tang}. x) \frac{dx}{\operatorname{Cos}. x \sqrt{\operatorname{Sin}. 2 x}} = \frac{1}{2} \sqrt{\frac{\pi}{p}} \quad \text{V. T. 224. N}^\circ 5.$

- $$\left. \begin{aligned} 1) \int \sin x \, dx &= 2 \\ 2) \int \sin^3 x \, dx &= \frac{4}{3} \end{aligned} \right\} \text{Poisson, Chaleur. 82.}$$
- $$3) \int \sin^{2a+1} x \, dx = \frac{(1^{a/1})^2}{2^{2a/1}} 2^{2a} \quad \text{Oettinger, Cr. 38. 162.}$$
- $$4) \int \sin^a x \, dx = \frac{\pi}{2^a} \frac{\Gamma(a+1)}{\left\{ \Gamma\left(\frac{a}{2} - 1\right) \right\}^2} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211.}$$
- $$5) \int \sin^{p-1} x \, dx = \frac{\Gamma\left(\frac{1}{2} p\right)}{\Gamma\left(\frac{1+p}{2}\right)} \sqrt{\pi} \quad \text{V. T. 53. N° 20.}$$
- $$6) \quad = \frac{\left\{ \Gamma\left(\frac{1}{2} p\right) \right\}^2}{\Gamma(p)} 2^{p-1} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 1.}$$
- $$\left. \begin{aligned} 7) \int \cos^{2a+1} x \, dx &= 0 \\ 8) \int \cos^{2a} x \, dx &= \frac{1^{a/2}}{2^{a/2}} \pi \end{aligned} \right\} \text{Cauchy, Exerc. 1826. p. 205.}$$
- $$\left. \begin{aligned} 9) \int \sin^2 p x \, dx &= \frac{1}{2} \pi \\ 10) \int \cos^2 p x \, dx &= \frac{1}{2} \pi \end{aligned} \right\} \text{Euler, Calc. Int. 4. S. 4. 94. — Poisson, Chal. 92.}$$
- $$\left. \begin{aligned} 11) \int \sin a x \cdot \sin b x \, dx &= 0 \\ 12) \int \cos a x \cdot \cos b x \, dx &= 0 \end{aligned} \right\} \text{Poisson, Chal. 92.}, \quad a > \text{ ou } < b;$$
- $$\left. \begin{aligned} 13) \int \sin p x \cdot \sin a x \, dx &= (-1)^{a-1} \frac{a \sin p \pi}{a^2 - p^2} \\ 14) \int \cos p x \cdot \sin x \, dx &= \frac{\pi \cos p \pi}{1 - p^2} + \frac{2 p \sin p \pi}{(1 - p^2)^2} \\ 15) \int \cos p x \cdot \cos a x \, dx &= (-1)^a \frac{p \sin p \pi}{a^2 - p^2} \end{aligned} \right\} \text{Schlömlich, Beitr. I. § 5.}$$

$$16) \int \sin^q x \cdot \sin q x dx = \frac{\pi}{2^q} \sin \frac{q\pi}{2} \left\{ \begin{array}{l} \text{Lobatschewsky, Mém. Kasan. 1835. 211.} \end{array} \right.$$

$$17) \int \sin^q x \cdot \cos q x dx = \frac{\pi}{2^q} \cos \frac{q\pi}{2} \left\{ \right.$$

$$18) \int \sin^q x \cdot \sin p x dx = \frac{\pi}{2^q} \frac{\sin \frac{1}{2} p \pi \Gamma(q+1)}{\Gamma\left(\frac{p+q}{2}+1\right) \Gamma\left(\frac{q-p}{2}+1\right)} \left\{ \begin{array}{l} \text{Kummer, Cr. 17. 210. — Lo-} \\ \text{batschewsky, Mém. Kasan. 1835.} \\ \text{211.} \end{array} \right.$$

$$19) \int \sin^q x \cdot \cos p x dx = \frac{\pi}{2^q} \frac{\cos \frac{1}{2} p \pi \Gamma(q+1)}{\Gamma\left(\frac{p+q}{2}+1\right) \Gamma\left(\frac{q-p}{2}+1\right)} \left\{ \right.$$

$$20) \int \sin^{q-1} x \cdot \sin p x dx = 2^{q-1} \sin \frac{1}{2} p \pi \frac{\Gamma\left(\frac{q-p}{2}\right) \Gamma\left(\frac{q+p}{2}\right) \Gamma(q)}{\Gamma(q-p) \Gamma(q+p)} \left\{ \begin{array}{l} \text{Serret, L. 8. 1.} \end{array} \right.$$

$$21) \int \sin^{q-1} x \cdot \cos p x dx = 2^{q-1} \cos \frac{1}{2} p \pi \frac{\Gamma\left(\frac{q-p}{2}\right) \Gamma\left(\frac{q+p}{2}\right) \Gamma(q)}{\Gamma(q-p) \Gamma(q+p)} \left\{ \right.$$

$$22) \int \sin^{q-1} x \cdot \cos \left\{ p \left(\frac{\pi}{2} - x \right) \right\} dx = 2^{q-1} \frac{\Gamma\left(\frac{q-p}{2}\right) \Gamma\left(\frac{q+p}{2}\right) \Gamma(q)}{\Gamma(q-p) \Gamma(q+p)} \left\{ \right.$$

$$23) \int \cos^{2a-1} x \cdot \sin^{2b-1} x dx = 0 \quad \text{Poisson, Chaleur. 80.}$$

$$24) \int \sin^b x \cdot \cos^{2a} x dx = \frac{\Gamma\left(\frac{b+1}{2}\right) \Gamma\left(\frac{2a+1}{2}\right)}{\Gamma\left(a + \frac{b}{2} + 1\right)} \quad \text{Lobatschewsky, Mém. Kasan. 1836. 1.}$$

$$25) \int \cos^{a+b} \frac{1}{2} x \cdot \cos \frac{1}{2} b x \cdot \cos \frac{1}{2} a x dx = \frac{\pi}{2^{a+b+1}} \left\{ 2 + \sum_1^{\infty} \binom{a}{n} \binom{b}{n} \right\} \left\{ \begin{array}{l} \text{Smaasen, Cr. 42. 222.} \end{array} \right.$$

$$26) \int \cos^a \frac{1}{2} x \cdot \cos \frac{1}{2} a x dx = \frac{\pi}{2^a}$$

$$\begin{aligned}
 1) & \int (1 + p^2 - 2 p \cos. x) dx = (1 + p^2) \pi \\
 2) & \int (1 + p^2 - 2 p \cos. x) \cos. x dx = - p \pi \\
 3) & \int (1 + p^2 - 2 p \cos. x)^2 dx = (1 + 4 p^2 + p^4) \pi \\
 4) & \int (1 + p^2 - 2 p \cos. x)^2 \cos. x dx = - 2 p (1 + p^2) \pi \\
 5) & \int (1 + p^2 - 2 p \cos. x)^2 \cos. 2 x dx = p^2 \pi \\
 6) & \int (1 + p^2 - 2 p \cos. x)^a \cos. a x dx = (-1)^a p^a \pi \quad \text{Euler, Calc. Int. 4. S. 4. 30. — Legendre, Exerc. 3. 64.} \\
 7) & \int (1 + p^2 - 2 p \cos. x)^a dx = \pi \sum_0^a \binom{a}{n}^2 p^{2n} \quad \text{Euler, Calc. Int. 4. S. 4. 31, 67. — Legendre, Exerc. 3. 64.} \\
 8) & \int (1 + p^2 - 2 p \cos. x)^a \cos. b x dx = \pi (-p)^b \frac{a^{b/1}}{1^{b/1}} \left\{ 1 + \binom{a}{1} \frac{a-b}{b+1} p^2 + \binom{a}{2} \frac{a-b, a-b-1}{b+1, b+2} p^4 + \dots \right\} \quad \text{Legendre, Exerc. 3. 63.}
 \end{aligned}$$

$$\begin{aligned}
 1) & \int \cos. (a \sin. x) dx = \pi \sum_0^\infty \frac{(-a^2)^n}{(2^{n+1/2})^2} \quad \text{Fourier, Chal. 314.} \\
 2) & \int \cos. (q \cos. x) \sin. x dx = \frac{2}{q} \sin. q \\
 3) & \int \cos. (q \cos. x) \sin.^3 x dx = \frac{4}{q^3} (\sin. q - q \cos. q) \\
 4) & \int \cos. (q \sin. x) \cos. \{(2b+1)x\} dx = 0 \\
 5) & \int \sin. (q \sin. x) \sin. 2bx dx = 0 \\
 6) & \int \cos. (q \sin. x) \cos. 2bx dx = \left(\frac{q}{2}\right)^{2b} \frac{\pi}{1^{2b/1}} \left[1 + \sum_1^\infty (-1)^n \frac{(\frac{1}{2} q)^{2n}}{1^{n/1} (2b+1)^{n/1}} \right] \\
 7) & \int \sin. (q \sin. x) \sin. \{(2b+1)x\} dx = \left(\frac{q}{2}\right)^{2b+1} \frac{\pi}{1^{2b+1/1}} \left[1 + \sum_1^\infty (-1)^n \frac{(\frac{1}{2} q)^{2n}}{1^{n/1} (2b+2)^{n/1}} \right]
 \end{aligned}$$

Poisson,
Conn. des
Temps.
1836. 1. —
Lefort, L.
11. 142.

$$\begin{aligned}
 8) \int \cos. \{a(x - q \sin. x)\} dx &= \frac{\pi (\frac{1}{2} a q)^a}{1^{a/1}} \left\{ 1 + \sum_1^{\infty} (-1)^n \frac{(\frac{1}{2} a q)^{2n}}{1^{n/1} (1+a)^{n/1}} \right\} \left\{ \begin{array}{l} \text{Bessel, Abh. Ber-} \\ \text{lin. 1816. 49. —} \\ \text{Id., ib. 1824. 1. —} \\ \text{Poisson, Conn. des} \\ \text{Temps. 1836. 1. —} \\ \text{Lefort, L. 11. 142.} \end{array} \right\} \\
 9) \int (1 - q \cos. x)^2 \cos. \{a(x - q \sin. x)\} dx &= -\frac{\pi (\frac{1}{2} a q)^a}{a \cdot 1^{a/1}} \left\{ 1 + \sum_1^{\infty} (-1)^n (\frac{1}{2} a q)^{2n} \frac{a+2n}{1^{n/1} (1+a)^{n/1}} \right\} \\
 10) \int \frac{\sin. \{a(x - q \sin. x)\}}{(1 - q \cos. x)^2} \sin. x dx &= \frac{\pi a^2 (\frac{1}{2} a q)^{a-1}}{2 \cdot 1^{a/1}} \left\{ 1 + \sum_1^{\infty} (-1)^n \frac{(\frac{1}{2} a q)^{2n}}{1^{n/1} (1+a)^{n/1}} \right\} \left\{ \begin{array}{l} \text{Poisson,} \\ \text{Conn.} \\ \text{des Temps.} \\ \text{1836. 1.} \end{array} \right\} \\
 11) \int \frac{q - \cos. x}{(1 - q \cos. x)^2} \cos. \{a(x - q \sin. x)\} dx &= -\frac{\pi (\frac{1}{2} a q)^a}{a \cdot 1^{a/1}} \left\{ 1 + \sum_1^{\infty} (-1)^n (\frac{1}{2} a q)^{2n} \frac{a+2n}{1^{n/1} (1+a)^{n/1}} \right\}
 \end{aligned}$$

$$\begin{aligned}
 1) \int \frac{dx}{\cos. x} &= 0 \quad \text{V. T. 19. N}^\circ. 13. \\
 2) \int \frac{\sin. x}{\cos. x^2} dx &= \infty \quad \text{Schlömlehl, Int. 21.} \\
 3) \int \sin. x^{2a+1} \frac{dx}{\text{Tang. } \frac{1}{2} x} &= (-1)^a \left(-\frac{1}{a} \right) \quad \text{Raabe, Cr. 25. 160.}
 \end{aligned}$$

$$\begin{aligned}
 1) \int \frac{dx}{2 - \sin. x} &= \frac{4\pi}{3\sqrt{3}} \quad \text{V. T. 65. N}^\circ. 3. \\
 2) \int \frac{dx}{2 + \sin. x} &= \frac{2\pi}{3\sqrt{3}} \quad \text{V. T. 65. N}^\circ. 4. \\
 3) \int \frac{dx}{1 - \sin. x \cdot \cos. \frac{2a\pi}{b}} &= \frac{1}{b} \text{Cosec. } \frac{2a\pi}{b} \sum_1^{b-1} \sin. \frac{2n\pi}{b} \cdot \text{Cot. } \frac{n\pi}{b} \quad \text{V. T. 25. N}^\circ. 14. \\
 4) \int \frac{dx}{1 - \sin. x \cdot \cos. \lambda} &= 2(\pi - \lambda) \text{Cosec. } \lambda \quad \text{V. T. 25. N}^\circ. 2. \\
 5) \int \frac{dx}{1 + \sin. x \cdot \cos. \lambda} &= 2\lambda \text{Cosec. } \lambda \quad \text{V. T. 25. N}^\circ. 3. \\
 6) \int \frac{dx}{p + q \cos. x} &= \frac{\pi}{\sqrt{(p^2 - q^2)}}, p^2 > q^2; \quad \text{Euler, Calc. Int. T. 4. S. 6. 22. — Schlömlehl, Cr. 33. 268. — Ramus, Danske Afh. 6. 265. — Björ-} \\
 &\quad \text{ling, Gr. 21. 26.}
 \end{aligned}$$

- 7) $\int \frac{dx}{p + q \cos x} = 0$ (valeur princ.) , $p^2 < q^2$;
- 8) $\int \frac{dx}{-p + q \cos x} = \frac{-\pi}{\sqrt{p^2 - q^2}}$, $p^2 > q^2$;
- 9) $= 0$ (valeur princ.) , $p^2 < q^2$;
- 10) $\int \frac{\sin^a x}{p + q \cos x} dx = \frac{2\sqrt{\pi}}{a(p^2 - q^2)^{\frac{a+1}{2}}} \frac{\Gamma\left(\frac{a+1}{2}\right)}{\Gamma\left(\frac{a}{2}\right)}$ Lobatschewsky, Mém. Kasan. 1835. 1.
- 11) $\int \frac{\cos ax}{1 + p \cos x} dx = \frac{\pi}{\sqrt{1-p^2}} \left\{ \frac{\sqrt{1-p^2}-1}{p} \right\}^a$, $p < 1$;
- 12) $\int \frac{\cos ax}{1 - p \cos x} dx = \frac{\pi}{\sqrt{1-p^2}} \left\{ \frac{1-\sqrt{1-p^2}}{p} \right\}^a$ Ohm, Ausw. 26.
- 13) $\int \frac{dx}{a - b \cos x} = \frac{\pi}{\sqrt{a^2 + b^2}}$, $1 > p > 0$; Bonnet, L. 17. 265.
- 14) $\int \frac{dx}{(1 + \sin \lambda \cos x)^{a+1}} = \frac{1^{a/2}}{1^{a/1}} \pi \sum_0^\infty \left\{ (-1)^n \frac{(n+1)^{n/1}}{(2a-1)^{n/2-2}} \binom{a}{2n} \frac{1}{2^n} \sec^{2(a-n)+1} \lambda \right\}$
- 15) $\int \frac{\cos ax}{(1 + \sin \lambda \cos x)^{a+1}} dx = \frac{1^{a/2}}{1^{a/1}} \frac{(-1)^a \pi}{\sin^{a+1} \lambda} \sum_0^\infty \left\{ (-1)^n \frac{(n+1)^{n/1}}{(2a-1)^{n/2-2}} \binom{a}{2n} \frac{1}{2^n} \tan^{2(a-n)+1} \lambda \right\}$
- 16) $\int \frac{\sin^a x}{(p + q \cos x)^{a+1}} dx = \frac{1^{a-1/1}}{2^{a-2} (p^2 - q^2)^{\frac{a+1}{2}}} \frac{\pi}{\left\{ \Gamma\left(\frac{a}{2}\right) \right\}^2 a}$ Lobatschewsky, Mém. Kasan. 1835. 1.
- 17) $= \frac{\Gamma\left(\frac{a+1}{2}\right) \sqrt{\pi}}{(p^2 - q^2)^{\frac{a+1}{2}} \Gamma\left(\frac{1}{2}a + 1\right)}$
- 18) $\int \frac{\sin^a x}{(p + q \cos x)^b} dx = \frac{\Gamma\left(\frac{a+1}{2}\right) \sqrt{\pi}}{p^b \Gamma\left(\frac{a}{2} + 1\right)} \sum_0^\infty \frac{\left(\frac{1}{2}n - \frac{1}{2}\right)^{n/1-1} \left(\frac{1}{2}n\right)^{n/1}}{\left(\frac{1}{2}a + 1\right)^{n/1} 1^{n/1}} \left(\frac{q}{p}\right)^{2n}$
- 19) $= \frac{\left\{ \Gamma\left(\frac{a+1}{2}\right) \right\}^3}{1^{a/1}} \frac{a+1}{(p^2 - q^2)^{\frac{a+1}{2}}}$ Schlömilch, Häh. Anal. 85.
- 20) $\int \frac{dx}{1 \pm \sin x \cos x \sqrt{3}} = 2\pi$ V. T. 30. N°. 15.

- 1) $\int \frac{\sin x}{1 + \cos^2 x} dx = -\frac{1}{2} \pi$ Grunert, Gr. 4. 113.
- 2) $\int \frac{dx}{4 - \sin^2 x} = \frac{\pi}{2\sqrt{3}}$ V. T. 66. N^o. 6.
- 3) $\int \frac{\sin x}{4 - \sin^2 x} dx = \frac{\pi}{3\sqrt{3}}$ V. T. 66. N^o. 7.
- 4) $\int \frac{dx}{4 - 3 \sin^2 x} = \frac{1}{2} \pi$ V. T. 30. N^o. 16.
- 5) $\int \frac{dx}{1 - \cos^2 \lambda \sin^2 x} = \pi \operatorname{Cosec} \lambda$ V. T. 82. N^o. 4. 5.
- 6) $\int \frac{\sin x}{1 - \cos^2 \lambda \sin^2 x} dx = 2(\pi - 2\lambda) \operatorname{Cosec} 2\lambda$ V. T. 82. N^o. 4. 5.
- 7) $\int \frac{dx}{p^2 \sin^2 x + q^2 \cos^2 x} = \frac{\pi}{pq}$ Grunert, Cr. 8. 146. — Lobatto, Cr. 11. 169.
- 8) $\int \frac{dx}{(p^2 \sin^2 x + q^2 \cos^2 x)^2} = \frac{p^2 + q^2}{2p^3 q^3} \pi$ V. T. 83. N^o. 7. 9.
- 9) $\int \frac{\sin^2 x}{(p^2 \sin^2 x + q^2 \cos^2 x)^2} dx = \frac{\pi}{2p^3 q}$ Grunert, Cr. 8. 146.
- 10) $\int \frac{\cos^2 x}{(p^2 \sin^2 x + q^2 \cos^2 x)^2} dx = \frac{\pi}{2pq^3}$ V. T. 83. N^o. 7. 9.
- 11) $\int \frac{\cos 2x}{(p^2 \sin^2 x + q^2 \cos^2 x)^2} dx = \frac{p^2 - q^2}{2p^3 q^3} \pi$ V. T. 82. N^o. 9. 10.
- 12) $\int \frac{\cot^q x}{1 - (2p - p^2) \cos^2 x} dx = \frac{1}{(1-p)^{q+1}} \frac{\pi}{2} \operatorname{Sec} \frac{1}{2} q \pi, q^2 < 1; \left. \vphantom{\int} \right\} p^2 < 1;$
- 13) $\int \frac{\operatorname{Tang}^{1-q} x}{1 - (2p - p^2) \cos^2 x} dx = \frac{1}{(1-p)^q} \frac{\pi}{2} \operatorname{Cosec} \frac{1}{2} q \pi, 0 < q < 2; \left. \vphantom{\int} \right\} \operatorname{Meyer, Int. Déf. 376.}$
- 14) $\int \frac{\cos ax}{1 - p^2 \sin^2 \frac{1}{2} x} dx = \frac{(-1)^a \pi}{\sqrt{1-p^2}} \left\{ \frac{1 - \sqrt{1-p^2}}{p} \right\}^{2a}$ Plana, Mém. Turin. 1820. 389. N^o 4.

- 1) $\int \frac{dx}{1+p^2-2p \cos. x} = \frac{\pi}{1-p^2}, p^2 < 1;$ Euler, Calc. Int. 4. S. 4. 22. — Schlömilch, Beitr. II. § 1.
- 2) $= \frac{\pi}{p^2-1}, p^2 > 1;$
- 3) $\int \frac{\cos. ax}{1+p^2-2p \cos. x} dx = \frac{\pi p^a}{1-p^2}, p^2 < 1;$ Euler, Calc. Int. 4. S. 4. 22, 45. — Legendre, Exerc. 3. 61. — Poisson, P. 19. 404. N°. 75. — Plana, Mém. Turin. 1817. 7. Art. 2, 14. — Schlömilch, Beitr. II. § 1.
- 4) $= \frac{\pi p^{-a}}{p^2-1}, p^2 > 1;$
- 5) $\int \frac{\sin. ax. \sin. x}{1+p^2-2p \cos. x} dx = \frac{1}{2} \pi p^{a-1}, p^2 < 1;$ Poisson, P. 19. 404. N°. 95. — Schlömilch, Beitr. II. § 1.
- 6) $= \frac{1}{2} \pi \frac{1}{p^{1+a}}, p^2 > 1;$
- 7) $\int \frac{\cos. ax. \cos. x}{1+p^2-2p \cos. x} dx = \frac{\pi}{2} \frac{1+p^2}{1-p^2} p^{a-1}, p^2 < 1;$ Bierens de Haan, Gr. 13. 193.
- 8) $= \frac{\pi}{2} \frac{p^2+1}{p^2-1} \frac{1}{p^{a+1}}, p^2 > 1;$
- 9) $\int \frac{dx}{1+p^2+2p \cos. x} = \frac{\pi}{1-p^2} \left\{ , p < 1; \right.$
- 10) $\int \frac{\cos. x}{1+p^2+2p \cos. x} dx = \frac{p \pi}{p^2-1} \left\{ \right. \text{Raabe, Int. 161.}$
- 11) $\int \frac{\cos. kx}{1+p^2-2p \cos. x} dx = 0 \left\{ , \text{pour } k = \infty ; \right.$
- 12) $\int \frac{\sin. kx. \text{Tang. } x}{1+p^2-2p \cos. x} dx = \infty \left\{ \begin{array}{l} \text{Meyer, Int. Déf. 220; il trouve pour 12) faut.} = 0; \text{ encore} \\ k \text{ y doit être de la forme } 2k+1. \end{array} \right.$
- 13) $\int \frac{dx}{1-p \cos. \lambda - p i \sin. \lambda. \cos. x} = \frac{\pi}{\sqrt{(1-2p \cos. \lambda + p^2)}}, 0 < p < 1; \text{ Bonnet, L. 17. 265.}$
- 14) $\int \frac{\sin. 2ax. \sin. x}{1+p^2-2p \cos. 2x} dx = 0, p^2 < 1 \text{ et } p^2 > 1;$
- 15) $\int \frac{\sin. \{(2a-1)x\}. \sin. x}{1+p^2-2p \cos. 2x} dx = \frac{\pi}{2} \frac{p^a}{1+p}, p^2 < 1;$ Bierens de Haan, Gr. 13. 193.
- 16) $= \frac{\pi}{2} \frac{p^{-a}}{1+p}, p^2 > 1;$

$$\left. \begin{aligned} 17) \int \frac{\sin. \{(2a-1)x\} \cdot \sin. 2x}{1+p^2-2p \cos. 2x} dx &= 0, \\ 18) \int \frac{\cos. \{(2a-1)x\}}{1+p^2-2p \cos. 2x} dx &= 0 \\ 19) \int \frac{\cos. 2ax \cdot \cos. x}{1+p^2-2p \cos. 2x} dx &= 0 \\ 20) \int \frac{\cos. \{(2a-1)x\} \cdot \cos. x}{1+p^2-2p \cos. 2x} dx &= \frac{\pi}{2} \frac{p^a}{1-p}, p^2 < 1; \\ 21) &= \frac{\pi}{2} \frac{p^{-a}}{p-1}, p^2 > 1; \\ 22) \int \frac{\cos. \{(2a-1)x\} \cdot \cos. 2x}{1+p^2-2p \cos. 2x} dx &= 0, p^2 < 1 \text{ et } p^2 > 1; \end{aligned} \right\}$$

Bierens de Haan, Gr. 13. 193.

$$23) \int \frac{1-p \cos. 2x}{1+p^2-2p \cos. 2x} dx = \pi \quad \text{Smaasen, Cr. 42. 222.}$$

$$24) \int \frac{\cos. x \cos. \{(2c+1)x\}}{1+(a \sin. x + b)^2} dx = \frac{\pi}{a} \cos. \left\{ (2c+1) \operatorname{Arctg}. 1 \sqrt{\frac{q}{2}} \right\} \cdot \operatorname{Tang}.^{2c+1} \left\{ \frac{1}{2} \operatorname{Arccos}. 1 \sqrt{\frac{q}{2b^2}} \right\}$$

$$25) \int \frac{\cos. x \cdot \cos. 2cx}{1+(a \sin. x + b)^2} dx = -\frac{\pi}{a} \sin. \left\{ 2c \operatorname{Arctg}. 1 \sqrt{\frac{q}{2}} \right\} \cdot \operatorname{Tang}.^{2c} \left\{ \frac{1}{2} \operatorname{Arccos}. 1 \sqrt{\frac{q}{2b^2}} \right\}$$

De ces deux formules, où $q = -(1+a^2-b^2) + \sqrt{(1+a^2-b^2)^2 + 4b^2}$, voyez: Legendre, Exerc. 5. 121.

$$26) \int \frac{1-p^b \cos. bx}{1-2p^b \cos. bx + p^{2b}} \cos.^{\frac{c}{2}} x \cdot \cos.^{\frac{1}{2}} cx dx = \frac{\pi}{2^{c+1}} \sum_1^{\infty} \binom{c}{nb} p^{nb} \quad \text{Smaasen, Cr. 42. 222.}$$

$$1) \int \frac{\sin. x}{1+p^2-2p \cos. x} \frac{dx}{\operatorname{Tang}.^{\frac{1}{2}} x} = \frac{\pi}{1-p}, p^2 < 1;$$

$$2) = \frac{\pi}{p-1}, p^2 > 1;$$

$$3) \int \frac{\sin. x \cdot \operatorname{Tang}.^{\frac{1}{2}} x}{1+p^2-2p \cos. x} dx = \frac{\pi}{1+p}, p^2 < 1 \text{ et } p^2 > 1;$$

$$4) \int \frac{1}{1+p^2-2p \cos. x} \frac{dx}{\cos. x} = \infty, p^2 < 1;$$

$$5) = \infty, p^2 > 1;$$

Schlömilch, Beitr. II. § 1.
Il trouve faut.:

pour 4) $\frac{2\pi p}{1-p^4}, p^2 < 1;$

pour 5) $\frac{2\pi p}{p^4-1}, p^2 > 1.$

$$\begin{aligned}
 6) \int \frac{dx}{(1+p^2-2p \cos. x)^2} &= \frac{1+p^2}{(1-p^2)^3} \pi, p^2 < 1; \\
 7) &= \frac{1+p^2}{(p^2-1)^3} \pi, p^2 > 1; \\
 8) \int \frac{dx}{(1+p^2-2p \cos. x)^3} &= \frac{1+4p^2+p^4}{(1-p^2)^5} \pi, p^2 < 1; \\
 9) &= \frac{1+4p^2+p^4}{(p^2-1)^5} \pi, p^2 > 1; \\
 10) \int \frac{dx}{(1+p^2-2p \cos. x)^4} &= \frac{1+9p^2+9p^4+p^6}{(1-p^2)^7}, p^2 < 1; \\
 11) &= \frac{1+9p^2+9p^4+p^6}{(p^2-1)^7}, p^2 > 1; \\
 12) \int \frac{dx}{(1+p^2-2p \cos. x)^{c+1}} &= \frac{\pi}{(1-p^2)^{2c+1}} \sum_{n=0}^c \binom{c}{n}^2 a^{2n}, p^2 < 1; \\
 13) &= \frac{\pi}{(p^2-1)^{2c+1}} \sum_{n=0}^c \binom{c}{n}^2 a^{2n}, p^2 > 1; \\
 14) \int \frac{\cos. ax}{(1+p^2-2p \cos. x)^2} dx &= \frac{\pi p^a}{(1-p^2)^3} \{a+1-(a-1)p^2\}, p^2 < 1; \\
 15) &= \frac{\pi p^{-a}}{(p^2-1)^3} \{(a+1)p^2-(a-1)\}, p^2 > 1; \\
 16) \int \frac{\cos. ax}{(1+p^2-2p \cos. x)^3} dx &= \binom{a+2}{2} \frac{\pi p^a}{(1-p^2)^5} \left\{ 1 - \frac{a-2}{a+1} 2p^2 + \frac{a-2}{a+1} \frac{a-1}{a+2} p^4 \right\}, p^2 < 1; \\
 17) &= \binom{a+2}{2} \frac{\pi p^{-a}}{(p^2-1)^5} \left\{ p^4 - \frac{a-2}{a+1} 2p^2 + \frac{a-2}{a+1} \frac{a-1}{a+2} p^6 \right\}, p^2 > 1; \\
 18) \int \frac{\cos. ax}{(1+p^2-2p \cos. x)^4} dx &= \binom{a+3}{3} \frac{\pi p^a}{(1-p^2)^7} \\
 &\quad \left\{ 1 - \frac{a-3}{a+1} 3p^2 + \frac{a-3}{a+1} \frac{a-2}{a+2} 3p^4 - \frac{a-3}{a+1} \frac{a-2}{a+2} \frac{a-1}{a+3} p^6 \right\}, p^2 < 1;
 \end{aligned}$$

Euler, Calc. Int. 4. S. 4. 22.

Euler, Calc. Int. 4. S. 4. 31, 67. — Legendre, Exerc. 3. 62.

Euler, Calc. Int. 4. S. 4. 22, 50. — Legendre, Exerc. 3. 62.

Euler, Calc. Int. 4. S. 4. 22.

$$\begin{aligned}
 19) \int \frac{\cos. ax}{(1+p^2-2p \cos. x)^3} dx &= \binom{a+3}{3} \frac{\pi p^{-a}}{(p^2-1)^7} \\
 &\left\{ p^6 - \frac{a-3}{a+1} 3p^4 + \frac{a-3}{a+1} \frac{a-2}{a+2} 3p^2 - \frac{a-3}{a+1} \frac{a-2}{a+2} \frac{a-1}{a+3} \right\}, p^2 > 1; \left. \begin{array}{l} \text{Euler, Calc. Int. 4.} \\ \text{S. 4. 22.} \end{array} \right\} \\
 20) \int \frac{\cos. ax}{(1+p^2-2p \cos. x)^{c+1}} dx &= \binom{a+c}{c} \frac{\pi p^a}{(1-p^2)^{2c+1}} \\
 &\left\{ 1 - \binom{c}{1} \frac{a-c}{a+1} p^2 + \binom{c}{2} \frac{a-c}{a+1} \frac{a-c+1}{a+2} p^4 - \dots \right\}, p^2 < 1; \left. \begin{array}{l} \text{Euler, Calc. Int.} \\ \text{T. 4. S. 4. 81. —} \\ \text{Legendre, Exerc.} \\ \text{3. 62.} \end{array} \right\} \\
 21) \int \frac{\cos. ax}{(1+p^2-2p \cos. x)^{c+1}} dx &= \binom{a+c}{c} \frac{\pi p^{-a}}{(p^2-1)^{2c+1}} \\
 &\left\{ p^{2c} - \binom{c}{1} \frac{a-c}{a+1} p^{2c-2} + \binom{c}{2} \frac{a-c}{a+1} \frac{a-c+1}{a+2} p^{2c-4} - \dots \right\}, p^2 > 1; \\
 22) \int \frac{\cos. ax}{(1+p^2-2p \cos. x)^{a+1}} dx &= \frac{\pi p^a}{(1-p^2)^{2a+1}} \frac{(a+1)^{a/1}}{1^{a/1}}, p^2 < 1; \left. \begin{array}{l} \text{Euler, Calc. Int. T. 4.} \\ \text{S. 4. 30. — Legendre,} \\ \text{Exerc. 3. 62.} \end{array} \right\} \\
 23) &= \frac{\pi p^a}{(p^2-1)^{2a+1}} \frac{(a+1)^{a/1}}{1^{a/1}}, p^2 > 1; \\
 24) \int \frac{\sin.^{2a} x}{(1-p^2 \cos. x + p^2)^{2a}} dx &= \frac{1^{2a/2}}{2^{2a/2}} \pi, p^2 < 1; \\
 25) &= \frac{1^{2a/2}}{2^{2a/2}} \frac{\pi}{p^{2a}}, p^2 > 1; \left. \begin{array}{l} \text{de Morgan, Definite Integrals. (Enc. Me-} \\ \text{trop.).} \end{array} \right\} \\
 26) \int \frac{\cos. ax}{(1+p-2 \cos. x \sqrt{p})^b} dx &= \frac{\pi}{1^{b-1/1} p^{1/2}} \frac{d^{b-1}}{dp^{b-1}} \cdot \frac{p^{a+b-1}}{(1-p)^b} \quad \text{Boole, Phil. Trans. 1844.} \\
 27) \int \frac{1}{1+p^2-2p \cos. x} \frac{dx}{1+q^2-2q \cos. x} &= \frac{\pi}{(1-p^2)(1-q^2)} \frac{1+pq}{1-pq}, p^2 < 1, q^2 < 1; \\
 28) &= \frac{\pi}{(p^2-1)(q^2-1)} \frac{pq+1}{pq-1}, p^2 > 1, q^2 > 1; \left. \begin{array}{l} \text{Schlöm-} \\ \text{milch,} \\ \text{Beitr.} \\ \text{II. 2.} \end{array} \right\} \\
 29) \int \frac{\sin.^2 x}{1+p^2-2p \cos. x} \frac{dx}{1+q^2-2q \cos. x} &= \frac{\pi}{2} \frac{1}{1-pq}, p^2 < 1, q^2 < 1; \\
 30) &= \frac{\pi}{2} \frac{1}{pq-1}, p^2 > 1, q^2 > 1;
 \end{aligned}$$

$$31) \int \frac{\cos. a x}{(1-2p_1 \cos. x + p_1^2)^l (1-2p_2 \cos. x + p_2^2)^m \dots (\text{à } h \text{ facteurs})} dx = \frac{\pi}{\Gamma(l)\Gamma(m)\dots} \frac{d^{l-1}}{dy_1^{l-1}} \frac{d^{m-1}}{dy_2^{m-1}} \dots$$

$$\frac{y_1^{l-1} y_2^{m-1} \dots}{(1-y_1)^l (1-y_2)^m \dots} \times \left\{ Y_1 \left(\frac{y_1}{p_1} \right)^{h+a-1} + Y_2 \left(\frac{y_2}{p_2} \right)^{h+a-1} + \dots \right\}$$

$$, \text{ où les fonctions } Y_q = \frac{\left(1 - \frac{y_1}{p_1}\right)^2 \left(1 - \frac{y_2}{p_2}\right)^2 \dots \left(1 - \frac{y_h}{p_h}\right)^2}{\left(1 - \frac{y_q}{p_q}\right)^2 \times \left(\frac{y_q}{p_q} - \frac{y_1}{p_1}\right) \left(\frac{y_q}{p_q} - \frac{y_2}{p_2}\right) \dots \left(\frac{y_q}{p_q} - \frac{y_h}{p_h}\right)}$$

Après la différentiation mettez $p_1^2, p_2^2, \dots, p_h^2$ au lieu de $y_1, y_2, \dots, y_h$.

Voyez de cette intégrale: Boole, Phil. Trans. 1844.

$$32) \int \frac{\cos. 2 k x}{1 - 2 p \cos. x + p^2} \frac{dx}{\cos. x} = 0, \text{ pour } k = \infty; \text{ Schlömilch. Beitr. II. § 1.}$$

$$1) \int \frac{\cos^2. \frac{1}{2} x}{\sqrt{1 - p^2 \cos^2. x}} dx = F(p) \text{ Raabe, Cr. 25. 160.}$$

$$2) \int \frac{\sin. x}{\sqrt{1 - \cos. x}} dx = \sqrt{2}$$

$$3) \int \frac{\sin. x}{\sqrt{1 + p^2 - 2 p \cos. x}} dx = 2, \quad p^2 < 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Poisson, Mém. Ac. 1823. 571. N°. 12.}$$

$$4) \quad = \frac{2}{p}, \quad p^2 > 1; \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$5) \int \frac{\cos. x}{\sqrt{1 + p^2 - 2 p \cos. x}} dx = \frac{2}{p} \{F'(p) - E'(p)\}, p < 1; \text{ Ramus, Danske Afh. 6. 365.}$$

$$6) \int \frac{\cos. a x}{\sqrt{1 + p^2 - 2 p \cos. x}} dx = \frac{1^{a/2}}{2^{a/2}} \pi p^a \sum_1 \frac{1^{n/2} (2a+1)^{n/2}}{2^{n/2} (2a+2)^{n/2}} p^{2n}, \quad p < 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Schlömilch,}$$

$$7) \quad = \frac{1^{a/2}}{2^{a/2}} \frac{\pi p^a}{\sqrt{1-p^2}} \sum_0 \frac{(1^{n/2})^2}{2^{n/2} (2a+2)^{n/2}} \left(\frac{p^2}{p^2-1}\right)^n, p < \sqrt{\frac{1}{2}}; \left. \begin{array}{l} \text{Stud.} \\ \text{II. 7.} \end{array} \right\}$$

$$8) \int \frac{\sin. x}{\sqrt{1 + p^2 - 2 p \cos. x}^3} dx = \frac{2}{1-p^2}, \quad p^2 < 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Poisson, P. 19. 145. N°. 4. — Id.,}$$

$$9) \int \quad = \frac{2}{p(p^2-1)}, \quad p^2 > 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Chaleur. 107, 178.}$$

$$10) \int \frac{\sin. x}{\sqrt{(1+p^2-2p \cos. x)^3}} dx = \frac{2}{p^2-1}, p^2 = 1; \text{ Meyer, Int. Déf. 279.}$$

$$11) \int \frac{p - \cos. x}{\sqrt{(1+p^2-2p \cos. x)^3}} \sin. x dx = 0, p^2 < 1; \left. \begin{array}{l} \\ \\ 12) \end{array} \right\} = \frac{2}{p^2}, p^2 > 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Poisson, Mém. Ac. 1823. 571. N°. 12.}$$

$$13) \int \frac{dx}{\sqrt{(1+p^2-2p \cos. 2x)}} = \frac{2}{1+p} E' \left(\frac{2 \sqrt{p}}{1+p} \right) \text{Smaasen, Cr. 42. 222.}$$

$$14) \int \frac{dx}{\sqrt{(p^2-q^2 \cos. x)^3}} = \frac{2}{\sqrt{(p^2+q)}} \frac{1}{p^2 q} E' \left(\frac{2q}{p^2+q} \right) \left. \begin{array}{l} \\ \\ 15) \end{array} \right\} \begin{array}{l} \text{Plana,} \\ \text{Mém. Tu-} \\ \text{rin. 1820.} \\ \text{389.} \end{array}$$

$$15) \int \frac{\cos. x}{\sqrt{(p^2-q^2 \cos. x)^3}} dx = \frac{2}{q \sqrt{(p^2+q)}} \left\{ -E' \left(\frac{2q}{p^2+q} \right) + \frac{p^2}{p^2-q} E' \left(\frac{2q}{p^2+q} \right) \right\}$$

$$16) \int \frac{\sin. 2^a x}{\sqrt{(1-p^2 \sin. 2^a x)}} dx = \frac{1^{a/2}}{2^{a/2}} \pi \sum_1 \frac{1^{n/2} (2a+1)^{n-2}}{2^{n/2} (2a+2)^{n-2}} p^{2n}, p < 1; \left. \begin{array}{l} \\ \\ 17) \end{array} \right\} \begin{array}{l} \text{Schlömleli,} \\ \text{Stud. II. 7.} \end{array}$$

$$17) = \frac{1^{a/2}}{2^{a/2}} \frac{\pi}{\sqrt{(1-p^2)}} \sum_0 \frac{(1^{n/2})^2}{2^{n/2} (2a+2)^{n-2}} \left(\frac{p^2}{p^2-1} \right)^n, p < 1 \frac{1}{2};$$

$$\left. \begin{array}{l} 1) \int \sin. 2^{a+1} x dx = 0 \\ 2) \int \sin. 2^{a+1} x \cos. bx dx = 0 \\ 3) \int \sin. 2^a x \cos. x dx = 0 \\ 4) \int (\cos. x - \cos. ax) dx = 0 \\ 5) \int (\cos. x - \cos. ax) \cos. x dx = \pi \\ 6) \int (\cos. x - \cos. ax) \cos. ax dx = -\pi \end{array} \right\} \text{Raabe, Cr. 23. 105.}$$

$$7) \int (\cos. x - \cos. ax) \cos. bx \, dx = 0, \text{ où } b > 1 \text{ n'est pas facteur de } a. \text{ Raabe; Cr. 23. 105.}$$

$$8) \int \sin.^{2a} x \, dx = \frac{1^{a/2}}{2^{a/2}} 2\pi \quad \text{Schubert, Samml. 118.}$$

$$9) \int \cos. \{a(x - q \sin. x)\} \cdot \cos. x \, dx = \frac{2\pi (\frac{1}{2} a q)^a}{q \cdot 1^{a/1}} \left\{ 1 + \sum_1^{\infty} (-1)^n \frac{(\frac{1}{2} a q)^{2n}}{1^{n/1} (1+a)^{n/1}} \right\}$$

$$10) \int \cos.(ax - p \cos. x - q \sin. x) dx = 2\pi \cos. \left(a \operatorname{Arctg.} \frac{q}{p} \right) \frac{(p^2 + q^2)^{\frac{1}{2}a}}{2^a \cdot 1^{a/1}} \left\{ 1 + \sum_1^{\infty} \frac{(-1)^n}{1^{n/1} (1+a)^{n/1}} \left(\frac{p^2 + q^2}{4} \right)^n \right\}$$

$$11) \int \cos.(p \cos. x + q \sin. x) \cdot \cos. 2ax \, dx = 2\pi \cos. \left(2a \operatorname{Arctg.} \frac{q}{p} \right) \frac{(p^2 + q^2)^a}{2^{2a} \cdot 1^{2a/1}} \left\{ 1 + \sum_1^{\infty} (-1)^n \frac{(p^2 + q^2)^n}{2^{2n} \cdot 1^{n/1} (1+2a)^{n/1}} \right\}$$

$$12) \int \cos. (p \cos. x + q \sin. x) \cdot \cos. \{(2a+1)x\} \, dx = 0$$

$$13) \int \sin. (p \cos. x + q \sin. x) \cdot \sin. 2ax \, dx = 0$$

$$14) \int \sin.(p \cos. x + q \sin. x) \cdot \sin. \{(2a-1)x\} \, dx = 2\pi \cos. \left\{ (2a-1) \operatorname{Arctg.} \frac{q}{p} \right\} \frac{(p^2 + q^2)^{2a-1}}{2^{2a-1} \cdot 1^{2a-1/1}} \left\{ 1 + \sum_1^{\infty} (-1)^n \frac{(p^2 + q^2)^n}{2^{2n} \cdot 1^{n/1} (2a)^{n/1}} \right\}$$

$$15) \int \cos. (p \sin. x) \cdot \cos. 2^a x \, dx = \frac{1^{a/2}}{2^{a-1} \cdot 1^{a/1}} \left\{ 1 + \sum_1^{\infty} \frac{(-1)^n}{1^{n/1} (a+1)^{n/1}} \left(\frac{p}{2} \right)^{2n} \right\}$$

Sur les Integrales 9 à 15 voyez: Bessel, Abhandl. Berlin. 1824. 1.

$$1) \int \sin.^{2a+1} x \frac{dx}{\operatorname{Tang.} \frac{1}{2} x} = \frac{1^{a/2}}{2^{a/2}} 2\pi \quad \text{Raabe, Cr. 23. 105.}$$

$$2) \int \frac{\sin. ax \cdot \sin. x}{1 + \cos. x} \, dx = (-1)^a 2\pi \quad \text{V. T. 356. N°. 5.}$$

$$3) \int \frac{\sin. ax}{1 - p \cos. x} \, dx = 0 \quad \left\{ \begin{array}{l} p < 1; \\ \end{array} \right.$$

$$4) \int \frac{\cos. ax}{1 - p \cos. x} \, dx = \frac{2\pi}{1 \swarrow (1-p^2)} \left\{ \frac{1 - \swarrow (1-p^2)}{p} \right\}^a \quad \left. \begin{array}{l} \text{Raabe, Int. 172. — Ohm, Ausw. 26.} \end{array} \right\}$$

$$5) \int \frac{\sin. ax}{1 + p \cos. x} \, dx = 0, \quad p < 1; \quad \text{Ohm, Ausw. 26.}$$

$$6) \int \frac{\cos. ax}{1 + p \cos. x} dx = \frac{(-1)^a 2\pi}{\sqrt{1-p^2}} \left\{ \frac{1 - \sqrt{1-p^2}}{p} \right\}^a, p < 1; \quad \text{Ohm, Ausw. 26.}$$

$$7) \int \frac{dx}{1 - \cos.^2 \lambda. \sin.^2 x} = 2\pi \operatorname{Cosec.} \lambda. \quad \text{V. T. 30. N° 17.}$$

$$8) \int \frac{\sin. x}{1 - \cos.^2 \lambda. \sin.^2 x} dx = 4(\pi - 2\lambda) \operatorname{Cosec.} 2\lambda \quad \text{V. T. 30. N° 17.}$$

$$\left. \begin{aligned} 9) \int \frac{dx}{p + q \cos. x} &= \frac{2\pi}{\sqrt{p^2 - q^2}}, p^2 > q^2; \\ 10) &= 0 \text{ (val. princ.)}, p^2 < q^2; \\ 11) \int \frac{dx}{-p + q \cos. x} &= \frac{-2\pi}{\sqrt{p^2 - q^2}}, p^2 > q^2; \\ 12) &= 0 \text{ (val. princ.)}, p^2 < q^2; \end{aligned} \right\} \quad \text{Björling, Gr. 20. 26.}$$

$$13) \int \frac{\cos. x - \cos. ax}{\operatorname{Tang.} \frac{1}{2} x} dx = 0 \quad \text{Raabe, Cr. 23. 105.}$$

$$14) \int \frac{\sin. ax}{(1-p \cos. x)(1-q \cos. x)(1-r \cos. x) \dots} dx = 0, \text{ pour toute } p, q, r, \dots < 1; \quad \text{Raabe, Int. 172.}$$

$$\left. \begin{aligned} 1) \int \frac{dx}{1 + p^2 - 2p \cos. x} &= \frac{2\pi}{1 - p^2}, p < 1; \\ 2) &= \frac{2\pi}{p^2 - 1}, p > 1; \end{aligned} \right\} \quad \text{Bierens de Haan, Gr. 13. 193. — Ohm, Ausw. 26.}$$

$$\left. \begin{aligned} 3) \int \frac{\sin. x}{1 + p^2 - 2p \cos. x} dx &= 0 \\ 4) \int \frac{\sin. ax}{1 + p^2 - 2p \cos. x} dx &= 0 \end{aligned} \right\} \quad p < 1; \quad \text{Raabe, Int. 172.}$$

$$\left. \begin{aligned} 5) \int \frac{\cos. x}{1 + p^2 - 2p \cos. x} &= \frac{2\pi p}{1 - p^2}, p < 1; \\ 6) &= \frac{2\pi}{p^2 - 1}, p > 1; \end{aligned} \right\} \quad \text{Bierens de Haan, Gr. 13. 193.}$$

$$\begin{aligned}
7) \int \frac{\text{Cos. } ax}{1+p^2-2p\text{Cos. } x} dx &= \frac{2\pi p^a}{1-p^2}, p < 1; \\
8) &= \frac{2\pi p^{-a}}{p^2-1}, p > 1; \left. \vphantom{\int} \right\} \text{Raabe, Int. 172. — Bierens de Haan, Gr. 13. 193.} \\
9) \int \frac{\text{Sin. } ax \cdot \text{Sin. } x}{1+p^2-2p\text{Cos. } x} dx &= \pi p^{a-1}, p < 1; \\
10) &= \frac{\pi}{p^{a-1}}, p > 1; \left. \vphantom{\int} \right\} \text{Bierens de Haan, Gr. 13. 193.} \\
11) \int \frac{\text{Cos. } ax \cdot \text{Cos. } x}{1+p^2-2p\text{Cos. } x} dx &= \pi p^{a-1} \frac{1+p^2}{1-p^2}, p < 1; \\
12) &= \frac{\pi}{p^{a-1}} \frac{p^2+1}{p^2-1}, p > 1; \left. \vphantom{\int} \right\} \\
13) \int \frac{\text{Sin. } ax - p \text{Sin. } \{(a+1)x\}}{1+p^2-2p\text{Cos. } x} dx &= 0 \left. \vphantom{\int} \right\}, p < 1; \\
14) \int \frac{\text{Cos. } ax - p \text{Cos. } \{(a+1)x\}}{1+p^2-2p\text{Cos. } x} dx &= 2\pi p^a \left. \vphantom{\int} \right\} \text{Ohm, Ausw. 26. — Raabe, Int. 172.} \\
15) \int \frac{\text{Cos. } x}{e^a + e^{-a} - 2\text{Cos. } x} dx &= 2\pi a \frac{e^{-a}}{e^a - e^{-a}} \text{Poisson, P. 17. 612. N}^\circ \text{ 20.} \\
16) \int \frac{1 - p \text{Cos. } x + pi \text{Sin. } x}{1 - 2p\text{Cos. } x + p^2} &= 2\pi, p < 1; \text{Moigno, Calc. Int. 138.}
\end{aligned}$$

$$\begin{aligned}
1) \int \frac{dx}{a+b\text{Cos. } x+c\text{Sin. } x} &= \frac{2\pi}{\sqrt{a^2-b^2-c^2}}, a^2 > b^2+c^2; \left. \vphantom{\int} \right\} \text{Dienger, Gr. 12. 409.} \\
2) &= 0, a^2 < b^2+c^2; \\
3) &= 0 \text{ (val. princ.)}, a^2 < b^2+c^2; \\
4) &= \infty, a^2 = b^2+c^2; \\
5) \int \frac{dx}{-a+b\text{Cos. } x+c\text{Sin. } x} &= \frac{-2\pi}{\sqrt{a^2-b^2-c^2}}, a^2 > b^2+c^2; \left. \vphantom{\int} \right\} \text{Björling, Gr. 21. 26.} \\
6) &= 0 \text{ (val. princ.)}, a^2 < b^2+c^2; \\
7) &= -\infty, a^2 = b^2+c^2;
\end{aligned}$$

$$8) \int \frac{dx}{1 - p \cos x - p i \sin x} = 2\pi, p < 1; \text{ Moigno, Calc. Int. 138.}$$

$$9) \int \frac{dx}{a + bi \cos x + ci \sin x} = \frac{2\pi}{\sqrt{a^2 + b^2 + c^2}} \quad \text{Jacobi, L. 10. 229.}$$

$$10) \int \frac{dx}{1 - (p + qi) \cos x - (r + si) \sin x} = 0, (ps - qr)^2 > q^2 + s^2;$$

$$11) \int \frac{dx}{1 - (p + qi) \cos x - (r + si) \sin x} = \frac{2\pi}{\sqrt{1 - GH}}, (ps - qr)^2 < q^2 + s^2;$$

$$\left. \begin{aligned} 12) \int \frac{\cos x}{1 - (p + qi) \cos x - (r + si) \sin x} dx &= -\frac{2\pi}{G} \\ 13) \int \frac{\sin x}{1 - (p + qi) \cos x - (r + si) \sin x} dx &= \frac{2\pi i}{G} \end{aligned} \right\}, (ps - qr)^2 > q^2 + s^2;$$

$$14) \int \frac{\sin ax}{1 - (p + qi) \cos x - (r + si) \sin x} dx = \frac{\pi i}{\sqrt{1 - GH}} \frac{\{1 + \sqrt{1 - GH}\}^a - \{1 - \sqrt{1 - GH}\}^a}{G^a}, (ps - qr)^2 > q^2 + s^2;$$

$$15) \int \frac{\sin ax}{1 - (p + qi) \cos x - (r + si) \sin x} dx = \frac{\pi i}{\sqrt{1 - GH}} \frac{G^a - H^a}{\{1 + \sqrt{1 - GH}\}^a}, (ps - qr)^2 < q^2 + s^2;$$

$$16) \int \frac{\cos ax}{1 - (p + qi) \cos x - (r + si) \sin x} dx = \frac{-\pi}{\sqrt{1 - GH}} \frac{\{1 + \sqrt{1 - GH}\}^a - \{1 - \sqrt{1 - GH}\}^a}{G^a}, (ps - qr)^2 > q^2 + s^2;$$

$$17) \int \frac{\cos ax}{1 - (p + qi) \cos x - (r + si) \sin x} dx = \frac{\pi}{\sqrt{1 - GH}} \frac{G^a + H^a}{\{1 + \sqrt{1 - GH}\}^a}, (ps - qr)^2 < q^2 + s^2;$$

$$18) \int \frac{dx}{p + qi - (r + si) \cos x - (t + ui) \sin x} = 0, (ru - st)^2 > (ps - qr)^2 + (pu - qt)^2;$$

Dans les formules (10) à (18), trouvées par Jacobi. Cr. 32. 8, on a p, q, r, s réels, $(ps - qr)^2 \geq q^2 + s^2$, $ps - qr > 0$, a entier et > 0 , $G = p + s + (q - r)i$, $H = p - s + (q + r)i$, $\sqrt{1 - GH}$ positive.

$$19) \int \frac{dx}{(a + b \cos x + c \sin x)^2} = \frac{2a\pi}{\sqrt{a^2 - b^2 - c^2}^3}, a^2 > b^2 + c^2; \left. \begin{aligned} 20) \int \frac{dx}{(a + b \cos x + c \sin x)^2} &= 0, a^2 < b^2 + c^2; \end{aligned} \right\} \text{Dienger, Gr. 12. 409.}$$

$$\begin{aligned}
 21) \int \frac{dx}{(a + b \cos x + c \sin x)^3} &= \frac{2a^2 + b^2 + c^2}{\sqrt{(a^2 - b^2 - c^2)^5}} \frac{\pi}{2}, a^2 > b^2 + c^2; \\
 22) &= 0, a^2 < b^2 + c^2; \\
 23) \int \frac{dx}{(a + b \cos x + c \sin x)^4} &= \frac{2a^2 + 3b^2 + 3c^2}{\sqrt{(a^2 - b^2 - c^2)^7}} \frac{\pi}{2}, a^2 > b^2 + c^2; \\
 24) &= 0, a^2 < b^2 + c^2; \\
 25) \int \frac{dx}{(a + bi \cos x + ci \sin x)^2} &= \frac{2a\pi}{\sqrt{(a^2 + b^2 + c^2)^3}} \\
 26) \int \frac{dx}{(r - q \cos \lambda + qi \sin \lambda \cos x)^2} &= 2\pi \frac{r - q \cos \lambda}{\sqrt{(r^2 - 2rq \cos \lambda + q^2)}}
 \end{aligned}
 \left. \vphantom{\begin{aligned} 21) \\ 22) \\ 23) \\ 24) \\ 25) \\ 26) \end{aligned}} \right\} \text{Dienger, Gr. 12. 409.}$$

$$\begin{aligned}
 1) \int \frac{dx}{\sqrt{(a - b \cos x)}} &= \frac{4}{\sqrt{(a + b)}} F' \left(\sqrt{\frac{2b}{a + b}} \right) \\
 2) \int \frac{\cos x}{\sqrt{(a - b \cos x)}} dx &= \frac{4}{b \sqrt{(a + b)}} \left\{ a F' \left(\sqrt{\frac{2b}{a + b}} \right) - (a + b) E' \left(\sqrt{\frac{2b}{a + b}} \right) \right\} \\
 3) \int \frac{dx}{\sqrt{(a + b \cos x)}} &= \frac{4}{\sqrt{(a + b)}} F' \left(\sqrt{\frac{2b}{a + b}} \right) \\
 4) \int \frac{\cos x}{\sqrt{(a + b \cos x)}} dx &= \frac{4}{b \sqrt{(a + b)}} \left\{ (a + b) E' \left(\sqrt{\frac{2b}{a + b}} \right) - a F' \left(\sqrt{\frac{2b}{a + b}} \right) \right\} \\
 5) \int \frac{dx}{\sqrt{(a + b \cos x)^3}} &= \frac{4 \sqrt{(a + b)}}{a^2 - b^2} E' \left(\sqrt{\frac{2b}{a + b}} \right)
 \end{aligned}
 \left. \vphantom{\begin{aligned} 1) \\ 2) \\ 3) \\ 4) \\ 5) \end{aligned}} \right\} \text{Dienger, Gr. 13. 424.}$$

$$\begin{aligned}
 1) \int \frac{\text{Tang.}^{p-1} x - \text{Tang.}^{1-p} x}{\text{Cos. } 2x} dx &= \frac{1}{2} \pi \cot. \frac{1}{2} p \pi \quad \text{V. T. 5. N}^\circ 12. \\
 2) \int (\text{Tang.}^p x + \text{Cot.}^p x) (\text{Tang.}^q x + \text{Cot.}^q x) dx &= 2\pi \frac{\text{Cos. } \frac{1}{2} p \pi \cdot \text{Cos. } \frac{1}{2} q \pi}{\text{Cos. } p \pi + \text{Cos. } q \pi} \quad \text{V. T. 5. N}^\circ 10. \\
 3) \int (\text{Tang.}^p x - \text{Cot.}^p x) (\text{Tang.}^q x - \text{Cot.}^q x) dx &= 2\pi \frac{\text{Sin. } \frac{1}{2} p \pi \cdot \text{Sin. } \frac{1}{2} q \pi}{\text{Cos. } p \pi + \text{Cos. } q \pi} \quad \text{V. T. 5. N}^\circ 9.
 \end{aligned}$$

- 4) $\int (Tang.^p x - Cot.^p x) (Tang.^q x + Cot.^q x) \frac{dx}{Cos. 2x} = \frac{-\pi Sin. p \pi}{Cos. p \pi + Cos. q \pi} \quad V. T. 5. N^{\circ}. 15.$
- 5) $\int \frac{dx}{1 - Sin. x. Cos. x} = \frac{2\pi}{3 \sqrt{3}} \quad V. T. 7. N^{\circ}. 1.$
- 6) $\int \frac{dx}{1 + Sin. x. Cos. x} = \frac{\pi}{3 \sqrt{3}} \quad V. T. 7. N^{\circ}. 2.$
- 7) $\int \frac{Tang.^p x + Cot.^p x}{1 + Sin. 2x. Cos. \lambda} dx = \frac{\pi Sin. p \lambda}{Sin. p \pi. Sin. \lambda}, p < 1; \quad V. T. 7. N^{\circ}. 7.$
- 8) $\int \frac{dx}{1 - 3 Sin.^2 x. Cos.^2 x} = \frac{1}{2} \pi \quad V. T. 7. N^{\circ}. 19.$
- 9) $\int \frac{Sin.^{p-1} 2x}{(Cos. x + Sin. x)^{2p}} dx = \frac{1}{2^{p+1}} \frac{\Gamma(p) \Gamma(\frac{1}{2})}{\Gamma(p + \frac{1}{2})} \quad V. T. 4. N^{\circ}. 3.$
- 10) $\int \frac{1}{Tang.^p x + Cot.^p x} \frac{dx}{Sin. 2x} = \frac{\pi}{8p} \quad V. T. 5. N^{\circ}. 23.$
- 11) $\int \frac{Tang.^p x + Tang.^q x}{Tang.^{p+q} x + 1} \frac{dx}{Sin. 2x} = \frac{1}{2} \frac{\pi}{p+q} Sec. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\} \quad V. T. 31. N^{\circ}. 17.$
- 12) $\int \frac{Tang.^p x - Tang.^q x}{Tang.^{p+q} x - 1} \frac{dx}{Sin. 2x} = \frac{1}{2} \frac{\pi}{p+q} Tang. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\} \quad V. T. 31. N^{\circ}. 18.$
- 13) $\int (\sqrt{Tang. x} + \sqrt{Cot. x}) dx = \frac{1}{2} \pi \sqrt{2} \quad V. T. 15. N^{\circ}. 2.$

- 1) $\int Cos.^{b-1} x. Cos. a x dx = 2^{b-2} \frac{\Gamma\left(\frac{b-a}{2}\right) \Gamma\left(\frac{a+b}{2}\right) \Gamma(b)}{\Gamma(b-a) \Gamma(b+a)} \quad Serret, L. 8. 1.$
- 2) $\int \frac{\pi \Gamma(b)}{2^{b-1} \Gamma\left(\frac{a+b}{2} - 1\right) \Gamma\left(\frac{b-a}{2} - 1\right)} \left\{ \right. \quad Kummer, Cr. 17. 210.$
- 3) $\int Cos.^b x. Sin. a x dx = 0$

$$\left. \begin{aligned} 4) \int \cos.^b x \cdot \cos. \left(\frac{1}{2} a \pi - a x \right) dx &= \frac{\pi \Gamma(b+1) \cos. \frac{1}{2} a \pi}{2^b \Gamma\left(\frac{a+b}{2} + 1\right) \Gamma\left(\frac{b-a}{2} + 1\right)} \\ 5) \int \cos.^b x \cdot \sin. \left(\frac{1}{2} a \pi - a x \right) dx &= \frac{\pi \Gamma(b+1) \sin. \frac{1}{2} a \pi}{2^b \Gamma\left(\frac{a+b}{2} + 1\right) \Gamma\left(\frac{b-a}{2} + 1\right)} \end{aligned} \right\} \text{Kummer, Cr. 17. 210.}$$

$$6) \int \cos.^p x \cdot \cos. \{q(x-\lambda)\} dx = \frac{\pi \cos. p \lambda}{2^p} \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+q}{2} + 1\right) \Gamma\left(\frac{q-p}{2} + 1\right)} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211.}$$

$$7) \int \frac{dx}{1 \pm \sin. x \cdot \cos. x} = \frac{2\pi}{\sqrt{3}} \quad \text{V. T. 30. N°. 10, 11.}$$

$$8) \int \frac{dx}{1 \pm \sin. x \cdot \cos. x \sqrt{3}} = 2\pi \quad \text{V. T. 30. N°. 15.}$$

$$9) \int \frac{\sin.^2 x}{(p^2 \sin.^2 x + q^2 \cos.^2 x)^2} dx = \frac{\pi}{2 p^3 q} \quad \text{Grunert, Cr. S. 146.}$$

$$1) \int_{-\frac{1}{2}\pi}^0 \frac{\sin.^2 x}{(p^2 \sin.^2 x + q^2 \cos.^2 x)^2} dx = \frac{\pi}{4 p^3 q} \quad \text{Grunert, Cr. S. 146.}$$

$$2) \int_{-2\pi}^0 \frac{\sin. \left\{ \left(a + \frac{1}{2} \right) x \right\}}{\sin. \frac{1}{2} x} dx = 2\pi \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211.}$$

$$3) \int_0^{\frac{4}{3}\pi} \frac{\sin. x}{1 + \cos.^2 x} dx = \frac{3}{4} \pi - \text{Arctang. } \sqrt{2} \quad \text{Cauchy, Sav. Etr. 1827. 599. P. 2. § 2.}$$

$$4) \int_{-\pi}^{\pi} \cos.^a x \cdot \cos. \{b(x-\lambda)\} dx = \frac{\pi}{2^a} \frac{1 + (-1)^{a+b}}{\Gamma\left(\frac{a+b}{2} + 1\right) \Gamma\left(\frac{a-b}{2} + 1\right)} 1^{a/1} \cos. b \lambda \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211.}$$

$$5) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos. x dx \sqrt{1 - q^2 \cos.^2 x} = -\frac{1}{2} \left\{ 1 + \frac{q^2 - 1}{2q} \ln \frac{1-q}{1+q} \right\} \quad \text{Rognier, Material.}$$

$$6) \int_0^{2h\pi} \frac{dx}{a + b \cos. x + c \sin. x} = \frac{2h\pi}{\sqrt{a^2 - b^2 - c^2}}, \quad a^2 > b^2 + c^2; \quad \left\{ \begin{array}{l} \text{Dienger, Gr. 12. 409.} \\ 7) \int = 0, \quad a^2 < b^2 + c^2; \end{array} \right.$$

$$8) \int_{2k\pi}^{2(h+k)\pi} \frac{dx}{a + b \cos x + c \sin x} = \frac{2h\pi}{\sqrt{a^2 - b^2 - c^2}}, \quad a^2 > b^2 + c^2;$$

$$9) \quad = 0, \quad a^2 < b^2 + c^2;$$

$$10) \int_0^{2h\pi} \frac{dx}{(a + b \cos x + c \sin x)^2} = \frac{2ah\pi}{\sqrt{a^2 - b^2 - c^2}^3}, \quad a^2 > b^2 + c^2;$$

$$11) \quad = 0, \quad a^2 < b^2 + c^2;$$

$$12) \int_0^{2h\pi} \frac{dx}{(a + b \cos x + c \sin x)^3} = \frac{2a^2 + b^2 + c^2}{\sqrt{a^2 - b^2 - c^2}^5} h\pi, \quad a^2 > b^2 + c^2;$$

$$13) \quad = 0, \quad a^2 < b^2 + c^2;$$

$$14) \int_0^{2h\pi} \frac{dx}{(a + b \cos x + c \sin x)^4} = \frac{2a^2 + 3(b^2 + c^2)}{\sqrt{a^2 - b^2 - c^2}^7} h\pi, \quad a^2 > b^2 + c^2;$$

$$15) \quad = 0, \quad a^2 < b^2 + c^2;$$

$$16) \int_0^{2h\pi} \frac{dx}{a + (b \cos x + c \sin x)i} = \frac{2h\pi}{\sqrt{a^2 + b^2 + c^2}}$$

$$17) \int_0^{2h\pi} \frac{dx}{\{a + (b \cos x + c \sin x)i\}^2} = \frac{2ah\pi}{\sqrt{a^2 + b^2 + c^2}^3}$$

$$18) \int_{\frac{a\pi}{b}}^{\frac{a+1}{b}\pi} \sin bx \, dx = (-1)^a \frac{2}{b} \quad \text{Ohm, Ausw. 55.}$$

Dienger, Gr. 12.
409.

$$\left. \begin{aligned} 1) \int \cos q x \, dx &= \frac{1}{q} \sin q \\ 2) \int \sin q x \, dx &= \frac{1}{q} (1 - \cos q) \end{aligned} \right\} \quad \begin{aligned} &, q^2 < \pi^2; \\ &\text{Dienger, Cr. 28. 331.} \end{aligned}$$

$$\left. \begin{aligned} 3) \int \sin^2 (2\pi x) \, dx &= \frac{1}{2} \\ 4) \int \cos^2 (2\pi x) \, dx &= \frac{1}{2} \end{aligned} \right\} \quad \text{Abria, L. 4. 248.}$$

$$5) \int \sin. 2 a x. \cos. (2 \pi x) dx = 0 \quad \text{Abria, L. 4. 248.}$$

$$6) \int \frac{\sin (2 a \pi x)}{\text{Tang. } \pi x} dx = 1 \quad \text{V. T. 301. N° 5.}$$

$$7) \int \frac{\cos. (2 a \pi x)}{\text{Tang. } \pi x} dx = \infty \quad \text{V. T. 301. N° 4.}$$

$$8) \int \sin. (2 a \pi x) dx = 0 \quad \text{Kummer, Cr. 35. 1.}$$

$$9) \int \cos. (p \vee x) dx = \frac{1}{p^2} (p \sin. p + \cos. p - 1) \quad \text{Dicneger, Cr. 46. 119.}$$

$$1) \int \sin. x dx = \alpha, \text{ où } \alpha \text{ indéterminé; Meyer, Int. Déf. 98.}$$

$$2) \int \quad = 1 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Raabe, Cr. 15. 355. — Id., Cr. 16. 219. — Boncompagni, Cr. 25. 74.} \\ \\ \end{array}$$

$$3) \int \cos. x dx = 0$$

$$4) \int \quad = \alpha, \text{ où } \alpha \text{ indéterminé; Meyer, Int. Déf. 98. — Lejeune-Dirichlet, Cr. 17. 57.}$$

$$5) \int \sin. p x dx = \frac{1}{p} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Poisson, P. 16. 215. N° 2. — Cisa de Grésy, Mém. Turin. 1821. 209. II.} \\ \text{Art. 53. — Plana, Mém. Brux. T. 10. — Oettinger, Cr. 38. 216.} \\ \end{array}$$

$$6) \int \cos. p x dx = 0$$

$$7) \int \sin. (x^2) dx = \frac{1}{4} \vee 2 \pi \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Cauchy, Sav. Etr. 1827. 599. P. I. § 6.}$$

$$8) \int \cos. (x^2) dx = \frac{1}{4} \vee 2 \pi$$

$$9) \int \sin. \left(\frac{x^2}{2 q^4} \right) dx = \frac{1}{2} q \vee \pi \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Bidone, Mém. Turin. 1812. 231. Art. I. N° 4. — Schlömilch,} \\ \text{Stud. I. 13. — Helmling, Transf. 51. — Mascheroni, Adn. 57.} \end{array}$$

$$10) \int \cos. \left(\frac{x^2}{2 q^2} \right) dx = \frac{1}{2} q \vee \pi \quad \text{les trouve fautiv. = } q \vee \pi.$$

$$\left. \begin{aligned} 11) \int \text{Sin.}^{2a} x dx &= \infty \\ 12) \int \text{Sin.}^{2a+1} x dx &= \frac{2^{a/2}}{3^{a/2}} \end{aligned} \right\} \text{Raabe, Int. 149.}$$

$$\left. \begin{aligned} 13) \int \text{Cos.}^{2a} x dx &= \infty \\ 14) \int \text{Cos.}^{2a+1} x dx &= 0 \end{aligned} \right\} \text{Raabe, Int. 152.}$$

$$\left. \begin{aligned} 1) \int \text{Sin.}^{2a} x. \text{Sin.} p x dx &= \frac{1}{p} \frac{1^{2a/1}}{2^2 - p^2 \cdot 4^2 - p^2 \dots (2a)^2 - p^2} \\ &\text{, où } p \text{ tout nombre, hors la forme } 2k, \text{ où } k \leq a; \\ 2) \int \text{Sin.}^{2a} x. \text{Cos.} p x dx &= 0 \\ 3) \int \text{Sin.}^{2a+1} x. \text{Sin.} p x dx &= 0 \\ 4) \int \text{Sin.}^{2a+1} x. \text{Cos.} p x dx &= \frac{1^{2a+1/1}}{1^2 - p^2 \cdot 3^2 - p^2 \dots (2a+1)^2 - p^2} \\ &\text{, où } p \text{ tout nombre, hors la forme } 2k+1, \text{ où } k \leq a; \end{aligned} \right\} \text{Raabe, Int. 150.}$$

$$\left. \begin{aligned} 5) \int \text{Cos.}^{2a} x. \text{Sin.} p x dx &= \frac{1}{p} \frac{1^{2a/1}}{2^2 - p^2 \cdot 4^2 - p^2 \dots (2a)^2 - p^2} \\ &\left\{ 1 - \frac{p^2}{1 \cdot 2} - \frac{p^2 \cdot 2^2 - p^2}{1 \cdot 2 \cdot 3 \cdot 4} - \dots - \frac{p^2 \cdot 2^2 - p^2 \dots (2a-2)^2 - p^2}{1^{2a/1}} \right\} \\ &\text{, où } p \text{ tout nombre, hors la forme } 2k, \text{ où } k \leq a; \\ 6) \int \text{Cos.}^{2a} x. \text{Cos.} p x dx &= 0 \\ 7) \int \text{Cos.}^{2a+1} x. \text{Sin.} p x dx &= \frac{1^{2a+1/1}}{1^2 - p^2 \cdot 3^2 - p^2 \dots (2a+1)^2 - p^2} \\ &\left\{ -\frac{p^2}{1} - \frac{p^2 \cdot 1^2 - p^2}{1 \cdot 2 \cdot 3} - \dots - \frac{p^2 \cdot 1^2 - p^2 \dots (2a-1)^2 - p^2}{1^{2a+1/1}} \right\} \\ &\text{, où } p \text{ tout nombre, hors la forme } 2k+1, \text{ où } k \leq a; \end{aligned} \right\} \text{Raabe, Int. 153.}$$

$$\begin{aligned}
 8) \int \text{Cos.}^{2a+1} x. \text{Cos. } p x \, dx &= 0 \quad \text{Raabe, Int. 153.} \\
 9) \int \text{Sin. } (x^2). \text{Cos. } 2 q x \, dx &= \frac{1}{4} \{ \text{Cos. } (q^2) - \text{Sin. } (q^2) \} \vee 2 \pi \\
 10) \int \text{Cos. } (x^2). \text{Cos. } 2 q x \, dx &= \frac{1}{4} \{ \text{Cos. } (q^2) + \text{Sin. } (q^2) \} \vee 2 \pi \\
 11) \int \text{Sin. } (q^2 + x^2). \text{Cos. } 2 q x \, dx &= \frac{1}{4} \vee 2 \pi \\
 12) \int \text{Cos. } (q^2 + x^2). \text{Cos. } 2 q x \, dx &= \frac{1}{4} \vee 2 \pi \\
 13) \int \{ \text{Cos. } (x^2) + \text{Sin. } (x^2) \} \text{Cos. } 2 q x \, dx &= \frac{1}{2} \text{Cos. } (q^2) \vee 2 \pi \\
 14) \int \{ \text{Cos. } (x^2) - \text{Sin. } (x^2) \} \text{Cos. } 2 q x \, dx &= \frac{1}{2} \text{Sin. } (q^2) \vee 2 \pi \\
 15) \int \text{Sin. } (a x^2). \text{Cos. } b x \, dx &= \frac{1}{2} \left\{ \text{Cos. } \left(\frac{b^2}{4 a} \right) - \text{Sin. } \left(\frac{b^2}{4 a} \right) \right\} \vee \frac{\pi}{2 a} \\
 16) \int \text{Cos. } (a x^2). \text{Cos. } b x \, dx &= \frac{1}{2} \left\{ \text{Cos. } \left(\frac{b^2}{4 a} \right) + \text{Sin. } \left(\frac{b^2}{4 a} \right) \right\} \vee \frac{\pi}{2 a}
 \end{aligned}$$

Cauchy, Sav. Etr. 1827. 124. Note 2.

Raabe, Int. 168.

$$\begin{aligned}
 1) \int \text{Sin. } \left(x^2 - q + \frac{q^2}{4 x^2} \right) dx &= \frac{1}{4} \vee 2 \pi \\
 2) \int \text{Cos. } \left(x^2 - q + \frac{q^2}{4 x^2} \right) dx &= \frac{1}{4} \vee 2 \pi \\
 3) \int \text{Sin. } \left(x^2 + \frac{q^2}{4 x^2} \right) dx &= \frac{1}{4} (\text{Cos. } q + \text{Sin. } q) \vee 2 \pi \\
 4) \int \text{Cos. } \left(x^2 + \frac{q^2}{4 x^2} \right) dx &= \frac{1}{4} (\text{Cos. } q - \text{Sin. } q) \vee 2 \pi \\
 5) \int \left\{ \text{Sin. } \left(x^2 + \frac{q^2}{4 x^2} \right) + \text{Cos. } \left(x^2 + \frac{q^2}{4 x^2} \right) \right\} dx &= \frac{1}{2} \text{Cos. } q \vee 2 \pi \\
 6) \int \left\{ \text{Sin. } \left(x^2 + \frac{q^2}{4 x^2} \right) - \text{Cos. } \left(x^2 + \frac{q^2}{4 x^2} \right) \right\} dx &= \frac{1}{2} \text{Sin. } q \vee 2 \pi
 \end{aligned}$$

Cauchy, Sav. Etr. 1827. 124. Note 2.

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$$\begin{aligned}
 7) \int \frac{\sin. b x}{\sin. a x} dx &= 0, b < a; \text{ Cisa de Grésy, Mém. Turin. 1821. 209. II. 59.} \\
 8) \int \frac{\sin. x}{1 - p \cos. x} dx &= 0 \\
 9) \int \frac{\sin. a x}{1 - p \cos. x} dx &= \frac{1}{\sqrt{1-p^2}} \left[\left\{ \frac{1 + \sqrt{1-p^2}}{p} \right\}^a - \left\{ \frac{1 - \sqrt{1-p^2}}{p} \right\}^a \right] \frac{\sqrt{1+p} + \sqrt{1-p}}{2 \sqrt{1-p}} \\
 &\quad - \frac{1}{\sqrt{1-p^2}} \sum_{i=1}^{a-1} \frac{1}{n-a} \left[\left\{ \frac{1 + \sqrt{1-p^2}}{p} \right\}^n - \left\{ \frac{1 - \sqrt{1-p^2}}{p} \right\}^n \right] \left. \vphantom{\int} \right\} \text{Raabe,} \\
 &\quad \text{Int. 189.} \\
 10) \int \frac{\cos. a x}{1 - p \cos. x} dx &= \infty \\
 11) \int \frac{\cos. x}{a^2 + 4 k^2 \sin.^2 \frac{x}{2 k}} dx &= \frac{\pi}{2 a} e^{-a}, \text{ où } k = \infty; \\
 12) \int \frac{\sin. \frac{x}{k} \cdot \sin. x}{a^2 + 4 k^2 \sin.^2 \frac{x}{2 k}} dx &= \frac{\pi}{2 k} e^{-a}, \text{ où } k = \infty; \left. \vphantom{\int} \right\} \text{Poisson, P. 19. 404. N° 75.}
 \end{aligned}$$

$$\begin{aligned}
 1) \int \sin. x dx \vee \sin. 2 q x &= \frac{1}{2} \left(\sin. \frac{1}{2} q + \cos. \frac{1}{2} q \right) \vee q \pi \\
 2) \int \cos. x dx \vee \sin. 2 q x &= \frac{1}{2} \left(\sin. \frac{1}{2} q - \cos. \frac{1}{2} q \right) \vee q \pi \left. \vphantom{\int} \right\} \text{Cauchy, Sav. Etr. 1827. 124.} \\
 &\quad \text{Note 3.} \\
 3) \int \sin. x dx \vee \cos. 2 q x &= \sum_0^{\infty} (-1)^n \frac{1}{(2n+1)^{2n/1}} (2 q)^{2n} \\
 4) \int \cos. x dx \vee \cos. 2 q x &= \sum_0^{\infty} (-1)^n \frac{1}{(2n+2)^{2n+1/1}} (2 q)^{2n+1} \left. \vphantom{\int} \right\} \text{Cauchy, Sav. Etr. 1827. 124.} \\
 &\quad \text{Note 2.} \\
 5) \int \frac{\cos. a x}{\sqrt{1-p^2 \sin.^2 x}} dx &= 0 \\
 6) \int \frac{\cos. a x}{(1-p^2 \sin.^2 x)^{\frac{2b+1}{2}}} dx &= 0 \left. \vphantom{\int} \right\} \begin{aligned} &, p^2 < 1; \\ &\text{Raabe, Int. 196.} \end{aligned}
 \end{aligned}$$

$$1) \int \text{Sin.}(x^2) dx = \frac{1}{2} \sqrt{2\pi}$$

$$2) \int \text{Cos.}(x^2) dx = \frac{1}{2} \sqrt{2\pi}$$

Fourier, Chaleur. 407. — Lejeune-Dirichlet, Cr. 17. 57. — Id., Abh. Berlin. 1835.

$$3) \int \text{Sin.}(p x^2) dx = \sqrt{\frac{\pi}{2p}}$$

$$4) \int \text{Cos.}(p x^2) dx = \sqrt{\frac{\pi}{2p}}$$

Schlömilch, Stud. I. 13.

$$5) \int \text{Sin.}\left(\frac{\pi x^2}{2}\right) dx = 1$$

$$6) \int \text{Cos.}\left(\frac{\pi x^2}{2}\right) dx = 1$$

Abria, L. 4. 248.

$$7) \int \text{Cos.}\{(x+p)^2\} dx = \sqrt{\frac{\pi}{2}} \quad \text{Lejeune-Dirichlet, Abh. Berlin. 1835. — Id., Cr. 17. 57.}$$

$$8) \int \text{Sin.}\left\{\left(px - \frac{q}{x}\right)^2\right\} dx = \frac{1}{p} \sqrt{\frac{\pi}{2}}$$

$$9) \int \text{Cos.}\left\{\left(px - \frac{q}{x}\right)^2\right\} dx = \frac{1}{p} \sqrt{\frac{\pi}{2}}$$

Ohm, Ausw. 24.

$$10) \int \text{Sin.}\left(p^2 x^2 + \frac{q^2}{x^2}\right) dx = \text{Sin.}\left(\frac{1}{4}\pi + 2pq\right) \sqrt{\frac{\pi}{p}}$$

$$11) \int \text{Cos.}\left(p^2 x^2 + \frac{q^2}{x^2}\right) dx = \text{Cos.}\left(\frac{1}{4}\pi + 2pq\right) \sqrt{\frac{\pi}{p}}$$

$$12) \int \text{Sin.}(p x^2 + q x) dx = \text{Sin.}\left(\frac{1}{4}\pi - \frac{q^2}{4p}\right) \sqrt{\frac{\pi}{p}}$$

$$13) \int \text{Cos.}(p x^2 + q x) dx = \text{Cos.}\left(\frac{1}{4}\pi - \frac{q^2}{4p}\right) \sqrt{\frac{\pi}{p}}$$

$$14) \int \text{Sin.}(p x^2 + q x + r) dx = \text{Sin.}\left(\frac{1}{4}\pi - \frac{q^2 - 4pr}{4p}\right) \sqrt{\frac{\pi}{p}}$$

$$15) \int \text{Cos.}(p x^2 + q x + r) dx = \text{Cos.}\left(\frac{1}{4}\pi - \frac{q^2 - 4pr}{4p}\right) \sqrt{\frac{\pi}{p}}$$

Ohm, Ausw. 25.

1) $\int \text{Sin.}(qx^2). \text{Cos.} p x dx = \text{Sin.} \left(\frac{\pi}{4} - \frac{p^2}{4q^2} \right) \vee \frac{\pi}{q}$	Fourier, Chaleur. 407.
2) $\int \text{Cos.}(qx^2). \text{Cos.} p x dx = \text{Sin.} \left(\frac{\pi}{4} + \frac{p^2}{4q^2} \right) \vee \frac{\pi}{q}$	
3) $\int \text{Sin.}(x^2). \text{Cos.} 2 p x dx = \frac{1}{2} \{ \text{Cos.}(p^2) - \text{Sin.}(p^2) \} \vee 2 \pi$	Lejeune-Dirichlet, Cr. 17. 57. — Id., Abh. Berlin. 1835. — Schlö- milch, Stud. II. 9.
4) $\int \text{Cos.}(x^2). \text{Cos.} 2 p x dx = \frac{1}{2} \{ \text{Cos.}(p^2) + \text{Sin.}(p^2) \} \vee 2 \pi$	
5) $\int \text{Sin.} \left(\frac{x^2}{4q} \right). \text{Cos.} p x dx = \{ \text{Cos.}(p^2 q) - \text{Sin.}(p^2 q) \} \vee 2 q \pi$	Schlömilch, Stud. II. 9.
6) $\int \text{Cos.} \left(\frac{x^2}{4q} \right). \text{Cos.} p x dx = \{ \text{Cos.}(p^2 q) + \text{Sin.}(p^2 q) \} \vee 2 q \pi$	
7) $\int \text{Sin.}(p x^2). \text{Sin.} q x dx = 0$	Ohm, Ausw. 25.
8) $\int \text{Cos.}(p x^2). \text{Sin.} q x dx = 0$	
9) $\int \text{Sin.}(p^2 + x^2). \text{Sin.} 2 p x dx = 0$	Lejeune-Dirichlet, Cr. 17. 57. — Schlömilch, Stud. II. 9.
10) $\int \text{Cos.}(p^2 + x^2). \text{Sin.} 2 p x dx = 0$	Schlömilch, Stud. II. 9.
11) $\int \text{Sin.}(p^2 + x^2). \text{Cos.} 2 p x dx = \vee \frac{\pi}{2}$	
12) $\int \text{Cos.}(p^2 + x^2). \text{Cos.} 2 p x dx = \vee \frac{\pi}{2}$	

$$1) \int \text{Cos.} x dx = -1$$

$$2) \int \text{Cos.}^{2a} x dx = \infty$$

$$3) \int \text{Cos.}^{2a+1} x dx = -\frac{2^{a/2}}{3^{a/2}}$$

$$4) \int \text{Sin.}^{2a} x dx = \infty$$

$$5) \int \text{Sin.}^{2a+1} x dx = 0$$

$$6) \int \text{Sin.}^{2a} x. \text{Sin.} p x dx = \frac{1}{p} \text{Cos.} \frac{1}{2} p \pi \frac{1^{2a/1}}{2^2 - p^2. 4^2 - p^2 \dots (2a)^2 - p^2} \\ \left\{ 1 - \frac{p^2}{1.2} - \frac{p^2. 2^2 - p^2}{1.2.3.4} - \dots - \frac{p^2. 2^2 - p^2 \dots (2a-2)^2 - p^2}{1^{2a/1}} \right\}$$

$$7) \int \text{Sin.}^{2a} x. \text{Cos.} p x dx = -\frac{1}{p} \text{Sin.} \frac{1}{2} p \pi \frac{1^{2a/1}}{2^2 - p^2. 4^2 - p^2 \dots (2a)^2 - p^2} \\ \left\{ 1 - \frac{p^2}{1.2} - \frac{p^2. 2^2 - p^2}{1.2.3.4} - \dots - \frac{p^2. 2^2 - p^2 \dots (2a-2)^2 - p^2}{1^{2a/1}} \right\}$$

$$8) \int \text{Sin.}^{2a+1} x. \text{Sin.} p x dx = \frac{1}{p} \text{Cos.} \frac{1}{2} p \pi \frac{1^{2a+1/1}}{1^2 - p^2. 3^2 - p^2 \dots (2a+1)^2 - p^2} \\ \left\{ -\frac{p^2}{1} - \frac{p^2. 1^2 - p^2}{1.2.3} - \dots - \frac{p^2. 1^2 - p^2 \dots (2a-1)^2 - p^2}{1^{2a+1/1}} \right\}$$

$$9) \int \text{Sin.}^{2a+1} x. \text{Cos.} p x dx = -\frac{1}{p} \text{Sin.} \frac{1}{2} p \pi \frac{1^{2a+1/1}}{1^2 - p^2. 3^2 - p^2 \dots (2a+1)^2 - p^2} \\ \left\{ -\frac{p^2}{1} - \frac{p^2. 1^2 - p^2}{1.2.3} - \dots - \frac{p^2. 1^2 - p^2 \dots (2a-1)^2 - p^2}{1^{2a+1/1}} \right\}$$

$$10) \int \text{Cos.}^{2a} x. \text{Sin.} p x dx = \frac{1}{p} \text{Cos.} \frac{1}{2} p \pi \frac{1^{2a/1}}{2^2 - p^2. 4^2 - p^2 \dots (2a)^2 - p^2}$$

$$11) \int \text{Cos.}^{2a} x. \text{Cos.} p x dx = -\frac{1}{p} \text{Sin.} \frac{1}{2} p \pi \frac{1^{2a/1}}{2^2 - p^2. 4^2 - p^2 \dots (2a)^2 - p^2}$$

$$12) \int \text{Cos.}^{2a+1} x. \text{Sin.} p x dx = \text{Sin.} \frac{1}{2} p \pi \frac{1^{2a+1/1}}{1^2 - p^2. 3^2 - p^2 \dots (2a+1)^2 - p^2}$$

$$13) \int \text{Cos.}^{2a+1} x. \text{Cos.} p x dx = -\text{Cos.} \frac{1}{2} p \pi \frac{1^{2a+1/1}}{1^2 - p^2. 3^2 - p^2 \dots (2a+1)^2 - p^2}$$

Toutes ces formules sont déduites par Raabe, Int. 253.

$$\begin{aligned}
 & 1) \int_0^a \sin.^2 \frac{2b\pi x}{a} dx = \frac{1}{2}a \\
 & 2) \int_0^a \cos.^2 \frac{2b\pi x}{a} dx = \frac{1}{2}a \\
 & 3) \int_0^a \sin. \frac{2b\pi x}{a} \cdot \cos. \frac{2b\pi x}{a} dx = 0 \\
 & 4) \int_0^a \sin. \frac{2b\pi x}{a} \cdot \sin. \frac{2c\pi x}{a} dx = 0 \\
 & 5) \int_0^a \sin. \frac{2b\pi x}{a} \cdot \cos. \frac{2c\pi x}{a} dx = 0 \\
 & 6) \int_0^a \cos. \frac{2b\pi x}{a} \cdot \cos. \frac{2c\pi x}{a} dx = 0 \\
 & 7) \int_0^a \cos. \frac{2b\pi x}{a} \cdot \sin. \frac{2c\pi x}{a} dx = 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} 1) \\ 2) \\ 3) \\ 4) \\ 5) \\ 6) \\ 7) \end{aligned}} \right\} \begin{array}{l} \text{Poisson, Chaleur. 134.} \\ , b > c; \end{array}$$

$$8) \int_0^1 \frac{\sin. p x}{\sin. x} dx = \frac{1}{2}\pi, p < \pi; \text{ Ohm, Ausw. 67.}$$

$$9) \int_0^{\frac{1}{2} \text{Arccos. } p} dx \sqrt{\frac{\cos. 2x - p}{\cos. 2x - 1}} = 2\pi \left\{ 1 - \sqrt{\frac{1+p}{2}} \right\} \text{ Catalan, L. 6. 419.}$$

$$10) \int_0^{\lambda} \frac{dx}{\cos.^2 x \sqrt{(1-p^2 \sin.^2 x)}} = \frac{1}{1-p^2} \{ \text{Tang. } \lambda \sqrt{(1-p^2 \sin.^2 \lambda)} + (1-p^2) \text{F}(p, \lambda) - \text{E}(p, \lambda) \}$$

$$11) \int_0^{\lambda} \frac{dx}{\sqrt{(1-p^2 \sin.^2 x)^3}} = \frac{1}{1-p^2} \left\{ \text{E}(p, \lambda) - \frac{p^2 \sin. \lambda \cdot \cos. \lambda}{\sqrt{(1-p^2 \sin.^2 \lambda)}} \right\}$$

$$\left. \vphantom{\begin{aligned} 10) \\ 11) \end{aligned}} \right\} \begin{array}{l} \text{Catalan,} \\ \text{L. 4.} \\ \text{323.} \end{array}$$

Les formules 1 à 15 de cette Table sont déduites par Legendre, Exerc. Supplém. Tome I, aux numéros indiqués; on y a partout:

$$c = \frac{\sin. \lambda}{\sin. \mu}, \quad \cos. \nu = \cos. \lambda \cdot \cos. \mu, \quad \text{Tang. } \vartheta = \sin. \lambda \cdot \cot. \mu, \quad \cot. \varphi = \sin. \mu \cdot \cot. \lambda.$$

- $$\begin{aligned}
 1) & \int \frac{\sin x}{\sqrt{(\cos^2 x - \cos^2 \lambda)}} dx = \frac{1}{2} l \frac{1 + \sin \lambda}{1 - \sin \lambda} \\
 2) & \int \frac{\sin^3 x}{\sqrt{(\cos^2 x - \cos^2 \lambda)}} dx = \frac{1 + \sin^2 \lambda}{4} l \frac{1 + \sin \lambda}{1 - \sin \lambda} - \frac{1}{2} \sin \lambda \left. \vphantom{\int} \right\} \text{N}^\circ. 11. \\
 3) & \int \frac{\sin x \cos x}{\sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} dx = \eta \sec \mu \\
 4) & \int \frac{\sin x \cos^3 x}{\sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} dx = \frac{1 + \cos^2 \mu}{2 \cos^2 \mu} \eta - \frac{\sin \mu \sin \lambda}{2 \cos^2 \mu} \\
 5) & \int \frac{\sin x}{\sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} \frac{dx}{\cos x} = \varphi \sec \lambda \\
 6) & \int \frac{\sin x}{\sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} \frac{dx}{\cos^3 x} = \frac{1 + \cos^2 \mu}{2 \cos^3 \lambda} \varphi + \frac{\sin \mu \sin \lambda}{2 \cos^2 \lambda} \\
 7) & \int \frac{\sin x}{\sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} dx = F(\sin \mu, \varphi) \\
 8) & \int \frac{\sin x}{\sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} \frac{dx}{\cos^2 x} = \sec^2 \lambda \cdot E(\sin \mu, \varphi) \\
 9) & \int \frac{\sin x \cos^2 x}{\sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} dx = \sec^2 \mu \cdot E(\sin \mu, \varphi) - \frac{\sin \lambda \sin \mu}{\cos^2 \mu} \\
 10) & \int \frac{dx}{\sqrt{(\cos^2 x - \sin^2 \lambda)}} = F'(\sin \lambda) \\
 11) & \int \frac{dx}{\cos^2 x \sqrt{(\cos^2 x - \sin^2 \lambda)}} = \sec^2 \lambda \cdot E'(\sin \lambda) \\
 12) & \int \frac{\cos^2 x}{\sqrt{(\cos^2 x - \sin^2 \lambda)}} dx = E'(\sin \lambda) \\
 13) & \int \frac{dx}{\sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} = \operatorname{Cosec} \mu \cdot F(c) \\
 14) & \int \frac{dx}{\cos^2 x \sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} = \frac{\cos^2 \mu}{\sin \mu} F(c) + \frac{\sin \mu}{\cos^2 \lambda} E'(c) \\
 15) & \int \frac{\cos^2 x}{\sqrt{\{(\cos^2 x - \cos^2 \lambda)(1 - \cos^2 \mu \cos^2 x)\}}} dx = \frac{\cos^2 \lambda}{\sin \mu} F'(c) + \frac{\cos \lambda}{\cos \mu} \{F'(c) E(c, \mu) - E(c) F(c, \mu)\} \\
 16) & \int \frac{\cos \frac{1}{2} x}{1 + p^2 - 2p \cos x \sqrt{2(\cos x - \cos \lambda)}} \frac{dx}{\sqrt{2(\cos x - \cos \lambda)}} = \frac{x}{2(1-p) \sqrt{1-2p \cos \lambda + p^2}} \quad \left. \vphantom{\int} \right\} \text{Vielle, Exerc. 164.}
 \end{aligned}$$

$$\begin{aligned}
 1) \int \sin. x. \cos. x dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi}{16} (\sin.^2 \mu - \sin.^2 \lambda)^2 \\
 2) \int \sin.^3 x. \cos. x dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi}{32} (\sin.^2 \mu - \sin.^2 \lambda)^2 (\sin.^2 \lambda + \sin.^2 \mu) \\
 3) \int \sin.^{2a+1} x. \cos. x dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \\
 &= \frac{\pi}{4} \frac{(\sin.^2 \mu - \sin.^2 \lambda)^2}{\sin.^4 \mu} \sin.^{2a+4} \mu \sum_0^{\infty} (-1)^n \binom{a}{n} \frac{1^{n/2}}{4^{n/2}} \frac{(\sin.^2 \mu - \sin.^2 \lambda)^n}{\sin.^{2n} \mu}
 \end{aligned}$$

Voyez de ces formules: Legendre, Exerc. Suppl. N°. 6.

Toutes les formules de cette Table sont déduites par Legendre, Exerc. Suppl. Tome I, les formules 1 à 9 dans le N°. 5, les formules 10 à 16 dans le N°. 7; on a dans ces formules:

$$k = \frac{\sin.^2 \mu - \sin.^2 \lambda}{\sin.^2 \mu} \quad h = \frac{\cos.^2 \lambda - \cos.^2 \mu}{\cos.^2 \lambda}$$

$$\begin{aligned}
 1) \int \frac{\cos. x}{\sin. x} dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi}{4} (\sin. \mu - \sin. \lambda)^2 \\
 2) \int \frac{\cos. x}{\sin.^3 x} dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi}{4} \frac{(\sin. \mu - \sin. \lambda)^2}{\sin. \lambda. \sin. \mu} \\
 3) \int \frac{\cos. x}{\sin.^5 x} dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi}{16} \frac{(\sin.^2 \mu - \sin.^2 \lambda)^2}{\sin.^3 \lambda. \sin.^3 \mu} \\
 4) \int \frac{\cos. x}{\sin.^7 x} dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi}{32} \frac{(\sin.^2 \mu - \sin.^2 \lambda)^2}{\sin.^5 \lambda. \sin.^5 \mu} (\sin.^2 \lambda + \sin.^2 \mu) \\
 5) \int \frac{\cos. x dx}{\sin.^{2a+1} x} \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi k^2 \sin. \mu}{4 \sin.^{2a+1} \lambda} \sum_0^{\infty} (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}} k^n \\
 6) \int \frac{\sin. x}{\cos. x} dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi}{4} (\cos. \lambda - \cos. \mu)^2 \\
 7) \int \frac{\sin. x}{\cos.^3 x} dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi}{4} \frac{(\cos. \lambda - \cos. \mu)^2}{\cos. \lambda. \cos. \mu} \\
 8) \int \frac{\sin. x}{\cos.^5 x} dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi}{16} \frac{(\cos.^2 \lambda - \cos.^2 \mu)^2}{\cos.^3 \lambda. \cos.^3 \mu} \\
 9) \int \frac{\sin. x}{\cos.^{2a+1} x} dx \sqrt{\{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)\}} &= \frac{\pi h^2 \cos. \lambda}{4 \cos.^{2a+1} \mu} \sum_0^{\infty} (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}} h^n
 \end{aligned}$$

$$10) \int \frac{dx}{\sin x \cos x} \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} = \frac{\pi}{2} \{1 - \cos(\mu - \lambda)\}$$

$$11) \int \frac{dx}{\sin^3 x \cos x} \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} = \frac{\pi \sin^2(\mu - \lambda)}{4 \sin \lambda \sin \mu}$$

$$12) \int \frac{dx}{\sin^{2a+1} x \cos x} \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} = \int_{\lambda}^{\mu} \frac{dx}{\sin^{2a-1} x \cos x} \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} \\ + \frac{\pi k^2 \sin \mu}{4 \sin^{2a-1} \lambda} \sum_0^{\infty} (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}}$$

$$13) \int \frac{dx}{\sin x \cos^3 x} \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} = \frac{\pi \sin^2(\mu - \lambda)}{4 \cos \lambda \cos \mu}$$

$$14) \int \frac{dx}{\sin x \cos^{2a+1} x} \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} = \int_{\lambda}^{\mu} \frac{dx}{\sin x \cos^{2a-1} x} \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} \\ + \frac{\pi h^2 \cos \lambda}{4 \cos^{2a-1} \mu} \sum_0^{\infty} (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}}$$

$$15) \int \frac{\sin^3 x}{\cos x} dx \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} = \frac{\pi}{2} (\cos \lambda - \cos \mu)^2 - \frac{\pi}{16} (\sin^2 \mu - \sin^2 \lambda)^2$$

$$16) \int \frac{\sin^{2a+1} x}{\cos x} dx \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} = \int_{\lambda}^{\mu} \frac{\sin^{2a-1} x}{\cos x} dx \sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}} \\ - \frac{\pi}{16} k^2 \sin^{2a} \mu \sum_0^{\infty} (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}}$$

Toutes les formules de cette Table sont trouvées par Legendre, Exerc. Suppl. I, aux numéros indiqués; on a dans ces intégrales partout:

$$k = \frac{\sin^2 \mu - \sin^2 \lambda}{\sin^2 \mu}, \quad h = \frac{\cos^2 \lambda - \cos^2 \mu}{\cos^2 \lambda}, \quad \cos \theta = \frac{\cos \mu}{\cos \lambda}.$$

$$\left. \begin{aligned} 1) \int \frac{\sin x \cos x}{\sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}}} dx &= \frac{1}{2} \pi \\ 2) \int \frac{\sin^3 x \cos x}{\sqrt{\{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)\}}} dx &= \frac{1}{4} \pi (\sin^2 \lambda + \sin^2 \mu) \end{aligned} \right\} \quad N^{\circ} 2.$$

$$\begin{aligned}
3) \int \frac{\text{Sin.}^5 x \cdot \text{Cos. } x}{\sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{1}{2} \pi \left(\frac{1.3}{2.4} \text{Sin.}^4 \lambda + \frac{1}{2} \text{Sin.}^2 \lambda \cdot \text{Sin.}^2 \mu + \frac{1.3}{2.4} \text{Sin.}^4 \mu \right) \left\{ \begin{array}{l} N^2. \\ 2. \end{array} \right. \\
4) \int \frac{\text{Sin.}^{2a+1} x \cdot \text{Cos. } x}{\sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{1}{2} \pi \text{Sin.}^{2a} \mu \sum_0^{\infty} (-1)^n \binom{a}{n} \frac{1^{n/2}}{2^{n/2}} k^n \\
5) \int \frac{\text{Cos. } x}{\text{Sin. } x \sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{\pi}{2 \text{Sin. } \lambda \cdot \text{Sin. } \mu} \\
6) \int \frac{\text{Cos. } x}{\text{Sin.}^3 x \sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{\pi}{4 \text{Sin.}^3 \lambda \cdot \text{Sin.}^3 \mu} (\text{Sin.}^2 \lambda + \text{Sin.}^2 \mu) \left\{ \begin{array}{l} N^2. \\ 3. \end{array} \right. \\
7) \int \frac{\text{Cos. } x}{\text{Sin.}^{2a+1} x \sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{\pi}{2 \text{Sin.}^{2a+1} \lambda \cdot \text{Sin. } \mu} \sum_0^{\infty} (-1)^n \binom{a}{n} \frac{1^{n/2}}{2^{n/2}} k^n \\
8) \int \frac{\text{Sin. } x \cdot \text{Cos.}^3 x}{\sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{\pi}{4} (\text{Cos.}^2 \lambda + \text{Cos.}^2 \mu) \\
9) \int \frac{\text{Sin. } x \cdot \text{Cos.}^{2a+1} x}{\sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{1}{2} \pi \text{Cos.}^{2a} \lambda \sum_0^{\infty} (-1)^n \binom{a}{n} \frac{1^{n/2}}{2^{n/2}} h^n \\
10) \int \frac{\text{Sin. } x}{\text{Cos. } x \sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{\pi}{2 \text{Cos. } \lambda \cdot \text{Cos. } \mu} \left\{ \begin{array}{l} N^2. \\ 4. \end{array} \right. \\
11) \int \frac{\text{Sin. } x}{\text{Cos.}^3 x \sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{\pi}{4 \text{Cos.}^3 \lambda \cdot \text{Cos.}^3 \mu} (\text{Cos.}^2 \lambda + \text{Cos.}^2 \mu) \\
12) \int \frac{\text{Sin. } x}{\text{Cos.}^{2a+1} x \sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{\pi}{2 \text{Cos.}^{2a+1} \mu \cdot \text{Cos. } \lambda} \sum_0^{\infty} (-1)^n \binom{a}{n} \frac{1^{n/2}}{2^{n/2}} h^n \\
13) \int \frac{dx}{\sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} &= \frac{1}{\text{Cos. } \lambda \cdot \text{Sin. } \mu} F' \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu} \right) \\
14) \int \frac{dx}{\text{Sin.}^2 x \sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} &= \frac{1}{\text{Cos. } \lambda \cdot \text{Sin. } \mu} F' \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu} \right) + \frac{\text{Cos. } \lambda}{\text{Sin.}^2 \lambda \cdot \text{Sin. } \mu} E' \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu} \right) \left\{ \begin{array}{l} N^2. \\ 9. \end{array} \right. \\
15) \int \frac{dx}{\text{Cos.}^2 x \sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} &= \frac{1}{\text{Cos. } \lambda \cdot \text{Sin. } \mu} F' \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu} \right) + \frac{\text{Sin. } \mu}{\text{Cos. } \lambda \cdot \text{Cos.}^2 \mu} E' \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu} \right) \\
16) \int \frac{\text{Sin.}^2 x}{\sqrt{\{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)(\text{Sin.}^2 \mu - \text{Sin.}^2 x)\}}} dx &= \frac{\text{Sin. } \mu}{\text{Cos. } \lambda} F' \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu} \right) + \\
&\quad + E' \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu} \right) F \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu}, \mu \right) - F' \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu} \right) E \left(\frac{\text{Sin. } \theta}{\text{Sin. } \mu}, \mu \right) \left\{ \begin{array}{l} N^2. \\ 10. \end{array} \right.
\end{aligned}$$

$$\begin{aligned}
 17) \int \frac{\text{Sin.}^4 x}{\sqrt{\{\text{Sin.}^2 x - \text{Sin.}^2 \lambda\} \{\text{Sin.}^2 \mu - \text{Sin.}^2 x\}}} dx &= \frac{1 + \text{Sin.}^2 \lambda + \text{Sin.}^2 \mu}{2} \left[E' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu} \right) F' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu}, \mu \right) - \right. \\
 &\quad \left. - F' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu} \right) E \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu}, \mu \right) + \frac{1 + \text{Sin.}^2 \mu}{2 \text{Cos.} \lambda} \text{Sin.} \mu F' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu} \right) - \frac{\text{Sin.} \mu \text{Cos.} \lambda}{2} E' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu} \right) \right] \\
 18) \int dx \sqrt{\frac{\text{Sin.}^2 x - \text{Sin.}^2 \lambda}{\text{Sin.}^2 \mu - \text{Sin.}^2 x}} &= \frac{\text{Sin.}^2 \mu - \text{Sin.}^2 \lambda}{\text{Sin.} \mu \text{Cos.} \lambda} F' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu} \right) + E' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu} \right) F' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu}, \mu \right) - F' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu} \right) E \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu}, \mu \right) \\
 19) \int dx \sqrt{\frac{\text{Sin.}^2 \mu - \text{Sin.}^2 x}{\text{Sin.}^2 x - \text{Sin.}^2 \lambda}} &= F' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu} \right) E \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu}, \mu \right) - E' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu} \right) F' \left(\frac{\text{Sin.} \theta}{\text{Sin.} \mu}, \mu \right) \\
 20) \int \frac{\text{Cos.} x}{\sqrt{\{\text{Sin.}^2 x - \text{Sin.}^2 \lambda\} \{\text{Sin.}^2 \mu - \text{Sin.}^2 x\}}} dx &= \frac{1}{\text{Sin.} \mu} F' \left(\sqrt{\frac{\text{Sin.}^2 \mu - \text{Sin.}^2 \lambda}{\text{Sin.}^2 \mu}} \right) \\
 21) \int \frac{\text{Sin.}^2 x \text{Cos.} x}{\sqrt{\{\text{Sin.}^2 x - \text{Sin.}^2 \lambda\} \{\text{Sin.}^2 \mu - \text{Sin.}^2 x\}}} dx &= \frac{1}{\text{Sin.} \mu} E' \left(\sqrt{\frac{\text{Sin.}^2 \mu - \text{Sin.}^2 \lambda}{\text{Sin.}^2 \mu}} \right) \\
 22) \int \frac{\text{Cos.} x}{\text{Sin.}^2 x \sqrt{\{\text{Sin.}^2 x - \text{Sin.}^2 \lambda\} \{\text{Sin.}^2 \mu - \text{Sin.}^2 x\}}} dx &= \frac{1}{\text{Sin.}^2 \lambda \text{Sin.}^2 \mu} E' \left(\sqrt{\frac{\text{Sin.}^2 \mu - \text{Sin.}^2 \lambda}{\text{Sin.}^2 \mu}} \right) \\
 23) \int \frac{dx}{\text{Cos.} x \sqrt{\{\text{Sin.}^2 x - \text{Sin.}^2 \lambda\} \{\text{Sin.}^2 \mu - \text{Sin.}^2 x\}}} &= \frac{1}{\text{Sin.} \mu \text{Cos.}^2 \mu} \Pi' \left(\frac{\text{Sin.}^2 \mu - \text{Sin.}^2 \lambda}{\text{Sin.}^2 \mu}, \text{Tang.}^2 \mu, \sqrt{\frac{\text{Sin.}^2 \mu - \text{Sin.}^2 \lambda}{\text{Sin.}^2 \mu}} \right)
 \end{aligned}$$

N°. 10.

N°. 50.

$$\begin{aligned}
 1) \int \text{Arctang.} x dx &= \frac{1}{4} \pi - \frac{1}{2} l 2 \\
 2) \int \text{Arctang.} p x dx &= \text{Arctang.} p - \frac{1}{2p} l(1 + p^2) \\
 3) \int \text{Arcsin.} x dx &= \frac{1}{2} \pi - 1 \\
 4) \int \text{Arcsin.} p x dx &= \text{Arcsin.} p + \frac{1}{p} \sqrt{1 - p^2} - \frac{1}{p} \\
 5) \int \text{Arctang.}(x e^{pi}) dx &= \frac{1}{4} \pi - \frac{1}{2} p \text{Sin.} p - \frac{1}{2} \text{Cos.} p l(2 \text{Cos.} p) + \frac{i}{4} \left\{ l \frac{1 + \text{Sin.} p}{1 - \text{Sin.} p} + 2 \text{Sin.} p l(2 \text{Cos.} p) - p \text{Cos.} p \right\}, p < \frac{1}{2} \pi; \\
 6) \int \text{Arcsin.}(x e^{pi}) dx &= \text{Arcsin.} \left(\frac{\text{Cos.} p}{\sqrt{1 + \text{Sin.} p}} \right) - \text{Cos.} p + \left(\text{Cos.} \frac{\pi + 2p}{4} - i \text{Sin.} \frac{\pi + 2p}{4} \right) l(2 \text{Sin.} p) + i \text{Sin.} p + \\
 &\quad + i l \left\{ \text{Sin.} \frac{1}{2} p + \sqrt{1 - \text{Sin.} p} \right\}, p \leq \frac{1}{2} \pi;
 \end{aligned}$$

Sur les intégrales 1 à 6 voyez: Dienger, Cr. 38. 331.

- 7) $\int \operatorname{Arccot}. x \, dx = \frac{1}{4} \pi + \frac{1}{2} l \, 2$ V. T. 108. N°. 1.
- 8) $\int \operatorname{Arcsin}. (\surd x) \, dx = \frac{\pi}{4}$ V. T. 9. N°. 4.
- 9) $\int \operatorname{Arccos}. (\surd x) \, dx = \frac{\pi}{4}$ V. T. 9. N°. 4.
- 10) $\int (\operatorname{Arccot}. x)^2 \, dx = -\frac{\pi}{16} + \frac{3}{4} \pi l \, 2 - \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 27.

- 1) $\int \operatorname{Arctang}. x \, dx = \infty$ V. T. 109. N°. 2.
- 2) $\int \operatorname{Arccot}. x \, dx = \frac{1}{2} \pi l \, 2$ V. T. 264. N°. 2.
- 3) $\int (\operatorname{Arccos}. x)^2 \, dx = \frac{1}{2} \pi l \, 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.
- 4) $\int (\operatorname{Arccot}. x)^2 \, dx = \pi l \, 2$ Cauchy, Lim. Imag. Add § 31. — Id., Sav. Etr. 1827. 599. P. 2. § 5.
— Mosta, Gr. 10. 449.
- 5) $\int (\operatorname{Arccot}. x)^p \, dx = p \left(\frac{\pi}{2} \right)^{p-1} \left\{ 1 - \sum_1^{\infty} \frac{2}{p+2m-1} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\}$ V. T. 265. N°. 26.
- 6) $\int \operatorname{Arctang}. p x. \operatorname{Arctang}. q x \, dx = \infty$ V. T. 109. N°. 1. 10.
- 7) $\int \operatorname{Arctang}. x. \operatorname{Arccot}. x \, dx = \frac{\pi-4}{4} \pi l \, 2$ V. T. 109. N°. 2, 4.
- 8) $\int \operatorname{Arctang}. x. \operatorname{Arccot}. \frac{x}{q} \, dx = \frac{\pi^2 q}{2} l \, 2 + \frac{\pi q}{2} l \, 2 - \frac{1+q}{2} \pi l (1+q)$ V. T. 109. N°. 2, 11.
- 9) $\int \operatorname{Arctang}. \frac{x}{q} \operatorname{Arccot}. x \, dx = \frac{\pi^2}{2} l \, 2 + \frac{\pi q}{2} l \, 2 - \frac{1+q}{2} \pi l (1+q)$ V. T. 109. N°. 2, 11.
- 10) $\int \operatorname{Arctang}. p x. \operatorname{Arccot}. q x \, dx = \frac{\pi}{2} \left\{ \frac{\pi}{2 q} l \, 2 + \frac{1}{p} l \frac{q}{p+q} + \frac{1}{q} l \frac{p+q}{p} \right\}$ V. T. 109. N°. 2, 12.
- 11) $\int \operatorname{Arccot}. x. \operatorname{Arccot}. q x \, dx = \frac{\pi}{2} \left\{ \frac{1+q}{q} l (1+q) - l q \right\}$ V. T. 264. N°. 13.
- 12) $\int \operatorname{Arccot}. p x. \operatorname{Arccot}. q x \, dx = \frac{\pi}{2} \left\{ \frac{1}{p} l \left(1 + \frac{p}{q} \right) + \frac{1}{q} l \left(1 + \frac{q}{p} \right) \right\}$ V. T. 264. N°. 14.

- 1) $\int_1^{\infty} \text{Arctang. } x \, dx = \infty$ V. T. 108. N°. 1 et T. 109. N°. 1.
- 2) $\int_1^{\infty} \text{Arccot. } x \, dx = \frac{\pi-1}{2} l 2 - \frac{\pi}{4}$ V. T. 108. N°. 7 et T. 109. N°. 2.
- 3) $\int_1^{\infty} (\text{Arccot. } x)^2 \, dx = \frac{\pi^2}{16} + \frac{\pi}{4} l 2 + \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 7.
- 4) $\int_1^{\infty} (\text{Arccot. } x)^p \, dx = \left(\frac{\pi}{4}\right)^p + \frac{p}{2} \left(\frac{\pi}{2}\right)^{p-1} \left\{1 - \sum_1^{\infty} \frac{1}{p+2m} \sum_1^{\infty} \frac{1}{(4n)^{2m}}\right\}$ V. T. 270. N°. 11.
- 5) $\int_0^p \text{Arcsin. } \left(\frac{x}{p}\right) \, dx = \frac{\pi-2}{2} p$ V. T. 34. N°. 7.
- 6) $\int_0^p \text{Arccos. } \left(\frac{x}{p}\right) \, dx = p$ V. T. 34. N°. 7.
- 7) $\int_0^p \text{Arcsin. } \left(1 \setminus \frac{x}{p}\right) \, dx = \frac{1}{4} p \pi$ V. T. 34. N°. 8.
- 8) $\int_0^p \text{Arccos. } \left(1 \setminus \frac{x}{p}\right) \, dx = \frac{1}{4} p \pi$ V. T. 34. N°. 8.

- 1) $\int_0^1 B'(x) \, dx = \frac{(-1)^{a-1}}{2a+2} B_{2a+1}$
 - 2) $\int_0^1 B''(x) \, dx = 0$
 - 3) $\int_0^1 \{B'(x)\}^2 \, dx = \frac{1^{2a+1/1}}{(2a+2)^{2a+3/2}} B_{4a+3} + \left(\frac{1}{2a+2} B_{2a+1}\right)^2$
 - 4) $\int_0^1 \{B''(x)\}^2 \, dx = \frac{1^{2a+1}}{(2a+1)^{2a+2/1}} B_{4a+1}$
 - 5) $\int_0^1 dx \, li.(x) = -l 2$ V. T. 300. N°. 1.
- Raabe, Cr. 42. 348.

- $$\left. \begin{aligned} 1) \int e^{ax} x dx &= \frac{1}{a^3} \{(a-1)e^a + 1\} \\ 2) \int e^{-x^2} x dx &= \frac{e-1}{2e} \end{aligned} \right\} \text{Dienger, Cr. 46. 119.}$$
- $$3) \int e^{-x^2} x^3 dx = \frac{e-2}{2e} \quad \text{V. T. 115. N}^\circ. 2.$$
- $$4) \int \frac{e^{-x} dx}{x} = \infty \quad \text{Cisa de Grésy, Mém. Turin. 1821. 209. I. 27.}$$
- $$5) \int \frac{e^x x dx}{(1+x)^2} = \frac{1}{2} e - 1 \quad \text{Rogner, Mater.}$$
- $$6) \int e^{-x} dx \swarrow x = \frac{1}{2} \swarrow \pi \quad \text{Plana, Cr. 17. 1.}$$
- $$7) \int \frac{e^{-x} dx}{1 \swarrow x} = \frac{2 + e \swarrow \pi}{4e} \quad \text{V. T. 115. N}^\circ. 6.$$
- $$8) \int (e^{1-\frac{1}{x}} - x^q) \frac{dx}{x(1-x)} = Z'(q) \quad \text{Lejeune-Dirichlet, Cr. 15. 258. — Stern, Cr. 21. 377. — Grunert, Gr. 2. 266.}$$
- $$9) \int \left(\frac{b e^{1-\frac{1}{x}}}{1-x} - \frac{x^a}{1-x^{\frac{1}{b}}} \right) \frac{dx}{x} = \sum_1^b Z' \left(a + \frac{n-1}{n} \right)$$
- $$10) \int \left(\frac{b e^{1-x^{-b}}}{1-x^b} - \frac{x^{ab}}{1-x} \right) \frac{dx}{x} = \frac{1}{b} \sum_1^b Z' \left(a + \frac{n-1}{n} \right)$$
- $$11) \int \left(\frac{b e^{1-x^{-b}}}{1-x^b} - \frac{e^{1-\frac{1}{x}}}{1-x} \right) \frac{dx}{x} = \frac{1}{b} \frac{d}{da} \frac{\Gamma(a) \Gamma\left(a + \frac{1}{b}\right) \dots \Gamma\left(a + \frac{b-1}{b}\right)}{\Gamma(ab)}$$
- $$12) \int \frac{e^{px} - 2px - e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{dx}{x} = p \swarrow \pi - l - \frac{\Gamma\left(\frac{\pi+p}{2\pi}\right)}{\Gamma\left(\frac{\pi-p}{2\pi}\right)}, \quad p < \pi; \quad \text{Malmsten, Cr. 38. 1.}$$

$$1) \int e^{-ax} x dx = \frac{1}{a^2} \quad \text{V. T. 152. N}^\circ. 1.$$

- 2) $\int e^{-x} x^{a-1} dx = 1^{a-1/1} = 1.2.3 \dots a-1$ Euler, Calc. Int. 4. S. 5. 129. — Legendre, Exerc. 3. 31. — Poisson, P. 19. 404. N°. 68. — Binet, P. 27. 123. — Liouville, Cr. 11. 1. — Oettinger, Cr. 35. 13. — Lejeune-Dirichlet, Cr. 15. 258. — Schaar, Mém. Cour. Brux. T. 22. — Lobatschewsky, Mém. Kasan. 1835. 211. — Id., ib. 1836. I. II. form. (12).
- 3) $\int e^{-x} x^{p-1} dx = \Gamma(p)$ $\infty > p > -1$; C'est la fonction Eulérienne de seconde espèce.
- 4) $\int e^{-qx} x^{a-1} dx = \frac{1^{a-1/1}}{q^a}$ Euler, Calc. Int. 4. S. 5. 131. — Lejeune-Dirichlet, Cr. 15. 258. — Oettinger, Cr. 35. 13. — Grunert, Gr. 2. 266
- 5) $\int e^{-qx} x^{p-1} dx = \frac{\Gamma(p)}{q^p}$ Cauchy, Cours. Leç. 32. — Kummer, Cr. 17. 228. — Serret, L. 8. 1.
- 6) $\int e^{-cx} x^{a+p-1} dx = \frac{p^{a/1}}{c^{a+p}} \Gamma(p)$ Schlömilch, Stud. I. 1.
- 7) $\int r^{-x} x^{p-1} dx = \frac{\Gamma(p)}{(lr)^p}$, $\infty > p > -1$, $r > 1$; Lejeune-Dirichlet, Cr. 4. 94.
- 8) $\int x^k e^{-x} dx = e^{-k} k! \sqrt{2k\pi}$, pour $k = \infty$; Liouville, L. 11. 464.
- 9) $\int \{e^x x^{q-1} - e^{-px} (1 - e^{-x})^{q-1}\} dx = \frac{\Gamma(p+q) - \Gamma(p)}{q} \frac{\Gamma(1+q)}{\Gamma(p+q)}$ Schaar, Mém. Cour. Brux. T. 23.
- 10) $\int e^{-px} (1 - e^{-qx})^a x^b dx = (-1)^b 1^{b,1} \sum_0^{\infty} \binom{a}{n} \frac{(-1)^n}{(p+nq)^{b+1}}$ V. T. 151. N°. 8.
- 11) $\int e^{-qx} (x+x^2)^{p-1} dx = \Gamma(2p-1) q^{1-2p} e^{\frac{1}{2}q} \psi\left(\frac{3}{2}-p, \frac{1}{16}q^2\right) + \frac{\Gamma(1-2p)\Gamma(p)}{\Gamma(1-p)} e^{\frac{1}{2}q} \psi\left(\frac{1}{2}+p, \frac{1}{16}q^2\right)$ Kummer, Cr. 17. 228.
- 12) $\int e^{-axi} x^{p-1} dx = \frac{\Gamma(p)}{a^p} e^{-\frac{1}{2}p\pi i}$ Moigno, Calc. Int. 132.
- 13) $\int e^{axi} x^{p-1} dx = \frac{\Gamma(p)}{a^p} e^{\frac{1}{2}p\pi i}$, $1 > p > 0$; Lejeune-Dirichlet, C. R. 8. 157. — Schlömilch, St. I. 13.
- 14) $\int e^{-(1+qi)x} x^{p-1} dx = \frac{\Gamma(p)}{(1+qi)^p}$ Schlömilch, Stud. I. 13.
- 15) $\int e^{-(a+bi)x} x^{p-1} dx = \frac{\Gamma(p)}{(a^2+b^2)^{\frac{1}{2}p}} e^{p \cdot i \cdot \text{Arctang} \frac{b}{a}}$ Moigno, Calc. Int. 132.
- 16) $\int e^{-(p+qi)x} x^a dx = \frac{1^{a/1}}{(p+qi)^{a+1}}$, où il y a faut.: $(p+qi)^a$ Meyer, Int. Déf. 117.

$$17) \int e^{-(p+qi)x} x^{r-1} dx = \frac{\Gamma(r)}{(p+qi)^r} \quad \text{Cauchy, Cours. Leç. 39. — Lejeune-Dirichlet, Cr. 4. 94.}$$

$$18) \quad = \frac{\Gamma(r)}{\sqrt{(p^2+q^2)^r}} e^{-ri \operatorname{Arctang} \frac{q}{p}} \quad \text{Serret, L. 8. 1.}$$

$$19) \int (1 - e^{-x}) x e^{-x} dx = \frac{3}{4} \quad \text{V. T. 151. N°. 12.}$$

$$\left. \begin{aligned} 1) \int e^{-x^2} x^2 dx &= \frac{1}{4} \sqrt{\pi} \\ 2) \int e^{-x^2} x^4 dx &= \frac{3}{8} \sqrt{\pi} \\ 3) \int e^{-x^2} x^6 dx &= \frac{15}{16} \sqrt{\pi} \end{aligned} \right\} \quad \text{Krampe, Réfr. 3. N°. 70. — Boncompagni, Cr. 25. 74.}$$

$$4) \int e^{-x^2} x^{2a+1} dx = 3^{a-2/1} \quad \text{Oettinger, Cr. 35. 13.}$$

$$5) \int \quad = 0 \text{ (fautive)} \quad \text{Boncompagni, Cr. 25. 74.}$$

$$6) \int e^{-x^2} x^{2a} dx = \frac{1}{2} \Gamma\left(\frac{2a+1}{2}\right) \quad \text{Legendre, Exerc. 3. 29.}$$

$$7) \int \quad = \frac{1^{a/2}}{2^{a+1}} \sqrt{\pi} \quad \text{Krampe, Réfr. 3. N°. 70. — Laplace, Mém. Inst. 1809. 253. § 3. — Boncompagni, Cr. 25. 74. — Oettinger, Cr. 35. 13.}$$

$$8) \int e^{-px^2} x^{2a} dx = \frac{1^{a/2}}{(2p)^a} \frac{1}{2} \sqrt{\frac{\pi}{p}} \quad \text{Schlömlich, Gr. 5. 90. — Id., Gr. 5. 100. — Id., Beitr. III. 14. — Id., Stud. 1. 12.}$$

$$9) \int e^{-px^2} x^{2a+1} dx = \frac{1}{(2p)^{a+1}} 1^{a/1} \quad \text{Schlömlich, Beitr. III. 14.}$$

$$10) \int e^{-p^2 x^2} x dx = \frac{1}{2p^2} \quad \text{Schlömlich, Gr. 9. 379.}$$

$$11) \int e^{-px^2} x^2 dx = \frac{1}{4p} \sqrt{\frac{\pi}{p}} \quad \text{Olin, Ausw. 20.}$$

$$12) \int e^{-px^2} x^4 dx = \frac{3}{8p^2} \sqrt{\frac{\pi}{p}} \quad \text{V. T. 114. N}^\circ 11.$$

$$13) \int r^{-p^2 x^2} x^{2a} dx = \frac{1^{a/2}}{2^{a+1} (p \sqrt{lr})^{2a+1}} \sqrt{\pi} \quad \text{Meyer, Int. déf. 116.}$$

$$14) \int e^{px^2} x^{2q-1} dx = \frac{\Gamma(q)}{2p^q} e^{\frac{1}{2}\pi qi} \quad \text{Schaar, Mém. Brux. T. 24.}$$

$$15) \int e^{-x^4} x^2 dx = \frac{\pi^{\frac{3}{4}}}{4 \sqrt{(2\pi_1)} \sqrt{2}}, \text{ où } \pi_1 = 1,31102877714605987 \quad \text{Laplace, Mém. Acad. 1782. 1. § 5.}$$

$$\left. \begin{aligned} 16) \int e^{-x^6} x^2 dx &= \frac{1}{6} \sqrt{\pi} \\ 17) \int e^{-x^6} x^3 dx &= \frac{1}{6} \frac{3.6.9...}{2.5.8...} = \frac{1^{-\frac{1}{2}}}{6} \end{aligned} \right\} \quad \text{Oettinger, Cr. 35. 13.}$$

$$\left. \begin{aligned} 1) \int e^{-x^b} x^{a-1} dx &= \frac{1}{a} 1^{\frac{a}{b}} \\ 2) \int e^{-x^b} x^{ab-1} dx &= \frac{1}{ab} 1^{a/1} \end{aligned} \right\} \quad \text{Kramp, Réfr. 3. N. 62, 64. — Oettinger, Cr. 35. 13.}$$

$$\left. \begin{aligned} 3) \int e^{-x^p} x^{p-1} dx &= \frac{1}{p} \\ 4) \int e^{-x^p} x^{2p-1} dx &= \frac{1}{p} \\ 5) \int e^{-x^{2p}} x^{p-1} dx &= \frac{1}{2p} \sqrt{\pi} \end{aligned} \right\} \quad \text{Kramp, Réfr. 3. N}^\circ 65, 66.$$

$$6) \int e^{-x^a} x^b dx = \frac{1}{a} \Gamma \frac{b+1}{a} \quad \text{Legendre, Mém. Inst. 1809. 416. N}^\circ 81. — \text{Id., Exerc. 2. 81.}$$

$$7) \int e^{-x^a} x^{\frac{1}{2}a-1} dx = \frac{1}{a} \sqrt{\pi} \quad \text{Laplace, Mém. Acad. 1782. 1. § 5.}$$

$$8) \int e^{-x^{ac}} x^{ab-1} dx = \frac{1}{ab} 1^{\frac{b}{c}} \quad \text{Kramp, Réfr. 3. N}^\circ 68.$$

$$9) \int e^{-qx^a} x^{ap-1} dx = \frac{1}{a q^p} \Gamma(p) \quad \text{Boncompagni, Cr. 25. 74.}$$

$$10) \int e^{-c^b x^b} x^{a-1} dx = \frac{1}{a c^a} e^{c^b} 1^{\frac{a}{b}+1} \quad \text{Oettinger, Cr. 35. 13.}$$

$$1) \int e^{-x^2+px} x dx = \frac{1}{2} (-1 + \frac{1}{2} p e^{\frac{1}{4}p^2} \sqrt{\pi}) \quad \text{V. T. 37. N°. 5.}$$

$$2) \int (e^{px} - e^{-px}) e^{-x^2} x dx = \frac{1}{2} p e^{\frac{1}{4}p^2} \sqrt{\pi} \quad \text{V. T. 37. N°. 9.}$$

$$3) \int e^{-(x+\frac{q}{x})} x^{p-1} dx = \Gamma(p) \psi(1-p, q) + \Gamma(-p) q^p \psi(1+p, q), q > 0; \quad \text{Kummer, Cr. 17. 228.}$$

$$4) \int e^{-(ax^q+bx)} x^{pq-1} dx = \frac{\Gamma(p)}{q a^p} \left(1 + \frac{b}{a}\right)^{-p} \quad \text{Boncompagni, Cr. 25. 74. Elle ne vaut que pour } q=1.$$

$$\left. \begin{aligned} 5) \int e^{-\frac{1}{x^6}} x dx &= \frac{1}{2} 1^{\frac{1}{1}} \\ 6) \int e^{-\frac{1}{x^6}} x^2 dx &= \frac{1}{3} 1^{\frac{1}{1}} \sqrt{\pi} \\ 7) \int e^{-\frac{1}{x^a}} x^{b-1} dx &= \frac{1}{b} 1^{-\frac{b}{a}+1} \end{aligned} \right\} \quad \text{Oettinger, Cr. 35. 13.}$$

$$8) \int e^{-q(x^2+\frac{1}{x^2})} x^{2a} dx = \frac{1}{2} e^{-2q} \sqrt{\frac{\pi}{q}} \sum_{n=0}^{\infty} (-1)^n \frac{(a-n+1)^{2n/1}}{2^n 1^{n/1}} \left(\frac{1}{2q}\right)^n \quad \text{Cauchy, P. 28. 147. P. 2. — Id., Exerc. 1826. p. 54.}$$

$$9) \int e^{-xe^{qi}} x^{a-1} dx = e^{-pqi} \Gamma(p) \quad \text{Serret, L. S. 1.}$$

$$10) \int \left\{ \frac{d^n}{dy^n} \left(y e^{-x^2 y^2} \right) \right\} e^{-x^2} dx = \frac{1}{2} \sqrt{\pi} \frac{d^n}{dy^n} \left(\frac{y}{1+(1+y^2)} \right) \quad \text{Schlömilch, Gr. 4. 364.}$$

$$11) \int e^{-x^{\frac{2a}{1+b}} x^{a-1}} dx = \frac{1+\frac{2}{a}b}{a \cdot 2^{\frac{b}{b+1}}} 1^{\frac{b}{b+1}} \sqrt{\pi} \quad \text{Kramp, Réfr. 3. 67.}$$

$$12) \int (e^{-x}-1)^a e^{-px} x^{b-1} dx = 1^{\frac{b}{b+1}} \Delta^a (p^{-b}) \quad \text{Cauchy, P. 28. 147. P. 3. § 1.}$$

$$1) \int \frac{x}{e^x + 1} dx = \frac{1}{12} \pi^2 \quad \text{V. T. 152. N}^\circ. 3.$$

$$2) \int \frac{x^3}{e^x + 1} dx = \frac{7}{120} \pi^4 \quad \text{V. T. 154. N}^\circ. 10.$$

$$3) \int \frac{x^5}{e^x + 1} dx = \frac{31}{256} \pi^6 \quad \text{V. T. 155. N}^\circ. 2.$$

$$4) \int \frac{x^7}{e^x + 1} dx = \frac{127}{1680} \pi^8 \quad \text{V. T. 155. N}^\circ. 9.$$

$$5) \int \frac{x}{e^x - 1} dx = \frac{1}{6} \pi^2 \quad \text{Cauchy, Mém. Paris. 1823. 603. -- Id., Sav. Etr. 1827. 599. P. 2. § 5.}$$

$$\left. \begin{aligned} 6) \int \frac{x^3}{e^x - 1} dx &= \frac{1}{15} \pi^4 \\ 7) \int \frac{x^5}{e^x - 1} dx &= \frac{8}{63} \pi^6 \end{aligned} \right\} \text{Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.}$$

$$8) \int \frac{x}{e^{2x} - 1} dx = \frac{1}{24} \pi^2 \quad \text{V. T. 152. N}^\circ. 14.$$

$$9) \int \frac{x}{e^{2x} + 1} dx = \frac{1}{48} \pi^2 \quad \text{V. T. 152. N}^\circ. 12.$$

$$10) \int \frac{x^3}{e^{2x} + 1} dx = \frac{7}{1920} \pi^4 \quad \text{V. T. 154. N}^\circ. 13.$$

$$11) \int \frac{x^{p+1}}{e^x - q} dx = \frac{p+1}{q} \Gamma(p+1) \sum_0^\infty \frac{q^n}{n^{p+2}} \quad \text{Hoppe, Cr. 40. 139.}$$

$$12) \int \frac{x^{2a}}{e^x + 1} dx = \frac{2^{2a} - 1}{2^{2a}} 1^{2a/1} \sum_0^\infty \frac{1}{n^{2a+1}} \quad \text{V. T. 157. N}^\circ. 2.$$

$$13) \int \frac{x^{2a-1}}{e^x + 1} dx = \frac{2^{2a-1} - 1}{2^a} \pi^{2a} B_{2a-1} \quad \text{V. T. 157. N}^\circ. 5.$$

$$14) \int \frac{x^{2a}}{e^x - 1} dx = 1^{2a/1} \sum_0^\infty \frac{1}{n^{2a+1}} \quad \text{V. T. 157. N}^\circ. 3.$$

$$15) \int \frac{x^{2a-1}}{e^x - 1} dx = \frac{2^{2a-1} \pi^{2a}}{2^a} B_{2a-1} \quad \text{Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. -- Raabe, Cr. 42. 348.}$$

$$16) \int \frac{x^{p-1}}{e^x + 1} dx = \Gamma(p) \sum_0^{\infty} \frac{(-1)^n}{(n+1)^p} \quad \text{V. T. 157. N}^\circ. 8.$$

$$17) \int \frac{x^{p-1}}{e^x - 1} dx = \Gamma(p) \sum_0^{\infty} \frac{1}{(n+1)^p} \quad \text{V. T. 157. N}^\circ. 9.$$

$$\left. \begin{aligned} 18) \int \frac{x}{e^{2\pi x} - 1} dx &= \frac{1}{24} \\ 19) \int \frac{x^2}{e^{2\pi x} - 1} dx &= \frac{1}{240} \end{aligned} \right\} \text{Poisson, Mém. Inst. 1811. 163. N}^\circ. 40.$$

$$20) \int \frac{x}{e^{2\pi x} - 1} dx = \frac{1}{24} a^2 \quad \text{Cauchy, Mém. Ac. 1823. 603.}$$

$$21) \int \frac{x^{2a-1}}{e^{\pi x} + 1} dx = \frac{2^{2a-1} - 1}{2^a} B_{2a-1} \quad \text{Schlömlich, Gr. 3. 9.}$$

$$22) \int \frac{x^{2a-1}}{e^{\pi x} - 1} dx = \frac{2^{2a-1}}{2^a} B_{2a-1} \quad \text{Malmsten, Cr. 35. 55.}$$

$$23) \int \frac{x^{2a-1}}{e^{2\pi x} - 1} dx = \frac{1}{4^a} B_{2a-1} \quad \begin{array}{l} \text{Binet, P. 27. 123. — Plana, Mém. Turin. 1820. 1. — Malmsten,} \\ \text{Cr. 35. 55. — Schlömlich, Gr. 3. 9. — Id., Gr. 12. 130.} \end{array}$$

$$1) \int \frac{x e^{-x}}{e^x - 1} dx = \frac{1}{6} \pi^2 - 1 \quad \text{V. T. 152. N}^\circ. 8.$$

$$2) \int \frac{e^{-2x} x}{1 + e^{-x}} dx = -\frac{1}{12} \pi^2 + 1 \quad \text{V. T. 152. N}^\circ. 4.$$

$$3) \int \frac{e^{-3x} x}{1 + e^{-x}} dx = \frac{1}{12} \pi^2 - \frac{3}{4} \quad \text{V. T. 152. N}^\circ. 5.$$

$$4) \int \frac{e^{-x} x^{p+1}}{1 - q e^{-x}} dx = \frac{p+1}{q} \Gamma(p+1) \sum_1^{\infty} \frac{q^n}{n^{p+2}} \quad \text{V. T. 157. N}^\circ. 10.$$

$$5) \int \frac{1 - e^{-px}}{1 - e^{-x}} e^{-x} x^{a-1} dx = \Gamma(a) \sum_1^b \frac{1}{n^a} \quad \text{V. T. 157. N}^\circ. 14.$$

$$6) \int \frac{e^{-ax} x^{p-1}}{1 + e^{-x}} dx = \Gamma(p) \sum_0^{\infty} \frac{(-1)^n}{(a+n)^p} \quad \text{V. T. 157. N}^\circ. 11.$$

$$\left. \begin{aligned} 7) \int \frac{e^{-px} x}{1 - e^{-x}} dx &= \sum_0^{\infty} \frac{1}{(n+p)^2} \\ 8) \int \frac{e^{-ax} x^{p-1}}{1 - e^x} dx &= -\Gamma(p) \sum_1^{\infty} \frac{1}{(a+n)^p} \end{aligned} \right\} \quad \text{Binet, P. 27. 123.}$$

$$9) \int \frac{e^{-px} x^r}{1 - e^{-qx}} dx = 1^{r/1} \sum_0^{\infty} \frac{1}{(p+nq)^{r+1}} \quad \text{V. T. 158. N}^\circ. 10.$$

$$10) \int \frac{e^{-x} + 1}{e^x - 1} x dx = -1 + 2 \sum_1^{\infty} \frac{1}{n^2} \quad \text{Euler, N. C. P. 14. 129.}$$

$$11) \int \frac{1 - e^{-x}}{1 + e^{-3x}} x e^{-x} dx = \frac{2}{27} \pi^3 \quad \text{V. T. 152. N}^\circ. 15.$$

$$12) \int \frac{e^{-ax} + e^{(a-b)x}}{1 - e^{-bx}} x dx = \left(\frac{\pi}{b}\right)^2 \text{Cosec.}^2 \frac{a\pi}{b} \quad \text{V. T. 152. N}^\circ. 21.$$

$$13) \int \frac{e^{-qx} - e^{(q-p)x}}{1 - e^{-px}} x^2 dx = \left(\frac{\pi}{p \text{Sin.} \frac{q\pi}{p}}\right)^3 2 \text{Cos.} \frac{q\pi}{p} \quad \text{V. T. 154. N}^\circ. 7.$$

$$14) \int \frac{e^{-qx} + e^{(q-p)x}}{1 - e^{-px}} x^3 dx = \left(\frac{\pi}{p \text{Sin.} \frac{q\pi}{p}}\right)^4 \left(2 + 4 \text{Cos.}^2 \frac{q\pi}{p}\right) \quad \text{V. T. 154. N}^\circ. 8.$$

$$15) \int \frac{e^{\pi x} + 1}{e^{\gamma x} - 1} x^{2a-1} dx = \frac{2^{2a-1}}{a} B_{2a-1} \quad \text{Schlömlich, Gr. 1. 360.}$$

$$16) \int \frac{1 - e^{(c-b)x}}{1 - e^{-bx}} e^{-\frac{1}{2}cx} x^{2a} dx = \frac{(-1)^{a+1}}{b} (2\pi)^{2a+1} \sum_1^b B'' \left(\frac{n}{2b}\right) \text{Sin.} \frac{ne\pi}{b} \quad \text{V. T. 164. N}^\circ. 6.$$

$$17) \int \frac{1 + e^{(c-b)x}}{1 + e^{-bx}} e^{-\frac{1}{2}cx} x^{2a} dx = \frac{(-1)^{a+1}}{b} (2\pi)^{2a+1} \sum_1^b B'' \left(\frac{2n-1}{4b}\right) \text{Sin.} \left(\frac{2n-1}{2b} e\pi\right) \quad \text{V. T. 164. N}^\circ. 5.$$

$$18) \int e^{-px} (e^{-x} - 1)^c \left(p + \frac{e e^{-x}}{e^{-x} - 1}\right) x^q dx = \Gamma(q) \Delta^c(p^{-q}) \quad \text{Cauchy, P. 28. 147. P. III. § 1.}$$

Exp. binôme $(e^{ax} \pm 1)^2$ en dénom.

- 1) $\int \frac{e^x - e^{-x} + 2}{(e^x - 1)^2} x^2 dx = -1 + 2 \sum_1^{\infty} \frac{1}{n^2}$ V. T. 118. N°. 10.
- 2) $\int \frac{e^x - 2}{(e^x - 1)^2} x^2 dx = \frac{1}{3} \pi^2$ V. T. 117. N°. 4.
- 3) $\int \frac{e^x x^q}{(e^x + 1)^2} dx = \Gamma(q) \sum_0^{\infty} \frac{(-1)^n}{(1+n)^q}$ V. T. 117. N°. 15.
- 4) $\int \frac{e^x x^q}{(e^x - 1)^2} dx = \Gamma(q) \sum_0^{\infty} \frac{1}{(1+n)^q}$ V. T. 117. N°. 16.
- 5) $\int \frac{e^x x^2}{(e^x + 1)^2} dx = \frac{1}{6} \pi^2$ V. T. 117. N°. 1.
- 6) $\int \frac{e^{-x} x^{q+1}}{(p e^{-x} - 1)^2} dx = \frac{q}{(q+1)p} \Gamma(q) \sum_1^{\infty} \frac{p^n}{n^{q+1}}$ V. T. 117. N°. 10.
- 7) $\int \frac{(a-1)e^{-x} + a}{(1+e^{-x})^2} e^{-ax} x^p dx = p \Gamma(p) \sum_0^{\infty} \frac{(-1)^n}{(a+n)^p}$ V. T. 118. N°. 6.
- 8) $\int \frac{(a+1)e^{-x} + a}{(1-e^x)^2} e^{-ax} x^p dx = p \Gamma(p) \sum_1^{\infty} \frac{-1}{(a+n)^p}$ V. T. 118. N°. 8.
- 9) $\int \frac{e^{\pi x} x^{2a}}{(e^{\pi x} + 1)^2} dx = \frac{2^{2a-1} - 1}{\pi} B_{2a-1}$ V. T. 117. N°. 20.
- 10) $\int \frac{e^{7x} x^{2a}}{(e^{7x} - 1)^2} dx = \frac{2^{2a-1}}{\pi} B_{2a-1}$ V. T. 117. N°. 21.

Exp. bin. $(e^{ax} \pm e^{-ax})$ en dén.

Numér. algèbr.

- 1) $\int \frac{x}{e^x + e^{-x}} dx = - \sum_1^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 187. N°. 2.
- 2) $= \frac{1}{2} \pi \log 2 - 2 \operatorname{Li} \left(\frac{\pi}{4} \right)$ Lobatschewsky, Mém. Kasan. 1836. 1. 1. form. (103).
- 3) $\int \frac{x^2}{e^x + e^{-x}} dx = \frac{1}{16} \pi^3$ V. T. 154. N°. 1.
- 4) $\int \frac{x^4}{e^x + e^{-x}} dx = \frac{5}{64} \pi^5$ V. T. 155. N°. 1.

- 5) $\int \frac{x^6}{e^x + e^{-x}} dx = \frac{61}{256} \pi^7$ V. T. 155. N°. 8.
- 6) $\int \frac{x}{e^x - e^{-x}} dx = \frac{3}{4} \frac{1}{6} \pi^2$
- 7) $\int \frac{x^3}{e^x - e^{-x}} dx = \frac{15}{8} \frac{1}{30} \pi^4$
- 8) $\int \frac{x^5}{e^x - e^{-x}} dx = \frac{63}{16} \frac{1}{42} \pi^6$ } Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.
- 9) $\int \frac{x^7}{e^x - e^{-x}} dx = \frac{17}{32} \pi^8$ V. T. 155. N°. 10
- 10) $\int \frac{x^{2a}}{e^x + e^{-x}} dx = \frac{1}{2} (-1)^{a+1} (2\pi)^{2a+1} B'' \left(\frac{1}{4} \right)$ Raabe, Cr. 42. 348.
- 11) $\int \frac{x^q}{e^x + e^{-x}} dx = \Gamma(q+1) \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^{q+1}}$ V. T. 187. N°. 7.
- 12) $\int \frac{x^{2a+1}}{e^x - e^{-x}} dx = \frac{1}{1} (-1)^{a+1} (2\pi)^{2a+2} B' \left(\frac{1}{2} \right)$ Raabe, Cr. 42. 348.
- 13) $\int \frac{x^{2a}}{e^x - e^{-x}} dx = \frac{2^{2a+1} - 1}{2^{2a+1}} 1^{2a+1} \sum_1^{\infty} \frac{1}{n^{2a+1}}$ V. T. 158. N°. 4.
- 14) $\int \frac{x^{2a}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} dx = \frac{1}{2} B_{2a}$ Schlömilch, Gr. 1. 360. — Id., Gr. 12. 130. — Id., Beitr. II. § 6.
- 15) $\int \frac{x^{2a}}{e^{qx} + e^{-qx}} dx = \frac{(-1)^{a+1}}{2} \left(\frac{2\pi}{q} \right)^{2a+1} B'' \left(\frac{1}{4} \right)$ Raabe, Cr. 42. 348.
- 16) $\int \frac{x^{2a}}{e^{\frac{1}{2}bx} + e^{-\frac{1}{2}bx}} dx = \frac{(-1)^{a+1}}{b} (2\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b} \right)$ V. T. 164. N°. 4.
- 17) $\int \frac{x}{e^{ax} - e^{-ax}} dx = \frac{\pi^2}{8a^2}$ V. T. 152. N°. 18.
- 18) $\int \frac{x^{2a-1}}{e^{\pi x} - e^{-\pi x}} dx = \frac{2^{2a} - 1}{4a} B_{2a-1}$ Schlömilch, Beitr. II. § 6. — Id., Gr. 12. 130.
- 19) $\int \frac{x^{2a-1}}{e^{qx} - e^{-qx}} dx = \frac{(-1)^a}{4} \left(\frac{2\pi}{q} \right)^{2a} B' \left(\frac{1}{2} \right)$ Raabe, Cr. 42. 348.
- 20) $\int \frac{x^{2a-1}}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} dx = \frac{2^{2a} - 1}{2a} 2^{2a} B_{2a-1}$ Schlömilch, Gr. 3. 9.

- 1) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} x dx = \infty$ V. T. 153. N°. 10.
- 2) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} x dx = \infty$ V. T. 153. N°. 11.
- 3) $\int \frac{e^x - e^{-x}}{e^{px} + e^{-px}} x dx = \frac{\pi^2}{4p^2} \text{Sin.} \frac{\pi}{2p} \cdot \text{Sec.}^2 \frac{\pi}{2p}, p < 1;$ V. T. 152. N°. 19.
- 4) $\int \frac{e^x + e^{-x}}{e^{px} - e^{-px}} x dx = \frac{\pi^2}{4p^2} \text{Sec.}^2 \frac{\pi}{2p}, p < 1;$ V. T. 152. N°. 20.
- 5) $\int \frac{e^{qx} + e^{-qx}}{e^{px} + e^{-px}} x dx = -\frac{\pi^2}{4p^2} \text{Sin.} \frac{q\pi}{2p} \cdot \text{Sec.}^2 \frac{q\pi}{2p}, p < q;$ V. T. 153. N°. 12.
- 6) $\int \frac{e^{qx} - e^{-qx}}{e^{px} - e^{-px}} x dx = \frac{\pi^2}{4p^2} \text{Sec.}^2 \frac{q\pi}{2p}, p < q;$ V. T. 153. N°. 13.
- 7) $\int \frac{e^x + e^{-x}}{e^{2x} + e^{-2x}} x^2 dx = \frac{3}{64} \pi^3 \swarrow 2$ V. T. 154. N°. 2.
- 8) $\int \frac{e^{qx} + e^{-qx}}{e^{px} + e^{-px}} x^2 dx = \frac{\pi^3}{8p^3} \left\{ 3 \text{Sec.}^3 \frac{q\pi}{2p} - \text{Sec.} \frac{q\pi}{2p} \right\}, p < q;$ V. T. 154. N°. 5.
- 9) $\int \frac{e^{qx} - e^{-qx}}{e^{px} - e^{-px}} x^2 dx = \frac{3\pi^3}{8p^3} \text{Sin.} \frac{q\pi}{2p} \cdot \text{Sec.}^3 \frac{q\pi}{2p}, p < q;$ V. T. 154. N°. 6.
- 10) $\int \frac{e^x - e^{-x}}{e^{3x} - e^{-3x}} x^2 dx = \frac{\pi^3}{81 \swarrow 3}$ V. T. 154. N°. 2.
- 11) $\int \frac{e^{2x} - e^{-2x}}{e^{3x} - e^{-3x}} x^2 dx = \frac{\pi^3}{9 \swarrow 3}$ V. T. 154. N°. 3.
- 12) $\int \frac{e^{-x}}{e^{1x} - e^{-1x}} x dx = 2\pi^2$ V. T. 163. N°. 18.
- 13) $\int \frac{e^{px} + e^{-px}}{e^{1\pi x} + e^{-1\pi x}} x^{2a} dx = \frac{d^{2a}}{dp^{2a}} \cdot \text{Sec.} p$
- 14) $\int \frac{e^{px} - e^{-px}}{e^{1\pi x} + e^{-1\pi x}} x^{2a+1} dx = \frac{d^{2a+1}}{dp^{2a+1}} \cdot \text{Sec.} p$
- 15) $\int \frac{e^{-1x}}{e^{1x} + e^{-1x}} x^{2a+1} dx = -\frac{1}{2} \frac{(2\pi)^{2a+1}}{2a+2} B_{2a+1} + \frac{1}{4} (-1)^{a+1} (2\pi)^{2a+2} B\left(\frac{1}{2}\right)$
- 16) $\int \frac{e^{px} - e^{-px}}{e^{1\pi x} - e^{-1\pi x}} x^{2a} dx = \frac{d^{2a}}{dp^{2a}} \cdot \text{Tang.} p, p < \frac{1}{2} \pi;$

Ranbe, Cr.
12 448.

$$\begin{aligned}
 17) \int \frac{e^{px} + e^{-px}}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} x^{2a+1} dx &= \frac{d^{2a+1}}{d p^{2a+1}} \cdot \text{Tang. } p, p < \frac{1}{2} \pi; \\
 18) \int \frac{e^{-\frac{1}{2}x}}{e^{\frac{1}{2}x} - e^{-\frac{1}{2}x}} x^{2a+1} dx &= \frac{1}{2} \frac{(2\pi)^{2a+2}}{2a+2} B_{2a+1} \left. \vphantom{\int} \right\} \text{Raabe, Cr. 42. 348.} \\
 19) \int \frac{e^{bx} + e^{-bx}}{e^x + e^{-x}} x^{2a} dx &= \frac{(-1)^{a+1} \left(\frac{2\pi}{b}\right)^{2a+1}}{b} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b}\right) \text{Sin.} \left\{ \frac{2n-1}{2} b\pi \right\} \text{ V. T. 158. N}^\circ. 3. \\
 20) \int \frac{e^{(c-b)x} + e^{(b-c)x}}{e^{cx} + e^{-cx}} x^{2a} dx &= \frac{(-1)^{a+1}}{c} (2\pi)^{2a+1} \sum_1^c B'' \left(\frac{2n-1}{4c}\right) \text{Sin.} \left\{ \frac{2n-1}{2c} b\pi \right\} \text{ V. T. 158. N}^\circ. 12. \\
 21) \int \frac{e^{bx} - e^{-bx}}{e^x - e^{-x}} x^{2a} dx &= \frac{(-1)^{a+1}}{b} \left(\frac{2\pi}{b}\right)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{n}{2b}\right) \text{Sin. } n b \pi \text{ V. T. 158. N}^\circ. 9. \\
 22) \int \frac{e^{(c-b)x} - e^{(b-c)x}}{e^{cx} - e^{-cx}} x^{2a} dx &= \frac{(-1)^{a+1}}{c} (2\pi)^{2a+1} \sum_1^{c-1} B'' \left(\frac{n}{2c}\right) \text{Sin.} \frac{n b \pi}{c} \text{ V. T. 158. N}^\circ. 11.
 \end{aligned}$$

$$\begin{aligned}
 1) \int \frac{x}{(e^x - e^{-x})^2} dx &= \frac{1}{4} l 2 \quad \text{Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Lobatschewsky, Mém. Kasan. 1836. I. I. form. (100)} \\
 2) \int \frac{x}{(e^{px} + e^{-px})^2} dx &= \frac{1}{4 p^2} l 2 \quad \text{Lobatschewsky, Mém. Kasan. 1836. I. I. form. (100), II. form. (20).} \\
 3) \int \frac{e^{px} - e^{-px}}{(e^{px} + e^{-px})^2} x dx &= \frac{\pi}{4 p^2} \text{ V. T. 38. N}^\circ. 8. \\
 4) \int \frac{e^x - e^{-x}}{(e^x + e^{-x})^{2p+1}} x dx &= \frac{\{\Gamma(p)\}^2}{8 p \Gamma(2p)}, p < 1; \text{ V. T. 39. N}^\circ. 9. \\
 5) \int \frac{e^{px} - e^{-px}}{(e^{px} + e^{-px})^3} x^2 dx &= \frac{1}{4 p^3} l 2 \text{ V. T. 122. N}^\circ. 2. \\
 6) \int \frac{e^{ax} + e^{-ax}}{(e^{ax} - e^{-ax})^2} x^2 dx &= \frac{\pi}{4 a^3} \text{ V. T. 120. N}^\circ. 17. \\
 7) \int \frac{(p+q)(e^{(p+q)x} - e^{-(p+q)x}) + (p-q)(e^{(p-q)x} - e^{-(p-q)x})}{(e^{px} + e^{-px})^2} x^2 dx &= \frac{\pi^2}{2 p^2} \text{Sin.} \frac{q\pi}{2p} \text{Sec.} \frac{q\pi}{2p}, p < q; \text{ V. T. 121. N}^\circ. 5. \\
 8) \int \frac{(p-q)(e^{(p-q)x} - e^{-(p-q)x}) - (p+q)(e^{(p+q)x} - e^{-(p+q)x})}{(e^{px} - e^{-px})^2} x^2 dx &= \frac{\pi^2}{2 p^2} \text{Sec.} \frac{q\pi}{2p}, p < q; \text{ V. T. 121. N}^\circ. 6.
 \end{aligned}$$

- 9) $\int \frac{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x})^2} x^{2a+1} dx = \frac{2a+1}{\pi} B_{2a} \quad \text{V. T. 120. N}^\circ. 14.$
- 10) $\int \frac{e^{\pi x} - e^{-\pi x}}{(e^{\pi x} + e^{-\pi x})^2} x^{2a+1} dx = \frac{2a+1}{\pi} \frac{2^{2a+1} - 1}{(2\pi)^{2a+1}} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}} \quad \text{V. T. 120. N}^\circ. 13.$
- 11) $\int \frac{x^{2a}}{(e^{\pi x} - e^{-\pi x})^2} dx = \frac{1}{4\pi} B_{2a-1} \quad \text{V. T. 117. N}^\circ. 22.$
- 12) $\int \frac{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})^2} x^{2a} dx = \frac{2^{2a} - 1}{\pi} 2^{2a+1} B_{2a-1} \quad \text{V. T. 120. N}^\circ. 20.$
- 13) $\int \frac{e^{\pi x} + e^{-\pi x}}{(e^{\pi x} - e^{-\pi x})^2} x^{2a} dx = \frac{2^{2a} - 1}{2\pi} B_{2a-1} \quad \text{V. T. 120. N}^\circ. 18.$
- 14) $\int \frac{e^{qx} - e^{-qx}}{(e^{qx} + e^{-qx})^2} x^{2a+1} dx = \frac{2a+1}{2q} (-1)^{a+1} \left(\frac{2\pi}{q}\right)^{2a+1} B' \left(\frac{1}{1}\right) \quad \text{V. T. 120. N}^\circ. 15.$
- 15) $\int \frac{e^{qx} + e^{-qx}}{(e^{qx} - e^{-qx})^2} x^{2a} dx = \frac{a}{2q} (-1)^a \left(\frac{2\pi}{q}\right)^{2a} B' \left(\frac{1}{2}\right) \quad \text{V. T. 120. N}^\circ. 19.$
- 16) $\int \frac{e^x - e^{-x}}{(e^x + e^{-x})^2} x^n dx = p \Gamma(p) \sum_0^\infty \frac{(-1)^n}{(2n+1)^p} \quad \text{V. T. 120. N}^\circ. 11.$

- 1) $\int \frac{(1+xi)^{2a} - (1-xi)^{2a}}{i} \frac{dx}{e^{\pi x} + 1} = \frac{1}{2^{2a+1}}$
- 2) $\int \frac{(1+xi)^{2a-1} - (1-xi)^{2a-1}}{i} \frac{dx}{e^{\pi x} + 1} = \frac{1}{2^a} \left\{ 1 + (-1)^a 2^{2a} B_{2a-1} \right\}$
- 3) $\int \frac{(1+xi)^{2a} - (1-xi)^{2a}}{i} \frac{dx}{e^{2\pi x} - 1} = \frac{1}{2} \frac{2^a - 1}{2^{2a+1}}$
- 4) $\int \frac{(1+xi)^{2a-1} - (1-xi)^{2a-1}}{i} \frac{dx}{e^{2\pi x} - 1} = \frac{a-1}{2^a} + (-1)^{a+1} \frac{1}{2^a} B_{2a-1}$
- 5) $\int \frac{(1+xi)^{2a-1} - (1-xi)^{2a-1}}{i} \frac{dx}{e^{\pi x} - 1} = 2^a - 1 + (-1)^a \frac{2^{2a-1} - 1}{2^a} B_{2a-1}$
- 6) $\int \frac{(1+xi)^{2a} - (1-xi)^{2a}}{i} \frac{dx}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} = (-1)^{a+1} B_{2a} + 1$

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55. -- Schlömilch.
Gr. 3. 9.

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$$7) \int \frac{(1+x i)^{2a-1} - (1-x i)^{2a-1}}{i} \frac{dx}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} = 1 \quad \text{Schlömlich, Gr. 3. 9.}$$

$$8) \int \frac{(1+x i)^{2a-1} + (1-x i)^{2a-1}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} dx = (-1)^{a+1} \frac{2^{2a}-1}{2a} 2^{2a} B_{2a+1} \quad \text{Schlömlich, Gr. 1. 360.}$$

$$1) \int \frac{x}{e^x + e^{-x} - 1} dx = \frac{4}{27} \pi^2 \quad \text{V. T. 153. N°. 3.}$$

$$2) \int \frac{x}{e^{2x} + e^{-2x} - 1} dx = \frac{1}{27} \pi^2 \quad \text{V. T. 153. N°. 7.}$$

$$3) \int \frac{x}{e^x + e^{-x} - 2 \cos. \lambda} dx = \frac{\lambda \log 2 - \text{L}(\lambda)}{\sin. \lambda \cos. \lambda} \quad \text{Lobatschewsky, Mém. Kasan. 1836. 1. II. form. (31).}$$

$$4) \int \frac{x}{e^{2x} + e^{-2x} - 2 \cos. 2\lambda} dx = \frac{1}{2 \sin. 2\lambda} \left\{ \left(\frac{1}{2} \pi - \lambda \right) \log 2 - \text{L} \left(\frac{1}{2} \pi - \lambda \right) \right\} \quad \text{Lobatschewsky, Mém. Kasan. 1836. 1. I. form. (95).}$$

$$5) \int \frac{x^2}{e^x + e^{-x} + 1} dx = \frac{8}{243} \pi^3 \log 3 \quad \text{V. T. 156. N°. 1.}$$

$$6) \int \frac{x^2}{e^x + e^{-x} - 1} dx = \frac{10}{243} \pi^3 \log 3 \quad \text{V. T. 156. N°. 2.}$$

$$7) \int \frac{x^2}{e^x + e^{-x} + 2 \cos. \lambda} dx = \frac{\lambda}{2 \sin. \lambda} \frac{\pi^2 - \lambda^2}{3} \quad \text{V. T. 156. N°. 3.}$$

$$8) \int \frac{x^2}{e^x + e^{-x} - 2 \cos. \lambda} dx = \frac{2\lambda}{\sin. \lambda} \left\{ \frac{1}{6} \pi^2 - \frac{1}{4} \pi \lambda + \frac{1}{12} \lambda^2 \right\} \quad \text{V. T. 156. N°. 4.}$$

$$9) \int \frac{x^4}{e^x + e^{-x} - 2 \cos. \lambda} dx = \frac{\lambda}{\sin. \lambda} \frac{\pi^2 - \lambda^2}{3} \frac{7\pi^2 - 3\lambda^2}{5} \quad \text{V. T. 156. N°. 5.}$$

$$10) \int \frac{x^{2a}}{e^x + e^{-x} - 2 \cos. 2p\pi} dx = \frac{(-1)^{a+1}}{2 \sin. 2p\pi} (2\pi)^{2a+1} B''(p), p < 1; \quad \text{Raabe, Cr. 42. 348.}$$

$$11) \int \frac{x^{2a}}{e^x + e^{-x} + 1} dx = \frac{(-1)^{a+1}}{\log 3} (2\pi)^{2a+1} B'' \left(\frac{1}{3} \right) \quad \text{V. T. 159. N°. 1.}$$

$$12) \int \frac{x^{2a}}{e^x + e^{-x} - 1} dx = \frac{(-1)^{a+1}}{\log 3} (2\pi)^{2a+1} B'' \left(\frac{1}{6} \right) \quad \text{V. T. 159. N°. 2.}$$

- 1) $\int \frac{1 + 2e^{-x}}{e^x + e^{-x} + 1} x dx = \frac{1}{9} \pi^2$ V. T. 153. N°. 1.
- 2) $\int \frac{1 - 2e^{-x}}{e^x + e^{-x} - 1} x dx = \frac{1}{18} \pi^2$ V. T. 153. N°. 2.
- 3) $\int \frac{e^x \text{Cos. } \lambda - 1}{e^{2x} + 1 - 2e^x \text{Cos. } \lambda} x dx = \frac{1}{6} \pi^2 - \frac{1}{2} \pi \lambda + \frac{1}{4} \lambda^2$ Cauchy, Sav. Etr. 1827. 599. P. 1. § 5.
- 4) $\int \frac{e^x - e^{-x}}{e^{2x} + e^{-2x} + 2p} x dx = \frac{\pi}{2 \sqrt{\{2(p-1)\}}} l \frac{\sqrt{(p-1)} + \sqrt{(p+1)} - \sqrt{2}}{\sqrt{(p-1)} - \sqrt{(p+1)} + \sqrt{2}}$
- 5) $\int \frac{e^x - e^{-x}}{e^{2x} + e^{-2x} + e^{2p} + e^{-2p}} x dx = \frac{p}{2} \frac{\pi}{e^p - e^{-p}}$
- 6) $\int \frac{e^x - e^{-x}}{e^{2x} + e^{-2x} + 2 \text{Cos. } 2\lambda} x dx = \frac{1}{4} \pi \lambda \text{Cosec. } \lambda$
- 7) $\int \frac{e^x + e^{-x}}{e^{2x} + e^{-2x} - 2 \text{Cos. } 2\lambda} x dx = \frac{1}{\text{Sin. } \lambda} \left\{ \frac{1}{2} \pi l 2 - L \left(\frac{\lambda}{2} \right) - L \left(\frac{2\pi - \lambda}{2} \right) \right\}$ Lobatschewsky, Mém. Kasan. 1836. I. I. form. (103), II form. (33).
- 8) $\int \frac{e^x}{e^{2x} + 1 - 2e^x \text{Cos. } \lambda} x^2 dx = \left(\frac{1}{3} \pi^2 \lambda + \frac{1}{2} \pi \lambda^2 + \frac{1}{6} \lambda^3 \right) \text{Cosec. } \lambda$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.
- 9) $\int \frac{\text{Cos. } 2p\pi - e^{-x}}{e^x + e^{-x} - 2 \text{Cos. } 2p\pi} x^{2a+1} dx = \frac{(-1)^a}{2} (2\pi)^{2a+2} B'(p) + \frac{(2\pi)^{2a+2}}{2(2a+2)} B_{2a+1} \cdot p < 1;$ Raabe, Cr. 42. 348.
- 10) $\int \frac{\text{Cos. } \lambda - p e^{-x}}{e^x + p^2 e^{-x} - 2p \text{Cos. } \lambda} e^{(1-q)x} x^{r-1} dx = \Gamma(r) \sum_1 \frac{p^{n-1} \text{Cos. } n\lambda}{(q+n-1)^r}$ V. T. 159. N°. 7.
- 11) $\int \frac{e^x - e^{-x}}{(e^x - 2 \text{Cos. } \lambda + e^{-x})^2} x dx = \frac{\pi - \lambda}{2 \text{Sin. } \lambda}$ V. T. 39. N°. 1.
- 12) $\int \frac{e^x - e^{-x}}{(e^x + e^{-x} - 1)^2} x^2 dx = \frac{8}{27} \pi^3$ V. T. 124. N°. 1.
- 13) $\int \frac{e^x - e^{-x}}{(e^x + e^{-x} + 1)^2} x^3 dx = \frac{8}{81} \pi^3 \sqrt{3}$ V. T. 124. N°. 5.
- 14) $\int \frac{e^x - e^{-x}}{(e^x + e^{-x} - 1)^2} x^3 dx = \frac{10}{81} \pi^3 \sqrt{3}$ V. T. 124. N°. 6.
- 15) $\int \frac{e^x - e^{-x}}{(e^x + 2 \text{Cos. } \lambda + e^{-x})^2} x^3 dx = \lambda \frac{\pi^2 - \lambda^2}{2 \text{Sin. } \lambda}$ V. T. 124. N°. 7.

$$16) \int \frac{e^x - e^{-x}}{(e^x + e^{-x} + 1)^2} x^{2a+1} dx = \frac{2a+1}{1 \swarrow 3} (-1)^{a+1} (2\pi)^{2a+1} B'' \left(\frac{1}{3} \right) \quad \text{V. T. 124. N}^\circ 11.$$

$$17) \int \frac{e^x - e^{-x}}{(e^x + e^{-x} - 1)^2} x^{2a+1} dx = \frac{2a+1}{1 \swarrow 3} (-1)^{a+1} (2\pi)^{2a+1} B'' \left(\frac{1}{6} \right) \quad \text{V. T. 124. N}^\circ 12.$$

$$1) \int \frac{e^{-x}}{x} dx = \infty \quad \text{Cisa de Grésky, Mém. Turin. 1821. 209. I. 26.}$$

$$2) \int \frac{e^{-px}}{x} dx = -A - lo - lp = \infty \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N}^\circ 20. - \text{Cisa de Grésky, ib. 1821. 209. II. 34.}$$

$$3) \int \frac{e^{-x^2}}{x^2} dx = -1 \swarrow \pi \quad \text{Kramp, Réfr. 3. 72. — Oettinger, Cr. 35. 13.}$$

$$\left. \begin{aligned} 4) \int \frac{e^{-x^2}}{x^4} dx &= \frac{2}{3} 1 \swarrow \pi \\ 5) \int \frac{e^{-x^2}}{x^6} dx &= -\frac{4}{3 \cdot 5} 1 \swarrow \pi \\ 6) \int \frac{e^{-x^2}}{x^8} dx &= \frac{8}{3 \cdot 5 \cdot 7} 1 \swarrow \pi \\ 7) \int \frac{e^{-x^2}}{x^{2a}} dx &= \frac{(-1)^a 2^{a-1} 1 \swarrow \pi}{1^{a/2}} \end{aligned} \right\} \text{Kramp, Réfr. 3. 72.}$$

$$8) \int \frac{e^x}{x^p} dx = \frac{\pi}{\Gamma(p)} \text{Cosec. } p\pi \quad \text{Cauchy, P. 28. 147. P. 1. § 2.}$$

$$9) \quad = \Gamma(1-p) \quad \text{Svanberg, Transf. 3.}$$

$$\left. \begin{aligned} 10) \int \frac{e^{-px}}{x^{a+1}} dx &= p^a \Gamma(-a) \\ 11) \int \frac{e^{-x}}{x^{a+1}} dx &= \Gamma(-a) \end{aligned} \right\} \begin{array}{l} \text{, valeurs extraordinaires;} \\ \text{Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 38.} \end{array}$$

$$12) \int \frac{e^{-x^a}}{x^{b+1}} dx = -\frac{1 - \frac{b}{a}}{b} \quad \text{Oettinger, Cr. 35. 13.}$$

- 13) $\int \frac{e^{-x^a}}{x^{ab+1}} dx = \infty$
- 14) $\int \frac{e^{-x^{-a}}}{x^{b+1}} dx = -\frac{a}{b} \cdot \frac{2a}{a+b} \cdot \frac{3a}{2a+b} \dots$
- 15) $\int e^{xi} \frac{dx}{x^p} = \frac{q}{1-p} (-1)^{\frac{1-p}{2}}$, où $q = 0, 9 \ 0 \ 6 \ 4 \ 0 \ 2$; Laplace, P. 15. 229.
- 16) $\int e^{-q^2 x^2 - \frac{1}{x^2}} \frac{dx}{x^2} = \frac{1}{2} e^{-2q} \sqrt{\pi}$ Bonnet, L. 14. 249.
- 17) $\int e^{-(px^2 + \frac{q}{x^2})i} \frac{dx}{x^{a+1}} = \left(\frac{p}{q}\right)^{\frac{a-1}{2}} e^{-2i\sqrt{pq}} \frac{1-i}{2} \sqrt{\frac{\pi}{2p}} \sum_0^{\infty} \frac{(a-2n-1)^{2n/2}}{4^{n/4}} \frac{1}{(4i\sqrt{pq})^n}$ Cauchy, P. 19. 511.

- 1) $\int \frac{1-e^{px}}{x} e^{-x} dx = -l(1-p)$, $p^2 < 1$; Dienger, Cr. 46. 119.
- 2) $\int \frac{e^{-px}-1}{x} e^{-x} dx = l \frac{1}{1+p}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 36.
- 3) $\int \frac{e^{-x}-e^{-px}}{x} dx = lp$ Lejeune-Dirichlet, Cr. 15. 253. — Liouville, L. 4. 317. — Grunert, Gr. 2. 266. — Arndt, Gr. 10. 253. — Schlömilch, Stud. I. § 6.
- 4) $\int \frac{e^{-px}-e^{-qx}}{x} dx = l \frac{q}{p}$ Cauchy, Cours. Leç. 33. — Id., Exerc. 1827. p. 112. — Id., C. R. 16. 422. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 20. — Cisa de Grésy, ib. 1821. 209. II. 34. — Pioch, Mém. Cour. Bruxelles. T. 15. P. 2. — Grunert, Gr. 2. 266. — Schlömilch, Stud. I. § 6.
- 5) $\int \frac{e^{-px}-e^{-qx}}{x} e^{-rx} dx = l \frac{q+ri}{p+ri}$ Meyer, Int. Déf. 109.
- 6) $\int e^{-ax} (e^{-x}-1)^b \frac{dx}{x} = -\Delta^b l a$ Laplace, Prob. I. 41.
- 7) $\int \frac{e^{-(q+r)x}-e^{-qx}-e^{-rx}+1}{x} e^{-px} dx = l \frac{(p+q)(p+r)}{p(p+q+r)}$ V. T. 167. N°. 7.
- 8) $\int \frac{(e^{-px}-e^{-qx})(e^{-rx}-e^{-sx})}{x} e^{-x} dx = l \frac{(p+s+1)(q+r+1)}{(p+r+1)(q+s+1)}$ V. T. 167. N°. 8.
- 9) $\int \frac{e^{-x}-e^{-ax}}{x} e^{-px} x^{q-1} dx = \frac{(p+a)^{1-q}-(p+1)^{1-q}}{1-q} \Gamma(q)$ Cauchy, C. R. 16. 422.

$$10) \int \frac{(e^{-qx} - 1)^2}{x^2} e^{-x} dx = (2q + 1)l(2q + 1) - 2(q + 1)l(q + 1) \quad \text{V. T. 168. N}^\circ. 4.$$

$$11) \int \frac{(e^{-qx} - 1)^2}{x^2} e^{-px} dx = (p + 2q)l(p + 2q) - 2(p + q)l(p + q) + plp \quad \text{V. T. 168. N}^\circ. 5.$$

$$12) \int e^{-px} (e^{-x} - 1)^a \frac{dx}{x^2} = \Delta^a . plp \quad \text{Meyer, Int. Déf. 181.}$$

$$13) \int \frac{(b-c)e^{-ax} + (c-a)e^{-bx} + (a-b)e^{-cx}}{x^2} dx = (b-a)ala + (c-a)blb + (a-b)c lc \quad \text{V. T. 168. N}^\circ. 6.$$

$$14) \int e^{-px} (e^{-x} - 1)^b \frac{dx}{x^{q+1}} = \frac{\pi}{\Gamma(q+1)} \text{Cosec.}\{(q+1)\pi\} \Delta^b p^q \quad \text{Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 58; pour } q < b, \text{ mais valeur extraordinaire pour } q > b.$$

$$15) \int \frac{(-1)^{q+1}}{1^{q!}} \Delta^b . p^q lp, \text{ pour } q \text{ entier, } < b; \text{ Cauchy, Exerc. 1826. p. 58.}$$

$$16) \int \left\{ \left(\frac{1}{x} + \frac{1}{2} \right) e^{-x} - \frac{1}{x} e^{-\frac{1}{2}x} \right\} \frac{dx}{x} = \frac{1}{2} (l2 - 1)$$

$$17) \int \left\{ e^{-x} + \frac{e^{-x}}{x} - \frac{1}{x} \right\} \frac{dx}{x} = 1$$

$$18) \int \left\{ p e^{-x} + \frac{e^{-px}}{x} - q e^{-x} - \frac{e^{-qx}}{x} \right\} \frac{dx}{x} = plp - p - qlq + q$$

Stern, Gött. Stud. 1847.

$$19) \int \left\{ (p - \frac{1}{2}) e^{-x} + \frac{x+2}{2x} (e^{-px} - e^{-\frac{1}{2}x}) \right\} \frac{dx}{x} = (p - \frac{1}{2})(lp - 1) \quad \text{Meyer, Int. Déf. 121.}$$

$$20) \int \left\{ 1 - \frac{x+2}{2x} (1 - e^{-x}) \right\} e^{-qx} \frac{dx}{x} = -1 + \left(q + \frac{1}{2} \right) l \frac{q+1}{q} \quad \text{Cauchy, C. R. 16. 422.}$$

$$21) \int \left(\frac{e^{-x} - e^{-2x}}{x} - \frac{e^{-2x}}{x^2} \right) dx = 1 - l2. \quad \text{Meyer, Int. Déf. 123.}$$

$$22) \int \frac{(1 - e^{-x})^2}{x^2} e^{-2x} dx = 2l \frac{32}{27} \quad \text{V. T. 168. N}^\circ. 2.$$

$$23) \int (e^{-x} - 1)^2 \frac{e^{-qx}}{x^2} dx = (q+2)l(q+2) - 2(q+1)l(q+1) + qlq \quad \text{V. T. 168. N}^\circ. 3.$$

$$24) \int \frac{(1 - e^{-px})(1 - e^{-qx})(1 - e^{-rx})}{x^2} e^{-x} dx = (p+q+1)l(p+q+1) + (p+r+1)l(p+r+1) + (q+r+1)l(q+r+1) - (p+1)l(p+1) - (q+1)l(q+1) - (r+1)l(r+1) - (p+q+r)l(p+q+r) \quad \text{V. T. 168. N}^\circ. 8.$$

$$25) \int (1 - e^{-px})^a \frac{e^{-x}}{x^2} dx = \sum_1^{\infty} \binom{a}{2n} (2np+1) l(2np+1) - \sum_1^{\infty} \binom{a}{2n-1} \{(2n-1)p+1\} l\{(2n-1)p+1\} \quad \text{V. T. 168. N}^\circ 9.$$

$$26) \int (1 - e^{-px})^a \frac{e^{-qx}}{x^2} dx = \sum_0^{\infty} (-1)^n \binom{a}{n} (q+np) l(q+np) \quad \text{V. T. 168. N}^\circ 10.$$

$$27) \int \left\{ \frac{e^{-ax}}{(a-b)(a-c)(a-d)} + \frac{e^{-bx}}{(b-a)(b-c)(b-d)} + \frac{e^{-cx}}{(c-a)(c-b)(c-d)} + \frac{e^{-dx}}{(d-a)(d-b)(d-c)} \right\} \frac{dx}{x^3} = \frac{a^2 l a}{(a-b)(a-c)(a-d)} + \frac{b^2 l b}{(b-a)(b-c)(b-d)} + \frac{c^2 l c}{(c-a)(c-b)(c-d)} + \frac{d^2 l d}{(d-a)(d-b)(d-c)} \quad \text{V. T. 168. N}^\circ 11.$$

$$28) \int (1 - e^{-px})^a \frac{e^{-x}}{x^3} dx = \frac{1}{2} \sum_1^{\infty} (-1)^{n-1} \binom{a}{n} (pn+1)^2 l(pn+1) \quad \text{V. T. 168. N}^\circ 13.$$

$$29) \int (1 - e^{-px})^a \frac{e^{-qx}}{x^3} dx = \frac{1}{2} \sum_1^{\infty} (-1)^{n-1} \binom{a}{n} (q+np)^2 l(q+np) \quad \text{V. T. 168. N}^\circ 14.$$

$$30) \int \frac{(1 - e^{-px})^a (1 - e^{-qx})}{x^3} e^{-x} dx = \frac{1}{2} \sum_0^{\infty} (-1)^n \binom{a}{n} (q+pn+1)^2 l(q+pn+1) + \frac{1}{2} \sum_1^{\infty} (-1)^{n-1} \binom{a}{n} (pn+1)^2 l(pn+1) \quad \text{V. T. 168. N}^\circ 15.$$

$$\left. \begin{aligned} 31) \int \left\{ \frac{q e^{-x}}{x} - \frac{1 - e^{-qx}}{x^2} \right\} dx &= q l q - q \\ 32) \int \left\{ \frac{q^2 e^{-x}}{2x} - \frac{q}{x^2} + \frac{1 - e^{-qx}}{x^3} \right\} dx &= \frac{1}{2} q^2 l q - \frac{3}{4} q^2 \\ 33) \int \left\{ \frac{q^2 e^{-x}}{6x} - \frac{q^2}{2x^2} + \frac{q}{x^3} - \frac{1 - e^{-qx}}{x^4} \right\} dx &= \frac{1}{6} q^2 l q - \frac{11}{36} q^3 \end{aligned} \right\} \text{Sohnke, Samml.}$$

$$\left. \begin{aligned} 1) \int \frac{e^{-qx} - e^{-rx}}{x^{p+1}} dx &= \frac{\Gamma(1-p)}{p} (r^p - q^p), \quad p < 1; \\ 2) \int \frac{e^{-ax^c} - e^{-bx^c}}{x^c} dx &= \frac{\Gamma\left(\frac{1}{c}\right)}{c-1} \left\{ b^{\frac{c-1}{c}} - a^{\frac{c-1}{c}} \right\}, \quad b > a > 0; \\ 3) \int e^{-bx} (e^{-x} - 1)^c \frac{dx}{x^{q+1}} &= -\frac{\pi}{\sin q \pi \Gamma(q+1)} \Delta^c b q, \quad a < c; \\ 4) \int &= -\frac{(-1)^{q+c}}{\Gamma(q+1)} \Delta^c b q l b, \text{ pour } q \text{ entier;} \end{aligned} \right\} \begin{aligned} &\text{Lindmann, Stockh. Handl. 1850. IV.} \\ &\text{Cauchy, P. 28. 147. P. 1.} \\ &\text{§ 2. — Laplace, Prob. 41. —} \\ &\text{Id., Mém. Acad. 1781. 29.} \end{aligned}$$

$$5) \int \left(\frac{e^{-qx} - 1}{x} \right)^c e^{-px} dx = \frac{(-1)^c}{1^{c-1/1}} \frac{\Delta^c}{\Delta p} p^{c-1} l p, \text{ après la différentiation mettez } \Delta p = q; \text{ V. T. 168. N}^\circ 17.$$

$$6) \int \frac{(e^{-rx} - 1)^c}{x^{q+1}} e^{-px} dx = \frac{(-1)^c r^q}{\Gamma(q+1)} \Delta^c p^q l p \quad \text{V. T. 168. N}^\circ 18.$$

$$7) \int \frac{e^{-bx} (e^{-x} - 1)^c - (-x)^c}{x^{q+1}} dx = -\frac{\pi}{\Gamma(q+1)} \operatorname{Cosec}. q \pi \Delta^c b^q, \quad q < c;$$

$$8) \int \frac{e^{-bx} (e^{-x} - 1)^{c-1} - (-x)^{c-1} \left(1 - \frac{2b+c-1}{2} x \right)}{x^{q+1}} dx = -\frac{\pi}{\Gamma(q+1)} \operatorname{Cosec}. q \pi \Delta^{c-1} b^q, \quad c < q < c+1;$$

$$9) \int \frac{e^{-bx} (e^{-x} - 1)^{c-2} - (-x)^{c-2} \left(1 - \frac{2b+c-2}{2} x + \frac{6b(b+c-2) + (c-2)(3c-7)}{12} x^2 \right)}{x^{q+1}} dx =$$

$$= -\frac{\pi}{\Gamma(q+1)} \operatorname{Cosec}. q \pi \Delta^{c-2} b^q, \quad c < q < c+1;$$

Cau-
chy,
P. 18.
147.
P. 3.
§. 2.

$$1) \int \frac{e^{-px}}{x+1} dx = -e^p li.(e^{-p}) \quad \text{Schlömilch, Beitr. III. 5. — Id., Gr. 5. 204.}$$

$$2) \int \frac{e^{-x}}{x+q} dx = -e^q li.(e^{-q}) \quad \text{Winckler, Cr. 45. 102. — Schlömilch, Stud. I. 18. — Id., Gr. 5. 204.}$$

$$3) \int \frac{e^{-px}}{x+q} dx = -e^{pq} li.(e^{-pq}) \quad \text{Schlömilch, Stud. I. 18.}$$

$$4) \quad = -e^{pq} Ei.(-pq) \quad \text{Arndt, Gr. 10. 247.}$$

$$5) \int \frac{e^{pxi}}{x+q} dx = \pi e^{-pq} + i e^{-pq} li.(e^{pq}) \quad \text{Meyer, Int. Déf. 264.}$$

$$6) \int \frac{e^{-px}}{x+q} x^a dx = (-1)^{a+1} q^a e^{pq} Ei.(-pq) + \frac{1}{p^a} \sum_1^a 1^{a-n/1} (-pq)^{n-1} \quad \text{Bierens de Haan, Verh. K. Akad. v. W. Dl. II. blad 19.}$$

$$7) \int \frac{e^{-px}}{1-x} dx = e^{-p} li.(e^p) \quad \text{Schlömilch, Beitr. III. 5. — Id., Gr. 5. 204.}$$

$$8) \int \frac{e^{-x}}{q-x} dx = e^{-q} li.(e^q) \quad \text{Schlömilch, Gr. 5. 204.}$$

- 9) $\int \frac{e^{-px}}{q-x} dx = e^{-pq} \text{li.}(e^{pq})$ Schlömilch, Stud. II. 20.
- 10) $= e^{-pq} \left\{ \frac{1}{2} l \alpha^2 + E i.(pq) \right\}$, où α indéterminé;
Arndt, Gr. 10. 247.
- 11) $\int \frac{e^{pxi}}{q+xi} dx = i e^{pq} \text{li.}(e^{-pq})$ Meyer, Int. Déf. 264.
- 12) $\int \frac{e^{-px}}{x-q} x^a dx = -q^a e^{-pq} E i.(-pq) + \frac{1}{p^a} \sum_1^a 1^{a-n+1} (-pq)^{n-1}$ Bierens de Haan, Verh. v. K.
Ak. v. Wet. Dl. II. blad 19.
- 13) $\int \frac{e^{-x}}{lp-x} dx = \frac{1}{p} \text{li.}(p)$ Schlömilch, Gr. 5. 204.

- 1) $\int \frac{x e^{-px}}{1+x^2} dx = \sum_0^\infty (-1)^n \frac{1^{2n+1/1}}{p^{2n+2}} \left\{ \begin{array}{l} \text{Bidone, Mém. Turin. 1812. 231. Art. 2. 33.} \\ \text{Arndt, Gr. 10. 225. — Schlömilch, Cr. 33. 325.} \end{array} \right.$
- 2) $\int \frac{e^{-px}}{1+x^2} dx = \sum_0^\infty (-1)^n \frac{1^{2n/1}}{p^{2n+1}} \left\{ \begin{array}{l} \text{Arndt, Gr. 10. 225. — Schlömilch, Cr. 33. 325.} \\ \text{— Id., Stud. II. 21. — Arndt, Gr. 10. 225.} \end{array} \right.$
- 3) $= \text{Sin. } q. \text{Ci.}(q) + \text{Cos. } q \left\{ \frac{1}{2} \pi - \text{Si.}(q) \right\}$ Arndt, Gr. 10. 225. — Schlömilch, Cr. 33. 325.
- 4) $\int \frac{e^{-px}}{q^2+x^2} dx = \frac{1}{q} \left\{ \text{Ci.}(pq). \text{Sin. } pq - \text{Si.}(pq). \text{Cos. } pq + \frac{1}{2} \pi \text{Cos. } pq \right\}$, $0 < q < \infty$; Schlömilch, Cr. 33. 325. — Id., Stud. II. 21. — Arndt, Gr. 10. 225.
- 5) $\int \frac{x e^{-px}}{q^2+x^2} dx = -\text{Ci.}(pq). \text{Cos. } pq - \text{Si.}(pq). \text{Sin. } pq + \frac{1}{2} \pi \text{Sin. } pq$, $0 < q < \infty$; Arndt, Gr. 10. 225. — Schlömilch, Stud. II. 21. — Id., Cr. 33. 325, la trouvait fautivement négative.
- 6) $\int \frac{e^{pxi}}{q^2+x^2} dx = \frac{\pi}{2q} e^{-pq} - \frac{1}{2qi} \left\{ e^{-pq} \text{li.}(e^{pq}) - e^{pq} \text{li.}(e^{-pq}) \right\}$ Meyer, Int. Déf. 267.
- 7) $\int \frac{x e^{pxi}}{q^2+x^2} dx = \frac{\pi i}{2} e^{-pq} - \frac{1}{2} \left\{ e^{-pq} \text{li.}(e^{pq}) + e^{pq} \text{li.}(e^{-pq}) \right\}$
- 8) $\int \frac{e^{-px}}{x^2+q^2} x^{2a} dx = (-1)^a q^{2a-1} \left\{ \text{Ci.}(pq). \text{Sin. } pq - \text{Si.}(pq). \text{Cos. } pq + \frac{1}{2} \pi \text{Cos. } pq \right\} + \frac{1}{p^{2a-1}} \sum_1^a 1^{2a-2n+1} (-p^2 q^2)^{n-1}$
- 9) $\int \frac{e^{-px}}{x^2+q^2} x^{2a+1} dx = (-1)^{a+1} q^{2a} \left\{ \text{Ci.}(pq). \text{Cos. } pq + \text{Si.}(pq). \text{Sin. } pq - \frac{1}{2} \pi \text{Sin. } pq \right\} + \frac{1}{p^{2a}} \sum_1^a 1^{2a-2n+1/1} (-p^2 q^2)^{n-1}$

Bierens de Haan, Verh. v. K. Akad. v. Wet. Dl. II. blad 19.

- 10) $\int \frac{e^{-px}}{q^2 - x^2} dx = \frac{1}{2q} \{e^{-pq} \text{li.}(e^{pq}) - e^{pq} \text{li.}(e^{-pq})\}$ Schlömilch, Cr. 33. 325. — Id., Stud. II. 20.
- 11) $= \frac{1}{2q} \left\{ -e^{pq} \text{Ei.}(-pq) - e^{-pq} \left(\frac{1}{2} l \alpha^2 - \text{Ei.}(pq) \right) \right\}$, où α indéterminé; Arndt, Gr. 10. 247.
- 12) $\int \frac{x e^{-px}}{q^2 - x^2} dx = \frac{1}{2} \{e^{-pq} \text{li.}(e^{pq}) + e^{pq} \text{li.}(e^{-pq})\}$ Schlömilch, Cr. 33. 325. — Id., Stud. II. 20.
- 13) $= \frac{1}{2} \left\{ e^{pq} \text{Ei.}(-pq) - e^{-pq} \left(\frac{1}{2} l \alpha^2 - \text{Ei.}(pq) \right) \right\}$, où α indéterminé; Arndt, Gr. 10. 247.
- 14) $\int \frac{e^{-px}}{x^2 - q^2} x^{2a} dx = \frac{1}{2} q^{2a-1} \{e^{pq} \text{Ei.}(-pq) - e^{-pq} \text{Ei.}(pq)\} + \frac{1}{p^{2a-1}} \sum_1^a 1^{2a-2n+1} (p^2 q^2)^{n-1}$ $\left\{ \begin{array}{l} \text{Bierens de} \\ \text{Haan, Verh.} \\ \text{v. K. Ak. v.} \\ \text{Wet. Dl. II.} \\ \text{blad 19.} \end{array} \right.$
- 15) $\int \frac{e^{-px}}{x^2 - q^2} x^{2a+1} dx = -\frac{1}{2} q^{2a} \{e^{pq} \text{Ei.}(-pq) + e^{-pq} \text{Ei.}(pq)\} + \frac{1}{p^{2a}} \sum_1^a 1^{2a-2n+1} (p^2 q^2)^{n-1}$ $\left\{ \begin{array}{l} \text{Bierens de} \\ \text{Haan, Verh.} \\ \text{v. K. Ak. v.} \\ \text{Wet. Dl. II.} \\ \text{blad 19.} \end{array} \right.$

- 1) $\int \frac{e^{-x}}{(q+x)^2} dx = \frac{1}{q} + e^q \text{li.}(e^{-q})$ V. T. 43. N°. 18.
- 2) $\int \frac{e^{-x}}{(q-x)^2} dx = -\frac{1}{q} + e^{-q} \text{li.}(e^q)$ V. T. 43. N°. 19.
- 3) $\int \frac{e^{-ax} x^{p-1}}{(1+x)^b} dx = \frac{\Gamma(p)}{a^p} \chi(p, b, a)$ Kummer, Cr. 17. 228. — Boncompagni, Cr. 25. 74.
- 4) $\int \frac{e^{-ax^c} x^{p-1}}{(1+x)^b} dx = \frac{\Gamma(p)}{c a^p} \chi(p, b, a)$ Boncompagni, Cr. 25. 74. Elle ne vaut que pour $c=1$.
- 5) $\int \frac{e^{-px} x^a}{(x+q)^{c+1}} dx = \frac{(-1)^{a+c+1}}{1^{c+1}} \frac{d^{a+c}}{d p^a, d q^c} \{e^{pq} \text{li.}(e^{-pq})\}$ Schlömilch, Stud. I. 18.
- 6) $\int \frac{e^{-px}}{(x+q)^a} dx = (-1)^a \frac{p^{a-1}}{1^{a-1/1}} e^{pq} \text{Ei.}(-pq) + \frac{1}{1^{a-1/1} q^{a-1}} \sum_1^{a-1} 1^{a-n-1/1} (-pq)^{n-1}$ $\left\{ \begin{array}{l} \text{Bierens de} \\ \text{Haan, Verh.} \\ \text{v. K. Ak. v.} \\ \text{Wet. Dl. II.} \\ \text{blad 19.} \end{array} \right.$
- 7) $\int \frac{e^{-px}}{(x-q)^a} dx = (-1)^a \frac{p^{a-1}}{1^{a-1/1}} e^{-pq} \text{Ei.}(pq) + \frac{(-1)^{a-1}}{1^{a-1/1} q^{a-1}} \sum_1^{a-1} 1^{a-n-1/1} (pq)^{n-1}$ $\left\{ \begin{array}{l} \text{Bierens de} \\ \text{Haan, Verh.} \\ \text{v. K. Ak. v.} \\ \text{Wet. Dl. II.} \\ \text{blad 19.} \end{array} \right.$
- 8) $\int \frac{e^{xi}}{(q+xi)^p} dx = \frac{\pi e^{-q}}{\Gamma(p)}$ Poisson, P. 19. 404. N°. 68.

$$\left. \begin{aligned} 9) \int \frac{e^{-px}}{(x^2 + q^2)^2} dx &= \frac{1}{2q^3} \left[Ci.(pq).Sin.pq - Si.(pq).Cos.pq + \frac{1}{2}\pi Cos.pq - \right. \\ &\quad \left. - pq \left\{ Ci.(pq).Cos.pq + Si.(pq).Sin.pq - \frac{1}{2}\pi Sin.pq \right\} \right] \\ 10) \int \frac{x e^{-px}}{(x^2 + q^2)^2} dx &= \frac{1}{2q^2} \left[1 + pq \left\{ Ci.(pq).Sin.pq - Si.(pq).Cos.pq + \frac{1}{2}\pi Cos.pq \right\} \right] \\ 11) \int \frac{e^{-px}}{(x^2 - q^2)^2} dx &= \frac{1}{4q^3} \left[(pq - 1) e^{pq} Ei.(-pq) + (1 + pq) e^{-pq} Ei.(pq) \right] \\ 12) \int \frac{x e^{-px}}{(x^2 - q^2)^2} dx &= \frac{1}{4q^2} \left[1 + pq \{ e^{-pq} Ei.(pq) - e^{pq} Ei.(-pq) \} \right] \end{aligned} \right\} \begin{array}{l} \text{Bierens de} \\ \text{Haan, Verh.} \\ \text{v. K. Ak. v.} \\ \text{Wet. Dl. II.} \\ \text{blad 19.} \end{array}$$

$$\begin{aligned} 1) \int \frac{e^{-px}}{x^4 - q^4} dx &= \frac{1}{4q^3} \left[e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) - 2Ci.(pq).Sin.pq + 2Si.(pq).Cos.pq - \pi Cos.pq \right] \\ 2) \int \frac{x e^{-px}}{x^4 - q^4} dx &= \frac{1}{4q^2} \left[-e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) + 2Ci.(pq).Cos.pq + 2Si.(pq).Sin.pq - \pi Sin.pq \right] \\ 3) \int \frac{x^2 e^{-px}}{x^4 - q^4} dx &= \frac{1}{4q} \left[e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) + 2Ci.(pq).Sin.pq - 2Si.(pq).Cos.pq + \pi Cos.pq \right] \\ 4) \int \frac{x^3 e^{-px}}{x^4 - q^4} dx &= \frac{1}{4} \left[-e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) - 2Ci.(pq).Cos.pq - 2Si.(pq).Sin.pq + \pi Sin.pq \right] \\ 5) \int \frac{x^{4a} e^{-px}}{x^4 - q^4} dx &= \frac{1}{4} q^{4a-1} \left[e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) - 2Ci.(pq).Sin.pq + 2Si.(pq).Cos.pq - \pi Cos.pq \right] \\ &\quad + \frac{1}{p^{4a-3}} \sum_1^a 1^{4a-4n+1} (p^4 q^4)^{n-1} \\ 6) \int \frac{x^{4a+1} e^{-px}}{x^4 - q^4} dx &= \frac{1}{4} q^{4a-2} \left[-e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) + 2Ci.(pq).Cos.pq + 2Si.(pq).Sin.pq - \pi Sin.pq \right] \\ &\quad + \frac{1}{p^{4a-2}} \sum_1^a 1^{4a-4n+1} (p^4 q^4)^{n-1} \\ 7) \int \frac{x^{4a+2} e^{-px}}{x^4 - q^4} dx &= \frac{1}{4} q^{4a-1} \left[e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) + 2Ci.(pq).Sin.pq - 2Si.(pq).Cos.pq + \pi Cos.pq \right] \\ &\quad + \frac{1}{p^{4a-1}} \sum_1^a 1^{4a-4n+1} (p^4 q^4)^{n-1} \end{aligned}$$

Sur les formules (1) à (7) voyez: Bierens de Haan, Verh. v. K. Akad. v. Wet. Dl. II. blad 19.

$$8) \int \frac{x^{4a+3} e^{-px}}{x^4 - q^4} dx = \frac{1}{4} q^{4a} \left[-e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) - 2 Ci.(pq) \cdot Cos.pq - 2 Si.(pq) \cdot Sin.pq + \pi Sin.pq \right] \\ + \frac{1}{p^{4a}} \sum_1^a 1^{4a-4n+3/1} (p^4 q^4)^{n-1}$$

Bierens de Haan, Verh. v. K. Akad. v. Wet. Dl. II. blad 19.

$$\left. \begin{aligned} 9) \int \frac{e^{-px}}{q^3 - q^2 x + q x^2 - x^3} dx &= \frac{1}{2 q^2} \left[Ci.(pq) \{ Sin.pq + Cos.pq \} + \right. \\ &\quad \left. + \{ Si.(pq) - \frac{1}{2} \pi \} (Sin.pq - Cos.pq) - e^{pq} Ei.(-pq) \right] \\ 10) \int \frac{x e^{-px}}{q^3 - q^2 x + q x^2 - x^3} dx &= \frac{1}{2 q} \left[Ci.(pq) \{ Cos.pq - Sin.pq \} + \right. \\ &\quad \left. + \{ Si.(pq) - \frac{1}{2} \pi \} (Sin.pq + Cos.pq) - e^{-pq} Ei.(pq) \right] \\ 11) \int \frac{x^2 e^{-px}}{q^3 - q^2 x + q x^2 - x^3} dx &= \frac{1}{2} \left[Ci.(pq) (Cos.pq - Sin.pq) + \right. \\ &\quad \left. + \{ Si.(pq) - \frac{1}{2} \pi \} (Sin.pq + Cos.pq) + e^{-pq} Ei.(pq) \right] \\ 12) \int \frac{e^{-px}}{q^3 + q^2 x + q x^2 + x^3} dx &= \frac{1}{2 q^2} \left[Ci.(pq) (Sin.pq - Cos.pq) - \right. \\ &\quad \left. - \{ Si.(pq) - \frac{1}{2} \pi \} (Sin.pq + Cos.pq) + e^{-pq} Ei.(pq) \right] \\ 13) \int \frac{x e^{-px}}{q^3 + q^2 x + q x^2 + x^3} dx &= \frac{1}{2 q} \left[Ci.(pq) (Sin.pq + Cos.pq) + \right. \\ &\quad \left. + \{ Si.(pq) - \frac{1}{2} \pi \} (Sin.pq - Cos.pq) - e^{-pq} Ei.(pq) \right] \\ 14) \int \frac{x^2 e^{-px}}{q^3 + q^2 x + q x^2 + x^3} dx &= \frac{1}{2} \left[-Ci.(pq) (Sin.pq + Cos.pq) + \right. \\ &\quad \left. + \{ Si.(pq) - \frac{1}{2} \pi \} (Cos.pq - Sin.pq) + e^{pq} Ei.(-pq) \right] \end{aligned} \right\} \begin{array}{l} \text{V. T. 129.} \\ \text{N}^\circ. 9. \\ \text{\& T. 131.} \\ \text{N}^\circ. 6, 7. \end{array}$$

$$1) \int \left(e^{-x} - \frac{1}{1+x} \right) \frac{dx}{x} = -A \quad \text{Schlömlich, Gr. 9. 5. — Id., Stud. I. 4. — Arndt, Gr. 10. 225. — Id., Gr. 10. 233.}$$

$$2) \int \left(e^{-qx} - \frac{1}{1+x} \right) \frac{dx}{x} = -A - l q \quad \text{Cauchy, P. 28. 147. P. 1. § 6.}$$

$$3) \int \left(e^{-x} - \frac{1}{(1+x)^p} \right) \frac{dx}{x} = Z'(p) \quad \text{Lejeune-Dirichlet, Cr. 15. 258. — Grunert, Gr. 2. 266. — Schlömilch, Stud. 1. 4.}$$

$$4) \int \left(\frac{e^{-x}-1}{x} + \frac{1}{1+x} \right) \frac{dx}{x} = A-1 \quad \text{Arndt, Gr. 10. 233.}$$

$$5) \int \left(e^{-x} - \frac{1}{1+x^2} \right) \frac{dx}{x} = -A \quad \text{Arndt, Gr. 10. 225.}$$

$$6) \int \left\{ \frac{e^{-xi}}{\left(1 - \frac{xi}{q}\right)^q} + \frac{e^{xi}}{\left(1 + \frac{xi}{q}\right)^q} \right\} dx = \frac{\pi}{\Gamma(q)} \left(\frac{e}{q}\right)^q \quad \text{Legendre, Exerc. 3. 40.}$$

$$1) \int \frac{1-e^{-x}}{e^x+1} \frac{dx}{x} = l \frac{\pi}{2} \quad \text{V. T. 171. N°. 1.}$$

$$2) \int \frac{1-e^{(1-q)x}}{e^x+1} \frac{dx}{x} = -Z' \left(\frac{q+1}{2} \right) + Z' \left(\frac{q}{2} \right) - A \quad \text{V. T. 171. N°. 2.}$$

$$3) \int \frac{e^{-qx}-e^{(q-1)x}}{e^{-x}+1} \frac{dx}{x} = l \text{Cot.} \frac{q\pi}{2p} \quad \text{V. T. 175. N°. 2.}$$

$$4) \int \frac{e^{-qx}-e^{-px}}{e^{-x}+1} \frac{dx}{x} = l \frac{\Gamma\left(\frac{q}{2}\right) \Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\frac{p}{2}\right) \Gamma\left(\frac{q+1}{2}\right)} \quad \text{V. T. 171. N°. 4.}$$

$$5) \int \frac{e^{-px}-e^{(p-q)x}}{1+e^{-qx}} \frac{dx}{x} = l \text{Cot.} \frac{p\pi}{2q} \quad \text{V. T. 175. N°. 15.}$$

$$6) \int \frac{1-e^{-px}}{1+e^{-x}} \frac{e^{-(q+1)x}}{x} dx = l \frac{\Gamma\left(\frac{p}{2}+1\right) \Gamma\left(\frac{p+q+1}{2}\right)}{\Gamma\left(\frac{p+1}{2}\right) \Gamma\left(\frac{p+q}{2}+1\right)} \quad \text{V. T. 171. N°. 3.}$$

$$7) \int \frac{e^{qx}+e^{-qx}-2}{e^x-1} \frac{dx}{x} = l(q\pi \text{Cosec.} q\pi) \quad \text{V. T. 175. N°. 4.}$$

$$8) \int \frac{(e^{qx}-e^{-qx})^2}{e^x+1} \frac{dx}{x} = -l(q\pi \text{Cot.} q\pi) \quad \text{V. T. 175. N°. 3.}$$

- 9) $\int \frac{(e^{qx} - e^{-qx})^2}{e^{2x} - 1} \frac{dx}{x} = l(q\pi \operatorname{Cosec} q\pi)$ V. T. 175. N°. 7.
- 10) $\int \frac{(e^{qx} - e^{-qx})^2}{1 - e^{2px}} \frac{dx}{x} = l\left(p \operatorname{Sin} \frac{q\pi}{p}\right) - lq\pi$ V. T. 172. N°. 8.
- 11) $\int \frac{(e^{px} - e^{-px})^2}{1 - e^{-x}} \frac{e^{qx}}{x} dx = l \frac{\Gamma(q-2p)\Gamma(q+2p)}{\{\Gamma(q)\}^2}$ Malmsten, Cr. 35. 55.
- 12) $\int \frac{1 - e^{-qx}}{1 - e^{-x}} \frac{1 - e^{-px}}{x} e^{-x} dx = l \frac{\Gamma(p+q+1)}{\Gamma(p+1)\Gamma(q+1)}$ V. T. 171. N°. 17.
- 13) $\int \frac{1 - e^{-qx}}{e^x - 1} \frac{1 - e^{-px}}{x} dx = l \frac{p+q}{pq} - lB(p, q)$ V. T. 171. N°. 14.
- 14) $\int \frac{e^{-ax} - e^{-bx}}{1 + e^{-x}} \frac{1 + e^{-(2c+1)x}}{x} dx = -l \left[\left(\frac{c+1}{c} \right)^{\frac{b-1}{2}} \frac{\left(\frac{a+1}{2} \right)^{c/1} \left(\frac{b}{2} \right)^{c+1/1}}{\left(\frac{a}{2} \right)^{c+1/1} \left(\frac{b+1}{2} \right)^{c/1}} \right]$ V. T. 171. N°. 5.
- 15) $\int \frac{1 - e^{-qx}}{1 - e^{-x}} \frac{1 - e^{-px}}{x} e^{-rx} dx = l \frac{\Gamma(r)\Gamma(p+q+r)}{\Gamma(p+r)\Gamma(q+r)}$ V. T. 171. N°. 18.
- 16) $\int \frac{e^{-px} + e^{-qx}}{1 + e^{-rx}} \frac{1 + e^{(p+q-r)x}}{x} dx = l \left(\operatorname{Tang} \frac{q\pi}{2r}, \operatorname{Cot} \frac{p\pi}{2r} \right)$ V. T. 175. N°. 16.
- 17) $\int \frac{(1 - e^{-px})(1 - e^{-qx})(1 - e^{-rx})}{1 - e^{-x}} \frac{e^{-sx}}{x} dx = l \frac{\Gamma(p+q+s)\Gamma(p+r+s)\Gamma(q+r+s)\Gamma(s)}{\Gamma(p+s)\Gamma(q+s)\Gamma(r+s)\Gamma(p+q+r+s)}$ V. T. 171. N°. 21.
- 18) $\int \frac{1 - e^{(1-q)x}}{1 - e^{-x}} \frac{1 - e^{(1-q)x}}{e^{1/2x}} \frac{dx}{x} = l 2^{2q-2}$ V. T. 177. N°. 15.
- 19) $\int \frac{1 - e^{-x}}{1 + e^{-x}} \frac{1}{e^x + e^{-x}} \frac{dx}{x} = \frac{1}{2} l 2$ V. T. 175. N°. 18.
- 20) $\int \frac{1 - e^{-x}}{1 + e^{-x}} \frac{e^{-2x}}{e^x + e^{-x}} \frac{dx}{x} = l \frac{\pi}{24 \cdot 2}$ V. T. 175. N°. 17.

- 1) $\int \left\{ \frac{1}{1 - e^{-x}} - \frac{1}{x} \right\} e^{-x} dx = A$ V. T. 171. N°. 7.
- 2) $\int \left\{ \frac{e^{-x}}{x} - \frac{e^{-qx}}{e^x - 1} \right\} dx = Z'(1+q)$ Schlömilch, Stud. I. 10. — Schaar, Mém. Cour. Brux. T. 22.

- 3) $\int \left\{ \frac{e^{-qx}}{1 - e^{-x}} - \frac{e^{-px}}{x} \right\} dx = l p - Z'(q)$ V. T. 171. N°. 9.
- 4) $\int \left\{ \frac{b e^{-bqx}}{1 - e^{-x}} - \frac{b e^{-bx}}{x} \right\} dx = - \sum_1^b Z' \left(q + \frac{b-n}{b} \right)$ V. T. 171. N°. 10.
- 5) $\int \left\{ \frac{1}{2} - \frac{1}{1 + e^{-4x}} \right\} \frac{e^{-x}}{x} dx = \frac{1}{2} l \frac{\pi}{4}$
- 6) $\int \left\{ \frac{1}{e^x + 1} - \frac{1}{2} e^{-2x} \right\} \frac{dx}{x} = \frac{1}{2} l \pi$ Stern, Gött. Stud. 1847.
- 7) $\int \left\{ a - \frac{1 - e^{-ax}}{1 - e^{-x}} \right\} \frac{e^{-x}}{x} dx = l(1^a/l)$ Liouville, L. 4. 317.
- 8) $\int \left\{ \frac{e^{-px} - e^{-(p+q)x}}{e^x - 1} - q e^{-x} \right\} \frac{dx}{x} = l \frac{\Gamma(p+1)}{\Gamma(p+q+1)}$ V. T. 171. N°. 12.
- 9) $\int \left\{ 1 - e^{-x} - \frac{(1 - e^{-qx})(1 - e^{-px})}{1 - e^{-x}} \right\} \frac{dx}{x} = l B(p, q)$ V. T. 175. N°. 1.
- 10) $\int \left\{ p - 1 - \frac{1 - e^{(1-p)x}}{1 - e^{-x}} \right\} \frac{e^{-x}}{x} dx = l \Gamma(p)$ Malmsten, Cr. 35. 55.
- 11) $\int \left\{ \frac{b}{x} - \frac{e^{(1-q)x}}{1 - e^{-x/b}} \right\} e^{-x} dx = \sum_1^b Z' \left(q + \frac{b-n}{b} \right)$ V. T. 177. N°. 22.
- 12) $\int \left\{ \frac{1}{1 - e^{-2x}} - \frac{2 - e^{-x}}{2x} + \frac{1 - e^{-x}}{2} \right\} \frac{e^{-x}}{x} dx = 0$
- 13) $\int \left\{ \frac{1}{1 - e^{-x}} - \frac{1}{x} - \frac{1}{2} \right\} e^{-x} \frac{dx}{x} = \frac{1}{2} (1 - l 2)$
- 14) $\int \left\{ \frac{1}{1 - e^{-2x}} - \frac{1}{2x} - \frac{1}{2} \right\} e^{-x} \frac{dx}{x} = \frac{1}{2} (1 - l 2)$
- 15) $\int \left\{ \left[(p-1) - \frac{1}{1 - e^{-x}} \right] e^{-x} + \left(\frac{1}{2} + \frac{1}{x} \right) e^{-px} \right\} \frac{dx}{x} = \left(p - \frac{1}{2} \right) l p - p + \frac{1}{2} l 2 \pi$
- 16) $\int \left\{ \left(\frac{1}{x} + \frac{1}{2} \right) e^{-4x} - \left(\frac{1}{2} + \frac{1}{1 - e^{-x}} \right) e^{-x} \right\} \frac{dx}{x} = \frac{1}{2} l 2 \pi - \frac{1}{2}$
- 17) $\int \left\{ p e^{-x} - \frac{e^{-px}}{x} - \frac{1}{2} e^{-px} - \frac{1}{e^x - 1} \right\} \frac{dx}{x} = \left(p + \frac{1}{2} \right) l p - p + \frac{1}{2} l 2 \pi$ Stern, Gött. Stud. 1847.

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$$18) \int \left\{ \frac{e^{-x}}{x} - \frac{e^{-ax} - e^{-bx} - e^{(b-a)x}}{1 - e^{-x}} e^{-px} \right\} dx = l \frac{\Gamma(b+p) \Gamma(a-b+p)}{\Gamma(a+p)} \quad \text{Malmsten, Cr. 35. 55.}$$

$$19) \int \left\{ \frac{1}{e^x - 1} - \frac{1}{e^{2x} - 1} - \frac{e^{-\frac{1}{2}x}}{x} - \frac{e^{-x}}{2x} \right\} \frac{dx}{x} = 0$$

$$20) \int \left\{ \frac{1}{2} e^{-x} + \frac{1}{x} e^{-x} - \frac{1}{e^x - 1} \right\} \frac{dx}{x} = \frac{1}{2} l 2\pi - 1$$

$$21) \int \left\{ \frac{e^{-x}}{x} - \frac{1}{2} \frac{e^{-x} + 1}{e^x - 1} \right\} \frac{dx}{x} = \frac{1}{2} l 2\pi - 1$$

$$22) \int \left\{ \frac{e^{-x}}{x} - x e^{-x} - \frac{1}{2} \frac{e^{-x} + 1}{e^x - 1} \right\} \frac{dx}{x} = \frac{1}{2} l 2\pi$$

$$23) \int \left\{ \frac{1}{x} - \frac{1}{2} e^{-x} - \frac{1}{e^x - 1} \right\} \frac{dx}{x} = \frac{1}{2} l 2\pi$$

$$24) \int \left\{ ap - \frac{1}{2}(a-1) - \frac{a}{1-e^{-x}} - \frac{e^{-(a-1)x}}{1-e^a} \right\} \frac{e^{-x}}{x} dx = l \left\{ \Gamma(p+1) \Gamma\left(p - \frac{1}{a} + 1\right) \Gamma\left(p - \frac{2}{a} + 1\right) \dots \Gamma\left(p - \frac{a+1}{a}\right) \right\}$$

$$25) \int \left\{ \frac{a-1}{2} + \frac{a-1}{1-e^{-x}} + \frac{e^{(1-a)x}}{1-e^a} + \frac{e^{-apx}}{1-e^{-x}} \right\} \frac{e^{-x}}{x} dx = l \frac{\Gamma(pa+1)}{\left\{ \Gamma(p+1) \Gamma\left(p - \frac{1}{a} + 1\right) \Gamma\left(p - \frac{2}{a} + 1\right) \dots \Gamma\left(p - \frac{a+1}{a}\right) \right\}}$$

$$26) = \frac{1}{2} (a-1) l 2\pi - \left(ap + \frac{1}{2} \right) la$$

$$27) \int \left\{ \frac{e^{(1-p)x}}{1-e^x} - \frac{e^{(1-p)qx}}{1-e^{qx}} - \frac{e^x}{1-e^x} + \frac{e^{qx}}{1-e^{qx}} \right\} \frac{dx}{x} = q l p$$

$$28) \int \left\{ \frac{1}{e^x - 1} - \frac{p e^{-px}}{1-e^{-px}} + \left(pq + \frac{p+1}{2} \right) e^{-px} + (1-pq) e^{-x} \right\} \frac{dx}{x} = \frac{p-1}{2} l 2\pi + \left(\frac{1}{2} - pq \right) l p \quad \text{V. T. 175. N° 19.}$$

$$29) \int \left\{ \frac{e^{-qx}}{1-e^{-x}} - \frac{e^{-pqx}}{1-e^{-px}} - \frac{p-1}{1-e^{-px}} e^{-px} - \frac{p-1}{2} e^{-px} \right\} \frac{dx}{x} = \frac{p-1}{2} l 2\pi + \left(\frac{1}{2} - pq \right) l p \quad \text{V. T. 175. N° 20.}$$

$$30) \int \left\{ q - \frac{1-e^{-rx}-e^{-px}}{2x} + \frac{p e^{-pqx}}{1-e^{-px}} - \frac{r e^{-rqx}}{1-e^{-rx}} \right\} \frac{dx}{x} = (p-r) \left\{ \frac{1}{2} - q + \frac{1}{2} l \pi - l \Gamma(q) + \frac{1}{2} l 2 \right\} \quad \text{V. T. 176. N° 3.}$$

$$31) \int \left\{ \frac{e^{-qx}}{1-e^{-x}} - \frac{e^{-pqx} + (p-1) e^{lpx}}{1-e^{-px}} \right\} \frac{dx}{x} = \frac{1}{2} (p-1) l 2 + \left(\frac{1}{2} - pq \right) l p \quad \text{V. T. 177. N° 24.}$$

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- 1) $\int \frac{1}{e^x - e^{-x}} \frac{dx}{x} = \frac{1}{2} l 2$ V. T. 172. N°. 2.
- 2) $\int \frac{e^x - e^{-x}}{e^{2x} + e^{-2x}} \frac{dx}{x} = l \text{Tang.} \frac{3\pi}{8}$ V. T. 172. N°. 4.
- 3) $\int \frac{(e^{qx} - e^{-qx})^2}{e^x - e^{-x}} \frac{dx}{x} = l \text{Cos.} q \pi$ V. T. 175. N°. 6.
- 4) $\int \frac{1 - e^{2(q-p)x}}{e^{qx} + e^{(q-2p)x}} \frac{dx}{x} = l \text{Cot.} \frac{q\pi}{2p}$ V. T. 172. N°. 9.
- 5) $\int \frac{e^{qx} - e^{-qx}}{e^x - e^{-x}} \frac{e^{-x}}{x} dx = -l(q\pi \text{Cosec.} q\pi)$ V. T. 175. N°. 7.
- 6) $\int \frac{1 - e^{-qx}}{e^x - e^{-x}} \frac{1 - e^{-(q+1)x}}{x} dx = q l 2, q > 1;$ V. T. 172. N°. 3.
- 7) $\int \frac{(1 - e^{-x})^2}{e^x + e^{-x}} \frac{dx}{x} = l \frac{4}{\pi}$ V. T. 172. N°. 1.
- 8) $\int \frac{\{1 - e^{(q-p)x}\}^2}{e^{qx} - e^{(q-2p)x}} \frac{dx}{x} = l \text{Cosec.} \frac{q\pi}{2p}$ V. T. 172. N°. 10.
- 9) $\int \frac{e^{qx} - e^{-qx}}{e^{px} + e^{-px}} \frac{dx}{x} = l \text{Tang.} \left(\frac{p+q}{4p} \pi \right)$ V. T. 172. N°. 6.
- 10) $\int \frac{e^{qx} + e^{-qx}}{e^{px} - e^{-px}} \frac{dx}{x} = l \text{Sec.} \frac{q\pi}{2p}$ V. T. 172. N°. 7.
- 11) $\int \frac{e^{qx} - e^{-qx}}{e^{\pi x} + e^{-\pi x}} \frac{dx}{x} = l \text{Tang.} \left(\frac{\pi}{4} + \frac{q}{4} \right)$
- 12) $\int \frac{e^{qx} + e^{-qx} - 2}{e^{\pi x} - e^{-\pi x}} \frac{dx}{x} = -l \text{Cos.} \frac{1}{2} q$ } Legendre, Exerc. 5. 45.
- 13) $\int \frac{e^{(p-q)x} + e^{(q-p)x} - 2}{e^{px} - e^{-px}} \frac{dx}{x} = l \text{Cosec.} \frac{q\pi}{2p}$ V. T. 175. N°. 10.
- 14) $\int \frac{e^{qx} + e^{-qx} - 2}{e^{px} - e^{-px}} \frac{dx}{x} = l \text{Sec.} \frac{q\pi}{2p}$ V. T. 175. N°. 9.
- 15) $\int \frac{e^{qx} + e^{-qx} - e^{rx} - e^{-rx}}{e^{px} - e^{-px}} \frac{dx}{x} = l \left(\text{Cos.} \frac{r\pi}{2p} \cdot \text{Sec.} \frac{q\pi}{2p} \right)$ V. T. 175. N°. 14.

$$16) \int \frac{e^{(p-q)x} - e^{(p-r)x} - e^{(r-p)x} + e^{(q-p)x}}{e^{px} - e^{-px}} \frac{dx}{x} = l \left(\text{Sin.} \frac{r\pi}{2p} \cdot \text{Cosec.} \frac{q\pi}{2p} \right) \quad \text{V. T. 175. N}^\circ. 13.$$

$$17) \int \frac{e^{qx} - e^{-qx}}{e^{\pi x} - e^{-\pi x}} \frac{dx}{x^p} = \Gamma(1-p) \sum_0^\infty \left[\frac{1}{\{(2n+1)\pi - q\}^{1-p}} - \frac{1}{\{(2n+1)\pi + q\}^{1-p}} \right] \quad \left. \begin{array}{l} \text{Malmsten,} \\ \text{Cr. 38. 1.} \end{array} \right\}$$

$$18) \int \frac{e^{qx} + e^{-qx}}{e^{\pi x} + e^{-\pi x}} dx = \Gamma(1-p) \sum_0^\infty (-1)^n \left[\frac{1}{\{(2n+1)\pi - q\}^{1-p}} - \frac{1}{\{(2n+1)\pi + q\}^{1-p}} \right]$$

$$1) \int \frac{1}{e^x + 1 + e^{-x}} \frac{dx}{x} = \frac{1}{2} l 3 \quad \text{V. T. 174. N}^\circ. 2.$$

$$2) \int \frac{e^x}{e^{2x} - 1 + e^{-2x}} \frac{dx}{x} = l \frac{2}{\sqrt{3}} \quad \text{V. T. 174. N}^\circ. 3.$$

$$3) \int \frac{1}{e^x + e^{-x} + 2 \text{Cos.} \frac{a\pi}{b}} \frac{dx}{x} = \frac{1 + \text{Cos.} \frac{a\pi}{b}}{\text{Sin.} \frac{a\pi}{b}} \left\{ \text{Tang.} \frac{a\pi}{2b} l 2b + 2 \sum_1^{b-1} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+n+1}{2b}\right)}{\Gamma\left(\frac{n+1}{2b}\right)} \right\}, \text{ pour } a+b \text{ impair;}$$

$$4) \int \frac{1}{e^x + e^{-x} + 2 \text{Cos.} \frac{a\pi}{b}} \frac{dx}{x} = \frac{1 + \text{Cos.} \frac{a\pi}{b}}{\text{Sin.} \frac{a\pi}{b}} \left\{ \text{Tang.} \frac{a\pi}{2b} l b + 2 \sum_1^{\frac{b-1}{2}} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b-n+1}{b}\right)}{\Gamma\left(\frac{n+1}{b}\right)} \right\}, \text{ pour } a+b \text{ pair;}$$

V. T. 174. N^o. 5, 6.

$$5) \int \frac{1}{e^x + e^{-x} + 2 \text{Cos.} \lambda} \frac{dx}{x^{1-q}} = \text{Cosec.} \lambda \Gamma(q) \sum_1^\infty (-1)^{n-1} \frac{\text{Sin.} n\lambda}{n^q} \quad \text{V. T. 174. N}^\circ. 4.$$

$$6) \int \frac{e^x + e^{-x}}{e^{2x} + e^{-2x} + 2 \text{Cos.} 2\lambda} \frac{dx}{x^{1-q}} = \text{Sec.}^2 \lambda \Gamma(q) \sum_1^\infty (-1)^n \frac{\text{Cos.} \{(2n+1)\lambda\}}{(2n+1)^q} \quad \text{V. T. 174. N}^\circ. 13.$$

$$7) \int \frac{1 - e^{p-q}x}{e^x + e^{-x} + 2 \text{Cos.} \frac{a\pi}{b}} \frac{e^{-px}}{x} dx = \text{Cosec.} \frac{a\pi}{b} \sum_1^{b-1} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+q+n}{2b}\right) \Gamma\left(\frac{p+n}{2b}\right)}{\Gamma\left(\frac{b+p+n}{2b}\right) \Gamma\left(\frac{q+n}{2b}\right)}, \text{ pour } a+b \text{ impair;}$$

$$8) \int \frac{1 - e^{p-q}x}{e^x + e^{-x} + 2 \text{Cos.} \frac{a\pi}{b}} \frac{e^{-px}}{x} dx = \text{Cosec.} \frac{a\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+q-n}{b}\right) \Gamma\left(\frac{p+n}{b}\right)}{\Gamma\left(\frac{b+p-n}{b}\right) \Gamma\left(\frac{q+n}{b}\right)}, \text{ pour } a+b \text{ pair;}$$

V. T. 174. N^o. 7, 8.

$$\begin{aligned}
 9) \int \frac{(1-e^{-x})^2}{e^x + e^{-x} + 2 \cos \frac{a\pi}{b}} \frac{dx}{x} &= \operatorname{Cosec} \frac{a\pi}{b} \sum_{i=1}^{b-1} (-1)^{n-1} \operatorname{Sin} \frac{na\pi}{b} l \frac{\Gamma\left\{\left(\frac{b+n+1}{2b}\right)\right\}^2 \Gamma\left(\frac{n+2}{2b}\right) \Gamma\left(\frac{n}{2b}\right)}{\Gamma\left(\frac{n+1}{2b}\right)^2 \Gamma\left(\frac{n+b}{2b}\right) \Gamma\left(\frac{n+b+2}{2b}\right)}, \text{ pour } a+b \text{ impair;} \\
 &\quad \text{V. T. 174. N° 9, 10.} \\
 10) &= \operatorname{Cosec} \frac{a\pi}{b} \sum_{i=1}^{b-1} (-1)^{n-1} \operatorname{Sin} \frac{na\pi}{b} l \frac{\left\{\Gamma\left(\frac{b-n+1}{b}\right)\right\}^2 \Gamma\left(\frac{n+2}{b}\right) \Gamma\left(\frac{n}{b}\right)}{\Gamma\left(\frac{n+1}{b}\right)^2 \Gamma\left(\frac{b-n}{b}\right) \Gamma\left(\frac{b-n+2}{b}\right)}, \text{ pour } a+b \text{ pair;} \\
 11) \int \left\{ e^{-x} \operatorname{Tang} \frac{a\pi}{2b} - \frac{2e^{-px} \operatorname{Sin} \frac{a\pi}{b}}{e^x + e^{-x} + 2 \cos \frac{a\pi}{2b}} \right\} \frac{dx}{x} &= \operatorname{Tang} \frac{a\pi}{2b} l 2b + 2 \sum_{i=1}^{b-1} (-1)^{n-1} \operatorname{Sin} \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+p+n}{2b}\right)}{\Gamma\left(\frac{p+n}{2b}\right)}, \text{ pour } a+b \text{ pair;} \\
 &\quad \text{V. T. 174. N° 11, 12.} \\
 12) &= \operatorname{Tang} \frac{a\pi}{2b} l b + 2 \sum_{i=1}^{b-1} (-1)^{n-1} \operatorname{Sin} \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+p-n}{b}\right)}{\Gamma\left(\frac{p-n}{b}\right)}, \text{ pour } a+b \text{ impair.} \\
 13) \int \left\{ q - \frac{1}{2} + \frac{(1-e^{-x})(1-qx) - xe^{-x}}{e^{-2x} + 1 - 2e^{-x}} e^{(1-q)x} \right\} \frac{e^{-x}}{x} dx &= q - \frac{1}{2} - l \Gamma\left(\frac{1}{2}\right) + l \Gamma(q) - \frac{1}{2} l 2 \text{ V. T. 174. N° 1.}
 \end{aligned}$$

$$\begin{aligned}
 1) \int \frac{1}{e^{\pi x} + e^{-\pi x}} \frac{dx}{q^2 + x^2} &= \frac{1}{4q} \left\{ Z' \left(\frac{q}{2} + \frac{3}{4} \right) - Z' \left(\frac{q}{2} + \frac{1}{4} \right) \right\} \\
 2) \int \frac{1}{e^{\pi x} + e^{-\pi x}} \frac{dx}{1 + x^2} &= 1 - \frac{1}{4} \pi \\
 3) \int \frac{1}{e^{\pi x} + e^{-\pi x}} \frac{dx}{1 + 4x^2} &= \frac{1}{4} l 2 \\
 4) \int \frac{e^{px} - e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{dx}{1 + x^2} &= -\frac{1}{2} p \operatorname{Cos} p + \frac{1}{2} \operatorname{Sin} p l \{ 2(1 + \operatorname{Cos} p) \}, p < \pi; \text{ Legendre, Exerc. 5. 46. — Schlömilch, Beitr. II. § 9.} \\
 5) \int \frac{e^{\pi-\lambda)x} - e^{(\lambda-\pi)x}}{e^{\pi x} - e^{-\pi x}} \frac{dx}{q^2 + x^2} &= \frac{1}{q} \sum_{i=1}^{\infty} \frac{\operatorname{Sin} n\lambda}{q+n}, \lambda^2 < \pi^2; \text{ Schlömilch, Cr. 42. 125. il trouve fautivement } x dx \text{ au lieu de } dx \\
 6) \int \frac{e^{px} + e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{x}{1 + x^2} dx &= \frac{1}{2} (p \operatorname{Sin} p - 1) + \frac{1}{2} \operatorname{Cos} p l \{ 2(1 + \operatorname{Cos} p) \} \text{ Legendre, Exerc. 5. 46.}
 \end{aligned}$$

- 7) $\int \frac{e^{px} + e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{x}{1+x^2} dx = \frac{1}{2} \text{Cos. } p l(2 + 2 \text{Cos. } p) + \frac{1}{4} p \text{Sin. } p - \frac{1}{2}$, valeur fautive; (V. N°. 6.)
Poisson, Mém. Inst. 1811. 163. N°. 27.
- 8) $\int \frac{e^{(\pi-\lambda)x} + e^{(\lambda-\pi)x}}{e^{\pi x} - e^{-\pi x}} \frac{x}{q^2 + x^2} dx = \frac{1}{2q} + \sum_1 \frac{\text{Cos. } n\lambda}{q+n}, \lambda^2 \leq \pi^2$; Schlömilch, Cr. 42. 125.
- 9) $\int \frac{x}{e^{2\pi x} - 1} \frac{dx}{q^2 + x^2} = -\frac{1}{4}q + \frac{1}{2}lq - \frac{1}{2}Z'(q)$ Legendre, Exerc. 5. 49.
- 10) $\int \frac{x}{e^{2\pi x} - 1} \frac{dx}{1+x^2} = \frac{1}{2}A - \frac{1}{4}$ Poisson, P. 18. 295. N°. 25. — Id., Mém. Inst. 1811. 163. N°. 29. — Legendre, Exerc. 5. 49.
- 11) $\int \frac{1}{e^{2\pi qx} - 1} \frac{x}{1+x^2} dx = \frac{1}{2}lq + \frac{1}{4}q - \frac{1}{2}Z'(1+q)$ Schaar, Mém. Cour. Brux. T. 22.
- 12) $\int \frac{x}{e^{\pi x} - e^{-\pi x}} \frac{dx}{1+x^2} = l\frac{1}{2} - \frac{1}{2}$ V. T. 387. N°. 3.
- 13) $\int \frac{e^{px} - e^{-px}}{e^{i\pi x} - e^{-i\pi x}} \frac{dx}{1+x^2} = \frac{1}{2}\pi \text{Sin. } p - \frac{1}{2}\text{Cos. } p l \frac{1+\text{Sin. } p}{1-\text{Sin. } p}$, $p < \frac{1}{2}\pi$;
Schlömilch, Beitr. II. 9. — Id., Stud. II. 19.
- 14) $\int \frac{1}{e^{i\pi x} + e^{-i\pi x}} \frac{dx}{1+x^2} = \frac{1}{2}l\frac{1}{2}$
- 15) $\int \frac{e^{px} + e^{-px}}{e^{i\pi x} - e^{-i\pi x}} \frac{x}{1+x^2} dx = -1 + \frac{1}{2}\pi \text{Cos. } p + \frac{1}{2}\text{Sin. } p l \frac{1+\text{Sin. } p}{1-\text{Sin. } p}$, $\frac{1}{2}\pi \geq p \geq 0$;
Schlömilch, Beitr. II. 7.
- 16) $\int \frac{1}{e^{i\pi x} - e^{-i\pi x}} \frac{x}{1+x^2} dx = \frac{1}{4}\pi - \frac{1}{2}$
- 17) $\int \frac{1}{e^{i\pi x} + e^{-i\pi x}} \frac{dx}{1+x^2} = \frac{1}{2\sqrt{2}} \left(\pi - l \frac{\sqrt{2}+1}{\sqrt{2}-1} \right)$ Schlömilch, Beitr. II. 9. — Id., Stud. II. 19.
- 18) $\int \frac{1}{e^{i\pi x} - e^{-i\pi x}} \frac{x}{1+x^2} dx = \frac{1}{4}\pi \sqrt{2} - 1 + \frac{1}{2\sqrt{2}} l \frac{\sqrt{2}+1}{\sqrt{2}-1}$ Schlömilch, Beitr. II. 7.
- 19) $\int \frac{1}{e^{2\pi x} - 1} \frac{x}{(q^2 + x^2)^2} dx = -\frac{1}{8q^3} - \frac{1}{4q^2} + \frac{1}{4q} \frac{dZ'(q)}{dq}$ Cisa de Grésy, Mém. Turin. 1811. 209. II. 62.
- 20) $= \frac{1}{4q^4} \sum_0 \frac{B_{2n+1}}{q^{2n}}$ Cisa de Grésy, Mém. Turin. 1811. 209. II. 61. — Plana, Mém. Turin. 1820.
- 21) $\int \frac{1}{e^{2\pi x} - 1} \frac{x}{q^2 - x^2} dx = \frac{1}{4q^4} \sum_0 (-1)^n \frac{B_{2n+1}}{n+1} q^{-2n}$
Plana, Mém. Turin. 1820.
- 22) $\int \frac{1}{e^{2\pi x} - 1} \frac{x}{(q^2 - x^2)^2} dx = \frac{1}{4q^4} \sum_0 (-1)^n \frac{B_{2n+1}}{q^{2n}}$

- 1) $\int e^{-x} dx \vee x = \frac{1}{2} \vee \pi$ Euler, Calc. Int. 4. S. 5. 211. — Plana, Cr. 17. 1.
- 2) $\int e^{-px} dx \vee x = \frac{1}{2p} \vee \frac{\pi}{p}$ Dienger, Cr. 46. 119.
- 3) $\int e^{-x} dx \vee x^b = \frac{q \cdot 2q \cdot 3q \cdot \dots}{b+q \cdot b+2q \cdot b+3q \cdot \dots}$ Oettinger, Cr. 35. 13.
- 4) $\int e^{-bx} x^{a-1} dx = \frac{1^{a/2}}{(2b)^a} \vee \frac{\pi}{b}$ Schlömilch, Stud. I. 12.
- 5) $\int e^{-4x} \vee^b (x+x^2)^{q-1} dx = \frac{\Gamma(q-\frac{1}{2})}{2^{2q+1} \pi} \frac{e^{21'b}}{b^q} \left\{ \Gamma(q) \psi(1-q, b) + \Gamma(-q) b^q q (1+q, b) \right\}$ Kummer, Cr. 17. 228.
- 6) $\int e^{-a^2(x+\frac{1}{x})} dx \vee x = \frac{1}{a} \left(1 + \frac{1}{2a^2} \right) e^{-2a^2} \vee \pi$ Cauchy, P. 28. 147. P. I. § 4.
- 7) $\int e^{-\frac{1+x^2}{2qx}} dx \vee x = \frac{1+q}{\vee^q e} \vee 2q\pi$ Legendre, Exerc. 3. 51.
- 8) $\int e^{-(px+\frac{q}{x})} x^{c-1} dx = \left(\frac{q}{p} \right)^{\frac{c}{2}} \vee \frac{\pi}{p} e^{-2\vee pq} \sum_0^{\infty} \frac{(c-n)^{2n/1}}{2^{n/2} (2 \vee pq)^n}$ Schlömilch, Cr. 33. 263. — Id., Stud. I. 17.
- 9) $\int e^{-x^a} x^{(b+1)a-1} dx = \frac{1^{b/2}}{2ba} \vee \pi$ Kramp, Réf. 3. 67.
- 10) $\int e^{-\frac{1+x^2}{2qx}} x^{\frac{2b+1}{2}} dx = \frac{\vee 2q\pi}{\vee^q e} \sum_0^{\infty} \frac{(b-n+1)^{2n/1}}{2^n 1^{n/1}} (-q)^n$ Legendre, Exerc. 3. 52.
- 11) $\int \frac{dx \vee x}{e^x + e^{-x}} = \frac{1}{2} \vee \pi \sum_0^{\infty} (-1)^n \frac{1}{\vee (2n+1)^3}$ V. T. 187. N°. 18.
- 12) $\int \frac{e^x - e^{-x}}{(e^x + e^{-x})^2} dx \vee x = \frac{1}{2} \vee \pi \sum_0^{\infty} (-1)^n \frac{1}{\vee (2n+1)}$ V. T. 140. N°. 19.
- 13) $\int \frac{e^x - e^{-x}}{(e^x + e^{-x} + 1)^2} dx \vee x = \frac{\vee \pi}{2 \sin. \frac{1}{3} \pi} \sum_1^{\infty} (-1)^{n-1} \frac{\sin. \frac{n\pi}{3}}{\vee n}$ V. T. 140. N°. 20.

- 1) $\int \frac{e^{-x}}{\vee x} dx = \vee \pi$ Euler, Calc. Int. 4. S. 5. 211. — Cauchy, Cours. Leç. 33. — Bidone, Mém. Turin. 1812. 231. Art. 1. 20. — Binet, P. 27. 123. — Plana, Cr. 17. 1. — Grunert, Gr. 2. 266.

- 2) $\int \frac{e^{-qx}}{\sqrt{x}} dx = \sqrt{\frac{\pi}{q}}$ Bidone, Mém. Turin. 1812. 231. Tableau. — Cisa de Grézy, Mém. Turin. 1821. 209. II. 34. — Dicnger, Cr. 46. 119.
- 3) $\int \frac{e^{pxi}}{\sqrt{x}} dx = e^{i\pi} \sqrt{\frac{\pi}{p}}$ Schlömilch, Stud. I. 13.
- 4) $\int e^{-qx} \frac{x^a}{\sqrt{x}} dx = \frac{1^{a/2}}{(2q)^a} \sqrt{\frac{\pi}{q}}$ Raabe, Int. 165.
- 5) $\int e^{-\frac{1+x^2}{2qx}} \frac{dx}{\sqrt{x}} = \frac{\sqrt{2q\pi}}{\sqrt{q}e}$ Legendre, Exerc. 3. 50. — Bidone, Mém. Turin. 1812. 231. Art. 2. 34. — Cisa de Grézy, Mém. Turin. 1821. 209. II. 38.
- 6) $\int e^{-q^2(x+\frac{1}{x})} \frac{dx}{\sqrt{x}} = \frac{1}{q} e^{-2q^2} \sqrt{\pi}$ Cauchy, P. 28. 147. P. I. 4.
- 7) $\int e^{-\frac{1+x^2}{2qx}} \frac{dx}{x\sqrt{x}} = \frac{2q}{\sqrt{q}e} \sqrt{\frac{\pi}{2q}}$
- 8) $\int e^{-\frac{1+x^2}{2qx}} \frac{dx}{x^2\sqrt{x}} = \frac{1+q}{\sqrt{q}e} \sqrt{2q\pi}$ } Legendre, Exerc. 3. 53.
- 9) $\int e^{-(px+\frac{q}{x})} \frac{dx}{\sqrt{x}} = e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{p}}$ Cauchy, Sav. Etr. 1827. 124. Note. 16.
- 10) $\int e^{-x} \sqrt{x}^b \frac{dx}{x} = \frac{2q \cdot 3q \cdot 4q \cdots}{b \cdot b+q \cdot b+2q \cdots}$ Oettinger, Cr. 35. 13.
- 11) $\int e^{-\frac{1+x^2}{2qx}} \frac{dx}{x^{\frac{2b+1}{2}}} = \frac{\sqrt{2q\pi}}{\sqrt{q}e} \sum_0^{\infty} \frac{b^{2n/1}}{2^{n/2}} q^n$ Legendre, Exerc. 3. 52.
- 12) $\int e^{-(px+\frac{q}{x})} \frac{dx}{x^{c+1}} = \left(\frac{p}{q}\right)^{\frac{c}{2}} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{p}} \sum_0^{\infty} \frac{(c-n)^{2n/1}}{2^{n/2} (2\sqrt{pq})^n}$ Schlömilch, Cr. 33. 268. — Id., Stud. I. 17.
- 13) $\int \frac{e^{-qx}-1}{\sqrt{x}} e^{-x} dx = \left\{1 - \frac{1}{\sqrt{(1+q)}}\right\} \sqrt{\pi}$ V. T. 178. N°. 2.
- 14) $\int \frac{e^{-px}-e^{-qx}}{\sqrt{x}} e^{-x} dx = \left\{\frac{1}{\sqrt{(1+q)}} - \frac{1}{\sqrt{(1+p)}}\right\} \sqrt{\pi}$ V. T. 178. N°. 3.
- 15) $\int \frac{e^{xp\sqrt{x}} + e^{-p\sqrt{x}}}{\sqrt{x}} e^{-x} dx = 2e^{1/2} \sqrt{\pi}$ V. T. 37. N°. 14.
- 16) $\int \frac{e^{-px}-e^{-qx}}{x^{\frac{2a-1}{a}}} dx = \frac{a\Gamma\left(\frac{1}{a}\right)}{a-1} \left(q^{\frac{a-1}{a}} - p^{\frac{a-1}{a}}\right), q > p > 0;$ Lindmann, Stock. Handl. 1850. IV.

$$\begin{aligned}
 17) & \int \frac{\text{Sin. } p \sqrt{\sqrt{(p^2 + x^2) + x}} - \text{Cos. } p \sqrt{\sqrt{(p^2 + x^2) - x}}}{\sqrt{(p^2 + x^2)}} e^{-x} dx = 0 \\
 18) & \int \frac{\text{Sin. } p \sqrt{\sqrt{(p^2 + x^2) - x}} + \text{Cos. } p \sqrt{\sqrt{(p^2 + x^2) + x}}}{\sqrt{(p^2 + x^2)}} e^{-x} dx = 0 \\
 19) & \int \frac{1}{e^x + e^{-x}} \frac{dx}{\sqrt{x}} = \sqrt{\pi} \sum_0^{\infty} \frac{(-1)^n}{\sqrt{(2n+1)}} \\
 20) & \int \frac{1}{e^x + e^{-x} + 1} \frac{dx}{\sqrt{x}} = \frac{\sqrt{\pi}}{\text{Sin. } \frac{1}{3}\pi} \sum_1^{\infty} (-1)^{n-1} \frac{\text{Sin. } \frac{1}{3}n\pi}{\sqrt{n}} \left. \vphantom{\int} \right\} \text{Malmsten, Cr. 38. 1.} \\
 & \hspace{15em} , \text{ dans 19) il a faut. } \frac{\sum_1^{\infty}}{1}; \\
 21) & \int \frac{\text{Cos. } \lambda - e^{-x} - e^{-ax} \text{Cos. } \{(a+1)\lambda\} + e^{-(a+1)x} \text{Cos. } a\lambda}{e^x + e^{-x} - 2 \text{Cos. } \lambda} \frac{dx}{\sqrt{x}} = \sqrt{\pi} \sum_1^a \frac{\text{Cos. } n\lambda}{\sqrt{n}} \quad \text{V. T. 178. N}^\circ. 6. \\
 22) & \int \frac{\text{Sin. } \lambda - e^{-ax} \text{Sin. } \{(a+1)\lambda\} + e^{-(a+1)x} \text{Sin. } a\lambda}{e^x + e^{-x} - 2 \text{Cos. } \lambda} \frac{dx}{\sqrt{x}} = \sqrt{\pi} \sum_1^a \frac{\text{Sin. } n\lambda}{\sqrt{n}} \quad \text{V. T. 178. N}^\circ. 7.
 \end{aligned}$$

$$\begin{aligned}
 1) & \int e^{-x} x dx \sqrt{(1 - e^{-2x})} = \frac{1}{4} \pi \left(\frac{1}{2} + l 2 \right) \quad \text{V. T. 162. N}^\circ. 1. \\
 2) & \int e^{-2x} x dx \sqrt{(1 - e^{-2x})} = \frac{1}{3} \left(\frac{4}{3} - l 2 \right) \quad \text{V. T. 162. N}^\circ. 2. \\
 3) & \int e^{-x} x dx (1 - e^{-2x})^{\frac{2a-1}{2}} = \frac{1^{a/2} \pi}{2^{a+2} 1^{a/2}} \{ \Lambda + Z'(a+1) + 2 l 2 \} \quad \text{V. T. 162. N}^\circ. 3. \\
 4) & \int \frac{x}{\sqrt{(e^x - 1)}} dx = 2 \pi l 2 \quad \text{V. T. 181. N}^\circ. 1. \\
 5) & \int \frac{x^2}{\sqrt{(e^{2x} - 1)}} dx = \frac{1}{2} \pi \left\{ (l 2)^2 + \frac{1}{12} \pi^2 \right\} \quad \text{V. T. 164. N}^\circ. 1. \\
 6) & \int \frac{x}{\sqrt{(e^{2x} - 1)}} dx = \frac{1}{2} \pi l 2 \quad \text{V. T. 163. N}^\circ. 2. \\
 7) & \int \frac{x e^{-x}}{\sqrt{(e^{2x} - 1)}} dx = 1 - l 2 \quad \text{V. T. 163. N}^\circ. 3. \\
 8) & \int \frac{x e^{-2x}}{\sqrt{(e^{2x} - 1)}} dx = \frac{1}{8} \pi (2 l 2 - 1) \quad \text{V. T. 163. N}^\circ. 4.
 \end{aligned}$$

- 9) $\int \frac{x e^{-3x}}{\sqrt[3]{(e^{2x}-1)}} dx = \frac{1}{9} (5 - 6 l 2) \quad \text{V. T. 163. N}^\circ. 5.$
- 10) $\int \frac{x e^{-4x}}{\sqrt[3]{(e^{2x}-1)}} dx = \frac{3}{16} \pi \left(l 2 - \frac{7}{12} \right) \quad \text{V. T. 163. N}^\circ. 6.$
- 11) $\int \frac{x e^{-5x}}{\sqrt[3]{(e^{2x}-1)}} dx = \frac{8}{15} \left(\frac{47}{60} - l 2 \right) \quad \text{V. T. 163. N}^\circ. 7.$
- 12) $\int \frac{x e^{-x}}{\sqrt[3]{(e^{2x}-e^{-2x})}} dx = \frac{1}{8} \pi l 2 \quad \text{V. T. 163. N}^\circ. 12.$
- 13) $\int \frac{x e^{-3x}}{\sqrt[3]{(e^{2x}-e^{-2x})}} dx = \frac{1}{4} (1 - l 2) \quad \text{V. T. 163. N}^\circ. 13.$
- 14) $\int \frac{e^{2x} x^2}{\sqrt[3]{(e^{2x}-1)^3}} dx = \pi l 2 \quad \text{V. T. 141. N}^\circ. 4.$
- 15) $\int \frac{e^{2x} x^3}{\sqrt[3]{(e^{2x}-1)^3}} dx = \frac{3\pi}{2} \left\{ (l 2)^2 + \frac{1}{12} \pi^2 \right\} \quad \text{V. T. 141. N}^\circ. 5.$
- 16) $\int \frac{e^{-x} x}{\sqrt[3]{(1-e^{-2x})}} dx = \frac{\pi^2}{54} + \frac{5\pi l 3}{18 \sqrt[3]{3}} \quad \text{V. T. 163. N}^\circ. 8.$
- 17) $\int \frac{e^{-2x} x}{\sqrt[3]{(1-e^{-2x})}} dx = -\frac{\pi^3}{54} + \frac{5\pi l 3}{18 \sqrt[3]{3}} \quad \text{V. T. 163. N}^\circ. 9.$
- 18) $\int \frac{x}{\sqrt[3]{(e^{3x}-1)}} dx = \frac{\pi}{3 \sqrt[3]{3}} \left(l 3 + \frac{\pi}{3 \sqrt[3]{3}} \right) \quad \text{V. T. 163. N}^\circ. 10.$
- 19) $\int \frac{x}{\sqrt[3]{(e^{3x}-1)^2}} dx = \frac{\pi}{3 \sqrt[3]{3}} \left(l 3 - \frac{\pi}{3 \sqrt[3]{3}} \right) \quad \text{V. T. 163. N}^\circ. 11.$
- 20) $\int \frac{e^{-ax} x^h}{\sqrt[3]{(1-e^{-bx})^{b-c}}} dx = l^h l \sum_0^{\infty} \frac{(b-c)^{n/b}}{b^{n/b}} \frac{1}{(a+bn)^{h+1}} \quad \text{V. T. 164. N}^\circ. 2.$
- 21) $\int \frac{x}{e^x + p^2 - 1} \frac{e^x}{\sqrt[3]{(e^x-1)}} dx = \frac{2\pi}{p} l (1+p) \quad \text{V. T. 181. N}^\circ. 3.$
- 22) $\int \frac{x}{p^2 e^x + (q^2 - p^2)} \frac{e^x}{\sqrt[3]{(e^x-1)}} dx = \frac{\pi}{pq} l \frac{p}{p+q} \quad \text{V. T. 181. N}^\circ. 10.$
- 23) $\int \frac{x}{p^2 e^x - (p^2 + q^2)} \frac{e^x}{\sqrt[3]{(e^x-1)}} dx = -\frac{\pi}{pq} \text{Arctang.} \frac{q}{p} \quad \text{V. T. 181. N}^\circ. 11.$
- 24) $\int \frac{\{a \sqrt[3]{(e^{-x}-1)-bi}\}^{-p} + \{a \sqrt[3]{(e^{-x}-1)+bi}\}^{-b}}{(e^{-x}-1)^{\frac{3-p}{2}}} x e^{-x} dx = \frac{4}{b} \frac{\pi}{p-1} \{a^{-p-(a+b)-p}\} \quad \text{V. T. 184. N}^\circ. 18.$

$$\begin{aligned}
 & \left. \begin{aligned}
 1) \int e^{ix} (ix)^{p-1} dx &= 2 \sin. p \pi \Gamma(p) \\
 2) \int e^{ix} (-ix)^{p-1} dx &= 0 \\
 3) \int e^{ix} (r+ix)^{q-1} dx &= \frac{\pi e^{-r}}{\Gamma(1-q)} \\
 4) \int e^{ix} (r-ix)^{q-1} dx &= 0 \\
 5) \int e^{ix} (ix)^{p-1} (-ix)^{q-1} dx &= 2 \sin. p \pi \Gamma(p+q-1)
 \end{aligned} \right\} \begin{aligned}
 & , 0 < p < 1, 1 \leq q < \dots, r > 0; \\
 & \text{Cayley, L. 12. 231.}
 \end{aligned} \\
 & \left. \begin{aligned}
 6) \int e^{-x^2} x^2 dx &= \frac{1}{2} \sqrt{\pi} \\
 7) \int e^{-px^2} x^2 dx &= \frac{1}{2p} \sqrt{\frac{\pi}{p}} \\
 8) \int e^{-x^2} x^{2a} dx &= \frac{1}{2^a} \sqrt{\pi} \text{ Fourier, Chal. 370. — Poisson, Chal. 75.} \\
 9) \int e^{-x^2} x^{2a+1} dx &= 0 \text{ Poisson, Chal. 75.}
 \end{aligned} \right\} \text{Ohm, Ausw. 20.} \\
 & \left. \begin{aligned}
 10) \int e^{-x^2+2px} x dx &= p e^{p^2} \sqrt{\pi} \\
 11) \int e^{-x^2+2px} x^2 dx &= \frac{1+2p^2}{2} e^{p^2} \sqrt{\pi} \\
 12) \int e^{-px^2+2qx} x^{a+1} dx &= \frac{\sqrt{\pi}}{2^a p \sqrt{p}} \frac{d^a}{d q^a} q e^{q^2/p} \\
 13) \int e^{-px^2+2qx} x dx &= \frac{q}{p} \sqrt{\frac{\pi}{p}} e^{\frac{q^2}{p}}
 \end{aligned} \right\} \text{Dienger, Cr. 46. 119.} \\
 & \left. \begin{aligned}
 14) \int e^{-px^2+qx} x^a dx &= (-1)^a \left(\frac{q}{2p}\right)^a e^{\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}} \sum_0^{\infty} \frac{a^{2n-1}}{1^{n/1}} \left(\frac{p}{q^2}\right)^n \\
 15) \int e^{(px^2+qx)i} x^a dx &= (-1)^a (1+i) \left(\frac{q}{2p}\right)^a e^{-\frac{q^2 i}{4p}} \sqrt{\frac{\pi}{2p}} \sum_0^{\infty} \frac{a^{2n-1}}{1^{n/1}} \left(\frac{p i}{q^2}\right)^n \\
 16) \int e^{-(px^2+qx)i} x^a dx &= (-1)^a (1-i) \left(\frac{q}{2p}\right)^a e^{\frac{q^2 i}{4p}} \sqrt{\frac{\pi}{2p}} \sum_0^{\infty} \frac{a^{2n-1}}{1^{n/1}} \left(\frac{p}{q^2 i}\right)^n
 \end{aligned} \right\} \text{Cauchy, P. 19. 511.}
 \end{aligned}$$

$$17) \int e^{-x} x e^x dx = -A \quad \text{V. T. 273. N}^\circ. 1.$$

$$18) \int e^{-q e^x} x e^x dx = -\frac{1}{q} (A + l q) \quad \text{V. T. 273. N}^\circ. 2.$$

$$19) \int e^{-e^{2x}} x e^x dx = -\frac{1}{4} (A + l 4) \sqrt{\pi} \quad \text{V. T. 273. N}^\circ. 3.$$

$$20) \int e^{-q e^{2x}} x e^x dx = -\frac{1}{4} (A + l 4 q) \sqrt{\frac{\pi}{q}} \quad \text{V. T. 273. N}^\circ. 4.$$

$$1) \int \frac{e^{p x} x}{e^x + q} dx = \frac{\pi q^{p-1}}{\text{Sin. } p \pi} (l q - \pi \text{Cot. } p \pi), p < 1; \quad \text{V. T. 180. N}^\circ. 1.$$

$$2) \int \frac{e^{p x} x}{e^x - 1} dx = (\pi \text{Cosec. } p \pi)^2, 0 < p < 1; \quad \text{V. T. 183. N}^\circ. 1.$$

$$3) \int \frac{x}{e^x - 1} \frac{dx}{e^{[p-1]x}} = (\pi \text{Cosec. } p \pi)^2 \quad \text{V. T. 183. N}^\circ. 2.$$

$$4) \int \frac{1 - e^{-x}}{1 - e^{2ax}} e^{-(a-1)x} x dx = \left(\frac{\pi}{2a} \text{Tang. } \frac{\pi}{2a} \right)^2, a > 2; \quad \text{V. T. 180. N}^\circ. 14.$$

$$5) \int \frac{1 - e^x}{1 - e^{2ax}} e^{-(a-2)x} x dx = \left(\frac{\pi}{2a} \text{Tang. } \frac{\pi}{2a} \right)^2, a > 2; \quad \text{V. T. 180. N}^\circ. 15.$$

$$6) \int \frac{1 - e^{2x}}{1 - e^{2bx}} e^{ax} x dx = \left(\frac{\pi}{2b} \right)^2 \frac{\text{Sin. } \left\{ \frac{a+1}{b} \pi \right\} \cdot \text{Sin. } \frac{\pi}{b}}{\text{Sin.}^2 \frac{a\pi}{2b} \cdot \text{Sin.}^2 \left\{ \frac{a+2}{2b} \pi \right\}} \quad \text{V. T. 180. N}^\circ. 16.$$

$$7) \int \frac{x}{e^x + e^{-x}} dx = 0 \quad \text{V. T. 180. N}^\circ. 2.$$

$$8) \int \frac{x}{q e^{-x} + e^x} dx = \frac{\pi}{2q} l \frac{1}{q} \quad \text{V. T. 180. N}^\circ. 8.$$

$$9) \int \frac{x}{p^2 e^{-x} + q^2 e^x} dx = \frac{\pi}{2pq} l \frac{q}{p} \quad \text{V. T. 180. N}^\circ. 10.$$

$$10) \int \frac{x}{e^x - e^{-x}} dx = \frac{1}{4} \pi^2 \quad \text{V. T. 180. N}^\circ. 11.$$

$$11) \int \frac{e^{-px} x}{e^x - e^{-x}} dx = \left(\frac{1}{2} \pi \operatorname{Cosec.} \left\{ \frac{p+1}{2} \pi \right\} \right)^2, p^2 < 1; \text{ V. T. 180. N}^\circ. 12.$$

$$12) \int \frac{1 - e^{-\frac{2x}{b}}}{e^x - e^{-x}} x dx = - \left(\frac{1}{2} \pi \operatorname{Tang.} \frac{\pi}{b} \right)^2, b > 2; \text{ V. T. 185. N}^\circ. 7.$$

$$13) \int \frac{1 - e^{\frac{2x}{b}}}{e^x - e^{-x}} x dx = - \left(\frac{1}{2} \pi \operatorname{Tang.} \frac{\pi}{b} \right)^2, b > 2; \text{ V. T. 185. N}^\circ. 8.$$

$$14) \int \frac{1 - e^{px}}{e^x - e^{-x}} x dx = - \left(\frac{1}{2} \pi \operatorname{Tang.} \frac{1}{2} p \pi \right)^2, p < 1; \text{ V. T. 183. N}^\circ. 4.$$

$$15) \int \frac{1 - e^{\frac{2x}{b}}}{e^x - e^{-x}} e^{\left(\frac{a}{b}-1\right)x} x dx = \frac{1}{4} \pi^2 \frac{\operatorname{Sin.} \left\{ \frac{a+1}{b} \pi \right\} \cdot \operatorname{Sin.} \frac{\pi}{b}}{\operatorname{Sin.}^2 \frac{a\pi}{b} \cdot \operatorname{Sin.}^2 \left\{ \frac{a+2}{b} \pi \right\}} \text{ V. T. 185. N}^\circ. 9.$$

$$1) \int \frac{x e^x}{(q + e^x)^{p+1}} dx = \frac{1}{p q^p} \{ l q - \Lambda - Z'(p) \} \text{ V. T. 182. N}^\circ. 4.$$

$$2) \int \frac{x e^x}{(q + e^x)^2} dx = \frac{1}{q} l q \text{ V. T. 182. N}^\circ. 1.$$

$$3) \int \frac{x e^x}{(q + e^x)^{b+\frac{1}{2}}} dx = \frac{2}{2b-1} q^{\frac{1}{2}-b} \left\{ l(q-1) + 2 l 2 - \sum_1^{\frac{b-2}{2}} \frac{1}{n} - 2 \sum_{b-1}^{\frac{2b-3}{2}} \frac{1}{n} \right\} \text{ V. T. 185. N}^\circ. 10.$$

$$4) \int \frac{x e^x}{(a^2 + b^2 e^{2x})^p} dx = \frac{\Gamma(p - \frac{1}{2}) \sqrt{\pi}}{4 a^{2p-1} b \Gamma(p)} \left\{ 2 l \frac{a}{2p} - \Lambda - Z'(p - \frac{1}{2}) \right\} \text{ V. T. 182. N}^\circ. 6.$$

$$5) \int \left(\frac{1}{e^x + e^{-x}} \right)^{2a+1} x dx = 0 \text{ V. T. 183. N}^\circ. 7.$$

$$6) \int \frac{x}{(q^2 e^{-x} + e^x)^{2a+1}} dx = \frac{1^{a/2}}{2^{a/2}} \frac{\pi l q}{2 q^{2a+1}} \text{ V. T. 183. N}^\circ. 8.$$

$$7) \int \frac{x}{(q^2 e^{-x} + e^x)^{2a}} dx = \frac{1^{a/1}}{2^{a/1}} \frac{l q}{2 q^{2a}} \text{ V. T. 183. N}^\circ. 9.$$

$$8) \int \frac{x}{(q^2 e^x + e^{-x})^p} dx = \frac{l q}{2 q^p} \frac{\{ \Gamma(\frac{1}{2} p) \}^2}{\Gamma(p)} \text{ V. T. 183. N}^\circ. 10.$$

- 9) $\int \frac{x e^{-x}}{(a + e^x)^{b+1}} dx = \frac{1}{b a^b} \left\{ -la + \sum_1^a \frac{1}{n} \right\}$ V. T. 180. N°. 7.
- 10) $\int \frac{x}{e^x + q} \frac{dx}{e^{-x} + 1} = \frac{(lq)^2}{2(q+1)}$ V. T. 183. N°. 12.
- 11) $\int \frac{x}{q e^{-x} + 1} \frac{dx}{e^x - 1} = \frac{\pi^2 + (lq)^2}{2(q+1)}$ V. T. 183. N°. 14.
- 12) $\int \frac{e^{p-1} x}{e^x + q e^x + 1} dx = \frac{\pi}{q-1} \frac{q^p l q \sin. p \pi - (1 - q^p) \pi \cos. p \pi}{\sin.^2 p \pi}, p^2 < 1;$ V. T. 183. N°. 13.
- 13) $\int \frac{e^{p x}}{q e^{-x} + 1} \frac{x}{e^x - 1} dx = \frac{\pi}{1+q} \frac{\pi + q^p (\sin. p \pi l q - \pi \cos. p \pi)}{\sin.^2 p \pi}, p^2 < 1;$ V. T. 183. N°. 15.

- 1) $\int \left(\frac{x}{e^{\frac{1}{2}x} - e^{-\frac{1}{2}x}} \right)^2 dx = \frac{2}{3} \pi^2$ V. T. 182. N°. 3.
- 2) $\int \frac{e^x - e^{-x}}{(e^x + e^{-x})^2} x^2 dx = 0$ V. T. 143. N°. 7.
- 3) $\int \frac{e^x + e^{-x}}{(e^x - e^{-x})^2} x^2 dx = 0$ V. T. 143. N°. 10.
- 4) $\int \frac{p^2 e^x - q^2 e^{-x}}{(p^2 e^x + q^2 e^{-x})^2} x^2 dx = \frac{\pi}{p q} l \frac{p}{q}$ V. T. 143. N°. 9.
- 5) $\int \frac{p + (1-p) e^{-x}}{(1 - e^{-x})^2} e^{-p x} x^2 dx = 2 \pi^2 \operatorname{Cosec}.^2 p \pi$ V. T. 143. N°. 3.
- 6) $\int \frac{e^x - q^2 e^{-x}}{(q^2 e^{-x} + e^x)^{2a}} x^2 dx = \frac{1^{a-1/2}}{2^{a-1/2}} \frac{4 l q}{q^{2a-1}} \frac{\pi}{2a+1}$ V. T. 144. N°. 6.
- 7) $\int \frac{e^x - q^2 e^{-x}}{(q^2 e^{-x} + e^x)^{2a+1}} x^2 dx = \frac{1^{a/1}}{2^{a/2}} \frac{l q}{2 a q^{2a}}$ V. T. 144 N°. 7.
- 8) $\int \frac{q^2 e^x - e^{-x}}{(q^2 e^x + e^{-x})^{p+1}} x^2 dx = \frac{l q}{q^p} \frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma(p+1)}$ V. T. 144. N°. 8.
- 9) $\int \frac{x^2}{e^x - 1} \frac{dx}{1 + q e^{-x}} = \frac{\pi^2 + (lq)^2}{6(1+q)} l q$ V. T. 184. N°. 1.

- 10) $\int \frac{x-lq}{e^x-1} \frac{x}{1-qe^{-x}} dx = \frac{4\pi^2 + (lq)^2}{6(q-1)} lq$ V. T. 184. N°. 8.
- 11) $\int \frac{x-lq}{e^x-1} \frac{x e^{px}}{1-qe^{-x}} dx = \frac{\pi^2 (q^p+1) lq - 2\pi (q^p-1) \text{Cot. } p\pi}{q-1 \text{Sin.}^2 p\pi}, p^2 < 1;$ V. T. 184. N°. 7.
- 12) $\int \frac{x^3}{e^x-1} \frac{dx}{1+qe^{-x}} = \frac{\{\pi^2 + (lq)^2\}^2}{24(1+q)}$ V. T. 184. N°. 2.
- 13) $\int \frac{x^4}{e^x-1} \frac{dx}{1+qe^{-x}} = \frac{(\pi^2 + (lq)^2)^2}{360} \frac{7\pi^2 + 3(lq)^2}{1+q} lq$ V. T. 184. N°. 3.
- 14) $\int \frac{x^5}{e^x-1} \frac{dx}{1+qe^{-x}} = \frac{\{\pi + (lq)^2\}^2}{720} \frac{\{3\pi^2 + (lq)^2\}^2}{1+q}$ V. T. 184. N°. 4.
- 15) $\int \frac{e^x - qe^{-x}}{(e^x + q)^2} \frac{x^2}{(1+e^{-x})^2} dx = \frac{1}{q+1} (lq)^2$ V. T. 144. N°. 10.
- 16) $\int \frac{e^x + qe^{-x}}{(qe^{-x} + 1)^2} \frac{x^2}{(1-e^x)^2} dx = \frac{1}{q+1} \{\pi^2 + (lq)^2\}$ V. T. 144. N°. 11.
- 17) $\int \frac{x^{2a+1}}{e^x + e^{-x}} dx = 0$ V. T. 180. N°. 3.
- 18) $\int \frac{x^{2a}}{e^x + e^{-x}} dx = 2 \cdot 12a! \sum_0^{\infty} (-1)^n \frac{1}{(2n+1)^{2a+1}}$ V. T. 180. N°. 4.
- 19) $\int \frac{x^{2a}}{e^{1bx} + e^{-1bx}} dx = \frac{(-1)^{a+1}}{b} (4\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b} \right)$ V. T. 185. N°. 12.
- 20) $\int \frac{e^{qx} x^a}{e^x + e^{-x}} dx = \frac{1}{2} \pi (-1)^a \frac{d^a}{dq^a} \text{Sec. } \frac{1}{2} q\pi$ V. T. 180. N°. 6.
- 21) $\int \frac{e^{bx} + e^{-bx}}{e^x + e^{-x}} x^{2a} dx = \frac{(-1)^{a+1}}{b} 2 \left(\frac{2\pi}{b} \right)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b} \right) \text{Cos. } \left\{ \frac{2n-1}{2} a\pi \right\}$ V. T. 184. N°. 5.
- 22) $\int \frac{e^{bx} - e^{-bx}}{e^x - e^{-x}} x^{2a} dx = \frac{(-1)^{a+1}}{b} 2 \left(\frac{2\pi}{b} \right)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{n}{2b} \right) \text{Sin. } n a\pi$ V. T. 184. N°. 6.
- 23) $\int \frac{x^2}{e^x + 2 \text{Cos. } \lambda + e^{-x}} dx = \lambda \text{Cosec. } \lambda \frac{\pi^2 - \lambda^2}{3}$ V. T. 184. N°. 9.
- 24) $\int \frac{x^4}{e^x + 2 \text{Cos. } \lambda + e^{-x}} dx = 2\lambda \frac{\pi^2 - \lambda^2}{5} - \frac{7\pi^2 - 3\lambda^2}{\text{Sin. } \lambda}$ V. T. 184. N°. 10.
- 25) $\int \frac{x^{2a}}{e^x + e^{-x} + 1} dx = \frac{(-1)^{a+1}}{1/3} 2 (2\pi)^{2a+1} B'' \left(\frac{1}{3} \right)$ V. T. 184. N°. 11.

$$26) \int \frac{x^{2a}}{e^x + e^{-x} - 1} dx = \frac{(-1)^{a+1}}{\sqrt{3}} 2 (2\pi)^{2a+1} B'' \left(\frac{1}{6} \right) \quad \text{V. T. 184. N° 12.}$$

$$27) \int \frac{x^{2a}}{e^x + e^{-x} - 2 \cos. 2p\pi} dx = (-1)^{a+1} (2\pi)^{a+1} \frac{B''(p)}{\sin. 2p\pi}, \quad 0 < p < 1; \quad \text{Raabe, Cr. 42. 348.}$$

F. Algèbr. rat. fract. } Dén. à fact. x^a . TABLE 146.
Expon.

Lim. — ∞ et ∞ .

$$1) \int \frac{e^{-x^2 - \frac{p}{4x^2}}}{x^2} dx = \frac{2}{p} e^{-p} \sqrt{\pi} \quad \text{Meyer, Int. Déf. 152.}$$

$$2) \int \frac{e^{-px^2 - \frac{q}{x^2}}}{x^2} dx = e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{q}}$$

$$3) \int \frac{e^{-px^2 - \frac{q}{x^2}}}{x^{2a}} dx = \left(\frac{p}{q} \right)^{\frac{a}{2}} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{p}} \sum_0^{\infty} \frac{(a-n)^{2n/1}}{1^{n/1}} \left(\frac{1}{4pq} \right)^n$$

$$4) \int \frac{e^{(px^2 + \frac{q}{x^2})i}}{x^{2a}} dx = \left(\frac{p}{q} \right)^{\frac{a}{2}} e^{2i\sqrt{pq}} (1+i) \sqrt{\frac{\pi}{2p}} \sum_0^{\infty} \frac{a^{2n/1-1}}{1^{n/1}} \left(\frac{i}{4pq} \right)^n$$

$$5) \int \frac{e^{-(px^2 + \frac{q}{x^2})i}}{x^{2a}} dx = \left(\frac{p}{q} \right)^{\frac{a}{2}} e^{-2i\sqrt{pq}} (1-i) \sqrt{\frac{\pi}{2p}} \sum_0^{\infty} \frac{a^{2n/1-1}}{1^{n/1}} \left(\frac{1}{4pqi} \right)^n$$

$$6) \int \frac{e^{-pxi}}{q^2 + x^2} \frac{dx}{x^r} = \frac{\pi e^{-pq} e^{ir\pi i}}{q^{r+1}}$$

$$7) \int \frac{e^{-pxi}}{q^2 + x^2} \frac{dx}{x^{2a}} = (-1)^a \frac{\pi}{q^{2a+1}} e^{-pq}$$

$$8) \int \frac{e^{-pxi}}{q^2 + x^2} \frac{dx}{x^{2a-1}} = (-1)^{a-1} \frac{\pi i}{q^{2a}} e^{-pq}$$

$$9) \int \frac{e^{-pxi}}{1+x^2} \frac{dx}{(xi)^{1-q}} = (-1)^{q-1} \pi e^p$$

$$10) \int \frac{e^{-pxi}}{1-x^2} \frac{dx}{(xi)^{1-q}} = -\frac{1}{2} \pi \cos. \left(\frac{1}{2} q \pi - p \right) \quad , \quad 0 < q < 1; \quad \text{Meyer, Int. Déf. 156.}$$

$$11) \int \frac{e^{(p-1)x} - e^{(q-1)x}}{e^x + e^{-x}} \frac{dx}{x} = \pi l \left(\text{Tang. } \frac{1}{4} p \pi . \text{Cot. } \frac{1}{4} q \pi \right) \quad \text{V. T. 180. N° 7.}$$

$$12) \int \frac{e^{(p-1)x} - e^{(q-1)x}}{e^x - e^{-x}} \frac{dx}{x} = \pi l \left(\text{Sin. } \frac{1}{2} p \pi . \text{Cosec. } \frac{1}{2} q \pi \right) \quad \text{V. T. 180. N° 13.}$$

F. Algèbr. rat. fract. } Den. à fact x^a . TABLE 146 suite. Lim. — ∞ et ∞ .
Expon.

$$13) \int \frac{e^{-px} - e^{-qx}}{1 + e^{-2qx}} \frac{dx}{x} = l \cot. \frac{p\pi}{4q} \quad \text{V. T. 183. N}^\circ. 17.$$

$$14) \int \frac{e^{-px} - e^{-qx}}{1 + e^{-rx}} \frac{dx}{x} = l \left(\text{Tang.} \frac{q\pi}{2r} \cdot \cot. \frac{p\pi}{2r} \right) \quad \text{V. T. 183. N}^\circ. 18.$$

F. Algèbr. rat. fract. } Dén. sans fact. x^a . TABLE 147. Lim. — ∞ et ∞ .
Expon.

$$1) \int \frac{e^{xi}}{q + xi} dx = 2\pi e^{-q} \quad \text{Poisson, P. 19. 404. N}^\circ. 68. — \text{Liouville, Cr. 13. 219.}$$

$$\left. \begin{aligned} 2) \int \frac{(-xi)^p}{q + xi} e^{xi} dx &= 2\pi q^p e^{-q} \\ 3) \int \frac{(xi)^p}{q + xi} e^{-xi} dx &= 0 \end{aligned} \right\} \quad \text{Cayley, L. 12. 231.}$$

$$4) \int \frac{e^{-pxi}}{(q + xi)^q} dx = 0 \quad \text{Ohm, Ausw. 23.}$$

$$5) \int \frac{e^{xi}}{(q + xi)^p} dx = \frac{2\pi e^{-q}}{\Gamma(p)} \quad \text{Poisson, P. 19. 404. N}^\circ. 73. — \text{Laplace, Prob. 33. — Liouville, Cr. 13. 219. — Lobatschewsky, Mém. Kasan. 1835. 211.}$$

$$6) \int \frac{e^{pxi}}{(q + xi)^r} dx = \frac{2\pi}{\Gamma(r)} p^{r-1} e^{-pq} \quad \text{Cauchy, Lim. Imag. 101. — Id., Exerc. 1827. p. 141.}$$

$$7) \int \frac{e^{pxi}}{(q - xi)^r} dx = 0 \quad \text{Cauchy, Lim. Imag. 105. — Id., Exerc. 1827. p. 141.}$$

$$\left. \begin{aligned} 8) \int \frac{e^{pxi}}{1 + x^2} dx &= \pi e^{-p} \\ 9) \int \frac{e^{pxi}}{1 + x^2} (-xi)^{q-1} dx &= \pi e^{-p}, q < 1; \end{aligned} \right\} \quad \text{Cauchy, Cours. Lec. 39.}$$

$$10) \int \frac{e^{-pxi}}{q^2 + x^2} dx = \frac{\pi}{q} e^{-pq} \quad \text{Lejeune-Dirichlet, Cr. 4. 94. — Schlömilch, Stud. II. 17.}$$

$$11) \int \frac{e^{pxi}}{q^2 + x^2} dx = \frac{\pi}{q} e^{-pq} \quad \text{Minding, Taf. 6.}$$

$$\left. \begin{aligned} 12) \int \frac{e^{(p-r)xi}}{q^2 + x^2} dx &= \frac{\pi}{q} e^{p-r)q}, \text{ pour } 0 < r < p; \\ 13) &= \frac{\pi}{q} e^{(r-p)q}, \text{ pour } p < r < \infty; \end{aligned} \right\} \quad \text{Ohm, Ausw. 23.}$$

- 14) $\int (-xi)^p e^{xi} \frac{dx}{q^2 + x^2} = \pi q^{p-1} e^{-q}$ Cayley, L. 12. 231.
- 15) $\int (xi)^{p+1} e^{-rxi} \frac{dx}{q^2 + x^2} = \pi q^p e^{-rq}$ Cauchy, P. 19. 511.
- 16) $\int \frac{e^{-pxi}}{(s + xi)^r} \frac{dx}{q^2 + x^2} = \frac{\pi}{q} \frac{e^{-pq}}{(q + s)^r}$
- 17) $\int \frac{e^{-pxi}}{(a + xi)^r (b + xi)^s \dots} \frac{dx}{q^2 + x^2} = \frac{\pi e^{-pq}}{q} \frac{1}{(a + q)^r (b + q)^s \dots}$ } Lejeune-Dirichlet, Cr. 4.
94. — Schlömilch, Stud. II. 27.
- , où $a, b, \dots$ peuvent être aussi des fractions;
- 18) $\int \frac{e^{-pxi}}{(a + xi)^s (b + xi)^t \dots} \frac{dx}{q^2 + r^2 x^2} = \frac{\pi}{q} e^{\frac{-pq}{r}} (q + ar)^{-s} (q + br)^{-t} \dots$ Ohm, Ausw. 93.
- , où $a, b \dots$ peuvent être aussi des fractions;
- 19) $\int (xi)^{r+1} e^{-pxi} \frac{dx}{q^2 - x^2} = \pi q^r \text{Cos.} \left\{ \frac{r+2}{2} \pi - pq \right\}$ Cauchy, P. 19. 511.

- 1) $\int e^{-px} dx \sqrt{x} = \frac{1}{2p} \sqrt{\frac{\pi}{p}}$ Ohm, Ausw. 20.
- 2) $\int e^{xi} \frac{dx}{\sqrt{(q + xi)}} = 2 e^{-q} \sqrt{\pi}$
- 3) $\int e^{q+xi} \frac{dx}{\sqrt{(q + xi)}} = 2 \sqrt{\pi}$
- 4) $\int e^{q+xi+\frac{pi}{4(q+xi)}} \frac{dx}{\sqrt{(q + xi)}} = (e^{\sqrt{pi}} + e^{-\sqrt{pi}}) \sqrt{\pi}$
- 5) $\int e^{q+xi+\frac{pi}{4(q+xi)}} \frac{dx}{\sqrt{(q + xi)}} = \frac{4}{2 - \sqrt{pi}} e^{-\sqrt{pi}} \sqrt{\pi}$
- } Cauchy, P. 19. 511.

- 1) $\int_0^{\frac{1}{\infty}} \frac{k}{k^2 + (b - x)^2} x^{p-1} e^{-qx} dx = \frac{\pi}{2 \Gamma(p)} b^{p-1} e^{-bq}$, où il faut mettre $k=0$ après l'intégration :
Cauchy, P. 28. 147. P. 1. 3.

$$2) \int_{-1}^0 \frac{e^{-x}}{x} dx = -\infty \quad \text{V. T. 45. N}^{\circ} 7.$$

$$3) \int_{-\infty}^0 \frac{e^{-x}}{x} dx = -\infty \quad \text{V. T. 45. N}^{\circ} 9.$$

$$4) \int_0^a \frac{e^{(bc-1)x} - e^{-(bc+1)x}}{x} dx = \frac{1}{2} l \left(\frac{1+bc}{1-bc} \right)^2 + Ei \left(-\frac{a}{c} + ab \right) - Ei \left(-\frac{a}{c} - ab \right) \quad \text{Arndt, Gr. 11. 70.}$$

$$5) \int_0^{l2} (e^x - 1)^{a-1} x e^x dx = \frac{1}{a} \left\{ l2 + (-1)^a \sum_{n=1}^{\infty} \frac{1}{a+n+1} \right\} \quad \text{V. T. 151. N}^{\circ} 11.$$

$$6) \int_0^{l2} \frac{x}{1-e^{-x}} dx = \frac{1}{12} \pi^2 \quad \text{V. T. 160. N}^{\circ} 1.$$

$$7) \int_0^{l2} \frac{e^x x^2}{(e^x - 1)^2} dx = \frac{1}{6} \pi^2 - 2(l2)^2 \quad \text{V. T. 149. N}^{\circ} 6.$$

$$8) \int_0^{l2} \frac{e^x - e^{-x}}{(e^{4x} - e^{-4x})^4} x^2 dx = \frac{\pi}{4} l2 - 2(l2)^2 \quad \text{V. T. 149. N}^{\circ} 9$$

$$9) \int_0^{l2} \frac{x}{e^x + e^{-x} - 2} dx = \frac{1}{8} \pi l2 \quad \text{V. T. 160. N}^{\circ} 2.$$

$$10) \int_0^{l(1+p)} (1+x) e^x dx = (1+p) l(1+p) \quad \text{V. T. 42. N}^{\circ} 4.$$

$$11) \int_0^{l \frac{1+p}{1-p}} \frac{x}{(p^2-1)(e^x + e^{-x}) - 2(p^2+1)} dx = -\frac{1}{p} \text{Arcsin. } p, p^2 < 1; \quad \text{V. T. 186. N}^{\circ} 2.$$

$$12) \int_0^{l \frac{1+p}{1-p}} \frac{e^x - 1}{(p^2-1)(e^{2x}+1) + 2(p^2+1)e^x} \cdot \frac{x e^x}{\{(p^2-1)(e^{2x}+1) + 2(p^2+1)e^x\}} dx = \frac{\pi}{2p} \text{Arcsin. } p, p \leq 1; \quad \text{V. T. 160. N}^{\circ} 17.$$

$$13) \int_0^{l \frac{1+p}{1-p}} \frac{1 - e^x}{(p^2 - q^2)(1 + e^{2x}) + 2(p^2 + q^2)e^x} \cdot \frac{x e^x dx}{\{(p^2-1)(e^{2x}+1) + 2(p^2+1)e^x\}} \quad \text{V. T. 166. N}^{\circ} 17.$$

$$= \frac{\pi}{2pq \sqrt{(1-q^2)}} \frac{l^p q + \{1 - \sqrt{(1-q^2)}\} \{1 - \sqrt{(1-p^2)}\}}{p q - \{1 - \sqrt{(1-q^2)}\} \{1 - \sqrt{(1-p^2)}\}}$$

$$14) \int_0^{2l \operatorname{Cot} \frac{1}{2} \lambda} \frac{x}{\sqrt{\{2(1 + \operatorname{Cos}^2 \lambda) e^x - \operatorname{Sin}^2 \lambda (1 + e^{2x})\}}} \frac{e^x}{1 - e^x} dx = \frac{1}{4} \pi \frac{\pi - 2\lambda}{\operatorname{Cos} \lambda} \quad \text{V. T. 166. N}^\circ 7.$$

$$15) \int_0^q \frac{e^{-px}}{x - q} dx = -\infty \quad \text{Arndt, Gr. 10. 247.}$$

$$16) \int_0^p (e^{-kqx} - e^{-krx}) \frac{dx}{x} = l \frac{r}{q}, \text{ pour } k = \infty; \quad \text{Schlömilch, Gr. 11. 63.}$$

$$1) \int_1^\infty \frac{e^{-\frac{x}{q}}}{\sqrt{(x-1)}} \frac{dx}{\sqrt{q} e} = \frac{\sqrt{q} \pi}{\sqrt{e}} \quad \text{Legendre, Exerc. 3. 50.}$$

$$2) \int_{-\infty}^{-1} \frac{e^{-x}}{x} dx = -\infty \quad \text{V. T. 45. N}^\circ 8.$$

$$3) \int_{-p}^\infty \frac{e^{-x}}{x} dx = -\frac{1}{2} l \alpha^2 - li.(p), \text{ où } \alpha \text{ est indéterminé; } \quad \text{Arndt, Gr. 10. 247.}$$

$$4) \int_a^\infty \frac{e^{-x}}{x} dx = -A - la - \sum_1^\infty (-1)^n \frac{a^n}{n 1^{n/1}} \quad \text{Clausius, Cr. 34. 123.}$$

$$5) \quad = -Ei.(-a) \quad \text{Beez, Gr. 19. 419.}$$

$$6) \int_q^\infty \frac{e^{-x}}{x^2} dx = Ei.(-q) + \frac{1}{q} e^{-q} \quad \text{V. T. 150. N}^\circ 8.$$

$$7) \int_p^\infty \frac{e^{-x}}{x^a} dx = \frac{(-1)^a}{1^{a/1}} \left\{ A - \sum_1^{a-1} \frac{1}{n} \right\} + \sum_1^{a-1} \frac{1}{1^{n-1/1}} \frac{(-1)^n}{(a-n)p^{a-n}} + \frac{(-1)^{a-1}}{1^{a-1/1}} \left\{ lp + \sum_1^\infty \frac{(-p)^n}{a^{n/1}} \right\} \quad \text{Arndt, Gr. 10. 233.}$$

$$8) \int_p^\infty \frac{e^{!qx}}{e^{!qx} - e^{-qx}} \frac{dx}{x} = \frac{1}{2pq} + \frac{1}{\pi} \sum_1^\infty (-1)^n \frac{1}{2n} \operatorname{Arctang} \frac{2n\pi}{pq} + \frac{1}{2\pi} \sum_0^\infty \frac{(-1)^n}{2n+1} l \left\{ 1 + \left(\frac{2n+1}{pq} \pi \right)^2 \right\}$$

$$9) \int_p^\infty \frac{1}{e^{!qx} - e^{-!qx}} \frac{dx}{x} = \frac{1}{pq} + \frac{2}{\pi} \sum_1^\infty \frac{(-1)^n}{2n} \operatorname{Arctang} \frac{2n\pi}{pq}$$

$$10) \int_p^\infty \frac{1}{e^{!qx} + e^{-!qx}} \frac{dx}{x} = \frac{1}{\pi} \sum_0^\infty \frac{(-1)^n}{2n+1} l \left\{ 1 + \left(\frac{2n+1}{pq} \pi \right)^2 \right\}$$

Raabe,
Int.
419.

$$11) \int_q^\infty \frac{x}{\sqrt{(e^x - q^2)}} \frac{e^x}{e^x - q^2 + 1} dx = 2\pi l(1+q) \quad \text{V. T. 181. N}^\circ 2.$$

$$12) \int_q^\infty \frac{e^{-px}}{x-q} dx = \infty \quad \text{Arndt, Gr. 10. 247.}$$

$$13) \int_{lp}^\infty \frac{e^{-x}}{x^2} dx = li(p) - \frac{1}{p lp} \quad \text{V. T. 150. N}^\circ 14.$$

$$14) \int_{-lp}^\infty \frac{e^{-x}}{x} dx = -li(p) \quad \text{V. T. 45. N}^\circ 5.$$

$$15) \int_{-1 \cot^2 \lambda}^\infty \frac{e^{-a^2 x}}{\sqrt{(\cos^2 \lambda + 2x \sin^2 \lambda)}} dx = \frac{1}{2a \sin \lambda} e^{\frac{1}{2} a^2 \cot^2 \lambda} \sqrt{2\pi} \quad \text{Vieille, Exere. p. 165.}$$

$$1) \int x^{2a} l x dx = \frac{1}{(2a+1)^2} \quad \text{Arndt, Gr. 6. 187.}$$

$$2) \int x^{p-1} \left(l \frac{1}{x} \right)^a dx = \frac{1^{a/1}}{p^{a+1}} \quad \text{Euler, Calc. Int. 4. S. 3 § 7. — Legendre, Mém. Inst. 1809. 416. N}^\circ 41. — \text{Id., Exerc. 2. 40. — Id., Exere. 4. 147.}$$

$$3) \int x^p (l x)^a dx = (-1)^a \frac{1^{a/1}}{(1+p)^{a+1}} \quad \text{Euler, N. C. Petr. 14. 129. — Id., Calc. Int. 4. S. 5. N}^\circ 13. — \text{Oettinger, Cr. 35. 13.}$$

$$4) \int x^{q+ri-1} \left(l \frac{1}{x} \right)^{p-1} dx = \frac{\Gamma(p)}{(q+ri)^p} \quad \text{V. T. 113. N}^\circ 17.$$

$$5) \int x^{p-1} dx \sqrt{\left(l \frac{1}{x} \right)} = \frac{1}{2p} \sqrt{\frac{\pi}{p}} \quad \text{V. T. 139. N}^\circ 2.$$

$$6) \int (1-x)^{q-1} x^{p-1} l x dx = \frac{Z'(p) - Z'(p+q)}{\Gamma(p+q)} \Gamma(p) \Gamma(q) \quad \text{Raabe, Int. 228. — Féaux, Funct. Transc. p. 36.}$$

$$6') = - \frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)} \sum_{n=0}^q \frac{1}{n+p-1}, \text{ pour } q \text{ entier; } \quad \text{Féaux, Funct. Transc. p. 36.}$$

$$7) \int (1-x)^a x^{p-1} (l x)^b dx = (-1)^{a+b} 1^{b/1} \Delta^a \frac{1}{q p^{b+1}} \left. \begin{aligned} &= (-1)^b 1^{b/1} \sum_{n=0}^{\infty} \binom{a}{n} \frac{(-1)^n}{(p+nq)^{b+1}} \end{aligned} \right\} \quad \text{Oettinger, Cr. 38. 162.}$$

8)

$$= (-1)^b 1^{b/1} \sum_{n=0}^{\infty} \binom{a}{n} \frac{(-1)^n}{(p+nq)^{b+1}}$$

- 9) $\int (x-1)^c x^{b-1} \left(l \frac{1}{x}\right)^{q-1} dx = \Gamma(q) \Delta^c . b^{-q}$ Legendre, Exerc. 4. 147.
- 10) $\int \left\{ \left(l \frac{1}{x}\right)^{q-1} - x^{p-1} (1-x)^{q-1} \right\} dx = \frac{\Gamma(p+q) - \Gamma(p)}{q \Gamma(p+q)} \Gamma(1+q)$ V. T. 113. N°. 9.
- 11) $\int l(1+x) x^{a-1} dx = \frac{1}{a} \left\{ 2 + (-1)^a \sum_0^{\infty} \frac{1}{a+1+n} \right\}$ V. T. 3. N°. 2.
- 12) $\int l(1-x) x dx = -\frac{3}{4}$ Euler, N. C. Petr. 14. 129.
- 13) $\int l(1+x^2) x^{p-1} dx = \frac{1}{2p} \left\{ 2l2 + Z' \left(\frac{p+1}{4} \right) - Z' \left(\frac{p+3}{4} \right) \right\}$ V. T. 3. N°. 13.
- 14) $\int (p x^{p-1} - q x^{q-1}) l(1-x^2) dx = Z' \left(\frac{1}{2} q + 1 \right) - Z' \left(\frac{1}{2} p + 1 \right)$ V. T. 3. N°. 14.
- 15) $\int l(q+lx) x^{p-1} dx = \frac{1}{p} \{ e^{-pq} Ei.(pq) + lq \}$ V. T. 169. N°. 1.
- 16) $\int l(q-lx) x^{p-1} dx = \frac{1}{p} \{ e^{pq} Ei.(-pq) + lq \}$ V. T. 169. N°. 2.

- 1) $\int lx \frac{dx}{x^{a-1}} = -\frac{1}{a^2}$ Euler, Mém. Petersb. 1814.
- 2) $\int lx \frac{dx}{1+x} = -\frac{1}{2} \sum_1^{\infty} \frac{1}{n^2}$ Euler, Calc. Int. 4. S. 5. § 47. — Id., N. C. Petr. 19. 66.
- 3) $\int = -\frac{1}{12} \pi^2$ Euler, Calc. Int. 4. S. 5. § 78. — Kausler, Mém. Petersb. T. 3. — Plana, Mém. Turin. 1818. 7. IV. 21.
- 4) $\int lx \frac{x}{1+x} dx = -1 + \frac{1}{12} \pi^2$ V. T. 42. N°. 1, et T. 152. N°. 3. — Kausler, Mém. Petersb. T. 3. p. 414. trouve faut. $\frac{1}{12} \pi^2$.
- 5) $\int lx \frac{x^2}{1+x} dx = \frac{3}{4} - \frac{1}{12} \pi^2$ V. T. 42. N°. 1, et T. 152. N°. 4. — Kausler, Mém. Petersb. T. 3. p. 114. trouve faut. $-\frac{1}{12} \pi^2$.
- 6) $\int lx \frac{dx}{1-x} = -\sum_1^{\infty} \frac{1}{n^2}$ Euler, Calc. Int. 4. S. 5. § 47. — Id., N. C. Petr. 19. 66.
- 7) $= -\frac{1}{6} \pi^2$ Euler, Calc. Int. 4. S. 3. § 78. — Plana, Mém. Turin. 1818. 7. IV. 21. — Schaeffer, Cr. 30. 277.

- 8) $\int l x \frac{x}{1-x} dx = 1 - \frac{1}{6} \pi^2$ V. T. 42. N°. 1 et T. 152. N°. 7.
- 9) $\int l x \frac{1+x}{1-x} dx = 1 - 2 \sum_1^{\infty} \frac{1}{n^2}$ Euler, N. C. P. 14. 129.
- 10) $\int l x \frac{x^{p-1}}{1-x} dx = - \sum_0^{\infty} \frac{1}{(p+n)^2}$ Binet, P. 27. 123.
- 11) $\int l x \frac{dx}{1+x^2} = - \sum_0^{\infty} (-1)^n \frac{1}{(2n+1)^2}$ Bidone, Mém. Turin. 1812. 231. Art. 13. N°. 37.
- 12) $\int l x \frac{x}{1+x^2} dx = - \frac{1}{48} \pi^2$ Euler, Calc. Int. 4. S. 5. 49. — Id., N. C. P. 19. 66.
- 13) $\int l x \frac{dx}{1-x^2} = - \frac{1}{8} \pi^2$ Euler, Calc. Int. 4. S. 3. 78. — Id., N. C. P. 19. 30. — Poisson, Mém. Inst. 1811. 163. N°. 27. $\left(\text{fautiv.} - \frac{1}{8} \pi \right)$ — Plana, Mém. Turin. 1818. 7. IV. 21.
- 14) $\int l x \frac{x}{1-x^2} dx = - \frac{1}{24} \pi^2$ Euler, Calc. Int. 4. S. 3. 78. — Plana, Mém. Turin. 1818. 7. IV. 21.
- 15) $\int l x \frac{1-x}{1+x^3} dx = - \frac{2}{27} \pi^3$ Euler, Calc. Int. 4. S. 3. 80. — Id., N. C. P. 19. 30.
- 16) $\int l x \frac{x}{1-x^4} dx = - \frac{1}{32} \pi^2$ V. T. 152. N°. 12, 14.
- 17) $\int l x \frac{x^3}{1-x^4} dx = - \frac{1}{96} \pi^2$ V. T. 152. N°. 12, 14.
- 18) $\int l x \frac{x^{a-1}}{1-x^{2a}} dx = - \frac{\pi^2}{8a^2}$ Euler, Calc. Int. 4. S. 3. 77. — Id., N. C. P. 19. 30.
- 19) $\int l x \frac{1-x^2}{1+x^{2p}} x^{p-2} dx = - \frac{1}{4} \left(\frac{\pi}{p} \right)^2 \text{Sin.} \frac{\pi}{2p} \cdot \text{Sec.}^2 \frac{\pi}{2p}$
- 20) $\int l x \frac{1+x^2}{1-x^{2p}} x^{p-2} dx = - \frac{1}{4} \left(\frac{\pi}{p} \right)^2 \text{Sec.}^2 \frac{\pi}{2p}$
- 21) $\int l \left(\frac{1}{x} \right) \frac{x^{a-1} + x^{b-a-1}}{1-x^b} dx = \left(\frac{\pi}{b} \right)^2 \text{Cosec.}^2 \frac{a\pi}{b}$ Legendre, Exerc. 2. 44. — Id., Mém. Inst. 1809. 416. N°. 45.

- $$\left. \begin{aligned} 1) \int lx \frac{1+2x}{1+x+x^2} dx &= -\frac{1}{9} \pi^2 \\ 2) \int lx \frac{1-2x}{1-x+x^2} dx &= -\frac{1}{18} \pi^2 \end{aligned} \right\} \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. 105. — Id., ib. S. 5. 50. — Id.,} \\ \text{N. C. P. 19. 30.} \end{array}$$
- $$3) \int lx \frac{dx}{1-x+x^2} = -\frac{4}{27} \pi^2 \quad \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. 80. — Id., N. C. P. 19. 30. —} \\ \text{Legendre, Mém. Inst. 1809. 416. N°. 51.} \end{array}$$
- $$4) \int lx \frac{x}{1-x+x^2} dx = -\frac{5}{108} \pi^2 \quad \text{V. T. 153. N°. 2, 3.}$$
- $$5) \int lx \frac{1-x^2}{1+(e^{2p}+e^{-2p})x^2+x^4} dx = \frac{p}{2} \frac{\pi}{e^p-e^{-p}} \quad \text{V. T. 125. N°. 5.}$$
- $$6) \int lx \frac{\cos \lambda - x}{1-2x \cos \lambda + x^2} dx = \frac{1}{6} \pi^2 - \frac{1}{2} \pi \lambda + \frac{1}{4} \lambda^2 \quad \begin{array}{l} \text{Euler, N. C. P. 19. 66. — Id., Calc.} \\ \text{Int. 4. S. 5. 46.} \end{array}$$
- $$7) \int lx \frac{x}{1-x^2+x^4} dx = -\frac{1}{27} \pi^2 \quad \text{Euler, Calc. Int. 4. S. 3. 80. — Id., N. C. P. 19. 30.}$$
- $$8) \int lx \frac{1-x^2}{1+2px^2+x^4} dx = \frac{\pi}{2\sqrt{\{2(p-1)\}}} l \frac{\sqrt{(p-1)} - \sqrt{(p+1)} + \sqrt{2}}{\sqrt{(p-1)} + \sqrt{(p+1)} - \sqrt{2}} \quad \text{V. T. 125. N°. 4.}$$
- $$9) \int lx \frac{1-x^2}{1+2x^2 \cos 2\lambda + x^4} dx = -\frac{1}{4} \pi \lambda \operatorname{Cosec} \lambda \quad \text{V. T. 125. N°. 6.}$$
- $$\left. \begin{aligned} 10) \int lx \frac{1-x^2}{1+x^2} \frac{dx}{x} &= -\infty \\ 11) \int lx \frac{1+x^2}{1-x^2} \frac{dx}{x} &= -\infty \end{aligned} \right\} \quad \text{Euler, N. C. P. 19. 30. — Id., Calc. Int. 4. S. 3. 81.}$$
- $$\left. \begin{aligned} 12) \int lx \frac{x^{p-q} + x^{p+q}}{1+x^{2p}} \frac{dx}{x} &= \frac{\pi^2}{4p^2} \operatorname{Sin} \frac{q\pi}{2p} \cdot \operatorname{Sec}^2 \frac{q\pi}{2p} \\ 13) \int lx \frac{x^{p-q} - x^{p+q}}{1-x^{2p}} \frac{dx}{x} &= -\frac{\pi^2}{4p^2} \operatorname{Sec}^2 \frac{q\pi}{2p} \end{aligned} \right\} \quad \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. 74. — Id., N. C.} \\ \text{P. 19. 30.} \end{array}$$
- $$14) \int lx \frac{dx}{(1+x)^2} = \frac{1}{2} l \frac{1}{2} \quad \text{V. T. 122. N°. 1.}$$
- $$15) \int lx \frac{x}{(1+x^2)^2} dx = \frac{1}{4} l \frac{1}{2} \quad \text{V. T. 335. N°. 1.}$$
- $$16) \int lx \frac{x^2}{1-x^2} \frac{dx}{1+x^4} = -\frac{\pi^2}{16(2+\sqrt{2})} \quad \text{Euler, Calc. Int. 4. S. 3. 80. — Id., N. C. P. 19. 30.}$$

- 17) $\int l x \frac{(p+q)(x^{p-q} - x^{q-p}) + (p-q)(x^{p+q} - x^{-(p+q)})}{(x^p + x^{-p})^2} \frac{dx}{x} = \frac{\pi}{2p} \text{Sec.} \frac{q\pi}{2p}, p > q; \quad \text{V. T. 5. N}^\circ \text{ 18.}$
- 18) $\int l x \frac{(p+q)(x^{p-q} - x^{q-p}) + (q-p)(x^{p+q} - x^{-(p+q)})}{(x^p - x^{-p})^2} \frac{dx}{x} = -\frac{\pi}{2p} \text{Tang.} \frac{q\pi}{2p}, p > q; \quad \text{V. T. 5. N}^\circ \text{ 17.}$
- 19) $\int l x \frac{x^p - x^{-p}}{(x^p + x^{-p})^2} \frac{dx}{x} = \frac{\pi}{4p^2} \quad \text{V. T. 5. N}^\circ \text{ 23.}$
- 20) $\int l x \frac{1-x^2}{1+x^2} \frac{dx}{x \left(x + \frac{1}{x}\right)^{2p}} = \frac{\{\Gamma(p)\}^2}{8p \Gamma(2p)} \quad \text{V. T. 5. N}^\circ \text{ 24.}$
- 21) $\int \left\{ \frac{1+plx}{1-x} + \frac{x lx}{(1-x)^2} \right\} x^{p-1} dx = -1 \quad \text{Arndt, Gr. 10. 253.}$

- 1) $\int (lx)^2 \frac{dx}{1+x^2} = \frac{1}{16} \pi^3 \quad \left\{ \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. 84. — Id., ib. S. 5. 49. — Id.,} \\ \text{N. C. P. 19. 30.} \end{array} \right.$
- 2) $\int (lx)^2 \frac{1+x^2}{1+x^4} dx = \frac{3 \sqrt{2}}{64} \pi^3 \quad \left\{ \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. 84. — Id., ib. S. 5. 49. — Id.,} \\ \text{N. C. P. 19. 30.} \end{array} \right.$
- 3) $\int (lx)^2 \frac{1-x^4}{1-x^6} dx = \frac{1}{27} \pi^3 \sqrt{3} \quad \left\{ \begin{array}{l} \text{Legendre, Mém. Inst. 1809. 416. N}^\circ \text{ 50. — Id., Exerc.} \\ \text{2. 49.} \end{array} \right.$
- 4) $\int (lx)^2 \frac{1-x^2}{1-x^6} x dx = \frac{1}{243} \pi^3 \sqrt{3} \quad \left\{ \begin{array}{l} \text{Legendre, Mém. Inst. 1809. 416. N}^\circ \text{ 50. — Id., Exerc.} \\ \text{2. 49.} \end{array} \right.$
- 5) $\int (lx)^2 \frac{x^{p-q-1} + x^{p+q-1}}{1+x^{2p}} dx = \frac{\pi^3}{8p^3} \left(2 \text{Sec.}^3 \frac{q\pi}{2p} - \text{Sec.} \frac{q\pi}{2p} \right) \quad \left\{ \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. 82. —} \\ \text{Id., N. C. P. 19. 30.} \end{array} \right.$
- 6) $\int (lx)^2 \frac{x^{p-q-1} - x^{p+q-1}}{1-x^{2p}} dx = \frac{\pi^3}{4p^3} \text{Sin.} \frac{q\pi}{2p} \cdot \text{Sec.}^3 \frac{q\pi}{2p} \quad \left\{ \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. 82. —} \\ \text{Id., N. C. P. 19. 30.} \end{array} \right.$
- 7) $\int (lx)^2 \frac{x^{q-1} - x^{p-q-1}}{1-x^p} dx = 2 \left(\frac{\pi}{p} \text{Cosec.} \frac{q\pi}{p} \right)^3 \cdot \text{Cos.} \frac{q\pi}{p} \quad \left\{ \begin{array}{l} \text{Legendre, Exerc. 2. 44. —} \\ \text{Id., Mém. Inst. 1809. 416.} \\ \text{N}^\circ \text{ 45.} \end{array} \right.$
- 8) $\int (lx)^3 \frac{x^{q-1} + x^{p-q-1}}{1-x^p} dx = \left(\frac{\pi}{p} \text{Cosec.} \frac{q\pi}{p} \right)^4 \left(2 + 4 \text{Cos.}^2 \frac{q\pi}{p} \right) \quad \left\{ \begin{array}{l} \text{Legendre, Exerc. 2. 44. —} \\ \text{Id., Mém. Inst. 1809. 416.} \\ \text{N}^\circ \text{ 45.} \end{array} \right.$
- 9) $\int (lx)^3 \frac{dx}{1+x} = -\frac{21}{4} \sum_{n=1}^{\infty} \frac{1}{n^4} \quad \text{Euler, Calc. Int. 4. S. 5. 47. — Id., N. C. P. 19. 66.}$

- 10) $\int (lx)^3 \frac{dx}{1+x} = -\frac{7}{120} \pi^4$ Euler, Calc. Int. 4. S. 3. 96. — Id., N. C. P. 19. 30.
- 11) $\int (lx)^3 \frac{dx}{1-x} = -\frac{1}{15} \pi^4$
- 12) $= -6 \sum_1^{\infty} \frac{1}{n^4}$ Euler, Calc. Int. 4. S. 5. 47. — Id., N. C. P. 19. 66.
- 13) $\int (lx)^3 \frac{x}{1+x^2} dx = -\frac{7}{1920} \pi^4$ Euler, Calc. Int. 4. S. 5. 49. — Id., N. C. P. 19. 66.
- 14) $\int (lx)^3 \frac{dx}{1-x^2} = -\frac{1}{16} \pi^4$ Euler, Calc. Int. 4. S. 3. 95. — Id., N. C. P. 19. 30.
- 15) $\int (lx)^3 \frac{x}{1-x^2} dx = -\frac{1}{240} \pi^4$ V. T. 154. N°. 10, 12.
- 16) $\int (lx)^3 \frac{x}{1-x^4} dx = -\frac{1}{256} \pi^4$ V. T. 154. N°. 13, 15.
- 17) $\int (lx)^3 \frac{x^3}{1-x^4} dx = -\frac{1}{3840} \pi^4$ V. T. 154. N°. 13, 15.

- 1) $\int (lx)^4 \frac{dx}{1+x^2} = \frac{5}{64} \pi^5$ Euler, Calc. Int. 4. S. 3. 94. — Id., N. C. P. 19. 30.
- 2) $\int (lx)^5 \frac{dx}{1+x} = -\frac{31}{252} \pi^6$
- 3) $\int (lx)^5 \frac{dx}{1-x} = -\frac{8}{63} \pi^6$ Euler, Calc. Int. 4. S. 3. 97. — Id., N. C. P. 19. 30.
- 4) $\int (lx)^5 \frac{dx}{1-x^2} = -\frac{1}{8} \pi^6$ Euler, Calc. Int. 4. S. 3. 95. — Id., N. C. P. 19. 30.
- 5) $\int (lx)^5 \frac{x}{1-x^2} dx = -\frac{1}{504} \pi^6$ V. T. 155. N°. 2, 3.
- 6) $\int (lx)^5 \frac{dx}{1+x} = -\frac{465}{4} \sum_1^{\infty} \frac{1}{n^6}$
- 7) $\int (lx)^5 \frac{dx}{1-x} = -120 \sum_1^{\infty} \frac{1}{n^6}$ Euler, Calc. Int. 4. S. 5. 47. — Id., N. C. P. 19. 66.

F. Alg. rat. fract. à dén. binôme.
Log. en num. $(lx)^4, (lx)^5, (lx)^6, (lx)^7$. TABLE 155 suite.

Lim. 0 et 1.

$$8) \int (lx)^6 \frac{dx}{1+x^2} = \frac{61}{256} \pi^7 \quad \text{Euler, Calc. Int. 4. S. 3. 94. — Id., N. C. P. 19. 30}$$

$$9) \int (lx)^7 \frac{dx}{1+x} = -\frac{127}{1680} \pi^8 \quad \text{Euler, Calc. Int. 4. S. 5. 47.}$$

$$10) \int (lx)^7 \frac{dx}{1-x^2} = -\frac{17}{32} \pi^8 \quad \text{Euler, N. C. P. 19. 30.}$$

F. Alg. rat. fract. à dén. trinôme.
Log. en num. $(lx)^a$ pour a spécial. TABLE 156.

Lim. 0 et 1.

$$\left. \begin{aligned} 1) \int (lx)^2 \frac{dx}{1+x+x^2} &= \frac{8}{243} \pi^3 \checkmark 3 \\ 2) \int (lx)^2 \frac{dx}{1-x+x^2} &= \frac{10}{243} \pi^3 \checkmark 3 \end{aligned} \right\} \begin{aligned} &\text{Euler, Calc. Int. 4. S. 3. 105. — Id., N. C. P. 19. 30. —} \\ &\text{Legendre, Exerc. 2. 49. — Id., Mém. Inst. 1809. 416.} \\ &\text{N°. 50.} \end{aligned}$$

$$3) \int (lx)^2 \frac{dx}{1+2x \cos \lambda + x^2} = \frac{1}{2} \lambda \operatorname{Cosec} \lambda \frac{\pi^2 - \lambda^2}{3} \quad \text{Legendre, Exerc. 4. 105.}$$

$$4) \int (lx)^2 \frac{dx}{1-2x \cos \lambda + x^2} = 2 \lambda \operatorname{Cosec} \lambda \left(\frac{1}{6} \pi^2 - \frac{1}{4} \pi \lambda + \frac{1}{12} \lambda^2 \right) \quad \text{Euler, N. C. P. 19. 66.}$$

$$5) \int (lx)^4 \frac{dx}{1+2x \cos \lambda + x^2} = \frac{1}{5} \lambda \operatorname{Cosec} \lambda (\pi^2 - \lambda^2) (7 \pi^2 - 3 \lambda^2) \quad \text{Legendre, Exerc. 4. 105.}$$

F. Alg. rat. fract. à dén. binôme $x \pm b$
Log. en num. $(lx)^a$ pour a général. TABLE 157.

Lim. 0 et 1.

$$1) \int (lx)^{2a} \frac{dx}{1 \pm x} = 0 \text{ (fautive)} \quad \text{Euler, Calc. Int. 4. S. 5. 47.}$$

$$\left. \begin{aligned} 2) \int (lx)^{2a} \frac{dx}{1+x} &= \frac{2^{2a}-1}{2^{2a}} 1^{2a/1} \sum_1^{\infty} \frac{1}{n^{2a+1}} \\ 3) \int (lx)^{2a} \frac{dx}{1-x} &= 1^{2a/1} \sum_1^{\infty} \frac{1}{n^{2a+1}} \end{aligned} \right\} \quad \text{Raabe, Cr. 42. 348.}$$

$$4) \int (lx)^{2a-1} \frac{dx}{1+x} = 1^{2a-1/1} \left(\frac{1}{2^{2a-1}} - 1 \right) \sum_1^{\infty} \frac{1}{n^{2a}} \quad \text{Euler, Calc. Int. 4. S. 5. 47.}$$

$$5) \quad = -\frac{2^{2a-1}-1}{2^a} \pi^{2a} B_{2a-1} \quad \text{Arndt, Gr. 6. 434.}$$

- 6) $\int (lx)^{2a-1} \frac{dx}{1-x} = -\frac{2^{2a-2}}{a} \pi^{2a} B_{2a-1}$ Arndt, Gr. 6. 434.
- 7) $= -1^{2a-1} \sum_1^{\infty} \frac{1}{n^{2a}}$ Euler, Calc. Int. 4. S. 5. 47.
- 8) $\int \left(l \frac{1}{x}\right)^{a-1} \frac{dx}{1+x} = 1^{a/1} \sum_0^{\infty} \frac{(-1)^n}{(1+n)^a}$ Arndt, Gr. 6. 434.
- 9) $\int \left(l \frac{1}{x}\right)^{a-1} \frac{dx}{1-x} = 1^{a/1} \sum_0^{\infty} \frac{1}{(1+n)^a}$ Euler, N. C. P. 14. 129. — Arndt, Gr. 6. 434.
- 10) $\int \left(l \frac{1}{x}\right)^{p+1} \frac{dx}{1-qx} = \frac{p+1}{q} \Gamma(p+1) \sum_0^{\infty} \frac{q^n}{n^{p+2}}$ V. T. 117. N°. 11.
- 11) $\int \left(l \frac{1}{x}\right)^{b-1} \frac{x^a}{1+x} dx = 1^{b/1} \sum_0^{\infty} \frac{(-1)^n}{(a+n+1)^b}, b > 1;$ Arndt, Gr. 6. 434.
- 12) $\int \left(l \frac{1}{x}\right)^{b-1} \frac{x^a}{1-x} dx = 1^{b/1} \sum_0^{\infty} \frac{1}{(a+n+1)^b}$ Binet, P. 27. 123. — Arndt, Gr. 6. 434.
- 13) $\int \left(l \frac{1}{x}\right)^q x^{b-1} (x-1)^c \left(b + \frac{cx}{x-1}\right) dx = \Gamma(q) \Delta^c b^{-q}$ V. T. 118. N°. 18.
- 14) $\int \left(l \frac{1}{x}\right)^{a-1} \frac{1-x^b}{1-x} dx = 1^{a/1} \sum_1^b \frac{1}{n^a}$ Euler, N. C. Petr. 14. 129.

- 1) $\int (lx)^a \frac{dx}{1+x^2} = (-1)^a 1^{a/1} \sum_1^{\infty} \frac{(-1)^n}{(2n+1)^{a+1}}$ Bidone, Mém. Turin. 1812. 231. Art. 3. 37.
- 2) $\int (lx)^{2a} \frac{dx}{1+x^2} = \frac{(-1)^{a+1}}{2} (2\pi)^{2a+1} B''\left(\frac{1}{4}\right)$
- 3) $\int (lx)^{2a} \frac{x^b + x^{-b}}{1+x^2} dx = \frac{(-1)^{a+1}}{b} \left(\frac{2\pi}{b}\right)^{2a+1} \sum_1^b (-1)^{n+1} B''\left(\frac{2n-1}{4b}\right) \cos\left(\frac{2n-1}{2} b\pi\right)$ Raabe, Cr. 42. 348.
- 4) $\int (lx)^{2a} \frac{dx}{1-x^2} = \frac{2^{2a+1}-1}{2^{2a+1}} 1^{2a/1} \sum_1^{\infty} \frac{1}{n^{2a+1}}$
- 5) $\int \left(l \frac{1}{x}\right)^{2a-1} \frac{dx}{1-x^2} = \frac{2^{2a}-1}{4a} \pi^{2a} B_{2a-1}$ V. T. 120. N°. 18.

- 6) $\int (lx)^{2a-1} \frac{x}{1-x^2} dx = (-1)^a \frac{\pi^{2a}}{4a} B_{2a-1}$ Plana, Mém. Turin. 1820.
- 7) $\int (lx)^{p-1} \frac{dx}{1-x^2} = \frac{\Gamma(p)}{(-1)^{p-1}} \sum_0^{\infty} \frac{1}{(2n+1)^p}$ V. T. 336. N°. 17.
- 8) $\int (lx)^{p-1} \frac{x^q}{1-x^2} dx = \frac{\Gamma(p)}{(-1)^{p-1}} \sum_0^{\infty} \frac{1}{(q+1+2n)^p}$ V. T. 336. N°. 18.
- 9) $\int (lx)^{2a} \frac{x^{-b}-x^b}{1-x^2} dx = \frac{(-1)^{a+1}}{b} \left(\frac{2\pi}{b}\right)^{2a+1} \sum_1^b (-1)^{n-1} B''\left(\frac{n}{2b}\right) \text{Sin. } nb\pi$ Raabe, Cr. 42. 348.
- 10) $\int \left(\frac{1}{x}\right)^r \frac{x^{p-1}}{1-x^q} dx = 1^{r/1} \sum_0^{\infty} \frac{1}{(p+nq)^{r+1}}$ Oettinger, Cr. 38. 162.
- 11) $\int (lx)^{2a} \frac{x^{b-1}-x^{2c-b-1}}{1-x^{2c}} dx = \frac{(-1)^{a+1}}{c} (2\pi)^{2a+1} \sum_1^{c-1} B''\left(\frac{n}{2c}\right) \text{Sin. } \frac{nb\pi}{c}$
- 12) $\int (lx)^{2a} \frac{x^{-1}+x^{2c-b-1}}{1+x^{2c}} dx = \frac{(-1)^{a+1}}{c} (2\pi)^{2a+1} \sum_1^c B''\left(\frac{2n-1}{4c}\right) \text{Sin. } \left(\frac{2n-1}{2c}b\pi\right)$
- 13) $\int \frac{x^p-x^{-p}}{1-x^2} (lx)^{2a} dx = \left(\frac{\pi}{2}\right)^{2a+1} \frac{d^{2a}}{dp^{2a}} \cdot \text{Tang. } 2p\pi, p < 1;$ V. T. 121. N°. 15.
- 14) $\int \left(\frac{1}{x}\right)^{2a-1} \frac{1+x}{1-x} \frac{dx}{x} = \frac{2^{2a-1}}{a} \pi^{2a} B_{2a-1}$ V. T. 118. N°. 15.
- 15) $\int (lx)^{2a} \frac{dx}{x^{1+q}+x^{1-q}} = \frac{1}{2} (-1)^{a+1} \left(\frac{2\pi}{q}\right)^{2a+1} B''\left(\frac{1}{4}\right)$ V. T. 120. N°. 15.
- 16) $\int (lx)^{2a-1} \frac{dx}{x^{1+q}-x^{1-q}} = \frac{1}{4} (-1)^a \left(\frac{2\pi}{q}\right)^{2a} B'\left(\frac{1}{2}\right)$ V. T. 120. N°. 19.

- 1) $\int (lx)^{2a} \frac{dx}{1+x+x^2} = \frac{(-1)^{a+1}}{1\sqrt{3}} (2\pi)^{2a+1} B''\left(\frac{1}{3}\right)$
- 2) $\int (lx)^{2a} \frac{dx}{1-x+x^2} = \frac{(-1)^{a+1}}{1\sqrt{3}} (2\pi)^{2a+1} B''\left(\frac{1}{6}\right)$

Raabe, Cr. 42. 348.

$$\begin{aligned}
 3) \int (lx)^{2a} \frac{dx}{1+x^2-2x \cos \lambda} &= \operatorname{Cosec} \lambda \left[1^{2a/1} \left\{ \lambda \sum_1^{\infty} \frac{1}{n^{4a}} - \frac{\lambda^3}{1.2.3} \sum_1^{\infty} \frac{1}{n^{4a-2}} + \dots \right\} \right. \\
 &\quad \left. + \frac{(-1)^a}{2a+1} \lambda^{2a-1} \left\{ 2a(2a+1) \sum_1^{\infty} \frac{1}{n^2} - \frac{2a+1}{2} \lambda \pi + \frac{\lambda^2}{2} \right\} \right] \left. \vphantom{\int} \right\} \begin{array}{l} \text{Euler, Calc.} \\ \text{Int. 4. S. 4.} \\ 46. \end{array} \\
 4) \int (lx)^{2a+1} \frac{\cos \lambda + x}{1+x^2-2x \cos \lambda} dx &= 1^{2a+1/1} \left\{ \sum_1^{\infty} \frac{1}{n^{4a+2}} - \frac{\lambda^2}{1.2} \sum_1^{\infty} \frac{1}{n^{4a}} + \frac{\lambda^4}{1.2.3.4} \sum_1^{\infty} \frac{1}{n^{4a-2}} - \dots \right\} \\
 &\quad + \frac{(-1)^a}{2a+2} \lambda^{2a} \left\{ (2a+1)(2a+2) \sum_1^{\infty} \frac{1}{n^2} - \frac{2a+2}{2} \lambda \pi + \frac{1}{2} \lambda^2 \right\} \left. \vphantom{\int} \right\} . \\
 5) \int (lx)^{2b} \frac{dx}{1+x^2-2x \cos 2p\pi} &= \frac{(-1)^{b+1}}{2} (2\pi)^{2b+1} \operatorname{Cosec} 2p\pi \cdot B''(p), 0 < p < 1; \quad \begin{array}{l} \text{Raabe, Cr.} \\ 42. 348. \end{array} \\
 6) \int (lx)^{2b-1} \frac{\cos 2p\pi - x}{1+x^2-2x \cos 2p\pi} dx &= \frac{(-1)^a}{2} (2\pi)^{2b} B'(p) - \frac{(2\pi)^{2b}}{4b} B_{2b-1} \quad \text{V. T. 125. N}^\circ. 9. \\
 7) \int \left(l \frac{1}{x} \right)^{r-1} \frac{p \cos \lambda - p^2 x}{1-2px \cos \lambda + p^2 x^2} x^{r-1} dx &= \Gamma(r) \sum_1^{\infty} \frac{p^n \cos n\lambda}{(q+n-1)^r} \quad \text{Kummer, Cr. 17. 210.}
 \end{aligned}$$

$$\begin{aligned}
 1) \int l(1+x) \frac{dx}{x} &= \frac{1}{12} \pi^2 \quad \text{Ohm, Ausw. 16.} \\
 2) \int l(1+x) \frac{dx}{1+x^2} &= \frac{1}{8} \pi l 2 \quad \begin{array}{l} \text{Bertrand, L. 8. 110. — Serret, L. 9. 436. — Grunert, Gr. 4. 113 —} \\ \text{Id., Gr. 6. 448. — Mill, Cr. 3. 102.} \end{array} \\
 3) \int l(1+x) \frac{(p-1)x^{p-1} - px^{-p}}{x} dx &= 2l 2 - \pi \operatorname{Cosec} p\pi, p < 1 \quad \text{V. T. 5. N}^\circ. 1. \\
 4) \int l(1+px) \frac{1-x^2}{1+x^2} dx &= \frac{1}{2} \frac{(1+p)^2}{1+p^2} l(1+p) - \frac{1}{2} \frac{p}{1+p^2} l 2 - \frac{\pi}{4} \frac{p^2}{1+p^2} \quad \text{V. T. 6. N}^\circ. 1. \\
 5) \int l(1-x) \frac{dx}{x} &= -\frac{1}{6} \pi^2 \quad \text{Euler, N. C. Petr. 14. 129. — Schaeffer, Cr. 30. 277.} \\
 6) \int l(1+x^2) \frac{dx}{x} &= \frac{1}{24} \pi^2 \quad \text{V. T. 152. N}^\circ. 12. \\
 7) \int l(1+x^2) \frac{dx}{1+x^2} &= \frac{1}{2} \pi l 2 - \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 157. N. 21.}
 \end{aligned}$$

$$8) \int l(1+x^2) \frac{dx}{(1+px^2)^2} = \frac{1}{p} \left\{ \frac{p}{1+p^2} l \frac{1+p}{\sqrt{2}} - \frac{1}{1+p} l 2 + \frac{1}{4} p^2 \pi \right\} \quad \text{V. T. 6. N}^\circ. 1.$$

$$9) \int l(1+x^2) \frac{dx}{x(1+x^2)} = \frac{1}{2} \left\{ (l 2)^2 - \frac{1}{12} \pi^2 \right\} \quad \text{V. T. 160. N}^\circ. 17.$$

$$10) \int l(1-x^2) \frac{dx}{x} = -\frac{1}{12} \pi^2 \quad \text{Ohm, Ausw. 16.}$$

$$11) \int l(1-x^4) \frac{dx}{x} = -\frac{1}{24} \pi^2 \quad \text{V. T. 152. N}^\circ. 17.$$

$$12) \int l(1+x+x^2) \frac{dx}{x} = \frac{1}{9} \pi^2 \quad \text{V. T. 153. N}^\circ. 1.$$

$$13) \int l(1-x+x^2) \frac{dx}{x} = \frac{1}{18} \pi^2 \quad \text{V. T. 153. N}^\circ. 2.$$

$$14) \int l(1-2x \cos \lambda + x^2) \frac{dx}{x} = \frac{1}{3} \pi^2 - \pi \lambda + \frac{1}{2} \lambda^2 \quad \text{V. T. 153. N}^\circ. 6.$$

$$15) \int l \frac{1+x}{1-x} \frac{dx}{x} = \frac{1}{4} \pi^2 \quad \text{Ohm, Ausw. 16.}$$

$$16) \int l \frac{1-x^2 \operatorname{Cot} h p^2 \lambda}{1+x^2 \operatorname{Cot} h p^2 \lambda} \frac{dx}{1-(1-x^2) \operatorname{Cos} h p^2 \lambda} = \frac{2 \lambda l \operatorname{Sin} h p \lambda}{\operatorname{Sin} h p \lambda \cdot \operatorname{Cos} h p \lambda} \quad \text{V. T. 343. N}^\circ. 12.$$

$$17) \int l \frac{1+p \sqrt{1-x^2}}{1-p \sqrt{1-x^2}} \frac{dx}{1+x^2} = \pi \operatorname{Arcsin} p, p \leq 1; \text{Raabe, Int. 421.}$$

$$18) \int l \frac{1+\operatorname{Cos} \mu \sqrt{1-x^2}}{1-\operatorname{Cos} \mu \sqrt{1-x^2}} \frac{dx}{\operatorname{Tang}^2 \lambda + x^2} = 2 \pi \operatorname{Cot} \lambda \cdot l \left[\operatorname{Cos} \left\{ \frac{1}{2} (\lambda - \mu) \right\} \cdot \operatorname{Cosec} \left\{ \frac{1}{2} (\lambda + \mu) \right\} \right] \quad \text{V. T. 343. N}^\circ. 20.$$

$$19) \int l \frac{1+\operatorname{Sin} \lambda \sqrt{1-x^2}}{1-\operatorname{Sin} \lambda \sqrt{1-x^2}} \frac{dx}{1-x^2} = \pi \lambda \quad \text{V. T. 340. N}^\circ. 10.$$

$$20) \int l \frac{x+\sqrt{1-x^2}}{x-\sqrt{1-x^2}} \frac{dx}{1-x^2} = \frac{1}{4} \pi^2 \quad \text{V. T. 340. N}^\circ. 14.$$

- 1) $\int (lx)^2 . l(1+x) \frac{dx}{x} = \frac{7}{360} \pi^4$ V. T. 154. N°. 10.
- 2) $\int (lx)^2 . l(1-x) \frac{dx}{x} = -\frac{1}{45} \pi^4$ V. T. 154. N°. 11.
- 3) $\int (lx)^2 . l(1+x^2) \frac{dx}{x} = \frac{7}{2880} \pi^4$ V. T. 154. N°. 13.
- 4) $\int (lx)^2 . l(1-x^2) \frac{dx}{x} = -\frac{1}{360} \pi^4$ V. T. 154. N°. 15.
- 5) $\int (lx)^2 . l(1-x^4) \frac{dx}{x} = -\frac{1}{2880} \pi^4$ V. T. 154. N°. 17.
- 6) $\int (lx)^4 . l(1+x) \frac{dx}{x} = \frac{31}{1260} \pi^6$ V. T. 155. N°. 2.
- 7) $\int (lx)^4 . l(1-x) \frac{dx}{x} = -\frac{8}{315} \pi^6$ V. T. 155. N°. 3.
- 8) $\int (lx)^4 . l(1-x^2) \frac{dx}{x} = -\frac{1}{2560} \pi^6$ V. T. 155. N°. 5.
- 9) $\int (lx)^{2a} . l(1+x) \frac{dx}{x} = \frac{2^{2a+1} - 1}{(2a+1)(2a+2)} \pi^{2a+2} B_{2a+1}$ V. T. 157. N°. 5.
- 10) $\int (lx)^{2a} . l(1-x) \frac{dx}{x} = \frac{-2^{2a}}{(a+1)(2a+1)} \pi^{2a+2} B_{2a+1}$ V. T. 157. N°. 6.
- 11) $\int (lx)^{2a} . l(1-x^2) \frac{dx}{x} = \frac{-1}{(2a+1)(2a+2)} \pi^{2a+2} B_{2a+1}$ V. T. 158. N°. 6.
- 12) $\int (lx)^{2a} . l(1-2x \cos. 2p\pi + x^2) \frac{dx}{x} = \frac{(2\pi)^{2a+2}}{2a+1} \left\{ (-1)^{a+1} B(p) - \frac{1}{2^{a+2}} B_{2a+1} \right\}$, $p < 1$; Raabe, Cr. 42.348.
- 13) $\int \left(l \frac{1}{x} \right)^{r-1} . l(1-2px \cos. \lambda + p^2 x^2) \frac{dx}{x} = \frac{2\Gamma(r+1)}{r} \sum_1 \frac{p^n \cos. n\lambda}{(n+1)^{r+1}}$ V. T. 159. N°. 7.
- 14) $\int \left(l \frac{1}{x} \right)^{a-1} . l(1+x) \frac{dx}{x} = \frac{1^{a+1/1}}{a} \sum_0 \frac{(-1)^n}{(1+n)^{a+1}}$ V. T. 157. N°. 8.
- 15) $\int \left(l \frac{1}{x} \right)^{a-1} . l(1-x) \frac{dx}{x} = -\frac{1^{a+1/1}}{a} \sum_0 \frac{1}{(1+n)^{a+1}}$ V. T. 157. N°. 9.

F. Algèbr. rat. fract. Log. en num. de forme diverse. (deux fact.).	TABLE 161 suite.	Lim. 0 et 1.
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- 16) $\int \left(\frac{1}{x}\right)^p \cdot l(1-qx) \frac{dx}{x} = -\Gamma(p+1) \sum_0^{\infty} \frac{q^n}{n^{p+1}}$ V. T. 157. N°. 10.
- 17) $\int \left(\frac{1}{x}\right)^{a-1} l(1-x^2) \frac{dx}{x} = \frac{1^{a+1/1}}{2^{a+1}} \sum_0^{\infty} \frac{1}{(1+n)^{a+1}}$ V. T. 157. N°. 8, 9.
- 18) $\int l(1+x^2) \frac{2x^2 l x - (1+x^2) dx}{(1+x^2)^2} = -\frac{1}{2} l 2$ V. T. 153. N°. 15.
- 19) $\int l(1-x^2) \frac{(1-3x^4) l x + (1+x^4)}{(1+x^4)^2} dx = \frac{-\pi^2}{8(2+\sqrt{2})}$ V. T. 153. N°. 16.

F. Algèbr. irrat. ent. Logar. en numér.	TABLE 162.	Lim. 0 et 1.
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- 1) $\int l x dx \vee (1-x^2) = -\frac{1}{4} \pi \left(\frac{1}{2} + l 2 \right)$ Euler, Calc. Int. 4. S. 3. 152, 154. — Id., Act. Petr. 1777. II. 3.
- 2) $\int x l x dx \vee (1-x^2) = -\frac{1}{3} \left(\frac{3}{4} - l 2 \right)$
- 3) $\int l x dx \vee (1-x^2)^{2a-1} = -\frac{1^{a/2}}{2^{a+1} 1^{a/1}} \frac{\pi}{2} (A + Z'(a+1) + 2 l 2)$ Lindmann, Stockh. Handl. 1850. III.
- 4) $\int x^{p-1} \left(\frac{1}{x}\right)^{a-\frac{1}{2}} dx = \frac{1^{a/2}}{(2p)^a} \vee \frac{\pi}{p}$ V. T. 114. N°. 8.
- 5) $\int l(1+p^2 x^2) dx \vee (1-x^2) = \frac{1}{2} \pi \left\{ l \frac{1+\vee(1+p^2)}{2} + \frac{\vee(1+p^2)-1}{p^2} - \frac{1}{2} \right\}$ V. T. 335. N°. 6.
- 6) $\int l(1+p^2 x^2 - p^2) dx \vee (1-x^2) = \frac{1}{2} \pi \left\{ l \frac{1+\vee(1-p^2)}{2} - \frac{1}{2} \frac{1-\vee(1-p^2)}{1+\vee(1-p^2)} \right\}$ V. T. 335. N°. 5.

F. Algèbr. irrat. fract. Logar. en num. $l x$.	TABLE 165.	Lim. 0 et 1.
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- 1) $\int l x \frac{dx}{\vee(1-x^2)} = -1 - \sum_1^{\infty} \frac{1^{n^2}}{2^{n^2}} \frac{1}{(2n+1)^2}$ Arndt, Gr. 6. 187.
- 2) $= -\frac{1}{2} \pi l 2$ Euler, Calc. Int. 4. S. 3. 117. — Id., N. C. Petr. 14. 129. — Id., Act. Petr. 1777. II. 3. — Id., Mém. Petersb. T. 6. p. 30. — Legendre, Exerc. 2. 43. — Id., Mém. Inst. 1809. 416. N°. 11. — Kausler, Mém. Pétersb. T. 3. — Oettinger, Cr. 38. 162. — Arndt, Gr. 6. 187.
- 3) $\int l x \frac{x}{\vee(1-x^2)} dx = l 2 - 1$ Euler, Calc. Int. 4. S. 3. 144, 164. — Id., Act. Petr. 1777. II. 3. — Kausler, Mém. Pétersb. T. 3. — Oettinger, Cr. 38. 162.

- $$\left. \begin{aligned} 4) \int l x \frac{x^2}{\sqrt{(1-x^2)}} dx &= -\frac{\pi}{4} \left(l 2 - \frac{1}{2} \right) \\ 5) \int l x \frac{x^3}{\sqrt{(1-x^2)}} dx &= -\frac{2}{3} \left(\frac{5}{6} - l 2 \right) \\ 6) \int l x \frac{x^4}{\sqrt{(1-x^2)}} dx &= -\frac{3}{16} \pi \left(l 2 - \frac{7}{12} \right) \\ 7) \int l x \frac{x^5}{\sqrt{(1-x^2)}} dx &= -\frac{8}{15} \left(\frac{47}{60} - l 2 \right) \\ 8) \int l x \frac{dx}{\sqrt{(1-x^2)}} &= -\frac{\pi^2}{54} - \frac{5\pi}{18\sqrt{3}} l 3 \\ 9) \int l x \frac{x}{\sqrt{(1-x^2)}} dx &= \frac{\pi^2}{54} - \frac{5\pi}{18\sqrt{3}} l 3 \end{aligned} \right\} \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. 147, sqq. — Id., Act.} \\ \text{Petr. 1777. II. 3. — Kausler, Mém. Pétersb.} \\ \text{T. 3.} \end{array}$$
- $$\left. \begin{aligned} 10) \int l x \frac{dx}{\sqrt{(1-x^3)}} &= -\frac{\pi}{3\sqrt{3}} \left(l 3 + \frac{\pi}{3\sqrt{3}} \right) \\ 11) \int l x \frac{x}{\sqrt{(1-x^3)^2}} dx &= -\frac{\pi}{3\sqrt{3}} \left(l 3 - \frac{\pi}{3\sqrt{3}} \right) \\ 12) \int l x \frac{x}{\sqrt{(1-x^4)}} dx &= -\frac{1}{8} \pi l 2 \\ 13) \int l x \frac{x^3}{\sqrt{(1-x^4)}} dx &= \frac{1}{4} (l 2 - 1) \end{aligned} \right\} \begin{array}{l} \text{Euler, Calc. Int. 4. S. 3. 157, sqq. —} \\ \text{Id., Act. Petr. 1777. II. 3. (où les formu-} \\ \text{les 10 et 11 sont fautives).} \end{array}$$
- $$14) \int l x \frac{dx}{\sqrt{(1-p^2 x^2)(1-x^2)}} = -\frac{1}{2} l p \cdot F'(p) - \frac{1}{4} \pi F' \{ \sqrt{(1-p^4)} \} \quad \text{V. T. 347. N}^\circ. 4.$$
- $$15) \int l x \frac{1}{(1-p)^2 + 4 p x^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{\pi}{2(1-p^2)} l \frac{1+p}{2}, p^2 < 1; \quad \text{V. T. 346. N}^\circ. 4.$$
- $$16) \int l x \frac{1+q-2 q x^2}{(1+q)^2 + 4 q x^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{4} \pi l \frac{1+q}{4} \quad \text{V. T. 346. N}^\circ. 5.$$
- $$17) \int l x \frac{1-q+2 q x^2}{(1-q)^2 + 4 q x^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{4} \pi l \frac{1-q}{4}, q^2 < 1; \quad \text{V. T. 346. N}^\circ. 1.$$
- $$18) \int l \frac{x^{-1} + x^{-i}}{1-x} dx = 2 \pi^2 \quad \text{Poisson, Mém. Inst. 1811. 163. N}^\circ. 54.$$
- $$19) \int l \frac{1}{x} \frac{x^{a-1}}{\sqrt{(1-x^b)^{b-c}}} dx = \sum_0^{\infty} \frac{(b-c)^{n/b}}{b^{n/b}} \frac{1}{(a+bn)^2} \quad \text{Legendre, Mém. Inst. 1809. 416. N}^\circ. 24.$$

- 1) $\int \left(\frac{1}{x}\right)^2 \frac{dx}{\sqrt{1-x^2}} = \frac{1}{2} \pi \left\{ (l2)^2 + \frac{1}{12} \pi^2 \right\}$ Legendre, Exerc. 2. 43. — Id., Mém. Inst. 1809. 416. N°. 44.
 - 2) $\int \left(\frac{1}{x}\right)^h \frac{x^{a-1}}{\sqrt{b}(1-x^b)^{b-c}} dx = \frac{1}{b} \sum_0^{\infty} \frac{(b-c)^{n/b}}{b^{n/b}} \frac{1}{(nb+a)^{h+1}}$ Legendre, Mém. Inst. 1809. 416. N°. 24.
 - 3) $\int \left(\frac{1}{x}\right)^{2a-1} \frac{dx}{(1-x)\sqrt{x}} = \frac{2^{2a}-1}{4a} (2\pi)^{2a} B_{2a-1}$ V. T. 120. N°. 20.
 - 4) $\int (lx)^{2a} \frac{x^{b-1}}{1+x^b} dx = \frac{(-1)^{a+1}}{2b} (2\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b} \right)$
 - 5) $\int (lx)^{2a} \frac{1+x^{b-c}}{1+x^b} x^{c-1} dx = \frac{(-1)^{a+1}}{b} (2\pi)^{2a+1} \sum_1^b B'' \left(\frac{2n-1}{4b} \right) \text{Sin.} \left\{ \frac{2n-1}{2b} c\pi \right\}$
 - 6) $\int (lx)^{2a} \frac{1-x^{b-c}}{1-x^b} x^{c-1} dx = \frac{(-1)^{a+1}}{b} (2\pi)^{2a+1} \sum_1^b B'' \left(\frac{n}{2b} \right) \text{Sin.} \frac{nc\pi}{b}$
- Raabe, Cr. 42. 348.

- 1) $\int l(1+x) \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2} l2 - 2 \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 11.
- 2) $\int l(1-x) \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2} l2 + 2 \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 12.
- 3) $\int l(1+px) \frac{dx}{x\sqrt{1-x^2}} = \frac{1}{8} \{ \pi^2 - 4(\text{Arccos. } p)^2 \}, p^2 < 1;$ V. T. 339. N°. 28.
- 4) $\int l(r+px) \frac{x}{1-qx^2} \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{1'q(1-q)} \left\{ p1'q - \{1-\sqrt{1-q}\} \{r+1(r^2-p^2)\} \right\}$ Lobatschewsky, Mém. Kasan. 1835. 1.
- 5) $\int l(1+x^2) \frac{dx\sqrt{1-x^2}}{x} = \frac{2\pi\sqrt{2}}{\{\Gamma(\frac{1}{2})\}^2} - \frac{\{\Gamma(\frac{1}{2})\}^2}{2\sqrt{2}\pi}$ V. T. 12. N°. 9.
- 6) $\int l(1+qx^2) \frac{dx}{\sqrt{1-x^2}} = \pi l \frac{1+\sqrt{1+q^2}}{2}$ V. T. 334. N°. 8.
- 7) $\int l(1+x^2 \text{Tang.}^2 \lambda) \frac{dx}{\sqrt{1-x^2}} = \pi l \left(\text{Cos.}^2 \frac{1}{2} \lambda, \text{Sec.} \lambda \right)$ V. T. 334. N°. 9.
- 8) $\int l(1+q^2 x^2) \frac{x^2}{\sqrt{1-x^2}} dx = \frac{1}{2} \pi \left\{ l \frac{1+\sqrt{1+q^2}}{2} + \frac{1-l}{q^2} \frac{(1-q^2)}{2} + \frac{1}{2} \right\}$ $q < 1;$ V. T. 335. N°. 7.

- 9) $\int l(1+px^2) \frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}} = \frac{1}{2} l \frac{2+2p}{\sqrt{p}} \cdot F'(p) - \frac{1}{8} \pi F' \{ \sqrt{1-p^2} \}, p^2 < 1; \begin{matrix} \text{V. T. 348.} \\ \text{N}^\circ. 10. \end{matrix}$
- 10) $\int l(1+x^2 \text{Cot.}^2 \lambda) \frac{dx}{\sqrt{(1-x^2)(1-p^2x^2)}} = \pi F' \{ \sqrt{1-p^2}, \lambda \} - 2 F'(p) Y \{ \sqrt{1-p^2}, \lambda \} -$
 $- 2 F'(p) l \text{Sin.} \lambda - \frac{1}{2} \pi F' \{ \sqrt{1-p^2} \} - F'(p) l p - \{ E'(p) - F'(p) \} [F \{ \sqrt{1-p^2}, \lambda \}]^2, p^2 < 1;$
 $\begin{matrix} \text{V. T. 348.} \\ \text{N}^\circ. 14. \end{matrix}$
- 11) $\int l(1-x^2) \frac{dx}{\sqrt{(1-x^2)}} = \pi l 2 \quad \text{V. T. 163. N}^\circ. 2.$
- 12) $\int l(1-x^2) \frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}} = l \frac{\sqrt{1-p^2}}{p} \cdot F'(p) - \frac{1}{2} \pi F' \{ \sqrt{1-p^2} \}, p < 1; \begin{matrix} \text{V. T. 347.} \\ \text{N}^\circ. 11. \end{matrix}$
- 13) $\int l(1-p^2x^2 \text{Sin.}^2 \lambda) \frac{dx}{\sqrt{(1-x^2)(1-p^2x^2)}} = E'(p) F(p, \lambda) - 2 F'(p) Y(p, \lambda), p < 1; \begin{matrix} \text{V. T. 348.} \\ \text{N}^\circ. 17. \end{matrix}$
- 14) $\int l(1-x^2 \text{Sin.} \lambda) \frac{dx}{\sqrt{(1-x^2)}} = 2 \pi l \text{Cos.} \frac{1}{2} \lambda \quad \text{V. T. 334. N}^\circ. 13.$
- 15) $\int l(1-x^2 \text{Sin} h p. \lambda) \frac{dx}{\sqrt{(1-x^2)}} = 2 \pi l \text{Cos} h p. \frac{1}{2} \lambda \quad \text{V. T. 334. N}^\circ. 10.$
- 16) $\int l(1-x^2 \text{Cos} h p. \lambda) \frac{dx}{\sqrt{(1-x^2)}} = \pi l \frac{1 + \text{Sin} h p. \lambda}{2} \quad \text{V. T. 334. N}^\circ. 12.$
- 17) $\int l(1-p^2x^2) \frac{x^2}{\sqrt{(1-x^2)}} dx = \frac{1}{2} \pi \left\{ l \frac{1 + \sqrt{1-p^2}}{2} - \frac{11 - \sqrt{1-p^2}}{21 + \sqrt{1-p^2}} \right\}, p^2 < 1; \begin{matrix} \text{V. T. 333.} \\ \text{N}^\circ. 5. \end{matrix}$
- 18) $\int l(1-p^2x^2) \frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}} = \frac{1}{2} l(1-p^2) \cdot F'(p), p < 1; \quad \text{V. T. 348. N}^\circ. 12.$
- 19) $\int l(1-px^2) \frac{dx}{\sqrt{(1-p^2x^2)(1-x)}} = \frac{1}{2} l \frac{2-2p}{\sqrt{p}} F'(p) - \frac{1}{8} \pi F' \{ \sqrt{1-p^2} \}, p < 1; \begin{matrix} \text{V. T. 348.} \\ \text{N}^\circ. 11. \end{matrix}$
- 20) $\int l(1-p^2x^2) \frac{dx}{(1-p^2x^2) \sqrt{(1-p^2x^2)(1-x)}} = \frac{1}{1-p^2} \left[(p^2-2) F'(p) + \left\{ 2 + \frac{1}{2} l(1-p^2) \right\} E'(p) \right], p^2 < 1; \begin{matrix} \text{V. T. 348.} \\ \text{N}^\circ. 13. \end{matrix}$
- 21) $\int l(1-p^2x^2) \frac{dx}{1-p^2x^2} \frac{1-x^2}{1-p^2x^2} = \frac{1}{p^2} \left[\left\{ 2-p^2 + \frac{1}{2} l(1-p^2) \right\} F'(p) + \left\{ 2 + \frac{1}{2} l(1-p^2) \right\} E'(p) \right], p^2 < 1; \begin{matrix} \text{V. T. 348.} \\ \text{N}^\circ. 20. \end{matrix}$
- 22) $\int l(\text{Sec.}^2 \lambda - x^2 \text{Tang.}^2 \lambda) \frac{dx}{\sqrt{(1-x)}} = \pi l \left(\text{Cos.}^2 \frac{1}{2} \lambda \cdot \text{Sec.} \lambda \right) \quad \text{V. T. 334. N}^\circ. 9.$

- 23) $\int l(1-p^2 x^2) \frac{x^2}{1-p^2 x^2} \frac{dx}{\sqrt{(1-p^2 x^2)(1-x^2)}} = \frac{1}{p^2(1-p^2)}$, $p^2 < 1$;
V. T. 348.
N°. 19.
- 24) $\int l(1-p^2 x^4) \frac{dx}{\sqrt{(1-p^2 x^2)(1-x^2)}} = \frac{1}{2} l \frac{4(1-p^2)}{p} \cdot F'(p) - \frac{1}{4} \pi F'\{\sqrt{1-p^2}\}$, $p^2 < 1$; V. T. 348.
N°. 16.
- 25) $\int l(1+p^2+2px) \frac{dx}{\sqrt{(1-x^2)}} = \sum_0^{\infty} \frac{1}{1+2n} \frac{2^{n/2}}{1^{n/2}} \left(\frac{2p}{1+p^2}\right)^{2n+1}$, $p \leq 1$; V. T. 334. N°. 19.
- 26) $\int l(1+p^2-p^2 x^2) \frac{x^2}{\sqrt{(1-x^2)}} dx = \frac{1}{2} \pi \left\{ l \frac{1+\sqrt{1+p^2}}{2} + \frac{\sqrt{1+p^2}-1}{p^2} - \frac{1}{2} \right\}$ V. T. 335.
N°. 6.
- 27) $\int l\{1-(\cos.^2 \lambda + p^2 \sin.^2 \lambda) x^2\} \frac{dx}{\sqrt{(1-x^2)(1-p^2 x^2)}} = \pi F\{\sqrt{1-p^2}, \lambda\} - 2F'(p)Y\{\sqrt{1-p^2}, \lambda\} +$, $p < 1$;
V. T. 348.
N°. 15.
 $+ F'(p)l \frac{\sqrt{1-p^2}}{p} - \frac{1}{2} \pi F'\{\sqrt{1-p^2}\} - \{E'(p) - F'(p)\} \{F(p, \lambda)\}^2$
- 28) $\int l\{1-x^2+x^2 \sqrt{1-p^2}\} \frac{dx}{\sqrt{(1-x^2)(1-p^2 x^2)}} = \frac{1}{2} l \frac{2 \sqrt{1-p^2}^3}{1+\sqrt{1-p^2}} \cdot F'(p)$, $p^2 < 1$; V. T. 348.
N°. 13.

- 1) $\int l \frac{1+x^2 \sin. \lambda}{1-x^2 \sin. \lambda} \frac{dx}{\sqrt{(1-x^2)}} = \pi l \frac{1+\sin. \frac{1}{2} \lambda}{\cos. \frac{1}{2} \lambda}$ V. T. 334. N°. 22.
- 2) $\int l \frac{1-x^2 \sin.^2 \lambda}{1-x^2 \sin.^2 \mu} \frac{dx}{\sqrt{(1-x^2)}} = 2 \pi l \left(\cos. \frac{1}{2} \lambda. \sec. \frac{1}{2} \mu \right)$ V. T. 334. N°. 23.
- 3) $\int l \frac{\sin.^2 \lambda - x^2 \cos.^2 \mu}{1-x^2 \sin.^2 \mu} \frac{dx}{\sqrt{(1-x^2)}} = \pi l \cot. \frac{1}{2} \mu. \text{Tang.} \left(\frac{1}{2} \text{Arcsin.} \frac{\sin. \mu}{\sin. \lambda} \right)$ V. T. 334.
N°. 24.
- 4) $\int l \frac{\cos.^2 \lambda + x^2 \sin.^2 \lambda}{\cos.^2 \mu + x^2 \sin.^2 \mu} \frac{dx}{\sqrt{1-x^2}} = 2 \pi l \left(\cos. \frac{1}{2} \lambda. \sec. \frac{1}{2} \mu \right)$ V. T. 334. N°. 23.
- 5) $\int l \frac{1+x}{1-x} \frac{dx}{x \sqrt{(1-x^2)}} = \frac{1}{2} \pi^2$ V. T. 340. N°. 2.
- 6) $\int l \frac{1+qx}{1-qx} \frac{dx}{x \sqrt{(1-x^2)}} = \pi \text{Arcsin. } q$ V. T. 340. N°. 3.

- 7) $\int l \frac{1+x \operatorname{Sin} . \lambda}{1-x \operatorname{Sin} . \lambda} \frac{dx}{x \sqrt{(1-x^2)}} = \pi \lambda$ Lobatschewsky, Mém. Kasan. 1835. 1.
- 8) $\int l \left\{ \frac{1+x}{1-x} - 2x \right\} \frac{dx}{x^3 \sqrt{(1-x^2)}} = \frac{1}{4} \pi^2$ V. T. 340. N°. 6.
- 9) $\int l \frac{1+\sqrt{(1-x^2)}(\operatorname{Sin} .^2 \lambda - x^2 \operatorname{Sin} .^2 \mu)}{1-\sqrt{(1-x^2)}(\operatorname{Sin} .^2 \lambda - x^2 \operatorname{Sin} .^2 \mu)} \frac{dx}{\sqrt{(1-x^2)}} = \pi l \frac{1}{2} \left\{ \operatorname{Cos} .^2 \frac{1}{2} \lambda + \sqrt{\left(\operatorname{Cos} .^2 \frac{1}{2} \lambda + \operatorname{Sin} .^2 \frac{1}{2} \mu \operatorname{Cos} .^2 \frac{1}{2} \mu \right)} \right\}$ V. T. 348. N°. 3.
- 10) $\int l \frac{1-x \operatorname{Cos} h p . \lambda . \operatorname{Cos} h p . \mu \sqrt{(1-x^2 \operatorname{Cos} h p .^2 \lambda . \operatorname{Tang} h p .^2 \mu)}}{1+x \operatorname{Cos} h p . \lambda . \operatorname{Cos} h p . \mu \sqrt{(1-x^2 \operatorname{Cos} h p .^2 \lambda . \operatorname{Tang} h p .^2 \lambda)}} \frac{dx}{\sqrt{(1-x^2)}} =$

$$= \pi l \frac{4 \operatorname{Sin} h p . \lambda}{\{\operatorname{Sin} h p . \lambda + \sqrt{(1-\operatorname{Cos} h p .^2 \lambda . \operatorname{Cos} h p .^2 \mu)}\} (1+\operatorname{Sin} h p . \lambda)}$$
 V. T. 348. N°. 3.
- 11) $\int l \frac{1+q \sqrt{(1-p^2 x^2)}}{1-q \sqrt{(1-p^2 x^2)} \sqrt{(1-p^2 x^2)} (1-x^2)} \frac{dx}{\sqrt{(1-x^2)}} = \pi \operatorname{F}\{ \sqrt{(1-p^2)}, \operatorname{Arcsin} . q \}$ V. T. 348. N°. 22.
- 12) $\int l \frac{1+x \operatorname{Cos} . \mu}{1-x \operatorname{Cos} . \mu} \frac{1}{1+x \operatorname{Cos} . \lambda} \frac{dx}{\sqrt{(1-x^2)}} = \frac{2 \pi}{\operatorname{Sin} . \lambda} l \frac{\operatorname{Cos} . \left\{ \frac{1}{4} (\pi - 2 \lambda) \right\}}{\operatorname{Cos} . \left\{ \frac{1}{2} (\lambda - \mu) \right\}}$ V. T. 343. N°. 13.
- 13) $\int l \frac{1+x \operatorname{Cos} . \mu}{1-x \operatorname{Cos} . \mu} \frac{1}{1-x^2 \operatorname{Cos} . \lambda} \frac{dx}{\sqrt{(1-x^2)}} = \frac{\pi}{\operatorname{Sin} . \lambda} l \frac{1+\operatorname{Sin} . \lambda}{\operatorname{Sin} . \lambda + \operatorname{Sin} . \mu}$ V. T. 343. N°. 19.
- 14) $\int l \frac{1+q x \operatorname{Cos} . \lambda}{1-q x \operatorname{Cos} . \lambda} \frac{1}{1-x^2 \operatorname{Cos} .^2 \lambda} \frac{dx}{\sqrt{(1-x^2)}} = \frac{\pi}{\operatorname{Sin} . \lambda} l \frac{1+\operatorname{Sin} . \lambda}{\operatorname{Sin} . \lambda + \sqrt{(1-q^2 \operatorname{Cos} .^2 \lambda)}}$ V. T. 343. N°. 17.
- 15) $\int l \frac{1+x \operatorname{Cos} . \lambda}{1-x \operatorname{Cos} . \lambda} \frac{x}{1-x^2 \operatorname{Cos} .^2 \lambda} \frac{dx}{\sqrt{(1-x^2)}} = 2 \pi \operatorname{Cosec} . 2 \lambda . l \operatorname{Sin} . \lambda$ V. T. 343. N°. 14.
- 16) $\int l \frac{1+x \operatorname{Cos} . \mu}{1-x \operatorname{Cos} . \mu} \frac{x}{1-x^2 \operatorname{Cos} .^2 \lambda} \frac{dx}{\sqrt{(1-x^2)}} = \frac{2 \pi}{\operatorname{Sin} . 2 \lambda} l \frac{\operatorname{Sin} . \left\{ \frac{1}{2} (\mu + \lambda) \right\}}{\operatorname{Cos} . \left\{ \frac{1}{2} (\mu - \lambda) \right\}}$ V. T. 343. N°. 10.
- 17) $\int l \frac{1+px}{1-px} \frac{x}{1-qx^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{\pi}{\sqrt{q(1-q)}} l \frac{p \sqrt{q} + \{1-\sqrt{(1-q)}\} \{1-\sqrt{(1-p^2)}\}}{p \sqrt{q} - \{1-\sqrt{(1-q)}\} \{1-\sqrt{(1-p^2)}\}}$ Lobatschewsky, Mém. Kasan. 1835. 1.
- 18) $\int l \frac{1+x \operatorname{Cos} h p . \lambda}{1-x \operatorname{Cos} h p . \lambda} \frac{x}{1-x^2 \operatorname{Cos} h p .^2 \lambda} \frac{dx}{\sqrt{(1-x^2)}} = -2 \pi \frac{l \operatorname{Sin} h p . \lambda}{\operatorname{Sin} h p . \lambda . \operatorname{Cos} h p . \lambda}$ V. T. 343. N°. 15.
- 19) $\int l \frac{1+x}{1-x} \frac{x}{1-\operatorname{Cos} .^2 \lambda . \operatorname{Cos} .^2 \mu - x^2 \operatorname{Sin} .^2 \mu} \frac{dx}{\sqrt{(x^2 - \operatorname{Cos} .^2 \lambda)}} =$

$$= \frac{\pi}{2 \operatorname{Sin} . \lambda . \operatorname{Sin} . \mu} l \frac{\operatorname{Sin} . \mu + \sqrt{(1-\operatorname{Cos} .^2 \lambda . \operatorname{Cos} .^2 \mu)}}{\operatorname{Sin} . \mu (1+\operatorname{Sin} . \lambda)}$$
 V. T. 347. N°. 14.

$$20) \int l \frac{1+x \operatorname{Cos} h p . u}{1-x \operatorname{Cos} h p . u} \frac{x}{1-x^2 \operatorname{Cos}^2 \lambda} \frac{dx}{\sqrt{(1-x^2)}} =$$

$$= \frac{2 \pi}{\operatorname{Sin} . 2 \lambda} l \left[\operatorname{Cot} h p . \left(\frac{1}{2} \operatorname{Arccos} h p . \frac{\operatorname{Cos} h p . u}{\operatorname{Cos} . \lambda} \right) \operatorname{Tang} h p . \left\{ \frac{1}{2} \operatorname{Arccot} h p . \frac{\operatorname{Tang} . \lambda}{\operatorname{Tang} h p . u} \right\} \right]$$

V. T. 343.
N^o. 21.

$$21) \int l x \frac{x dx}{\sqrt{(1-x^2)}} \left\{ \frac{1}{1-x^2} l \frac{1-x \operatorname{Sin} . \lambda}{1+x \operatorname{Sin} . \lambda} + \frac{2 \operatorname{Sin}^2 . \lambda}{(1-x^2 \operatorname{Sin}^2 . \lambda)^2} \right\} = \pi \lambda \quad \text{V. T. 166. N^o. 7.}$$

$$1) \int \frac{x^2}{l x} dx = \infty \quad \text{Euler, Calc. Int. 4. S. 5. 21. — Legendre, Exere. 3. 57.}$$

$$2) \int \frac{1-x}{l x} dx = -l 2 \quad \text{Euler, Calc. Int. 4 S. 5. 5. — Id., N. C. Petr. 19. 66. — Legendre, Exere. 5. 3.}$$

$$3) \int \frac{1-x^p}{l x} dx = -l(p+1) \quad \begin{array}{l} \text{Euler, Calc. Int. 4. S. 5. 5. — Id., N. C. Petr. 19. 66. — Poisson,} \\ \text{P. 18. 295. N^o. 25. — Legendre, Exere. 3. 57. — Id., ib. 5. 3. —} \\ \text{Bidone, Mém. Turin. 1812. 231. Art. 3. N^o. 36. — Plana, Mém.} \\ \text{Turin. 1818. 7. Art. 14. Add. — Cisa de Grésy, Mém. Turin. 1821.} \\ \text{209. 1. 29. — Arndt, Gr. 10. 253.} \end{array}$$

$$4) \int \frac{x^p - x^q}{l x} dx = l \frac{p+1}{q+1} \quad \begin{array}{l} \text{Euler, Act. Petr. 1777. 2. 29. — Id., Calc. Int. 4. S. 5. 5. 22. —} \\ \text{Id., N. C. P. 19. 66. — Bidone, Mém. Turin. 1812. 231. Art. 3. N^o. 36.} \end{array}$$

$$5) \int \frac{x^q - 1}{l x} x^p dx = l \frac{p+q+1}{q+1} \quad \begin{array}{l} \text{Legendre, Exere. 3. 57. — Cisa de Grésy, Mém. Turin. 1821.} \\ \text{209. 1. 29.} \end{array}$$

$$6) \int \frac{x^{p-1} - x^{q-1}}{l x} x^r dx = l \frac{p+r}{q+r} \quad \text{Meyer, Int. Déf. 109.}$$

$$7) \int \frac{(x^q - 1)(x^r - 1)}{l x} x^{p-1} dx = l \frac{p+q+r}{p+q} - l \frac{p+r}{p} \quad \text{Binet, P. 27. 123.}$$

$$8) \int \frac{(x^p - x^q)(x^r - x^s)}{l x} dx = l \frac{(p+r+1)(q+s+1)}{(p+s+1)(q+r+1)} \quad \text{Euler, N. C. Petr. 20. 59.}$$

$$9) \int x^{a-1} (x-1)^b \frac{dx}{l x} = \Delta^b . l a \quad \text{V. T. 127. N^o. 6.}$$

$$10) \int (r-1)^a \frac{dx}{l x} = \sum_0^{\infty} \binom{a}{n} l(a-n+1) \quad \begin{array}{l} \text{Euler, Calc. Int. 4. S. 5. 29. — Id., N. C. Petr. 19.} \\ \text{66. — Stern, Gött. Stud. 1847.} \end{array}$$

- $$\left. \begin{aligned} 1) \int \left(\frac{1-x}{lx} \right)^2 dx &= l \frac{27}{16} \\ 2) \int \left(\frac{1-x}{lx} \right)^2 x dx &= 2l \frac{32}{27} \end{aligned} \right\} \text{Euler, N. C. Petr. 19. 66.}$$
- $$3) \int \frac{x^{q-1}(x-1)^2}{(lx)^2} dx = (q+2)l(q+2) - 2(q+1)l(q+1) + qlq \quad \text{Euler, N. C. P. 20. 59.}$$
- $$4) \int \left(\frac{x^q-1}{lx} \right)^2 dx = (2q+1)l(2q+1) - 2(q+1)l(q+1)$$
- $$5) \int \left(\frac{x^q-1}{lx} \right)^2 x^p dx = (2q+p+1)l(2q+p+1) - 2(q+p+1)l(q+p+1) + (p+1)l(p+1) \quad \left. \begin{aligned} &\text{Legendre,} \\ &\text{Exerc. 3.} \\ &58. \end{aligned} \right\}$$
- $$6) \int \frac{(q-r)x^{p-1} + (r-p)x^{q-1} + (p-q)x^{r-1}}{(lx)^2} dx = \begin{aligned} &\text{Euler, Calc. Int. 4. S. 5. 23. — Id.,} \\ &\text{N. C. Petr. 19. 66. — Id., N. C. Petr.} \\ &= (q-r)p lp + (r-p)q lq + (p-q)r lr \quad 20. 59. \end{aligned}$$
- $$7) \int \left\{ 1 - \left(\frac{1}{2} - \frac{1}{lx} \right) (1-x) \right\} x^{q-1} \frac{dx}{lx} = 1 + \left(q + \frac{1}{2} \right) l \frac{q}{q+1} \quad \text{V. T. 124. N°. 20.}$$
- $$8) \int (1-x^p)(1-x^q)(1-x^r) \frac{dx}{(lx)^2} = (p+q+1)l(p+q+1) + (p+r+1)l(p+r+1) \\ + (q+r+1)l(q+r+1) - (p+1)l(p+1) - (q+1)l(q+1) \\ - (r+1)l(r+1) - (p+q+r)l(p+q+r) \quad \left. \begin{aligned} &\text{Stern,} \\ &\text{Gött.} \\ &\text{Stud.} \\ &1847. \end{aligned} \right\}$$
- $$9) \int (1-x^p)^a \frac{dx}{(lx)^2} = \sum_1^{\infty} \binom{a}{2n} (2np+1)l(2np+1) - \sum_0^{\infty} \binom{a}{2n-1} \{ (2n-1)p+1 \} l \{ (2n-1)p+1 \}$$
- $$10) \int (1-x^p)^a \frac{x^q dx}{(lx)^2} = \sum_0^{\infty} (-1)^n \binom{a}{n} (q+np+1)l(q+np+1)$$
- $$11) \int \left\{ \frac{x^p}{(p-q)(p-r)(p-s)} + \frac{x^q}{(q-p)(q-r)(q-s)} + \frac{x^r}{(r-p)(r-q)(r-s)} + \frac{x^s}{(s-p)(s-q)(s-r)} \right\} \frac{dx}{x(lx)^3} = \begin{aligned} &\text{Euler,} \\ &\text{N. C.} \\ &\text{Petr.} \\ &= \frac{p^2 lp}{(p-q)(p-r)(p-s)} + \frac{q^2 lq}{(q-p)(q-r)(q-s)} + \frac{r^2 lr}{(r-p)(r-q)(r-s)} + \frac{s^2 ls}{(s-p)(s-q)(s-r)} \quad 20. 59. \end{aligned}$$
- $$12) \int \left\{ \frac{1-x}{lx} + \frac{x}{(lx)^2} \right\} dx = l2 - 1 \quad \text{V. T. 127. N°. 21.}$$
- $$13) \int (1-x^p)^a \frac{dx}{(lx)^3} = \frac{1}{2} \sum_1^{\infty} (-1)^n \binom{a}{n} (pn+1)^2 l(pn+1) \quad \text{Stern, Gött. Stud. 1847.}$$

- $$\begin{aligned}
 14) \int (1-x^p)^a \frac{x^q}{(lx)^3} dx &= \frac{1}{2} \sum_0^{\infty} (-1)^n \binom{a}{n} (pn+q+1)^2 l(pn+q+1) \\
 15) \int (1-x^p)^a (1-x^q) \frac{dx}{(lx)^3} &= -\frac{1}{2} \sum_0^{\infty} (-1)^n \binom{a}{n} (q+pn+1)^2 l(q+pn+1) \\
 &\quad + \frac{1}{2} \sum_1^{\infty} (-1)^n \binom{a}{n} (pn+1) l(pn+1) \left\{ \begin{array}{l} \text{Stern,} \\ \text{Gött.} \\ \text{Stud.} \\ 1847. \end{array} \right. \\
 16) \int \frac{x^{a-1}}{(lx)^q} dx &= \frac{1q!}{q^{a+1}} \text{ (faufif) } \text{Oettinger, Cr. 35. 13.} \\
 17) \int \left(\frac{x^q-1}{lx} \right)^a x^{p-1} dx &= \frac{1}{1^{a-1}l} \frac{\Delta^a}{\Delta p} \cdot p^{a-1}lp, \text{ en posant } \Delta p = q; \text{ Legendre, Exerc. 3. 58.} \\
 18) \int \left(\frac{x^q-1}{lx} \right)^a x^{pq-1} dx &= \frac{1}{1^{a-1}l} \Delta^a \cdot \{(pq)^{a-1}lpq\} \\
 19) \int \frac{x^{p-1}}{(lx)^{b+1}} (x-1)^a dx &= \frac{1}{1^{b+1}} \Delta^a \cdot p^b lp \\
 20) \int \frac{x^{pr-1}}{(lx)^{b+1}} (x^r-1)^a dx &= \frac{r-b}{1^{b+1}} \Delta^a \cdot p^b lp \\
 21) \int \frac{x^{q-1}-x^{r-1}}{(lx)^{p+1}} dx &= (-1)^{p+1} \Gamma(1-p) \frac{q^p-r^p}{p}, p < 1; \text{ V. T. 128. N}^\circ \text{ 1.} \\
 22) \int \frac{x^{b-1}}{\left(\frac{1}{x}\right)^{q+1}} (x-1)^c dx &= -\frac{\pi}{\Gamma(q+1)} \text{Cosec. } p\pi \Delta^c \cdot bq \\
 23) &= \frac{(-1)^{q+1}}{1^{q+1}} \Delta^c \cdot bq lb, \text{ pour } q \text{ entier;} \left\{ \begin{array}{l} , q < b; \\ \text{V. T. 128. N}^\circ \text{ 3, 4.} \end{array} \right. \\
 24) \int \frac{dx}{lx} \left\{ (p-q) + \frac{x^{q-1}-x^{p-1}}{lx} \right\} &= p-q+qlq-plp \text{ V. T. 127. N}^\circ \text{ 18.}
 \end{aligned}$$

- $$\begin{aligned}
 1) \int \frac{x^{p-1}}{q+lx} dx &= -e^{-pq} Ei.(pq) \text{ V. T. 129. N}^\circ \text{ 9.} \\
 2) \int \frac{x^{p-1}}{q-lx} dx &= e^{pq} Ei.(-pq) \text{ V. T. 129. N}^\circ \text{ 4.}
 \end{aligned}$$

- 3) $\int \frac{x}{q^2 + (lx)^2} dx = \sum_1^{\infty} (-1)^n \frac{1^{2n+1}}{q^{2n+1}} \quad \text{V. T. 130. N}^\circ. 2.$
- 4) $\int \frac{x lx}{q^2 + (lx)^2} dx = - \sum_1^{\infty} (-1)^n \frac{1^{2n+1}}{q^{2n-1}} \quad \text{V. T. 130. N}^\circ. 1.$
- 5) $\int \frac{x^{q-1}}{1 + (lx)^2} dx = \sum_0^{\infty} (-1)^n \frac{1^{2n+1}}{q^{2n+1}} \quad \text{V. T. 130. N}^\circ. 2.$
- 6) $\int \frac{x^{q-1} lx}{1 + (lx)^2} dx = - \sum_0^{\infty} (-1)^n \frac{1^{2n+1}}{q^{2n-2}} \quad \text{V. T. 130. N}^\circ. 1.$
- 7) $\int \frac{x^{p-1}}{q^2 + (lx)^2} dx = \frac{1}{q} \left\{ Ci.(pq). Sin.pq - Si.(pq). Cos.pq + \frac{1}{2} \pi Cos.pq \right\} \quad \text{V. T. 130. N}^\circ. 4.$
- 8) $\int \frac{x^{p-1} lx}{q^2 + (lx)^2} dx = Ci.(pq). Cos.pq + Si.(pq). Sin.pq - \frac{1}{2} \pi Sin.pq \quad \text{V. T. 130. N}^\circ. 5.$
- 9) $\int \frac{x^{p-1}}{q^2 - (lx)^2} dx = \frac{1}{2q} \{ e^{-pq} Ei.(pq) - e^{pq} Ei.(-pq) \} \quad \text{V. T. 130. N}^\circ. 10.$
- 10) $\int \frac{x^{p-1} lx}{q^2 - (lx)^2} dx = - \frac{1}{2} \{ e^{-pq} Ei.(pq) + e^{pq} Ei.(-pq) \} \quad \text{V. T. 130. N}^\circ. 12.$
- 11) $\int \frac{x^{p-1}}{q^4 - (lx)^4} dx = \frac{1}{4q^3} \{ e^{-pq} Ei.(pq) - e^{pq} Ei.(-pq) + 2 Ci.(pq). Sin.pq - 2 Si.(pq). Cos.pq + \pi Cos.pq \} \quad \text{V. T. 132. N}^\circ. 1.$
- 12) $\int \frac{x^{p-1} lx}{q^4 - (lx)^4} dx = \frac{1}{4q^2} \{ -e^{-pq} Ei.(pq) - e^{pq} Ei.(-pq) + 2 Ci.(pq). Cos.pq + 2 Si.(pq). Sin.pq - \pi Sin.pq \} \quad \text{V. T. 132. N}^\circ. 2.$
- 13) $\int \frac{x^{p-1} (lx)^2}{q^4 - (lx)^4} dx = \frac{1}{4q} \{ e^{-pq} Ei.(pq) - e^{pq} Ei.(-pq) - 2 Ci.(pq). Sin.pq + 2 Si.(pq). Cos.pq - \pi Cos.pq \} \quad \text{V. T. 132. N}^\circ. 3.$
- 14) $\int \frac{x^{p-1} (lx)^3}{q^4 - (lx)^4} dx = \frac{1}{4} \{ -e^{-pq} Ei.(pq) - e^{pq} Ei.(pq) - 2 Ci.(pq). Cos.pq - 2 Si.(pq). Sin.pq + \pi Sin.pq \} \quad \text{V. T. 132. N}^\circ. 4.$
- 15) $\int \frac{x^{q-1}}{(p + lx)^2} dx = - \frac{1}{p} \{ 1 + pq e^{-pq} Ei.(pq) \} \quad \text{V. T. 169. N}^\circ. 1.$
- 16) $\int \frac{x^{q-1}}{(p - lx)^2} dx = \frac{1}{p} \{ 1 - pq e^{pq} Ei.(-pq) \} \quad \text{V. T. 169. N}^\circ. 2.$
- 17) $\int \frac{x^{p-1}}{\{q^2 + (lx)^2\}^2} dx = \frac{1}{2q^3} \left\{ Ci.(pq). Sin.pq - Si.(pq). Cos.pq + \frac{1}{2} \pi Cos.pq \right\} - \frac{p}{2q^2} \left\{ Ci.(pq). Cos.pq + Si.(pq). Sin.pq - \frac{1}{2} \pi Sin.pq \right\} \quad \left. \begin{array}{l} \text{V. T. 169.} \\ \text{N}^\circ. 7. \end{array} \right\}$

- 1) $\int \frac{x^p - x}{x} \frac{dx}{lx} = lq$ Binet, P. 27. 123.
- 2) $\int \frac{x^p - x^q}{x} \frac{dx}{lx} = l \frac{p}{q}$ Cauchy, Cours. Leç. 33.
- 3) $\int \left\{ 1 + \frac{1}{lx} + \frac{1}{xlx} \right\} \frac{dx}{lx} = -1$ V. T. 127. N°. 17.
- 4) $\int \left\{ \frac{x^q - 1}{x(lx)^2} - \frac{q}{lx} \right\} dx = qlq - q$
- 5) $\int \left\{ \frac{x^q - 1}{x(lx)^3} - \frac{q}{x(lx)^2} - \frac{q^2}{2lx} \right\} dx = \frac{1}{2} q^2 lq - \frac{3}{4} q^2$
- 6) $\int \left\{ \frac{x^q - 1}{x(lx)^4} - \frac{q}{x(lx)^3} - \frac{q^2}{2x(lx)^2} - \frac{q^3}{6lx} \right\} dx = \frac{1}{6} q^3 lq - \frac{11}{36} q^3$
- 7) $\int \left\{ x - \left(\frac{1}{1-lx} \right)^q \right\} \frac{dx}{xlx} = -Z'(q)$ V. T. 112. N°. 8.
- 8) $\int \frac{l(1-x^a)}{1+(lx)^2} \frac{dx}{x} = \pi \left\{ l\Gamma\left(\frac{a}{2\pi} + 1\right) + \frac{1}{2} la + \frac{a}{2\pi} \left(l\frac{a}{2\pi} - 1 \right) \right\}$ V. T. 378. N°. 4.
- 9) $\int \left\{ x - \frac{1}{1-lx} \right\} \frac{dx}{xlx} = A$ V. T. 133. N°. 1.
- 10) $\int \left\{ x - \frac{1}{1+(lx)^2} \right\} \frac{dx}{xlx} = A$ V. T. 133. N°. 5.
- 11) $\int \left\{ x^q - \frac{1}{(1-lx)} \right\} \frac{dx}{xlx} = lq + A$ V. T. 133. N°. 2.
- 12) $\int \left\{ x - \frac{1}{(1-lx)^p} \right\} \frac{dx}{xlx} = -Z'(p)$ V. T. 133. N°. 3.
- 13) $\int \left\{ \frac{x-1}{lx} - \frac{1}{1-lx} \right\} \frac{dx}{xlx} = -1 + A$ V. T. 133. N°. 4.
- 14) $\int \frac{xlx + 1-x}{x(lx)^2} l(1+x) dx = l \frac{2}{\pi}$ V. T. 171. N°. 1.
- 15) $\int l(1+x)^2 \frac{2qlx(x^q + x^{-q}) - (x^q - x^{-q})}{(lx)^2} \frac{x^q - x^{-q}}{x} dx = 2l \frac{Tang. q \pi}{q \pi}$ V. T. 175. N°. 3.

Sohnke, Samml.

- 16) $\int l(1-x)^2 \frac{2^q l x (x^q + x^{-q}) - (x^q - x^{-q})}{(l x)^2} \frac{x^q - x^{-q}}{x} dx = 2 l \frac{\text{Sin. } 2^q \pi}{2^q \pi}$ V. T. 175. N°. 4.
- 17) $\int l(1-x^2)^2 \frac{2^q l x (x^q + x^{-q}) - (x^q - x^{-q})}{(l x)^2} \frac{x^q - x^{-q}}{x} dx = 4 l \frac{\text{Sin. } q \pi}{q \pi}$ V. T. 175. N°. 7.
- 18) $\int \frac{l x}{1-x^a} \frac{\{1+(l x)^2\} a x^a - (1-x^2) 2 l x l(1-x^2)}{\{1+(l x)^2\}^2} \frac{dx}{x} = \pi \left\{ -l \Gamma\left(\frac{a}{2\pi}\right) + 2la + \frac{a}{2\pi} \left(l \frac{a}{2\pi} - 1\right) \right\}$ V. T. 170. N°. 8.

- 1) $\int \frac{1-x}{1+x} \frac{dx}{l x} = \frac{2}{\pi}$ Euler, N. C. Petr. 20. 59. — Legendre, Exerc. 5. 3.
- 2) $\int \frac{1-x^{q-1}}{1+x} \frac{dx}{l x} = Z'\left(\frac{q}{2}\right) - Z'\left(\frac{q+1}{2}\right) - Z'\left(\frac{1}{2}\right)$ Legendre, Exerc. 5. 3.
- 3) $\int \frac{1-x^p}{1+x} \frac{x^q}{l x} dx = l \frac{\Gamma\left(\frac{1}{2}p+1\right) \Gamma\left(\frac{p+q+1}{2}\right)}{\Gamma\left(\frac{p+1}{2}\right) \Gamma\left(\frac{p+q}{2}+1\right)}$ Stern, Gött. Stud. 1847.
- 4) $\int \frac{x^{p-1}-x^{q-1}}{1+x} \frac{dx}{l x} = l \frac{\Gamma\left(\frac{q}{2}\right) \Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\frac{p}{2}\right) \Gamma\left(\frac{q+1}{2}\right)}$
- 5) $\int \frac{x^{p-1}-x^{q-1}}{1+x} \frac{1+x^{2a+1}}{l x} dx = l \left\{ \left(\frac{a}{a+1}\right)^{\frac{a-1}{2}} \frac{\left(\frac{p}{2}\right)^{a+1/2} \left(\frac{q+1}{2}\right)^{a/2}}{\left(\frac{p+1}{2}\right)^{a/2} \left(\frac{q}{2}\right)^{a+1/2}} \right\}$ Kummer, Cr. 17. 210.
- 6) $\int \left\{ \frac{1}{1+x} - \frac{x}{2} \right\} \frac{dx}{l x} = -\frac{1}{2} l \pi$ V. T. 135. N°. 6.
- 7) $\int \left(\frac{1}{l x} + \frac{1}{1-x} \right) dx = A$ Legendre, Exerc. 5. 12. — Cauchy, P. 28. 147. P. 1. § 6.
- 8) $\int \left(\frac{1}{l x} + \frac{x^{q-1}}{1-x} \right) dx = -Z'(q)$ Cauchy, P. 28. 147. P. 1. § 6. — Gauss, 1812. — Lejeune-Dirichlet, Cr. 15. 258. — Schaar, Mém. Cour. Brux. T. 22.
- 9) $\int \left(\frac{x^{p-1}}{l x} + \frac{x^{q-1}}{1-x} \right) dx = l p - Z'(q)$ Legendre, Exerc. 5. 12. — Arndt, Gr. 10. 250.

$$10) \int \left(\frac{b x^{bq-1}}{1-x} + \frac{b x^{b-1}}{lx} \right) dx = - \sum_1^b Z' \left(q + \frac{b-n}{b} \right) \quad \text{Arndt, Gr. 10. 253.}$$

$$11) \int \left(\frac{1-x^{q-1}}{1-x} - q + 1 \right) \frac{dx}{lx} = l \Gamma(q) \quad \text{Kummer, Cr. 35. 1.}$$

$$12) \int \left(\frac{x^p - x^{p+q}}{1-x} - q \right) \frac{dx}{lx} = l \frac{\Gamma(p+q+1)}{\Gamma(p+1)} \quad \text{Plana, Mém. Turin. 1818. 7. Art 4. Add.}$$

$$13) \int \left\{ \frac{1}{lx} + \frac{x^a - x^b - x^{a-b}}{1-x} x^{p-1} \right\} dx = l \frac{\Gamma(a+p)}{\Gamma(b+p) \Gamma(a+b+p)} \quad \text{V. T. 135. N°. 18.}$$

$$14) \int \frac{1-x^q}{1-x} \frac{1-x^p}{lx} dx = l B(p, q) - l \frac{p+q}{pq} \quad \text{Binet, P. 27. 123.}$$

$$15) \int \frac{1-x^a}{1-x} \frac{1-x^b}{lx} dx = l \frac{1^{a/1}}{(b+1)^{a/1}}, \text{ pour } a \text{ entier, } b \text{ fraction;} \left\{ \begin{array}{l} \text{Cisa de Grésy, Mém. Turin. 1821.} \\ \text{209. I. 30.} \end{array} \right.$$

$$16) \quad = -l \frac{1^{a+b/1}}{1^{a/1} 1^{b/1}}, \text{ pour } a \text{ et } b \text{ entiers;} \left\{ \begin{array}{l} \text{Cisa de Grésy, Mém. Turin. 1821.} \\ \text{209. I. 30.} \end{array} \right.$$

$$17) \quad = -l \frac{\Gamma(a+b+1)}{\Gamma(a+1) \Gamma(b+1)}, \text{ pour } a \text{ et } b \text{ fractions;} \left\{ \begin{array}{l} \text{Legendre, Exerc. 4. 111.} \\ \text{111. — Cisa de Grésy,} \\ \text{Mém. Turin. 1821. 209.} \\ \text{I. 30.} \end{array} \right.$$

$$18) \int \frac{1-x^q}{1-x} \frac{1-x^p}{lx} x^{r-1} dx = l \frac{\Gamma(q+r) \Gamma(p+r)}{\Gamma(r) \Gamma(p+q+r)} \quad \text{Legendre, Exerc. 4. 111.}$$

$$19) \int \frac{x^p - x^q}{1-x} \frac{x^r - x^s}{lx} dx = l \frac{(1+p+s)(1+q+r)}{(1+p+r)(1+q+s)} \quad \text{Schlömlich, Gr. 4. 167.}$$

$$20) \int \frac{(1-x^p)(1-x^q)(1-x^r)}{1-x} \frac{dx}{lx} = l \frac{\Gamma(p+1) \Gamma(q+1) \Gamma(r+1) \Gamma(p+q+r+1)}{\Gamma(p+q+1) \Gamma(p+r+1) \Gamma(q+r+1)}$$

$$21) \int \frac{(1-x^p)(1-x^q)(1-x^r)}{1-x} x^{s-1} \frac{dx}{lx} = l \frac{\Gamma(p+s) \Gamma(q+s) \Gamma(r+s) \Gamma(p+q+r+s)}{\Gamma(p+q+s) \Gamma(p+r+s) \Gamma(q+r+s) \Gamma(s)}$$

$$22) \int \frac{(1-x^p)(1-x^q)(1-x^r)(1-x^s)}{1-x} \frac{dx}{lx} = \left\{ \begin{array}{l} \text{Stern,} \\ \text{Gott.} \\ \text{Stud.} \\ \text{1847} \end{array} \right. \\ = l \frac{\Gamma(p+1) \Gamma(q+1) \Gamma(r+1) \Gamma(s+1) \Gamma(p+q+r+1) \Gamma(p+q+s+1) \Gamma(p+r+s+1) \Gamma(q+r+s+1)}{\Gamma(p+q+1) \Gamma(p+r+1) \Gamma(p+s+1) \Gamma(q+r+1) \Gamma(q+s+1) \Gamma(r+s+1) \Gamma(p+q+r+s+1)}$$

$$23) \int \left(\frac{1}{lx} + \frac{1}{1-x} - \frac{1}{2} \right) \frac{dx}{lx} = -1 + \frac{1}{2} l 2\pi \quad \text{V. T. 135. N°. 20.}$$

$$24) \int \left(\frac{1}{lx} + \frac{1}{2} \frac{1+x}{1-x} \right) \frac{dx}{lx} = \frac{1}{2} l 2 \pi - 1 \quad \text{V. T. 135. N}^\circ. 21.$$

$$25) \int \left(\frac{1}{lx} - lx + \frac{1}{2} \frac{1+x}{1-x} \right) \frac{dx}{lx} = \frac{1}{2} l 2 \pi \quad \text{V. T. 135. N}^\circ. 22.$$

$$26) \int \left(\frac{1}{lx} + \frac{1}{2} x + \frac{1}{1-x} \right) \frac{dx}{lx} = \frac{1}{2} l 2 \pi \quad \text{V. T. 135. N}^\circ. 23.$$

$$27) \int \left(p + \frac{x^{p-1}}{lx} - \frac{1}{2} x^{p-1} - \frac{1}{1-x} \right) \frac{dx}{lx} = - \left(p + \frac{1}{2} \right) lp + p - \frac{1}{2} l 2 \pi \quad \text{V. T. 135. N}^\circ. 17.$$

$$1) \int \frac{(1-x)^2}{1+x^2} \frac{dx}{lx} = l \frac{\pi}{4} \quad \text{Legendre, Exere. 5. 3.}$$

$$2) \int \frac{1}{1-x^2} \frac{dx}{lx} = - \frac{1}{2} l 2 \quad \text{Euler, Calc. Int. 4. S. 3. 115.}$$

$$3) \int \frac{1-xq}{1-x^2} \frac{1-x^{q+1}}{lx} dx = - q l 2, \quad q > -1; \quad \text{Legendre, Exere. 4. 113.}$$

$$4) \int \frac{1-x^2}{1+x^4} \frac{dx}{lx} = l \text{Cot.} \frac{3\pi}{8} \quad \text{Euler, Calc. Int. 4. S. 3. 111.}$$

$$5) \int \left\{ \frac{1}{1-x^2} + \frac{1}{2lx} - \frac{1}{2} \right\} \frac{dx}{lx} = \frac{1}{2} (l 2 - 1) \quad \text{V. T. 135. N}^\circ. 14.$$

$$\left. \begin{aligned} 6) \int \frac{x^{p+q-1} - x^{p-q-1}}{1+x^{2p}} \frac{dx}{lx} &= l \text{Tang.} \left\{ \frac{p+q}{4p} \pi \right\} \\ 7) \int \frac{x^{p+q-1} + x^{p-q-1}}{1-x^{2p}} \frac{dx}{lx} &= l \text{Cos.} \frac{q\pi}{4p} \end{aligned} \right\} \quad \text{Euler, N. C. Petr. 19. 30. — Id., Calc. Int. 4. S. 5. 30.}$$

$$8) \int \frac{(1-xq)^2}{1-x^p} x^{p-q-1} \frac{dx}{lx} = l \left(\frac{p}{q\pi} \text{Sin.} \frac{q\pi}{p} \right), \quad p > q; \quad \text{Euler, N. Act. Petr. I. P. 2. 29.}$$

$$\left. \begin{aligned} 9) \int \frac{1-x^{2p-2q}}{1+x^{2p}} \frac{x^{q-1}}{lx} dx &= l \text{Tang.} \frac{q\pi}{2p} \\ 10) \int \frac{(1-x^{p-q})^2}{1-x^{2p}} \frac{x^{q-1}}{lx} dx &= l \text{Sin.} \frac{q\pi}{2p} \end{aligned} \right\} \quad \text{Euler, N. A. Petr. 7. 64.}$$

- 1) $\int \frac{lx}{4\pi^2 q^2 + (lx)^2} \frac{dx}{1-x^2} = -\frac{1}{2} lq - \frac{1}{4q} + \frac{1}{2} Z'(1+q)$ V. T. 138. N°. 11.
- 2) $\int \frac{1}{q^2 + (lx)^2} \frac{dx}{1-x} = \frac{1}{2} \left\{ -\frac{q}{4\pi} + l \frac{q}{2\pi} - Z' \left(\frac{q}{2\pi} \right) \right\}$ V. T. 138. N°. 9.
- 3) $\int \frac{lx}{4\pi^2 + (lx)^2} \frac{dx}{1-x} = \frac{1}{4} - \frac{1}{2} \Lambda$ V. T. 138. N°. 10.
- 4) $\int \frac{lx}{q^2 - (lx)^2} \frac{dx}{1-x} = \frac{4\pi^4}{q^4} \sum_0^{\infty} (-1)^{n-1} \frac{B_{2n+1}}{(n+1)q^{2n}} \pi^{2n}$ V. T. 138. N°. 22.
- 5) $\int \frac{lx}{\{q^2 + (lx)^2\}^2} \frac{dx}{1-x} = -\frac{\pi^2}{q^4} \sum_0^{\infty} \frac{B_{2n+1}}{q^{2n}} (2\pi)^{2n}$ V. T. 138. N°. 21.
- 6) $\int \frac{lx}{\{q^2 - (lx)^2\}^2} \frac{dx}{1-x} = \frac{\pi^2}{q^4} \sum_0^{\infty} (-1)^{n-1} \frac{B_{2n+1}}{q^{2n}} (2\pi)^{2n}$ V. T. 138. N°. 23.
- 7) $\int \frac{1}{\pi^2 + (lx)^2} \frac{dx}{1+x^2} = \frac{4-\pi}{4\pi}$ V. T. 138. N°. 2.
- 8) $\int \frac{1}{\pi^2 + 4(lx)^2} \frac{dx}{1+x^2} = \frac{1}{4} \pi l 2$ V. T. 138. N°. 3.
- 9) $\int \frac{1}{q^2 + (lx)^2} \frac{dx}{1+x^2} = \frac{1}{4q} \left\{ Z' \left(\frac{2q+3}{4\pi} \right) - Z' \left(\frac{2q+1}{4\pi} \right) \right\}$ V. T. 138. N°. 1.
- 10) $\int \frac{lx}{\pi^2 + 4(lx)^2} \frac{dx}{1+x^2} = \frac{2-\pi}{16}$ V. T. 138. N°. 16.
- 11) $\int \frac{1}{\pi^2 + (lx)^2} \frac{lx}{1-x^2} dx = \frac{1}{2\pi} - l 2$ V. T. 138. N°. 12.
- 12) $\int \frac{1}{q^2 + (lx)^2} \frac{x}{1-x^2} dx = -\frac{1}{2} \left\{ \frac{\pi}{2q} + l \frac{\pi}{q} + Z' \left(\frac{q}{\pi} \right) \right\}$ V. T. 138. N°. 9.
- 13) $\int \frac{lx}{\pi^2 + (lx)^2} \frac{x}{1-x^2} dx = \frac{1}{4} - \frac{1}{2} \Lambda$ V. T. 138. N°. 10.
- 14) $\int \frac{lx}{q^2 \pi^2 + (lx)^2} \frac{x}{1-x^2} dx = \frac{1}{4q^4} \sum_0^{\infty} (-1)^n \frac{B_{2n+1}}{(n+1)q^{2n}}$ V. T. 138. N°. 22.
- 15) $\int \frac{lx}{\{q^2 + (lx)^2\}^2} \frac{x}{1-x^2} dx = -\frac{\pi^2}{4q^4} \sum_0^{\infty} \frac{B_{2n+1}}{q^{2n}} \pi^{2n}$ V. T. 138. N°. 21.

$$16) \int \frac{lx}{\{q^2 - (lx)^2\}^2} \frac{x}{1-x^2} dx = \frac{\pi^2}{4q^4} \sum_0^{\infty} (-1)^{n+1} \frac{B_{2n+1}}{q^{2n}} \pi^{2n} \quad \text{V. T. 138. N}^\circ. 23.$$

$$17) \int \frac{1}{\frac{1}{16}\pi^2 + (lx)^2} \frac{dx}{1-x^2} = \frac{1}{\pi \frac{1}{2}} \left\{ \pi + l \frac{1 \vee 2 - 1}{1 \vee 2 + 1} \right\} \quad \text{V. T. 138. N}^\circ. 18.$$

$$18) \int \frac{lx}{\frac{1}{16}\pi^2 + (lx)^2} \frac{dx}{1-x^2} = -\frac{1}{4} \frac{1 \vee \pi + 1}{2} + \frac{1}{2} \frac{1 \vee 2 - 1}{1 \vee 2 + 1} \quad \text{V. T. 138. N}^\circ. 19.$$

$$1) \int \left\{ q - \frac{1}{2} + \frac{(1-x)(1+qlx) + xlx}{(1-x)^2} x^{q-1} \right\} \frac{dx}{lx} = \frac{1}{2} - q + l\Gamma\left(\frac{1}{2}\right) - l\Gamma(q) + \frac{1}{2} l2 \quad \text{Arndt, Gr. 10. 455.}$$

$$2) \int \frac{x}{1+x^2+x^4} \frac{dx}{lx} = -\frac{1}{2} l3 \quad \text{Euler, Calc. Int. T. 4. S. 3. 112.}$$

$$3) \int \frac{x}{1-x^2+x^4} \frac{dx}{lx} = l \left(\frac{1}{2} \vee 3 \right) \quad \text{Euler, Calc. Int. T. 4. S. 3. 116.}$$

$$4) \int \frac{1}{1+x^2+2x \cos. \lambda \left(\frac{1}{x} \right)^{1-q}} \frac{dx}{x} = \text{Cosec. } \lambda \cdot \Gamma(q) \sum_1^{\infty} (-1)^{n-1} \frac{\text{Sin. } n\lambda}{n^q}$$

$$5) \int \frac{1}{1+x^2+2x \cos. \frac{a\pi}{b} l \frac{1}{x}} \frac{dx}{l \frac{1}{x}} = \frac{1 + \cos. \frac{a\pi}{b}}{\sin. \frac{a\pi}{b}} \left\{ \text{Tang. } \frac{a\pi}{2b} lb + 2 \sum_1^{\frac{b-1}{2}} (-1)^{n-1} \sin. \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+n+1}{2b}\right)}{\Gamma\left(\frac{n+1}{2b}\right)} \right\} \begin{matrix} \text{, pour} \\ a+b \\ \text{impair;} \end{matrix}$$

$$6) = \frac{1 + \cos. \frac{a\pi}{b}}{\sin. \frac{a\pi}{b}} \left\{ \text{Tang. } \frac{a\pi}{2b} lb + 2 \sum_1^{\frac{b-1}{2}} (-1)^{n-1} \sin. \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b-n+1}{b}\right)}{\Gamma\left(\frac{n+1}{b}\right)} \right\} \begin{matrix} \text{, pour} \\ a+b \\ \text{pair;} \end{matrix}$$

$$7) \int \frac{1-x^{p-q}}{1+x^2+2x \cos. \frac{a\pi}{b} l \frac{1}{x}} \frac{x^q}{x} dx = \text{Cosec. } \frac{a\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} \sin. \frac{na\pi}{b} l \frac{\Gamma\left(\frac{p+b+n}{2b}\right) \Gamma\left(\frac{q+n}{2b}\right)}{\Gamma\left(\frac{q+b+n}{2b}\right) \Gamma\left(\frac{p+n}{2b}\right)} \begin{matrix} \text{, pour } a+b \\ \text{impair;} \end{matrix}$$

Les formules (4) à (7) sont déduites par Malmsten, Cr. 38. 1. Partout $a < b$.

$$\begin{aligned}
 8) \int \frac{1-x^{p-q}}{1+x^2+2x \cos \frac{a\pi}{b}} \frac{x^q}{l \frac{1}{x}} dx &= \text{Cosec.} \frac{a\pi}{b} \sum_1^{b-1} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} \cdot l \frac{\Gamma\left(\frac{p+b-n}{b}\right) \Gamma\left(\frac{q+n}{b}\right)}{\Gamma\left(\frac{q+b-n}{b}\right) \Gamma\left(\frac{p+n}{b}\right)}, \text{ pour } a+b \\
 &\quad \text{pair;} \\
 9) \int \frac{(1-a)^2}{1+x^2+2x \cos \frac{a\pi}{b}} \frac{dx}{l \frac{1}{x}} &= \text{Cosec.} \frac{a\pi}{b} \sum_1^{b-1} (-1)^{n-1} \text{Sin.} \frac{n\pi}{b} \cdot l \frac{\left\{ \Gamma\left(\frac{b+n+1}{2b}\right) \right\}^2 \Gamma\left(\frac{n+2}{2b}\right) \Gamma\left(\frac{n}{2b}\right)}{\left\{ \Gamma\left(\frac{n+1}{2b}\right) \right\}^2 \Gamma\left(\frac{b+n}{2b}\right) \Gamma\left(\frac{b+n+2}{2b}\right)}, \text{ pour } \\
 &\quad \text{a+b pair;} \\
 10) &= \text{Cosec.} \frac{a\pi}{b} \sum_1^{b-1} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} \cdot l \frac{\left\{ \Gamma\left(\frac{b-n+1}{b}\right) \right\}^2 \Gamma\left(\frac{n+2}{b}\right) \Gamma\left(\frac{n}{b}\right)}{\left\{ \Gamma\left(\frac{n+1}{b}\right) \right\}^2 \Gamma\left(\frac{b-n}{b}\right) \Gamma\left(\frac{b-n+2}{b}\right)}, \text{ pour } \\
 &\quad \text{a+b pair;} \\
 11) \int \left(\text{Tang.} \frac{a\pi}{2b} - \frac{2x^q \text{Sin.} \frac{a\pi}{b}}{1+x^2+2x \cos \frac{a\pi}{b}} \right) \frac{dx}{l \frac{1}{x}} &= \text{Tang.} \frac{a\pi}{2b} l 2b + 2 \sum_1^{b-1} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} \cdot l \frac{\Gamma\left(\frac{q+b+n}{2b}\right)}{\Gamma\left(\frac{q+n}{2b}\right)}, \text{ pour } \\
 &\quad \text{a+b pair;} \\
 12) &= \text{Tang.} \frac{a\pi}{2b} l b + 2 \sum_1^{b-1} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} \cdot l \frac{\Gamma\left(\frac{q+b-n}{b}\right)}{\Gamma\left(\frac{q+n}{b}\right)}, \text{ pour } \\
 &\quad \text{a+b pair;} \\
 13) \int \frac{1+x^2}{1+x^4+2x^2 \cos \frac{a\pi}{b}} \frac{dx}{(lx)^{1-q}} &= \text{Sec.} \frac{1}{2} \lambda \Gamma(q) \sum_1^{\infty} (-1)^n \frac{\text{Cos.} \left\{ (n+\frac{1}{2})\lambda \right\}}{(2n+1)^q}
 \end{aligned}$$

Les formules (8) à (13) sont déduites par Malmsten, Cr. 38. 1. Partout $a < b$.

$$\begin{aligned}
 1) \int \frac{(1-x^q)(1-x^p) - (1-x)^2}{1-x} \frac{dx}{xlx} &= lB(p, q) \quad \text{Binet, P. 27. 123. — Id., C. R. 9. 39.} \\
 2) \int \frac{x^{q-1} - x^{-q}}{1+x} \frac{dx}{lx} &= l \text{Tang.} \frac{1}{2} q \pi \quad \text{Kummer, Cr. 17. 210.} \\
 3) \int \frac{(x^q - x^{-q})^2}{1+x} \frac{dx}{lx} &= l(q\pi \text{Cot.} q\pi) \quad \text{Binet, P. 27. 123.} \\
 4) \int \frac{x^q + x^{-q} - 2}{1-x} \frac{dx}{lx} &= l \frac{\text{Sin.} q\pi}{q\pi} \quad \text{Legendre, Exerc. 5. 3. — Binet, P. 27. 123.}
 \end{aligned}$$

- 5) $\int \frac{x^q - x^{-q}}{1 + x^2} \frac{dx}{lx} = l \text{ Tang. } \left(\frac{q+1}{4} \pi \right)$ V. T. 136. N°. 11.
- 6) $\int \frac{(x^q - x^{-q})^2}{1 - x^2} \frac{dx}{lx} = l \text{ Cos } q \pi$
- 7) $\int \frac{(x^q - x^{-q})^2}{1 - x^2} \frac{x}{lx} dx = l \frac{\text{Sin. } q \pi}{q \pi}$
- 8) $\int \frac{x^q + x^{-q} - 2}{1 - x^2} \frac{dx}{lx} = l \text{ Cos. } \frac{1}{2} q \pi$ V. T. 136 N°. 12.
- 9) $\int \frac{x^{p+q} - x^{2p} + x^{p+q}}{1 - x^{2p}} \frac{dx}{x lx} = l \text{ Cos. } \frac{q \pi}{2 p}$
- 10) $\int \frac{x^q - 2 x^p + x^{2p+q}}{1 - x^{2p}} \frac{dx}{x lx} = l \text{ Sin. } \frac{q \pi}{2 p}$
- 11) $\int \frac{x^{p+q} - x^{p-q}}{1 + x^{2p}} \frac{dx}{x lx} = l \text{ Tang. } \left(\frac{p+q}{4 p} \pi \right)$ Euler, Calc. Int. 4. S. 3. 110.
- 12) $\int \left\{ \frac{x^{p-1}}{1-x} - \frac{x^{p-1}}{1-x^q} - \frac{1}{x(1-x)} + \frac{1}{x(1-x^2)} \right\} \frac{dx}{lx} = q l p$ V. T. 135. N°. 17.
- 13) $\int \frac{x^q - x^r - x^{2p-r} + x^{2p-q}}{1 - x^{2p}} \frac{dx}{x lx} = l \left(\text{Sin. } \frac{q \pi}{2 p} \cdot \text{Cosec. } \frac{r \pi}{2 p} \right)$
- 14) $\int \frac{x^{p+q} - x^{p-r} - x^{p+r} + x^{p+q}}{1 - x^{2p}} \frac{dx}{x lx} = l \left(\text{Cos. } \frac{q \pi}{2 p} \cdot \text{Sec. } \frac{r \pi}{2 p} \right)$
- 15) $\int \frac{x^p - x^{r-p}}{1 + x^r} \frac{dx}{x lx} = l \text{ Tang. } \frac{p \pi}{2 r}$ Euler, N. A. Petr. 1777. P. 2. 29.
- 16) $\int \frac{x^p - x^q}{1 + x^r} \frac{1 + x^{r-p-q}}{x} \frac{dx}{lx} = l \left\{ \text{Tang. } \frac{p \pi}{2 r} \cdot \text{Cot. } \frac{q \pi}{2 r} \right\}$ Euler, N. A. Petr. 7. 64.
- 17) $\int \frac{1-x}{1+x} \frac{x^2}{1+x^2} \frac{dx}{lx} = l \frac{2 \sqrt{2}}{\pi}$ Legendre, Exere. 5. 3.
- 18) $\int \frac{1-x}{1+x} \frac{1}{1+x^2} \frac{dx}{lx} = -\frac{1}{2} l 2$ V. T. 171. N°. 1. et T. 175. N°. 17.
- 19) $\int \left\{ \frac{1}{1-x} - \frac{p x^{p-1}}{1-x^p} + \left(p q - \frac{p+1}{2} \right) x^{p-1} + (1-pq) \right\} \frac{dx}{lx} = \frac{1-p}{2} l 2 \pi + \left(p q - \frac{1}{2} \right) l p$
- 20) $\int \left\{ \frac{x^{q-1}}{1-x} - \frac{x^{p-1}}{1-x^p} - \frac{p-1}{1-x^p} x^{p-1} - \frac{1}{2} (p-1) x^{p-1} \right\} \frac{dx}{lx} = \frac{1-p}{2} l 2 \pi + \left(p q - \frac{1}{2} \right) l p$

Arndt, Gr.
10. 455.

- 1) $\int \left(\frac{1}{1-x^2} + \frac{2-x}{2lx} - \frac{1-x}{x} \right) \frac{dx}{lx} = 0$ V. T. 135. N°. 12.
- 2) $\int \left\{ p-1 - \frac{1}{1-x} + \left(\frac{1}{2} - \frac{1}{lx} \right) x^{p-1} \right\} \frac{dx}{lx} = \left(\frac{1}{2} - p \right) lp + p - \frac{1}{2} l 2\pi$ V. T. 135. N°. 15.
- 3) $\int \left\{ \left(q - \frac{1}{2} \right) \frac{x^{p-1} - x^{r-1}}{lx} + \frac{p x^{pq-1}}{1-x^p} - \frac{r x^{r-1}}{1-x^r} \right\} \frac{dx}{lx} = (p-r) \left\{ \frac{1}{2} - q + l \Gamma \left(\frac{1}{2} \right) - l \Gamma(q) + \frac{1}{2} l 2 \right\}$ Arndt. Gr. 10. 455.
- 4) $\int \frac{x^{-q} + x^q}{1-x^2} \frac{dx}{(lx)^p} = \frac{\Gamma(1-p)}{\pi^{p-1}} \sum_0^{\infty} (-1)^{p+n} \left\{ \frac{1}{(2n+1-q)^{1-p}} - \frac{1}{(2n+1+q)^{1-p}} \right\}$ V. T. 136. N°. 18.
- 5) $\int \frac{x^{-q} - x^q}{1-x^2} \frac{dx}{(lx)^p} = (-1)^p \frac{\Gamma(1-p)}{\pi^{p-1}} \sum_0^{\infty} \left\{ \frac{1}{(2n-1-q)^{1-p}} - \frac{1}{(2n-1+q)^{1-p}} \right\}$ V. T. 136. N°. 17.
- 6) $\int \frac{x^{-p} - x^p}{1-x^2} \frac{dx}{\frac{1}{4}\pi^2 + (lx)^2} = \text{Sin.} \frac{1}{2} p\pi - \pi \text{Cos.} \frac{1}{2} p\pi. l \frac{1 + \text{Sin.} \frac{1}{2} p\pi}{1 - \text{Sin.} \frac{1}{2} p\pi}, p < 1;$ V. T. 138. N°. 13.
- 7) $\int \frac{x^{-p} + x^p}{1-x^2} \frac{lx}{\frac{1}{4}\pi^2 + (lx)^2} dx = 1 - \frac{1}{2} \pi \text{Cos.} \frac{1}{2} p\pi + \frac{1}{2} \text{Sin.} \frac{1}{2} p\pi. l \frac{1 - \text{Sin.} \frac{1}{2} p\pi}{1 + \text{Sin.} \frac{1}{2} p\pi}, p < 1;$ V. T. 138. N°. 15.
- 8) $\int \frac{x^{-p} - x^p}{1-x^2} \frac{dx}{\pi^2 + (lx)^2} = \frac{1}{2\pi} [-p\pi \text{Cos.} p\pi + \text{Sin.} p\pi. l \{ 2(1 + \text{Cos.} p\pi) \}], p < 1;$ V. T. 138. N°. 4.
- 9) $\int \frac{x^{-p} + x^p}{1-x^2} \frac{lx}{\pi^2 + (lx)^2} dx = \frac{1}{2} [1 - p\pi \text{Sin.} p\pi - \text{Cos.} p\pi. l \{ 2(1 + \text{Cos.} p\pi) \}]$ V. T. 138. N°. 6.
- 10) $\int \frac{x^{p-1} - x^{1-p}}{1-x^2} \frac{dx}{q^2 + (lx)^2} = \frac{\pi}{q} \sum_1^{\infty} \frac{\text{Sin.} n p \pi}{q + n \pi}, p^2 < 1;$ V. T. 138. N°. 5.
- 11) $\int \frac{x^{p-1} + x^{1-p}}{1-x^2} \frac{lx}{q^2 + (lx)^2} dx = -\frac{\pi}{2q} - \sum_1^{\infty} \frac{\text{Cos.} n p \pi}{q + n \pi}, p^2 < 1;$ V. T. 138. N°. 8.
- 12) $\int \frac{lx}{x(1-x^2q)} \frac{x^2 q}{\pi^2 + (lx)^2} dx = -\frac{1}{2} l q - \frac{1}{4q} + \frac{1}{2} Z(1+q)$ V. T. 138. N°. 11.
- 13) $\int \frac{lx}{x(1-x^2q)} \frac{x^2}{\pi^2 + (lx)^2} dx = -\frac{1}{2} l q - \frac{1}{4q} + \frac{1}{2} Z(1+q)$ V. T. 138. N°. 11.

- 1) $\int \frac{1}{(1+x) \sqrt{x \pi^2 + (lx)^2}} dx = \frac{1}{2\pi} l \cdot 2$ V. T. 138. N°. 3.
- 2) $\int \frac{1}{(1+x) \sqrt{x \pi^2 + (lx)^2}} dx = \frac{4-\pi}{8\pi}$ V. T. 138. N°. 2.
- 3) $\int \frac{1}{(1+x) \sqrt{x q^2 + (lx)^2}} dx = \frac{1}{4q} \left\{ Z' \left(\frac{q+3\pi}{4\pi} \right) - Z' \left(\frac{q+\pi}{4\pi} \right) \right\}$ V. T. 138. N°. 1.
- 4) $\int \frac{1}{(1+x) \sqrt{x \pi^2 + 4(lx)^2}} dx = \frac{1}{4\pi \sqrt{2}} \left\{ \pi - l \frac{\sqrt{2}+1}{\sqrt{2}-1} \right\}$ V. T. 138. N°. 18.
- 5) $\int \frac{lx}{(1-x) \sqrt{x \pi^2 + (lx)^2}} dx = \frac{1}{2} - \frac{1}{4}\pi$ V. T. 138. N°. 16.
- 6) $\int \frac{lx}{(1-x) \sqrt{x \pi^2 + 4(lx)^2}} dx = -\frac{\pi}{2\sqrt{2}} + 1 + \frac{1}{2\sqrt{2}} l \frac{\sqrt{2}-1}{\sqrt{2}+1}$ V. T. 138. N°. 19.
- 7) $\int \frac{1}{(1+lx) \sqrt{x^3 \pi^2 + (lx)^2}} dx = \frac{1}{2\pi \sqrt{2}} \left\{ \pi - l \frac{\sqrt{2}+1}{\sqrt{2}-1} \right\}$ V. T. 138. N°. 18.
- 8) $\int \frac{lx}{(1-lx) \sqrt{x^3 \pi^2 + (lx)^2}} dx = -\frac{\pi}{2\sqrt{2}} + 1 + \frac{1}{2\sqrt{2}} l \frac{\sqrt{2}-1}{\sqrt{2}+1}$ V. T. 138. N°. 19.
- 9) $\int \frac{x^{-p} - x^p}{(1-x) \sqrt{x \pi^2 + (lx)^2}} dx = \frac{1}{2} \text{Sin. } p\pi + \frac{1}{2} \pi \text{Cos. } p\pi \cdot l \frac{1 - \text{Sin. } p\pi}{1 + \text{Sin. } p\pi}, p < \frac{1}{2};$ V.T. 138. N°. 13.
- 10) $\int \frac{x^{-p} + x^p}{(1-x) \sqrt{x \pi^2 + (lx)^2}} dx = 1 - \frac{1}{2} \pi \text{Cos. } p\pi + \frac{1}{2} \text{Sin. } p\pi \cdot l \frac{1 - \text{Sin. } p\pi}{1 + \text{Sin. } p\pi}, p < \frac{1}{2};$ V. T. 138. N°. 15.
- 11) $\int \frac{x^{-p} - x^p}{(1-x) \sqrt{x \pi^2 + (lx)^2}} dx = \frac{1}{4\pi} [2p\pi \text{Cos. } 2p\pi + \text{Sin. } 2p\pi \cdot l \{ (1 + \text{Cos. } 2p\pi) \}]$ V. T. 138. N°. 4.
- 12) $\int \frac{x^{-p} + x^p}{(1-x) \sqrt{x \pi^2 + (lx)^2}} dx = \frac{1}{2} [1 - 2p\pi \text{Sin. } 2p\pi - \text{Cos. } 2p\pi \cdot l \{ 2(1 + \text{Cos. } 2p\pi) \}]$ V. T. 138. N°. 6.
- 13) $\int \frac{x^{\frac{p-1}{2}} - x^{\frac{1-p}{2}}}{(1-x) \sqrt{x q^2 + (lx)^2}} dx = \frac{2\pi}{q} \sum_{n=1}^{\infty} \frac{\text{Sin. } 2np\pi}{q + 2n\pi}, p < 1;$ V. T. 138. N°. 5.
- 14) $\int \frac{x^{\frac{p-1}{2}} + x^{\frac{1-p}{2}}}{(1-x) \sqrt{x q^2 + (lx)^2}} dx = -\frac{\pi}{q} - 2\pi \sum_{n=1}^{\infty} \frac{\text{Cos. } 2np\pi}{q + 2n\pi}, p < 1;$ V. T. 138. N°. 8.
- 15) $\int \frac{1-x^{\eta-1}}{1-x} \frac{1-x_l^{-1}}{\sqrt{x}} \frac{dx}{lx} = -l 2^{2\eta-2}$ Legendre, Exerc. 4. 113. — Cisa de Grésy, Mém. Turin. 1821. 209. I. 31.

- 16) $\int \left(\frac{1}{1-x} + \frac{1}{lx} - \frac{1}{2} \right) \frac{dx}{lx \sqrt{x}} = \frac{1}{2} (l2 - 1) \quad \text{V. T. 135. N}^\circ. 13.$
- 17) $\int \left\{ \frac{1}{lx} - \frac{1}{2} - \frac{\sqrt{x}}{lx} \right\} \frac{dx}{lx} = \frac{1}{2} (l2 - 1) \quad \text{V. T. 127. N}^\circ. 16.$
- 18) $\int \left\{ \left(\frac{1}{lx} - \frac{1}{2} \right) \sqrt{x} + \left(\frac{1}{2} + \frac{1}{1-x} \right) x \right\} \frac{dx}{x lx} = \frac{1}{2} l2 \pi - \frac{1}{2} \quad \text{V. T. 135. N}^\circ. 16.$
- 19) $\int \left\{ \frac{1}{x} - \frac{1}{1 + \sqrt{x}} \right\} \frac{dx}{lx} = \frac{1}{2} l \frac{4}{\pi} \quad \text{V. T. 135. N}^\circ. 5.$
- 20) $\int \left\{ \frac{1}{1-x} - \frac{2}{1-x^2} + \frac{\sqrt{\frac{1}{x}}}{lx} - \frac{1}{2 lx} \right\} \frac{dx}{lx} = 0 \quad \text{V. T. 135. N}^\circ. 19.$
- 21) $\int \left\{ \frac{a-1}{2} + \frac{a-1}{1-x} + \frac{x^{a-1}}{1 - \sqrt{\frac{1}{x}}} + \frac{x^{ap}}{1-x} \right\} \frac{dx}{lx} = \left(ap + \frac{1}{2} \right) l a - \frac{1}{2} (a-1) l2 \pi \quad \text{V. T. 135. N}^\circ. 26.$
- 22) $\int \left(\frac{b}{lx} + \frac{x^{q-1}}{1 - \sqrt{x}} \right) dx = - \sum_1^b Z' \left(q + \frac{b-n}{b} \right) \quad \text{Arndt, Gr. 10. 253.}$
- 23) $\int \left\{ \left(p - \frac{1}{2} \right) x + \left(\frac{1}{2} - \frac{1}{lx} \right) \left(x^{p-1} - \sqrt{\frac{1}{x}} \right) \right\} \frac{dx}{lx} = \left(\frac{1}{2} - p \right) (lp - 1) \quad \text{V. T. 127. N}^\circ. 19.$
- 24) $\int \left\{ \frac{x^{q-1}}{1-x} - \frac{x^{pq-1} + (p-1)x^{lp-1}}{1-x^p} \right\} \frac{dx}{lx} = \frac{1}{2} (1-p) l2 + \left(pq - \frac{1}{2} \right) lp \quad \text{Arndt, Gr. 10. 455.}$
- 25) $\int l(1-x)^{q lx} \frac{\{x^{lq} + x^{-lq}\} - (x^{lq} - x^{-lq})}{x(lx)^2} dx = l \frac{\text{Sin. } q \pi}{q \pi} \quad \text{V. T. 175. N}^\circ. 4.$

- 1) $\int \frac{x^{p-1}}{\sqrt[l]{l} \frac{1}{x}} dx = \sqrt[l]{\frac{\pi}{p}} \quad \text{Bonnet, L. 17. 265.}$
- 2) $\int \frac{x^q - 1}{\sqrt[l]{lx}} dx = \left\{ \frac{1}{\sqrt[l]{1+q}} - 1 \right\} \sqrt[l]{- \pi}$
- 3) $\int \frac{x^p - x^q}{\sqrt[l]{lx}} dx = \left\{ \frac{1}{\sqrt[l]{1+p}} - \frac{1}{\sqrt[l]{1+q}} \right\} \sqrt[l]{- \pi}$

Bidone, Mém. Turin. 1812. 231. Art. 3. N^o. 36.

Log. en dén. sous forme irrat.

$$1) \int \frac{1}{1+x^2} \frac{dx}{\sqrt[n]{l} \frac{1}{x}} = \frac{1}{\sqrt[n]{l}} \pi \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)} \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 37.}$$

$$2) \int \frac{1}{1+x+x^2} \frac{dx}{\sqrt[n]{l} \frac{1}{x}} = \frac{1}{\sqrt[n]{l}} \frac{\pi}{\sin \frac{1}{3} \pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sin \frac{1}{3} n \pi}{\sqrt[n]{n}} \quad \text{V. T. 140. N°. 20.}$$

$$3) \int \frac{\cos \lambda - x - x^a \cos \{(a+1)\lambda\} + x^{a+1} \cos a\lambda}{1 - 2x \cos \lambda + x^2} \frac{dx}{\sqrt[n]{l} x} = \Gamma\left(\frac{1}{2}\right) \sum_{n=1}^a \frac{\cos n\lambda}{\sqrt[n]{n}} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Bonnet, L. 17. 265.}$$

$$4) \int \frac{\sin \lambda - x^a \sin \{(a+1)\lambda\} + x^{a+1} \sin a\lambda}{1 - 2x \cos \lambda + x^2} \frac{dx}{\sqrt[n]{l} x} = \Gamma\left(\frac{1}{2}\right) \sum_{n=1}^a \frac{\sin n\lambda}{\sqrt[n]{n}} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$5) \int \frac{x^{p-1} - x^{q-1}}{\left(\frac{1}{x}\right)^{2-\frac{1}{c}}} dx = \frac{c}{c-1} \Gamma\left(\frac{1}{c}\right) \left(q^{\frac{c-1}{c}} - p^{\frac{c-1}{c}}\right), q > p > 0; \quad \text{V. T. 140. N°. 16.}$$

Log.

$$1) \int l(1+x) \frac{dx}{x^{2-p}} = \frac{\pi \operatorname{Cosec} p \pi}{1-p}, 0 < p < 1; \quad \text{V. T. 18. N°. 5.}$$

$$2) \int l(1+x) \frac{dx}{x^{1+p}} = \frac{\pi}{p} \operatorname{Cosec} p \pi, 0 < p < 1; \quad \text{V. T. 22. N°. 1.}$$

$$3) \int l(1+qx) \frac{dx}{x^{1-p}} = -\frac{\pi}{(p+1)q^{p-1}} \operatorname{Cosec} p \pi, p < 1; \quad \text{V. T. 18. N°. 7.}$$

$$4) \int l(1-x) \frac{dx}{x^{2-p}} = \frac{\pi}{p-1} \cot p \pi, 0 < p < 1; \quad \text{V. T. 18. N°. 8.}$$

$$5) \int l(1+x^2) \frac{dx}{x^{2-p}} = \frac{\pi}{1-p} \sec \frac{1}{2} p \pi, 0 < p < 1; \quad \text{V. T. 19. N°. 8.}$$

$$6) \int l(1-x^2) \frac{dx}{x^{3-p}} = \frac{\pi}{p-2} \cot \frac{1}{2} p \pi, 0 < p < 1; \quad \text{V. T. 19. N°. 9.}$$

$$7) \int l(1+x^3) \frac{dx}{x^3} = \frac{\pi}{3} \sqrt[3]{3} \quad \text{V. T. 19. N°. 10.}$$

$$8) \int l(1+x^1) \frac{dx}{x^1} = \frac{1}{3} \pi \vee 2 \quad \text{V. T. 19. N}^\circ. 13.$$

$$9) \int l(1+x^6) \frac{dx}{x^2} = 2 \pi \quad \text{V. T. 19. N}^\circ. 17.$$

$$10) \int l(1+x^6) \frac{dx}{x^4} = \frac{1}{3} \pi \quad \text{V. T. 19. N}^\circ. 16.$$

$$11) \int l(1+x^6) \frac{dx}{x^6} = \frac{2}{5} \pi \quad \text{V. T. 19. N}^\circ. 15.$$

$$12) \int l \frac{(x+1)(x+q^2)}{(x+q)^2} \frac{dx}{x} = (lq)^2 \quad \left. \begin{array}{l} , q > 1; \\ \text{Minding, Tafeln. II.} \end{array} \right\}$$

$$13) \int l \frac{1+x}{1 \vee (x^2 + 2x \cos. \lambda + 1)} \frac{dx}{x} = \frac{1}{2} \lambda^2, 0 < \lambda < \pi; \quad \left. \begin{array}{l} , p < 1; \\ \text{Legendre, Exerc. 4. 104.} \end{array} \right\}$$

$$14) \int l(1-x^2) \frac{(p-1)x^p + (p+1)x^{-p}}{x^2} dx = -\pi \text{Tang. } \frac{1}{2} p \pi, 1 > p > 0; \quad \text{V. T. 22. N}^\circ. 7.$$

$$15) \int l \frac{1+2x+x^2}{1+2x \cos. \lambda + x^2} \frac{dx}{x^{1-p}} = 2 \pi \frac{1 - \cos. p \lambda}{p \sin. p \pi} \quad \left. \begin{array}{l} , p < 1; \\ \text{Legendre, Exerc. 4. 104.} \end{array} \right\}$$

$$16) \int l \frac{1+2x+x^2}{1+2x \cos. \lambda + x^2} \frac{x^p + x^{-p}}{x} dx = \frac{4 \pi}{p \sin. p \pi} (1 - \cos. p \lambda) \quad \left. \begin{array}{l} , p^2 < 1, q > 1; \\ \text{Minding Tafeln. II.} \end{array} \right\}$$

$$17) \int l \frac{(x+1)(x+q^2)}{(x+q)^2} \frac{dx}{x^{1-p}} = \frac{\pi}{p} \frac{(q^p - 1)^2}{\sin. p \pi}, p^2 < 1, q > 1; \quad \left. \begin{array}{l} , 0 < \lambda < \pi, \\ p^2 < 1; \end{array} \right\} \text{Minding Tafeln. II.}$$

$$18) \int l \frac{x+1}{1 \vee (x^2 + 2x \cos. \lambda + 1)} \frac{dx}{x^{1-p}} = \frac{\pi}{p \sin. p \pi} (1 - \cos. p \lambda), 0 < \lambda < \pi, \quad \left. \begin{array}{l} , 0 < \lambda < \pi, \\ p^2 < 1; \end{array} \right\}$$

$$19) \int l x \cdot l \frac{a^2 + 2bx + x^2}{a^2 - 2bx + x^2} \frac{dx}{x} = 2 \pi l a \cdot \text{Arcsin. } \frac{b}{a}, a \geq b; \quad \text{Schlömilch, Gr. 4. 316.}$$

$$20) \int l x \cdot l \frac{1+b^2 x^2}{1+a^2 x^2} \frac{dx}{x^2} = \pi (b-a) + \pi l \frac{a^a}{b^b} \quad \text{V. T. 45. N}^\circ. 1.$$

$$21) \int l \left(1 + \frac{x^2}{p^2} \right) \cdot l \left(1 + \frac{q^2}{x^2} \right) \frac{dx}{x^2} = 2 \pi \frac{p+q}{pq} l \frac{p+q}{p} - \frac{2}{p} \quad \text{Schlömilch, Gr. 4. 71.}$$

$$22) \int l(1-x) \cdot \{p l x - 1\} \frac{dx}{x^{p+1}} = (\pi \text{Cosec. } p \pi)^2, p < 1; \quad \text{V. T. 183. N}^\circ. 2.$$

$$23) \int l(1+x^2) \frac{\{p-2\}x^p - \{q-2\}x^q\} l x + x q - x^p dx}{(l x)^2} \frac{dx}{x^3} = 2 \pi l \left\{ \text{Tang. } \frac{1}{4} q \pi, \text{Cot. } \frac{1}{4} p \pi \right\}, p < 1, \text{V. T. 183. N}^\circ. 7.$$

$$24) \int \frac{1}{l(1-x^2)} \left\{ \frac{(p-2)x^p - (q-2)x^q}{(lx)^2} \right\} \frac{dx}{x^3} = 2\pi l \left\{ \text{Sin.} \frac{1}{2} q\pi \cdot \text{Cosec.} \frac{1}{2} p\pi \right\}, p < 1, \text{V. T. 180.}$$

$$25) \int l(1-x) \left\{ (p-1)lx + 1 \right\} \frac{dx}{x^{2-p}} = -(\pi \text{Cosec. } p\pi)^2, 0 < p < 1; \text{V. T. 183. N}^\circ. 1.$$

$$1) \int \frac{x^{p-1}}{x+q} lx dx = \pi q^{p-1} \text{Cosec. } p\pi (lq - \pi \text{Cot. } p\pi), 0 < p < 1, \text{Minding, Taf. II.}$$

$$2) \int \frac{lx}{1+x^2} dx = 0 \text{ Hill, Cr. 3. 101. — Bidone, Mém. Turin. 1812. 231. Art. 3. 37.}$$

$$3) \int \frac{(lx)^{2a+1}}{1+x^2} dx = 0$$

$$4) \int \frac{(lx)^{2a}}{1+x^2} dx = 2 \cdot 1^{2a/1} \sum_0^{\infty} (-1)^n \frac{1}{(2n+1)^{2a+1}} \left\{ \text{Bidone, Mém. Turin. 1812. 231. Art. 3. 37.} \right.$$

$$5) = (-1)^{a+1} (2\pi)^{2a+1} B' \left(\frac{1}{2} \right) \text{ Raabe, Cr. 42. 348.}$$

$$6) \int \frac{x^q}{1+x^2} (lx)^a dx = \frac{1}{2} \pi \frac{d^a}{dq^a} \cdot \text{Sec.} \frac{1}{2} q\pi \text{ Moigno, Calc. Int. 136.}$$

$$7) \int \frac{x^{p-1} - x^{q-1}}{1+x^2} \frac{dx}{lx} = \pi l \left(\text{Tang.} \frac{1}{4} p\pi \cdot \text{Cot.} \frac{1}{4} q\pi \right) \text{ Cauchy, Lim. Imag. 128.}$$

$$8) \int \frac{lx}{q^2+x^2} dx = \frac{\pi}{2q} lq \text{ Schlömilch, Gr. 4. 316. — Arndt, Gr. 11. 70.}$$

$$9) \int \frac{lp}{q^2+x^2} dx = \frac{\pi}{2q} lpq \text{ Arndt, Gr. 11. 70.}$$

$$10) \int \frac{lx}{p^2+q^2x^2} dx = \frac{\pi}{2pq} l \frac{p}{q} \text{ Lindmann, Stockh. Handl. 1850. II.}$$

$$11) \int \frac{lx}{1-x^2} dx = -\frac{1}{4} \pi^2$$

$$12) \int \frac{x^p lx}{1-x^2} dx = -\frac{1}{4} \left\{ \pi \text{Cosec.} \left(\frac{p+1}{2} \pi \right) \right\}^2, p^2 < 1; \left\{ \text{Ohm, Ausw. 16.} \right.$$

- 13) $\int \frac{x^{p-1} - x^{q-1}}{1 - x^2} \frac{dx}{lx} = \pi l \left(\text{Sin.} \frac{1}{2} p \pi. \text{Cosec.} \frac{1}{2} q \pi \right)$ Cauchy, Lim. Imag. 129.
- 14) $\int \frac{1-x}{1-x^{2a}} x^{a-2} lx dx = - \left(\frac{\pi}{2a} \text{Tang.} \frac{\pi}{2a} \right)^2, a > 1;$
- 15) $\int \frac{1-x^2}{1-x^{2a}} x^{a-3} lx dx = - \left(\frac{\pi}{2a} \text{Tang.} \frac{\pi}{a} \right)^2, a > 2;$
- 16) $\int \frac{1-x^2}{1-x^{2b}} x^{a-1} lx dx = - \left(\frac{\pi}{2b} \right)^2 \frac{\text{Sin.} \left\{ \frac{a+1}{b} \pi \right\} \cdot \text{Sin.} \frac{\pi}{b}}{\text{Sin.}^2 \frac{a\pi}{2b} \cdot \text{Sin.}^2 \left\{ \frac{a+2}{2b} \pi \right\}}$ Schlömilch, Gr. 12. 208.
Lindmann, Gr. 14. 94.

- 1) $\int l(1+x^2) \frac{dx}{1+x^2} = \pi l 2$
- 2) $\int l(q^2+x^2) \frac{dx}{1+x^2} = \pi l(1+q)$
- 3) $\int l(1+p^2 x^2) \frac{dx}{1+x^2} = \pi l(1+p)$
- 4) $\int l \left(1 + \frac{p^2}{x^2} \right) \frac{dx}{1+x^2} = \pi l(1+p)$
- 5) $\int l \left(\frac{x+1}{x-1} \right)^2 \frac{x}{1+x^2} dx = \frac{1}{2} \pi^2$ V. T. 340. N°. 14.
- 6) $\int l \left(1 + \frac{1}{x^2} \right) \frac{dx}{1+p^2 x^2} = \frac{\pi}{p} l(1+p)$ Schlömilch, Gr. 4. 71.
- 7) $\int l(x^2+q^2) \frac{dx}{p^2+x^2} = \frac{\pi}{p} l(p+q)$ V. T. 265. N°. 12.
- 8) $\int l(x^2-q^2)^2 \frac{dx}{p^2+x^2} = \frac{\pi}{p} l(p^2+q^2)$ V. T. 265. N°. 13.
- 9) $\int l(x^4-q^4)^2 \frac{dx}{p^2+x^2} = \frac{\pi}{p} l(p^2+q^2)(p+q)^2$ V. T. 265. N°. 15.

- $$\left. \begin{aligned} 10) \int l \left(1 + \frac{q^2}{x^2} \right) \frac{dx}{p^2 + x^2} &= \frac{\pi}{p} l \frac{p+q}{p} \\ 11) \int l \left(1 + \frac{q^2}{x^2} \right) \frac{dx}{p^2 - x^2} &= \frac{\pi}{p} \text{Arctang.} \frac{q}{p} \end{aligned} \right\} \text{Cauchy, Lim. Imag. Add. 28, 30.}$$
- $$12) \int l \frac{x^2 + \text{Sec.}^2 \lambda}{1 + x^2} \frac{dx}{1 + x^2} = \pi l \left(\text{Cos.}^2 \frac{1}{2} \lambda, \text{Sec.} \lambda \right) \quad \text{V. T. 334. N}^\circ. 9.$$
- $$13) \int l \frac{\text{Cos.}^2 \lambda + x^2}{1 + x^2} \frac{dx}{1 + x^2} = 2 \pi l \text{Cos.} \frac{1}{2} \lambda \quad \text{V. T. 334. N}^\circ. 13.$$
- $$14) \int l \frac{x^2 + \text{Cos.}^2 \lambda}{x^2 + \text{Cos.}^2 \mu} \frac{dx}{1 + x^2} = 2 \pi l \left(\text{Cos.} \frac{1}{2} \lambda, \text{Sec.} \frac{1}{2} \mu \right) \quad \text{V. T. 334. N}^\circ. 23.$$
- $$15) \int l \frac{1 + \text{Sin.} 2 \lambda + x^2}{1 - \text{Sin.} 2 \lambda + x^2} \frac{dx}{1 + x^2} = \pi l \frac{1 + \text{Sin.} \lambda}{\text{Cos.} \lambda} \quad \text{V. T. 334. N}^\circ. 22.$$
- $$16) \int l \frac{\text{Cos.}^2 \mu - \text{Cos.}^2 \lambda + x^2 \text{Sin.}^2 \lambda}{\text{Cos.}^2 \mu + x^2} \frac{dx}{1 + x^2} = \pi l \left\{ \text{Cot.} \frac{1}{2} \mu, \text{Tang.} \left(\frac{1}{2} \text{Arcsin.} \left(\frac{\text{Sin.} \mu}{\text{Sin.} \lambda} \right) \right) \right\} \quad \text{V. T. 334. N}^\circ. 24.$$
- $$17) \int l (1 - e^{-2\pi x}) \frac{dx}{p^2 + x^2} = - \frac{\pi}{2p} l \left[\left(\frac{e}{p} \right)^p \Gamma(p) \vee \frac{p}{2\pi} \right] \quad \text{Liouville, L. 17. 448.}$$

- $$1) \int l x \frac{dx}{(q+x)^2} = \frac{1}{q} l q, q < 1; \quad \text{Schlömilch, Beitr. III. § 10.}$$
- $$2) \int (l x)^2 \frac{x}{(1+x^2)^2} dx = 0 \quad \text{V. T. 153. N}^\circ. 15 \text{ et T. 187. N}^\circ. 5.$$
- $$3) \int (l x)^2 \frac{dx}{(1-x)^2} = \frac{2}{3} \pi^2 \quad \text{Minding, Taf. II.}$$
- $$4) \int l x \frac{dx}{(q+x)^{p+1}} = \frac{1}{p q^p} \{ l q - \Lambda - Z'(p) \} \quad \text{Schlömilch, Beitr. III. § 9.}$$
- $$5) \int (l x)^2 \frac{dx}{(1+x^2)^q} = \frac{\Gamma(q-\frac{1}{2})}{2 \Gamma(q)} \vee \pi \left\{ \Lambda + 2 l 2 + Z' \left(\frac{2q-1}{2} \right) \right\} \quad \text{V. T. 333. N}^\circ. 7.$$
- $$6) \int l x \frac{dx}{(a^2 + b^2 x^2)^p} = \frac{\Gamma(p-\frac{1}{2})}{a^{2p-1} b \Gamma(p)} \vee \pi \left\{ 2 l \frac{a}{2p} - \Lambda - Z' \left(p - \frac{1}{2} \right) \right\} \quad \text{Lindmann, Gr. 16. 94.}$$
- $$7) \int l x \frac{dx}{(a+x)^{b+2}} = \frac{1}{(b+1) a^{b+1}} \left\{ l a - \sum_1^n \frac{1}{n} \right\} \quad \text{Schlömilch, Beitr. III. § 10.}$$

- 8) $\int l(1+x) \frac{x}{(q^2+x^2)^2} dx = \frac{1}{2(1+q^2)} \left(lq + \frac{\pi}{2q} \right)$ V. T. 24. N°. 3.
- 9) $\int l(1-x)^2 \frac{x}{(q^2+x^2)^2} dx = \frac{1}{1+q^2} \left(lq - \frac{\pi}{2q} \right)$ V. T. 24. N°. 4.
- 10) $\int l(1+x) \frac{q^2-x^2}{(q^2+x^2)^2} dx = \frac{1}{1+q^2} \left(lq - \frac{1}{2}q\pi \right)$ V. T. 24. N°. 1.
- 11) $\int l(1-x)^2 \frac{q^2-x^2}{(q^2+x^2)^2} dx = \frac{-2}{1+q^2} \left(lq + \frac{1}{2}q\pi \right)$ V. T. 24. N°. 2.
- 12) $\int l(1+x^2) \frac{dx}{(1+x^2)^2} = \frac{\pi}{2} \left(l2 - \frac{1}{2} \right)$ V. T. 163. N°. 4.
- 13) $\int l(q+x) \frac{dx}{(1-x)^2} = \frac{2qlq}{1+q}$ V. T. 24. N°. 12.
- 14) $\int l(1-x)^2 \frac{dx}{(q+x)^2} = \frac{2}{1+q} lq$ V. T. 24. N°. 12.
- 15) $\int l(q^2+x^2) \frac{dx}{(1+x)^2} = \frac{2q^2}{1+q^2} lq - \frac{q\pi}{1+q^2}, q < 1;$ V. T. 24. N°. 1.
- 16) $\int l(q^2+x^2) \frac{dx}{(1-x)^2} = \frac{-2q^2}{1+q^2} lq + \frac{q\pi}{1+q^2}$ V. T. 24. N°. 2.
- 17) $\int l(q^2+x^2) \frac{x}{(1-x^2)^2} dx = \frac{2q^2 lq}{1+q^2}$ V. T. 24. N°. 12.
- 18) $\int l(q^2+x^2) \frac{p^2-x^2}{(p^2+x^2)^2} dx = -\frac{\pi}{p+q}$ V. T. 24. N°. 8.
- 19) $\int l(q^2+x^2) \frac{p^2+x^2}{(p^2-x^2)^2} dx = -\frac{\pi q}{p^2+q^2}$ V. T. 24. N°. 11.
- 20) $\int l(1-x^2)^2 \frac{x}{(q^2+x^2)^2} dx = \frac{2}{1+q^2} lq$ V. T. 24. N°. 12.
- 21) $\int l(p^2-x^2)^2 \frac{q^2-x^2}{(q^2+x^2)^2} dx = -\frac{2q\pi}{p^2+q^2}$ V. T. 24. N°. 11.
- 22) $\int l \left(\frac{1+x}{1-x} \right)^2 \frac{x}{(q^2+x^2)^2} dx = \frac{1}{1+q^2} \frac{\pi}{2q}$ V. T. 24. N°. 13.

$$1) \int \frac{l x}{x} \frac{x^p}{x-1} dx = (\pi \operatorname{Cosec} p \pi)^2, 0 < p < 1; \text{ Minding, Tafeln. II.}$$

$$2) \int \frac{l x}{x-1} \frac{dx}{x^p} = (\pi \operatorname{Cosec} p \pi)^2 \text{ Svanberg, Transf. § 5.}$$

$$3) \int l \left(\frac{1+x}{1-x} \right)^2 \frac{dx}{x(1+x^2)} = \frac{1}{2} \pi^2 \text{ Schlömilch, Gr. 4. 316.}$$

$$4) \int l x \frac{1-x^{-p}}{1-x^2} dx = \left(\frac{1}{2} \pi \operatorname{Tang} \frac{1}{2} p \pi \right)^2, p < 1; \text{ Schlömilch, Gr. 12. 203.}$$

$$5) \int l x \frac{(p+q)(x^{q-p} - x^{p-q}) + (p-q)(x^{p+q} - x^{p-q})}{(x^p - x^{-p})^2} \frac{dx}{x} = \frac{\pi}{p} \operatorname{Tang} \frac{q \pi}{2 p} \text{ V. T. 22. N°. 14.}$$

$$6) \int l x \frac{(p-q)(x^{p+q} - x^{p-q}) + (p+q)(x^{p-q} - x^{q-p})}{(x^p + x^{-p})^2} \frac{dx}{x} = \frac{\pi}{p} \operatorname{Sec} \frac{q \pi}{2 p} \text{ V. T. 22. N°. 15.}$$

$$\left. \begin{aligned} 7) \int l x \left(\frac{x}{1+x^2} \right)^{2a+1} \frac{dx}{x} &= 0 \\ 8) \int l x \left(\frac{x}{q^2+x^2} \right)^{2a+1} \frac{dx}{x} &= \frac{1^{a/2}}{2^{a/2}} \frac{\pi l q}{2 q^{2a+1}} \\ 9) \int l x \left(\frac{x}{q^2+x^2} \right)^{2a} \frac{dx}{x} &= \frac{1^{a-1/2}}{a^{a/2}} \frac{l q}{2 q^{2a}} \\ 10) \int l x \left(\frac{x}{q^2+x^2} \right)^p dx &= \frac{p}{2 q^p} \frac{\{\Gamma(\frac{1}{2} p)\}^2}{\Gamma(p)} \\ 11) \int l \frac{x}{q} \left(\frac{x}{q^2+x^2} \right)^p \frac{dx}{x} &= 0 \end{aligned} \right\} \text{ Schlömilch, Gr. 4. 316.}$$

$$\left. \begin{aligned} 12) \int \frac{l x}{x+q} \frac{dx}{x+1} &= \frac{1}{2} \frac{(l q)^2}{q-1} \\ 13) \int \frac{l x}{x+q} \frac{x^p}{x+1} dx &= \frac{\pi}{q-1} \frac{q^p l q \cdot \operatorname{Sin} p \pi + (1-q^p) \pi \operatorname{Cos} p \pi}{\operatorname{Sin}^2 p \pi} \\ 14) \int \frac{l x}{x+q} \frac{dx}{x-1} &= \frac{1}{2(1+q)} \{\pi^2 + (l q^2)\} \\ 15) \int \frac{l x}{x+q} \frac{x^p}{x-1} dx &= \frac{\pi}{1+q} \frac{\pi + q^p (\operatorname{Sin} p \pi l q - \pi \operatorname{Cos} p \pi)}{\operatorname{Sin}^2 p \pi} \end{aligned} \right\} , p^2 < 1, q > 1; \text{ Minding, Taf. II.}$$

$$16) \int l x \frac{p+x^2}{p^2+x^2} \frac{dx}{1+x^2} = \frac{1}{2} \frac{\pi}{1+p} l p \quad \text{V. T. 346. N}^{\circ} 7.$$

$$\left. \begin{aligned} 17) \int \frac{x^{p-1} - x^{q-1}}{1+x^{2q}} \frac{dx}{l x} &= l \text{Tang.} \frac{p-q}{4q} \\ 18) \int \frac{x^{p-1} - x^{q-1}}{1+x^r} \frac{dx}{l x} &= l \left(\text{Tang.} \frac{p\pi}{2r} \cdot \text{Cot.} \frac{q\pi}{2r} \right) \end{aligned} \right\} \text{Euler, N. A. Petr. 7. 64.}$$

$$\left. \begin{aligned} 1) \int \frac{(l x)^2}{x-1} \frac{dx}{x+q} &= \frac{1}{3(1+q)} l q \{ \pi^2 + (l q)^2 \} \\ 2) \int \frac{(l x)^3}{x-1} \frac{dx}{x+q} &= \frac{1}{4(1+q)} \{ \pi^2 + (l q)^2 \}^2 \\ 3) \int \frac{(l x)^4}{x-1} \frac{dx}{x+q} &= \frac{1}{15(1+q)} \{ \pi^2 + (l q)^2 \}^2 \{ 7\pi^2 + 2(l q)^2 \} l q \\ 4) \int \frac{(l x)^5}{x-1} \frac{dx}{x+q} &= \frac{1}{6(1+q)} \{ \pi^2 + (l q)^2 \}^2 \{ 3\pi^2 + (l q)^2 \}^2 \end{aligned} \right\} , q > 1 ;$$

Dedekind, Eul. Int. S. 23. —
Minding, Taf. II; il a fau-
tivement dans les dénomi-
nateurs 6, 24, 360, 720,
au lieu de 3, 4, 15, 6;

$$\left. \begin{aligned} 5) \int \frac{(l x)^{2a}}{1+x^2} (x^{-b} + x^b) dx &= \frac{(-1)^{a+1}}{b} 2(2q\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b} \right) \text{Cos.} \left(\frac{2n-1}{2} a\pi \right) \\ 6) \int \frac{(l x)^{2a}}{1-x^2} (x^{-b} - x^b) dx &= \frac{(-1)^{a+1}}{b} 2(2q\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{n}{2b} \right) \text{Sin.} n a \pi \end{aligned} \right\} \left. \begin{array}{l} \text{Raabe,} \\ \text{Cr. 42.} \\ 348. \end{array} \right\}$$

$$\left. \begin{aligned} 7) \int \frac{l x \cdot l \frac{x}{q}}{x-q} \frac{x^p}{x-1} dx &= \frac{\pi^2}{q-1} \frac{(q^p+1) l q - 2\pi (q^p-1) \text{Cot.} p\pi}{\text{Sin.}^2 p\pi} \\ 8) \int \frac{l x \cdot l \frac{x}{q}}{x-q} \frac{dx}{x-1} &= \frac{4\pi^2 + (l q)^2}{6(q-1)} l q \end{aligned} \right\} , p^2 < 1, q > 1 ;$$

Minding, Taf. II.

$$9) \int (l x)^2 \frac{dx}{1+2x \text{Cos.} \lambda + x^2} = \lambda \text{Cosec.} \lambda \frac{\pi^2 - \lambda^2}{3} \quad \text{Legendre, Exere. 4. 105.}$$

$$10) \int (l x)^4 \frac{dx}{1+2x \text{Cos.} \lambda + x^2} = \frac{2}{5} \lambda \text{Cosec.} \lambda (\pi^2 - \lambda^2) (7\pi^2 - 3\lambda^2) \quad \text{Legendre, Exere. 4. 105.}$$

$$\left. \begin{aligned} 11) \int (lx)^{2a} \frac{dx}{1+x+x^2} &= \frac{(-1)^{a+1}}{1-3} 2(2\pi)^{2a+1} B''\left(\frac{1}{3}\right) \\ 12) \int (lx)^{2a} \frac{dx}{1-x+x^2} &= \frac{(-1)^{a+1}}{1-3} 2(2\pi)^{2a+1} B''\left(\frac{1}{6}\right) \\ 13) \int (lx)^{2a} \frac{dx}{1-2x \cos 2p\pi + x^2} &= (-1)^{a+1} (2\pi)^{2a+1} \operatorname{Coscc}. 2p\pi B''(p) \end{aligned} \right\} , p < 1; \text{ Raabe, Cr. 42. 348. }^{\circ}$$

$$14) \int l(1+x^2) \frac{dx}{x(1+x^2)} = \frac{1}{12} \pi^2 \quad \text{V. T. 152. N}^{\circ}. 14.$$

$$15) \int l(1+x) \frac{x lx - x - q}{(x+q)^2} \frac{dx}{x} = \frac{1}{q-1} lq \quad \text{V. T. 183. N}^{\circ}. 12.$$

$$16) \int l(1-x) \frac{x lx - x - q}{(x+q)^2} \frac{dx}{x} = \frac{1}{2(1-q)} \{\pi^2 + (lq)^2\} \quad \text{V. T. 183. N}^{\circ}. 14.$$

$$17) \int l(1-x) \frac{x}{1-x^2} dx = \infty \quad \text{V. T. 181. N}^{\circ}. 5.$$

$$18) \int l\left(1 + \frac{b^2}{x^2}\right) \frac{(a-xi)^{-p} + (a+xi)^{-p}}{2} dx = \frac{\pi}{p-1} \left\{ \left(\frac{1}{a}\right)^p - \left(\frac{1}{a+b}\right)^p \right\} \quad \text{Cauchy, Lim. Imag. Add. 29.}$$

$$19) \int \left\{ \frac{p-1}{(1+x)^2} - \frac{(1+x)^{-1} - (1+x)^{-p}}{x} \right\} \frac{dx}{l(1+x)} = l\Gamma(p) \quad \text{Féaux, Fonct. Trause.}$$

$$1) \int lx \frac{dx}{x(1-x)} = 2\pi \quad \text{V. T. 28. N}^{\circ}. 1.$$

$$2) \int lx \frac{1-x}{(1+x)^2} \frac{dx}{1-x} = -\frac{1}{2} \pi \quad \text{V. T. 28. N}^{\circ}. 1.$$

$$3) \int lx \frac{1+x}{(1-x)^2} \frac{dx}{1-x} = 0 \quad \text{V. T. 28. N}^{\circ}. 2.$$

$$4) \int lx \frac{dx}{1-(1+x^2)\{1+(1-p^2)x^2\}} = -\frac{1}{2} F'(p) l(1-p^2), p^2 < 1; \quad \text{V. T. 347. N}^{\circ}. 13.$$

$$5) \int lx \frac{dx}{1-(1+x^2)(x^2+1-p^2)} = \frac{1}{2} F'(p) l(1-p^2), p^2 < 1; \quad \text{V. T. 347. N}^{\circ}. 13.$$

Page 256.

- $$\left. \begin{aligned} 6) \int l x \frac{1-x^{\frac{2}{a}}}{1-x^2} dx &= \left(\frac{1}{2} \pi \operatorname{Tang.} \frac{\pi}{a} \right)^2, \quad a > 2; \\ 7) \int l x \frac{1-x^{-\frac{2}{a}}}{1-x^2} dx &= \left(\frac{1}{2} \pi \operatorname{Tang.} \frac{\pi}{a} \right)^2, \quad a > 2; \\ 8) \int l x \frac{1-x^{\frac{2}{a}}}{1-x^2} x^{\frac{b}{a}-1} dx &= - \left(\frac{\pi}{2} \right)^2 \frac{\operatorname{Sin.} \left\{ \frac{b+1}{a} \pi \right\} \cdot \operatorname{Sin.} \frac{\pi}{a}}{\operatorname{Sin.}^2 \frac{b\pi}{2a} \cdot \operatorname{Sin.}^2 \left\{ \frac{b+2}{2a} \pi \right\}} \end{aligned} \right\} \text{Lindmann, Gr. 16. 94.}$$
- $$9) \int l x \frac{dx}{(q+x)^{b+\frac{3}{2}}} = \frac{2}{(2b+1)q^{b+\frac{1}{2}}} \left\{ lq + 2l2 - \sum_1^{b-1} \frac{1}{n} - 2 \sum_b^{2b-1} \frac{1}{n} \right\} \text{Schlömlich, Beitr. III. § 10.}$$
- $$10) \int l x \frac{dx}{(1-x^2)^{\frac{1}{2}+a}} = - \frac{1a^2}{2^{a+1} 1a^{\frac{1}{2}}} \frac{\pi}{2} \left\{ 1 + 2l2 + \sum_1^a \frac{1}{n} \right\} \text{V. T. 331. N}^\circ \text{ 10.}$$
- $$11) \int (lx)^{2a} \frac{x^{4b-1}}{1+x^b} dx = \frac{(-1)^{a+1}}{b} (1\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b} \right) \text{Raabe, Cr. 42. 348.}$$
- $$12) \int l(1-x)^2 \frac{dx}{x \sqrt{x}} = 0 \quad \text{V. T. 28. N}^\circ \text{ 2.}$$
- $$13) \int l(1+x) \frac{dx}{x^{p+\frac{1}{2}}} = \frac{2}{2p-1} \pi \operatorname{Sec.} p\pi, \quad p^2 < \frac{1}{4}; \quad \text{V. T. 28. N}^\circ \text{ 5.}$$
- $$14) \int l \frac{1 - \operatorname{Coth} p^2 \cdot \lambda + x^2}{1 + \operatorname{Coth} p^2 \cdot \lambda + x^2} \frac{2}{1 + (1 - \operatorname{Cosh} p^2 \cdot \lambda) x^2} \sqrt{\frac{dx}{(1+x^2)}} = \frac{2\lambda l \operatorname{Sin} h p \cdot \lambda}{\operatorname{Sin} h p \cdot \lambda \cdot \operatorname{Cosh} p \cdot \lambda} \text{V. T. 343. N}^\circ \text{ 12.}$$

- $$\left. \begin{aligned} 1) \int l \frac{1 \sqrt{(1+x^2)} + 1 \sqrt{(1-p^2)}}{1 \sqrt{(1+x^2)} - 1 \sqrt{(1-p^2)}} \frac{dx}{1 \sqrt{(1+x^2)}} &= \pi \operatorname{Arccos.} p \\ 2) \int l \frac{1 \sqrt{(1+x^2)} + p}{1 \sqrt{(1+x^2)} - p} \frac{dx}{1 \sqrt{(1+x^2)}} &= \pi \operatorname{Arcsin.} p \end{aligned} \right\} \text{Raabe, Int. 421.}$$
- $$3) \int l(1+x^2) \frac{dx}{(q+x)^2} = \frac{1}{1+q} \left(lq + \frac{\pi}{1 \sqrt{q}} \right) \quad \text{V. T. 24. N}^\circ \text{ 3.}$$
- $$4) \int l(1-x^2) \frac{dx}{(q+x)^2} = \frac{1}{1+q} \left\{ lq - \frac{\pi}{1 \sqrt{q}} \right\} \quad \text{V. T. 24. N}^\circ \text{ 4.}$$

$\left. \begin{aligned} 5) \int l \frac{x}{\sqrt[3]{1+x^3}} \frac{dx}{1+x^3} &= -\frac{\pi l 3}{3 \sqrt[3]{3}} - \frac{\pi^2}{27} \\ 6) \int l \frac{x}{\sqrt[3]{1+x^3}} \frac{x}{1+x^3} dx &= -\frac{\pi l 3}{3 \sqrt[3]{3}} + \frac{\pi^2}{27} \\ 7) \int l \frac{x}{\sqrt[3]{1+x^3}} \frac{dx}{1-x+x^2} &= -\frac{2 \pi l 3}{3 \sqrt[3]{3}} \\ 8) \int l \frac{x}{\sqrt[3]{1+x^3}} \frac{1-x}{1+x^3} dx &= -\frac{2}{27} \pi^2 \end{aligned} \right\}$	Euler, Calc. Int. 4. S. 3. 161. — Id., Act. Petr. 1777. II. 3.
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$$\begin{aligned} 1) \int (lx)^p \frac{dx}{x^2} &= \Gamma(1+p) \quad \text{V. T. 42. N}^\circ 2. \\ 2) \int lx \frac{dx}{1+x^2} &= \sum_0 \frac{1}{(2n+1)^2} \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 3. N}^\circ 37. \\ 3) \int lx \frac{dx}{1-x^2} &= -\frac{1}{8} \pi^2 \quad \text{Ohm, Ausw. 16.} \\ 4) \int lx \frac{x}{(1+x^2)^2} dx &= \frac{1}{4} l \frac{1}{2} \quad \text{V. T. 335. N}^\circ 1. \\ 5) \int lx \frac{dx}{x^2 \sqrt{x^2-1}} &= 1 - l 2 \quad \text{V. T. 103. N}^\circ 3. \\ 6) \int (lx)^a \frac{dx}{1+x^2} &= l^{a/2} \sum_0 (-1)^n \frac{1}{(2n+1)^{a+1}} \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 3. N}^\circ 37 \\ 7) \int l(1+x^2) \frac{dx}{1+x^2} &= \frac{\pi}{2} l 2 + \sum_0 \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 270. N}^\circ 7. \\ 8) \int l(1+x^2) \frac{dx}{x(1+x^2)} &= \frac{1}{2} \left\{ \frac{1}{6} \pi^2 - (l 2)^2 \right\} \quad \text{V. T. 160. N}^\circ 17 \text{ et T. 184. N}^\circ 15. \\ 9) \int \frac{1}{lp - lx} \frac{dx}{x^2} &= \frac{1}{p} li.p \quad \text{V. T. 43. N}^\circ 5. \\ 10) \int \frac{1}{q + lx} \frac{dx}{x^2} &= -e^q Ei.(-q) \quad \text{V. T. 129. N}^\circ 9. \\ 11) \int \frac{1}{q - lx} \frac{dx}{x^2} &= e^{-q} Ei.(q) \quad \text{V. T. 129. N}^\circ 4. \end{aligned}$$

- 12) $\int \frac{1}{q^2 + (lx)^2} \frac{dx}{x^2} = Ci.(q).Sin q - Si.(q).Cos. q + \frac{1}{2} \pi Cos. q$ V. T. 130. N°. 4
- 13) $\int \frac{lx}{q^2 + (lx)^2} \frac{dx}{x^2} = Ci.(q).Cos. q + Si.(q).Sin. q - \frac{1}{2} \pi Sin. q$ V. T. 130. N°. 5.
- 14) $\int \frac{1}{q^2 - (lx)^2} \frac{dx}{x^2} = \frac{1}{2q} \{e^{-q} Ei.(q) - e^q Ei.(-q)\}$ V. T. 130. N°. 10.
- 15) $\int \frac{lx}{q^2 - (lx)^2} \frac{dx}{x^2} = -\frac{1}{2q} \{e^{-q} Ei.(q) + e^q Ei.(-q)\}$ V. T. 130. N°. 12.
- 16) $\int \frac{(lx)^b}{1 + (lx)^a} \frac{dx}{x^2} = \frac{b}{a} Cosec. \frac{b\pi}{a}$ V. T. 43. N°. 17.
- 17) $\int \frac{dx \sqrt{lx}}{1 + x^2} = \frac{1}{2} \sqrt{\pi} \sum_0^{\infty} (-1)^n \frac{1}{\sqrt{(2n+1)}}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 37.
- 18) $\int \frac{dx}{x^2 \sqrt{lx}} = \sqrt{\pi}$ Euler, Calc. Int. 4. S. 5. 211.
- 19) $\int \frac{1}{1 + x^2} \frac{dx}{\sqrt{lx}} = \sqrt{\pi} \sum_0^{\infty} (-1)^n \frac{1}{\sqrt{(2n+1)}}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 37.

- 1) $\int_{-1}^0 l(1-x) \frac{dx}{x} = \frac{1}{12} \pi^2$ Schaeffer, Cr. 30. 277.
- 2) $\int_{-1}^0 l(x-1)^2 \frac{dx}{1+x^2} = -\frac{1}{4} \pi l 2$ Hill, Cr. 3. 101.
- 3) $\int_0^{1/\sqrt{2}} lx \frac{dx}{\sqrt{(1-x^2)}} = -\frac{1}{4} \pi l 2 - \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 271 N°. 4.
- 4) $\int_0^{1/\sqrt{2}} l(1-x^2)^2 \frac{dx}{\sqrt{(1-x^2)}} = \pi l 2 + 2 \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 271. N°. 6.
- 5) $\int_0^{\frac{1}{2}} l(1-x) \frac{dx}{x} = \frac{1}{2} (l 2)^2 - \frac{1}{12} \pi^2$
- 6) $\int_0^2 l(1-x) \frac{dx}{x} = -\frac{1}{4} \pi^2 + \pi i l 2$
- Schaeffer, Cr. 30. 277.

$$7) \int_0^{\frac{1}{e}} \frac{dx \sqrt{x}}{x \sqrt{-(1+lx)}} = \frac{\sqrt{q} \pi}{\sqrt{e}} \quad \text{V. T. 150. N}^\circ. 1.$$

$$8) \int_0^{\frac{-1+\sqrt{5}}{2}} l(1-x) \frac{dx}{x} = -\frac{1}{10} \pi^2 + \frac{1}{5} \left(l \frac{1+\sqrt{5}}{2} \right)^2 + \frac{2}{5} \left(l \frac{-1+\sqrt{5}}{2} \right)^2 l \frac{3-\sqrt{5}}{2}$$

$$9) \int_0^{\frac{3-\sqrt{5}}{2}} l(1-x) \frac{dx}{x} = -\frac{1}{15} \pi^2 - \frac{1}{5} \left(l \frac{1+\sqrt{5}}{2} \right)^2 + \frac{3}{5} l \frac{1+\sqrt{5}}{2} l \frac{3-\sqrt{5}}{2}$$

$$10) \int_0^{\frac{1-\sqrt{5}}{2}} l(1-x) \frac{dx}{x} = \frac{1}{15} \pi^2 - \frac{3}{10} \left(l \frac{1+\sqrt{5}}{2} \right)^2 + \frac{2}{5} l \frac{-1+\sqrt{5}}{2} l \frac{3-\sqrt{5}}{2} + l \frac{-1+\sqrt{5}}{2} l \frac{1+\sqrt{5}}{2}$$

$$11) \int_0^{\frac{1+\sqrt{5}}{2}} l(1-x) \frac{dx}{x} = -\frac{7}{30} \pi^2 + \frac{3}{10} \left(l \frac{1+\sqrt{5}}{2} \right)^2 - \frac{2}{5} l \frac{-1+\sqrt{5}}{2} l \frac{3-\sqrt{5}}{2} + \pi i l \frac{1+\sqrt{5}}{2}$$

$$12) \int_0^{\frac{-1-\sqrt{5}}{2}} l(1-x) \frac{dx}{x} = \frac{1}{10} \pi^2 + \frac{4}{5} \left(l \frac{1+\sqrt{5}}{2} \right)^2 - \frac{2}{5} l \frac{-1+\sqrt{5}}{2} l \frac{3-\sqrt{5}}{2} + l \frac{-1+\sqrt{5}}{2} l \frac{1+\sqrt{5}}{2}$$

$$13) \int_0^{\frac{3+\sqrt{5}}{2}} l(1-x) \frac{dx}{x} = -\frac{4}{15} \pi^2 + \frac{1}{2} \left(l \frac{3+\sqrt{5}}{2} \right)^2 + \frac{1}{5} \left(l \frac{1+\sqrt{5}}{2} \right)^2 - \frac{3}{5} l \frac{-1+\sqrt{5}}{2} l \frac{3-\sqrt{5}}{2} + \pi i l \frac{3+\sqrt{5}}{2}$$

$$14) \int_{\frac{1}{q}}^0 l(q+x)^2 \frac{dx}{1+x^2} = \text{Arctang.} \frac{1}{q} \cdot l(1+q^2) \quad \text{Hill, Cr. 3. 101.}$$

$$15) \int_0^q l(1+qx) \frac{dx}{1+x^2} = l(1+q^2) \cdot \text{Arctang.} q \quad \text{Bertrand, L. 8. 110. — Grunert, Gr. 4. 113.}$$

$$16) \int_0^{e^{-q}} \frac{x^{a-1}}{q+lx} dx = -\infty \quad \text{V. T. 150. N}^\circ. 12.$$

$$17) \int_0^1 l \left(2l \frac{1}{x} - 1 \right) \frac{x^{2a-1}}{lx} dx = -\frac{1}{2} \{ \text{Ei.}(-a) \}^2 \quad \text{V. T. 383. N}^\circ. 3.$$

Schaeffer,
Cr. 30.
277.

$$1) \int_{e^{-q}}^1 \frac{x^{a-1}}{q + lx} dx = \infty \quad \text{V. T. 149. N}^\circ. 15.$$

$$2) \int_1^e \frac{lx}{(1-lx)^2} \frac{dx}{x} = \frac{1}{2} e - 1 \quad \text{V. T. 112. N}^\circ. 5.$$

$$3) \int_1^{e^{-\frac{a}{c}}} \frac{x^{bc} - x^{-bc}}{lx} dx = \frac{1}{2} l \left(\frac{1+bc}{1-bc} \right)^2 + Ei. \left(-\frac{a}{c} + ab \right) - Ei. \left(-\frac{a}{c} - ab \right) \quad \text{V. T. 149. N}^\circ. 4.$$

$$4) \int_1^e \frac{dx}{x^2 lx} = \infty \quad \text{V. T. 112. N}^\circ. 4.$$

$$5) \int_{-1}^{+1} l(px-q) \frac{x}{1-rx^2} \frac{dx}{(1-x^2)} = \frac{\pi}{r(1-r)} l \frac{p\sqrt{r} - \{1 - \sqrt{1-r}\} \{q + \sqrt{q^2 - p^2}\}}{p + \{1 - \sqrt{1-r}\} \{q + \sqrt{q^2 - p^2}\}}$$

$$6) \int_{-1}^{+1} l \left(\frac{1-x^a}{1-x} \right) \frac{x}{1-x^2 \text{Sin.}^2 \lambda \sqrt{1-x^2}} dx = 2\pi \text{Cosec.} \lambda \sum_1^{\frac{1}{2}(a-1)} l \frac{1-2g \text{Tang.} \frac{1}{2} \lambda + h \text{Tang.}^2 \frac{1}{2} \lambda}{1+2g \text{Tang.} \frac{1}{2} \lambda + h \text{Tang.}^2 \frac{1}{2} \lambda}$$

$$, \text{ où } g = \text{Cos.} \left(\frac{2n+1}{a} \pi \right) + \text{Cos.} \left(\frac{1}{4} \pi + \frac{2n+1}{2a} \pi \right) \sqrt{2 \text{Sin.} \left(\frac{2n+1}{a} \pi \right)},$$

$$h = 1 + 2 \text{Sin.} \left(\frac{2n+1}{a} \pi \right) + 2 \text{Sin.} \left(\frac{1}{4} \pi + \frac{2n+1}{2a} \pi \right) \sqrt{2 \text{Sin.} \left(\frac{2n+1}{a} \pi \right)};$$

$$7) \int_{-1}^{+1} l \left(\frac{1-x^a}{1-x} \right) \frac{x}{\sqrt{1-x^2}} dx = \pi - 2\pi \sum_1^{\frac{1}{2}(a-1)} \left[\text{Cos.} \left(\frac{1}{4} \pi - \frac{2n+1}{a} \pi \right) \sqrt{2 \text{Sin.} \left(\frac{2n+1}{a} \pi \right)} \right]$$

$$8) \int_{-\infty}^{\infty} l \left(1 + \frac{pi}{x} \right) \frac{dx}{q + xi} = 2\pi l \frac{p+q}{q}$$

$$9) \int_{-\infty}^{\infty} l \left(1 + \frac{pi}{x} \right) \frac{dx}{q - xi} = 0$$

Cauchy, Lim. Imag. Add. N^o. 27.

$$10) \int_p^{\infty} l(1+x) \frac{dx}{x^2} = \frac{1}{p} l(1+p) - l \frac{1+p}{p} \quad \text{Schlömlich, Gr. 4. 71.}$$

$$11) \int_{-q}^q l(p-x) dx \sqrt{q^2 - x^2} = \frac{1}{2} q \pi \left[lp - l \frac{pq - \sqrt{p^2 - q^2}}{q^2} + \frac{1}{2} l \frac{pq - \sqrt{p^2 - q^2}}{q^2} \right]^2$$

Plana, Mém.
Turin. 1820.
389.

F. Alg.
Log.

TABLE 189 suite.

Lim. diverses p et q.

$$12) \int_{-q}^q l(1-pr) dx \sqrt{q^2 - x^2} = \frac{1}{2} q^2 \pi l \left\{ 2 \frac{1 - \sqrt{1 - p^2 q^2}}{p^2 q^2} \right\} + \frac{\pi}{4 p^2} \{1 - \sqrt{1 - p^2 q^2}\}^2 \quad \text{Plana, Mém. Turin. 1820. 389.}$$

$$13) \int_{-q}^q l(x-r) \frac{x}{q - p x^2} \frac{dx}{\sqrt{q^2 - x^2}} = \frac{\pi q}{\sqrt{p(1-p)}} l \frac{q \sqrt{p} - \{1 - \sqrt{1-p}\} \{r + \sqrt{r^2 - q^2}\}}{q \sqrt{p} + \{1 - \sqrt{1-p}\} \{r + \sqrt{r^2 - q^2}\}} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 1.}$$

$$\left. \begin{aligned} 14) \int_p^q \frac{(r+lx)^s}{x} dx &= \frac{1}{s+1} \{(r+lq)^{s+1} - (r+lp)^{s+1}\} \\ 15) \int_p^q (lx + 2a\pi i)^s \frac{dx}{x} &= \frac{1}{1+s} \{(lq + 2a\pi i)^{s+1} - (lp + 2a\pi i)^{s+1}\} \\ 16) \int_p^q \frac{1}{lx+r} \frac{dx}{x} &= l \{(r+lq)\} - l \{(r+lp)\} \\ 17) \int_p^q \frac{1}{lx+2a\pi i} \frac{dx}{x} &= l \{(lq + 2a\pi i)\} - l \{(lp + 2a\pi i)\} \end{aligned} \right\} \quad \text{Ohm, Ausw. 5, 6.}$$

$$18) \int_a^b (lx)^2 \frac{dx}{\sqrt{(b^2 - x^2)(x^2 - a^2)}} = \frac{1}{b} l a b. F' \left\{ \sqrt{\frac{a^2 - b^2}{a^2}} \right\} \quad \text{Roberts, L. 14. 238.}$$

F. Alg.
Log. de Log.

TABLE 190.

Lim. 0 et 1.

$$1) \int l l \frac{1}{x} x^{a-1} dx = -\frac{1}{a} (\Lambda + l a) \quad \text{Malmsten, Cr. 38. 1.}$$

$$2) \int l l \frac{1}{x} \cdot \left(l \frac{1}{x} \right)^{p-1} x^{a-1} dx = -\frac{\Gamma(p)}{a^p} \{Z'(p) - l a\} \quad \text{V. T. 377. N°. 2.}$$

$$3) \int l l \frac{1}{x} x^{a-1} \frac{dx}{\sqrt{l \frac{1}{x}}} = -\sqrt{\frac{\pi}{a}} \{\Lambda + 2 l 2 + l a\} \quad \text{V. T. 273. N°. 4.}$$

$$4) \int l l \frac{1}{x} \frac{1}{1+x^2} \frac{dx}{\sqrt{l \frac{1}{x}}} = \sqrt{\pi} \sum_0^{\infty} (-1)^{n+1} \frac{l(2n+1) + 2 l 2 + \Lambda}{\sqrt{(l 2 n + 1)}} \quad \text{V. T. 381. N°. 4.}$$

$$5) \int l l \frac{1}{x} \frac{x^{-\frac{b}{c}} + x^{\frac{b}{c}}}{1+x^2} dx = \frac{1}{2} \pi \text{Sec.} \frac{b\pi}{2c} (l\pi - A) - \pi \sum_{n=0}^{\infty} \left\{ \frac{l \left\{ (2n+1)\pi - \frac{b\pi}{c} \right\}}{(2n+1)\pi - \frac{b\pi}{c}} - \frac{l \left\{ (2n+1)\pi + \frac{b\pi}{c} \right\}}{(2n+1)\pi + \frac{b\pi}{c}} \right\} \quad \text{V. T. 274. N}^{\circ} 1.$$

$$6) \int l l \frac{1}{x} \frac{x^{-\frac{b}{c}} - x^{\frac{b}{c}}}{1-x^2} dx = \frac{1}{2} \pi \text{Tang.} \frac{b\pi}{2c} (l\pi + A) - \pi \sum_{n=0}^{\infty} \left\{ \frac{l \left\{ (2n+1)\pi - \frac{b\pi}{c} \right\}}{(2n+1)\pi - \frac{b\pi}{c}} - \frac{l \left\{ (2n+1)\pi + \frac{b\pi}{c} \right\}}{(2n+1)\pi + \frac{b\pi}{c}} \right\} \quad \text{V. T. 274. N}^{\circ} 4.$$

$$7) \int l \frac{1}{x} \frac{dx}{(1+x)^2} = \frac{1}{2} \left\{ Z' \left(\frac{1}{2} \right) + l 2\pi \right\} \quad \text{Malmsten, Cr. 38. 1.}$$

$$8) \int l \frac{1}{x} \frac{1}{1+x+x^2} \frac{dx}{\sqrt{l \frac{1}{x}}} = \frac{\sqrt{\pi}}{\text{Sin.} \frac{1}{3}\pi} \sum_{n=1}^{\infty} (-1)^n \text{Sin.} \frac{1}{3} n \pi \frac{l n + 2 l 2 + A}{\sqrt{(2n+1)}} \quad \text{V. T. 331. N}^{\circ} 15.$$

$$9) \int l \frac{1}{x} \frac{dx}{1+2x \text{Cos.} \lambda + x^2} = \frac{1}{2} \pi \text{Cosec.} \lambda l \frac{(2\pi)^{\frac{\lambda}{\pi}} \Gamma \left(\frac{1}{2} + \frac{\lambda}{2\pi} \right)}{\Gamma \left(\frac{1}{2} - \frac{\lambda}{2\pi} \right)} \quad \text{Malmsten, Cr. 38. 1.}$$

$$10) \int l \{a^2 + (lx)^2\} \frac{dx}{1+x^2} = \pi l \frac{2 \Gamma \left(\frac{2a+3\pi}{4\pi} \right)}{\Gamma \left(\frac{2a+\pi}{4\pi} \right)} - \frac{1}{2} \pi l \frac{\pi}{2} \quad \text{V. T. 275. N}^{\circ} 17.$$

$$11) \int l \{a^2 + (lx)^2\} \frac{x^{-\frac{b}{c}} + x^{\frac{b}{c}}}{1+x^2} dx = \pi \text{Sec.} \frac{b\pi}{2c} l 2c\pi + 2\pi \sum_{n=1}^{\infty} (-1)^{n-1} \text{Cos.} \left\{ \left(n - \frac{1}{2} \right) \frac{b\pi}{c} \right\} l \frac{\Gamma \left(\frac{2a+2\pi n - \pi}{4c\pi} + \frac{1}{2} \right)}{\Gamma \left(\frac{2a+2\pi n - \pi}{4c\pi} \right)}, \text{ pour } b+c \text{ impair;}$$

$$12) = \pi \text{Sec.} \frac{b\pi}{2c} l c\pi + 2\pi \sum_{n=1}^{c-1} (-1)^{n-1} \text{Cos.} \left\{ \left(n - \frac{1}{2} \right) \frac{b\pi}{c} \right\} l \frac{\Gamma \left(\frac{2a-2\pi n + \pi}{2c\pi} + 1 \right)}{\Gamma \left(\frac{2a+2\pi n - \pi}{2c\pi} \right)}, \text{ pour } b+c \text{ pair;}$$

$$13) \int l \left\{ \frac{1}{4} \pi^2 c^2 + (lx)^2 \right\} \frac{x^{-\frac{b}{c}} + x^{\frac{b}{c}}}{1+x^2} dx = \pi \text{Sec.} \frac{b\pi}{2c} l \pi + \pi \sum_{n=1}^c (-1)^{n-1} \text{Cos.} \left\{ \left(n - \frac{1}{2} \right) \frac{b\pi}{c} \right\} l \left\{ \left(\frac{c+1}{2} - n \right) \text{Cot} \left(\frac{\pi}{4} - \frac{2n-1}{4c} \pi \right) \right\} \quad \text{V. T. 275. N}^{\circ} 16.$$

$$14) \int l \left\{ \frac{\pi^2}{4} + (lx)^2 \right\} \frac{dx}{1+x^2} = \frac{1}{2} \pi l \frac{\pi^2}{8} \quad \text{V. T. 275. N}^\circ. 1.$$

$$15) \int l \{a^2 + (lx)^2\} \frac{x^{-\frac{b}{c}} - x^{\frac{b}{c}}}{1-x^2} dx = \pi \text{Tang.} \frac{b\pi}{2c} l 2c\pi + 2\pi \sum_1^{c-1} (-1)^{n-1} \text{Sin.} \frac{nb\pi}{c} l \frac{\Gamma\left(\frac{a+\pi n}{2c\pi} + \frac{1}{2}\right)}{\Gamma\left(\frac{a+\pi n}{2c\pi}\right)}, \text{ pour } b+c \text{ impair;}$$

V. T. 275.
N^o. 14, 15.

$$16) \quad = \pi \text{Tang.} \frac{b\pi}{2c} l c\pi + 2\pi \sum_1^{\frac{c-1}{2}} (-1)^{n-1} \text{Sin.} \frac{nb\pi}{c} l \frac{\Gamma\left(\frac{a-\pi n}{c\pi} + 1\right)}{\Gamma\left(\frac{a+\pi n}{c\pi}\right)}, \text{ pour } b+c \text{ pair;}$$

$$17) \int l \left\{ \frac{1}{4} \pi^2 c^2 + (lx)^2 \right\} \frac{x^{-\frac{b}{c}} - x^{\frac{b}{c}}}{1-x^2} dx = -\pi \text{Tang.} \frac{b\pi}{2c} l \pi + \quad , \text{ pour } b+c \text{ impair;}$$

$$+ \pi \sum_0^{c-1} (-1)^{n-1} \text{Sin.} \frac{bn\pi}{c} l \left\{ \left(\frac{1}{2} c - n \right) \text{Cot.} \left(\frac{\pi}{4} - \frac{n\pi}{2c} \right) \right\} \quad \text{V. T. 275. N}^\circ. 17.$$

$$18) \int l \{a^2 + (lx)^2\} \frac{dx}{(1+x) \vee x} = 2\pi l \frac{\Gamma\left(\frac{a+3\pi}{4\pi}\right)}{\Gamma\left(\frac{a+\pi}{4\pi}\right)} + \pi l \pi \quad \text{V. T. 275. N}^\circ. 18.$$

$$19) \int l \{a^2 + (lx)^2\} \frac{1+x^{\frac{2}{3}}}{1+x^{\frac{2}{3}}+x^{\frac{4}{3}}} \frac{dx}{\vee x^2} = -\pi l \pi - 2\pi \text{Sin.} \frac{\pi}{3} l \frac{6\Gamma\left(\frac{a+4\pi}{6\pi}\right)\Gamma\left(\frac{a+5\pi}{6\pi}\right)}{\Gamma\left(\frac{a+\pi}{6\pi}\right)\Gamma\left(\frac{a+2\pi}{6\pi}\right)} \quad \text{V. T. 275. N}^\circ. 19.$$

$$20) \int \left\{ (p-1)x + \frac{(1-lx)^{-1} - (1-lx)^{-p}}{l(1-lx)} \right\} \frac{dx}{x l x} = -l \Gamma(p) \quad \text{V. T. 378. N}^\circ. 10.$$

$$21) \int \left\{ \frac{x}{l x} + \frac{1}{(1-lx)^2 l(1-lx)} \right\} \frac{dx}{x} = 0 \quad \text{V. T. 378. N}^\circ. 9.$$

$$22) \int \left\{ x - \frac{(1-lx)^{-(p+1)}}{l(1-lx)} \right\} \frac{dx}{x l x} = -lp \quad \text{V. T. 378. N}^\circ. 11.$$

$$1) \int_0^\infty l x \frac{dx}{1+x^2} = \frac{1}{2} \pi l \left(\frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \vee 2\pi \right) \quad \text{Malmsten, Cr. 38. 1.}$$

$$2) \int_0^\infty l l x \frac{dx}{1+x+x^2} = \frac{\pi}{\sqrt{3}} l \left(\frac{\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{3})} \sqrt{2\pi} \right)$$

$$3) \int_0^\infty l l x \frac{x^{a-1} - x^{-a-1}}{x^b - x^{-b}} dx = \frac{\pi}{2b} \text{Tang.} \frac{a\pi}{2b} l 2\pi + \frac{\pi^{b-1}}{b} \sum_1 (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} l \frac{\Gamma(\frac{b+n}{2b})}{\Gamma(\frac{n}{2b})}, \text{ pour } a+b \text{ impair;}$$

$$4) = \frac{\pi}{2b} \text{Tang.} \frac{a\pi}{2b} l \pi + \frac{\pi^{\frac{b-1}{2}}}{b} \sum_1 (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} l \frac{\Gamma(\frac{b-n}{b})}{\Gamma(\frac{n}{b})}, \text{ pour } a+b \text{ pair;}$$

$$5) \int_0^\infty l l x \frac{x^{a-2}}{1+x^2+x^4+\dots+x^{2a-2}} dx = \frac{\pi}{2a} \text{Tang.} \frac{\pi}{2a} l 2\pi + \frac{\pi^{a-1}}{a} \sum_1 (-1)^{n-1} \text{Sin.} \frac{n\pi}{a} l \frac{\Gamma(\frac{a+n}{2a})}{\Gamma(\frac{n}{2a})}, \text{ pour } a \text{ pair;}$$

$$6) = \frac{\pi}{2a} \text{Tang.} \frac{\pi}{2a} l \pi + \frac{\pi^{\frac{a-1}{2}}}{a} \sum_1 (-1)^{n-1} \text{Sin.} \frac{n\pi}{a} l \frac{\Gamma(\frac{a-n}{a})}{\Gamma(\frac{n}{a})}, \text{ pour } a \text{ impair;}$$

$$7) \int_1^\infty l l x \frac{dx}{1-x+x^2} = \frac{2\pi}{\sqrt{3}} \left\{ \frac{5}{6} l 2\pi - l \Gamma\left(\frac{1}{6}\right) \right\}$$

$$8) \int_1^\infty l l x \frac{x^{a-1} + x^{-a-1}}{x^b + x^{-b}} dx = \frac{\pi}{2b} \text{Sec.} \frac{a\pi}{2b} l 2\pi + \frac{\pi^{\frac{b}{2}}}{b} \sum_1 (-1)^{n-1} \text{Cos.} \frac{(n-\frac{1}{2})a\pi}{b} l \frac{\Gamma(\frac{b+n-\frac{1}{2}}{2b})}{\Gamma(\frac{n-\frac{1}{2}}{2b})}, \text{ pour } a+b \text{ impair;}$$

$$9) = \frac{\pi}{2b} \text{Sec.} \frac{a\pi}{2b} l \pi + \frac{\pi^{\frac{b-1}{2}}}{b} \sum_1 (-1)^{n-1} \text{Cos.} \frac{(n-\frac{1}{2})a\pi}{b} l \frac{\Gamma(\frac{b-n+\frac{1}{2}}{b})}{\Gamma(\frac{n-\frac{1}{2}}{b})}, \text{ pour } a+b \text{ pair;}$$

Les integrales 2 à 9 sont déduites par Malmsten, Cr. 38. 1, où il y a plusieurs fautes.

- 1) $\int x \sin. a x dx = \frac{1}{a^2} (\sin. a - a \cos. a)$ Kummer, Cr. 35. 1.
- 2) $\int x \cos. a x dx = \frac{1}{a^2} (a \sin. a + \cos. a - 1)$ Dicneger, Cr. 46. 119.
- 3) $\int x^2 \cos. a x dx = \frac{1}{a^3} \{ (a^2 - 2) \sin. a + 2 a \cos. a \}$ V. T. 192. N°. 1.
- 4) $\int x^{l-1} \sin. 2 p \pi x dx = \frac{1-q}{(l q)^2 + 4 p^2 \pi^2} \frac{2 p \pi}{q}$ Kummer, Cr. 35. 1.
- 5) $\int \frac{\sin. p x}{x} dx = Si. (p) = \sum_1^{\infty} \frac{1}{2 n - 1} \frac{p^{2n-1}}{1^{2n-1/1}}$ Arndt, Gr. 10. 225. — Schlömilch, Cr. 33. 316.
- 6) $\int (1-x^2)^{\frac{2a-1}{2}} \cos. p x dx = \frac{1^{a/2}}{2^{a+2} 1^{a/1}} \left\{ 1 + \sum_1^{\infty} (-1)^n \frac{(\frac{1}{2} p)^{2n}}{1^{n/1} (a+1)^{n/1}} \right\}$ Bessel, Abb. Berlin. 1824. I.
- 7) $\int (1-x^2)^{a-\frac{3}{2}} \cos. 2 p x dx = \frac{\Gamma(a-\frac{1}{2})}{2 \cdot 1^{a/1}} \sqrt{\pi} \sum_0^{\infty} (-1)^n \frac{p^{2n}}{1^{n/1} a^{n/1}}$
- 8) $\int (1-x^2)^{a-b-1} x^{2b-1} \cos. p x dx = \frac{1^{b/1} 1^{a-b/1}}{2 \cdot 1^{a/1}} \sum_0^{\infty} (-1)^n \frac{b^{n/1}}{2^{2n/1} a^{n/1}} p^{2n}$
- 9) $\int x^{b-1} (1-x)^{a-b-1} \cos. (\sqrt{p} x) dx = \frac{1^{b/1} 1^{a-b/1}}{1^{a/1}} \sum_0^{\infty} \frac{b^{n/1}}{2^{2n/1} a^{n/1}} p^n$
- 10) $\int \sin. \left\{ \frac{1}{2} p \left(x + \frac{1}{x} \right) \right\} \cdot \sin. \left\{ \frac{1}{2} p \left(x - \frac{1}{x} \right) \right\} \frac{dx}{1-x^2} = -\frac{1}{2} \pi \sin. p$ Cauchy, Lim. Imag. Add. 17.
- 11) $\int \frac{\cos. q x - \cos. \left(\frac{q}{x} \right)}{x - \frac{1}{x}} \frac{dx}{x} = -\frac{1}{2} \pi \sin. q$
- 12) $\int \left\{ \frac{\cos. q x}{x - \frac{1}{x}} - b \frac{\cos. \left(\frac{q}{x^b} \right)}{x^b - \frac{1}{x^b}} \right\} \frac{dx}{x} = \frac{1}{2} \pi (\cos. q \cdot l b - \sin. q)$
- 13) $\int \left\{ \frac{\sin. \lambda}{1 + 2 x \cos. \lambda + x^2} - \frac{\lambda}{(1+x)^2} \right\} dx = 0$ Malmsten, Cr. 38. 1.

- $$\left. \begin{aligned} 1) \int x \sin. x \, dx &= 0 \\ 2) \int x \cos. x \, dx &= -1 \end{aligned} \right\} \text{Boncompagni, Cr. 25. 74.}$$
- $$3) \int x^2 \sin. q \, x \, dx = -\frac{2}{q^3} \quad \text{Cisa de Grézy, Mém. Turin. 1821. 209. II. 50. — Oettinger, Cr. 38. 216.}$$
- $$\left. \begin{aligned} 4) \int x^3 \sin. q \, x \, dx &= \frac{24}{q^4} \\ 5) \int x \cos. q \, x \, dx &= -\frac{1}{q^2} \\ 6) \int x^3 \cos. q \, x \, dx &= \frac{6}{q^4} \\ 7) \int x^5 \cos. q \, x \, dx &= -\frac{120}{q^6} \\ 8) \int x^{2a} \sin. q \, x \, dx &= (-1)^a \frac{1^{2a+1}}{q^{2a+1}} \\ 9) \int x^{2a} \cos. q \, x \, dx &= 0 \\ 10) \int x^{2a-1} \sin. q \, x \, dx &= 0 \\ 11) \int x^{2a-1} \cos. q \, x \, dx &= (-1)^a \frac{1^{2a-1}}{q^{2a}} \end{aligned} \right\} \text{Oettinger, Cr. 38. 216.}$$
- $$\left. \begin{aligned} 12) \int x^{p-1} \sin. x \, dx &= \Gamma(p) \sin. \frac{1}{2} p \pi \\ 13) \int x^{p-1} \cos. x \, dx &= \Gamma(p) \cos. \frac{1}{2} p \pi \end{aligned} \right\} , p^2 < 1; \quad \text{Cauchy, Sav. Etr. 1827. 124. Note 3. — Plana, Mém. Bruxelles. 1837. — Boncompagni, Cr. 25. 74.}$$
- $$\left. \begin{aligned} 14) \int x^{p-1} \sin. q \, x \, dx &= \frac{\Gamma(p)}{q^p} \sin. \frac{1}{2} p \pi \\ 15) \int x^{p-1} \cos. q \, x \, dx &= \frac{\Gamma(p)}{q^p} \cos. \frac{1}{2} p \pi \end{aligned} \right\} \quad \text{Legendre, Exerc. 3 55. — Plana, Mém. Bruxelles. 1837. — Oettinger, Cr. 38. 216. — Schlömilch, Stud. I. 13. (pour } 1 > q > 0 \text{ et } q^2 < 1 \text{ resp.). — Raabe, Int. 416. (pour tout } p \text{ et } q).$$

$$16) \int x^{p-2} \sin. q x dx = \frac{1}{(1-p)q^{p-1}} \Gamma(p) \cos. \frac{1}{2} p \pi \quad \text{Plana, Mém. Brux. 1837.}$$

$$17) \int x \sin. (x^2) \sin. p x dx = \frac{1}{8} p \sqrt{2\pi} \left\{ \cos. \left(\frac{1}{4} p^2 \right) + \sin. \left(\frac{1}{4} p^2 \right) \right\} \left\{ \begin{array}{l} \text{Cauchy, Lim. Imag. § 192. —} \\ \text{Id., Sav. Etr. 1827. 124.} \end{array} \right.$$

$$18) \int x \cos. (x^2) \sin. p x dx = -\frac{1}{8} p \sqrt{2\pi} \left\{ \cos. \left(\frac{1}{4} p^2 \right) - \sin. \left(\frac{1}{4} p^2 \right) \right\} \left\{ \begin{array}{l} \text{Note 2.} \end{array} \right.$$

$$19) \int x^{p-1} \sin. (q x^r) dx = \frac{\Gamma\left(\frac{p}{r}\right)}{r \sqrt[r]{q^p}} \sin. \frac{p \pi}{2r} \left\{ \begin{array}{l} \text{Raa'c, Int. 416.} \end{array} \right.$$

$$20) \int x^{p-1} \cos. (q x^r) dx = \frac{\Gamma\left(\frac{p}{r}\right)}{r \sqrt[r]{q^p}} \cos. \frac{p \pi}{2r}$$

$$21) \int \frac{x \sin. x}{\cos. \lambda - \cos. x} dx = 2 \pi l (1 - \cos. \lambda + i \sin. \lambda) \left\{ \begin{array}{l} \text{Poisson, P. 18. 295. N°. 37.} \end{array} \right.$$

$$22) \int \frac{x \sin. x}{e^q + e^{-q} - 2 \cos. x} dx = \pi l (1 - e^{-q})$$

$$23) \int \frac{x \sin. x}{p - \cos. x} dx = \pi l \{2(1+p)\} - i \operatorname{Arctang.} \frac{1 - (1-p^2)}{1+p} \quad \text{Plana, Mém. Turin. 1820.}$$

$$1) \int \frac{\sin. x}{x} dx = \frac{1}{2} \pi \quad \left\{ \begin{array}{l} \text{Mascheroni, Adn. 52. — Euler, Calc. Int. T. 4. S. 5. 139. — Bidone, Mém.} \\ \text{Turin. 1812. 231. Art. 1. N°. 2. — Fourier, Chal. 415. — Laplace, P. 15.} \\ \text{229. — Lobatschewsky, Mém. Kasan. 1835. 211. — Id., Cr. 24. 164. —} \\ \text{Schlömilch, Gr. 1. 417. — Id., Cr. 36. 268. — Id., Beitr. III. § 4.} \end{array} \right.$$

$$2) \int \frac{\cos. x}{x} dx = A \text{ (fautive)} \quad \text{Mascheroni, Adn. 45. — Boncompagni, Cr. 24. 75.}$$

$$3) \quad = \infty \quad \text{Laplace, P. 15. 229.}$$

$$4) \quad = -A - l 0 \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 5.}$$

$$5) \int \frac{\sin. p x}{x} dx = \frac{1}{2} \pi, p > 0; \left\{ \begin{array}{l} \text{Poisson, Chaleur. 102, 158. — Id., Mém. Acad. 1823. 571. N°.} \\ \text{12. — Cauchy, Cours. Leç. 33. — Id., Exerc. 1826. P. 95. —} \\ \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19. — Cisa de} \\ \text{Grézy, Mém. Turin. 1821. 209. II. 53. — Pioch, Mém. Courr.} \\ \text{Brux. T. 15. P. 2. — Libri, Cr. 7. 224. — Besge, L. 14. 31.} \end{array} \right.$$

$$6) \quad = 0, p = 0;$$

$$7) \quad = -\frac{1}{2} \pi, p < 0;$$

$$8) \int \frac{\sin. p x}{x} dx = \frac{1}{2} \pi, p \text{ très-petit}; \text{ Cauchy, Sav. Etr. 1827. 599. S. 2.}$$

Sur la formule (5) seule voyez encore: Legendre, Exerc. 3. 46. — Cauchy, Lim. Imag. Add. 16. — Id., Cours. Leç. 33. — Laplace, Probab. L. 1. 25. — Bidone, Mém. Turin. 1812. 231. Art. 2. 18. — Poisson, P. 16. 215. N°. 2. — Lobatto, Cr. 11. 171. — Raabe, Cr. 23. 105. — Oettinger, Cr. 38. 216. — Bonnet, L. 14. 249. — Schlömilch, Gr. 1. 417. — Id., Cr. 36. 268. — Id., Stud. 1. 13. — Lindmann, Gr. 16. 94.

$$9) \int \frac{\cos. p x}{x} dx = \infty \quad \text{Legendre, Exerc. 3. 46. — Cauchy, Cours. Leç. 33. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 53.}$$

$$10) \quad = -\infty \quad \text{Lobatto, Cr. 11. 171. (fautive).}$$

$$11) \quad = -A - la - l0 \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 6.}$$

$$12) \int \frac{\text{Tang. } x}{x} dx = \frac{1}{2} \pi \quad \text{Schlömilch, Gr. 4. 316.}$$

$$13) \int \frac{\text{Tang. } p x}{x} dx = \frac{1}{2} \pi \quad \text{Legendre, Exerc. 5. 35. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 38. — Plana, Mém. Turin. 1818. 7. 1. 13.}$$

$$14) \int \frac{\sin. 2a+1 x}{x} dx = (-1)^a \left(-\frac{1}{a} \right) \frac{\pi}{2} \quad \text{Raabe, Cr. 23. 105. — Id., Cr. 25. 160.}$$

$$15) \quad = \frac{1}{2} \pi \frac{1^{a/2}}{2^{a/2}} \quad \text{Raabe, Cr. 23. 105. — Schlömilch, Gr. 4. 316 (pour } a \text{ fraction à dénominateur et numérateur impairs).}$$

$$16) \int \frac{\sin. 3 q x}{x} dx = \frac{1}{4} \pi$$

$$17) \int \frac{\sin. 5 q x}{x} dx = \frac{3}{16} \pi$$

$$18) \int \frac{\sin. 2a q x}{x} dx = \infty$$

$$19) \int \frac{\sin. 2a+1 q x}{x} dx = \frac{\pi}{2^{2a+1}} \sum_0^a (-1)^n \left(\frac{2a+1}{a+n+1} \right)$$

$$20) \int \frac{\sin. \{(p-q)x\}}{x} dx = \frac{1}{2} \pi, p > q;$$

$$21) \quad = 0, p = q;$$

$$22) \quad = -\frac{1}{2} \pi, p < q;$$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 10, 15.

Poich, Mém. Courr. Brux. T. 15. P. 2.

1)	$\int \frac{\sin x \cos qx}{x} dx = \frac{1}{2} \pi, q^2 \leq 1;$	} Fourier, Chal. 357. — Schlömilch, Stud. I. 21.
2)	$= 0, q^2 > 1;$	
3)	$\int \frac{\sin qx \cos x}{x} dx = \frac{1}{2} \pi, q > 1;$	} Serret, L. 8. 489.
4)	$= 0, q < 1;$	
5)	$\int \frac{\sin qx \cos px}{x} dx = \frac{1}{2} \pi, q > p;$	} Legendre, Exerc. 3. 46. — Schlömilch, Cr. 36. 268. — Id., Stud. I. 21. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.
6)	$= 0, q < p;$	
7)	$= \frac{1}{4} \pi, q = p;$	
Libri, Cr. 7. 224 et Arndt, Gr. 11. 70 trouvent les mêmes formules (5) à (7) pour les limites respectives $q > p > -q, -q > p > -\infty$ et $+q < p < \infty, q = \pm p$.		
8)	$\int \frac{\sin qx \sin^2 px}{x} dx = 0, -\infty < q < -2p;$	} Dienger, Gr. 12. 416.
9)	$= -\frac{1}{8} \pi, q = -2p;$	
10)	$= -\frac{1}{4} \pi, -2p < q < 0;$	
11)	$= 0, q = 0;$	
12)	$= \frac{1}{4} \pi, 0 < q < 2p;$	
13)	$= \frac{1}{8} \pi, q = 2p;$	
14)	$= 0, 2p < q < \infty;$	
15)	$\int \frac{\sin qx \cos^2 px}{x} dx = \frac{1}{2} \pi, p > 2q;$	} Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.
16)	$= \frac{3}{8} \pi, p = 2q;$	
17)	$= \frac{1}{4} \pi, p < 2q;$	

$$\begin{aligned}
 18) \int \frac{\text{Sin.}^2 q x. \text{Cos.}^3 p x}{x} dx &= \frac{1}{16} l \frac{(2q+p)^3 (p-2q)^3 (2q+3p) (3p-2q)}{9 p^8}, \quad p > 2q; \\
 19) &= \frac{1}{16} l \frac{(2q+p)^3 (2q-p)^3 (2q+3p) (3p-2q)}{9 p^8}, \quad 3p > 2q > p; \\
 20) &= \frac{1}{16} l \frac{(2q+p)^3 (2q-p)^3 (2q+3p) (2q-3p)}{9 p^8}, \quad 3p < 2q; \\
 21) \int \frac{\text{Sin.}^2 q x. \text{Cos.}^3 q x}{x} dx &= \frac{1}{16} l 15 \\
 22) \int \frac{\text{Sin.}^2 x. \text{Cos.}^3 \frac{2}{3} x}{x} dx &= \infty \\
 23) \int \frac{\text{Sin.}^{2a+1} x. \text{Cos.}^{2b} x}{x} dx &= \frac{\Gamma(a + \frac{1}{2}) \Gamma(b + \frac{1}{2})}{2 \Gamma(a + b + 1)} \\
 24) \int \frac{\text{Sin.}^{2a+1} x. \text{Cos.}^{2b-1} x}{x} dx &= \frac{\Gamma(a + \frac{1}{2}) \Gamma(b + \frac{1}{2})}{2 \Gamma(a + b + 1)} \\
 25) \int \frac{\text{Sin.}^{2a+1} x. \text{Cos.}^{2b} x}{x} dx &= \frac{\pi}{2} \frac{1^{a/2} 1^{b/2}}{2^{a+b/2}} \\
 26) \int \frac{\text{Sin.}^{2a+1} x. \text{Cos.}^{2b-1} x}{x} dx &= \frac{\pi}{2} \frac{1^{a/2} 1^{b/2}}{2^{a+b/2}} \\
 27) \int \frac{\text{Sin.} p x. \text{Sin.} q x. \text{Sin.} r x}{x} dx &= 0, \quad -\infty < p < -(q+r); \\
 28) &= -\frac{1}{8} \pi, \quad p = -(q+r); \\
 29) &= -\frac{1}{4} \pi, \quad -(q+r) < p < q-r; \\
 30) &= -\frac{1}{8} \pi, \quad p = q-r; \\
 31) &= 0, \quad q-r < p < r-q; \\
 32) &= \frac{1}{8} \pi, \quad p = r-q; \\
 33) &= \frac{1}{4} \pi, \quad r-q < p < r+q; \\
 34) &= \frac{1}{8} \pi, \quad p = r+q; \\
 35) &= 0, \quad r+q < p \leq \infty;
 \end{aligned}$$

Bidone,
 Mém.
 Turin.
 1812.
 231.
 Art. 1.
 N^o. 19.

Schlömilch, Gr. 4. 316, où a et b peuvent
 aussi être des fractions à numérateur et
 à dénominateur impairs.

, $p < q < r$;
 Dienger, Cr. 12. 210.

- 1) $\int \frac{\text{Cos. } x - \text{Cos. } qx}{x} dx = \frac{1}{2} l q^2$ Raabe, Cr. 23. 103. — Arndt, Gr. 11. 70.
 - 2) $\int \frac{\text{Cos. } qx - \text{Cos. } px}{x} dx = \frac{1}{2} l \frac{p^2}{q^2}$ Poisson, P. 16. 215 N°. 2. — Bidone, Mém. Turin. 1812. 231. Art. 1. 6. — Cisa de Grésy, Mém. Turin. 1821. 209. II. § 54. — Raabe, Cr. 23. 105 (pour q et p aussi des fractions).
 - 3) $\int \frac{\text{Sin.}^2 qx - \text{Sin.}^2 px}{x} dx = \frac{1}{2} l \frac{q}{p}$
 - 4) $\int \frac{\text{Sin.}^{2a} qx - \text{Sin.}^{2a} px}{x} dx = \frac{1}{2^{2a+1}} l \frac{q}{p} \sum_1^a (-1)^{n-1} \binom{2a}{a+n}$
 - 5) $\int \frac{\text{Cos.}^{2a} qx - \text{Cos.}^{2a} px}{x} dx = \frac{1}{2} l \left(\frac{p}{q} \right)^2 \left\{ 1 - \frac{(a+1)^{a,1}}{4^{a/4}} \right\}$
 - 6) $\int \frac{\text{Cos.}^{2a+1} qx - \text{Cos.}^{2a+1} px}{x} dx = \frac{1}{2} l \left(\frac{p}{q} \right)^2$
 - 7) $\int \frac{\text{Cos. } \lambda - \text{Cos. } b\lambda}{x} \text{Sin. } ax dx = \frac{1}{2} \pi (\text{Cos. } \lambda - 1), \frac{a}{b} > \lambda > 0;$
 - 8) $\quad \quad \quad = \frac{1}{2} \pi \text{Cos. } \lambda, \quad \frac{a}{b} < \lambda < \infty;$
 - 9) $\int \frac{3 - 4 \text{Sin.}^2 qx}{x} \text{Sin.}^2 qx dx = \frac{1}{2} l 2$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 10.
- Bidone, Mém. Turin. 1812. 231. Tableau.
- Schlömilch, Gr. 5. 152.
- Arndt, Gr. 11. 70.

- 1) $\int \frac{\text{Sin. } qx}{x^2} dx = q(1 - \Lambda - lq - lo) = \infty$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.
 - 2) $\int \frac{\text{Cos. } qx}{x^2} dx = \infty$
 - 3) $\quad \quad \quad = \infty - \frac{1}{2} q \pi$
 - 4) $\int \frac{\text{Sin. } qx}{x^3} dx = -\infty - \frac{1}{4} q \pi^2$
 - 5) $\int \frac{\text{Cos. } qx}{x^3} dx = \infty^2 - \frac{3}{4} q^2 + \frac{1}{2} q^2 \{ \Lambda + lq + lo \}$
- Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7, 8.

$$6) \int \frac{\text{Sin.}^2 x}{x^2} dx = \frac{1}{2} \pi \quad \text{Lindmann, Gr. 16. 94.}$$

$$\left. \begin{aligned} 7) \int \frac{\text{Sin.}^2 q x}{x^2} dx &= \frac{1}{2} q \pi \\ 8) \int \frac{\text{Sin.}^3 q x}{x^2} dx &= \frac{3}{4} q l 3 \\ 9) \int \frac{\text{Sin.}^4 q x}{x^2} dx &= \frac{1}{4} q \pi \\ 10) \int \frac{\text{Sin.}^5 q x}{x^2} dx &= 5 q \frac{3 l 3 - l 5}{16} \\ 11) \int \frac{\text{Sin.}^6 q x}{x^2} dx &= \frac{3}{16} q \pi \\ 12) \int \frac{\text{Sin.}^{10} q x}{x^2} dx &= \frac{35}{256} q \pi \end{aligned} \right\}$$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 11, 14.

$$\left. \begin{aligned} 13) \int \frac{\text{Sin.}^3 q x}{x^3} dx &= \frac{3}{8} q^2 \pi \\ 14) \int \frac{\text{Sin.}^4 q x}{x^3} dx &= q^2 l 2 \\ 15) \int \frac{\text{Sin.}^5 q x}{x^3} dx &= \frac{5}{32} q^2 \pi \\ 16) \int \frac{\text{Sin.}^6 q x}{x^3} dx &= 3 q^2 \frac{8 l 2 - 3 l 3}{16} \end{aligned} \right\}$$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 12, 15.

$$\left. \begin{aligned} 17) \int \frac{\text{Sin.}^4 q x}{x^4} dx &= \frac{1}{2} q^3 \pi \\ 18) \int \frac{\text{Sin.}^5 q x}{x^4} dx &= 5 q^3 \frac{25 l 5 - 27 l 3}{96} \end{aligned} \right\}$$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 11, 14.

$$\left. \begin{aligned} 19) \int \frac{\text{Sin.}^3 q x}{x^5} dx &= \pi \\ 20) \int \frac{\text{Sin.}^5 q x}{x^5} dx &= \frac{125}{384} q^4 \pi \\ 21) \int \frac{\text{Sin.}^6 q x}{x^5} dx &= \frac{27 l 3 - 32 l 2}{16} q^4 \end{aligned} \right\}$$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 18, 15, 12.

1)	$\int \frac{\text{Sin. } p x . \text{Sin. } q x}{x^2} dx = \frac{1}{2} p \pi, p \leq q;$	} Ohm, Ausw. 18.
2)	$= \frac{1}{2} q \pi, p \geq q;$	
3)	$\int \frac{\text{Sin.}^2 q x . \text{Cos.}^2 p x}{x^2} dx = \frac{2 q - p}{4} \pi, q > p;$	} Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.
4)	$= \frac{1}{4} q \pi, q = p;$	
5)	$= \frac{1}{4} q \pi, q < p;$	
6)	$\int \frac{\text{Sin.}^2 p x . \text{Sin. } q x}{x^3} dx = \frac{1}{2} p^2 \pi, q \geq 2 p;$	} Ohm, Ausw. 18.
7)	$= \frac{1}{8} \pi (4 p q - q^2), q \leq 2 p;$	
8)	$\int \frac{\text{Sin. } p x . \text{Sin } q x . \text{Sin } r x}{x^3} dx = \frac{1}{2} p q \pi, r \geq p + q;$	
9)	$= \frac{1}{4} \pi (p q + p r + q r) - \frac{1}{8} \pi (p^2 + q^2 + r^2), r < p + q;$	
10)	$\int \frac{\text{Sin.}^2 p x . \text{Sin.}^2 q x}{x^4} dx = \frac{1}{6} p^2 \pi (3 q - p), p \leq q;$	
11)	$\int \frac{\text{Sin.}^3 p x . \text{Sin. } q x}{x^4} dx = \frac{1}{2} p^3 \pi, q > 3 p;$	
12)	$= \frac{1}{48} \pi \{24 p^3 - (3 p - q)^3\}, p \leq q \leq 3 p;$	
13)	$= \frac{1}{48} \pi \{24 p^3 q - (p + q)^3\}, q \leq p;$	

1)	$\int \frac{1 - \text{Cos. } q x}{x^2} dx = \frac{1}{2} \pi q, q > 0;$	} Poisson, Mém. Acad. 1816. 71. N°. 16. — Id., Chal. 100.
2)	$= -\frac{1}{2} \pi q, q < 0;$	

- 3) $\int \frac{\cos. q x - \cos. p x}{x^2} dx = \frac{p - q}{2} \pi$ Poisson, P. 16. 215. N°. 7. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 9.
- 4) $\int \frac{\sin. x - x \cos. x}{x^2} dx = 1$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.
- 5) $\int \frac{p \cos. q x - r x \sin. q x + s}{x^2} dx = (r - p q) \frac{\pi}{2}$ Cellérier, L. 8. 255.
- 6) $\int \frac{\sin. q x - q x \cos. q x}{x^3} dx = \frac{1}{4} \pi q^2, \quad q > 0; \quad \left\{ \begin{array}{l} \text{Poisson, Mém. Acad. 1816. 71. N°. 16.} \\ \text{7) } = -\frac{1}{4} \pi q^2, \quad q < 0; \end{array} \right.$
- 8) $\int \frac{\sin. x - x \cos. x}{x^3} dx = \frac{1}{4} \pi$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 9.
- 9) $\int \frac{x^3 - \sin.^3 x}{x^5} dx = \frac{13}{32} \pi$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 18.

- 1) $\int \frac{\sin. x}{x^{p+1}} dx = \frac{C}{1-p} \frac{1}{p} \sin. \frac{1}{2} p \pi$
- 2) $\int \frac{\sin. x}{x^p} dx = \frac{C}{1-p} \sin. \left\{ (2\alpha + 1) (1-p) \frac{\pi}{2} \right\}, \text{ où } \alpha \text{ entier arbitraire;}$
- 3) $= \frac{C}{1-p} \cos. \frac{1}{2} p \pi$
- 4) $= 1, \text{ pour } p \text{ très-petit;}$
- 5) $= \frac{\pi}{2 \Gamma(p)} \operatorname{Cosec.} \frac{1}{2} p \pi, \quad 2 > p > 0; \text{ Schlömilch, Cr. 33. 353. — Id., Beitr. III. § 4.}$
- 6) $\int \frac{\sin. x}{x^{a+1+p}} dx = \frac{(-1)^{a+2}}{p^{a+1/2}} \sin. \left\{ \frac{a+p}{2} \pi \right\} \Gamma(1-p), \quad p < 1; \text{ Flana, Mém. Brux. 1837.}$
- 7) $\int \frac{\sin. q x}{x^p} dx = \frac{q^{p-1}}{2 \Gamma(p)} \pi \operatorname{Cosec.} \frac{1}{2} p \pi, \quad 2 > p > 0; \text{ Schlömilch, Gr. 6. 200. — Id., Stud. I. 13.}$
- 8) $= \frac{\Gamma(1-p)}{q^{1-p}} \cos. \frac{1}{2} p \pi \quad \text{Lobatto, Int. 74.}$

- 9) $\int \frac{\text{Sin.}^q x}{x^p} dx = \infty, p \geq 2$; Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.
- 10) $\int \frac{\text{Sin.}^q x}{x^{a+1}} dx = q^a \frac{i^{-a} - i^a}{2i} \infty$ Cisa de Grésy, Mém. Turin. 1821. 209. II. 51.
- 11) $= -1^{-1-a/1} q^a \text{Sin.} \frac{1}{2} a \pi, a < 1$; Oettinger, Cr. 38. 216.
- 12) $= q^a \Gamma(-a) \text{Sin.} \frac{1}{2} a \pi$ (val. extraord.) Cauchy, P. 28. 147. P. III. Suppl. —
ld., Exerc. 1826. p. 53.
- 13) $\int \frac{\text{Sin.}(x-a)}{x^p} dx = \Gamma(1-p) \text{Cos.} \left(\frac{1}{2} p \pi + a \right), 0 < p < 1$; Svanberg, L. 11. 197.
- 14) $\int \frac{\text{Sin.}^{2b} q x}{x^{2a}} dx = (-1)^a \frac{\pi}{2^{2b}} \frac{q^{2a-1}}{1^{2a-1/1}} \sum_1^b (-1)^n \binom{2b}{b+n} (2n)^{2a-1} \left\{ \begin{array}{l} 1 < a < b; \\ 15) \int \frac{\text{Sin.}^{2b} q x}{x^{2a+1}} dx = (-1)^a \frac{1}{2^{2b-1}} \frac{q^{2a}}{1^{2a/1}} \sum_1^b (-1)^{n-1} \binom{2b}{b+n} (2n)^{2a} l 2 n \end{array} \right\}$
- 16) $\int \frac{\text{Sin.}^{2b+1} q x}{x^{2a}} dx = (-1)^{a+1} \frac{1}{2^{2b}} \frac{q^{2a-1}}{1^{2a-1/1}} \sum_1^b (-1)^{n-1} \binom{2b+1}{b+n+1} (2n+1)^{2a-1} l (2n+1) \left\{ \begin{array}{l} 0 < a < b+1; \\ 17) \int \frac{\text{Sin.}^{2b+1} q x}{x^{2a+1}} dx = (-1)^a \frac{\pi}{2^{2b+1}} \frac{q^{2a}}{1^{2a/1}} \sum_1^{b+1} (-1)^{n-1} \binom{2b+1}{b+n} (2n-1)^{2a} \end{array} \right\}$
- 18) $\int \frac{\text{Sin.}^b x}{x^a} dx = \frac{(-1)^{\frac{a+b-1}{2}}}{2^{b-1} 1^{a-1/1}} \frac{\pi}{2} \sum_1^{\frac{b-1}{2}} (-1)^n \binom{b}{n} \binom{b-2}{n}^{a-1}, a \text{ et } b \text{ impairs; } \left\{ \begin{array}{l} a \leq b; \\ 19) = \frac{(-1)^{\frac{a+b}{2}}}{2^{b-1} 1^{a-1/1}} \frac{\pi}{2} \sum_0^{\frac{b}{2}-1} (-1)^n \binom{b}{n} \binom{b-2}{n}^{a-1}, a \text{ et } b \text{ pairs; } \end{array} \right\}$
- 20) $\int \frac{\text{Sin.}^b x}{x^{a+1}} dx = \frac{\pi}{2^b 1^{a/1}} \text{Cossec.} \left\{ \frac{a+b}{2} \pi \right\} \left\{ b^a - \binom{b}{1} (b-2)^a + \binom{b}{2} (b-4)^a + \dots + \binom{b}{\frac{1}{2}b-1} 1^a \right\}, b \text{ pair, } a \text{ impair;}$
- 21) $= \frac{\pi}{2^b 1^{a/1}} \text{Cosec.} \left\{ \frac{a+b}{2} \pi \right\} \left\{ b^a - \binom{b}{1} (b-2)^a + \binom{b}{2} (b-4)^a + \dots + \binom{b}{\frac{1}{2}(b-1)} 1^a \right\}, b \text{ impair, } a \text{ pair;}$
- 22) $= \frac{(-1)^{\frac{a+b}{2}}}{2^b 1^{a/1}} \left\{ b^a l b - \binom{b}{1} (b-2)^a l (b-2) + \dots \right\}, a \text{ et } b \text{ pairs, ou } a \text{ et } b \text{ impairs;}$

Bidone,
Mém.
Turin.
1812.
231. Art.
1. N°. 11,
12, 14.
15.

Les formules (20) à (22) se trouvent Cauchy, P. 28. 147. P. 1. § 2.

- $$\begin{aligned} 1) \int \frac{\text{Cos. } x}{x^p} dx &= \frac{C}{1-p} \text{Cos.} \left\{ (2\alpha+1)(1-p) \frac{\pi}{2} \right\}, \text{ où } \alpha \text{ entier arbitraire;} \\ 2) &= \frac{C}{1-p} \text{Sin.} \frac{1}{2} p \pi \\ 3) &= \frac{1}{2} p \pi, \text{ pour } p \text{ très-petit;} \\ 4) &= \frac{\pi}{2 \Gamma(p)} \text{Sec.} \frac{1}{2} p \pi, 1 > p > 0; \text{ Schlömilch, Cr. 33. 353. — Id., Beitr. III. § 4.} \\ 5) \int \frac{\text{Cos. } x}{x^{a+1+p}} dx &= \frac{(-1)^{a+1}}{p^{a-1/1}} \text{Cos.} \left\{ \frac{a+p}{2} \pi \right\} \Gamma(1-p), p < 1; \text{ Plana, Mém. Brux. 1837.} \\ 6) \int \frac{\text{Cos. } q x}{x^p} dx &= \frac{q^{p-1}}{2 \Gamma(p)} \pi \text{Sec.} \frac{1}{2} p \pi, 1 > p > 0; \text{ Schlömilch, Gr. 6. 200. — Id., Stud. I. 13.} \\ 7) &= \frac{\Gamma(1-p)}{q^{1-p}} \text{Sin.} \frac{1}{2} p \pi \text{ Lobatto, Int. 74.} \\ 8) &= \infty, p \geq 1; \text{ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.} \\ 9) \int \frac{\text{Cos. } 2 q x}{x^{2a}} dx &= (-1)^a \frac{(2q)^{2a-1}}{1^{2a-1/1}} \frac{\pi}{2} \text{ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 11.} \\ 10) \int \frac{\text{Cos. } q x}{x^{a+1}} dx &= q^a \frac{i^{-a} + i^a}{2} \infty \text{ Cisa de Grésy, Mém. Turin. 1821. 209. II. N°. 51.} \\ 11) &= 1^{-a-1/1} q^a \text{Cos.} \frac{1}{2} a \pi, a < 1; \text{ Oettinger, Cr. 33. 216.} \\ 12) &= q^a \Gamma(-a) \text{Cos.} \frac{1}{2} a \pi \text{ (val. extraord.) Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 57.} \\ 13) \int \frac{\text{Cos.}^a x}{x^b} dx &= \frac{(-1)^{\frac{b+a-1}{2}}}{2^{a-1} 1^{b-1/1}} \frac{\pi}{2} \sum_{n=0}^{\frac{a-1}{2}} (-1)^n \binom{a}{n} \left(\frac{a-2}{n} \right)^{b-1}, a \text{ et } b \text{ impairs;} \\ 14) &= \frac{(-1)^{\frac{b+a}{2}}}{2^{a-1} 1^{b-1/1}} \frac{\pi}{2} \sum_{n=0}^{\frac{a-1}{2}} (-1)^n \binom{a}{n} \left(\frac{a-2}{n} \right)^{b-1}, a \text{ et } b \text{ pairs;} \end{aligned} \left. \vphantom{\int \frac{\text{Cos.}^a x}{x^b} dx} \right\} \begin{aligned} &\text{, } a \geq b > 0, \\ &\text{Lindmann, Gr. 17. 455.} \end{aligned}$$

$$\begin{aligned}
 1) \int \frac{\sin. x. \sin. qx}{x^p} dx &= \frac{\pi}{4 \Gamma(p)} \operatorname{Sec}. \frac{1}{2} p \pi \{ (1-q)^{p-1} - (1+q)^{p-1} \}, q < 1; \\
 2) &= \frac{\pi}{4 \Gamma(p)} \operatorname{Sec}. \frac{1}{2} p \pi \{ (q-1)^{p-1} - (1+q)^{p-1} \}, q > 1; \\
 3) \int \frac{\sin. x. \cos. qx}{x^p} dx &= \frac{\pi}{4 \Gamma(p)} \operatorname{Cosec}. \frac{1}{2} p \pi \{ (1-q)^{p-1} + (1+q)^{p-1} \}, q < 1; \\
 4) &= \frac{\pi}{4 \Gamma(p)} \operatorname{Cosec}. \frac{1}{2} p \pi \{ (q+1)^{p-1} - (q-1)^{p-1} \}, q > 1; \\
 5) \int \frac{\sin. ax}{x} \left(\frac{\sin. x}{x} \right)^a dx &= \frac{1}{2} \pi \quad \text{Hoppe, Cr. 40. 142.} \\
 6) \int \frac{\sin. bqx}{x} \left(\frac{\sin. x}{x} \right)^b dx &= \frac{1}{2} \pi \left\{ 1 - \frac{1}{2^{b-1} 1^{b/1}} \sum_0^{\frac{a-bq}{2}} (-1)^n \frac{b^{n/1-1}}{1^{n/1}} (b-bq-2n)^b \right\} \\
 7) \int \cos. bqx \left(\frac{\sin. x}{x} \right)^b dx &= \frac{\pi}{2^{b-1}} \sum_0^{\frac{a \pm bq}{2}} (-1)^n \frac{b^{n/1-1}}{1^{n/1}} (b \pm bq - 2n)^{b-1} \\
 8) \int \cos. ax \left(\frac{\sin. x}{x} \right)^b dx &= 0, a \geq b; \quad \text{Hoppe, Cr. 40. 142.} \\
 9) \int \cos. qx \left(\frac{\sin. x}{x} \right)^b dx &= \frac{\pi}{1^{b/1} 2^b} \sum_0^{\infty} (-1)^n \binom{b}{n} (q + b - 2n)^{b-1} \quad \text{Laplace, Probab. L. 1. 42.} \\
 10) \int \cos. \left(qx - \frac{1}{4} a \pi \right) \frac{\sin. x}{x^p} dx &= \frac{\pi}{4 \Gamma(p)} \left\{ \left(\frac{\sin. \frac{1}{4} a \pi}{\cos. \frac{1}{2} p \pi} + \frac{\cos. \frac{1}{4} a \pi}{\sin. \frac{1}{2} p \pi} \right) (1-q)^{p-1} + \right. \\
 &\quad \left. + \left(\frac{\cos. \frac{1}{4} a \pi}{\sin. \frac{1}{2} p \pi} - \frac{\sin. \frac{1}{4} a \pi}{\cos. \frac{1}{2} p \pi} \right) (1+q)^{p-1} \right\}, q < 1; \\
 11) &= \frac{\pi}{4 \Gamma(p)} \left\{ \left(\frac{\sin. \frac{1}{4} a \pi}{\cos. \frac{1}{2} p \pi} - \frac{\cos. \frac{1}{4} a \pi}{\sin. \frac{1}{2} p \pi} \right) (q-1)^{p-1} + \right. \\
 &\quad \left. + \left(\frac{\cos. \frac{1}{4} a \pi}{\sin. \frac{1}{2} p \pi} - \frac{\sin. \frac{1}{4} a \pi}{\cos. \frac{1}{2} p \pi} \right) (1+q)^{p-1} \right\}, q > 1;
 \end{aligned}$$

Ces form. (10) et (11) se trouvent Meyer, Int. Déf. 448.

$$\begin{aligned}
 12) \int \cos. \left(\frac{a+1}{2} \pi + px \right) \frac{dx}{x^{q+1}} &= 0 \\
 13) \int \cos. \left(\frac{a+1}{2} \pi - px \right) \frac{dx}{x^{q+1}} &= \frac{\pi pq}{\Gamma(q+1)}
 \end{aligned}
 \left. \vphantom{\int} \right\} \begin{array}{l} \text{valeurs extraord.;} \\ \text{Canehy, P. 28. 147. P. III. Suppl. — Id., Exerc.} \\ \text{1826. p. 57.} \end{array}$$

$$14) \int \frac{\cos. \left\{ \frac{r-1}{2} \pi + 2(p+q)x \right\} + \cos. \left\{ \frac{r+1}{2} \pi + 2(p-q)x \right\}}{x^{r+1}} dx = 0 \quad , p > q \left\{ \begin{array}{l} \text{, val. extraord.;} \\ \text{Cauchy, P. 28. 147.} \\ \text{P. III. Suppl. —} \\ \text{Id., Exerc. 1826.} \end{array} \right.$$

$$15) \quad = \frac{(q-p)^r}{\Gamma(r+1)} \pi, p < q; \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{p. 57.}$$

$$16) \int \frac{\sin. 2q x. \sin^b x}{x^{b+1}} dx = (-1)^b \frac{\pi}{2^{b+1}}, 2q < b;$$

$$17) \quad = 0, \quad 2q > b, \text{ et } q \text{ entier;}$$

$$18) \int \frac{\cos. 2ax. \sin^b x}{x^{b+1}} dx = 0 \quad a + b \text{ impair;}$$

$$19) \int \frac{\sin. 2cx. \sin^b x}{x^{a+1}} dx = \frac{\pi}{2^{b+1} \Gamma(a+1)} \sec. \left\{ \frac{a+b}{2} \pi \right\} \Delta^b. (2c-b)^a, 2c < b;$$

$$20) \quad = \frac{\pi}{2^{b+1} \Gamma(a+1)} \sec. \left\{ \frac{a+b}{2} \pi \right\} \left\{ \sum_0^\infty (-1)^n \binom{b}{n} (b+2c-2n)^a - \sum_0^\infty (-1)^n \binom{b}{n} (b-2c-2n)^a \right\}, \left. \begin{array}{l} a < b+1, \\ a+b \text{ pair;} \end{array} \right\}$$

$$21) \quad = \frac{(-1)^{\frac{a+b-1}{2}}}{2^b \Gamma(a+1)} \Delta^b. \{ (2c-b)^a \Gamma(2c-b) \}, \quad a + b \text{ impair;}$$

$$22) \int \frac{\cos. 2cx. \sin^b x}{x^{a+1}} dx = (-1)^{\frac{b-1}{2}} \frac{\pi}{2^{b+1} \Gamma(a+1)} \operatorname{Cosec}. \frac{1}{2} a \pi \Delta^b. (2c-b)^a, b \text{ pair;}$$

$$23) \quad = (-1)^{\frac{b-1}{2}} \frac{\pi}{2^{b+1} \Gamma(a+1)} \sec. \frac{1}{2} a \pi \Delta^b. (2c-b)^a, b \text{ impair;}$$

$$24) \quad = - \frac{\pi}{2^{b+1} \Gamma(a+1)} \operatorname{Cosec}. \left\{ \frac{a+b}{2} \pi \right\} \Delta^b. (2c-b)^a, \text{ tout } b;$$

$$25) \quad = - \frac{\pi}{2^{b+1} \Gamma(a+1)} \operatorname{Cosec}. \left\{ \frac{a+b}{2} \pi \right\} \left\{ \sum_0^\infty (-1)^n \binom{b}{n} (b+2c-2n)^a - \sum_0^\infty (-1)^n \binom{b}{n} (b-2c-2n)^a \right\}, 2c < b;$$

$$26) \quad = 0 \quad , a + b \text{ impair} \quad , 2c > b;$$

Cauchy,
P. 28.
147. P.
I. § 2.

$$\begin{aligned}
 27) \int \frac{\cos 2cx \cdot \sin^b x}{x^{a+1}} dx &= (-1)^{\frac{a+b+1}{2}} \frac{1}{2^b 1^{a/1}} \Delta^b \{(2c-b)^a l (2c-b)\}, a+b \text{ pair}, 2c > b; \\
 28) &= (-1)^{\frac{a+b-1}{2}} \frac{1}{2^b 1^{a/1}} \sum_0^\infty (-1)^n \binom{b}{n} (b \pm 2c - 2n)^a, a+b \text{ impair}; \\
 29) &= (-1)^{\frac{a+b}{2}+1} \frac{1}{2^b 1^{a/1}} \Delta^b \{(2c-b)^a l (2c-b)\}, a+b \text{ pair}; \\
 30) \int \frac{\sin bx \cdot \cos x \left(\frac{\sin x}{x}\right)^{b-1}}{x} dx &= \frac{1}{2} \pi \quad \text{Hoppe, Cr. 40. 142.} \\
 31) \int \frac{\sin^b x}{x^{a+1}} \cos \left\{ 2cx + (a-b+1) \frac{\pi}{2} \right\} dx &= \frac{\pi}{2^b 1^{a/1}} \sum_0^\infty (-1)^n \binom{b}{n} (b-2c-2n)^a, b > 2c; \\
 32) \int \frac{\sin^b x}{x^{a+1}} \cos \left\{ 2cx - (a-b+1) \frac{\pi}{2} \right\} dx &= \frac{\pi}{2^b 1^{a/1}} \sum_0^\infty (-1)^n \binom{b}{n} (b+2c-2n)^a \\
 33) \int \frac{\sin \{(2c+b)x\} \cdot \sin^b x}{x^{a+1}} dx &= \frac{2^{a-b-1} \pi}{b} \sec. \frac{1}{2} a \pi \Delta^b . c^a, b \text{ pair}; \\
 34) &= \frac{2^{a-b-1} \pi}{(-1)^{\frac{b+1}{2}} 1^{a/1}} \operatorname{Cosec}. \frac{1}{2} a \pi \Delta^b . c^a, b \text{ impair}; \\
 35) &= \frac{2^{a-b-1} \pi}{1^{a/1}} \sec. \left\{ \frac{a+b}{2} \pi \right\} \Delta^b . c^a, \text{ tout } b; \\
 36) \int \frac{\cos \{(2c+b)x\} \cdot \sin^b x}{x^{a+1}} dx &= \frac{2^{a-b-1} \pi}{b} \operatorname{Cosec}. \frac{1}{2} a \pi \Delta^b . c^a, b \text{ pair}; \\
 37) &= \frac{2^{a-b-1} \pi}{(-1)^{\frac{b+1}{2}} 1^{a/1}} \sec. \frac{1}{2} a \pi \Delta^b . c^a, b \text{ impair}; \\
 38) &= - \frac{2^{a-b-1} \pi}{1^{a/1}} \operatorname{Cosec}. \left\{ \frac{a+b}{2} \pi \right\} \Delta^b . c^a, \text{ tout } b; \\
 39) \int \cos (bx \vee a) \left(\frac{\sin x}{x} \right)^a dx &= \frac{\pi}{2^a 1^{a/1}} \sum_0^\infty (-1)^n \binom{a}{n} (a+b \vee a-2n)^{a-1} \\
 40) \int \sin^a \frac{1}{2} x \cdot \sin \left(\frac{2p+q}{2} x + \frac{1}{2} a \pi \right) \frac{dx}{x^{q+1}} &= \frac{\pi}{2^{a+1} \Gamma(q+1)} \operatorname{Cosec}. \left\{ \frac{q+1}{2} \pi \right\} \Delta^a . p^q \\
 41) \int \sin^a \frac{1}{2} x \cdot \cos \left(\frac{2p+q}{2} x + \frac{1}{2} a \pi \right) \frac{dx}{x^{q+1}} &= \frac{\pi}{2^{a+1} \Gamma(q+1)} \sec. \left\{ \frac{q+1}{2} \pi \right\} \Delta^a . p^q
 \end{aligned}$$

Cauchy, P. 28. 147. P. I. § 2.
 Dans 28, on peut prendre le double signe à volonté.
 Cauchy, P. 28. 147. P. III. § 3.
 Cauchy, P. 28. 147. P. III. § 2.
 Laplace, Mém. Inst. 1809. 353. § 10.
 Cauchy, P. 28. 147. P. III. § 2.

- $$\left. \begin{aligned} 1) \int \frac{\text{Sin. } q x}{1+x} dx &= \text{Sin. } q. \text{Ci. } (q) + \text{Cos. } q. \left\{ \frac{1}{2} \pi - \text{Si. } (q) \right\} \\ 2) \int \frac{\text{Cos. } q x}{1+x} dx &= -\text{Cos. } q. \text{Ci. } (q) + \text{Sin. } q. \left\{ \frac{1}{2} \pi - \text{Si. } (q) \right\} \end{aligned} \right\} \text{Arndt, Gr. 10. 225. — Schlömilch, St. II. 21.}$$
- $$\left. \begin{aligned} 3) \int \frac{\text{Sin. } k x}{1+x} dx &= 0 \\ 4) \int \frac{\text{Cos. } k x}{1+x} dx &= 0 \end{aligned} \right\} , k = \infty ; \text{Raabe, Int. 202.}$$
- $$\left. \begin{aligned} 5) \int \frac{\text{Sin. } p x}{x+q} dx &= \frac{1}{2} \pi \text{Cos. } p q + (\Lambda + l p q) \text{Sin. } p q + \text{Cos. } p q \sum_1^{\infty} (-1)^n \frac{(p q)^{2n-1}}{(2n-1) 1^{2n-1/1}} + \\ &\quad + \text{Sin. } p q \sum_1^{\infty} (-1)^n \frac{(p q)^{2n}}{2 n. 1^{2n/1}} \\ 6) \int \frac{\text{Cos. } p x}{x+q} dx &= \frac{1}{2} \pi \text{Sin. } p q - (\Lambda + l p q) \text{Cos. } p q + \text{Sin. } p q \sum_1^{\infty} (-1)^n \frac{(p q)^{2n-1}}{(2n-1) 1^{2n-1/1}} - \\ &\quad - \text{Cos. } p q \sum_1^{\infty} (-1)^n \frac{(p q)^{2n}}{2 n. 1^{2n/1}} \end{aligned} \right\} \text{Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 27.}$$
- $$\left. \begin{aligned} 7) \int \frac{\text{Sin. } p x}{x+q} dx &= \text{Sin. } p q. \text{Ci. } (p q) + \text{Cos. } p q. \left\{ \frac{1}{2} \pi - \text{Si. } (p q) \right\} \\ 8) \int \frac{\text{Cos. } p x}{x+q} dx &= -\text{Cos. } p q. \text{Ci. } (p q) + \text{Sin. } p q. \left\{ \frac{1}{2} \pi - \text{Si. } (p q) \right\} \end{aligned} \right\} \text{Arndt, Gr. 10. 225. — Schlömilch, Stud. II. 21.}$$
- $$\left. \begin{aligned} 9) \int \frac{\text{Sin. } p x}{x-q} dx &= \frac{1}{2} \pi \text{Cos. } p q - (\Lambda + l p q) \text{Sin. } p q - \text{Cos. } p q \sum_1^{\infty} (-1)^n \frac{(p q)^{2n-1}}{(2n-1) 1^{2n-1/1}} - \\ &\quad - \text{Sin. } p q \sum_1^{\infty} (-1)^n \frac{(p q)^{2n}}{2 n. 1^{2n/1}} \\ 10) \int \frac{\text{Cos. } p x}{x-q} dx &= -\frac{1}{2} \pi \text{Sin. } p q - (\Lambda + l p q) \text{Cos. } p q + \text{Sin. } p q \sum_1^{\infty} (-1)^n \frac{(p q)^{2n-1}}{(2n-1) 1^{2n-1/1}} - \\ &\quad - \text{Cos. } p q \sum_1^{\infty} (-1)^n \frac{(p q)^{2n}}{2 n. 1^{2n/1}} \end{aligned} \right\} \text{Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 28.}$$
- $$11) \int \frac{\text{Sin. } p x}{x-q} dx = -\text{Sin. } p q. \text{Ci. } (p q) + \text{Cos. } p q. \left\{ \frac{1}{2} \pi + \text{Si. } (p q) \right\} + \text{Sin. } p q. l \alpha \left. \vphantom{\int} \right\} , \text{ où } \alpha \text{ arbitraire,}$$
- $$12) \int \frac{\text{Cos. } p x}{x-q} dx = -\text{Cos. } p q. \text{Ci. } (p q) - \text{Sin. } p q. \left\{ \frac{1}{2} \pi + \text{Si. } (p q) \right\} + \text{Cos. } p q. l \alpha \left. \vphantom{\int} \right\} \text{Arndt, Gr. 10. 240.}$$

- 1) $\int \frac{\cos. p x}{1+x^2} dx = \frac{1}{2} \pi e^p, p < 0$; Fourier, Chal. 358.
- 2) $= \frac{1}{2} \pi e^{-p}, p > 0$; $\left\{ \begin{array}{l} \text{Laplace, Bull. Soc. Phil. Avr. 1811. — Id., Prob. L. 1.} \\ \text{N}^\circ. 26. — \text{Poisson, P. 19. 60. — Id., P. 16. 215. N}^\circ. 7. —} \\ \text{Cauchy, Sav. Etr. 1827. 124. Note 18. — Id., Sav. Etr. 1824.} \\ \text{599. P. II. § 7. 1. — Legendre, Exerc. 3. 42. — Schlömilch,} \\ \text{Gr. 5. 204. — Arndt, Gr. 11. 70. — Fourier, Chal. 358.} \end{array} \right\}$
- 3) $\int \frac{x \sin. p x}{1+x^2} dx = \frac{1}{2} \pi e^{-p}, p > 0$; $\left. \begin{array}{l} \text{Sur la form. (2) seule voyez: Serret, L. 8. 1. — Id., L. 8. 189. — Poisson,} \\ \text{P. 17. 612. N}^\circ. 19.} \end{array} \right\}$
- 4) $= -\frac{1}{2} \pi e^p, p < 0$; Fourier, Chal. 358.
- 5) $= \frac{1}{2} \pi, p \text{ très-petit}$; $\left\{ \begin{array}{l} \text{Cauchy, Sav. Etr. 1827. 599. Suppl. 2.} \end{array} \right\}$
- 6) $= 0, p = 0$; $\left. \begin{array}{l} \end{array} \right\}$
- 7) $\int \frac{\sin. p x}{1+x^2} dx = \frac{1}{2} \{e^{-p} \text{li.}(ep) - ep \text{li.}(e^{-p})\}$ $\left. \begin{array}{l} \end{array} \right\}$ Schlömilch, Gr. 5. 204.
- 8) $\int \frac{x \cos. p x}{1+x^2} dx = -\frac{1}{2} \{ep \text{li.}(e^{-p}) + e^{-p} \text{li.}(ep)\}$ $\left. \begin{array}{l} \end{array} \right\}$
- 9) $\int \frac{x \text{Tang.} p x}{1+x^2} dx = \pi \frac{e^{-p}}{ep + e^{-p}}$ Legendre, Exerc. 5. 35.
- 10) $\int \frac{x \text{Cot.} p x}{1+x^2} dx = \pi \frac{e^{-p}}{ep - e^{-p}}$ Cauchy, Sav. Etr. 1827. 599. S. 2. — Legendre, Exerc. 5. 33.
- 11) $\int \frac{\cos. p x}{1+q x^2} dx = \frac{\pi}{2 \sqrt{q}} e^{-\frac{p}{\sqrt{q}}}$ Poisson, P. 19. 404. N^o. 56.
- 12) $\int \frac{\sin. \{(a+k)x\} \cos. \{(a-k)x\}}{1+x^2} x dx = \frac{1+e^{-2a}}{4} \pi, k \text{ très petit}$; $\left. \begin{array}{l} \end{array} \right\}$ Cauchy, Sav. Etr. 1827. 599.
- 13) $= \frac{1}{4} \pi e^{-2a}, k = 0$; $\left. \begin{array}{l} \end{array} \right\}$ S. 2.
- 14) $\int \sin. \left(\frac{1}{2} a \pi - q x \right) \frac{x^{a-1}}{1+x^2} dx = \frac{1}{2} \pi e^{-q}, a < 3$; Svanberg, L. 11. 197.
- 15) $\int \cos. \left(\frac{1}{2} p \pi - q x \right) \frac{x^p}{1+x^2} dx = \frac{1}{2} \pi e^{-q}, -1 < p < 1$; Liouville, Cr. 11. 1.

F. Alg. rat. fract. à dén. $1 \pm x^2$.
Circ. Dir. en num.

TABLE 204 suite.

Lim. 0 et ∞ .

- 16) $\int \left\{ \cos. \left(qx - \frac{1}{2} p \pi \right) - \cos. \frac{1}{2} p \pi \right\} \frac{x^p}{1+x^2} dx = \frac{1-e^q}{2e^q} \pi, -2 < p \leq -1$; Liouville, C r. 11. 1.
- 17) $\int \frac{\cos. x + x \sin. x}{1+x^2} dx = \frac{\pi}{e}$ Legendre, Exerc. 3. 41. — Laplace, Probab. L. 1. 33.
- 18) $\int \frac{\cos. qx + x \sin. qx}{1+x^2} dx = \pi e^{-q}$ Bidone, Mém. Turin. 1812. 231. Art. 2. 22.
- 19) $\int \sin. \left(\frac{1}{2} p x - qx \right) \frac{x^{p-1}}{1+x^2} dx = \frac{1}{2} \pi e^{-q}, p < 1$; Cauchy, Cours. Leç. 39. — Svanberg, L. 11. 197.
- 20) $\int \frac{x \sin. px}{1+q^2 x^2} dx = \frac{\pi}{2q^2} e^{-\frac{p}{q}}$ Raabe, Int. 169.
- 21) $\int \frac{\cos. qx}{1-x^2} dx = \frac{1}{2} \pi \sin. q$ Cauchy, Lim. Imag. Add. 17. — Id., Sav. Etr. 1827. 124. Note 18.
- 22) $\int \frac{x \sin. qx}{1-x^2} dx = -\frac{1}{2} \pi \cos. q$ Cauchy, Sav. Etr. 1827. 124. Note 18.
- 23) $\int \frac{\sin. qx \sin. \pi x}{1-x^2} dx = \frac{1}{2} \pi \sin. q, 0 < q < \pi$; Fourier, Chaleur. 358.
- 24) $\qquad \qquad \qquad = 0, q > \pi$

F. Alg. rat. fract. à dén. $a^2 + x^2$.
Circ. Dir. en num. à une fonct.

TABLE 205.

Lim. 0 et ∞ .

- 1) $\int \frac{\cos. x}{q^2 + x^2} dx = \frac{\pi}{2q} e^{-q}$ Poisson, P. 17. 612. N°. 20. — Id., P. 18. 295. N°. 32. — Id., P. 19. 404. N°. 75. — Liouville, Cr. 13. 219. — Schlömilch, Cr. 36. 271.
- 2) $\int \frac{x \sin. x}{q^2 + x^2} dx = \frac{1}{2} \pi e^{-q}$ Poisson, P. 18. 295. N°. 32. — Id., P. 19. 404. N°. 75. — Schlömilch, Cr. 36. 271.
- 3) $\int \frac{x \text{Tang. } x}{q^2 + x^2} dx = \frac{\pi}{e^{2q} + 1}$
- 4) $\int \frac{x \text{Cot. } x}{q^2 + x^2} dx = \frac{\pi}{e^{2q} - 1}$ Schlömilch, Gr. 10. 440.
- 5) $\int \frac{\cos. px}{q^2 + x^2} dx = \frac{\pi}{2q} e^{-pq}, p \geq 0$; Laplace, Nouv. Bull. de la Soc. Phil. N°. 43, 49. — Poisson, Chal. 135. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 21, 22. — Plana, Mém. Turin. 1818. 7. I. 6. — Cisa de Grézy, Mém. Turin. 1821. 209. II. 55. — Cauchy, Lim. Imag. Add. 14. — Id., P. 19. 511. — Id., P. 28. 147. I. § 3. — Id., Sav. Etr. 1827. 124. Note 6. — Id., Sav. Etr. 1827. 599. P. II. § 7. — Id., Exerc. 1826. p. 95. — Id., ib. 1827. p. 141. — Legendre, Exerc. 3. 42. — v. Schmidten, Cr. 5. 388. — Schlömilch, Gr. 5. 204. — Id., Gr. 9. 379. — Id., Gr. 10. 440. — Id., Beitr. II § 3. — Id., Sud. II. 14. — Arndt, Gr. 11. 70. — Schellbach, Cr. 48. 207. — Sur la form. 5) seule voyez: Schlömilch, Cr. 36. 268.
- 6) $\int \frac{x \sin. px}{q^2 + x^2} dx = \frac{1}{2} \pi e^{-pq}, p > 0$

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- 7) $\int \frac{\cos. p x}{q^2 + x^2} dx = \frac{\pi}{2 \sqrt{q}} e^{-p \sqrt{q}}$ Poisson, P. 16. 215. N°. 7.
- 8) $\int \frac{\sin. p x}{q^2 + x^2} dx = -\frac{e^{pq} - e^{-pq}}{2q} (\lambda + l p q) + \frac{e^{pq} + e^{-pq}}{2q} \sum_1^{\infty} \frac{(p q)^{2n-1}}{(2n-1) 1^{2n-1/l}} - \frac{e^{pq} - e^{-pq}}{2q} \sum_1^{\infty} \frac{(p q)^{2n}}{2n. 1^{2n/l}}$ } Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 27.
- 9) $\int \frac{x \cos. p x}{q^2 + x^2} dx = -\frac{e^{pq} + e^{-pq}}{2} (\lambda + l p q) + \frac{e^{pq} - e^{-pq}}{2} \sum_1^{\infty} \frac{(p q)^{2n-1}}{(2n-1) 1^{2n-1/l}} - \frac{e^{pq} + e^{-pq}}{2} \sum_1^{\infty} \frac{(p q)^{2n}}{2n. 1^{2n/l}}$ }
- 10) $\int \frac{\sin. p x}{q^2 + x^2} dx = \frac{1}{2q} \left\{ e^{-pq} \text{Ei.}(pq) - e^{pq} \text{Ei.}(-pq) \right\}$ } Schlömilch, Gr. 5. 204 (où la form. (10) est fautive). — Id., Stud. II. 20. — Arndt, Gr. 11. 70. — Sur la formule (10) seule voyez: Schlömilch, Gr. 11. 174.
- 11) $\int \frac{x \cos. p x}{q^2 + x^2} dx = -\frac{1}{2} \left\{ e^{pq} \text{Ei.}(-pq) + e^{-pq} \text{Ei.}(pq) \right\}$ }
- 12) $\int \frac{x^2 \cos. p x}{q^2 + x^2} dx = -\frac{1}{2} p q e^{-pq}$ Plana, Mém. Turin. 1818. 7. IV. 17. La valeur en est infinie.
- 13) $\int \frac{x \text{Tang. } p x}{q^2 + x^2} dx = \frac{1}{2} \pi, z < 1;$ } , dans le cas de x complexe $= y + z i$;
- 14) $= \frac{\pi}{e^{2pq} + 1}, z > 1;$ } Poisson, P. 18. 295. N°. 42.
- 15) $= \frac{\pi}{e^{2pq} + 1}$ Legendre, Exerc. 4. 131. — Bidone, Mém. Turin. 1812. 231. Art. 3. 39. — Schlömilch, Gr. 10. 440. — Id., Beitr. II. 4.
- 16) $\int \frac{x \cot. p x}{q^2 + x^2} dx = \frac{\pi}{e^{2pq} - 1}$ Cauchy, Exerc. 1827. p. 141. — Legendre, Exerc. 4. 131. — Bidone, Mém. Turin. 1812. 231. Art. 3. 39. — Schlömilch, Gr. 10. 440. — Id., Beitr. II. 4.
- 17) $\int \frac{x \text{Tang. } \frac{1}{2} x}{q^2 + x^2} dx = \frac{\pi}{e^q + 1}$ } Schlömilch, Stud. II. 18. — Id. Cr. 36. 271.
- 18) $\int \frac{x \cot. \frac{1}{2} x}{q^2 + x^2} dx = \frac{\pi}{e^q - 1}$ }
- 19) $\int \frac{x \sin. 2 x}{q^2 + x^2} dx = \frac{\pi}{2} e^{-2q}$ } Schlömilch, Gr. 10. 440.
- 20) $\int \frac{\cos. 2 x}{q^2 + x^2} dx = \frac{\pi}{2} e^{-2q}$ }
- 21) $\int \frac{\sin. 2 x}{q^2 + x^2} dx = \frac{\pi}{4} \frac{1 - e^{-2q}}{q}$ V. T. 205. N°. 20, 22.
- 22) $\int \frac{\cos. 2 x}{q^2 + x^2} dx = \frac{\pi}{4} \frac{1 + e^{-2q}}{q}$ Schlömilch, Gr. 10. 440.

F. Alg. rat. fract. à dén. $a^2 + x^2$.
Circ. Dir. en num. à une fonct.

TABLE 205 suite.

Lim. 0 et ∞ .

$$23) \int \frac{\cos. p x}{q^2 + x^2} dx = \frac{\pi}{2q} \frac{1 + e^{-2pq}}{2} \quad \text{Dienger, Gr. 12. 97.}$$

$$24) \int \sin \left(\frac{1}{2} r x - p x \right) \frac{x^{r-1}}{q^2 + x^2} dx = \frac{1}{2} \pi q^{r-2} e^{-pq}, \quad r < 2; \quad \text{Cauchy, Lim. Imag. Add. N°. 22. — Id., P. 19. 511. — Id., Exerc. 1826. p. 95.}$$

$$25) \int \cos. \left(\frac{1}{2} a \pi - x \right) \frac{x^a}{q^2 + x^2} dx = \frac{1}{2} \pi q^{a-1} e^{-q} \quad \text{Cayley, L. 12. 231.}$$

$$26) \int \frac{x^{2a} \cos. p x}{q^2 + x^2} dx = (-1)^a \frac{1}{2} \pi q^{2a-1} e^{-pq} \quad \left\{ \begin{array}{l} \text{Schlömilch, Cr. 33. 353. — Arndt, Gr. 11. 70. les} \\ \text{trouve indéterminées. — Elles sont infinies pour } a > 0. \end{array} \right.$$

$$27) \int \frac{x^{2a+1} \sin. p x}{q^2 + x^2} dx = (-1)^a \frac{1}{2} \pi q^{2a} e^{-pq}$$

$$28) \int \frac{\cos. p x}{(r + q i)^2 + x^2} dx = \frac{1}{2} \frac{\pi}{r + q i} e^{-p(r+qi)}, \quad p \geq 0; \quad \left\{ \begin{array}{l} \text{Poisson, P. 18. 295. N°. 44.} \end{array} \right.$$

$$29) \int \frac{x \sin. p x}{(r + q i)^2 + x^2} dx = \frac{1}{2} \pi e^{-p(r+qi)}, \quad p > 0;$$

F. Alg. rat. fract. à dén. $a^2 - x^2$.
Circ. Dir. en num. à une fonct.

TABLE 206.

Lim. 0 et ∞ .

$$1) \int \frac{x \sin. p x}{q^2 - x^2} dx = -\frac{1}{2} \pi \cos. p q \quad \left\{ \begin{array}{l} \text{Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 29. — Cauchy,} \\ \text{Sav. Etr. 1827. 124. Note 6. — Id., Sav. Etr. 1827. 599.} \\ \text{P. 2. § 7. — Id., Lim. Imag. Add. 15. — Plana, Mém.} \\ \text{Turin. 1818. 7. I. 3. — Schlömilch, Cr. 33. 316. — Id.,} \\ \text{Gr. 7. 270. — Id., Stud. II. 15. — Mosta, Gr. 10. 419.} \end{array} \right.$$

$$2) \int \frac{\cos. p x}{q^2 - x^2} dx = \frac{\pi}{2q} \sin. p q$$

Sur la form. (1) voyez: Cisa de Grésy, Mém. Turin. 1821. 209. II. 57.

$$3) \int \frac{x \sin. p x}{q^2 - x^2} dx = -\frac{1}{2} \sin. p q l \alpha - \frac{1}{2} \pi \cos. p q \quad \left\{ \begin{array}{l} \text{, où } \alpha \text{ est indéterminé;} \end{array} \right.$$

$$4) \int \frac{\cos. p x}{q^2 - x^2} dx = \frac{1}{2q} \{ \pi \sin. p q - \cos. p q l \alpha \} \quad \left\{ \begin{array}{l} \text{Arndt, Gr. 10. 240. — Cauchy, P. 19. 511.} \\ \text{trouve ces formules pour valeurs générales.} \\ \text{mais dans (4) il a } \pi l \alpha \text{ au lieu de } l \alpha. \end{array} \right.$$

$$5) \int \frac{x \sin. p x}{q^2 - x^2} dx = -\frac{1}{2} \pi \cos. p q + \frac{1}{2} \sin. p q l (-1)$$

$$6) \quad = -\frac{1}{2} \pi e^{-pq} \quad \left\{ \begin{array}{l} \text{Poisson, P. 18. 295. N°. 38.} \end{array} \right.$$

$$7) \int \frac{\cos. p x}{q^2 - x^2} dx = \frac{\pi}{2q} \sin. p q + \frac{1}{2q} \cos. p q l (-1)$$

- 8) $\int \frac{\text{Cos. } p x}{q^2 - x^2} dx = \frac{\pi}{2 q i} e^{-p q i}$ Poisson, P. 18. 295. N°. 38.
- 9) $\int \frac{\text{Sin. } p x}{q^2 - x^2} dx = \frac{1}{q} \{ \text{Ci. } (p q) \cdot \text{Sin. } p q - \text{Si. } (p q) \cdot \text{Cos. } p q \}$
- 10) $\int \frac{x \text{Cos. } p x}{q^2 - x^2} dx = \text{Ci. } (p q) \cdot \text{Cos. } p q + \text{Si. } (p q) \cdot \text{Sin. } p q$
- 11) $\int \frac{\text{Sin. } p x}{q^2 - x^2} dx = -\frac{1}{q} \text{Si. } (p q) \cdot \text{Cos. } p q - \frac{1}{q} \text{Sin. } p q \cdot \left\{ \frac{1}{2} l \alpha^2 - \text{Ci. } (p q) \right\}$, où α arbitraire;
- 12) $\int \frac{x \text{Cos. } p x}{q^2 - x^2} dx = \text{Si. } (p q) \cdot \text{Sin. } p q - \text{Cos. } p q \cdot \left\{ \frac{1}{2} l \alpha^2 - \text{Ci. } (p q) \right\}$ Arndt, Gr. 10. 210.
- 13) $\int x \text{Sin. } \frac{b \pi x}{a} \frac{dx}{a^2 - x^2} = \frac{1}{2} \pi \text{Cos. } b \pi$
- 14) $\int \text{Cos. } \left\{ \frac{2b-1}{2a} \pi x \right\} \frac{dx}{a^2 - x^2} = -\frac{\pi}{2a} \text{Sin. } \left\{ \left(b - \frac{1}{2} \right) \pi \right\}$
- 15) $\int \frac{x \text{Tang. } p x}{q^2 - x^2} dx = -\frac{1}{2} \pi$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 39.
- 16) $= -\frac{1}{2} \pi + \frac{1}{2} \text{Tang. } \frac{1}{2} p q \cdot l(-1)$
- 17) $\int \text{Tang. } \frac{2 b \pi x}{a} \frac{x}{a^2 - x^2} dx = -\frac{1}{2} \pi$
- 18) $\int \frac{x \text{Cosce. } p x}{q^2 - x^2} dx = 0$
- 19) $\int \frac{x \text{Cot. } p x}{q^2 - x^2} dx = \frac{1}{2} \pi$
- 20) $\int \text{Sin. } \left(\frac{1}{2} a \pi - p x \right) \frac{x^{a-1}}{q^2 - x^2} dx = \frac{1}{2} \pi q^{a-2} \text{Cos. } \left(\frac{1}{2} a \pi - p q \right)$ Cauchy, Lim. Imag. 23. — Id., P. 19. 511.
- 21) $\int \frac{\text{Cos.}^2 p x}{q^2 - x^2} dx = \frac{\pi}{4 q} \text{Sin. } 2 p q$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 31.

- 1) $\int \frac{x \text{Tang. } p x}{x^4 + q^4} dx = \frac{\pi e^{-pq\sqrt[4]{2}}}{q^2} \frac{\text{Sin.}(pq\sqrt[4]{2})}{1 + 2e^{-pq\sqrt[4]{2}} \text{Cos.}(pq\sqrt[4]{2}) + e^{-2pq\sqrt[4]{2}}}$
- 2) $\int \frac{x \text{Cot. } p x}{x^4 + q^4} dx = \frac{\pi e^{-pq\sqrt[4]{2}}}{q^2} \frac{\text{Sin.}(pq\sqrt[4]{2})}{1 - 2e^{-pq\sqrt[4]{2}} \text{Cos.}(pq\sqrt[4]{2}) + e^{-2pq\sqrt[4]{2}}}$
- 3) $\int \frac{x \text{Cosec. } p x}{x^4 + q^4} dx = \frac{\pi e^{-\frac{1}{2}pq\sqrt[4]{2}}}{q^2} \frac{1 + e^{-pq\sqrt[4]{2}}}{1 - 2e^{-pq\sqrt[4]{2}} \text{Cos.}(pq\sqrt[4]{2}) + e^{-2pq\sqrt[4]{2}}} \text{Sin.}\left(\frac{1}{2}pq\sqrt[4]{2}\right)$
- 4) $\int \frac{x^2 \text{Cos. } p x}{x^4 + 4q^4} dx = \frac{\pi}{4q} e^{-pq} (\text{Cos. } pq - \text{Sin. } pq)$ Schlömilch, Stud. II. 16.
- 5) $\int \frac{\text{Cos. } 2 p x}{q^4 + x^4} dx = \frac{\pi}{4q^3} \sqrt[4]{2} \cdot e^{-pq\sqrt[4]{2}} \{ \text{Cos.}(pq\sqrt[4]{2}) + \text{Sin.}(pq\sqrt[4]{2}) \}$
- 6) $\int \frac{x^2 \text{Cos. } 2 p x}{q^4 + x^4} dx = \frac{\pi}{4q} \sqrt[4]{2} \cdot e^{-pq\sqrt[4]{2}} \{ \text{Cos.}(pq\sqrt[4]{2}) - \text{Sin.}(pq\sqrt[4]{2}) \}$
- 7) $\int \frac{x \text{Sin. } 2 p x}{q^4 + x^4} dx = \frac{\pi}{2q^2} e^{-pq\sqrt[4]{2}} \text{Sin.}(pq\sqrt[4]{2})$
- 8) $\int \frac{x^3 \text{Sin. } 2 p x}{q^4 + x^4} dx = \frac{\pi}{2} e^{-pq\sqrt[4]{2}} \text{Cos.}(pq\sqrt[4]{2})$
- 9) $\int \frac{\text{Sin. } p x}{q^4 - x^4} dx = \frac{1}{4q^3} \{ 2 \text{Ci.}(pq) \cdot \text{Sin. } pq - 2 \text{Si.}(pq) \cdot \text{Cos. } pq + e^{-pq} \text{Ei.}(pq) - e^{pq} \text{Ei.}(-pq) \}$ V. T. 205.
N°. 10 et T. 206. N°. 9.
- 10) $\int \frac{x \text{Sin. } p x}{q^4 - x^4} dx = \frac{\pi}{4q^2} (e^{-pq} - \text{Cos. } pq)$ V. T. 205. N°. 6 et T. 206. N°. 1.
- 11) $\int \frac{x^2 \text{Sin. } p x}{q^4 - x^4} dx = \frac{1}{4q} \{ 2 \text{Ci.}(pq) \cdot \text{Sin. } pq - 2 \text{Si.}(pq) \cdot \text{Cos. } pq - e^{-pq} \text{Ei.}(pq) + e^{pq} \text{Ei.}(-pq) \}$ V. T. 205. N°. 10 et T. 206. N°. 9.
- 12) $\int \frac{x^3 \text{Sin. } p x}{q^4 - x^4} dx = -\frac{\pi}{4} (e^{-pq} + \text{Cos. } pq)$ V. T. 205. N°. 6 et T. 206. N°. 1.
- 13) $\int \frac{\text{Cos. } p x}{q^4 - x^4} dx = \frac{\pi}{4q^3} (e^{-pq} + \text{Sin. } pq)$ V. T. 205. N°. 5 et T. 206. N°. 2.
- 14) $\int \frac{x \text{Cos. } p x}{q^4 - x^4} dx = \frac{1}{4q^2} \{ 2 \text{Ci.}(pq) \cdot \text{Cos. } pq + 2 \text{Si.}(pq) \cdot \text{Sin. } pq - e^{-pq} \text{Ei.}(pq) - e^{pq} \text{Ei.}(-pq) \}$ V. T. 205. N°. 11 et T. 206. N°. 10.
- 15) $\int \frac{x^2 \text{Cos. } p x}{q^4 - x^4} dx = \frac{\pi}{4q} (\text{Sin. } pq - e^{-pq})$ V. T. 205. N°. 5 et T. 206. N°. 2.

$$16) \int \frac{x^3 \text{Cos. } px}{q^4 - x^4} dx = \frac{1}{4} \{ 2 \text{Ci.}(pq) \cdot \text{Cos. } pq + 2 \text{Si.}(pq) \cdot \text{Sin. } pq + e^{-pq} \text{Ei.}(pq) + e^{pq} \text{Ei.}(-pq) \} \quad \left. \begin{array}{l} \text{V. T. 205. N}^\circ. \\ 11 \text{ et T. 206.} \\ \text{N}^\circ. 10. \end{array} \right\}$$

$$17) \int \frac{x \text{Sin. } ax}{1 + x^{2b}} dx = \frac{\pi}{2b} e^{-a} - \frac{\pi}{b} \sum_1^{b-1} e^{-a \text{Cos. } \frac{n\pi}{b}} \text{Cos.} \left(\frac{n\pi}{b} - a \text{Sin. } \frac{n\pi}{b} \right), b \text{ impair};$$

$$18) = \frac{\pi}{b} \sum_1^{b-1} e^{-a \text{Cos.} \left(\frac{2n+1}{2b} \pi \right)} \text{Cos.} \left\{ \frac{2n+1}{2b} \pi - a \text{Sin.} \left(\frac{2n+1}{2b} \pi \right) \right\}, b \text{ pair}; \quad \left. \begin{array}{l} \text{Plana,} \\ \text{Mém. Tu-} \\ \text{rin. 1818.} \\ 7. \text{ II. } 16, \\ 15. \end{array} \right\}$$

$$19) \int \frac{\text{Cos. } ax}{1 + x^{2b}} dx = \frac{\pi}{2b} e^{-a} - \frac{\pi}{b} \sum_1^{b-1} e^{-a \text{Cos. } \frac{n\pi}{b}} \text{Cos.} \left(\frac{n\pi}{b} - a \text{Sin. } \frac{n\pi}{b} \right), b \text{ impair};$$

$$20) = \frac{\pi}{b} \sum_1^{b-1} e^{-a \text{Cos.} \left(\frac{2n+1}{2b} \pi \right)} \text{Cos.} \left\{ \frac{2n+1}{2b} \pi - a \text{Sin.} \left(\frac{2n+1}{2b} \pi \right) \right\}, b \text{ pair}; \quad \left. \begin{array}{l} \text{Poisson, P.} \\ 16. 215. \text{N}^\circ. \\ 4, 5. - \text{Pla-} \\ \text{na, Mém.} \\ \text{Turin. 1818.} \\ 7. \text{ II. } 16, \\ 15. \end{array} \right\}$$

$$21) \int \frac{\text{Cos. } px}{1 + x^b} x^{a-1} dx = \frac{\pi}{b} \sum_1^{\frac{b}{2}} e^{-p \text{Sin. } \frac{2n-1}{b} \pi} \text{Sin.} \left\{ \frac{2n-1}{b} a\pi + p \text{Cos.} \left(\frac{2n-1}{b} \pi \right) \right\}, a \text{ im-}$$

$$22) = 0, a \text{ pair};$$

Dans (21) et (22) b est pair et $a < b + 1$.

$$23) \int \frac{\text{Cos. } px}{b^c + x^c} x^{a-1} dx = \frac{\pi}{c b^{c-a}} \sum_1^{\frac{c}{2}} e^{-bp \text{Sin.} \left(\frac{2n-1}{c} \pi \right)} \text{Sin.} \left\{ \frac{2n-1}{c} a\pi + bp \text{Cos.} \left(\frac{2n-1}{c} \pi \right) \right\} \quad \left. \begin{array}{l} \text{Schlömle,} \\ \text{Stud. II.} \\ 16. \end{array} \right\}$$

$$24) \int \frac{\text{Cos. } px}{b^c - x^c} x^{a-1} dx = \frac{\pi}{c b^{c-a}} \sum_0^{\frac{c}{2}-1} e^{-bp \text{Sin.} \frac{2n\pi}{c}} \text{Sin.} \left(\frac{2na\pi}{c} + bp \text{Cos.} \frac{2n\pi}{c} \right)$$

Dans (23) et (24) a est impair, c est pair, et $a < c + 1$.

$$25) \int \frac{\text{Cos.}(px^2) - \text{Sin.}(px^2)}{1 - x^4} dx = \frac{1}{4} \pi (\text{Sin. } p + \text{Cos. } p) \quad \text{Cauchy, Sav. Etr. 1827. 124. Note 18.}$$

$$26) \int \frac{\text{Cos. } 2px}{1 + x^4} dx = \frac{\pi}{2} e^{-p\sqrt{2}} \{ \text{Sin.}(p\sqrt{2}) + \text{Cos.}(p\sqrt{2}) \} \quad \text{Helmiling, Transf. II. S. 131.}$$

$$27) \int \frac{\text{Sin.}(p\pi - r^a x^a)}{q^2 + x^{2a}} dx = \frac{1}{2} e^{-qr^a} q^{2p-1} p \pi (1 + \text{Cot. } p\pi)$$

$$28) \int \frac{\text{Sin.}(p\pi - r^a x^a)}{q^2 e^{2\lambda i} + x^{2a}} dx = \frac{1}{2} e^{-f+gi} e^{p-1} 2\lambda i q^{2p-1} p \pi (1 + \text{Cot. } p\pi)$$

, où $f = qr^a \text{Cos. } \lambda, g = qr^a \text{Sin. } \lambda$;

- $$\left. \begin{aligned} 1) \int \frac{b^2 (p+x)^2 + a(a+1)}{(p+x)^{a+2}} \sin. b x \, dx &= \frac{b}{p^a} \\ 2) \int \frac{b^2 (p+x)^2 + a(a+1)}{(p+x)^{a+2}} \cos. b x \, dx &= \frac{a}{p^{a+1}} \end{aligned} \right\} \text{Meyer, Int. Déf. 368.}$$
- $$3) \int \frac{x \sin. p x}{(q^2 + x^2)^2} \, dx = \frac{\pi}{4q} p e^{-pq} \quad \text{Legendre, Exere. 3. 43. — Helmling, Transf. II. S. 62.}$$
- $$4) \int \frac{x^3 \sin. p x}{(q^2 + x^2)^2} \, dx = \frac{2-pq}{4} \pi e^{-pq} \quad \text{Helmling, Transf. II. S. 62.}$$
- $$\left. \begin{aligned} 5) \int \frac{x \sin. p x}{(q^2 + x^2)^3} \, dx &= \frac{p^2 q + p}{16 q^3} \pi e^{-pq} \\ 6) \int \frac{x \sin. p x}{(q^2 + x^2)^4} \, dx &= \frac{3p + 3p^2 q + p^3 q^2}{96 q^5} \pi e^{-pq} \end{aligned} \right\} \text{Legendre, Exere. 3. 43.}$$
- $$7) \int \frac{\cos. p x}{(q^2 + x^2)^2} \, dx = \frac{1+pq}{4 q^3} \pi e^{-pq} \quad \text{Legendre, Exere. 3. 43. — Plana, Mém. Turin. 1818. 7. I. 6. — Helmling, Transf. II. S. 62.}$$
- $$8) \int \frac{x^2 \cos. p x}{(q^2 + x^2)^2} \, dx = \frac{1-pq}{4 q} \pi e^{-pq} \quad \text{Plana, Mém. Turin. 1818. 7. I. 6.}$$
- $$9) \int \frac{\cos. p x}{(q^2 + x^2)^3} \, dx = \frac{3 + 3pq + p^2 q^2}{16 q^5} \pi e^{-pq} \quad \text{Legendre, Exere. 3. 43.}$$
- $$\left. \begin{aligned} 10) \int \frac{x \sin x}{(q^2 + x^2)^{a+1}} \, dx &= \frac{\pi e^{-q}}{1^{a/1} 2 (2q)^a} \sum_0^{\infty} \frac{(a-n)^{2n/1}}{1^{n/1} (2q)^n} \\ 11) \int \frac{\cos. x}{(q^2 + x^2)^{a+1}} \, dx &= \frac{\pi e^{-q}}{1^{a/1} 2 (2q)^{a+1}} \sum_0^{\infty} \frac{(a-n+1)^{2n/1}}{1^{n/1} (2q)^n} \end{aligned} \right\} \text{Schlömle, Cr. 33. 268.}$$
- $$\left. \begin{aligned} 12) \int \frac{x \sin. p x}{(q^2 + x^2)^{a+1}} \, dx &= \frac{\pi}{1^{a/1}} \frac{p e^{-pq}}{2^{a+1}} \sum_0^{\infty} \frac{(a-n)^{2n/1}}{2^{n/2}} \frac{p^{a-n-1}}{q^{a+n}} \\ 13) \int \frac{\cos. p x}{(q^2 + x^2)^a} \, dx &= \frac{\pi}{1^{a-1/1}} \frac{e^{-pq}}{2^a} \sum_0^{\infty} \frac{(a-n+1)^{2n/1}}{2^{n/2}} \frac{p^{a-n-1}}{q^{a+n}} \end{aligned} \right\} \text{Legendre, Exere. 3. 43. — Schlömle, Cr. 33. 353. — Id., Stetl. II. 14. — Id., Beitr. II. 3.}$$
- $$14) \int \frac{\cos. p x}{(q + x^2)^a} \, dx = \frac{(-1)^a \pi}{2} \frac{d^a}{d q^a} \cdot \frac{e^{-p \sqrt{q}}}{\sqrt{q}} \quad \text{Poisson, P. 16. 215. N°. 7.}$$
- $$15) \quad = \frac{(-1)^a \pi}{1^{a-1/1}} \frac{d^a}{p \, d q^a} \cdot e^{-p \sqrt{q}} \quad \text{Schlömle, Int. 27.}$$

$$16) \int \frac{x \sin. p x}{(q^2 + x^2)^a} dx = \frac{(-1)^{a-1}}{1^{a-1/1}} \frac{\pi}{2} \frac{d^{a-1}}{dq^{a-1}} \cdot e^{-pV'q} \quad \text{Schlömlich, Int. 27.}$$

$$17) \int \frac{\cos. p x}{(q^2 - x^2)^2} dx = \frac{\pi}{4q^3} (\sin. p q - p q \cos. p q) \quad \text{Plana, Mém. Turin. 1818. 7. I. 3.}$$

$$1) \int \frac{\sin. a x. \sin. b x}{q^2 + x^2} dx = \pi e^{-aq} \frac{e^{bq} - e^{-bq}}{4q}, \quad 0 < b < a;$$

$$2) \int \frac{x \sin. a x. \cos. b x}{q^2 + x^2} dx = \pi e^{-aq} \frac{e^{-bq} + e^{bq}}{4}, \quad 0 < b < a;$$

$$3) \quad = \pi e^{-bq} \frac{e^{-aq} - e^{aq}}{4}, \quad a < b < \infty;$$

$$4) \int \frac{\cos. a x. \cos. b x}{q^2 + x^2} dx = \pi e^{-aq} \frac{e^{bq} + e^{-bq}}{4q}, \quad 0 < b < a;$$

$$5) \quad = \pi e^{-bq} \frac{e^{aq} + e^{-aq}}{4q}, \quad a < b < \infty;$$

$$6) \int \frac{\sin.^2 a x. \cos.^2 b x}{q^2 + x^2} dx = \frac{\pi}{8q} \left\{ -\frac{1}{2} e^{-2q(a+b)} + e^{-2bq} - \frac{1}{2} e^{-2q(a-b)} - e^{-2aq} + 1 \right\}, \quad a > b;$$

$$7) \quad = \frac{\pi}{16q} (1 - e^{-4aq}), \quad a = b;$$

$$8) \quad = \frac{\pi}{8q} \left\{ -\frac{1}{2} e^{-2q(a+b)} + e^{-2bq} - \frac{1}{2} e^{-2q(b-a)} - e^{-2aq} + 1 \right\}, \quad a < b;$$

$$9) \int \frac{x \sin. a x. \cos.^2 b x}{q^2 + x^2} dx = \frac{\pi}{4} \left\{ \frac{1}{2} e^{-q(a+2b)} + e^{-q(a-2b)} + e^{-aq} \right\}, \quad a > 2b;$$

$$10) \quad = \frac{\pi}{4} \left\{ \frac{1}{2} e^{-3aq} + e^{-aq} \right\}, \quad a = 2b;$$

$$11) \quad = \frac{\pi}{4} \left\{ \frac{1}{2} e^{-q(a+2b)} + e^{-q(2b-a)} + e^{-aq} \right\}, \quad a < 2b;$$

$$12) \int \frac{\cos. \lambda - \cos. b \lambda x}{q^2 + x^2} \cos. a x dx = \frac{\pi}{2q} e^{-aq} \left\{ \cos. \lambda - \frac{1}{2} (e^{bq} + e^{-bq}) \right\}, \quad 0 < \lambda < \frac{a}{b};$$

Boncompagni, Cr. 25. 74. — Arndt, Gr. 11. 70. — Dienger, Gr. 12. 97 (les form. 2) à 4)). — Schlömlich, Stud. 1. 18. (les form. 1) à 4))

Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 22.

- 13) $\int \frac{\cos.\lambda - \cos.b\lambda x}{q^2 + x^2} \cos.ax dx = \frac{\pi}{2q} e^{-aq} \cos.\lambda - \frac{\pi}{4q} e^{-bq\lambda} (e^{aq} + e^{-aq}), \frac{a}{b} < \lambda < \infty;$
- 14) $\int \frac{\cos.\lambda - \cos.b\lambda x}{q^2 + x^2} x \sin ax dx = \frac{1}{2} \pi e^{-aq} \left\{ \cos.\lambda - \frac{1}{2} (e^{bq\lambda} + e^{-bq\lambda}) \right\}, 0 < \lambda < \frac{a}{b};$
- 15) $= \frac{1}{2} \pi e^{-aq} \cos.\lambda - \frac{1}{4} \pi e^{-bq\lambda} (e^{aq} - e^{-aq}), \frac{a}{b} < \lambda < \infty;$
- 16) $\int \frac{x \sin.x + q \cos.x}{q^2 + x^2} dx = \pi e^{-q} \text{ Poisson, P. 19. 404. N}^\circ. 68.$
- 17) $\int \frac{x \sin.px - q \cos.px}{q^2 + x^2} dx = 0 \text{ Poisson, Chal. 153.}$
- 18) $\int \frac{\cos.\lambda x - \cos.q\lambda}{q^2 - x^2} dx = \frac{\pi}{2q} \sin.q\lambda \text{ Poisson, P. 18. 295. N}^\circ. 38. — Schellbach, Cr. 48. 207.$
- 19) $\int \frac{(1-x^2) \cos.2x + 2x \sin.2x}{(1+x^2)^2} dx = \frac{2\pi}{e^2} \text{ Legendre, Exerc. 3. 41.}$
- 20) $\int \frac{\cos.(a^2 x^2) - \sin.(a^2 x^2)}{q^4 + x^4} dx = \frac{\pi}{2q^3 \sqrt{2}} e^{-a^2 q^2} \text{ Schlömilch, Gr. 11. 174.}$

- 1) $\int \frac{a+bx}{y+2hx+x^2} \sin.px dx = \left\{ b \cos.ph - \frac{a-bh}{\sqrt{(y^2-h^2)}} \sin.ph \right\} \pi e^{-p\sqrt{(y^2-h^2)}}, y > h;$
- 2) $\int \frac{a+bx}{y+2hx+x^2} \cos.px dx = \left\{ \frac{a-bh}{\sqrt{(y^2-h^2)}} \cos.ph + b \sin.ph \right\} \pi e^{-p\sqrt{(y^2-h^2)}} \left\{ \begin{array}{l} \text{Laplace,} \\ \text{Prob. L.} \\ \text{1. 26.} \end{array} \right.$
- 3) $\int \frac{(e^{ac} + e^{-ac}) \cos.ax - (e^{ac} - e^{-ac}) i \sin.ax}{b^2 + (x+ci)^2} dx = \frac{\pi}{b} (e^{-ab} - e^{ab}), c > b;$
- 4) $= \frac{2\pi}{b} e^{-ab}, c < b;$
- 5) $\int \frac{a^2+x^2}{b^2+(a^2+x^2)^2} \sin.2ax dx = \frac{\pi e^{-2aq}}{2b^2 \sqrt{(b^2+a^4)}} \{ (pa^2 + pq^2 - p^2q) \cos.2ap + (a^2q - p^3 - q^3) \sin.2ap \}$
- 6) $\int \frac{\cos.2ax}{b^2+(a^2+x^2)^2} dx = \frac{\pi e^{-2aq}}{2b \sqrt{(b^2+a^4)}} (q \sin.2ap + p \cos.2ap)$
- , où $2p = \sqrt{\{\sqrt{(b^2+a^4)}+b\}} - \sqrt{\{\sqrt{(b^2+a^4)}-b\}}$, $2q = \sqrt{\{\sqrt{(b^2+a^4)}+b\}} + \sqrt{\{\sqrt{(b^2+a^4)}-b\}}$;

$$\begin{aligned}
 7) \int \frac{x \operatorname{Sin}. a x}{x^4 + 2 b^2 x^2 \operatorname{Cos}. 2 \lambda + b^4} dx &= \frac{\pi}{2 b^2} e^{-ab \operatorname{Cos}. \lambda} \operatorname{Cosec}. 2 \lambda. \operatorname{Sin}. (a b \operatorname{Sin}. \lambda) \\
 8) \int \frac{\operatorname{Cos}. a x}{x^4 + 2 b^2 x^2 \operatorname{Cos}. 2 \lambda + b^4} dx &= \frac{\pi}{2 b^3} e^{-ab \operatorname{Cos}. \lambda} \operatorname{Cosec}. 2 \lambda. \operatorname{Sin}. (\lambda + a b \operatorname{Sin}. \lambda) \\
 9) \int \frac{x^3 \operatorname{Sin}. a x}{x^4 + 2 b^2 x^2 \operatorname{Cos}. 2 \lambda + b^4} dx &= \frac{\pi}{2} e^{-ab \operatorname{Cos}. \lambda} \operatorname{Cosec}. 2 \lambda. \operatorname{Sin}. (2 \lambda - a b \operatorname{Sin}. \lambda) \\
 10) \int \frac{x^2 \operatorname{Cos}. a x}{x^4 + 2 b^2 x^2 \operatorname{Cos}. 2 \lambda + b^4} dx &= \frac{\pi}{2 b} e^{-ab \operatorname{Cos}. \lambda} \operatorname{Cosec}. 2 \lambda. \operatorname{Sin}. (\lambda - a b \operatorname{Sin}. \lambda) \\
 11) \int \frac{x \operatorname{Tang}. a x}{x^4 + 2 b^2 x^2 \operatorname{Cos}. 2 \lambda + b^4} dx &= \frac{\pi}{b^2} \frac{e^{-2ab \operatorname{Cos}. \lambda} \operatorname{Sin}. (2 a b \operatorname{Sin}. \lambda). \operatorname{Cosec}. 2 \lambda}{1 + 2 e^{-2ab \operatorname{Cos}. \lambda} \operatorname{Cos}. (2 a b \operatorname{Sin}. \lambda) + e^{-4ab \operatorname{Cos}. \lambda}} \\
 12) \int \frac{x \operatorname{Cot}. a x}{x^4 + 2 b^2 x^2 \operatorname{Cos}. 2 \lambda + b^4} dx &= \frac{\pi}{b^2} \frac{e^{-2ab \operatorname{Cos}. \lambda} \operatorname{Sin}. (2 a b \operatorname{Sin}. \lambda). \operatorname{Cosec}. 2 \lambda}{1 - 2 e^{-2ab \operatorname{Cos}. \lambda} \operatorname{Cos}. (2 a b \operatorname{Sin}. \lambda) + e^{-4ab \operatorname{Cos}. \lambda}} \\
 13) \int \frac{x \operatorname{Cosec}. a x}{x^4 + 2 b^2 x^2 \operatorname{Cos}. 2 \lambda + b^4} dx &= \frac{\pi}{b^2} \frac{(1 + e^{-2ab \operatorname{Cos}. \lambda}) \operatorname{Cosec}. 2 \lambda}{1 - 2 e^{-2ab \operatorname{Cos}. \lambda} \operatorname{Cos}. (2 a b \operatorname{Sin}. \lambda) + e^{-4ab \operatorname{Cos}. \lambda}}
 \end{aligned}$$

Plana, Mém. Turin. 1818. 7. I.
 7. — Legendre, Exerc. 3. 44. — Helmling, Transf. 64, 63.
 Legendre, Exerc. 3. 44. — Helmling, Transf. II. S. 62.
 Legendre, Exerc. 3. 44. — Helmling, Transf. 65.
 Plana, Mém. Turin. 1818. 7. II. 10.

$$\begin{aligned}
 1) \int \frac{\operatorname{Sin}. p x}{q^3 + q^2 x + q x^2 + x^3} dx &= \frac{1}{4 q^2} \{e^{-pq} \operatorname{Ei}. (pq) - e^{pq} \operatorname{Ei}. (-pq) + \\
 &\quad + 2 \operatorname{Ci}. (pq). \operatorname{Sin}. pq - 2 \operatorname{Si}. (pq). \operatorname{Cos}. pq - \pi (e^{-pq} - \operatorname{Cos}. pq)\} \\
 2) \int \frac{x \operatorname{Sin}. p x}{q^3 + q^2 x + q x^2 + x^3} dx &= \frac{1}{4 q} \{e^{-pq} \operatorname{Ei}. (pq) - e^{pq} \operatorname{Ei}. (-pq) - \\
 &\quad - 2 \operatorname{Ci}. (pq). \operatorname{Sin}. pq + 2 \operatorname{Si}. (pq). \operatorname{Cos}. pq + \pi (e^{-pq} - \operatorname{Cos}. pq)\} \\
 3) \int \frac{x^2 \operatorname{Sin}. p x}{q^3 + q^2 x + q x^2 + x^3} dx &= \frac{1}{4} \{-e^{-pq} \operatorname{Ei}. (pq) + e^{pq} \operatorname{Ei}. (-pq) + \\
 &\quad + 2 \operatorname{Ci}. (pq). \operatorname{Sin}. pq - 2 \operatorname{Si}. (pq). \operatorname{Cos}. pq + \pi (e^{-pq} + \operatorname{Cos}. pq)\} \\
 4) \int \frac{\operatorname{Cos}. p x}{q^3 + q^2 x + q x^2 + x^3} dx &= \frac{1}{4 q^2} \{e^{-pq} \operatorname{Ei}. (pq) + e^{pq} \operatorname{Ei}. (-pq) - \\
 &\quad - 2 \operatorname{Ci}. (pq). \operatorname{Cos}. pq - 2 \operatorname{Si}. (pq). \operatorname{Sin}. pq + \pi (e^{-pq} + \operatorname{Sin}. pq)\} \\
 5) \int \frac{x \operatorname{Cos}. p x}{q^3 + q^2 x + q x^2 + x^3} dx &= \frac{1}{4 q} \{-e^{-pq} \operatorname{Ei}. (pq) - e^{pq} \operatorname{Ei}. (-pq) + \\
 &\quad + 2 \operatorname{Ci}. (pq). \operatorname{Cos}. pq + 2 \operatorname{Si}. (pq). \operatorname{Sin}. pq + \pi (e^{-pq} - \operatorname{Sin}. pq)\}
 \end{aligned}$$

V. T. 203. N°. 7 et T. 205. N°. 6, 10.
 V. T. 203. N°. 8 et T. 205. N°. 5, 11.

$$\begin{aligned}
 6) \int \frac{x^2 \cos. p x}{q^3 + q^2 x + q x^2 + x^3} dx &= \frac{-1}{4} \{ e^{-pq} Ei. (p q) + e^{pq} Ei. (-p q) + \\
 &\quad + 2 Ci. (p q). \cos. p q + 2 Si. (p q). \sin. p q + \pi (e^{-pq} - \sin. p q) \} \\
 7) \int \frac{\sin. p x}{q^3 - q^2 x + q x^2 - x^3} dx &= \frac{1}{4 q^2} \{ e^{-pq} Ei. (p q) - e^{pq} Ei. (-p q) + \\
 &\quad + 2 Ci. (p q). \sin. p q - 2 Si. (p q). \cos. p q + \pi (e^{-pq} - \cos. p q) \} \\
 8) \int \frac{x \sin. p x}{q^3 - q^2 x + q x^2 - x^3} dx &= \frac{1}{4 q} \{ -e^{-pq} Ei. (p q) + e^{pq} Ei. (-p q) + \\
 &\quad + 2 Ci. (p q). \sin. p q - 2 Si. (p q). \cos. p q + \pi (e^{-pq} - \cos. p q) \} \\
 9) \int \frac{x^2 \sin. p x}{q^3 - q^2 x + q x^2 - x^3} dx &= \frac{1}{4} \{ -e^{-pq} Ei. (p q) + e^{pq} Ei. (-p q) + \\
 &\quad + 2 Ci. (p q). \sin. p q - 2 Si. (p q). \cos. p q - \pi (e^{-pq} + \cos. p q) \} \\
 10) \int \frac{\cos. p x}{q^3 - q^2 x + q x^2 - x^3} dx &= \frac{1}{4 q^2} \{ -e^{-pq} Ei. (p q) - e^{pq} Ei. (-p q) + \\
 &\quad + 2 Ci. (p q). \cos. p q + 2 Si. (p q). \sin. p q + \pi (e^{-pq} + \sin. p q) \} \\
 11) \int \frac{x \cos. p x}{q^3 - q^2 x + q x^2 - x^3} dx &= \frac{1}{4 q} \{ -e^{-pq} Ei. (p q) - e^{pq} Ei. (-p q) + \\
 &\quad + 2 Ci. (p q). \cos. p q + 2 Si. (p q). \sin. p q - \pi (e^{-pq} - \sin. p q) \} \\
 12) \int \frac{x^2 \cos. p x}{q^3 - q^2 x + q x^2 - x^3} dx &= \frac{1}{4} \{ e^{-pq} Ei. (p q) + e^{pq} Ei. (-p q) + \\
 &\quad + 2 Ci. (p q). \cos. p q + 2 Si. (p q). \sin. p q - \pi (e^{-pq} - \sin. p q) \}
 \end{aligned}$$

V. T. 203.
N°. 8 et T.
205. N°. 5,
11.

V. T. 203.
N°. 11 et T.
205. N°. 6,
10.

V. T. 203.
N°. 12 et
T. 205. N°. 5,
11.

$$1) \int \left\{ \cos. x - \frac{1}{1+x} \right\} \frac{dx}{x} = -\Lambda \quad \text{Arndt, Gr. 10. 225. — Id., Gr. 10. 233.}$$

$$2) \int \left\{ \frac{\sin. x}{x} - \frac{1}{1+x} \right\} \frac{dx}{x} = 1 - \Lambda$$

$$3) \int \left\{ \frac{\cos. x - 1}{x^2} + \frac{1}{2(1+x)} \right\} \frac{dx}{x} = \frac{1}{2} \Lambda - \frac{3}{4}$$

Arndt, Gr. 10. 253.

$$4) \int \frac{\sin. p x}{1+x^2} \frac{dx}{x} = \frac{1}{2} \pi (1 - e^{-p}) \quad \begin{array}{l} \text{Legendre, Exerc. 3. 46. — Cauchy, Sav. Extr. 1827. 519. P. 11} \\ \text{§ 7. — Poisson, P. 16. 215. N°. 7. — Serret, L. 8. 1.} \end{array}$$

- 5) $\int \frac{Tang. p x}{1+x^2} \frac{dx}{x} = \frac{1}{2} \pi \frac{e^p - e^{-p}}{e^p + e^{-p}}$ Legendre, Exerc. 5. 35. — Cauchy, Sav. Etr. 1827. 599. Suppl. 2.
- 6) $\int \left\{ Cos. x - \frac{1}{1+x^2} \right\} \frac{dx}{x} = -\Lambda$ Arndt, Gr. 10. 225.
- 7) $\int \frac{Cos. q x - Cos. p x}{1+x^2} \frac{dx}{x^2} = \frac{1}{2} \pi (e^{-p} - e^{-q}) + \frac{1}{2} \pi (p - q)$ Poisson, P. 16. 215. N°. 7.
- 8) $\int \frac{Cos. q x}{1+x^2} \frac{dx}{x^{1-p}} = \frac{1}{4} (-1)^p \pi e^q Cosec. \left(\frac{p-1}{2} \pi \right), 0 < p;$ Meyer, Int. Déf. 156.
- 9) $\int \frac{Sin. q x}{1+x^2} \frac{dx}{x^{1-p}} = \frac{1}{4} (-1)^{p-1} \pi e^q Cosec. \left(\frac{p-1}{2} \pi \right)$ V. T. 212. N°. 8.
- 10) $\int \frac{Sin. q x}{1-x^2} \frac{dx}{x^{2-p}} = -\frac{1}{8} \pi Cos. \left(\frac{p-1}{2} \pi - q \right). Cosec. \left(\frac{p-1}{2} \pi \right)$ Meyer, Int. Déf. 156.
- 11) $\int \frac{Cos. q x}{1-x^2} \frac{dx}{x^{1-p}} = \frac{1}{8} \pi Sin. \left(\frac{p-1}{2} \pi - q \right). Cosec. \left(\frac{p-1}{2} \pi \right)$ V. T. 212. N°. 10.
- 12) $\int \frac{Sin. p x}{q^2 + x^2} \frac{dx}{x} = \frac{\pi}{2 q^2} (1 - e^{-pq})$ Cauchy, P. 19. 511. — Id., P. 28. 147. I. § 5. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 22. — Schlömilch, Stud. II. 14. — Schellbach, Cr. 48. 207.
- 13) $\int \frac{1 - Cos. p x}{q^2 + x^2} \frac{dx}{x^2} = \frac{\pi}{2 q^2} \left\{ p - \frac{1 - e^{-pq}}{q} \right\}$ Minding, Taf. 11.
- 14) $\int \frac{Sin. p x}{q^2 + x^2} \frac{dx}{x^{2a-1}} = (-1)^a \frac{\pi}{2 q^{2a}} e^{-pq}$
- 15) $\int \frac{Cos. p x}{q^2 + x^2} \frac{dx}{x^{2a}} = (-1)^a \frac{\pi}{2 q^{2a+1}} e^{-pq}$ Meyer, Int. Déf. 274.
- 16) $\int \frac{Cos. (p x + \frac{1}{2} r \pi)}{q^2 + x^2} \frac{dx}{x^r} = \frac{\pi e^{-pq}}{4 q^{r+1}}$ Schlömilch, Gr. 11. 174.
- 17) $\int \frac{Sin. p x}{q^2 - x^2} \frac{dx}{x} = \frac{\pi}{2 q^2} (1 - Cos. p q)$ Cauchy, P. 19. 511. — Schlömilch, Stud. II. 15.

- 1) $\int \left\{ \frac{b+x}{a^2 + (b+x)^2} - \frac{b-x}{a^2 + (b-x)^2} \right\} Sin. q x dx = \pi e^{-bq} Cos. b q$
- 2) $\int \left\{ \frac{b+x}{a^2 + (b+x)^2} + \frac{b-x}{a^2 + (b-x)^2} \right\} Cos. q x dx = \pi e^{-bq} Sin. b q$

Cauchy, Sav. Etr. 1827. 124. Note 6.

$$\begin{aligned}
 3) \int \frac{x \sin. p x}{(q^2 + x^2)(q^4 - x^4)} dx &= \frac{\pi}{8 q^4} \left\{ (1 + p q) e^{-pq} - \cos. p q \right\} \\
 4) \int \frac{x^3 \sin. p x}{(q^2 + x^2)(q^4 - x^4)} dx &= \frac{\pi}{8 q^2} \left\{ (1 - p q) e^{-pq} - \cos. p q \right\} \\
 5) \int \frac{x^5 \sin. p x}{(q^2 + x^2)(q^4 - x^4)} dx &= \frac{\pi}{8} \left\{ (p q - 3) e^{-pq} - \cos. p q \right\} \\
 6) \int \frac{\cos. p x}{(q^2 + x^2)(q^4 - x^4)} dx &= \frac{\pi}{8 q^5} \left\{ \sin. p q + (p q + 2) e^{-pq} \right\} \\
 7) \int \frac{x^2 \cos. p x}{(q^2 + x^2)(q^4 - x^4)} dx &= \frac{\pi}{8 q^3} \left\{ \sin. p q - p q e^{-pq} \right\} \\
 8) \int \frac{x^4 \cos. p x}{(q^2 + x^2)(q^4 - x^4)} dx &= \frac{\pi}{8 q} \left\{ \sin. p q + (p q - 2) e^{-pq} \right\} \\
 9) \int \left\{ \frac{b^2}{(a+x)^c} + \frac{c(c+1)}{(a+x)^{c+2}} \right\} \sin. b x dx &= b a^{-c} \\
 10) \int \left\{ \frac{b^2}{(a+x)^c} + \frac{c(c+1)}{(a+x)^{c+2}} \right\} \cos. b x dx &= \frac{c}{a^{c+1}} \\
 11) \int \frac{(a-xi)^{-p} - (a+xi)^{-p}}{2i} \sin. q x dx &= \frac{\pi}{2 \Gamma(p)} q^{p-1} e^{-aq} \\
 12) \int \frac{(a-xi)^{-p} + (a+xi)^{-p}}{2} \cos. q x dx &= \frac{\pi}{2 \Gamma(p)} q^{p-1} e^{-aq} \\
 13) \int \frac{\sin. a x}{x(x^2+2^2)(x^2+4^2)\dots(x^2+4b^2)} dx &= \frac{\pi(-1)^b}{2^{2b-1} 1^{2b/1}} \sum_0^b (-1)^n \binom{2b}{n} e^{2(n-b)a} \\
 14) \int \frac{\cos. a x}{(x^2+1^2)(x^2+3^2)\dots(x^2+(2b+1)^2)} dx &= \frac{\pi(-1)^b}{2^{2b} 1^{2b+1}} \sum_0^b (-1)^n \binom{2b+1}{n} e^{(2n-2b-1)a} \\
 15) \int \frac{(a-xi)^{-p} + (a+xi)^{-p}}{2} x^{2c-1} \sin. q x dx &= \frac{(-1)^c}{2} \frac{\pi}{\Gamma(p)} \frac{d^{2c-1}}{dq^{2c-1}} \cdot q^{p-1} e^{-aq} \\
 16) \int \frac{(a-xi)^{-p} + (a+xi)^{-p}}{2} x^{2c} \cos. q x dx &= \frac{(-1)^c}{2} \frac{\pi}{\Gamma(p)} \frac{d^{2c}}{dq^{2c}} \cdot q^{p-1} e^{-aq} \\
 17) \int \frac{(a-xi)^{-p} - (a+xi)^{-p}}{2i} x^{2c} \sin. q x dx &= \frac{(-1)^c}{2} \frac{\pi}{\Gamma(p)} \frac{d^{2c}}{dq^{2c}} \cdot q^{p-1} e^{-aq}
 \end{aligned}$$

Schlömlich, Beitr. III. § 4.
Cauchy, Lim. Imag. N°. 107. —
Id., P. 28. 147. P. 1. § 3. —
Id., Sav. Etr. 1827. 124. Note 6. —
Id., Exerc. 1827. p. 141.
Schlömlich, Gr. 1. 38.
Cauchy, P. 28. 147 l. 1. § 3.

$$\begin{aligned}
 18) \int \frac{(a-xi)^{-p} - (a+xi)^{-p}}{2i} x^{2c-1} \text{Cos.} q x dx &= \frac{(-1)^{\frac{2c-1}{2}} \pi}{2} \frac{d^{2c-1}}{\Gamma(p) d q^{2c-1}} \cdot q^{p-1} e^{-aq} \quad \left. \begin{array}{l} \text{Cauchy, P. 28.} \\ 147. \text{ P. I. } \S 3. \end{array} \right\} \\
 19) \int \left\{ \frac{(b-xi)^{-a} + (b+xi)^{-a}}{2} \text{Cos.} e x + \frac{(b-xi)^{-a} - (b+xi)^{-a}}{2i} \text{Sin.} e x \right\} dx &= \frac{\pi c^a}{1^{a/1}} e^{-bc}, e > 0; \left. \begin{array}{l} \text{Cauchy,} \\ \text{P. 28.} \\ 147. \text{ P.} \\ \text{III. } \S 3. \end{array} \right\} \\
 20) &= 0, e < 0; \\
 21) \int \frac{(q-xi)^{-a} - (q+xi)^{-a}}{2i} x^b \text{Sin.} \left(\frac{1}{2} b \pi + p x \right) dx &= \frac{\pi}{2 \cdot 1^{a/1}} \frac{d^b}{d p^b} p^{a-1} e^{-pq} \left. \begin{array}{l} \text{Cauchy, Exerc.} \\ 1827. \text{ p. 141.} \end{array} \right\} \\
 22) \int \frac{(q-xi)^{-a} + (q+xi)^{-a}}{2} x^b \text{Cos.} \left(\frac{1}{2} b \pi + p x \right) dx &= \frac{\pi}{2 \cdot 1^{a/1}} \frac{d^b}{d p^b} p^{a-1} e^{-pq}
 \end{aligned}$$

$$\begin{aligned}
 1) \int \frac{\text{Sin.} a x}{\text{Cos.} b x} \frac{dx}{x} &= 0, a < b; \\
 2) &= \frac{1 - \text{Cos.} h \pi}{2} \pi, a = 2 b h + e; \left. \begin{array}{l} \text{Legendre, Exerc. 5. 37, 39. — Cisa de Grésy,} \\ \text{Mém. Turin. 1821. 209. II. 59.} \end{array} \right\} \\
 3) \int \frac{\text{Sin.} \{(b-a) x\}}{\text{Cos.} b x} \frac{dx}{x} &= 0, a \text{ très-petit}; \\
 4) &= \frac{1}{2} \pi, a = 0; \left. \begin{array}{l} \text{Cauchy, Sav. Etr. 1827. 599. S. 2.} \end{array} \right\} \\
 5) \int \frac{\text{Sin.} \{(2a+1) b x\}}{\text{Cos.} b x} \frac{dx}{x} &= \frac{1}{2} \pi \quad \text{Legendre, Exerc. 5. 40.} \\
 6) \int \frac{dx}{x \text{Tang.} x} &= \infty \quad \text{Plana, Mém. Turin. 1818. 7. 2. N° 13.}
 \end{aligned}$$

$$\begin{aligned}
 1) \int \frac{\text{Sin.} b x}{\text{Sin.} a x} \frac{dx}{1+x^2} &= \frac{1}{2} \pi \frac{e^b - e^{-b}}{e^a - e^{-a}}, b < a; \\
 2) \int \frac{\text{Cos.} b x}{\text{Sin.} a x} \frac{x dx}{1+x^2} &= \frac{1}{2} \pi \frac{e^b + e^{-b}}{e^a - e^{-a}} \left. \begin{array}{l} \text{Cauchy, Sav. Etr. 1827. 599. P. II. } \S 5. \text{ — Id., Sav. Etr. 1827.} \\ 599. \text{ Suppl. 2. — Cisa de Grésy, Mém. Turin. 1821. 209. II.} \\ 60. — Legendre, Exerc. 5. 29. — Boncompagni, Cr. 25. 74.} \end{array} \right\}
 \end{aligned}$$

- 3) $\int \frac{\text{Sin. } b x}{\text{Sin. } a x} \frac{dx}{1+x^2} = \frac{1}{2} \pi \frac{e^{ar} + e^{-ar} - 2e^{-b}}{e^a - e^{-a}}$ } où $\frac{b}{2a}$ est égal à un nombre entier $+\frac{1}{2}r$; si r
 4) $\int \frac{\text{Cos. } b x}{\text{Sin. } a x} \frac{x dx}{1+x^2} = \frac{1}{2} \pi \frac{e^{ar} - e^{-ar} + 2e^{-a}}{e^a - e^{-a}}$ } est négatif, il faut changer les signes de e^{ar} et de
 } e^{-ar} dans la formule (4).
 Cauchy, Sav. Etr. 1827. 599. P. II. § 7.
- 5) $\int \frac{\text{Sin. } \{(c+2ha)x\}}{\text{Sin. } a x} \frac{dx}{1+x^2} = \frac{1}{2} \pi \frac{e^c + e^{-c} - 2e^{-(c+2ha)}}{e^a - e^{-a}}$ Legendre, Exerc. 5. 31.
- 6) $\int \frac{\text{Sin. } \{(2h+1)ax\}}{\text{Sin. } a x} \frac{dx}{1+x^2} = \frac{1}{2} \pi \frac{e^{2a} + 1 - 2e^{-2ha}}{e^{2a} - 1}$ } Legendre, Exerc. 5. 36. Cauchy, Sav.
 7) $\int \frac{\text{Sin. } 2ha x}{\text{Sin. } a x} \frac{dx}{1+x^2} = \pi \frac{1 - e^{-2ha}}{e^a - e^{-a}}$ } Etr. 1827. 599. S. 1.
- 8) $\int \frac{\text{Cos. } \{(c+2ha)x\}}{\text{Sin. } a x} \frac{x dx}{1+x^2} = \frac{1}{2} \pi \frac{e^c - e^{-c} + 2e^{-(c+2ha)}}{e^a - e^{-a}}$ Legendre, Exerc. 5. 32.
- 9) $\int \frac{\text{Cos. } \{(2h+1)ax\}}{\text{Sin. } a x} \frac{x dx}{1+x^2} = \pi \frac{e^{-(2h+1)a}}{e^a - e^{-a}}$ } Legendre, Exerc. 5. 33, 36. — Cauchy, Sav. Etr.
 10) $\int \frac{\text{Cos. } 2ha x}{\text{Sin. } a x} \frac{x dx}{1+x^2} = \pi \frac{e^{-2ha}}{e^a - e^{-a}}$ } 1827. 599. S. 2.
- 11) $\int \frac{\text{Cos. } \{(a-b)x\}}{\text{Sin. } a x} \frac{x dx}{1+x^2} = \frac{1}{2} \pi \frac{e^a + e^{-a}}{e^a - e^{-a}}, b \text{ très-petit;}$
- 12) $= \pi \frac{e^{-a}}{e^a - e^{-a}}, b = 0;$
- 13) $\int \frac{\text{Cos. } \{(a+b)x\}}{\text{Sin. } a x} \frac{x dx}{1+x^2} = \frac{\pi e^{-a}}{e^a - e^{-a}} - \frac{1}{2} \pi, b \text{ très-petit;}$
- 14) $= \frac{\pi e^{-a}}{e^a - e^{-a}}, b = 0;$
- 15) $\int \frac{\text{Cos. } \{(2c+1)a \pm b\} x}{\text{Sin. } a x} \frac{x dx}{1+x^2} = \pi \frac{e^{-(2c+1)a}}{e^a - e^{-a}} \mp \frac{\pi}{2}, b \text{ très-petit;}$
- 16) $= \pi \frac{e^{-(2c+1)a}}{e^a - e^{-a}}, b = 0;$
- 17) $\int \frac{\text{Sin. } b x \text{ Sin. } \{(2c+1)ax\}}{\text{Sin. } a x} \frac{x dx}{1+x^2} = \frac{1}{2} \pi, b \text{ très-petit;}$
- 18) $= 0, b = 0;$

Cauchy, Sav. Etr. 1827.
 599. S. 2.

Lim. 0 et ∞ .

$$\begin{aligned} 1^o) & \int \frac{\text{Sin.} \{(2c+1)ax\} - e^{-a} \text{Sin.} 2acx}{\text{Sin.} ax} \frac{dx}{1+x^2} = \frac{1}{2} \pi \left\{ \begin{array}{l} \text{où } \frac{b}{2a} \text{ est égal à un nombre pair } + \frac{1}{2} r; \\ \text{Cauchy, Sav. Etr. 1827. 569. Suppl. 1.} \end{array} \right. \\ 2^o) & \int \frac{\text{Sin.} \{(a+b)x\} - e^{-a} \text{Sin.} bx}{\text{Sin.} ax} \frac{dx}{1+x^2} = \frac{1}{2} \pi e^{-ar} \\ 3^o) & \int \frac{x}{\text{Sin.} ax} \frac{dx}{1+x^2} = \frac{\pi}{e^a - e^{-a}} \quad \text{Legendre, Exerc. 4. 132. — Cauchy, Sav. Etr. 1827. 599. P. II.} \\ & \quad \S 5. — \text{Id, Sav. Etr. 1827. 599. S. 2.} \end{aligned}$$

F. Alg. rat. fract. à dén. $1 + x^2$.
Circ. Dir. en dén. mon. $\text{Cos. } x$. } Val. princ. TABLE 216.

Lim. 0 et ∞ .

$$\begin{aligned}
1) & \int \frac{1}{\cos. a x} \frac{dx}{1+x^2} = \frac{\pi}{e^a + e^{-a}} \quad \text{Cauchy, Sav. Etr. 1827. 599. P. II. § 5.} \\
2) & \int \frac{\sin. \{(c+2ha)x\}}{\cos. a x} \frac{x dx}{1+x^2} = -\frac{1}{2} \pi \cos. h \pi \frac{e^c + e^{-c}}{e^a + e^{-a}} + \pi \frac{e^{-(c+2ha)}}{e^a + e^{-a}} \quad \text{Legendre, Exerc. 5. 37.} \\
3) & \int \frac{\sin. \{(2h+1)ax\}}{\cos. a x} \frac{x dx}{1+x^2} = \pi \frac{e^{-(2h+1)a}}{e^a + e^{-a}} \left. \vphantom{\int} \right\} \text{Legendre, Exerc. 5. 37, 41.} \\
4) & \int \frac{\sin. 2 h a x}{\cos. a x} \frac{x dx}{1+x^2} = \pi \frac{e^{-ha} - \cos. h \pi}{e^a + e^{-a}} \left. \vphantom{\int} \right\} \\
5) & \int \frac{\cos. b x}{\cos. a x} \frac{dx}{1+x^2} = \frac{1}{2} \pi \frac{e^b + e^{-b}}{e^a + e^{-a}}, a > b; \quad \text{Cauchy, Sav. Etr. 1827. 599. P. II. § 5. — Legendre, Exerc. 5. 2J. — Cisa de Gréssy, Mém. de Turin. 1821. 209. II. 60. — Bonecompagni, Cr. 25. 74.} \\
6) & = \frac{1}{2} \pi \frac{e^{ar} - e^{-ar} + 2e^{-b}}{e^a + e^{-a}}, \text{ où } \frac{b}{2a} \text{ est égal à un nombre pair } + \frac{1}{2} r; \text{ si ce nombre est impair, ou si } r \text{ est négatif, il faut changer les signes de } e^{ar} \text{ et de } e^{-ar}; \text{ Cauchy, Sav. Etr. 1827. 599. P. II. § 7.} \\
7) & \int \frac{\cos. \{(c+2ha)x\}}{\cos. a x} \frac{dx}{1+x^2} = \frac{1}{2} \pi \cos. h \pi \frac{e^c - e^{-c}}{e^a + e^{-a}} + \pi \frac{e^{-(c+2ha)}}{e^a + e^{-a}}, a < c; \quad \text{Legendre, Exerc. 5. 34.} \\
8) & \int \frac{\cos. \{(2h+1)ax\}}{\cos. a x} \frac{dx}{1+x^2} = \frac{1}{2} \pi \frac{(e^{2a} - 1) \cos. h \pi + 2e^{-2ha}}{e^{2a} + 1} \left. \vphantom{\int} \right\} \text{Legendre, Exerc. 5. 36.} \\
9) & \int \frac{\cos. 2 h a x}{\cos. a x} \frac{dx}{1+x^2} = \pi \frac{e^{-2ha}}{e^a + e^{-a}} \left. \vphantom{\int} \right\}
\end{aligned}$$

- 1) $\int \frac{1}{\sin. a x q^2 + x^2} \frac{x dx}{x} = \frac{\pi e^{aq}}{e^{2aq} - 1}$ Legendre, Exerc. 4. 133. — B. done, Mém. Turin. 1812. 231.
Art. 3. N°. 39. — Schlömilch, Beitr. II. § 4.
- 2) $\int \frac{\sin. b x}{\sin. a x q^2 + x^2} \frac{dx}{x} = \frac{\pi}{2 q} \frac{e^{bq} - e^{-bq}}{e^{aq} - e^{-aq}}$
- 3) $\int \frac{\cos. b x}{\sin. a x q^2 + x^2} \frac{x dx}{x} = \frac{\pi}{2} \frac{e^{bq} + e^{-bq}}{e^{aq} - e^{-aq}}$ } Cauchy, Lim. Imag. Add. 33. — Boncompagni, Cr. 25. 74.
- 4) $\int \frac{\sin. k x}{\sin. x} \frac{x \text{Tang. } x}{p^2 + 4x^2} dx = 0$ } , $k = \infty$; Schlömilch, Beitr. II. 4.
- 5) $\int \frac{\sin. k x}{\sin. x} \frac{\text{Tang. } x \sin. x}{p^2 + x^2} dx = 0$ } Elles sont fautives: au lieu de k mettez $2k + 1$; leurs valeurs sont alors ∞ .
- 6) $\int \frac{\cos. k x}{\sin. x} \frac{x \text{Cot. } x}{p^2 + 4x^2} dx = 0$, $k = \infty$; Meyer, Int. Déf. 221.
Elle est fautive: mettez $2k$ au lieu de k , alors la valeur en est ∞ .
- 7) $\int \frac{\cos. k x}{\sin. x} \frac{x}{p^2 + 4x^2} dx = 0$ }
- 8) $\int \frac{\cos. k x}{\cos. x} \frac{dx}{p^2 + x^2} = 0$ } , $k = \infty$;
Schlömilch, Beitr. II. 4.
- 9) $\int \frac{1}{\cos. x} \frac{dx}{q^2 + x^2} = \frac{\pi}{q} \frac{1}{e^q + e^{-q}}$ }
- 10) $\int \frac{1}{\cos. a x} \frac{dx}{x^2 + q^2} = \frac{\pi}{q} \frac{1}{e^{aq} + e^{-aq}}$ Schlömilch, Beitr. II. § 4.
- 11) $\int \frac{\sin. b x}{\cos. a x} \frac{x}{x^2 + q^2} dx = -\frac{1}{2} \pi \frac{e^{bq} - e^{-bq}}{e^{aq} + e^{-aq}}$ }
- 12) $\int \frac{\cos. b x}{\cos. a x} \frac{dx}{x^2 + q^2} = \frac{\pi}{2 q} \frac{e^{bq} + e^{-bq}}{e^{aq} + e^{-aq}}$ } Cauchy, Lim. Imag. Add. 33. — Boncompagni, Cr. 25. 74.
- 13) $\int \frac{1}{\cos. a x} \frac{dx}{1 + x^2} = \frac{\pi}{2} \frac{(e^{aV^2} + e^{-aV^2}) \cos. \left(\frac{a}{1/2} \right) + (e^{aV^2} - e^{-aV^2}) \sin. \left(\frac{a}{1/2} \right)}{e^{aV^2} + e^{-aV^2} + 2 \cos. (a/2)}$ } Cauchy, Sav. Etr. 1827. 599. P. II. § 5.
- 14) $\int \frac{x}{\sin. a x} \frac{dx}{1 + x^2} = \pi \sin. \left(\frac{a}{1/2} \right) \frac{e^{aV^2} + e^{-aV^2}}{e^{aV^2} + e^{-aV^2} - 2 \cos. (a/2)}$ }

$$\begin{aligned}
 1) \int \frac{\sin. b x}{\cos. a x} \frac{1}{x} \frac{dx}{1+x^2} &= \frac{1}{2} \pi \frac{e^b - e^{-b}}{e^a + e^{-a}}, a > b; \\
 2) &= \frac{1}{2} \pi \frac{e^{ar} + e^{-ar} - 2e^{-b}}{e^a + e^{-a}}, \text{ où } \frac{b}{2a} \text{ est égal à un nombre pair } + \frac{1}{2}r; \text{ si le nom-} \\
 &\text{bre est impair, ou si } r \text{ est négatif, il faut changer les} \\
 &\text{signes de } e^{ar} \text{ et de } e^{-ar}; \\
 &\text{Cauchy, Sav. Etr. 1827. 599. P. II. § 5. — Le-} \\
 &\text{gendre, Exerc. 5. 29. — Cisa de Grésy, Mem. Turin.} \\
 &\text{1821. 209. II. 60. — Boncompagni, Cr. 25. 74.} \\
 3) \int \frac{\sin. \{(c+2ha)x\}}{\cos. a x} \frac{1}{x} \frac{dx}{1+x^2} &= \frac{1}{2} \pi (1 - \cos. h\pi) - \frac{\pi e^{-(c+2ha)}}{e^a + e^{-a}} + \frac{1}{2} \pi \cos. h\pi \frac{e^c + e^{-c}}{e^a + e^{-a}} \\
 4) \int \frac{\sin. \{(2h+1)ax\}}{\cos. a x} \frac{1}{x} \frac{dx}{1+x^2} &= \frac{1}{2} \pi - \pi \frac{e^{-(2h+1)a}}{e^a + e^{-a}} \\
 5) \int \frac{\sin. 2hax}{\cos. a x} \frac{1}{x} \frac{dx}{1+x^2} &= \frac{1}{2} \pi \frac{e^a + e^{-a} - 2e^{ha}}{e^a + e^{-a}} - \frac{1}{2} \pi \cos. h\pi \frac{e^a + e^{-a} - 2}{e^a + e^{-a}} \\
 6) \int \frac{\cos. b x}{\sin. a x} \frac{1}{x} \frac{dx}{q^2 + x^2} &= \frac{\pi}{2q^2} \frac{e^{bq} + e^{-bq}}{e^{aq} - e^{-aq}} \\
 7) \int \frac{\sin. b x}{\cos. a x} \frac{1}{x} \frac{dx}{q^2 + x^2} &= \frac{\pi}{2q^2} \frac{e^{bq} - e^{-bq}}{e^{aq} + e^{-aq}}
 \end{aligned}$$

$\left. \begin{array}{l} a < c; \\ \text{Legendre,} \\ \text{Exerc. 5.} \\ 35, 36. \end{array} \right\}$
 Boncompagni, Cr. 25. 74.

$$\begin{aligned}
 1) \int \frac{\sin. x}{1+2p\cos. 2x+p^2} \frac{dx}{x} &= \frac{1}{2} \pi \frac{1}{1-p^2}, p < 1; \\
 2) &= \frac{1}{2} \pi \frac{1}{p^2-1}, p > 1; \\
 3) \int \frac{\text{Tang. } x}{1+2p\cos. 2x+p^2} \frac{dx}{x} &= \frac{1}{2} \pi \frac{1}{1-p^2}, p < 1; \\
 4) &= \frac{1}{2} \pi \frac{1}{p^2-1}, p > 1; \\
 5) \int \frac{\sin. ax}{1-2p\cos. ax+p^2} \frac{dx}{x} &= \frac{1}{2} \pi \frac{1}{1-p}, p < 1; \\
 6) &= \frac{1}{2p} \pi \frac{1}{1-p}, p > 1; \\
 7) \int \frac{\sin. ax}{1+2p\cos. ax+p^2} \frac{dx}{x} &= \frac{1}{2} \pi \frac{1}{1+p}, p < 1; \\
 8) &= \frac{1}{2p} \pi \frac{1}{1+p}, p > 1;
 \end{aligned}$$

Schlömilch, Gr. 4. 316.
 Plana, Mém. Turin. 1818. 7. II. 13.

Circ. Dir. en dén. trin. $a + b \cos. x + c$.

$$\begin{aligned}
 1) & \int \frac{\sin. x}{1 + 2p \cos. x + p^2} \frac{x}{q^2 + x^2} dx = \frac{\pi}{2} \frac{1}{eq + p} \left. \vphantom{\int} \right\} p^2 < 1; \\
 2) & \int \frac{1 + p \cos. x}{1 + 2p \cos. x + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} \frac{1}{1 + pe^{-q}} \left. \vphantom{\int} \right\} \text{Schlömleib, Stud. II. 18.} \\
 3) & \int \frac{\sin. rx}{1 + 2p \cos. rx + p^2} \frac{x}{q^2 + x^2} dx = \frac{1}{2} \frac{\pi}{e^{qr} + p}, p < 1; \text{Legendre, Exerc. 4. 131.} \\
 4) & = \frac{1}{2p} \frac{\pi}{pe^{qr} + 1}, p > 1; \text{Ohm, Ausw. 26.} \\
 5) & \int \frac{1}{e^{-2ac} + 2 \cos. 2ax + e^{2ac}} \frac{dx}{q^2 + x^2} = \frac{1}{e^{2ac} - e^{-2ac}} \left\{ \frac{\pi}{2q} - \frac{\pi}{q} \frac{e^{-2ac}}{e^{2ac} + e^{-2ac}} \right\} \left. \vphantom{\int} \right\} \text{Poisson, P. 18. 295.} \\
 6) & \int \frac{\sin. 2ax}{e^{-2ac} + 2 \cos. 2ax + e^{2ac}} \frac{x}{q^2 + x^2} dx = \frac{\pi}{2} \frac{e^{-2ac}}{e^{2ac} + e^{-2ac}} \left. \vphantom{\int} \right\} \text{N. 42, il a faut. dans 6) pour dénom.} \\
 7) & \int \frac{\cos. 2ax}{1 + 2p \cos. 2ax + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} \frac{1}{e^{2aq} + p} \text{Legendre, Exerc. 4. 134.} \\
 8) & \int \frac{\sin. ax \sin. bx}{1 + 2p \cos. ax + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{4pq} e^{-bq} \left\{ \frac{pe^{aq}}{1 + pe^{aq}} - \frac{pe^{-aq}}{1 + pe^{-aq}} \right\} \left. \vphantom{\int} \right\} p^2 < 1; \\
 9) & \int \frac{\sin. ax \cos. bx}{1 + 2p \cos. ax + p^2} \frac{x}{q^2 + x^2} dx = \frac{\pi}{4p} e^{-bq} \left\{ \frac{pe^{-aq}}{1 + pe^{-aq}} - \frac{pe^{aq}}{1 + pe^{aq}} \right\} \left. \vphantom{\int} \right\} \text{Bonecompagni, Cr. 25. 74.} \\
 10) & \int \frac{\sin. ax}{1 + 2p \cos. ax + p^2} \frac{dx}{1 + x^{2b}} = \frac{\pi}{2b} \frac{e^{-a}}{1 + pe^{-a}} - \frac{\pi}{b} \sum_{i=1}^{\frac{b-1}{2}} \frac{e^{-a \cos. \frac{n\pi}{b}} \cos. \frac{2n\pi}{b} \cos. \left(a \sin. \frac{n\pi}{b} \right) + pe^{-a \cos. \frac{n\pi}{b}}}{1 + 2pe^{-a \cos. \frac{n\pi}{b}} \cos. \left(a \sin. \frac{n\pi}{b} \right) + p^2 e^{-2a \cos. \frac{n\pi}{b}}} \\
 & - \frac{\pi}{b} \sum_{i=1}^{\frac{b-1}{2}} \frac{e^{-a \cos. \frac{n\pi}{b}} \sin. \frac{2n\pi}{b} \sin. \left(a \sin. \frac{n\pi}{b} \right)}{1 + 2pe^{-a \cos. \frac{n\pi}{b}} \cos. \left(a \sin. \frac{n\pi}{b} \right) + p^2 e^{-2a \cos. \frac{n\pi}{b}}}, b \text{ impair:} \\
 11) & = \frac{\pi}{b} \sum_{i=1}^{\frac{b}{2}-1} \frac{e^{-a \cos. \left(\frac{2n+1}{2b} \pi \right)} \cos. \left(a \sin. \left\{ \frac{2n+1}{2b} \pi \right\} \right) + pe^{-a \cos. \left(\frac{2n+1}{2b} \pi \right)}}{1 + 2pe^{-a \cos. \left(\frac{2n+1}{2b} \pi \right)} \cos. \left(a \sin. \left\{ \frac{2n+1}{2b} \pi \right\} \right) + p^2 e^{-2a \cos. \left(\frac{2n+1}{2b} \pi \right)}} \cos. \left(\frac{2n+1}{b} \pi \right) \\
 & + \frac{\pi}{b} \sum_{i=0}^{\frac{b}{2}-1} \frac{e^{-a \cos. \left(\frac{2n+1}{2b} \pi \right)} \sin. \left(a \sin. \left\{ \frac{2n+1}{2b} \pi \right\} \right) \sin. \left(\frac{2n+1}{b} \pi \right)}{1 + 2pe^{-a \cos. \left(\frac{2n+1}{2b} \pi \right)} \cos. \left(a \sin. \left\{ \frac{2n+1}{2b} \pi \right\} \right) + p^2 e^{-2a \cos. \left(\frac{2n+1}{2b} \pi \right)}}, b \text{ pair:}
 \end{aligned}$$

$$\begin{aligned}
 12) \int \frac{p + \cos ax}{1 + 2p \cos ax + p^2} \frac{dx}{1 + x^{2b}} &= \frac{\pi}{2b} \frac{e^{-a}}{1 + pe^{-a}} - \frac{\pi}{b} \sum_{n=1}^{\frac{b-1}{2}} \frac{e^{-a \cos \frac{n\pi}{b}} \sin \frac{n\pi}{b} \cdot \sin \left(a \sin \frac{n\pi}{b} \right)}{1 + 2pe^{-a \cos \frac{n\pi}{b}} \cos \left(a \sin \frac{n\pi}{b} \right) + p^2 e^{-2a \cos \frac{n\pi}{b}}} \\
 &- \frac{\pi}{b} \sum_{n=1}^{\frac{b-1}{2}} \cos \frac{n\pi}{b} \frac{e^{-a \cos \frac{n\pi}{b}} \cos \left(a \sin \frac{n\pi}{b} \right) + pe^{-2a \cos \frac{n\pi}{b}}}{1 + 2pe^{-a \cos \frac{n\pi}{b}} \cos \left(a \sin \frac{n\pi}{b} \right) + p^2 e^{-2a \cos \frac{n\pi}{b}}}, \text{ } b \text{ impair;} \\
 13) &= \frac{\pi}{b} \sum_{n=1}^{\frac{b-1}{2}} \frac{e^{-a \cos \left(\frac{2n+1}{2b} \pi \right)} \sin \left(a \sin \left\{ \frac{2n+1}{2b} \pi \right\} \right) \cdot \sin \left(\frac{2n+1}{2b} \pi \right)}{1 + 2pe^{-a \cos \left(\frac{2n+1}{2b} \pi \right)} \cos \left(a \sin \left\{ \frac{2n+1}{2b} \pi \right\} \right) + p^2 e^{-2a \cos \left(\frac{2n+1}{2b} \pi \right)}} \\
 &+ \frac{\pi}{b} \sum_{n=1}^{\frac{b-1}{2}} \cos \left(\frac{2n+1}{2b} \pi \right) \frac{e^{-a \cos \left(\frac{2n+1}{2b} \pi \right)} \cos \left(a \sin \left\{ \frac{2n+1}{2b} \pi \right\} \right) + p^2 e^{-2a \cos \left(\frac{2n+1}{2b} \pi \right)}}{1 + 2pe^{-a \cos \left(\frac{2n+1}{2b} \pi \right)} \cos \left(a \sin \left\{ \frac{2n+1}{2b} \pi \right\} \right) + p^2 e^{-2a \cos \left(\frac{2n+1}{2b} \pi \right)}}, \text{ } b \text{ pair;}
 \end{aligned}$$

Les formules (10) à (13) valent pour $p < 1$; voyez: Plana, Mém. Turin. 1818. 7 III. 16.

$$\begin{aligned}
 1) \int \frac{\sin ax \sin bx}{1 - 2p \cos ax + p^2} \frac{dx}{1 + x^2} &= \frac{1}{4} \pi \frac{e^b - e^{-b}}{e^a - p}, \quad b < a; \\
 2) &= \frac{1}{4} \pi \left\{ \frac{p^c e^q - e^{-b}}{e^a - p} + \frac{e^{-b} - p^c e^{-q}}{e^{-a} - p} \right\}, \quad b = ac + q; \\
 3) &= \frac{1}{4} \pi \left\{ \frac{p^r e^{b-ar} - e^{-b}}{e^a - p} + e^{-b} \frac{1 - p^r e^{ar}}{e^{-a} - p} \right\}, \quad b = ra + q; \\
 4) \int \frac{\cos ax - p \cos bx}{1 - 2p \cos ax + p^2} \frac{dx}{1 + x^2} &= \frac{1}{4} \pi \left\{ \frac{p^r e^{b-ra} + e^{-b}}{e^a - p} + e^{-b} \frac{1 - p^r e^{ra}}{e^{-a} - p} \right\}, \quad b = ra + q; \\
 5) \int \frac{\sin x}{1 - 2p \cos x + p^2} \frac{x}{q^2 + x^2} dx &= \frac{\pi}{2} \frac{1}{e^q - p}, \quad p^2 \leq 1; \quad \text{Schlömilch, Beitr. II. § 4.} \\
 6) \int \frac{1}{1 - 2p \cos x + p^2} \frac{dx}{q^2 + x^2} &= \frac{\pi}{2q} \frac{1}{1 - p^2} \frac{e^q + p}{e^q - p} \quad \left. \vphantom{\int} \right\} p^2 < 1; \\
 7) \int \frac{\cos x}{1 - 2p \cos x + p^2} \frac{dx}{q^2 + x^2} &= \frac{\pi}{2q} \frac{1}{1 - p^2} \frac{1 - pe^q}{e^q - p} \quad \left. \vphantom{\int} \right\} \begin{array}{l} \text{V. T. 19. N}^\circ 2 \text{ et T. 221. N}^\circ 5. \end{array}
 \end{aligned}$$

- 8) $\int \frac{1 - p \cos. x}{1 - 2p \cos. x + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} \frac{e^q}{e^q - p}$ Schlömilch, Beitr. II. 4.
- 9) $\int \frac{\sin. rx}{1 - 2p \cos. rx + p^2} \frac{x}{q^2 + x^2} dx = \frac{\pi}{2} \frac{1}{e^{qr} - p}$, $p < 1$; Legendre, Exerc. 4. 131. — Boncompagni, Cr. 25. 74.
- 10) $= \frac{\pi}{2q} \frac{1}{p e^{qr} - 1}$, $p > 1$;
- 11) $\int \frac{\cos. rx - p}{1 - 2p \cos. rx + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} \frac{1}{e^{rq} - p}$, $p^2 < 1$;
- 12) $= \frac{\pi}{2q} \frac{1}{e^{-rq} - p}$, $p^2 > 1$;
- 13) $\int \frac{1 - p \cos. rx}{1 - 2p \cos. rx + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} \frac{1}{1 - p e^{-rq}}$, $p^2 < 1$; Boncompagni, Cr. 25. 74.
- 14) $= \frac{\pi}{2q} \frac{1}{1 - p e^{rq}}$, $p^2 > 1$; Ohm, Ausw. 26.
- 15) $\int \frac{\sin. rx}{1 - 2p \cos. rx + p^2} \frac{x}{q^2 + x^2} dx = \frac{1}{2} \frac{\pi}{1 + p} \frac{e^{rq}}{e^{2qr} - 1}$, $p < 1$; Legendre, Exerc. 4. 132.
- 16) $= \frac{1}{2} \frac{\pi}{1 + p} \frac{e^{qr}}{e^{2qr} - 1}$, $p > 1$; Ohm, Ausw. 26.
- 17) $\int \frac{\cos. 2rx - p}{1 - 2p \cos. 2rx + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} \frac{1}{e^{2qr} - p}$, $p^2 < 1$; Legendre, Exerc. 4. 134.
- 18) $\int \frac{\sin. ax \sin. bx}{1 - 2p \cos. ax + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{4pq} e^{-bq} \left\{ \frac{p e^{aq}}{1 - p e^{aq}} - \frac{p e^{-aq}}{1 - e^{-aq}} \right\}$, $p^2 < 1$;
- 19) $\int \frac{\sin. ax \cos. bx}{1 - 2p \cos. ax + p^2} \frac{x}{q^2 + x^2} dx = \frac{\pi}{4p} e^{-bq} \left\{ \frac{p e^{-aq}}{1 - p e^{-aq}} - \frac{p e^{aq}}{1 - e^{aq}} \right\}$ Boncompagni, Cr. 25. 74.
- 20) $\int \frac{\sin. 2ax}{1 - 2p \cos. 2ax + p^2} \frac{x}{q^2 + x^2} dx = -\frac{\pi}{4p} + \frac{1}{4p} \frac{(1 - p^2)(1 - 2p \sin 2aq)(-1)}{1 - 2p \cos. 2aq + p^2}$ Poisson, P. 18. 295. N°. 13 (où il y a faut. aq au lieu de $2aq$).

$$1) \int \frac{\sin. ax}{1 + 2p \cos. ax + p^2} \frac{x}{q^2 + x^2} dx =$$

$$= \frac{\pi}{2q^2} \frac{e^{-aq \cos. \lambda}}{1 + 2p e^{-aq \cos. \lambda} \cos. (aq \sin. \lambda) + p^2 e^{-2aq \cos. \lambda}} \frac{\sin. (aq \sin. \lambda)}{\sin. 2\lambda}$$

$$2) \int \frac{\cos. a x + c}{1 + 2 p \cos. a x + p^2 x^4 + 2 q^2 x^2 \cos. 2 \lambda + q^4} dx =$$

$$= \frac{\pi}{4 q^3} \frac{e^{-a q \cos. \lambda}}{1 + 2 p e^{-a q \cos. \lambda} \cos. (a q \sin. \lambda) + p^2 e^{-2 a q \cos. \lambda}} \left\{ \frac{\cos. (a q \sin. \lambda) + c e^{-q}}{\cos. \lambda} + \frac{\sin. (a q \sin. \lambda)}{\sin. \lambda} \right\}$$

$$3) \int \frac{\sin. a x}{1 - 2 p \cos. a x + p^2 x^4 + 2 q^2 x^2 \cos. 2 \lambda + q^4} dx =$$

$$= \frac{\pi}{2 q} \frac{e^{-a q \cos. \lambda}}{1 - 2 p e^{-a q \cos. \lambda} \cos. (a q \sin. \lambda) + p^2 e^{-2 a q \cos. \lambda}} \frac{\sin. (a q \sin. \lambda)}{\sin. 2 \lambda}$$

$$4) \int \frac{\sin. a x}{1 - 2 p \cos. 2 a x + p^2 x^4 + 2 q^2 x^2 \cos. 2 \lambda + q^4} dx =$$

$$\frac{\pi}{2 q^2} \frac{e^{-a q \cos. \lambda} \sin. (a q \sin. \lambda)}{1 - 2 p e^{-2 a q \cos. \lambda} \cos. (2 a q \sin. \lambda) + p^2 e^{-4 a q \cos. \lambda}} \frac{1 + p e^{-2 a q \cos. \lambda}}{(1 + p) \sin. 2 \lambda}$$

Les formules (1) à (4) se trouvent Plana, Mém. Turin. 1818. 7. II. 10.

$$5) \int \frac{\sin. 2 a x}{e^{-2 a c} + 2 \cos. 2 a x + e^{2 a c} x^2 + (b + c)^2} dx = \frac{\pi}{2} \frac{e^{-2 a c}}{c^2 a (b + c) + e^{-2 a c}}$$

$$6) \int \frac{e^{-2 a c} - e^{2 a c}}{e^{-2 a c} + 2 \cos. 2 a x + e^{2 a c} x^2 + (b + c)^2} dx = \frac{1}{2} \frac{\pi}{b + c} - \frac{\pi}{b + c} \frac{e^{-2 a c}}{e^{2 a (b + c)} + e^{-2 a c}}$$

$$7) \int \frac{\sin. 2 a x}{e^{-2 a c} + 2 \cos. 2 a x + e^{2 a c} x^2 + (b - c)^2} dx = \frac{\pi}{e^{2 a b} + 1}, b > c;$$

$$8) = \frac{\pi e^{-2 a c}}{e^{2 a (c - b)} - e^{-2 a c}}, b < c;$$

$$9) \int \frac{e^{2 a c} - e^{-2 a c}}{e^{-2 a c} + 2 \cos. 2 a x + e^{2 a c} x^2 + (b - c)^2} dx = \frac{1}{2} \frac{\pi}{b - c} - \frac{\pi}{b - c} \frac{1}{e^{2 a b} + 1}, b > c;$$

$$10) = \frac{1}{2} \frac{\pi}{c - b} + \frac{\pi}{b - c} \frac{e^{-2 a c}}{e^{2 a (c - b)} - e^{-2 a c}}, b < c;$$

$$11) \int \frac{(b^2 + c^2 + x^2) 2 x \sin. 2 a x - c (b^2 - c^2 - x^2) (e^{2 a c} - e^{-2 a c})}{e^{-2 a c} + 2 \cos. 2 a x + e^{2 a c}}$$

$$\frac{dx}{\{x^2 + (b - c)^2\} \{x^2 + (b + c)^2\}} = \pi, c > b;$$

$$12) = \frac{2 \pi}{e^{2 a b} + 1}, c < b;$$

Poisson, P.
18.295. N°. 42.

$$\begin{aligned} 13) \int \frac{\text{Sin.}\{(a+b)x\} \cdot \text{Sin. } 2ax}{1 + \text{Cos. } 2ax} \frac{dx}{q^2 + x^2} &= \frac{\pi}{2q} e^{-(a+b)q} \left\{ \frac{e^{2ar}}{1 + e^{2ar}} - \frac{e^{-2ar}}{1 + e^{-2ar}} \right\} \\ 14) \int \frac{\text{Sin.}\{(a+b)x\} \cdot \text{Sin. } 2ax}{1 - \text{Cos. } 2ax} \frac{dx}{q^2 + x^2} &= \frac{\pi}{2q} e^{-(a+b)q} \left\{ \frac{e^{2ar}}{1 - e^{2ar}} - \frac{e^{-2ar}}{1 - e^{-2ar}} \right\} \\ 15) \int \frac{\text{Sin. } 2ax}{1 + p \text{Cos. } 2ax + p^2} \frac{dx}{x(1 + x^2)} &= \frac{1}{2} \frac{\pi}{1 + p} \frac{e^a - e^{-a}}{e^a + p e^{-a}}, p < 1; \end{aligned}$$

Boncompagni, Cr.
25. 74.

Cauchy, Sav. Etr. 1327.
599, S. 2.

$$\begin{aligned} 1) \int \text{Sin. } qx \cdot dx \vee x &= \frac{1}{4q} \vee \frac{2\pi}{q} \\ 2) \int \text{Sin. } qx \cdot x dx \vee x &= -\frac{3}{8q^2} \vee \frac{2\pi}{q} \\ 3) \int \text{Sin. } qx \cdot x^2 dx \vee x &= -\frac{15}{16q^3} \vee \frac{2\pi}{q} \\ 4) \int \text{Cos. } qx \cdot dx \vee x &= \frac{1}{4q} \vee \frac{2\pi}{q} \\ 5) \int \text{Cos. } qx \cdot x dx \vee x &= \frac{3}{8q^2} \vee \frac{2\pi}{q} \\ 6) \int \text{Cos. } qx \cdot x^2 dx \vee x &= \frac{15}{16q^3} \vee \frac{2\pi}{q} \\ 7) \int \text{Sin. } x \cdot x^{2a-1} dx \vee x &= (-1)^a \Gamma\left(\frac{4a+1}{2}\right) \frac{1}{\vee 2} \\ 8) \int \text{Sin. } x \cdot x^{2a} dx \vee x &= (-1)^a \Gamma\left(\frac{4a+3}{2}\right) \frac{1}{\vee 2} \\ 9) \int \text{Cos. } x \cdot x^{2a-1} dx \vee x &= (-1)^a \Gamma\left(\frac{4a+1}{2}\right) \frac{1}{\vee 2} \\ 10) \int \text{Cos. } x \cdot x^{2a} dx \vee x &= (-1)^{a-1} \Gamma\left(\frac{4a+3}{2}\right) \frac{1}{\vee 2} \\ 11) \int \text{Sin. } ax \cdot x^{2b-1} dx \vee x &= (-1)^b \frac{4^{2b-1} 3^{1/2}}{3^{2b-1} 2 a^{2b} \vee a} \\ 12) \int \text{Sin. } ax \cdot x^{2b} dx \vee x &= (-1)^b \frac{4^{2b} 3^{1/2} \vee 3}{3^{2b} 2 a^{2b+1} \vee a} \end{aligned}$$

Oettinger, Cr. 38. 216.

Cauchy, Sav. Etr. 1827. 124, Note 3.

Oettinger, Cr. 38. 216.

$$\left. \begin{aligned} 13) \int \cos. a x. x^{2b-1} dx \sqrt{x} &= (-1)^b \frac{4^{2b-1/3} 1^{1/3} \sqrt{3}}{3^{2b-1} 2 a^{2b} \sqrt{a}} \\ 14) \int \cos. a x. x^{2b} dx \sqrt{x} &= (-1)^{b+1} \frac{4^{2b/3} 1^{1/3}}{3^{2b} 2 a^{2b+1} \sqrt{a}} \end{aligned} \right\} \text{Oettinger, Cr. 38. 216.}$$

$$\left. \begin{aligned} 1) \int \sin. x \frac{dx}{\sqrt{x}} &= \sqrt{\frac{\pi}{2}} \left\{ \begin{array}{l} \text{Euler, Calc. Int. 4. S. 5. § 127. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 2, 24. — Fourier, Chal. 360. — Laplace, P. 15. 229. — Cauchy, Sav. Etr. 1827. 124. Note 16. — Id., Sav. Etr. 1827. 599. P. 1. § 6. — Boncompagni, Cr. 25. 74. — Schlömilch, Beitr. III. 4. — Id., Stud. I. 13.} \end{array} \right. \\ 2) \int \cos. x \frac{dx}{\sqrt{x}} &= \sqrt{\frac{\pi}{2}} \end{aligned} \right\}$$

$$3) \text{ toutes deux } = \sqrt{2} \pi \text{ (fautes par faute de calcul) } \quad \text{Mascheroni, Adm. p. 57, 58.}$$

$$= \frac{1}{2} \sqrt{\pi} \text{ (fautes) } \quad \text{Fusz, Mém. Pétersb. 1830.}$$

$$\left. \begin{aligned} 4) \int \cos. p x \frac{dx}{\sqrt{x}} &= \sqrt{\frac{\pi}{2p}} \left\{ \begin{array}{l} \text{Legendre, Exerc. 3. 55. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 53. — Plana, Mém. Brux. 1837. — Oettinger, Cr. 38. 216.} \end{array} \right. \\ 5) \int \sin. p x \frac{dx}{\sqrt{x}} &= \sqrt{\frac{\pi}{2p}} \end{aligned} \right\}$$

$$\left. \begin{aligned} 6) &= 0, p = 0; \\ 7) &= -\sqrt{\frac{\pi}{2p}}, p < 0; \end{aligned} \right\} \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.}$$

$$8) \int \text{Tang. } p x \frac{dx}{\sqrt{x}} = \sqrt{\frac{\pi}{p}} \sum_1 \frac{(-1)^{n+1}}{\sqrt{n}} \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 38.}$$

$$\left. \begin{aligned} 9) \int \sin.^2 p x \frac{dx}{\sqrt{x}} &= \sqrt{x} - \frac{1}{4} \sqrt{\frac{\pi}{p}} = \infty \\ 10) \int \sin.^3 p x \frac{dx}{\sqrt{x}} &= \frac{3 \sqrt{3-1}}{4 \sqrt{3}} \sqrt{\frac{\pi}{2p}} \end{aligned} \right\} \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 10, 16.}$$

$$11) \int \cos.^2 p x \frac{dx}{\sqrt{x}} = \infty \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 17.}$$

$$12) \int \cos.^3 p x \frac{dx}{\sqrt{x}} = \frac{3 \sqrt{3+1}}{4 \sqrt{3}} \sqrt{\frac{\pi}{2p}} \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 17.}$$

$$\begin{aligned}
 13) \int \cos. p x \frac{dx}{\sqrt{x}} &= \frac{1}{16} \left(10 + \frac{5}{\sqrt{3}} + \frac{1}{\sqrt{5}} \right) \sqrt{\frac{\pi}{2p}} \\
 14) \int \sin. 2b+1 p x \frac{dx}{\sqrt{x}} &= \frac{1}{2^{2b}} \sqrt{\frac{\pi}{2p}} \sum_0^b (-1)^n \binom{2b+1}{b+n+1} \frac{1}{\sqrt{(2n+1)}} \\
 15) \int \cos. 2b+1 p x \frac{dx}{\sqrt{x}} &= \frac{1}{2^{2b}} \sqrt{\frac{\pi}{2p}} \sum_0^b \binom{2b+1}{b+n+1} \frac{1}{\sqrt{(2n+1)}} \\
 16) \int x^{2a} \sin. q x \frac{dx}{\sqrt{x}} &= (-1)^a \sqrt{\frac{2\pi}{q}} \frac{1^{2a/2}}{2^{2a+1} q^{2a}} \\
 17) &= (-1)^a \left(\frac{-\frac{1}{2}}{2a} \right) \frac{1^{2a/1}}{q^{2a}} \sqrt{\frac{\pi}{2q}} \\
 18) \int x^{2a+1} \sin. q x \frac{dx}{\sqrt{x}} &= (-1)^a \sqrt{\frac{2\pi}{q}} \frac{1^{2a+1/2}}{2^{2a+2} q^{2a+1}} \\
 19) &= (-1)^{a+1} \left(\frac{-\frac{1}{2}}{2a+1} \right) \frac{1^{2a+1/1}}{q^{2a+1}} \sqrt{\frac{\pi}{2q}} \\
 20) \int x^{2a} \cos. q x \frac{dx}{\sqrt{x}} &= (-1)^a \sqrt{\frac{2\pi}{q}} \frac{1^{2a/2}}{2^{2a+1} q^{2a}} \\
 21) &= (-1)^a \left(\frac{-\frac{1}{2}}{2a} \right) \frac{1^{2a/1}}{q^{2a}} \sqrt{\frac{\pi}{2q}} \\
 22) \int x^{2a+1} \cos. q x \frac{dx}{\sqrt{x}} &= (-1)^{a+1} \sqrt{\frac{2\pi}{q}} \frac{1^{2a+1/2}}{2^{2a+2} q^{2a+1}} \\
 23) &= (-1)^a \left(\frac{-\frac{1}{2}}{2a+1} \right) \frac{1^{2a+1/1}}{q^{2a+1}} \sqrt{\frac{\pi}{2q}}
 \end{aligned}
 \left. \vphantom{\int} \right\} \begin{array}{l} \text{Bidone, Mém. Turin.} \\ 1812. 231. \text{ Art. 1.} \\ \text{N}^\circ. 17, 16. \end{array}$$

Sur les intégrales 16, 18, 20, 22 voyez: Oettinger, Cr. 38. 216.

Sur les intégrales 17, 19, 21, 23 voyez: Raabe, Int. 167.

$$\begin{aligned}
 1) \int \frac{\sin. x}{x \sqrt{x}} dx &= \sqrt{2\pi} \quad \text{Laplace, P. 15. 229. — Bidone, Mém. Turin. 1812. 231. Art. 1. N}^\circ. 4 \text{ —} \\
 &\quad \text{Plana, Mém. Brux. 1837.} \\
 2) \int \frac{\sin. p x}{x \sqrt{x}} dx &= \sqrt{2p\pi} \quad \text{Cisa de Grésy, Mém. Turin. 1821. 209. II. 53.} \\
 3) &= -\sqrt{2p\pi} \quad \text{Oettinger, Cr. 38. 216. (faut.).}
 \end{aligned}$$

$$1) \int \frac{\text{Sin. } p x}{x^2 \sqrt{x}} dx = \infty \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.}$$

$$5) \quad = \frac{2p}{3} \sqrt{2p\pi}$$

$$6) \int \frac{\text{Sin. } p x}{x^3 \sqrt{x}} dx = -\frac{4p^2}{15} \sqrt{2p\pi}$$

$$7) \int \frac{\text{Sin. } p x}{x^{2a} \sqrt{x}} dx = (-1)^a \frac{(2p)^{2a-1}}{1^{2a/2}} \sqrt{2p\pi}$$

$$8) \int \frac{\text{Sin. } p x}{x^{2a+1} \sqrt{x}} dx = (-1)^a \frac{(2p)^{2a}}{1^{2a+1/2}} \sqrt{2p\pi}$$

$$9) \int \frac{\text{Sin. } p x}{x^{2a} \sqrt[3]{x}} dx = (-1)^a \frac{1^{1/3} 3^{2a} p^{2a-1} \sqrt[3]{p}}{2 \cdot 1^{2a/3}} \sqrt[3]{3}$$

$$10) \int \frac{\text{Sin. } p x}{x^{2a+1} \sqrt[3]{x}} dx = (-1)^a \frac{1^{1/3} 3^{2a+1} p^{2a} \sqrt[3]{p}}{2 \cdot 1^{2a+1/3}}$$

$$11) \int \frac{\text{Sin.}^2 p x}{x \sqrt{x}} dx = \sqrt{p\pi}$$

$$12) \int \frac{\text{Sin.}^2 p x}{x^2 \sqrt{x}} dx = \frac{4}{3} p \sqrt{p\pi}$$

$$13) \int \frac{\text{Sin.}^3 p x}{x \sqrt{x}} dx = \frac{3 - \sqrt{3}}{4} \sqrt{2p\pi}$$

$$14) \int \frac{\text{Sin.}^3 p x}{x^2 \sqrt{x}} dx = \frac{\sqrt{3} - 1}{2} p \sqrt{2p\pi}$$

$$15) \int \frac{\text{Sin.}^4 p x}{x \sqrt{x}} dx = \frac{4 - \sqrt{2}}{4} \sqrt{p\pi}$$

$$16) \int \frac{\text{Sin.}^4 p x}{x^2 \sqrt{x}} dx = \frac{4 - 2\sqrt{2}}{2} p \sqrt{p\pi}$$

$$17) \int \frac{\text{Sin.}^6 p x}{x^5 \sqrt{x}} dx = \frac{5 - 32\sqrt{2} + 271 - 3}{315} 16 p^4 \sqrt{p\pi}$$

$$18) \int \frac{\text{Sin.}^{2b} p x}{x^{a+1/2}} dx = \frac{p^{a-1/2} \sqrt{\pi}}{1^{a/2} 2^{2b-2a}} \sum_1^b (-1)^n \binom{2b}{b+n} n^{a-1}, a \text{ de la forme } 4h \text{ et } 4h+1;$$

$$19) \quad = \frac{p^{a-1/2} \sqrt{\pi}}{1^{a/2} 2^{2b-2a}} \sum_1^b (-1)^{n-1} \binom{2b}{b+n} n^{a-1}, a \text{ de la forme } 4h+2 \text{ et } 4h+3;$$

Oettinger, Cr. 38. 216.

Bidone,
Mém. Tu-
rin. 1812.
231. Art. 1.
N°. 13, 16.

$$20) \int \frac{\sin. 2b+1 px}{x^{a+1}} dx = \frac{p^{a-\frac{1}{2}} \sqrt{x} \pi}{1 \cdot 2 \cdot 2^{2b-a+1}} \sum_1^{b+1} (-1)^{n-1} \binom{2b+1}{b+n} (2n-1)^{a-1}, a \text{ de la forme } 4k \text{ et } 4k+1; \left. \begin{array}{l} \text{Bidone,} \\ \text{Mém. Tu-} \\ \text{rin. 1812.} \\ \text{231. Art.} \\ \text{1. N}^\circ \text{ 13,} \\ \text{16.} \end{array} \right\}$$

$$21) = \frac{p^{a-\frac{1}{2}} \sqrt{x} \pi}{1 \cdot 2 \cdot 2^{2b-a+1}} \sum_1^{b+1} (-1)^n \binom{2b+1}{b+n} (2n-1)^{a-1}, a \text{ de la forme } 4k+2 \text{ et } 4k+3;$$

$$22) \int \frac{\cos. x}{x \sqrt{x}} dx = -\sqrt{2} \pi \quad \text{Plana, Mém. Brux. 1837.}$$

$$23) = \infty \text{ (faut.) Cisa de Grésy, Mém. Turin. 1821. 209. II. 53.}$$

$$24) \int \frac{\cos. px}{x \sqrt{x}} dx = -\sqrt{2} p \pi \quad \text{Oettinger, Cr. 38. 216.}$$

$$25) \int \frac{\cos. px}{x^2 \sqrt{x}} dx = \infty \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N}^\circ \text{ 8. — Cisa de Grésy, Mém. Tu-} \\ \text{rin. 1821. 209. II. 53.}$$

$$26) = \frac{2p}{3} \sqrt{2} p \pi$$

$$27) \int \frac{\cos. px}{x^3 \sqrt{x}} dx = \frac{4p^2}{15} \sqrt{2} p \pi$$

$$28) \int \frac{\cos. px}{x^4 \sqrt{x}} dx = \frac{8p^3}{105} \sqrt{2} p \pi$$

$$29) \int \frac{\cos. px}{x^{2a} \sqrt{x}} dx = (-1)^{a+1} \frac{(2p)^{2a-1}}{1 \cdot 2^{a/2}} \sqrt{2} p \pi$$

$$30) \int \frac{\cos. px}{x^{2a+1} \sqrt{x}} dx = (-1)^{a+1} \frac{(2p)^{2a}}{1 \cdot 2^{a+1/2}} \sqrt{2} p \pi$$

$$31) \int \frac{\cos. px}{x^{2a} \sqrt{x}} dx = (-1)^a \frac{1-\frac{1}{2} 1 \cdot 3^{2a} p^{2a-1} \sqrt{x} p}{2 \cdot 1^{2a/3}}$$

$$32) \int \frac{\cos. px}{x^{2a+1} \sqrt{x}} dx = (-1)^a \frac{1-\frac{1}{2} 1 \cdot 3^{2a-1} p^{2a-2} \sqrt{x} p}{2 \cdot 1^{2a-1/3}}$$

$$33) \int \frac{\text{Tang. } px}{x \sqrt{x}} dx = +\sqrt{x} p \pi \sum_1^{\infty} (-1)^{n+1} \sqrt{x} n \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 3. N}^\circ \text{ 5}$$

Oettinger, Cr. 38. 216.

$$\begin{aligned}
 1) \int \frac{\sin. a x. \cos. b x}{\sqrt{x}} dx &= \left\{ \frac{1}{2\sqrt{a+b}} + \frac{1}{2\sqrt{a-b}} \right\} \sqrt{\frac{\pi}{2}}, a > b; \\
 2) &= \frac{1}{4} \sqrt{\frac{\pi}{a}}, a = b; \\
 3) &= \left\{ \frac{1}{2\sqrt{a+b}} - \frac{1}{2\sqrt{b-a}} \right\} \sqrt{\frac{\pi}{2}}, a < b; \\
 4) \int \frac{\sin.^2 a x. \cos.^3 b x}{\sqrt{x}} dx &= \frac{1}{8} \sqrt{\frac{\pi}{2}} \left\{ -\frac{1}{2\sqrt{2a+3b}} + \frac{1}{\sqrt{3b}} - \frac{1}{2\sqrt{2a-3b}} - \right. \\
 &\quad \left. -\frac{3}{2\sqrt{2a+b}} + \frac{3}{\sqrt{b}} - \frac{3}{2\sqrt{2a-b}} \right\}, 2a > 3b; \\
 5) &= \frac{1}{8} \sqrt{\frac{\pi}{2}} \left\{ -\frac{1}{2\sqrt{2a+3b}} + \frac{1}{\sqrt{3b}} + \frac{1}{2\sqrt{3b-2a}} - \right. \\
 &\quad \left. -\frac{3}{2\sqrt{2a+b}} + \frac{3}{\sqrt{b}} - \frac{3}{2\sqrt{2a-b}} \right\}, 3b > 2a > b; \\
 6) &= \frac{1}{8} \sqrt{\frac{\pi}{2}} \left\{ -\frac{1}{2\sqrt{2a+3b}} + \frac{1}{\sqrt{3b}} + \frac{1}{2\sqrt{3b-2a}} - \right. \\
 &\quad \left. -\frac{3}{2\sqrt{2a+b}} + \frac{3}{\sqrt{b}} + \frac{3}{2\sqrt{b-2a}} \right\}, b > 2a; \\
 7) \int \frac{\sin. a x. \cos. b x}{x\sqrt{x}} dx &= \sqrt{\pi \frac{a+b}{2}} + \sqrt{\pi \frac{a-b}{2}}, a > b; \\
 8) &= \sqrt{a\pi}, a = b; \\
 9) &= \sqrt{\pi \frac{a+b}{2}} - \sqrt{\pi \frac{b-a}{2}}, a < b;
 \end{aligned}$$

Bidone,
Mém. Tu-
rin. 1812.
231. Art. 1.
N°. 19.

$$\begin{aligned}
 1) \int \frac{\sin.^2 a x - \sin.^2 b x}{\sqrt{x}} dx &= \frac{1}{4} \left(\sqrt{\frac{\pi}{b}} - \sqrt{\frac{\pi}{a}} \right) \\
 2) \int \frac{\sin.^4 a x - \sin.^4 b x}{\sqrt{x}} dx &= \frac{1}{4} \left(\sqrt{\frac{\pi}{b}} - \sqrt{\frac{\pi}{a}} \right) - \frac{1}{16} \left(\sqrt{\frac{\pi}{2b}} - \sqrt{\frac{\pi}{2a}} \right) \\
 3) \int \frac{\cos.^2 a x - \sin.^2 b x}{\sqrt{x}} dx &= \frac{1}{4} \left(\sqrt{\frac{\pi}{a}} + \sqrt{\frac{\pi}{b}} \right)
 \end{aligned}$$

Bidone, Mém. Turin. 1812.
231. Tableau.

- $$\begin{aligned} 4) \int \frac{\cos^4 ax - \sin^4 bx}{\sqrt{x}} dx &= \frac{1}{4} \left(\sqrt{\frac{\pi}{a}} + \sqrt{\frac{\pi}{b}} \right) + \frac{1}{16} \left(\sqrt{\frac{\pi}{2a}} + \sqrt{\frac{\pi}{2b}} \right) \\ 5) \int \frac{\cos^2 ax - \cos^2 bx}{\sqrt{x}} dx &= \frac{1}{4} \left(\sqrt{\frac{\pi}{a}} - \sqrt{\frac{\pi}{b}} \right) \\ 6) \int \frac{\cos^4 ax - \cos^4 bx}{\sqrt{x}} dx &= \frac{1}{4} \left(\sqrt{\frac{\pi}{a}} - \sqrt{\frac{\pi}{b}} \right) + \frac{1}{16} \left(\sqrt{\frac{\pi}{2a}} - \sqrt{\frac{\pi}{2b}} \right) \\ 7) \int \frac{\sin^4(a-x) + \cos^4(a-x)}{\sqrt{x}} dx &= \sin a \sqrt{2\pi} \quad \text{Cauchy, Sav. Etr. 1827. 124. Note 16.} \\ 8) \int \frac{\sin x - x \cos x}{x^2 \sqrt{x}} dx &= \frac{1}{3} \sqrt{2\pi} \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 9.} \\ 9) \int \frac{\cos(bx\sqrt{a}) + \sin(bx\sqrt{a})}{\sqrt{x}} \left(\frac{\sin x}{x} \right)^a dx &= \frac{\sqrt{2\pi}}{1^{a/2}} \sum_0^{\infty} (-1)^n \binom{a}{n} (a + b\sqrt{a} - 2n)^{a-1} \quad \left. \begin{array}{l} a \text{ de la forme } 4h \\ \text{et } 4h + 3; \end{array} \right\} \\ 10) \int \frac{\cos(bx\sqrt{a}) - \sin(bx\sqrt{a})}{\sqrt{x}} \left(\frac{\sin x}{x} \right)^a dx &= \frac{\sqrt{2\pi}}{1^{a/2}} \sum_0^{\infty} (-1)^n \binom{a}{n} (a - b\sqrt{a} - 2n)^{a-1} \quad \left. \begin{array}{l} a \text{ de la forme } 4h + 1 \\ \text{et } 4h + 2; \end{array} \right\} \\ &\text{, où } 0 \leq 2a < 4b + 1; \text{ voyez sur ces deux intégrales: Laplace, Mém. Inst. 1809. 353. § 10.} \end{aligned}$$

- $$\begin{aligned} 1) \int \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} &= e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{Cauchy, P. 28. 147. P. 1. § 3.} \\ 2) \int \left(x - \frac{1}{x} \right) \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} &= \frac{1 + 4a}{2a} e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 228. N°. 7.} \\ 3) \int \left(x - \frac{1}{x} \right)^2 \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} &= - \frac{3 + 8a + 16a^2}{4a^2} e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 228. N°. 1.} \\ 4) \int \left(x - \frac{1}{x} \right)^3 \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} &= - \frac{15 + 36a + 48a^2 + 64a^3}{8a^3} e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 228. N°. 7.} \\ 5) \int \left(x - \frac{1}{x} \right)^{2b} \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} &= (-1)^b \frac{\pi}{2} \frac{d^{2b}}{da^{2b}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 228. N°. 1.} \\ 6) \int \left(x - \frac{1}{x} \right)^{2b-1} \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} &= (-1)^{b-1} \sqrt{\frac{\pi}{2}} \frac{d^{2b-1}}{da^{2b-1}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 228. N°. 7.} \end{aligned}$$

Circ. Dir. en num. circ. de $x - \frac{1}{x}$.

- 7) $\int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} = e^{-2a} \sqrt{\frac{\pi}{2a}}$ Cauchy, P. 28, 117. P. 1. § 3.
- 8) $\int \left(x - \frac{1}{x} \right) \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} = -\frac{1+4a}{2a} e^{-2a} \sqrt{\frac{\pi}{2a}}$ V. T. 228. N°. 1.
- 9) $\int \left(x - \frac{1}{x} \right)^2 \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} = -\frac{3+8a+16a^2}{4a^2} e^{-2a} \sqrt{\frac{\pi}{2a}}$ V. T. 228. N°. 7.
- 10) $\int \left(x - \frac{1}{x} \right)^3 \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} = \frac{15+36a+48a^2+64a^3}{8a^3} e^{-2a} \sqrt{\frac{\pi}{2a}}$ V. T. 228. N°. 1.
- 11) $\int \left(x - \frac{1}{x} \right)^{2b} \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} = (-1)^b \sqrt{\frac{\pi}{2}} \frac{d^{2b}}{da^{2b}} \cdot \frac{e^{-2a}}{\sqrt{a}}$ V. T. 228. N°. 7.
- 12) $\int \left(x - \frac{1}{x} \right)^{2b-1} \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{dx}{\sqrt{x}} = (-1)^b \sqrt{\frac{\pi}{2}} \frac{d^{2b-1}}{da^{2b-1}} \cdot \frac{e^{-2a}}{\sqrt{a}}$ V. T. 228. N°. 1.

Circ. Dir. en num.

- 1) $\int \frac{\text{Sin. } x}{\sqrt{x^3}} dx = -p \text{Sin.} \frac{1}{8} \pi \left\{ \right.$, où $p = 3,625608$;
- 2) $\int \frac{\text{Cos. } x}{\sqrt{x^3}} dx = p \text{Cos.} \frac{1}{8} \pi \left\{ \right.$ Laplace, P. 15. 229.
- 3) $\int \frac{\text{Sin. } px}{\sqrt{x^{q-1}}} dx = \frac{\Gamma\left(\frac{1}{q}\right)}{\sqrt[q]{p}} \text{Sin.} \frac{\pi}{2q} \left\{ \right.$, $q \geq 1$; Raabe, Int. 416.
- 4) $\int \frac{\text{Cos. } px}{\sqrt{x^{q-1}}} dx = \frac{\Gamma\left(\frac{1}{q}\right)}{\sqrt[q]{p}} \text{Cos.} \frac{\pi}{2q} \left\{ \right.$
- 5) $\int \frac{\text{Sin. } px}{a+bx} \frac{dx}{\sqrt{x}} = \frac{\pi}{\sqrt{ab}} \text{Sin.} \frac{ap}{b} + \frac{1}{a} \sqrt{\frac{\pi}{2b}} \sum_{n=1}^{\infty} \frac{1}{1^{n^2}} \left(\frac{2ap}{b} \right)^n \left\{ \right.$ Bidone, Mém. Turin, 1812. 231.
- 6) $\int \frac{\text{Cos. } px}{a+bx} \frac{dx}{\sqrt{x}} = \frac{\pi}{\sqrt{ab}} \text{Cos.} \frac{ap}{b} + \frac{1}{b} \sqrt{\frac{\pi}{2b}} \sum_{n=1}^{\infty} \frac{(-1)^n}{1^{n^2}} \left(\frac{2ap}{b} \right)^n \left\{ \right.$ Art. 2. 25.
- 7) $\int \frac{\text{Cos. } bx - \text{Sin. } bx}{a^2+x^2} \frac{dx}{\sqrt{x}} = \frac{\pi}{4a} e^{-ab} \sqrt{\frac{2}{a}}$ Schlömilch, Gr. 11. 174.

$$\left. \begin{aligned} 8) \int \frac{\sin bx - \cos bx}{1+x^2} dx \sqrt{x} &= \frac{1}{2} \pi e^{-b} \\ 9) \int \frac{\cos bx - \sin bx}{(q^2+x^2)^2} dx \sqrt{x} &= \frac{\pi}{4q} e^{-bq} \left(1 + \frac{1}{q^2}\right) \\ 10) \int \frac{\sin bx - \cos bx}{(q^2+x^2)^2} dx \sqrt{x} &= \frac{\pi}{4\sqrt{q}} e^{-bq} \left(b - \frac{1}{2q}\right) \end{aligned} \right\} \text{Helmling, Transf. II. S. 116, 117.}$$

Circ. Dir. en num. circ. de $x - \frac{1}{x}$.

$$\begin{aligned} 1) \int \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{3+x}{\left(x + \frac{1}{x} \right)^2} dx \sqrt{x} &= e^{-2a} \sqrt{2\pi} \quad \text{Cauchy, P. 28. 147. P. 1. § 3.} \\ 2) \int \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{3-x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right) dx \sqrt{x} &= -\frac{1-4a}{2} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 230. N°. 7.} \\ 3) \int \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{3+x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right)^2 dx \sqrt{x} &= \frac{1+8a-16a^2}{4a} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 230. N°. 1.} \\ 4) \int \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{3-x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right)^3 dx \sqrt{x} &= \frac{3+12a+48a^2-64a}{8a^2} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 230. N°. 7.} \\ 5) \int \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{3+x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right)^{2b} dx \sqrt{x} &= (-1)^b \sqrt{2\pi} \frac{d^{2b}}{da^{2b}} \cdot e^{-2a} \sqrt{a} \quad \text{V. T. 230. N°. 1.} \\ 6) \int \sin \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{3-x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right)^{2b+1} dx \sqrt{x} &= (-1)^{b+1} \sqrt{2\pi} \frac{d^{2b+1}}{da^{2b+1}} \cdot e^{-2a} \sqrt{a} \quad \text{V. T. 230. N°. 7.} \\ 7) \int \cos \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{3-x}{\left(x + \frac{1}{x} \right)^2} dx \sqrt{x} &= e^{-2a} \sqrt{2\pi} \quad \text{Cauchy, P. 28. 147. P. 1. § 3.} \\ 8) \int \cos \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{3+x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right) dx \sqrt{x} &= \frac{1-4a}{2} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 230. N°. 1.} \end{aligned}$$

Circ. Dir. en num. circ. de $x - \frac{1}{x}$.

- 9) $\int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left\{ \frac{3-x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right)^2 dx \sqrt{x} \right\} = \frac{1+8a-16a^2}{4a} e^{-2a} \sqrt{\frac{2\pi}{a}} \text{ V. T. 230. N}^\circ 7.$
- 10) $\int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left\{ \frac{3+x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right)^3 dx \sqrt{x} \right\} = -\frac{3+12a+48a^2-64a^3}{8a^2} e^{-2a} \sqrt{\frac{2\pi}{a}} \text{ V. T. 230. N}^\circ 1.$
- 11) $\int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left\{ \frac{3-x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right)^{2b} dx \sqrt{x} \right\} = (-1)^{b-1} \sqrt{2\pi} \frac{d^{2b}}{da^{2b}} \cdot e^{-2a} \sqrt{a} \text{ V. T. 230. N}^\circ 7.$
- 12) $\int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left\{ \frac{3+x}{\left(x + \frac{1}{x} \right)^2} \left(x - \frac{1}{x} \right)^{2b+1} dx \sqrt{x} \right\} = (-1)^{b-1} \sqrt{2\pi} \frac{d^{2b+1}}{da^{2b+1}} \cdot e^{-2a} \sqrt{a} \text{ V. T. 230. N}^\circ 1.$

Circ. Dir. en dén.

- 1) $\int \frac{\text{Cos. } 2px}{\text{Sin. } px + \text{Cos. } px} \frac{x dx \sqrt{x}}{1+x^2} = \frac{1}{2} \pi e^{-p}$
- 2) $\int \frac{\text{Cos. } 2px}{\text{Sin. } px + \text{Cos. } px} \frac{dx \sqrt{x}}{q^2+x^2} = \frac{\pi}{2q} e^{-pq}$
- 3) $\int \frac{\text{Cos. } 2px}{\text{Sin. } px + \text{Cos. } px} \frac{dx \sqrt{x}}{(q^2+x^2)^2} = \frac{1+q^2}{q^2} \frac{\pi}{4q} e^{-pq}$
- 4) $\int \frac{\text{Cos. } 2px}{\text{Sin. } px + \text{Cos. } px} \frac{x dx \sqrt{x}}{(q^2+x^2)^2} = \left(p - \frac{1}{2q} \right) \frac{\pi}{4\sqrt{q}} e^{-pq}$
- 5) $\int \frac{\text{Sin. } x}{\sqrt{1-p^2 \text{Sin.}^2 x}} \frac{dx}{x} = \text{F}'(p)$
- 6) $\int \frac{\text{Sin. } x}{\sqrt{1-p^2 \text{Cos.}^2 x}} \frac{dx}{x} = \text{F}'(p)$
- 7) $\int \frac{\text{Sin. } 2x}{\sqrt{1-p^2 \text{Cos.}^2 x}} \frac{dx}{x} = \frac{2}{p^2} \{ \text{F}'(p) - \text{E}'(p) \}$
- Helmling, Transf. 87, 88, 92, 93.
- , $p < 1$; Raabe, Cr. 25. 160.

- 1) $\int \frac{\text{Sin. } a x}{x} dx = \pi$ Poisson, P. 18. 295. N°. 42.
- 2) $\int \frac{\text{Sin. } p x}{x + q} dx = \pi \text{ Cos. } p q$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 32.
- 3) $\int \frac{\text{Sin. } p x}{x + q i} dx = \pi e^{-pq}$
- 4) $\int \frac{\text{Sin. } p x}{x + (q i - r)} dx = \pi e^{-p(r+qi)}$ } Ohm, Ausw. 23.
- 5) $\int \frac{\text{Sin. } p x}{x - q} dx = \pi \text{ Cos. } p q$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 32.
- 6) $\int \frac{\text{Sin. } p x}{x - q i} dx = \pi e^{-pq}$
- 7) $\int \frac{\text{Sin. } p x}{x - (q i + r)} dx = \pi e^{-p(r-qi)}$ } Ohm, Ausw. 23.
- 8) $\int \frac{\text{Sin. } x}{(a + x i)^{1-p}} dx = -e^{-a} \Gamma(p) i \text{ Sin. } p \pi$
- 9) $\int \frac{\text{Sin. } x}{(a - x i)^{1-p}} dx = e^{-a} \Gamma(p) i \text{ Sin. } p \pi$ } Cayley, L. 12. 231.
- 10) $\int \frac{\text{Sin. } p x}{1 + x^2} dx = 0$ Moigno, Int. 133.
- 11) $\int \frac{\text{Sin. } p x}{q^2 + x^2} dx = 0$ Lejeune-Dirichlet, Cr. 4. 94.
- 12) $\int \frac{\text{Sin. } \{a(b-x)\}}{1 + x^2} dx = \pi e^{-a} \text{ Sin. } a b$ Poisson, P. 19. 404. N°. 66.
- 13) $\int \frac{x \text{ Sin. } p x}{q^2 + x^2} dx = \pi e^{-pq}$
- 14) $\int \frac{p + q x}{r + 2 s x + x^2} \text{ Sin. } t x dx = \left(\frac{p - q s}{\sqrt{(r - s^2)}} \text{ Sin. } r t + q \text{ Cos. } r t \right) \pi e^{-t \sqrt{(r - s^2)}}$ } Ohm, Ausw. 25.
- 15) $\int \frac{x \text{ Sin. } p x}{x^2 + (q i + r)^2} dx = \pi e^{-p(r+qi)}$
- 16) $\int \frac{x \text{ Sin. } p x}{x^2 + (q i - r)^2} dx = \pi e^{-p(r-qi)}$ } Ohm, Ausw. 23.

$$\begin{aligned}
 17) \int \frac{\sin. p x}{q^2 + x^2} \frac{dx}{x^{2a}} &= 0 \\
 18) \int \frac{\sin. p x}{q^2 + x^2} \frac{dx}{x^{2a-1}} &= (-1)^a \frac{\pi}{q^{2a}} e^{-pq} \\
 19) \int \frac{\sin. p x}{x^2 + (qi + r)^2} \frac{dx}{x} &= \frac{\pi}{(r + qi)^3} \{1 - e^{-p(r+qi)}\} \\
 20) \int \frac{\sin. p x}{x^2 + (qi - r)^2} \frac{dx}{x} &= \frac{\pi}{(r - qi)^3} \{1 - e^{-p(r-qi)}\}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Meyer, Int. Déf. 274.} \\ \\ \text{Ohm, Ausw. 23.} \end{array}$$

$$\begin{aligned}
 1) \int \frac{\cos. p x}{x + q} dx &= \pi \sin. p q \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 32.} \\
 2) \int \frac{\cos. p x}{x + qi} dx &= -i \pi e^{-pq} \\
 3) \int \frac{\cos. p x}{x + (qi - r)} dx &= -i \pi e^{-p(r+qi)} \\
 4) \int \frac{\cos. p x}{1 - x} dx &= \pi \sin. p \quad \text{Schlömlich, Stud. II. 16.} \\
 5) \int \frac{\cos. p x}{x - q} dx &= -\pi \sin. p q \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 32.} \\
 6) \int \frac{\cos. p x}{x - qi} dx &= i \pi e^{-pq} \\
 7) \int \frac{\cos. p x}{x - (r + qi)} dx &= i \pi e^{-p(r-qi)} \\
 8) \int \frac{\cos. x}{(a + xi)^{1-p}} dx &= e^{-a} \Gamma(p) \sin. p \pi \\
 9) \int \frac{\cos. x}{(a - xi)^{1-p}} dx &= e^{-a} \Gamma(p) \sin. p \pi \\
 10) \int \frac{\cos. p x}{1 + x^2} dx &= \pi e^{-p} \quad \text{Cauchy, Cours. Lec. 39. — Moigno, Int. 133.}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \\ \\ \\ \\ \text{Ohm, Ausw. 23.} \\ \\ \text{Cayley, L. 12. 231.} \end{array}$$

$$11) \int \frac{\text{Cos.} \left\{ \frac{a(b-x)}{1+x^2} \right\}}{1+x^2} dx = \pi e^{-a} \text{Cos. } ab \quad \text{Poisson, P. 19. 104. N°. 66.}$$

$$12) \int \frac{\text{Cos. } p x}{x^2 + q^2} dx = \frac{\pi}{q} e^{-pq} \quad \text{Ohm, Ausw. 25.}$$

$$13) \quad = -\pi \frac{e^{-pq} - e^{pq}}{2q}, \quad q < z; \quad \left. \vphantom{\int} \right\} \text{, dans le cas, où } x = y + z i;$$

$$14) \quad = \frac{\pi}{q} e^{-pq}, \quad q > z; \quad \left. \vphantom{\int} \right\} \quad \text{Poisson, P. 18. 295. N°. 40.}$$

$$15) \int \frac{x \text{Cos. } p x}{x^2 + q^2} dx = 0$$

$$16) \int \frac{\text{Cos. } p x}{x^2 + (q i + r)^2} dx = \frac{\pi}{r + q i} e^{-p(r + q i)} \quad \left. \vphantom{\int} \right\} \text{Ohm, Ausw. 23.}$$

$$17) \int \frac{\text{Cos. } p x}{x^2 + (q i - r)^2} dx = \frac{\pi}{r - q i} e^{-p(r - q i)}$$

$$18) \int \frac{x^{a-1}}{1+x^{2b}} \text{Cos. } p x dx = \frac{\pi}{b} \sum_1^b e^{-p \text{Sin.} \left(\frac{2a-1}{2b} \pi \right)} \text{Sin.} \left\{ \frac{2n-1}{2b} a \pi + p \text{Cos.} \left(\frac{2n-1}{2b} \pi \right) \right\} \quad \left. \vphantom{\int} \right\} , a < 2b + 1;$$

$$19) \int \frac{x^{a-1}}{1-x^{2b}} \text{Cos. } p x dx = \frac{\pi}{b} \sum_0^{b-1} e^{-p \text{Sin.} \frac{n\pi}{b}} \text{Sin.} \left(\frac{n a \pi}{b} + p \text{Cos.} \frac{n \pi}{b} \right) \quad \left. \vphantom{\int} \right\} \text{Schlömilch, Stud. II. 16.}$$

$$20) \int \frac{p + q x}{r + 2s x + x^2} \text{Cos. } t x dx = \left(q \text{Sin. } r t - \frac{p - q s}{\sqrt{(r - s^2)}} \text{Cos. } r t \right) \pi e^{-t \sqrt{(r - s^2)}} \quad \text{Ohm, Ausw. 25.}$$

$$21) \int \frac{p + q x}{a^2 - 2b x + x^2} \text{Cos. } c x dx = \pi e^{-c \sqrt{(a^2 - b^2)}} \left\{ \frac{p + b q}{\sqrt{(a^2 - b^2)}} \text{Cos. } b c - q \text{Sin. } b c \right\} \quad \left. \vphantom{\int} \right\} \text{Schlömilch, Stud. II. 16.}$$

$$22) \int \frac{\text{Cos.} \{ (b-c) \lambda \} - x \text{Cos. } b \lambda}{1 - 2x \text{Cos. } \lambda + x^2} \text{Cos. } c x dx = \pi e^{-c \text{Sin. } \lambda} \text{Sin.} (b \lambda + c \text{Cos. } \lambda)$$

$$23) \int \frac{a + b x}{p^2 - 2p x \text{Cos. } \lambda + x^2} \text{Cos. } c x dx = \pi e^{-p \text{Sin. } \lambda} \left\{ \frac{a - b p \text{Cos. } \lambda}{p \text{Sin. } \lambda} \text{Cos.} (p c \text{Cos. } \lambda) + b \text{Sin.} (p c \text{Cos. } \lambda) \right\} \quad \left. \vphantom{\int} \right\} \begin{array}{l} \text{Laplace, Prob.} \\ 26. — \text{Piana.} \\ \text{Mém. Turin.} \\ 1818. 7. II. 12. \end{array}$$

$$24) \int \frac{\text{Cos. } p x}{q^2 + x^2} \frac{dx}{x^{2a}} = (-1)^a \frac{\pi}{q^{a+1}} e^{-pq} \quad \left. \vphantom{\int} \right\} \text{Meyer, Int. Déf. 274.}$$

$$25) \int \frac{\text{Cos. } p x}{q^2 + x^2} \frac{dx}{x^{2a-1}} = 0$$

$$\begin{aligned}
 1) & \int \frac{Tang. p x}{x} dx = \pi \\
 2) & \int \frac{x Tang. p x}{x^2 + q^2} dx = \pi, q < z, \\
 3) & = \frac{2\pi}{e^{2pq} + 1}, q > z; \left. \vphantom{\int} \right\} \text{Poisson, P. 18. 295. N}^\circ. 42. \\
 & \text{, où } x = y + z i; \\
 4) & \int \frac{x Tang. p x}{x^2 - q^2} dx = \pi \\
 5) & \int \frac{Cos. a x - Cos. a \lambda}{x^2 - \lambda^2} dx = -\frac{\pi}{\lambda} Sin. a \lambda \quad \text{Poisson, P. 18. 295. N}^\circ. 38. \\
 6) & \int \frac{(e^{ac} + e^{-ac}) Cos. a x - (e^{ac} - e^{-ac}) i Sin. a x}{b^2 + x^2 - c^2 + 2 c x i} dx = \pi \frac{e^{-ab} - e^{ab}}{b}, c > b; \\
 7) & = \frac{2\pi}{b} e^{-ab}, c < b; \\
 8) & \int \frac{(b^2 + c^2 + x^2) 2 x Sin. 2 a x - c (b^2 - c^2 - x^2) (e^{2ac} - e^{-2ac})}{e^{2ac} + 2 Cos. 2 a x + e^{-2ac}} dx \\
 & \frac{dx}{\{x^2 + (b-c)^2\} \{x^2 + (b+c)^2\}} = \pi, c > b; \\
 9) & = \frac{2\pi}{e^{2ab} + 1}, c < b; \left. \vphantom{\int} \right\} \text{Poisson, P. 18. 295. N}^\circ. 41.
 \end{aligned}$$

$$\begin{aligned}
 1) & \int \frac{Sin. \{p(x-1)\}}{x} dx = Ci.(p). Sin. p + Cos. p \left\{ \frac{1}{2} \pi - Si.(p) \right\} \quad \text{Arndt, Gr. 10. 225.} \\
 2) & \int Sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right) dx \sqrt{x} = e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{Cauchy, P. 28. 147. P. 1. § 3.} \\
 3) & \int Sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right) \left(x + \frac{1}{x} \right) dx \sqrt{x} = \frac{1+4a}{2a} e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 236. N}^\circ. 3. \\
 4) & \int Sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right)^3 dx \sqrt{x} = -\frac{3+8a+16a^2}{4a^2} e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 235. N}^\circ. 2.
 \end{aligned}$$

$$5) \int \sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right)^3 \left(x + \frac{1}{x} \right) dx \sqrt{x} = - \frac{15 + 36a + 48a^2 + 64a^3}{8a^3} e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 236, N}^\circ 3.$$

$$6) \int \sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right)^{2b-1} \left(x + \frac{1}{x} \right) dx \sqrt{x} = (-1)^{b-1} \sqrt{\frac{\pi}{2}} \frac{d^{2b-1}}{da^{2b-1}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 236, N}^\circ 3.$$

$$7) \int \sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right)^{2b+1} dx \sqrt{x} = (-1)^b \sqrt{\frac{\pi}{2}} \frac{d^{2b}}{da^{2b}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 235, N}^\circ 2.$$

$$8) \int \sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4+x+\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) \frac{dx}{x} = e^{-2a} \sqrt{2a} \pi \quad \text{Cauchy, P. 28, 147, P. 1, § 5.}$$

$$9) \int \sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4-x-\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right) \frac{dx}{x} = - \frac{1-4a}{2} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 236, N}^\circ 9.$$

$$10) \int \sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4+x+\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right)^2 \frac{dx}{x} = \frac{1+8a-16a^2}{4a} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 235, N}^\circ 8.$$

$$11) \int \sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4-x-\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right)^3 \frac{dx}{x} = - \frac{3+12a+18a^2-64a^3}{8a^2} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 236, N}^\circ 9.$$

$$12) \int \sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4+x+\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right)^{2b} \frac{dx}{x} = (-1)^b \sqrt{2} \cdot \frac{d^{2b}}{da^{2b}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 235, N}^\circ 8.$$

$$13) \int \sin. \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4-x-\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right)^{2b-1} \frac{dx}{x} = (-1)^b \sqrt{2} \cdot \frac{d^{2b-1}}{da^{2b-1}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 236, N}^\circ 9.$$

$$14) \int \sin. \left\{ u \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} - \{x+1+(x-1)i\}^{-b}}{2i} \left(x - \frac{1}{x} \right) x^{2b-1} dx = \frac{\pi A^{1-b} - B^{1-b}}{2^{b+1} \Gamma^{1/2}(b)} \quad \text{Cauchy, P. 28, 147, P. 1, § 5.}$$

- 15) $\int \text{Sin.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} + \{x+1+(x-1)i\}^{-b}}{2}$
 $\left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right) x^{4b-1} dx = - \frac{\pi a^{4b-2} e^{-2a}}{2^{4b+1} \Gamma(\frac{1}{2}b)} \left(\frac{b-2}{2} - 2a \right)$ V. T. 236.
 N°. 15.
- 16) $\int \text{Sin.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} - \{x+1+(x-1)i\}^{-b}}{2i}$
 $\left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)^2 x^{4b-1} dx = - \frac{\pi a^{4b-2} e^{-2a}}{2^{4b+1} \Gamma(\frac{1}{2}b)} \left\{ \frac{1}{4}(b-4)(b-2) - (b-2)2a + 4a^2 \right\}$ V. T. 235.
 N°. 14.
- 17) $\int \text{Sin.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} + \{x+1+(x-1)i\}^{-b}}{2} \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)^3 x^{4b-1} dx =$
 $= - \frac{\pi a^{4b-2} e^{-2a}}{2^{4b+1} \Gamma(\frac{1}{2}b)} \left\{ \frac{1}{8}(b-6)(b-4)(b-2) + \frac{3}{2}(b-4)(b-2)a + 6(b-2)a^2 - 8a^3 \right\}$ V. T. 236.
 N°. 15.
- 18) $\int \text{Sin.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} + \{x+1+(x-1)i\}^{-b}}{2}$
 $\left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)^{2c-1} x^{4b-1} dx = \frac{(-1)^c \pi}{2^{4b+1} \Gamma(\frac{1}{2}b)} \frac{d^{2c-1}}{da^{2c-1}} \cdot a^{4b-1} e^{-2a}$ V. T. 236.
 N°. 15.
- 19) $\int \text{Sin.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} - \{x+1+(x-1)i\}^{-b}}{2i}$
 $\left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)^{2c} x^{4b-1} dx = \frac{(-1)^c \pi}{2^{4b+1} \Gamma(\frac{1}{2}b)} \frac{d^{2c}}{da^{2c}} \cdot a^{4b-1} e^{-2a}$ V. T. 235.
 N°. 14.
- 20) $\int \text{Sin.} ax dx \sqrt{\frac{x}{x^2-1}} = (\text{Cos.} a + \text{Sin.} a) \sqrt{\frac{\pi}{4a}}$, pour a très-petit; V. T. 77. N°. 1.

- 1) $\int \text{Cos.} p x \frac{dx}{x} = - \text{Ci.}(p)$
 2) $\int \frac{\text{Cos.} \{p(x-1)\}}{x} dx = - \text{Ci.}(p) \cdot \text{Cos.} p + \text{Sin.} p \left\{ \frac{1}{2} \pi - \text{Si.}(p) \right\}$
 3) $\int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x + \frac{1}{x} \right) dx \sqrt{x} = e^{-2a} \sqrt{\frac{\pi}{2a}}$ Cauchy, P. 28. 147. P. 1. § 3.

$$1) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right)^2 dx \sqrt{x} = - \frac{1-4a}{2a} e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 235. N}^\circ 2.$$

$$5) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right)^2 \left(x + \frac{1}{x} \right) dx \sqrt{x} = - \frac{3+8a+16a^2}{4a^2} e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 235. N}^\circ 3.$$

$$6) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right)^4 dx \sqrt{x} = \frac{15+36a+48a^2+64a^3}{8a^3} e^{-2a} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 235. N}^\circ 2.$$

$$7) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right)^{2b} \left(x + \frac{1}{x} \right) dx \sqrt{x} = (-1)^b \sqrt{\frac{\pi}{2}} \frac{d^{2b}}{da^{2b}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 236. N}^\circ.$$

$$8) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \left(x - \frac{1}{x} \right)^{2b} dx \sqrt{x} = (-1)^b \sqrt{\frac{\pi}{2}} \frac{d^{2b-1}}{da^{2b-1}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 235. N}^\circ 2.$$

$$9) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4-x-\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \frac{dx}{x} = e^{-2a} \sqrt{2a} \pi \quad \text{Cauchy, P. 28. 147. P. 1. § 3.}$$

$$10) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4+x+\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right) \frac{dx}{x} = \frac{1-4a}{2} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 235. N}^\circ 5.$$

$$11) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4-x-\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right)^2 \frac{dx}{x} = \frac{1+8a-16a^2}{4a} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 235. N}^\circ 9.$$

$$12) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4+x+\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right)^3 \frac{dx}{x} = - \frac{3+12a+48a^2-64a^3}{8a^2} e^{-2a} \sqrt{\frac{2\pi}{a}} \quad \text{V. T. 235. N}^\circ 8.$$

$$13) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4-x-\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right)^{2b} \frac{dx}{x} = (-1)^b \sqrt{2\pi} \frac{d^{2b}}{da^{2b}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 235. N}^\circ 9.$$

$$14) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{4+x+\frac{1}{x}}{\left(x + \frac{1}{x} \right)^2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) \left(x - \frac{1}{x} \right)^{2b+1} \frac{dx}{x} = (-1)^b \sqrt{2\pi} \frac{d^{2b+1}}{da^{2b+1}} \cdot \frac{e^{-2a}}{\sqrt{a}} \quad \text{V. T. 235. N}^\circ 8.$$

$$15) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} + \{x+1+(x-1)i\}^{-b}}{2} \left(x + \frac{1}{x} \right) x^{1b-1} dx = \frac{\pi a^{1b-1} e^{-2a}}{2^{1b+1} \Gamma(\frac{1}{2}b)} \quad \begin{matrix} \text{Cauchy,} \\ \text{P. 28. 147.} \\ \text{P. 1. § 2.} \end{matrix}$$

$$16) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} - \{x+1+(x-1)i\}^{-b}}{2i} \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right) x^{1b-1} dx = \quad \begin{matrix} \text{V. T.} \\ 235. \end{matrix}$$

$$= \frac{\pi a^{1b-2} e^{-2a}}{2^{1b+1} \Gamma(\frac{1}{2}b)} \left\{ \frac{1}{2} (b-2) - 2a \right\} \quad \begin{matrix} \text{N}^{\circ}. 11. \end{matrix}$$

$$17) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} + \{x+1+(x-1)i\}^{-b}}{2} \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)^2 x^{1b-1} dx = \quad \begin{matrix} \text{V. T.} \\ 236. \end{matrix}$$

$$= - \frac{\pi a^{1b-2} e^{-2a}}{2^{1b+1} \Gamma(\frac{1}{2}b)} \left\{ \frac{1}{4} (b-4)(b-2) - (b-2)2a + 4a^2 \right\} \quad \begin{matrix} \text{N}^{\circ}. 15. \end{matrix}$$

$$18) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} - \{x+1+(x-1)i\}^{-b}}{2i} \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)^3 x^{1b-1} dx = \quad \begin{matrix} \text{V. T.} \\ 235. \end{matrix}$$

$$= - \frac{\pi a^{1b-2} e^{-2a}}{2^{1b+1} \Gamma(\frac{1}{2}b)} \left\{ \frac{1}{8} (b-6)(b-4)(b-2) - \frac{3}{2} (b-4)(b-2)a + 6(b-2)a^2 - 8a^3 \right\} \quad \begin{matrix} \text{N}^{\circ}. 14. \end{matrix}$$

$$19) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} + \{x+1+(x-1)i\}^{-b}}{2} \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)^{2c} x^{1b-1} dx = \quad \begin{matrix} \text{V. T.} \\ 236. \end{matrix}$$

$$= \frac{(-1)^c \pi}{2^{1b+1} \Gamma(\frac{1}{2}b)} \frac{d^{2c}}{da^{2c}} \cdot a^{1b-1} e^{-2a} \quad \begin{matrix} \text{N}^{\circ}. 15. \end{matrix}$$

$$20) \int \text{Cos.} \left\{ a \left(x - \frac{1}{x} \right) \right\} \frac{\{x+1-(x-1)i\}^{-b} - \{x+1+(x-1)i\}^{-b}}{2i} \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)^{2c+1} x^{1b-1} dx = \quad \begin{matrix} \text{V. T.} \\ 235. \end{matrix}$$

$$= \frac{(-1)^c \pi}{2^{1b+1} \Gamma(\frac{1}{2}b)} \frac{d^{2c+1}}{da^{2c+1}} \cdot a^{1b-1} e^{-2a} \quad \begin{matrix} \text{N}^{\circ}. 14. \end{matrix}$$

$$21) \int \text{Cos.} a x dx \sqrt{\frac{x}{x^2-1}} = (\text{Cos.} a - \text{Sin.} a) \sqrt{\frac{\pi}{4a}}, \text{ pour } a \text{ très petit; } \quad \begin{matrix} \text{V. T. 77.} \\ \text{N}^{\circ}. 2. \end{matrix}$$

$$1) \int x \text{Tang.} x dx = -\frac{\pi}{8} l 2 + \frac{1}{2} \sum_0 \frac{(-1)^n}{(2n+1)^2} \quad \begin{matrix} \text{V. T. 304.} \\ \text{N}^{\circ}. 1. \end{matrix}$$

$$2) \int x \text{Co'.} x dx = \frac{\pi}{8} l 2 + \frac{1}{2} \sum_0 \frac{(-1)^n}{(2n+1)^2} \quad \begin{matrix} \text{V. T. 303.} \\ \text{N}^{\circ}. 1. \end{matrix}$$

$$3) \int x \text{Tang.}^2 x dx = \frac{1}{4} \pi - \frac{1}{32} \pi^2 - \frac{1}{2} l 2 \quad \begin{matrix} \text{V. T. 237.} \\ \text{N}^{\circ}. 5. \end{matrix}$$

- $$4) \int \frac{x}{\sin. 2 x} dx = \frac{1}{2} \sum_0^{\infty} (-1)^n \frac{1}{(2n+1)^2} \quad \text{V. T. 305. N}^\circ. 1.$$
- $$5) \int \frac{x}{\cos.^2 x} dx = \frac{1}{4} \pi - \frac{1}{2} l 2 \quad \text{V. T. 46. N}^\circ. 1.$$
- $$6) \int \frac{x^2}{\sin.^2 x} dx = \frac{1}{4} \pi l 2 - \frac{1}{16} \pi^2 + \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 238. N}^\circ. 4.$$
- $$7) \int \frac{x^2 \text{Tang. } x}{\cos.^2 x} dx = \frac{1}{2} l 2 - \frac{1}{4} \pi + \frac{1}{16} \pi^2 \quad \text{V. T. 237. N}^\circ. 3.$$
- $$8) \int \frac{x^{p+1}}{\sin.^2 x} dx = - \left(\frac{1}{4} \pi \right)^{p+1} + \frac{p+1}{2} \left(\frac{\pi}{4} \right)^p \left\{ 1 - \sum_1^{\infty} \frac{4}{p+2m} \sum_1^{\infty} \frac{1}{(4n)^{2m}} \right\} \quad \text{V. T. 238. N}^\circ. 20.$$
- $$9) \int \frac{x \sin.^{a-1} x}{\cos.^{a+1} x} dx = \frac{\pi}{4} + \sum_0^{\infty} \frac{(-1)^{n-1}}{a+2n+1} \quad \text{V. T. 46. N}^\circ. 2.$$
- $$10) \int \frac{(c - \frac{1}{2} \pi) \text{Tang.}^2 x + x}{\cos. 2 x} \frac{dx}{\cos.^2 x} = \frac{1}{4} \pi l 2 \quad \text{V. T. 258. N}^\circ. 28.$$
- $$11) \int \frac{x}{(\cos. x + p \sin. x)^2} dx = \frac{1}{1+p^2} l \frac{1 \vee 2}{1+p} + \frac{\pi}{4} \frac{1-p}{(1+p)(1+p^2)} \quad \text{V. T. 48. N}^\circ. 1.$$
- $$12) \int \frac{x \cos. 2 x}{(1 + \sin. x \cos. x)^2} dx = \pi \frac{2 - 1 \vee 3}{6 \vee 3} \quad \text{V. T. 48. N}^\circ. 3.$$
- $$13) \int \frac{x \cos. 2 x}{(1 - \sin. x \cos. x)^2} dx = \pi \frac{3 \vee 3 - 4}{6 \vee 3} \quad \text{V. T. 48. N}^\circ. 2.$$
- $$14) \int \frac{x \sin. 4 x}{(1 - \sin.^2 x \cos.^2 x)^2} dx = \pi \frac{1 \vee 3 - 1}{3 \vee 3} \quad \text{V. T. 48. N}^\circ. 4.$$
- $$15) \int \frac{x}{\sin. x + \cos. x} \frac{dx}{\cos. x} = \frac{1}{8} \pi l 2 \quad \text{V. T. 306. N}^\circ. 1.$$
- $$16) \int \frac{1 - 2 \cos. \lambda \sin. 2 x \sin.^2 x}{(1 - \cos. \lambda \sin. 2 x)^2} \frac{x}{\cos.^2 x} dx = \frac{\pi}{4(1 - \cos. \lambda)} + \frac{\lambda - \pi}{2 \text{Tang. } \lambda} + l \left\{ 2 \sin. \frac{1}{2} \lambda \right\} \quad \begin{matrix} \text{V. T. 48.} \\ \text{N}^\circ. 8. \end{matrix}$$
- $$17) \int \frac{1 \vee \text{Tang. } x - \vee \text{Cot. } x}{\sin. 2 x} x dx = \frac{1}{2} \pi (1 - \vee 2) \quad \text{V. T. 50. N}^\circ. 15.$$
- $$18) \int \frac{x \text{Tang.}^3 x}{1 \vee \cos. 2 x} dx = \frac{\left\{ \Gamma \left(\frac{1}{4} \right) \right\}^2}{8 \vee 2 \pi} - \frac{\pi 1 \vee 2 \pi}{2 \left\{ \Gamma \left(\frac{1}{4} \right) \right\}^2} \quad \text{V. T. 50. N}^\circ. 1.$$
- $$19) \int \frac{x}{\sin. x \vee \cos. 2 x} dx = \frac{1}{2} \pi l (1 + \vee 2) \quad \text{V. T. 261. N}^\circ. 14.$$

- 1) $\int x \cos. x dx = \frac{1}{2} \pi - 1$ V. T. 103. N°. 3.
- 2) $\int x \sin. 2x dx = \frac{1}{2} \pi l 2$ V. T. 267. N°. 22.
- 3) $\int x \cot. x dx = \frac{1}{2} \pi l 2$ Legendre, Exerc. 5. 61. — Poisson, P. 17. 612. N°. 16. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5 7. — Mosta, Gr. 10. 449.
- 4) $\int x \cot. \frac{1}{2} x dx = \frac{1}{2} \pi l 2 + 2 \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ Legendre, Exerc. 5. 64.
- 5) $\int \left(\frac{\pi}{2} - x \right) \text{Tang. } x dx = \frac{1}{2} \pi l 2$ Cauchy, Sav. Etr. 1827. 599. Suppl. 1.
- 6) $\int x \text{Tang. } x dx = \infty$ V. T. 333. N°. 1.
- 7) $\int x \cos. p x \cdot \text{Tang. } x dx = \frac{\pi}{p \cdot 2^{p+1}} \frac{\Gamma(p+1)}{\{\Gamma(\frac{1}{2}p+1)\}^2}$ V. T. 53 N°. 21.
- 8) $\int \cos. a-1 x \cdot \sin. \{(a+1)x\} \cdot x dx = \frac{\pi}{a \cdot 2^{a+1}}$
- 9) $\int (2 \cos. x)^{p-1} \cdot \sin. q x \cdot x dx = \frac{1}{4} \pi \Gamma(p) \frac{Z' \left(\frac{p+q+1}{2} \right) - Z' \left(\frac{p-q+1}{2} \right)}{\Gamma \left(\frac{p+q+1}{2} \right) \Gamma \left(\frac{p-q+1}{2} \right)} \left. \vphantom{\int} \right\} \text{Kummer, Cr. 20. 1.}$
- 10) $\int \sin. (p \text{Tang. } x) \cdot x dx = \frac{1}{4} \pi e^{-p} \{ \Lambda + l 2 p + e^{2p} \text{Ei.} (-2p) \}$ V. T. 431. N°. 5.
- 11) $\int \cos. (p \text{Tang. } x) \text{Tang. } x \cdot x dx = -\frac{1}{4} \pi e^{-p} \{ \Lambda + l 2 p + e^{2p} \text{Ei.} (-2p) \}$ V. T. 431. N°. 7.
- 12) $\int x^2 \sin. x dx = \pi - 2$ V. T. 238. N°. 1.
- 13) $\int x^2 \cot. x dx = \frac{1}{4} \pi^2 l 2 - 2 \sum_1^{\infty} \left\{ \frac{1}{n^3} - \frac{(-1)^{n-1}}{n^3} - 2 \frac{(-1)^{n-1}}{(2n)^3} \right\}$
- 14) $\int x^3 \cot. x dx = \frac{1}{8} \pi^3 l 2 - 6 \pi \sum_1^{\infty} \frac{(-1)^{n-1}}{(2n)^3}$
- 15) $\int (9 \pi x^2 - 14 x^3) \cot. x dx = \frac{1}{2} \pi^3 l 2$

Legendre, Exerc. 5. 61.

$$\begin{aligned}
 16) \int x^n \operatorname{Tang.} \frac{1}{2} x dx &= -\left(\frac{1}{2}\pi\right)^a l 2 + 2 \operatorname{Cos.} \frac{1}{2} a \pi \cdot l a' l \sum_1 \frac{(-1)^{n-1}}{n^{a+1}} + \\
 &+ 2 \sum_1 (-1)^{n-1} \left\{ a^{2n-1} l^{-1} \left(\frac{\pi}{2}\right)^{a-2n+1} \sum_0 \frac{(-1)^n}{(2m+1)^{2n}} + a^{2n-1} \left(\frac{\pi}{2}\right)^{a-2n} \sum_0 \frac{(-1)^{n-1}}{(2m)^{2n+1}} \right\} \\
 17) \int x^n \operatorname{Cot.} \frac{1}{2} x dx &= \left(\frac{1}{2}\pi\right)^a l 2 + 2 \operatorname{Cos.} \frac{1}{2} a \pi \cdot l a' l \sum_1 \frac{1}{n^{a+1}} + \\
 &+ 2 \sum_1 (-1)^{n-1} \left\{ a^{2n-1} l^{-1} \left(\frac{\pi}{2}\right)^{a-2n+1} \sum_0 \frac{(-1)^n}{(2m+1)^{2n}} - a^{2n-1} \left(\frac{\pi}{2}\right)^{a-2n} \sum_0 \frac{(-1)^{n-1}}{(2m)^{2n+1}} \right\} \\
 18) \int x^p \operatorname{Cot.} x dx &= \left(\frac{\pi}{2}\right)^p \left\{ 1 - \sum_1 \frac{2}{p+2m} \sum_1 \frac{1}{(2n)^{2m}} \right\} \quad , \text{ où } p \text{ fraction;} \\
 19) \int x^p \operatorname{Cot.} \frac{1}{2} x dx &= \left(\frac{\pi}{2}\right)^p \left\{ 1 - \sum_1 \frac{4}{p+2m} \sum_1 \frac{1}{(4n)^{2m}} \right\} \quad \text{Legendre, Exerc. 5. 63.}
 \end{aligned}
 \left. \vphantom{\int x^n \operatorname{Tang.} \frac{1}{2} x dx} \right\} \begin{array}{l} \text{Legendre,} \\ \text{Exerc. 5.} \\ \text{58, 59.} \end{array}$$

$$\begin{aligned}
 1) \int \frac{x}{\operatorname{Sin.} x} dx &= 2 \sum_0 \frac{(-1)^n}{(2n+1)^2} \\
 2) \int \frac{x^2}{\operatorname{Sin.} x} dx &= -2 \sum_1 \left\{ \frac{1}{n^3} + \frac{(-1)^{n-1}}{n^3} + \pi \frac{(-1)^{n-1}}{(2n-1)^2} \right\} \\
 3) \int \frac{x^3}{\operatorname{Sin.} x} dx &= 3 \sum_1 (-1)^n \left\{ \frac{1}{2} \pi^2 \frac{1}{(2n-1)^2} - \frac{4}{(2n-1)^4} \right\} \\
 4) \int \frac{x^a}{\operatorname{Sin.} x} dx &= \operatorname{Cos.} \frac{1}{2} a \pi \cdot l a' l \sum_1 \left\{ \frac{1}{n^{a+1}} + \frac{(-1)^{n-1}}{n^{a+1}} \right\} + 2 \sum_1 (-1)^{n-1} a^{2n-1} l^{-1} \left(\frac{\pi}{2}\right)^{a-2n-1} \sum_0 \frac{(-1)^{2n-1}}{(2m-1)^{2n}} \\
 5) \int \frac{x^p}{\operatorname{Sin.} x} dx &= \left(\frac{\pi}{2}\right)^p \left\{ 1 + \sum_1 \frac{1}{2^{2m-2}} \frac{2^{2m-1}-1}{p+2m} \sum_1 \frac{1}{(4n^2)^m} \right\} \quad , \text{ où } p \text{ fraction;} \\
 6) \int \frac{x \operatorname{Cos.} x}{\operatorname{Sin.} x} dx &= \frac{1}{2} \pi l 2 \quad \text{Legendre, Exerc. Suppl. 28. — Cauchy, Exerc. 1826, p. 205.} \\
 7) \int \frac{x}{\operatorname{Sin.}^2 x} dx &= \frac{1}{2} \pi l 2 \quad \text{V. T. 264. N°. 2.} \\
 8) \int \frac{x^2}{\operatorname{Sin.}^2 x} dx &= \pi l 2 \quad \text{Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Mosta, Gr. 10. 449.}
 \end{aligned}
 \left. \vphantom{\int \frac{x}{\operatorname{Sin.} x} dx} \right\} \begin{array}{l} \text{Legendre, Exerc. 5. 61.} \\ \text{Legendre, Exerc. 5. 63.} \\ \text{Legendre, Ex. 5. 60.} \end{array}$$

- 9) $\int \frac{x^{p+1}}{\sin^2 x} dx = (p+1) \left(\frac{1}{2}\pi\right)^p \left\{1 - \sum_1^{\infty} \frac{2}{p+2m} \sum_1^{\infty} \frac{1}{(1+n^2)^m}\right\}$ V. T. 238. N°. 18.
- 10) $\int \frac{x^2 \cos x}{\sin^2 x} dx = -\frac{1}{4}\pi^2 + 4 \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 239. N°. 1.
- 11) $\int \frac{x^3 \cos x}{\sin^2 x} dx = -\frac{1}{16}\pi^3 + \frac{3}{2}\pi l 2$ V. T. 239. N°. 8.
- 12) $\int \frac{1-x \cot x}{\sin^2 x} dx = \frac{1}{4}\pi$ Legendre, Exerc. Suppl. 17.
- 13) $\int \frac{1+x^2 \cos x + (2\pi-x)x}{\sin x} dx = \pi^2 l 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.
- 14) $\int \frac{x}{\sin 2x} dx = \infty$ Cauchy, Exerc 1826. p. 205.
- 15) $\int \frac{x^a}{\text{Tang. } x} dx = \left(\frac{1}{2}\pi\right)^a l 2 + \cos. \frac{1}{2} a \pi \cdot 1^{a/1} \sum_1^{\infty} \left\{ \frac{1}{n^{a+1}} + \frac{(-1)^n}{n^{a+1}} + \right.$
 $\left. + (-1)^n 2^a \sum_1^{\infty} (-1)^{m-1} (a-1)^{2m-1/-1} \left(\frac{\pi}{2}\right)^{a-2m} \frac{1}{(2n-1)^{2m+1}} \right\}$ Legendre, Exerc. 5.
60.
- 16) $\int \sin. (q \cot x) \frac{x}{\sin^2 x} dx = \frac{e^{-q}-1}{2q} \pi$ V. T. 374. N°. 1.
- 17) $\int \sin. (q \text{Tang. } x) \frac{x}{\cos^2 x} dx = \frac{\pi}{2q} e^{-q}$ V. T. 374. N°. 2.
- 18) $\int \cos. (q \text{Tang. } x) \frac{x}{\sin 2x} dx = -\frac{1}{4}\pi \text{Ei. } (-q)$ V. T. 431. N°. 1.
- 19) $\int \frac{x}{\cos^2 x \sin x} dx = -\infty$ V. T. 334. N°. 1.
- 20) $\int \frac{x \sin^p x}{\text{Tang. } x} dx = \frac{\pi}{2p} - 2^{p-2} \frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma(p+1)}$ V. T. 53. N°. 18.
- 21) $\int \frac{x}{\text{Tang. } x \cos 2x} dx = \frac{1}{4}\pi l 2$ V. T. 265. N°. 13.
- 22) $\int \frac{x}{\text{Tang. }^p x \sin 2x} dx = \frac{\pi}{p} \text{Sec. } p\pi$, $p < 1$; V. T. 63. N°. 4.

- 1) $\int \frac{x \sin x}{\cos^2 \lambda - \sin^2 x} dx = -2 \operatorname{Cosec} \lambda \sum_0^{\infty} \frac{\sin \{(2n+1)\lambda\}}{(2n+1)^2}$ Legendre, Exerc. 5. 55.
- 2) $\int \frac{x \sin 2x}{1 + \operatorname{Tang}^2 \lambda \cos^2 x} dx = -\pi \operatorname{Cot}^2 \lambda l \left(\cos^2 \frac{1}{2} \lambda, \operatorname{Sec} \lambda \right)$ V. T. 334. N°. 9.
- 3) $\int \frac{x^2}{1 - \cos x} dx = \pi l 2 - \frac{\pi^2}{4} + \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 238. N°. 4.
- 4) $\int \frac{x^{p+1}}{1 - \cos x} dx = -\left(\frac{1}{2}\pi\right)^{p+1} + (p+1) \left(\frac{1}{2}\pi\right)^p \left\{ 1 - \sum_1^{\infty} \frac{4}{p+2m} \sum_1^{\infty} \frac{1}{(4n)^{2m}} \right\}$ V. T. 238. N. 19.
- 5) $\int \frac{x^a \sin x}{\cos x + \cos \lambda} dx = -\left(\frac{1}{2}\pi\right)^a l(2 \cos \lambda) + 2 \cdot 1^{a/1} \cos \frac{1}{2} a \pi \sum_1^{\infty} \frac{\cos n \lambda}{n^{a+1}} + 2 \sum_1^{\infty} (-1)^{n-1}$
 $\left\{ \cos \{(2n-1)\lambda\} \sum_1^{\infty} (-1)^{m-1} \frac{a^{2m-1} l^{-1} \left(\frac{\pi}{2}\right)^{a+1-2m}}{(2n-1)^{2m}} + \cos 2n \lambda \sum_1^{\infty} (-1)^{m-1} \frac{a^{2m-1} \left(\frac{\pi}{2}\right)^{a-2m}}{(2n)^{2m+1}} \right\}$ Legendre, Exerc. 5. 57.
- 6) $\int \frac{x^a \sin x}{\cos x - \cos \lambda} dx = -\left(\frac{1}{2}\pi\right)^a l(2 \cos \lambda) - 2 \cdot 1^{a/1} \cos \frac{1}{2} a \pi \sum_1^{\infty} \frac{\cos n \lambda}{n^{a+1}} - 2 \sum_1^{\infty} (-1)^{n-1}$
 $\left\{ \cos \{(2n-1)\lambda\} \sum_1^{\infty} (-1)^{m-1} \frac{a^{2m-1} l^{-1} \left(\frac{\pi}{2}\right)^{a+1-2m}}{(2n-1)^{2m}} - \cos 2n \lambda \sum_1^{\infty} (-1)^{m-1} \frac{a^{2m-1} \left(\frac{\pi}{2}\right)^{a-2m}}{(2n)^{2m+1}} \right\}$ Legendre, Exerc. 5. 55.
- 7) $\int \frac{x^a \sin x}{q + \cos x} dx = 2 \cos \frac{1}{2} a \pi \cdot 1^{a/1} \sum_1^{\infty} (-1)^{n-1} \frac{e^n}{n^{a+1}} - 2 \sum_1^{\infty} \left\{ e^{2n} \sum_0^{\infty} \binom{a}{2m} (-1)^m \left(\frac{1}{2}\pi\right)^{a-2m} \frac{1}{(2n)^{2m-1}} - \right.$
 $\left. - e^{2n-1} \sum_0^{\infty} \binom{a}{2m+1} (-1)^m \left(\frac{\pi}{2}\right)^{a-2m-1} \frac{1}{(2n)^{2m-2}} \right\}$
- 8) $\int \frac{x^a \sin x}{q - \cos x} dx = 2 \cos \frac{1}{2} a \pi \cdot 1^{a/1} \sum_1^{\infty} \frac{e^n}{n^{a+1}} + 2 \sum_1^{\infty} \left\{ e^{2n} \sum_0^{\infty} \binom{a}{2m} (-1)^m \left(\frac{1}{2}\pi\right)^{a-2m} \frac{1}{(2n)^{2m+1}} + \right.$
 $\left. + e^{2n-1} \sum_0^{\infty} \binom{a}{2m+1} (-1)^m \left(\frac{1}{2}\pi\right)^{a-2m-1} \frac{1}{(2n)^{2m-2}} \right\}$

Sur les form. 7), 8), où $c = a - 1$ ($a^2 - 1$), voyez: Legendre, Exerc. 5. 73.

- 9) $\int \frac{x \sin 2x}{p + \cos 2x} dx = -\frac{1}{4} \pi l \{2(1-p)\}$, $p < 1$;
 - 10) $\int \frac{x \sin 2x}{p - \cos 2x} dx = \frac{1}{4} \pi l \frac{p+1}{2(p-1)} \{p^2-1\}$, $p > 1$;
 - 11) $\int \frac{x \sin 2x}{p - \cos 2x} dx = \frac{1}{4} \pi l \{2(1+p)\}$, $p < 1$;
- Cauchy, Sav. Fir. 1827. 599. S. 1

$$12) \int \frac{x \sin. 2x}{p - \cos. 2x} dx = \frac{1}{4} \pi l \frac{2(1+p)}{p+1 \sqrt{(p^2-1)}}, p > 1; \text{ Cauchy, Sav. Etr. 1827. 599. S. 1.}$$

$$13) \int \frac{x \sin. 2x}{1 - \sin. 2x \cos. 2x} dx = -2 \pi \operatorname{Cosec.} 2x \cdot l \cos. \frac{1}{2} x \quad \text{V. T. 334. N°. 13.}$$

$$14) \int \frac{x \sin. 2x}{1 + q^2 \cos. 2x} dx = \frac{\pi}{q^2} l \frac{1 + \sqrt{(1+q)}}{2} \quad \text{V. T. 334. N°. 8.}$$

$$15) \int \frac{x \sin. 2x}{p^2 - \cos. 2x} dx = \frac{\pi}{8p} l \frac{1+p}{1-p}, p < 1; \left\{ \quad \text{V. T. 240. N°. 9-12.} \right.$$

$$16) \quad = \frac{\pi}{8p} l \frac{p+1}{p-1}, p > 1; \left\{ \right.$$

$$17) \int \frac{x \sin. 4x}{p^2 - \cos. 2x} dx = \frac{1}{4} \pi l \{4(1-p^2)\}, p < 1; \left\{ \quad \text{V. T. 210. N°. 9-12.} \right.$$

$$18) \quad = \frac{1}{2} \pi l \frac{2 \sqrt{(p^2-1)}}{p + \sqrt{(p^2-1)}}, p > 1; \left\{ \right.$$

$$19) \int \frac{x \sin. 2x}{1 - \sin. 2x \cos. 4x} dx = \frac{1}{2} \pi \operatorname{Cosec.} x \cdot l \frac{1 + \sin. \frac{1}{2} x}{\cos. \frac{1}{2} x} \quad \text{V. T. 334. N°. 22.}$$

$$20) \int \frac{x \operatorname{Tang.} x}{p^2 \sin. 2x + \cos. 2x} dx = -\frac{\pi}{p^2} l(1+p) \quad \text{V. T. 334. N°. 14.}$$

$$1) \int \frac{x \sin. x}{1 - 2p \cos. x + p^2} dx = \frac{\pi}{p} l(1-p) \quad \text{V. T. 356. N°. 2.}$$

$$2) \int \frac{x \cos. x}{1 + 2p \sin. x + p^2} dx = \frac{\pi}{2p} l(1+p) - \frac{1}{2p} \sum_1 \frac{1}{2n-1} \frac{2^{n/2}}{3^{n/2}} \left(\frac{2p}{1+p^2} \right)^{2n-1} \quad \text{V. T. 334. N°. 19.}$$

$$3) \int \frac{x \sin. 2x}{1 - 2p \cos. 2x + p^2} dx = \frac{\pi}{4p} l(1+p), p^2 < 1; \left\{ \quad \text{Cauchy, Lim. Imag. 115, 121. — Poisson, P. 19. 404. N°. 76. 171.} \right.$$

$$4) \quad = \frac{\pi}{4p} l \frac{1+p}{p}, p^2 > 1; \left\{ \right.$$

$$5) \int \frac{x \cot. x}{1 - (1-q) \cos. 2x + q} dx = \frac{\pi}{8q} l \frac{1 + \sqrt{(1+q)}}{2} \quad \text{V. T. 313. N°. 16.}$$

- 6) $\int \frac{x}{1 - q^4 \text{Tang.}^4 x \text{Sin.}^2 x} dx = \frac{1}{16} \pi l \frac{(1+q)^2 (1+q^4)}{q^4} \quad \text{V. T. 265. N}^\circ. 15.$
- 7) $\int \frac{\text{Sin.} x}{1 - q^4 \text{Tang.}^4 x \text{Cos.}^2 x} dx = \frac{\pi}{8 q^2} l \frac{1+q^2}{(1+q)^2} \quad \text{V. T. 265. N}^\circ. 14.$
- 8) $\int \frac{x \text{Cos.} x}{(1 + \text{Sin.} x. \text{Cos.} x)^2} dx = 2 \lambda \text{Cosec.} 2 \lambda - \frac{1}{2 \text{Cos.} \lambda} - \frac{\pi}{1 + \text{Cos.} \lambda} \quad \text{V. T. 65. N}^\circ. 6.$
- 9) $\int \frac{x \text{Cos.} 2 x}{(1 + \text{Sin.} x. \text{Cos.} x)^2} dx = \frac{2}{9} \pi \wedge 3 - \frac{1}{2} \pi \quad \text{V. T. 65. N}^\circ. 4.$
- 10) $\int \frac{x \text{Cos.} x}{(1 - \text{Sin.} x. \text{Cos.} x)^2} dx = 2 (\lambda - \pi) \text{Cosec.} 2 \lambda + \frac{\pi}{2 \text{Cos.} \lambda (1 - \text{Cos.} \lambda)} \quad \text{V. T. 65. N}^\circ. 5.$
- 11) $\int \frac{x \text{Cos.} 2 x}{(1 - \text{Sin.} x. \text{Cos.} x)^2} dx = \frac{1}{2} \pi - \frac{4}{9} \pi \wedge 3 \quad \text{V. T. 65. N}^\circ. 3.$
- 12) $\int \frac{x \text{Sin.} 2 x}{(1 - \text{Cos.}^2 \lambda. \text{Sin.}^2 x)^2} dx = -2 \pi \text{Cosec.}^2 2 \lambda (1 - \text{Sin.} \lambda) \quad \text{V. T. 66. N}^\circ. 8.$
- 13) $\int \frac{x}{(\text{Sin.} x \pm q \text{Cos.} x)^2} dx = \pm \frac{\pi}{2} \frac{q}{1+q^2} - \frac{1}{1+q^2} l q \quad \text{V. T. 65. N}^\circ. 7, 8.$
- 14) $\int \frac{x \text{Sin.} x}{(p+q \text{Cos.} x)^2} dx = \frac{\pi}{2 p} - \frac{1}{\sqrt{(p^2-q^2)}} \text{Arccos.} \frac{q}{p} \quad , q < p; \left\{ \begin{array}{l} \text{V. T. 65. N}^\circ. 9, 10. \\ \\ \end{array} \right.$
- 15) $= \frac{\pi}{2 p} + \frac{1}{\sqrt{(q^2-p^2)}} l \frac{p}{q + \sqrt{(q^2-p^2)}}, q > p; \left\{ \right.$
- 16) $\int \frac{x \text{Sin.} 4 x}{(1 - \text{Sin.}^2 x. \text{Cos.}^2 x)^2} dx = \frac{2}{1 \wedge 3} \pi \quad \text{V. T. 66. N}^\circ. 6.$
- 17) $\int \frac{x \text{Sin.} 2 x}{(\text{Cos.}^2 x + p^2 \text{Sin.}^2 x)^2} dx = \frac{\pi}{2 p^2 (1+p)} \quad \text{V. T. 66. N}^\circ. 2.$
- 18) $\int \frac{x \text{Sin.} 2 x}{(p^2 \text{Sin.}^2 x + q^2 \text{Cos.}^2 x)^3} dx = \frac{\pi}{8 p^4 q^3} \frac{p^2 + p q + 2 q^2}{p+q} \quad \text{V. T. 67. N}^\circ. 7.$
- 19) $\int \frac{x \text{Sin.} 2 x}{(p^2 \text{Sin.}^2 x + q^2 \text{Cos.}^2 x)^4} dx = \frac{\pi}{48 p^6 q^5} \frac{3 p^4 + 3 p^3 q + 5 p^2 q^2 + 5 p q^3 + 8 q^4}{p+q} \quad \text{V. T. 67. N}^\circ. 10.$
- 20) $\int \frac{x \text{Sin.} 2 x}{(p^2 \text{Sin.}^2 x + q^2 \text{Cos.}^2 x)^5} dx = \frac{\pi}{128 p^8 q^7} \frac{5 p^6 + 5 p^5 q + 8 p^4 q^2 + 8 p^3 q^3 + 11 p^2 q^4 + 11 p q^5 + 16 q^6}{p+q} \quad \text{V. T. 67. N}^\circ. 13.$
- 21) $\int \frac{\text{Cos.}^2 \lambda + \text{Sin.}^2 x}{(\text{Cos.}^2 \lambda - \text{Sin.}^2 x)^2} x^2 \text{Cos.} x dx = -\frac{\pi^2}{4 \text{Sin.}^2 \lambda} + 4 \text{Cosec.} \lambda \sum_0^{\infty} \frac{\text{Sin.} \{(2n+1)\lambda\}}{(2n+1)^2} \quad \text{V. T. 240. N}^\circ. 1.$

$$22) \int \frac{Tang.^2 x}{(p^2 + Tang.^2 x)^2} \frac{x}{Sin. 2 x} dx = \frac{\pi}{8 p (p + 1)} \quad \text{V. T. 66. N}^\circ. 11.$$

$$23) \int \frac{x}{(Tang. x + Cot. x)^3} \frac{dx}{Tang. 2x. Sin. 2x} = -\frac{\pi}{128} \quad \text{V. T. 67. N}^\circ. 24.$$

$$24) \int \frac{Sin. x. Cos. x}{1 - Sin.^2 \lambda. Cos.^2 x} \frac{x}{1 - Sin.^2 \mu. Cos.^2 x} dx = \frac{\pi}{Cos.^2 \lambda - Cos.^2 \mu} l \left(Cos. \frac{1}{2} \lambda. Sec. \frac{1}{2} \mu \right) \quad \text{Lobatschewsky, Mém. Kasan. 1835. 1.}$$

$$25) \int \frac{Sin. 2x}{Sin.^2 \lambda - Sin.^2 \mu. Cos.^2 x} \frac{x}{1 - Sin.^2 \mu. Cos.^2 x} dx = \frac{\pi}{Sin.^2 \mu. Cos.^2 \lambda} l \left(\frac{Cot. \frac{1}{2} \mu}{Sin. \lambda} Tang. \left(\frac{1}{2} Arcsin. \frac{Sin. \mu}{Sin. \lambda} \right) \right) \quad \text{V. T. 334. N}^\circ. 24.$$

$$26) \int \left[\frac{p^2 x Sin. 2px}{Cos. p\pi - Cos. 2px} - \frac{(1-p)^2 x - (1-p)^{\frac{1}{2}} \pi}{Cos. p\pi - Cos. \{(1-p)^{\frac{1}{2}} x\}} Sin. \{2(1-p)x\} \right] dx = \frac{\pi}{4} l \{2(1 + Cos. p\pi)\} \quad \text{Cauchy, Sav. Etr. 1827. 599. S. 1.}$$

$$1) \int x Sin. 2x dx \vee (1 - p^2 Sin.^2 x) = \frac{2}{3p^2} \left[\frac{4-2p^2}{3} E'(p) - \frac{1-p^2}{3} F'(p) - \frac{\pi}{2} \vee (1-p^2)^3 \right] \quad \text{V. T. 72. N}^\circ. 10.$$

$$2) \int x Tang. x dx \vee Cos. x = \vee 27 \left\{ (1 - \vee 3) F' \left(Cos. \frac{\pi}{12} \right) + 2 \vee 3 E' \left(Cos. \frac{\pi}{12} \right) \right\} \quad \text{V. T. 72. N}^\circ. 22.$$

$$3) \int \frac{x dx}{Tang. x} \vee Sin. x = \frac{3}{2} \pi + \vee 27 \left\{ (\vee 3 - 1) F' \left(Cos. \frac{\pi}{12} \right) - 2 \vee 3 E' \left(Cos. \frac{\pi}{12} \right) \right\} \quad \text{V. T. 72. N}^\circ. 20.$$

$$4) \int \frac{\vee Tang. x - \vee Cot. x}{Sin. 2x} x dx = -\infty \quad \text{V. T. 73. N}^\circ. 4.$$

$$5) \int \frac{x Cos. x}{\vee Sin.^3 x} dx = -\pi + 2 \vee 2. F' \left(Sin. \frac{\pi}{4} \right) \quad \text{V. T. 73. N}^\circ. 3.$$

$$6) \int \frac{x Sin. x}{\vee Cos.^3 x} dx = -\infty \quad \text{V. T. 73. N}^\circ. 3.$$

$$7) \int \frac{x Cos. x}{\vee Sin. x} dx = \frac{3}{4} \pi + \frac{9}{2} \vee 3 \left\{ \frac{3 + \vee 3}{2} F' \left(Sin. \frac{\pi}{12} \right) - 3 E' \left(Sin. \frac{\pi}{12} \right) \right\} \quad \text{V. T. 72. N}^\circ. 21.$$

$$8) \int \frac{x Sin. x}{\vee Cos. x} dx = \frac{3}{2} \vee 3 \left\{ 3 E' \left(Sin. \frac{\pi}{12} \right) - \frac{3 + 3\vee 3}{2} F' \left(Sin. \frac{\pi}{12} \right) \right\} \quad \text{V. T. 72. N}^\circ. 23.$$

$$9) \int \frac{x Tang. x}{\vee Cos. x} dx = \infty \quad \text{V. T. 73. N}^\circ. 2.$$

$$10) \int \frac{x \operatorname{Tang.} x}{\sqrt{\cos.^2 x}} dx = \infty \quad \text{V. T. 73. N}^\circ. 10.$$

$$11) \int \frac{x}{\operatorname{Tang.} x \sqrt{\sin. x}} dx = \frac{3}{\sqrt{3}} F' \left(\cos. \frac{\pi}{12} \right) - \frac{3}{2} \pi \quad \text{V. T. 73. N}^\circ. 7.$$

$$12) \int \frac{x}{\operatorname{Tang.} x \sqrt{\sin.^2 x}} dx = \frac{3}{2} \sqrt{27} \cdot F' \left(\sin. \frac{\pi}{12} \right) - \frac{3}{4} \pi \quad \text{V. T. 73. N}^\circ. 8.$$

$$13) \int \frac{x dx \sqrt[2q-2]{\sin.^{-2} x}}{\cos.^{\frac{q}{q}} x} = \frac{q \pi}{2(2-q)} \operatorname{Cosec.} \frac{\pi}{q} \quad \text{V. T. 73. N}^\circ. 12.$$

$$1) \int \frac{x \sin. 2x}{\sqrt{(1-p^2 \sin.^2 x)}} dx = -\frac{\pi}{p^2} \sqrt{(1-p^2)} + \frac{2}{p^2} E'(p) \quad \text{V. T. 72. N}^\circ. 2.$$

$$2) \int \frac{x \sin. 2x}{\sqrt{(1-p^2 \cos.^2 x)}} dx = \frac{\pi}{p^2} - \frac{2}{p^2} E'(p) \quad \text{V. T. 72. N}^\circ. 11.$$

$$3) \int \frac{x \sin.^2 x \sin. 2x}{\sqrt{(1-p^2 \sin.^2 x)}} dx = \frac{2}{3p^4} \left\{ \frac{2p^2+5}{3} E'(p) + \frac{1-p^2}{3} F'(p) - \frac{2+p^2}{2} \pi \sqrt{(1-p^2)} \right\} \quad \text{V. T. 72. N}^\circ. 1, 4.$$

$$4) \int \frac{x \sin. x}{\sqrt{(p^2 + \cos.^2 x)}^3} dx = \frac{1}{p^2} \operatorname{Arccot.} p \quad \text{V. T. 75. N}^\circ. 8.$$

$$5) \int \frac{x \cos. x}{\sqrt{(p^2 + \sin.^2 x)}^3} dx = \frac{1}{p^2} \left\{ -\operatorname{Arccot.} p + \frac{\pi}{2 \sqrt{(1+p^2)}} \right\} \quad \text{V. T. 75. N}^\circ. 5.$$

$$6) \int \frac{x \sin. 2x}{\sqrt{(1 + \sin.^2 x)}^3} dx = \frac{\pi}{2} - \frac{1}{2} \sqrt{2} \cdot F' \left(\sin. \frac{\pi}{4} \right) \quad \text{V. T. 75. N}^\circ. 1.$$

$$7) \int \frac{x \sin. 2x}{\sqrt{(1 + \cos.^2 x)}^3} dx = \pi - \sqrt{2} \cdot F' \left(\sin. \frac{\pi}{4} \right) \quad \text{V. T. 74. N}^\circ. 5.$$

$$8) \int \frac{x \sin. x}{\sqrt{(a+b \cos. x)}^3} dx = \frac{1}{b} \left[\frac{\pi}{\sqrt{a}} - \frac{4}{\sqrt{(a+b)}} F \left(\frac{1}{4} \pi, \sqrt{\frac{2b}{a+b}} \right) \right] \quad \text{V. T. 74. N}^\circ. 1.$$

$$9) \int \frac{x \sin. x}{\sqrt{(a-b \cos. x)}^3} dx = \frac{1}{b} \left[\frac{-\pi}{\sqrt{a}} + \frac{4}{\sqrt{(a+b)}} \left\{ F \left(\frac{1}{2} \pi, \sqrt{\frac{2b}{a+b}} \right) - F \left(\frac{1}{4} \pi, \sqrt{\frac{2b}{a+b}} \right) \right\} \right] \quad \text{V. T. 71. N}^\circ. 2.$$

$$10) \int \frac{x \sin 2x}{1 \sqrt{(a-b \cos x)^3}} dx = \frac{4}{b^2} \left[-\pi \sqrt{a} + \frac{2a}{1 \sqrt{(a+b)}} \left\{ F\left(\frac{\pi}{2}, \sqrt{\frac{2b}{a+b}}\right) - F\left(\frac{1}{4}\pi, \sqrt{\frac{2b}{a+b}}\right) \right\} + \right. \\ \left. + 2a \sqrt{(a+b)} \left\{ E\left(\frac{1}{2}\pi, \sqrt{\frac{2b}{a+b}}\right) - E\left(\frac{1}{4}\pi, \sqrt{\frac{2b}{a+b}}\right) \right\} \right] \quad \text{V. T. 74. N}^\circ 2 \text{ et } 3.$$

$$11) \int \frac{x \sin 2x}{1 \sqrt{(a+b \cos x)^3}} dx = \frac{4}{b^2} \left[-\pi \sqrt{a} + \frac{2}{1 \sqrt{(a+b)}} \left\{ (a+b) E\left(\frac{\pi}{4}, \sqrt{\frac{2b}{a+b}}\right) + a F\left(\frac{\pi}{4}, \sqrt{\frac{2b}{a+b}}\right) \right\} \right] \quad \text{V. T. 74. N}^\circ 1 \text{ et } 4.$$

$$12) \int \frac{x \cos x}{1 \sqrt{(1-p^2 \sin^2 x)^3}} dx = \frac{\pi}{2 \sqrt{(1-p^2)}} + \frac{1}{2p} l \frac{1-p}{1+p}, p < 1; \quad \text{V. T. 75. N}^\circ 10.$$

$$13) \int \frac{x \sin 2x}{1 \sqrt{(1-p^2 \sin^2 x)^3}} dx = \frac{1}{p^2} \left\{ \frac{\pi}{1 \sqrt{(1-p^2)}} - 2 F'(p) \right\}, p < 1; \quad \text{V. T. 75. N}^\circ 9.$$

$$14) \int \frac{x \sin 2x}{1 \sqrt{(1-p^2 \sin^2 x)^5}} dx = \frac{2}{3p^2} \frac{1}{1-p^2} \left\{ \frac{\pi}{2 \sqrt{(1-p^2)}} - E'(p) \right\}, p < 1; \quad \text{V. T. 75. N}^\circ 18.$$

$$15) \int \frac{x \sin^2 x \cos x}{1 \sqrt{(1-p^2 \sin^2 x)^5}} dx = \frac{1}{3p^2(1-p^2)} \left[-1 - \frac{p^2 \pi}{1 \sqrt{(1-p^2)}} + \frac{1-p^2}{2p} l \frac{1+p}{1-p} \right] \quad \text{V. T. 75. N}^\circ 10 \text{ et } 19.$$

$$16) \int \frac{x \sin^2 x \sin 2x}{1 \sqrt{(1-p^2 \sin^2 x)^5}} dx = \frac{1}{3p^4} \left[\pi \frac{3p^2-2}{1 \sqrt{(1-p^2)^3}} - \frac{2}{1-p^2} E'(p) + 6 F'(p) \right] \quad \text{V. T. 75. N}^\circ 9 \text{ et } 20.$$

$$17) \int \frac{x}{\sin x + \cos x} \frac{dx}{1 \sqrt{\sin 2x}} = \frac{1}{8} \pi^2 \sqrt{2} \quad \text{V. T. 268. N}^\circ 1.$$

$$18) \int \frac{1-x \cot x}{1 \sqrt{(1-\cos^2 \lambda \sin^2 x)} \sin x} \frac{dx}{\sin x} = \frac{1}{2} \frac{\pi}{1+\cos \lambda} + \frac{\lambda \cot \lambda - 1}{\sin \lambda} \quad \text{Legendre, Exerc. Suppl. 49.}$$

$$19) \int \frac{\sin 2x}{1 \sqrt{(1-p^2 \sin^2 x)} \sqrt{\sin x}} \frac{xdx}{\sin x} = \frac{2}{p^2} \left[-\pi \sqrt{(1-p^2)} + 4 \frac{cF'(c)+bF'(b)}{(b+c)^2} + 4 \frac{b-c}{(b+c)^2} \{ E'(b) - E'(c) \} \right] \\ , \text{ où } 2b^2 = \frac{(1+\sqrt{p})^2}{1+p}, 2c^2 = \frac{(1-\sqrt{p})^2}{1+p}; \quad \text{V. T. 73. N}^\circ 13.$$

$$20) \int \frac{x}{1 \sqrt{\sin^2 x} + 1 \sqrt{\cos^2 x} \sqrt{\sin^2 x \cos^2 x}} \frac{dx}{1 \sqrt{\sin^2 x \cos^2 x}} = \frac{3}{8} \pi^2 \quad \text{V. T. 268. N}^\circ 4.$$

$$1) \int x \cos ax dx = \frac{1}{a^2} (\cos a\pi - 1) \quad \text{Dienger, Cr. 34. 75. — Schlömilch, Höh. An. 80. — Id., Beitr. I. § 8.}$$

$$2) \int x \sin ax dx = \frac{\pi}{a} \cos \{(a+1)\pi\} \quad \text{Schlömilch, Höh. Anal. 80. — Id., Beitr. I. § 8.}$$

- 3) $\int x \operatorname{Sin.} \left\{ \frac{2a-1}{2} x \right\} dx = \frac{4}{(2a-1)^2} \operatorname{Sin.} \left\{ \frac{2a-1}{2} \pi \right\}$ Dienger, Cr. 34. 75.
- 4) $\int x \operatorname{Cot.} \frac{1}{2} x dx = 2\pi l 2$ Legendre, Exerc. 5. 68.
- 5) $\int x \operatorname{Tang.} x dx = -\pi l 2$ V. T. 346. N°. 6.
- 6) $\int x \operatorname{Sin.}^q x dx = \frac{x^2}{2q+1} \frac{\Gamma(q+1)}{\{\Gamma(\frac{1}{2}q+1)\}^2}$ Lobatschewsky, Mém. Kasan. 1835. 211. — Grunert, Gr. 4. 113.
- 7) $\int x \operatorname{Sin.} x \operatorname{Cos.} a x dx = (-1)^{a+1} \frac{\pi}{a^2-1}$
- 8) $\int x \operatorname{Sin.} a x \operatorname{Cos.} x dx = (-1)^a \frac{a\pi}{a^2-1}$ Schlömilch, Beitr. 1. § 8, 10.
- 9) $\int x \operatorname{Sin.} x \operatorname{Cos.}^{2a} x dx = \frac{\pi}{2a+1}$ Poisson, P. 17. 612. N°. 17.
- 10) $\int x \operatorname{Tang.} x \operatorname{Sec.} x dx = -\pi$ V. T. 81. N°. 1.
- 11) $\int \left(\frac{\pi}{2} - x \right) \operatorname{Tang.} x dx = \pi l 2$ V. T. 271. N°. 3.
- 12) $\int x^2 \operatorname{Sin.} q x dx = \frac{1}{q^2} \{ 2 - q^2 \pi^2 \} \operatorname{Cos.} q\pi - 2$ V. T. 244. N°. 1.
- 13) $\int x^2 \operatorname{Cos.} a x dx = \frac{2\pi}{a^3} \operatorname{Cos.} a\pi$ Dienger, Cr. 34. 75.
- 14) $\int x^2 \operatorname{Cot.} \frac{1}{2} x dx = 2\pi^2 l 2 + 4 \sum_1 \left\{ \frac{1}{n^3} + \frac{(-1)^{n-1}}{n^3} \right\}$
- 15) $\int x^3 \operatorname{Cot.} \frac{1}{2} x dx = 2\pi^3 l 2 - 9\pi \sum_1 \frac{1}{n^3}$
- 16) $\int x^4 \operatorname{Cot.} \frac{1}{2} x dx = 2\pi^4 l 2 + 24 \sum_1 \left\{ \pi^2 \frac{(-1)^n}{n^3} + \frac{2}{n^3} + 2 \frac{(-1)^{n-1}}{n^3} \right\}$
- 17) $\int x^5 \operatorname{Cot.} \frac{1}{2} x dx = 2\pi^5 l 2 + 10 \sum_1 \left\{ \pi^3 \frac{(-1)^n}{n^3} + 6\pi \frac{(-1)^{n-1}}{n^3} \right\}$

Legendre Exerc.
5. 68.

- 1) $\int \frac{x \sin. x}{1 + \cos. x} dx = -\infty$ V. T. 353. N°. 7.
- 2) $\int \frac{x \sin. x}{2 + \cos. x} dx = -2\pi l(\sqrt{3} - 1)$ Poisson, P. 17. 612. N°. 17.
- 3) $\int \frac{x}{p + \cos. x} dx = \frac{\pi^2}{2\sqrt{p^2 - 1}} + \frac{4}{\sqrt{p^2 - 1}} \sum_0^\infty \frac{\{p - \sqrt{p^2 - 1}\}^{2n+1}}{(2n+1)^2}, p > 1$ Legendre, Exerc. 5. 84.
- 4) $\int \frac{x \sin. x}{p + \cos. x} dx = -\pi l\{2(1-p)\}, p < 1;$
 5) $= -2\pi l\{1 - p + \sqrt{p^2 - 1}\}, p > 1;$ Legendre, Exerc. 5. 75.
- 6) $\int \frac{x \sin. x}{i + \cos. x} dx = -2\pi l\{1 - (1 - \sqrt{2})i\}$ Poisson, P. 17. 612. N°. 17.
- 7) $\int \frac{x \sin. x}{1 + p \cos. x} dx = \frac{\pi}{p} l \frac{1 + \sqrt{1 - p^2}}{2(1 - p)}, p < 1;$ V. T. 353. N°. 9.
- 8) $\int \frac{x}{\cos. x + \cos. \lambda} dx = -4 \operatorname{Cosec.} \lambda \sum_0^\infty \frac{\sin. \{(2n+1)\lambda\}}{(2n+1)^2}$ Legendre, Exerc. 5. 85.
- 9) $\int \frac{x^a \sin. x}{\cos. x + \cos. \lambda} dx = -\pi^a l\{2(1 - \cos. \lambda)\} + 2.1^a \cos. \frac{1}{2} a \pi \sum_0^\infty (-1)^{n-1} \frac{\cos. n \lambda}{n^{a+1}} -$
 $- 2 \sum_1^\infty \left\{ \frac{\cos. n \lambda}{n} \sum_1^\infty (-1)^m a^{2m-1} \pi^{a-2m} \frac{1}{n^{2m}} \right\}$ Legendre, Exerc. 5. 66.
- 10) $\int \frac{x^p \sin. x}{q + \cos. x} dx = 2 \cos. \frac{1}{2} p \pi \Gamma(1+p) \sum_1^\infty (-1)^{n-1} \frac{c^n}{n^{p+1}} - 2 \pi^p l(1 - c) -$
 $- 2 \sum_1^\infty \left\{ \frac{c^n}{n} \sum_1^\infty (-1)^{m-1} p^{2m-1} \pi^{a-2m} \frac{1}{n^{2m}} \right\}$ Legendre, Exerc. 5. 74.
 , où $c = q - \sqrt{q^2 - 1}$;
- 11) $\int \frac{x \cos. x}{1 + 2q^2 \sin. x} dx = \frac{\pi}{q^2} l \frac{2}{1 + \sqrt{1 + 4q^2}}$ V. T. 327. N°. 10.
- 12) $\int \frac{x \sin. x}{1 + \cos.^2 x} dx = \frac{1}{4} \pi^2$ Poisson, P. 17. 612. N°. 17. — Grunert, Gr. 4. 113.
- 13) $\int \frac{p \cos. x + q}{\cos.^2 x + \cot.^2 \lambda} x \sin. x dx = 2p \pi l \cos. \frac{1}{2} \lambda + \pi q \operatorname{Tang.} \lambda$ Legendre, Exerc. 5. 77.

- 1) $\int \frac{x^2}{1 - \cos x} dx = 4 \pi l 2$ V. T. 238. N°. 3.
- 2) $\int \frac{x \sin x}{1 - \cos x} dx = 2 \pi l 2$
- 3) $\int \frac{x \sin x}{2 - \cos x} dx = 2 \pi l (3 - \sqrt{3})$
- 4) $\int \frac{x}{p - \cos x} dx = \frac{\pi^2}{2 \sqrt{p^2 - 1}} - \frac{4}{\sqrt{p^2 - 1}} \sum_0^{\infty} \frac{\{p - \sqrt{p^2 - 1}\}^{2n-1}}{(2n+1)^2}, p > 1;$
- 5) $\int \frac{x}{\cos x - \cos \lambda} dx = -4 \operatorname{Cosec} \lambda \sum_0^{\infty} \frac{\sin \{(2n+1)\lambda\}}{(2n+1)^2}$
- 6) $\int \frac{x \sin x}{p - \cos x} dx = \pi l \{2(1+p)\}$, $p < 1;$
- 7) $= 2 \pi l \{1 + p - \sqrt{p^2 - 1}\}$, $p > 1;$
- 8) $\int \frac{x \sin x}{\cos \lambda - \cos x} dx = 2 \pi l \{1 + \cos \lambda - i \sin \lambda\}$
- 9) $\int \frac{x \sin x}{e^p + e^{-p} - 2 \cos x} dx = 2 \pi l \{1 + e^{-p}\}$
- 10) $\int \frac{x \sin x}{i - \cos x} dx = 2 \pi l \{1 + (1 - \sqrt{2})i\}$
- 11) $\int \frac{x \sin x}{1 - p \cos x} dx = \frac{\pi l}{p} \frac{2(1+p)}{1 + \sqrt{1-p^2}}, p < 1;$ V. T. 353. N°. 9.
- 12) $\int \frac{x^a \sin x}{\cos x - \cos \lambda} dx = -\pi^a l \{2(1 + \cos \lambda)\} - 2.1^{a/1} \cos \frac{1}{2} a \pi \sum_0^{\infty} \frac{\cos n \lambda}{n^{a+1}} +$
- $+ 2 \sum_1^{\infty} \left\{ \frac{\cos n \lambda}{n} (-1)^n \sum_1^{\infty} (-1)^n a^{2m-1} \pi^{a-2m} \frac{1}{n^{2m}} \right\}$
- 13) $\int \frac{x^p \sin x}{q - \cos x} dx = 2 \cos \frac{1}{2} p \pi \Gamma(1+p) \sum_1^{\infty} \frac{c^n}{n^{p+1}} + 2 \pi p l (1+c) +$
- $+ 2 \sum_1^{\infty} (-1)^n \frac{c^n}{n} \sum_1^{\infty} (-1)^{m-1} p^{2m-1} \frac{\pi^{a-2m}}{n^{2m}}$
- 14) $\int \frac{x}{p^2 - \cos^2 x} dx = \frac{\pi^2}{2 p \sqrt{p^2 - 1}}, p > 1;$ V. T. 245. N°. 3 et T. 246. N°. 1.

$$15) \int \frac{x}{\cos.^2 \lambda - \cos.^2 x} dx = 0 \quad \text{V. T. 245. N}^\circ. 8 \text{ et T. 246. N}^\circ. 5.$$

$$16) \int \frac{x \sin. x}{p^2 - \cos.^2 x} dx = \frac{\pi}{2p} l \frac{1+p}{1-p}, p < 1; \quad \left. \begin{array}{l} \text{V. T. 245. N}^\circ. 4, 5 \text{ et T. 246. N}^\circ. 6, 7. \end{array} \right\}$$

$$17) \quad = \frac{\pi}{2p} l \frac{p+1}{p-1}, p > 1; \quad \left. \begin{array}{l} \end{array} \right\}$$

$$18) \int \frac{x \cos. x}{p^2 - \cos.^2 x} dx = \frac{-4}{\sqrt{(p^2-1)}} \sum_0^{\infty} \frac{\{p-1\sqrt{(p^2-1)}\}^{2n+1}}{(2n+1)^2}, p > 1; \quad \left. \begin{array}{l} \text{V. T. 245. N}^\circ. 3 \text{ et} \\ \text{T. 246. N}^\circ. 4. \end{array} \right\}$$

$$19) \int \frac{x \cos. x}{\cos.^2 \lambda - \cos.^2 x} dx = 4 \operatorname{Cosec.} \lambda \sum_0^{\infty} \frac{\sin. \{(2n+1)\lambda\}}{(2n+1)^2} \quad \text{V. T. 245. N}^\circ. 8 \text{ et T. 246. N}^\circ. 5.$$

$$20) \int \frac{x \sin. 2x}{p^2 - \cos.^2 x} dx = \pi l \{1(1-p^2)\} \quad , p < 1; \quad \left. \begin{array}{l} \text{V. T. 245. N}^\circ. 4, 5 \text{ et T.} \\ \text{246. N}^\circ. 6, 7. \end{array} \right\}$$

$$21) \quad = 2\pi l \{2\{(p^2-1+p\sqrt{p^2-1})\}\}, p > 1; \quad \left. \begin{array}{l} \end{array} \right\}$$

$$1) \int \frac{x \sin. x}{(1 - \cos. \lambda. \cos. x)^2} dx = \frac{1}{2} \pi \sqrt{2. \operatorname{Cosec.} \lambda. \operatorname{Sec.} \frac{1}{2} \lambda. \operatorname{Cosec.} \frac{\pi+2\lambda}{4}} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 1.}$$

$$2) \int \frac{x \cos. x}{(1 + \cos. \lambda. \sin. x)^2} dx = 4\lambda \operatorname{Cosec.} 2\lambda - \frac{\pi}{\cos. \lambda} \quad \text{V. T. 82. N}^\circ. 5.$$

$$3) \int \frac{x \cos. x}{(1 - \cos. \lambda. \sin. x)^2} dx = 4(\lambda - \pi) \operatorname{Cosec.} 2\lambda + \frac{\pi}{\cos. \lambda} \quad \text{V. T. 82. N}^\circ. 4.$$

$$4) \int \frac{x \cos. x}{\left(1 - \sin. x. \cos. \frac{2a\pi}{b}\right)^2} dx = \pi \operatorname{Sec.} \frac{2a\pi}{b} - \frac{2}{b} \operatorname{Cosec.} \frac{4a\pi^{b-1}}{b} \sum_1^{\infty} \sin. \frac{2na\pi}{b} \cot. \frac{n\pi}{b} \quad \left. \begin{array}{l} \text{V. T. 82.} \\ \text{N}^\circ. 3. \end{array} \right\}$$

$$5) \int \frac{x \sin. x}{(\cos. x - a)^2} dx = \infty \quad \text{Legendre, Exerc. 5. 80.}$$

$$6) \int \frac{x^2 \sin. x}{(p + \cos. x)^2} dx = \frac{-\pi^2}{\sqrt{(p^2-1)}} + \frac{\pi^2}{p-1} - \frac{8}{\sqrt{(p^2-1)}} \sum_0^{\infty} \frac{\{p-1\sqrt{(p^2-1)}\}^{2n+1}}{(2n+1)^2}, p > 1; \quad \left. \begin{array}{l} \text{V. T. 245.} \\ \text{N}^\circ. 3. \end{array} \right\}$$

- 7) $\int \frac{x^2 \sin. x}{(\cos. x + \cos. \lambda)^2} dx = \frac{-\pi^2}{1 - \cos. \lambda} + 8 \operatorname{Cosec.} \lambda \sum_0^{\infty} \frac{\sin. \{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 245. N°. 8.
- 8) $\int \frac{x^2 \sin. x}{(p - \cos. x)^2} dx = \frac{\pi^2}{\sqrt{p^2 - 1}} - \frac{\pi^2}{p+1} - \frac{8}{\sqrt{p^2 - 1}} \sum_0^{\infty} \frac{\{p - \sqrt{p^2 - 1}\}^{2n+1}}{(2n+1)^2}, p > 1;$ V. T. 246. N°. 4.
- 9) $\int \frac{x^2 \sin. x}{(\cos. x - \cos. \lambda)^2} dx = -\frac{\pi^2}{1 + \cos. \lambda} + 8 \operatorname{Cosec.} \lambda \sum_0^{\infty} \frac{\sin. \{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 246. N°. 5.
- 10) $\int \frac{p \cos. x + 1}{(p + \cos. x)^2} x^2 dx = 4\pi l \{1 - p - \sqrt{p^2 - 1}\}, p \geq 1;$
- 11) $= 2\pi l \{2(1 - p)\}, p < 1;$ } V. T. 245. N°. 4, 5.
- 12) $\int \frac{p \cos. x - 1}{(p - \cos. x)^2} x^2 dx = -2\pi l \{2(1 + p)\}, p < 1;$
- 13) $= -4\pi l \{1 + p - \sqrt{p^2 - 1}\}, p \geq 1;$ } V. T. 246. N°. 6, 7.
- 14) $\int \frac{x \sin. x}{(p + q \cos. x)^2} dx = \frac{\pi}{q} \left\{ \frac{1}{p - q} - \frac{1}{\sqrt{p^2 - q^2}} \right\}, p^2 > q^2;$ V. T. 82. N°. 6
- 15) $\int \frac{q \cos. 2x + \sin.^2 x}{(q - \sin.^2 x)^2} dx = \pi l (-4q), q < 0;$
- 16) $\int \frac{q \cos. 2x - \sin.^2 x}{(q + \sin.^2 x)^2} dx = -2\pi l [2\{-q + \sqrt{q(q+1)}\}], q > 0;$ } V. T. 246. N°. 20, 21.
- 17) $\int \frac{p^2 - 1 - \sin.^2 x}{(p^2 - \cos.^2 x)^2} \cos. x dx = \frac{\pi l}{p} \frac{1 - p}{1 + p}, p < 1;$
- 18) $= \frac{\pi l}{p} \frac{p - 1}{p + 1}, p > 1;$ } V. T. 246. N°. 16, 17.
- 19) $\int \frac{x^2 \sin. 2x}{(p^2 - \cos.^2 x)^2} dx = \frac{\pi^2}{p} \frac{\sqrt{p^2 - 1} - p}{p^2 - 1}, p > 1;$ V. T. 246. N°. 14.
- 20) $\int \frac{x \sin. 2x}{(1 - \cos.^2 \lambda \sin.^2 x)^2} dx = \frac{\sin. \lambda - 1}{\cos. \lambda \sin. 2\lambda} 2\pi$ V. T. 83. N°. 5.
- 21) $\int \frac{x^2 \sin. 2x}{(\cos.^2 \lambda - \cos.^2 x)^2} dx = \pi^2 \operatorname{Cosec.}^2 \lambda$ V. T. 246. N°. 15.

$$22) \int \frac{1 + \text{Cos.}^2 \lambda \cdot \text{Sin.}^2 x}{(1 - \text{Cos.}^2 \lambda \cdot \text{Sin.}^2 x)^2} x \text{Cos.} x dx = 2 \text{Cosec.} 2 \lambda (2 \lambda - \pi) \quad \text{V. T. 83. N}^\circ. 6.$$

$$23) \int \frac{x \text{Sin.} 2x}{(p^2 \text{Sin.}^2 x + q^2 \text{Cos.}^2 x)^2} dx = \frac{-\pi}{p q^2 (q + p)} \quad \text{V. T. 83. N}^\circ. 7.$$

$$24) \int \frac{x \text{Sin.} 2x}{(p^2 \text{Sin.}^2 x + q^2 \text{Cos.}^2 x)^3} dx = \frac{-\pi}{4 p^3 q^4} \frac{p^2 + p q + q^2}{q + p} \quad \text{V. T. 83. N}^\circ. 8.$$

$$25) \int \frac{x \text{Sin.} x}{(\text{Cos.} x + \text{Cos.} \lambda)^a} dx = \infty \quad \left. \vphantom{\int} \right\} \text{Legendre, Exerc. 5. 79.}$$

$$26) \int \frac{x \text{Sin.} x}{(\text{Cos.} x - \text{Cos.} \lambda)^a} dx = \infty \quad \left. \vphantom{\int} \right\} , a \geq 2;$$

$$27) \int \frac{\text{Cos.} k x}{(q + 2 p \text{Cos.} x)^a} dx = \frac{a^{k/1}}{1 q^2} (q^2 - 4 p^2)^{-\frac{a}{2}} \left\{ \frac{-4 p}{q + \sqrt{(q^2 - 4 p^2)}} \right\}^k \sqrt{\frac{\pi}{k}}, k = \infty; \text{Jacobi, Cr. 15. 1.}$$

$$1) \int \frac{x \text{Sin.} x}{1 - 2 p \text{Cos.} x + p^2} dx = \frac{\pi}{p} l (1 + p), p < 1; \quad \left. \vphantom{\int} \right\} \text{Poisson, P. 17. 612. N}^\circ. 16.$$

$$2) \quad = \frac{\pi}{p} l \frac{1 + p}{p}, p > 1; \quad \left. \vphantom{\int} \right\}$$

$$3) \int \frac{x \text{Sin.} x}{1 - \text{Sin.}^2 \lambda \cdot \text{Cos.} x + \text{Cos.}^2 \lambda} dx = -4 \pi \text{Cosec.}^2 \lambda l \text{Cos.} \frac{1}{2} \lambda \quad \text{V. T. 334. N}^\circ. 13.$$

$$4) \int \frac{\text{Cos.} b x}{1 - 2 p \text{Cos.} x + p^2} x^{\pm 2a} dx = (-1)^a \pi \frac{p^b}{1 - p^2} (l p)^{\pm 2a}$$

$$5) \int \frac{\text{Cos.} b x \text{Cos.} x}{1 - 2 p \text{Cos.} x + p^2} x^{\pm 2a} dx = (-1)^a \frac{1}{2} \pi \frac{1 + p^2}{1 - p^2} p^{b-1} (l p)^{\pm 2a}$$

$$6) \int \frac{\text{Cos.} b x \text{Sin.} x}{1 - 2 p \text{Cos.} x + p^2} x^{\pm (2a+1)} dx = \pm (-1)^a \frac{1}{2} \pi p^{b-1} (l p)^{\pm (2a+1)}$$

$$7) \int \frac{\text{Sin.} b x}{1 - 2 p \text{Cos.} x + p^2} x^{\pm (2a+1)} dx = \pm (-1)^{a+1} \pi \frac{p^b}{1 - p^2} (l p)^{\pm (2a+1)}$$

$$8) \int \frac{\text{Sin.} b x \text{Sin.} x}{1 - 2 p \text{Cos.} x + p^2} x^{\pm 2a} dx = (-1)^a \frac{1}{2} \pi p^{b-1} (l p)^{\pm 2a}$$

$$9) \int \frac{\text{Sin.} b x \text{Cos.} x}{1 - 2 p \text{Cos.} x + p^2} x^{\pm (2a+1)} dx = \pm (-1)^{a+1} \frac{1}{2} \pi \frac{1 + p^2}{1 - p^2} p^{b-1} (l p)^{\pm (2a+1)}$$

Circ. Dir. endén. trin. $1 - a \operatorname{Cos}. x + b$.

$$10) \int \frac{\operatorname{Cos}. \{(2b-1)x\}}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a} dx = 0$$

$$11) \int \frac{\operatorname{Sin}. \{(2b-1)x\}}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a+1} dx = 0$$

$$12) \int \frac{\operatorname{Cos}. 2bx \cdot \operatorname{Cos}. x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a} dx = 0$$

$$13) \int \frac{\operatorname{Cos}. 2bx \cdot \operatorname{Sin}. x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a+1} dx = 0$$

$$14) \int \frac{\operatorname{Sin}. 2bx \cdot \operatorname{Sin}. x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a} dx = 0$$

$$15) \int \frac{\operatorname{Sin}. 2bx \cdot \operatorname{Cos}. x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a+1} dx = 0$$

$$16) \int \frac{\operatorname{Cos}. \{(2b-1)x\} \cdot \operatorname{Cos}. 2x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a} dx = 0$$

$$17) \int \frac{\operatorname{Cos}. \{(2b-1)x\} \cdot \operatorname{Sin}. 2x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a+1} dx = 0$$

$$18) \int \frac{\operatorname{Sin}. \{(2b-1)x\} \cdot \operatorname{Sin}. 2x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a} dx = 0$$

$$19) \int \frac{\operatorname{Sin}. \{(2b-1)x\} \cdot \operatorname{Cos}. 2x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a+1} dx = 0$$

$$20) \int \frac{\operatorname{Cos}. \{(2b+1)x\} \cdot \operatorname{Cos}. x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a} dx = (-1)^a \frac{\pi}{2^{+2a+2}} \frac{q^b}{1-q} (lq)^{+2a}$$

$$21) \int \frac{\operatorname{Cos}. \{(2b+1)x\} \cdot \operatorname{Sin}. x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+(2a+1)} dx = \pm (-1)^a \frac{\pi}{2^{+(2a+1)+2}} \frac{q^b}{1+q} (lq)^{+(2a+1)}$$

$$22) \int \frac{\operatorname{Sin}. \{(2b+1)x\} \cdot \operatorname{Sin}. x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+2a} dx = (-1)^a \frac{\pi}{2^{+2a+2}} \frac{q^b}{1+q} (lq)^{+2a}$$

$$23) \int \frac{\operatorname{Sin}. \{(2b+1)x\} \cdot \operatorname{Cos}. x}{1 - 2q \operatorname{Cos}. 2x + q^2} x^{+(2a+1)} dx = \pm (-1)^{a+1} \frac{\pi}{2^{+(2a+1)+2}} \frac{q^b}{1-q} (lq)^{+(2a+1)}$$

Les intégrales (4) à (23) où $p^2 \leq 1$, $0 \leq q < 1$, se trouvent: Bierens de Haan, Gr. 13. 193

$$24) \int \frac{x \operatorname{Sin}. 2x}{1 - 2p \operatorname{Cos}. 2x + p^2} dx = \frac{\pi}{2p} l(1-p), p < 1; \text{ V. T. 353. N}^\circ. 23.$$

- 1) $\int \frac{x \sin x}{1 + 2p \cos x + p^2} dx = -\frac{\pi}{p} l(1-p), p \leq 1;$
 2) $= \frac{\pi}{p} l \frac{p}{p-1}, p > 1;$ V. T. 333. N°. 19, 20.
- 3) $\int \frac{x \sin x}{2 + p \cos x + p} dx = \frac{2\pi}{p} l \frac{1 + \sqrt{1 + p}}{2}$ V. T. 334. N°. 8.
- 4) $\int \frac{\cos x + q \cos \lambda}{q^2 + 2q \cos \lambda \cos x + \cos^2 x} x \sin x dx = -\pi l \{ 1 - 2q \cos \lambda + 2h \cos \theta +$
 $+ q^2 + h^2 - 2gh \cos(\lambda - \theta) \}$ Legendre, Exerc. 5. 77.
- 5) $\int \frac{x \sin x}{q^2 + 2q \cos \lambda \cos x + \cos^2 x} dx = \frac{2}{q} \pi \operatorname{Cosec} \lambda \operatorname{Arctang} \frac{h \sin \theta - q \sin \lambda}{1 - q \cos \lambda + h \cos \theta}$
- 6) $\int \frac{p \cos x + r}{q^2 + 2q \cos \lambda \cos x + \cos^2 x} x \sin x dx = -2\pi p l \{ 1 - 2q \cos \lambda + 2h \cos \theta +$
 $+ q^2 + h^2 - 2gh \cos(\lambda - \theta) \} + \frac{r - pq \cos \lambda}{q \sin \lambda} 2\pi \operatorname{Arctang} \frac{h \sin \theta - q \sin \lambda}{1 - q \cos \lambda + h \cos \theta}$ où $\operatorname{Tang} 2\theta = \frac{q^2 \sin 2\lambda}{q^2 \cos 2\lambda - 1}$, $h^2 = 1 - 2q^2 \cos 2\lambda + q^4$.
- 7) $\int \frac{x \sin x}{(\cos x - q)^2 - k^2} dx = -\frac{\pi}{1 + q}, k$ infiniment petit > 0 ; Legendre, Exerc. 5. 81.
- 8) $\int \frac{\frac{\pi}{2} + x}{\sin^2 x + (a \sin x + b \cos x)^2} dx = \frac{\pi}{b} \left[\frac{1}{2} \operatorname{Arctang} \left(\frac{2ab}{1 + a^2 - b^2} \right) - \right.$
 $\left. - \operatorname{Arctang} \frac{\operatorname{Tang} \left\{ \frac{1}{2} \operatorname{Arctang} \left(\frac{2ab}{1 + a^2 - b^2} \right) \right\}}{a \operatorname{Tang} \left\{ \frac{1}{2} \operatorname{Arctang} \left(\frac{2ab}{1 + a^2 - b^2} \right) \right\} - b} \right]$ V. T. 271. N°. 2.
- 9) $\int \frac{\sin x}{1 - \cos \lambda \cos x} \frac{x}{1 - \cos \mu \cos x} dx = \pi \operatorname{Cosec} \left\{ \frac{1}{2} (\lambda + \mu) \right\} \cdot \operatorname{Cosec} \left\{ \frac{1}{2} (\lambda - \mu) \right\} \cdot l \frac{1 + \operatorname{Tang} \frac{1}{2} \lambda}{1 + \operatorname{Tang} \frac{1}{2} \mu}$ Lobatschewsky, Mém. Kasan. 1835. 1
- 10) $\int \frac{(1 + p^2) \cos x - 2p}{(1 - 2p \cos x + p^2)^2} x^2 dx = 2 \frac{\pi}{p} l \frac{p}{1 + p}, p \geq 1;$
 11) $= -2 \frac{\pi}{p} l(1 + p), p < 1;$ V. T. 243. N°. 1, 2.
- 12) $\int \frac{x \sin x}{(1 + p^2 - 2p \cos x)^2} dx = \frac{\pi}{(1 - p)(1 + p)^2}, p < 1;$ V. T. 84. N°. 1.

$$13) \int \frac{x \sin. x}{(1+p^2-2p \cos. x)^2} dx = \frac{\pi}{p(p-1)(1+p)^2}, p > 1; \quad \text{V. T. 84. N}^\circ. 2.$$

$$14) \int \frac{x \sin. x}{(1+p^2+2p \cos. x)^2} dx = \frac{\pi}{(1+p)(1-p)^2}, p < 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{V. T. 84. N}^\circ. 3.$$

$$15) \quad = \frac{\pi}{p(1+p)(p-1)^2}, p > 1; \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$16) \int \frac{x \sin. x}{(1+p^2-2p \cos. x)^3} dx = \frac{p^2-p+2}{2(1+p)^4(1-p)^3}, p < 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{V. T. 85. N}^\circ. 6, 7.$$

$$17) \quad = \frac{2p^2-p+1}{2p(1+p)^4(p-1)^3}, p > 1; \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$18) \int \frac{x \sin. x}{(1+p^2-2p \cos. x)^{a+1}} dx = \frac{\pi}{2pa} \left\{ \frac{-1}{(1+p)^{2a}} + \frac{1}{(1-p^2)^{2a-1}} \sum_0^{a-1} \binom{a-1}{n} p^{2n} \right\}, p^2 < 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{V. T. 85. N}^\circ. 12, 13.$$

$$19) \quad = \frac{\pi}{2pa} \left\{ \frac{-1}{(1+p)^{2a}} + \frac{1}{(p^2-1)^{2a-1}} \sum_0^{a-1} \binom{a-1}{n} p^{2n} \right\}, p^2 > 1; \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$20) \int \frac{x \text{Tang. } x}{(1+p^2-2p \cos. x)^2} \frac{dx}{\cos. x} = -\pi \frac{1+p+3p^2-p^3}{(1+p)^2(1+p^2)^2(1-p)}, p^2 < 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{V. T. 85. N}^\circ. 4, 5.$$

$$21) \quad = \pi \frac{1-3p-p^2-p^3}{(1+p)^2(1+p^2)^2(p-1)}, p^2 > 1; \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{d'après Schlömilch; elles sont } \infty.$$

$$22) \int \frac{x \sin. x}{1 \sqrt{(1-2p \cos. x + p^2)}} dx = \frac{1+p}{p} \pi + 2 \frac{1-p^2}{p} F'(p) - \frac{1}{p} E'(p), p < 1; \quad \text{Ramus, Danske Afh. 6. 265.}$$

$$23) \int \frac{p \cos.^2 x - (1+p^2) \cos. x + p}{1 \sqrt{(1-2p \cos. x + p^2)^3}} dx = \frac{1+p}{p} 2\pi + 4 \frac{1-p^2}{p} F'(p) - \frac{8}{p} E'(p), p < 1; \quad \text{V. T. 249. N}^\circ. 22.$$

$$24) \int \frac{\sin. x}{x} \cos. \frac{2a\pi x}{b} dx = \frac{1}{2} a, a < b; \quad \text{Schaar, Mém. Cour. Brux. T. 22.}$$

$$1) \int x \sin. x dx = -2\pi \left\{ \begin{array}{l} \\ \\ \end{array} \right. \quad \text{Raabe, Cr. 15 355.}$$

$$2) \int x \cos. x dx = 0 \left\{ \begin{array}{l} \\ \\ \end{array} \right.$$

- 3) $\int \frac{\text{Sin. } ax}{1+p \text{Cos. } x} x dx = \frac{2\pi}{\sqrt{1-p^2}} \left[\frac{\{-1(1-p^2)-1\}^a - \{\sqrt{1-p^2}-1\}^a}{p^a} l \frac{2\sqrt{1+p}}{\sqrt{1+p}+\sqrt{1-p}} + \sum_{n=1}^{a-1} \frac{1}{a-n} \frac{\{-\sqrt{1-p^2}-1\}^n - \{\sqrt{1-p^2}-1\}^n}{a^n} \right], p < 1; \text{ Ohm, Ausw. 26.}$
- 4) $\int \frac{\text{Sin. } ax}{1-p \text{Cos. } x} x dx = \frac{2\pi}{\sqrt{1-p^2}} \left[\frac{\{1+\sqrt{1-p^2}\}^a - \{1-\sqrt{1-p^2}\}^a}{p^a} l \frac{2\sqrt{1-p}}{\sqrt{1+p}+\sqrt{1-p}} + \sum_{n=1}^{a-1} \frac{1}{a-n} \frac{\{1+\sqrt{1-p^2}\}^n - \{1-\sqrt{1-p^2}\}^n}{a^n} \right], p < 1; \text{ Raabe, Int. 173. — Ohm, Ausw. 26.}$
- 5) $\int \frac{\text{Cos. } ax}{1+p \text{Cos. } x} x dx = \frac{2\pi^2}{\sqrt{1-p^2}} \left\{ \frac{1-\sqrt{1-p^2}}{p} \right\}^a, p < 1;$
- 6) $\int \frac{\text{Cos. } ax}{1-p \text{Cos. } x} x dx = \frac{2\pi^2}{\sqrt{1-p^2}} \left\{ \frac{\sqrt{1-p^2}-1}{p} \right\}^a \text{ Raabe, Int. 173. — Ohm, Ausw. 26.}$
- 7) $\int \frac{x \text{Sin. } x}{1-2p \text{Cos. } x + p^2} dx = \frac{2\pi}{p} l(1-p), p < 1; \text{ Raabe, Int. 173.}$
- 8) $\int \frac{x \text{Sin. } ax}{1-2p \text{Cos. } x + p^2} dx = \frac{2\pi}{1-p^2} \left\{ (p^{-a} - p^a) l(1-p) + \sum_{n=1}^{a-1} \frac{p^{-n} - p^n}{a-n} \right\} \text{ Raabe, Int. 173.}$
- 9) $\int \frac{\text{Cos. } bx}{1-2p \text{Cos. } x + p^2} x^{+2a} dx = (-1)^a \pi \frac{p^b}{1-p^2} (lp)^{+2a}$
- 10) $\int \frac{\text{Cos. } bx \text{Cos. } x}{1-2p \text{Cos. } x + p^2} x^{+2a} dx = (-1)^a \frac{1}{2} \pi \frac{1+p^2}{1-p^2} p^{b-1} (lp)^{+2a}$
- 11) $\int \frac{\text{Cos. } bx \text{Sin. } x}{1-2p \text{Cos. } x + p^2} x^{+(2a+1)} dx = \pm (-1)^a \frac{1}{2} \pi p^{b-1} (lp)^{+(2a+1)}$
- 12) $\int \frac{\text{Sin. } bx}{1-2p \text{Cos. } x + p^2} x^{+(2a+1)} dx = \pm (-1)^{a-1} \pi \frac{p^b}{1-p^2} (lp)^{+(2a+1)}$
- 13) $\int \frac{\text{Sin. } bx \text{Sin. } x}{1-2p \text{Cos. } x + p^2} x^{+2a} dx = (-1)^a \frac{1}{2} \pi p^{b-1} (lp)^{+2a}$
- 14) $\int \frac{\text{Sin. } bx \text{Cos. } x}{1-2p \text{Cos. } x + p^2} x^{+(2a+1)} dx = (-1)^{a+1} \frac{1}{2} \pi \frac{1+p^2}{1-p^2} p^{b-1} (lp)^{+(2a+1)}$
- 15) $\int \frac{\text{Cos. } ax - p \text{Cos. } \{(a+1)x\}}{1-2p \text{Cos. } x + p^2} x dx = 2\pi^2 p^a, p < 1; \text{ Raabe, Int. 173.}$

, $0 \leq p \leq 1;$ Bierens de Haan,
Gr. 13. 193.

- $$\left. \begin{aligned} 1) \int x \operatorname{Sin.}(2 a \pi x^2) dx &= \frac{1}{2 a \pi} \left\{ 1 - \operatorname{Cos.}(2 a \pi r^2) \right\} \\ 2) \int x \operatorname{Cos.}(2 a \pi x^2) dx &= \frac{1}{2 a \pi} \operatorname{Sin.}(2 a \pi r^2) \end{aligned} \right\} \text{Abria, L. 4. 248.}$$
- $$3) \int \frac{\operatorname{Sin.} x}{x} dx = \operatorname{Si.}(r) \quad \text{Arndt, Gr. 10. 225. — Schlömilch, Gr. 11. 389.}$$
- $$4) \int \frac{\operatorname{Sin.} q x}{x} dx = \operatorname{Si.}(q r) \quad \text{Schlömilch, Gr. 11. 389.}$$
- $$\left. \begin{aligned} 5) \int \frac{1 - \operatorname{Cos.} r x}{x^2} dx &= \frac{1}{2} r \pi, \quad r > 0; \\ 6) &= -\frac{1}{2} r \pi, \quad r < 0; \\ 7) \int \frac{\operatorname{Sin.} r x - r x \operatorname{Cos.} r x}{x^3} dx &= \frac{1}{4} r^2 \pi, \quad r > 0; \\ 8) &= -\frac{1}{4} r^2 \pi, \quad r < 0; \end{aligned} \right\} \text{Poisson, Mém. Inst. 1816. 71.}$$
- $$9) \int \frac{\frac{r}{x} \operatorname{Sin.} \frac{r \lambda}{x} - \frac{r}{x} \operatorname{Sin.} \frac{\lambda x}{r}}{\frac{x}{r} - \frac{r}{x}} \frac{dx}{x} = \frac{1}{2} \pi (1 - \operatorname{Cos.} \lambda)$$
- $$10) \int \frac{\left(\frac{x}{r}\right)^{1-p} \operatorname{Sin.} \left(\frac{1}{2} p \pi - \frac{r \lambda}{x}\right) - \left(\frac{r}{x}\right)^{1-p} \operatorname{Sin.} \left(\frac{1}{2} p \pi - \frac{\lambda x}{r}\right)}{\frac{x}{r} - \frac{r}{x}} \frac{dx}{x} = \frac{1}{2} \pi \operatorname{Cos.} \left(\frac{1}{2} p \pi - \lambda\right) \quad \left. \begin{array}{l} \text{Cauchy,} \\ \text{P. 29. 511.} \end{array} \right\}$$
- $$11) \int \frac{x}{\operatorname{Cos.} x \cdot \operatorname{Cos.}(r-x)} dx = r \operatorname{Cosec.} r \cdot \operatorname{tSec.} r, \quad r < \frac{1}{2} \pi; \quad \text{Lindman, Gr. 16. 94.}$$
- $$12) \int \frac{x^2 \operatorname{Sin.}(2 x - r)}{\operatorname{Cos.}^2 x \cdot \operatorname{Cos.}^2(r-x)} dx = r^2 \operatorname{Sec.} r + 2 r \operatorname{Cosec.} r \cdot \operatorname{tCos.} r, \quad r < \frac{1}{2} \pi; \quad \text{V. T. 251. N. 11.}$$
- $$13) \int \frac{x \operatorname{Sin.} x}{(\operatorname{Cos.} x - \operatorname{Cos.} \mu)^2} dx = \frac{-\frac{1}{2} r}{\operatorname{Cos.} r - \operatorname{Cos.} \mu} + \frac{\operatorname{Sin.} r}{2 \operatorname{Sin.}^2 \mu (\operatorname{Cos.} r - \operatorname{Cos.} \mu)} + \frac{\operatorname{Cos.} \mu}{2 \operatorname{Sin.}^3 \mu} \operatorname{t} \frac{\operatorname{Sin.} \left\{ \frac{1}{2} (\mu + r) \right\}}{\operatorname{Sin.} \left\{ \frac{1}{2} (\mu - r) \right\}}, \quad r < \mu. \quad \left. \begin{array}{l} \text{Legendre,} \\ \text{Exerc. 5.} \\ \text{83.} \end{array} \right\}$$

- $$1) \int x \sin x \sqrt{(\sin^2 \lambda - \sin^2 x)} = \frac{1}{8} \pi \sin^2 \lambda + \frac{1}{4} \pi \cos^2 \lambda \cdot l \cos \lambda$$
- $$2) \int \frac{x dx \sqrt{(\sin^2 \lambda - \sin^2 x)}}{\sin x} = \pi \frac{1 + \sin \lambda}{4} l(1 + \sin \lambda) + \pi \frac{1 - \sin \lambda}{4} l(1 - \sin \lambda)$$
- $$3) \int \frac{x \sin x}{\sqrt{(\sin^2 \lambda - \sin^2 x)}} dx = \frac{1}{2} \pi l \sec \lambda$$
- $$4) \int \frac{x \sin^3 x}{\sqrt{(\sin^2 \lambda - \sin^2 x)}} dx = -\pi \frac{1 + \sin^2 \lambda}{4} l \cos \lambda - \frac{1}{8} \pi \sin^2 \lambda$$
- $$5) \int \frac{x}{\sin x \sqrt{(\sin^2 \lambda - \sin^2 x)}} dx = \frac{1}{4} \pi \operatorname{Cosec} \lambda \cdot l \frac{1 + \sin \lambda}{1 - \sin \lambda} \quad \text{Legendre, Exerc. Suppl. 23.}$$
- $$6) \int \frac{x \sin x}{\cos^2 x \sqrt{(\sin^2 \lambda - \sin^2 x)}} dx = \frac{1}{2} \pi \sec^2 \lambda (1 - \cos \lambda) \quad \begin{array}{l} \text{Legendre, Exerc. Suppl. 23. —} \\ \text{Lobatschewsky, Mém. Kasan. 1835.} \\ \text{1. — Id., Mém. Kasan. 1836. 1.} \\ \text{I. form. (82). — Id., ib. II. form (18).} \end{array}$$
- $$7) \int \frac{x \sin x}{1 - \sin^2 \mu \sin^2 x} \frac{dx}{\sqrt{(\sin^2 \lambda - \sin^2 x)}} = \frac{\pi}{2 \cos \mu} \frac{1}{1 - \sin^2 \lambda \sin^2 \mu} l \frac{\cos \mu + 1}{2 \cos \lambda \sin^2 \frac{1}{2} \mu} (1 - \sin^2 \lambda \sin^2 \mu)$$
- Lobatschewsky, Mém. Kasan. 1835. 1. — Id., Mém. Kasan. 1836. I. I form. (79). — Id., ib. II. form. (17).
- $$8) \int \frac{x \sin x}{\sin^2 \mu - \sin^2 x} \frac{dx}{\sqrt{(\sin^2 \lambda - \sin^2 x)}} = \frac{\pi \sec \mu}{2 \sqrt{(\sin^2 \mu - \sin^2 \lambda)}} \left\{ {}^n - \operatorname{Arccos} \frac{\cos \mu}{\cos \lambda} \right\}$$
- Lobatschewsky, Mém. Kasan. 1835. I. — Id., Mém. Kasan. 1836. I. I form. (81). — Id., ib. II. form. (19).
- $$9) \int \frac{1 - x \cot x}{\sin^2 x \sqrt{(\sin^2 \lambda - \sin^2 x)}} \cos x dx = \frac{1}{4} \pi \operatorname{Cosec}^3 \lambda - \frac{1}{8} \pi \cos^2 \lambda \operatorname{Cosec}^2 \lambda \cdot l \frac{1 + \sin \lambda}{1 - \sin \lambda}$$
- Legendre, Exerc. Suppl. 17.

- $$1) \int \frac{x}{\sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} dx = \frac{1}{2} \pi \sec \lambda \cdot \operatorname{Cosec} \mu F'(c, \mu)$$
- $$2) \int \frac{x}{\sin^2 x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} dx = \frac{\pi \sin \lambda - \sin \mu}{2 \sin^2 \lambda \sin^2 \mu} + \frac{\pi}{2 \cos \lambda \sin \mu} F(c, \mu) + \frac{\pi \cos \lambda}{2 \sin^2 \lambda \sin \mu} E(c, \mu)$$
- Legendre, Exerc. Suppl. 15. — Roberts, L. 11. 157.

$$\begin{aligned}
3) & (2a+1) \operatorname{Sin}^2 \lambda \cdot \operatorname{Sin}^2 \mu \int_{\lambda}^{\mu} \frac{x}{\operatorname{Sin}^{2a+2} x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} dx = \\
& 2a(\operatorname{Sin}^2 \lambda + \operatorname{Sin}^2 \mu + \operatorname{Sin}^2 \lambda \cdot \operatorname{Sin}^2 \mu) \int_{\lambda}^{\mu} \frac{x dx}{\operatorname{Sin}^{2a} x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} \\
& - (2a-1)(1 + \operatorname{Sin}^2 \lambda + \operatorname{Sin}^2 \mu) \int_{\lambda}^{\mu} \frac{x dx}{\operatorname{Sin}^{2a-2} x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} \quad \text{Legendre, Exerc. S. 15.} \\
& + (2a-2) \int_{\lambda}^{\mu} \frac{x dx}{\operatorname{Sin}^{2a-4} x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} \\
& - \frac{\pi(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 \lambda)^2}{4 \operatorname{Sin}^{2a-1} \lambda \cdot \operatorname{Sin}^3 \mu} \sum_0^n (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}} \left(\frac{\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 \lambda}{\operatorname{Sin}^2 \mu} \right)^n \\
4) & \int \frac{x}{\operatorname{Cos}^2 x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} dx = \frac{\pi}{2} \frac{\operatorname{Cos} \mu - \operatorname{Cos} \lambda}{\operatorname{Cos}^2 \lambda \cdot \operatorname{Cos} \mu} + \quad \text{Legendre, Exerc. Suppl. 19. — Roberts, L. 11 157.} \\
& + \frac{\pi}{2 \operatorname{Cos} \lambda \cdot \operatorname{Sin} \mu} \operatorname{F}(c, \mu) + \frac{\pi \operatorname{Sin} \mu}{2 \operatorname{Cos} \lambda \cdot \operatorname{Cos}^2 \mu} \operatorname{E}(c, \mu) \\
5) & (2a+1) \operatorname{Cos}^2 \lambda \cdot \operatorname{Cos}^2 \mu \int_{\lambda}^{\mu} \frac{x}{\operatorname{Cos}^{2a+2} x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} dx = \\
& 2a(\operatorname{Cos}^2 \lambda + \operatorname{Cos}^2 \mu + \operatorname{Cos}^2 \lambda \cdot \operatorname{Cos}^2 \mu) \int_{\lambda}^{\mu} \frac{x dx}{\operatorname{Cos}^{2a} x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} \\
& - (2a-1)(1 + \operatorname{Cos}^2 \lambda + \operatorname{Cos}^2 \mu) \int_{\lambda}^{\mu} \frac{x dx}{\operatorname{Cos}^{2a-2} x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} \quad \text{Legendre, Exerc. S. 21.} \\
& + (2a-2) \int_{\lambda}^{\mu} \frac{x dx}{\operatorname{Cos}^{2a-4} x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} \\
& + \frac{\pi \operatorname{Cos} \mu (\operatorname{Cos}^2 \lambda - \operatorname{Cos}^2 \mu)^2}{4 \operatorname{Cos}^{2a-1} \mu \cdot \operatorname{Cos}^3 \lambda} \sum_0^n (-1)^n \binom{a-2}{n} \frac{1^{n+1/2}}{4^{n+1/2}} \left(\frac{\operatorname{Cos}^2 \lambda - \operatorname{Cos}^2 \mu}{\operatorname{Cos}^2 \lambda} \right)^n \\
6) & \int \frac{x \operatorname{Sin}^2 x}{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda)(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)} dx = \frac{\pi}{2 \operatorname{Cos} \lambda \cdot \operatorname{Sin} \mu} \operatorname{F}(c, \mu) - \quad \text{Legendre, Exerc. S. 27.} \\
& - \frac{\pi \operatorname{Cos}^2 \mu}{2 \operatorname{Cos} \lambda \cdot \operatorname{Sin} \mu} \Pi(-\operatorname{Sin}^2 \vartheta, c, \mu) - \frac{1}{4} \pi l (1 + \operatorname{Sin}^2 \mu - \operatorname{Sin}^2 \lambda)
\end{aligned}$$

$$7) \int \frac{x \operatorname{Sin}^2 x}{\sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda) (\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} dx = \frac{\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 \lambda}{8} \pi +$$

$$+ \frac{\pi^2 - \operatorname{Cos}^2 \lambda \operatorname{Cos}^2 \mu}{4 \operatorname{Cos} \lambda \operatorname{Sin} \mu} \operatorname{F}(c, \mu) - \frac{\pi \operatorname{Cos} \lambda \operatorname{Sin} \mu \operatorname{E}(c, \mu)}{4} - \frac{1 + \operatorname{Sin}^2 \lambda + \operatorname{Sin}^2 \mu}{8} \pi l(1 - \operatorname{Sin}^2 \lambda + \operatorname{Sin}^2 \mu)$$

$$- \frac{\pi \operatorname{Cos}^2 \mu}{4 \operatorname{Cos} \lambda \operatorname{Sin} \mu} (1 + \operatorname{Sin}^2 \lambda + \operatorname{Sin}^2 \mu) \Pi(-\operatorname{Sin}^2 \theta, c, \mu)$$

$$8) 2a \int_{\lambda}^{\mu} \frac{x \operatorname{Sin}^{2a+2} x}{\sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda) (\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} dx =$$

$$(2a-1)(1 + \operatorname{Sin}^2 \lambda + \operatorname{Sin}^2 \mu) \int_{\lambda}^{\mu} \frac{x \operatorname{Sin}^{2a} x dx}{\sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda) (\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}}$$

$$- (2a-2)(\operatorname{Sin}^2 \lambda + \operatorname{Sin}^2 \mu + \operatorname{Sin}^2 \lambda \operatorname{Sin}^2 \mu) \int_{\lambda}^{\mu} \frac{x \operatorname{Sin}^{2a-2} x dx}{\sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda) (\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}}$$

$$+ (2a-3) \operatorname{Sin}^2 \lambda \operatorname{Sin}^2 \mu \int_{\lambda}^{\mu} \frac{x \operatorname{Sin}^{2a-4} x dx}{\sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda) (\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}}$$

$$- \frac{\pi}{4} \frac{(\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 \lambda)^2}{\operatorname{Sin}^{3-2a} \mu} \sum_0^n (-1)^n \binom{a-2}{n} \frac{1^{n+1/2}}{4^{n+1/2}} \left(\frac{\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 \lambda}{\operatorname{Sin}^2 \mu} \right)^n$$

Legendre,
Exerc.
Suppl.
27.

$$9) \int x dx \sqrt{\frac{\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x}{\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda}} = \frac{1}{4} \pi l(1 - \operatorname{Sin}^2 \lambda + \operatorname{Sin}^2 \mu) +$$

$$+ \frac{\pi \operatorname{Cos}^2 \mu}{2 \operatorname{Cos} \lambda \operatorname{Sin} \mu} \Pi(-\operatorname{Sin}^2 \theta, c, \mu) - \frac{\pi \operatorname{Cos}^2 \mu}{2 \operatorname{Cos} \lambda \operatorname{Sin} \mu} \operatorname{F}(c, \mu)$$

Legendre, Exerc.
Suppl. 25.

$$10) \int x dx \sqrt{\frac{\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda}{\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x}} = -\frac{1}{4} \pi l(1 - \operatorname{Sin}^2 \lambda + \operatorname{Sin}^2 \mu) -$$

$$- \frac{\pi \operatorname{Cos}^2 \mu}{2 \operatorname{Cos} \lambda \operatorname{Sin} \mu} \Pi(-\operatorname{Sin}^2 \theta, c, \mu) + \frac{\pi \operatorname{Cos} \lambda}{2 \operatorname{Sin} \mu} \operatorname{F}(c, \mu)$$

Legendre, Exerc.
Suppl. 26.

$$11) \int \frac{x \operatorname{Tang}^2 x}{\sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda) (\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} dx = \frac{\pi \operatorname{Sin} \mu}{2 \operatorname{Cos} \lambda \operatorname{Cos}^2 \mu} \{ \operatorname{E}(c, \mu) - \operatorname{Cot} \mu + \operatorname{Cot} \mu \operatorname{Cos} \mu \operatorname{Sec} \lambda \}$$

$$12) \int \frac{x}{\operatorname{Tg}^2 x \sqrt{(\operatorname{Sin}^2 x - \operatorname{Sin}^2 \lambda) (\operatorname{Sin}^2 \mu - \operatorname{Sin}^2 x)}} dx = \frac{\pi}{2 \operatorname{Sin} \lambda \operatorname{Tg} \lambda \operatorname{Sin} \mu} \left\{ \operatorname{E}(c, \mu) + \frac{\operatorname{Sin} \lambda - \operatorname{Sin} \mu}{\operatorname{Cos} \lambda} \right\}$$

$$13) \int \frac{x}{\cos.^3 x \sqrt{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)}} dx = \frac{\pi}{\cos.^2 \lambda \cos.^2 \mu} \left\{ \frac{\cos.^2 \lambda + \cos.^2 \mu + \cos.^2 \lambda \cos.^2 \mu}{3 \cos.^2 \lambda} \right. \\ \left. - \frac{3 \cos.^2 \lambda + 3 \cos.^2 \mu + 4 \cos.^2 \lambda \cos.^2 \mu + 2 \cos \lambda \cos \mu}{12 \cos \lambda \cos \mu} + \frac{2 \cos.^2 \lambda \cos.^2 \mu + \cos.^2 \lambda + \cos.^2 \mu - 1}{6 \cos \lambda \sin \mu} F(c\mu) \right. \\ \left. + \frac{\cos.^2 \lambda + \cos.^2 \mu + \cos.^2 \lambda \cos.^2 \mu}{3 \cos \lambda \cos \mu} \operatorname{Tang.} \mu V(c, \mu) \right\}$$

$$14) \int \frac{\sin x \cos x}{1 - p^2 \sin.^2 x \sqrt{\{\sin.^2 x - \sin.^2 \lambda\} \{\sin.^2 \mu - \sin.^2 x\}}} dx = \frac{\pi}{2 \sqrt{\{1 - p^2 \sin.^2 \lambda\} \{1 - p^2 \sin.^2 \mu\}}}$$

Sur les formules (11) à (14) voyez: Roberts, L. 11. 157.
Partout on a ici: $\cos. \vartheta = \cos. \mu. \sec. \lambda, c = \sin. \vartheta. \operatorname{cosec.} \mu$.

$$1) \int_1^\infty \frac{\cos. p x}{x} dx = - \operatorname{Ci.} (p)$$

$$2) \quad \quad \quad = - \Lambda - \frac{1}{2} l p^2 - \sum_1^\infty (-1)^n \frac{p^{2n}}{2n \cdot 1^{2n} l}$$

Arndt, Gr. 10. 225.

$$3) \int_1^\infty \frac{\cos. k x}{x} dx = 0, k = \infty; \text{ Raabe, Int. 202.}$$

$$4) \int_p^\infty \frac{\sin. x}{x} dx = \frac{1}{2} \pi - \operatorname{Si.} (p) \quad \text{Schlömilch, Stud. II. 21.}$$

$$5) \int_p^\infty \frac{\cos. x}{x} dx = - \operatorname{Ci.} (p)$$

$$6) \quad \quad \quad = - \Lambda - \frac{1}{2} l p^2 + \sum_1^\infty (-1)^{n-1} \frac{1}{2n} \frac{p^{2n}}{1^{2n} l}$$

Arndt, Gr. 10. 225. — Schlömilch, Gr. 11. 389.

$$7) \int_p^\infty \frac{\cos. q x}{x} dx = - \operatorname{Ci.} (p q) \quad \text{Schlömilch, Gr. 11. 389.}$$

$$8) \int_p^\infty \frac{\sin. x}{x^{2a}} dx = \frac{(-1)^a}{1^{2a-1} l} \left(\Lambda - \sum_1^{2a-1} \frac{1}{n} \right) - \sum_1^{a-1} \frac{(-1)^n}{1^{2n-1} l} \frac{1}{2(a-n) p^{2(a-n)}} + \frac{(-1)^a}{1^{2a-1} l} l p + \sum_1^\infty \frac{(-1)^{a+n}}{1^{2a+2n-1} l} \frac{p^{2n-1}}{2n-1}$$

Arndt, Gr. 10. 233.

$$9) \int_p^\infty \frac{\cos. x}{x^{2a+1}} dx = \frac{(-1)^{a+1}}{1^{2a+1} l} \left(\Lambda - \sum_1^{2a+1} \frac{1}{n} \right) - \sum_1^a \frac{(-1)^n}{1^{2n-1} l} \frac{1}{2(a+1-n) p^{2(a-n+1)}} + \frac{(-1)^{a+1}}{1^{2a+1} l} l p - \sum_1^\infty \frac{(-1)^{a+n}}{1^{2a+2n+1} l} \frac{p^{2n}}{2n}$$

F. Alg. rat. fract.
Circ. Dir.

TABLE 254 suite.

Lim. diverses p et $\pm \infty$.

$$\left. \begin{aligned} 10) \int_p^\infty \frac{\text{Sin. } p x}{x - q} dx &= \frac{1}{2} \pi \text{Cos. } p q - (\Lambda + l q + l 0) \text{Sin. } p q \\ 11) \int_{-x}^p \frac{\text{Sin. } p x}{x - q} dx &= \frac{1}{2} \pi \text{Cos. } p q + (\Lambda + l p + l 0) \text{Sin. } p q \end{aligned} \right\} \text{Bidone, Mém. Turin. 1812. 231. N. 32.}$$

F. Alg. rat.
Circ. Dir.

TABLE 255.

Lim. diverses.

$$\begin{aligned} 1) \int_0^{2a\pi} x^b \text{Cos. } q x dx &= - \sum_{n=0}^{b-1} \frac{1}{n!} \binom{b}{n} (2a\pi)^{b-n} \text{Cos. } \left\{ \frac{n+1}{2} \pi \right\} \quad \text{Hoppe, Cr. 40. 139.} \\ 2) \int_{b\pi}^{c\pi} x \text{Cos. } 2a x dx &= \frac{c^2 - b^2}{2} \pi^2 \frac{1}{2a^2} \quad \text{Arndt, Gr. 6. 187.} \\ 3) \int_\lambda^{\frac{\pi}{2}} \frac{x \text{Sin. } x}{(\text{Cos. } x - \text{Cos. } \mu)^3} dx &= -\frac{1}{2} \frac{\pi}{(1 + \text{Cos. } \mu)^2} + \frac{1}{2} \frac{\lambda}{(\text{Cos. } \lambda - \text{Cos. } \mu)^2} - \frac{\text{Sin. } \lambda}{2 \text{Sin. }^2 \mu (\text{Cos. } \lambda - \text{Cos. } \mu)} \\ &\quad - \frac{\text{Cos. } \mu}{2 \text{Sin. }^2 \mu} l \frac{\text{Sin. } \left\{ \frac{1}{2} (\lambda + \mu) \right\}}{\text{Sin. } \left\{ \frac{1}{2} (\lambda - \mu) \right\}}, \mu < \lambda; \quad \text{Legendre, Exerc. 5. 83.} \\ 4) \int_\lambda^{\frac{\pi}{2}} \frac{x \text{Cos. } x}{\sqrt{(\text{Sin. }^2 x - \text{Sin. }^2 \lambda)}} dx &= \frac{1}{2} \pi l (1 + \text{Cos. } \lambda) \quad \text{Legendre, Exerc. Suppl. 29.} \\ 5) \int_\lambda^{\frac{\pi}{2}} \frac{x \text{Cos. } x}{\text{Sin. }^2 x \sqrt{(\text{Sin. }^2 x - \text{Sin. }^2 \lambda)}} dx &= \frac{1}{2} \pi \text{Cosec. } \lambda \left(1 - \text{Tang. } \frac{1}{2} \lambda \right) \quad \text{Legendre, Exerc. Suppl. 17.} \\ 6) \int_0^\infty \frac{\text{Sin. } p x}{x} dx &= \frac{1}{2} \pi \quad \text{Fourier, Chaleur. 415.} \end{aligned}$$

F. Alg. rat. ent.
Circ. Inv. de x .

TABLE 256.

Lim. 0 et 1.

$$\begin{aligned} 1) \int x \text{Arcsin. } x dx &= \frac{1}{8} \pi \quad \text{V. T. 9. N°. 4.} \\ 2) \int x^{2a} \text{Arcsin. } x dx &= \frac{1}{2a+1} \left\{ \frac{1}{2} \pi - \frac{2^{a+1/2}}{1^{a+1/2}} \right\} \quad \text{V. T. 12. N°. 13.} \end{aligned}$$

$$3) \int x^{2a-1} \operatorname{Arcsin} x \, dx = \frac{\pi}{4a} \left\{ 1 - \frac{3^{a-1/2}}{2^{a/2}} \right\} \quad \text{V. T. 12. N}^\circ. 12.$$

$$4) \int (1-x^2)^{a-1} x \operatorname{Arcsin} x \, dx = \frac{\pi}{2^{2a+1}} \frac{(a+1)^{a-1/1}}{1^{a/1}} \quad \text{V. T. 9. N}^\circ. 5.$$

$$5) \int \operatorname{Arccos} x \cdot x^{2a-1} \, dx = \frac{\pi}{4a} \frac{3^{a-1/2}}{2^{a/2}} \quad \text{V. T. 12. N}^\circ. 12.$$

$$6) \int \operatorname{Arccos} x \cdot x^{2a} \, dx = \frac{2^{a-1/2}}{1^{a+1/2}} \quad \text{V. T. 12. N}^\circ. 13.$$

$$7) \int x^{p-1} \operatorname{Arctang} x \, dx = \frac{1}{4p} \left\{ \pi + Z' \left(\frac{p+1}{4} \right) - Z' \left(\frac{p+3}{4} \right) \right\} \quad \text{V. T. 3. N}^\circ. 13.$$

$$8) \int \operatorname{Arccot} x \cdot x^{p-1} \, dx = \frac{1}{4p} \left\{ \pi + Z' \left(\frac{p+3}{4} \right) - Z' \left(\frac{p+1}{4} \right) \right\} \quad \text{V. T. 3. N}^\circ. 13.$$

$$1) \int \operatorname{Arcsin} x \frac{dx}{x} = \frac{1}{2} \pi \ln 2 \quad \text{Euler, N. A. Petr. 14. 129. — Arndt, Gr. 6. 187.}$$

$$2) \int (\operatorname{Arcsin} x)^p \frac{dx}{x} = \left(\frac{1}{2} \pi \right)^p \left\{ 1 - \sum_{n=1}^{\infty} \frac{2}{p+2n} \sum_{m=1}^{\infty} \frac{1}{(2n)^{2m}} \right\} \quad \text{V. T. 238. N}^\circ. 18$$

$$3) \int \operatorname{Arctang} x \frac{dx}{x} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)^2} \quad \text{V. T. 152. N}^\circ. 11.$$

$$4) \int \operatorname{Arctang} x \frac{dx}{x^2} = \infty \quad \text{V. T. 110. N}^\circ. 2.$$

$$5) \int \operatorname{Arctang} x \frac{x^p - x^{-p}}{x} \, dx = \frac{\pi}{2p} \left(1 - \operatorname{Sec} \frac{1}{2} p \pi \right) \quad \text{V. T. 5. N}^\circ. 11.$$

$$6) \int \operatorname{Arccot} x \frac{(x^p - x^{-p})}{x} \, dx = \frac{\pi}{2} \left\{ 1 + \operatorname{Sec} \frac{1}{2} p \pi \right\} \quad \text{V. T. 5. N}^\circ. 11.$$

$$7) \int \operatorname{Arctang} x \frac{(p-q)(x^{p+q} - x^{q-p}) - (p+q)(x^{p+q} - x^{-p-q})}{x} \, dx = \frac{2\pi \operatorname{Sin} \frac{1}{2} p \pi \cdot \operatorname{Sin} \frac{1}{2} q \pi}{\operatorname{Cos} p \pi + \operatorname{Cos} q \pi} \quad p+q < 1, q < p < 1; \left\{ \begin{array}{l} \text{V. T.} \\ 5. \\ \text{N}^\circ. \\ 9. \end{array} \right.$$

$$8) \quad = \infty \quad , p+q > 1,$$

$$\begin{aligned}
 9) \int \operatorname{Arctang.} x \frac{(p+q)(x^{p+q} - x^{-p-q}) + (p-q)(x^{p-q} - x^{q-p})}{x} dx &= \\
 &= \pi \frac{\cos. p \pi + \cos. q \pi - 2 \cos. \frac{1}{2} p \pi \cos. \frac{1}{2} q \pi}{\cos. p \pi + \cos. q \pi}, p+q < 1, q < p < 1; \left. \begin{array}{l} \text{V. T. 5.} \\ \text{N}^\circ. 10. \end{array} \right\} \\
 10) &= \infty, p+q > 1; \\
 11) \int (\operatorname{Arcsin.} x)^2 \frac{dx}{x^2} &= -\frac{1}{4} \pi^2 + 4 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 261. N}^\circ. 6. \\
 12) \int (\operatorname{Arctang.} x)^2 \frac{dx}{x^2} &= -\frac{1}{16} \pi^2 + \frac{1}{4} \pi l 2 + \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 260. N}^\circ. 4. \\
 13) \int (\operatorname{Arcsin.} x)^p \frac{dx}{x^2} &= p \left(\frac{\pi}{2}\right)^{p-1} \left[1 + \sum_1^\infty \left\{ \frac{1}{4^{m-1}} \frac{2^{2m-1}-1}{p+2m-1} \sum_1^\infty \frac{1}{(2n)^{2m}} \right\} \right] - \frac{1}{p} \left(\frac{\pi}{2}\right)^p \quad \text{V. T. 261. N}^\circ. 24. \\
 14) \int (\operatorname{Arctang.} x)^{p+1} \frac{dx}{x^2} &= -\left(\frac{\pi}{4}\right)^{p+1} + \frac{p+1}{2^{2p+2}} \pi^p \left[1 - \sum_1^\infty \frac{4}{p+2m} \sum_1^\infty \frac{1}{(4n)^{2m}} \right] \quad \text{V. T. 260. N}^\circ. 20. \\
 15) \int (\operatorname{Arcsin.} x)^3 \frac{dx}{x^3} &= \frac{3}{2} \pi l 2 - \frac{1}{16} \pi^3 \quad \text{V. T. 261. N}^\circ. 22.
 \end{aligned}$$

$$\begin{aligned}
 1) \int \operatorname{Arcsin.} x \frac{x}{1-x^2 \operatorname{Sin.}^2 \lambda} dx &= \frac{1}{2} \pi \operatorname{Cosec.}^2 \lambda l \left(\operatorname{Cos.}^2 \frac{1}{2} \lambda, \operatorname{Sec.} \lambda \right) \quad \text{V. T. 165. N}^\circ. 22. \\
 2) \int \operatorname{Arcsin.} x \frac{x}{1+q x^2} dx &= \frac{\pi}{2q} l \frac{2 l \vee (1+q)}{1+\vee (1+q)} \quad \text{V. T. 165. N}^\circ. 6. \\
 3) \int \operatorname{Arcsin.} x \frac{x}{1+x^2 \operatorname{Tang.}^2 \lambda} dx &= -\pi \operatorname{Cot.}^2 \lambda l \operatorname{Cos.} \frac{1}{2} \lambda \quad \text{V. T. 165. N}^\circ. 7. \\
 4) \int \operatorname{Arcsin.} x \frac{x}{1-x^4 \operatorname{Sin.}^2 \lambda} dx &= -\frac{1}{4} \pi \operatorname{Cosec.} \lambda l \left\{ \frac{1+\operatorname{Sin.} \frac{1}{2} \lambda \operatorname{Cos.} \lambda}{1+\operatorname{Sin.} \lambda \operatorname{Cos.} \frac{1}{2} \lambda} \right\} \quad \text{V. T. 166. N}^\circ. 1. \\
 5) \int \operatorname{Arcsin.} x \frac{x}{\frac{1}{2}(p+1)-x^2} dx &= -\frac{\pi}{4} l \{ 2(1-p) \}, p^2 < 1; \left. \begin{array}{l} \\ \text{V. T. 240. N}^\circ. 9, 10. \end{array} \right\} \\
 6) &= \frac{\pi}{4} l \frac{p+1 \vee (p^2-1)}{2(p-1)}, p^2 > 1; \left. \begin{array}{l} \\ \end{array} \right\}
 \end{aligned}$$

- $$\begin{aligned} 7) \int \operatorname{Arcsin} x \frac{x}{\frac{1}{2}(p-1)+x^2} dx &= \frac{\pi}{4} l \{2(1+p)\} \quad , p^2 < 1; \\ 8) &= \frac{\pi}{4} l \frac{2(1+p)}{p+1 \vee (p^2-1)} \quad , p^2 > 1; \end{aligned} \left. \vphantom{\int} \right\} \text{V. T. 240. N}^\circ \text{ 11, 12.}$$
- $$\begin{aligned} 9) \int \operatorname{Arcsin} x \frac{x}{(1-p)^2+4px^2} dx &= \frac{\pi}{8p} l(1+p) \quad , 0 < p < 1; \\ 10) &= \frac{\pi}{8p} l \frac{1+p}{p} \quad , p > 1; \end{aligned} \left. \vphantom{\int} \right\} \text{V. T. 241. N}^\circ \text{ 3, 4.}$$
- $$11) \int \operatorname{Arccos} x \frac{dx}{1+x} = -\frac{1}{2} \pi l 2 + 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 258. N}^\circ \text{ 13, 14.}$$
- $$12) \int \operatorname{Arccos} x \frac{dx}{1-x} = \frac{1}{2} \pi l 2 + 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 258. N}^\circ \text{ 13, 14.}$$
- $$13) \int \operatorname{Arccos} x \frac{dx}{1-x^2} = 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 239. N}^\circ \text{ 1.}$$
- $$14) \int \operatorname{Arccos} x \frac{x}{1-x^2} dx = \frac{1}{2} \pi l 2 \quad \text{V. T. 165. N}^\circ \text{ 11.}$$
- $$15) \int \operatorname{Arccos} x \frac{dx}{\operatorname{Sin}^2 \lambda - x^2} = 2 \operatorname{Cosec} \lambda \sum_0^\infty \frac{\operatorname{Sin} \{(2n+1)\lambda\}}{(2n+1)^2} \quad \text{V. T. 240. N}^\circ \text{ 1.}$$
- $$16) \int \operatorname{Arccos} x \frac{x}{1-x^2 \operatorname{Sin}^2 \lambda} dx = -\pi \operatorname{Cosec}^2 \lambda l \operatorname{Cos} \frac{1}{2} \lambda \quad \text{V. T. 165. N}^\circ \text{ 14.}$$
- $$17) \int \operatorname{Arccos} x \frac{x}{1+qx^2} dx = \frac{\pi}{2q} l \frac{1+1 \vee (1+q)}{2} \quad \text{V. T. 165. N}^\circ \text{ 6.}$$
- $$18) \int \operatorname{Arccos} x \frac{x}{1+x^2 \operatorname{Tang}^2 \lambda} dx = \frac{1}{2} \pi \operatorname{Cot}^2 \lambda l \left(\operatorname{Cos}^2 \frac{1}{2} \lambda \operatorname{Sec} \lambda \right) \quad \text{V. T. 163. N}^\circ \text{ 7.}$$
- $$19) \int \operatorname{Arccos} x \frac{x}{1-x^4 \operatorname{Sin}^2 \lambda} dx = \frac{1}{4} \pi \operatorname{Cosec} \lambda l \frac{1+\operatorname{Sin} \frac{1}{2} \lambda}{\operatorname{Cos} \frac{1}{4} \lambda} \quad \text{V. T. 166. N}^\circ \text{ 1.}$$
- $$\begin{aligned} 20) \int \operatorname{Arccos} x \frac{x}{\frac{1}{2}(p+1)-x^2} dx &= \frac{\pi}{4} l \{2(1+p)\} \quad , p^2 < 1; \\ 21) &= \frac{\pi}{4} l \frac{2(1+p)}{p+1 \vee (p^2-1)} \quad , p^2 > 1; \end{aligned} \left. \vphantom{\int} \right\} \text{V. T. 240. N}^\circ \text{ 11, 12.}$$

$$\begin{aligned}
 22) \int \operatorname{Arccos.} x \frac{x}{\frac{1}{2}(p-1) + x^2} dx &= -\frac{\pi}{4} l\{2(1-p)\} \quad , p^2 < 1; \\
 23) &= \frac{\pi}{4} l \frac{p+1}{2(p-1)} \quad , p^2 > 1; \left. \vphantom{\int} \right\} \text{V. T. 240. N}^\circ 9, 10. \\
 24) \int \operatorname{Arccos.} x \frac{x}{(1+p)^2 - 4px^2} dx &= \frac{\pi}{8p} l(1+p), 0 < p < 1; \\
 25) &= \frac{\pi}{8p} l \frac{1+p}{p} \quad p > 1; \left. \vphantom{\int} \right\} \text{V. T. 241. N}^\circ 3, 4. \\
 26) \int \operatorname{Arctang.} x \frac{dx}{1+x} &= \frac{1}{8} \pi l 2 \quad \text{V. T. 160. N}^\circ 2. \\
 27) \int \operatorname{Arccot.} x \frac{x}{1+x^2} dx &= \frac{3}{8} \pi l 2 - \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 265. N}^\circ 90 \text{ et T. 270. N}^\circ 7. \\
 28) \int \left(x \operatorname{Arccot.} x - \frac{1}{x} \operatorname{Arctang.} x \right) \frac{dx}{x - \frac{1}{x}} &= \frac{1}{4} \pi l 2 \quad \text{Cauchy, Lim. Imag. Add. 32.} \\
 29) \int \operatorname{Arcsin.} x \frac{dx}{1+p^2+2px} &= \frac{1}{2p} \left\{ \pi l(1+p) - \sum_0^{\infty} \frac{1}{2n+1} \frac{2^{n/2}}{3^{n/2}} \left(\frac{2p}{1+p^2} \right)^{2n+1} \right\} \\
 30) \int \operatorname{Arccos.} x \frac{dx}{1+p^2+2px} &= \frac{1}{2p} \left\{ -\frac{\pi}{2} l(1+p^2) + \sum_0^{\infty} \frac{1}{2n+1} \frac{2^{n/2}}{3^{n/2}} \left(\frac{2p}{1+p^2} \right)^{2n+1} \right\} \left. \vphantom{\int} \right\} \text{V. T. 165. N}^\circ 25.
 \end{aligned}$$

$$\begin{aligned}
 1) \int (\operatorname{Arccos.} x)^2 \frac{dx}{1-x^2} &= -2 \sum_1^{\infty} \left\{ \frac{1}{n^3} - \frac{(-1)^n}{n^3} - \pi \frac{(-1)^n}{(2n-1)^2} \right\} \quad \text{V. T. 239. N}^\circ 2. \\
 2) \int (\operatorname{Arccos.} x)^3 \frac{dx}{1-x^2} &= 3 \sum_1^{\infty} (-1)^n \left\{ \frac{\pi^2}{2} \frac{1}{(2n-1)^2} - \frac{4}{(2n-1)^4} \right\} \quad \text{V. T. 239. N}^\circ 3. \\
 3) \int (\operatorname{Arccos.} x)^p \frac{dx}{1+x} &= \left(\frac{\pi}{2} \right)^p \sum_1^{\infty} \left\{ \frac{2^{2m}-1}{4^{m-1}} \frac{1}{p+2m} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\} \\
 4) \int (\operatorname{Arccos.} x)^p \frac{dx}{1-x} &= \left(\frac{\pi}{2} \right)^p \left[2 - \sum_1^{\infty} \left\{ \frac{1}{4^{m-1}} \frac{1}{p+2m} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\} \right] \left. \vphantom{\int} \right\} \text{V. T. 238. N}^\circ 18. \\
 &\quad \text{et T. 239. N}^\circ 5. \\
 5) \int (\operatorname{Arccos.} x)^b \frac{dx}{q+x} &= -2 \cos. \frac{1}{2} b \pi \cdot 1^b \cdot 1 \sum_1^{\infty} \frac{(-c)^n}{n^{b+1}} - 2 \sum_1^{\infty} \left[c^{2n} \sum_0^{\infty} \left(\frac{b}{2m} \right) (-1)^m \left(\frac{\pi}{2} \right)^{b-2m} \frac{1}{(2n)^{2m+1}} \right. \\
 &\quad \left. - c^{2n-1} \sum_0^{\infty} \left(\frac{b}{2m+1} \right) (-1)^m \left(\frac{\pi}{2} \right)^{b-2m-1} \frac{1}{(2n)^{2m+2}} \right] \quad \text{V. T. 240. N}^\circ 7.
 \end{aligned}$$

F. Alg. rat. fract. à dén. polynôme.
Circ. Inv. de x à plus fact.

TABLE 259 suite.

Lim. 0 et 1.

- 6) $\int (\operatorname{Arccos}.x)^b \frac{dx}{q-x} = 2 \operatorname{Cos}.\frac{1}{2} b \pi. 1^{b/1} \sum_1^{\infty} \frac{c^n}{n^{b+1}} + 2 \sum_1^{\infty} \left[c^{2n} \sum_0^{\infty} \binom{b}{2m} (-1)^m \left(\frac{\pi}{2}\right)^{b-2m} \frac{1}{(2n)^{2m+1}} \right]$ V. T. 240. N°. 8.
+ $c^{2n-1} \sum_0^{\infty} \binom{b}{2m+1} (-1)^m \left(\frac{\pi}{2}\right)^{b-2m-1} \frac{1}{(2n-1)^{2m+2}}$ et (6) $c = q-1$ (q^2-1);
- 7) $\int (\operatorname{Arccos}.x)^p \frac{dx}{1-x^2} = \left(\frac{\pi}{2}\right)^p \left[1 + \sum_1^{\infty} \left\{ \frac{1}{4^{m-1}} \frac{2^{2m-1}-1}{p+2m} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\} \right]$ V. T. 239. N°. 5.
- 8) $\int (\operatorname{Arccos}.x)^p \frac{x}{1-x^2} dx = \left(\frac{\pi}{2}\right)^p \left[1 - \sum_1^{\infty} \left\{ \frac{2}{p+2m} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\} \right]$ V. T. 238. N°. 13.
- 9) $\int \operatorname{Arctang}.x \frac{(1+x)(1+2x \operatorname{Arctang}.x) - (1+x^2)}{(1+x)^2} dx = \frac{\pi}{16} (1-2l2)$ V. T. 258. N°. 26.
- 10) $\int \operatorname{Arctang}.x \frac{(1+x^2) \operatorname{Arccot}.x - x}{1+x^2} dx = \frac{1}{16} \pi^2 - \frac{3}{8} \pi l2 + \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 27.

F. Alg. rat. fract. à dén. prod. de fact.
Circ. Inv. de x .

TABLE 260.

Lim. 0 et 1.

- 1) $\int \operatorname{Arcsin}.x \frac{dx}{x(1-x^2)} = \infty$ V. T. 239. N°. 14.
- 2) $\int \operatorname{Arcsin}.x \frac{x - (1+x^2) \operatorname{Arctang}.x}{x^2} \frac{dx}{1+x^2} = \frac{1}{8} \pi^2 - \frac{1}{2} \pi l(1+\sqrt{2})$ V. T. 261. N°. 14.
- 3) $\int \operatorname{Arcsin}.x \frac{qx - (1+q^2 x^2) \operatorname{Arctg}.qx}{x^2} \frac{dx}{1+q^2 x^2} = \frac{1}{2} \pi \operatorname{Arctang}.q - \frac{1}{2} \pi l\{q + \sqrt{q^2+1}\}$ V. T. 261. N°. 15.
- 4) $\int \operatorname{Arctang}.x \frac{dx}{x(1+x^2)} = \frac{1}{8} \pi l2 + \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 235. N°. 4.
- 5) $\int \operatorname{Arctang}.x \frac{x - (1+x^2) \operatorname{Arctang}.x}{x^2} \frac{dx}{1+x^2} = \frac{1}{16} \pi^2 - \frac{1}{8} \pi l2 - \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 260. N°. 4.
- 6) $\int \operatorname{Arcsin}.x \frac{x}{\operatorname{Cos}^2 \lambda + x^2 \operatorname{Sin}^2 \lambda} \frac{dx}{\operatorname{Cos}^2 \mu + x^2 \operatorname{Sin}^2 \mu} = \frac{\pi}{\operatorname{Sin}(\lambda-\mu) \operatorname{Sin}(\lambda+\mu)} l \operatorname{Cos} \frac{1}{2} \mu \operatorname{Sec} \frac{1}{2} \lambda$ V. T. 166. N°. 1.
- 7) $\int \operatorname{Arcsin}.x \frac{x}{1-x^2 \operatorname{Sin}^2 \lambda} \frac{dx}{1-x^2 \operatorname{Sin}^2 \mu} = \frac{\pi}{\operatorname{Sin}^2 \lambda - \operatorname{Sin}^2 \mu} l \frac{\operatorname{Cos} \frac{1}{2} \lambda.1 - \operatorname{Cos} \mu}{\operatorname{Cos} \frac{1}{2} \mu.1 - \operatorname{Cos} \lambda}$ V. T. 166. N°. 2.
- 8) $\int \operatorname{Arcsin}.x \frac{x}{\operatorname{Sin}^2 \lambda - x^2 \operatorname{Sin}^2 \mu} \frac{dx}{1-x^2 \operatorname{Sin}^2 \mu} = \frac{\pi}{2 \operatorname{Sin}^2 \mu \operatorname{Cos}^2 \lambda} l \operatorname{Cot} \frac{1}{2} \mu.1 - \operatorname{Sin}^2 \lambda - \operatorname{Sin}^2 \mu$ V. T. 166. N°. 1.

- 9) $\int \operatorname{Arccos} x \frac{x}{\sin^2 \lambda - x^2 \sin^2 \mu} \frac{dx}{1 - x^2 \sin^2 \mu} = \frac{\pi}{2 \sin^2 \mu \cos^2 \lambda} l \frac{\sin \lambda \cot \frac{1}{2} \mu}{\operatorname{Tang} \left\{ \frac{1}{2} \operatorname{Arcsin} \left(\frac{\sin \mu}{\sin \lambda} \right) \right\}}$ V. T. 166. N°. 3.
- 10) $\int \operatorname{Arccos} x \frac{x}{\cos^2 \lambda + x^2 \sin^2 \lambda} \frac{dx}{\cos^2 \mu + x^2 \sin^2 \mu} = \frac{1}{2} \frac{\pi}{\sin(\lambda + \mu) \sin(\lambda - \mu)} l \frac{1 + \operatorname{Sec} \lambda}{1 + \operatorname{Sec} \mu}$ V. T. 166. N°. 4.
- 11) $\int \operatorname{Arccos} x \frac{x}{1 - x^2 \sin^2 \lambda} \frac{dx}{1 - x^2 \sin^2 \mu} = \frac{\pi}{\sin^2 \lambda - \sin^2 \mu} l \frac{\cos \frac{1}{2} \mu}{\cos \frac{1}{2} \lambda}$ V. T. 166. N°. 2.
- 12) $\int \operatorname{Arcsin} x \frac{x^{2p-1}}{(1-x^2)^{p+1}} dx = \infty$ V. T. 12. N°. 17.
- 13) $\int \operatorname{Arccos} x \frac{x^{2p-1}}{(1-x^2)^{p+1}} dx = \frac{\pi}{4p} \operatorname{Sec} p \pi$ V. T. 12. N°. 17.
- 14) $\int \operatorname{Arctang} x \frac{2x+x^2}{(1+x)^2} dx = \frac{1}{4} \pi - \frac{3}{4} l 2$ V. T. 3. N°. 16.
- 15) $\int \operatorname{Arctang} x \frac{dx}{(1+px)^2} = \frac{1}{1+p^2} l \frac{1+p}{1/2} + \frac{\pi}{4p} \frac{1-p}{(1+p)(1+p^2)}$ V. T. 6. N°. 1.
- 16) $\int \operatorname{Arccot} x \frac{x+2}{(1+x)^2} x dx = \frac{3}{4} l 2$ V. T. 3. N°. 16.
- 17) $\int \operatorname{Arccot} x \frac{x}{(1+x^2)^2} dx = \frac{1}{16} \left\{ \pi + 2 + Z' \left(\frac{3}{4} \right) - Z' \left(\frac{5}{4} \right) \right\}$ V. T. 4. N°. 18.
- 18) $\int \operatorname{Arctang} x \frac{1-x^2}{(1+x^2)^{2p}} x^{2p-2} dx = \frac{1}{2p-1} \left\{ \frac{\pi}{2^{2p+1}} - \frac{\{\Gamma(p)\}^2}{4 \Gamma(2p)} \right\}$ V. T. 5. N°. 24.
- 19) $\int (\operatorname{Arccos} x)^3 \frac{x}{(1-x^2)^2} dx = \frac{3}{2} \pi l 2 - \frac{1}{16} \pi^3$ V. T. 261. N°. 25.
- 20) $\int (\operatorname{Arctang} x)^p \frac{dx}{x(1+x^2)} = \frac{\pi p}{2^{2p+1}} \left\{ 2 - \sum_{1}^{\infty} \frac{4}{p+2m} \sum_{1}^{\infty} \frac{1}{(4n)^{2m}} \right\}$ V. T. 238. N°. 19.
- 21) $\int (\operatorname{Arctang} x)^p \frac{px - (1+x^2) \operatorname{Arctang} x}{x^2} \frac{dx}{1+x^2} = \left(\frac{1}{4} \pi \right)^{p+1} - \frac{\pi p}{2^{2p+1}} \left\{ 2 - \sum_{1}^{\infty} \frac{4}{p+2m} \sum_{1}^{\infty} \frac{1}{(4n)^{2m}} \right\}$ V. T. 260. N°. 20.

- 1) $\int \operatorname{Arcsin} x \frac{dx}{\sqrt{x}} = \frac{3}{2} \left\{ \frac{1}{2} \pi - \frac{3 \sqrt{3}}{\sqrt{3}} E' \left(\sin \frac{\pi}{12} \right) + \frac{3 + 3 \sqrt{3}}{2 \sqrt{3}} F' \left(\sin \frac{\pi}{12} \right) \right\}$ V. T. 12. N°. 16.
- 2) $\int \operatorname{Arcsin} x \frac{dx}{\sqrt{x^2}} = 3 \left\{ \frac{1}{2} \pi + \frac{\sqrt{3}-1}{\sqrt{3}} F' \left(\cos \frac{\pi}{12} \right) - 2 \sqrt{3} E' \left(\cos \frac{\pi}{12} \right) \right\}$ V. T. 12. N°. 15.

$$3) \int \operatorname{Arcsin} x \frac{dx}{x \sqrt{x}} = \sqrt[3]{27} \cdot \Gamma' \left(\cos. \frac{\pi}{12} \right) - \frac{3}{2} \pi \quad \text{V. T. 15. N}^{\circ}. 12.$$

$$4) \int \operatorname{Arcsin} x \frac{dx}{x \sqrt{x^2}} = \frac{3}{2} \sqrt[3]{27} \cdot \Gamma' \left(\sin. \frac{\pi}{12} \right) - \frac{3}{4} \pi \quad \text{V. T. 15. N}^{\circ}. 11.$$

$$5) \int \operatorname{Arcsin} x \frac{x}{\sqrt{(1-p^2 x^2)}} dx = \frac{1}{p^2} \left\{ 1 - \frac{1}{2} \pi \sqrt{(1-p^2)} - \sum_1^{\infty} \left\{ \frac{1^{n-1/2}}{2^{n/2}} \right\}^2 (2n-1) p^{2n} \right\} \quad \begin{matrix} \text{V. T. 12.} \\ \text{N}^{\circ}. 14. \end{matrix}$$

$$6) \int \operatorname{Arcsin} x \frac{dx}{x \sqrt{(1-x^2)}} = 2 \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 239. N}^{\circ}. 1.$$

$$7) \int \operatorname{Arcsin} x \frac{x}{\sqrt{(1+x^2)^3}} dx = -\frac{1}{4} \pi \sqrt[3]{2} + \frac{1}{2} \sqrt[3]{2} \cdot \Gamma' \left(\sin. \frac{\pi}{4} \right) \quad \text{V. T. 13. N}^{\circ}. 6.$$

$$8) \int \operatorname{Arcsin} x \frac{dx}{\sqrt{(p^2+x^2)^3}} = \frac{1}{p^2} \left(\frac{1-p}{2p} \pi + \operatorname{Arctang} p \right) \quad \text{V. T. 16. N}^{\circ}. 6.$$

$$9) \int \operatorname{Arcsin} x \frac{x}{x^2 - \cos^2 \lambda} \frac{dx}{\sqrt{(1-x^2)}} = 2 \operatorname{Cosec} \lambda \sum_0^{\infty} \frac{\sin. \{(2n+1)\lambda\}}{(2n+1)^2} \quad \text{V. T. 240. N}^{\circ}. 1.$$

$$10) \int \operatorname{Arccos} x \frac{dx}{x \sqrt{x}} = \infty \quad \text{V. T. 15. N}^{\circ}. 12.$$

$$11) \int \operatorname{Arccos} x \frac{dx}{x \sqrt{x^2}} = \infty \quad \text{V. T. 15. N}^{\circ}. 11.$$

$$12) \int \operatorname{Arccos} x \frac{x}{\sqrt{(1+x^2)^3}} dx = \frac{\pi}{2} - \frac{1}{2} \sqrt[3]{2} \Gamma \left(\sin. \frac{\pi}{4} \right) \quad \text{V. T. 13. N}^{\circ}. 6.$$

$$13) \int \operatorname{Arctang} x \frac{1-x}{x} \frac{dx}{\sqrt{x}} = \pi (\sqrt[3]{2} - 1) \quad \text{V. T. 15. N}^{\circ}. 2.$$

$$\left. \begin{aligned} 14) \int \operatorname{Arctang} x \frac{dx}{x \sqrt{(1-x^2)}} &= \frac{1}{2} \pi l(1 + \sqrt[3]{2}) \\ 15) \int \operatorname{Arctang} q x \frac{dx}{x \sqrt{(1-x^2)}} &= \frac{1}{2} \pi l \{ q + \sqrt{(1+q^2)} \} \end{aligned} \right\} \quad \text{Schlömiltch, Int. § 26.}$$

$$16) \int \operatorname{Arctang} x \frac{x^3}{\sqrt{(1-x^2)}} dx = \frac{\left\{ \Gamma \left(\frac{1}{4} \right) \right\}^2}{8 \sqrt[3]{2} \pi} - \frac{\pi \sqrt[3]{2} \pi}{2 \left\{ \Gamma \left(\frac{1}{4} \right) \right\}^2} \quad \text{V. T. 12. N}^{\circ}. 9.$$

$$17) \int \operatorname{Arctang} x \frac{x}{\sqrt{(1-x^2)}} \frac{dx}{\operatorname{Tang}^2 \mu + x^2} = \frac{\pi}{2} \operatorname{Cos} \mu l \left\{ \operatorname{Cos} \frac{\pi - 4\mu}{8} \cdot \operatorname{Cosec} \frac{\pi + 4\mu}{8} \right\} \quad \begin{matrix} \text{V. T. 160.} \\ \text{N}^{\circ}. 18. \end{matrix}$$

- 18) $\int \text{Arctang. } x \frac{x}{1-x^2} \frac{dx}{\sqrt{1-x^4}} = \pi$ V. T. 13. N°. 6.
- 19) $\int \frac{\text{Arcsin. } x \sqrt{1-x^2} - x}{x^3} \frac{dx}{\sqrt{1-x^2}} = -\frac{1}{4}\pi$ V. T. 239. N°. 12.
- 20) $\int \frac{x \text{Arccos. } x - \sqrt{1-x^2}}{\sqrt{1-x^2}^3} dx = -\frac{1}{4}\pi$ V. T. 239. N°. 12.
- 21) $\int (\text{Arcsin. } x)^2 \frac{dx}{x \sqrt{1-x^2}} = 2 \sum_1^\infty \left\{ -\frac{1}{n^3} + \frac{(-1)^n}{n^3} - \pi \frac{(-1)^n}{(2n-1)^2} \right\}$ V. T. 239. N°. 2.
- 22) $\int (\text{Arcsin. } x)^2 \frac{dx}{x^2 \sqrt{1-x^2}} = \pi l 2$ V. T. 239. N°. 8
- 23) $\int (\text{Arcsin. } x)^3 \frac{dx}{x \sqrt{1-x^2}} = 3 \sum_1^\infty \frac{(-1)^{n-1}}{(2n-1)^2} \left\{ \frac{1}{2}\pi^2 - \frac{4}{(2n-1)^2} \right\}$ V. T. 239. N°. 3.
- 24) $\int (\text{Arcsin. } x)^p \frac{dx}{x \sqrt{1-x^2}} = \left(\frac{1}{2}\pi\right)^p \left\{ 1 + \sum_1^\infty \frac{1}{4^{m-1}} \frac{2^{2m-1}-1}{p+2m} \sum_1^\infty \frac{1}{(2n)^{2m}} \right\}$ V. T. 239. N°. 5.
- 25) $\int (\text{Arccos. } x)^2 \frac{dx}{\sqrt{1-x^2}^3} = \pi l 2$ V. T. 239. N°. 8.

- 1) $\int \text{Arctang. } \{\sqrt{1-x^2}\} \frac{dx}{1-x^2 \text{Cos.}^2 \mu} = \frac{\pi}{\text{Cos.} \mu} l \left\{ \text{Cos.} \frac{\pi-4\mu}{8}, \text{Cosec.} \frac{\pi+4\mu}{8} \right\}$ V. T. 166
N°. 16.
- 2) $\int \text{Arctang. } \{p \sqrt{1-x^2}\} \frac{dx}{1-x^2} = \frac{1}{2}\pi l \{p + \sqrt{1+p^2}\}$ Raabe, Int. p. 421.
- 3) $\int \text{Arctang. } \{\sqrt{1-x}\} \frac{dx}{(1-x \text{Cos.}^2 \mu) \sqrt{x}} = \frac{2\pi}{\text{Cos.} \mu} l \left\{ \text{Cos.} \frac{\pi-4\mu}{8}, \text{Cosec.} \frac{\pi+4\mu}{8} \right\}$ V. T. 166.
N°. 16.
- 4) $\int \text{Arctang. } \{\text{Tang. } \lambda. \sqrt{1-p^2 x^2}\} \frac{dx}{\sqrt{1-x^2} (1-p^2 x^2)} = \frac{1}{2}\pi F(p, \lambda)$ V. T. 369. N°. 14.
- 5) $\int \text{Arctang. } \{\text{Tang. } \lambda. \sqrt{1-p^2 x^2}\} \frac{dx}{(1-p^2 x^2) \sqrt{1-x^2} (1-p^2 x^2)} =$
 $= \frac{1}{2} \frac{\pi}{1-p^2} E(p, \lambda) - \frac{\pi \text{Tang. } \lambda}{2} \frac{1}{1-p^2} \{ \sqrt{1-p^2 \text{Sin.}^2 \lambda} - \sqrt{1-p^2} \}$
V. T. 369.
N°. 16.

F. Alg. fract. Circ. Inv. d'autre forme.	TABLE 262 suite.	Lim. 0 et 1.
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$$6) \int \text{Arctang.} \left\{ \text{Tang.} \lambda \sqrt{1-p^2 x^2} \right\} dx \sqrt{\frac{1-p^2 x^2}{1-x^2}} = \frac{1}{2} \pi \text{E}(p, \lambda) - \frac{1}{2} \pi \text{Cot.} \lambda \left\{ 1 - \sqrt{1-p^2 \text{Sin.}^2 \lambda} \right\} \quad \text{V. T. 368. N}^\circ 11.$$

$$7) \int \text{Arctang.} \left\{ \frac{\text{Cot.} \lambda}{\sqrt{1-p^2 x^2}} \right\} \frac{dx}{\sqrt{(1-x^2)(1-p^2 x^2)}} = \frac{1}{2} \pi \text{F}(p, q) \quad \text{V. T. 369. N}^\circ 15.$$

$$8) \int \text{Arctang.} \left\{ \frac{\text{Cot.} \lambda}{\sqrt{1-p^2 x^2}} \right\} \frac{dx}{(1-p^2 x^2) \sqrt{(1-x^2)(1-p^2 x^2)}} = \frac{1}{2} \frac{\pi}{1-p^2} \text{E}(p, q) - \frac{\pi}{2} \frac{\text{Tang.} \lambda}{\sqrt{1-p^2}} \left\{ 1 - \sqrt{1-p^2 \text{Sin.}^2 q} \right\} \quad \text{V. T. 369. N}^\circ 17.$$

$$9) \int \text{Arctang.} \left\{ \frac{\text{Cot.} \lambda}{\sqrt{1-p^2 x^2}} \right\} dx \sqrt{\frac{1-p^2 x^2}{1-x^2}} = \frac{1}{2} \pi \text{E}(p, q) - \frac{1}{2} \pi \frac{\text{Cot.} \lambda}{\sqrt{1-p^2}} \left\{ \sqrt{1-p^2 \text{Sin.}^2 q} - \sqrt{1-p^2} \right\} \quad \text{V. T. 368. N}^\circ 13.$$

Dans les intégrales (7) à (9) q est donné par l'équation: $\text{Cot.} q = \text{Tang.} \lambda \sqrt{1-p^2}$

Dans les intégrales (4) à (9) on a $p^2 < 1$.

F. Alg. rat. ent. Circ. Inv. de x .	TABLE 265.	Lim. 0 et ∞
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$$1) \int x^{p-1} \text{Arccot.} x dx = \frac{\pi}{2p} \text{Sec.} \frac{1}{2} p \pi, 0 < p < 1; \quad \text{V. T. 19. N}^\circ 8.$$

$$2) \int x^{p-2} \text{Arctang.} x dx = \frac{1}{1-p} \frac{\pi}{2} \text{Cosec.} \frac{1}{2} p \pi, 0 < p < 1; \quad \text{V. T. 19. N}^\circ 6.$$

$$3) \int (1-x \text{Arccot.} x) dx = \frac{1}{4} \pi \quad \text{V. T. 239. N}^\circ 12.$$

F. Alg. rat. fract. à dén. monôme. Circ. Inv. de x .	TABLE 264.	Lim. 0 et ∞
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$$1) \int \text{Arctang.} \frac{x dx}{q x} = \infty \quad \text{V. T. 180. N}^\circ 10.$$

$$2) \int \text{Arctang.} x \frac{dx}{x^2} = \frac{1}{2} \pi l 2 \quad \text{Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. fautive elle est } x$$

$$3) \int \text{Arctang.} x \frac{dx}{x^p} = \frac{1}{2} \frac{\pi}{p-1} \text{Sec.} \left(\frac{p-1}{2} \pi \right), p < 1; \quad \text{V. T. 19. N}^\circ 8.$$

$$4) \int (\text{Arctang.} x)^2 \frac{dx}{x^2} = \pi l 2 \quad \text{Schlömilch, Gr. 4. 71. — Mosta, Gr. 10. 439.}$$

- 5) $\int (\text{Arctang. } x)^3 \frac{dx}{x^2} = \frac{3}{4} \pi^2 l 2 - 6 \sum_1^\infty \left\{ \frac{1}{n^3} + \frac{(-1)^n}{n^3} + 2 \frac{(-1)^n}{(2n)^3} \right\}$ V. T. 266. N°. 8.
- 6) $\int (\text{Arctang. } x)^{p+1} \frac{dx}{x^2} = (p+1) \left(\frac{1}{2} \pi \right)^p \left\{ 1 - \sum_1^\infty \frac{2}{(p+2m)} \sum_1^\infty \frac{1}{(2n)^{2m}} \right\}$ V. T. 266. N°. 10.
- 7) $\int \text{Arccot. } p x \frac{dx}{x} = x$ V. T. 180. N°. 8.
- 8) $\int \text{Arccot. } x \frac{dx}{x^2} = \infty$ V. T. 109. N°. 1.
- 9) $\int \text{Arccot. } x \frac{dx}{x^p} = \frac{\pi}{2(1-p)} \text{Cosec. } \frac{1}{2} p \pi, p < 1;$ V. T. 19. N°. 6.
- 10) $\int (\text{Arcsec. } x)^2 \frac{dx}{x^2} = \frac{1}{2} \pi l 2$ V. T. 109. N°. 3; mais elle est fautive, avec celle-ci.
- 11) $\int \frac{\text{Arctang. } x - x}{x^3} dx = -\frac{1}{4} \pi$ V. T. 239. N°. 12.
- 12) $\int \text{Arctang. } p x. \text{Arctang. } x \frac{dx}{x^2} = \frac{1+p}{2} \pi l (1+p) - \frac{p\pi}{2} l p$ V. T. 266. N°. 2, 3.
- 13) $\int \text{Arctang. } x. \text{Arctang. } \frac{x}{q} \frac{dx}{x^2} = \frac{1}{2} \pi \left\{ \frac{1}{q} l (1+q) + l \frac{1+q}{q} \right\}$
- 14) $\int \text{Arctang. } \frac{x}{p}. \text{Arctang. } \frac{x}{q} \frac{dx}{x^2} = \frac{1}{2} \pi \left\{ \frac{1}{p} l \left(1 + \frac{p}{q} \right) + \frac{1}{q} l \left(1 + \frac{q}{p} \right) \right\}$ } Schlömilch, Gr. 4. 71.
- 15) $\int \text{Arctang. } x. \text{Arccos. } x \frac{dx}{x^2} = \infty$ V. T. 109. N°. 7. (corrigée).
- 16) $\int \text{Arctang. } x. \text{Arccot. } q x \frac{dx}{x^2} = \infty$ V. T. 109. N°. 9. (corrigée).
- 17) $\int \text{Arctang. } q x. \text{Arccot. } x \frac{dx}{x^2} = \infty$ V. T. 109. N°. 8. (corrigée).
- 18) $\int \text{Arctang. } \frac{x}{p}. \text{Arccot. } \frac{x}{q} \frac{dx}{x^2} = \infty$ V. T. 109. N°. 10. (corrigée).

- 1) $\int \text{Arctang. } x \frac{dx}{1+x^2} = \frac{1}{8} \pi^2$ Schlömilch, Gr. 4. 316.
- 2) $\int \text{Arctang. } x \frac{x}{q^2+x^2} dx = \infty$ V. T. 181. N°. 2.
- 3) $\int \text{Arctang. } x \frac{x}{1+x^4} dx = \frac{1}{16} \pi^2$ V. T. 268. N°. 1.
- 4) $\int \text{Arctang. } p x \frac{x}{x^4-q^4} dx = \frac{\pi}{8 q^2} l \frac{(p q + 1)^2}{p^2 q^2 + 1}$ V. T. 265. N°. 11.
- 5) $\int \text{Arctang. } x \frac{dx}{(1+q x)^2} = \frac{1}{1+q^2} \left\{ l q + \frac{\pi}{2 q} \right\}$ V. T. 24. N°. 1.
- 6) $\int \text{Arctang. } x \frac{x}{(1+x^2)^3} dx = \frac{3}{64} \pi$ V. T. 21. N°. 4.
- 7) $\int \text{Arctang. } x \frac{x^3}{(1+x^2)^3} dx = \frac{1}{64} \pi$ V. T. 21. N°. 6.
- 8) $\int \text{Arctang. } x \frac{x}{(1+x^2)^2 - \text{Sin.}^2 2 \lambda} dx = \frac{\pi}{4 \text{Sin. } 2 \lambda} l \frac{1 + \text{Sin. } \lambda}{\text{Cos. } \lambda}$ V. T. 181. N°. 15.
- 9) $\int \text{Arccot. } x \frac{dx}{1+x^2} = \frac{1}{8} \pi^2$ V. T. 265. N°. 1.
- 10) $\int \text{Arccot. } x \frac{x}{1+x^2} dx = \frac{1}{2} \pi l 2$ Cauchy, Lim. Imag. Add. N°. 31.
- 11) $\int \text{Arccot. } x \frac{x}{1-x^2} dx = -\frac{1}{4} \pi l 2$ V. T. 265. N°. 13.
- 12) $\int \text{Arccot. } \frac{x}{p} \frac{x}{x^2+q^2} dx = \frac{1}{2} \pi l \frac{p+q}{q}$
- 13) $\int \text{Arccot. } \frac{x}{p} \frac{x}{x^2-q^2} dx = \frac{1}{4} \pi l \frac{p^2+q^2}{q^2}$
- 14) $\int \text{Arccot. } \frac{x}{p x^4-q^4} dx = \frac{\pi}{8 q^2} l \frac{p^2+q^2}{(p+q)^2}$
- 15) $\int \text{Arccot. } \frac{x}{p} \frac{x^3}{x^4-q^4} dx = \frac{1}{8} \pi l \frac{(p+q)^2 (p^2+q^2)}{q^4}$
- 16) $\int \text{Arccot. } p x \frac{x}{1+x^2} dx = \frac{1}{2} \pi l \frac{1+p}{p}$ V. T. 266. N°. 3.

- 17) $\int \operatorname{Arccot} x \frac{dx}{(1+x^2)^2} = \frac{1}{1+q^2} \left\{ \frac{1}{2} q \pi - l q \right\}$ V. T. 24. N°. 1.
- 18) $\int \operatorname{Arccot} x \frac{x}{(1+x^2)^3} dx = \frac{1}{64} \pi$ V. T. 21. N°. 4.
- 19) $\int \operatorname{Arccot} x \frac{x^3}{(1+x^2)^3} dx = \frac{3}{64} \pi$ V. T. 21. N°. 6.
- 20) $\int \operatorname{Arccot} x \frac{x}{(1+x^2)^2 - \sin^2 2\lambda} dx = \frac{\pi}{4 \sin 2\lambda} l \frac{(1 + \sin 2\lambda)(1 - \sin 2\lambda)}{(1 - \sin 2\lambda)(1 + \sin 2\lambda)}$ V. T. 181. N°. 15.
- 21) $\int (\operatorname{Arctang} x)^q \frac{dx}{1+x^2} = \frac{1}{q+1} \left(\frac{1}{2} \pi \right)^{q+1}$
- 22) $\int (p + \operatorname{Arctang} x)^q \frac{dx}{1+x^2} = \frac{1}{q+1} \left\{ \left(p + \frac{1}{2} \pi \right)^{q+1} - p^{q+1} \right\}$ } Raabe, Int. 138. — Ohm, Ausw. 5.
- 23) $\int (p + \operatorname{Arctang} x)^{q-1} \operatorname{Arctang} x \frac{dx}{1+x^2} = \frac{1}{q} \left[\left(p + \frac{1}{2} \pi \right)^q - \frac{1}{2} \frac{1}{q+1} \left\{ \left(p + \frac{1}{2} \pi \right)^{q+1} - p^{q+1} \right\} \right]$ V. T. 265. N°. 22.
- 24) $\int (\operatorname{Arccot} x)^2 \frac{x}{1+x^2} dx = \frac{1}{4} \pi^2 l 2 - 2 \sum_1 \left\{ \frac{1}{n^3} + \frac{(-1)^n}{n^3} + 2 \frac{(-1)^n}{(2n)^3} \right\}$ V. T. 238. N°. 13.
- 25) $\int (\operatorname{Arccot} x)^3 \frac{x}{1+x^2} dx = \frac{1}{8} \pi^3 l 2 + 6 \pi \sum_1 \frac{(-1)^n}{(2n)^3}$ V. T. 238. N°. 14.
- 26) $\int (\operatorname{Arccot} x)^p \frac{x}{1+x^2} dx = \left(\frac{1}{2} \pi \right)^p \left\{ 1 - \sum_1 \frac{2}{p+2m} \sum_1 \frac{1}{(2n)^{2m}} \right\}$ V. T. 266. N°. 10.
- 27) $\int (\operatorname{Arccot} x)^p \frac{dx}{1+x^2} = \frac{1}{p+1} \left(\frac{1}{2} \pi \right)^{p+1}$ V. T. 265. N°. 21.
- 28) $\int (q + \operatorname{Arccot} x)^p \frac{x}{1+x^2} dx = \frac{1}{p+1} \left\{ \left(q + \frac{1}{2} \pi \right)^{p+1} - q^{p+1} \right\}$ V. T. 265. N°. 22.

- 1) $\int \operatorname{Arctang} x \frac{dx}{x(1+x^2)} = \frac{1}{2} \pi l 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.
- 2) $\int \operatorname{Arctang} x \frac{dx}{x(q^2+x^2)} = \frac{\pi}{2q^2} l(1+q)$ Schlömilch, Cr. 33. 268. — Id., Gr. 4. 71.

- 3) $\int \text{Arctang. } p x \frac{dx}{x(1+x^2)} = \frac{1}{2} \pi l(1+p)$ Schlömilch, Cr. 33. 268. — Id., Gr. 4. 71.
- 4) $\int \text{Arctang. } p x \frac{dx}{x(1+q^2 x^2)} = \frac{1}{2} \pi l \frac{p+q}{q}$ V. T. 265. N°. 12.
- 5) $\int \text{Arctang. } \frac{x}{p} \frac{dx}{x(q^2+x^2)} = \frac{1}{2 q^2} \pi l \frac{p+q}{p}$ V. T. 181. N°. 10.
- 6) $\int \text{Arctang. } p x \frac{dx}{x(1-q^2 x^2)} = \frac{1}{4} \pi l \frac{p^2+q^2}{q^2}$ V. T. 265. N°. 13.
- 7) $\int \text{Arctang. } p x \frac{x dx}{x^4 - q^4} = \frac{\pi}{8 q^2} l \frac{(p q + 1)^2}{p^2 q^2 + 1}$ V. T. 265. N°. 14.
- 8) $\int (\text{Arctang. } x)^2 \frac{dx}{x(1+x^2)} = \frac{1}{4} \pi^2 l 2 - 2 \sum_1 \left\{ \frac{1}{n^3} + \frac{(-1)^n}{n^3} + 2 \frac{(-1)^n}{(2n)^3} \right\}$ V. T. 238. N°. 13.
- 9) $\int (\text{Arctang. } x)^3 \frac{dx}{x(1+x^2)} = \frac{1}{8} \pi^3 l 2 + \frac{3}{4} \pi \sum_1 \frac{(-1)^n}{n^3}$ V. T. 238. N°. 14.
- 10) $\int (\text{Arctang. } x)^p \frac{dx}{x(1+x^2)} = \left(\frac{1}{2} \pi \right)^p \left\{ 1 - \sum_1 \frac{2}{p+2m} \sum_1 \frac{1}{(2n)^{2m}} \right\}$ V. T. 238. N°. 18.

- 1) $\int \text{Arctang. } \frac{x}{q} \frac{dx}{(1+x^2)^2} = \frac{q}{1+q^2} \left(l q + \frac{\pi}{2 q} \right)$ V. T. 24. N°. 3.
- 2) $\int \text{Arctang. } x \frac{x}{(1+x^2)^2} dx = \frac{1}{8} \pi$ Schlömilch, Gr. 4. 316.
- 3) $\int \text{Arctang. } x \frac{x}{(p^2+x^2)^2} dx = \frac{\pi}{4 p(p+1)} \left\{ \begin{array}{l} \end{array} \right.$
- 4) $\int \text{Arctang. } p x \frac{x}{(1+x^2)^2} dx = \frac{\pi p}{4(p+1)} \left\{ \begin{array}{l} \end{array} \right.$ V. T. 24. N°. 7.
- 5) $\int \text{Arctang. } \frac{x}{q} \frac{x}{(x^2+p^2)^2} dx = \frac{\pi}{4 p(p+q)}$ V. T. 24. N°. 8.
- 6) $\int \text{Arctang. } \frac{x}{q} \frac{dx}{(1-x^2)^2} = \frac{q}{1+q^2} \left(l q - \frac{\pi}{2 q} \right)$ V. T. 24. N°. 4.

- $$7) \int \text{Arctang.} \frac{x}{q(1-x^2)^2} dx = -\frac{\pi}{4(1+q^2)} \left\{ \begin{array}{l} \text{V. T. 267. N}^\circ. 1, 6. \\ 8) \int \text{Arctang.} \frac{x}{q(1-x^2)^2} dx = \frac{q}{1+q^2} lq \end{array} \right.$$
- $$9) \int \text{Arctang.} \frac{x}{q(x^2-p^2)^2} dx = -\frac{\pi}{4(p^2+q^2)} \quad \text{V. T. 24. N}^\circ. 11.$$
- $$10) \int \text{Arctang.} x \frac{x}{(q^2-x^2)^2} dx = -\frac{\pi}{4(1+q^2)} \quad \text{V. T. 24. N}^\circ. 13.$$
- $$11) \int \text{Arctang.} x \frac{x}{1+x^2} \frac{dx}{1+x^2 \cos^2 \lambda} = \frac{1}{2} \pi \text{Cosec.}^2 \lambda l \left(\cos^2 \frac{1}{2} \lambda, \text{Sec.} \frac{1}{2} \lambda \right) \quad \text{V. T. 181. N}^\circ. 12.$$
- $$12) \int \text{Arctang.} x \frac{x}{1+x^2} \frac{dx}{x^2 + \cos^2 \lambda} = \pi \text{Cosec.}^2 \lambda l \text{Sec.} \frac{1}{2} \lambda \quad \text{V. T. 178. N}^\circ. 13.$$
- $$13) \int \text{Arctang.} x \frac{x}{\cos^2 \mu - \cos^2 \lambda + x^2 \sin^2 \lambda} \frac{dx}{x^2 + \cos^2 \mu} = \frac{\pi}{2 \cos^2 \lambda \sin^2 \mu} \times$$
- $$\times l \left[\sin \lambda, \cot \frac{1}{2} \mu, \cot \left\{ \frac{1}{2} \text{Arcsin.} \left(\frac{\sin \mu}{\sin \lambda} \right) \right\} \right] \quad \begin{array}{l} \text{V. T. 181.} \\ \text{N}^\circ. 16. \end{array}$$
- $$14) \int \text{Arctang.} x \frac{x}{x^2 + \cos^2 \lambda} \frac{dx}{x^2 + \cos^2 \mu} = \frac{\pi}{\cos^2 \lambda - \cos^2 \mu} l \left(\cos \frac{1}{2} \lambda, \text{Sec.} \frac{1}{2} \mu \right) \quad \begin{array}{l} \text{V. T. 241.} \\ \text{N}^\circ. 24. \end{array}$$
- $$15) \int \text{Arccot.} q x \frac{dx}{(1+x)^2} = \frac{q}{1+q^2} \left\{ \frac{\pi}{2q} + lq \right\} \quad \text{V. T. 24. N}^\circ. 3.$$
- $$16) \int \text{Arccot.} q x \frac{dx}{(1-x)^2} = \frac{q}{1+q^2} \left\{ lq - \frac{\pi}{2q} \right\} \quad \text{V. T. 24. N}^\circ. 4.$$
- $$17) \int \text{Arccot.} x \frac{x}{(1+x^2)^2} dx = \frac{1}{8} \pi \quad \text{V. T. 267. N}^\circ. 2.$$
- $$18) \int \text{Arccot.} \frac{x}{q} \frac{x}{(p^2+x^2)^2} dx = \frac{\pi q}{4p^2(p+q)} \quad \text{V. T. 24. N}^\circ. 8.$$
- $$19) \int \text{Arccot.} x \frac{x}{(q^2-x^2)^2} dx = -\frac{\pi}{4q^2(1+q^2)} \quad \text{V. T. 24. N}^\circ. 14.$$
- $$20) \int \text{Arccot.} x \frac{x}{1+x^2} \frac{dx}{1+x^2 \cos^2 \lambda} = \pi \text{Cosec.}^2 \lambda l \text{Sec.} \frac{1}{2} \lambda \quad \text{V. T. 181. N}^\circ. 12.$$
- $$21) \int \text{Arccot.} x \frac{x}{1+x^2} \frac{dx}{x^2 + \cos^2 \lambda} = \frac{1}{2} \pi \text{Cosec.}^2 \lambda l \left(\text{Sec.} \lambda, \cos^2 \frac{1}{2} \lambda \right) \quad \text{V. T. 181. N}^\circ. 13.$$

$$22) \int \operatorname{Arccot} x \frac{x}{1+x^2} \frac{dx}{\cos^2 \lambda + x^2 \cos^2 \mu} = \frac{\pi}{\cos^2 \lambda - \cos^2 \mu} l \left(\cos \frac{1}{2} \lambda \sec \frac{1}{2} \mu \right) \quad \text{V. T. 241. N° 24.}$$

$$23) \int \operatorname{Arccot} x \frac{x}{x^2 + \cos^2 \lambda} \frac{dx}{x^2 + \cos^2 \mu} = \frac{1}{2} \frac{\pi}{\cos^2 \lambda - \cos^2 \mu} l \frac{\cos^2 \frac{1}{2} \mu \cos \lambda}{\cos^2 \frac{1}{2} \lambda \cos \mu} \quad \text{V. T. 181. N° 11.}$$

$$24) \int \operatorname{Arccot} x \frac{x}{\cos^2 \mu - \cos^2 \lambda + x^2 \sin^2 \lambda} \frac{dx}{x^2 + \cos^2 \mu} = \frac{\pi}{2 \sin^2 \lambda \sin^2 \mu} l \frac{\cos \mu \operatorname{Tang} \left\{ \frac{1}{2} \operatorname{Arcsin} \frac{\sin \mu}{\sin \lambda} \right\}}{\cot \frac{1}{2} \mu \sqrt{\cos^2 \mu - \cos^2 \lambda}} \quad \text{V. T. 181. N° 16.}$$

$$25) \int \operatorname{Arccot} \frac{x}{p} \frac{(b-xi)^{-a} - (b+xi)^{-a}}{2i} dx = \frac{\pi}{2(a-1)} \left\{ \left(\frac{1}{b} \right)^{a-1} - \left(\frac{1}{b+p} \right)^{a-1} \right\} \quad \text{Cauchy, Lim. Imag. Add N° 29.}$$

où les puissances a de $\frac{1}{b}$ et de $\frac{1}{b+p}$ sont fautives.

$$26) \int (\operatorname{Arctang} x)^2 \frac{1-x^2}{(1+x^2)^2} dx = \frac{\pi-1}{4} \pi \quad \text{V. T. 267. N° 2.}$$

$$1) \int \operatorname{Arctang} x \frac{dx}{(1+x)\sqrt{x}} = \frac{1}{4} \pi^2 \quad \text{Schlömlich, Gr. 4. 316.}$$

$$2) \int \operatorname{Arctang} x \frac{dx}{x\sqrt{(1+x^2)}} = 2 \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 239. N° 1.}$$

$$3) \int \operatorname{Arctang} x \frac{x}{\cos^2 \lambda - x^2 \sin^2 \lambda} \frac{dx}{\sqrt{(1+x^2)}} = -2 \operatorname{Cosec} \lambda \sum_0^{\infty} \frac{\sin \{(2n+1)\lambda\}}{(2n+1)^2} \quad \text{V. T. 240. N° 1.}$$

$$4) \int \operatorname{Arctang} x \frac{dx}{\sqrt{x^2 + 1} \sqrt{x^2 + 1}} = \frac{3}{8} \pi^2 \quad \text{Schlömlich, Gr. 4. 316.}$$

$$5) \int \operatorname{Arctang} \frac{x}{q} \frac{x}{\sqrt{(p^2 + x^2)^3}} dx = \frac{1}{\sqrt{(p^2 - q^2)}} \operatorname{Arctang} \frac{\sqrt{(p^2 - q^2)}}{q}, q < p; \quad \left. \begin{array}{l} \text{V. T. 28.} \\ \text{N° 9, 10.} \end{array} \right\}$$

$$6) \quad = \frac{1}{\sqrt{(q^2 - p^2)}} l \frac{q + \sqrt{(q^2 - p^2)}}{p}, q > p;$$

$$7) \int \operatorname{Arctg} \frac{x}{p} \frac{x^2 + 2p^2 - q^2}{\sqrt{(p^2 + x^2)}} \frac{xdx}{(q^2 + x^2)^2} = -\frac{\pi}{2} + \frac{p}{q\sqrt{(p^2 - q^2)}} \operatorname{Arctg} \frac{\sqrt{(p^2 - q^2)}}{q}, p < q; \quad \left. \begin{array}{l} \text{V. T. 28.} \\ \text{N° 9, 10.} \end{array} \right\}$$

$$8) \quad = -\frac{\pi}{2} + \frac{p}{q\sqrt{(q^2 - p^2)}} l \frac{q + \sqrt{(q^2 - p^2)}}{p}, p < q; \quad \left. \begin{array}{l} \text{V. T. 28.} \\ \text{N° 9, 10.} \end{array} \right\}$$

$$9) \int \operatorname{Arccot} x \frac{dx}{(1+x)\sqrt{1-x}} = \frac{1}{4} \pi^2 \quad \text{V. T. 268. N° 1.}$$

- 10) $\int \operatorname{Arccot} . x \frac{d x}{\sqrt{1+x^2}} = 2 \sum_0^{\infty} \frac{(-1)^n}{(2 n+1)^2}$ V. T. 239. N°. 1.
- 11) $\int \operatorname{Arccot} . \frac{x}{q} \frac{x}{\sqrt{(p^2+x^2)^3}} d x = \frac{1}{2} \left[\frac{\pi}{2 p} - \frac{1}{\sqrt{(p^2-q^2)}} \operatorname{Arctang} . \frac{\sqrt{(p^2-q^2)}}{q} \right], p > q;$ V. T. 28.
 12) $= \frac{1}{2} \left[\frac{\pi}{2 p} + \frac{1}{\sqrt{(q^2-p^2)}} l \frac{p}{q} + \sqrt{(q^2-p^2)} \right], p < q;$ N°. 9, 10.
- 13) $\int \operatorname{Arccot} . x \frac{d x}{(\operatorname{Sin} .^2 \lambda - x^2 \operatorname{Cos} .^2 \lambda) \sqrt{1+x^2}} = 2 \operatorname{Cosec} . \lambda \sum_0^{\infty} \frac{\operatorname{Sin} .\{(2 n+1) \lambda\}}{(2 n+1)^2}$ V. T. 240. N°. 1.
- 14) $\int \operatorname{Arccot} . x \frac{x^2+2 p^2-q^2}{\sqrt{(p^2+x^2)}} \frac{x d x}{(q^2+x^2)^2} = -\frac{p}{2 q} \left[\frac{\pi}{2 q} + \frac{1}{\sqrt{(p^2-q^2)}} \operatorname{Arctg} . \frac{\sqrt{(p^2-q^2)}}{q} \right], p > q;$ V. T. 28.
 15) $= \frac{p}{2 q} \left[\frac{\pi}{2 q} + \frac{1}{\sqrt{(q^2-p^2)}} l \frac{q+\sqrt{(q^2-p^2)}}{p} \right], p < q;$ N°. 9, 10.
- 16) $\int (\operatorname{Arctang} . x)^2 \frac{d x}{x^2 \sqrt{1+x^2}} = -\frac{1}{4} \pi^2 + 4 \sum_0^{\infty} \frac{(-1)^n}{(2 n+1)^2}$ V. T. 268. N°. 2.
- 17) $\int (\operatorname{Arccot} . x)^2 \frac{x}{\sqrt{1+x^2}} d x = -\frac{1}{4} \pi^2 + 4 \sum_0^{\infty} \frac{(-1)^n}{(2 n+1)^2}$ V. T. 268. N°. 10.

- 1) $\int \operatorname{Arctang} . (x^2) \frac{d x}{1+x} = \frac{1}{8} \pi^2$ V. T. 268. N°. 1.
- 2) $\int \operatorname{Arctang} . (x^3) \frac{d x}{1+x^2} = \frac{1}{8} \pi^2$ V. T. 268. N°. 4.
- 3) $\int \operatorname{Arctang} . (\sqrt{x}) \frac{d x}{(1+x)^2} = \frac{1}{4} \pi$ V. T. 267. N°. 2.
- 4) $\int \operatorname{Arccot} . (x^2) \frac{d x}{1+x^2} = \frac{1}{8} \pi^2$ V. T. 268. N°. 9.
- 5) $\int \operatorname{Arccot} . (x^3) \frac{d x}{1+x^2} = \frac{1}{8} \pi^2$ V. T. 269. N°. 2.
- 6) $\int \operatorname{Arccot} . (\sqrt{x}) \frac{d x}{(1+x)^2} = \frac{1}{4} \pi$ V. T. 267. N°. 17.

F. Alg. fract. Circ. Inv. d'autre forme.	TABLE 269 suite.	Lim. 0 et ∞ .
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- 7) $\int \{ \operatorname{Arccot}(\sqrt{x}) \}^2 \frac{dx}{\sqrt{1+x}} = -\frac{1}{2} \pi^2 + 8 \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 268. N°. 17.
- 8) $\int \operatorname{Arctang} \left(\frac{p}{\sqrt{1+x^2}} \right) \frac{dx}{\sqrt{1+x^2}} = \frac{1}{2} \pi l \{ p + \sqrt{1+p^2} \}$, $p \geq 1$;
- 9) $\int \operatorname{Arctang} \left(\sqrt{\frac{p^2-1}{1+x^2}} \right) \frac{dx}{\sqrt{1+x^2}} = \frac{1}{2} \pi l \{ p + \sqrt{p^2-1} \}$ } Raabe, Int. 421.
- 10) $\int \operatorname{Arctang} \left(x \sqrt{\frac{p^2-1}{1+x^2}} \right) \frac{dx}{x \sqrt{1+x^2}} = \frac{1}{2} \pi l \{ p + \sqrt{p^2-1} \}$, $p \geq 1$; V. T. 269. N°. 9.
- 11) $\int \operatorname{Arctang} \left(\frac{px}{\sqrt{1+x^2}} \right) \frac{dx}{x \sqrt{1+x^2}} = \frac{1}{2} \pi l \{ p + \sqrt{1+p^2} \}$, $p \geq 1$; V. T. 269. N°. 8.
- 12) $\int \operatorname{Arctang}(\sqrt{x}) \left\{ \frac{1}{\sqrt{1+x}} \operatorname{Arctang} \left(\sqrt{\frac{p^2-1}{1+x}} \right) - \frac{\sqrt{p^2-1}}{x+p^2} \right\} dx = \left\{ \begin{array}{l} \text{V. T. 269.} \\ \text{N°. 9.} \end{array} \right.$
 $= -\pi l \{ p + \sqrt{p^2-1} \}$

F. Alg. fract. Circ. Inv.	TABLE 270.	Lim. 1 et ∞ .
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- 1) $\int \operatorname{Arctang} x \frac{dx}{x} = \infty$ V. T. 187. N°. 2.
- 2) $\int \operatorname{Arctang} x \frac{dx}{x(1+x^2)} = \frac{3}{8} \pi l 2 - \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 260. N°. 4 et T. 266. N°. 1.
- 3) $\int \operatorname{Arctang} x \frac{x - (1+x^2) \operatorname{Arctang} x}{x^2} \frac{dx}{1+x^2} = -\frac{1}{16} \pi^2 - \frac{3}{8} \pi l 2 + \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 2.
- 4) $\int \operatorname{Arccot} x \frac{dx}{x} = \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 257. N°. 3.
- 5) $\int \operatorname{Arccot} x \frac{dx}{x^2} = \frac{1}{4} \pi - \frac{1}{2} l 2$ V. T. 108. N°. 1.
- 6) $\int \operatorname{Arccot} \frac{x}{p} \frac{dx}{x^2} = \operatorname{Arctang} p - \frac{1}{2p} l(1+p^2)$ V. T. 108. N°. 2.
- 7) $\int \operatorname{Arccot} x \frac{x}{1+x^2} dx = \frac{1}{8} \pi l 2 + \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 236. N°. 4.

- 8) $\int \operatorname{Arccosec}. x \frac{dx}{x^2} = \frac{1}{2} \pi - \frac{1}{x} \quad \text{V. T. 108. N}^\circ. 3.$
- 9) $\int \operatorname{Arccosec}. \frac{x}{p} \frac{dx}{x^2} = \operatorname{Arcsin}. p + \frac{1}{p} \sqrt{1 - p^2} - \frac{1}{p} \quad \text{V. T. 108. N}^\circ. 4.$
- 10) $\int (\operatorname{Arctang}. x)^2 \frac{dx}{x^2} = \frac{\pi^2}{16} + \frac{3}{4} \pi l 2 - \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 270. N}^\circ. 2.$
- 11) $\int (\operatorname{Arccot}. x)^p \frac{x}{1+x^2} dx = \frac{\pi^p}{2^{2p+1}} \left\{ 2 - \sum_1^\infty \frac{4}{p+2m} \sum_1^\infty \frac{1}{(4n)^{2m}} \right\} \quad \text{V. T. 238. N}^\circ. 19.$
- 12) $\int \operatorname{Arctang}. x \frac{(1+x^2) \operatorname{Arccot}. x - x}{1+x^2} dx = \frac{1}{16} \pi^2 - \frac{1}{8} \pi l 2 - \frac{1}{2} \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 270. N}^\circ. 7.$
- 13) $\int \operatorname{Arctg}. x (\operatorname{Arccot}. x)^{p-1} \frac{(1+x^2) \operatorname{Arccot}. x - px}{1+x^2} dx = \left(\frac{1}{4} \pi \right)^{p+1} - \frac{\pi^p}{2^{2p+1}} \left\{ 2 - \sum_1^\infty \frac{1}{p+2m} \sum_1^\infty \frac{1}{(4n)^{2m}} \right\} \quad \text{V. T. 270. N}^\circ. 11.$
- 14) $\int \operatorname{Arccot}. x \left(\operatorname{Arctang}. x - \frac{x}{1+x^2} \right) \frac{dx}{x^2} = \frac{\pi^2}{16} - \frac{3}{8} \pi l 2 + \frac{1}{2} \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 270. N}^\circ. 2.$

- 1) $\int_{-\infty}^{\infty} \operatorname{Arctg}. (a+bx) \frac{dx}{1+x^2} = -\pi \left\{ \frac{1}{2} \operatorname{Arctg}. \frac{2ab}{1+a^2-b^2} - \operatorname{Arctg}. \frac{\operatorname{Tang}. \left(\frac{1}{2} \operatorname{Arctang}. \frac{2ab}{1+a^2-b^2} \right)}{\operatorname{Arctg}. \left(\frac{1}{2} \operatorname{Arctg}. \frac{2ab}{1+a^2-b^2} \right) - b} \right\} \quad \left. \begin{array}{l} \text{Hill,} \\ \text{Cr. 3.} \\ 101. \end{array} \right\}$
- 2) $\int_{-\infty}^{\infty} \operatorname{Arctg}. x \frac{dx}{1+(a+bx)^2} = \frac{\pi}{b} \left\{ \frac{1}{2} \operatorname{Arctg}. \frac{2ab}{1+a^2-b^2} - \operatorname{Arctg}. \frac{\operatorname{Tang}. \left(\frac{1}{2} \operatorname{Arctang}. \frac{2ab}{1+a^2-b^2} \right)}{\operatorname{Arctg}. \left(\frac{1}{2} \operatorname{Arctg}. \frac{2ab}{1+a^2-b^2} \right) - b} \right\} \quad \left. \begin{array}{l} \text{V. T.} \\ 271. \\ \text{N}^\circ. 1. \end{array} \right\}$
- 3) $\int_{-\infty}^{\infty} \operatorname{Arctang}. x \frac{dx}{x(1+x^2)} = \pi l 2 \quad \text{Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.}$
- 4) $\int_0^{1/\frac{1}{2}} \operatorname{Arcsin}. x \frac{dx}{x} = \frac{1}{8} \pi l 2 + \frac{1}{2} \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 238. N}^\circ. 4.$
- 5) $\int_0^{1/\frac{1}{2}} (\operatorname{Arcsin}. x)^p \frac{dx}{x} = \frac{\pi^p}{2^{2p+1}} \left\{ 2 - \sum_1^\infty \frac{4}{p+2m} \sum_1^\infty \frac{1}{(4n)^{2m}} \right\} \quad \text{V. T. 238. N}^\circ. 19.$

- 6) $\int_{\sqrt{\frac{1}{2}}}^1 \operatorname{Arccos}.x \frac{x}{1-x^2} dx = \frac{1}{8} \pi l 2 + \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 238. N}^\circ. 4.$
- 7) $\int_{\sqrt{\frac{1}{2}}}^1 (\operatorname{Arccos}.x)^p \frac{x}{1-x^2} dx = \frac{\pi^p}{2^{2p+1}} \left\{ 2 - \sum_1^{\infty} \frac{4}{p+2m} \sum_1^{\infty} \frac{1}{(4n)^{2m}} \right\} \quad \text{V. T. 238. N}^\circ. 19.$
- 8) $\int_{-1}^0 \operatorname{Arctang}.x \frac{dx}{1-x} = -\frac{1}{4} \pi l 2 \quad \text{V. T. 188. N}^\circ. 2.$
- 9) $\int_{-1}^0 \operatorname{Arccot}.x \frac{dx}{1-x} = -\frac{3}{4} \pi l 2 \quad \text{V. T. 188. N}^\circ. 2.$
- 10) $\int_0^p \operatorname{Arcsin}.\frac{x}{p} x dx = \frac{1}{8} p^2 \pi \quad \text{V. T. 34. N}^\circ. 8.$
- 11) $\int_0^p \operatorname{Arcsin}.\frac{x}{p} x^{2a-1} dx = \frac{\pi}{4a} p^{2a} \left[1 - \frac{1^{a/2}}{2^{a/2}} \right] \quad \text{V. T. 34. N}^\circ. 10.$
- 12) $\int_0^p \operatorname{Arcsin}.\frac{x}{p} x^{2a} dx = \frac{p^{2a+1}}{2a+1} \left[\frac{\pi}{2} - \frac{2^{a/2}}{3^{a/2}} \right] \quad \text{V. T. 34. N}^\circ. 11.$
- 13) $\int_0^p \operatorname{Arccos}.\frac{x}{p} x dx = \frac{1}{8} p^2 \pi \quad \text{V. T. 34. N}^\circ. 8.$
- 14) $\int_0^p \operatorname{Arccos}.\frac{x}{p} x^{2a-1} dx = \frac{1^{a/2}}{2^{a/2}} p^{2a} \frac{\pi}{4a} \quad \text{V. T. 34. N}^\circ. 10.$
- 15) $\int_0^p \operatorname{Arccos}.\frac{x}{p} x^{2a} dx = \frac{p^{2a+1}}{2a+1} \frac{2^{a/2}}{3^{a/2}} \quad \text{V. T. 34. N}^\circ. 11.$
- 16) $\int_0^p \operatorname{Arcsin}.\frac{x}{p} \frac{x}{\sqrt{(q^2-x^2)^3}} dx = \frac{\pi}{2\sqrt{(q^2-p^2)}} - \frac{1}{q} \operatorname{F}'\left(\frac{p}{q}\right), p < q; \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{V. T. 34. N}^\circ. 12.$
- 17) $\int_0^p \operatorname{Arccos}.\frac{x}{p} \frac{x}{\sqrt{(q^2-x^2)^3}} dx = \frac{1}{q} \operatorname{F}'\left(\frac{p}{q}\right) - \frac{\pi}{2q}, p < q;$
- 18) $\int_0^p \operatorname{Arctang}.x \frac{dx}{1+p^2x} = \frac{1}{2p} \operatorname{Arctang}.p.l(1+p^2) \quad \text{V. T. 188. N}^\circ. 15.$

F. Alg. fract.
Circ. Inv.

TABLE 271 suite.

Lim. diverses.

$$19) \int_0^{\frac{1}{p}} \text{Arctang. } x \frac{dx}{p+x} = \frac{1}{2} \text{Arccol. } p, l \frac{1+p^2}{p^2} \quad \text{V. T. 188. N}^\circ 14.$$

$$20) \int_p^q \text{Arcsin. } (\sqrt{x}) \frac{r-x}{(r+x)^2} \frac{dx}{\sqrt{x}} = \frac{2\sqrt{q}}{q+r} \text{Arcsin. } (\sqrt{q}) - \frac{2\sqrt{p}}{p+r} \text{Arcsin. } (\sqrt{p}) - \\ - \frac{1}{(\sqrt{1-r})} l \left\{ \frac{\sqrt{(1+r)} + \sqrt{(1-p)}}{\sqrt{(1+r)} + \sqrt{(1-q)}} \cdot \frac{\sqrt{(1+r)} - \sqrt{(1-q)}}{\sqrt{(1+r)} - \sqrt{(1-p)}} \right\} \quad \text{V. T. 35. N}^\circ 22.$$

F. Algèbr.

Autres fonctions.

TABLE 272.

Lim. diverses.

$$1) \int_0^1 l i \left(\frac{1}{x} \right) x dx = 0 \quad \text{V. T. 300. N}^\circ 2.$$

$$2) \int_0^1 l i (x) x^{p-1} dx = \frac{1}{p} l(1+p), p^2 \geq -1; \quad \text{V. T. 300. N}^\circ 3.$$

$$3) \int_0^p E(x) \frac{x}{1-x^2} \frac{dx}{\sqrt{(p^2-x^2)}} = \frac{p\pi}{2\sqrt{(1-p^2)}} \quad \left. \begin{array}{l} \text{Roberts, L. 10. 454.} \\ \text{La formule 4) est fautive, et ne vaut que} \\ \text{pour } E(x) \text{ au lieu de } F(x). \end{array} \right\}$$

$$4) \int_0^p F(x) \frac{x}{1-x^2} \frac{dx}{\sqrt{(p^2-x^2)}} = \frac{p\pi}{2\sqrt{(1-p^2)}} \quad \left. \begin{array}{l} \text{La formule 4) est fautive, et ne vaut que} \\ \text{pour } E(x) \text{ au lieu de } F(x). \end{array} \right\}$$

F. Exponentielle. } F. entière.
Logarithme. }

TABLE 275.

Lim. 0 et ∞ .

$$1) \int e^{-x} l x dx = -A \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Schlömle, Beitr. III. § 9. — Id., Gr. 4. 167. — Id., Gr. 9. 5.}$$

$$2) \int e^{-qx} l x dx = -\frac{1}{q} (A + l q)$$

$$3) \int e^{-x^2} l \frac{1}{x} dx = \frac{1}{4} \pi (A + 2 l 2) \quad \text{Meyer, Int. déf. 373.}$$

$$4) \int e^{-qx^2} l x dx = -\frac{\sqrt{\pi} q}{4 q} (A + l q + 2 l 2) \quad \text{Schlömle, Gr. 4. 167. — Id., Stud. I. 14.}$$

$$5) \int e^{-px} l (q+x)^2 dx = \frac{1}{p} \{ i q^2 - 2 e p q Ei. (-p q) \} \quad \text{Bierens de Haan, Verh. der K. Ak. van Wet. 1854. bl. 19. — Winckler, Cr. 50. 1.}$$

$$\begin{aligned} 6) \int e^{-px} l(q-x)^2 dx &= \frac{1}{p} \{ lq^2 - 2e^{-pq} Ei.(pq) \} \\ 7) \int e^{-px} l(q^2-x^2)^2 dx &= \frac{2}{p} \{ lq^2 - e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) \} \\ 8) \int e^{-px} l(q^2+x^2)^2 dx &= \frac{2}{p} \{ lq^2 - 2Ci.(pq).Cos.pq - 2Si.(pq).Sin.pq + \pi Sin.pq \} \\ 9) \int e^{-px} l(q^4-x^4)^2 dx &= \frac{2}{p} \{ 2lq^2 - e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) - 2Ci.(pq).Cos.pq \\ &\quad - 2Si.(pq).Sin.pq + \pi Sin.pq \} \end{aligned}$$

Bierens
de Haan,
Verh. der
K. Ak. van
Wet. 1854.
bl. 19. —
Winckler,
Cr. 50. 1.

$$\begin{aligned} 1) \int lx \frac{e^{\frac{bx}{c}} + e^{-\frac{bx}{c}}}{e^{\frac{bx}{c}} + e^{-\pi x}} dx &= -\frac{1}{2} A Sec. \frac{b\pi}{2c} - \sum_0^{\infty} (-1)^n \left\{ \frac{l \left\{ (2n+1)\pi - \frac{b\pi}{c} \right\}}{(2n+1)\pi - \frac{b\pi}{c}} + \frac{l \left\{ (2n+1)\pi + \frac{b\pi}{c} \right\}}{(2n+1)\pi + \frac{b\pi}{c}} \right\}, b < c, \\ &\quad \left. \begin{array}{l} \text{Malmsten,} \\ \text{Cr. 38. 1.} \\ \text{la trouve} \\ \text{fautive.} \end{array} \right\} \\ 2) \int lx \frac{e^{ax} + e^{-ax}}{e^{bx} + e^{-bx}} dx &= \frac{\pi}{2b} Sec. \frac{a\pi}{2b} l2\pi + \frac{\pi}{b} \sum_1^b (-1)^{n-1} Cos. \left(\frac{2n-1}{2b} a\pi \right) l \frac{\Gamma \left(\frac{2b+2n-1}{4b} \right)}{\Gamma \left(\frac{2n-1}{4b} \right)}, a+b \\ &\quad \left. \begin{array}{l} \text{impair; } \\ \text{V. T.} \\ \text{191.} \\ \text{N° 8, 9.} \end{array} \right\} \\ 3) &= \frac{\pi}{2b} Sec. \frac{a\pi}{2b} l\pi + \frac{\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} Cos. \left(\frac{2n-1}{2b} a\pi \right) l \frac{\Gamma \left(\frac{2b-2n+1}{2b} \right)}{\Gamma \left(\frac{2n-1}{2b} \right)}, a+b \\ &\quad \left. \begin{array}{l} \text{pair; } \end{array} \right\} \\ 4) \int lx \frac{e^{\frac{bx}{c}} - e^{-\frac{bx}{c}}}{e^{\frac{bx}{c}} - e^{-\pi x}} dx &= -\frac{1}{2} A Tang. \frac{b\pi}{2c} - \sum_0^{\infty} \left\{ \frac{l \left\{ (2n+1)\pi - \frac{b\pi}{c} \right\}}{(2n+1)\pi - \frac{b\pi}{c}} - \frac{l \left\{ (2n+1)\pi + \frac{b\pi}{c} \right\}}{(2n+1)\pi + \frac{b\pi}{c}} \right\}, b < c, \\ &\quad \left. \begin{array}{l} \text{Malmsten,} \\ \text{Cr. 38. 1.} \\ \text{la trouve} \\ \text{fautive.} \end{array} \right\} \\ 5) \int lx \frac{e^x - e^{-x} - 2}{(1+e^{-x})^2} dx &= l \frac{\pi}{2} \quad \text{V. T. 134. N° 1.} \\ 6) \int lx \frac{(q-p) \{ e^{p+q} x + e^{-(p+q)x} \} + (p+q) \{ e^{p-q} x + e^{q-p} x \}}{(e^{px} + e^{-px})^2} dx &= l Tang. \left\{ \frac{p+q}{4p} \pi \right\} \quad \left. \begin{array}{l} \text{V. T. 136.} \\ \text{N° 9.} \end{array} \right\} \end{aligned}$$

F. Exponent. polynôme en dén.
Logar. en num. lx .

TABLE 274 suite.

Lim. 0 et ∞ .

- 7) $\int lx \frac{(2q-1) e^{(q+1)x} + (2q+1) e^{(1-q)x} + 2q(e^{qx} + e^{-qx})}{(e^x + 1)^2} (e^{qx} - e^{-qx}) dx = l(q\pi \text{Cot.} q\pi) \quad \begin{matrix} \text{V. T. 134.} \\ \text{N}^\circ. 8. \end{matrix}$
- 8) $\int lx \frac{e^x}{(e^x + 1)^2} dx = \frac{1}{2} Z' \left(\frac{1}{2} \right) + \frac{1}{2} l 2\pi \quad \text{V. T. 190. N}^\circ. 7.$
- 9) $\int lx (e^{qx} - e^{-qx}) \frac{(p+q) e^{-qx} - (p-q) e^{qx} - q e^{-2px} (e^{qx} + e^{-qx})}{(e^{px} - e^{-px})^2} dx = \frac{1}{2} l \left\{ \frac{q\pi}{p} \text{Cosec.} \frac{q\pi}{p} \right\} \quad \begin{matrix} \text{V. T. 136.} \\ \text{N}^\circ. 5. \end{matrix}$
- 10) $\int lx (e^{qx} - e^{-qx}) \frac{(p-2q) \{e^{(q+p)x} - e^{-(q+p)x}\} + (p+2q) \{e^{(q-p)x} - e^{-(q-p)x}\}}{(e^{px} - e^{-px})^2} dx = l \text{Sec.} \frac{q\pi}{p} \quad \begin{matrix} \text{V. T. 136.} \\ \text{N}^\circ. 14. \end{matrix}$
- 11) $\int lx \frac{dx}{e^x + e^x - 1} = \frac{2\pi}{\sqrt{3}} \left\{ \frac{5}{6} l 2\pi - l \Gamma \left(\frac{1}{6} \right) \right\} \quad \text{V. T. 191. N}^\circ. 7.$
- 12) $\int lx \frac{dx}{e^x + e^{-x} + 2 \text{Cos.} \lambda} = \frac{1}{2} \pi \text{Cosec.} \lambda l \frac{(2\pi)^\lambda \Gamma \left(\frac{1}{2} + \frac{\lambda}{2\pi} \right)}{\Gamma \left(\frac{1}{2} - \frac{\lambda}{2\pi} \right)} \quad \text{V. T. 190. N}^\circ. 9.$

F. Exponent. polynôme en dén.
Logar. en num. $l(p^2 \pm x^2)$.

TABLE 275.

Lim. 0 et ∞ .

- 1) $\int l(1+x^2) \frac{dx}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} = l \frac{4}{\pi} \quad \text{Malmsten, Cr. 38. 1.}$
- 2) $\int l(1+x^2) \frac{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})^2} dx = 2\sqrt{2} - \frac{8}{\pi} + \frac{2\sqrt{2}}{\pi} l \frac{\sqrt{2}+1}{\sqrt{2}-1} \quad \text{V. T. 138. N}^\circ. 19.$
- 3) $\int l(1+x^2) \frac{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})^2} dx = \frac{\pi-2}{\pi} \quad \text{V. T. 138. N}^\circ. 16.$
- 4) $\int l(1+x^2) \frac{dx}{(e^{\pi x} - e^{-\pi x})^2} = \frac{1}{4\pi} (2\Lambda - 1) \quad \text{V. T. 138. N}^\circ. 10.$
- 5) $\int l(1+x^2) \frac{e^{\pi x} + e^{-\pi x}}{(e^{\pi x} - e^{-\pi x})^2} dx = \frac{2\sqrt{2}-1}{2\pi} \quad \text{V. T. 138. N}^\circ. 12.$
- 6) $\int l(1+x^2) \frac{dx}{(e^{qx} - e^{-qx})^2} = \frac{1}{2q} \left\{ l \frac{q}{\pi} + \frac{\pi}{2q} - Z' \left(\frac{\pi+q}{\pi} \right) \right\} \quad \text{V. T. 138. N}^\circ. 11.$

$$7) \int l(1+x^2) \frac{\pi(e^{px} + e^{-px})(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}) - 2p(e^{px} - e^{-px})(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})}{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})^2} dx = -4 + \\ + 2\pi \cos p + 2 \sin p l \frac{1 + \sin p}{1 - \sin p}, 0 \leq p \leq \frac{1}{2}\pi; \quad \text{V. T. 138.} \\ \text{N}^\circ. 15.$$

$$8) \int l(1+x^2) \frac{\lambda(e^{(2\pi-\lambda)x} + e^{(\lambda-2\pi)x}) + (2\pi-\lambda)(e^{\lambda x} + e^{-\lambda x})}{(e^{\pi x} - e^{-\pi x})^2} dx = 1 + 2 \sum_1^{\infty} \frac{\cos n\lambda}{n+1} \quad \text{V. T. 138.} \\ \text{N}^\circ. 8.$$

$$9) \int l(1+x^2) \frac{\pi(e^{\pi x} + e^{-\pi x})(e^{px} + e^{-px}) - p(e^{px} - e^{-px})(e^{\pi x} - e^{-\pi x})}{(e^{\pi x} - e^{-\pi x})^2} dx = \quad \text{V. T. 138.} \\ \text{N}^\circ. 6. \\ = p \sin p - 1 + \cos p l \{2(1 + \cos p)\}$$

$$10) \int l\left(\frac{9}{4} + x^2\right) \frac{e^{\frac{3}{2}\pi x} - e^{-\frac{3}{2}\pi x}}{e^{\pi x} - e^{-\pi x}} dx = 2 \sin \frac{1}{3}\pi \cdot l\left(\frac{1}{2} \cot \frac{\pi}{12}\right)$$

$$11) \int l(a^2+x^2) \frac{e^{\frac{b\pi x}{c}} + e^{-\frac{b\pi x}{c}}}{e^{\pi x} + e^{-\pi x}} dx = \sec \frac{b\pi}{2c} l 2c + 2 \sum_1^c (-1)^{n-1} \cos \left\{ \left(n - \frac{1}{2}\right) \frac{b\pi}{c} \right\} l \frac{\Gamma\left(\frac{a+c+n-\frac{1}{2}}{2c}\right)}{\Gamma\left(\frac{a+n-\frac{1}{2}}{2c}\right)}, b+c \\ \text{impair;}$$

$$12) = \sec \frac{b\pi}{2c} l c + 2 \sum_1^{\frac{c-1}{2}} (-1)^{n-1} \cos \left\{ \left(n - \frac{1}{2}\right) \frac{b\pi}{c} \right\} l \frac{\Gamma\left(\frac{a+c-n+\frac{1}{2}}{c}\right)}{\Gamma\left(\frac{a+n-\frac{1}{2}}{c}\right)}, b+c \text{ pair;}$$

$$13) \int l(a^2+x^2) \frac{e^{\frac{b\pi x}{c}} - e^{-\frac{b\pi x}{c}}}{e^{\pi x} - e^{-\pi x}} dx = \tan \frac{b\pi}{2c} l 2c + 2 \sum_1^{\frac{c-1}{2}} (-1)^{n-1} \sin \frac{n b \pi}{c} l \frac{\Gamma\left(\frac{a+c+n}{2c}\right)}{\Gamma\left(\frac{a+n}{2c}\right)}, b+c \\ \text{impair;}$$

$$14) = \tan \frac{b\pi}{2c} l c + 2 \sum_1^{\frac{c-1}{2}} (-1)^{n-1} \sin \frac{n b \pi}{c} l \frac{\Gamma\left(\frac{a+c-n}{c}\right)}{\Gamma\left(\frac{a+n}{c}\right)}, b+c \\ \text{pair;}$$

$$15) \int l\left(\frac{1}{4}c^2+x^2\right) \frac{e^{\frac{b\pi x}{c}} + e^{-\frac{b\pi x}{c}}}{e^{\pi x} + e^{-\pi x}} dx = \sum_1^c (-1)^{n-1} \cos \left\{ \left(n - \frac{1}{2}\right) \frac{b\pi}{c} \right\} l \left\{ \left(\frac{c+1}{2} - n\right) \cot \left(\frac{\pi}{4} - \frac{2n-1}{4c}\pi\right) \right\}, b+c \\ \text{impair;}$$

$$16) \int l\left(\frac{1}{4}c^2+x^2\right) \frac{e^{\frac{b\pi x}{c}} - e^{-\frac{b\pi x}{c}}}{e^{\pi x} - e^{-\pi x}} dx = \sum_1^{\frac{c-1}{2}} (-1)^{n-1} \sin \frac{n b \pi}{c} l \left\{ \left(\frac{1}{2}c - n\right) \cot \left(\frac{\pi}{4} - \frac{n\pi}{2c}\right) \right\}, b+c \\ \text{impair;}$$

$$17) \int l(a^2 + x^2) \frac{e^{i\pi x} - e^{-i\pi x}}{e^{\pi x} - e^{-\pi x}} dx = 2l \frac{\Gamma\left(\frac{a+3}{4}\right)}{\Gamma\left(\frac{a+1}{4}\right)}$$

$$18) \int l(a^2 + x^2) \frac{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}}{e^{\pi x} - e^{-\pi x}} dx = 2 \text{Sin.} \frac{1}{3} \pi l \frac{\Gamma\left(\frac{a+4}{6}\right) \Gamma\left(\frac{a+5}{6}\right)}{\Gamma\left(\frac{a+1}{6}\right) \Gamma\left(\frac{a+2}{6}\right)}$$

Les intégrales 10) à 18) sont trouvées par Malmsten, Cr. 38. 1; (où il y a plusieurs fautes).

$$19) \int l(1-x^2) \frac{dx}{(e^{\pi x} - e^{-\pi x})^2} = \frac{1}{4\pi} \sum_0^{\infty} (-1)^{n-1} \frac{B_{2n+1}}{n+1} \quad \text{V. T. 138. N}^\circ. 21.$$

$$1) \int l x \frac{dx}{e^x + e^{-x}} = \frac{1}{2} \pi l \left\{ \frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \vee 2\pi \right\} \quad \text{V. T. 191. N}^\circ. 1.$$

$$2) \int l x \frac{e^{ax} - e^{-ax}}{e^{bx} - e^{-bx}} dx = \frac{\pi}{2b} \text{Tang.} \frac{a\pi}{2b} l 2\pi + \frac{\pi}{b} \sum_1^{b-1} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+n}{2b}\right)}{\Gamma\left(\frac{n}{2b}\right)}, a+b \text{ impair;}$$

$$3) \quad = \frac{\pi}{2b} \text{Tang.} \frac{a\pi}{2b} l \pi + \frac{\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} \text{Sin.} \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b-n}{b}\right)}{\Gamma\left(\frac{n}{b}\right)}, a+b \text{ pair;}$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \text{V. T. 191. N}^\circ. 3, 4.$$

$$4) \int l x \frac{dx}{e^x + e^{-x} + 1} = \frac{\pi}{3} l \left\{ \frac{\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{3}\right)} \vee 2\pi \right\} \quad \text{V. T. 191. N}^\circ. 2.$$

$$5) \int l x \frac{e^{(a-1)x} dx}{1 + e^{2x} + e^{4x} + \dots + e^{2(a-1)x}} = \frac{\pi}{2a} \text{Tang.} \frac{\pi}{2a} l 2\pi + \frac{\pi}{a} \sum_1^{a-1} (-1)^{n-1} \text{Sin.} \frac{n\pi}{a} l \frac{\Gamma\left(\frac{a+n}{2a}\right)}{\Gamma\left(\frac{n}{2a}\right)}, a \text{ pair;}$$

$$6) \quad = \frac{\pi}{2a} \text{Tang.} \frac{\pi}{2a} l \pi + \frac{\pi}{a} \sum_1^{\frac{a-1}{2}} (-1)^{n-1} \text{Sin.} \frac{n\pi}{a} l \frac{\Gamma\left(\frac{a-n}{a}\right)}{\Gamma\left(\frac{n}{a}\right)}, a \text{ impair;}$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \text{V. T. 191. N}^\circ. 5, 6.$$

F. Expon.
Logar.

TABLE 276 suite.

Lim. — ∞ et ∞ .

$$\begin{aligned}
 7) \int l x \frac{(p+1) e^{(p-1)x} + (p-1) e^{(p+1)x} - (q+1) e^{(q-1)x} - (q-1) e^{(q+1)x}}{(e^x + e^{-x})^2} dx &= \text{V. T. 146. N}^\circ 11. \\
 &= \pi l \left\{ \text{Tang.} \left(\frac{p+1}{4} \pi \right) . \text{Cot.} \left(\frac{q+1}{4} \pi \right) \right\} \\
 8) \int l x \frac{(p+1) e^{(p-1)x} - (p-1) e^{(p+1)x} - (q+1) e^{(q-1)x} + (q-1) e^{(q+1)x}}{(e^x - e^{-x})^2} dx &= \text{V. T. 146. N}^\circ 12. \\
 &= \pi l \left\{ \text{Sin.} \left(\frac{q+1}{2} \pi \right) . \text{Cosec.} \left(\frac{p+1}{2} \pi \right) \right\}
 \end{aligned}$$

F. Expon.
Logar.

TABLE 277.

Lim. diverses.

$$\begin{aligned}
 1) \int_0^{2\pi} l(1 - p e^{xi}) dx &= 0 \\
 2) \int_0^{2\pi} l(1 - p e^{-xi}) dx &= 0 \\
 3) \int_{-\pi}^{\pi} e^{-\eta xi} l(1 - p e^{xi}) dx &= -\frac{2\pi}{q} p^\eta \\
 4) \int_{-\pi}^{\pi} e^{\eta xi} l(1 - p e^{xi}) dx &= 0 \\
 5) \int_0^\infty \frac{e^{-x}}{lx} dx &= 0 \quad \text{V. T. 43. N}^\circ 4. \\
 6) \int_p^\infty e^{-x} l x dx &= e^{-p} l p - \text{Ei.}(-p) \quad \text{V. T. 150. N}^\circ 5. \\
 7) \int_p^\infty l x \frac{e^{\eta x} - e^{-\eta x}}{(e^{\eta x} + e^{-\eta x})^2} dx &= \frac{1}{q} \frac{l p}{e^{\eta p} + e^{-\eta p}} + \frac{1}{q \pi} \sum_0^\infty \left[\frac{(-1)^n}{2n+1} l \left\{ 1 + \left(\frac{2n+1}{2pq} \pi \right)^2 \right\} \right] \quad \text{V. T. 150. N}^\circ 10. \\
 8) \int_p^\infty l x \frac{e^{\eta x} + e^{-\eta x}}{(e^{\eta x} - e^{-\eta x})^2} dx &= \frac{1}{q} \frac{l p}{e^{\eta p} - e^{-\eta p}} + \frac{1}{2pq^2} + \frac{1}{q \pi} \sum_1^\infty \left\{ \frac{(-1)^n}{n} \text{Arctang.} \left(\frac{n\pi}{pq} \right) \right\} \quad \text{V. T. 150. N}^\circ 9. \\
 9) \int_{-lp}^\infty e^{-x} l x dx &= p l l \frac{1}{p} - \text{li}(p) \quad \text{V. T. 150. N}^\circ 14. \\
 10) \int_0^1 e^{l'x-1} l(1-l/x) dx &= 2 \frac{1-e}{e} \quad \text{V. T. 376. N}^\circ 3.
 \end{aligned}$$

- $$\left. \begin{aligned} 1) \int e^{-x} \sin. x \, dx &= \frac{1}{2} \\ 2) \int e^{-qx} \sin. qx \, dx &= \frac{1}{2q} \\ 3) \int e^{-qx} \cos. qx \, dx &= \frac{1}{2q} \end{aligned} \right\} \text{Oettinger, Cr. 38. 216.}$$
- $$\left. \begin{aligned} 4) \int e^{-px} \sin. x \, dx &= \frac{1}{p^2 + 1} \\ 5) \int e^{-px} \cos. x \, dx &= \frac{p}{1 + p^2} \end{aligned} \right\} \text{Dienger, Cr. 38. 231. — Raabe, Int. 152.}$$
- $$6) \int e^{-x} \sin. qx \, dx = \frac{q}{1 + q^2} \quad \text{Poisson, P. 19. 60. — Dienger, Cr. 46. 119. — Schlömilch, Gr. 5. 204.}$$
- $$7) \int e^{-x} \cos. qx \, dx = \frac{1}{1 + q^2} \quad \text{Dienger, Cr. 46. 119. — Schlömilch, Gr. 5. 204.}$$
- $$\left. \begin{aligned} 8) \int e^{-px} \sin. qx \, dx &= \frac{q}{p^2 + q^2} \\ 9) \int e^{-px} \cos. qx \, dx &= \frac{p}{p^2 + q^2} \end{aligned} \right\} \text{Poisson, P. 16. 215. N° 2. — Cauchy, Cours. Leç. 32. — Grunert, Cr. 8. 146. — Lobatto, Cr. 11. 169. — Bonecompagni, Cr. 25. 74. — Oettinger, Cr. 38. 216.}$$
- Sur la formule (9) voyez encore: Poisson, Mém. Inst. 1811. 163. N° 25. — Id., P. 18. 295. N° 21. — Dienger, Cr. 38. 331.
- $$10) \int e^{-px} \sin. (qx + \lambda) \, dx = \frac{1}{p^2 + q^2} (q \cos. \lambda + p \sin. \lambda) \quad \text{Poisson, Chal. 158.}$$
- $$11) \int e^{bx\pi} \sin. qxi \, dx = \frac{qi}{b^2 \pi^2 - q^2} \quad \text{Schlömilch, Gr. 3. 9.}$$
- $$12) \int e^{-px} \cot. \frac{1}{2} qx \, dx = 2q \sum_1^{\infty} \frac{n}{p^2 + q^2 n^2} \quad \text{Cauchy, Exerc. 1827. p. 141.}$$
- $$13) \int e^{-x} \sin. (2p \vee x) \, dx = p e^{-p^2} \vee \pi \quad \text{Helmling, Transf. 14.}$$
- $$14) \int e^{-x} \text{Tang.} (q \vee x) \, dx = 2q \vee \pi \sum_1^{\infty} (-1)^n n e^{-(nq)^2} \quad \text{V. T. 388. N° 20.}$$
- $$15) \int e^{-x} \cot. (q \vee x) \, dx = -2q \vee \pi \sum_1^{\infty} n e^{-(nq)^2} \quad \text{V. T. 388. N° 21.}$$
- $$16) \int e^{-x} \text{Cosec.} (2q \vee x) \, dx = -2q \vee \pi \sum_1^{\infty} (2n - 1) e^{-(2n-1)^2 q^2} \quad \text{V. T. 388. N° 22.}$$

- $$1) \int e^{-px} \sin. 2a x dx = \frac{2a \cdot 2a-1}{p^2 + (2a)^2} \cdot \frac{2a-2 \cdot 2a-3}{p^2 + (2a+2)^2} \cdots \frac{2 \cdot 1}{p^2 + 2^2} \frac{1}{p} \left\{ \begin{array}{l} , a > 1; \\ \text{Dienger, Cr. 38. 331.} \\ \text{Schlömlich, Gr. 7. 38.} \end{array} \right.$$
- $$2) \int e^{-px} \sin. 2a+1 x dx = \frac{2a+1 \cdot 2a}{p^2 + (2a+1)^2} \cdot \frac{2a-1 \cdot 2a-2}{p^2 + (2a-1)^2} \cdots \frac{3 \cdot 2}{p^2 + 3^2} \frac{1}{p^2 + 1} \left\{ \begin{array}{l} \text{Dienger, Cr. 38. 331.} \\ \text{Schlömlich, Gr. 7. 38.} \end{array} \right.$$
- $$3) \int e^{-px} \cos. 2a x dx = \frac{1}{p} \frac{1^{2a/1}}{p^2 + 2^2 \cdot p^2 + 4^2 \cdots p^2 + (2a)^2} \left\{ 1 + \frac{p^2}{1 \cdot 2} + \frac{p^2 \cdot p^2 + 2^2}{1 \cdot 2 \cdot 3 \cdot 4} + \cdots + \frac{p^2 \cdot p^2 + 2^2 \cdots p^2 + (2a-2)^2}{1^{2a-1}} \right\} \left\{ \begin{array}{l} \text{Dienger,} \\ \text{Cr. 38.} \\ \text{331.} \end{array} \right.$$
- $$4) \int e^{-px} \cos. 2a+1 x dx = p \frac{1^{2a+1}}{p^2 + 1^2 \cdot p^2 + 3^2 \cdots p^2 + (2a+1)^2} \left\{ 1 + \frac{p^2 + 1^2}{1 \cdot 2 \cdot 3} + \cdots + \frac{p^2 + 1^2 \cdot p^2 + 3^2 \cdots p^2 + (2a-1)^2}{1^{2a+1/1}} \right\} \left\{ \begin{array}{l} \text{Dienger,} \\ \text{Cr. 38.} \\ \text{331.} \end{array} \right.$$
- $$5) \int e^{-px} \cos. q x \cdot \cos. r x dx = \frac{1}{2} \frac{p}{p^2 + (q-r)^2} p \text{ très-petit; } \left\{ \begin{array}{l} \text{Cauchy, Sav. Etr. 1827. 124. Note} \\ \text{6. — Id., P. 19. 511.} \end{array} \right.$$
- $$6) = \frac{p}{2} \left\{ \frac{1}{p^2 + (q-r)^2} + \frac{1}{p^2 + (q+r)^2} \right\} \text{Poisson, P. 18. 295. No. 26.}$$
- $$7) = p \frac{p^2 + q^2 + r^2}{(p^2 + q^2 + r^2)^2 - 4 q^2 r^2} \left\{ \begin{array}{l} \text{Dienger, Gr. 12. 97.} \end{array} \right.$$
- $$8) \int e^{-px} \cos. q x \cdot \sin. r x dx = 2 \frac{p^2 - q^2 + r^2}{(p^2 + q^2 + r^2)^2 - 4 q^2 r^2} \left\{ \begin{array}{l} \text{Dienger, Gr. 12. 97.} \end{array} \right.$$
- $$9) = \frac{1}{2} \left\{ \frac{q+r}{p^2 + (q+r)^2} - \frac{q-r}{p^2 + (q-r)^2} \right\} \left\{ \begin{array}{l} \text{Cauchy, Sav. Etr. 1827. 124.} \\ \text{Note 6.} \end{array} \right.$$
- $$10) = r \frac{p^2 - q^2 + r^2}{\{p^2 + (q-r)^2\} \{p^2 + (q+r)^2\}} \left\{ \begin{array}{l} \text{Dienger, Cr. 41. 137.} \end{array} \right.$$
- $$11) \int e^{-px} \sin. q x \cdot \sin. r x dx = \frac{2 p q r}{\{p^2 + (q-r)^2\} \{p^2 + (q+r)^2\}} \left\{ \begin{array}{l} \text{Dienger, Cr. 41. 137.} \end{array} \right.$$
- $$12) = \frac{2 p q r}{(p^2 + q^2 + r^2)^2 - 4 q^2 r^2} \text{Dienger, Gr. 12. 97.}$$
- $$13) = \frac{1}{2} p \left\{ \frac{-1}{p^2 + (q+r)^2} + \frac{1}{p^2 + (q-r)^2} \right\} \left\{ \begin{array}{l} \text{Cauchy, Sav. Etr. 1827.} \\ \text{124. Note 6.} \end{array} \right.$$
- $$14) = \frac{1}{2} \frac{p}{p^2 + (q-r)^2}, p \text{ très-petit;}$$
- $$15) \int e^{-px} \cos. x dx \vee \cos. 2 b x = \sum_0^{\infty} \frac{(-2b)^n}{n^{n-1/1}} \frac{\cos. (n \operatorname{Arccot}. p)}{\vee (1+p^2)^n} \left\{ \begin{array}{l} \text{Cauchy, Sav. Etr. 1827. 124.} \\ \text{Note 3.} \end{array} \right.$$
- $$16) \int (e^{-qx} \cos. p x - e^{-px} \sin. q x) dx = 0 \text{ Lobatto, Cr. 11. 169.}$$

$$\begin{aligned}
 17) \int (e^{-qx} \sin. px - e^{-px} \cos. qx) dx &= 0 \quad \text{Lobatto, Cr. 11. 169.} \\
 18) \int e^{-2px} \sin. (q^2 x^2) dx &= \frac{1}{4q} \left\{ \cos. \left(\frac{p^2}{q^2} \right) + \sin. \left(\frac{p^2}{q^2} \right) \right\} \sqrt{2\pi} - \\
 &\quad - \frac{1}{q^2} \left\{ \cos. \left(\frac{p^2}{q^2} \right) \sum_0^{\infty} (-1)^n \frac{p}{(4n+1)1^{2n+1}} \left(\frac{p}{q} \right)^{4n} + \sin. \left(\frac{p^2}{q^2} \right) \sum_0^{\infty} (-1)^n \frac{p}{(4n-1)1^{2n-1}} \left(\frac{p}{q} \right)^{4n-2} \right\} \\
 19) \int e^{-2px} \cos. (q^2 x^2) dx &= \frac{1}{4q} \left\{ \cos. \left(\frac{p^2}{q^2} \right) - \sin. \left(\frac{p^2}{q^2} \right) \right\} \sqrt{2\pi} - \\
 &\quad - \frac{1}{q^2} \left\{ \sin. \left(\frac{p^2}{q^2} \right) \sum_0^{\infty} (-1)^n \frac{p}{(4n+1)1^{2n+1}} \left(\frac{p}{q} \right)^{4n} - \cos. \left(\frac{p^2}{q^2} \right) \sum_0^{\infty} (-1)^n \frac{p}{(4n-1)1^{2n-1}} \left(\frac{p}{q} \right)^{4n-2} \right\}
 \end{aligned}$$

Helm-
ling,
Transf.
18, 19.

$$\begin{aligned}
 1) \int e^{-x^2} \cos. px dx &= \frac{1}{2} e^{-\frac{p^2}{4}} \sqrt{\pi} \quad \text{Cauchy, Sav. Etr. 1827. 124. Note 2. — Id., Cours. Leç. 40. —} \\
 &\quad \text{Id., Sav. Etr. 1827. 599. P. 1. § 2. — Id., Lim. Imag. 91. —} \\
 &\quad \text{Legendre, Exerc. 3. 48. — Bidone, Mém. Turin. 1812. 231.} \\
 &\quad \text{Art. 3. N° 34. — Kummer, Cr. 17. 210. — Schlömilch, Stud. I. 25.} \\
 2) \int e^{-px^2} \cos. x dx &= \frac{1}{2} e^{-\frac{1}{4p}} \sqrt{\frac{\pi}{p}} \quad \text{Laplace,} \\
 3) \int e^{-px^2} \cos. qx dx &= \frac{1}{2} e^{-\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}} \quad \text{Poisson, Chal. 103. — Cauchy, Exerc. 1827. p. 233. — La-} \\
 &\quad \text{place, Probab. 1. 25. — Schlömilch, Beitr. III. § 16. — Id.,} \\
 &\quad \text{Stud. I. 12. — Id., Gr. 5. 90. — Id., Gr. 9. 379. — Helm-} \\
 &\quad \text{ling, Transf. 12. — Raabe, Cr. 48. 178.} \\
 4) \int e^{-p^2 x^2} \cos. qx dx &= \frac{1}{2p} e^{-\frac{q^2}{4p^2}} \sqrt{\pi} \quad \left\{ \begin{array}{l} \text{Oettinger, Cr. 38. 216.} \\ \text{La valeur de (4) est fautive.} \end{array} \right. \\
 5) \int e^{-q^2 x^2} \cos. qx dx &= \frac{1}{qe} \sqrt{\pi} \\
 6) \int e^{-\frac{a}{b} x^2} \cos. (bx \vee a) dx &= \frac{1}{2} e^{-\frac{b^2}{4a}} \sqrt{\frac{b\pi}{a}} \quad \text{Laplace, Mém. Inst. 1809. 353. § 3., où elle est} \\
 &\quad \text{fautive.} \\
 7) \int e^{-x^2} \sin. ax dx &= \frac{1}{2} \sum_0^{\infty} (-1)^n \frac{a^{2n+1}}{(n+1)^{n+1/2}} \quad \text{Legendre, Exerc. 3. 49.} \\
 8) \int e^{-px^2} \sin. qxdx &= 0 \quad \text{Meyer, Int. Déf. 119. (fautive).} \\
 9) &= \frac{1}{\sqrt{2p}} \sum_0^{\infty} (-1)^n \frac{1}{1^{n+1/2}} \left(\frac{q}{\sqrt{2p}} \right)^{2n+1} \quad \text{Schubert, Samml. 117. (fautive)}
 \end{aligned}$$

- $$\left. \begin{aligned} 10) \int e^{-px^2} \text{Sin. } qx \, dx &= \sum_0^{\infty} (-1)^n \frac{1}{(n+2)^{n+1/2}} \cdot \frac{q^{2n+1}}{p^{n+1}} \\ 11) \int e^{-q^2x^2} \text{Sin. } qx \, dx &= \frac{1}{q} \sum_0^{\infty} \frac{(-1)^n}{(n+2)^{n+1/2}} \end{aligned} \right\} \text{Oettinger, Cr. 38. 216.}$$
- $$12) \int e^{x^2 i} \text{Cos. } qx \, dx = \frac{1+i}{2} e^{-\frac{q^2 i}{4}} \sqrt{\frac{\pi}{2}} \quad \text{Cauchy, Lim. Imag. 190.}$$
- $$13) \int e^{-rx^2} \text{Sin. } px \cdot \text{Sin. } qx \, dx = \frac{1}{4} \sqrt{\frac{\pi}{r}} \left\{ e^{-\frac{(q-p)^2}{4r}} - e^{-\frac{(p+q)^2}{4r}} \right\} \quad \text{Poisson, Chal. 143.}$$
- $$14) \int e^{-rx^2} \text{Cos. } px \cdot \text{Cos. } qx \, dx = \frac{1}{4} \sqrt{\frac{\pi}{r}} \left\{ e^{-\frac{(q-p)^2}{4r}} + e^{-\frac{(p+q)^2}{4r}} \right\} \quad \text{V. T. 280. N°. 3, 13.}$$
- $$\left. \begin{aligned} 15) \int e^{-px^2} \text{Sin. } (qx^2) \cdot \text{Cos. } rx \, dx &= \frac{1}{2} \sqrt{\frac{\pi}{p^2+q^2}} \cdot e^{-ab} (b \text{Sin. } ac + c \text{Cos. } ac) \\ 16) \int e^{-px^2} \text{Cos. } (qx^2) \cdot \text{Cos. } rx \, dx &= \frac{1}{2} \sqrt{\frac{\pi}{p^2+q^2}} \cdot e^{-ab} (b \text{Cos. } ac + c \text{Sin. } ac) \end{aligned} \right\} \begin{aligned} &\text{où } a = \frac{r^2}{4(p^2+q^2)}, \\ &b = \sqrt{\frac{p + \sqrt{p^2+q^2}}{2}}, \\ &c = \sqrt{\frac{-p + \sqrt{p^2+q^2}}{2}} \end{aligned}$$
- $$\left. \begin{aligned} 17) \int e^{-x^2} \text{Sin. } \left(\frac{2p^2}{x^2} \right) dx &= \frac{1}{2} e^{-2p} \text{Sin. } (2p) \sqrt{\pi} \\ 18) \int e^{-x^2} \text{Cos. } \left(\frac{2p^2}{x^2} \right) dx &= \frac{1}{2} e^{-2p} \text{Cos. } (2p) \sqrt{\pi} \end{aligned} \right\} \begin{aligned} &\text{Helmling, Transf. 21, 22,} \\ &29, 30. \end{aligned}$$
- $$\left. \begin{aligned} 19) \int e^{-x^2} \text{Sin.}^2 x \, dx &= \frac{e-1}{4e} \sqrt{\pi} \\ 20) \int e^{-x^2} \text{Sin.}^2 (x \sqrt{p}) \, dx &= \frac{1-e^{-p}}{4} \sqrt{\pi} \end{aligned} \right\} \text{Dienger, Cr. 46. 119.}$$
- $$21) \int e^{bx} \text{Sin.}^a x \, dx = \frac{1 - (-1)^a e^{b\pi}}{\Gamma\left(\frac{a+bi}{2} + 1\right) \Gamma\left(\frac{a-bi}{2} + 1\right)} \frac{\pi}{2^a} 1^{a/1} e^{b\pi} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211.}$$
- $$22) \int e^{-x^2} \text{Cot. } px \, dx = \sqrt{\pi} \sum_1^{\infty} e^{-(np)^2} \quad \text{Cauchy, Exerc. 1827. p. 141.}$$
- $$\left. \begin{aligned} 23) \int e^{-px^2} \text{Sin. } (qx^2) \, dx &= \frac{\sqrt{\pi}}{2 \sqrt{p^2+q^2}} \text{Sin. } \left(\frac{1}{2} \text{Arctang. } \frac{q}{p} \right) \\ 24) \int e^{-px^2} \text{Cos. } (qx^2) \, dx &= \frac{\sqrt{\pi}}{2 \sqrt{p^2+q^2}} \text{Cos. } \left(\frac{1}{2} \text{Arctang. } \frac{q}{p} \right) \end{aligned} \right\} \text{Helmling, Transf. II. S. 63.}$$

F. Exponent. e^{-ax^2} .
Circ. Dir. ent.

TABLE 280 suite.

Lim. 0 et ∞ .

$$\begin{aligned}
 25) \int e^{-px^2} \text{Sin.}(rx^2) \cdot \text{Cos.} 2sx dx &= \frac{\sqrt{\pi}}{2\sqrt{(p^2+r^2)}} e^{-b^2(p \text{Sin.} b p - c \text{Cos.} b p)} \left\{ \begin{array}{l} \text{Helmling, Transf. II. S. 82,} \\ 69, 70. \\ \text{où } a^2 = p^2 + r^2, \\ 4b = \frac{s^2}{p^2 + r^2}, \\ c = \sqrt{\frac{-p + \sqrt{(p^2+r^2)}}{2}}, \\ p = \sqrt{\frac{p + \sqrt{(p^2+r^2)}}{2}}, \\ \psi = \frac{1}{2} \text{Arctang. } \frac{r}{p}. \end{array} \right. \\
 26) &= \frac{\sqrt{\pi}}{a} e^{-\frac{s^2}{a^2} \text{Cos.} 2\psi} \cdot \text{Sin.} \left(\psi - \frac{s^2}{a^2} \text{Sin.} 2\psi \right) \\
 27) \int e^{-px^2} \text{Cos.}(rx^2) \cdot \text{Cos.} 2sx dx &= \frac{\sqrt{\pi}}{a} e^{-\frac{s^2}{a^2} \text{Cos.} 2\psi} \cdot \text{Cos.} \left(\psi - \frac{s^2}{a^2} \text{Sin.} 2\psi \right) \\
 28) &= \frac{1}{2} \sqrt{\frac{\pi}{p^2+r^2}} e^{-b^2(p \text{Cos.} b p + c \text{Sin.} b p)}
 \end{aligned}$$

F. Expon. en dén. binôme à exp. $e^{\pm ax}$.
Circ. Dir. en num.

TABLE 281.

Lim. 0 et ∞ .

$$\begin{aligned}
 1) \int \frac{\text{Sin.} p x}{e^x - 1} dx &= -\frac{1}{2}\pi - \frac{1}{2p} - \frac{\pi}{e^{-2p} - 1} \quad \text{Plana, Mém. Turin. 1818. 7. IV. N°. 18. — Id.,} \\
 &\quad \text{Mém. Turin. 1820.} \\
 2) \int \frac{\text{Sin.} p x}{e^x - e^{-x}} dx &= -\frac{1}{4}\pi + \frac{1}{2} \frac{\pi}{1 + e^{-p\pi}} \quad \text{Plana, Mém. Turin. 1818. 7. IV. 19.} \\
 3) \int \frac{\text{Cos.} p x}{e^x - 1} dx &= \infty \quad \text{Plana, Mém. Turin. 1820.} \\
 4) \int \frac{\text{Cos.} p x}{e^{\pi x} + e^{-\pi x}} dx &= \frac{1}{2} \frac{e^{ip}}{e^p + 1}, \quad p < \pi; \quad \text{Legendre, Exerc. 5. 45.} \\
 5) \int \frac{\text{Sin.} p x i}{i} \frac{dx}{e^{\pi x} + 1} &= \frac{1}{2} \text{Cosec.} p - \frac{1}{2p} \quad \text{Schlömilch, Gr. 3. 9.} \\
 6) \int \frac{\text{Cos.} p x}{e^{i\pi x} + e^{-i\pi x}} dx &= \frac{1}{e^p + e^{-p}} \quad \text{Schlömilch, Gr. 1. 360. — Id., Beitr. II. § 6. — Id., Stud. II. 19.} \\
 7) \int \frac{\text{Sin.} p x i}{i} \frac{dx}{e^{i\pi x} - e^{-i\pi x}} &= \frac{1}{2} \text{Tang.} p \quad \text{Schlömilch, Gr. 3. 9.} \\
 8) \int \frac{\text{Sin.} p x}{e^{\pi x} - e^{-\pi x}} dx &= \frac{1}{4} \frac{e^p - 1}{e^p + 1}, \quad p < \pi; \quad \text{Legendre, Exerc. 5. 45. — Poisson, Mém. Inst. 1811.} \\
 &\quad 163. N°. 28. — Plana, Mém. Turin. 1818. 7. IV. 19. — Schlömilch, Beitr. II. § 6. — Id., Stud. II. 19. \\
 9) \int \frac{\text{Sin.} p x}{e^{2\pi x} - 1} dx &= \frac{1}{4} \frac{e^p + 1}{e^p - 1} - \frac{1}{2p} \quad \text{Legendre, Exerc. 5. 48. — Poisson, P. 18. 295. N°. 25. —} \\
 &\quad \text{Id., P. 20. 222. — Id., Mém. Inst. 1811. 163. N°. 18. — Plana, Mém. Turin. 1818. 7. IV. 18. — Dienger, Gr. 14. 223.} \\
 10) \int \frac{\text{Sin.} p x i}{i} \frac{dx}{e^{2\pi x} - 1} &= \frac{1}{2p} - \frac{1}{4} \text{Cot.} \frac{1}{2} p \quad \text{Schlömilch, Gr. 3. 9.}
 \end{aligned}$$

F. Expon. en dén. binôme à exp. $e^{\pm ax}$. TABLE 281 suite.
Circ. Dir. en num.

Lim. 0 et ∞ .

- 11) $\int \frac{\text{Sin. } p x}{e^{ax} - e^{(a-1)x}} dx = \frac{1}{2} \pi - \frac{1}{2} p + \frac{\pi}{e^{2p\pi} - 1} - \sum_0^{\infty} \frac{n}{n^2 + (a-1)^2}$ Plana, Mém. Turin. 1818. 7. Add.
- 12) $\int \frac{\text{Sin.}^2 p x}{e^{\pi x} + e^{-\pi x}} dx = \frac{1}{8} \frac{(e^p - 1)^2}{e^{2p} + 1}$ V. T. 38. N°. 7 et T. 281. N°. 4.
- 13) $\int \frac{\text{Cos.}^2 p x}{e^{\pi x} + e^{-\pi x}} dx = \frac{1}{8} \frac{(e^p + 1)^2}{e^{2p} + 1}$ V. T. 38. N°. 7 et T. 281. N°. 4.
- 14) $\int \frac{\text{Sin. } 2 p x. \text{Sin. } 2 q x}{e^{! \pi x} + e^{-! \pi x}} dx = \frac{e^{2p} - e^{-2p}}{2} \frac{e^{2q} - e^{-2q}}{e^{4p} + e^{-4p} + e^{4q} + e^{-4q}}$
- 15) $\int \frac{\text{Cos. } 2 p x. \text{Cos. } 2 q x}{e^{! \pi x} + e^{-! \pi x}} dx = \frac{e^{2p} + e^{-2p}}{2} \frac{e^{2q} + e^{-2q}}{e^{4p} + e^{-4p} + e^{4q} + e^{-4q}}$
- 16) $\int \frac{\text{Sin. } 2 p x. \text{Sin. } 2 q x}{e^{\pi x} + e^{-\pi x}} dx = \frac{e^p - e^{-p}}{4} \frac{e^q - e^{-q}}{e^{2p} + e^{-2p} + e^{2q} + e^{-2q}}$
- 17) $\int \frac{\text{Cos. } 2 p x. \text{Cos. } 2 q x}{e^{\pi x} + e^{-\pi x}} dx = \frac{e^p + e^{-p}}{4} \frac{e^q + e^{-q}}{e^{2p} + e^{-2p} + e^{2q} + e^{-2q}}$
- 18) $\int \frac{\text{Sin. } p x. \text{Cos. } q x}{e^{\pi x} - e^{-\pi x}} dx = \frac{1}{4} \frac{e^p - e^{-p}}{e^p + e^{-p} + e^q + e^{-q}}, p + q < \pi;$ Poisson, P. 18. 295. N°. 21. — Id., Mém. Inst. 1811. 163. N°. 28. — Plana, Mém. Turin. 1818. 7. IV. 20.

F. Exp. en num. et en dén. binôme à exp. $e^{\pm ax}$. TABLE 282.
Circ. Dir. en num.

Lim. 0 et ∞ .

- 1) $\int \frac{e^{qx}}{e^{\pi x} - e^{-\pi x}} \text{Sin. } p x dx = \sum_1^{\infty} \frac{p}{\{(2n-1)\pi - q\}^2 + p^2}, q < \pi;$ Poisson, P. 18. 295. N°. 21.
- 2) $\int \frac{e^{! \pi x} - e^{-! \pi x}}{e^{! \pi x} + e^{-! \pi x}} \text{Sin. } p x dx = \frac{2}{e^p - e^{-p}}$ Poisson, P. 18. 295. N°. 25. — Schlömilch, Beitr. II. 6. — Id., Stud. II. 19.
- 3) $\int \frac{e^{2px} + e^{-2px}}{e^{! \pi x} + e^{-! \pi x}} \text{Cos. } q x dx = \frac{e^q + e^{-q}}{e^{2q} + e^{-2q} + 2 \text{Cos. } 4p} 2 \text{Cos. } 2 p, p < \frac{1}{2} \pi;$ Poisson, P. 18. 295. N°. 21.
- 4) $\int \frac{e^{2px} + e^{-2px}}{e^{\pi x} + e^{-\pi x}} \text{Cos. } 2 q x dx = \frac{e^q + e^{-q}}{e^{2q} + e^{-2q} + 2 \text{Cos. } 4p} \text{Cos. } p, p < \frac{1}{2} \pi;$ Legendre, Exerc. 5. 44. — Poisson, P. 18. 295. N°. 21.
- 5) $\int \frac{e^{2px} - e^{-2px}}{e^{! \pi x} + e^{-! \pi x}} \text{Sin. } q x dx = \frac{e^q - e^{-q}}{e^{2q} + e^{-2q} + 2 \text{Cos. } 4p} 2 \text{Sin. } 2 p, p < \frac{1}{2} \pi;$ Poisson, P. 18. 295. N°. 21. — Id., Mém. Inst. 1811. 163. N°. 28.
- 6) $\int \frac{e^{2px} - e^{-2px}}{e^{\pi x} + e^{-\pi x}} \text{Sin. } 2 q x dx = \frac{e^q - e^{-q}}{e^{2q} + e^{-2q} + 2 \text{Cos. } 2p} \text{Sin. } p, p < \frac{1}{2} \pi;$ Legendre, Exerc. 5. 43. — Poisson, P. 18. 295. N°. 21. — Id., Mém. Inst. 1811. 163. N°. 28.

- 7) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} \sin. q x dx = -\frac{1}{2} \pi + \frac{\pi}{1 - e^{-q\pi}}$ Plana, Mém. Turin. 1818. 7. IV. 18.
- 8) $\int \frac{e^{\pi x} + e^{-\pi x}}{e^{\pi x} - e^{-\pi x}} \sin. q x dx = \frac{1}{2} \frac{e^q + 1}{e^q - 1}$ Poisson, P. 18. 295. N°. 25. — Schlömilch, Beitr. II. 6. — Id., Stud. II. 19.
- 9) $\int \frac{e^{\pi x} + 1}{e^{\pi x} - 1} \sin. q x dx = \frac{e^q + e^{-q}}{e^q - e^{-q}}$ Schlömilch, Gr. 1. 360.
- 10) $\int \frac{e^{px} + e^{-px}}{e^{\pi x} - e^{-\pi x}} \sin. q x dx = \frac{1}{2} \frac{e^q - e^{-q}}{e^q + e^{-q} + 2 \cos. p}, p^2 \leq \pi^2$; 26. — Legendre, Exerc. 5. 42. — Plana, Mém. Turin. 1818. 7. IV. 20. — Schlömilch, Beitr. II. 6. — Id., Stud. II. 19.
- 11) $\int \frac{e^{px} + e^{-px}}{e^{2\pi x} - 1} \sin. q x dx = \frac{1}{2} \frac{e^q - e^{-q}}{e^q + e^{-q} - 2 \cos. p} - \frac{q}{q^2 + p^2}$, $p < 2\pi$;
- 12) $\int \frac{e^{-px} + e^{(p-2\pi)x}}{1 - e^{-2\pi x}} \sin. q x dx = \frac{1}{2} \frac{e^q - e^{-q}}{e^q + e^{-q} - 2 \cos. p}$ Poisson, Mém. Inst. 1811. 163. N°. 25.
- 13) $\int \frac{e^{qx} \sin. p x}{e^{2\pi x} - 1} dx = \sum_1 \frac{p}{(2n\pi - q)^2 + p^2}$
- 14) $\int \frac{e^{-qx} \sin. p x}{e^{2\pi x} - 1} dx = \sum_1 \frac{p}{(q + 2n\pi)^2 + p^2}$ Poisson, Mém. Inst. 1811. 163. N°. 25.
- 15) $\int \frac{e^{-qx} \sin. p x}{1 - e^{-x}} dx = q - \frac{1}{2p} \sin. q - \sum_1 \frac{\sin. 2nq \sin. 2np}{2n p^{2n}} B_{2n-1}$, où $\cot. q = \frac{q-1}{p}$; Plana, Mém. Turin. 1818. 7. Add.
- 16) $\int \frac{e^{px} - e^{-px}}{e^{\pi x} - e^{-\pi x}} \cos. q x dx = \frac{\sin. p}{e^q + e^{-q} + 2 \cos. p}, p^2 \leq \pi^2$; Legendre, Exerc. 5. 44. — Poisson, P. 18. 295. N°. 21. — Id., Mém. Inst. 1811. 163. N°. 26 (qui la trouve fautive) — Plana, Mém. Turin. 1818. 7. IV. 20. — Schlömilch, Beitr. II. 6. — Id., Stud. II. 19.

- 1) $\int \frac{\cos. \left\{ x \sqrt{l \frac{1}{q}} \right\}}{1 - 2q \cos. \left\{ 2x \sqrt{l \frac{1}{q}} \right\} + q^2} e^{-x^2} dx = \frac{\sqrt{\pi}}{2(1-q)\sqrt{q^2}} \sum_1 q^{n^2}$
- 2) $\int \frac{\cos. \left\{ x \sqrt{l \frac{1}{q}} \right\}}{1 + 2q \cos. \left\{ 2x \sqrt{l \frac{1}{q}} \right\} + q^2} e^{-x^2} dx = \frac{\sqrt{\pi}}{2(1+q)\sqrt{q^2}} \sum_1 (-1)^n q^{n^2}$
- 3) $\int \frac{e^{-px^2}}{1 - 2q \cos. x + q^2} dx = \frac{1}{1-q^2} \left\{ \frac{1}{2} + \sum_1 q^n e^{-\frac{n^2}{4p}} \right\} \sqrt{\frac{\pi}{p}}$ Poisson, P. 19. 404. N°. 51.

$$\begin{aligned}
4) \int \frac{\text{Cos.} \left\{ x \sqrt{l \frac{1}{q}} \right\} - q \text{Cos.} \left\{ 3x \sqrt{l \frac{1}{q}} \right\}}{1 - 2q \text{Cos.} \left\{ 2x \sqrt{l \frac{1}{q}} \right\} + q^2} e^{-x^2} dx &= \frac{1}{2} \sqrt{\pi} \sqrt[4]{q} \sum_0^{\infty} q^{n^2} \\
5) &= \frac{1}{4} \sqrt{\pi} \sqrt[4]{q} \left[1 + \sqrt{\frac{2}{\pi}} \text{F}'(p) \right] \\
6) \int \frac{\text{Cos.} \left\{ x \sqrt{l \frac{1}{q}} \right\} + q \text{Cos.} \left\{ 3x \sqrt{l \frac{1}{q}} \right\}}{1 + 2q \text{Cos.} \left\{ 2x \sqrt{l \frac{1}{q}} \right\} + q^2} e^{-x^2} dx &= \frac{1}{2} \sqrt{\pi} \sqrt[4]{q} \sum_0^{\infty} (-1)^n q^{n^2} \\
7) &= \frac{1}{4} \sqrt{\pi} \sqrt[4]{q} \left[1 + \sqrt{\frac{2}{\pi}} \sqrt{(1-p^2)} \text{F}'(p) \right] \quad \text{Kummer, Cr. 17. 210.} \\
8) \int \frac{1 - q \text{Cos.} \left\{ 2x \sqrt{l \frac{1}{q}} \right\}}{1 - 2q \text{Cos.} \left\{ 2x \sqrt{l \frac{1}{q}} \right\} + q^2} e^{-x^2} dx &= \frac{1}{2} \sqrt{\pi} \sqrt[4]{q} \sum_0^{\infty} q^{\left(\frac{2n+1}{2}\right)^2} \\
9) &= \frac{1}{2} \sqrt{\pi} \sqrt[4]{q} \sqrt{\frac{p}{2\pi}} \text{F}'(p) \quad \left. \begin{array}{l} \text{, où partout on} \\ \text{trouve } p \text{ par} \\ \text{l'équation} \\ \frac{1}{\pi} \text{F}'(p) = \\ \frac{q}{\pi} \sqrt{(1-p^2)}. \end{array} \right\} \\
10) \int \frac{1 + q \text{Cos.} \left\{ 2x \sqrt{l \frac{1}{q}} \right\}}{1 + 2q \text{Cos.} \left\{ 2x \sqrt{l \frac{1}{q}} \right\} + q^2} e^{-x^2} dx &= \frac{1}{2} \sqrt{\pi} \sqrt[4]{q} \sum_0^{\infty} (-1)^n q^{\left(\frac{2n+1}{2}\right)^2} \\
11) \int \frac{\text{Cos.} \left\{ 2ax \sqrt{l \frac{1}{q}} \right\} - r \text{Cos.} \left\{ 2(a+1)x \sqrt{l \frac{1}{q}} \right\}}{1 - 2r \text{Cos.} \left\{ 2x \sqrt{l \frac{1}{q}} \right\} + r^2} e^{-x^2} dx &= \frac{1}{2} q^{a^2} \sqrt{\pi} \sum_0^{\infty} r^n q^{n^2 - 2an}, r^2 < 1; \\
12) \int \frac{\text{Cos.} \left\{ 2(a-1)x \sqrt{l \frac{1}{q}} \right\} - r \text{Cos.} \left\{ 2(a+1)x \sqrt{l \frac{1}{q}} \right\}}{1 - 2r \text{Cos.} \left\{ 2x \sqrt{l \frac{1}{q}} \right\} + r^2} e^{-x^2} dx &= \frac{1}{2} q^{a^2} \sqrt{\pi} \sum_0^{\infty} r^n q^{(2n+1)^2 - 2a(2n+1)}, r^2 < 1;
\end{aligned}$$

$$\begin{aligned}
1) \int \frac{\text{Cos. } qx}{e^x + e^{-x} + 2 \text{Cos. } p} dx &= \frac{1}{2} \pi \text{Cosec. } p \frac{e^{pq} - e^{-pq}}{e^{q\pi} - e^{-q\pi}}, p \leq \pi; \text{Poisson, P. 18. 295. N}^\circ. 26. - \\
&\quad \text{Schellbach, Cr. 48. 107.} \\
2) \int \frac{\text{Cos. } qx}{e^x + e^{-x} + e^p + e^{-p}} dx &= \frac{2\pi}{e^p - e^{-p}} \frac{\text{Sin. } pq}{e^{q\pi} - e^{-q\pi}}, p \leq \pi; \text{Poisson, P. 18. 295. N}^\circ. 26.
\end{aligned}$$

- 3) $\int \frac{e^x - e^{-x}}{e^x + e^{-x} - 2 \cos p} \sin qx \, dx = \pi \frac{e^{pq} + e^{2q\pi - pq}}{e^{2q\pi} - 1}$ Plana, Mém. Turin. 1818. 7. IV. 22.
- 4) $\int \frac{e^x + e^{-x}}{e^x + e^{-x} + 2 \cos p} \cos qx \, dx = -\pi \cot p \frac{e^{pq} - e^{-pq}}{e^{q\pi} - e^{-q\pi}}$
- 5) $\int \frac{e^x - e^{-x}}{e^x + e^{-x} + 2 \cos p} \sin qx \, dx = \pi \frac{e^{pq} + e^{-pq}}{e^{q\pi} - e^{-q\pi}}$ } $p \leq \pi$;
Poisson, P. 18. 295. N°. 26.
- 6) $\int \frac{e^{px} - e^{-px}}{e^{2px} + e^{-2px} + 2 \cos 2qx} \sin qx \, dx = \frac{q}{p^2 + q^2} \frac{\pi}{4}$
- 7) $\int \frac{e^{px} + e^{-px}}{e^{2px} + e^{-2px} + 2 \cos 2qx} \cos qx \, dx = \frac{p}{p^2 + q^2} \frac{\pi}{4}$ } Raabe, Int. 144. — Ohm, Ausw. 10.
- 8) $\int \frac{\sin \{(2k+1)x\}}{\sin x} e^{-px} \, dx = \frac{\pi}{2} \frac{1 + e^{-p\pi}}{1 - e^{-p\pi}}, k = \infty$; Schlömilch, Beitr. I. § 4.
- 9) $\int \frac{\cos \{(2k+1)x\}}{\cos x} e^{-px} \, dx = (-1)^k \pi \frac{e^{-4p\pi}}{1 - e^{-p\pi}}, k = \infty$; Raabe, Int. 180.
- 10) $\int \frac{\sin \{(2a+1)x\}}{\sin x} e^{-2px} \, dx = \frac{1}{2p} + \sum_1^a \frac{p}{a^2 + p^2}$ Schlömilch, Beitr. I. § 4.
- 11) $\int \frac{\cos \{(2a+1)x\}}{\sin x} e^{-px} \sin x \, dx = -\frac{2a+1}{p^2 + (2a+1)^2} + 2 \sum_0^a (-1)^n \frac{2n+1}{p^2 + (2n+1)^2}$
- 12) $= \frac{\pi e^{-4p\pi}}{1 + e^{-p\pi}}, a = \infty$; } Schlömilch, Beitr. I. § 6.
- 13) $\int \frac{\sin \{(2b+1)x\}}{\sin x} e^{-a^2 x^2} \, dx = \frac{\sqrt{\pi}}{a} \left\{ \frac{1}{2} + \sum_1^b e^{-\left(\frac{n}{a}\right)^2} \right\}$
- 14) $\int \frac{\cos \{(4b+1)x\}}{\cos x} e^{-a^2 x^2} \, dx = \frac{\sqrt{\pi}}{a} \left\{ \frac{1}{2} + \sum_1^{2b} (-1)^n e^{-\left(\frac{n}{a}\right)^2} \right\}$ } Schlömilch, Stud. II. 3, 4.
- 15) $\int \frac{\sin qx - p \sin \{(q-r)x\}}{1 - 2p \cos rx + p^2} \frac{dx}{e^{\pi x} - e^{-\pi x}} = \frac{1}{4(1-p)} - \frac{1}{2} \sum_0^\infty \frac{p^n}{1 + e^{q+n\pi}}$
- 16) $\int \frac{\sin qx - p \sin \{(q-r)x\}}{1 - 2p \cos rx + p^2} \frac{dx}{e^{2\pi x} - 1} = \frac{1}{4(1-p)} - \frac{1}{2} \sum_0^\infty \frac{p^n}{n r + q} - \frac{1}{2} \sum_0^\infty \frac{p^n}{1 - e^{q+n\pi}}$
- 17) $\int \frac{\sin qx - p \sin \{(q-r)x\}}{1 - 2p \cos rx + p^2} \frac{e^{\lambda x} + e^{-\lambda x}}{e^{\pi x} - e^{-\pi x}} \, dx = \frac{1}{2(1-p)} - \sum_0^\infty \frac{1 + e^{q+n\pi} \cos \lambda}{1 + 2e^{q+n\pi} \cos \lambda + e^{2q+2n\pi}} p^n$
- 18) $\int \frac{\cos qx - p \cos \{(q-r)x\}}{1 - 2p \cos rx + p^2} \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\pi x} - e^{-\pi x}} \, dx = \sum_0^\infty \frac{e^{q+n\pi} p^n \sin \lambda}{1 + 2e^{q+n\pi} \cos \lambda + e^{2q+2n\pi}}$ } $\lambda < \pi$;
Poisson, P. 20. 222.
où 15
était
fautive;

F. Exp. $e^{\pm ax}$ ou $e^{\pm ax^2}$. } d'autre forme. TABLE 284 suite.

Lim. 0 et ∞ .

$$\begin{aligned}
 19) & \int \frac{1 - p \cos. rx}{1 - 2p \cos. rx + p^2} \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\pi x} - e^{-\pi x}} dx = \sum_0^{\infty} \frac{p^n \sin. \lambda}{e^{nr} + 2 \cos. \lambda + e^{-nr}} \\
 20) & \int \frac{e^{\lambda x} - e^{-\lambda x}}{1 - 2p \cos. rx + p^2} \frac{dx}{e^{\pi x} - e^{-\pi x}} = \frac{-1}{2(1-p^2)} \text{Tang.} \frac{1}{2} \lambda + \frac{2}{1-p^2} \sum_0^{\infty} \frac{p^n \sin. \lambda}{e^{nr} + 2 \cos. \lambda + e^{-nr}} \\
 21) & \int \frac{e^{\lambda x} - e^{-\lambda x}}{1 - 2p \cos. 2rx + p^2} \frac{\cos. rx}{e^{\pi x} - e^{-\pi x}} dx = \frac{1}{1-p^2} \sum_0^{\infty} \frac{p^n \sin. \lambda}{e^{(2n+1)r} + 2 \cos. \lambda + e^{-(2n+1)r}} \\
 22) & \int \frac{(e^{\lambda x} + e^{-\lambda x}) \sin. rx \sin. \lambda - (e^{\lambda x} - e^{-\lambda x})(e^r - \cos. rx) \cos. \lambda}{e^r - 2 \cos. rx + e^{-r}} \frac{dx}{e^{\pi x} - e^{-\pi x}} = \frac{\sin. \lambda}{2(e^r - 1)} + \sum_0^{\infty} \frac{\sin. \lambda}{e^{nr} + 2 \cos. \lambda + e^{-nr}} \\
 23) & \int \frac{(e^{\lambda x} + e^{-\lambda x}) \sin. rx \sin. \lambda + (e^{\lambda x} - e^{-\lambda x})(e^r + \cos. rx) \cos. \lambda}{e^r + 2 \cos. rx + e^{-r}} \frac{dx}{e^{\pi x} - e^{-\pi x}} = \frac{\sin. \lambda}{2(e^r + 1)} - \sum_0^{\infty} \frac{(-1)^n \sin. \lambda}{e^{nr} + 2 \cos. \lambda + e^{-nr}}
 \end{aligned}$$

$\lambda < \pi$;
 Poisson,
 P. 20,
 222; où
 21 était
 fautive.

F. Expon. d'autre forme.
Circ. Dir.

TABLE 285.

Lim. 0 et ∞ .

$$\begin{aligned}
 1) & \int e^{-\sqrt{2}qx} \sin. x dx = \frac{\sin. \frac{1}{2} q - \cos. \frac{1}{2} q}{2} \sqrt{q} \pi + \sum_0^{\infty} (-1)^n \frac{(2q)^{2n}}{(2n+1)^{2n+1}} \\
 2) & \int e^{-\sqrt{2}qx} \cos. x dx = \frac{\sin. \frac{1}{2} q + \cos. \frac{1}{2} q}{2} \sqrt{q} \pi - \sum_0^{\infty} (-1)^n \frac{(2q)^{2n+1}}{(2n+2)^{2n+1}} \\
 3) & \int e^{-\frac{p^2}{x^2}} \sin. (2q^2 x^2) dx = e^{-2pq} \sqrt{\pi} \frac{\sin. 2pq + \cos. 2pq}{4q} \\
 4) & \int e^{-\frac{p^2}{x^2}} \cos. (2q^2 x^2) dx = e^{-2pq} \sqrt{\pi} \frac{\sin. 2pq - \cos. 2pq}{4q} \\
 5) & \int e^{-x^2 - \frac{pr^2}{x^2(p^2+q^2)}} \sin. \left\{ \frac{p^2 q}{x^2(p^2+q^2)} \right\} dx = \frac{1}{2} \sqrt{\pi} e^{-2ap} \sin. 2bp \\
 6) & \int e^{-x^2 - \frac{pr^2}{x^2(p^2+q^2)}} \cos. \left\{ \frac{p^2 q}{x^2(p^2+q^2)} \right\} dx = \frac{1}{2} \sqrt{\pi} e^{-2ap} \cos. 2bp \\
 7) & \int e^{-px^2 - \frac{q^2}{x^2}} \sin. (rx^2) dx = \frac{1}{2} e^{-2aq} \sqrt{\frac{\pi}{p^2+r^2}} (b \cos. 2bq + a \sin. 2bq) \\
 8) & = \frac{1}{2} \sqrt{\frac{\pi}{p}} e^{-2aq} \sin. \left(2bq + \frac{1}{2} \varphi \right) \sqrt{\cos. \varphi} \\
 9) & \int e^{-px^2 - \frac{q^2}{x^2}} \cos. (rx^2) dx = \frac{1}{2} e^{-2aq} \sqrt{\frac{\pi}{p^2+r^2}} (a \cos. 2bq - b \sin. 2bq) \\
 10) & = \frac{1}{2} \sqrt{\frac{\pi}{p}} e^{-2aq} \cos. \left(2bq + \frac{1}{2} \varphi \right) \sqrt{\cos. \varphi}
 \end{aligned}$$

Cauchy, Sav. Etr.
 1827. 124. Note 3.

 Helmling, Transf.
 31-33, 65*, 75, 76.
 , où partout
 $a = \sqrt{\frac{p+1}{2} (p^2+r^2)}$,
 $b = \sqrt{\frac{-p+1}{2} (p^2+r^2)}$.
 $\text{Tang. } \varphi = \frac{r}{p}$.

$$\begin{aligned}
 11) \int e^{-p^2 x^2 \cos. 2\lambda - \frac{q^2}{4x^2}} \sin. (p^2 x^2 \sin. 2\lambda) dx &= \frac{\sqrt{\pi}}{2p} e^{-pq \cos. \lambda} \sin. (\lambda + pq \sin. \lambda) \\
 12) \int e^{-p^2 x^2 \cos. 2\lambda - \frac{q^2}{4x^2}} \cos. (p^2 x^2 \sin. 2\lambda) dx &= \frac{\sqrt{\pi}}{2p} e^{-pq \cos. \lambda} \cos. (\lambda + pq \sin. \lambda) \\
 13) \int e^{-p \frac{1+x^4}{x^2} - \frac{q^2 x^2}{(1-x^2)^2}} \sin. \left(r \frac{1+x^4}{x^2} \right) dx &= \frac{1}{2} \sqrt{\frac{\pi \cos. \varphi}{p}} \cdot e^{-2(aq+p)} \sin. \left\{ \frac{1}{2}(bq+r) + \frac{1}{2} \varphi \right\} \\
 14) \int e^{-p \frac{1+x^4}{x^2} - \frac{q^2 x^2}{(1-x^2)^2}} \cos. \left(r \frac{1+x^4}{x^2} \right) dx &= \frac{1}{2} \sqrt{\frac{\pi \cos. \varphi}{p}} \cdot e^{-2(aq+p)} \cos. \left\{ \frac{1}{2}(bq+r) + \frac{1}{2} \varphi \right\} \\
 15) \int e^{-p \left(x^2 + \frac{1}{x^2} \right)} \sin. \left\{ q \left(x^2 + \frac{1}{x^2} \right) \right\} dx &= \frac{1}{2} \sqrt{\frac{\pi \cos. 2\varphi}{p}} \cdot e^{-2p} \sin. (\varphi + 2 \text{Tang. } 2\varphi) \\
 16) \int e^{-p \left(x^2 + \frac{1}{x^2} \right)} \cos. \left\{ q \left(x^2 + \frac{1}{x^2} \right) \right\} dx &= \frac{1}{2} \sqrt{\frac{\pi \cos. 2\varphi}{p}} \cdot e^{-2p} \cos. (\varphi + 2 \text{Tang. } 2\varphi) \\
 17) \int \{ e^{-x} \cos. (2p \sqrt{x}) - 4p e^{-x^2} \sin. 2px \} dx &= 1 \quad \text{Helmmling, Transf. 16.} \\
 18) \int e^{-\frac{(x+bi)^{2a} + (x-bi)^{2a}}{2}} \cos. \left\{ \frac{(x+bi)^{2a} - (x-bi)^{2a}}{2} \right\} dx &= \frac{1}{2a} \Gamma \left(\frac{1}{2a} \right) \\
 19) \int e^{-\left(px^2 + \frac{q}{x^2} \right)} \sin. \left(rx^2 + \frac{s}{x^2} \right) dx &= \frac{\sqrt{\pi}}{2a} e^{-c} \sin. \varphi \\
 20) \int e^{-\left(px^2 + \frac{q}{x^2} \right)} \cos. \left(rx^2 + \frac{s}{x^2} \right) dx &= \frac{\sqrt{\pi}}{2a} e^{-c} \cos. \varphi \\
 21) \int e^{-\left(px^2 + \frac{q}{x^2} \right)} \sin. \left(rx^2 - \frac{s}{x^2} \right) dx &= \frac{\sqrt{\pi}}{2a} e^{-f} \sin. \psi \\
 22) \int e^{-\left(px^2 + \frac{q}{x^2} \right)} \cos. \left(rx^2 - \frac{s}{x^2} \right) dx &= \frac{\sqrt{\pi}}{2a} e^{-f} \cos. \psi \\
 23) \int e^{-\left(px^2 + \frac{q}{x^2} \right)} \sin. (rx^2) \cdot \sin. \left(\frac{s}{x^2} \right) dx &= \frac{\sqrt{\pi}}{4a} (e^{-f} \cos. \psi - e^{-c} \cos. \varphi) \\
 24) \int e^{-\left(px^2 + \frac{q}{x^2} \right)} \sin. (rx^2) \cdot \cos. \left(\frac{s}{x^2} \right) dx &= \frac{\sqrt{\pi}}{4a} (e^{-c} \sin. \varphi + e^{-f} \sin. \psi) \\
 25) \int e^{-\left(px^2 + \frac{q}{x^2} \right)} \cos. (rx^2) \cdot \sin. \left(\frac{s}{x^2} \right) dx &= \frac{\sqrt{\pi}}{4a} (e^{-c} \sin. \varphi - e^{-f} \sin. \psi) \\
 26) \int e^{-\left(px^2 + \frac{q}{x^2} \right)} \cos. (rx^2) \cdot \cos. \left(\frac{s}{x^2} \right) dx &= \frac{\sqrt{\pi}}{4a} (e^{-c} \cos. \varphi + e^{-f} \cos. \psi)
 \end{aligned}$$

Helmmling, Transf. 31—38, 65*, 75, 76., où partout
 $a = \sqrt{\frac{p + \sqrt{p^2 + r^2}}{2}}$,
 $b = \sqrt{\frac{-p + \sqrt{p^2 + r^2}}{2}}$,
 $\text{Tang. } \varphi = \frac{r}{p}$.
 Helmmling, Transf. II. S. 64.
 Cauchy, Sav. Etr. 1827. 599. P. 1. § 2. d'après T. 36. N°. 12.
 Helmmling, Transf. II. S. 87, 88.
 où
 $a^4 = p^2 + r^2$,
 $b^4 = q^2 + s^2$,
 $\alpha = \frac{1}{2} \text{Arctang. } \frac{r}{p}$,
 $\beta = \frac{1}{2} \text{Arctang. } \frac{s}{q}$,
 $e = 2ab \cos. (\alpha + \beta)$,
 $f = 2ab \cos. (\alpha - \beta)$,
 $\varphi = 2ab \sin. (\alpha + \beta) + \alpha$,
 $\psi = 2ab \sin. (\alpha - \beta) + \alpha$.

$$27) \int e^{-px^2} (e^{2qx} + e^{-2qx}) \text{Sin.}(rx^2) dx = \frac{\sqrt{\pi}}{a} e^{-\frac{q^2}{a^2} \text{Cos.} 2\alpha} \text{Sin.} \left(\frac{q^2}{a^2} \text{Sin.} 2\alpha \right) \left. \begin{array}{l} \text{Helmling, Transf. II.} \\ \text{S. 68, 69.} \end{array} \right\}$$

$$28) \int e^{-px^2} (e^{2qx} + e^{-2qx}) \text{Cos.}(rx^2) dx = \frac{\sqrt{\pi}}{a} e^{-\frac{q^2}{a^2} \text{Cos.} 2\alpha} \text{Cos.} \left(\frac{q^2}{a^2} \text{Sin.} 2\alpha \right) \left. \begin{array}{l} \text{Helmling, Transf. II.} \\ \text{S. 68, 69.} \end{array} \right\}$$

, où a et α ont les mêmes valeurs qu'auparavant.

$$29) \int e^{-px^2} \{e^{2qx} \text{Sin.}(rx^2 - 2sx) + e^{-2qx} \text{Sin.}(rx^2 + 2sx)\} dx = \frac{\sqrt{\pi}}{a} e^b \text{Sin.} q \left. \begin{array}{l} \text{Helmling, Transf. II.} \\ \text{S. 65.} \end{array} \right\}$$

$$30) \int e^{-px^2} \{e^{2qx} \text{Cos.}(rx^2 - 2sx) + e^{-2qx} \text{Cos.}(rx^2 + 2sx)\} dx = \frac{\sqrt{\pi}}{a} e^b \text{Cos.} q \left. \begin{array}{l} \text{Helmling, Transf. II.} \\ \text{S. 65.} \end{array} \right\}$$

$$, \text{ où } a^2 = p^2 + r^2, b = \frac{q^2 + s^2}{\sqrt{(p^2 + r^2)}} \text{Cos.} \left\{ \text{Arctang.} \frac{r}{p} - 2 \text{Arctang.} \frac{s}{q} \right\},$$

$$q = \frac{q^2 + s^2}{\sqrt{(p^2 + r^2)}} \text{Sin.} \left\{ \text{Arctang.} \frac{r}{p} - 2 \text{Arctang.} \frac{s}{q} \right\} + \text{Arctang.} \frac{s}{q}.$$

$$1) \int e^{-p\sqrt{x^2}} \text{Cos.} qx \text{Cos.} rx dx = \frac{p}{p^2 + (r-q)^2} + \frac{p}{p^2 + (r+q)^2} \quad \text{Cauchy, P. 19. 511.}$$

$$2) \int e^{-q^2 x^2} \text{Cos.} px dx = \frac{1}{q} e^{-\frac{p^2}{4q^2}} \sqrt{\pi} \quad \text{Cauchy, Exerc. 1827. p. 233.}$$

$$3) \int e^{-q^2 x^2} \text{Sin.} px dx = 0 \quad \text{Cauchy, Exerc. 1827. p. 233. — Lobatto, Int. 68.}$$

$$4) \int e^{-q^2 x^2} \text{Sin.}\{p(x+\lambda)\} dx = \frac{\sqrt{\pi}}{q} e^{-\frac{p^2}{4q^2}} \text{Sin.} p\lambda \left. \begin{array}{l} \\ \end{array} \right\}$$

$$5) \int e^{-q^2 x^2} \text{Cos.}\{p(x+\lambda)\} dx = \frac{\sqrt{\pi}}{q} e^{-\frac{p^2}{4q^2}} \text{Cos.} p\lambda \left. \begin{array}{l} \\ \end{array} \right\} \quad \text{Lobatto, Int. 68. où elles sont fautives.}$$

$$6) \int e^{-x^2} \text{Cos.} 2px dx = e^{-p^2} \sqrt{\pi} \left. \begin{array}{l} \\ \end{array} \right\}$$

$$7) \int e^{-qx^2} \text{Cos.} px dx = e^{-\frac{p^2}{4q}} \sqrt{\frac{\pi}{q}} \left. \begin{array}{l} \\ \end{array} \right\} \quad \text{Fourier, Chal. 375. — Cauchy, P. 19. 511.}$$

$$8) \int e^{-q^2(x^2 - 2\lambda x)} \text{Sin.} px dx = \frac{\sqrt{\pi}}{q} e^{-\frac{p^2}{4q^2} + q^2 \lambda^2} \text{Sin.} p\lambda \left. \begin{array}{l} \\ \end{array} \right\}$$

$$9) \int e^{-q^2(x^2 - 2\lambda x)} \text{Cos.} px dx = \frac{\sqrt{\pi}}{q} e^{-\frac{p^2}{4q^2} + q^2 \lambda^2} \text{Cos.} p\lambda \left. \begin{array}{l} \\ \end{array} \right\} \quad \text{V. T. 286. N°. 4, 5.}$$

F. Exp. Circ. Dir. ent.	TABLE 286 suite.	Lim. — ∞ et ∞ .
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$$10) \int e^{px^2 i} \text{Cos. } a x dx = (1+i) e^{-\frac{a^2 i}{4p}} \sqrt{\frac{\pi}{2p}} \quad \text{Cauchy, P. 19. 511. — Schaar, Mém. Brux. T. 25.}$$

$$11) \int e^{-(px^2 \text{Cos. } \lambda + 2qx \text{Cos. } \mu + r \text{Cos. } \nu)} \text{Sin. } (px^2 \text{Sin. } \lambda + 2qx \text{Sin. } \mu + r \text{Sin. } \nu) dx = \\ = e^{-\frac{q^2}{p} \text{Cos. } (\lambda - 2\mu) - r \text{Cos. } \nu} \text{Sin. } \left\{ \frac{1}{2} \lambda + \frac{q^2}{p} \text{Sin. } (\lambda - 2\mu) + r \text{Sin. } \nu \right\} \sqrt{\frac{\pi}{p}}$$

$$12) \int e^{-(px^2 \text{Cos. } \lambda + 2qx \text{Cos. } \mu + r \text{Cos. } \nu)} \text{Cos. } (px^2 \text{Sin. } \lambda + 2qx \text{Sin. } \mu + r \text{Sin. } \nu) dx = \\ = e^{-\frac{q^2}{p} \text{Cos. } (\lambda - 2\mu) - r \text{Cos. } \nu} \text{Cos. } \left\{ \frac{1}{2} \lambda + \frac{q^2}{p} \text{Sin. } (\lambda - 2\mu) + r \text{Sin. } \nu \right\} \sqrt{\frac{\pi}{p}}$$

$$13) \int e^{-(px^2 + qx + r)} \text{Sin. } (sx^2 + tx + u) dx = e^{-r + \frac{p(q^2 - t^2) + 2qst}{4(p^2 - s^2)}} \text{Sin. } \left\{ u + \frac{(q^2 - t^2)s - 2pqt}{4(p^2 + s^2)} + \frac{1}{2} \text{Arctang. } \frac{s}{p} \right\} \sqrt{\frac{\pi}{p^2 + s^2}}$$

$$14) \int e^{-(px^2 + qx + r)} \text{Cos. } (sx^2 + tx + u) dx = e^{-r + \frac{p(q^2 - t^2) + 2qst}{4(p^2 - s^2)}} \text{Cos. } \left\{ u + \frac{(q^2 - t^2)s - 2pqt}{4(p^2 + s^2)} + \frac{1}{2} \text{Arctang. } \frac{s}{p} \right\} \sqrt{\frac{\pi}{p^2 + s^2}}$$

$$15) \int e^{-px^2 \text{Cos. } \lambda} \text{Sin. } (px^2 \text{Sin. } \lambda) dx = \text{Sin. } \frac{1}{2} \lambda \sqrt{\frac{\pi}{p}}$$

$$16) \int e^{-px^2 \text{Cos. } \lambda} \text{Cos. } (px^2 \text{Sin. } \lambda) dx = \text{Cos. } \frac{1}{2} \lambda \sqrt{\frac{\pi}{p}}$$

Sur form. 11) à 16) voyez: Cauchy, Exerc. 1827. p. 233.

F. Exp. $e^{\pm ax}$. Circ. Dir.	TABLE 287.	Lim. 0 et $\frac{\pi}{2}$.
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$$1) \int e^{(q+1)xi} \text{Sin. } q^{-1} x dx = \frac{1}{q} e^{iq\pi i} \quad \text{Kummer, Cr. 20. 1}$$

$$2) \int e^{(p+q)xi} \text{Sin. } q^{-1} x \text{Cos. } p^{-1} x dx = e^{iq\pi i} B(p, q) \quad \text{Serret, L. 3. 1.}$$

$$3) \int e^{2x} \text{Sin. }^2 x dx = \frac{1}{8} (3e^\pi - 1) \quad \text{Rogner, Mat.}$$

$$4) \int (e^{2qx} + e^{-2qx}) \text{Cos. }^2 b x dx = \frac{\pi}{2^{2b+1}} \frac{1^{2b/1}}{\Gamma(b+qi+1) \Gamma(b-qi+1)} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211. où elle est fautive.}$$

F. Exp. à exp. de Circ. Dir.
Circ. Dir. ent.

TABLE 288.

Lim. 0 et $\frac{\pi}{2}$.

$$1) \int e^{-a \sin x} \sin 2x dx = \frac{2}{a^2} \{ (a-1) e^a + 1 \} \quad \text{V. T. 112. N}^\circ. 1.$$

$$2) \int e^{-\sin^2 x} \sin 2x dx = 1 - \frac{1}{e} \quad \text{V. T. 112. N}^\circ. 2.$$

$$3) \int e^{-\cos x} \tan x dx = \infty \quad \text{V. T. 112. N}^\circ. 4.$$

$$4) \int e^{-q \tan x} dx = \text{Ci.}(q) \cdot \sin q + \cos q \left\{ \frac{\pi}{2} - \text{Si.}(q) \right\} \quad \text{V. T. 130. N}^\circ. 3.$$

$$5) \int e^{-q \tan x} \tan x dx = -\text{Ci.}(q) \cdot \cos q + \sin q \left\{ \frac{\pi}{2} - \text{Si.}(q) \right\} \quad \text{V. T. 130. N}^\circ. 5.$$

$$\left. \begin{aligned} 6) \int (e^{q \sin x} - e^{-q \sin x}) \sin(q \cos x) \sin 2ax dx &= \frac{1}{2} \pi \frac{(-1)^{a-1} q^{2a}}{1^{2a/1}} \\ 7) \int (e^{q \sin x} - e^{-q \sin x}) \cos(q \cos x) \sin \{(2a-1)x\} dx &= \frac{1}{2} \pi \frac{(-1)^{a-1} q^{2a-1}}{1^{2a-1/1}} \\ 8) \int (e^{q \sin x} + e^{-q \sin x}) \sin(q \cos x) \cos \{(2a-1)x\} dx &= \frac{1}{2} \pi \frac{(-1)^{a-1} q^{2a-1}}{1^{2a-1/1}} \\ 9) \int (e^{q \sin x} + e^{-q \sin x}) \cos(q \cos x) \cos 2ax dx &= \frac{1}{2} \pi \frac{(-1)^a q^{2a}}{1^{2a/1}} \end{aligned} \right\} \quad \text{Poisson, P. 19. 404. N}^\circ. 77.$$

F. Exp. à exp. de Circ. Dir.
Circ. Dir. en den. $\sin 2x$.

TABLE 289.

Lim. 0 et $\frac{\pi}{2}$.

$$1) \int e^{-p \tan x} \frac{dx}{\sin 2x} = \infty \quad \text{V. T. 126. N}^\circ. 2.$$

$$2) \int e^{-\tan x} \frac{\tan^p x}{\sin 2x} dx = \frac{1}{2} \Gamma(p), \quad \infty > p > -1; \quad \text{V. T. 113. N}^\circ. 3.$$

$$3) \int e^{-q \tan x} \frac{\tan^p x}{\sin 2x} dx = \frac{1}{2 q^p} \Gamma(p) \quad \text{V. T. 113. N}^\circ. 5.$$

$$4) \int e^{-p \tan^2 x} \frac{\tan^{2a} x}{\sin 2x} dx = \frac{1}{2^{a+1} p^a} 1^{a-1/1} \quad \text{V. T. 114. N}^\circ. 9.$$

$$5) \int e^{-p \tan^2 x} \frac{\tan^{2a+1} x}{\sin 2x} dx = \frac{1}{4} \frac{1^{a/2}}{(2p)^a} \sqrt{\frac{\pi}{p}} \quad \text{V. T. 114. N}^\circ. 8.$$

$$6) \int e^{-\tan^p x} \frac{\tan^p x}{\sin 2x} dx = \frac{1}{2p} \quad \text{V. T. 115. N}^\circ. 3.$$

- 7) $\int e^{-Tang.^{2p}x} \frac{Tang.^p x}{Sin.^2 x} dx = \frac{1}{4p} \sqrt{\pi} \quad \text{V. T. 115. N}^\circ. 5.$
- 8) $\int e^{-q Tang.^{ap}x} \frac{Tang.^{ap} x}{Sin.^2 x} dx = \frac{1}{2 a q^p} \Gamma(p) \quad \text{V. T. 115. N}^\circ. 9.$
- 9) $\int e^{-Tang.^{ac}x} \frac{Tang.^{ab} x}{Sin.^2 x} dx = \frac{1}{2 a b} 1^{\frac{b}{c}/1} \quad \text{V. T. 115. N}^\circ. 8.$
- 10) $\int e^{-(Tang.^{.x}+q Cot.^{.x})} \frac{Tang.^p x}{Sin.^2 x} dx = \frac{1}{2} \Gamma(p) \psi(1-p, q) + \frac{1}{2} \Gamma(-p) q^p \psi(1+p, q) \quad \text{V. T. 116. N}^\circ. 3.$
- 11) $\int e^{-q(Tang.^{2x}+Cot.^{2x})} \frac{Tang.^{2a+1} x}{Sin.^2 x} dx = \frac{1}{4} e^{-2q} \sqrt{\frac{\pi}{q}} \sum_{n=0}^a \frac{1}{(2q)^n} \frac{(a-n+1)^{2n,1}}{2^n 1^{n,1}} \quad \text{V. T. 116. N}^\circ. 8.$
- 12) $\int \frac{e^{-Tang.^{.x} - Cos.^2 x}}{Sin.^2 x} dx = -\frac{1}{2} A \quad \text{V. T. 133. N}^\circ. 5.$
- 13) $\int \frac{e^{-Tang.^{.x} - e^{-p} Tang.^{.x}}}{Sin.^2 x} dx = \frac{1}{2} l p \quad \text{V. T. 127. N}^\circ. 3.$
- 14) $\int \frac{e^{-p Tang.^{.x} - e^{-q} Tang.^{.x}}}{Sin.^2 x} dx = \frac{1}{2} l \frac{q}{p} \quad \text{V. T. 127. N}^\circ. 4.$
- 15) $\int e^{-Tang.^{2x}} \frac{dx}{Sin.^2 2x} = \frac{3}{8} \sqrt{\pi} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{V. T. 290. N}^\circ. 2, 3.$
- 16) $\int e^{-Tang.^{2x}} \frac{Cos.^2 x}{Sin.^2 2x} dx = \frac{1}{8} \sqrt{\pi} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$

- 1) $\int e^{-Cot.^2 x} \frac{dx}{Sin.^2 x} = \frac{1}{2} \sqrt{\pi} \quad \text{V. T. 36. N}^\circ. 7.$
- 2) $\int e^{-Tang.^2 x} \frac{dx}{Sin.^2 x} = \sqrt{\pi} \quad \text{V. T. 126. N}^\circ. 3.$
- 3) $\int e^{-Tang.^2 x} \frac{dx}{Cos.^2 x} = \frac{1}{2} \sqrt{\pi} \quad \text{V. T. 36. N}^\circ. 7.$
- 4) $\int e^{-Tang.^2 x} \frac{Tang.^{2a} x}{Cos.^2 x} dx = \frac{1^{a/2}}{2^{a+1}} \sqrt{\pi} \quad \text{V. T. 114. N}^\circ. 7.$
- 5) $\int e^{-Cot.^2 x} \frac{dx}{Cos.^2 x} = \sqrt{\pi} \quad \text{V. T. 126. N}^\circ. 3.$

- $$6) \int e^{-Tang.^2 x} \frac{Sin.^2 x}{Cos.^4 x} dx = \frac{1}{4} \vee \pi \quad \text{V. T. 290. N}^\circ. 3.$$
- $$7) \int e^{-p Tang.^2 x} \frac{Sin.^2 x}{Cos.^4 x} dx = \frac{1}{4p} \vee \frac{\pi}{p} \quad \text{V. T. 114. N}^\circ. 11.$$
- $$8) \int e^{-Tang.^2 x} \frac{dx}{Cos.^4 x} = \frac{3}{4} \vee \pi$$
- $$9) \int e^{-Tang.^2 x} \frac{Cos.^2 x}{Cos.^4 x} dx = \frac{1}{4} \vee \pi \quad \left. \vphantom{\int} \right\} \text{V. T. 290. N}^\circ. 3, 6.$$
- $$10) \int e^{-q Cot.x} \frac{dx}{Tang. x} = -Ci.(q).Cos.q + Sin.q \left\{ \frac{\pi}{2} - Si.(q) \right\} \quad \text{V. T. 130. N}^\circ. 5.$$
- $$11) \int e^{-Sin.x} \frac{dx}{Tang. x} = \infty \quad \text{V. T. 112. N}^\circ. 4.$$
- $$12) \int e^{-p Tang.x} \frac{dx}{Cos.^2 x} = \frac{1}{2} \{ e^{-p} Ei.(p) - e^p Ei.(-p) \} \quad \text{V. T. 130. N}^\circ. 10.$$
- $$13) \int e^{-p Tang.x} \frac{Tang. x}{Cos.^2 x} = \frac{1}{2} \{ e^{-p} Ei.(p) + e^p Ei.(-p) \} \quad \text{V. T. 130. N}^\circ. 12.$$

- $$1) \int e^{-p Cot.x} \frac{dx}{Cos.^2 x. Tang. x} = -\frac{1}{2} \{ e^{-p} Ei.(p) + e^p Ei.(-p) \} \quad \text{V. T. 130. N}^\circ. 12.$$
- $$2) \int e^{-Cot.x} \frac{dx}{Sin.^2 x. Tang.^p x} = \frac{1}{2} \Gamma(p), \infty > p > -1 \quad \text{V. T. 113. N}^\circ. 3.$$
- $$3) \int e^{-q Cot.x} \frac{dx}{Sin.^2 x. Tang.^p x} = \frac{1}{2q^p} \Gamma(p) \quad \text{V. T. 113. N}^\circ. 5.$$
- $$4) \int e^{-Cot.^2 x} \frac{dx}{Sin.^2 x. Tang.^{2a} x} = \frac{1}{2} 3^{a-2/1} \quad \text{V. T. 114. N}^\circ. 4.$$
- $$5) \int e^{-p Cot.^2 x} \frac{dx}{Sin.^2 x. Tang.^{2a+1} x} = \frac{1}{4} \frac{1^{a2}}{(2p)^a} \vee \frac{\pi}{p} \quad \text{V. T. 114. N}^\circ. 8.$$
- $$6) \int e^{-p Cot.^2 x} \frac{dx}{Sin.^2 x. Tang.^{2a} x} = \frac{1^{a-11}}{2^{a+1} p^a} \vee \pi \quad \text{V. T. 114. N}^\circ. 9.$$

- 7) $\int e^{-\text{Cot.}^{2p}x} \frac{dx}{\text{Sin. } 2x \cdot \text{Tang.}^p x} = \frac{1}{4p} \sqrt{\pi} \quad \text{V. T. 115. N}^\circ. 5.$
- 8) $\int e^{-\text{Cot.}^q x} \frac{dx}{\text{Tang.}^q x \cdot \text{Sin. } 2x} = \frac{1}{2q} \quad \text{V. T. 115 N}^\circ. 3.$
- 9) $\int e^{-q \text{Cot.}^a x} \frac{dx}{\text{Tang.}^{ap} x \cdot \text{Sin. } 2x} = \frac{1}{2 a q^p} \Gamma(p) \quad \text{V. T. 115. N}^\circ. 9.$
- 10) $\int e^{-\text{Cot.}^{ac} x} \frac{dx}{\text{Tang.}^{ab} x \cdot \text{Sin. } 2x} = \frac{1}{2 a b} 1^{\frac{b}{c}} \quad \text{V. T. 115. N}^\circ. 8.$
- 11) $\int e^{-(q \text{Tang.} x + \text{Cot.} x)} \frac{dx}{\text{Tang.}^p x \cdot \text{Sin. } 2x} = \frac{1}{2} \Gamma(p) \psi(1-p, q) + \frac{1}{2} \Gamma(-p) q^p \psi(1+p, q) \quad \text{V. T. 116. N}^\circ. 3.$
- 12) $\int e^{-q(\text{Tang.}^2 x + \text{Cot.}^2 x)} \frac{dx}{\text{Tang.}^{2a+1} x \cdot \text{Sin. } 2x} = \frac{1}{4} e^{-2q} \sqrt{\frac{\pi}{q}} \sum_{n=0}^a \frac{1}{(2q)^n} \frac{(a-n+1)^{2n/1}}{2^n 1^{n/1}} \quad \text{V. T. 116. N}^\circ. 8.$

- 1) $\int \frac{dx}{e^{i\pi \text{Tang.} x} + e^{-i\pi \text{Tang.} x}} = \frac{1}{2\sqrt{2}} \left\{ \pi - l \sqrt{\frac{2+1}{2-1}} \right\} \quad \text{V. T. 138. N}^\circ. 17.$
- 2) $\int \frac{dx}{e^{i\pi \text{Tang.} x} + e^{-i\pi \text{Tang.} x}} = \frac{1}{2} l 2 \quad \text{V. T. 138. N}^\circ. 14.$
- 3) $\int \frac{dx}{e^{\pi \text{Tang.} x} + e^{-\pi \text{Tang.} x}} = \frac{4-\pi}{4} \quad \text{V. T. 138. N}^\circ. 2.$
- 4) $\int \frac{e^{p \text{Tang.} x} - e^{-p \text{Tang.} x}}{e^{\pi \text{Tang.} x} - e^{-\pi \text{Tang.} x}} dx = -\frac{1}{2} p \text{Cos.} p + \frac{1}{2} \text{Sin.} p l \{2(1 + \text{Cos.} p)\}, \pi > p > 0; \quad \text{V. T. 138. N}^\circ. 4.$
- 5) $\int \frac{\text{Tang.} x}{e^{i\pi \text{Tang.} x} - e^{-i\pi \text{Tang.} x}} dx = \frac{1}{4} \pi \sqrt{2-1} + \frac{1}{4} l 2 \sqrt{\frac{2+1}{2-1}} \quad \text{V. T. 138. N}^\circ. 18.$
- 6) $\int \frac{\text{Tang.} x}{e^{i\pi \text{Tang.} x} - e^{-i\pi \text{Tang.} x}} dx = \frac{\pi-2}{4} \quad \text{V. T. 138. N}^\circ. 16.$
- 7) $\int \frac{e^{p \text{Tang.} x} + e^{-p \text{Tang.} x}}{e^{i\pi \text{Tang.} x} - e^{-i\pi \text{Tang.} x}} \text{Tang.} x dx = -1 + \frac{1}{2} \pi \text{Cos.} p + \frac{1}{2} \text{Sin.} p l \frac{1 + \text{Sin.} p}{1 - \text{Sin.} p}, 0 \leq p \leq \frac{1}{2} \pi, \quad \text{V. T. 138. N}^\circ. 15.$
- 8) $\int \frac{e^{p \text{Tang.} x} - e^{-p \text{Tang.} x}}{e^{i\pi \text{Tang.} x} - e^{-i\pi \text{Tang.} x}} dx = \frac{1}{2} \pi \text{Sin.} p - \frac{1}{2} \text{Cos.} p l \frac{1 + \text{Sin.} p}{1 - \text{Sin.} p}, 0 \leq p \leq \frac{1}{2} \pi, \quad \text{V. T. 138. N}^\circ. 13.$

F. Exp. en dén. polynôme.
Circ. Dir. en num.

TABLE 292 suite.

Lim. 0 et $\frac{\pi}{2}$.

- 9) $\int \frac{e^{p \text{Tang. } x} + e^{-p \text{Tang. } x}}{e^{\pi \text{Tang. } x} - e^{-\pi \text{Tang. } x}} \text{Tang. } x \, dx = \frac{1}{2}(p \text{Sin. } p - 1) + \frac{1}{2} \text{Cos. } pl \{2(1 + \text{Cos. } p)\}, 0 < p < \pi; \text{ V. T. 138. N}^\circ. 6.$
- 10) $\int \frac{\text{Tang. } x}{e^{\pi \text{Tang. } x} - e^{-\pi \text{Tang. } x}} \, dx = \frac{1}{2} \left(-\frac{1}{2} + l 2\right) \text{ V. T. 138. N}^\circ. 12.$
- 11) $\int \frac{e^{(\pi-\lambda) \text{Tang. } x} - e^{(\lambda-\pi) \text{Tang. } x}}{e^{\pi \text{Tang. } x} - e^{-\pi \text{Tang. } x}} \, dx = \sum_1 \frac{\text{Sin. } n \lambda}{n+1}, \lambda^2 < \pi^2; \text{ V. T. 138. N}^\circ. 5.$
- 12) $\int \frac{e^{(\pi-\lambda) \text{Tang. } x} + e^{(\lambda-\pi) \text{Tang. } x}}{e^{\pi \text{Tang. } x} - e^{-\pi \text{Tang. } x}} \text{Tang. } x \, dx = \frac{1}{2} + \sum_1 \frac{\text{Cos. } n \lambda}{n+1}, \lambda^2 \leq \pi^2; \text{ V. T. 138. N}^\circ. 8.$
- 13) $\int \frac{\text{Tang. } x}{e^{2\pi \text{Tang. } x} - 1} \, dx = \frac{1}{2} A - \frac{1}{4} \text{ V. T. 138. N}^\circ. 10.$
- 14) $\int \frac{\text{Tang. } x}{e^{2\pi q \text{Tang. } x} - 1} \, dx = \frac{1}{2} l q + \frac{1}{4 q} - \frac{1}{2} Z'(1+q) \text{ V. T. 138. N}^\circ. 11.$

F. Exp. en dén. polynôme.
Circ. Dir. en dén.

TABLE 295.

Lim. 0 et $\frac{\pi}{2}$.

- 1) $\int \frac{e^{\frac{\pi \text{Cos. } x}{2b}} + e^{-\frac{\pi \text{Cos. } x}{2b}}}{e^{\frac{\pi \text{Cos. } x}{b}} + 2 \text{Cos.} \left(\frac{\pi \text{Sin. } x}{2b}\right) + e^{-\frac{\pi \text{Cos. } x}{b}}} \text{Cos.} \left(\frac{\pi \text{Sin. } x}{2b}\right) \, dx = \frac{1}{2} \pi$
- 2) $\int \frac{e^{\frac{\pi \text{Cos. } x}{2b}} + e^{-\frac{\pi \text{Cos. } x}{2b}}}{e^{\frac{\pi \text{Cos. } x}{b}} + 2 \text{Cos.} \left(\frac{\pi \text{Sin. } x}{b}\right) + e^{-\frac{\pi \text{Cos. } x}{b}}} \text{Cos.} \left(\frac{\pi \text{Sin. } x}{2b}\right) \text{Cos. } 2ax \, dx = \frac{(-1)^a b}{4 \cdot 1^{2a+1}} \left(\frac{\pi}{2b}\right)^{2a+1} B_{2a}$
- 3) $\int \frac{\text{Sin.} \left(\frac{\pi \text{Sin. } x}{b}\right) \text{Sin.} \{(2a-1)x\}}{e^{\frac{\pi \text{Cos. } x}{b}} + 2 \text{Cos.} \left(\frac{\pi \text{Sin. } x}{b}\right) + e^{-\frac{\pi \text{Cos. } x}{b}}} \, dx = \frac{(-1)^{a-1}}{1^{2a-1}} \frac{2^{2a}-1}{S a} b \left(\frac{\pi}{b}\right)^{2a} B_{2a-1}$
- 4) $\int \frac{e^{\frac{\pi \text{Cos. } x}{2b}} - e^{-\frac{\pi \text{Cos. } x}{2b}}}{e^{\frac{\pi \text{Cos. } x}{b}} + 2 \text{Cos.} \left(\frac{\pi \text{Sin. } x}{b}\right) + e^{-\frac{\pi \text{Cos. } x}{b}}} \text{Sin.} \left(\frac{\pi \text{Sin. } x}{2b}\right) \text{Sin. } 2ax \, dx = \frac{(-1)^{a-1} b}{4 \cdot 1^{2a+1}} \left(\frac{\pi}{2b}\right)^{2a+1} B_{2a}$
- 5) $\int \frac{e^{\frac{\pi \text{Cos. } x}{b}} - e^{-\frac{\pi \text{Cos. } x}{b}}}{e^{\frac{\pi \text{Cos. } x}{b}} + 2 \text{Cos.} \left(\frac{\pi \text{Sin. } x}{b}\right) + e^{-\frac{\pi \text{Cos. } x}{b}}} \text{Cos.} \{(2a-1)x\} \, dx = \frac{(-1)^{a-1}}{1^{2a-1}} \frac{2^{2a}-1}{S a} b \left(\frac{\pi}{b}\right)^{2a} B_{2a-1}$

Poisson,
P. 19.
404.
N^o. 77.
d'après
T. 120.
N^o. 14,
18.

- $$6) \int \frac{Tang.^q x}{e^{Tang.x} + 1} \frac{dx}{Sin. 2x} = \frac{1}{2} \Gamma(q) \sum_0^{\infty} \frac{(-1)^n}{(n+1)^q} \quad \text{V. T. 117. N}^{\circ} 16.$$
- $$7) \int \frac{Tang.^q x}{e^{Tang.x} - 1} \frac{dx}{Sin. 2x} = \frac{1}{2} \Gamma(q) \sum_0^{\infty} \frac{1}{(n+1)^q} \quad \text{V. T. 117. N}^{\circ} 17.$$
- $$8) \int \frac{1}{e^{qTang.x} - 1} \frac{Tang. x}{Cos. 2x} dx = \frac{4\pi^4}{q^4} \sum_0^{\infty} (-1)^n \left(\frac{2\pi}{q}\right)^{2n} \frac{B_{2n+1}}{n+1} \quad \text{V. T. 138. N}^{\circ} 21.$$
- $$9) \int \frac{1}{e^{qTang.x} - 1} \frac{Sin. 2x}{Cos.^2 2x} dx = \frac{\pi^2}{q^2} \sum_0^{\infty} (-1)^n \left(\frac{2\pi}{q}\right)^{2n} B_{2n+1} \quad \text{V. T. 138. N}^{\circ} 22.$$
- $$10) \int \frac{1}{e^{qTang.x} - 1} \frac{Sin.^2 x}{Tang. x} dx = \frac{\pi^2}{q^2} \sum_0^{\infty} \left(\frac{2\pi}{q}\right)^{2n} B_{2n+1} \quad \text{V. T. 138. N}^{\circ} 20.$$
- $$11) \int \frac{e^{-pTang.x} - e^{-qTang.x}}{e^{-Tang.x} + 1} \frac{dx}{Sin. 2x} = \frac{1}{2} l \frac{\Gamma\left(\frac{1}{2}p\right) \Gamma\left(\frac{q+1}{2}\right)}{\Gamma\left(\frac{1}{2}q\right) \Gamma\left(\frac{p+1}{2}\right)} \quad \text{V. T. 134. N}^{\circ} 4.$$
- $$12) \int \frac{(e^{qTang.x} - e^{-qTang.x})^2}{e^{Tang.x} + 1} \frac{dx}{Sin. 2x} = -\frac{1}{2} l (q\pi \text{Cot. } q\pi) \quad \text{V. T. 134. N}^{\circ} 8.$$
- $$13) \int \frac{1}{e^{Cot.x} + 1} \frac{dx}{Tang.^q x. Sin. 2x} = \frac{1}{2} \Gamma(q) \sum_0^{\infty} \frac{(-1)^n}{(n+1)^q} \quad \text{V. T. 117. N}^{\circ} 16.$$
- $$14) \int \frac{1}{e^{Cot.x} - 1} \frac{dx}{Tang.^q x. Sin. 2x} = \frac{1}{2} \Gamma(q) \sum_0^{\infty} \frac{1}{(n+1)^q} \quad \text{V. T. 117. N}^{\circ} 17.$$
- $$15) \int \frac{e^{qTang.x} - e^{-qTang.x}}{e^{pTang.x} + e^{-pTang.x}} \frac{dx}{Sin. 2x} = \frac{1}{2} l Tang. \left\{ \frac{p+q}{4p} \pi \right\} \quad \text{V. T. 136. N}^{\circ} 9.$$
- $$16) \int \frac{e^{qTang.x} + e^{-qTang.x} - 2}{e^{\pi Tang.x} - e^{-\pi Tang.x}} \frac{dx}{Sin. 2x} = \frac{1}{2} l Sec. \frac{q}{2} \quad \text{V. T. 136. N}^{\circ} 12.$$
- $$17) \int \frac{(e^{qTang.x} - e^{-qTang.x})^2}{e^{Tang.x} - e^{-Tang.x}} \frac{dx}{Sin. 2x} = \frac{1}{2} l Cos. q\pi \quad \text{V. T. 136. N}^{\circ} 3.$$
- $$18) \int \frac{Tang.^q x}{e^{Tang.x} + e^{-Tang.x} + 2 Cos. \lambda} \frac{dx}{Sin. 2x} = \frac{\Gamma(q)}{2 Sin. \lambda} \sum_1^{\infty} (-1)^{n-1} \frac{Sin. n\lambda}{n^q} \quad \text{V. T. 137. N}^{\circ} 5.$$

1)	$\int \frac{(e^{xi} \cos. x)^p + (e^{-xi} \cos. x)^p}{\cos.^2 x + q^2 \sin.^2 x} dx = \frac{\pi}{q} \left(\frac{q}{q+1} \right)^p$	Serret, L. 8. 489.
2)	$\int \frac{e^{-p \text{Tang.} x}}{\sin. 2 x + q \cos. 2 x + q} dx = -\frac{1}{2} e^{pq} \text{Ei.} (-pq)$	V. T. 129. N°. 3.
3)	$\int \frac{e^{-p \text{Tang.} x}}{\sin. 2 x - q \cos. 2 x - q} dx = -\frac{1}{2} e^{-pq} \text{Ei.} (pq)$	V. T. 129. N°. 9.
4)	$\int \frac{e^{-p \text{Cot.} x}}{\sin. 2 x + q \cos. 2 x - q} dx = -\frac{1}{2} e^{-pq} \text{Ei.} (pq)$	
5)	$\int \frac{e^{-p \text{Cot.} x}}{\sin. 2 x - q \cos. 2 x + q} dx = -\frac{1}{2} e^{pq} \text{Ei.} (-pq)$	V. T. 129. N°. 3.
6)	$\int \frac{e^{-p \text{Tang.} x} \sin. 2 x}{(1-q^2) - 2 q^2 \cos. 2 x - (1+q^2) \cos.^2 2 x} dx = -\frac{1}{4} \{e^{-pq} \text{Ei.}(pq) + e^{pq} \text{Ei.}(-pq)\}$	V. T. 294. N°. 2, 3.
7)	$\int \frac{e^{-p \text{Cot.} x} \sin. 2 x}{(1-q^2) + 2 q^2 \cos. 2 x - (1+q^2) \cos.^2 2 x} dx = -\frac{1}{4} \{e^{-pq} \text{Ei.}(pq) + e^{pq} \text{Ei.}(-pq)\}$	V. T. 294. N°. 4, 5.

1)	$\int e^{-\text{Tang.} x} \frac{dx \surd \sin. 2 x}{\cos.^3 x} = \surd \frac{\pi}{2}$	V. T. 112. N°. 6.
2)	$\int e^{-\text{Cot.} x} \frac{dx \surd \sin. 2 x}{\sin.^3 x} = \surd \frac{\pi}{2}$	V. T. 112. N°. 6.
3)	$\int e^{-\frac{1}{q} \text{Cosec.} 2x} \frac{\surd \sin. 2 x}{\cos.^3 x} dx = \frac{1+q}{\surd e} 2 \surd q \pi$	V. T. 139. N°. 7.
4)	$\int e^{-q \text{Tang.} x} \frac{dx}{\cos. x \surd \sin. 2 x} = \surd \frac{\pi}{2 q}$	V. T. 140. N°. 2.
5)	$\int e^{-\frac{1}{q} \text{Cosec.} 2x} \frac{dx}{\cos. x \surd \sin. 2 x} = \frac{\surd q \pi}{\surd e}$	V. T. 140. N°. 5.
6)	$\int e^{-\frac{1}{q} \text{Cosec.} 2x} \frac{\text{Tang.}^b x}{\sin. x \surd \sin. 2 x} dx = \frac{\surd \pi q}{\surd e} \sum_0^{\infty} \frac{b^{2n+1}}{2^{n+2}} (-q)^n$	V. T. 140. N°. 11.
7)	$\int e^{-\frac{1}{q} \text{Cosec.} 2x} \frac{dx}{\sin. 2 x. \text{Tang.}^{b-1} x} = \frac{\surd 2 \pi q}{2 \surd e} \sum_0^{\infty} \frac{b^{2n+1}}{2^{n+2}} (-q)^n$	

$$\begin{aligned}
 8) \int e^{-\frac{1}{q} \text{Cosec.} x} \frac{dx}{\text{Tang.} x \sqrt{\{\text{Sin.} x (1 - \text{Sin.} x)\}}} &= \frac{\sqrt{q\pi}}{\sqrt{e}} \left. \vphantom{\int} \right\} \text{V. T. 150. N}^\circ. 1. \\
 9) \int e^{-\frac{1}{q} \text{Sec.} x} \frac{\text{Tang.} x}{\sqrt{\{\text{Cos.} x (1 - \text{Cos.} x)\}}} dx &= \frac{\sqrt{q\pi}}{\sqrt{e}} \\
 10) \int e^{-q^2 (\text{Tang.} x + \text{Cot.} x)} \frac{dx}{\text{Cos.} x \sqrt{\text{Sin.} 2x}} &= \frac{1}{2q} e^{-2q^2} \sqrt{2\pi} \quad \text{V. T. 140. N}^\circ. 6. \\
 11) \int e^{-p \text{Tang.} x - q \text{Cot.} x} \frac{dx}{\text{Cos.} x \sqrt{\text{Sin.} 2x}} &= e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{2p}} \quad \text{V. T. 140. N}^\circ. 9. \\
 12) \int e^{-q^2 (\text{Tang.} x + \text{Cot.} x)} \frac{dx}{\text{Sin.} x \sqrt{\text{Sin.} 2x}} &= \frac{1}{2q} e^{-2q^2} \sqrt{2\pi} \quad \text{V. T. 140. N}^\circ. 6. \\
 13) \int e^{-p \text{Tang.} x - q \text{Cot.} x} \frac{dx}{\text{Tang.}^a x \cdot \text{Cos.} x \sqrt{\text{Sin.} 2x}} &= \left(\frac{p}{q}\right)^{1a} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{2p}} \sum_0^{\infty} \frac{(a-n)^{2n-1}}{2^{n/2} (2\sqrt{pq})^n} \quad \text{V. T. 140. N}^\circ. 12. \\
 14) \int \frac{1}{e^{\text{Tang.} x} + e^{-\text{Tang.} x}} \frac{dx}{\text{Cos.} x \sqrt{\text{Sin.} 2x}} &= \sqrt{\frac{\pi}{2}} \sum_0^{\infty} \frac{(-1)^n}{\sqrt{(2n+1)}} \quad \text{V. T. 140. N}^\circ. 19. \\
 15) \int \frac{1}{e^{\text{Tang.} x} + e^{-\text{Tang.} x} + 1} \frac{dx}{\text{Cos.} x \sqrt{\text{Sin.} 2x}} &= \frac{\sqrt{2\pi}}{2 \text{Sin.} \frac{1}{3}\pi} \sum_1^{\infty} (-1)^{n-1} \frac{\text{Sin.} \frac{1}{3} n \pi}{1^n} \quad \text{V. T. 140. N}^\circ. 20.
 \end{aligned}$$

$$\begin{aligned}
 1) \int e^{\eta x} \text{Sin.} p x dx &= \frac{p}{p^2 + q^2} (1 - e^{\eta\pi} \text{Cos.} p\pi) \left. \vphantom{\int} \right\} \text{Dienger, Cr. 34. 75. — Schlömilch, Beitr. I. § 8, 10.} \\
 2) \int e^{\eta x} \text{Cos.} p x dx &= \frac{q}{p^2 + q^2} (e^{\eta\pi} \text{Cos.} p\pi - 1) \\
 3) \int (e^{\eta x} + e^{-\eta x}) \text{Cos.} p x dx &= \frac{q}{p^2 + q^2} \text{Cos.} p\pi (e^{\eta\pi} - e^{-\eta\pi}) \quad \text{Schlömilch, Stud. II. 6. — Id., Beitr. I. § 8.} \\
 4) \int (e^{\eta x} - e^{-\eta x}) \text{Sin.} p x dx &= (-1)^{p-1} p \frac{e^{\eta\pi} - e^{-\eta\pi}}{p^2 + q^2} \quad \text{Schlömilch, Beitr. I. § 10.} \\
 5) \int e^{ax} \text{Sin.}^b x dx &= \frac{\pi}{2^b} \frac{e^{1a\pi} 1^{b/1}}{\Gamma\left(\frac{a+bi}{2} + 1\right) \Gamma\left(\frac{a-bi}{2} + 1\right)} \quad \text{Lobatschewsky, Mém. Kasan. 1835. 211.} \\
 6) \int e^{p \text{Cos.} x} \text{Cos.} (p \text{Sin.} x) dx &= \pi, p^2 \leq 1; \quad \text{Poisson, P. 19. 404. N}^\circ. 77. — Serret, L. 8. 489. — Schlömilch, Beitr. II. 1.
 \end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
7) \int e^{p \cos x} \sin.(p \sin. x). \sin. a x dx &= \frac{1}{2} \pi \frac{p^a}{1^{a/1}} \\
8) \int e^{p \cos x} \cos.(p \sin. x). \cos. a x dx &= \frac{1}{2} \pi \frac{p^a}{1^{a/1}} \\
9) \int e^{p \cos x} \sin.(p \sin. x) \operatorname{Tang.} \frac{1}{2} x dx &= \pi (1 - e^{-p}) \\
10) \int e^{p \cos x} \sin.(p \sin. x) \operatorname{Cot.} \frac{1}{2} x dx &= \pi (e^p - 1)
\end{aligned} \right\} , p \leq 1; \\
& \left. \begin{aligned}
11) \int e^{p \cos x} \sin.(p \sin. x) \frac{dx}{\sin. x} &= \frac{1}{2} \pi (e^p - e^{-p}) \\
12) \int e^{p \cos x} \cos.(p \sin. x) \frac{dx}{\cos. x} &= \infty
\end{aligned} \right\} \begin{aligned}
& \text{Poisson, P. 19. 404. N}^\circ 77. - \text{Schlömlich,} \\
& \text{Beitr. II. 1.} \\
& , p^2 \leq 1; \text{ Schlömlich, Beitr. II. 1.} \\
& \text{il trouve faut. pour } 12) \pi \sin. p.
\end{aligned} \\
13) \int e^{-\operatorname{Cot.}^2 x} \frac{dx}{\operatorname{Tang.}^{2a} x. \sin. 2x} &= 0 \quad \text{V. T. 142. N}^\circ 9. \\
14) \int e^{-\operatorname{Cot.}^2 x} \frac{dx}{\operatorname{Tang.}^{2a+1} x. \sin. 2x} &= \frac{1^{a/2}}{2^{a+1}} \sqrt{\pi} \quad \text{V. T. 142. N}^\circ 8. \\
15) \int (e^{p \sin x} + e^{-p \sin x}) \left\{ e^{\frac{b \sin x}{p}} \sin. \left(x + \frac{b \cos x}{p} \right) - e^{-\frac{b \sin x}{p}} \sin. \left(x - \frac{b \cos x}{p} \right) \right\} \cos. (p \cos x) dx &= \left. \begin{aligned}
& = \frac{2b}{p} \left\{ 2 + \sum_1 \frac{p^{2n}}{(2n+1)(1^{2n/1})^2} \right\} \\
& \text{Smaasen, Cr. 42. 222.}
\end{aligned} \right\} \\
16) \int e^{a^p (\cos. px + \cos. qx)} \cos. (a^p \sin. px). \cos. (a^p \sin. qx) dx &= \frac{1}{2} \pi \left\{ 2 + \sum_1 \frac{a^{np(p+q)}}{1^{np/1} 1^{nq/1}} \right\} \\
17) \int e^{-p \operatorname{Tang.}^2 x - q \operatorname{Cot.}^2 x} \frac{\operatorname{Tang.}^{2a} x}{\sin.^2 x} dx &= \left(\frac{q}{p} \right)^{1a} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{q}} \sum_0 \frac{(a-n)^{2n/1}}{1^{n/1} (1 \sqrt{pq})^n} \quad \text{V. T. 146. N}^\circ 3. \\
18) \int \frac{e^{p \cos x} \cos. (p \sin. x)}{\cos.^2 \frac{1}{2} x + q^2 \sin.^2 \frac{1}{2} x} dx &= \frac{\pi}{q} e^{\frac{p^q - 1}{q+1}} \quad \text{Serret, L. S. 489.} \\
19) \int \frac{\sin. (q \sin. x). \sin. x}{1 - 2p \cos. x + p^2} e^{q \cos x} dx &= \frac{\pi}{2p} (e^{pq} - 1) \\
20) \int \frac{\cos. (q \sin. x)}{1 - 2p \cos. x + p^2} e^{q \cos x} dx &= \frac{\pi}{1 - p^2} e^{pq} \\
& \left. \begin{aligned}
& , p^2 < 1; \\
& \text{Poisson, P. 19. 404. N}^\circ 77. - \text{Schlömlich,} \\
& \text{Beitr. II. 2.}
\end{aligned} \right\} \\
21) \int \frac{1 - p^b \cos. bx}{1 - 2p^b \cos. bx + p^{2b}} e^{\frac{c \cos x}{p}} \cos. \left(\frac{c \sin. x}{p} \right) dx &= \frac{1}{2} \pi \left\{ 2 + \sum_1 \frac{e^{nb}}{1^{nb/1}} \right\} \quad \text{Smaasen, Cr. 42. 222.}
\end{aligned}$$

F. Expon.
Circ. Dir.

TABLE 296 suite.

Lim 0 et π .

$$22) \int \frac{\sin \frac{3x}{2} - p e^{\cos x} \sin \left(\frac{5x}{2} - \sin x \right)}{1 - 2 p e^{\cos x} \cos (x - \sin x) + p^2 e^{2 \cos x}} \sin \frac{1}{2} x dx = \sum_1 \frac{n^{n-1}}{1^{n/1}} q^n \quad \text{Poisson, P. 19. 404. N}^\circ 80.$$

$$23) \int e^{-p \cot^2 x - q \operatorname{Tang}^2 x} \frac{dx}{\cos^2 x} = \frac{1}{2} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{q}} \quad \text{V. T. 146. N}^\circ 2.$$

F. Expon.
Circ. Dir.

TABLE 297.

Lim. — $\frac{\pi}{2}$ et $\frac{\pi}{2}$.

$$1) \int e^{-px} \cos^{2a} x dx = \frac{1^{2a/1}}{p^2 + 2^2 \cdot p^2 + 4^2 \dots p^2 + 4a^2} \frac{1}{p} (e^{4p\pi} - e^{-4p\pi}) \quad \left. \vphantom{\int} \right\} \text{Ohm, Ausw. 13.}$$

$$2) \int e^{-px} \cos^{2a+1} x dx = \frac{1^{2a+1/1}}{p^2 + 1^2 \cdot p^2 + 3^2 \dots p^2 + (2a+1)^2} (e^{4p\pi} + e^{-4p\pi})$$

$$3) \int e^{-\operatorname{Tang}^2 x} \frac{\operatorname{Tang}^{2a+1} x}{\sin^2 x} dx = \frac{1^{a/2}}{2^{a+1}} \sqrt{\pi} \quad \text{V. T. 142. N}^\circ 5.$$

$$4) \int e^{-\operatorname{Tang}^2 x} \frac{\operatorname{Tang}^{2a} x}{\sin^2 x} dx = 0 \quad \text{V. T. 142. N}^\circ 9.$$

$$5) \int e^{-(p \operatorname{Tang}^2 x + q \cot^2 x)} \frac{dx}{\sin^2 x} = e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{q}} \quad \text{V. T. 146. N}^\circ 2.$$

$$6) \int e^{-(p \operatorname{Tang}^2 x + q \cot^2 x)} \frac{dx}{\operatorname{Tang}^{2a} x \cos^2 x} = \left(\frac{p}{q} \right)^{\frac{a}{2}} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{p}} \sum_0 \frac{(a-n)^{2n/1-1}}{1^{n/1}} \left(\frac{1}{4\sqrt{pq}} \right)^n \quad \text{V. T. 146. N}^\circ 3.$$

$$7) \int (2 \cos x)^{q-1} e^{(q+1)xi + 4be^{-xi}} \sec x dx = \frac{\pi b^q}{e^b \Gamma(q+1)} \quad \text{Kummer, Cr. 20. 1. où il y a faut. } e^{-(q+1)xi}.$$

F. Expon.
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TABLE 298.

Lim. diverses.

$$1) \int_0^1 \frac{e^{\frac{\pi}{b}\sqrt{1-x^2}} - e^{-\frac{\pi}{b}\sqrt{1-x^2}}}{e^{\frac{\pi}{b}\sqrt{1-x^2}} + e^{-\frac{\pi}{b}\sqrt{1-x^2}} + 2 \cos \left(\frac{\pi x}{b} \right)} dx = \frac{\pi^2}{16b} \quad \text{V. T. 293. N}^\circ 5.$$

$$2) \int_0^1 \frac{\sin \left\{ \frac{\pi}{b} \sqrt{1-x^2} \right\}}{e^{\frac{\pi x}{b}} + e^{-\frac{\pi x}{b}} + 2 \cos \left\{ \frac{\pi}{b} \sqrt{1-x^2} \right\}} dx = \frac{\pi^2}{16b} \quad \text{V. T. 293. N}^\circ 3.$$

- 3) $\int_0^{\lambda} p^{x-1} \text{Sin. } 2a\pi x \, dx = \frac{1-p}{(lp)^2 + 4a^2\pi^2} \frac{2a\pi}{p}$ Kummer, Cr. 35. 1.
- 4) $\int_0^{\lambda} e^{-p \text{Cos. } x} \text{Sin. } (x - p \text{Sin. } x) \, dx = \frac{1}{p} \{e^{-p \text{Cos. } \lambda} \text{Cos. } (p \text{Sin. } \lambda) - e^{-p}\}$
- 5) $\int_0^{\lambda} e^{-p \text{Cos. } x} \text{Cos. } (x - p \text{Sin. } x) \, dx = \frac{1}{p} \{e^{-p \text{Cos. } \lambda} \text{Sin. } (p \text{Sin. } \lambda)\}$
- 6) $\int_0^{\lambda} e^{-p \text{Cos. } x} \text{Sin. } (ax - p \text{Sin. } x) \, dx = (-1)^a \frac{d^a}{d p^a} \cdot \frac{e^{-p \text{Cos. } \lambda} \text{Cos. } (p \text{Sin. } \lambda) - e^{-p}}{p}$
- 7) $\int_0^{\lambda} e^{-p \text{Cos. } x} \text{Cos. } (ax - p \text{Sin. } x) \, dx = (-1)^a \frac{d^a}{d p^a} \cdot \frac{e^{-p \text{Cos. } \lambda} \text{Sin. } (p \text{Sin. } \lambda)}{p}$
- 8) $\int_a^{a+1} e^{-a(x-1)} \text{Sin. } \{q(x-p)\} \, dx = q e^{-ap} \frac{e^q - 1}{a^2 + q^2}$ Schaar, Mém. Cour. Brux. T. 23.
- 9) $\int_0^{\pi} e^{-\text{Tang. } x} \frac{dx}{\text{Sin. } 2x} = \infty$ V. T. 112. N°. 4.
- 10) $\int_0^{\pi} e^{\text{Tang. } x} \frac{\text{Tang. } x}{(\text{Sin. } x + \text{Cos. } x)^2} \, dx = \frac{1}{2} e - 1$ V. T. 112. N°. 5.
- 11) $\int_0^{\pi} e^{\text{Cos. } 2x} \frac{\text{Sin. }^3 2x}{\text{Cos. }^2 x} \, dx = 2(e - 2)$ V. T. 112. N°. 3.
- 12) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{\text{Cot. } x} \frac{\text{Cot. } x}{(\text{Cos. } x + \text{Sin. } x)^2} \, dx = \frac{1}{2} e - 1$ V. T. 112. N°. 5.
- 13) $\int_{-\pi}^{\pi} e^{\eta x} \text{Cos. } \frac{1}{2} p x \, dx = \frac{e^{\eta \pi} - e^{-\eta \pi}}{4q^2 + p^2} 4q \text{Cos. } \frac{1}{2} p \pi + \frac{e^{\eta \pi} + e^{-\eta \pi}}{4q^2 + p^2} 2p \text{Sin. } \frac{1}{2} p \pi$ Dienger, Cr. 34. 75.
- 14) $\int_{-\pi}^{\pi} e^{\eta x} \text{Sin. } \frac{1}{2} p x \, dx = \frac{2}{4q^2 + p^2} \left\{ (e^{\eta \pi} + e^{-\eta \pi}) 2q \text{Sin. } \frac{1}{2} p \pi - (e^{\eta \pi} - e^{-\eta \pi}) p \text{Cos. } \frac{1}{2} p \pi \right\}$ V. T. 298. N°. 13.
- 15) $\int_{-\pi}^{\pi} \frac{(1 - e^{-xi})(p + e^{-xi})}{1 - q e^{p + \text{Cos. } x} e^{(x - \text{Sin. } x)i}} \, dx = 2\pi \left\{ p + \sum_1^{\infty} \frac{n^{n-1}}{1^{n/1}} q^n e^{np} \right\}$ Poisson, P. 19. 404. N°. 80.

$$16) \int_{-\pi}^{\pi} \frac{e^{-\frac{1}{2}xi} \sin. \frac{1}{2}x}{1 - q e^{p+Cos.x} e^{(x-\sin.x)i}} dx = \frac{\pi}{i} \sum_{n=1}^{\infty} \frac{n^{n-1}}{1^{n-1}} q^n e^{np} \quad \text{Poisson, P. 19. 404. N°. 80.}$$

$$17) \int_{-\pi}^{\pi} \frac{e^{b\sin.x} \sin. \{(2a+1)x\} - \sin. \{(2a+1)x - b\cos.x\}}{e^{b\sin.x} - 2\cos.(b\cos.x) + e^{-b\sin.x}} dx = \left(\frac{b}{2\pi}\right)^{2a-1} \sum_1^b n^{2a}$$

$$18) \int_{-\pi}^{\pi} \frac{e^{b\sin.x} \sin.\left(x + \frac{ab}{4\pi^2} \sin.2x\right) - \sin.\left(x + \frac{ab}{4\pi^2} \sin.2x - b\cos.x\right)}{e^{b\sin.x} - 2\cos.(b\cos.x) + e^{-b\sin.x}} e^{\frac{ab}{4\pi^2} \cos.2x} dx = \frac{b}{2\pi} \left\{ \frac{1}{2} + \sum_1^b e^{n^2 a} \right\}$$

Sur (17) et (18) voyez: Cauchy, Exerc. 1826. p. 205.

$$\left. \begin{aligned} 19) \int_{\frac{1}{2}\pi}^{\infty} e^{-px} \cos.2ax dx &= \frac{1^{2a/1}}{p^2 + 2^2.p^2 + 4^2 \dots p^2 + 4a^2} \frac{1}{p} e^{-\frac{1}{2}p\pi} \\ 20) \int_{\frac{1}{2}\pi}^{\infty} e^{-px} \cos.2a+1x dx &= \frac{1^{2a+1/1}}{p^2 + 1^2.p^2 + 3^2 \dots p^2 + (2a+1)^2} e^{-\frac{1}{2}p\pi} \\ 21) \int_{-\frac{1}{2}\pi}^{\infty} e^{-px} \cos.2ax dx &= \frac{1^{2a/1}}{p^2 + 2^2.p^2 + 4^2 \dots p^2 + 4a^2} \frac{1}{p} e^{\frac{1}{2}p\pi} \\ 22) \int_{-\frac{1}{2}\pi}^{\infty} e^{-px} \cos.2a+1x dx &= \frac{1^{2a+1/1}}{p^2 + 1^2.p^2 + 3^2 \dots p^2 + (2a+1)^2} e^{\frac{1}{2}p\pi} \end{aligned} \right\} \quad \text{Ohm, Ausw. 23.}$$

$$1) \int \text{Arctang.} \frac{\pi}{q} e^{-px} dx = \frac{1}{p} \left\{ \text{Ci.}(pq) \sin.pq - \text{Si.}(pq) \cos.pq + \frac{1}{2}\pi \cos.pq \right\} \quad \left. \begin{array}{l} \text{Bierens de Haan, Verh. v.} \\ \text{K. Aead. van Wet. 1854,} \\ \text{bl. 19.} \end{array} \right\}$$

$$2) \int \text{Arctang.} x \frac{dx}{e^{2\pi x} - 1} = \frac{1}{2} - \frac{1}{4} l 2\pi \quad \text{Plana, Mém. Turin. 1820.}$$

$$3) \int \text{Arctang.} \frac{x}{p} \frac{dx}{e^{2\pi x} - 1} = \frac{1}{2} l \left\{ \left(\frac{e}{p}\right)^p \Gamma(p) - \frac{p}{2\pi} \right\} \quad \text{Binet, P. 27. 123.}$$

$$4) \int \text{Arctang.} x \frac{dx}{e^{2\pi qx} - 1} = \frac{1}{2q} \left\{ l \Gamma(q+1) - \frac{1}{2} l 2q\pi + q(1-lq) \right\} \quad \text{V. T. 378. N°. 4.}$$

$$5) \int \text{Arctang.} 2x \frac{e^{\pi x} - e^{-\pi x}}{(e^{\pi x} + e^{-\pi x})^2} dx = \frac{1}{2\pi} l 2 \quad \text{V. T. 138. N°. 3.}$$

F. Expon. Circ. Inv.	TABLE 299 suite.	Lim. 0 et ∞ .
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- $$6) \int \text{Arctang. } x \frac{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x})^2} dx = \frac{\sqrt{2}}{\pi} \left\{ \pi - l \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right\} \quad \text{V. T. 138. N}^\circ. 17.$$
- $$7) \int \text{Arctang. } x \frac{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x})^2} dx = \frac{1}{\pi} l 2 \quad \text{V. T. 138. N}^\circ. 14.$$
- $$8) \int \text{Arctang. } x \frac{e^{\pi x} - e^{-\pi x}}{(e^{\pi x} + e^{-\pi x})^2} dx = \frac{4 - \pi}{4\pi} \quad \text{V. T. 138. N}^\circ. 2.$$
- $$9) \int \text{Arctang. } \frac{x}{p} \frac{e^{\pi x} - e^{-\pi x}}{(e^{\pi x} + e^{-\pi x})^2} dx = \frac{1}{4\pi} \left\{ Z' \left(\frac{2q+3}{4} \right) - Z' \left(\frac{2q+1}{4} \right) \right\} \quad \text{V. T. 138. N}^\circ. 1.$$
- $$10) \int \text{Arctang. } x \frac{\pi (e^{px} - e^{-px}) (e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}) - 2p (e^{px} + e^{-px}) (e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})}{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})^2} dx =$$
- $$= \pi \text{Sin. } p - \text{Cos. } p l \frac{1 + \text{Sin. } p}{1 - \text{Sin. } p}, 0 \leq p \leq \frac{1}{2}\pi; \quad \text{V. T. 138. N}^\circ. 13.$$
- $$11) \int \text{Arctang. } \frac{x}{q} \frac{p (e^{\pi x} - e^{-\pi x}) (e^{(p-p)x} + e^{(p-\pi)x}) - 2\pi (e^{px} - e^{-px})}{(e^{\pi x} - e^{-\pi x})^2} dx = \sum_{n=1}^{\infty} \frac{\text{Sin. } np}{q+n} p^2 < \pi^2; \quad \text{V. T. 138. N}^\circ. 5.$$
- $$12) \int \text{Arctang. } x \frac{\pi (e^{px} - e^{-px}) (e^{\pi x} + e^{-\pi x}) - p (e^{px} + e^{-px}) (e^{\pi x} - e^{-\pi x})}{(e^{\pi x} - e^{-\pi x})^2} dx =$$
- $$- \frac{1}{2} p \text{Cos. } p + \frac{1}{2} \text{Sin. } p l \{ 2(1 + \text{Cos. } p) \} \quad \text{V. T. 138. N}^\circ. 4.$$

F. Expon. Autres fonctions.	TABLE 500.	Lim. 0 et ∞ .
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- $$1) \int e^{-x} li.(e^{-x}) dx = -l 2 \quad \text{Schl\"omilch, Gr. 9. 5.}$$
- $$2) \int e^{-2x} li.(e^x) dx = 0 \quad \text{Schl\"omilch, Beitr. III. 6.}$$
- $$3) \int e^{-px} li.(e^{-x}) dx = -\frac{1}{p} l(1+p), p \geq -1;$$
- $$4) \int e^{-px^2} li.(e^{-x^2}) dx = -\sqrt{\frac{\pi}{p}} l \{ \sqrt{p} + \sqrt{1+p} \}, p > 0;$$
- $$5) \int e^{px^2} li.(e^{-x^2}) dx = -\sqrt{\frac{\pi}{p}} \text{Arcsin.}(\sqrt{p}), p < 1;$$
- $$\left. \begin{array}{l} 4) \\ 5) \end{array} \right\} \text{Schl\"omilch, Beitr. III. 7.}$$

- 1) $\int l x . \text{Cos. } p x . d x = -\frac{1}{p} \text{Si.}(p) \quad \text{V. T. 192. N}^{\circ} . 5.$
 - 2) $\int \text{Sin. } (q l x) d x = -\frac{q}{1+q^2} \quad \text{V. T. 278. N}^{\circ} . 6.$
 - 3) $\int \text{Cos. } (q l x) d x = \frac{1}{1+q^2} \quad \text{V. T. 278. N}^{\circ} . 7.$
 - 4) $\int \text{Sin. } 2 a \pi x . l \text{Sin. } \pi x d x = 0$
 - 5) $\int \text{Cos. } 2 a \pi x . l \text{Sin. } \pi x d x = -\frac{1}{2 a}$
 - 6) $\int \text{Cos. } \{2 b \pi (x-a)\} . l \text{Sin. } \pi x d x = -\frac{1}{2 b} e^{-2 \pi a b i}$
- Schaar, Mém. Cour. Brux. T. 23.
-
- 7) $\int \text{Sin. } \left(2 p \sqrt{l \frac{1}{x}}\right) d x = p e^{-p^2} \sqrt{\pi} \quad \text{V. T. 388. N}^{\circ} . 1.$
- 8) $\int \text{Sin. } (l x) d x \sqrt{l \frac{1}{x}} = -\text{Sin. } \frac{3}{8} \pi . \sqrt{\frac{\pi}{8}} \quad \text{V. T. 397. N}^{\circ} . 1.$
- 9) $\int \text{Cos. } (l x) d x \sqrt{l \frac{1}{x}} = \text{Cos. } \frac{3}{8} \pi . \sqrt{\frac{\pi}{8}} \quad \text{V. T. 397. N}^{\circ} . 2.$
- 10) $\int \text{Sin. } (l x) . d x l l \frac{1}{x} = -\frac{1}{8} \pi + \frac{1}{4} l 2 + \frac{1}{2} \Lambda \quad \text{V. T. 439. N}^{\circ} . 1.$
- 11) $\int \text{Sin. } (l x) \frac{d x}{l x} = \frac{1}{4} \pi$
- 12) $\int \text{Sin. } (q l x) \frac{d x}{l x} = \text{Arctang. } q$
- 13) $\int \text{Sin. } (p l x) . \text{Sin. } (q l x) \frac{d x}{l x} = \frac{1}{4} l \frac{1+(p-q)^2}{1+(p+q)^2}$
- 14) $\int \text{Sin. } (p l x) . \text{Cos. } (q l x) \frac{d x}{l x} = \frac{1}{2} \text{Arctang. } \left\{ \frac{2 p}{1-p^2+q^2} \right\}$
- 15) $\int \text{Sin. }^2 \left(\frac{1}{2} p l x \right) \frac{d x}{l x} = \frac{1}{4} l (1+p^2) \quad \text{V. T. 392. N}^{\circ} . 9.$
- 16) $\int \text{Sin. } (l x) \frac{d x}{\sqrt{l \frac{1}{x}}} = -\sqrt{\frac{\pi}{2} \sqrt{\frac{2-1}{2}}} \quad \text{V. T. 398. N}^{\circ} . 3.$

$$17) \int \text{Cos.}(lx) \frac{dx}{\sqrt{l \frac{1}{x}}} = \sqrt{\frac{\pi \sqrt{2+1}}{2}} \quad \text{V. T. 398. N}^\circ. 4.$$

$$18) \int \text{Cos.} \left\{ 2 \sqrt{l \frac{1}{x}} \right\} \frac{dx}{\sqrt{l \frac{1}{x}}} = e^{-q} \sqrt{\pi} \quad \text{V. T. 396. N}^\circ. 7.$$

$$19) \int \frac{\text{Cos.}(plx) - \text{Cos.}(qlx)}{lx} dx = \frac{1}{2} l \frac{1+p^2}{1+q^2} \quad \text{Euler, N. C. Petr. 20. 59.}$$

$$20) \int \text{Cos.} \left\{ q \sqrt{l \frac{1}{x}} \right\} dx = \sum_0^{\infty} (-1)^n \frac{q^{2n}}{(n+1)^{n/1}} \quad \text{V. T. 388. N}^\circ. 13.$$

$$21) \int \text{Tang.} \left\{ q \sqrt{l \frac{1}{x}} \right\} dx = 2a \sqrt{\pi} \sum_1^{\infty} (-1)^n n e^{-n^2 a^2} \quad \text{V. T. 388. N}^\circ. 20.$$

$$22) \int \text{Cot.} \left\{ q \sqrt{l \frac{1}{x}} \right\} dx = 2a \sqrt{\pi} \sum_1^{\infty} n e^{-n^2 a^2} \quad \text{V. T. 388. N}^\circ. 21.$$

$$23) \int \text{Cosec.} \left\{ q \sqrt{l \frac{1}{x}} \right\} dx = 4a \sqrt{\pi} \sum_1^{\infty} (2n-1) e^{-(2n-1)^2 a^2} \quad \text{V. T. 388. N}^\circ. 22.$$

$$24) \int \text{Sin.} \left\{ q \sqrt{l \frac{1}{x}} \right\} \frac{dx}{lx} = \frac{1}{2} q \sqrt{\pi} \sum_0^{\infty} \frac{(-1)^{n-1}}{(2n+1) 1^{n/1}} \left(\frac{q}{2} \right)^{2n} \quad \text{V. T. 392. N}^\circ. 15.$$

$$25) \int l \text{Sin.}(-qlx) dx = -\frac{1}{2} l 2 - \frac{1}{2} \sum_1^{\infty} \frac{1}{n} \frac{1}{1+n^2 q^2} \quad \text{V. T. 439. N}^\circ. 6.$$

$$26) \int l \text{Cos.}(-qlx) dx = -\frac{1}{2} l 2 - \frac{1}{2} \sum_1^{\infty} \frac{(-1)^n}{n} \frac{1}{1+n^2 q^2} \quad \text{V. T. 439. N}^\circ. 7.$$

$$27) \int l \text{Tang.}(-qlx) dx = -\sum_1^{\infty} \frac{1}{2n-1} \frac{1}{1+(2n-1)^2 q^4} \quad \text{V. T. 439. N}^\circ. 8.$$

$$\left. \begin{aligned} 1) \int l \text{Sin.}^2 x dx &= \infty \\ 2) \int l \text{Cos.}^2 x dx &= \infty \\ 3) \int l(1 + \text{Cos.} x) dx &= \infty \\ 4) \int l(1 - \text{Cos.} x) dx &= \infty \end{aligned} \right\} \text{Raabe, Int. 188.}$$

$$5) \int l(1 + p^2 + 2p \cos. x) dx = \infty, p > 1; \quad \text{Raabe, Int. 188.}$$

$$6) \quad \quad \quad = 0, p \leq 1;$$

$$7) \int l(1 + p^2 + 2p \sin. x) dx = \sum_0^{\infty} \frac{1}{2n+1} \frac{2^{n/2}}{3^{n/2}} \left(\frac{2p}{1+p^2} \right)^{2n+1}, p \leq 1; \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Raabe, Cr. 15. 355.}$$

$$8) \int l \frac{p^2 + x^2}{q^2 + x^2} \cos. rx dx = \frac{\pi}{r} (e^{-qr} - e^{-pr})$$

$$9) \int l \frac{x^2}{q^2 + x^2} \cos. rx dx = \frac{\pi}{r} (e^{-qr} - 1)$$

$$10) \int l \left(1 + \frac{p^2}{x^2} \right) \cos. rx dx = \frac{\pi}{r} (1 - e^{-pr})$$

Raabe, Int. 170.

$$1) \int l \sin. x dx = -\frac{1}{4} \pi l 2 - \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 238. N°. 4.}$$

$$2) \int l \sin. x \cos. a 2 x \sin. 2 x dx = \frac{-1}{4(a+1)} \left\{ l 2 + \sum_0^a \frac{1}{n+1} \right\} \quad \text{V. T. 47. N°. 8.}$$

$$3) \int l (2 \sin.^2 x) \text{Tang. } 2 x dx = -\frac{1}{12} \pi^2 \quad \text{V. T. 160. N°. 5.}$$

$$4) \int l \sin. 2 x \text{Tang.} \left(\frac{\pi}{4} + x \right) dx = -\frac{1}{12} \pi^2 \quad \text{V. T. 316. N°. 8.}$$

$$5) \int l \sin. 2 x \text{Tang.} \left(\frac{\pi}{4} - x \right) dx = -\frac{1}{24} \pi^2 \quad \text{V. T. 316. N°. 4.}$$

$$6) \int l \sin. 2 x \text{Tang.} \left(\frac{\pi}{4} + x \right) \sin. 2 x dx = \frac{6 - \pi^2}{12} \quad \text{V. T. 152. N°. 8.}$$

$$7) \int l \sin. 2 x \text{Tang.}^2 \left(\frac{\pi}{4} + x \right) \cos. 2 x dx = \frac{3 - \pi^2}{6} \quad \text{V. T. 152. N°. 9.}$$

$$8) \int (l \sin. 2 x)^3 \text{Tang.} \left(\frac{\pi}{4} + x \right) dx = -\frac{1}{30} \pi^4 \quad \text{V. T. 154. N°. 11.}$$

- 9) $\int (l \sin. 2x)^3 \text{ Tang. } \left(\frac{\pi}{4} - x \right) dx = -\frac{7}{240} \pi^4$ V. T. 154. N°. 10.
- 10) $\int (l \sin. 2x)^5 \text{ Tang. } \left(\frac{\pi}{4} + x \right) dx = -\frac{4}{63} \pi^6$ V. T. 155. N°. 3.
- 11) $\int (l \sin. 2x)^5 \text{ Tang. } \left(\frac{\pi}{4} - x \right) dx = -\frac{31}{504} \pi^6$ V. T. 155. N°. 2.
- 12) $\int (l \sin. 2x)^{2a} \text{ Tang. } \left(\frac{\pi}{4} + x \right) dx = \frac{1^{2a+1}}{2} \sum_1^{\infty} \frac{1}{n^{2a+1}}$ V. T. 157. N°. 3.
- 13) $\int (l \sin. 2x)^{a-1} \text{ Tang. } \left(\frac{\pi}{4} - x \right) dx = (-1)^{a-1} 1^{a-1/1} \frac{1}{2} \sum_0^{\infty} \frac{1}{(1+n)^a}$ V. T. 157. N°. 9.
- 14) $\int (l \sin. 2x)^{a-1} \text{ Tang. } \left(\frac{\pi}{4} - x \right) dx = (-1)^{a-1} 1^{a-1/1} \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(1+n)^a}$ V. T. 157. N°. 8.
- 15) $\int (l \sin. 2x)^{2a-1} \text{ Tang. } \left(\frac{\pi}{4} + x \right) dx = -\frac{1}{8a} (2\pi)^{2a} B_{2a-1}$ V. T. 157. N°. 6.
- 16) $\int (l \sin. 2x)^{2a-1} \text{ Tang. } \left(\frac{\pi}{4} - x \right) dx = \frac{1 - 2^{2a-1}}{4a} \pi^{2a} B_{2a-1}$ V. T. 157. N°. 5.
- 17) $\int (l \sin. 2x)^{b-1} \text{ Tang. } \left(\frac{\pi}{4} + x \right) \cdot \sin. a 2x dx = \frac{1}{2} (-1)^{b-1} 1^{b-1/1} \sum_0^{\infty} \frac{1}{(a+n+1)^b}$ V. T. 157. N°. 12.
- 18) $\int (l \sin. 2x)^{b-1} \text{ Tang. } \left(\frac{\pi}{4} - x \right) \cdot \sin. a 2x dx = \frac{1}{2} (-1)^{b-1} 1^{b-1/1} \sum_0^{\infty} \frac{(-1)^n}{(a+n+1)^b}$ V. T. 157. N°. 11.

- 1) $\int l \cos. x dx = -\frac{1}{1} \pi l 2 + \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 303. N°. 1 et T. 305. N°. 1.
- 2) $\int l \cos. x \cdot \cos. p-1 2x \cdot \text{Tang. } 2x dx = \frac{-1}{S(p-1)} \left\{ Z' \left(\frac{p+1}{2} \right) - Z' \left(\frac{p}{2} \right) \right\}$ V. T. 46. N°. 11.
- 3) $\int l (2 \cos. x)^2 \cdot \text{Tang. } 2x dx = \frac{1}{24} \pi^2$ V. T. 160. N°. 1.
- 4) $\int l \cos. 2x \cdot \text{Tang. } x dx = -\frac{1}{48} \pi^2$ V. T. 304. N°. 3.

- 5) $\int (l \text{ Cos. } 2x)^3 \text{ Tang. } x dx = -\frac{7}{240} \pi^4$ V. T. 154. N°. 10.
- 6) $\int (l \text{ Cos. } 2x)^5 \text{ Tang. } x dx = -\frac{31}{504} \pi^6$ V. T. 154. N°. 2.
- 7) $\int (l \text{ Cos. } 2x)^{a-1} \text{ Tang. } x dx = (-1)^{a-1} 1^{a-1/1} \sum_0^{\infty} \frac{(-1)^n}{(1+n)^a}$ V. T. 157. N°. 8.
- 8) $\int (l \text{ Cos. } 2x)^{2a-1} \text{ Tang. } x dx = \frac{1-2^{2a-1}}{4a} \pi^{2a} B_{2a-1}$ V. T. 157. N°. 5.
- 9) $\int (l \text{ Cos. } 2x)^{2a} \text{ Tang. } x dx = \frac{2^{2a}-1}{2^{2a+1}} 1^{2a/1} \sum_1^{\infty} \frac{1}{n^{2a+1}}$ V. T. 157. N°. 2.
- 10) $\int (l \text{ Cos. } 2x)^{b-1} \text{ Tang. } x. \text{ Cos. }^a 2x dx = \frac{1}{2} (-1)^{b-1} 1^{b-1/1} \sum_0^{\infty} \frac{(-1)^n}{(a+n+1)^b}$ V. T. 157. N°. 11.

- 1) $\int l \text{ Tang. } x dx = -\sum_0^{\infty} (-1)^n \frac{1}{(2n+1)^2}$ V. T. 237. N°. 4.
- 2) $\int l \text{ Tang. } x. \text{ Tang. } x dx = -\frac{1}{48} \pi^2$ V. T. 152. N°. 12.
- 3) $\int l \text{ Tang. } x. \text{ Tang. } 2x dx = -\frac{1}{16} \pi^2$ V. T. 160. N°. 15.
- 4) $\int l \text{ Tang. } x. \text{ Cos. } 2x. \text{ Sin. }^{2p-1} 2x dx = -2^{2p-4} \frac{\{\Gamma(p)\}^2}{p \Gamma(2p)}$ V. T. 153. N°. 20.
- 5) $\int (l \text{ Tang. } x)^2 dx = \frac{1}{16} \pi^3$ V. T. 154. N°. 1.
- 6) $\int (l \text{ Tang. } x)^3 \text{ Tang. } x dx = -\frac{7}{1920} \pi^4$ V. T. 154. N°. 13.
- 7) $\int (l \text{ Tang. } x)^3 \text{ Tang. } 2x dx = -\frac{1}{128} \pi^4$ V. T. 154. N°. 16.
- 8) $\int (l \text{ Tang. } x)^4 dx = \frac{5}{64} \pi^5$ V. T. 155. N°. 1.

$$\begin{aligned}
 9) \int (l Tang. x)^6 dx &= \frac{61}{256} \pi^7 \quad \text{V. T. 155. N}^\circ. 8. \\
 10) \int (l Tang. x)^{q-1} dx &= \frac{\Gamma(q)}{(-1)^{q-1}} \sum_0^\infty \frac{(-1)^n}{(2n+1)^q} \left. \vphantom{\int} \right\} \quad \text{Arndt, Gr. 6. 434.} \\
 11) \int (l Tang. x)^{2a} dx &= \frac{1}{2^{2a+2}} 1^{2a/1} \pi^{2a+1} B_{2a+2} \left. \vphantom{\int} \right\} \\
 12) \int (lTg..x)^{2a} (Tg..x + Cot..x) dx &= \frac{(-1)^{a+1}}{b} \left(2b\pi \right)^{2a+1} \sum_1^b (-1)^{n+1} B'' \left(\frac{2n-1}{4b} \right) Cos. \left(\frac{2n-1}{2} q\pi \right) \left. \vphantom{\int} \right\} \begin{array}{l} \text{V. T. 158.} \\ \text{N}^\circ. 3. \\ \text{où } q^2 = \alpha^2 b^2 + 1, \alpha \text{ arbitraire.} \end{array} \\
 13) \int (l Tang. x)^{b-1} Tang..^a x dx &= \frac{1^{b-1/1}}{(-1)^{b-1}} \sum_0^\infty \frac{(-1)^n}{(a+1+2n)^b} \quad \text{Arndt, Gr. 6. 434.}
 \end{aligned}$$

$$\begin{aligned}
 1) \int l(1 + Tang. x) dx &= \frac{1}{8} \pi l 2 \quad \text{Serret, L. 9. 436. — Grunnert, Gr. 6. 448.} \\
 2) \int l Tang. \left(\frac{\pi}{4} + x \right). Sin. 2 x dx &= \frac{1}{4} \pi \\
 3) \int l Tang. \left(\frac{\pi}{4} + x \right). Tang..x (Sin. 2 x + Tang. x) dx &= \frac{3}{2} l 2 \left. \vphantom{\int} \right\} \quad \text{V. T. 47. N}^\circ. 5, 6. \\
 4) \int l Tang. x (l Cos. 2 x)^2 Tang. 2 x dx &= -\frac{1}{96} \pi^4 \quad \text{V. T. 337. N}^\circ. 15. \\
 5) \int l Tang. x (l Cos. 2 x)^4 Tang. 2 x dx &= -\frac{1}{80} \pi^6 \quad \text{V. T. 337. N}^\circ. 17. \\
 6) \int l Tang. x (l Cos. 2 x)^6 Tang. 2 x dx &= -\frac{17}{448} \pi^8 \quad \text{V. T. 337. N}^\circ. 19. \\
 7) \int l Tang. x (l Cos. 2 x)^{2a} Tang. 2 x dx &= -\frac{2^{2a+1}-1}{(2a+1)(2a+2)} \pi^{2a+2} B_{2a+1} \quad \text{V. T. 337. N}^\circ. 20. \\
 8) \int l Tang. x (l Cos. 2 x)^{2a-1} Tang. 2 x dx &= \frac{2^{2a+1}-1}{2^{2a+2}} 1^{2a-1/1} \sum_0^\infty \frac{n^{2a+1}}{1} \quad \text{V. T. 337. N}^\circ. 22. \\
 9) \int l Tang. x (l Cos. 2 x)^a Tang. 2 x dx &= (-1)^{a+1} \frac{1^{a/1}}{2^a} \sum_0^\infty \frac{1}{(1+2n)^{a+1}} \quad \text{V. T. 317. N}^\circ. 7.
 \end{aligned}$$

- 1) $\int l \sin. x \frac{\sin. 2^a x}{\cos. 2^a + 2 x} dx = -\frac{1}{2^a + 1} \left\{ \frac{1}{2} l 2 + (-1)^a \frac{\pi}{4} + (-1)^a \sum_0^a \frac{(-1)^n}{2n-1} \right\}$ V. T. 46. N°. 4.
- 2) $\int l \sin. x \frac{\sin. 2^a - 1 x}{\cos. 2^a + 1 x} dx = \frac{1}{2^a} \left\{ -\frac{1}{2} l 2 + (-1)^a \frac{1}{2} l 2 + (-1)^a \sum_1^{a-1} \frac{(-1)^n}{2n} \right\}$ V. T. 46. N°. 5.
- 3) $\int l \sin. x \frac{\sin. 2 x}{\cos^{p+1} 2 x} dx = \frac{1}{4^p} \{ \Lambda + Z'(1-p) \}$, $0 > p > -1$; V. T. 47. N°. 11.
- 4) $\int l \sin. x (\cos^p 2 x + \sec^p 2 x) \text{Tang.} 2 x dx = \frac{1}{4^p} \left\{ \pi \cot. p \pi - \frac{1}{p} \right\}$, $0 > p > -1$; V. T. 47. N°. 12.
- 5) $\int l \sin. x \frac{(p-1) \cos. 2^{p-1} 2 x + p}{\cos. p 2 x} \text{Tang.} 2 x dx = \frac{1}{4} \pi \cot. p \pi$, $0 > p > -1$; V. T. 47. N°. 10.
- 6) $\int l \cos. x \frac{dx}{\sin. 2 x} = -\frac{1}{96} \pi^2$ V. T. 305. N°. 2.
- 7) $\int l \cos. 2 x \frac{dx}{\text{Tang.} x} = -\frac{1}{12} \pi^2$ V. T. 303. N°. 3.
- 8) $\int l \cos. 2 x \frac{\sin. 2 x}{\text{Tang.} x} dx = -\frac{1}{4}$ V. T. 307. N°. 9, 10.
- 9) $\int l \cos. 2 x \frac{\cos. 3 x}{\text{Tang.} x} dx = \frac{1}{4} - \frac{1}{12} \pi^2$ V. T. 152. N°. 9.
- 10) $\int l \cos. 2 x \frac{\cos. 2 x}{\text{Tang.} x} = \frac{1}{12} (6 - \pi^2)$ V. T. 152. N°. 8.
- 11) $\int l \cos. 2 x \frac{\sin. 2 x}{\text{Tang.}^2 x} dx = \frac{1}{6} (3 - \pi^2)$ V. T. 152. N°. 9.
- 12) $\int l \cos. x \frac{\sin. 2^a x}{\cos. 2^a + 2 x} dx = \frac{1}{2^a + 1} \left\{ -\frac{1}{2} l 2 + (-1)^{a+1} \frac{\pi}{4} + (-1)^{a+1} \sum_1^{a+1} \frac{(-1)^n}{2n-1} \right\}$ V. T. 46. N°. 4.
- 13) $\int l \cos. x \frac{\sin. 2^a - 1 x}{\cos. 2^a + 1 x} dx = \frac{1}{2^a} \left\{ -\frac{1}{2} l 2 + (-1)^a \frac{1}{2} l 2 + (-1)^a \sum_1^a \frac{(-1)^n}{2n} \right\}$ V. T. 46. N°. 5.
- 14) $\int l \cos. x \frac{\text{Tang.}^p x}{\sin. 2 x} dx = \frac{1}{8^p} \left\{ 2 l \frac{1}{2} + Z' \left(\frac{p+1}{4} \right) - Z' \left(\frac{p+2}{4} \right) \right\}$ V. T. 151. N°. 13.
- 15) $\int l \cos. 2 x \frac{\cos. p-1 2 x}{\text{Tang.} x} dx = -\frac{1}{2} \sum_0^{\infty} \frac{1}{(p+n)^2}$ V. T. 152. N°. 10.

F. Log. en num. $(l \text{ Sin. } ax)^b, (l \text{ Cos. } ax)^b$. TABLE 508.
Circ. Dir. rat. en dén. monôme.

Lim. 0 et $\frac{\pi}{4}$.

- 1) $\int (l \text{ Sin. } 2x)^{q-1} \frac{\text{Sin. } 2x}{\text{Tang.} \left(\frac{\pi}{4} - x \right)} dx = \frac{1}{2} \frac{\Gamma(q)}{(-1)^{q-1}} \sum_0^{\infty} \frac{1}{(a+n+1)^q}$ V. T. 157. N°. 12.
- 2) $\int (l \text{ Sin. } 2x)^{2a-1} \text{Tang.}^2 \left(\frac{\pi}{4} + x \right) \frac{dx}{\text{Tang. } 2x} = \frac{1}{2a} (-2)^{2a-1} \pi^{2a} B_{2a-1}$ V. T. 158. N°. 14.
- 3) $\int (l \text{ Cos. } 2x)^3 \frac{dx}{\text{Tang. } x} = -\frac{1}{30} \pi^4$ V. T. 154. N°. 11.
- 4) $\int (l \text{ Cos. } 2x)^5 \frac{dx}{\text{Tang. } x} = -\frac{1}{63} \pi^6$ V. T. 155. N°. 3.
- 5) $\int (l \text{ Cos. } 2x)^{a-1} \frac{dx}{\text{Tang. } x} = \frac{1}{2} (-1)^{a-1} 1^{a-1/1} \sum_0^{\infty} \frac{1}{(1+n)^a}$ V. T. 157. N°. 9.
- 6) $\int (l \text{ Cos. } 2x)^{2a-1} \frac{\text{Tang. } 2x}{\text{Tang.}^2 x} dx = \frac{1}{2a} (-2)^{2a-1} \pi^{2a} B_{2a-1}$ V. T. 158. N°. 14.
- 7) $\int (l \text{ Cos. } 2x)^{2a-1} \frac{dx}{\text{Tang. } x} = -\frac{1}{a} 2^{2a-3} \pi^{2a} B_{2a-1}$ V. T. 157. N°. 6.
- 8) $\int (l \text{ Cos. } 2x)^{q-1} \frac{\text{Cos. } 2x}{\text{Tang. } x} dx = \frac{1}{2} \frac{\Gamma(q)}{(-1)^{q-1}} \sum_0^{\infty} \frac{1}{(a+n+1)^q}$ V. T. 157. N°. 12.

F. Log. en num. $l \text{ Tang. } ax$. TABLE 509.
Circ. Dir. rat. en dén. monôme.

Lim. 0 et $\frac{\pi}{4}$.

- 1) $\int l \text{ Tang. } x \frac{dx}{\text{Cos. } 2x} = -\frac{1}{8} \pi^2$ V. T. 152. N°. 13.
- 2) $\int l \text{ Tang. } x \frac{dx}{\text{Sin. } 4x} = -\infty$ V. T. 153. N°. 11.
- 3) $\int l \text{ Tang. } x \frac{dx}{\text{Tang. } 2x} = -\infty$ V. T. 153. N°. 10.
- 4) $\int l \text{ Tang. } x \frac{\text{Tang. } 2x}{\text{Cos.}^2 x} dx = -\frac{1}{12} \pi^2$ V. T. 340. N°. 4.
- 5) $\int l \text{ Tang. } x \frac{\text{Tang. } x}{\text{Cos. } 2x} dx = -\frac{1}{24} \pi^2$ V. T. 152. N°. 14.
- 6) $\int l \text{ Tang. } x \frac{\text{Sin.}^{2a} x}{\text{Cos.}^{2a+2} x} dx = -\left(\frac{1}{2a+1} \right)^2$ V. T. 307. N°. 1, 12.

$$7) \int l \text{ Tang. } x \frac{\text{Sin.}^{2a-1} x}{\text{Cos.}^{2a+1} x} dx = -\frac{1}{4a} \quad \text{V. T. 307. N}^\circ. 2, 13.$$

$$8) \int l \text{ Tang. } x \cdot \text{Sin. } (p \text{ Cot. } x) \frac{dx}{\text{Sin.}^2 x} = \infty \quad \text{V. T. 47. N}^\circ. 23.$$

$$9) \int l \text{ Tang. } x \cdot \text{Cos. } (p \text{ Tang. } x) \frac{dx}{\text{Cos.}^2 x} = -\frac{1}{p} \text{Si. } (p) \quad \text{V. T. 47. N}^\circ. 22.$$

$$10) \int l \text{ Tang. } x \cdot \text{Tang. } \left(\frac{\pi}{4} + x \right) \frac{dx}{\text{Cos.}^2 x} = \frac{1}{2} (3 - \pi^2) \quad \text{V. T. 152. N}^\circ. 9.$$

$$11) \int l \text{ Tang. } x \frac{\text{Sin.}^3 x}{\text{Cos. } 2x \cdot \text{Cos. } x} dx = -\frac{1}{96} \pi^2 \quad \text{V. T. 152. N}^\circ. 17.$$

$$12) \int l \text{ Tang. } x \left(\frac{\text{Cos. } x - \text{Sin. } x}{\text{Sin. } x} \right)^p \frac{dx}{\text{Sin.}^2 x} = -\frac{\pi}{p} \text{Cosec. } p \pi, 0 > p > -1; \quad \text{V. T. 47. N}^\circ. 18.$$

$$13) \int l \text{ Tang. } x \frac{dx}{\text{Tang.}^a x \cdot \text{Sin. } 2x} = \frac{-1}{2(a+2)^2} \quad \text{V. T. 152. N}^\circ. 1.$$

$$1) \int (l \text{ Tang. } x)^3 \frac{dx}{\text{Cos. } 2x} = -\frac{1}{16} \pi^4 \quad \text{V. T. 154. N}^\circ. 14.$$

$$2) \int (l \text{ Tang. } x)^3 \frac{\text{Tang. } x}{\text{Cos. } 2x} dx = -\frac{1}{240} \pi^4 \quad \text{V. T. 154. N}^\circ. 15.$$

$$3) \int (l \text{ Tang. } x)^3 \frac{\text{Sin. } x \cdot \text{Cos. } x}{\text{Cos. } 2x} dx = -\frac{1}{256} \pi^4 \quad \text{V. T. 154. N}^\circ. 16.$$

$$4) \int (l \text{ Tang. } x)^3 \frac{\text{Sin.}^3 x}{\text{Cos. } 2x \cdot \text{Cos. } x} dx = -\frac{1}{3840} \pi^4 \quad \text{V. T. 154. N}^\circ. 17.$$

$$5) \int (l \text{ Tang. } x)^5 \frac{dx}{\text{Cos. } 2x} = -\frac{1}{8} \pi^6 \quad \text{V. T. 155. N}^\circ. 4.$$

$$\left. \begin{aligned} 6) \int (l \text{ Tang. } x)^5 \frac{\text{Tang. } x}{\text{Cos. } 2x} dx &= -\frac{1}{504} \pi^6 \\ 7) \int (l \text{ Tang. } x)^5 \frac{\text{Sin. } x \cdot \text{Cos. } x}{\text{Cos. } 2x} dx &= -\frac{1}{512} \pi^6 \end{aligned} \right\} \quad \text{V. T. 155. N}^\circ. 5.$$

- 8) $\int (l \text{ Tang. } x)^7 \frac{dx}{\text{Cos. } 2x} = -\frac{17}{32} \pi^3$ V. T. 153. N°. 10.
- 9) $\int (l \text{ Tang. } x)^{2a-1} \frac{dx}{\text{Cos. } 2x} = \frac{1-2^{2a}}{4a} \pi^{2a} B_{2a-1}$ V. T. 158. N°. 5.
- 10) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{\text{Cos. } 2x} = \frac{2^{2a+1}-1}{2^{2a+1}} 1^{2a/1} \sum_1^{\infty} \frac{1}{n^{2a+1}}$ V. T. 158. N°. 4.
- 11) $\int (l \text{ Tang. } x)^{2a-1} \frac{\text{Tang. } x}{\text{Cos. } 2x} dx = -\frac{1}{4a} \pi^{2a} B_{2a-1}$ V. T. 158. N°. 6.
- 12) $\int (l \text{ Tang. } x)^{2a} \frac{\text{Tang. } x + \text{Cot. } x}{\text{Cos. } 2x} dx = \frac{(-1)^a}{b} (2b\pi)^{2a+1} \sum_0^b (-1)^{n-1} B' \left(\frac{n}{2b} \right) \text{Sin. } nq\pi$ V. T. 158. N°. 9.
où $q^2 = \alpha^2 b^2 < 1$, α arbitraire.
- 13) $\int (l \text{ Tang. } x)^a \text{Tang. }^{p-1} x \frac{dx}{\text{Sin. } 2x} = \frac{(-1)^a}{2p^{a+1}} 1^{a+1}$ V. T. 151. N°. 2.
- 14) $\int l (\text{Tang. } x)^{2a-1} \text{Tang. } \left(\frac{\pi}{4} + x \right) \frac{dx}{\text{Sin. } 2x} = -\frac{2^{2a-2}}{a} \pi^{2a} B_{2a-1}$ V. T. 158. N°. 14.
- 15) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{\text{Cos. }^2 \left(\frac{\pi}{4} + x \right)} = (2\pi)^{2a} B_{2a-1}$ V. T. 310. N°. 14.

- 1) $\int l \text{ Tang. } x \frac{dx}{2 - \text{Sin. } 2x} = -\frac{2}{27} \pi^2$ V. T. 153. N°. 3.
- 2) $\int l \text{ Tang. } x \frac{\text{Cos. } 2x}{1 + p \text{Sin. } 2x} dx = \frac{1}{16p} \{4 (\text{Arccos. } p)^2 - \pi^2\}$, $p \leq 1$; V. T. 339. N°. 30.
- 3) $\int l \text{ Tang. } x \frac{\text{Cos. } 2x}{1 - p \text{Sin. } 2x} dx = -\frac{1}{4p} \{ \pi + \text{Arcsin. } p \} \text{Arcsin. } p$, $p < 1$; V. T. 311. N°. 2, 10.
- 4) $\int l \text{ Tang. } x \frac{\text{Tang. } x}{1 - \text{Sin. } x \text{Cos. } x} dx = -\frac{5}{108} \pi^2$ V. T. 153. N°. 4.
- 5) $\int l \text{ Tang. } x \frac{\text{Cos. } \lambda - \text{Tang. } x}{1 - \text{Sin. } 2x \text{Cos. } \lambda} dx = \frac{1}{6} \pi^2 - \frac{1}{2} \pi \lambda + \frac{1}{4} \lambda^2$ V. T. 153. N°. 6.
- 6) $\int l \text{ Tang. } x \frac{\text{Sin. } 2x}{4 - 3 \text{Sin. }^2 2x} dx = -\frac{1}{54} \pi^2$ V. T. 153. N°. 7.
- 7) $\int l \text{ Tang. } x \frac{\text{Cos. } 2x}{1 - \text{Sin. }^2 \lambda \text{Sin. }^2 2x} dx = -\frac{1}{4} \pi \lambda \text{Cosec } \lambda$ V. T. 153. N°. 9.

- 8) $\int l \text{ Tang. } x \frac{\cos. 2x}{1 - \sin.^2 2x \sin.^2 \lambda} dx = -\frac{1}{4} \pi \lambda \operatorname{Cosec.} \lambda \quad \text{V. T. 340. N}^\circ. 16.$
- 9) $\int l \text{ Tang. } x \frac{\sin. 4x}{1 - p^2 \sin.^2 2x} dx = -\frac{1}{4 p^2} (\operatorname{Arcsin.} p)^2, p < 1; \quad \text{V. T. 311. N}^\circ. 2, 10.$
- 10) $\int l \text{ Tang. } x \frac{\cos. 2x}{1 - p^2 \sin.^2 2x} dx = -\frac{\pi}{4 p} \operatorname{Arcsin.} p, p \leq 1; \quad \text{V. T. 340. N}^\circ. 3.$
- 11) $\int l \text{ Tang. } x \frac{\cos. 2x}{1 + p^2 \sin.^2 2x} dx = -\frac{\pi}{4 p} l \{p + \sqrt{1 + p^2}\} \quad \text{V. T. 369 N}^\circ. 3.$
- 12) $\int l \text{ Tang. } x \frac{\cos. 2x}{\cos.^2 2x + p^2 \sin.^2 2x} dx = -\frac{\pi}{4 \sqrt{1 - p^2}} \operatorname{Arccos.} p, p < 1; \quad \text{V. T. 340. N}^\circ. 5.$
- 13) $\int l \text{ Tang. } x \frac{\cos. 2x}{1 + \left(\frac{e^p - e^{-p}}{2}\right)^2 \sin.^2 2x} dx = -\frac{1}{2} p \frac{\pi}{e^p - e^{-p}} \quad \text{V. T. 153. N}^\circ. 5.$

- 1) $\int (l \text{ Tang. } x)^2 \frac{dx}{2 + \sin. 2x} = \frac{4}{243} \pi^3 \sqrt{3} \quad \text{V. T. 156. N}^\circ. 1.$
- 2) $\int (l \text{ Tang. } x)^2 \frac{dx}{1 + \sin. 2x \cos. \lambda} = \frac{\pi^2 - \lambda^2}{6} \lambda \operatorname{Cosec.} \lambda \quad \text{V. T. 156. N}^\circ. 3.$
- 3) $\int (l \text{ Tang. } x)^2 \frac{dx}{2 - \sin. 2x} = \frac{5}{243} \pi^3 \sqrt{3} \quad \text{V. T. 156. N}^\circ. 2.$
- 4) $\int (l \text{ Tang. } x)^2 \frac{dx}{1 - \sin. 2x \cos. \lambda} = 2 \lambda \operatorname{Cosec.} \lambda \left\{ \frac{1}{6} \pi^2 - \frac{1}{4} \pi \lambda + \frac{1}{12} \lambda^2 \right\} \quad \text{V. T. 156. N}^\circ. 4.$
- 5) $\int (l \text{ Tang. } x)^2 \frac{dx}{\sin.^4 x + \cos.^4 x} = \frac{3}{64} \pi^3 \sqrt{2} \quad \text{V. T. 154. N}^\circ. 2.$
- 6) $\int (l \text{ Tang. } x)^2 \frac{dx}{1 - \sin.^2 x \cos.^2 x} = \frac{1}{27} \pi^3 \sqrt{3} \quad \text{V. T. 154. N}^\circ. 3.$
- 7) $\int (l \text{ Tang. } x)^2 \frac{\sin. 2x}{1 - \sin.^2 x \cos.^2 x} dx = \frac{2}{243} \pi^3 \sqrt{3} \quad \text{V. T. 154. N}^\circ. 4.$
- 8) $\int (l \text{ Tang. } x)^4 \frac{dx}{1 + \sin. 2x \cos. \lambda} = \frac{\pi^2 - \lambda^2}{\sin. \lambda} \frac{7 \pi^2 - 3 \lambda^2}{5} \lambda \quad \text{V. T. 156. N}^\circ. 5.$

F. Log. en num. (*l Tang. a x*)^b. TABLE 512 suite. Lim 0 et $\frac{\pi}{4}$.
Circ. Dir. rat. en dén. binôme.

$$\begin{aligned} 9) \int (l \text{ Tang. } x)^{2a} \frac{dx}{2 + \text{Sin. } 2x} &= \frac{(-1)^{a+1}}{2 \sqrt{3}} (2\pi)^{2a+1} B''\left(\frac{1}{3}\right) \quad \text{V. T. 159. N}^\circ. 1. \\ 10) \int (l \text{ Tang. } x)^{2a} \frac{dx}{2 - \text{Sin. } 2x} &= \frac{(-1)^{a+1}}{2 \sqrt{3}} (2\pi)^{2a+2} B''\left(\frac{1}{6}\right) \quad \text{V. T. 159. N}^\circ. 2. \\ 11) \int (l \text{ Tang. } x)^{2a} \frac{dx}{1 - \text{Sin. } 2x \text{ Cos. } 2p\pi} &= \frac{1}{2} \text{Cosec. } 2p\pi. (-1)^{a+1} (2\pi)^{2a-1} B''(p) \quad \text{V. T. 159. N}^\circ. 5. \end{aligned}$$

F. Log. en num. *l Tang. a x*. TABLE 515. Lim. 0 et $\frac{\pi}{4}$.
Circ. Dir. rat. en dén. à fact. mon. et bin.

$$\begin{aligned} 1) \int l \text{ Tang. } x \frac{dx}{\text{Cos. } x (\text{Sin. } x + \text{Cos. } x)} &= -\frac{1}{12} \pi^2 \quad \text{V. T. 316. N}^\circ. 6. \\ 2) \int l \text{ Tang. } x \frac{dx}{\text{Cos. } x (\text{Cos. } x - \text{Sin. } x)} &= -\frac{1}{6} \pi^2 \quad \text{V. T. 316. N}^\circ. 7. \\ 3) \int l \text{ Tang. } x \frac{\text{Tang. } p x}{\text{Cos. } x - \text{Sin. } x} \frac{dx}{\text{Sin. } 2x} &= -\frac{1}{2} \sum_0^\infty \frac{1}{(p+n)^2} \quad \text{V. T. 152. N}^\circ. 10. \\ 4) \int l \text{ Tang. } x \frac{\text{Sin. }^a 2x}{\text{Cos. }^{2a} x - \text{Sin. }^{2a} x} \frac{dx}{\text{Sin. } 2x} &= -2^{a-4} \frac{\pi^2}{a^2} \quad \text{V. T. 152. N}^\circ. 18. \\ 5) \int l \text{ Tang. } x \frac{\text{Sin. }^2 2x}{\text{Sin. }^4 x + \text{Cos. }^4 x} \frac{dx}{\text{Cos. } 2x} &= -\frac{\pi}{4(2 + \sqrt{2})} \quad \text{V. T. 153. N}^\circ. 16. \\ 6) \int l \text{ Tang. } x \frac{\text{Cos. } 2x}{\text{Tang. }^p x + \text{Cot. }^p x} \frac{dx}{\text{Sin. }^2 2x} &= -\frac{\pi^2}{16p^2} \text{Sin. } \frac{\pi}{2p} \text{Sec. }^2 \frac{\pi}{2p} \quad \text{V. T. 152. N}^\circ. 19. \\ 7) \int l \text{ Tang. } x \frac{dx}{\text{Sin. }^2 2x (\text{Tang. }^p x - \text{Cot. }^p x)} &= \frac{\pi^2}{16p^2} \text{Sec. }^2 \frac{\pi}{2p} \quad \text{V. T. 152. N}^\circ. 20. \\ 8) \int l \text{ Tang. } x \frac{\text{Tang. }^q x - \text{Cot. }^q x}{\text{Tang. }^p x + \text{Cot. }^p x} \frac{dx}{\text{Sin. } 2x} &= \frac{\pi^2}{8p^2} \text{Sin. } \frac{q\pi}{2p} \text{Sec. }^2 \frac{q\pi}{2p} \quad \text{V. T. 153. N}^\circ. 12. \\ 9) \int l \text{ Tang. } x \frac{\text{Tang. }^q x + \text{Cot. }^q x}{\text{Tang. }^p x - \text{Cot. }^p x} \frac{dx}{\text{Sin. } 2x} &= \frac{\pi^2}{8p^2} \text{Sec. }^2 \frac{q\pi}{2p} \quad \text{V. T. 153. N}^\circ. 13. \\ 10) \int l \text{ Tang. } x \frac{dx}{(\text{Cos. } x + \text{Sin. } x)^2} &= -l2 \quad \text{V. T. 153. N}^\circ. 14. \\ 11) \int l \text{ Tang. } x \frac{\text{Sin. }^{p-1} x}{(\text{Cos. } x - \text{Sin. } x)^{p+1}} dx &= -\frac{1}{p} \pi \text{Cosec. } p\pi, p < 1; \quad \text{V. T. 49. N}^\circ. 24. \end{aligned}$$

- 12) $\int l \text{ Tang. } x \frac{\text{Sin. } p-1 \text{ } 2x \cdot \text{Cos. } 2x}{(1 + \text{Sin. } 2x)^{p+1}} dx = -\frac{1}{p} \frac{\Gamma(p) \sqrt{\pi}}{2^{p+1} \Gamma(p + \frac{1}{2})}, p \leq 1; \text{ V. T. 49. N}^\circ. 5.$
- 13) $\int l \text{ Tang. } x \frac{\text{Tang. } p x - \text{Cot. } p x}{(\text{Tang. } p x + \text{Cot. } p x)^2} \frac{dx}{\text{Sin. } 2x} = \frac{\pi}{8p^2} \text{ V. T. 49. N}^\circ. 19.$
- 14) $\int l \text{ Tang. } x \frac{p \text{ Tang. } p x (1 - \text{Tang. }^{2q} x) + q \text{ Tang. } q x (1 - \text{Tang. }^{2p} x)}{(\text{Tang. } p+q x + 1)^2} \frac{dx}{\text{Sin. } 2x} =$
 $-\frac{1}{2} \frac{\pi}{p+q} \text{Sec. } \left\{ \frac{q-p}{p+q} \frac{\pi}{2} \right\}, p < q; \text{ V. T. 49. N}^\circ. 13.$
- 15) $\int l \text{ Tang. } x \frac{p \text{ Tang. } p x (1 - \text{Tang. }^{2q} x) - q \text{ Tang. } q x (1 - \text{Tang. }^{2p} x)}{(\text{Tang. } p+q x - 1)^2} \frac{dx}{\text{Sin. } 2x} =$
 $-\frac{1}{2} \frac{\pi}{p+q} \text{Tang. } \left\{ \frac{q-p}{p+q} \frac{\pi}{2} \right\}, p < q; \text{ V. T. 49. N}^\circ. 14.$
- 16) $\int l \text{ Tang. } x \frac{(q-p)(\text{Tang. } p+q x - \text{Cot. } p+q x) - (p+q)(\text{Tang. } p-q x - \text{Cot. } p-q x)}{(\text{Tang. } p x + \text{Cot. } p x)^2} \frac{dx}{\text{Sin. } 2x} =$
 $-\frac{\pi}{4p} \text{Sec. } \frac{q\pi}{2p}, p > q; \text{ V. T. 49. N}^\circ. 18.$
- 17) $\int l \text{ Tang. } x \frac{(q-p)(\text{Tang. } p+q x - \text{Cot. } p+q x) - (p+q)(\text{Tang. } p-q x - \text{Cot. } p-q x)}{(\text{Tang. } p x - \text{Cot. } p x)^2} \frac{dx}{\text{Sin. } 2x} =$
 $-\frac{\pi}{4p} \text{Tang. } \frac{q\pi}{2p}, p > q; \text{ V. T. 49. N}^\circ. 17.$
- 18) $\int l \text{ Tang. } x \frac{dx}{(\text{Tang. } x + \text{Cot. } x)^{2p+1} \cdot \text{Tang. } 2x \cdot \text{Sin. } 2x} = -\frac{\{\Gamma(p)\}^2}{32p\Gamma(2p)} \text{ V. T. 49. N}^\circ. 21.$
- 19) $\int l \text{ Tang. } 2x \frac{\text{Sin. } (pl \text{ Tang. } x)}{\text{Sin. } 2x} dx = \infty \text{ V. T. 329. N}^\circ. 13.$
- 20) $\int l \text{ Tang. } \left(\frac{\pi}{4} \pm x \right) \frac{\text{Cos. } (pl \text{ Tang. } x)}{\text{Sin. } 2x} dx = \infty \text{ V. T. 329. N}^\circ. 7.$

- 1) $\int (l \text{ Tang. } x)^2 \frac{\text{Tang. } q x + \text{Cot. } q x}{\text{Tang. } p x + \text{Cot. } p x} \frac{dx}{\text{Sin. } 2x} = \frac{\pi^3}{16p^3} \left\{ 2 \text{Sec. }^3 \frac{q\pi}{2p} - \text{Sec. } \frac{q\pi}{2p} \right\} \text{ V. T. 154. N}^\circ. 5.$
- 2) $\int (l \text{ Tang. } x)^2 \frac{\text{Tang. } q x - \text{Cot. } q x}{\text{Tang. } p x - \text{Cot. } p x} \frac{dx}{\text{Sin. } 2x} = \frac{\pi^3}{16p^3} \text{Sin. } \frac{q\pi}{2p} \cdot \text{Sec. }^3 \frac{q\pi}{2p} \text{ V. T. 154. N}^\circ. 6.$

$$3) \int (l \text{ Tang. } x)^2 \frac{\text{Tang.}^a x + \text{Cot.}^a x}{(\text{Tang.}^a x - \text{Cot.}^a x)^2} \frac{dx}{\text{Sin. } 2x} = \frac{\pi}{8a^3} \quad \text{V. T. 313. N}^\circ. 4.$$

$$4) \int (l \text{ Tang. } x)^3 \frac{dx}{\text{Cos. } x (\text{Cos. } x + \text{Sin. } x)} = -\frac{7}{120} \pi^4 \quad \text{V. T. 154. N}^\circ. 10.$$

$$5) \int (l \text{ Tang. } x)^3 \frac{dx}{\text{Cos. } x (\text{Cos. } x - \text{Sin. } x)} = -\frac{1}{15} \pi^4 \quad \text{V. T. 154. N}^\circ. 11.$$

$$6) \int (l \text{ Tang. } x)^5 \frac{dx}{\text{Cos. } x (\text{Cos. } x - \text{Sin. } x)} = -\frac{8}{63} \pi^6 \quad \text{V. T. 155. N}^\circ. 3.$$

$$7) \int (l \text{ Tang. } x)^5 \frac{dx}{\text{Cos. } x (\text{Cos. } x + \text{Sin. } x)} = -\frac{31}{252} \pi^6 \quad \text{V. T. 155. N}^\circ. 2.$$

$$8) \int (l \text{ Tang. } x)^7 \frac{dx}{\text{Cos. } x (\text{Cos. } x + \text{Sin. } x)} = -\frac{127}{240} \pi^8 \quad \text{V. T. 155. N}^\circ. 9.$$

$$9) \int (l \text{ Tang. } x)^a \frac{dx}{\text{Cos. } x (\text{Cos. } x + \text{Sin. } x)} = \frac{2^a - 1}{2^a} (-1)^a 1^{a/1} \sum_1^{\infty} \frac{1}{n^{a+1}} \quad \text{V. T. 157. N}^\circ. 2.$$

$$10) \int (l \text{ Tang. } x)^a \frac{dx}{\text{Cos. } x (\text{Cos. } x - \text{Sin. } x)} = (-1)^a 1^{a/1} \sum_1^{\infty} \frac{1}{n^{a+1}} \quad \text{V. T. 157. N}^\circ. 3.$$

$$11) \int (l \text{ Tang. } x)^{b-1} \frac{\text{Tang.}^a x}{\text{Cos. } x + \text{Sin. } x} \frac{dx}{\text{Cos. } x} = \frac{1^{b-1/1}}{(-1)^{b-1}} \sum_0^{\infty} \frac{(-1)^n}{(a+n+1)^b} \quad \text{V. T. 157. N}^\circ. 11.$$

$$12) \int (l \text{ Tang. } x)^{b-1} \frac{\text{Tang.}^a x}{\text{Cos. } x - \text{Sin. } x} \frac{dx}{\text{Cos. } x} = (-1)^{b-1} 1^{b-1/1} \sum_0^{\infty} \frac{1}{(a+n+1)^b} \quad \text{V. T. 157. N}^\circ. 12.$$

$$13) \int (l \text{ Tang. } x)^{2a-1} \frac{dx}{\text{Cos. } x (\text{Cos. } x + \text{Sin. } x)} = \frac{1 - 2^{2a-1}}{2^a} \pi^{2a} B_{2a-1} \quad \text{V. T. 157. N}^\circ. 5.$$

$$14) \int (l \text{ Tang. } x)^{2a-1} \frac{dx}{\text{Cos. } x (\text{Cos. } x - \text{Sin. } x)} = -\frac{1}{a} 2^{2a-2} \pi^{2a} B_{2a-1} \quad \text{V. T. 157. N}^\circ. 6.$$

$$15) \int (l \text{ Tang. } x)^{2a} \frac{1}{\text{Tang.}^q x + \text{Cot.}^q x} \frac{dx}{\text{Sin. } 2x} = \frac{1}{4} (-1)^{a+1} \left(\frac{2\pi}{q} \right)^{2a+1} B' \left(\frac{1}{4} \right) \quad \text{V. T. 158. N}^\circ. 15.$$

$$16) \int (l \text{ Tang. } x)^{2a-1} \frac{1}{\text{Tang.}^q x - \text{Cot.}^q x} \frac{dx}{\text{Sin. } 2x} = \frac{1}{8} (-1)^a \left(\frac{2\pi}{q} \right)^{2a} B' \left(\frac{1}{2} \right) \quad \text{V. T. 158. N}^\circ. 16.$$

$$17) \int (l \text{ Tang. } x)^{p-1} \frac{\text{Cos. } \lambda - \text{Tang. } x}{1 - \text{Cos. } \lambda} \frac{\text{Tang.}^q x}{\text{Sin. } 2x} dx = \frac{1}{2} (-1)^{p-1} \Gamma(p) \sum_1^{\infty} \frac{\text{Cos. } n\lambda}{(q+n-1)^p} \quad \text{V. T. 159. N}^\circ. 7.$$

- 1) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{dx}{\operatorname{Sin}. 2x} = \pm \frac{1}{8} \pi^2$ V. T. 309. N°. 6.
- 2) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{dx}{\operatorname{Tang}. 2x} = \pm \frac{1}{16} \pi^2$ V. T. 336. N°. 2.
- 3) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Tang}^{p-1} x + \operatorname{Cot}^{p-1} x}{\operatorname{Sin}. 2x} dx = \mp \frac{\pi}{2(p-1)} \operatorname{Cot}.\frac{1}{2} p \pi, p < 1$; V. T. 47. N°. 7.
- 4) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{(p+q)(\operatorname{Tang}^{p+q} x - \operatorname{Cot}^{p+q} x) + (p-q)(\operatorname{Tang}^{p-q} x - \operatorname{Cot}^{p-q} x)}{\operatorname{Sin}. 2x} dx =$
 $= \pm \frac{\pi \operatorname{Sin}. p \pi}{\operatorname{Cos}. p \pi + \operatorname{Cos}. q \pi}, p+q < 1$; V. T. 47. N°. 20.
- 5) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{(\operatorname{Cos}. 2^{p-1} 2x - 1)(p \operatorname{Sin}.^2 2x + \operatorname{Cos}. 2x) - \operatorname{Sin}.^2 2x}{\operatorname{Sin}.^2 x. \operatorname{Cos}.^p 2x} dx = \pm \pi \operatorname{Cot}. p \pi$ V. T. 47. N°. 10.
- 6) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Sin}. 2x}{1+p \operatorname{Cos}. 2x} dx = \pm \frac{1}{16p} \{\pi^2 - 4(\operatorname{Arccos}. p)^2\}, p^2 \leq 1$; V. T. 339. N°. 28.
- 7) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Sin}. 2x}{1-p \operatorname{Cos}. 2x} dx = \pm \frac{1}{4p} \operatorname{Arcsin}. p (\pi + \operatorname{Arcsin}. p), p^2 \leq 1$; V. T. 315. N°. 6, 8.
- 8) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Sin}. 2x}{1-p^2 \operatorname{Cos}.^2 2x} dx = \pm \frac{\pi}{4p} \operatorname{Arcsin}. p, p^2 \leq 1$; V. T. 340. N°. 9.
- 9) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Sin}. 4x}{1-p^2 \operatorname{Cos}.^2 2x} dx = \pm \frac{1}{4p^2} (\operatorname{Arcsin}. p)^2, p^2 \leq 1$, V. T. 315. N°. 6, 8.
- 10) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Sin}. 2x}{1-\operatorname{Cos}.^2 2x. \operatorname{Sin}.^2 \lambda} dx = \pm \frac{1}{4} \pi \lambda \operatorname{Cosec}. \lambda$ V. T. 345. N°. 11.
- 11) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Sin}. 2x}{1+p^2 \operatorname{Cos}.^2 2x} dx = \pm \frac{\pi}{4p} l\{p + \sqrt{1+p^2}\}$ V. T. 368. N°. 6.
- 12) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Sin}. 2x}{\operatorname{Sin}.^2 2x + p^2 \operatorname{Cos}.^2 2x} dx = \pm \frac{\pi}{4\sqrt{1-p^2}} \operatorname{Arccos} p, p < 1$; V. T. 340. N°. 12.
- 13) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Sin}. 2x}{1-p^4 \operatorname{Cos}.^4 2x} dx = \pm \frac{\pi}{8p} [l\{p + \sqrt{1+p^2}\} + \operatorname{Arcsin} p], p < 1$; $\left. \begin{array}{l} \text{V. T. 315.} \\ \text{N°. 8, 11.} \end{array} \right\}$
- 14) $\int l \operatorname{Tang}.\left(\frac{\pi}{4} \pm x\right) \frac{\operatorname{Sin}. 4x. \operatorname{Cos}. 2x}{1-p^4 \operatorname{Cos}.^4 2x} dx = \pm \frac{\pi}{4p^3} [\operatorname{Arcsin}. p - l\{p + \sqrt{1+p^2}\}], p < 1$; $\left. \begin{array}{l} \text{V. T. 315.} \\ \text{N°. 8, 11.} \end{array} \right\}$

- 1) $\int l \cos. x \frac{dx}{(\cos. x + p \sin. x)^2} = \frac{1}{1+p^2} \left\{ -\frac{\pi}{4} + \frac{1}{p} l(1+p) - \frac{1-p^3}{2(p+1)p} l2 \right\}, p < 1; \text{V. T. 48. N}^\circ. 1.$
- 2) $\int l \cos. x \frac{\cos. 2x dx}{(1-p \sin. 2x)^2} = \frac{1}{4p} l(1-p) - \frac{1}{4(1-p)} l2 + \frac{1}{4} \text{Arccos.}(-p). \text{Cosec.} \{ \text{Arccos.}(-p) \}, p < 1; \text{V. T. 48. N}^\circ. 10.$
- 3) $\int l \cos. x \frac{\cos. 2x}{(1 - \cos. \lambda. \sin. 2x)^2} dx = \frac{\pi - \lambda}{4 \sin. \lambda} + \frac{l \sin. \frac{1}{2} \lambda}{2 \cos. \lambda} - \frac{1}{4} \frac{1 + \cos. \lambda}{1 - \cos. \lambda} \text{Sec.} \lambda l2 \text{ V. T. 48. N}^\circ. 8.$
- 4) $\int l \left\{ 2 \sin.^2 \left(\frac{\pi}{4} + x \right) \right\} \frac{dx}{\text{Tang.} 2x} = \frac{1}{24} \pi^2 \text{ V. T. 160. N}^\circ. 1.$
- 5) $\int l \left\{ 2 \sin.^2 \left(\frac{\pi}{4} - x \right) \right\} \frac{dx}{\text{Tang.} 2x} = -\frac{1}{12} \pi^2 \text{ V. T. 160. N}^\circ. 5.$
- 6) $\int l(1 + \text{Tang.} x) \frac{dx}{\sin. 2x} = \frac{1}{24} \pi^2 \text{ V. T. 160. N}^\circ. 1.$
- 7) $\int l(1 - \text{Tang.} x) \frac{dx}{\sin. 2x} = -\frac{1}{12} \pi^2 \text{ V. T. 160. N}^\circ. 5.$
- 8) $\int l(\sin. x \cos. x) \frac{\sin. 2^a x}{\cos. 2^a + 2, x} dx = \frac{1}{2a+1} \left\{ (-1)^{a+1} \frac{\pi}{2} - l2 + \frac{1}{2a+1} + 2(-1)^{a+1} \sum_1^a \frac{(-1)^n}{2n-1} \right\} \text{V. T. 307. N}^\circ. 1, 12.$
- 9) $\int l(\sin. x \cos. x) \frac{\sin. 2^{a-1} x}{\cos. 2^{a-1} x} dx = \frac{1}{2a} \left\{ \frac{\pi}{2} (-1)^a l2 - l2 + \frac{1}{2a} + (-1)^a \sum_1^{a-1} \frac{(-1)^n}{n} \right\} \text{V. T. 307. N}^\circ. 2, 13.$
- 10) $\int l \left\{ \frac{\cos. 2x}{\cos.^2 x} \right\} \frac{dx}{\sin. 2x} = -\frac{1}{24} \pi^2 \text{ V. T. 160. N}^\circ. 10.$
- 11) $\int l \left\{ \frac{1 - \sin. 2x \cos. \lambda}{\cos.^2 x} \right\} \frac{dx}{\sin. 2x} = \frac{1}{6} \pi^2 - \frac{1}{2} \pi \lambda + \frac{1}{4} \lambda^2 \text{ V. T. 160. N}^\circ. 14.$

- 1) $\int l \cos. x. (l \text{Tang.} x)^2 \frac{dx}{\sin. 2x} = -\frac{7}{11520} \pi^4 \text{ V. T. 305. N}^\circ. 6.$
- 2) $\int l \cos. x. (l \text{Tang.} x)^{2a} (2a+1 + \sin. 2x. \text{Tang.} 2x. l \text{Tang.} x) \frac{dx}{\sin. 4x} = \frac{1}{16(a+1)} \pi^{2a+2} B_{2a+1} \text{ V. T. 310. N}^\circ. 11.$
- 3) $\int l \cos. 2x. (l \text{Tang.} x)^2 \frac{dx}{\sin. 2x} = -\frac{1}{384} \pi^4 \text{ V. T. 305. N}^\circ. 7.$
- 4) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right). (l \sin. 2x)^2 \frac{dx}{\text{Tang.} 2x} = \pm \frac{1}{96} \pi^4 \text{ V. T. 336. N}^\circ. 13.$

F. Log. en num. Autre forme : deux fact. log. TABLE 517 suite.
Circ. Dir. rat. en dén.

Lim. 0 et $\frac{\pi}{4}$.

- 5) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Sin. } 2x)^4 \frac{dx}{\text{Tang. } 2x} = \pm \frac{1}{80} \pi^6 \quad \text{V. T. 336. N}^\circ. 13.$
- 6) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Sin. } 2x)^6 \frac{dx}{\text{Tang. } 2x} = \pm \frac{17}{448} \pi^8 \quad \text{V. T. 336. N}^\circ. 14.$
- 7) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Sin. } 2x)^{a-1} \frac{dx}{\text{Tang. } 2x} = \pm \frac{1}{2a} (-1)^{a+1} \sum_0^{\infty} \frac{1}{(2n+1)^{a+1}} \quad \text{V. T. 336. N}^\circ. 17.$
- 8) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Sin. } 2x)^{2a} \frac{dx}{\text{Tang. } 2x} = \pm \frac{2^{2a+2} - 1}{8 \cdot a + 1 \cdot 2a + 1} \pi^{2a+2} B_{2a+1} \quad \text{V. T. 336. N}^\circ. 16.$
- 9) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Sin. } 2x)^{2a-1} \frac{dx}{\text{Tang. } 2x} = \pm \frac{1 - 2^{2a+1}}{a \cdot 2^{2a+3}} 1^{2a+1} \sum_1^{\infty} \frac{1}{n^{2a+1}} \quad \text{V. T. 336. N}^\circ. 15.$
- 10) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Tang. } x)^2 \frac{dx}{\text{Sin. } 2x} = \pm \frac{1}{48} \pi^4 \quad \text{V. T. 310. N}^\circ. 1.$
- 11) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Tang. } x)^4 \frac{dx}{\text{Sin. } 2x} = \pm \frac{1}{40} \pi^6 \quad \text{V. T. 310. N}^\circ. 5.$
- 12) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Tang. } x)^6 \frac{dx}{\text{Sin. } 2x} = \pm \frac{17}{224} \pi^8 \quad \text{V. T. 310. N}^\circ. 8.$
- 13) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Tang. } x)^{2a} \frac{dx}{\text{Sin. } 2x} = \pm \frac{2^{2a+2} - 1}{4 \cdot a + 1 \cdot 2a + 1} \pi^{2a+2} B_{2a+1} \quad \text{V. T. 310. N}^\circ. 9.$
- 14) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \cdot (l \text{Tang. } x)^{2a-1} \frac{dx}{\text{Sin. } 2x} = \pm \frac{1 - 2^{2a+1}}{2^{2a+2} a} 1^{2a+1} \sum_1^{\infty} \frac{1}{n^{2a+1}} \quad \text{V. T. 310. N}^\circ. 10.$
- 15) $\int (l \text{Tang. } x)^{2b} \cdot l \frac{1 - \text{Sin. } 2x \cdot \text{Cos. } 2p\pi}{\text{Cos. }^2 x} \frac{dx}{\text{Sin. } 2x} = \frac{(2\pi)^{2b+1}}{2(2b+1)} \left\{ (-1)^{b+1} B'(p) - \frac{1}{2b+2} B_{2+2b} \right\} \quad \text{V. T. 161. N}^\circ. 12.$

F. Log. en num. Log. de Log.
Circ. Dir. rat. en dén.

TABLE 518.

Lim. 0 et $\frac{\pi}{4}$.

- 1) $\int l \text{Cot. } x \frac{\text{Tang. } x}{\text{Sin. } 2x} dx = -\frac{1}{2q} (A + lq) \quad \text{V. T. 190. N}^\circ. 1.$
- 2) $\int l \text{Cot. } x \frac{dx}{2 - \text{Sin. } 2x} = \frac{\pi}{\sqrt{3}} \left\{ \frac{5}{6} l 2\pi - l \Gamma \left(\frac{1}{6} \right) \right\} \quad \text{V. T. 191. N}^\circ. 7.$
- 3) $\int l \text{Cot. } x \frac{dx}{1 + \text{Sin. } 2x \cdot \text{Cos. } \lambda} = \frac{1}{2} \pi \text{Cosec. } \lambda \cdot l \frac{(2\pi)^{\frac{\lambda}{\pi}} \Gamma \left(\frac{\pi + \lambda}{2\pi} \right)}{\Gamma \left(\frac{\pi - \lambda}{2\pi} \right)} \quad \text{V. T. 190. N}^\circ. 9.$

$$4) \int l \cot x \frac{dx}{(\sin x + \cos x)^2} = \frac{1}{2} Z' \left(\frac{1}{2} \right) + \frac{1}{2} l 2 \pi \quad \text{V. T. 190. N}^\circ. 7.$$

$$5) \int l \cot x \frac{\text{Tang.}^a x + \text{Cot.}^a x}{\text{Tang.}^b x + \text{Cot.}^b x \sin 2x} dx = \frac{\pi}{4b} \text{Sec.} \frac{a\pi}{2b} l 2\pi + \frac{\pi}{2b} \sum_1^b (-1)^{n-1} \cos \left(\frac{n-\frac{1}{2}}{b} a\pi \right) l \frac{\Gamma \left(\frac{b+n-\frac{1}{2}}{2b} \right)}{\Gamma \left(\frac{n-\frac{1}{2}}{2b} \right)}, \quad \left. \begin{array}{l} a+b \\ \text{impair;} \end{array} \right\} \text{V. T. 191. N}^\circ. 8, 9.$$

$$6) \quad = \frac{\pi}{4b} \text{Sec.} \frac{a\pi}{2b} l \pi + \frac{\pi}{2b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} \cos \left(\frac{n-\frac{1}{2}}{b} a\pi \right) l \frac{\Gamma \left(\frac{b-n+\frac{1}{2}}{b} \right)}{\Gamma \left(\frac{n-\frac{1}{2}}{b} \right)}, \quad \left. \begin{array}{l} a+b \\ \text{pair;} \end{array} \right\}$$

$$7) \int l(p + l \text{Tang.} x) \frac{\text{Tang.}^q x}{\sin 2x} dx = \frac{1}{2q} \left\{ lp - e^{-pq} \text{Ei.}(pq) \right\} \quad \text{V. T. 325. N}^\circ. 6.$$

$$8) \int l(p - l \text{Tang.} x) \frac{\text{Tang.}^q x}{\sin 2x} dx = \frac{1}{2q} \left\{ lp + e^{pq} \text{Ei.}(-pq) \right\} \quad \text{V. T. 325. N}^\circ. 7.$$

$$9) \int l \{a^2 + (l \text{Tang.} x)^2\} dx = \pi l \frac{2\Gamma \left(\frac{2a+3\pi}{4\pi} \right)}{\Gamma \left(\frac{2a+\pi}{4\pi} \right)} - \frac{1}{2} \pi l \frac{\pi}{2} \quad \text{V. T. 190. N}^\circ. 10.$$

$$10) \int l \cot x \frac{\text{Tg.}^p x - \text{Cot.}^p x}{\cos 2x} dx = -\frac{\pi}{2} (l\pi - \Lambda) \text{Tg.} \frac{p\pi}{2} + \pi \sum_{n=1}^{\infty} \left[\frac{l\{(2n+1)\pi - p\pi\}}{(2n+1)\pi - p\pi} - \frac{l\{(2n+1)\pi + p\pi\}}{(2n+1)\pi + p\pi} \right] \quad \text{V. T. 190. N}^\circ. 6.$$

$$11) \int l \cot x (l \cot x)^{p-1} \frac{\text{Tang.}^q x}{\sin 2x} dx = \frac{\Gamma(p)}{2q^p} \left\{ lq - Z'(p) \right\} \quad \text{V. T. 190. N}^\circ. 2.$$

$$1) \int l \text{Tang.} x \frac{dx \vee \cos 2x}{\cos^3 x} = -\frac{1}{4} \pi \left(\frac{1}{2} + l 2 \right) \quad \text{V. T. 162. N}^\circ. 1.$$

$$2) \int l \text{Tang.} x \frac{\sin x dx \vee \cos 2x}{\cos^4 x} = \frac{1}{3} \left(l 2 - \frac{4}{3} \right) \quad \text{V. T. 162. N}^\circ. 2.$$

$$3) \int l \text{Tang.} x \frac{(\cos 2x)^{a-1}}{\cos^{2a+1} x} dx = -\frac{1^{a/2} \pi}{2^{a+2} 1^{a/1}} \{ \Lambda + Z'(a+1) + 2l 2 \} \quad \text{V. T. 162. N}^\circ. 3.$$

$$4) \int l \text{Tang.} x \frac{\text{Tang.} x}{\vee \cos 2x} dx = -\frac{1}{8} \pi l 2 \quad \text{V. T. 163. N}^\circ. 12.$$

- 5) $\int l \text{ Tang. } x \frac{\text{Tang.}^3 x}{\sqrt{\text{Cos. } 2x}} dx = \frac{1}{4} (l2 - 1) \quad \text{V. T. 163 N}^\circ. 13.$
- 6) $\int l \text{ Tang. } x \frac{dx}{\text{Cos. } x \sqrt{\text{Cos. } 2x}} = -\frac{1}{2} \pi l2 \quad \text{V. T. 163. N}^\circ. 2.$
- 7) $\int l \text{ Tang. } x \frac{\text{Sin. } x}{\text{Cos.}^2 x \sqrt{\text{Cos. } 2x}} dx = l2 - 1 \quad \text{V. T. 163. N}^\circ. 3.$
- 8) $\int l \text{ Tang. } x \frac{\text{Sin.}^2 x}{\text{Cos.}^3 x \sqrt{\text{Cos. } 2x}} dx = \frac{1}{4} \pi \left(\frac{1}{2} - l2 \right) \quad \text{V. T. 163. N}^\circ. 4.$
- 9) $\int l \text{ Tang. } x \frac{\text{Sin. } x}{\text{Cos.}^4 x \sqrt{\text{Cos. } 2x}} dx = \frac{2}{3} \left(l2 - \frac{5}{6} \right) \quad \text{V. T. 163. N}^\circ. 5.$
- 10) $\int l \text{ Tang. } x \frac{\text{Sin.}^4 x}{\text{Cos.}^5 x \sqrt{\text{Cos. } 2x}} dx = \frac{3}{16} \pi \left(\frac{7}{12} - l2 \right) \quad \text{V. T. 163. N}^\circ. 6.$
- 11) $\int l \text{ Tang. } x \frac{\text{Sin.}^5 x}{\text{Cos.}^6 x \sqrt{\text{Cos. } 2x}} dx = \frac{8}{15} \left(l2 - \frac{47}{60} \right) \quad \text{V. T. 163. N}^\circ. 7.$
- 12) $\int l \text{ Tang. } x \frac{dx}{\text{Cos. } x \sqrt{\text{Cos.}^3 x - \text{Sin.}^3 x}} = -\frac{1}{27} \pi^2 - \frac{\pi l3}{3 \sqrt{3}} \quad \text{V. T. 163. N}^\circ. 10.$
- 13) $\int l \text{ Tang. } x \frac{\text{Sin. } x}{\text{Cos.}^2 x \sqrt{\text{Cos.}^3 x - \text{Sin.}^3 x}} dx = \frac{1}{27} \pi^2 - \frac{\pi l3}{3 \sqrt{3}} \quad \text{V. T. 163. N}^\circ. 11.$
- 14) $\int l \text{ Tang. } x \frac{(\text{Cot. } x - 1)^{p-1}}{\text{Sin.}^2 x} dx = -\frac{2}{2p+1} \pi \text{Sec. } p\pi, p < \frac{1}{2}; \quad \text{V. T. 52. N}^\circ. 15.$
- 15) $\int l \text{ Tang. } x \frac{1}{(\text{Cot. } x - 1)^{1+p}} \frac{dx}{\text{Sin.}^2 x} = \frac{2}{2p-1} \pi \text{Sec. } p\pi, p < \frac{1}{2}; \quad \text{V. T. 50. N}^\circ. 12.$
- 16) $\int l \text{ Tang. } x \frac{dx}{\sqrt{\text{Cos. } x (\text{Cos. } x - \text{Sin. } x)^3}} = -4l2 \quad \text{V. T. 52. N}^\circ. 9.$
- 17) $\int l \text{ Tang. } x \frac{\sqrt{\text{Tang. } x} + \sqrt{\text{Cot. } x}}{\text{Cos. } x - \text{Sin. } x} \frac{dx}{\sqrt{\text{Sin. } 2x}} = -\pi^2 \sqrt{2} \quad \text{V. T. 163. N}^\circ. 18.$
- 18) $\int (l \text{ Tang. } x)^2 \frac{dx}{\text{Cos. } x \sqrt{\text{Cos. } 2x}} = \frac{\pi}{2} \left\{ (l2)^2 + \frac{1}{12} \pi^2 \right\} \quad \text{V. T. 164. N}^\circ. 1.$
- 19) $\int (l \text{ Tang. } x)^{2a-1} \frac{1}{\text{Cos. } x - \text{Sin. } x} \frac{dx}{\sqrt{\text{Sin. } 2x}} = \frac{1 - 2^{2a}}{4a \sqrt{2}} (2\pi)^{2a} B_{2a-1} \quad \text{V. T. 164. N}^\circ. 3.$
- 20) $\int (l \text{ Tang. } x)^{2a} \frac{\text{Sin. } x + \text{Cos. } x}{(\text{Sin. } x - \text{Cos. } x)^2 \sqrt{\text{Sin. } 2x}} dx = \frac{2^{2a} - 1}{\sqrt{2}} (2\pi)^{2a} B_{2a-1} \quad \text{V. T. 319. N}^\circ. 19.$

$$1) \int l \sin. x \frac{3 \sqrt{\text{Tang. } x} + \sqrt{\text{Cot. } x}}{\cos.^2 x} dx = -\pi \sqrt{2} - 2 l 2 \quad \text{V. T. 50. N}^\circ. 15.$$

$$2) \int l \sin. 2 x \frac{1 + \sqrt{\sin. 2 x}}{\sqrt{\sin.^3 2 x}} \text{Tang.} \left(\frac{\pi}{4} + x \right) dx = -\pi^2 \quad \text{V. T. 163. N}^\circ. 18.$$

$$3) \int (l \sin. 2 x)^{2a-1} \frac{\text{Tang.} \left(\frac{\pi}{4} + x \right)}{\sqrt{\sin. 2 x}} dx = \frac{1-2^{2a}}{8a} 2\pi^{2a} B_{2a-1} \quad \text{V. T. 164. N}^\circ. 3.$$

$$4) \int l \cos. x \frac{\sqrt{\text{Tang. } x} + 3 \sqrt{\text{Cot. } x}}{\sin.^2 x} dx = -\pi \sqrt{2} + 2 l 2 \quad \text{V. T. 50. N}^\circ. 15.$$

$$5) \int l \cos. x \frac{\cos. 3 x}{\sin.^2 x \cos. 2 x \sqrt{\cos. 2 x}} = \frac{1}{\sqrt{2}} l 2 - \frac{1}{2} \pi \quad \text{V. T. 51. N}^\circ. 2.$$

$$6) \int l \cos. x \frac{1 + \cos.^2 2 x}{\sin.^2 2 x} \frac{dx}{\sqrt{\cos. 2 x}} = \frac{\pi \sqrt{2} \pi}{2 \{\Gamma(\frac{1}{4})\}^2} - \frac{\{\Gamma(\frac{1}{4})\}^2}{8 \sqrt{2} \pi} \quad \text{V. T. 50. N}^\circ. 1.$$

$$7) \int l \cos. x \frac{\sqrt{\cos. 2 x}}{\sin. 2 x \cos.^2 x} dx = \frac{\{\Gamma(\frac{1}{4})\}^2}{8 \sqrt{2} \pi} - \frac{\pi \sqrt{2} \pi}{2 \{\Gamma(\frac{1}{4})\}^2} \quad \text{V. T. 165. N}^\circ. 5.$$

$$8) \int l \cos. x \frac{2 \text{Cot. } 2 x + \sin. 2 x l \text{Tang. } x}{\cos. 2 x \sqrt{\cos. 2 x}} dx = -\frac{1}{8} \pi l 2 \quad \text{V. T. 319. N}^\circ. 4.$$

$$9) \int l \cos. 2 x \frac{1 + \sqrt{\cos. 2 x}}{\text{Tang. } x} \frac{dx}{\sqrt{\cos.^3 2 x}} = -\pi^2 \quad \text{V. T. 163. N}^\circ. 18.$$

$$10) \int (l \cos. 2 x)^{2a-1} \frac{dx}{\text{Tang. } x \sqrt{\cos. 2 x}} = \frac{1-2^{2a}}{8a} 2\pi^{2a} B_{2a-1} \quad \text{V. T. 164. N}^\circ. 3.$$

$$11) \int (l \text{Cot. } x)^{a-1} \frac{\text{Tang.}^p x}{\sin. 2 x} dx = \frac{1 a^2}{(2 p)^{a+1}} \sqrt{p} \pi \quad \text{V. T. 162. N}^\circ. 4.$$

$$12) \int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \frac{\sin. x}{\cos.^2 x \sqrt{\cos. 2 x}} dx = \pm \pi \quad \text{V. T. 51. N}^\circ. 2.$$

$$13) \int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \frac{1 - 2 \text{Tang.}^2 x}{\cos. x \sqrt{\cos. 2 x}} dx = \mp 2 \quad \text{V. T. 51. N}^\circ. 3.$$

$$14) \int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \frac{2 \sin.^2 x \cos.^2 x - 3 \cos. 2 x}{\cos.^4 x \sqrt{\cos. 2 x}} \sin.^2 x dx = \pm 1 \quad \text{V. T. 50. N}^\circ. 2.$$

$$15) \int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \frac{\sin. x}{\cos.^2 x + p^2 \cos. 2 x} \frac{dx}{\sqrt{\cos. 2 x}} = \pm \frac{\pi}{p} l \{p + \sqrt{1+p^2}\} \quad \text{V. T. 374. N}^\circ. 5.$$

$$16) \int dx \sqrt{l \cot x} = \frac{1}{2} \sqrt{l} \pi \sum_0^{\infty} (-1)^n \frac{1}{(2n+1)^3} \quad \text{V. T. 187. N}^\circ. 17.$$

$$17) \int l^{\frac{\cos 2x}{\cos^2 x}} \frac{dx}{\cos x \sqrt{\cos 2x}} = -\pi l^2 \quad \text{V. T. 165. N}^\circ. 11.$$

$$18) \int dx l^{\frac{\cos x + p \sqrt{\cos 2x}}{\cos x - p \sqrt{\cos 2x}}} = \pi \operatorname{Arcsin} p, p \leq 1; \quad \text{V. T. 160. N}^\circ. 17.$$

$$19) \int l^{\frac{\cos x + \sin x \sin \lambda}{\cos x - \sin x \sin \lambda}} \frac{dx}{\sin x \sqrt{\cos 2x}} = \pi \lambda \quad \text{V. T. 166. N}^\circ. 7.$$

$$20) \int l \cot x \sqrt[2]{\operatorname{Tg}^b x + \cot^b x} dx = \frac{1}{4} \pi (l\pi - \lambda) \operatorname{Sec} \frac{b\pi}{2c} - \pi \sum_0^{\infty} (-1)^n \left\{ \frac{l \left\{ (2n+1)\pi - \frac{b\pi}{c} \right\}}{(2n+1)\pi - \frac{b\pi}{c}} + \frac{l \left\{ (2n+1)\pi + \frac{b\pi}{c} \right\}}{(2n+1)\pi + \frac{b\pi}{c}} \right\} \quad \text{V. T. 190. N}^\circ. 5.$$

$$1) \int \sin^2 \left(\frac{\pi}{4} - x \right) \cdot \operatorname{Tang} \left(\frac{\pi}{4} - x \right) \frac{dx}{l \sin 2x} = \frac{1}{4} l \frac{2}{\pi} \quad \text{V. T. 171. N}^\circ. 1.$$

$$\left. \begin{aligned} 2) \int \sin^4 \left(\frac{\pi}{4} - x \right) \cdot \operatorname{Tang} \left(\frac{\pi}{4} - x \right) \frac{dx}{l \sin 2x} &= \frac{1}{8} l \frac{8}{\pi^2} \\ 3) \int \sin^2 \left(\frac{\pi}{4} - x \right) \cdot \sin 2x \cdot \operatorname{Tang} \left(\frac{\pi}{4} - x \right) \frac{dx}{l \sin 2x} &= \frac{1}{4} l \frac{\pi}{4} \end{aligned} \right\} \quad \text{V. T. 321. N}^\circ. 1, 4.$$

$$4) \int \sin^2 \left(\frac{\pi}{4} - x \right) \cdot \cos 2x \frac{dx}{l \sin 2x} = -\frac{1}{4} l^2 \quad \text{V. T. 167. N}^\circ. 2.$$

$$5) \int (1 - \sin^{q-1} 2x) \operatorname{Tang} \left(\frac{\pi}{4} - x \right) \frac{dx}{l \sin 2x} = \frac{1}{2} l \frac{\Gamma \left(\frac{q}{2} \right)}{\Gamma \left(\frac{1}{2} \right) \Gamma \left(\frac{q+1}{2} \right)} \quad \text{V. T. 171. N}^\circ. 2.$$

$$6) \int (1 - \sin^p 2x) (1 - \sin^q 2x) \operatorname{Tang} \left(\frac{\pi}{4} + x \right) \frac{dx}{l \sin 2x} = \frac{1}{2} l \frac{\Gamma(p+1) \Gamma(q+1)}{\Gamma(p+q+1)} \quad \text{V. T. 171. N}^\circ. 17.$$

$$7) \int \sin^2 x \cdot \operatorname{Tang} x \frac{dx}{l \cos 2x} = \frac{1}{4} l \frac{2}{\pi} \quad \text{V. T. 171. N}^\circ. 1.$$

$$8) \int \sin^2 x \cdot \sin 2x \frac{dx}{l \cos 2x} = -\frac{1}{4} l^2 \quad \text{V. T. 167. N}^\circ. 2.$$

$$9) \int \sin^2 x \cdot \cos 2x \cdot \operatorname{Tang} x \frac{dx}{l \cos 2x} = \frac{1}{4} l \frac{\pi}{4} \quad \text{V. T. 321. N}^\circ. 7, 8.$$

- 10) $\int \sin^4 x \cdot \text{Tang. } x \frac{dx}{l \cos 2x} = \frac{1}{8} l \frac{8}{\pi^2}$ V. T. 321. N°. 7, 8.
- 11) $\int \cos^2 2x \cdot \sin^4 x \cdot \text{Tang. } 2x \frac{dx}{(l \cos 2x)^2} = \frac{1}{8} \{(q+2)l(q+2) - 2(q+1)l(q+1) + q l q\}$ V. T. 168. N°. 3.
- 12) $\int \text{Tang. } \left(\frac{\pi}{4} - x\right) \frac{dx}{\cos^2 x l \text{Tang. } x} = l \frac{2}{\pi}$ V. T. 171. N°. 1.
- 13) $\int (1 - \text{Tang. } x)^2 \frac{dx}{l \text{Tang. } x} = l \frac{\pi}{4}$ V. T. 172. N°. 1.
- 14) $\int \text{Tang. } \left(\frac{\pi}{4} - x\right) \frac{\text{Tang.}^2 x}{l \text{Tang. } x} dx = l \frac{2 \sqrt{2}}{\pi}$ V. T. 175. N°. 17.
- 15) $\int \text{Tang. } \left(\frac{\pi}{4} - x\right) \frac{dx}{l \text{Tang. } x} = -\frac{1}{2} l 2$ V. T. 175. N°. 18.
- 16) $\int l \text{Tang. } \left(\frac{\pi}{4} \pm x\right) \frac{dx}{\cos^2 x l \text{Tang. } x} = \pm l \frac{\pi}{2}$ V. T. 321. N°. 14, 15.

- 1) $\int (\sin^{q-1} 2x - \text{Cosec.}^{q-1} 2x) \text{Tang. } \left(\frac{\pi}{4} - x\right) \frac{dx}{l \sin 2x} = \frac{1}{2} l \text{Tang. } \frac{1}{2} q \pi$ V. T. 175. N°. 2.
- 2) $\int (\sin^q 2x - \text{Cosec.}^q 2x)^2 \text{Tang. } \left(\frac{\pi}{4} + x\right) \frac{dx}{l \sin 2x} = \frac{1}{2} l \frac{\sin 2 q \pi}{2 q \pi}$ V. T. 175. N°. 4.
- 3) $\int (\sin^q 2x - \text{Cosec.}^q 2x)^2 \text{Tang. } \left(\frac{\pi}{4} - x\right) \frac{dx}{l \sin 2x} = \frac{1}{2} l q \pi \cot q \pi$ V. T. 175. N°. 3.
- 4) $\int \frac{\sin^q 2x \cdot \sin^2 \left(\frac{\pi}{4} - x\right)}{\text{Tang. } 2x} \frac{dx}{(l \sin 2x)^2} = \frac{1}{8} \{(q+2)l(q+2) - 2(q+1)l(q+1) + q l q\}$ V. T. 168. N°. 3.
- 5) $\int \frac{1 - \cos^{q-1} 2x}{\cot x} \frac{dx}{l \cos 2x} = \frac{1}{2} l \frac{\Gamma\left(\frac{q}{2}\right)}{\Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{q+1}{2}\right)}$ V. T. 171. N°. 2.
- 6) $\int (\cos^{q-1} 2x - \sec^q 2x) \text{Tang. } x \frac{dx}{l \cos 2x} = \frac{1}{2} l \text{Tang. } \frac{1}{2} q \pi$ V. T. 175. N°. 2.
- 7) $\int \frac{(1 - \cos^p 2x)(1 - \cos^q 2x)}{\text{Tang. } x} \frac{dx}{l \cos 2x} = \frac{1}{2} l \frac{\Gamma(p+1) \Gamma(q+1)}{\Gamma(p+q+1)}$ V. T. 171. N°. 17.

- 8) $\int (Cos.^q 2x - Sec.^q 2x)^2 Tang.^x \frac{dx}{l Cos.^2 x} = \frac{1}{2} l(q\pi Cot.q\pi)$ V. T. 175. N°. 3.
- 9) $\int \frac{Cos.^q 2x + Sec.^q 2x - 2}{Tang.^x} \frac{dx}{l Cos.^2 x} = \frac{1}{2} l \frac{Sin.q\pi}{q\pi}$ V. T. 175. N°. 4.
- 10) $\int \frac{1 - Sin.2x + (1 - Tang.x)Cos.2x}{Cos.^2 x.Cos.2x} \frac{dx}{l Tang.^x} = -l\pi$ V. T. 172. N°. 6.
- 11) $\int \frac{Tang.(\frac{\pi}{4} - x)}{Cos.^2 x} \frac{dx}{l Tang.^x} = l \frac{2}{\pi}$ V. T. 171. N°. 1.
- 12) $\int (Tang.^p x - Cot.^p x) \frac{dx}{l Tang.^x} = l Tang. \left\{ \frac{1+p}{4} \pi \right\}$ V. T. 175. N°. 5.
- 13) $\int \frac{Cos.x - Sin.x}{Cos.^3 x} \frac{dx}{l Tang.^x} = -l2$ V. T. 167. N°. 2.
- 14) $\int \frac{Tang.^q x - Tang.^p x}{Sin.2x} \frac{dx}{l Tang.^x} = \frac{1}{2} l \frac{q}{p}$ V. T. 170. N°. 2.
- 15) $\int \frac{(Tang.^q x - Cot.^q x)^2}{Cos.2x} \frac{dx}{l Tang.^x} = l Cos.q\pi$ V. T. 175. N°. 6.
- 16) $\int \frac{(Tang.^q x - Cot.^q x)^2}{Cos.2x} Tang.^x \frac{dx}{l Tang.^x} = l \frac{Sin.q\pi}{q\pi}$ V. T. 175. N°. 7.
- 17) $\int \frac{(1 - Tang.^q x)(1 - Tang.^{q+1} x)}{Cos.2x} \frac{dx}{l Tang.^x} = -ql2$ V. T. 172. N°. 3.
- 18) $\int \left(\frac{Cos.x - Sin.x}{Cos.^2 x} \right)^2 \frac{dx}{(l Tang.^x)^2} = l \frac{27}{16}$ V. T. 168. N°. 1.
- 19) $\int \left(\frac{Cos.x - Sin.x}{Cos.^3 x} \right)^2 \frac{Sin.2x}{(l Tang.^x)^2} dx = 4l \frac{32}{27}$ V. T. 168. N°. 2.
- 20) $\int (Tang.^q x + Cot.^q x) \frac{dx}{(l Tang.^x)^p} = (-1)^p \Gamma(1-p) \sum_0^{\infty} (-1)^n \left\{ \frac{1}{(2n+1-q)^{1-p}} - \frac{1}{(2n+1+q)^{1-p}} \right\}$ V.T. 176. N°. 4.
- 21) $\int \frac{Tang.^q x - Cot.^q x}{Cos.2x} \frac{dx}{(l Tang.^x)^p} = (-1)^{p-1} \Gamma(1-p) \sum_0^{\infty} \left\{ \frac{1}{(2n+1-q)^{1-p}} - \frac{1}{(2n+1+q)^{1-p}} \right\}$ V.T. 176. N°. 5.
- 22) $\int \frac{l Tang.(\frac{\pi}{4} \pm x)}{(l Sin.2x)^2} \frac{dx}{Tang.2x} = \mp \frac{1}{4} l2$ V. T. 350. N°. 5.

- 1) $\int \frac{\text{Sin.}^2 x \cdot \text{Tang. } x}{1 + \text{Cos.}^2 2x} \frac{dx}{l \text{Cos. } 2x} = -\frac{1}{4} l 2 \quad \text{V. T. 175. N}^\circ 18.$
- 2) $\int \frac{\text{Sin.}^2 x \cdot \text{Tang. } x}{1 + \text{Sec.}^2 2x} \frac{dx}{l \text{Cos. } 2x} = \frac{1}{2} l \frac{2 \sqrt{2}}{\pi} \quad \text{V. T. 175. N}^\circ 17.$
- 3) $\int \frac{(1 - \text{Tang.}^q x)(1 - \text{Tang.}^p x) - (1 - \text{Tang. } x)^2}{\text{Cos. } x - \text{Sin. } x} \frac{dx}{\text{Sin. } x l \text{Tang. } x} = l B(p, q) \quad \text{V. T. 175. N}^\circ 1.$
- 4) $\int \frac{\text{Cos. } 2x}{1 - 2 \text{Sin.}^2 x \cdot \text{Cos.}^2 x} \frac{dx}{l \text{Tang. } x} = l \text{Cot. } \frac{3\pi}{8} \quad \text{V. T. 172. N}^\circ 4.$
- 5) $\int \frac{1 - \text{Tang.}^q x}{\text{Sin. } x + \text{Cos. } x} \frac{\text{Tang.}^p x dx}{\text{Cos. } x l \text{Tang. } x} = l \frac{\Gamma\left(\frac{1}{2}p + 1\right) \Gamma\left(\frac{p+q+1}{2}\right)}{\Gamma\left(\frac{p+1}{2}\right) \Gamma\left(\frac{p+q}{2} + 1\right)} \quad \text{V. T. 171. N}^\circ 3.$
- 6) $\int \frac{\text{Tang.}^p x - \text{Tang.}^q x}{\text{Sin. } x + \text{Cos. } x} \frac{dx}{\text{Sin. } x l \text{Tang. } x} = l \frac{\Gamma\left(\frac{q}{2}\right) \Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\frac{q+1}{2}\right) \Gamma\left(\frac{p}{2}\right)} \quad \text{V. T. 171. N}^\circ 4.$
- 7) $\int \frac{\text{Tang.}^q x - \text{Cot.}^q x}{\text{Tang.}^p x + \text{Cot.}^p x} \frac{dx}{\text{Sin. } 2x l \text{Tang. } x} = \frac{1}{2} l \text{Tang. } \left\{ \frac{p+q}{4p} \pi \right\} \quad \text{V. T. 172. N}^\circ 6.$
- 8) $\int \frac{\text{Tang.}^q x + \text{Cot.}^q x - 2}{\text{Tang.}^p x - \text{Cot.}^p x} \frac{dx}{\text{Sin. } 2x l \text{Tang. } x} = \frac{1}{2} l \text{Cos. } \frac{q\pi}{2p} \quad \text{V. T. 172. N}^\circ 7. \text{ corr.}$
- 9) $\int \frac{\text{Tang.}^{q-1} x - \text{Cot.}^q x}{\text{Sin. } x + \text{Cos. } x} \frac{dx}{\text{Cos. } x l \text{Tang. } x} = l \text{Tang. } \frac{1}{2} q \pi \quad \text{V. T. 175. N}^\circ 2.$
- 10) $\int \frac{(\text{Tang.}^p x - \text{Cot.}^p x)^2}{\text{Sin. } x + \text{Cos. } x} \frac{dx}{\text{Cos. } x l \text{Tang. } x} = l(q\pi \text{Cot. } q\pi) \quad \text{V. T. 175. N}^\circ 3.$
- 11) $\int \frac{(\text{Tang.}^p x - \text{Cot.}^p x)^2}{\text{Sin. } x - \text{Cos. } x} \frac{dx}{\text{Cos. } x l \text{Tang. } x} = l(2q\pi \text{Cosec. } 2q\pi) \quad \text{V. T. 175. N}^\circ 4.$
- 12) $\int \frac{1 - \text{Tang.}^q x}{\text{Cos. } x - \text{Sin. } x} \frac{1 - \text{Tang.}^p x}{\text{Sin. } x} \frac{\text{Tang.}^r x}{l \text{Tang. } x} dx = l \frac{\Gamma(p+r) \Gamma(q+r)}{\Gamma(p+q+r) \Gamma(r)} \quad \text{V. T. 171. N}^\circ 18.$
- 13) $\int \frac{1}{1 + \text{Sin. } 2x \cdot \text{Cos. } \lambda} \frac{dx}{(l \text{Cot. } x)^{1-q}} = \text{Cosec. } \lambda \Gamma(q) \sum_1^{\infty} (-1)^{n-1} \frac{\text{Sin. } n\lambda}{n^q} \quad \text{V. T. 174. N}^\circ 4.$
- 14) $\int \frac{\text{Cos. } 2x}{1 - \text{Sin.}^2 2x \cdot \text{Sin.}^2 \lambda} \frac{dx}{(l \text{Cot. } x)^{1-q}} = \text{Sec. } \lambda \Gamma(q) \sum_1^{\infty} (-1)^n \frac{\text{Cos. } \{(2n-1)\lambda\}}{(2n+1)^q} \quad \text{V. T. 174. N}^\circ 13.$
- 15) $\int \left\{ \frac{1}{\text{Cos. } 2x (1 + \text{Cos. } 2x)} + \frac{1}{2 \text{Cos.}^2 x l \text{Tang. } x} \right\} \frac{dx}{l \text{Tang. } x} = \frac{1}{2} (l 2 - 1) \quad \text{V. T. 172. N}^\circ 5.$

- 1) $\int \frac{dx}{\pi^2 + (l \operatorname{Tang}. x)^2} = \frac{1 - \pi}{4\pi}$ V. T. 173. N°. 7.
- 2) $\int \frac{dx}{\pi^2 + (l \operatorname{Tang}.^2 x)^2} = \frac{1}{4\pi} l^2$ V. T. 173. N°. 8.
- 3) $\int \frac{dx}{q^2 + (l \operatorname{Tang}. x)^2} = \frac{1}{4q} \left\{ Z' \left(\frac{2q + 3\pi}{4\pi} \right) - Z' \left(\frac{2q + \pi}{4\pi} \right) \right\}$ V. T. 173. N°. 9.
- 4) $\int \operatorname{Tang}. \left(\frac{\pi}{4} + x \right) \frac{l \operatorname{Sin}. 2x dx}{q^2 + (l \operatorname{Sin}. 2x)^2} = \frac{1}{4} \left\{ l \frac{2\pi}{q} + \frac{\pi}{q} + Z' \left(\frac{q}{2\pi} \right) \right\}$ V. T. 173. N°. 2.
- 5) $\int \operatorname{Tang}. \left(\frac{\pi}{4} + x \right) \frac{l \operatorname{Sin}. 2x}{4\pi^2 + (l \operatorname{Sin}. 2x)^2} dx = \frac{1}{8} (1 - 2A)$ V. T. 173. N°. 3.
- 6) $\int \operatorname{Tang}. \left(\frac{\pi}{4} + x \right) \frac{l \operatorname{Sin}. 2x}{q^2 - (l \operatorname{Sin}. 2x)^2} dx = \frac{2\pi^4}{q^4} \sum_0^{\infty} (-1)^{n+1} \left(\frac{2\pi}{q} \right)^{2n} \frac{B_{2n+1}}{n+1}$ V. T. 173. N°. 4.
- 7) $\int \operatorname{Tang}. \left(\frac{\pi}{4} + x \right) \frac{l \operatorname{Sin}. 2x}{\{q^2 + (l \operatorname{Sin}. 2x)^2\}^2} dx = \frac{-\pi^2}{2q^4} \sum_0^{\infty} B_{2n+1} \left(\frac{2\pi}{q} \right)^{2n}$ V. T. 173. N°. 5.
- 8) $\int \operatorname{Tang}. \left(\frac{\pi}{4} + x \right) \frac{l \operatorname{Sin}. 2x}{\{q^2 - (l \operatorname{Sin}. 2x)^2\}^2} dx = \frac{\pi^2}{2q^4} \sum_0^{\infty} (-1)^{n+1} \left(\frac{2\pi}{q} \right)^{2n} B_{2n+1}$ V. T. 173. N°. 6.
- 9) $\int \operatorname{Tang}. 2x l \operatorname{Sin}. x \frac{q^2 - (l \operatorname{Cos}. 2x)^2}{\{q^2 + (l \operatorname{Cos}. 2x)^2\}^2} dx = \frac{1}{8} \left\{ l \frac{2\pi}{q} + \frac{\pi}{q} + Z' \left(\frac{q}{2\pi} \right) \right\}$ V. T. 325. N°. 1.
- 10) $\int \operatorname{Tang}. 2x l \operatorname{Sin}. x \frac{4\pi^2 - (l \operatorname{Cos}. 2x)^2}{\{4\pi^2 + (l \operatorname{Cos}. 2x)^2\}^2} dx = \frac{1}{16} (1 - 2A)$ V. T. 325. N°. 2.
- 11) $\int \operatorname{Tang}. 2x l \operatorname{Sin}. x \frac{q^2 + (l \operatorname{Cos}. 2x)^2}{\{q^2 - (l \operatorname{Cos}. 2x)^2\}^2} dx = \frac{\pi^4}{q^4} \sum_0^{\infty} (-1)^{n+1} \left(\frac{2\pi}{q} \right)^{2n} \frac{B_{2n+1}}{n+1}$ V. T. 325. N°. 3.
- 12) $\int \operatorname{Tang}. 2x l \operatorname{Sin}. x \frac{q^2 - 3(l \operatorname{Cos}. 2x)^2}{\{q^2 + (l \operatorname{Cos}. 2x)^2\}^3} dx = \frac{-\pi^2}{4q^4} \sum_0^{\infty} \left(\frac{2\pi}{q} \right)^{2n} B_{2n+1}$ V. T. 325. N°. 4.
- 13) $\int \operatorname{Tang}. 2x l \operatorname{Sin}. x \frac{q^2 + 3(l \operatorname{Cos}. 2x)^2}{\{q^2 - (l \operatorname{Cos}. 2x)^2\}^3} dx = \frac{\pi^2}{4q^4} \sum_0^{\infty} (-1)^{n-1} \left(\frac{2\pi}{q} \right)^{2n} B_{2n+1}$ V. T. 325. N°. 5.

- 1) $\int \frac{1}{Tang. x} \frac{l Cos. 2 x}{q^2 + (l Cos. 2 x)^2} dx = \frac{1}{8} \left\{ l \frac{2\pi}{q} + \frac{\pi}{q} + Z' \left(\frac{q}{2\pi} \right) \right\}$ V. T. 173. N°. 2.
- 2) $\int \frac{1}{Tang. x} \frac{l Cos. 2 x}{1 - \pi^2 + (l Cos. 2 x)^2} dx = \frac{1}{8} (1 - 2 A)$ V. T. 173. N°. 3.
- 3) $\int \frac{1}{Tang. x} \frac{l Cos. 2 x}{q^2 - (l Cos. 2 x)^2} dx = \frac{2\pi^4}{q^4} \sum_0^{\infty} (-1)^{n+1} \left(\frac{2\pi}{q} \right)^{2n} \frac{B_{2n+1}}{n+1}$ V. T. 173. N°. 4.
- 4) $\int \frac{1}{Tang. x} \frac{l Cos. 2 x}{\{q^2 + (l Cos. 2 x)^2\}^2} dx = \frac{-\pi^2}{2q^4} \sum_0^{\infty} \left(\frac{2\pi}{q} \right)^{2n} B_{2n+1}$ V. T. 173. N°. 5.
- 5) $\int \frac{1}{Tang. x} \frac{l Cos. 2 x}{\{q^2 - (l Cos. 2 x)^2\}^2} dx = \frac{\pi^2}{2q^4} \sum_0^{\infty} (-1)^{n+1} \left(\frac{2\pi}{q} \right)^{2n} B_{2n+1}$ V. T. 173. N°. 6.
- 6) $\int \frac{Tang. x}{Sin. 2 x} \frac{dx}{p + l Tang. x} = \frac{1}{2} e^{-pq} Ei. (pq)$ V. T. 169. N°. 1.
- 7) $\int \frac{Tang. x}{Sin. 2 x} \frac{dx}{p - l Tang. x} = -\frac{1}{2} e^{pq} Ei. (-pq)$ V. T. 169. N°. 2.
- 8) $\int \frac{Tang. x}{Cos. 2 x} \frac{l Tang. x}{q^2 + (l Tang. x)^2} dx = \frac{\pi}{4q} + \frac{1}{2} l \frac{\pi}{q} + \frac{1}{2} Z' \left(\frac{q}{\pi} \right)$ V. T. 173. N°. 12.
- 9) $\int \frac{Tang. x}{Cos. 2 x} \frac{l Tang. x}{q^2 - (l Tang. x)^2} dx = \frac{\pi^4}{4q^4} \sum_0^{\infty} (-1)^{n-1} \frac{B_{2n+1}}{n+1} \left(\frac{\pi}{q} \right)^2$ V. T. 173. N°. 14.
- 10) $\int \frac{l Tang. x}{\pi^2 + (l Tang. x)^2} \frac{dx}{Cos. 2 x} = \frac{1}{2} \left\{ \frac{1}{2} - l 2 \right\}$ V. T. 173. N°. 11.
- 11) $\int \frac{l Tang. x}{\pi^2 + (l Tang. x)^2} \frac{Tang. x}{Cos. 2 x} dx = \frac{1}{4} - \frac{1}{2} \Lambda$ V. T. 173. N°. 13.
- 12) $\int \frac{l Tang. x}{\pi^2 + (l Tang. x)^2} \frac{dx}{Cos. 2 x} = \frac{2 - \pi}{16}$ V. T. 173. N°. 10.
- 13) $\int \frac{l Tang. x}{q^2 + (l Tang. x)^2} \frac{Tang. x}{Cos. 2 x} dx = \frac{1}{2} l \frac{q}{2\pi} - \frac{\pi}{2q} + \frac{1}{2} Z' \left(\frac{2\pi + q}{2\pi} \right)$ V. T. 173. N°. 1.
- 14) $\int \frac{l Tang. x}{\pi^2 + (l Tang. x)^2} \frac{dx}{Cos. 2 x} = -\frac{\pi \sqrt{2}}{64} + \frac{1}{16} + \frac{1}{32 \sqrt{2}} l \frac{\sqrt{2}-1}{\sqrt{2}+1}$ V. T. 173. N°. 18.
- 15) $\int \frac{Tang. p x - Cot. p x}{\pi^2 + (l Tang. x)^2} \frac{dx}{Cos. 2 x} = \frac{1}{2\pi} [p\pi Cos. p\pi - Sin. p\pi l \{2(1 + Cos. p\pi)\}] , p < 1;$ V. T. 176. N°. 8.
- 16) $\int \frac{Tang. p x + Cot. p x}{\pi^2 + (l Tang. x)^2} \frac{l Tang. x}{Cos. 2 x} dx = \frac{1}{2} [1 - p\pi Sin. p\pi - Cos. p\pi l \{2(1 + Cos. p\pi)\}] , p < 1;$ V. T. 176. N°. 9.

- 17) $\int \frac{Tang.^p x - Cot.^p x}{\pi^2 + (l Tang.^2 x)^2} \frac{dx}{Cos. 2x} = -\frac{1}{4} Sin. \frac{1}{2} p \pi + \frac{\pi}{4} Cos. \frac{1}{2} p \pi, l \frac{1 + Sin. \frac{1}{2} p \pi}{1 - Sin. \frac{1}{2} p \pi}, p < 1; \text{ V. T. 176. N}^\circ. 6.$
- 18) $\int \frac{Tang.^p x + Cot.^p x l Tang. x}{\pi^2 + (l Tang.^2 x)^2} \frac{dx}{Cos. 2x} = \frac{1}{4} - \frac{\pi}{8} Cos. \frac{1}{2} p \pi + \frac{1}{8} Sin. \frac{1}{2} p \pi, l \frac{1 - Sin. \frac{1}{2} p \pi}{1 + Sin. \frac{1}{2} p \pi}, p < 1; \text{ V. T. 176. N}^\circ. 7.$
- 19) $\int \frac{l Tang. x}{\{q^2 + l Tang. x\}^2} \frac{Tang. x}{Cos. 2x} dx = -\frac{\pi^2}{4 q^4} \sum_0^{\infty} \left(\frac{\pi}{q}\right)^{2n} B_{2n+1} \text{ V. T. 173. N}^\circ. 15.$
- 20) $\int \frac{l Tang. x}{\{q^2 - (l Tang. x)^2\}^2} \frac{Tang. x}{Cos. 2x} dx = \frac{\pi^2}{4 q^4} \sum_0^{\infty} (-1)^{n-1} \left(\frac{\pi}{q}\right)^{2n} B_{2n+1} \text{ V. T. 173. N}^\circ. 16.$
- 21) $\int l Tang. \left(\frac{\pi}{4} \pm x\right) \frac{\pi^2 - (l Tang. x)^2}{\{\pi^2 + (l Tang. x)^2\}^2} \frac{dx}{Sin. 2x} = \pm \frac{1}{2} \left\{ l 2 - \frac{1}{2} \right\} \text{ V. T. 325. N}^\circ. 10.$
- 22) $\int l Tang. \left(\frac{\pi}{4} \pm x\right) \frac{\pi^2 - (l Tang.^2 x)^2}{\{\pi^2 + (l Tang.^2 x)^2\}^2} \frac{dx}{Sin. 2x} = \pm \frac{\pi - 2}{16} \text{ V. T. 325. N}^\circ. 12.$
- 23) $\int l Tang. \left(\frac{\pi}{4} \pm x\right) \frac{\pi^2 - (l Tang.^4 x)^2}{\{\pi^2 + (l Tang.^4 x)^2\}^2} \frac{dx}{Sin. 2x} = \pm \left\{ \frac{\pi \sqrt{2}}{64} - \frac{1}{16} + \frac{1}{32 \sqrt{2}} l \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right\} \text{ V. T. 325. N}^\circ. 14.$
- 24) $\int \frac{l Tang. x}{4 \pi^2 + (l Tang. x)^2} \frac{dx}{Cos. x (Cos. x - Sin. x)} = \frac{1}{4} (1 - 2 A) \text{ V. T. 173. N}^\circ. 3.$
- 25) $\int \frac{l Tang. x}{q^2 - (l Tang. x)^2} \frac{dx}{Cos. x (Cos. x - Sin. x)} = \frac{4 \pi^4}{q^4} \sum_0^{\infty} (-1)^{n+1} \left(\frac{\pi}{q}\right)^{2n} \frac{B_{2n+1}}{n+1} \text{ V. T. 173. N}^\circ. 4.$

- 1) $\int \frac{Tang. \left(\frac{\pi}{4} - x\right)}{\pi^2 + (l Sin. 2x)^2} \frac{dx}{\sqrt{Sin. 2x}} = \frac{1}{4 \pi} l 2 \text{ V. T. 177. N}^\circ. 1.$
- 2) $\int \frac{Tang. \left(\frac{\pi}{4} - x\right)}{\pi^2 + 4 (l Sin. 2x)^2} \frac{dx}{\sqrt{Sin. 2x}} = \frac{1}{8 \pi \sqrt{2}} \left\{ \pi - l \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right\} \text{ V. T. 177. N}^\circ. 4.$
- 3) $\int \frac{Tang. \left(\frac{\pi}{4} - x\right)}{q^2 + (l Sin. 2x)^2} \frac{dx}{\sqrt{Sin. 2x}} = \frac{1}{8 q} \left\{ Z' \left(\frac{q + 3 \pi}{4 \pi} \right) - Z' \left(\frac{q + \pi}{4 \pi} \right) \right\} \text{ V. T. 177. N}^\circ. 3.$
- 4) $\int \frac{Tang. \left(\frac{\pi}{4} + x\right)}{\pi^2 + (l Sin. 2x)^2} \frac{Sin.^p 2x - Cosec.^p 2x}{\sqrt{Sin. 2x}} dx = \frac{1}{4} \left\{ \frac{1}{\pi} Cos. p \pi, l \frac{1 + Sin. p \pi}{1 - Sin. p \pi} - Sin. p \pi \right\} \text{ V. T. 177. N}^\circ. 9.$

- $$5) \int \frac{Tang. \left(\frac{\pi}{4} + x \right)}{q^2 + (l \sin. 2x)^2} \frac{\sin. p 2x - \operatorname{Cosec} p 2x}{\sqrt{\sin. 2x}} dx = \frac{\pi}{q} \sum_1^{\infty} \frac{\sin. \{(p+1)n\pi\}}{q + 2n\pi} \quad \text{V. T. 177. N}^\circ. 13.$$
- $$6) \int \frac{Tang. \left(\frac{\pi}{4} + x \right)}{\pi^2 + (l \sin. 2x)^2} \frac{l \sin. 2x}{\sqrt{\sin. 2x}} dx = \frac{2 - \pi}{8} \quad \text{V. T. 177. N}^\circ. 5.$$
- $$7) \int \frac{Tang. \left(\frac{\pi}{4} + x \right)}{\pi^2 + 4(l \sin. 2x)^2} \frac{l \sin. 2x}{\sqrt{\sin. 2x}} dx = \frac{1}{4\sqrt{2}} \left\{ \pi + 2\sqrt{2} - l \frac{\sqrt{2+1}}{\sqrt{2-1}} \right\} \quad \text{V. T. 177. N}^\circ. 6.$$
- $$8) \int \frac{Tang. \left(\frac{\pi}{4} + x \right)}{\pi^2 + (l \sin. 2x)^2} \frac{\sin. p 2x + \operatorname{Cosec} p 2x}{\sqrt{\sin. 2x}} l \sin. 2x dx = \frac{1}{2} - \frac{\pi}{4} \operatorname{Cos} p \pi - \frac{1}{4} \sin. p \pi l \frac{1 + \sin. p \pi}{1 - \sin. p \pi} \quad \text{V. T. 177. N}^\circ. 10.$$
- $$9) \int \frac{Tang. \left(\frac{\pi}{4} + x \right)}{q^2 + (l \sin. 2x)^2} \frac{\sin. p 2x + \operatorname{Cosec} p 2x}{\sqrt{\sin. 2x}} l \sin. 2x dx = -\frac{\pi}{2q} - \pi \sum_1^{\infty} \frac{\operatorname{Cos} \{(p+1)n\pi\}}{q + 2n\pi} \quad \text{V. T. 177. N}^\circ. 14.$$
- $$10) \int \frac{Tang. x}{\pi^2 + (l \cos. 2x)^2} \frac{dx}{\sqrt{\cos. 2x}} = \frac{1}{4\pi} l 2 \quad \text{V. T. 177. N}^\circ. 1.$$
- $$11) \int \frac{Tang. x}{\pi^2 + 4(l \cos. 2x)^2} \frac{dx}{\sqrt{\cos. 2x}} = \frac{1}{8\pi\sqrt{2}} \left\{ \pi - l \frac{\sqrt{2+1}}{\sqrt{2-1}} \right\} \quad \text{V. T. 177. N}^\circ. 4.$$
- $$12) \int \frac{Tang. x}{q^2 + (l \cos. 2x)^2} \frac{dx}{\sqrt{\cos. 2x}} = \frac{1}{8q} \left\{ Z' \left(\frac{q+3\pi}{4\pi} \right) - Z' \left(\frac{q+\pi}{4\pi} \right) \right\} \quad \text{V. T. 177. N}^\circ. 3.$$
- $$13) \int \frac{\cos p 2x - \sec p 2x}{\pi^2 + (l \cos. 2x)^2} \frac{dx}{Tang. x \sqrt{\cos. 2x}} = \frac{1}{4} \left\{ \frac{1}{\pi} \operatorname{Cos} p \pi l \frac{1 + \sin. p \pi}{1 - \sin. p \pi} - \sin. p \pi \right\} \quad \text{V. T. 177. N}^\circ. 9.$$
- $$14) \int \frac{\cos p 2x - \sec p 2x}{4\pi^2 + (l \cos. 2x)^2} \frac{dx}{Tang. x \sqrt{\cos. 2x}} = \frac{-1}{8\pi} [2p\pi \cos. 2p\pi + \sin. 2p\pi l \{2(1 + \cos. 2p\pi)\}] \quad \text{V. T. 177. N}^\circ. 11.$$
- $$15) \int \frac{\cos p 2x - \sec p 2x}{q^2 + (l \cos. 2x)^2} \frac{dx}{Tang. x \sqrt{\cos. 2x}} = \frac{\pi}{q} \sum_1^{\infty} \frac{\sin. \{(p+1)n\pi\}}{q + 2n\pi} \quad \text{V. T. 177. N}^\circ. 13.$$
- $$16) \int \frac{l \cos. 2x}{\pi^2 + (l \cos. 2x)^2} \frac{dx}{Tang. x \sqrt{\cos. 2x}} = \frac{2 - \pi}{8} \quad \text{V. T. 177. N}^\circ. 5.$$
- $$17) \int \frac{l \cos. 2x}{\pi^2 + 4(l \cos. 2x)^2} \frac{dx}{Tang. x \sqrt{\cos. 2x}} = \frac{1}{4\sqrt{2}} \left\{ \pi + 2\sqrt{2} - l \frac{\sqrt{2+1}}{\sqrt{2-1}} \right\} \quad \text{V. T. 177. N}^\circ. 6.$$

F. Log. en dén., Fonct. binôme. Circ. Dir. en dén. irrat. monôme.	TABLE 526 suite.	Lim. 0 et $\frac{\pi}{4}$.
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- 18) $\int \frac{\cos p 2x + \sec p 2x}{\pi^2 + (l \cos 2x)^2} \frac{l \cos 2x}{\text{Tang. } x \sqrt{\cos 2x}} dx = \frac{1}{2} - \frac{\pi}{4} \cos p \pi - \frac{1}{4} \sin p \pi \frac{1 + \sin p \pi}{1 - \sin p \pi}$ V. T. 177. N°. 10.
- 19) $\int \frac{\cos p 2x + \sec p 2x}{4\pi^2 + (l \cos 2x)^2} \frac{l \cos 2x}{\text{Tang. } x \sqrt{\cos 2x}} dx = \frac{1}{4} [1 - 2p\pi \sin 2p\pi - \cos 2p\pi \{2(1 + \cos 2p\pi)\}]$ V. T. 177. N°. 12.
- 20) $\int \frac{\cos p 2x + \sec p 2x}{q^2 + (l \cos 2x)^2} \frac{l \cos 2x}{\text{Tang. } x \sqrt{\cos 2x}} dx = -\frac{\pi}{2q} - \pi \sum_1^{\infty} \frac{\cos \{(p+1)n\pi\}}{q + 2n\pi}$ V. T. 177. N°. 14.

F. Log. en dén., Fonct. binôme. Circ. Dir. en dén. irrat. composée.	TABLE 527.	Lim. 0 et $\frac{\pi}{4}$.
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- 1) $\int \frac{1}{q^2 + (l \text{Tang. } x)^2} \frac{1}{\sin x + \cos x \sqrt{\sin 2x}} dx = \frac{1}{4q \sqrt{2}} \left\{ Z' \left(\frac{q+3\pi}{4\pi} \right) - Z' \left(\frac{q+\pi}{4\pi} \right) \right\}$ V. T. 177. N°. 3.
- 2) $\int \frac{1}{q^2 + (l \text{Tang. } x)^2} \frac{1}{\sin x - \cos x \sqrt{1 + \cos 2x}} dx = \frac{1}{2 \sqrt{2}} \left\{ \frac{\pi}{q} + l \frac{2\pi}{q} + Z' \left(\frac{q}{2\pi} \right) \right\}$ V. T. 173. N°. 2.
- 3) $\int \frac{\text{Tang. } \frac{p-1}{2} x - \text{Cot. } \frac{p-1}{2} x}{q^2 + (l \text{Tang. } x)^2} \frac{1}{\sin x - \cos x \sqrt{\sin 2x}} dx = -\frac{\pi \sqrt{2}}{q} \sum_1^{\infty} \frac{\sin np\pi}{q + 2n\pi}, p < 1;$ V. T. 177. N°. 13.
- 4) $\int \frac{\text{Tang. } \frac{p-1}{2} x + \text{Cot. } \frac{p-1}{2} x}{q^2 + (l \text{Tang. } x)^2} \frac{l \text{Tang. } x}{\sin x - \cos x \sqrt{\sin 2x}} dx = \frac{\pi}{q \sqrt{2}} + \pi \sqrt{2} \sum_1^{\infty} \frac{\cos np\pi}{q + 2n\pi}, p < 1;$ V. T. 177. N°. 14.
- 5) $\int \frac{1 - \text{Tang. } \frac{q-1}{2} x}{\sin x - \cos x \sqrt{\sin 2x}} \frac{1 - \text{Tang. } \frac{q-1}{2} x}{l \text{Tang. } x} dx = \frac{2q-2}{\sqrt{2}} l 2$ V. T. 177. N°. 15.

F. Log. sous forme irrat. Circ. Dir.	TABLE 528.	Lim. 0 et $\frac{\pi}{4}$.
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- 1) $\int \frac{dx}{\sqrt{l \text{Cot. } x}} = \sqrt{\pi} \sum_0^{\infty} \frac{(-1)^n}{\sqrt{(2n+1)}}$ V. T. 178. N°. 4.
- 2) $\int \frac{\sin x}{\sqrt{l \text{Sec. } x}} dx = \sqrt{\pi}$ V. T. 349. N°. 6.
- 3) $\int \frac{\cos x}{\sqrt{l \text{Cosec. } x}} dx = \sqrt{\pi}$ V. T. 349. N°. 1.
- 4) $\int \frac{\cos x}{\sqrt{(l \text{Cosec. } x)^{2a+1}}} dx = \frac{(-2)^a \sqrt{\pi}}{1^{a/2}}$ V. T. 44. N°. 6.

F. Log. sous forme irrat.
Circ. Dir.

TABLE 528 suite.

Lim. 0 et $\frac{\pi}{4}$.

$$5) \int \frac{l l \cot x}{\sqrt{l \cot x}} dx = \sqrt{\pi} \sum_0^{\infty} (-1)^{n-1} \frac{l(2n+1) + 2l2 + \Lambda}{\sqrt{(2n+1)}} \quad \text{V. T. 190. N}^\circ. 4.$$

$$6) \int \frac{dx}{\cos^2 x \sqrt{l \cot x}} = \sqrt{\pi} \quad \text{V. T. 187. N}^\circ. 18.$$

$$7) \int \frac{\text{Tang.}^p x}{\sin 2x \sqrt{l \cot x}} dx = \frac{1}{2} \sqrt{\frac{\pi}{p}} \quad \text{V. T. 178. N}^\circ. 1.$$

$$8) \int \frac{\text{Tang.}^q x \cdot l l \cot x}{\sin 2x \sqrt{l \cot x}} dx = -\frac{1}{2} \sqrt{\frac{\pi}{a}} \{ \Lambda + 2l2 + la \} \quad \text{V. T. 190. N}^\circ. 3.$$

$$9) \int \frac{1}{2 + \sin 2x} \frac{dx}{\sqrt{l \cot x}} = \frac{\sqrt{\pi}}{2 \sin \frac{\pi}{3}} \sum_1^{\infty} (-1)^{n-1} \frac{\sin \frac{n\pi}{3}}{\sqrt{n}} \quad \text{V. T. 178. N}^\circ. 5.$$

$$10) \int \frac{l l \cot x}{2 + \sin 2x} \frac{dx}{\sqrt{l \cot x}} = \frac{\sqrt{\pi}}{2 \sin \frac{\pi}{3}} \sum_1^{\infty} (-1)^n \sin \frac{n\pi}{3} \frac{ln + 2l2 + \Lambda}{\sqrt{(2n+1)}} \quad \text{V. T. 190. N}^\circ. 8.$$

F. Log. $\left\{ \begin{array}{l} \text{Circ. Dir.} \end{array} \right\}$ Autre forme.

TABLE 529.

Lim. 0 et $\frac{\pi}{4}$.

$$1) \int \sin(2pl \text{Tang.} x) \cdot l \text{Tang.} x dx = \frac{\pi^2}{4} \frac{ep\pi - e^{-p\pi}}{(ep\pi + e^{-p\pi})^2} \quad \text{V. T. 404. N}^\circ. 6.$$

$$2) \int \sin(pl \text{Tang.} x) (\text{Tang.}^q x - \cot^q x) dx = \pi \sin \frac{1}{2} q \pi \frac{e^{qp\pi} - e^{-qp\pi}}{ep\pi + 2\cos q\pi + e^{-p\pi}}, p^2 < 1, q^2 < 1; \quad \text{V. T. 404. N}^\circ. 8.$$

$$3) \int \sin^2(pl \text{Tang.} x) dx = \frac{\pi}{8} \frac{(ep\pi - 1)^2}{e^{2p\pi} + 1} \quad \text{V. T. 404. N}^\circ. 17.$$

$$4) \int \cos(pl \text{Tang.} x) dx = \frac{\pi}{2} \frac{e^{lp\pi}}{ep\pi + 1} \quad \text{V. T. 404. N}^\circ. 2.$$

$$5) \int \cos^2(pl \text{Tang.} x) dx = \frac{\pi}{8} \frac{(ep\pi + 1)^2}{e^{2p\pi} + 1} \quad \text{V. T. 404. N}^\circ. 8.$$

$$6) \int \cos(pl \text{Tang.} x) (\text{Tang.}^q x + \cot^q x) dx = \pi \cos \frac{1}{2} q \pi \frac{e^{lp\pi} + e^{-lp\pi}}{ep\pi + 2\cos q\pi + e^{-p\pi}}, p^2 < 1, q^2 < 1; \quad \text{V. T. 404. N}^\circ. 9.$$

$$7) \int \sin(pl \text{Tang.} x) \frac{dx}{\cos 2x} = \frac{\pi}{4} \frac{1 - ep\pi}{1 + ep\pi} \quad \text{V. T. 404. N}^\circ. 10.$$

$$8) \int \sin(pl \text{Tang.} x) \frac{dx}{\sin 4x} = \frac{\pi}{8} \frac{1 + ep\pi}{1 - ep\pi} \quad \text{V. T. 405. N}^\circ. 4.$$

- 9) $\int \text{Sin.}(pl \text{Tang. } x) \frac{\text{Tang.}^{q-1} x}{\text{Cos. } 2x} dx = - \sum_1^{\infty} \frac{p}{(2n+q)^2 + p^2} \quad \text{V. T. 404. N}^\circ. 12.$
- 10) $\int \text{Sin.}(pl \text{Tang. } x) \cdot \text{Tang.} \left(\frac{\pi}{4} + x \right) \frac{dx}{\text{Sin. } 2x} = \frac{\pi}{4} \frac{1 + e^{2p\pi}}{1 - e^{2p\pi}} \quad \text{V. T. 405. N}^\circ. 2.$
- 11) $\int \text{Sin.}(pl \text{Tang. } x) \cdot \text{Tang.} \left(\frac{\pi}{4} - x \right) \frac{dx}{\text{Sin. } 2x} = \frac{-\pi}{e^{p\pi} - e^{-p\pi}} \quad \text{V. T. 405. N}^\circ. 1.$
- 12) $\int \text{Sin.}(pl \text{Tang. } x) \frac{\text{Tang.}^q x + \text{Cot.}^q x}{\text{Cos. } 2x} dx = - \frac{\pi}{2} \frac{e^{p\pi} - e^{-p\pi}}{e^{p\pi} + 2 \text{Cos. } q\pi + e^{-p\pi}} \quad \text{V. T. 404. N}^\circ. 13.$
- 13) $\int \text{Cos.}(pl \text{Tang. } x) \frac{dx}{\text{Sin. } 4x} = \frac{1}{4} l(e^{4p\pi} - e^{-4p\pi}) \quad \text{V. T. 406. N}^\circ. 18.$
- 14) $\int \text{Cos.}(pl \text{Tang. } x) \frac{l \text{Tang. } x}{\text{Sin. } 4x} dx = \frac{1}{4} \pi^2 \frac{e^{p\pi}}{(1 - e^{p\pi})^2} \quad \text{V. T. 405. N}^\circ. 5.$
- 15) $\int \text{Cos.}(pl \text{Tang. } x) \frac{l \text{Tang. } x}{\text{Cos. } 2x} dx = \frac{1}{2} \pi^2 \frac{e^{p\pi}}{(e^{p\pi} + 1)^2} \quad \text{V. T. 404. N}^\circ. 14.$
- 16) $\int \text{Cos.}(pl \text{Tang. } x) \cdot \text{Tang.} \left(\frac{\pi}{4} - x \right) \frac{l \text{Tang. } x}{\text{Sin. } 2x} dx = \pi^2 e^{-p\pi} \frac{1 + e^{-2p\pi}}{(1 - e^{-2p\pi})^2} \quad \text{V. T. 405. N}^\circ. 3.$
- 17) $\int \text{Cos.}(pl \text{Tang. } x) \frac{\text{Tang.}^q x - \text{Cot.}^q x}{\text{Cos. } 2x} dx = \frac{\pi}{2} \frac{\text{Sin. } q\pi}{e^{p\pi} + 2 \text{Cos. } q\pi + e^{-p\pi}} \quad \text{V. T. 404. N}^\circ. 15.$
- 18) $\int \text{Sin.}(pl \text{Tang. } x) \frac{1}{1 - \text{Sin. } 2x \cdot \text{Cos. } \lambda} \frac{dx}{\text{Tang. } 2x} = - \frac{\pi}{2} \frac{e^{p\lambda} + e^{-p\lambda}}{e^{p\pi} - e^{-p\pi}} \quad \text{V. T. 405. N}^\circ. 12.$
- 19) $\int \text{Cos.}(pl \text{Tang. } x) \frac{dx}{1 + \text{Sin. } 2x \cdot \text{Cos. } \lambda} = \frac{\pi}{2} \text{Cosec. } \lambda \frac{e^{p\lambda} - e^{-p\lambda}}{e^{p\pi} - e^{-p\pi}} \quad \text{V. T. 405. N}^\circ. 7.$
- 20) $\int \text{Cos.}(pl \text{Tang. } x) \frac{1}{1 + \text{Sin. } 2x \cdot \text{Cos. } \lambda} \frac{dx}{\text{Sin. } 2x} = - \frac{\pi}{2} \text{Cot. } \lambda \frac{e^{p\lambda} - e^{-p\lambda}}{e^{p\pi} - e^{-p\pi}} \quad \text{V. T. 405. N}^\circ. 11.$
- 21) $\int \text{Sin.}(pl \text{Tang. } x) \frac{dx}{l \text{Tang. } x} = \text{Arctang.}(e^{lp\pi}) \quad \text{V. T. 406. N}^\circ. 15.$
- 22) $\int \text{Sin.}(2pl \text{Tang. } x) \frac{dx}{\text{Tang. } 2x \cdot l \text{Tang. } x} = \frac{1}{2} l \frac{1 + e^{-p\pi}}{1 - e^{-p\pi}} \quad \text{V. T. 406. N}^\circ. 17.$
- 23) $\int \text{Cos.}(2pl \text{Tang. } x) \frac{dx}{\text{Cos. } 2x \cdot l \text{Tang. } x} = - \frac{1}{2} l(e^{p\pi} + e^{-p\pi}) \quad \text{V. T. 406. N}^\circ. 16.$

$$1) \int l \sin. x dx = -\frac{1}{2} \pi l 2 \quad \left. \begin{array}{l} \text{Euler, Calc. Int. IV. S. 3. 123. — Id., N. C. Petr. 14. 129. — Cauchy,} \\ \text{Exerc. 1826. p. 205. — Id., Lim. Imag. 149. — Serret, L. 8. 1. —} \\ \text{Roberts, L. 11. 471. — Grunert, Gr. 4. 113. — Lindmann, Stockh.} \\ \text{Handl. 1850. III.} \end{array} \right\}$$

$$2) \int l((\sin. x)) dx = -\frac{1}{2} \pi (l 2 - 2 \alpha \pi i) \quad \text{Arndt, Gr. 6. 187.}$$

$$3) \quad = -\frac{1}{2} \pi l 2 + \alpha \pi^2 i \quad \left. \vphantom{\int} \right\} \text{Lindmann, Gr. 16. 94.}$$

$$4) \int l((- \sin. x)) dx = -\frac{1}{2} \pi l 2 + \frac{2 \alpha + 1}{2} \pi^2 i$$

$$5) \int l \sin. x. \sin. x dx = l 2 - 1 \quad \text{V. T. 163. N° 3.}$$

$$6) \int l \sin. x. \sin.^2 x dx = \frac{1}{8} \pi (1 - 2 l 2) \quad \text{V. T. 163. N° 4.}$$

$$7) \int l \sin. x. \sin.^3 x dx = \frac{2}{3} \left(l 2 - \frac{5}{6} \right) \quad \text{V. T. 163. N° 5.}$$

$$8) \int l \sin. x. \sin.^4 x dx = \frac{3}{16} \pi \left(\frac{7}{12} - l 2 \right) \quad \text{V. T. 163. N° 6.}$$

$$9) \int l \sin. x. \sin.^5 x dx = \frac{8}{15} \left(l 2 - \frac{47}{60} \right) \quad \text{V. T. 163. N° 7.}$$

$$10) \int l \sin. x. \cos.^2 x dx = -\frac{1}{8} \pi (1 + 2 l 2) \quad \text{V. T. 162. N° 1.}$$

$$11) \int l \sin. x. \sin. x. \cos.^2 x dx = -\frac{1}{9} (4 - 3 l 2) \quad \text{V. T. 162. N° 2.}$$

$$12) \int l \sin. x. \cos. 2 x dx = -\frac{1}{4} \pi \quad \text{V. T. 330. N° 6, 10.}$$

$$13) \int l \sin. x. \text{Tang. } x dx = -\frac{1}{24} \pi^2 \quad \text{V. T. 152. N° 14.}$$

$$14) \int l \sin. x. \cos.^{2a} x dx = -\frac{\pi 1^{a 2}}{2^{a+2} 1^{a/1}} \{A + Z' (a + 1) + 2 l 2\} \quad \text{V. T. 162. N° 3.}$$

$$15) \int l \sin. x. \sin.^{2a} x. \cos. x dx = -\frac{1}{(2^a + 1)^2} \quad \text{V. T. 151. N° 1.}$$

$$16) \int l \sin. x. \cos. (p \sin. x). \cos. x dx = -\frac{1}{p} \sum_{n=1}^{\infty} \frac{1}{2^n - 1} \frac{p^{2n-1}}{1^{2n-1/1}} \quad \text{V. T. 71. N° 16.}$$

- 1) $\int l \cos. x dx = -\frac{1}{2} \pi l 2$ Euler, Calc. Int. IV. S. 3. 126. — Id., N. C. Petr. 14. 129. — Cauchy, Exere. 1826. p. 205. — Id., Lim. Imag. 146. — Serret, L. S. 1. — Roberts, L. 11. 471. — Grunert, Gr. 4. 113. — Lindmann, Stockh. Handl. 1850. III.
- 2) $\int l \cos. x. \cos. x dx = l 2 - 1$ V. T. 163. N°. 3.
- 3) $\int l \cos. x. \cos.^2 x dx = \frac{1}{8} \pi (1 - 2 l 2)$ V. T. 163. N°. 4.
- 4) $\int l \cos. x. \cos.^3 x dx = \frac{2}{3} \left(l 2 - \frac{5}{6} \right)$ V. T. 163. N°. 5.
- 5) $\int l \cos. x. \cos.^4 x dx = \frac{3}{16} \pi \left(\frac{7}{12} - l 2 \right)$ V. T. 163. N°. 6.
- 6) $\int l \cos. x. \cos.^5 x dx = \frac{8}{15} \left(l 2 - \frac{47}{60} \right)$ V. T. 163. N°. 7.
- 7) $\int l \cos. x. \sin.^2 x dx = -\frac{1}{8} \pi (1 + 2 l 2)$ V. T. 162. N°. 1.
- 8) $\int l \cos. x. \sin.^2 x. \cos. x dx = -\frac{1}{9} (4 - 3 l 2)$ V. T. 162. N°. 2.
- 9) $\int l \cos. x. \cos. 2 x dx = \frac{1}{4} \pi$ V. T. 331. N°. 3, 7.
- 10) $\int l \cos. x. \sin.^{2a} x dx = -\frac{1^{a/2}}{2^{a+1} 1^{a/2}} \frac{\pi}{2} \left\{ 1 + 2 l 2 + \sum_0^a \frac{1}{2} \right\}$ Lindmann, Stockh. Handl. 1850. III.
- 11) $\int l \cos. x. \cos.^{2a} x. \sin. x dx = -\frac{1}{(2a+1)^2}$ V. T. 151. N°. 1.
- 12) $\int l \cos. x. \cos.^{p-1} x. \sin. p. \sin. p x dx = \frac{\pi}{2^{p+2}} \left\{ A + Z'(p) - \frac{1}{p} - 2 l 2 \right\}$ Lindmann, Stockh. Handl. 1850. III.
- 13) $\int l \cos. x. \cos. (p \cos. x). \sin. x dx = -\frac{1}{p} \sum_1^{\infty} \frac{1}{2n-1} \frac{p^{2n-1}}{1^{2n-1/2}}$ V. T. 60. N°. 4.
- 14) $\int l \cos. x. \cos. (p l \sin. x). \text{Tang. } x dx = \frac{1}{2 p^2} + \frac{\pi}{4 p} \frac{1 + e^{p\pi}}{1 - e^{p\pi}}$ V. T. 335. N°. 13.

$$1) \int (l \sin. x)^2 dx = \frac{\pi}{2} \left\{ (l 2)^2 + \frac{1}{12} \pi^2 \right\} \quad \text{V. T. 164. N}^\circ. 1.$$

$$2) \int (l \sin. x)^3 \text{Tang. } x dx = -\frac{1}{240} \pi^3 \quad \text{V. T. 154. N}^\circ. 15.$$

$$3) \int (l \sin. x)^5 \text{Tang. } x dx = -\frac{1}{504} \pi^5 \quad \text{V. T. 155. N}^\circ. 5.$$

$$4) \int (l \sin. x)^p \cos. x dx = (-1)^p \Gamma(p+1) \quad \text{V. T. 42. N}^\circ. 2.$$

$$5) \int (l \sin. x)^{2a-1} \text{Tang. } x dx = -\frac{1}{4a} \pi^{2a} B_{2a-1} \quad \text{V. T. 158. N}^\circ. 6.$$

$$6) \int (l \sin. x)^a \sin. p-1 x. \cos. x dx = \frac{(-1)^a 1^{a/1}}{p^{a+1}} \quad \text{V. T. 151. N}^\circ. 2.$$

$$7) \int (l \cos. x)^2 dx = \frac{1}{2} \pi \left\{ (l 2)^2 + \frac{1}{12} \pi^2 \right\} \quad \text{V. T. 164. N}^\circ. 1.$$

$$8) \int (l \cos. x)^p \sin. x dx = (-1)^p \Gamma(1+p) \quad \text{V. T. 42. N}^\circ. 2.$$

$$9) \int (l \cos. x)^a \cos. p-1 x. \sin. x dx = \frac{(-1)^a 1^{a/1}}{p^{a+1}} \quad \text{V. T. 151. N}^\circ. 2.$$

$$10) \int l \sin. x (l \cos. x)^2 \text{Tang. } x dx = -\frac{1}{720} \pi^3 \quad \text{V. T. 337. N}^\circ. 12.$$

$$11) \int l \sin. x (l \cos. x)^4 \text{Tang. } x dx = -\frac{1}{2520} \pi^5 \quad \text{V. T. 337. N}^\circ. 14.$$

$$12) \int l \sin. x (l \cos. x)^{2a} \text{Tang. } x dx = \frac{-1}{4} \frac{\pi^{2a+2}}{(a+1)(2a+1)} B_{2a+1} \quad \text{V. T. 337. N}^\circ. 17.$$

$$1) \int l \text{Tang. } x dx = 0 \quad \text{Euler. Calc. Int. T. 4. S. 2. 127. — Cauchy, Exerc. 1826. p. 205.}$$

$$2) \int l \text{Tang. } \frac{1}{2} x. \sin. x dx = -l 2 \quad \text{Lobatschewsky, Mém. Kasan. 1826 t. 1. 107.}$$

$$3) \int l \text{Tang. } x. \sin.^2 x dx = \frac{1}{4} \pi \quad \text{V. T. 333. N}^\circ. 1. 5.$$

F. Log. en num. $(l \text{ Tang. } x)^a$.
Circ. Dir. ent.

TABLE 555 suite.

Lim. 0 et $\frac{\pi}{2}$.

- 4) $\int l \text{ Tang. } x. \text{Cos.}^2 x dx = -\frac{1}{4}\pi$ V. T. 333. N°. 1, 5.
- 5) $\int l \text{ Tang. } x. \text{Cos.} 2x dx = -\frac{1}{2}\pi$ V. T. 330. N°. 12 et T. 331. N°. 9.
- 6) $\int l \text{ Tang. } x. \text{Sin.}^p 2x dx = 0$ V. T. 183. N°. 8, 9.
- 7) $\int l \text{ Tang. } x. \text{Cos.}^{2(q-1)} x dx = -\frac{\Gamma(q-\frac{1}{2})}{\Gamma(q)} \frac{1}{4} \pi \left\{ \Lambda + 2 l 2 + Z' \left(\frac{2q-1}{2} \right) \right\}$ Lindmann, Gr. 16. 94.
- 8) $\int l \text{ Tang. } x. \text{Cos.}^{q-1} x. \text{Cos.} \{(q-1)x\} dx = -\frac{\pi}{2q}$
- 9) $\int l \text{ Tang. } x. \text{Cos.}^{q-1} x. \text{Cot. } x. \text{Sin.} \{(q+1)x\} dx = -\frac{1}{2}\pi \{ \Lambda + Z'(q+1) \}$ } Lindmann, Stockh. Handl. 1850. II.
- 10) $\int l \text{ Tang. } x. \text{Sin.}^{2a-1} 2x. \text{Cos.} 2x dx = -\frac{(1^{a-1/1})^2}{1^{2a/1}} 2^{2a-3}$ V. T. 53. N°. 24.
- 11) $\int (l \text{ Tang. } x)^2 dx = \frac{1}{8}\pi^3$ V. T. 305. N°. 4 et T. 358. N°. 1.
- 12) $\int (l \text{ Tang. } x)^4 dx = \frac{5}{32}\pi^5$ V. T. 305. N°. 7 et T. 358. N°. 12.
- 13) $\int (l \text{ Tang. } x)^6 dx = \frac{61}{128}\pi^7$ V. T. 305. N°. 8 et T. 358. N°. 15.
- 14) $\int (l \text{ Tang. } x)^{2a-1} dx = 0$ V. T. 180. N°. 3.
- 15) $\int (l \text{ Tang. } x)^{2a} dx = 2. 1^{2a/1} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^{2a+1}}$ V. T. 180. N°. 4.
- 16) $\int (l \text{ Tang. } x)^a. \text{Tang.}^q x dx = \frac{1}{2}\pi \frac{d^a}{dq^a}. \text{Sec.} \frac{1}{2} q \pi$ V. T. 180. N°. 6.

F. Log. } Log. de Circ. Dir. d'autre forme.
Circ. Dir. } Sans fact. Circ. Dir. TABLE 554.

Lim. 0 et $\frac{\pi}{2}$.

- 1) $\int l(p \text{ Tang. } x) dx = \frac{\pi}{2p} l p$ V. T. 180. N°. 8.
- 2) $\int l \text{ Sin. } (p \text{ Tang. } x) dx = \frac{1}{2}\pi l \frac{e^{2p}-1}{2e^{2p}}$ V. T. 415. N°. 1.

$$3) \int l \cos.(p \text{ Tang. } x) dx = \frac{1}{2} \pi l \frac{e^{2p} + 1}{2 e^{2p}} \quad \text{V. T. 415. N}^\circ. 2.$$

$$4) \int l \text{Tang. } (p \text{ Tang. } x) dx = \frac{1}{2} \pi l \frac{e^{2p} - 1}{e^{2p} + 1} \quad \text{V. T. 415. N}^\circ. 3.$$

$$5) \int l \cot.(p \text{ Tang. } x) dx = \frac{1}{2} \pi l \frac{e^p + e^{-p}}{e^p - e^{-p}} \quad \text{V. T. 415. N}^\circ. 12.$$

$$6) \int l(1 + \cos.x) dx = -\frac{1}{2} \pi l 2 + 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 304. N}^\circ. 12.$$

$$7) \int l(1 - \cos.x) dx = -\frac{1}{2} \pi l 2 - 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 238. N}^\circ. 4.$$

$$\left. \begin{aligned} 8) \int l(1 + q \cos.^2 x) dx &= \pi l \frac{1 + \sqrt{1+q}}{2} \\ 9) \int l(1 + \text{Tang.}^2 \lambda. \cos.^2 x) dx &= \pi l \left(\cos.^2 \frac{1}{2} \lambda. \sec. \lambda \right) \end{aligned} \right\} \text{Lobatschewsky, Mém. Kasan. 1835. 1.}$$

$$\left. \begin{aligned} 10) \int l(1 - \sin h p.^2 \lambda. \sin.^2 x) dx &= 2 \pi l \cos h p. \frac{1}{2} \lambda \\ 11) \int l(1 + q^2 \sin. x. \cos. x) dx &= \pi l \frac{1 + \sqrt{1+q^2}}{2} \\ 12) \int l(1 - \cos h p.^2 \lambda. \cos.^2 x) dx &= \pi l \frac{1 + \sin h p. \lambda}{2} \end{aligned} \right\} \text{Lobatschewsky, Mém. Kasan. 1836. 1. I. 186, 74.}$$

$$13) \int l(1 - \sin.^2 \lambda. \cos.^2 x) dx = 2 \pi l \cos. \frac{1}{2} \lambda \quad \text{Lobatschewsky, Mém. Kasan. 1835. 1.}$$

$$14) \int l(1 + p^2 \text{Tang.}^2 x) dx = \pi l(1 + p) \quad \text{V. T. 181. N}^\circ. 3.$$

$$15) \int l(q^2 + \text{Tang.}^2 x) dx = \pi l(1 + q) \quad \text{V. T. 181. N}^\circ. 2.$$

$$16) \int l(1 + p^2 \cot.^2 x) dx = \pi l(1 + p) \quad \text{V. T. 181. N}^\circ. 3.$$

$$17) \int l \{ 1 + p^2 \text{Tang.}^2 (q \text{ Tang. } x) \} dx = \pi l \left[1 + p \frac{e^q - e^{-q}}{e^q + e^{-q}} \right] \quad \text{V. T. 416. N}^\circ. 1.$$

$$18) \int l \{ 1 + p^2 \cot.^2 (q \text{ Tang. } x) \} dx = \pi l \left[1 + p \frac{e^q + e^{-q}}{e^q - e^{-q}} \right] \quad \text{V. T. 416. N}^\circ. 2.$$

- 19) $\int l(1 + 2p \sin x + p^2) dx = \sum_0^{\infty} \frac{1}{2n+1} \frac{2^{n/2}}{3^{n/2}} \left(\frac{2p}{1+p^2} \right)^{2n+1}, p \leq 1; \text{ Raabe, Cr. 15. 355.}$
- 20) $\int l\{1 + 2p \cos(q \text{Tang. } x) + p^2\} dx = \pi l(1 + pe^{-q}), p^2 \leq 1;$
21) $= \pi l(p + e^{-q}), p^2 \geq 1.$ V. T. 416. N°. 5, 6.
- 22) $\int l \frac{1 + \sin. \lambda. \cos.^2 x}{1 - \sin. \lambda. \cos.^2 x} dx = \pi l \frac{1 + \sin. \frac{1}{2} \lambda}{\cos. \frac{1}{2} \lambda} \text{ Lobatschewsky, Mém. Kasan. 1836. I. II. 15.}$
- 23) $\int l \frac{1 - \sin.^2 \lambda. \cos.^2 x}{1 - \sin.^2 \mu. \cos.^2 x} dx = 2\pi l(\cos. \frac{1}{2} \lambda. \sec. \frac{1}{2} \mu)$
24) $\int l \frac{\sin.^2 \lambda - \sin.^2 \mu. \cos.^2 x}{1 - \sin.^2 \mu \cos.^2 x} dx = \pi l \left\{ \text{Tang.} \frac{1}{2} \lambda. \text{Tang.} \left(\frac{1}{2} \text{Arcsin.} \frac{\sin. \mu}{\sin. \lambda} \right) \right\}$ Lobatschewsky, Mém. Kasan. 1835. 1.
- 25) $\int l \left\{ \frac{1 + \sin. \lambda. \cos. x}{1 - \sin. \lambda. \cos. x} \cdot \frac{1 + \sin. \mu. \cos. x}{1 - \sin. \mu. \cos. x} \dots \right\} dx = -2\pi l(\cos. \frac{1}{2} \lambda. \cos. \frac{1}{2} \mu \dots)$
26) $\int l \left\{ \frac{1 + \sin. \lambda. \cos. x}{1 - \sin. \lambda. \cos. x} \dots \frac{1 - \sin. \mu. \cos. x}{1 + \sin. \mu. \cos. x} \dots \right\} dx = 2\pi l(\cos. \frac{1}{2} \mu \dots \sec. \frac{1}{2} \lambda \dots)$ Lobatschewsky, Mém. Kasan. 1836. I. II. 23.
- 27) $\int l l \text{Tang. } x dx = \frac{\pi}{2} l \left\{ \frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} \vee 2\pi \right\} \text{ V. T. 191. N°. 1.}$

- 1) $\int l \text{Tang.} \frac{1}{2} x. \sin. x dx = -l 2 \text{ Lobatschewsky, Mém. Kasan. 1836. II. 28.}$
- 2) $\int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \sin. 2x dx = \pm \frac{\pi}{2} \text{ V. T. 63. N°. 12.}$
- 3) $\int l(1 + \cos. x) \text{Tang. } x dx = \frac{1}{12} \pi^2 \text{ V. T. 160. N°. 1.}$
- 4) $\int l(1 - \cos. x) \text{Tang. } x dx = -\frac{1}{6} \pi^2 \text{ V. T. 160. N°. 5.}$
- 5) $\int l(1 - p^2 \sin.^2 x) \sin.^2 x dx = \frac{1}{2} \pi l \frac{1 + \vee(1 - p^2)}{2} - \frac{\pi}{4} \frac{1 - \vee(1 - p^2)}{1 + \vee(1 - p^2)}$
6) $\int l(1 + p^2 \cos.^2 x) \sin.^2 x dx = \frac{\pi}{2} l \frac{1 + \vee(1 + p^2)}{2} + \pi \frac{\vee(1 + p^2) - 1}{2p^2} - \frac{1}{4} \pi$ Roberts, L. 11. 201.

F. Log. } Log. de Circ. Dir. d'autre forme. TABLE 555 suite. Lim. 0 et $\frac{\pi}{2}$.
Circ. Dir. } Avec fact. Circ. Dir.

$$7) \int l(1 + p^2 \cos.^2 x) \cos.^2 x dx = \frac{1}{2} \pi l \frac{1 + \sqrt{1+p^2}}{2} + \pi \frac{1 - \sqrt{1+p^2}}{2p^2} + \frac{1}{4} \pi \text{ V. T. 334. N}^\circ. 8 \text{ et T. 335. N}^\circ. 6.$$

$$8) \int l(1 + p^2 \cos.^2 x) \cos. 2 x dx = \pi \frac{1 - \sqrt{1+p^2}}{p^2} + \frac{1}{2} \pi \text{ V. T. 334. N}^\circ. 8 \text{ et T. 335. N}^\circ. 6.$$

$$9) \int l(1 - \sin.^2 x) \text{Tang. } x dx = -\frac{1}{24} \pi^2 \text{ V. T. 160. N}^\circ. 11.$$

$$10) \int l(1 + \cos.^2 x) \text{Tang. } x dx = \frac{1}{24} \pi^2 \text{ V. T. 160. N}^\circ. 6.$$

$$11) \int l \text{Tang.} \left(\frac{\pi}{4} \pm x \right) \text{Tang. } x dx = \pm \frac{1}{4} \pi^2 \text{ V. T. 183. N}^\circ. 3.$$

$$12) \int l(p \text{Tang. } x) \sin. (q \text{Tang. } x) \cdot \text{Tang. } x dx = \frac{\pi}{4} e^{-q} \{2 l p - \text{Ei.}(q)\} - \frac{\pi}{4} e^q \text{Ei.}(-q) \text{ V. T. 417. N}^\circ. 5.$$

$$13) \int l(p \text{Tang. } x) \cos. (q \text{Tang. } x) dx = \frac{\pi}{4q} e^{-q} \{2 l p - \text{Ei.}(q)\} + \frac{\pi}{4q} e^q \text{Ei.}(-q) \text{ V. T. 417. N}^\circ. 6.$$

$$14) \int \sin. (p l \sin. x) \text{Tang. } x dx = \frac{\pi}{4} \frac{1 + e^{p\pi}}{1 - e^{p\pi}} + \frac{1}{2p} \text{ V. T. 404. N}^\circ. 11.$$

$$15) \int \sin. (p l \sin. x) \sin. q x \cdot \text{Tang. } x dx = -\sum_1 \frac{p}{(2n+q)^2 + p^2} \text{ V. T. 404. N}^\circ. 12.$$

$$16) \int l \text{Tang. } x \cdot \cos. (q \text{Tang. } x) dx = \frac{\pi}{4q} \{e^q \text{Ei.}(-q) - e^{-q} \text{Ei.}(q)\} \text{ V. T. 417. N}^\circ. 2.$$

$$17) \int l \cot. x \cdot \sin. (q \text{Tang. } x) dx = \frac{\pi}{4q} \{e^{-q} \text{Ei.}(q) + e^q \text{Ei.}(-q)\} \text{ V. T. 417. N}^\circ. 7.$$

$$18) \int l \cot. x \cdot \cos. (q \text{Tang. } x) dx = \frac{\pi}{4q} \{e^{-q} \text{Ei.}(q) - e^q \text{Ei.}(-q)\} \text{ V. T. 417. N}^\circ. 8.$$

F. Log. en num. ($l \sin. x$)^a. TABLE 556. Lim. 0 et $\frac{\pi}{2}$.
Circ. Dir. rat. en dén. monôme.

$$1) \int l \sin. x \frac{1 + \sin.^2 x}{\sin. 2 x} dx = -\infty \text{ V. T. 153. N}^\circ. 11.$$

$$2) \int l \sin. x \frac{dx}{\cos. x} = -\frac{1}{8} \pi^2 \text{ V. T. 152. N}^\circ. 13.$$

$$3) \int l \sin. x \frac{dx}{\cos. 2 x} = \frac{1}{8} \pi^2 \text{ V. T. 340. N}^\circ. 14.$$

$$4) \int l \sin. x \frac{dx}{\text{Tang}^p x} = \frac{1}{2} \frac{\pi}{p-1} \text{Sec.} \frac{1}{2} p \pi, p < 1; \quad \text{V. T. 63. N}^\circ. 4.$$

$$5) \int l \sin. x \frac{\text{Sin.}^{2p-1} x}{\text{Cos.}^{p+1} x} dx = -\frac{\pi}{4p} \text{Cosec.} \frac{1}{2} p \pi, 0 < p < \frac{1}{2} \quad \text{V. T. 53. N}^\circ. 25.$$

$$6) \int l \sin. x \frac{p-2 \text{Cos.}^2 x}{\text{Tang}^{p-1} x} dx = \frac{1}{4} p \pi \text{Cosec.} \frac{1}{2} p \pi, p < 2; \quad \text{V. T. 63. N}^\circ. 5.$$

$$7) \int l \sin. x \frac{p \text{Cos.} 2x + \text{Sin.}^2 2x}{\text{Tang}^p x} \frac{dx}{\text{Sin.} 2x} = -\frac{p+1}{4} l \text{Sec.} \left\{ \frac{p+1}{2} \pi \right\}, 0 < p < 1; \quad \text{V. T. 63. N}^\circ. 6.$$

$$8) \int l \sin. x \frac{(1 - \text{Sin.} x)^{p-1}}{\text{Sin}^p x} \frac{dx}{\text{Tang} x} = \infty \quad \text{V. T. 64. N}^\circ. 4.$$

$$9) \int l \sin. x \text{Cos.} (q \text{Tang.} x) \frac{dx}{\text{Cos.}^2 x} = \pi \frac{e^{-q} - 1}{2q} \quad \text{V. T. 70. N}^\circ. 2.$$

$$10) \int l \sin. x \text{Cos.} (p l \text{Cos.} x) \frac{dx}{\text{Tang.} x} = \frac{1}{2p^2} + \frac{\pi}{4p} \frac{1 + e^{p\pi}}{1 - e^{p\pi}} \quad \text{V. T. 335. N}^\circ. 14.$$

$$11) \int (l \sin. x)^2 \text{Sin.} (p l \text{Cos.} x) \frac{dx}{\text{Cot.} x} = \infty \quad \text{V. T. 336. N}^\circ. 10.$$

$$12) \int (l \sin. x)^3 \frac{dx}{\text{Cos.} x} = -\frac{1}{16} \pi^3 \quad \text{V. T. 154. N}^\circ. 14.$$

$$13) \int (l \sin. x)^5 \frac{dx}{\text{Cos.} x} = -\frac{1}{8} \pi^5 \quad \text{V. T. 155. N}^\circ. 4.$$

$$14) \int (l \sin. x)^7 \frac{dx}{\text{Cos.} x} = -\frac{17}{32} \pi^7 \quad \text{V. T. 155. N}^\circ. 10.$$

$$15) \int (l \sin. x)^{2a} \frac{dx}{\text{Cos.} x} = \frac{2^{2a+1} - 1}{2^{2a+1}} 1^{2a+1} \sum_1^{\infty} \frac{1}{n^{2a+1}} \quad \text{V. T. 158. N}^\circ. 4.$$

$$16) \int (l \sin. x)^{2a-1} \frac{dx}{\text{Cos.} x} = \frac{2^{2a} - 1}{4a} \pi^{2a} B_{2a-1}$$

$$17) \int (l \sin. x)^{a-1} \frac{dx}{\text{Cos.} x} = \frac{1^{a-1/1}}{(-1)^{a-1}} \sum_0^{\infty} \frac{1}{(2n+1)^a}$$

$$18) \int (l \sin. x)^{a-1} \frac{\text{Sin.}^q x}{\text{Cos.} x} dx = \frac{1^{a-1/1}}{(-1)^{a-1}} \sum_0^{\infty} \frac{1}{(2n+q+1)^a}$$

$$19) \int (l \sin. x)^{2a} \frac{\text{Sin.}^q x - \text{Cosec.}^q x}{\text{Cos.} x} dx = \frac{1}{b} (-1)^a (2b\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{n}{2b} \right) \text{Sin.} nq\pi, q^2 = a^2 b^2 < 1; \quad \text{V. T. 158. N}^\circ. 9.$$

- 1) $\int l \text{ Cos. } x \frac{dx}{\text{Sin. } x} = -\frac{1}{8} \pi^2$ V. T. 152. N°. 13.
- 2) $\int (l \text{ Cos. } x)^{p-1} \frac{\text{Cos. }^q x}{\text{Tang. } x} dx = \frac{\Gamma(p)}{(-1)^{p-1}} \sum_0^{\infty} \frac{1}{(q+1+2n)^p}$ V. T. 158. N°. 8.
- 3) $\int l \text{ Cos. } x \frac{dx}{\text{Tang. } x} = -\frac{1}{24} \pi^2$ V. T. 330. N°. 13.
- 4) $\int l \text{ Cos. } x \frac{\text{Tang. }^{p-1} x}{\text{Sin. } 2x} dx = \frac{\pi}{4(p-1)} \text{Sec. } \frac{1}{2} p \pi, p < 1;$ V. T. 179. N°. 5.
- 5) $\int l \text{ Cos. } x \frac{dx}{\text{Tang. }^{p+1} x \cdot \text{Sin. } 2x} = -\frac{\pi}{2(p+1)} \text{Sec. } \frac{1}{2} p \pi, p < 1;$ V. T. 63. N°. 4.
- 6) $\int l \text{ Cos. } x \frac{\text{Cos. } 2x - p}{\text{Tang. }^p x} dx = \frac{1-p}{4} \pi \text{Cosec. } \left(\frac{p-1}{2} \pi \right), p^2 < 1;$ V. T. 63. N°. 5.
- 7) $\int l \text{ Cos. } x \frac{(1 - \text{Cos. } x)^{p-1}}{\text{Cos. }^p x} \text{Tang. } x dx = \infty, p < 1;$ V. T. 64. N°. 5.
- 8) $\int l \text{ Cos. } x \cdot \text{Sin. } (p \text{Tang. } x) \frac{dx}{\text{Sin. } 2x} = -\frac{1}{4} \pi \text{Ei. } (-p)$ V. T. 414. N°. 3.
- 9) $\int l \text{ Cos. } x \cdot \text{Sin. } (p \text{Tang. } x) \frac{dx}{\text{Cos. }^2 x} = \infty$ V. T. 59. N°. 6.
- 10) $\int l \text{ Cos. } x \cdot \text{Cos. } (p \text{Tang. } x) \frac{dx}{\text{Cos. }^2 x} = \infty$ V. T. 59. N°. 5.
- 11) $\int l \text{ Cos. } x \cdot \text{Cos. } (p \text{Cot. } x) \frac{dx}{\text{Sin. }^2 x} = -\pi \frac{1 - e^{-q}}{2p}$ V. T. 60. N°. 5.
- 12) $\int l \text{ Cos. } x \cdot \text{Cosec. }^2 (q \text{Tang. } x) \frac{dx}{\text{Cos. }^2 x} = \frac{1}{2} \frac{\pi}{1 - e^{2q}}$ V. T. 70. N°. 12.
- 13) $\int l \text{ Cos. } x \cdot \text{Sec. }^2 (q \text{Tang. } x) \frac{dx}{\text{Cos. }^2 x} = \infty$ V. T. 59. N°. 7.
- 14) $\int l \text{ Cos. } x \cdot \text{Sec. }^2 (q \text{Cot. } x) \frac{dx}{\text{Sin. }^2 x} = \frac{\pi}{2p} \frac{1 - e^{2p}}{1 + e^{2p}}$ V. T. 60. N°. 6.
- 15) $\int (l \text{ Cos. } x)^3 \frac{dx}{\text{Sin. } x} = -\frac{1}{16} \pi^4$ V. T. 154. N°. 14.
- 16) $\int (l \text{ Cos. } x)^3 \frac{dx}{\text{Tang. } x} = -\frac{1}{240} \pi^4$ V. T. 154. N°. 15.

F. Log. en num. $(l \text{ Cos. } x)^a$.

Circ. Dir. rat. en dén. monôme.

TABLE 557 suite.

Lim. 0 et $\frac{\pi}{2}$.

$$17) \int (l \text{ Cos. } x)^5 \frac{dx}{\text{Sin. } x} = -\frac{1}{8} \pi^0 \quad \text{V. T. 155. N}^\circ. 4.$$

$$18) \int (l \text{ Cos. } x)^5 \frac{dx}{\text{Tang. } x} = -\frac{1}{504} \pi^6 \quad \text{V. T. 155. N}^\circ. 5.$$

$$19) \int (l \text{ Cos. } x)^7 \frac{dx}{\text{Sin. } x} = -\frac{17}{32} \pi^8 \quad \text{V. T. 155. N}^\circ. 10.$$

$$20) \int (l \text{ Cos. } x)^{2a-1} \frac{dx}{\text{Sin. } x} = \frac{1-2^{2a}}{4a} \pi^{2a} B_{2a-1} \quad \text{V. T. 158. N}^\circ. 5.$$

$$21) \int (l \text{ Cos. } x)^{2a-1} \frac{dx}{\text{Tang. } x} = -\frac{\pi^{2a}}{4a} B_{2a-1} \quad \text{V. T. 158. N}^\circ. 6.$$

$$22) \int (l \text{ Cos. } x)^{2a} \frac{dx}{\text{Sin. } x} = \frac{2^{2a+1}-1}{2^{2a+1}} 1^{2a+1} \sum_1^{\infty} \frac{1}{n^{2a+1}} \quad \text{V. T. 158. N}^\circ. 4.$$

$$23) \int (l \text{ Cos. } x)^{2a} \frac{\text{Cos. } qx - \text{Sec. } qx}{\text{Sin. } x} dx = \frac{1}{b} (-1)^a (2b\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{n}{2b} \right) \text{Sin. } nq\pi, q^2 = a^2 b^2 < 1; \quad \text{V. T. 155. N}^\circ. 9.$$

F. Log. en num. $(l \text{ Tang. } x)^a$.

Circ. Dir. rat. en dén. monôme.

TABLE 558.

Lim. 0 et $\frac{\pi}{2}$.

$$1) \int l \text{ Tang. } x \frac{dx}{\text{Sin. } 4x} = -\infty \quad \text{V. T. 309. N}^\circ. 2 \text{ et T. 357. N}^\circ. 2.$$

$$2) \int l \text{ Tang. } x \frac{dx}{\text{Cos. } 2x} = -\frac{1}{4} \pi^2 \quad \text{V. T. 180. N}^\circ. 11.$$

$$3) \int l \text{ Tang. } x \frac{dx}{\text{Tang. } 2x} = -\infty \quad \text{V. T. 309. N}^\circ. 3 \text{ et T. 357. N}^\circ. 4.$$

$$4) \int l \text{ Tang. } x \frac{\text{Tang. } px}{\text{Cos. } 2x} dx = -\left\{ \frac{1}{2} \pi \text{Cosec.} \left(\frac{p+1}{2} \pi \right) \right\}^2, p^2 < 1; \quad \text{V. T. 180. N}^\circ. 12.$$

$$5) \int l \text{ Tang. } x \frac{dx}{\text{Cos. } 2x \cdot \text{Tang. } px} = -\left\{ \frac{1}{2} \pi \text{Cosec.} \left(\frac{p+1}{2} \pi \right) \right\}^2, p^2 < 1; \quad \text{V. T. 180. N}^\circ. 12.$$

$$6) \int l(b \text{ Tang. } x) \cdot \text{Sin. } (a \text{ Tang. } x) \frac{dx}{\text{Sin. } 2x} = -\frac{1}{2} \pi \left\{ A + l \frac{a}{b} \right\} \quad \text{V. T. 414. N}^\circ. 2.$$

$$7) \int l \text{ Tang. } x \frac{1 - \text{Tang. }^p x}{\text{Cos. } 2x} dx = \left(\frac{1}{2} \pi \text{Tang. } \frac{1}{2} p \pi \right)^2, p^2 < 1; \quad \text{V. T. 185. N}^\circ. 7.$$

- 8) $\int (l \text{ Tang. } x)^3 \frac{dx}{\text{Cos. } 2x} = -\frac{1}{8} \pi^4$ V. T. 310. N°. 1 et T. 358. N°. 11.
- 9) $\int (l \text{ Tang. } x)^5 \frac{dx}{\text{Cos. } 2x} = -\frac{1}{4} \pi^6$ V. T. 310. N°. 5 et T. 358. N°. 14.
- 10) $\int (l \text{ Tang. } x)^7 \frac{dx}{\text{Cos. } 2x} = -\frac{17}{16} \pi^8$ V. T. 310. N°. 8 et T. 358. N°. 15.
- 11) $\int (l \text{ Tang. } x)^{2a-1} \frac{dx}{\text{Cos. } 2x} = \frac{1-2^{2a}}{2a} B_{2a-1} \pi^{2a}$ V. T. 310. N°. 9 et T. 359. N°. 3.
- 12) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{\text{Cos. } 2x} = 0$ V. T. 310. N°. 10 et T. 359. N°. 4.
- 13) $\int (l \text{ Ty. } x)^{2a} (\text{Tg. } qx + \text{Cot. } qx) dx = \frac{2}{b} (-1)^{a+1} (2b\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b} \right) \text{Cos.} \left(\frac{2n-1}{2} q\pi \right) \left\{ \begin{array}{l} \text{V. T. 184.} \\ \text{N°. 5, 6.} \end{array} \right.$
- 14) $\int (l \text{ Tang. } x)^{2a} \frac{(\text{Tang. } qx - \text{Cot. } qx)}{\text{Cos. } 2x} dx = \frac{2}{b} (-1)^{a+1} (2b\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{n}{2b} \right) \text{Sin. } qn\pi \left\{ \begin{array}{l} q^2 = a^2 b^2 < 1; \end{array} \right.$

- 1) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{dx}{\text{Sin. } 2x} = \pm \frac{1}{2} \pi^2$ V. T. 338. N°. 2.
- 2) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\text{Tang.}^{p-1} x}{\text{Sin. } 2x} dx = \pm \frac{\pi}{1-p} \text{Cot.} \frac{1}{2} p\pi$ V. T. 63. N°. 14.
- 3) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{dx}{\text{Sin. } 2x \cdot \text{Tang.}^{p-1} x} = \pm \frac{\pi}{1-p} \text{Cot.} \frac{1}{2} p\pi$ V. T. 63. N°. 15.
- 4) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{(p+q)(\text{Tang.}^{p+q} x + \text{Cot.}^{p+q} x) + (p-q)(\text{Tang.}^{p-q} x + \text{Cot.}^{p-q} x)}{\text{Sin. } 2x} dx =$
 $= \pm \frac{4\pi \text{Sin. } p\pi}{\text{Cos. } p\pi + \text{Cos. } q\pi}, p+q < 1; \text{ V. T. 64. N°. 15.}$
- 5) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{dx}{\text{Tang. } x} = \pm \frac{1}{2} \pi^2$ V. T. 183. N°. 3.
- 6) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\text{Tang.}^p x + \text{Cot.}^p x}{\text{Sin. } 2x} dx = \pm \frac{2\pi}{p+1} \text{Tang.} \frac{1}{2} p\pi$ V. T. 64. N°. 10
- 7) $\int l \text{ Sin. } (q \text{ Tang. } x) \frac{dx}{\text{Cos. } 2x} = \frac{1}{2} q\pi - \frac{1}{4} \pi^2$ V. T. 415. N°. 13.

- 8) $\int l \cos.(q \text{ Tang. } x) \frac{dx}{\cos. 2x} = \frac{1}{2} q \pi \quad \text{V. T. 415. N}^\circ. 14.$
- 9) $\int l \text{ Tang. } (q \text{ Tang. } x) \frac{dx}{\cos. 2x} = -\frac{1}{4} \pi^2 \quad \text{V. T. 415. N}^\circ. 15.$
- 10) $\int l(q \text{ Tang. } x). \sin^{2a} 2x dx = \frac{1^{a/2}}{2^{a/2}} \pi l q \quad \text{V. T. 183. N}^\circ. 8.$
- 11) $\int l(q \text{ Tang. } x). \sin^{2a-1} 2x dx = \frac{1^{a-1/2}}{a^{a/2}} 2^{2a-2} l q \quad \text{V. T. 183. N}^\circ. 9.$
- 12) $\int l(q \text{ Tang. } x). \sin^{p-1} 2x dx = 2^{2p-2} l q \frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma(p)} \quad \text{V. T. 183. N}^\circ. 10.$
- 13) $\int l \text{ Tang. } x. \sin.(q \text{ Cot. } x) \frac{dx}{\text{Tang. } x} = \frac{\pi}{4} \{e^{-q} \text{Ei.}(q) + e^q \text{Ei.}(-q)\} \quad \text{V. T. 417. N}^\circ. 7.$
- 14) $\int l \text{ Tang. } x. \sin.(q \text{ Cot. } x) \frac{dx}{\cos. 2x} = \frac{\pi}{2} \left\{ \text{Ci.}(q). \cos. q + \text{Si.}(q). \sin. q - \frac{1}{2} \sin. q \right\} \quad \text{V. T. 417. N}^\circ. 9.$
- 15) $\int l(p \text{ Cot. } x). \sin.(q \text{ Cot. } x) \frac{dx}{\text{Tang. } x} = \frac{\pi}{4} e^{-q} \{2lp - \text{Ei.}(q)\} - \frac{\pi}{4} e^q \text{Ei.}(-q) \quad \text{V. T. 417. N}^\circ. 5.$
- 16) $\int l \text{ Tang. } x. \cos.(q \text{ Tang. } x) \frac{dx}{\cos. 2x} = -\frac{\pi}{2} \left\{ \text{Ci.}(q). \sin. q - \text{Si.}(q). \cos. q + \frac{1}{2} \cos. q \right\} \quad \text{V. T. 417. N}^\circ. 10.$
- 17) $\int l \text{ Tang. }^2 \left(\frac{\pi}{4} \pm x \right). \sin.(q \text{ Tang. } x) \frac{dx}{\cos.^2 x} = \pm \frac{2\pi}{q} \sin. q \quad \text{V. T. 70. N}^\circ. 6.$
- 18) $\int l \text{ Tang. }^2 \left(\frac{\pi}{4} \pm x \right). \sin.(q \text{ Cot. } x) \frac{dx}{\sin.^2 x} = \pm \frac{2\pi}{q} \sin. q \quad \text{V. T. 71. N}^\circ. 15.$
- 19) $\int l \text{ Tang. }^2 \left(\frac{\pi}{4} \pm x \right). \cos.(q \text{ Tang. } x) \frac{dx}{\cos.^2 x} = \pm \frac{2}{q} \{ \text{Si.}(q). \cos. q - \text{Ci.}(q). \sin. q \} \quad \text{V. T. 70. N}^\circ. 7.$
- 20) $\int l \text{ Tang. }^2 \left(\frac{\pi}{4} \pm x \right). \text{Tang.}(q \text{ Tang. } x) \frac{dx}{\cos.^2 x} = \pm 2\pi \quad \text{V. T. 339. N}^\circ. 8.$
- 21) $\int l \text{ Tang. }^2 \left(\frac{\pi}{4} \pm x \right). \text{Cot.}(q \text{ Tang. } x) \frac{dx}{\cos.^2 x} = \pm \frac{\pi - 2q}{q} \pi \quad \text{V. T. 339. N}^\circ. 7.$
- 22) $\int l \text{ Tang. }^2 \left(\frac{\pi}{4} \pm x \right). \text{Cosec.}(q \text{ Tang. } x) \frac{dx}{\cos.^2 x} = \pm \frac{\pi^2}{q} \quad \text{V. T. 339. N}^\circ. 9.$

$$23) \int l(1 + \text{Sin. } x) \frac{dx}{\text{Tang. } x} = \frac{1}{12} \pi^2 \quad \text{V. T. 160. N}^\circ. 1.$$

$$24) \int l(1 - \text{Sin. } x) \frac{dx}{\text{Tang. } x} = -\frac{1}{6} \pi^2 \quad \text{V. T. 160. N}^\circ. 7.$$

$$25) \int l(1 + p \text{Sin. } x) \frac{\text{Cos.}^3 x}{(3 - \text{Cos. } 2x)^2} dx = \frac{1}{8} \left\{ (1 + p)^2 l(1 + p) - p l 2 - \frac{1}{2} p^2 \pi \right\} \frac{1}{1 + p^2} \quad \text{V. T. 160. N}^\circ. 4.$$

$$26) \int l(1 + \text{Sin.}^2 x) \frac{dx}{\text{Tang. } x} = \frac{1}{24} \pi^2 \quad \text{V. T. 160. N}^\circ. 6.$$

$$27) \int l(1 - \text{Sin.}^4 x) \frac{dx}{\text{Tang. } x} = -\frac{1}{24} \pi^2 \quad \text{V. T. 160. N}^\circ. 11.$$

$$28) \int l(1 + p \text{Cos. } x) \frac{dx}{\text{Cos. } x} = \frac{1}{5} \pi^2 - \frac{1}{2} (\text{Arcos. } p)^2, \quad p^2 < 1; \quad \text{Winckler, Cr. 45. 102.}$$

$$29) \int l(1 + \text{Cos. } x) \frac{\text{Sin. } x}{3 + \text{Cos. } 2x} dx = \frac{\pi}{16} l 2 \quad \text{V. T. 160. N}^\circ. 2.$$

$$\left. \begin{aligned} 30) \int l(1 + p^2 \text{Tang.}^2 x) \frac{dx}{\text{Cos. } 2x} &= -\pi \text{Arctang. } p \\ 31) \int l(1 + p^2 \text{Cot.}^2 x) \frac{dx}{\text{Cos. } 2x} &= \pi \text{Arctang. } p \end{aligned} \right\} \quad \text{V. T. 181. N}^\circ. 11.$$

$$32) \int \text{Sin. } (p l \text{Sin. } x) \frac{dx}{\text{Cos. } x} = \frac{\pi}{4} \frac{1 - e^{p\pi}}{1 + e^{p\pi}} \quad \text{V. T. 404. N}^\circ. 10.$$

$$33) \int \text{Sin. } (p l \text{Sin. } x) \frac{\text{Tang. } x}{\text{Sin.}^2 x} dx = -\sum_1^{\infty} \frac{p}{(2n - q)^2 + p^2} \quad \text{V. T. 404. N}^\circ. 16.$$

$$34) \int \text{Sin. } (p l \text{Cos. } x) \frac{dx}{\text{Tang. } x} = \frac{\pi}{4} \frac{1 + e^{p\pi}}{1 - e^{p\pi}} + \frac{1}{2p} \quad \text{V. T. 404. N}^\circ. 11.$$

$$35) \int \text{Sin. } (p l \text{Cos. } x) \frac{\text{Cos. } x}{\text{Tang. } x} dx = -\sum_1^{\infty} \frac{p}{(2n + q)^2 + p^2} \quad \text{V. T. 404. N}^\circ. 12.$$

$$36) \int \text{Cos. } (p l \text{Sin. } x) \frac{l \text{Sin. } x}{\text{Cos. } x} dx = \frac{\pi^2}{2} \frac{e^{p\pi}}{(e^{p\pi} + 1)^2} \quad \text{V. T. 404. N}^\circ. 14.$$

- 1) $\int_0^{\frac{\pi}{2}} l \frac{\cos. 2x}{\cos.^2 x} \frac{\text{Tang}^{p-2} x}{\sin. 2x} dx = \frac{\pi}{2(p-2)} \text{Cot.} \frac{1}{2} p \pi$ V. T. 179. N°. 6.
- 2) $\int_0^{\frac{\pi}{2}} l \frac{1 + \sin. x}{1 - \sin. x} \frac{dx}{\sin. x} = \frac{1}{2} \pi^2$ Legendre, Exerc. Suppl. 34. — Schlömilch, Gr. 4. 316.
- 3) $\int_0^{\frac{\pi}{2}} l \frac{1 + p \sin. x}{1 - p \sin. x} \frac{dx}{\sin. x} = \pi \text{Arcsin. } p, p \leq 1;$ Raabe, Int. 421.
- 4) $\int_0^{\frac{\pi}{2}} l \frac{2 \cos. x}{1 + \cos. x} \frac{dx}{\sin. x} = -\frac{1}{12} \pi^2$ V. T. 160. N°. 10.
- 5) $\int_0^{\frac{\pi}{2}} l \frac{1 + \sin. x \sqrt{1-p^2}}{1 - \sin. x \sqrt{1-p^2}} \frac{dx}{\sin. x} = \pi \text{Arccos. } p, p^2 \leq 1;$ V. T. 186. N°. 1.
- 6) $\int_0^{\frac{\pi}{2}} l \left\{ \frac{1 + \sin. x}{1 - \sin. x} - 2 \sin. x \right\} \frac{dx}{\sin.^3 x} = \frac{1}{4} \pi^2$ Legendre, Exerc. Suppl. 34.
- 7) $\int_0^{\frac{\pi}{2}} l \frac{1 + \sin. 2x}{1 + \cos. \lambda. \sin. 2x} \frac{\text{Tang}^p x + \text{Cot}^p x}{\sin. 2x} dx = \frac{2\pi}{p} \text{Cosec. } p \pi. (1 - \cos. p \lambda)$ V. T. 179. N°. 17.
- 8) $\int_0^{\frac{\pi}{2}} l \frac{(\sin. x + \cos. x)^2}{1 + \cos. \lambda. \sin. 2x} \frac{dx}{\sin. 2x} = \frac{1}{2} \lambda^2, 0 < \lambda < \pi;$ V. T. 179. N°. 13.
- 9) $\int_0^{\frac{\pi}{2}} l \frac{1 + q \cos. x}{1 - q \cos. x} \frac{dx}{\cos. x} = \pi \text{Arcsin. } q, q \leq 1;$ Raabe, Cr. 25. 169. — Id., Int. II. 421.
- 10) $\int_0^{\frac{\pi}{2}} l \frac{1 + \sin. \lambda. \cos. x}{1 - \sin. \lambda. \cos. x} \frac{dx}{\cos. x} = \pi \lambda$ V. T. 166. N°. 7.
- 11) $\int_0^{\frac{\pi}{2}} l \frac{1 + q \cos. a x}{1 - q \cos. a x} \frac{dx}{\cos. a x} = \pi \text{Arcsin. } q, q \leq 1;$ Raabe, Cr. 25. 169.
- 12) $\int_0^{\frac{\pi}{2}} l \frac{\text{Sec. } x + \sqrt{1-p^2}}{\text{Sec. } x - \sqrt{1-p^2}} \frac{dx}{\cos. x} = \pi \text{Arccos. } p, p^2 < 1;$ V. T. 186. N°. 1.
- 13) $\int_0^{\frac{\pi}{2}} l \frac{1 + \sin. 2x}{1 + \cos. \lambda. \sin. 2x} \frac{\sin^{p-1} x}{\cos^{p+1} x} dx = \frac{2\pi}{p} \text{Cosec. } p \pi (1 - \cos. p \lambda), p < 1;$ V. T. 179. N°. 16.
- 14) $\int_0^{\frac{\pi}{2}} l \frac{1 + \text{Tang. } x}{1 - \text{Tang. } x} \frac{dx}{\text{Tang. } x} = \frac{1}{4} \pi^2$ Schlömilch, Gr. 4. 316. — Id., Gr. 7. 100.
- 15) $\int_0^{\frac{\pi}{2}} l \frac{1 + \sin. (\text{Tang. } x)}{1 - \sin. (\text{Tang. } x)} \frac{dx}{\sin. 2x} = \frac{1}{4} \pi^2$ V. T. 414. N°. 4.
- 16) $\int_0^{\frac{\pi}{2}} l \left\{ \frac{1 + p \text{Tang. } (\text{Tang. } x)}{1 - p \text{Tang. } (\text{Tang. } x)} \right\}^2 \frac{dx}{\sin. 2x} = \frac{1}{4} \pi^2$ V. T. 414. N°. 5.

- 1) $\int l \cos. x. (l \sin. x)^2 \frac{dx}{\text{Tang. } x} = - \frac{1}{720} \pi^4 \quad \text{V. T. 332. N}^\circ. 2.$
- 2) $\int l \cos. x. (l \sin. x)^4 \frac{dx}{\text{Tang. } x} = - \frac{1}{2520} \pi^6 \quad \text{V. T. 332. N}^\circ. 3.$
- 3) $\int l \cos. x. (l \sin. x)^{2a} \frac{dx}{\text{Tang. } x} = - \frac{\pi^{2a+2}}{4(a+1)(2a+1)} B_{2a+1} \quad \text{V. T. 332. N}^\circ. 5.$
- 4) $\int l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right). (l \text{Tang. } x)^2 \frac{dx}{\sin. 2x} = \pm \frac{1}{12} \pi^4 \quad \text{V. T. 338. N}^\circ. 8.$
- 5) $\int l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right). (l \text{Tang. } x)^4 \frac{dx}{\sin. 2x} = \pm \frac{1}{10} \pi^6 \quad \text{V. T. 338. N}^\circ. 9.$
- 6) $\int l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right). (l \text{Tang. } x)^6 \frac{dx}{\sin. 2x} = \pm \frac{17}{56} \pi^8 \quad \text{V. T. 338. N}^\circ. 10.$
- 7) $\int l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right). (l \text{Tang. } x)^{2a} \frac{dx}{\sin. 2x} = \pm \frac{1 - 2^{2a+2}}{(a+1)(2a+1)} \pi^{2a+2} B_{2a+1} \quad \text{V. T. 338. N}^\circ. 11.$
- 8) $\int l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right). (l \text{Tang. } x)^{2a-1} \frac{dx}{\sin. 2x} = 0 \quad \text{V. T. 338. N}^\circ. 12.$
- 9) $\int l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{p l \text{Tang. } x + 1}{\sin. 2x} \text{Tang.}^p x dx = \pm \frac{1}{2} \pi^2 \text{Cosec.}^2 \left\{ \frac{p+1}{2} \pi \right\} \quad \text{V. T. 338. N}^\circ. 4.$
- 10) $\int l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{p l \text{Tang. } x - 1}{\text{Tang.}^p x. \sin. 2x} dx = \mp \frac{1}{2} \pi^2 \text{Cosec.}^2 \left\{ \frac{p+1}{2} \pi \right\} \quad \text{V. T. 338. N}^\circ. 5.$
- 11) $\int l \text{Tang. } x. l \frac{1+b \sin. 2x}{1-b \sin. 2x} \frac{dx}{\sin. 2x} = 0, b \leq 1; \quad \text{V. T. 179. N}^\circ. 19.$
- 12) $\int l(1+p^2 \text{Tang.}^2 x). l(1+q^2 \text{Cot.}^2 x) \frac{dx}{\sin.^2 x} = 2\pi \frac{pq+1}{q} l(1+pq) - 2p\pi \left\{ \begin{array}{l} \text{V. T. 179.} \\ \text{N}^\circ. 21. \end{array} \right.$
- 13) $\int l(1+p^2 \text{Tang.}^2 x). l(1+q^2 \text{Cot.}^2 x) \frac{dx}{\cos.^2 x} = 2\pi \frac{pq+1}{p} l(1+pq) - 2q\pi \left\{ \begin{array}{l} \text{V. T. 179.} \\ \text{N}^\circ. 21. \end{array} \right.$
- 14) $\int l(1+p^2 \text{Tang.}^2 x). l(1+q^2 \text{Cot.}^2 x) \frac{dx}{\sin.^2 2x} = \frac{p+q}{2} \pi \left\{ \frac{pq+1}{pq} l(1+pq) - 1 \right\} \left\{ \begin{array}{l} \text{V. T. 341.} \\ \text{N}^\circ. 12, 13. \end{array} \right.$
- 15) $\int l(1+p^2 \text{Tang.}^2 x). l(1+q^2 \text{Cot.}^2 x) \frac{\cos. 2x}{\sin.^2 2x} dx = \frac{p-q}{2} \pi \left\{ \frac{pq+1}{pq} l(1+pq) - 1 \right\} \left\{ \begin{array}{l} \text{V. T. 341.} \\ \text{N}^\circ. 12, 13. \end{array} \right.$

- 1) $\int l \sin. x \frac{\cos. x}{1 + \sin. x} dx = -\frac{1}{12} \pi^2$ V. T. 339. N°. 23.
- 2) $\int l \sin. x \frac{\cos. x}{1 - \sin. x} dx = -\frac{1}{6} \pi^2$ V. T. 339. N°. 24.
- 3) $\int l \sin. x \frac{\sin. 2x}{1 + \sin.^2 x} dx = -\frac{1}{24} \pi^2$ V. T. 339. N°. 26.
- 4) $\int l \sin. x \frac{\text{Tang.}^2 x. \sin. 2x}{1 + \sin.^2 x} dx = -\frac{1}{48} \pi^2$ V. T. 339. N°. 27.
- 5) $\int l \sin. x \frac{dx}{q^2 \sin.^2 x + \cos.^2 x} = -\frac{\pi}{2q} l(1+q)$ V. T. 369. N°. 10.
- 6) $\int l \sin. x \frac{1 + \cos.^2 \lambda. \sin.^2 x}{\{ \sin.^2 \lambda. \sec. x + \cos.^2 \lambda. \cos. x \}^2} \frac{dx}{\cos. x} = \sec. \lambda. l \text{Tang.} \frac{1}{2} \lambda$ V. T. 66. N°. 17.
- 7) $\int l \cos. x \frac{\sin. x}{1 + \cos. x} dx = -\frac{1}{12} \pi^2$ V. T. 335. N°. 3.
- 8) $\int l \cos. x \frac{\sin. x}{1 - \cos. x} dx = -\frac{1}{6} \pi^2$ V. T. 335. N°. 4.
- 9) $\int l \cos. x \frac{dx}{\sin.^2 x + q^2 \cos.^2 x} = -\frac{\pi}{2q} l(1+q)$ V. T. 368. N°. 8.
- 10) $\int l \text{Tang.} x \frac{dx}{2 - \sin. 2x} = 0$ V. T. 311. N°. 1 et T. 357. N°. 5.
- 11) $\int l \text{Tang.} x \frac{\sin. 2x}{4 \cos.^2 2x + \sin.^2 2x} dx = 0$ V. T. 311. N°. 6 et T. 357. N°. 6.
- 12) $\int l \text{Tang.} x \frac{dx}{p^2 \cos.^2 x + q^2 \sin.^2 x} = \frac{\pi}{2pq} l \frac{p}{q}$ V. T. 187. N°. 10.
- 13) $\int l \text{Tang.} \frac{1}{2} x \frac{\sin. x}{1 - \cos.^2 \lambda. \sin.^2 x} dx = \text{Coscc.} 2\lambda \{ 2L(\frac{1}{2}\pi - \lambda) - (\pi - 2\lambda) l 2 \}$ Lobatschewsky, Mém. Kasan. 1836. I. I. 106. — Id., ib. II. 39.
- 14) $\int (l \text{Tang.} x)^2 \frac{dx}{2 + \sin. 2x} = \frac{8}{243} \pi^3 \sqrt{3}$ V. T. 312. N°. 1 et T. 355. N°. 2.
- 15) $\int (l \text{Tang.} x)^2 \frac{dx}{2 - \sin. 2x} = \frac{10}{243} \pi^3 \sqrt{3}$ V. T. 312. N°. 3 et T. 358. N°. 3.
- 16) $\int (l \text{Tang.} x)^2 \frac{dx}{1 + \cos. \lambda. \sin. 2x} = \frac{\pi^2 - \lambda^2}{3} \lambda \text{Coscc.} \lambda$ V. T. 184. N°. 9.

- 17) $\int (l \text{ Tang. } x)^2 \frac{dx}{1 - \text{Cos. } \lambda. \text{Sin. } 2x} = 4\lambda \text{Cosec. } \lambda \left(\frac{1}{6} \pi^2 - \frac{1}{4} \pi \lambda + \frac{1}{12} \lambda^2 \right)$ V. T. 312. N°. 4
T. 358. N°. 5.
- 18) $\int (l \text{ Tang. } x)^2 \frac{dx}{1 - \text{Sin.}^2 x. \text{Cos.}^2 x} = \frac{2}{27} \pi^3 \text{V} \sqrt{3}$ V. T. 312. N°. 6 et T. 358 N°. 6.
- 19) $\int (l \text{ Tang. } x)^2 \frac{\text{Sin. } 2x}{1 - \text{Sin.}^2 x. \text{Cos.}^2 x} dx = \frac{4}{243} \pi^3 \text{V} \sqrt{3}$ V. T. 312. N°. 7 et T. 358. N°. 7.
- 20) $\int (l \text{ Tang. } x)^2 \frac{dx}{\text{Sin.}^4 x + \text{Cos.}^4 x} = \frac{3}{32} \pi^3 \text{V} \sqrt{2}$ V. T. 312. N°. 5 et T. 358. N°. 8.
- 21) $\int (l \text{ Tang. } x)^4 \frac{dx}{1 + \text{Cos. } \lambda. \text{Sin. } 2x} = \frac{\pi^2 - \lambda^2}{\text{Sin } \lambda} \frac{7\pi^2 - 3\lambda^2}{5} \lambda$ V. T. 312. N°. 8 et T. 358. N°. 13.
- 22) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{2 + \text{Sin. } 2x} = \frac{(-1)^{a+1}}{\text{V} \sqrt{3}} (2\pi)^{2a+1} \text{B}'' \left(\frac{1}{3} \right)$ V. T. 184. N°. 11.
- 23) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{2 - \text{Sin. } 2x} = \frac{(-1)^{a+1}}{\text{V} \sqrt{3}} (2\pi)^{2a+1} \text{B}'' \left(\frac{1}{6} \right)$ V. T. 184. N°. 12.
- 24) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{1 - \text{Cos. } 2p\pi. \text{Sin. } 2x} = \frac{1}{2} (-1)^{a+1} (2\pi)^{2a+1} \text{Cosec. } 2p\pi. \text{B}''(p)$ V. T. 184. N°. 13.
- 25) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\text{Sin. } 2x}{1 + p \text{Cos. } 2x} dx = \pm \frac{\pi}{p} \text{Arcsin. } p, p < 1;$ V. T. 355. N°. 1.
- 26) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\text{Tang. } x}{p^2 \text{Sin.}^2 x + \text{Cos.}^2 x} dx = \pm \frac{\pi}{p^2} \text{Arctang. } p$ V. T. 339. N°. 30.
- 27) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\text{Sin. } 2x}{(1-p)^2 + 4p \text{Sin.}^2 x} dx = \pm \frac{\pi}{p} \text{Arctang. } p, p^2 \leq 1;$ V. T. 355. N°. 3.
- 28) $\int l \text{ Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\text{Sin. } 4x}{(1-p)^2 + 4p \text{Sin.}^2 2x} dx = 0, p < 1;$ V. T. 355. N°. 4.
- 29) $\int l l \text{ Tang. } x \frac{dx}{2 + \text{Sin. } 2x} = \frac{\pi}{2 \text{V} \sqrt{3}} l \left(\frac{\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{3})} \text{V} \sqrt{2\pi} \right)$ V. T. 191. N°. 2.

- 1) $\int l(p \text{Sin. } x - r) \frac{\text{Sin. } x}{1 - q \text{Sin.}^2 x} dx = \frac{\pi}{\text{V} \sqrt{q(1-q)}} l \frac{p \text{V} \sqrt{q} - \{1 - \text{V} \sqrt{1-q}\} \{r + \text{V} \sqrt{r^2 - p^2}\}}{p \text{V} \sqrt{q} + \{1 - \text{V} \sqrt{1-q}\} \{r + \text{V} \sqrt{r^2 - p^2}\}}$ V. T. 165. N°. 4.
- 2) $\int l(1 + \text{Cothp.}^2 \lambda. \text{Sin.}^2 x) \frac{\text{Cos. } x}{1 - \text{Coshp.}^2 \lambda. \text{Cos.}^2 x} dx = \frac{2}{\text{Sinhp. } \lambda. \text{Coshp. } \lambda} l \{ \pi - \lambda \}$ Lobatschewsky,
Mém. Kasan. 1836
I. I. 121.

$$3) \int l(1 - \operatorname{Cot} h p \cdot {}^2 \lambda \cdot \operatorname{Sin}^2 x) \frac{\operatorname{Cos} x}{1 - \operatorname{Cos} h p \cdot {}^2 \lambda \cdot \operatorname{Cos}^2 x} dx = \frac{2}{\operatorname{Sin} h p \cdot \lambda \cdot \operatorname{Cos} h p \cdot \lambda} \{ \lambda l \operatorname{Sin} h p \cdot \lambda + \operatorname{I}_1(\lambda) \}$$

$$4) \int l(1 + \operatorname{Cos} h p \cdot \lambda \cdot \operatorname{Cos} x) \frac{\operatorname{Cos} x}{1 - \operatorname{Cos} h p \cdot {}^2 \lambda \cdot \operatorname{Cos}^2 x} dx = \frac{1}{\operatorname{Sin} h p \cdot \lambda \cdot \operatorname{Cos} h p \cdot \lambda} \left\{ \operatorname{I}_1(\lambda) + \left(\lambda - \frac{1}{2} \pi \right) l \operatorname{Sin} h p \cdot \lambda \right\}$$

$$5) \int l(1 - \operatorname{Cos} h p \cdot \lambda \cdot \operatorname{Cos} x) \frac{\operatorname{Cos} x}{1 - \operatorname{Cos} h p \cdot {}^2 \lambda \cdot \operatorname{Cos}^2 x} dx = \frac{1}{\operatorname{Sin} h p \cdot \lambda \cdot \operatorname{Cos} h p \cdot \lambda} \left\{ \operatorname{I}_1(\lambda) + \left(\lambda + \frac{1}{2} \pi \right) l \operatorname{Sin} h p \cdot \lambda \right\}$$

Lobatschewsky, Mém. Kasan. 1836. I. I. 122, 123, 124.

$$6) \int l(1 + \operatorname{Cos} \lambda \cdot \operatorname{Cos} x) \frac{\operatorname{Cos} x}{1 - \operatorname{Cos}^2 \lambda \cdot \operatorname{Cos}^2 x} dx = \frac{2}{\operatorname{Sin} 2 \lambda} \left\{ \operatorname{L} \left(\frac{\pi}{2} - \lambda \right) - \lambda l \operatorname{Sin} \lambda \right\} \left\{ \begin{array}{l} \text{Lobatschewsky, Mém.} \\ \text{Kasan, 1836. I. II.} \\ \text{40, 41.} \end{array} \right.$$

$$7) \int l(1 - \operatorname{Cos} \lambda \cdot \operatorname{Cos} x) \frac{\operatorname{Cos} x}{1 - \operatorname{Cos}^2 \lambda \cdot \operatorname{Cos}^2 x} dx = \frac{2}{\operatorname{Sin} 2 \lambda} \left\{ \operatorname{L} \left(\frac{\pi}{2} - \lambda \right) + (\pi - \lambda) l \operatorname{Sin} \lambda \right\} \left\{ \begin{array}{l} \text{Lobatschewsky, Mém.} \\ \text{Kasan, 1836. I. II.} \\ \text{40, 41.} \end{array} \right.$$

$$8) \int l(p \operatorname{Cos} x - r) \frac{\operatorname{Cos} x}{1 - q \operatorname{Cos}^2 x} dx = \frac{\pi}{\sqrt{q(1-q)}} l \frac{p \sqrt{q} - \{1 - \sqrt{1-q}\} \{r + \sqrt{r^2 - p^2}\}}{p \sqrt{q} + \{1 - \sqrt{1-q}\} \{r + \sqrt{r^2 - p^2}\}} \quad \left\{ \begin{array}{l} \text{V. T. 165.} \\ \text{N}^\circ. 4. \end{array} \right.$$

$$9) \int l \frac{1 + p \operatorname{Sin} x}{1 - p \operatorname{Sin} x} \frac{\operatorname{Sin} x}{1 - \operatorname{Cos}^2 x} dx = \pi \operatorname{Arcsin} p, \quad p \leq 1; \quad \text{V. T. 160. N}^\circ. 17.$$

$$10) \int l \frac{1 - \operatorname{Cos} \mu \cdot \operatorname{Sin} x}{1 + \operatorname{Cos} \mu \cdot \operatorname{Sin} x} \frac{\operatorname{Sin} x}{1 - \operatorname{Cos}^2 \lambda \cdot \operatorname{Sin}^2 x} dx = 2 \pi \operatorname{Cosec} 2 \lambda \cdot l \left\{ \operatorname{Sin} \frac{1}{2} (\mu + \lambda) \cdot \operatorname{Sec} \frac{1}{2} (\mu - \lambda) \right\} \left\{ \begin{array}{l} \text{Lobatschewsky,} \\ \text{Mém. Kasan.} \\ \text{1835. I.} \end{array} \right.$$

$$11) \int l \frac{1 + q \operatorname{Sin} x}{1 - q \operatorname{Sin} x} \frac{\operatorname{Sin} x}{1 - p \operatorname{Sin}^2 x} dx = \frac{\pi}{\sqrt{p(1-p)}} l \frac{q \sqrt{p} + \{1 - \sqrt{1-p}\} \{1 - \sqrt{1-q^2}\}}{q \sqrt{p} - \{1 - \sqrt{1-p}\} \{1 - \sqrt{1-q^2}\}} \left\{ \begin{array}{l} \text{Lobatschewsky,} \\ \text{Mém. Kasan.} \\ \text{1835. I.} \end{array} \right.$$

$$12) \int l \frac{1 - \operatorname{Cot} h p \cdot {}^2 \lambda \cdot \operatorname{Sin}^2 x}{1 + \operatorname{Cot} h p \cdot {}^2 \lambda \cdot \operatorname{Sin}^2 x} \frac{\operatorname{Cos} x}{1 - \operatorname{Cos}^2 h p \cdot {}^2 \lambda \cdot \operatorname{Cos}^2 x} dx = \frac{2 \lambda l \operatorname{Sin} h p \cdot \lambda}{\operatorname{Sin} h p \cdot \lambda \cdot \operatorname{Cos} h p \cdot \lambda} \quad \left\{ \begin{array}{l} \text{Lobatschewsky, Mém.} \\ \text{Kasan, 1836. I. I. 123.} \end{array} \right.$$

$$13) \int l \frac{1 + \operatorname{Cos} \mu \cdot \operatorname{Cos} x}{1 - \operatorname{Cos} \mu \cdot \operatorname{Cos} x} \frac{dx}{1 + \operatorname{Cos} \lambda \cdot \operatorname{Cos} x} = 2 \pi \operatorname{Cosec} \lambda \cdot l \left\{ \operatorname{Cos} \left(\frac{\pi}{4} - \frac{1}{2} \lambda \right) \operatorname{Sec} \frac{1}{2} (\lambda - \mu) \right\} \left\{ \begin{array}{l} \text{Lobatschewsky,} \\ \text{Mém. Kasan.} \\ \text{1836. II. 23, 41.} \end{array} \right.$$

$$14) \int l \frac{1 - \operatorname{Cos} \lambda \cdot \operatorname{Cos} x}{1 + \operatorname{Cos} \lambda \cdot \operatorname{Cos} x} \frac{\operatorname{Cos} x}{1 - \operatorname{Cos}^2 \lambda \cdot \operatorname{Cos}^2 x} dx = 2 \pi \operatorname{Cosec} 2 \lambda \cdot l \operatorname{Sin} \lambda$$

$$15) \int l \frac{1 + p \operatorname{Cos} x}{1 - p \operatorname{Cos} x} \frac{\operatorname{Cos} x}{1 + \operatorname{Sin}^2 x} dx = \pi \operatorname{Arcsin} p, \quad p \leq 1; \quad \text{V. T. 160. N}^\circ. 17$$

$$16) \int l \frac{1 + p \operatorname{Cos} x}{1 - p \operatorname{Cos} x} \frac{\operatorname{Cos} x}{1 - q \operatorname{Cos}^2 x} dx = \frac{\pi}{\sqrt{q(1-q)}} l \frac{p \sqrt{q} - \{1 + \sqrt{1-q}\} \{1 - \sqrt{1-p^2}\}}{p \sqrt{q} - \{1 + \sqrt{1-q}\} \{1 - \sqrt{1-p^2}\}} \quad \left\{ \begin{array}{l} \text{V. T. 170.} \\ \text{N}^\circ. 15.} \end{array} \right.$$

- 17) $\int l \frac{1+q \cos \lambda \cos x}{1-q \cos \lambda \cos x} \frac{dx}{1-\cos^2 \lambda \cos^2 x} = \pi \operatorname{Cosec} \lambda l \frac{1+\sin \lambda}{\sin \lambda + \sqrt{1-q^2 \cos^2 \lambda}}$
- 18) $\int l \frac{1+\operatorname{Cosh} p \lambda \cos x}{1-\operatorname{Cosh} p \lambda \cos x} \frac{\cos x}{1-\operatorname{Cosh} p^2 \lambda \cos^2 x} dx = \frac{-\pi l \sin h p \lambda}{\sin h p \lambda \operatorname{Cosh} p \lambda}$
- 19) $\int l \frac{1+\cos \mu \cos x}{1-\cos \mu \cos x} \frac{dx}{1-\cos^2 \lambda \cos^2 x} = \pi \operatorname{Cosec} \lambda l \frac{1+\sin \lambda}{\sin \lambda + \sin \mu}$
- 20) $\int l \frac{1+\cos \mu \cos x}{1-\cos \mu \cos x} \frac{\cos x}{1-\cos^2 \lambda \cos^2 x} dx = 2\pi \operatorname{Cosec} 2\lambda l [\cos \{\frac{1}{2}(\lambda-\mu)\} \operatorname{Cosec} \{\frac{1}{2}(\lambda+\mu)\}]$
- 21) $\int l \frac{1+\operatorname{Cosh} p \mu \cos x}{1-\operatorname{Cosh} p \mu \cos x} \frac{\cos x}{1-\cos^2 \lambda \cos^2 x} dx =$
 $= 2\pi \operatorname{Cosec} 2\lambda l \left\{ \operatorname{Coth} p \left(\frac{1}{2} \operatorname{Arccosh} p \frac{\operatorname{Cosh} p \mu}{\cos \lambda} \right) \operatorname{Tang} h p \left(\frac{1}{2} \operatorname{Arccosh} p \frac{\operatorname{Tang} \lambda}{\operatorname{Tang} h p \mu} \right) \right\}$
- Lobatschewsky,
Mém. Kasan.
1836. I. 71, 119.
- Lobatschewsky,
Mém. Kasan.
1836. II. 22, 16.
- Lobatschewsky,
Mém. Kasan.
1836. I. 78.

- 1) $\int l \sin x \frac{dx}{(\sin x \pm q \cos x)^2} = \frac{1}{q(1+q^2)} \left\{ \pm l q - \frac{1}{2} q \pi \right\}$ V. T. 65. N°. 8.
- 2) $\int l \sin x \frac{\sin^2 x - p^2 \cos^2 x}{(p^2 \cos^2 x + \sin^2 x)^2} dx = \frac{\pi}{2p(p+1)}$ V. T. 66. N°. 11.
- 3) $\int l \left(\frac{1}{2} \sin 2x \right) \frac{dx}{(\sin x \pm q \cos x)^2} = \mp \frac{\pi}{1+q^2} \pm \frac{1-q^2}{1+q^2} \frac{1}{q} l q$ V. T. 65. N°. 7, 8.
- 4) $\int l \left(\frac{1}{2} \sin 2x \right) \frac{\sin 2x}{(q^2 \sin^2 x + \cos^2 x)^2} dx = \frac{2}{q^2} \frac{1+q^2}{1-q^2} l q$ V. T. 66. N°. 21, 22.
- 5) $\int l \cos x \frac{\sin 2x}{(q^2 \sin^2 x + \cos^2 x)^2} dx = \frac{2}{q^2(1-q^2)} l q$ V. T. 66. N°. 21.
- 6) $\int l \cos x \frac{\sin 2x}{(\sin^2 x + q^2 \cos^2 x)^2} dx = \frac{2}{1-q^2} l q$ V. T. 66. N°. 22.
- 7) $\int l \cos x \frac{dx}{(\sin x \pm q \cos x)^2} = \frac{-q}{1+q^2} \left\{ \frac{\pi}{2q} \pm l q \right\}$ V. T. 65 N°. 7.
- 8) $\int l \cos x \frac{p^2 \sin x - \cos^2 x}{(p^2 \sin^2 x + \cos^2 x)^2} dx = -\frac{\pi}{2p(p+1)}$ V. T. 66. N°. 3.
- 9) $\int l \cos x \frac{\cos^p x - \sec^p x}{(\cos^p x + \sec^p x)^2} \operatorname{Tang} x dx = \frac{\pi}{4p^2}$ V. T. 66. N°. 14.

F. Log. ennum.

Circ. Dir. rat. en dén. : puissance de bin. TABLE 544 suite.

Lim. 0 et $\frac{\pi}{2}$.

$$10) \int l \cos. x \frac{\cos. p x}{(1 - \cos. x)^{p+1}} \text{Tang. } x dx = -\frac{\pi}{p} \text{Cosec. } p \pi \quad \text{V. T. 67. N}^\circ. 1.$$

$$11) \int l \text{Tang. } x \frac{dx}{(\cos. x + \sin. x)^2} = 0 \quad \text{V. T. 182. N}^\circ. 1.$$

$$12) \int l \text{Tang. } x \frac{dx}{(\sin. x \pm q \cos. x)^2} = \pm \frac{1}{q} l q \quad \text{V. T. 67. N}^\circ. 7, 8.$$

$$13) \int l \text{Tang. } x \frac{dx}{(q \sin. x + \cos. x)^2} = \frac{1}{q} l \frac{1}{q} \quad \text{V. T. 182. N}^\circ. 1.$$

$$14) \int l \text{Tang. } x \frac{\sin. 2x}{(\cos.^2 x + q^2 \sin.^2 x)^2} dx = -\frac{2}{q} l q \quad \text{V. T. 66. N}^\circ. 21, 22.$$

$$15) \int (l \text{Tang. } x)^2 \frac{dx}{(\sin. x - \cos. x)^2} = \frac{2}{3} \pi^2 \quad \text{V. T. 182. N}^\circ. 3.$$

$$16) \int l \text{Tang. } x^2 \left(\frac{\pi}{4} \pm x \right) \frac{\sin. 2x}{(q^2 \cos.^2 x + \sin.^2 x)^2} dx = \pm \frac{1}{1+q^2} \frac{\pi}{q} \quad \text{V. T. 182. N}^\circ. 22.$$

F. Log. en num.

Circ. Dir. en dén. à fact. bin. et autre. TABLE 545.

Lim. 0 et $\frac{\pi}{2}$.

$$1) \int l \sin. x \frac{\sin.^2 x}{1 + \sin.^4 x} \frac{dx}{\cos. x} = \frac{-\pi^2}{16(2 + \sqrt{2})} \quad \text{V. T. 153. N}^\circ. 16.$$

$$2) \int l \sin. x \frac{p \sin.^p x (\sin.^{2q} x - 1) + q \sin.^q x (\sin.^{2p} x - 1)}{(\sin.^{p+q} x + 1)^2} \frac{dx}{\text{Tang. } x} = \frac{\pi}{p+q} \text{Sec. } \left(\frac{q-p\pi}{q+p\pi} \right) \quad \text{V. T. 68. N}^\circ. 15.$$

$$3) \int l \sin. x \frac{p \sin.^p x (\sin.^{2q} x - 1) - q \sin.^q x (\sin.^{2p} x - 1)}{(\sin.^{p+q} x - 1)^2} \frac{dx}{\text{Tang. } x} = \frac{\pi}{p+q} \text{Tang. } \left(\frac{q-p\pi}{q+p\pi} \right) \quad \text{V. T. 68. N}^\circ. 16.$$

$$4) \int l \sin. x \frac{\sin.^p x - \text{Cosec.}^p x}{(\sin.^p x + \text{Cosec.}^p x)^2} \frac{dx}{\text{Tang. } x} = \frac{\pi}{4p^2} \quad \text{V. T. 68. N}^\circ. 14.$$

$$5) \int l \sin. x \frac{\sin.^p x}{(1 - \sin. x)^{p+1}} \frac{dx}{\text{Tang. } x} = -\frac{\pi}{p} \text{Cosec. } p \pi \quad \text{V. T. 68. N}^\circ. 23.$$

$$6) \int l \text{Tang. } x \frac{\text{Tang.}^p x}{\sin. x + \cos. x} \frac{dx}{\sin. x} = -\pi^2 \cos. p \pi. \text{Cosec.}^2 p \pi, p < 1; \quad \text{V. T. 338. N}^\circ. 4 \text{ et T. 345. N}^\circ. 7.$$

$$7) \int l \text{Tang. } x \frac{\text{Tang.}^p x}{\sin. x - \cos. x} \frac{dx}{\sin. x} = \pi^2 \text{Cosec.}^2 p \pi, p < 1; \quad \text{V. T. 183. N}^\circ. 1.$$

$$8) \int l \text{Tang. } x \frac{1}{\sin. x + \cos. x} \frac{dx}{\sin. x. \text{Tang.}^p x} = -\pi^2 \cos. p \pi. \text{Cosec.}^2 p \pi, p < 1; \quad \text{V. T. 338. N}^\circ. 5. \text{ et T. 345. N}^\circ. 7.$$

- 9) $\int l \operatorname{Tang} x \frac{1}{\sin x - \cos x} \frac{dx}{\cos x \operatorname{Tang}^p x} = \pi^2 \operatorname{Cosec}^2 p \pi, p < 1; \text{ V. T. 183 N}^\circ. 2.$
- 10) $\int l \operatorname{Tang} x \frac{\operatorname{Tang}^q x - \operatorname{Cot}^q x}{\operatorname{Tang}^p x + \operatorname{Cot}^p x} \frac{dx}{\sin 2x} = 0 \text{ V. T. 313. N}^\circ. 8 \text{ et T. 357. N}^\circ. 10.$
- 11) $\int l \operatorname{Tang} x \frac{\operatorname{Tang}^q x + \operatorname{Cot}^q x}{\operatorname{Tang}^p x - \operatorname{Cot}^p x} \frac{dx}{\sin 2x} = 0 \text{ V. T. 313. N}^\circ. 9 \text{ et T. 357. N}^\circ. 11.$
- 12) $\int l \operatorname{Tang} x \frac{\cos 2x}{\operatorname{Tang}^p x + \operatorname{Cot}^p x} \frac{dx}{\sin^2 2x} = -\frac{\pi^2}{8p^2} \sin \frac{\pi}{2p} \operatorname{Sec}^2 \frac{\pi}{2p} \text{ V. T. 313. N}^\circ. 6 \text{ et T. 357. N}^\circ. 9.$
- 13) $\int l \operatorname{Tang} x \frac{1}{\operatorname{Tang}^p x - \operatorname{Cot}^p x} \frac{dx}{\sin^2 2x} = \frac{\pi^2}{8p^2} \operatorname{Sec}^2 \frac{\pi}{2p} \text{ V. T. 313. N}^\circ. 7 \text{ et T. 357. N}^\circ. 8.$
- 14) $\int l \operatorname{Tang} x \frac{\sin^2 2x}{\sin^4 x + \cos^4 x} \frac{dx}{\cos 2x} = -\frac{\pi}{2(2 + \sqrt{2})} \text{ V. T. 313 N}^\circ. 5 \text{ et T. 357. N}^\circ. 7.$
- 15) $\int l \operatorname{Tang} x \frac{\operatorname{Tang}^p x - \operatorname{Cot}^p x}{(\operatorname{Tang}^p x + \operatorname{Cot}^p x)^2} \frac{dx}{\sin 2x} = \frac{\pi}{4p^2} \text{ V. T. 68. N}^\circ. 19.$
- 16) $\int l \operatorname{Tang} x \frac{p \operatorname{Tang}^p x (\operatorname{Tang}^{2q} x - 1) + q \operatorname{Tang}^q x (\operatorname{Tang}^{2p} x - 1)}{(\operatorname{Tang}^{p+q} x + 1)^2} \frac{dx}{\sin 2x} = \frac{\pi}{p+q} \operatorname{Sec} \left\{ \frac{q-p}{q+p} \pi \right\} \text{ V. T. 68. N}^\circ. 20.$
- 17) $\int l \operatorname{Tang} x \frac{p \operatorname{Tg}^p x (\operatorname{Tg}^{2q} x - 1) - q \operatorname{Tg}^q x (\operatorname{Tg}^{2p} x - 1)}{(\operatorname{Tang}^{p+q} x - 1)^2} \frac{dx}{\sin 2x} = \frac{\pi}{p+q} \operatorname{Tg} \left\{ \frac{q-p}{q+p} \pi \right\}, p < q; \text{ V. T. 68. N}^\circ. 21.$
- 18) $\int l \operatorname{Tang} x \frac{\cos 2x}{1 + \sin 2x} \frac{dx}{1 + \cos \lambda \sin 2x} = \frac{\lambda^2}{\cos \lambda - 1} \text{ V. T. 355. N}^\circ. 2.$
- 19) $\int l \operatorname{Tang}^2 \left(\frac{\pi}{4} \pm x \right) \frac{dx}{(\sin^2 x + q^2 \cos^2 x) \operatorname{Tang} x} = \pm \frac{2\pi}{p} \operatorname{Arc} \operatorname{Tang} p \text{ V. T. 339. N}^\circ. 31.$
- 20) $\int l \operatorname{Tang}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\sin 2x}{(q^2 \operatorname{Tang}^2 x + 1)^2} \frac{dx}{\cos^4 x} = \pm \frac{\pi}{q} \frac{2}{1 + q^2} \text{ V. T. 68. N}^\circ. 7.$
- 21) $\int l \operatorname{Tang}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\sin 2x}{(q^2 + \operatorname{Tang}^2 x)^2} \frac{dx}{\cos^4 x} = \pm \frac{\pi}{q} \frac{2}{1 + q^2} \text{ V. T. 68. N}^\circ. 6.$
- 22) $\int l \operatorname{Tang}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\sin 2x}{(q^2 \operatorname{Cot}^2 x + 1)^2} \frac{dx}{\sin^4 x} = \pm \frac{\pi}{q} \frac{2}{1 + q^2} \text{ V. T. 68. N}^\circ. 5.$
- 23) $\int l \operatorname{Tang}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\sin 2x}{(\operatorname{Cot}^2 x + q^2)^2} \frac{dx}{\sin^4 x} = \pm \frac{\pi}{q} \frac{2}{1 + q^2} \text{ V. T. 68. N}^\circ. 8.$

F. Log. en num.

Circ. Dir. en dén. à fact. bin. et autre.

TABLE 545 suite.

Lim. 0 et $\frac{\pi}{2}$.

$$24) \int (l \operatorname{Tang.} x)^2 \frac{\operatorname{Tang.}^q x + \operatorname{Cot.}^q x}{\operatorname{Tang.}^p x + \operatorname{Cot.}^p x} \frac{dx}{\operatorname{Sin.} 2x} = \frac{\pi^3}{8p^3} \left(2 \operatorname{Sec.}^3 \frac{q\pi}{2p} - \operatorname{Sec.} \frac{q\pi}{2p} \right) \quad \text{V. T. 314. N}^\circ 1 \text{ et T. 358. N}^\circ 9.$$

$$25) \int (l \operatorname{Tang.} x)^2 \frac{\operatorname{Tang.}^q x - \operatorname{Cot.}^q x}{\operatorname{Tang.}^p x - \operatorname{Cot.}^p x} \frac{dx}{\operatorname{Sin.} 2x} = \frac{\pi^3}{4p^3} \operatorname{Sin.} \frac{q\pi}{2p} \operatorname{Sec.}^3 \frac{q\pi}{2p} \quad \text{V. T. 314. N}^\circ 2 \text{ et T. 355. N}^\circ 10.$$

F. Log. en num.

Circ. Dir. rat. en dén. trinôme.

TABLE 546.

Lim. 0 et $\frac{\pi}{2}$.

$$1) \int l \operatorname{Sin.} x \frac{1 - p \operatorname{Cos.} 2x}{1 - 2p \operatorname{Cos.} 2x + p^2} dx = \frac{1}{4} \pi l \frac{1-p}{4}, \quad p < 1; \quad \text{Cauchy, Lim. Imag. 119.}$$

$$2) \int l \operatorname{Sin.} x \frac{\operatorname{Cos.} 2x - p}{1 - 2p \operatorname{Cos.} 2x + p^2} dx = -\frac{\pi}{4p} l \frac{1-p}{4}, \quad p^2 \leq 1; \quad \text{V. T. 371. N}^\circ 2.$$

$$3) \int l \operatorname{Sin.} x \frac{(1+q^2)(\operatorname{Sin.} 2x - p \operatorname{Sin.} 2px) + 2q(\operatorname{Sin.} 2x \operatorname{Cos.} 2px - p \operatorname{Cos.} 2x \operatorname{Sin.} 2px)}{(1 + 2q \operatorname{Cos.} x + q^2)(1 + 2q \operatorname{Cos.} 2px + q^2)} dx = 0 \quad \text{V. T. 355. N}^\circ 5.$$

$$4) \int l \operatorname{Cos.} x \frac{dx}{1 - 2p \operatorname{Cos.} 2x + p^2} = \frac{\pi}{2(1-p^2)} l \frac{1+p}{2}, \quad p^2 < 1; \quad \text{Poisson, P. 19. 404. N}^\circ 76.$$

$$5) \int l \operatorname{Cos.} x \frac{1 - p \operatorname{Cos.} 2x}{1 - 2p \operatorname{Cos.} 2x + p^2} dx = \frac{1}{4} \pi l \frac{1+p}{4}, \quad p < 1; \quad \text{Cauchy, Lim. Imag. 118.}$$

$$6) \int l \operatorname{Cos.} x \frac{\operatorname{Cos.} 2x - p}{1 - 2p \operatorname{Cos.} 2x + p^2} dx = \frac{\pi}{4p} l \frac{1+p}{4}, \quad p^2 \leq 1; \quad \text{V. T. 370. N}^\circ 23.$$

$$7) \int l \operatorname{Tang.} x \frac{1 - p \operatorname{Cos.} 2x}{1 - 2p \operatorname{Cos.} 2x + p^2} dx = \frac{\pi}{4} l \frac{1-p}{1+p}, \quad p < 1; \quad \text{Cauchy, Lim. Imag. 120.}$$

$$8) \int l \operatorname{Tang.} x \frac{\operatorname{Cos.} 2x - p}{1 - 2p \operatorname{Cos.} 2x + p^2} dx = \frac{\pi}{4p} l \frac{1-p}{1+p}, \quad p^2 \leq 1; \quad \text{V. T. 371. N}^\circ 1.$$

$$9) \int l \operatorname{Tang.} x \frac{\operatorname{Cos.} 2x}{1 - 2p^2 \operatorname{Cos.} 4x + p^4} dx = \frac{\pi}{4p(1-p^2)} l \frac{1-p}{1+p}, \quad p^2 \leq 1; \quad \text{V. T. 371. N}^\circ 5.$$

$$10) \int l \operatorname{Tang.} x \frac{\operatorname{Cos.} 4x - p}{1 - 2p \operatorname{Cos.} 4x + p^2} dx = 0, \quad p^2 \geq 1; \quad \text{V. T. 371. N}^\circ 8.$$

$$11) \int l \operatorname{Sin.} x \frac{(1+p^2) \operatorname{Cos.} 2x - 2p}{(1 - 2p \operatorname{Cos.} 2x + p^2)^2} dx = -\frac{\pi}{4(1-p)}, \quad p^2 < 1; \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{V. T. 85. N}^\circ 1.$$

$$12) \quad \quad \quad = -\frac{\pi}{4(p-1)}, \quad p^2 > 1; \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$13) \int l \operatorname{Cos.} x \frac{(1+p^2) \operatorname{Cos.} 2x - 2p}{(1 - 2p \operatorname{Cos.} 2x + p^2)^2} dx = \frac{\pi}{4(1+p)} \quad \text{V. T. 85. N}^\circ 2.$$

- 1) $\int l \sin. x \frac{\sin. x}{\sqrt{(1 + \sin.^2 x)}} dx = -\frac{1}{8} \pi l 2 \quad \text{V. T. 163. N}^\circ. 12.$
- 2) $\int l \sin. x \frac{\sin.^3 x}{\sqrt{(1 + \sin.^2 x)}} dx = \frac{1}{4} (l 2 - 1) \quad \text{V. T. 163. N}^\circ. 13.$
- 3) $\int l \sin. x \frac{\text{Tang.}^2 x}{\sin. 2 x} dx = \frac{1}{8} q \pi \text{Cosec.} \frac{\pi}{q} \quad \text{V. T. 73. N}^\circ. 11.$
- 4) $\int l \sin. x \frac{dx}{\sqrt{(1 - p^2 \sin.^2 x)}} = -\frac{1}{2} \Gamma'(p) l p - \frac{1}{4} \pi F\{\sqrt{(1 - p^2)}\}, p^2 < 1; \text{Roberts, L. 11 471.}$
- 5) $\int l \sin. x \frac{(1 - \sin. x)^{p-\frac{1}{2}}}{\sin.^{p-\frac{1}{2}} x. \text{Tang.} x} dx = -\frac{2 \pi}{2 p - 1} \text{Sec.} p \pi \quad \text{V. T. 73. N}^\circ. 18.$
- 6) $\int l \sin. x \frac{\sin.^{p-\frac{1}{2}} x}{(1 - \sin. x)^{p+\frac{1}{2}}} \frac{dx}{\text{Tang.} x} = -\frac{2 \pi}{1 - 2 p} \text{Sec.} p \pi, p < \frac{1}{2}; \text{V. T. 76. N}^\circ. 14.$
- 7) $\int l \cos. x \frac{\cos. x}{\sqrt{(1 + \cos.^2 x)}} dx = -\frac{1}{8} \pi l 2 \quad \text{V. T. 163. N}^\circ. 12.$
- 8) $\int l \cos. x \frac{\cos.^3 x}{\sqrt{(1 + \cos.^2 x)}} dx = \frac{1}{4} (l 2 - 1) \quad \text{V. T. 163. N}^\circ. 13.$
- 9) $\int l \cos. x \frac{x}{\text{Tang.}^2 x. \sin. 2 x} dx = \frac{1}{8} q \pi \text{Cosec.} \frac{\pi}{q} \quad \text{V. T. 73. N}^\circ. 12.$
- 10) $\int l \cos. x \frac{(1 - \cos. x)^{p-\frac{1}{2}}}{\cos.^{p-\frac{1}{2}} x} \text{Tang.} x dx = -\frac{2 \pi}{2 p - 1} \text{Sec.} p \pi \quad \text{V. T. 73. N}^\circ. 13.$
- 11) $\int l \cos. x \frac{dx}{\sqrt{(1 - p^2 \sin.^2 x)}} = \frac{1}{4} \Gamma'(p) l \frac{1 - p^2}{p^2} - \frac{1}{4} \pi F\{\sqrt{(1 - p^2)}\}, p^2 < 1; \text{Roberts, L. 11 471.}$
- 12) $\int l \cos. x \frac{\cos.^{p-\frac{1}{2}} x}{(1 - \cos. x)^{p+\frac{1}{2}}} \text{Tang.} x dx = -\frac{2 \pi}{2 p - 1} \text{Sec.} p \pi \quad \text{V. T. 74. N}^\circ. 10.$
- 13) $\int l \text{Tang.} x \frac{dx}{\sqrt{(1 - p^2 \sin.^2 x)}} = -\frac{1}{2} l (1 - p^2). \Gamma'(p), p^2 < 1; \text{Roberts, L. 11. 471.}$
- 14) $\int l \text{Cot.} \frac{1}{2} x \frac{\sin. x}{\sin.^2 \lambda + \text{Tang.}^2 \mu. \sin.^2 x} \frac{\cos. x}{\sqrt{(\sin.^2 \lambda - \sin.^2 x)}} dx =$

$$= \frac{1}{2} \pi \frac{\cos.^2 \mu}{\sin. \lambda. \sin. \mu} l \frac{\sin. \mu + \sqrt{(1 - \cos.^2 \lambda. \cos.^2 \mu)}}{\sin. \mu (1 + \sin. \lambda)}$$

Lobatschewsky, Mém. Kasan. 1836. I. II. 25.
- 15) $\int l \sin. x \frac{\cos. x}{1 - \sqrt{\sin. x} \sqrt{\sin.^3 x}} dx = -2 \pi^2 \quad \text{V. T. 163. N}^\circ. 18.$

$$16) \int l \cos x \frac{1 + \sqrt{\cos x}}{\sin x} \frac{1 + \cos x}{\sqrt{\cos^3 x}} dx = -2\pi^2 \quad \text{V. T. 163. N}^\circ 18.$$

$$17) \int (l \operatorname{Tang} x)^{2a-1} \frac{dx}{(\sin x - \cos x) \sqrt{\sin 2x}} = -\frac{2^{2a}-1}{2a} \frac{1}{\sqrt{2}} (2\pi)^{2a} B_{2a-1} \quad \text{V. T. 319. N}^\circ 19 \text{ et T. 359. N}^\circ 8.$$

$$1) \int dx l \{ \sin \lambda \cdot \sin x + \sqrt{(1 - \cos^2 \lambda \cdot \sin^2 x)} \} = \frac{1}{2} \pi l 2 - \frac{1}{2} I_1 \left(\frac{1}{2} \lambda \right) - 2 L \left(\frac{\pi - \lambda}{2} \right) \quad \left. \begin{array}{l} \text{Lobatschewsky,} \\ \text{Mém. Kasan. 1836.} \end{array} \right\}$$

$$2) \int dx l \frac{1 + \cos x \sqrt{(\sin^2 \lambda - \sin^2 \mu \cdot \sin^2 x)}}{1 - \cos x \sqrt{(\sin^2 \lambda - \sin^2 \mu \cdot \sin^2 x)}} = \pi l \frac{1}{2} \left\{ \cos^2 \frac{1}{2} \lambda + \left(\cos^4 \frac{1}{2} \lambda + \sin^2 \frac{1}{2} \lambda \cdot \cos^2 \frac{1}{2} \mu \right) \right\} \quad \left. \begin{array}{l} \text{Mém. Kasan. 1836.} \\ \text{I. II. 36.} \\ \text{13.} \end{array} \right\}$$

$$3) \int dx l \frac{1 - \cosh p \cdot \lambda \cdot \cosh p \cdot \mu \cdot \cos x \sqrt{(1 - \cot h p^2 \cdot \lambda \cdot \operatorname{Tang} h p^2 \cdot \mu \cdot \cos^2 x)}}{1 + \cosh p \cdot \lambda \cdot \cosh p \cdot \mu \cdot \cos x \sqrt{(1 - \cot h p^2 \cdot \lambda \cdot \operatorname{Tang} h p^2 \cdot \mu \cdot \cos^2 x)}} =$$

$$= \pi l \frac{4 \sinh p \cdot \lambda}{(1 + \sinh p \cdot \lambda) \{ \sinh p \cdot \lambda + \sqrt{(1 - \cosh p^2 \cdot \lambda \cdot \cosh p^2 \cdot \mu)} \}}$$

$$4) \int \frac{\cos x dx}{1 - \cosh p^2 \cdot \lambda \cdot \cos^2 x} l \frac{1 + \sqrt{(\cosh p^2 \cdot \mu - \sinh p^2 \cdot \mu \cdot \cot h p^2 \cdot \lambda \cdot \sin^2 x)}}{1 - \cosh p \cdot \lambda \cdot \cos x} =$$

$$= \frac{1}{2 \sinh p \cdot \lambda \cdot \cosh p \cdot \lambda} \{ L(\lambda + q) - L(\lambda - q) - 2 L(q) - (\pi - 2\lambda - 2q) l \sinh p \cdot \mu - 2\lambda l \sinh p \cdot \lambda \}$$

$$5) \int \frac{\cos x dx}{1 - \cosh p^2 \cdot \lambda \cdot \cos^2 x} l \frac{1 + \sqrt{(\cosh p^2 \cdot \mu - \sinh p^2 \cdot \mu \cdot \cot h p^2 \cdot \lambda \cdot \sin^2 x)}}{\sqrt{(1 + \cot h p^2 \cdot \lambda \cdot \sin^2 x)}} =$$

$$= \frac{1}{2 \sinh p \cdot \lambda \cdot \cosh p \cdot \lambda} \{ (\pi + 2\lambda + 2q) l \sinh p \cdot \mu + L(\lambda + q) - L(\lambda - q) - 2 L(q) \}$$

$$6) \int \frac{\cos x dx}{1 - \cosh p^2 \cdot \lambda \cdot \cos^2 x} l \{ 1 + \sqrt{(\cosh p^2 \cdot \mu - \sinh p^2 \cdot \mu \cdot \cot h p^2 \cdot \lambda \cdot \sin^2 x)} \} =$$

$$= \frac{1}{\sinh p \cdot \lambda \cdot \cosh p \cdot \lambda} \left\{ \left(\lambda + q - \frac{1}{2} \pi \right) l \sinh p \cdot \mu + L(\lambda) - L(q) + \frac{1}{2} L(\lambda + q) - \frac{1}{2} L(\lambda - q) \right\}$$

$$7) \int \frac{\cos x dx}{1 - \cosh p^2 \cdot \lambda \cdot \cos^2 x} l \{ 1 - \sqrt{(\cosh p^2 \cdot \mu - \sinh p^2 \cdot \mu \cdot \cot h p^2 \cdot \lambda \cdot \sin^2 x)} \} =$$

$$= \frac{1}{\sinh p \cdot \lambda \cdot \cosh p \cdot \lambda} \left\{ \left(\lambda - q + \frac{1}{2} \pi \right) l \sinh p \cdot \mu + L(\lambda) + L(q) - \frac{1}{2} L(\lambda + q) + \frac{1}{2} L(\lambda - q) \right\}$$

Dans les formules (3) à (7), trouvées par Lobatschewsky, Mém. Kasan. 1836. 1. I. 73. 114. 120, 125, 127, on a $\cos q = \frac{\cosh p \cdot \lambda}{\cosh p \cdot \mu}$.

$$\begin{aligned} 8) \int \frac{\cos. x \, dx}{1 - \sin.^2 \lambda. \cos.^2 x} &= \frac{1}{2} \{1 + \sqrt{(\sin.^2 \lambda - \cos.^2 \mu. \text{Tang.}^2 \lambda. \sin.^2 x)}\} = \\ &= \frac{2}{\sin. 2 \lambda} \{-(\psi + \lambda - \frac{1}{2} \pi) \ell \sin. \mu + \text{L}(\lambda) - \text{L}(\psi) + \frac{1}{2} \text{L}(\lambda + \psi) - \frac{1}{2} \text{L}(\lambda - \psi)\} \end{aligned}$$

$$9) \int \frac{\cos. x dx}{1 - \sin.^2 \lambda. \cos.^2 x} l \{1 - \sqrt{(\sin.^2 \mu - \cos.^2 \mu. \text{Tang.}^2 \lambda. \sin.^2 x)}\} =$$

$$= \frac{2}{\sin. 2 \lambda} \{ \lambda - \varphi + \frac{1}{2} \pi \} l \cos. \mu + L(\lambda) + L(\varphi) - \frac{1}{2} L(\lambda + \varphi) + \frac{1}{2} L(\lambda - \varphi) \}$$

Dans les formules (8), (9), trouvées par Lobatschewsky, Mém. Kasan. 1836. 1. II, 42, 43, on a

$$\cos. \varphi = \frac{\sin. \lambda}{\sin. \mu}.$$

$$10) \int \frac{dx}{\sqrt{(1-p^2 \sin^2 x)}} l(1+p \sin^2 x) = \frac{1}{2} l \left\{ \frac{2+p}{\sqrt{p}} \right\} F(p) - \frac{1}{8} \pi F\{\sqrt{(1-p^2)}\} \Bigg\}, p^2 < 1;$$

$$11) \int \frac{dx}{\sqrt{(1-p^2 \sin^2 x)}} l(1-p \sin^2 x) = \frac{1}{2} l \left\{ \frac{2-2p}{\sqrt{p}} \right\} F'(p) - \frac{1}{8} \pi F' \{ \sqrt{(1-p^2)} \} \left. \vphantom{\int} \right\} \text{L. 12. 449.} \quad \text{Roberts,}$$

$$12) \int \frac{dx}{\sqrt{(1-p^2 \sin^2 x)}} l \{ \sqrt{(1-p^2 \sin^2 x)} \} = \frac{1}{2} l \{ \sqrt{(1-p^2)} \} F'(p) \quad \left. \vphantom{\int} \right\} p^2 < 1; \quad \text{Roberts, L.}$$

$$13) \int \frac{dx}{\sqrt{(1-p^2 \sin^2 x)}} l \{ \cos^2 x + (1-p^2) \sin^2 x \} = \frac{1}{2} l \left\{ \frac{2 \sqrt{(1-p^2)^3}}{1 + \sqrt{(1-p^2)}} \right\} F(p) \begin{pmatrix} 11. 157. - \\ \text{Id., L. 11.} \\ 174. \end{pmatrix}$$

$$14) \int_{\sqrt{1-p^2} \sin x}^x \frac{dx}{\sqrt{1-p^2} \sin x} l \{1 + \cot^2 \lambda \sin^2 x\} = \pi F\{1'(1-p^2), \lambda\} - 2F'(p) Y\{1'(1-p^2), \lambda\} - 2F'(p) l \sin \lambda - \frac{1}{2} \pi F'\{\sqrt{1-p^2}\} - F'(p) l p - \{E'(p) - F'(p)\} [F\{\sqrt{1-p^2}, \lambda\}]^2, p^2 < 1;$$

$$15) \int_0^1 \frac{dx}{(1-p^2 \sin^2 x)} l[1 - \{1 - (1-p^2) \sin^2 x\} \sin^2 x] = \pi F\{\sqrt{1-p^2}, \lambda\} - 2F(p)Y\{\sqrt{1-p^2}, \lambda\} + \left. \begin{aligned} &+ F'(p)l \frac{\sqrt{1-p^2}}{p} - \frac{1}{2} \pi F'\{\sqrt{1-p^2}\} - \{E'(p) - F'(p)\} [F\{\sqrt{1-p^2}, \lambda\}]^2 \end{aligned} \right\} \text{Roberts, L. 11. 471.}$$

$$16) \int_0^1 \frac{dx}{(1-p^2 \sin^2 x)} \ln \{1-p^2 \sin^2 x\} = \frac{1}{2} \ln \left\{ \frac{4(1-p^2)}{p^2} \right\} F'(p) - \frac{1}{4} \pi F' \{1/(1-p^2)\}, p^2 < 1; \text{Roberts, L. 12. 449.}$$

$$17) \int \frac{dx}{1 - p^2 \sin^2 x} l(1 - p^2 \sin^2 x) = E'(p) \{F(p, x)\}^2 - 2F'(p) Y(p, x), p^2 < 1; \text{Roberts, L. 11. 471.}$$

$$18) \int \frac{dx}{\sqrt{(1-p^2 \sin^2 x)^3}} l(1-p^2 \sin^2 x) = \frac{1}{2(1-p^2)} [2(p^2-2)E'(p) + \{4+l(1-p^2)\}E(p)] \quad \begin{matrix} \text{V. T. 348.} \\ \text{N. 12, 19.} \end{matrix}$$

- 19) $\int \frac{\text{Sin.}^2 x dx}{\sqrt{(1-p^2 \text{Sin.}^2 x)^3}} l\{\sqrt{(1-p^2 \text{Sin.}^2 x)}\} = \frac{-1}{2p^2(1-p^2)} [\{2-p^2 + -(1-p^2)l[\sqrt{(1-p^2)}]\} F'(p) - \{2 + l[\sqrt{(1-p^2)}]\} E'(p)]$ Roberts, L. 11. 157.
- 20) $\int \frac{\text{Cos.}^2 x dx}{\sqrt{(1-p^2 \text{Sin.}^2 x)^3}} l(1-p^2 \text{Sin.}^2 x) = \frac{1}{2p^2} [\{2(2-p^2) + l(1-p^2)\} F'(p) - \{4 + l(1-p^2)\} E'(p)]$ V. T. 348. N°. 18, 19.
- 21) $\int \frac{\text{Cos.} 2x dx}{\sqrt{(1-p^2 \text{Sin.}^2 x)^3}} l(1-p^2 \text{Sin.}^2 x) = \frac{1}{2p^2(1-p^2)} [\{2-p^2 + (1-p^2)l(1-p^2)\} F'(p) - (2-p^2) \{4 + l(1-p^2)\} E'(p)]$ V. T. 348. N°. 18, 19.
- 22) $\int \frac{dx}{\sqrt{(1-p^2 \text{Sin.}^2 x)}} l \frac{1+q\sqrt{(1-p^2 \text{Sin.}^2 x)}}{1-q\sqrt{(1-p^2 \text{Sin.}^2 x)}} = \pi F\{\sqrt{(1-p^2)}, \text{Arcsin. } q\}$ Roberts, L. 11. 157.

- 1) $\int \text{Cos. } x dx \sqrt{l \text{Cosec. } x} = \frac{1}{2} \sqrt{\pi}$ V. T. 44. N°. 1.
- 2) $\int \text{Cos. } x (l \text{Cosec. } x)^{a+\frac{1}{2}} dx = \frac{3a^2}{2a} \sqrt{\pi}$ V. T. 44. N°. 2.
- 3) $\int \frac{\text{Sin.}^p x}{\text{Tang. } x} (l \text{Cosec. } x)^{a-\frac{1}{2}} dx = \frac{1a^2}{(2p)^a} \sqrt{\frac{\pi}{p}}$ V. T. 162. N°. 4.
- 4) $\int \frac{\text{Cos. } x}{\sqrt{l \text{Cosec. } x}} dx = \sqrt{\pi}$ V. T. 44. N°. 4.
- 5) $\int \frac{\text{Sin.}^p x}{\text{Tang. } x. \sqrt{l \text{Cosec. } x}} dx = \sqrt{\frac{\pi}{p}}$ V. T. 178. N°. 1.
- 6) $\int \text{Sin. } x dx \sqrt{l \text{Sec. } x} = \frac{1}{2} \sqrt{\pi}$ V. T. 44. N°. 1.
- 7) $\int \text{Sin. } x (l \text{Sec. } x)^{a+\frac{1}{2}} dx = \frac{3a^2}{2a} \sqrt{\pi}$ V. T. 44. N°. 2.
- 8) $\int \text{Cos.}^p x. \text{Tang. } x (l \text{Sec. } x)^{a-\frac{1}{2}} dx = \frac{1a^2}{(2p)^a} \sqrt{\frac{\pi}{p}}$ V. T. 162. N°. 4.
- 9) $\int \frac{\text{Sin. } x}{\sqrt{l \text{Sec. } x}} dx = \sqrt{\pi}$ V. T. 44. N°. 4.
- 10) $\int \frac{\text{Cos.}^{p-2} x. \text{Sin. } 2x}{\sqrt{l \text{Sec. } x}} dx = 2 \sqrt{\frac{\pi}{p}}$ V. T. 178. N°. 1.

- 1) $\int \frac{(\text{Sin.}^q x - \text{Cosec.}^q x)^2}{l \text{Sin.} x} \text{Tang.} x dx = l \frac{\text{Sin.} q \pi}{q \pi}$ V. T. 175. N°. 7.
- 2) $\int \frac{1 + \text{Sin.} x}{l \text{Sin.} x} \text{Sin.} 2 x. \text{Sin.} (l \text{Sin.} x) dx = \frac{1}{2} \pi$ V. T. 406. N°. 3.
- 3) $\int \frac{\text{Sin.}^q x - \text{Sin.}^p x}{l \text{Sin.} x} \text{Sin.} 2 x dx = 2 l \frac{q+2}{p+2}$ V. T. 167. N°. 4.
- 4) $\int \frac{(\text{Sin.}^p x - \text{Sin.}^q x)(\text{Sin.}^r x - \text{Sin.}^s x)}{l \text{Sin.} x} \text{Sin.} 2 x dx = 2 l \frac{(p+r+2)(q+s+2)}{(p+s+2)(q+r+2)}$ V. T. 167. N°. 8.
- 5) $\int \frac{\text{Sin.}^q x + \text{Cosec.}^q x}{\text{Sin.}^p x + \text{Cosec.}^p x} \frac{dx}{\text{Tang.} x. l \text{Sin.} x} = l \text{Tang.} \left\{ \frac{p+q}{4p} \pi \right\}$ V. T. 172. N°. 6.
- 6) $\int \frac{\text{Cos.} (2 p l \text{Sin.} x)}{l \text{Sin.} x} \frac{dx}{\text{Cos.} x} = \frac{1}{2} l \frac{1}{e^{p\pi} + e^{-p\pi}}$ V. T. 406. N°. 16.
- 7) $\int \frac{(1 - \text{Sin.}^{1-q} x)^2}{l \text{Sin.} x} \frac{\text{Sin.}^{q-1} x}{\text{Cos.} x} dx = l \text{Sin.} \frac{q \pi}{2}$ V. T. 172. N°. 10.
- 8) $\int \frac{1 - \text{Sin.}^q x}{l \text{Sin.} x} \frac{1 - \text{Sin.}^{q+1} x}{\text{Cos.} x} dx = -q l 2, q > -1;$ V. T. 172. N°. 3.
- 9) $\int \frac{(\text{Sin.}^q x - \text{Cosec.}^q x)^2}{l \text{Sin.} x} \frac{dx}{\text{Cos.} x} = l \text{Cos.} q \pi$ V. T. 175. N°. 6.
- 10) $\int \frac{\text{Cos.}^3 x}{l \text{Sin.} x 1 + \text{Sin.}^4 x} \frac{dx}{1 + \text{Sin.}^4 x} = l \text{Cot.} \frac{3 \pi}{8}$ V. T. 172. N°. 4.
- 11) $\int \left\{ \frac{1 + \text{Sin.}^2 x}{\text{Cos.} x} + \frac{\text{Cos.} x}{l \text{Sin.} x} \right\} \frac{dx}{l \text{Sin.} x} = l 2 - 1$ V. T. 172. N°. 5.
- 12) $\int \frac{\text{Cosec.}^q x - \text{Sin.}^q x}{(l \text{Sin.} x)^p} \frac{dx}{\text{Cos.} x} = (-1)^p \Gamma(1-p) \sum_1 \left\{ \frac{1}{(2n-1-q)^{1-p}} - \frac{1}{(2n-1+q)^{1-p}} \right\}$ V. T. 176. N°. 5.
- 13) $\int \frac{(\text{Cos.}^q x - \text{Sec.}^q x)^2}{l \text{Cos.} x} \frac{dx}{\text{Tang.} x} = l \frac{\text{Sin.} q \pi}{q \pi}$ V. T. 175. N°. 7.
- 14) $\int \frac{(\text{Cos.}^q x - \text{Sec.}^q x)^2}{l \text{Cos.} x} \frac{dx}{\text{Sin.} x} = l \text{Cos.} q \pi$ V. T. 175. N°. 6.
- 15) $\int \frac{1 + \text{Cos.} x}{l \text{Cos.} x} \text{Sin.} 2 x. \text{Sin.} (l \text{Cos.} x) dx = \frac{1}{2} \pi$ V. T. 406. N°. 3.
- 16) $\int \frac{\text{Cos.}^q x - \text{Cos.}^p x}{l \text{Cos.} x} \text{Sin.} 2 x dx = 2 l \frac{q+2}{p+2}$ V. T. 167. N°. 4.

- 17) $\int \frac{1 - \text{Cos.}^q x}{l \text{Cos.} x} \frac{1 - \text{Cos.}^{q+1} x}{\text{Sin.} x} dx = -q l 2, q > -1; \text{ V. T. 172. N}^\circ. 3.$
- 18) $\int \frac{(\text{Cos.}^p x - \text{Cos.}^q x)(\text{Cos.}^r x - \text{Cos.}^s x)}{l \text{Cos.} x} \text{Sin.} 2x dx = 2l \frac{(p+r+2)(q+s+2)}{(p+s+2)(q+r+2)} \text{ V. T. 167. N}^\circ. 8.$
- 19) $\int \frac{\text{Tang.}^{p-1} x - \text{Tang.}^{q-1} x}{l \text{Tang.} x} dx = \pi \left(l \text{Tang.} \frac{p\pi}{4} - l \text{Tang.} \frac{q\pi}{4} \right) \text{ V. T. 180. N}^\circ. 7.$
- 20) $\int \frac{\text{Tang.}^{p-1} x - \text{Tang.}^{q-1} x}{l \text{Tang.} x} \frac{dx}{\text{Cos.} 2x} = \pi (l \text{Sin.} \frac{1}{2} p \pi - l \text{Sin.} \frac{1}{2} q \pi) \text{ V. T. 150. N}^\circ. 13.$
- 21) $\int \frac{\text{Tang.}^q x - \text{Cot.}^q x}{l \text{Tang.} x} dx = 2 l \text{Tang.} \left\{ \frac{1+q}{4} \pi \right\} \text{ V. T. 322. N}^\circ. 12 \text{ et T. 360. N}^\circ. 1.$
- 22) $\int \frac{(\text{Tang.}^q x - \text{Cot.}^q x)^2}{l \text{Tang.} x} \frac{dx}{\text{Cos.} 2x} = 0 \text{ V. T. 322. N}^\circ. 15 \text{ et T. 360. N}^\circ. 2.$
- 23) $\int \frac{\text{Tang.}^p x - \text{Tang.}^q x}{\text{Sin.} x + \text{Cos.} x} \frac{dx}{\text{Sin.} x. l \text{Tang.} x} = l (\text{Tang.} \frac{1}{2} p \pi. \text{Cot.} \frac{1}{2} q \pi) \text{ V. T. 153. N}^\circ. 18.$
- 24) $\int \frac{\text{Tang.}^q x - \text{Cot.}^q x}{\text{Tang.}^p x + \text{Cot.}^p x \text{Sin.} 2x. l \text{Tang.} x} dx = l \text{Tang.} \left\{ \frac{p+q}{4p} \pi \right\} \text{ V. T. 323. N}^\circ. 7 \text{ et T. 360. N}^\circ. 4.$
- 25) $\int \frac{\text{Tang.}^q x + \text{Cot.}^q x - 2}{\text{Tang.}^p x - \text{Cot.}^p x} \frac{dx}{\text{Sin.} 2x. l \text{Tang.} x} = l \text{Cos.} \frac{q\pi}{2p} \text{ V. T. 323. N}^\circ. 8 \text{ et T. 360. N}^\circ. 5.$

- 1) $\int \frac{l \text{Cos.} x}{q^2 + (l \text{Sin.} x)^2} \frac{dx}{\text{Tang.} x} = \frac{\pi}{2q} l \Gamma \left(\frac{q+\pi}{\pi} \right) + \frac{\pi}{4q} l 2q + \frac{1}{2} \left(l \frac{q}{\pi} - 1 \right) \text{ V. T. 170. N}^\circ. 8.$
- 2) $\int \frac{l \text{Sin.} x}{\pi^2 + (l \text{Sin.} x)^2} \frac{dx}{\text{Cos.} x} = \frac{1}{2} \left\{ \frac{1}{2} - l 2 \right\} \text{ V. T. 173. N}^\circ. 11.$
- 3) $\int \frac{\text{Tang.} x. l \text{Sin.} x}{q^2 + (l \text{Sin.} x)^2} dx = -\frac{1}{2} \left\{ l \frac{q}{\pi} - \frac{\pi}{2q} - Z' \left(\frac{q}{\pi} \right) \right\} \text{ V. T. 173. N}^\circ. 12.$
- 4) $\int \frac{\text{Tang.} x. l \text{Sin.} x}{\pi^2 + (l \text{Sin.} x)^2} dx = \frac{1}{4} - \frac{1}{2} \Lambda \text{ V. T. 173. N}^\circ. 13.$
- 5) $\int \frac{\text{Tang.} x. l \text{Sin.} x}{q^2 - (l \text{Sin.} x)^2} dx = \frac{\pi^4}{4q^3} \sum_0^{\infty} (-1)^{n+1} \frac{B_{2n+1}}{n+1} \left(\frac{\pi}{q} \right)^{2n} \text{ V. T. 173. N}^\circ. 14.$

- 6) $\int \frac{l \sin. x}{\pi^2 + 4 (l \sin. x)^2} \frac{dx}{\cos. x} = \frac{1}{16} (2 - \pi)$ V. T. 173. N°. 10.
- 7) $\int \frac{\sin. p x - \operatorname{Cosec.} p x}{\pi^2 + (l \sin. x)^2} \frac{dx}{\cos. x} = \frac{1}{2\pi} [p\pi \cos p\pi - \sin. p\pi. l \{2(1 + \cos. p\pi)\}]$ V. T. 176. N°. 8.
- 8) $\int \frac{\sin. 1-p x - \sin. p-1 x}{q^2 + (l \sin. x)^2} \frac{dx}{\cos. x} = -\frac{\pi}{q} \sum_1 \frac{\sin. n p \pi}{q + n \pi}, p^2 < 1;$ V. T. 176. N°. 10.
- 9) $\int \frac{\sin. p x + \operatorname{Cosec.} p x}{\pi^2 + (l \sin. x)^2} \frac{l \sin. x}{\cos. x} dx = \frac{1}{2} [1 - p\pi \sin. p\pi - \cos. p\pi. l \{2(1 + \cos. p\pi)\}]$ V. T. 176. N°. 9.
- 10) $\int \frac{\sin. 1-p x + \sin. p-1 x}{q^2 + (l \sin. x)^2} \frac{l \sin. x}{\cos. x} dx = -\frac{\pi}{2q} - \pi \sum_1 \frac{\cos. n p \pi}{q + n \pi}, p^2 < 1;$ V. T. 176. N°. 11.
- 11) $\int \frac{\operatorname{Tang.} x. l \sin. x}{\{q^2 + (l \sin. x)^2\}^2} dx = -\frac{\pi^2}{4q^4} \sum_0 B_{2n+1} \left(\frac{\pi}{q}\right)^{2n}$ V. T. 173. N°. 15.
- 12) $\int \frac{\pi^2 - (l \sin. x)^2}{\{\pi^2 + (l \sin. x)^2\}^2} \frac{l \cos. x}{\operatorname{Tang.} x} dx = \frac{1}{4} (1 - 2A)$ V. T. 351. N°. 4.
- 13) $\int \frac{\operatorname{Tang.} x. l \sin. x}{\{q^2 - (l \sin. x)^2\}^2} dx = \frac{\pi^2}{4q^4} \sum_0 (-1)^{n+1} B_{2n+1} \left(\frac{\pi}{q}\right)^{2n}$ V. T. 173. N°. 16.
- 14) $\int \frac{q^2 - 3 l \sin. x^2}{\{q^2 + l \sin. x^2\}^3} \frac{l \cos. x}{\operatorname{Tang.} x} dx = -\frac{\pi^2}{4q^4} \sum_0 B_{2n+1} \left(\frac{\pi}{q}\right)^{2n}$ V. T. 351. N°. 11.
- 15) $\int \frac{q^2 + (l \sin. x)^2}{\{q^2 - (l \sin. x)^2\}^2} \frac{l \cos. x}{\operatorname{Tang.} x} dx = \frac{\pi^4}{4q^4} \sum_0 (-1)^{n+1} \frac{B_{2n+1}}{n+1} \left(\frac{\pi}{q}\right)^{2n}$ V. T. 351. N°. 5.
- 16) $\int \frac{q^2 + 3 (l \sin. x)^2}{\{q^2 - (l \sin. x)^2\}^3} \frac{l \cos. x}{\operatorname{Tang.} x} dx = \frac{\pi^2}{4q^4} \sum_0 (-1)^{n+1} B_{2n+1} \left(\frac{\pi}{q}\right)^{2n}$ V. T. 351. N°. 13.

- 1) $\int \frac{l \cos. x}{\pi^2 + (l \cos. x)^2} \frac{dx}{\sin. x} = \frac{1}{2} \left\{ \frac{1}{2} - l2 \right\}$ V. T. 173. N°. 11.
- 2) $\int \frac{l \cos. x}{q^2 + (l \cos. x)^2} \frac{dx}{\cot. x} = \frac{\pi}{2q} l \Gamma \left(q + \frac{\pi}{\pi} \right) + \frac{\pi}{4q} l2q + \frac{1}{2} \left(l \frac{q}{\pi} - 1 \right)$ V. T. 170. N°. 8.
- 3) $\int \frac{\cos. p x - \sec. p x}{\pi^2 + (l \cos. x)^2} \frac{dx}{\sin. x} = -\frac{1}{4} \sin. \frac{1}{2} p \pi + \frac{1}{4\pi} \cos. \frac{1}{2} p \pi. l \frac{1 + \sin. \frac{1}{2} p \pi}{1 - \sin. \frac{1}{2} p \pi}, p < 1;$ V. T. 176. N°. 6.

- 4) $\int \frac{\cos^p x + \sec^p x \, l \cos x}{\pi^2 + (l \cos x)^2 \sin x} dx = \frac{1}{4} - \frac{\pi}{8} \cos \frac{1}{2} p \pi + \frac{1}{8} \sin \frac{1}{2} p \pi \, l \frac{1 - \sin \frac{1}{2} p \pi}{1 + \sin \frac{1}{2} p \pi}, p < 1; \text{ V. T. 176. N}^\circ 7.$
- 5) $\int \frac{l \cos x}{q^2 + (l \cos x)^2} \frac{dx}{\text{Tang. } x} = -\frac{1}{2} \left\{ l \frac{q}{\pi} - \frac{\pi}{2q} - Z' \left(\frac{q}{\pi} \right) \right\} \text{ V. T. 173. N}^\circ 12.$
- 6) $\int \frac{l \cos x}{\pi^2 + (l \cos x)^2} \frac{dx}{\text{Tang. } x} = \frac{1}{4} (1 - 2\Lambda) \text{ V. T. 173 N}^\circ 13.$
- 7) $\int \frac{l \cos x}{q^2 - (l \cos x)^2} \frac{dx}{\text{Tang. } x} = \frac{\pi^4}{4q^4} \sum_0^{\infty} (-1)^{n+1} B_{2n+1} \left(\frac{\pi}{q} \right)^n \text{ V. T. 173. N}^\circ 14.$
- 8) $\int \frac{l \cos x}{\pi^2 + 4(l \cos x)^2} \frac{dx}{\sin x} = \frac{1}{16} (2 - \pi) \text{ V. T. 173. N}^\circ 10.$
- 9) $\int \frac{l \cos x}{\{q^2 + (l \cos x)^2\}^2} \frac{dx}{\text{Tang. } x} = -\frac{\pi^2}{4q^4} \sum_0^{\infty} B_{2n+1} \left(\frac{\pi}{q} \right)^{2n} \text{ V. T. 173. N}^\circ 15.$
- 10) $\int \frac{\pi^2 - (l \cos x)^2}{\{\pi^2 + (l \cos x)^2\}^2} \text{Tang. } x \, l \sin x \, dx = \frac{1}{4} (1 - 2\Lambda) \text{ V. T. 352. N}^\circ 6.$
- 11) $\int \frac{q^2 + (l \cos x)^2}{\{q^2 - (l \cos x)^2\}^2} \text{Tang. } x \, l \sin x \, dx = \frac{\pi^4}{4q^4} \sum_0^{\infty} (-1)^{n+1} \frac{B_{2n+1}}{n+1} \left(\frac{\pi}{q} \right)^{2n} \text{ V. T. 352. N}^\circ 7.$
- 12) $\int \frac{q^2 - 3(l \cos x)^2}{\{q^2 + (l \cos x)^2\}^3} \text{Tang. } x \, l \sin x \, dx = -\frac{\pi^2}{4q^4} \sum_0^{\infty} B_{2n+1} \left(\frac{\pi}{q} \right)^{2n} \text{ V. T. 352. N}^\circ 9.$
- 13) $\int \frac{l \cos x}{\{q^2 - (l \cos x)^2\}^2} \frac{dx}{\text{Tang. } x} = \frac{\pi^2}{4q^4} \sum_0^{\infty} (-1)^{n+1} B_{2n+1} \left(\frac{\pi}{q} \right)^{2n} \text{ V. T. 173. N}^\circ 16.$
- 14) $\int \frac{q^2 + 3(l \cos x)^2}{\{q^2 - (l \cos x)^2\}^3} \text{Tang. } x \, l \sin x \, dx = \frac{\pi^2}{4q^4} \sum_0^{\infty} (-1)^{n+1} B_{2n+1} \left(\frac{\pi}{q} \right)^{2n} \text{ V. T. 352. N}^\circ 13.$
- 15) $\int \frac{dx}{q^2 + (l \text{Tang. } x)^2} = \frac{1}{2q} \left\{ Z' \left(\frac{2q + 3\pi}{4\pi} \right) - Z' \left(\frac{2q - \pi}{4\pi} \right) \right\} \text{ V. T. 324. N}^\circ 3 \text{ et T. 363. N}^\circ 3.$
- 16) $\int \frac{1}{q^2 + (l \text{Tang. } x)^2} \frac{1}{\sin x + \cos x} \frac{dx}{\sin 2x} = \frac{1}{2q} \frac{1}{2} \left\{ Z' \left(\frac{q + 3\pi}{4\pi} \right) - Z' \left(\frac{q - \pi}{4\pi} \right) \right\} \text{ V. T. 327. N}^\circ 1 \text{ et T. 360. N}^\circ 6.$

- 1) $\int l \sin x \, dx = -\pi l 2 \text{ Grunert, Gr. 4. 113.}$
- 2) $= -\pi (l 2 - 2\alpha \pi i) \text{ Arndt, Gr. 6. 187.}$
- 3) $\int l ((\sin x)) \, dx = -\pi l 2 \pm 2\alpha \pi^2 i \text{ Lindmann, Gr. 16. 94.}$

$$4) \int l((- \sin. x)) dx = - \pi l 2 \pm (2 \alpha + 1) \pi^2 i \quad \text{Lindmann, Gr. 16. 94.}$$

$$5) \int l \text{Tang. } x dx = 0 \quad \text{Ohm, Ausw. 18.}$$

$$6) \int l \cos.^2 x dx = - 2 \pi l 2 \quad \text{V. T. 353. N°. 1, 5.}$$

$$\left. \begin{aligned} 7) \int l(1 + \cos. x) dx &= - \pi l 2 \\ 8) \int l(1 - \cos. x) dx &= - \pi l 2 \end{aligned} \right\} \text{Raabe, Int. 161. — Ohm, Ausw. 18.}$$

$$9) \int l(1 \pm p \cos. x) dx = \pi l \frac{1 + \sqrt{1-p^2}}{2}, \quad p < 1; \quad \text{Ohm, Ausw. 18.}$$

$$\left. \begin{aligned} 10) \int l(p + \cos. x) dx &= - \pi l 2, \quad p < 1; \\ 11) &= - 2 \pi l \{ \sqrt{p+1} - \sqrt{p-1} \}, \quad p > 1; \end{aligned} \right\} \begin{aligned} &\text{V. T. 245. N°. 4, 5.} \\ &\text{Lobatto, Cr. 9. 260 trouve fau-} \\ &\text{tivement } \pi l \{ p + \sqrt{p^2-1} \} \end{aligned}$$

$$\left. \begin{aligned} 12) \int l(p - \cos. x) dx &= - \pi l 2, \quad p^2 < 1; \\ 13) &= - 2 \pi l \{ \sqrt{p+1} - \sqrt{p-1} \}, \quad p^2 > 1; \end{aligned} \right\} \text{V. T. 124. N°. 6, 7.}$$

$$\left. \begin{aligned} 14) \int l(p^2 - \cos.^2 x)^2 dx &= - 4 \pi l 2, \quad p^2 < 1; \\ 15) &= - 8 \pi l \{ \sqrt{p+1} - \sqrt{p-1} \}, \quad p^2 > 1; \end{aligned} \right\} \text{V. T. 246. N°. 20, 21}$$

$$16) \int l(1 + q \cot.^2 \frac{1}{2} x) dx = \pi l \frac{1 + \sqrt{1+q}}{2} \quad \text{Ramus, Danske Afh. 6. 265.}$$

$$\left. \begin{aligned} 17) \int l(1 - 2p \cos. x + p^2) dx &= 0, \quad p \leq 1; \\ 18) &= 2 \pi l p, \quad p > 1; \end{aligned} \right\} \begin{aligned} &\text{Poisson, P. 17. 612. N°. 15. — Delaunay,} \\ &\text{L. 3. 355. — Grunert, Gr. 4. 113. — Lo-} \\ &\text{batschewsky, Mém. Kasan. 1835. 1.} \end{aligned}$$

$$\left. \begin{aligned} 19) \int l(1 + 2p \cos. x + p^2) dx &= 0, \quad p < 1; \\ 20) &= 2 \pi l p, \quad p > 1; \end{aligned} \right\} \text{Grunert, Gr. 4. 113.}$$

Les form. (17), (19) se trouvent aussi chez Schlömilch, Beitr. II. 1. — Bierens de Haan, Gr. 13. 193.
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$$\begin{aligned} 21) & \int l \left(1 + 2 \frac{b}{a} \cos x + \frac{b^2}{a^2} \right) dx = 0, \quad b \leq a; \\ 22) & = 2\pi l \frac{b}{a}, \quad b > a; \end{aligned} \quad \left. \vphantom{\int} \right\} \text{Ramus, Danske Afh. 6. 265.}$$

$$23) \int l(1 - 2p \cos 2x + p^2) dx = 0 \quad \text{Bierens de Haan, Gr. 13. 193.}$$

$$24) \int l \frac{1 + 2q \cos x + q^2}{1 + 2q \cos ax + q^2} dx = 0 \quad \text{Raabe, Cr. 23. 105.}$$

$$1) \int l \sin x \cos x dx = 0 \quad \text{V. T. 330. N°. 12 et T. 331. N°. 9.}$$

$$2) \int l \sin x \sin^{2a} 2x \cos 2x dx = \frac{-\pi}{4a+2} \frac{1^{a/2}}{2^{a/2}} \quad \text{V. T. 88. N°. 1.}$$

$$\begin{aligned} 3) & \int l \sin \frac{1}{2} x \cos qx dx = -\frac{1}{2} \pi q \\ 4) & = \frac{2\pi^{k-1}}{k} \sum_1 \cos \frac{2qn\pi}{k} \cdot l \sin \frac{n\pi}{2k}, k = \infty; \end{aligned} \quad \left. \vphantom{\int} \right\} \text{Raabe, Cr. 25. 160 (il trouve pour 4) faut. } \frac{2}{k} \Sigma.$$

$$5) \int l \tan x \tan 2x dx = -\frac{1}{4} \pi^2 \quad \text{V. T. 160. N°. 15.}$$

$$6) \int l(1 - 2p \cos x + p^2) \cos ax dx = -\frac{\pi}{a} p^a \quad \text{Schlömlich, Beitr. II. 1.}$$

$$7) \int l(1 - 2p \cos x + p^2) \sin ax \sin x dx = \frac{1}{2} \pi \left(\frac{p^{a+1}}{a+1} - \frac{p^{a-1}}{a-1} \right)$$

$$8) \int l(1 - 2p \cos x + p^2) \cos ax \cos x dx = -\frac{1}{2} \pi \left(\frac{p^{a+1}}{a+1} + \frac{p^{a-1}}{a-1} \right)$$

$$9) \int l(1 - 2p \cos 2x + p^2) \cos \{(2a-1)x\} dx = 0$$

$$10) \int l(1 - 2p \cos 2x + p^2) \sin 2ax \sin x dx = 0$$

$$11) \int l(1 - 2p \cos 2x + p^2) \cos 2ax \cos x dx = 0$$

Bierens de Haan, Gr. 13. 193.

F. Log. } Log. de Circ. Dir. avec fact. Circ. TABLE 354 suite. Lim. 0 et π
Circ. Dir.

$$12) \int l(1 - 2p \cos. 2x + p^2) \sin. \{(2a-1)x\} \sin. x \, dx = -\frac{1}{2} \pi \left(\frac{p^{a-1}}{a-1} - \frac{p^a}{a} \right) \quad \left. \begin{array}{l} \text{Bierens de Haan.} \\ \text{Gr. 13. 193.} \end{array} \right\}$$

$$13) \int l(1 - 2p \cos. 2x + p^2) \cos. \{(2a-1)x\} \cos. x \, dx = -\frac{1}{2} \pi \left(\frac{p^{a-1}}{a-1} + \frac{p^a}{a} \right)$$

$$14) \int l x \cdot \left(b \cos. x \cos. \frac{2a\pi x}{b} - 2a\pi \sin. x \sin. \frac{2a\pi x}{b} \right) dx = -\frac{1}{2} b \pi \quad \text{V. T. 249. N}^\circ 24.$$

F. Log. de Circ. Dir. TABLE 355. Lim. 0 et π
Circ. Dir. fract.

$$1) \int l(p \cos. x + 1) \frac{dx}{\cos. x} = \pi \operatorname{Arcsin}. p, \quad p^2 > 1; \quad \text{Winckler, Cr. 45. 102.}$$

$$2) \int l \frac{1 + \sin. x}{1 + \cos. x} \frac{dx}{\sin. x} = 2^2 \quad \text{V. T. 179. N}^\circ 13.$$

$$3) \int l(1 - 2p \cos. x + p^2) \frac{dx}{\cos. x} = \infty, \quad p^2 \leq 1; \quad \left. \begin{array}{l} \text{Schlömlich, Beitr. II. 1 trouve fautivement} \\ - 2\pi \operatorname{Arctang}. p. \end{array} \right\}$$

$$4) \int l(1 - 2p \cos. 2x + p^2) \frac{dx}{\cos. x} = 0, \quad p < 1; \quad \text{V. T. 348. N}^\circ 3.$$

$$5) \int l \frac{1 + 2q \cos. x + q^2}{1 + 2q \cos. ax + q^2} \frac{dx}{\operatorname{Tang}. \frac{1}{2} x} = 0 \quad \text{Raabe, Cr. 23. 105.}$$

$$6) \int l \sin. x \frac{dx}{1 \pm 2p \cos. x + p^2} = \frac{\pi}{1-p^2} l \frac{1-p^2}{2}, \quad p^2 < 1; \quad \left. \begin{array}{l} \\ \text{Raabe, Int. 161. — Ohm, Ausw. 18.} \end{array} \right\}$$

$$7) \quad = \frac{\pi}{p^2-1} l \frac{p^2-1}{2p^2}, \quad p^2 > 1; \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$8) \int l \sin. x \frac{p + \cos. x}{1 + 2p \cos. x + p^2} dx = \frac{\pi}{2p} l \frac{1}{1-p^2}, \quad p < 1; \quad \text{Raabe, Int. 161.}$$

$$9) \quad = \frac{\pi}{2p} l \frac{p^2-1}{4p^2}, \quad p^2 > 1; \quad \text{V. T. 373. N}^\circ 16.$$

$$10) \int l \sin. x \frac{\cos. x - p}{1 - 2p \cos. x + p^2} dx = \frac{\pi}{2p} l(1-p^2), \quad p^2 < 1; \quad \left. \begin{array}{l} \\ \text{V. T. 371. N}^\circ 11, 12. \end{array} \right\}$$

$$11) \quad = \frac{\pi}{2p} l \frac{4p^2}{p^2-1}, \quad p^2 > 1; \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

- 12) $\int l \sin. x \frac{p \cos. x + 1}{1 + 2p \cos. x + p^2} dx = \frac{1}{2} \pi l \frac{1 - p^2}{4}, p^2 < 1;$ V. T. 371. N°. 15, 16.
- 13) $= \frac{1}{2} \pi l \frac{p^2}{p^2 - 1}, p^2 > 1;$
- 14) $\int l \sin. x \frac{p \cos. x - 1}{1 - 2p \cos. x + p^2} dx = \frac{1}{2} \pi l \frac{4}{1 - p^2}, p^2 < 1;$ V. T. 371. N°. 13, 14.
- 15) $= \frac{1}{2} \pi l \frac{p^2 - 1}{p^2}, p^2 > 1;$
- 16) $\int l \sin. x \frac{\cos. x}{1 - 2p \cos. 2x + p^2} dx = 0, p > 0;$ V. T. 371. N°. 6.
- 17) $\int l \sin. x \frac{\cos. 2x - p}{1 - 2p \cos. 2x + p^2} dx = \frac{\pi}{2p} l(1 - p), p < 1;$ V. T. 371. N°. 9.
- 18) $\int l \cos. ax \frac{dx}{1 - 2p \cos. 2x + p^2} = \frac{\pi}{1 - p^2} l \frac{1 + p^{2a}}{2}$ Plana, Mém. Turin. 1818. 7. II. 14.
- 19) $\int l \cos. x \frac{\cos. 2x - p}{1 - 2p \cos. 2x + p^2} dx = \frac{\pi}{2p} l(1 + p), p^2 < 1;$ V. T. 315. N°. 17, 20.
- 20) $\int l \text{Tang. } x \frac{\cos. 2x - p}{1 - 2p \cos. 2x + p^2} dx = \frac{\pi}{2p} l \frac{1 - p}{1 + p}, p^2 < 1;$ V. T. 373. N°. 4.
- 21) $\int l(1 - 2p \cos. x + p^2) \frac{dx}{1 - 2q \cos. x + q^2} = \frac{2\pi}{1 - q^2} l(1 - pq), p^2 \leq 1;$ Schlömilch, Beitr. II. 2.
- 22) $\int l(1 + 2p \cos. ax + p^2) \frac{dx}{1 - 2q \cos. x + q^2} = \frac{2\pi}{1 - q^2} l(1 + pq^a)$ Plana, Mém. Turin. 1818. 7. II. 14.
- 23) $\int l(1 - p^2 \sin.^2 x) \frac{dx}{\sqrt{1 - p^2 \sin.^2 x}} = l(1 - p^2). F'(p)$ Roberts, L. 12. 449.

- 1) $\int l \sin. x dx = -2\pi \left(l 2 - 2\alpha \pi i - \frac{2\beta + 1}{2} \pi i \right)$ Arndt, Gr. 6. 187.
- 2) $\int l(1 - 2p \cos. x + p^2) dx = 0$ Bierens de Haan, Gr. 13. 193.
- 3) $\int l \frac{1 + 2p \cos. x + p^2}{1 + 2p \cos. ax + p^2} dx = 0$ Raabe, Cr. 23. 105.

- 4) $\int l \sin. \frac{1}{4} x. \cos. a x dx = \frac{2\pi}{k} \sum_1^{k-1} \cos. \frac{2an\pi}{k} l \sin. \frac{n\pi}{2k}, k = \infty; \text{Raabe, Cr. 25, 160. (trouve faut. } \frac{2}{k} \sum)$
- 5) $\int l(2 + 2 \cos. x) \cos. a x dx = \frac{2\pi}{a} (-1)^{a-1}$
- 6) $\int l \frac{1 + \cos. x}{1 + \cos. b x} \cos. a x dx = 2\pi \left\{ \frac{(-1)^{a-1}}{a} - \frac{(-1)^{\frac{a-1}{b}} b}{a} \right\}$
- 7) $\int l(1 + 2p \cos. b x + p^2) \cos. a x dx = 0$, où b indivisible par a ;
- 8) $\int l(1 + 2p \cos. x + p^2) \cos. a x dx = 2\pi (-1)^{a-1} \frac{1}{a} p^a, p^2 \leq 1;$
- 9) $\int l(1 + 2p \cos. x + p^2) \cos. a x dx = 2\pi (-1)^{a-1} \frac{1}{a p^a}, p^2 \geq 1;$
- 10) $\int l(1 - 2p \cos. x + p^2) \cos. a x dx = -\frac{2\pi}{a} p^a$
- 11) $\int l(1 - 2p \cos. x + p^2) \sin. a x. \sin. x dx = -\pi \left(\frac{p^{a-1}}{a-1} - \frac{p^{a+1}}{a+1} \right), p < 1;$
- 12) $\int l(1 - 2p \cos. x + p^2) \cos. a x. \cos. x dx = -\pi \left(\frac{p^{a+1}}{a+1} + \frac{p^{a-1}}{a-1} \right)$
- 13) $\int l \frac{1 + 2p \cos. x + p^2}{1 + 2p \cos. b x + p^2} \cos. a x dx = 2\pi \left\{ (-1)^{a-1} \frac{p^a}{a} - (-1)^{\frac{a-b}{b}} \frac{b p^{\frac{a}{b}}}{a} \right\}, p^2 \leq 1;$
- 14) $\int l \frac{1 + 2p \cos. x + p^2}{1 + 2p \cos. a x + p^2} \cos. a x dx = 2\pi \left\{ (-1)^{a-1} \frac{1}{a p^a} - (-1)^{\frac{a-b}{b}} \frac{b}{a p^{\frac{a}{b}}} \right\}, p^2 \geq 1;$
- 15) $\int l \frac{1 + 2p \cos. x + p^2}{1 + 2p \cos. a x + p^2} \text{Tang. } \frac{1}{2} x dx = 0 \quad \text{Raabe, Cr. 23, 105.}$
- 16) $\int l \sin. x. \frac{p - \cos. x}{1 - 2p \cos. x + p^2} dx = \frac{\pi}{p} l(1 - p^2), p^2 < 1; \quad \text{V. T. 373. N° 5.}$

- 1) $\int l \text{Tang. } x dx = \sum_0^{\infty} (-1)^n \frac{1}{(2n+1)^2} \quad \text{V. T. 152. N° 11.}$
- 2) $\int l \text{Tang. } x \frac{dx}{\sin. 4x} = -\infty \quad \text{V. T. 153. N° 11,}$

$$3) \int l \text{ Tang. } x \frac{dx}{\cos. 2x} = -\frac{1}{8} \pi^2 \quad \text{V. T. 187. N}^\circ. 13.$$

$$4) \int l \text{ Tang. } x \frac{dx}{\text{Tang. } 2x} = -\infty \quad \text{V. T. 153. N}^\circ. 10.$$

$$5) \int l \text{ Tang. } x \frac{dx}{2 - \sin. 2x} = \frac{2}{27} \pi^2 \quad \text{V. T. 153. N}^\circ. 3.$$

$$6) \int l \text{ Tang. } x \frac{\sin. 2x}{4 \cos.^2 2x + \sin.^2 2x} dx = \frac{1}{54} \pi^2 \quad \text{V. T. 153. N}^\circ. 7.$$

$$7) \int l \text{ Tang. } x \frac{\sin.^2 2x}{\sin.^4 x + \cos.^4 x} \frac{dx}{\cos. 2x} = -\frac{1}{4} \frac{\pi^2}{2 + \sqrt{2}} \quad \text{V. T. 153. N}^\circ. 19.$$

$$8) \int l \text{ Tang. } x \frac{1}{\text{Tang. }^p x - \cot.^p x} \frac{dx}{\sin.^2 2x} = \frac{\pi^2}{16 p^2} \sec.^2 \frac{\pi}{2p} \quad \text{V. T. 152. N}^\circ. 20.$$

$$9) \int l \text{ Tang. } x \frac{\cos. 2x}{\text{Tang. }^p x + \cot.^p x} \frac{dx}{\sin.^2 2x} = -\frac{\pi^2}{16 p^2} \sin. \frac{\pi}{2p} \sec.^2 \frac{\pi}{2p} \quad \text{V. T. 152. N}^\circ. 19.$$

$$10) \int l \text{ Tang. } x \frac{\text{Tang. }^q x - \cot.^q x}{\text{Tang. }^p x + \cot.^p x} \frac{dx}{\sin. 2x} = \frac{\pi^2}{8 p^2} \sin. \frac{q\pi}{2p} \sec.^2 \frac{q\pi}{2p} \quad \text{V. T. 153. N}^\circ. 12.$$

$$11) \int l \text{ Tang. } x \frac{\text{Tang. }^q x + \cot.^q x}{\text{Tang. }^p x - \cot.^p x} \frac{dx}{\sin. 2x} = \frac{\pi^2}{8 p^2} \sec.^2 \frac{q\pi}{2p} \quad \text{V. T. 153. N}^\circ. 13.$$

$$12) \int l \text{ Tang. }^2 \left(\frac{\pi}{4} \pm x \right) \frac{dx}{\sin. 2x} = \mp \frac{1}{4} \pi^2 \quad \text{V. T. 357. N}^\circ. 3.$$

$$1) \int (l \text{ Tang. } x)^2 dx = \frac{1}{16} \pi^3 \quad \text{V. T. 154. N}^\circ. 1.$$

$$2) \int (l \text{ Tang. } x)^2 \frac{dx}{2 + \sin. 2x} = \frac{4}{243} \pi^3 \sqrt{3} \quad \text{V. T. 156. N}^\circ. 1.$$

$$3) \int (l \text{ Tang. } x)^2 \frac{dx}{2 - \sin. 2x} = \frac{5}{243} \pi^3 \sqrt{3} \quad \text{V. T. 156. N}^\circ. 2.$$

$$4) \int (l \text{ Tang. } x)^2 \frac{dx}{1 + \cos. \lambda. \sin. 2x} = \lambda \operatorname{Cosec.} \lambda \frac{\pi^2 - \lambda^2}{6} \quad \text{V. T. 156. N}^\circ. 3.$$

$$5) \int (l \text{ Tang. } x)^2 \frac{dx}{1 - \cos. \lambda. \sin. 2x} = 2 \lambda \operatorname{Cosec.} \lambda \left(\frac{1}{6} \pi^2 - \frac{1}{4} \pi \lambda + \frac{1}{12} \lambda^2 \right) \quad \text{V. T. 156. N}^\circ. 4.$$

- 6) $\int (l \text{ Tang. } x)^2 \frac{dx}{1 - \text{Sin.}^2 x \cdot \text{Cos.}^2 x} = \frac{1}{27} \pi^3 \vee 3$ V. T. 154. N°. 3.
- 7) $\int (l \text{ Tang. } x)^2 \frac{\text{Sin. } 2x}{1 - \text{Sin.}^2 x \cdot \text{Cos.}^2 x} dx = \frac{2}{243} \pi^3 \vee 3$ V. T. 154. N°. 4.
- 8) $\int (l \text{ Tang. } x)^2 \frac{dx}{\text{Sin.}^4 x + \text{Cos.}^4 x} = \frac{3}{64} \pi^3 \vee 2$ V. T. 154. N°. 2.
- 9) $\int (l \text{ Tang. } x)^2 \frac{\text{Tang.}^q x + \text{Cot.}^q x}{\text{Tang.}^p x + \text{Cot.}^p x} \frac{dx}{\text{Sin. } 2x} = \frac{\pi}{16 p^3} \left\{ 2 \text{Sec.}^3 \frac{q\pi}{2p} - \text{Sec.} \frac{q\pi}{2p} \right\}$ V. T. 154. N°. 5.
- 10) $\int (l \text{ Tang. } x)^2 \frac{\text{Tang.}^q x - \text{Cot.}^q x}{\text{Tang.}^p x - \text{Cot.}^p x} \frac{dx}{\text{Sin. } 2x} = \frac{\pi^2}{8 p^3} \text{Sin.} \frac{q\pi}{2p} \cdot \text{Sec.}^3 \frac{q\pi}{2p}$ V. T. 154. N°. 6.
- 11) $\int (l \text{ Tang. } x)^3 \frac{dx}{\text{Cos. } 2x} = -\frac{1}{16} \pi^4$ V. T. 154. N°. 14.
- 12) $\int (l \text{ Tang. } x)^4 dx = \frac{5}{64} \pi^5$ V. T. 154. N°. 1.
- 13) $\int (l \text{ Tang. } x)^4 \frac{dx}{1 + \text{Cos. } \lambda \cdot \text{Sin. } 2x} = \lambda \frac{\pi^2 - \lambda^2}{\text{Sin. } \lambda} \frac{7\pi^2 - 3\lambda^2}{5}$ V. T. 156 N°. 5.
- 14) $\int (l \text{ Tang. } x)^5 \frac{dx}{\text{Cos. } 2x} = -\frac{1}{8} \pi^6$ V. T. 154. N°. 4.
- 15) $\int (l \text{ Tang. } x)^6 dx = \frac{61}{256} \pi^7$ V. T. 155. N°. 8.
- 16) $\int (l \text{ Tang. } x)^7 \frac{dx}{\text{Cos. } 2x} = -\frac{17}{32} \pi^8$ V. T. 155. N°. 10.

- 1) $\int (l \text{ Tang. } x)^q dx = \Gamma(q+1) \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^{q+1}}$ V. T. 187. N°. 7.
- 2) $\int (l \text{ Tang. } x)^{2a} dx = \frac{1}{2} (-1)^{a+1} (2\pi)^{2a+1} \text{B}'' \left(\frac{1}{4} \right)$ V. T. 158. N°. 2.
- 3) $\int (l \text{ Tang. } x)^{2a-1} \frac{dx}{\text{Cos. } 2x} = \frac{1 - 2^{2a}}{4a} \text{B}_{2a-1} \pi^{2a}$ V. T. 158. N°. 5.

- 4) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{\text{Cos. } 2x} = \frac{1 - 2^{2a+1}}{2^{2a+1}} 1^{2a/1} \sum_1^{\infty} \frac{1}{n^{2a+1}}$ V. T. 158. N°. 4.
- 5) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{2 + \text{Sin. } 2x} = \frac{(-1)^{a+1}}{2 \sqrt{3}} (2\pi)^{2a+1} B'' \left(\frac{1}{3} \right)$ V. T. 159. N°. 1.
- 6) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{2 - \text{Sin. } 2x} = \frac{(-1)^{a+1}}{2 \sqrt{3}} (2\pi)^{2a+1} B'' \left(\frac{1}{6} \right)$ V. T. 159. N°. 2.
- 7) $\int (l \text{ Tang. } x)^{2a} \frac{dx}{1 - \text{Cos. } 2p\pi \text{ Sin. } 2x} = \frac{1}{2} (-1)^{a+1} (2\pi)^{2a+1} \text{Cosec. } 2p\pi B''(p)$ V. T. 159. N°. 5.
- 8) $\int (l \text{ Tang. } x)^{2a-1} \frac{1}{\text{Sin. } x - \text{Cos. } x} \frac{dx}{\sqrt{\text{Sin. } 2x}} = \frac{2^{2a} - 1}{4a \sqrt{2}} (2\pi)^{2a} B_{2a-1}$ V. T. 164. N°. 3.

- 1) $\int \frac{\text{Tang. } q x - \text{Cot. } q x}{l \text{ Tang. } x} dx = l \text{ Tang. } \left\{ \frac{1+q}{4} \pi \right\}$ V. T. 175. N°. 5.
- 2) $\int \frac{(\text{Tang. } q x - \text{Cot. } q x)^2}{l \text{ Tang. } x} \frac{dx}{\text{Cos. } 2x} = l \text{ Cos. } q\pi$ V. T. 175. N°. 6.
- 3) $\int \frac{dx}{q^2 + (l \text{ Tang. } x)^2} = \frac{1}{4q} \left\{ Z' \left(\frac{2q+3\pi}{4\pi} \right) - Z' \left(\frac{2q+\pi}{4\pi} \right) \right\}$ V. T. 173. N°. 9.
- 4) $\int \frac{\text{Tang. } q x - \text{Cot. } q x}{\text{Tang. } p x + \text{Cot. } p x} \frac{dx}{\text{Sin. } 2x \cdot l \text{ Tang. } x} = \frac{1}{2} l \text{ Tang. } \left\{ \frac{p+q}{4p} \pi \right\}$ V. T. 172. N°. 6.
- 5) $\int \frac{\text{Tang. } q x + \text{Cot. } q x - 2}{\text{Tang. } p x - \text{Cot. } p x} \frac{dx}{\text{Sin. } 2x \cdot l \text{ Tang. } x} = \frac{1}{2} l \text{ Cos. } \frac{q\pi}{2p}$ V. T. 172. N°. 7. corr.
- 6) $\int \frac{1}{q^2 + (l \text{ Tang. } x)^2} \frac{1}{\text{Sin. } x + \text{Cos. } x} \frac{dx}{\sqrt{\text{Sin. } 2x}} = \frac{1}{4q \sqrt{2}} \left\{ Z' \left(\frac{q+3\pi}{4\pi} \right) - Z' \left(\frac{q+\pi}{4\pi} \right) \right\}$ V. T. 177. N°. 3.
- 7) $\int \frac{\text{Cos. } x}{\sqrt{l \text{ Cosec. } x}} dx = \sqrt{\pi}$ V. T. 349. N°. 6.
- 8) $\int \frac{\text{Sin. } x}{\sqrt{l \text{ Sec. } x}} dx = \sqrt{\pi}$ V. T. 349. N°. 1.

$$1) \int_0^{a\pi} l \sin x \, dx = -a\pi l 2 \quad \text{Clausen, Cr. 7. 309.}$$

$$2) \int_0^{2a\pi} l((\sin x)) \, dx = -2a\pi \left\{ l 2 - a\pi i - \frac{1}{2}(2\beta + 1)\pi i \right\}$$

$$3) \int_0^{(2a+1)\pi} l((\sin x)) \, dx = -(2a+1)\pi \left\{ l 2 - 2a\pi i - \frac{2\beta+1}{2a+1}a\pi i \right\} \left. \vphantom{\int_0^{(2a+1)\pi}} \right\} \begin{array}{l} \text{Arndt, Gr. 6. 187. —} \\ \text{Lindmann, Gr. 16. 94.} \\ \text{où fautives.} \end{array}$$

$$4) \int_0^{2a\pi} \cos \frac{bx}{a} \cdot l \sin \frac{x}{4a} \, dx = \frac{2a\pi}{k} \sum_1^{k-1} \cos \frac{2bn\pi}{k} \cdot l \sin \frac{n\pi}{2k}, \quad k = \infty; \quad \text{Raabe, Cr. 25. 160.}$$

$$5) \int_0^{1a\pi} l \operatorname{Tang}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\sin 2x}{1 - q^2 \cos^2 2x} \, dx = \pm \frac{a\pi}{2q} \operatorname{Arcsin} q, \quad q < 1; \quad \text{V. T. 361. N° 6.}$$

$$6) \int_0^{1a\pi} l \frac{1 + q \cos x}{1 - q \cos x} \frac{dx}{\cos x} = a\pi \operatorname{Arcsin} q, \quad q < 1; \quad \text{Raabe, Cr. 25. 169.}$$

$$1) \int l x \cos qx \, dx = \frac{1}{q} \{ \sin q\lambda \cdot l\lambda - \operatorname{Si}(q\lambda) \} \quad \text{V. T. 251. N° 4.}$$

$$2) \int dx l \{ \cos x + \sqrt{\cos^2 x - \sinh p^2 \mu} \} = \left(\lambda + q - \frac{1}{2}\pi \right) l \sinh p \cdot \mu +$$

$$+ \frac{1}{2} \operatorname{Li}(\lambda + q) - \frac{1}{2} \operatorname{Li}(\lambda - q) - \operatorname{Li}(q) \left. \vphantom{\int dx l} \right\} \begin{array}{l} \mu < \lambda, \cos q = \frac{\cosh p \cdot \lambda}{\cosh p \cdot \mu}; \end{array}$$

$$3) \int dx l \{ \cos x + \sqrt{\cos^2 x - \sinh p^2 \lambda} \} = \left(\lambda - \frac{1}{2}\pi \right) l \sinh p \cdot \lambda$$

$$4) \int dx l \frac{\cos x + \sqrt{\cos^2 x - \sinh p^2 \mu}}{\cos x - \sqrt{\cos^2 x - \sinh p^2 \lambda}} = \left(q + \lambda - \frac{1}{2}\pi \right) l \sinh p \cdot \mu -$$

$$- \left(\frac{1}{2}\pi + \lambda \right) l \sinh p \cdot \lambda + \frac{1}{2} \operatorname{Li}(\lambda + q) - \frac{1}{2} \operatorname{Li}(\lambda - q) - \operatorname{Li}(q) \left. \vphantom{\int dx l} \right\} \begin{array}{l} \text{Lobatschewsky, Mém. Ka-} \\ \text{san. 1836. I. I. 118, 117, 114.} \end{array}$$

$$5) \int dx l \{ \cos x + \sqrt{\cos^2 x - \cos^2 \lambda} \} = \left(\lambda - \frac{1}{2}\pi \right) l \cos \lambda$$

$$6) \int dx l \{ \cos x + \sqrt{\cos^2 x - \cos^2 \mu} \} = \left(\lambda + q - \frac{1}{2}\pi \right) l \cos \mu +$$

$$+ \frac{1}{2} \operatorname{Li}(\lambda + q) - \frac{1}{2} \operatorname{Li}(\lambda - q) - \operatorname{Li}(q) \left. \vphantom{\int dx l} \right\} \begin{array}{l} \cos q = \\ \sin \lambda, \operatorname{cosec} \mu, \mu < \lambda; \\ \text{Lobatschewsky, Mém. Ka-} \\ \text{san. 1836. I. II. 21, 39.} \end{array}$$

- 7) $\int dx l \frac{\cos. x + \cos. \lambda}{\cos. x - \cos. \lambda} = \pi l 2 - 2 L(\lambda) - 2 L\left(\frac{1}{2} \pi - \lambda\right)$
- 8) $\int dx l \frac{\sin. \lambda + \sin. \mu \cos. x \sqrt{(\sin.^2 \lambda - \sin.^2 x)}}{\sin. \lambda - \sin. \mu \cos. x \sqrt{(\sin.^2 \lambda - \sin.^2 x)}} =$
 $= \pi l \left\{ \text{Tang.} \frac{1}{2} \mu \sin. \lambda + \frac{1}{2} \left(\text{Tang.}^2 \frac{1}{2} \mu \sin.^2 \lambda + 1 \right) \right\}$ } Lobatschewsky, Mém. Kasan. 1836. I. II. 20, 14.
- 9) $\int l \frac{1 + \sin. x}{1 - \sin. x} \frac{\cos. x}{\sin. x \sqrt{(\sin.^2 \lambda - \sin. x)}} dx = \pi \lambda \text{Cosec.} \lambda$
- 10) $\int \left(l \frac{1 + \sin. x}{1 - \sin. x} - 2 \sin. x \right) \frac{\cos. x}{\sin.^3 x \sqrt{(\sin.^2 \lambda - \sin.^2 x)}} dx = \pi \text{Cosec.}^3 \lambda \frac{\lambda - \sin. \lambda \cos. \lambda}{2}$ } Legendre, Exerc. Suppl. 34.
- 11) $\int l \frac{1 + \sin. x}{1 - \sin. x} \frac{\sin. x \cos. x}{\sqrt{(\sin.^2 \lambda - \sin.^2 x)}} dx = \pi (1 - \cos. \lambda)$ Legendre, Exerc. Suppl. 35.
- 12) $\int l \frac{1 + \sin. x}{1 - \sin. x} \frac{\sin. x}{\cos. x \sqrt{(\sin.^2 \lambda - \sin.^2 x)}} dx = \pi \text{Sec} \lambda. l \text{Sec.} \lambda$
- 13) $\int l \frac{1 + \sin. x}{1 - \sin. x} \frac{\sin.^3 x}{\cos.^3 x \sqrt{(\sin.^2 \lambda - \sin.^2 x)}} dx = \frac{1}{4} \pi \sin.^2 \lambda. \text{Sec.}^3 \lambda - \frac{1}{2} \pi \text{Sec.}^3 \lambda. l \cos. \lambda$ } Legendre, Exerc. Suppl. 39.
- 14) $\int \left\{ l \frac{1 + \sin. x}{1 - \sin. x} - 2 \sin. x \right\} \frac{\cos. x}{\sin.^2 x \sqrt{(\sin.^2 \lambda - \sin.^2 x)}} dx = 2 \text{Cosec.} \lambda (1 - \lambda \cot. \lambda)$ Legendre, Exerc. Suppl. 49.

- 1) $\int l \cot. \frac{1}{2} x \frac{\sin. x \cos. x}{1 - \cos.^2 \lambda \cos.^2 x \sqrt{(\sin.^2 x - \sin.^2 \mu)}} dx =$
 $= \frac{\pi}{\sin. 2 \lambda} \sin. \left(\text{Arctang.} \frac{\text{Tang.} \lambda}{\sin. \mu} \right) l \left\{ \text{Tang.} \frac{1}{2} \lambda \cot. \left(\frac{1}{2} \text{Arctang.} \frac{\text{Tang.} \lambda}{\sin. \mu} \right) \right\}$ } Lobatschewsky, Mém. Kasan. 1835. I.
- 2) $\int l \cot. \frac{1}{2} x \frac{\sin. x \cos. x}{\sin.^2 x - \sin.^2 \mu \sqrt{(\sin.^2 x - \sin.^2 \lambda)}} dx = \frac{1}{2} \pi \text{Cosec.} \lambda. \text{Sec.} q l \left\{ \cot. \frac{1}{2} q. \text{Tg.} \frac{1}{2} \mu \right\}, \sin. q = \frac{\sin. \mu}{\sin. \lambda};$
 Lobatschewsky, Mém. Kasan. 1835. 1.
- 3) $\int l \cot. \frac{1}{2} x \frac{\sin. x \cos. x}{\sin.^2 \lambda + \text{Tg.}^2 \mu \sin.^2 x \sqrt{(\sin.^2 x - \sin.^2 \lambda)}} dx = \frac{\pi \cos.^2 \mu}{2 \sin. \lambda \sin. \mu} l \frac{\sin. \mu + 1 (1 - \cos.^2 \lambda \cos.^2 \mu)}{\sin. \mu (1 + \sin. \lambda)}$
 Lobatschewsky, Mém. Kasan. 1836. I. II. 25. où elle était fautive.
- 4) $\int l \frac{1 + \sin. x}{1 - \sin. x} \frac{dx}{\sqrt{(\sin.^2 x - \sin.^2 \lambda)}} = \pi F'(\sin. \lambda)$ Legendre, Exerc. Suppl. 34.

- 5) $\int l \frac{1 + \text{Sin. } x}{1 - \text{Sin. } x} \frac{\sqrt{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)}}{\text{Sin.}^2 x} dx = -\pi \text{Sin. } \lambda + \pi E'(\text{Sin. } \lambda)$ Legendre, Exerc. Suppl. 34.
- 6) $\int l \frac{1 + \text{Sin. } x}{1 - \text{Sin. } x} \frac{\text{Cos.}^2 x}{\sqrt{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)}} dx = -\pi + \pi E'(\text{Sin. } \lambda)$
- 7) $\int l \frac{1 + \text{Sin. } x}{1 - \text{Sin. } x} \frac{\text{Sin.}^4 x - \text{Sin.}^2 \lambda}{\text{Sin.}^2 x \sqrt{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)}} dx = \pi(1 - \text{Sin. } \lambda)$
- 8) $\int l \frac{1 + \text{Sin. } x}{1 - \text{Sin. } x} dx \sqrt{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)} = \pi + \text{Cos.}^2 \lambda F'(\text{Sin. } \lambda) - \pi E'(\text{Sin. } \lambda)$
- 9) $\int l \text{Cot. } \frac{1}{2} x \frac{\text{Sin. } x \cdot \text{Cos. } x}{\text{Sin.}^2 \lambda \cdot \text{Cos.}^2 \mu + \text{Sin.}^2 \mu \cdot \text{Sin.}^2 x} \frac{dx}{\sqrt{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)}} =$
 $= \frac{1}{\text{Sin. } \lambda \cdot \text{Sin. } \mu} \left\{ l \text{Sin. } \lambda - \frac{1}{2} \pi l \frac{1 + \text{Sin. } \mu}{\text{Sin. } \mu + \sqrt{(1 - \text{Cos.}^2 \lambda \cdot \text{Cos.}^2 \mu)}} \right\}$
- 10) $\int l \frac{\text{Sin. } x + \sqrt{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)}}{\text{Sin. } x - \sqrt{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)}} \frac{dx}{1 - \text{Cos.}^2 \mu \cdot \text{Cos.}^2 x} =$
 $= \text{Cosec. } \mu \left\{ -\pi l \text{Sin. } \lambda - \pi l \frac{1 + \text{Sin. } \mu}{\text{Sin. } \mu + \sqrt{(1 - \text{Cos.}^2 \lambda \cdot \text{Cos.}^2 \mu)}} \right\}$
- 11) $\int l \{ \text{Sin. } x + \sqrt{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)} \} \frac{dx}{1 - \text{Cos.}^2 \mu \cdot \text{Cos.}^2 x} =$
 $= \text{Cosec. } \mu \left\{ -\text{Arctang. } \frac{\text{Tang. } \lambda}{\text{Sin. } \mu} \cdot l \text{Sin. } \lambda - \frac{1}{2} \pi l \frac{1 + \text{Sin. } \mu}{\text{Sin. } \mu + \sqrt{(1 - \text{Cos.}^2 \lambda \cdot \text{Cos.}^2 \mu)}} \right\}$

} Legendre,
Exerc.
Suppl. 35.

} Lobatschewsky,
Mém. Kasan.
1836. I. I. 193,
191.

} Lobatschewsky,
Mém. Kasan.
1836. I. I. 193.
— Id., ib. II. 24.

- 1) $\int l \text{Sin. } x \cdot dx = I_1 \left(\frac{1}{2} \pi - \mu \right) - I_1 \left(\frac{1}{2} \pi - \lambda \right)$
- 2) $\int l \text{Cos. } x \cdot dx = I_1(\lambda) - I_1(\mu)$
- 3) $\int l \text{Tang. } x \cdot dx = I_1 \left(\frac{1}{2} \pi - \mu \right) + I_1(\mu) - I_1 \left(\frac{1}{2} \pi - \lambda \right) - I_1(\lambda)$
- 4) $\int l \frac{1 + \text{Sin. } x}{1 - \text{Sin. } x} \frac{\text{Cos. } x}{\sqrt{(\text{Sin.}^2 x - \text{Sin.}^2 \lambda)} (\text{Sin.}^2 \mu - \text{Sin.}^2 x)} dx = \pi \text{Cosec. } \mu F(c, \mu)$

} Lobatschewsky, Mém. Kasan.
1836. I. I. 17, 18, 19.

$$5) \int l \frac{1 - \sin x}{1 + \sin x} \frac{\cos x}{\sin^2 x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} dx = \frac{\pi}{\sin \lambda \sin \mu} +$$

$$+ \frac{\pi}{\sin^2 \lambda \sin \mu} F(c, \mu) - \frac{\pi}{\sin^2 \lambda \sin \mu} E(c, \mu)$$

$$6) (2a+1) \sin^2 \lambda \sin^2 \mu \int l \frac{1 + \sin x}{1 - \sin x} \frac{\cos x}{\sin^{2a+2} x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} dx =$$

$$= 2a(\sin^2 \lambda + \sin^2 \mu) \int_{\lambda}^{\mu} l \frac{1 + \sin x}{1 - \sin x} \frac{\cos x dx}{\sin^{2a} x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} -$$

$$- (2a-1) \int l \frac{1 + \sin x}{1 - \sin x} \frac{\cos x dx}{\sin^{2a-2} x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} -$$

$$- 2 \int_{\lambda}^{\mu} \frac{dx \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}}{\cos x \sin^{2a+1} x} \quad \text{Voir pour la dernière Intégrale T. 106. N°. 12.}$$

$$7) \int l \frac{1 + \sin x}{1 - \sin x} \frac{\cos x}{\sin^2 x} dx \sqrt{\frac{\sin^2 \mu - \sin^2 \lambda}{\sin^2 x - \sin^2 \lambda}} = \pi \sin \mu \operatorname{Cosec} \lambda + \pi \frac{\sin^2 \mu - \sin^2 \lambda}{\sin^2 \mu \sin^2 \lambda} F(c, \mu) - \frac{\pi \sin \mu}{\sin^2 \lambda} E(c, \mu)$$

$$8) \int l \frac{1 + \sin x}{1 - \sin x} \frac{\cos x}{\sin^2 x} dx \sqrt{\frac{\sin^2 x - \sin^2 \lambda}{\sin^2 \mu - \sin^2 x}} = \pi \sin \lambda \operatorname{Cosec} \mu + \pi \operatorname{Cosec} \mu E(c, \mu)$$

$$9) \int l \frac{1 + \sin x}{1 - \sin x} \frac{\cos x \sin^2 x}{\sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} dx = \pi(1 - \cos \lambda \cos \mu) +$$

$$+ \pi \sin \mu F(c, \mu) - \pi \sin \mu E(c, \mu)$$

$$10) (2a+1) \int l \frac{1 + \sin x}{1 - \sin x} \frac{\cos x \sin^{2a+2} x}{\sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} dx$$

$$= 2a(\sin^2 \lambda + \sin^2 \mu) \int_{\lambda}^{\mu} l \frac{1 + \sin x}{1 - \sin x} \frac{\cos x \sin^{2a} x dx}{\sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} -$$

$$- (2a-1) \sin^2 \lambda \sin^2 \mu \int_{\lambda}^{\mu} l \frac{1 + \sin x}{1 - \sin x} \frac{\cos x \sin^{2a-2} x dx}{\sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} +$$

$$+ 2 \int_{\lambda}^{\mu} \frac{\sin^{2a-1} x}{\cos x} dx \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)} \quad \text{Voir pour la dernière Intégrale T. 106. N°. 16.}$$

$$11) \int l \frac{1 + \sin x}{1 - \sin x} \cos x dx \sqrt{\frac{\sin^2 \mu - \sin^2 \lambda}{\sin^2 x - \sin^2 \lambda}} = \pi(\cos \lambda \cos \mu - 1) + \pi \sin \mu E(c, \mu)$$

$$12) \int l \frac{1 + \sin x}{1 - \sin x} \cos x dx \sqrt{\frac{\sin^2 x - \sin^2 \lambda}{\sin^2 \mu - \sin^2 x}} = \pi(1 - \cos \lambda \cdot \cos \mu) + \frac{\sin^2 \mu - \sin^2 \lambda}{\sin \mu} \pi F(c, \mu) - \pi \sin \mu E(c, \mu)$$

$$13) \int l \frac{1 + \sin x}{1 - \sin x} \frac{dx}{\cos x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} = \pi \operatorname{Cosec} \mu \Pi(-\sin^2 \lambda, c, \mu) + \frac{1}{2} \pi \sec \lambda \cdot \sec \mu l(1 + \operatorname{Tang}^2 \lambda + \operatorname{Tang}^2 \mu)$$

$$14) \int l \frac{1 + \sin x}{1 - \sin x} \frac{dx}{\cos^3 x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} =$$

$$= \frac{\pi \sin^2 \lambda \cdot \cos^2 \mu + \sin^2 \mu \cdot \cos^2 \lambda}{4 \cos^3 \lambda \cdot \cos^3 \mu} - \frac{1}{2} \pi \operatorname{Cosec} \mu \cdot \sec^2 \lambda F(c, \mu) - \frac{\pi \sin \mu}{2 \cos^2 \lambda \cdot \cos^2 \mu} E(c, \mu) +$$

$$+ \frac{\cos^2 \lambda + \cos^2 \mu + \cos^2 \lambda \cdot \cos^2 \mu}{\cos^2 \lambda \cdot \cos^2 \mu} \left\{ \frac{1}{2} \pi \operatorname{Cosec} \mu \Pi(-\sin^2 \lambda, c, \mu) + \right.$$

$$\left. + \frac{1}{4} \pi \sec \lambda \cdot \sec \mu l(1 + \operatorname{Tang}^2 \lambda + \operatorname{Tang}^2 \mu) \right\}$$

$$15) 2a \cos^2 \lambda \cdot \cos^2 \mu \int l \frac{1 + \sin x}{1 - \sin x} \frac{dx}{\cos^{2a+1} x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} =$$

$$= (2a-1)(\cos^2 \lambda + \cos^2 \mu + \cos^2 \lambda \cdot \cos^2 \mu) \int_{\lambda}^{\mu} l \frac{1 + \sin x}{1 - \sin x} \frac{dx}{\cos^{2a-1} x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} -$$

$$-(2a-2)(1 + \cos^2 \lambda + \cos^2 \mu) \int_{\lambda}^{\mu} l \frac{1 + \sin x}{1 - \sin x} \frac{dx}{\cos^{2a-3} x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} +$$

$$+ (2a-3) \int_{\lambda}^{\mu} l \frac{1 + \sin x}{1 - \sin x} \frac{dx}{\cos^{2a-5} x \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} +$$

$$+ 2 \int_{\lambda}^{\mu} \frac{\sin x}{\cos^{a+1} x} dx \sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)} \quad \left\{ \begin{array}{l} \text{Voir pour cette dernière Intégrale T. 106.} \\ \text{N}^{\circ} 9. \end{array} \right.$$

Ces formules 4—6, 7 et 8, 9—12, 13—15 se trouvent chez Legendre, Exerc. Suppl. N^o. 33, 34, 35, 38; on y a partout $c = \sin \lambda \cdot \operatorname{Cosec} \mu$.

$$16) \int l \frac{1 + a \sin x}{1 - a \sin x} \frac{\cos x dx}{\sqrt{(\sin^2 x - \sin^2 \lambda)(\sin^2 \mu - \sin^2 x)}} = \pi \operatorname{Cosec} \mu F\left\{ \frac{\sin \lambda}{\sin \mu}, \operatorname{Arcsin}(a \sin \mu) \right\} \quad \left\{ \begin{array}{l} \text{Roberts, L.} \\ \text{11. 157.} \end{array} \right.$$

$$1) \int_a^{a+1} l \sin \pi x dx = a\pi i - l2 \quad \text{Schaar, Mém. Cour. Brux. T. 23.}$$

$$2) \int_0^p \frac{\cos kx}{\cos x} l(1 - q \cos x) dx = 0, \quad k = \infty; \quad \text{Raabe, Int. 174.}$$

$$3) \int_0^{\operatorname{Arccos}(\operatorname{Tang} p, \lambda, \operatorname{Coth} p, \mu)} dx \, l \frac{1 - \operatorname{Cosh} p, \lambda, \operatorname{Cosh} p, \mu, \operatorname{Cos} x \sqrt{(1 - \operatorname{Coth} p, \lambda, \operatorname{Tang} p, \mu, \operatorname{Cos}^2 x)}}{1 + \operatorname{Cosh} p, \lambda, \operatorname{Cosh} p, \mu, \operatorname{Cos} x \sqrt{(1 - \operatorname{Coth} p, \lambda, \operatorname{Tang} p, \mu, \operatorname{Cos}^2 x)}} =$$

$$= \pi l \frac{\operatorname{Sin} h p, \mu (1 + \operatorname{Sin} h p, \lambda)}{\operatorname{Sin} h p, \lambda + \sqrt{(1 - \operatorname{Cosh} p, \lambda, \operatorname{Cosh} p, \mu)}} \quad \text{Lobatschewsky, Mém. Kasan. 1836. 1. 1. 75.}$$

$$4) \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \operatorname{Cos} p, x, \operatorname{Cos} p, x \, dx \, l \operatorname{Cos} x = -\frac{\pi}{2p} l 2 \quad \text{V. T. 446. N°. 26.}$$

$$5) \int_0^{\frac{1}{2}\pi - \lambda} dx \, l \{ \operatorname{Cos} x + \sqrt{(\operatorname{Cos}^2 x - \operatorname{Sin}^2 \lambda)} \} = -\lambda l \operatorname{Sin} \lambda \quad \text{Lobatschewsky, Mém. Kasan. 1836. 1. 1. 117.}$$

$$1) \int \operatorname{Arcsin} x, (lx + 1) dx = 1 - l 2 \quad \text{V. T. 163. N°. 3.}$$

$$2) \int \operatorname{Arccos} x, (1 + lx) dx = l 2 - 1 \quad \text{V. T. 163. N°. 3.}$$

$$3) \int \operatorname{Arctang} x, (lx + 1) dx = \frac{1}{48} \pi^2 \quad \text{V. T. 152. N°. 1.}$$

$$4) \int \operatorname{Arctang} x, (lx + 3) (lx)^2 dx = \frac{7}{1920} \pi^3 \quad \text{V. T. 154. N°. 13.}$$

$$5) \int \operatorname{Arctang} x, (lx + 5) (lx)^4 dx = \frac{31}{16128} \pi^6 \quad \text{V. T. 155. N°. 2.}$$

$$6) \int \operatorname{Arctang} x, (lx + a) (lx)^{a-1} dx = \frac{1^{a/1}}{(-2)^{a+1}} \sum_0^{\infty} \frac{(-1)^n}{(n+1)^{a+1}} \quad \text{V. T. 157. N°. 8.}$$

$$1) \int_0^1 dx \, li \left(\frac{1}{x} \right) \left(\frac{1}{x} \right)^{p-1} = -\pi \operatorname{Cot} p \pi \Gamma(p) \quad \text{V. T. 402. N°. 1.}$$

$$2) \int_0^1 l \Gamma(x) dx = \frac{1}{2} l 2 \pi \quad \text{Raabe, Cr. 25. 146. — Id., Cr. 28. 10. — Schaar, Mém. Cour. Brux. T. 22. — Id., Mém. Cour. Brux. T. 23.}$$

$$3) \int_0^1 l \Gamma(1+x) dx = -1 + \frac{1}{2} l 2 \pi \quad \text{Raabe, Cr. 25. 146.}$$

F. Log. Autres Fonctions.	TABLE 367 suite.	Lim. diverses.
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$$4) \int_0^1 l \Gamma(x+q) dx = \frac{1}{2} l 2\pi + q l q - q \quad \text{Raabe, Cr. 25. 146. — Id., Cr. 28. 10. — Stern, Gött. Stud. 1847.}$$

$$5) \int_a^{a+1} l \Gamma(x) dx = \frac{1}{2} l 2\pi + a(la-1) \quad \text{Schaar, Mém. Cour. Brux. T. 22. — Id., ib. T. 23.}$$

$$= \frac{1}{2} l 2\pi + la - 1 \text{ faute d'impression chez Raabe, Cr. 28. 10.}$$

$$6) \int_0^{\frac{1}{2}\pi} l \Theta(q, x) dx = \frac{\pi}{4} l \left(\frac{\Gamma(p)}{\pi} (2pp')^{\frac{1}{2}} q^{-\frac{1}{2}} \right), q = e^{-\pi \frac{\Gamma(p) \Gamma'(1-p^2)}{\Gamma(p)}}, p' = \sqrt{1-p^2} \quad \text{Roberts, L. 12. 449.}$$

$$7) \int_1^\infty dx l i \left(\frac{1}{x} \right) (lx)^{p-1} = -\pi \operatorname{Cosec}. p \pi \Gamma(p) \quad \text{V. T. 402. N°. 2.}$$

$$8) \int_0^\infty dx l i \left(\frac{1}{x} \right) (lx)^{p-1} = -\operatorname{Sin}. p \pi \Gamma(p) \quad \text{V. T. 367. N°. 1, 7.}$$

F. Circ. Dir. ent. Circ. Inv.	TABLE 368.	Lim. 0 et $\frac{\pi}{2}$.
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$$1) \int \operatorname{Arctang}. (Tang.^2 x) dx = \frac{1}{8} \pi^2 \quad \text{V. T. 269. N°. 1.}$$

$$2) \int \operatorname{Arctang}. (Tang.^3 x) dx = \frac{1}{8} \pi^2 \quad \text{V. T. 269. N°. 2.}$$

$$3) \int \operatorname{Arctang}. (\sqrt{Tang.} x) \frac{dx}{(\operatorname{Sin}. x + \operatorname{Cos}. x)^2} = \frac{1}{4} \pi^2 \quad \text{V. T. 269. N°. 3.}$$

$$4) \int \operatorname{Arccot}. (Tang.^2 x) dx = \frac{1}{8} \pi^2 \quad \text{V. T. 269. N°. 4.}$$

$$5) \int \operatorname{Arccot}. (Tang.^3 x) dx = \frac{1}{8} \pi^2 \quad \text{V. T. 269. N°. 5.}$$

$$6) \int \operatorname{Arccot}. (\sqrt{Tang.} x) \frac{dx}{(\operatorname{Sin}. x + \operatorname{Cos}. x)^2} = \frac{1}{4} \pi^2 \quad \text{V. T. 269. N°. 6.}$$

$$7) \int \operatorname{Arcsin}. (p \operatorname{Sin}. x) \operatorname{Cos}. x dx = \operatorname{Arcsin}. p + \frac{1}{p} \sqrt{1-p^2} - \frac{1}{p} \quad \text{V. T. 105. N°. 4.}$$

$$8) \int \operatorname{Arctang}. (p \operatorname{Cot}. x) \operatorname{Tang}. x dx = \frac{\pi}{2} l(1+p) \quad \text{V. T. 266. N°. 3.}$$

- 9) $\int \text{Arctang.}(p \text{ Tang. } x). \text{ Tang. } 2x dx = \frac{\pi}{4} l \frac{1+p^2}{(1+p)^2}$ V. T. 369. N°. 10, 11.
- 10) $\int \text{Arctang.}(p \text{ Cot. } x). \text{ Tang. } 2x dx = \frac{\pi}{4} l \frac{(1+p)^2}{1+p^2}$ V. T. 368. N°. 8 et T. 369. N° 9.
- 11) $\int \text{Arctang.} \{ \text{Tang. } \lambda. \sqrt{1-p^2 \text{ Sin.}^2 x} \} dx \sqrt{1-p^2 \text{ Sin.}^2 x} = \frac{\pi}{2} E(p, \lambda) -$
 $-\frac{\pi}{2} \text{Cot. } \lambda. \{ 1 - \sqrt{1-p^2 \text{ Sin.}^2 \lambda} \}$ Roberts, L. 11. 157.
- 12) $\int \text{Arccot.}(p \text{ Tang. } x). \text{ Tang. } x dx = \frac{\pi}{2} l \frac{1+p}{p}$ V. T. 265. N°. 12.
- 13) $\int \text{Arccot.} \{ \text{Tang. } \lambda. \sqrt{1-p^2 \text{ Sin.}^2 x} \} dx \sqrt{1-p^2 \text{ Sin.}^2 x} = \frac{\pi}{2} E(p, \varphi) -$
 $-\frac{\pi \text{Cot. } \lambda}{2 \sqrt{1-p^2}} \{ \sqrt{1-p^2 \text{ Sin.}^2 \varphi} - \sqrt{1-p^2} \}, \text{Cot. } \varphi = \text{Tang. } \lambda. \sqrt{1-p^2}, p < 1;$ Roberts, L. 11. 157.

- 1) $\int \text{Arctang.} \frac{p \text{ Sin.}(r \text{ Tang. } x)}{1+p \text{ Cos.}(r \text{ Tang. } x)}. \text{ Tang. } x dx = \frac{1}{2} \pi l (1+p e^{-r})$ V. T. 431. N°. 7.
- 2) $\int \text{Arctang.}(\text{Sin. } x) \frac{dx}{\text{Sin. } x} = \frac{1}{2} \pi l (1 + \sqrt{2})$ V. T. 261. N°. 14.
- 3) $\int \text{Arctang.}(p \text{ Sin. } x) \frac{dx}{\text{Sin. } x} = \frac{1}{2} \pi l \{ p + \sqrt{1+p^2} \}$, $p \geq 1$; Raabe, Int. 421.
- 4) $\int \text{Arctang.} \left(\frac{\text{Tang. } x}{a} \right). \text{Arctang.} \left(\frac{\text{Tang. } x}{b} \right) \frac{dx}{\text{Sin.}^2 x} = \frac{1}{2} \pi \left\{ \frac{1}{a} l \frac{a+b}{b} + \frac{1}{b} l \frac{a+b}{a} \right\}$ V. T. 264. N°. 14.
- 5) $\int \text{Arctang.}(\text{Cos. } x) \frac{dx}{\text{Cos. } x} = \frac{1}{2} \pi l (1 + \sqrt{2})$ V. T. 261. N°. 14.
- 6) $\int \text{Arctang.}(p \text{ Cos. } x) \frac{dx}{\text{Cos. } x} = \frac{1}{2} \pi l \{ p + \sqrt{1+p^2} \}$, $p \geq 1$; Raabe, Int. 421.
- 7) $\int \{ \text{Sin.}^2 x. \text{Arccot.}(\text{Sin. } x) - \text{Arctang.}(\text{Sin. } x) \} \frac{dx}{\text{Sin. } 2x} = \frac{1}{2} \pi l 2$ V. T. 258. N°. 28.
- 8) $\int \text{Arccot.}(a \text{ Tang. } x). \text{Arccot.}(b \text{ Tang. } x) \frac{dx}{\text{Cos.}^2 x} = \frac{\pi}{2} \left\{ \frac{1}{a} l \frac{a+b}{b} + \frac{1}{b} l \frac{a+b}{a} \right\}$ V. T. 264. N°. 14.

- 9) $\int \text{Arctang.}(p \text{ Cot. } x) \frac{\text{Tang. } x}{\text{Cos. } 2x} dx = -\frac{1}{4} \pi l(1+p^2) \quad \text{V. T. 265. N}^\circ. 13.$
 - 10) $\int \text{Arctang.}(p \text{ Tang. } x) \frac{dx}{\text{Tang. } x} = \frac{1}{2} \pi l(1+p) \quad \text{Mosta, Gr. 10. 449.}$
 - 11) $\int \text{Arctang.}(p \text{ Tang. } x) \frac{dx}{\text{Tang. } x. \text{Cos. } 2x} = \frac{1}{4} \pi l(1+p^2) \quad \text{V. T. 265. N}^\circ. 13.$
 - 12) $\int \text{Arctang.}(p \text{ Cot. } x) \frac{\text{Sin.}^3 x}{\text{Cos. } x. \text{Cos. } 2x} dx = -\frac{1}{8} \pi l\{(1+p^2)(1+p)^2\} \quad \text{V. T. 368. N}^\circ. 8 \text{ et T. 369. N}^\circ. 9.$
 - 13) $\int \text{Arctang.}(p \text{ Tang. } x) \frac{\text{Cos.}^3 x}{\text{Sin. } x. \text{Cos. } 2x} dx = \frac{1}{8} \pi l\{(1+p)^2(1+p^2)\} \quad \text{V. T. 369. N}^\circ. 10, 11.$
 - 14) $\int \text{Arctang.}\{ \text{Tang. } \lambda \vee (1-p^2 \text{ Sin.}^2 x) \} \frac{dx}{\vee (1-p^2 \text{ Sin.}^2 x)} = \frac{1}{2} \pi F(p, \lambda)$
 - 15) $\int \text{Arccot.}\{ \text{Tang. } \lambda \vee (1-p^2 \text{ Sin.}^2 x) \} \frac{dx}{\vee (1-p^2 \text{ Sin.}^2 x)} = \frac{1}{2} \pi F(p, q)$
 - 16) $\int \text{Arctang.}\{ \text{Tang. } \lambda \vee (1-p^2 \text{ Sin.}^2 x) \} \frac{dx}{\vee (1-p^2 \text{ Sin.}^2 x)^3} =$
 $= \frac{1}{2} \frac{\pi}{1-p^2} E(p, \lambda) - \frac{\pi}{2} \frac{\text{Tg. } \lambda}{1-p^2} \{ \vee (1-p^2 \text{ Sin.}^2 \lambda) - \vee (1-p^2) \}$
 - 17) $\int \text{Arccot.}\{ \text{Tang. } \lambda \vee (1-p^2 \text{ Sin.}^2 x) \} \frac{dx}{\vee (1-p^2 \text{ Sin.}^2 x)^3} =$
 $= \frac{1}{2} \frac{\pi}{1-p^2} E(p, q) - \frac{\pi}{2} \frac{\text{Tang. } \lambda}{\vee (1-p^2)} \{ 1 - \vee (1-p^2 \text{ Sin.}^2 q) \}$
- } $\text{Cot. } q = \text{Tg. } \lambda \vee (1-p^2)$
 Roberts, L. 11. 157.

- 1) $\int \text{Arctang.}(\text{Cos. } x) dx = 0 \quad \text{V. T. 245. N}^\circ. 12.$
- 2) $\int \text{Arctang.} \left\{ \frac{\text{Sin.}^2 x}{\vee (p^2 - 1)} \right\} dx = 4 \sum_0^{\infty} \frac{\{p - \vee (p^2 - 1)\}^{2n+1}}{(2n+1)^2}, p > 1; \quad \text{V. T. 246. N}^\circ. 18.$
- 3) $\int \text{Arctang.} \frac{p \text{ Sin. } x}{1-p \text{ Cos. } x} \cdot \text{Sin. } a x dx = \frac{\pi}{2a} p^a, p^2 \leq 1; \quad \text{Schlömlich, Beitr. II. § 1. — Bierens de Haan, Gr. 13. 193.}$

- 4) $\int \text{Arctang.} \frac{p \sin. x}{1 - p \cos. x} \cdot \sin. x dx = \frac{1}{2} p \pi$
- 5) $\int \text{Arctang.} \frac{p \sin. x}{1 - p \cos. x} \cdot \sin. ax \cdot \cos. x dx = \frac{1}{4} \pi \left(\frac{p^{a+1}}{a+1} + \frac{p^{a-1}}{a-1} \right)$
- 6) $\int \text{Arctang.} \frac{p \sin. x}{1 - p \cos. x} \cdot \cos. ax \cdot \sin. x dx = \frac{1}{4} \pi \left(\frac{p^{a+1}}{a+1} - \frac{p^{a-1}}{a-1} \right)$
- 7) $\int \text{Arctang.} \frac{2p \sin. x}{1 - p^2} \cdot \sin. 2ax dx = 0$
- 8) $\int \text{Arctang.} \frac{2p \sin. x}{1 - p^2} \cdot \sin. \{(2a-1)x\} dx = \frac{\pi}{2a-1} p^{2a-1}$
- 9) $\int \text{Arctang.} \frac{2p \sin. x}{1 - p^2} \cdot \sin. 2ax \cdot \cos. x dx = \frac{1}{2} \pi \left(\frac{p^{2a+1}}{2a+1} + \frac{p^{2a-1}}{2a-1} \right)$
- 10) $\int \text{Arctang.} \frac{2p \sin. x}{1 - p^2} \cdot \sin. \{(2a-1)x\} \cdot \cos. x dx = 0$
- 11) $\int \text{Arctang.} \frac{2p \sin. x}{1 - p^2} \cdot \cos. 2ax \cdot \sin. x dx = \frac{1}{2} \pi \left(\frac{p^{2a+1}}{2a+1} - \frac{p^{2a-1}}{2a-1} \right)$
- 12) $\int \text{Arctang.} \frac{2p \sin. x}{1 - p^2} \cdot \cos. \{(2a-1)x\} \cdot \sin. x dx = 0$
- 13) $\int \text{Arctang.} \frac{q \sin. 2x}{1 - q \cos. 2x} \cdot \sin. 2ax dx = \frac{\pi}{a} q^a$
- 14) $\int \text{Arctang.} \frac{q \sin. 2x}{1 - q \cos. 2x} \cdot \sin. \{(2a-1)x\} dx = 0$
- 15) $\int \text{Arctang.} \frac{q \sin. 2x}{1 - q \cos. 2x} \cdot \sin. 2ax \cdot \cos. x dx = 0$
- 16) $\int \text{Arctang.} \frac{q \sin. 2x}{1 - q \cos. 2x} \cdot \cos. \{(2a-1)x\} \cdot \sin. x dx = \frac{1}{4} \pi \left(\frac{1}{a} q^a + \frac{1}{a-1} q^{a-1} \right)$
- 17) $\int \text{Arctang.} \frac{q \sin. 2x}{1 - q \cos. 2x} \cdot \cos. 2ax \cdot \sin. x dx = 0$
- 18) $\int \text{Arctang.} \frac{q \sin. 2x}{1 - q \cos. 2x} \cdot \cos. \{(2a-1)x\} \cdot \sin. x dx = \frac{1}{4} \pi \left(\frac{1}{a} q^a - \frac{1}{a-1} q^{a-1} \right)$

, $p^2 < 1, 1 > q > 0$;

Bierens de Haan,
Gr. 13. 193.

$$19) \int (1 + 2a \cos x + a^2)^{\frac{1}{2}} (a^2 + 2ab \cos x + b^2)^{\frac{1}{2}} \sin \left\{ c \operatorname{Arccos} \frac{1 + a \cos x}{1 + 2a \cos x + a^2} \right\} \cdot \sin \left\{ g \operatorname{Arccos} \frac{a + b \cos x}{\sqrt{a^2 + 2ab \cos x + b^2}} \right\} dx =$$

$$= \frac{1}{2} \pi a g \sum_1 \binom{c}{n} \binom{g}{n} b^n$$

$$20) \int (1 + 2a \cos x + a^2)^{\frac{1}{2}} (a^2 + 2ab \cos x + b^2)^{\frac{1}{2}} \cos \left\{ c \operatorname{Arccos} \frac{1 + a \cos x}{1 + 2a \cos x + a^2} \right\} \cdot \cos \left\{ g \operatorname{Arccos} \frac{a + b \cos x}{\sqrt{a^2 + 2ab \cos x + b^2}} \right\} dx =$$

$$= \frac{1}{2} \pi a g \left\{ 2 + \sum_1 \binom{c}{n} \binom{g}{n} b^n \right\}$$

Sur les intégrales (19), (22) voyez Sinaasen Cr. 42. 222.

$$21) \int \operatorname{Arctang} \frac{p \sin x}{1 - p \cos x} \cdot \operatorname{Tang} \frac{1}{2} x dx = \pi l(1 + p), \quad p^2 \leq 1; \quad \text{Schlömleeh, Beitr. II. § 1.}$$

$$1) \int \operatorname{Arctang} \frac{p \sin x}{1 - p \cos x} \cdot \frac{dx}{\sin x} = \frac{1}{2} \pi l \frac{1 + p}{1 - p} \quad \left\{ \begin{array}{l} , p^2 \leq 1; \\ \text{Schlömleeh, Beitr. II. § 1.} \end{array} \right.$$

$$2) \int \operatorname{Arctang} \frac{p \sin x}{1 - p \cos x} \cdot \frac{dx}{\operatorname{Tang} \frac{1}{2} x} = -\pi l(1 - p)$$

$$3) \int \operatorname{Arctang} \frac{p \sin x}{1 - p \cos x} \cdot \frac{dx}{\operatorname{Tang} x} = -\frac{1}{2} \pi l(1 - p^2)$$

$$4) \int \operatorname{Arctang} \frac{p \sin x}{1 - p \cos x} \cdot \frac{\cos^2 x}{\sin x} dx = \frac{1}{2} \pi \left\{ l \frac{1 + p}{1 - p} - p \right\}$$

$$5) \int \operatorname{Arctang} \frac{2p \sin x}{1 - p^2} \cdot \frac{dx}{\sin x} = \pi l \frac{1 + p}{1 - p}$$

$$6) \int \operatorname{Arctang} \frac{2p \sin x}{1 - p^2} \cdot \frac{dx}{\operatorname{Tang} x} = 0$$

$$7) \int \operatorname{Arctang} \frac{2p \sin x \cos^2 x}{1 - p^2} \cdot \frac{dx}{\sin x} = \pi \left\{ l \frac{1 + p}{1 - p} - p \right\}$$

$$8) \int \operatorname{Arctang} \frac{q \sin 2x}{1 - q \cos 2x} \cdot \frac{dx}{\sin x} = 0$$

$$9) \int \operatorname{Arctang} \frac{q \sin 2x}{1 - q \cos 2x} \cdot \frac{dx}{\operatorname{Tang} x} = -\pi l(1 - q)$$

$$10) \int \operatorname{Arctang} \frac{q \sin 2x}{1 - q \cos 2x} \cdot \frac{\cos^2 x}{\sin x} dx = 0$$

, $p^2 < 1$, $1 > q \geq 0$;

Bierens de Haan, Gr. 13. 193.

$$\begin{aligned}
 11) \int \text{Arctang.} \frac{p \sin x}{1 + p \cos x} \cdot \frac{dx}{\text{Tang. } x} &= \frac{1}{2} \pi l (1 - p^2), p^2 < 1; \\
 12) &= \frac{1}{2} \pi l \frac{4p^2}{p^2 - 1}, p^2 > 1; \\
 13) \int \text{Arccot.} \frac{p + \cos x}{\sin x} \cdot \frac{dx}{\text{Tang. } x} &= \frac{1}{2} \pi l \frac{4}{1 - p^2}, p^2 < 1; \\
 14) &= \frac{1}{2} \pi l \frac{p^2 - 1}{p^2}, p^2 > 1; \\
 15) \int \text{Arccot.} \frac{p - \cos x}{\sin x} \cdot \frac{dx}{\text{Tang. } x} &= \frac{1}{2} \pi l \frac{1 - p^2}{4}, p^2 < 1; \\
 16) &= \frac{1}{2} \pi l \frac{p^2}{p^2 - 1}, p^2 > 1;
 \end{aligned}
 \left. \vphantom{\begin{aligned} 11) \\ 12) \\ 13) \\ 14) \\ 15) \\ 16) \end{aligned}} \right\} \begin{array}{l} \text{Ohm, Ausw. 18.} \\ \text{Pour les intégrales (13) et (15)} \\ \text{il trouve fautivement } \pi l \frac{2}{1 - p^2}. \end{array}$$

$$\begin{aligned}
 1) \int \text{Arctang.} \frac{p \sin x}{1 - p \cos x} \cdot \frac{\sin x}{1 - 2q \cos x + q^2} dx &= -\frac{\pi}{2q} l (1 - pq), p^2 \leq 1, q^2 \leq 1; \text{ Schlömilch, Beitr. II. § 2.} \\
 2) \int \text{Arctg.} \frac{a \sin x}{b + a \cos x} \cdot \frac{\sin x}{\sqrt{(1 + a^2 - 2a \cos x)}} dx &= \frac{1 + a a + b^2}{ab} \frac{a + b}{b - a} \frac{1 + b}{1 + b} F' \left\{ \frac{4ab}{(b-a)^2} \frac{2\sqrt{b}}{1+b} \right\} - \\
 &\quad - \frac{2}{b} E'(b) + \frac{1+b}{b} D \\
 \text{où } D &= \pi, \text{ pour } a < -b; \\
 3) &= \frac{1-b}{1+b} \frac{\pi}{2}, a = -b; \\
 4) &= 0, -b < a < b; \\
 5) &= \frac{1}{2} \pi, a = b; \\
 6) &= \pi, a > b; \\
 7) \int \text{Arctang.} \left\{ \frac{\text{Tang. } \lambda}{\sqrt{(1 - e^2 \sin^2 \lambda)}} \sqrt{(1 + p^2 - 2p \cos x)} \right\} \cdot \frac{dx}{\sqrt{(1 + p^2 - 2p \cos x)}} &= \pi F(p, \lambda)
 \end{aligned}
 \left. \vphantom{\begin{aligned} 1) \\ 2) \\ 3) \\ 4) \\ 5) \\ 6) \\ 7) \end{aligned}} \right\} \begin{array}{l} \text{Ramus,} \\ \text{Danske} \\ \text{Afh. 6.} \\ \text{265.} \end{array}$$

$$8) \int \text{Arctang.} \frac{b \cos. x}{\sqrt{(a - b^2 \cos.^2 x)}} \cdot \frac{\cos. x}{\sqrt{(a - b^2 \cos.^2 x)}} dx = \frac{\pi}{2b} l \frac{a}{a - b^2} \text{ Winckler, Cr. 45. 102.}$$

$$9) \int \cos. \left\{ c \text{ Arccos.} \frac{a + b \cos. x}{\sqrt{(a^2 + 2ab \cos. x + b^2)}} \right\} \frac{1 - a^g \cos. g x}{1 - 2a^g \cos. g x + a^{2g}} (a^2 + 2ab \cos. x + b^2)^{\frac{1}{2}c} dx =$$

$$= \frac{1}{2} \pi a^c \left\{ 2 + \sum_1^{\infty} \left(\frac{c}{ng} \right) b^{ng} \right\} \text{ Smaasen, Cr. 42. 222.}$$

$$\left. \begin{aligned} 1) \int \text{Arctang.} \frac{p \sin. x}{1 - p \cos. x} \cdot \sin. a x dx &= \frac{\pi}{a} p^a \\ 2) \int \text{Arctang.} \frac{p \sin. x}{1 - p \cos. x} \cdot \sin. a x \cdot \cos. x dx &= \frac{1}{2} \pi \left(\frac{p^{a+1}}{a+1} + \frac{p^{a-1}}{a-1} \right) \\ 3) \int \text{Arctang.} \frac{p \sin. x}{1 - p \cos. x} \cdot \cos. a x \cdot \sin. x dx &= \frac{1}{2} \pi \left(\frac{p^{a+1}}{a+1} - \frac{p^{a-1}}{a-1} \right) \\ 4) \int \text{Arctang.} \frac{p \sin. x}{1 - p \cos. x} \cdot \frac{dx}{\sin. x} &= \pi l \frac{1+p}{1-p} \\ 5) \int \text{Arctang.} \frac{p \sin. x}{1 - p \cos. x} \cdot \frac{dx}{\text{Tang.} x} &= -\pi l (1 - p^2) \\ 6) \int \text{Arctang.} \frac{p \sin. x}{1 - p \cos. x} \cdot \frac{\cos.^2 x}{\sin. x} dx &= \pi \left(l \frac{1+p}{1-p} - p \right) \end{aligned} \right\} , 1 > p \geq 0;$$

Bierens de Haan, Gr. 13. 193.

$$1) \int_0^{\infty} \text{Arctang.} \frac{a}{x} \cdot \sin. b x dx = \frac{\pi}{2b} (1 - e^{-ab}) \text{ Cauchy, P. 28. 147. I. § 5.}$$

$$2) \int_0^{\infty} \text{Arctang.} p x \cdot \sin. q x dx = \frac{\pi}{2q} e^{-q/p} \text{ Raabe, Int. 170.}$$

$$3) \int_0^{\infty} \cos.^{a+1} \left(\text{Arctang.} \frac{x}{q} \right) \cdot \sin. \left\{ (a+1) \text{Arctang.} \frac{x}{q} \right\} \cdot \sin. x dx = \frac{\pi q^a e^{-q}}{2 \Gamma(a+1)} \text{ V. T. 59. N°. 17.}$$

$$4) \int_0^{\infty} \cos.^{a+1} \left(\text{Arctang.} \frac{x}{q} \right) \cdot \cos. \left\{ (a+1) \text{Arctang.} \frac{x}{q} \right\} \cdot \cos. x dx = \frac{\pi q^a e^{-q}}{2 \Gamma(a+1)} \text{ V. T. 59. N°. 18.}$$

$$5) \int_0^{\frac{\pi}{4}} \text{Arctang.} \left(\frac{p \sqrt{\cos. 2x}}{\cos. x} \right) \cdot \frac{dx}{\cos. 2x} = \frac{1}{2} \pi l \{p + \sqrt{1+p^2}\} \quad \text{V. T. 262. N}^\circ. 2.$$

$$6) \int_0^{\frac{\pi}{4}} \text{Arcsin.} (\text{Tang.} x) \cdot \frac{dx}{\sin. 2x} = \frac{1}{4} \pi l 2 \quad \text{V. T. 257. N}^\circ. 1.$$

$$7) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \text{Arctg.} (a + b \text{Tg.} x) dx = -\pi \left\{ \frac{1}{2} \text{Arctg.} \frac{2ab}{1+a^2-b^2} - \text{Arctg.} \frac{\text{Tang.} \left(\frac{1}{2} \text{Arctang.} \frac{2ab}{1+a^2-b^2} \right)}{a \text{Tg.} \left(\frac{1}{2} \text{Arctg.} \frac{2ab}{1+a^2-b^2} \right) - b} \right\} \quad \text{V. T. 271. N}^\circ. 1.$$

$$1) \int_0^{\frac{\pi}{2}} E(\text{Co.} \lambda, \sin. x) dx = \frac{1}{2} \pi \text{Co.} \lambda \quad \text{Lobatschewsky, Mém. Kasan. 1835. 1.}$$

$$2) \int_0^{\frac{\pi}{2}} E(x) \frac{dx}{\sqrt{1-p^2 \sin.^2 x}} = \frac{1}{2} E'(p) \cdot F'(p) - \frac{1}{4} l(1-p^2) \quad \text{Roberts, L. 12. 449.}$$

$$3) \int_0^{\frac{\pi}{2}} E(p, \sin. x) \frac{\sin. x}{1-p^2 \sin.^2 x} dx = \frac{\pi}{2 \sqrt{1-p^2}} \quad \text{Roberts, L. 10. 453.}$$

$$4) \int_0^{\frac{\pi}{2}} F(p, x) \frac{\sin. x \cos. x}{1-p^2 \sin.^2 x} dx = \frac{-1}{4 p^2} l(1-p^2) \cdot F'(p)$$

$$5) \int_0^{\frac{\pi}{2}} F(1-p^2, x) \frac{\sin. x \cos. x}{\cos.^2 x + p \sin.^2 x} dx = \frac{1}{4(1-p)} l \frac{1}{(1+p) \sqrt{p}} \cdot F' \{ \sqrt{1-p^2} \}$$

$$6) \int_{\lambda}^{\mu} E(p, x) \frac{dx}{\sqrt{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)}} =$$

$$= \frac{1}{2 \cos. \lambda \sin. \mu} E'(p) \cdot F' \left\{ \sqrt{1 - \frac{\text{Tang.}^2 \lambda}{\text{Tang.}^2 \mu}} \right\} + \frac{p^2 \sin. \mu}{2 \cos. \lambda} F' \left\{ \sqrt{1 - \frac{\sin.^2 2 \lambda}{\sin.^2 2 \mu}} \right\}$$

$$7) \int_{\lambda}^{\mu} F(p, x) \frac{dx}{\sqrt{(\sin.^2 x - \sin.^2 \lambda)(\sin.^2 \mu - \sin.^2 x)}} =$$

$$= \frac{1}{2 \cos. \lambda \sin. \mu} F'(p) \cdot F' \{ \sqrt{1 - \text{Tang.}^2 \lambda \cdot \text{Co.}^2 \mu} \}$$

où dans 6) et 7) on a $p^2 = 1 - \text{Co.}^2 \lambda \cdot \text{Co.}^2 \mu$.

$$8) \int_0^{\frac{\pi}{2}} \gamma(p, x) \frac{dx}{\sqrt{(1-p^2 \sin^2 x)}} = \frac{1}{12} \pi F' \{ \sqrt{1-p^2} \} + \frac{1}{6} E'(p) \cdot \{ F'(p) \}^2 + \frac{1}{6} F'(p) \cdot E \frac{E}{\sqrt{1-p^2}}$$

$$9) \int_0^{\frac{\pi}{2}} \gamma(p, x) \frac{dx}{\sqrt{(1-p^2 \sin^2 x)}} = \frac{1}{6} \pi F' \{ \sqrt{1-p^2} \} + \frac{4}{3} E'(p) \cdot \{ F'(p) \}^2 + \frac{1}{3} F'(p) \cdot E \frac{E}{\sqrt{1-p^2}}$$

Sur les formules (8) et (9) voyez: Roberts, L. 12. 449. où $p < 1$.

$$10) \int_0^1 B'(x) \cdot \sin. 2c\pi x dx = 0$$

$$11) \int_0^1 B''(x) \cdot \cos. 2c\pi x dx = 0$$

$$12) \int_0^1 B'(x) \cdot \cos. 2c\pi x dx = \frac{(-1)^a}{(2\pi)^{2a+2}} \frac{1^{2a+1/2}}{c^{2a+2}}$$

$$13) \int_0^1 B''(x) \cdot \sin. 2c\pi x dx = \frac{(-1)^{a-1}}{(2\pi)^{2a+1}} \frac{1^{2a+1}}{c^{2a+1}}$$

Raabe, Cr 42. 348.

$$14) \int_0^{\lambda} E'(\sin x) \frac{\text{Tang. } x}{\sqrt{(\sin^2 \lambda - \sin^2 x)}} dx = \frac{p\pi}{2\sqrt{1-p^2}} \quad \text{V. T. 272. N°. 3.}$$

PARTIE TROISIÈME.

- 1) $\int e^{-x}(1-x) l x d x = \frac{1-e}{e}$ V. T. 112. N°. 2.
- 2) $\int e^{ax}(ax+2) x l x d x = -\frac{1}{a^2} \{(a-1) e^a + 1\}$ V. T. 112. N°. 1.
- 3) $\int e^{-x^2}(x^2-1) x l x d x = \frac{e-1}{4e}$ V. T. 112. N°. 2.
- 4) $\int e^{-(1-x)^2}(2-x)(1-x) x l(1-x) d x = \frac{1-e}{4e}$ V. T. 376. N°. 3.
- 5) $\int e^{x-1} x l(1-x) d x = \frac{e-1}{e}$ V. T. 112. N°. 2.
- 6) $\int e^x \frac{x^2+x+2}{(x+1)^3} x l x d x = 1 - \frac{1}{2} e$ V. T. 112. N°. 5.

- 1) $\int e^{-x} x^{p-1} l x d x = \frac{d \Gamma(p)}{d p}$ Cauchy, P. 28. 147. P. 1. § 6. — Lejeune-Dirichlet, Cr. 15. 258. — Grunert, Gr. 2. 266. — Lobatschewsky, Mém. Kasan. 1835. 211.
- 2) $\int e^{-ax} x^{p-1} l \frac{1}{x} d x = \frac{\Gamma(p)}{a^p} \{la - Z'(p)\}$ Cauchy, P. 28. 147. I. § 6. — Schlömilch, Stud. I. 14.
- 3) $\int e^{-ax} x^b l x d x = \frac{1^{b+1}}{a^{b+1}} \left\{ -A - la + \sum_1 \frac{1}{n} \right\}$ Schlömilch, Gr. 4. 167.
- 4) $\int e^{-x}(x-p) x^{p-1} l x d x = -\Gamma(p)$ V. T. 113. N°. 3.
- 5) $\int e^{-x^{2p}}(2x^{2p}-1) x^{p-1} l x d x = \frac{1}{2p^2} 1 \angle \pi$ V. T. 115. N°. 5.
- 6) $\int e^{-px^2}(px^2-a) x^{2a-1} l x d x = \frac{1}{2(2p)^a} 1^{a-1} 1$ V. T. 114. N°. 9.
- 7) $\int e^{-x^2}(2x^2-2a-1) x^{2a} l x d x = \frac{1}{2} \Gamma\left(\frac{2a+1}{2}\right)$ V. T. 114. N°. 6.
- 8) $\int e^{-px^2}(2px^2-2a-1) x^{2a} l x d x = \frac{1}{2} \frac{1}{(2p)^a} 1^{a/2} 1 \angle \frac{\pi}{p}$ V. T. 114. N°. 8.

$$9) \int e^{-px} x^a l(q+x)^2 dx = \frac{1}{p^{a+1}} \left[1^{a+1} \{lq^2 - 2e^{pq} Ei.(-pq)\} + \right. \\ \left. + 2 \{1 + pq e^{pq} Ei.(-pq)\} 2^{a-1/1} \sum_0^{a-1} 2^{n/1} (-pq)^n + 2 \cdot 3^{a-2/1} \sum_0^{a-2} \left\{ \frac{(pq)^n}{3^{n/1}} \sum_0^n \frac{1^{m+1/1}}{(-pq)^m} \right\} \right]$$

$$10) \int e^{-px} x^a l(q-x)^2 dx = \frac{1}{p^{a+1}} \left[1^{a+1} \{lq^2 - 2e^{-pq} Ei.(pq)\} + \right. \\ \left. + 2 \{1 - pq e^{-pq} Ei.(pq)\} 2^{a-1/1} \sum_0^{a-1} 2^{n/1} (pq)^n + 2 \cdot 3^{a-2/1} \sum_0^{a-2} \left\{ \frac{(-pq)^n}{3^{n/1}} \sum_0^n \frac{1^{m+1/1}}{(pq)^m} \right\} \right]$$

$$11) \int e^{-px} x^{2a} l(q^2 - x^2)^2 dx = \frac{2}{p^{2a+1}} \left[1^{2a+1} lq^2 - 1^{2a/1} e^{pq} Ei.(-pq) \sum_0^{2a-1} \frac{(-pq)^n}{1^{n/1}} - 1^{2a+1} e^{-pq} Ei.(pq) \sum_0^{2a} \frac{(pq)^n}{1^{n/1}} + \right. \\ \left. + 2^{2a-1/1} \sum_1^a \left\{ \frac{1}{1^{2n/1}} \sum_0^{n-1} 1^{2n-2m/1} (p^2 q^2)^m \right\} + 3^{2a-2/1} \sum_1^a \left\{ \frac{1}{1^{2n-1/1}} \sum_0^{n-1} 1^{2n-2m-1/1} (p^2 q^2)^m \right\} \right]$$

$$12) \int e^{-px} x^{2a+1} l(q^2 - x^2)^2 dx = \frac{2}{p^{2a+2}} \left[1^{2a+1/1} lq^2 - 1^{2a+1/1} e^{pq} Ei.(-pq) \sum_0^{2a} \frac{(-pq)^n}{1^{n/1}} - 1^{2a+1/1} e^{-pq} Ei.(pq) \sum_0^{2a} \frac{(pq)^n}{1^{n/1}} + \right. \\ \left. + 2^{2a/1} \sum_1^{a+1} \left\{ \frac{1}{1^{2n+1/1}} \sum_0^{n-1} 1^{2n-2m+1/1} (p^2 q^2)^m \right\} + 3^{2a-1/1} \sum_1^a \left\{ \frac{1}{1^{2n/1}} \sum_0^{n-1} 1^{2n-2m-1/1} (p^2 q^2)^m \right\} \right]$$

$$13) \int e^{-px} x^{2a} l(q^2 + x^2)^2 dx = \frac{2}{p^{2a+1}} \left[1^{2a/1} lq^2 - 1^{2a/1} \{2Ci.(pq).Cos.pq + 2Si.(pq).Sin.pq - \pi Sin.pq\} \sum_0^a \frac{(-p^2 q^2)^n}{1^{2n/1}} + \right. \\ \left. + 1^{2a+1} \{2Ci.(pq).Sin.pq - 2Si.(pq).Cos.pq + \pi Cos.pq\} \sum_0^a \frac{(pq)^{2n-1}}{1^{2n-1/1}} + \right. \\ \left. + 2^{2a-1/1} \sum_1^a \left\{ \frac{1}{1^{2n/1}} \sum_0^{n-1} 1^{2n-2m-1} (-p^2 q^2)^m \right\} + 3^{2a-2/1} \sum_1^a \left\{ \frac{1}{1^{2n-1/1}} \sum_0^{n-1} 1^{2n-2m-1/1} (-p^2 q^2)^m \right\} \right]$$

$$14) \int e^{-px} x^{2a+1} l(q^2 + x^2)^2 dx = \frac{2}{p^{2a+2}} \left[1^{2a+1/1} lq^2 - 1^{2a+1/1} \{2Ci.(pq).Cos.pq + 2Si.(pq).Sin.pq - \pi Sin.pq\} \sum_0^a \frac{(-p^2 q^2)^n}{1^{2n/1}} + \right. \\ \left. + 1^{2a+1/1} \{2Ci.(pq).Sin.pq - 2Si.(pq).Cos.pq + \pi Cos.pq\} \sum_1^{a+1} \frac{(pq)^{2n-1}}{1^{2n-1/1}} + \right. \\ \left. + 2^{2a/1} \sum_1^{a+1} \left\{ \frac{1}{1^{2n+1/1}} \sum_0^{n-1} 1^{2n-2m+1/1} (-p^2 q^2)^m \right\} + 3^{2a-1/1} \sum_1^a \left\{ \frac{1}{1^{2n/1}} \sum_0^{n-1} 1^{2n-2m-1} (-p^2 q^2)^m \right\} \right]$$

$$15) \int e^{-px} x l(q^4 - x^4)^2 dx = 8 + 4lq^2 + (pq-1) 2e^{pq} Ei.(-pq) + (pq+1) 2e^{-pq} Ei.(pq) - \\ - 2pq \{2Ci.(pq).Sin.pq - 2Si.(pq).Cos.pq + \pi Cos.pq\} - 2\{Ci.(pq).Cos.pq + 2Si.(pq).Sin.pq - \pi Sin.pq\}$$

F. Algèbr. ent.
Expon. monôme.
Logar.

TABLE 577 suite.

Lim. 0 et ∞ .

$$16) \int e^{-px} x^2 l(q^4 - x^4)^2 dx = 24 + 8 l q^2 - (p^2 q^2 - 2 p q + 2) 2 e^{pq} Ei.(-pq) - \\ - (p^2 q^2 + 2 p q + 2) 2 e^{-pq} Ei.(pq) - 4 p q \{ 2 Ci.(pq).Sin.pq - 2 Si.(pq).Cos.pq + \pi Cos.pq \} + \\ + (p^2 q^2 - 2) 2 \{ 2 Ci.(pq).Cos.pq + 2 Si.(pq).Sin.pq - \pi Sin.pq \}$$

$$17) \int e^{-px} x^3 l(q^4 - x^4)^2 dx = 83 + 24 l q^2 + (p^3 q^3 - 3 p^2 q^2 + 6 p q - 6) 2 e^{pq} Ei.(-pq) - \\ - (p^3 q^3 + 3 p^2 q^2 + 6 p q + 6) 2 e^{-pq} Ei.(pq) + (p^2 q^2 - 6) 2 p q \{ 2 Ci.(pq).Sin.pq - 2 Si.(pq).Cos.pq + \pi Cos.pq \} - \\ + (p^2 q^2 - 6) 2 \{ 2 Ci.(pq).Cos.pq + 2 Si.(pq).Sin.pq - \pi Sin.pq \}$$

Sur les intégrales 9) à 17) voyez Bierens de Haan, Verh. K. Ak. v. Wet. 1854, bl. 19.

$$18) \int e^{-qx^a} (q x^a - p) x^{ap-1} l x dx = \frac{1}{a^2 q^p} \Gamma p \quad \text{V. T. 115. N° 9.}$$

F. Algèbr. fract. à dén. mon. et bin.
Expon. monôme.
Logar.

TABLE 578.

Lim. 0 et ∞ .

$$1) \int e^{-x} l x \frac{x+p-1}{x^p} dx = \frac{\pi}{\Gamma(p)} \text{Cosec.} p \pi, p < 1; \quad \text{V. T. 126. N° 8.}$$

$$2) \int e^{-x^2} l x \frac{dx}{x^2} = \infty \quad \text{V. T. 126. N° 3 et T. 273. N° 3.}$$

$$3) \int e^{-px} l x \frac{p x + a}{x^{a+1}} dx = p^a \Gamma(-a), a < 0; \quad \text{V. T. 126. N° 10.}$$

$$4) \int l(1 - e^{-2a\pi x}) \frac{dx}{1+x^2} = \pi \left\{ \frac{1}{2} l 2 a \pi - l \Gamma(a+1) + a(la-1) \right\} \quad \text{Schaar, Mém. Cour. Brux. T. 22, 23.}$$

$$5) \int e^{-px} l(q+x)^2 \frac{p(x+q)l(q+x)^2-4}{x+q} = (lq^2)^2$$

$$6) \int e^{-px} l(q-x)^2 \frac{p(x-q)l(q-x)^2-4}{x-q} = (lq^2)^2$$

$$7) \int e^{-px} l(q^2+x^2)^2 \frac{p(x^2+q^2)l(q^2+x^2)^2-8}{x^2+q^2} = (lq^4)^2$$

$$8) \int e^{-px} l(q^2-x^2)^2 \frac{p(x^2-q^2)l(q^2-x^2)^2-8}{x^2-q^2} = (lq^4)^2$$

Bierens de Haan, Verh. K. Ak. v. Wet. 1854, bl. 19.

- 9) $\int \left\{ \frac{e^{-x}}{x} - \frac{1}{(1+x)^2 l(1+x)} \right\} dx = 0$ Cauchy, C. R. 16. 422. — Schlömilch, Stud. I. 10. — Féaux, Funct. Transe.
- 10) $\int \left\{ (p-1) e^{-x} + \frac{(1+x)^{-p} - (1+x)^{-1}}{l(1+x)} \right\} \frac{dx}{x} = l \Gamma(p)$ Féaux, Funct. Transe.
- 11) $\left\{ e^{-x} + \frac{(1+x)^{-p-1}}{l(1+x)} \right\} \frac{dx}{x} = lp$ Stern, Gött. Stud. 1817.
- 12) $\int e^{-q^2 x^2 - \frac{1}{x^2}} l \frac{2q^3 x^3 + x^2 - 2}{x^3} dx = \frac{1}{2} e^{-2q} \sqrt{\pi}$ V. T. 126. N°. 16.

- 1) $\int e^{-px} l(q+x)^2 \frac{px + pq + a - 1}{(x+q)^a} dx = \frac{lq^2}{q^{a-1}} - 2 \frac{(-p)^{a-1}}{1^{a-1/1}} e^{pq} Ei.(-pq) +$
 $+ \frac{2}{1^{a-1/1} q^{a-1}} \sum_1^{a-1} 1^{a-n-1/1} (-pq)^{n-1}$
- 2) $\int e^{-px} l(q-x)^2 \frac{px - pq + a - 1}{(x-q)^a} dx = (-1)^{a-1} \left\{ \frac{lq^2}{q^{a-1}} - 2 \frac{p^{a-1}}{1^{a-1/1}} e^{-pq} Ei.(pq) + \right.$
 $\left. + \frac{2}{1^{a-1/1} q^{a-1}} \sum_1^{a-1} 1^{a-n-1/1} (pq)^{n-1} \right\}$
- 3) $\int e^{-px} l(q+x)^2 \frac{px + pq + 1}{(x+q)^2} dx = 2p e^{pq} Ei.(-pq) + \frac{1}{q} (2 + lq^2)$
- 4) $\int e^{-px} l(q-x)^2 \frac{px - pq + 1}{(x-q)^2} dx = 2p e^{-pq} Ei.(pq) - \frac{1}{q} (2 + lq^2)$
- 5) $\int e^{-px} l(q+x)^2 \frac{px - pq + 1}{(x-q)^2} dx = \frac{1}{q} \{ e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) - lq^2 \}$
- 6) $\int e^{-px} l(q-x)^2 \frac{px + pq + 1}{(x+q)^2} dx = \frac{1}{q} \{ e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) + lq^2 \}$
- 7) $\int e^{-px} l(q^2 - x^2)^2 \frac{px + pq + 1}{(x+q)^2} dx = \frac{1}{q} \{ (2pq + 1) e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) + 2lq^2 + 2 \}$
- 8) $\int e^{-px} l(q^2 - x^2)^2 \frac{px - pq + 1}{(x-q)^2} dx = \frac{1}{q} \{ e^{pq} Ei.(-pq) + (2pq - 1) e^{-pq} Ei.(pq) - 2lq^2 - 2 \}$

F. Algèbr. fract. à dén. puiss. de bin.

Expon. monôme.

TABLE 579 suite.

Lim. 0 et ∞ .

Logar.

- $$9) \int e^{-px} l(q+x)^2 \frac{px^2 - (pq + 2a-1)x + 2aq}{(x-q)^2} x^{2a-1} dx = q^{2a-1} \{e^{pq} Ei(-pq) - e^{-pq} Ei(pq)\} +$$
- $$+ \frac{2}{p^{2a-1}} \sum_1^a 1^{2a-2n+1} (p^2 q^2)^{n-1}$$
- $$10) \int e^{-px} l(q+x)^2 \frac{px^2 - (pq + 2a)x + (2a+1)q}{(x-q)^2} x^{2a} dx = -q^{2a} \{e^{pq} Ei(-pq) + e^{-pq} Ei(pq)\} +$$
- $$+ \frac{2}{p^{2a}} \sum_1^a 1^{2a-2n+1} (p^2 q^2)^{n-1}$$
- $$11) \int e^{-px} l(q-x)^2 \frac{px^2 + (pq - 2a+1)x - 2aq}{(x+q)^2} x^{2a-1} dx = q^{2a-1} \{e^{pq} Ei(-pq) - e^{-pq} Ei(pq)\} +$$
- $$+ \frac{2}{p^{2a-1}} \sum_1^a 1^{2a-2n+1} (p^2 q^2)^{n-1}$$
- $$12) \int e^{-px} l(q-x)^2 \frac{px^2 + (pq - 2a)x - (2a+1)q}{(x+q)^2} x^{2a} dx = -q^{2a} \{e^{pq} Ei(-pq) + e^{-pq} Ei(pq)\} +$$
- $$+ \frac{2}{p^{2a}} \sum_1^a 1^{2a-2n+1} (p^2 q^2)^{n-1}$$
- $$13) \int e^{-px} l(q+x)^2 \frac{px^2 + 2x - pq^2}{(x^2 - q^2)^2} dx = \frac{1}{2q^2} \{2 - 4lq^2 - (2pq-1)e^{pq} Ei(-pq) - e^{-pq} Ei(pq)\}$$
- $$14) \int e^{-px} l(q-x)^2 \frac{px^2 + 2x - pq^2}{(x^2 - q^2)^2} dx = \frac{1}{2q^2} \{2 - 4lq^2 - e^{pq} Ei(-pq) + (2pq+1)e^{-pq} Ei(pq)\}$$
- $$15) \int e^{-px} l(q^2-x^2)^2 \frac{px^2 + 2x - pq^2}{(x^2 - q^2)^2} dx = \frac{1}{q^2} \{2 - 4lq^2 - pq e^{pq} Ei(-pq) + pq e^{-pq} Ei(pq)\}$$

Sur toutes ces intégrales voyez: Bierens de Haan, Verh. K. Ak. v. Wet. 1854. bl. 19.

F. Algèbr. rat.

Expon. en dén. polynôme.

TABLE 580.

Lim. 0 et ∞ .

Logar.

- $$1) \int l_x \frac{(x-q)e^x - q}{(e^x + 1)^2} x^{q-1} dx = \Gamma(q) \sum_0^\infty \frac{(-1)^n}{(1+n)^q} \quad \text{V. T. 117. N. 16.}$$
- $$2) \int l_x \frac{(x-2a-1)e^x - (x+2a+1)}{(e^x + 1)^3} x^{2a} e^x dx = (2^{2a+1} - 1) \Gamma_{2a+1} \quad \text{V. T. 119. N. 9.}$$
- $$3) \int l_x \frac{(ax-p)(1+e^{-x}) - x e^{-x}}{(1+e^{-x})^2} e^{-ax} x^{p-1} dx = \Gamma(p) \sum_0^\infty \frac{(-1)^n}{(a+n)^p} \quad \text{V. T. 115. N. 6.}$$

- $$4) \int l x \frac{(\pi x - 2a) e^{\pi x} - 2a}{(e^{\pi x} + 1)^2} x^{2a-1} dx = \frac{2^{2a-1} - 1}{2a} B_{2a-1} \quad \text{V. T. 117. N}^\circ 21.$$
- $$5) \int l x \frac{(x-q) e^x + q}{(e^x - 1)^2} x^{q-1} dx = \Gamma(q) \sum_0^\infty \frac{1}{(1+n)^q} \quad \text{V. T. 117. N}^\circ 17.$$
- $$6) \int l x \frac{(x-2a-1)e^x + (x+2a+1)}{(e^x - 1)^3} x^{2a} e^x dx = 2^{2a-1} \pi^{2a} B_{2a-1} \quad \text{V. T. 119. N}^\circ 10.$$
- $$7) \int l x \frac{(ax-p)(e^x-1) + x e^x}{(e^x - 1)^2} e^{-ax} x^{p-1} dx = \Gamma(p) \sum_0^\infty \frac{1}{(a+n)^p} \quad \text{V. T. 118. N}^\circ 8.$$
- $$8) \int l x \frac{\pi x e^{\pi x} - 2a(e^{2\pi x} - 1)}{(e^{\pi x} - 1)^2} x^{2a-1} dx = \frac{2^{2a-2}}{a} B_{2a-1} \quad \text{V. T. 117. N}^\circ 22.$$
- $$9) \int l x \frac{a e^{2\pi x} + \pi x e^{\pi x} - a}{(e^{\pi x} - 1)^2} x^{2a-1} dx = \frac{2^{2a-2}}{a} B_{2a-1} \quad \text{V. T. 118. N}^\circ 15.$$
- $$10) \int l x \frac{(q+1)(e^x + e^{-x}) - x(e^x - e^{-x})}{(e^x + e^{-x})^2} x^q dx = \Gamma(q+1) \sum_0^\infty \frac{(-1)^{n+1}}{(2n+1)^{q+1}} \quad \text{V. T. 120. N}^\circ 11.$$
- $$11) \int l x \frac{(4a+2)(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}) - \pi x(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})}{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x})^2} x^{2a} dx = -B_{2a} \quad \text{V. T. 120. N}^\circ 14.$$
- $$12) \int l(1+x^2) \frac{e^{\pi x}(1+\pi x) + e^{-\pi x}(1-\pi x)}{(e^{\pi x} + e^{-\pi x})^2} \frac{dx}{x^2} = 2 - \frac{1}{2}\pi \quad \text{V. T. 138. N}^\circ 2.$$
- $$13) \int l(1+4x^2) \frac{e^{\pi x}(1+\pi x) + e^{-\pi x}(1-\pi x)}{(e^{\pi x} + e^{-\pi x})^2} \frac{dx}{x^2} = 2l2 \quad \text{V. T. 138. N}^\circ 3.$$
- $$14) \int l x \frac{e^x(x-2a-1) + e^{-x}(x+2a+1)}{(e^x - e^{-x})^2} x^{2a} dx = \frac{2^{2a+1} - 1}{2^{2a+1}} 1^{2a+1} \sum_1^\infty \frac{1}{n^{2a+1}} \quad \text{V. T. 120. N}^\circ 13.$$
- $$15) \int l x \frac{e^{\pi x}(\pi x - 2a) + e^{-\pi x}(\pi x + 2a)}{(e^{\pi x} - e^{-\pi x})^2} x^{2a-1} dx = \frac{2^{2a} - 1}{4a} B_{2a-1} \quad \text{V. T. 120. N}^\circ 18.$$
- $$16) \int l x \frac{e^{\frac{1}{2}\pi x}(\pi x - 4a) + e^{\frac{1}{2}\pi x}(\pi x + 4a)}{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})^2} x^{2a-1} dx = \frac{2^{2a} - 1}{a} 2^{2a} B_{2a-1} \quad \text{V. T. 120. N}^\circ 20.$$
- $$17) \int l x \frac{(2a+1)(e^x + e^{-x} + 1) - x(e^x - e^{-x})}{(e^x + e^{-x} + 1)^2} x^{2a} dx = \frac{(-1)^a}{1/3} (2\pi)^{2a+1} B''\left(\frac{1}{3}\right) \quad \text{V. T. 124. N}^\circ 11.$$
- $$18) \int l x \frac{x(e^x - e^{-x}) - 3(e^{\frac{1}{2}x} - e^{-\frac{1}{2}x})^2 - 12 \cos^2 \frac{1}{2} \lambda}{(e^x + e^{-x} + 2 \cos \lambda)^2} x^2 dx = \frac{\lambda}{2 \sin \lambda} \frac{\pi^2 - \lambda^2}{3} \quad \text{V. T. 124. N}^\circ 7.$$

F. Algèbr. rat.

Expon. en dén. polynôme.

TABLE 580 suite.

Lim. 0 et ∞ .

Logar.

$$19) \int l x \frac{q(e^x + e^{-x} + 2 \cos. \lambda) - x(e^x - e^{-x})}{(e^x + e^{-x} + 2 \cos. \lambda)^2} x^{q-1} dx = \frac{\Gamma(q)}{\sin. \lambda} \sum_1^{\infty} (-1)^n \frac{\sin. n \lambda}{n^q} \quad \text{V. T. 137. N}^\circ. 5.$$

$$20) \int l x \frac{x(e^x - e^{-x}) - 2(e^{1/2} x - e^{-1/2} x)^2}{(e^x + e^{-x} - 1)^2} x dx = \frac{4}{27} \pi^2 \quad \text{V. T. 124. N}^\circ. 1.$$

$$21) \int l x \frac{2a(e^x + e^{-x} - 1) - x(e^x - e^{-x})}{(e^x + e^{-x} - 1)^2} x^{2a-1} dx = \frac{(-1)^a}{\sqrt{3}} (2\pi)^{2a+1} B'' \left(\frac{1}{6} \right) \quad \text{V. T. 124. N}^\circ. 12.$$

$$22) \int l x \frac{(x-2)e^{2x} + 2}{\sqrt{(e^{2x} - 1)^3}} x dx = \frac{\pi}{2} l 2 \quad \text{V. T. 141. N}^\circ. 6.$$

$$23) \int l x \frac{2(x-1)e^x + (2-x)e^{-x}}{\sqrt{(e^{2x} - 1)^3}} x dx = 1 - l 2 \quad \text{V. T. 141. N}^\circ. 7.$$

$$24) \int \frac{e^{px} + e^{-px}}{e^{\pi x} + e^{-\pi x}} \frac{l x}{x^q} dx = Z'(1-q) \Gamma(1-q) \sum_0^{\infty} (-1)^n \left[\frac{1}{\{(2n+1)\pi-p\}^{1-q}} + \frac{1}{\{(2n+1)\pi+p\}^{1-q}} \right] -$$

$$- \Gamma(1-q) \sum_0^{\infty} (-1)^n \left[\frac{l \{(2n+1)\pi-p\}}{\{(2n+1)\pi-p\}^{1-q}} + \frac{l \{(2n+1)\pi+p\}}{\{(2n+1)\pi+p\}^{1-q}} \right]$$

$$25) \int \frac{e^{px} - e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{l x}{x^q} dx = Z'(1-q) \Gamma(1-q) \sum_0^{\infty} \left[\frac{1}{\{(2n+1)\pi-p\}^{1-q}} - \frac{1}{\{(2n+1)\pi+p\}^{1-q}} \right] -$$

$$- \Gamma(1-q) \sum_0^{\infty} \left[\frac{l \{(2n+1)\pi-p\}}{\{(2n+1)\pi-p\}^{1-q}} - \frac{l \{(2n+1)\pi+p\}}{\{(2n+1)\pi+p\}^{1-q}} \right]$$

Malmsten,
Cr. 38. 1.

F. Algèbr. irrat.

Expon.

TABLE 581.

Lim. 0 et ∞ .

Logar.

$$1) \int e^{-x} l x dx \vee x = \left(1 - l 2 - \frac{1}{2} \Lambda \right) \vee \pi \quad \text{V. T. 140. N}^\circ. 1. \text{ et T. 381. N}^\circ. 6.$$

$$2) \int e^{-qx} l x dx \vee x = \frac{1}{2q} (2 - l q - 2 l 2 - \Lambda) \vee \frac{\pi}{q} \quad \text{V. T. 140. N}^\circ. 2. \text{ et T. 381. N}^\circ. 6.$$

$$3) \int e^{-qx} \left(qx - a - \frac{1}{2} \right) x^{a-1} dx l x = \frac{1^{a/2}}{(2q)^a} \vee \frac{\pi}{q} \quad \text{V. T. 140. N}^\circ. 4.$$

$$4) \int e^{-px} (2px - 3) l x dx \vee x = \frac{1}{p} \vee \frac{\pi}{p} \quad \text{V. T. 139. N}^\circ. 2.$$

$$5) \int e^{-\left(px + \frac{q}{x}\right)} \{2px^2 - (2c+1)x - 2q\} x^{c-1} dx l x = 2 \left(\frac{q}{p} \right)^{1/2} e^{-2\sqrt{pq}} \vee \frac{\pi}{p} \sum_0^{\infty} \frac{(c-n+1)^{2n/1}}{2^{n/2} (2 \vee pq)^n} \quad \text{V. T. 139. N}^\circ. 8.$$

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WIS- EN NATUURK. VERH. DER KONINKL. AKADEMIE. DEEL IV.

- 6) $\int e^{-qx} l x \frac{dx}{\sqrt{x}} = -(lq + 2l2 + A) \sqrt{\frac{\pi}{q}}$ Malmsten, Cr. 38. 1. — Schlömilch, Gr. 4. 167.
- 7) $\int e^{-q^2(x+\frac{1}{x})} l x \frac{2a^2x^2 - 3x - 2a^2}{\sqrt{x}} dx = \frac{2a^2 + 1}{a^3} e^{-2a^2} \sqrt{\pi}$ V. T. 139. N°. 6.
- 8) $\int e^{-(px+\frac{q}{x})} l x \frac{2px^2 - x - 2q}{x \sqrt{x}} dx = 2e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{p}}$ V. T. 140. N°. 9.
- 9) $\int e^{-\frac{1+x^2}{2qx}} l x \frac{1+qx-x^2}{x \sqrt{x}} dx = -\frac{\sqrt{2q\pi}}{\sqrt{e}} 2q$ V. T. 140. N°. 5.
- 10) $\int e^{-\frac{1+x^2}{2qx}} l x \frac{x^2+qx-1}{x^2 \sqrt{x}} dx = \frac{2q}{\sqrt{e}} \sqrt{2q\pi}$ V. T. 140. N°. 7.
- 11) $\int e^{-\frac{1+x^2}{2qx}} l x \frac{x^2+3qx-1}{x^3 \sqrt{x}} dx = \frac{1+q}{\sqrt{e}} 2q \sqrt{2q\pi}$ V. T. 140. N°. 8.
- 12) $\int e^{-px-\frac{q}{x}} l x \frac{2px^2+(2a-1)x-2q}{x^{a+1}} dx = 2\left(\frac{p}{q}\right)^{\frac{1}{2}} \sqrt{\frac{\pi}{p}} e^{-2\sqrt{pq}} \sum_0^{\infty} \frac{(a-n)^{2n/1}}{2^{n/2} (2\sqrt{pq})^n}$ V. T. 140. N°. 12.
- 13) $\int l x \frac{(2x-3)e^x - (2x+3)e^{-x}}{(e^x + e^{-x})^2} dx \sqrt{x} = \sqrt{\pi} \sum_0^{\infty} (-1)^n \frac{1}{\sqrt{(2n+1)^3}}$ V. T. 139. N°. 11.
- 14) $\int \frac{l x}{e^x + e^{-x}} \frac{dx}{\sqrt{x}} = \sqrt{\pi} \sum_0^{\infty} \left\{ (-1)^{n+1} \frac{l(2n+1) + 2l2 + A}{\sqrt{(2n+1)}} \right\}$
- 15) $\int \frac{l x}{e^x + 1 + e^{-x}} \frac{dx}{\sqrt{x}} = \text{Cosec.} \frac{1}{3} \pi. \sqrt{\pi} \sum_1^{\infty} \left\{ (-1)^n \text{Sin.} \frac{1}{3} n \pi \frac{l n + 2l2 + A}{\sqrt{n}} \right\}$
- 16) $\int l x \frac{(2x-1)e^x - (2x+1)e^{-x}}{(e^x + e^{-x})^2} \frac{dx}{\sqrt{x}} = 2 \sqrt{\pi} \sum_0^{\infty} \frac{(-1)^n}{\sqrt{(2n+1)}}$ V. T. 140. N°. 19.
- 17) $\int l x \frac{(2x-1)e^x - (2x+1)e^{-x} - 1}{(e^x + 1 + e^{-x})^2} \frac{dx}{\sqrt{x}} = 2 \text{Cosec.} \frac{1}{3} \pi. \sqrt{\pi} \sum_1^{\infty} (-1)^{n-1} \frac{\text{Sin.} \frac{1}{3} n \pi}{\sqrt{n}}$ V. T. 140. N°. 20.

- 1) $\int e^{-x^2} (x^2 - a) x^{2a-1} l x dx = 0$ V. T. 142. N°. 9.
- 2) $\int e^{-x^2} (2x^2 - 2a - 1) x^{2a} l x dx = \frac{1a^2}{2a} \sqrt{\pi}$ V. T. 142. N°. 8.

- 3) $\int e^{-px^2 - \frac{q}{x^2}} l x \frac{2px^4 + x^2 - 2q}{x^4} dx = e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{q}}$ V. T. 146. N°. 2.
- 4) $\int e^{-px^2 - \frac{q}{x^2}} l x \frac{2px^4 + (2a-1)x^2 - 2q}{x^{2a+2}} dx = \left(\frac{p}{q}\right)^{1/4} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{p}} \sum_{n=0}^{\infty} \frac{(a-n)^{2n/4}}{n!^{1/4} (4\sqrt{pq})^n}$ V. T. 146. N°. 3.
- 5) $\int e^{bxi} (-xi)^{a-1} l \left(1 + \frac{ci}{x}\right) \frac{dx}{p-xi} = 0$
- 6) $\int e^{bxi} (-xi)^{a-1} l \left(1 + \frac{ci}{x}\right) \frac{dx}{p+xi} = 2\pi p^{a-1} e^{-bp} l \left(1 + \frac{c}{p}\right)$
- 7) $\int e^{pxi} l(q+xi) \frac{dx}{(q+xi)^a} = \frac{2\pi}{1^{a/4}} p^{a-1} e^{-pq} \{Z'(a) - lp\}$
- 8) $\int e^{pxi} l(q-xi) \frac{dx}{(q-xi)^a} = 0$
- 9) $\int \frac{1 - e^{pxi}}{l(q-xi)} \frac{dx}{x} = \frac{2\pi i}{1-q} \{1 - e^{p(q-1)}\}, q < 1;$
- 10) $= 0, q > 1;$
- 11) $= \pi p i, q = 1;$
- 12) $\int \frac{e^{-bxi}}{l(1+xi)} \frac{dx}{c^2 + x^2} = \frac{\pi e^{-bc}}{cl(1+c)} - \frac{1}{c}$
- 13) $\int \frac{e^{-bxi}}{l(1+xi)} (xi)^a \frac{dx}{c^2 + x^2} = \frac{\pi c^{a-1} e^{-bc}}{l(1+c)}$
- 14) $\int \frac{e^{axi}}{l(1-pxi)} (-xi) \frac{dx}{1+x^2} = \frac{\pi e^{-a}}{l(1+p)}$ Cauchy, Cours. Lec. 39.
- 15) $\int \frac{e^{-axi}}{\{l(h+xi)\}^m} \frac{1}{(k+xi)^p (l+xi)^q \dots} \frac{dx}{b^2 + x^2} = \frac{\pi}{b} e^{-ab} \frac{1}{(b+k)^p (b+l)^q \dots} \frac{1}{\{l(b+h)\}^m}$
- 16) $\int \frac{e^{-axi}}{\{l(h+xi)\}^m \{l(g+xi)\}^n \dots} \frac{1}{(k+xi)^p (l+xi)^q \dots} \frac{dx}{b^2 + x^2} = \frac{\pi}{b} e^{-ab} \frac{1}{(b+k)^p (b+l)^q \dots} \frac{1}{\{l(b+h)\}^m \{l(b+g)\}^n \dots}$
- Les intégrales (15) (16) se trouvent: Lejeune-Dirichlet, Cr. 4. 94. — Schlömilch, Stud. II. 17.
- 17) $\int e^{-px^2 + qx} (2px^2 + qx - a - 1) x^a l x dx = \left(\frac{-q}{2p}\right)^a \frac{q^2}{e^{4p}} \sqrt{\frac{\pi}{p}} \sum_{n=0}^{\infty} \frac{a^{2n/4-1}}{n!^{1/4}} \left(\frac{p}{q^2}\right)^n$ V. T. 142. N°. 14.
- 18) $\int e^{-px^2 + 2qx} (px^2 - qx - 1) x l x dx = \frac{q}{2p} e^{\frac{q^2}{p}} \sqrt{\frac{\pi}{p}}$ V. T. 142. N°. 13.

F. Algébr.
Expon.
Logar.

TABLE 585.

Lim. diverses.

$$1) \int_0^l \frac{e^{-x} x}{\sqrt{b^2 - a^2 e^x}} \frac{dx}{(1 - e^{-x})^2} l \frac{b^2 - a^2 e^x}{e^x - 1} = -\frac{4\pi}{a+b} + \frac{4\pi}{a^2 - b^2} l \frac{a^2}{b^2} \quad \text{V. T. 45. N}^\circ. 1.$$

$$2) \int_1^\infty \frac{e^{-2ax} l x}{2x-1} \{a(2x-1)l(2x-1)-1\} dx = \frac{1}{4} \{li.(e^{-a})\}^2 \quad \text{V. T. 383. N}^\circ. 3.$$

$$3) \int_1^\infty e^{-2ax} l(2x-1) \frac{dx}{x} = \frac{1}{2} \{li.(e^{-a})\}^2 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Winckler, Cr. 45. 102.}$$

$$4) \int_{2a}^\infty e^{-x} l \frac{x-a}{a} \frac{dx}{x} = \frac{1}{2} \{li.(e^{-a})\}^2$$

F. Algébr. rat. ent.
Expon.
Circ. Dir.

TABLE 584.

Lim. 0 et $\frac{\pi}{2}$.

$$1) \int e^{-q \text{Tang.} x} \frac{x}{\text{Cos.}^2 x} dx = \frac{1}{q} \left\{ \text{Ci.}(q). \text{Sin.} q + \text{Cos.} q. \left(\frac{\pi}{2} - \text{Si.}(q) \right) \right\} \quad \text{V. T. 288. N}^\circ. 4.$$

$$2) \int e^{-\text{Tang.}^2 x} \frac{x \text{Sin.}^3 2x}{\text{Cos.}^3 x} dx = 2 \sqrt{\pi} \quad \text{V. T. 290. N}^\circ. 3.$$

$$3) \int e^{-\text{Tang.}^2 x} \frac{x \text{Sin.}^4 x}{\text{Cos.}^3 x} dx = \frac{3}{2} \sqrt{\pi} \quad \text{V. T. 290. N}^\circ. 8.$$

$$4) \int e^{-q \text{Cot.} x} \frac{\text{Sin.} x + \text{Cos.} x}{\text{Sin.}^3 x} x dx = \text{Sin.} q. \left\{ \frac{\pi}{2} - \text{Si.}(q) \right\} - \text{Ci.}(q). \text{Sin.} q \quad \text{V. T. 290. N}^\circ. 10.$$

$$5) \int \frac{e^{\pi \text{Tang.} x} - e^{-\pi \text{Tang.} x}}{(e^{\pi \text{Tang.} x} + e^{-\pi \text{Tang.} x})^2} \frac{x}{\text{Cos.}^2 x} dx = \frac{4-\pi}{4\pi} \quad \text{V. T. 292. N}^\circ. 3.$$

$$6) \int \frac{e^{\frac{1}{2}\pi \text{Tang.} x} - e^{-\frac{1}{2}\pi \text{Tang.} x}}{(e^{\frac{1}{2}\pi \text{Tang.} x} + e^{-\frac{1}{2}\pi \text{Tang.} x})^2} \frac{x}{\text{Cos.}^2 x} dx = \frac{\sqrt{2}}{\pi} \left\{ \pi + l \frac{\sqrt{2}-1}{\sqrt{2}+1} \right\} \quad \text{V. T. 292. N}^\circ. 1.$$

$$7) \int \frac{e^{\frac{1}{2}\pi \text{Tang.} x} - e^{-\frac{1}{2}\pi \text{Tang.} x}}{(e^{\frac{1}{2}\pi \text{Tang.} x} + e^{-\frac{1}{2}\pi \text{Tang.} x})^2} \frac{x}{\text{Cos.}^2 x} dx = \frac{1}{\pi} l 2 \quad \text{V. T. 292. N}^\circ. 2.$$

$$8) \int \frac{4(e^{\frac{1}{2}\pi \text{Tg.} x} - e^{-\frac{1}{2}\pi \text{Tg.} x}) - \pi \text{Tg.} x (e^{\frac{1}{2}\pi \text{Tg.} x} + e^{-\frac{1}{2}\pi \text{Tg.} x})}{(e^{\frac{1}{2}\pi \text{Tang.} x} - e^{-\frac{1}{2}\pi \text{Tang.} x})^2} \frac{x}{\text{Cos.}^2 x} dx = -\pi \frac{1}{2} + 4 + \frac{1}{2} l \frac{\sqrt{2}-1}{\sqrt{2}+1} \quad \text{V. T. 292. N}^\circ. 5.$$

$$9) \int \frac{2(e^{\frac{1}{2}\pi \text{Tang.} x} - e^{-\frac{1}{2}\pi \text{Tang.} x}) - \pi \text{Tang.} x (e^{\frac{1}{2}\pi \text{Tang.} x} + e^{-\frac{1}{2}\pi \text{Tang.} x})}{(e^{\frac{1}{2}\pi \text{Tang.} x} - e^{-\frac{1}{2}\pi \text{Tang.} x})^2} \frac{x}{\text{Cos.}^2 x} dx = \frac{2-\pi}{2} \quad \text{V. T. 292. N}^\circ. 6.$$

$$10) \int \frac{(e^{\pi \text{Tang.} x} - e^{-\pi \text{Tang.} x}) - \pi \text{Tang.} x (e^{\pi \text{Tang.} x} + e^{-\pi \text{Tang.} x})}{(e^{\pi \text{Tang.} x} - e^{-\pi \text{Tang.} x})^2} \frac{x}{\text{Cos.}^2 x} dx = \frac{1}{2} \left(l 2 - \frac{1}{2} \right) \quad \text{V. T. 292. N}^\circ. 10.$$

1) $\int e^{-qx} \sin. qx. x dx = \frac{1}{2q^2}$	}	Oettinger, Cr. 38. 216, où les intégrales 4), 5) sont fautives.	
2) $\int e^{-qx} \sin. qx. x^2 dx = \frac{1}{2q^3}$			
3) $\int e^{-qx} \sin. qx. x^3 dx = 0$			
4) $\int e^{-qx} \sin. qx. x^4 dx = -\frac{3}{q^5}$			
5) $\int e^{-qx} \sin. qx. x^5 dx = -\frac{15}{q^6}$			
6) $\int e^{-px} \sin. qx. x dx = \frac{2pq}{(p^2 + q^2)^2}$			Sohnke, Samml.
7) $\int e^{-px} \sin. qx. x^2 dx = 2 \frac{3p^2q - q^3}{(p^2 + q^2)^3}$			
8) $\int e^{-px} \sin. qx. x^3 dx = 24pq \frac{p^2 - q^2}{(p^2 + q^2)^4}$			
9) $\int e^{-px} \sin. qx. x^4 dx = 24 \frac{5p^4q - 10p^2q^3 + q^5}{(p^2 + q^2)^5}$			
10) $\int e^{-qx} \cos. qx. x dx = 0$	Poisson, Chal. 1. 159. — Oettinger, Cr. 38. 216.		
11) $\int e^{-qx} \cos. qx. x^2 dx = -\frac{1}{2q^3}$	}	Oettinger, Cr. 38. 216.	
12) $\int e^{-qx} \cos. qx. x^3 dx = -\frac{3}{2q^4}$			
13) $\int e^{-qx} \cos. qx. x^4 dx = -\frac{3}{q^5}$			
14) $\int e^{-px} \cos. qx. x dx = \frac{p^2 - q^2}{(p^2 + q^2)^2}$			
15) $\int e^{-px} \cos. qx. x^2 dx = 2 \frac{p^3 - 3pq^2}{(p^2 + q^2)^3}$			
16) $\int e^{-px} \cos. qx. x^3 dx = 6 \frac{p^4 - 6p^2q^2 + q^4}{(p^2 + q^2)^4}$			

F. Algèbr. rat. ent. x^a pour a spécial.

Expon. $e^{\pm p x}$.

TABLE 585 suite.

Lim. 0 et ∞ .

Circ. Dir. monôme.

$$17) \int e^{-p x} \cos. q x. x^3 dx = 6 \frac{p^4 - 5 q^4}{(p^2 + q^2)^4} \text{ (fautive) Sohuke, Samml.}$$

$$18) \int e^{-p x} \cos. q x. x^4 dx = 24 p \frac{p^4 - 10 p^2 q^2 + 5 q^4}{(p^2 + q^2)^5} \text{ Sohuke, Samml. où elle est fautive.}$$

F. Alg. rat. ent. x^a pour a général.

Expon. $e^{\pm p x}$.

TABLE 586.

Lim. 0 et ∞ .

Circ. Dir. monôme.

$$\left. \begin{aligned} 1) \int e^{-x} \sin. x. x^{p-1} dx &= \frac{1}{2^{1/p}} \sin. \frac{1}{4} p \pi. \Gamma(p) \\ 2) \int e^{-x} \cos. x. x^{p-1} dx &= \frac{1}{2^{1/p}} \cos. \frac{1}{4} p \pi. \Gamma(p) \end{aligned} \right\} \text{ Cauchy, Sav. Etr. 1827. 599. P. 1. § 3.}$$

$$\left. \begin{aligned} 3) \int e^{-x} \sin. (x \text{ Tang. } \lambda). x^{p-1} dx &= \Gamma(p) \cos. p \lambda. \sin. p \lambda \\ 4) \int e^{-x} \cos. (x \text{ Tang. } \lambda). x^{p-1} dx &= \Gamma(p) \cos. p \lambda. \cos. p \lambda \end{aligned} \right\} \text{ Kummer, Cr. 17. 210.}$$

$$\left. \begin{aligned} 5) \int e^{-x} \sin. \left(p x + \frac{1}{2} a \pi \right). x^a dx &= \frac{d^a}{d p^a} \frac{p}{1 + p^2} \\ 6) \int e^{-x} \cos. \left(p x + \frac{1}{2} a \pi \right). x^a dx &= \frac{d^a}{d p^a} \frac{1}{1 + p^2} \end{aligned} \right\} \text{ Dienger, Cr. 46. 119.}$$

$$\left. \begin{aligned} 7) \int e^{-a x} \cos. x. x^{p-1} dx &= \frac{\Gamma(p)}{(1 + a^2)^{1/p}} \sin. (p \text{ Arccot. } a) \\ 8) \int e^{-a x} \cos. x. x^{p-1} dx &= \frac{\Gamma(p)}{(1 + a^2)^{1/p}} \cos. (p \text{ Arccot. } a) \end{aligned} \right\} \text{ Boncompagni, Cr. 25. 74.}$$

$$9) \int e^{-k x} \sin. a x. x^{p-1} dx = \frac{\Gamma(p)}{a^p} \sin. \frac{1}{2} p \pi \left\{ \text{, pour } k \text{ très-petit:} \right.$$

$$10) \int e^{-k x} \cos. a x. x^{p-1} dx = \frac{\Gamma(p)}{a^p} \cos. \frac{1}{2} p \pi \left\{ \text{ Cauchy, P. 19. 511.} \right.$$

$$11) \int e^{-p x} \sin. q x. x^a dx = \frac{1^{a/1}}{p^{a+1}} \sum_0^{\infty} (-1)^n \frac{(a+1)^{2n/1}}{1^{2n+1/1}} \left(\frac{q}{p} \right)^{2n+1}, a \leq 1; \text{ Oettinger, Cr. 38. 216. où elle est fautive.}$$

F. Alg. rat. ent. x^a pour a général.

Expon. $e^{\pm px}$.

TABLE 586 suite.

Lim. 0 et ∞ .

Circ. Dir. monôme.

- 12) $\int e^{-px} \sin qx \cdot x^{a-1} dx = \frac{1^{a-1/1}}{(p^2 + q^2)^{1/2}} \sin \left(a \operatorname{Arctang} \frac{q}{p} \right)$ Euler, Calc. Int. 4. S. 5. 134. — Lacroix, Calc. Diff. T. 3. p. 490 (démonstration de Poisson.) — Legendre, Exerc. P. 3. 54. — Cauchy, P. 28. 147. P. 1. § 2. — Id., Cours. Leg. 32. — Id., Exerc. 1826. p. 58. —
- 13) $\int e^{-px} \cos qx \cdot x^{a-1} dx = \frac{1^{a-1/1}}{(p^2 + q^2)^{1/2}} \cos \left(a \operatorname{Arctang} \frac{q}{p} \right)$ Fuss, Mém. Pétersb. 1830. — Plana, Mém. Brux. 1837. — Grunert, Cr. 8. 146 — Liouville, Cr. 13. 210. — Schlömilch, Gr. 6. 200. —
Chez Oettinger, Cr. 38. 216 et Schlömilch, Stud. I. 13 ces deux formules valent pour a aussi fractionnaire.
- 14) $\int e^{-px} \sin qx \cdot x^{r-1} dx = \frac{(p - qi)^{-r} - (p + qi)^{-r}}{2i} \Gamma(r)$ Cauchy, P. 19. 511. — Id., Sav. Etr. 1827. 124. Note 6. — Id., P. 28. 147. I. § 2. — Plana, Mém. Brux. 1837.
- 15) $= \frac{(p + qi)^r - (p - qi)^r}{2i(p^2 + q^2)^r} \Gamma(r)$ Boncompagni, Cr. 25. 74, où faut. $e^{-qx} \sin px$.
- 16) $= \frac{\Gamma(r)}{p^r} \cos r \left(\operatorname{Arctg} \frac{q}{p} \right) \cdot \sin \left(r \operatorname{Arctg} \frac{q}{p} \right)$ Serret, L. 8. 489. — Boncompagni, Cr. 25. 74.
- 17) $= \frac{\Gamma(r)}{(p^2 + q^2)^{1/2}} \sin \left\{ r \operatorname{Arcsin} \frac{q}{\sqrt{p^2 + q^2}} \right\}$ Cauchy, Sav. Etr. 1827. 599. P. 1. § 3. — Fuss, Mém. Pétersb. 1830.
- 18) $\int e^{-px} \cos qx \cdot x^{r-1} dx = \frac{(p - qi)^{-r} + (p + qi)^{-r}}{2} \Gamma(r)$ Cauchy, P. 19. 511. — Id., Sav. Etr. 1827. 124. Note 6. — Id., P. 28. 147. I. § 2.
- 19) $= \frac{(p + qi)^r + (p - qi)^r}{2(p^2 + q^2)^r} \Gamma(r)$ Boncompagni, Cr. 25. 74, où faut. $e^{-qx} \cos px$.
- 20) $= \frac{\Gamma(r)}{p^r} \cos r \left(\operatorname{Arctg} \frac{q}{p} \right) \cdot \cos \left(r \operatorname{Arctg} \frac{q}{p} \right)$ Serret, L. 8. 489. — Boncompagni, Cr. 25. 74.
- 21) $= \frac{\Gamma(r)}{(p^2 + q^2)^{1/2}} \cos \left\{ r \operatorname{Arcsin} \frac{q}{\sqrt{p^2 + q^2}} \right\}$ Cauchy, Sav. Etr. 1827. 599. P. 1. § 2. — Fuss, Mém. Pétersb. 1830.

F. Alg. rat. ent.

Expon. $e^{\pm ax}$.

TABLE 587.

Lim. 0 et ∞ .

Circ. Dir. polynôme.

- 1) $\int e^{-cx} (b \sin ax + a \cos ax) dx = a \frac{b + c}{a^2 + c^2}$ Poisson, Chal. 154.
- 2) $\int e^{-qx} \cos(2x^2 + qx) \cdot x dx = 0$
- 3) $\int e^{-qx} \{ \sin(2x^2 + qx) + \cos(2x^2 + qx) \} x^2 dx = 0$
- Poisson, Chal. Suppl. Note B.

F. Alg. rat. ent.

Expon. $e^{\pm ax}$.

Circ. Dir. polynôme.

TABLE 587 suite.

Lim. 0 et ∞ .

$$\begin{aligned}
 1) \int e^{-qx} \{ \text{Sin.}(2x^2 - qx) - \text{Cos.}(2x^2 - qx) \} x^2 dx &= \frac{1}{16} (2 - q^2) e^{-\frac{1}{2}q^2} \sqrt{\pi} \\
 5) \int e^{-qx} \text{Cos.}(2x^2 - qx) . x dx &= \frac{1}{8} q e^{-\frac{1}{2}q^2} \sqrt{\pi} \\
 6) \int e^{-qx} \{ \text{Cos.}bx - i \text{Sin.}bx \} x^c dx &= \frac{\Gamma(c+1)}{(a+bi)^{c+1}} \quad \text{Cauchy, Cours. Leç. 32. — Moigno, Calc. Int. 42. où fautivement } + i \text{Sin.}bx. \\
 7) \int e^{-\frac{1}{2}x} \text{Cos.} \left\{ \frac{1}{2} x \text{Tang.} \lambda + b \lambda \right\} . x^{l-1} dx &= 2^l \Gamma(q) \text{Cos.} q \lambda . \text{Cos.} \{ (b+q) \lambda \} \quad \text{Kummer, Cr. 17. 228.}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Poisson, Chal.} \\ \text{Suppl. Note B.} \end{array}$$

F. Alg. rat. ent.

Expon. e^{-x^2} .

Circ. Dir.

TABLE 588.

Lim. 0 et ∞ .

$$\begin{aligned}
 1) \int e^{-x^2} \text{Sin.} ax . x dx &= \frac{1}{4} a e^{-\frac{1}{4}a^2} \sqrt{\pi} \quad \text{Legendre, Exerc. 3. 48. — Dienger, Cr. 46. 119. — Svanberg, Transf. 4.} \\
 2) \int e^{-x^2} \text{Cos.} ax . x dx &= \frac{1}{2} - \frac{1}{4} a \sum_0^{\infty} (-1)^n \frac{a^{2n+1}}{(n+1)^{n+1/2}} \quad \text{Legendre, Exerc. 3. 49.} \\
 3) \int e^{-x^2} \text{Sin.} 2x . x dx &= \frac{1}{2e} \sqrt{\pi} \\
 4) \int e^{-x^2} \text{Cos.} 2x . x^2 dx &= -\frac{1}{4e} \sqrt{\pi} \\
 5) \int e^{-x^2} \text{Cos.} 2px . x^2 dx &= \frac{1-2p^2}{4} e^{-p^2} \sqrt{\pi} \\
 6) \int e^{-x^2} \text{Sin.} ax . x^2 dx &= \frac{1}{4} a + \frac{2-a^2}{8} \sum_0^{\infty} (-1)^n \frac{a^{2n+1}}{(n+1)^{n+1/2}} \\
 7) \int e^{-x^2} \text{Sin.} ax . x^3 dx &= \frac{6a-a^3}{16} e^{-\frac{1}{4}a^2} \sqrt{\pi} \\
 8) \int e^{-x^2} \text{Sin.} ax . x^4 dx &= \frac{10a-a^3}{16} + \frac{12-12a^2+a^4}{32} \sum_0^{\infty} (-1)^n \frac{a^{2n+1}}{(n+1)^{n+1/2}} \\
 9) \int e^{-x^2} \text{Sin.} ax . x^5 dx &= \frac{60a-20a^3+a^5}{64} e^{-\frac{1}{4}a^2} \sqrt{\pi} \\
 10) \int e^{-x^2} \text{Cos.} ax . x^2 dx &= \frac{2-a^2}{8} e^{-\frac{1}{4}a^2} \sqrt{\pi}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Dienger, Cr. 46. 119. où dans 2) il est fautive-} \\ \text{ment } x^2 dx. \\ \\ \\ \\ \text{Legendre, Exerc. 3. 49.} \end{array}$$

- 11) $\int e^{-x^2} \text{Cos. } ax. x^3 dx = \frac{4-a^2}{8} - \frac{6a-a^3}{16} \sum_0^{\infty} (-1)^n \frac{a^{2n+1}}{(n+1)^{n+1/2}} \left. \vphantom{\int} \right\} \text{Legendre, Exerc. 3. 49.}$
- 12) $\int e^{-x^2} \text{Cos. } ax. x^4 dx = \frac{12-12a^2+a^4}{32} e^{-a^2} \sqrt{\pi} \left. \vphantom{\int} \right\}$
- 13) $\int e^{-p^2 x^2} \text{Cos. } qx. x dx = \frac{1}{2p^2} \sum_0^{\infty} (-1)^n \frac{1}{(n+1)^{n/2}} \left(\frac{q}{p}\right)^{2n} \text{Oettinger, Cr. 38. 216.}$
- 14) $\int e^{-x^2} \text{Sin. } ax. x^{2b-1} dx = \frac{1}{2} \sqrt{\pi} \frac{d^{b-1}}{dy^{b-1}} \cdot (y^{1(b-1)} e^{-y}) \text{ (après la différentiation mettez } y = \frac{1}{4} a^2.) \left. \vphantom{\int} \right\} \text{Liouville, L. 5. 311.}$
- 15) $= \frac{b^{b-1}}{2^{2b}} e^{-a^2} \sqrt{\pi} \sum_0^{\infty} (-1)^n \frac{(b-1)^{n/2-1}}{1^{2n+1/2}} a^{2n+1} \text{ Cauchy, Sav. Etr. 1827. 599. P. 1. § 2.}$
- 16) $\int e^{-x^2} \text{Cos. } ax. x^{2b} dx = \frac{1}{2a} \sqrt{\pi} \frac{d^b}{dy^b} \cdot (y^{b-1} e^{-y}) \text{ (après la différentiation mettez } y = \frac{1}{4} a^2.) \left. \vphantom{\int} \right\} \text{Liouville, L. 5. 311.}$
- 17) $= \frac{(b+1)^{b/2}}{2^{2b+1}} e^{-a^2} \sqrt{\pi} \sum_0^{\infty} (-1)^n \frac{b^{n/2-1}}{1^{2n+1}} a^{2n} \text{ Cauchy, Sav. Etr. 1827. 599. P. 1. § 2.}$
- 18) $\int e^{-x^2} \text{Sin. } 2ax. x^b dx = (-1)^{1(b-1)} \frac{1}{2} e^{-a^2} a^b \sqrt{\pi} \left\{ 1 + \sum_0^{\infty} (-1)^n \frac{b^{2n/2-1}}{1^{2n}} \left(\frac{1}{2a}\right)^{2n} \right\}, b \text{ impair; } \left. \vphantom{\int} \right\} \text{Cauchy, P. 28. 147. P. III. —}$
- 19) $\int e^{-x^2} \text{Cos. } 2ax. x^b dx = (-1)^{1b} \frac{1}{2} e^{-a^2} a^b \sqrt{\pi} \left\{ 1 + \sum_0^{\infty} (-1)^n \frac{b^{2n/2-1}}{1^{2n}} \left(\frac{1}{2a}\right)^{2n} \right\}, b \text{ pair; } \left. \vphantom{\int} \right\} \text{Id., Exerc. 1826. p. 57.}$
- 20) $\int e^{-x^2} \text{Tang. } ax. x dx = a \sqrt{\pi} \sum_1^{\infty} (-1)^{n-1} n e^{-(na)^2} \text{ V. T. 439. N°. 7.}$
- 21) $\int e^{-x^2} \text{Cot } ax. x dx = -a \sqrt{\pi} \sum_1^{\infty} n e^{-(na)^2} \text{ V. T. 439. N°. 6.}$
- 22) $\int e^{-x^2} \text{Cosec. } ax. x dx = -a \sqrt{\pi} \sum_1^{\infty} (2n-1) e^{-(2n-1)^2 a^2} \text{ V. T. 439. N°. 8.}$
- 23) $\int e^{-x^2} \text{Sin. } \left(2px + \frac{1}{2} a \pi\right). x^{a+1} dx = \frac{1}{2^{a+1}} \frac{d^a}{dp^a} p e^{-p^2} \text{ Dienger, Cr. 46. 119.}$
- 24) $\int e^{-x^2} \text{Cos. } \left(2px + \frac{1}{2} b \pi\right). x^b dx = \frac{(-1)^b}{2} e^{-p^2} \sqrt{\pi} \sum_0^{\infty} (-1)^n p^{b-2n} \binom{b}{2n} \frac{(n+1)^{n-1}}{2^{2n}} \text{ Schlömilch, Cr. 33. 268.}$
- 25) $\int e^{-x^2} (a \text{Sin. } ax + 2x \text{Cos. } ax) dx = 1 \left. \vphantom{\int} \right\} \text{Legendre, Exerc. 3. 49.}$
- 26) $\int e^{-x^2} (4x^2 + a^2 - 2) \text{Sin. } ax dx = a \left. \vphantom{\int} \right\}$

F. Alg. rat. ent.
Expon. e^{-ax^2} .
Circ. Dir.

TABLE 589.

Lim. 0 et ∞ .

- 1) $\int e^{x^2 i} \text{Sin. } a x . x d x = \frac{i-1}{4 i} a e^{-\frac{a^2 i}{4}} \sqrt{\pi}$ Cauchy, Lim. Imag. 191.
- 2) $\int e^{-la^2 x^2} \text{Sin. } a x . x d x = \frac{2 \sqrt{\pi}}{a^2 e}$ Oettinger, Cr. 33. 216.
- 3) $\int e^{-a^2 x^2} \text{Sin. } b x . x d x = \frac{b}{4 a^3} e^{-\frac{b^2}{4 a^2}} \sqrt{\pi}$ Oettinger, Cr. 33. 216. — Dienger, Cr. 46. 119.
- 4) $\int e^{-\frac{1}{2} b x^2} \text{Cos. } (a x \sqrt{b}) . x^4 d x = \frac{27}{b^2} e^{-\frac{1}{2} a^2} (1-6 a^3+3 a^4) \sqrt{\frac{3 \pi}{2 b}}$ Laplace, Mém. Inst. 1809. 353. § 3.
- 5) $\int e^{-a^2 x^2} \text{Sin. } b x . x^{2c+1} d x = (-1)^{c+1} \sqrt{\frac{\pi}{2 a}} \frac{d^{2c+1}}{d b^{2c+1}} e^{-\frac{b^2}{4 a^2}}$
- 6) $\int e^{-a^2 x^2} \text{Cos. } b x . x^{2c} d x = (-1)^c \sqrt{\frac{\pi}{2 a}} \frac{d^{2c}}{d b^{2c}} e^{-\frac{b^2}{4 a^2}}$ } Laplace, Probab. L. 1. N°. 25.
- 7) $\int e^{-p x^2} \text{Sin. } \left(2 q x + \frac{1}{2} a \pi \right) . x^{a+1} d x = \frac{\sqrt{\pi}}{2^{a+1} p \sqrt{p}} \frac{d^a}{d q^a} . q e^{-\frac{q^2}{p}}$ Dienger, Cr. 46. 119.
- 8) $\int e^{-q^2 x^2} \text{Cos. } \left(p x + \frac{1}{2} a \pi \right) . x^a d x = \frac{1}{2 q} \sqrt{\pi} \frac{d^a}{d p^a} . e^{-\frac{p^2}{4 q^2}}$ Schlömilch, Gr. 5. 90. — Id., Stud. I. 12.
- 9) $\int e^{-p x^2} (e^{2 q x \text{Sin. } \lambda} + e^{-2 q x \text{Sin. } \lambda}) \text{Sin. } (2 q x \text{Cos. } \lambda) . x d x = \frac{q}{p} e^{-\frac{q^2}{p} \text{Cos. } 2 \lambda} \sqrt{\frac{\pi}{p}} \text{Cos. } \left(\lambda - \frac{q^2}{p} \text{Sin. } 2 \lambda \right)$ } Dienger, Cr. 46. 119.
- 10) $\int e^{-p x^2} (e^{2 q x \text{Sin. } \lambda} - e^{-2 q x \text{Sin. } \lambda}) \text{Cos. } (2 q x \text{Cos. } \lambda) . x d x = \frac{q}{p} e^{-\frac{q^2}{p} \text{Cos. } 2 \lambda} \sqrt{\frac{\pi}{p}} \text{Sin. } \left(\lambda - \frac{q^2}{p} \text{Sin. } 2 \lambda \right)$ }

F. Alg. rat. ent.

Exp. d'autre forme monôme.

TABLE 590.

Lim. 0 et ∞ .

Circ. Dir.

- 1) $\int e^{-x \text{Cos. } \lambda} \text{Sin. } (x \text{Sin. } \lambda) . x^{p-1} d x = \text{Sin. } p \lambda . \Gamma(p)$
- 2) $\int e^{-x \text{Cos. } \lambda} \text{Cos. } (x \text{Sin. } \lambda) . x^{p-1} d x = \text{Cos. } p \lambda . \Gamma(p)$ } Cauchy, Lim. Imag. 162.
- 3) $\int e^{-a x^p} \text{Sin. } (b x^p) . x^{c-1} d x = \frac{1}{2 p i} \Gamma\left(\frac{c}{p}\right) \left\{ (a-b i)^{-\frac{c}{p}} - (a+b i)^{-\frac{c}{p}} \right\}$
- 4) $= \frac{1}{p} \Gamma\left(\frac{c}{p}\right) (a^2+b^2)^{-\frac{c}{2p}} \text{Sin. } \left\{ \frac{c}{p} \text{Arctang. } \frac{b}{a} \right\}$ } Plana, Mém. Brux. 1837.

- 5) $\int e^{-ax^n} \text{Cos.}(bx^n). x^{c-1} dx = \frac{1}{2p} \Gamma\left(\frac{c}{p}\right) \left\{ (a-bi)^{-\frac{c}{p}} + (a+bi)^{-\frac{c}{p}} \right\}$
 6) $= \frac{1}{p} \Gamma\left(\frac{c}{p}\right) (a^2 + b^2)^{-\frac{c}{2p}} \text{Cos.} \left\{ \frac{c}{p} \text{Arctg.} \frac{b}{a} \right\}$ Plana. Mém. Brux. 1847.
- 7) $\int e^{-ax^q} \text{Sin.}(ax \text{Tang.} \lambda). x^{pq-1} dx = \frac{\Gamma(p) \text{Sin.} p\lambda}{q a^p (1 + \text{Tang.}^2 \lambda)^{1/p}} \left\{ \begin{array}{l} \text{Bonecompagni, Cr. 25. 74.} \\ \text{ne valent que pour } q = 1. \end{array} \right.$
- 8) $\int e^{-ax^q} \text{Cos.}(ax \text{Tang.} \lambda). x^{pq-1} dx = \frac{\Gamma(p) \text{Cos.} p\lambda}{q a^p (1 + \text{Tang.}^2 \lambda)^{1/p}} \left\{ \begin{array}{l} \text{Bonecompagni, Cr. 25. 74.} \\ \text{ne valent que pour } q = 1. \end{array} \right.$
- 9) $\int e^{-px^2 - \frac{q^2}{x^2}} \text{Sin.}(rx^2). x^2 dx = \frac{1}{4} \sqrt{\pi} \left(\frac{1}{p} \text{Cos.} q \right)^{\frac{1}{2}} \text{Sin.} \left(2bq + \frac{3}{2}q \right). e^{-2aq} + \frac{q}{2p} \sqrt{\pi} \text{Cos.} q \text{Sin.} (2bq - 1). e^{-2aq}$
- 10) $\int e^{-px^2 - \frac{q^2}{x^2}} \text{Cos.}(rx^2). x^2 dx = \frac{1}{4} \sqrt{\pi} \left(\frac{1}{p} \text{Cos.} q \right)^{\frac{1}{2}} \text{Cos.} \left(2bq + \frac{3}{2}q \right). e^{-2aq} + \frac{q}{2p} \sqrt{\pi} \text{Cos.} q \text{Cos.} (2bq - 1). e^{-2aq}$
- 11) $\int e^{-px^2 - \frac{q^2}{x^2}} \text{Sin.}(rx^2). x^4 dx = \frac{1}{2} \sqrt{\pi} e^{-2aq} \left[\frac{3}{4} \left(\frac{1}{p} \text{Cos.} q \right)^{\frac{1}{2}} \text{Sin.} \left(2bq + \frac{5}{2}q \right) + \right.$
 $\left. + \frac{q}{p^2} \text{Cos.}^2 q. (\text{Cos.} 2bq + \text{Sin.} 2bq) + q^2 \left(\frac{1}{p} \text{Cos.} q \right)^{\frac{1}{2}} \text{Sin.} \left(2bq - \frac{5}{2}q \right) \right]$
- 12) $\int e^{-px^2 - \frac{q^2}{x^2}} \text{Cos.}(rx^2). x^4 dx = \frac{1}{2} \sqrt{\pi} e^{-2aq} \left[\frac{3}{4} \left(\frac{1}{p} \text{Cos.} q \right)^{\frac{1}{2}} \text{Cos.} \left(2bq + \frac{5}{2}q \right) + \right.$
 $\left. + \frac{q}{p^2} \text{Cos.}^2 q. (\text{Cos.} 2bq - \text{Sin.} 2bq) + q^2 \left(\frac{1}{p} \text{Cos.} q \right)^{\frac{1}{2}} \text{Cos.} \left(2bq - \frac{5}{2}q \right) \right]$

Dans les formules (8) à (12), dues à Helmling, Transf. 41-44, on a

$$\text{Tang.} q = \frac{r}{p}, u = \sqrt{\frac{p + 1'(p^2 + r^2)}{2}}, b = \frac{1' - p + 1'(p^2 + r^2)}{2}.$$

- 13) $\int e^{-a^2x^4 + b^2x^2} \{ 2ax \text{Cos.}(2abx^3) + b \text{Sin.}(2abx^3) \} dx = \frac{1}{2} \sqrt{\pi} \left\{ \begin{array}{l} \text{Cauchy, Sav. Etr. 1827.} \\ \text{599. P. 1. § 5.} \end{array} \right.$
- 14) $\int e^{-a^2x^4 + b^2x^2} \{ 2ax \text{Sin.}(2abx^3) - b \text{Cos.}(2abx^3) \} dx = 0$
- 15) $\int e^{b^2x^2 - (ax+c)^2} \text{Sin.} \{ 2bx(ax+c) \} . x^{h-1} dx = \frac{e^{-c^2} \Gamma(\frac{1}{2}h)}{\{(a^2+b^2)(1+2c)\}^h} \text{Sin.} \left\{ h \text{Arcsin.} \frac{b}{\sqrt{(a^2+b^2)}} \right\}$
- 16) $\int e^{b^2x^2 - (ax+c)^2} \text{Cos.} \{ 2bx(ax+c) \} . x^{h-1} dx = \frac{e^{-c^2} \Gamma(\frac{1}{2}h)}{\{(a^2+b^2)(1+2c)\}^h} \text{Cos.} \left\{ h \text{Arcsin.} \frac{b}{\sqrt{(a^2+b^2)}} \right\}$ $\left. \begin{array}{l} b \leq a, \\ b \geq a, \end{array} \right\}$

Sur 15) et 16) voir Cauchy, Sav. Etr. 1827. 599. P. 1. § 3. — Voir T. 116. N. 4.

F. Alg. rat. ent.

Exp. d'autre forme monôme. TABLE 590 suite.

Lim. 0 et ∞ .

Circ. Dir.

$$\left. \begin{aligned} 17) \int e^{(b^2-a^2)(x^2+\frac{c^2}{x^2})} \text{Sin.} \left\{ 2ab \left(x^2 - \frac{c^2}{x^2} \right) \right\} \cdot x^{h-1} dx &= \frac{1}{2 \sqrt{(a^2+b^2)^h}} \text{Cos.} \left\{ h \text{Arcsin.} \frac{b}{\sqrt{(a^2+b^2)}} \right\} \\ &\left[\Gamma \left(\frac{1}{2} h \right) \psi \left\{ 1 - \frac{1}{2} h, c^2 (a^2+b^2) \right\} - \Gamma \left(-\frac{1}{2} h \right) c^h \sqrt{(a^2+b^2)^h} \psi \left\{ 1 + \frac{1}{2} h, c^2 (a^2+b^2) \right\} \right] \\ 18) \int e^{(b^2-a^2)(x^2+\frac{c^2}{x^2})} \text{Cos.} \left\{ 2ab \left(x^2 - \frac{c^2}{x^2} \right) \right\} \cdot x^{h-1} dx &= \frac{1}{2 \sqrt{(a^2+b^2)^h}} \text{Sin.} \left\{ h \text{Arcsin.} \frac{b}{\sqrt{(a^2+b^2)}} \right\} \\ &\left[\Gamma \left(\frac{1}{2} h \right) \psi \left\{ 1 - \frac{1}{2} h, c^2 (a^2+b^2) \right\} - \Gamma \left(-\frac{1}{2} h \right) c^h \sqrt{(a^2+b^2)^h} \psi \left\{ 1 + \frac{1}{2} h, c^2 (a^2+b^2) \right\} \right] \end{aligned} \right\} a < b;$$

Sur 17) et 18) voir Cauchy, Sav. Etr. 1827. 599. P. 1. § 3. — Voir T. 116. N°. 3.

$$\left. \begin{aligned} 19) \int e^{(b^2-a^2)(x^2+\frac{c^2}{x^2})} \text{Sin.} \left\{ 2ab \left(x^2 - \frac{c^2}{x^2} \right) \right\} \cdot x^{2h} dx &= \\ &= \frac{1}{2c} e^{-2c \sqrt{(a^2+b^2)}} \text{Cos.} \left\{ (2h+1) \text{Arcsin.} \frac{b}{\sqrt{(a^2+b^2)}} \right\} \frac{\sqrt{\pi}}{(a^2+b^2)^{h+1}} \sum_{n=0}^{\infty} \frac{(h-n)^{n/1}}{2^{n/2} (2c)^n \sqrt{(a^2+b^2)^n}} \\ 20) \int e^{(b^2-a^2)(x^2+\frac{c^2}{x^2})} \text{Cos.} \left\{ 2ab \left(x^2 - \frac{c^2}{x^2} \right) \right\} \cdot x^{2h} dx &= \\ &= \frac{1}{2c} e^{-2c \sqrt{(a^2+b^2)}} \text{Sin.} \left\{ (2h+1) \text{Arcsin.} \frac{b}{\sqrt{(a^2+b^2)}} \right\} \frac{\sqrt{\pi}}{(a^2+b^2)^{h+1}} \sum_{n=0}^{\infty} \frac{(h-n)^{n/1}}{2^{n/2} (2c)^n \sqrt{(a^2+b^2)^n}} \end{aligned} \right\} a < b;$$

Sur 19) et 20) voir Cauchy, Sav. Etr. 1827. 599. P. 1. § 3. — Voir T. 116. N°. 8.

$$\left. \begin{aligned} 21) \int e^{-q(x^2+\frac{1}{x^2})} \text{Sin.} \left\{ p \left(x - \frac{1}{x} \right)^2 \right\} \cdot x^{2a} dx &= \frac{1}{2} e^{-2(q+pi)} \sqrt{\frac{\pi}{q+pi}} \sum_{n=0}^{\infty} \frac{(a+n)^{2n-1}}{2^{n/2}} \left\{ \frac{1}{2(p+qi)} \right\}^n \\ 22) \int e^{-q(x^2+\frac{1}{x^2})} \text{Cos.} \left\{ p \left(x - \frac{1}{x} \right)^2 \right\} \cdot x^{2a} dx &= \frac{1}{2} e^{-2(q+pi)} \sqrt{\frac{\pi}{q+pi}} \sum_{n=0}^{\infty} \frac{(a+n)^{2n-1}}{2^{n/2}} \left\{ \frac{1}{2(p+qi)} \right\}^n \\ 23) \int e^{-q(x^2+\frac{1}{x^2})} \text{Cos.} p a \cdot x^{2a} dx &= \frac{1}{2} e^{-2q} \sqrt{\frac{\pi}{q}} \sum_{n=0}^{\infty} \frac{(a+n)^{2n-1}}{1^{2n/1}} (-1)^n \frac{d^{2n}}{d p^{2n}} \cdot e^{-\frac{p^2}{4q}} \end{aligned} \right\} \begin{array}{l} \text{Cauchy,} \\ \text{Exerc.} \\ 1826. \\ \text{p. 54.} \end{array}$$

F. Alg. rat. ent.

Exp. en dén. binôme.

TABLE 591.

Lim. 0 et ∞ .

Circ. Dir.

$$1) \int \frac{e^x - e^{-x}}{e^x + e^{-x}} \text{Cos.} a x \cdot x dx = -\frac{1}{2} \pi^2 e^{-\frac{1}{2} a \pi} \frac{1 + e^{-a \pi}}{(1 - e^{-a \pi})^2} \quad \text{V. T. 391. N°. 2, 3.}$$

$$2) \int \frac{1}{e^x - e^{-x}} \text{Cos.} a x \cdot x dx = \frac{1}{2} \pi^2 \frac{e^{-a \pi}}{(1 + e^{-a \pi})^2} \quad \text{Piana, Mém. Turin. 1818. 7. IV. 18.}$$

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$$3) \int \frac{e^x + e^{-x}}{e^x - e^{-x}} \cos. a x . x d x = - \frac{\pi^2 e^{-a\pi}}{(1 - e^{-a\pi})^2} \quad \text{Plana, Mém. Turin. 1818. 7. IV. 19.}$$

$$4) \int \frac{e^x - 1}{e^x + 1} \cos. a x . x d x = - \frac{2 \pi^2 e^{-a\pi}}{(1 - e^{-2a\pi})^2} \frac{1 + e^{-2a\pi}}{(1 - e^{-2a\pi})^2} \quad \text{V. T. 391. N°. 2, 3.}$$

$$5) \int \frac{x \sin. a x}{e^{\pi x} + e^{-\pi x}} d x = \frac{1}{4} \frac{e^{1/4} - e^{-1/4}}{(e^{1/4} + e^{-1/4})^2} \left\{ \begin{array}{l} \text{Legendre, Exerc. 5. 45.} \\ \end{array} \right.$$

$$6) \int \frac{x \cos. a x}{e^{\pi x} + e^{-\pi x}} d x = \frac{1}{2} \frac{e^a}{(e^a + 1)^2} \left\{ \begin{array}{l} \end{array} \right.$$

$$1) \int e^{-x} \sin. a x \frac{d x}{x} = \text{Arctang. } a \quad \text{Arndt, Gr. 11. 70. — Dienger, Cr. 46. 119.}$$

$$2) \int e^{-ax} \sin. a x \frac{d x}{x} = \frac{1}{4} \pi \quad \text{Oettinger, Cr. 38. 216.}$$

$$3) \int e^{-px} \sin. q x \frac{d x}{x} = \text{Arctang. } \frac{q}{p} \quad \begin{array}{l} \text{Euler, Calc. Int. T. 4. S. 5 § 139. — Bidone, Mém. Turin. 1812.} \\ \text{231. Art. 3. N°. 34. — Poisson, P. 16. 215. N°. 2. — } \\ \text{Chal. 158. — Legendre, Exerc. 3. 55. — Plana, Mém. Turin.} \\ \text{1837. — Lobatto, Cr. 11. 169 — Oettinger, Cr. 38. 216. —} \\ \text{Hoppe, Cr. 40. 139. — Lindmann, Gr. 16. 94.} \end{array}$$

$$4) \int e^{-px} \cos. q x \frac{d x}{x} = \infty \quad \text{Poisson, P. 16. 215. N°. 2. — Legendre, Exerc. 3. 55.}$$

$$5) \quad = - \frac{1}{2} l \left(1 + \frac{b^2}{a^2} \right) \quad \text{Lobatto, Cr. 11. 169. (fautive).}$$

$$6) \quad = - l 0 - \frac{1}{2} l(a^2 + b^2) - A \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 3. 35.}$$

$$7) \quad = \frac{1}{2} l(\infty + b^2) - \frac{1}{2} l(a^2 + b^2) \quad \text{Plana, Mém. Turin. 1837.}$$

$$8) \int e^{-pxi} \sin. q x \frac{d x}{x} = \frac{i}{2} l \frac{p - q}{p + q} \quad \text{Oettinger, Cr. 38. 216.}$$

$$9) \int e^{-x} \sin.^2 p x \frac{d x}{x} = \frac{1}{4} l(1 + 4 p^2), p^2 < \frac{1}{4}; \quad \text{Dienger, Cr. 46. 119.}$$

$$10) \int e^{-ax} \sin.^2 \frac{1}{2} b x \frac{d x}{x} = \frac{1}{4} l \frac{a^2 + b^2}{a^2} \quad \text{Lindmann, Gr. 16. 94.}$$

F. Alg. rat. fract. à dén. x .

Exp. monôme.

Circ. Dir. monôme.

TABLE 592 suite.

Lim. 0 et ∞ .

$$11) \int e^{-x} \sin. p x. \sin. q x \frac{dx}{x} = \frac{1}{4} l \frac{1 + (p + q)^2}{1 + (p - q)^2} \quad \text{V. T. 301. N}^\circ. 13.$$

$$12) \int e^{-x} \sin. p x. \cos. q x \frac{dx}{x} = \frac{1}{2} \text{Arctang.} \frac{2p}{1 - p^2 + q^2} \quad \text{V. T. 301. N}^\circ. 14.$$

$$13) \int e^{-ax} \sin.^2 b x. \cos.^2 c x \frac{dx}{x} = \frac{1}{8} l \left\{ \frac{a^2 + 4b^2}{a^2} \cdot \frac{1}{a^2 + 4c^2} \cdot \frac{\{a^2 + 4(b+c)^2\} \cdot \{a^2 - 4(b-c)^2\}}{a^2 + 4c^2} \right\} \quad \left. \begin{array}{l} \text{Bidone, Mém.} \\ \text{Turin. 1812.} \\ \text{231. Art. 3. 35.} \end{array} \right\}$$

$$14) \int e^{ax} \cos. b x \sin. (a \sin. b x) \frac{dx}{x} = \frac{1}{2} \pi (e^a - 1) \quad \left. \begin{array}{l} \text{Cauchy, Exerc. 1826. p. 95. — Id., Lim. Imag. Add.} \\ \text{N}^\circ. 24. (\text{où faut.} = \frac{1}{2} \pi e^a). \end{array} \right\}$$

$$15) \int e^{-p^2 x^2} \sin. q x \frac{dx}{x} = \frac{q}{2p} \sqrt{\pi} \sum_0^\infty (-1)^n \frac{1}{(2n+1) 1^{n/1}} \left(\frac{q}{2p} \right)^{2n} \quad \text{Oettinger, Cr. 38. 216.}$$

F. Alg. rat. fract. à dén. x .

Exp.

Circ. Dir.

Fonct. polyn. en num. TABLE 595.

Lim. 0 et ∞ .

$$1) \int \frac{1 - e^{-qx}}{x} \sin. x dx = \text{Arctang. } q \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \text{Sohnke, Samml. — Minding, Tafeln. I.}$$

$$2) \int \frac{1 - e^{-qx}}{x} \cos. x dx = \frac{1}{2} l(1 + q^2) \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$3) \int \frac{e^{-qx} - e^{-rx}}{x} \sin. p x dx = \text{Arctang.} \frac{r}{p} - \text{Arctang.} \frac{q}{p} \quad \text{Piösch, Mém. Cour. Brux. T. 15. P. 2.}$$

$$4) \quad = \text{Arctang.} \frac{p}{q} - \text{Arctang.} \frac{p}{r} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \left. \begin{array}{l} \text{Cauchy, Cours. Lec. 33. — Lindmann,} \\ \text{Stockh. Handl. 1850. IV.} \end{array} \right\}$$

$$5) \int \frac{e^{-qx} - e^{-rx}}{x} \cos. p x dx = \frac{1}{2} l \frac{p^2 + r^2}{p^2 + q^2}$$

$$6) \int \frac{1 - \cos. p x}{x} e^{-x} dx = \frac{1}{2} l(1 + p^2) \quad \text{Dienger, Cr. 46. 119.}$$

$$7) \int \frac{1 - \cos. p x}{x} e^{-qx} dx = \frac{1}{2} l(p^2 + q^2) - lq \quad \text{Malmsten, Cr. 38. 1.}$$

$$8) \int \frac{\sin. p x - \sin. q x}{x} e^{-rx} dx = \text{Arctang.} \frac{p}{r} - \text{Arctang.} \frac{q}{r} \quad \text{Arndt, Gr. 11. 70.}$$

$$9) \int \frac{\cos. p x - \cos. q x}{x} e^{-x} dx = \frac{1}{2} l \frac{1 + q^2}{1 + p^2} \quad \text{V. T. 301. N}^\circ. 19.$$

F. Alg. rat. fract. à dén. x .

Exp. } Fonct. polyn. en num. TABLE 593 suite.
Circ. Dir. }

Lim. 0 et ∞ .

$$10) \int \frac{\cos. p x - \cos. q x}{x} e^{-rx} dx = \frac{1}{2} l \frac{r^2 + q^2}{r^2 + p^2} \quad \text{Poisson, P. 16. 215. N°. 2.}$$

$$11) \int \frac{e^{-x} - \cos. x}{x} dx = 0 \quad \text{Arndt, Gr. 10. 225.}$$

$$12) \int \frac{e^{-qx} - \cos. x}{x} dx = -lq \quad \text{Arndt, Gr. 11. 70.}$$

$$\left. \begin{aligned} 13) \int \frac{e^{-x} - \cos. q x}{x} dx &= lq \\ 14) \int \frac{e^{-qx} - \cos. q x}{x} dx &= 0 \end{aligned} \right\} \quad \text{Malmsten, Cr. 38. 1.}$$

$$15) \int \frac{e^{-x} - e^{-px} \cos. q x}{x} dx = \frac{1}{2} l (p^2 + q^2) \quad \text{Cauchy, Exerc. 1826. p. 95. — Malmsten, Cr. 38. 1.}$$

$$\left. \begin{aligned} 16) \int \frac{e^{-x} - e^{-\frac{x}{p}} \cos. x}{x} dx &= \frac{1}{2} l \frac{1+p^2}{p^2} \\ 17) \int \frac{e^{-px} - e^{-qx} \cos. r x}{x} dx &= \frac{1}{2} l \frac{q^2 + r^2}{p^2} \end{aligned} \right\} \quad \text{Schlömilch, Höh. Anal. 74.}$$

$$18) \int \frac{e^{-px} \sin. q x - e^{-rx} \sin. s x}{x} dx = \text{Arctang. } \frac{q r - p s}{p r - q s} \quad \text{Lindmann, Stockh. Handl. 1850. IV.}$$

$$19) \int \frac{e^{-px} \cos. q x - e^{-rx} \cos. s x}{x} dx = \frac{1}{2} l \frac{r^2 + s^2}{p^2 + q^2} \quad \text{Bidone, Mém. Turin. 1812. 231. Art. 3. 35. — Lindmann, Stockh. Handl. 1850. IV.}$$

F. Alg. rat. fract. à dén. x^a .

Exp. en num. } TABLE 594.
Circ. Dir. }

Lim. 0 et ∞ .

$$1) \int e^{-ax} \sin. b x \frac{dx}{x^2} = \infty \quad \text{Plana, Mém. Brux. 1837.}$$

$$2) \quad = b - a \text{Arctang. } \frac{b}{a} - b \left\{ \Lambda + \frac{1}{2} l (a^2 + b^2) + l o \right\} \quad \left. \begin{aligned} &\text{Bidone, Mém. Turin. 1812.} \\ &231. Art. 3. 35. \end{aligned} \right\}$$

$$3) \int e^{-ax} \cos. b x \frac{dx}{x^2} = a \left\{ \Lambda + \frac{1}{2} l (a^2 + b^2) + l o \right\} - b \text{Arctang. } \frac{b}{a} + \frac{1}{o}$$

$$4) \int e^{-qx} \sin.^2 \frac{1}{2} p x \frac{dx}{x^2} = \frac{1}{2} p \text{Arctang. } \frac{p}{q} - \frac{1}{4} q l \frac{p^2 + q^2}{q^2} \quad \text{Lindmann, Gr. 16. 94.}$$

$$\left. \begin{aligned} 5) \int e^{-\frac{1}{x^2}} \sin.(2p^2 x^2) \frac{dx}{x^2} &= \frac{1}{2} e^{-2p} \sin. 2p. \sqrt{\pi} \\ 6) \int e^{-\frac{1}{x^2}} \cos.(2p^2 x^2) \frac{dx}{x^2} &= \frac{1}{2} e^{-2p} \cos. 2p. \sqrt{\pi} \\ 7) \int e^{-px^2 - \frac{q^2}{x^2}} \sin.(rx^2) \frac{dx}{x^2} &= \frac{\sqrt{\pi}}{2q} e^{-2aq} \cos. q. \sin. 2bq \\ 8) \int e^{-px^2 - \frac{q^2}{x^2}} \cos.(rx^2) \frac{dx}{x^2} &= \frac{\sqrt{\pi}}{2q} e^{-2aq} \cos. q. \cos. 2bq \end{aligned} \right\} \begin{aligned} a &= \sqrt{\frac{p + \sqrt{p^2 + r^2}}{2}}, \\ b &= \sqrt{\frac{-p + \sqrt{p^2 + r^2}}{2}}, \\ \text{Tang. } q &= \frac{r}{p}; \end{aligned}$$

Helmking, Transf. 30*, 31*, 39, 40.

$$9) \int e^{-cx} \sin. bx \frac{dx}{x^{a+1}} = p \frac{b \cos. \left(\text{Arctang. } \frac{b}{c} \right) - c \sin. \left(\text{Arctang. } \frac{b}{c} \right)}{a(1-a) \sqrt{(b^2 + c^2)^{1-a}}}, p = 0,906402; \text{ Laplace, P. 15. 229.}$$

$$10) = \frac{(c-bi)^a - (c+bi)^a}{2i} \Gamma(-a) \text{ (val. extr.) } \begin{matrix} \text{Cauchy, P. 28. 147. P. III.} \\ \text{Suppl. — Id., Exerc. 1826. p. 58.} \end{matrix}$$

$$11) = (b^2 + c^2)^{\frac{1}{2}a} \sin. \left(a \text{ Arctang. } \frac{b}{c} \right) \Gamma(-a) \text{ Cauchy, Exerc. 1826. p. 58.}$$

$$12) = 1-a-1/1 (b^2 + c^2)^{\frac{1}{2}a} \sin. \left(a \text{ Arctang. } \frac{b}{c} \right) \left\{ \begin{array}{l} \text{Oettinger, Cr. 38. 216. où 12)} \\ \text{était fautive.} \end{array} \right.$$

$$13) \int e^{-cx} \cos. bx \frac{dx}{x^{a+1}} = 1-a-1/1 (b^2 + c^2)^{\frac{1}{2}a} \cos. \left(a \text{ Arctang. } \frac{b}{c} \right)$$

$$14) = \frac{(c-bi)^a + (c+bi)^a}{2} \Gamma(-a) \text{ (val. extr.) } \begin{matrix} \text{Cauchy, P. 28. 147. P. III.} \\ \text{Suppl. — Id., Exerc. 1826. p. 58.} \end{matrix}$$

$$15) = (b^2 + c^2)^{\frac{1}{2}a} \cos. \left(a \text{ Arctang. } \frac{b}{c} \right) \Gamma(-a) \text{ Cauchy, Exerc. 1826.}$$

$$16) \int e^{-cx} \sin. bx \frac{dx}{x^a} = p \frac{\sin. \left(\text{Arctang. } \frac{b}{c} \right)}{(1-a)(b^2 + c^2)^{\frac{1-a}{2}}}, p = 0,906402;$$

$$17) \int e^{-cx} \cos. bx \frac{dx}{x^a} = p \frac{\cos. \left(\text{Arctang. } \frac{b}{c} \right)}{(1-a)(b^2 + c^2)^{\frac{1-a}{2}}} \left\{ \begin{array}{l} \text{Laplace, P. 15. 229.} \end{array} \right.$$

F. Alg. rat. fract. à dén. x^a .

Exp. en num.

Circ. Dir.

TABLE 594 suite.

Lim. 0 et ∞ .

$$18) \int \frac{\cos. b x - \cos. c x}{x^2} e^{-ax} dx = \frac{1}{2} a l \frac{a^2 + b^2}{a^2 + c^2} + c \operatorname{Arctang} \frac{c}{a} - b \operatorname{Arctang} \frac{b}{a} \left\{ \begin{array}{l} \text{Bidone, Mém. Tu-} \\ \text{rin. 1812. 231. Art.} \\ \text{3. 35.} \end{array} \right.$$

$$19) \int \frac{e^{-cx} - e^{-bx}}{x^2} \sin. a x dx = \frac{1}{2} a l \frac{a^2 + b^2}{a^2 + c^2} + b \operatorname{Arctang} \frac{a}{b} - c \operatorname{Arctang} \frac{a}{c}$$

$$20) \int e^{-x^2} \frac{2 x \cos. x - \sin. x}{x^2} \sin. x dx = \frac{e-1}{2e} \sqrt{\pi} \quad \text{Dienger, Cr. 46. 119.}$$

$$21) \int e^{-cx} \frac{\cos. b x + i \sin. b x}{x^2} dx = \frac{p}{(1-a)(c-bi)^{1-a}}, \quad p = 0,906102; \quad \text{Laplace, P. 15. 229.}$$

$$22) \int \frac{e^{-px} \sin. qx - e^{-rx} \sin. sx}{x^{t+1}} dx = \frac{\Gamma(1-t)}{t} \left\{ (p^2 + q^2)^t \sin. \left(t \operatorname{Arctg} \frac{q}{p} \right) - (r^2 + s^2)^t \sin. \left(t \operatorname{Arctg} \frac{s}{r} \right) \right\} \left\{ \begin{array}{l} \text{Lind-} \\ \text{mann,} \\ \text{Stockh.} \\ \text{Hanbl.} \\ \text{1850.} \\ \text{IV.} \end{array} \right.$$

$$23) \int \frac{e^{-px} \cos. qx - e^{-rx} \cos. sx}{x^{t+1}} dx = \frac{\Gamma(1-t)}{t} \left\{ (p^2 + s^2)^t \cos. \left(t \operatorname{Arctg} \frac{s}{r} \right) - (p^2 + q^2)^t \cos. \left(t \operatorname{Arctg} \frac{q}{p} \right) \right\} \left\{ \begin{array}{l} \text{Lind-} \\ \text{mann,} \\ \text{Stockh.} \\ \text{Hanbl.} \\ \text{1850.} \\ \text{IV.} \end{array} \right.$$

F. Alg. rat. fract. à dén. $x^2 + a^2$.

Exp. en num.

Circ. Dir.

TABLE 595.

Lim. 0 et ∞ .

$$1) \int e^{p \cos. bx} \sin. (p \sin. b x) \frac{x}{x^2 + q^2} dx = \frac{1}{2} \pi \left(e^{p e^{-bq}} - 1 \right) \left\{ \begin{array}{l} \text{Cauchy, Lim. Imag. Add. N}^\circ, 25 \\ \text{où pour 1) faut. } \frac{\pi}{2} e^{p e^{-bq}}. - \text{Id.,} \end{array} \right.$$

$$2) \int e^{p \cos. bx} \cos. (p \sin. b x) \frac{dx}{x^2 + q^2} = \frac{1}{2q} \pi e^{p e^{-bq}} \left\{ \begin{array}{l} \text{Exerc. d. Math. 1. 95. — Boncom-} \\ \text{pagni, Cr. 25. 74.} \end{array} \right.$$

$$3) \int e^{\cos. bx} \sin. \left(\frac{1}{2} a \pi - \sin. b x \right) \frac{x^{a-1}}{x^2 + q^2} dx = \frac{\pi}{2q} q^{a-1} e^{e^{-bq}} \quad \text{Cauchy, Lim. Imag. Add. N}^\circ, 26.$$

$$4) \int \frac{e^{-p \sin. bx} + e^{p \sin. bx}}{q^2 + x^2} x \sin. (p \cos. b x) dx = \pi \{ \cos. (p e^{-bq}) - 1 \} \left\{ \begin{array}{l} \text{Boncompagni, Cr. 25. 74.} \\ \text{où pour 6) fautivement} \\ = - \pi \sin. (p e^{-bq}) \end{array} \right.$$

$$5) \int \frac{e^{-p \sin. bx} + e^{p \sin. bx}}{q^2 + x^2} \sin. (p \cos. b x) dx = \frac{\pi}{q} \sin. (p e^{-bq})$$

$$6) \int \frac{e^{-p \sin. bx} - e^{p \sin. bx}}{q^2 + x^2} x \cos. (p \cos. b x) dx = - \pi \{ \sin. (p e^{-bq}) - p e^{-bq} \}$$

$$7) \int \frac{e^{-p \sin. bx} + e^{p \sin. bx}}{q^2 + x^2} \cos. (p \cos. b x) dx = \frac{\pi}{q} \cos. (p e^{-bq})$$

- 1) $\int \frac{\text{Sin. } p x}{e^{\pi x} + e^{-\pi x}} \frac{d x}{x} = \text{Arctang.}(e^{4p}) \quad \text{V. T. 281. N}^{\circ} 4.$
- 2) $\int \frac{\text{Cos. } p x}{e^{\pi x} - e^{-\pi x}} \frac{d x}{x} = -\frac{1}{2} l(e^{4p} + e^{-4p}) \quad \text{V. T. 281. N}^{\circ} 8.$
- 3) $\int \frac{e^{\pi x} - e^{-\pi x}}{e^{\pi x} + e^{-\pi x}} \frac{\text{Cos. } p x}{x} d x = l \frac{1 + e^{-4p}}{1 - e^{-4p}} \quad \text{V. T. 282. N}^{\circ} 1.$
- 4) $\int \frac{e^{\pi x} + e^{-\pi x}}{e^{\pi x} - e^{-\pi x}} \frac{\text{Cos. } p x}{x} d x = -l(e^{4p} - e^{-4p}) \quad \text{V. T. 282. N}^{\circ} 8.$
- 5) $\int \frac{\text{Sin. } q x}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} \frac{x}{1+x^2} d x = -\frac{\pi}{2\sqrt{2}} e^{-q} - \frac{e^q - e^{-q}}{4\sqrt{2}} l \frac{e^q + 1}{2 + e^{-q}} + \frac{e^q + e^{-q}}{2\sqrt{2}} \text{Arctg.} \left(\frac{1}{e^q - e^{-q}} \right) \quad \text{V. T. 396. N}^{\circ} 16.$
- 6) $\int \frac{\text{Sin. } q x}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} \frac{x}{1+x^2} d x = -\frac{e^{-q}}{2\sqrt{2}} + \frac{e^q - e^{-q}}{4\sqrt{2}} l \frac{e^q + 1}{2 + e^{-q}} + \frac{e^q + e^{-q}}{2\sqrt{2}} \text{Arctg.} \left(\frac{1}{e^q - e^{-q}} \right) \quad \text{Schlömilch, Beitr. II. § 7.}$
- 7) $\int \frac{\text{Sin. } q x}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} \frac{x}{1+x^2} d x = -\frac{1}{2} q e^{-q} + \frac{e^q - e^{-q}}{4} l(1 + e^{-2q}) \quad \text{V. T. 396. N}^{\circ} 18.$
- 8) $\int \frac{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} \frac{\text{Sin. } q x}{1+x^2} d x = q e^{-q} - \frac{e^q - e^{-q}}{2} l(1 - e^{-2q}) \quad \text{V. T. 396. N}^{\circ} 17.$
- 9) $\int \frac{\text{Sin. } q x}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} \frac{d x}{1+x^2} = \frac{e^q + e^{-q}}{2} \text{Arctang.}(e^{-q}) - \frac{1}{4} \pi e^{-q}$
- 10) $\int \frac{e^{\frac{1}{2}\pi x} - 1}{e^{\frac{1}{2}\pi x} + 1} \frac{\text{Sin. } q x}{1+x^2} d x = -\frac{1}{2} \pi e^q + \frac{e^q - e^{-q}}{2} l \frac{e^q + 1}{e^q - 1} + (e^q + e^{-q}) \text{Arctang.}(e^q)$
- 11) $\int \frac{e^{\frac{1}{2}\pi x} + 1}{e^{\frac{1}{2}\pi x} - 1} \frac{\text{Sin. } q x}{1+x^2} d x = -\frac{1}{2} \pi e^{-q} + \frac{e^q - e^{-q}}{2} l \frac{e^q + 1}{e^q - 1} + (e^q + e^{-q}) \text{Arctang.}(e^{-q})$
- 12) $\int \frac{\text{Sin. } q x}{e^{\pi x} - e^{-\pi x}} \frac{d x}{1+x^2} = -\frac{1}{4} q e^{-q} + \frac{e^q - e^{-q}}{4} l(1 + e^{-q}) \quad \text{V. T. 396. N}^{\circ} 17.$
- 13) $\int \frac{e^{\pi x} + e^{-\pi x}}{e^{\pi x} - e^{-\pi x}} \frac{\text{Sin. } q x}{1+x^2} d x = \frac{1}{2} q e^{-q} - \frac{e^q - e^{-q}}{2} l(1 - e^{-q}) \quad \text{V. T. 396. N}^{\circ} 17.$
- 14) $\int \frac{e^{\pi x} + 1}{e^{\pi x} - 1} \frac{\text{Sin. } q x}{1+x^2} d x = \frac{e^q - e^{-q}}{2} l \frac{e^q + 1}{e^q - 1} \quad \text{Schlömilch, Beitr. II. § 7.}$
- 15) $\int \frac{e^{p x} + e^{-p x}}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} \frac{\text{Sin. } q x}{1+x^2} d x = -\frac{1}{2} \pi e^{-q} \text{Cos. } p + \frac{e^q - e^{-q}}{4} \text{Sin. } p. l \frac{e^q + 2 \text{Sin. } p + e^{-q}}{e^q - 2 \text{Sin. } p + e^{-q}} + \frac{e^q + e^{-q}}{2} \text{Cos. } p. \text{Arctang.} \left(\frac{2 \text{Cos. } p}{e^q - e^{-q}} \right), p^2 \leq \frac{1}{4} \pi^2; \quad \text{Schlömilch, Beitr. II. § 7. — Id., Stud. II. 19.}$

$$16) \int \frac{e^{px} - e^{-px}}{e^{i\pi x} - e^{-i\pi x}} \frac{x \sin qx}{1+x^2} dx = -\frac{1}{2} \pi e^{-q} \sin p - \frac{e^q - e^{-q}}{4} \cos p \cdot l \frac{e^q + 2 \sin p + e^{-q}}{e^q - 2 \sin p + e^{-q}} + \\ + \frac{e^q + e^{-q}}{2} \sin p \cdot \text{Arctang.} \left(\frac{2 \cos p}{e^q - e^{-q}} \right), p^2 < \frac{1}{4} \pi^2; \quad \text{V. T. 396. N}^\circ. 30.$$

$$17) \int \frac{e^{px} + e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{\sin qx}{1+x^2} dx = -\frac{1}{2} e^{-q} (q \cos p + p \sin p) + \frac{e^q - e^{-q}}{4} \cos p \cdot l (1 + 2 e^{-q} \cos p + e^{-2q}) + \\ + \frac{e^q + e^{-q}}{2} \sin p \cdot \text{Arctang.} \left(\frac{\sin p}{e^q + \cos p} \right), p^2 \leq \pi^2; \quad \text{Schlömlich, Beitr. II. § 8. — Id., Stud. II. 19.}$$

$$18) \int \frac{e^{px} - e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{x \sin qx}{1+x^2} dx = \frac{1}{2} e^{-q} (p \cos p - q \sin p) + \frac{e^q - e^{-q}}{4} \sin p \cdot l (1 + 2 e^{-q} \cos p + e^{-2q}) - \\ - \frac{e^q + e^{-q}}{2} \cos p \cdot \text{Arctang.} \left(\frac{\sin p}{e^q + \cos p} \right), p^2 < \pi^2; \quad \text{V. T. 396. N}^\circ. 32.$$

$$19) \int \frac{\cos qx}{e^{i\pi x} + e^{-i\pi x}} \frac{dx}{1+x^2} = \frac{\pi}{2} e^{-q} - \frac{e^q + e^{-q}}{4} l \frac{e^q + e^{-q} + 2}{e^q + e^{-q} - 1} + \frac{e^q - e^{-q}}{2} \text{Arctg.} \left(\frac{1+2}{e^q - e^{-q}} \right) \quad \text{V. T. 396. N}^\circ. 30.$$

$$20) \int \frac{\cos qx}{e^{i\pi x} + e^{-i\pi x}} \frac{dx}{1+x^2} = \frac{1}{2} q e^{-q} + \frac{e^q + e^{-q}}{4} l (1 + e^{-2q}) \quad \text{V. T. 396. N}^\circ. 32.$$

$$21) \int \frac{e^{i\pi x} - e^{-i\pi x}}{e^{i\pi x} + e^{-i\pi x}} \frac{x \cos qx}{1+x^2} dx = -q e^{-q} - \frac{e^q - e^{-q}}{2} l (1 - e^{-2q}) \quad \text{V. T. 396. N}^\circ. 31.$$

$$22) \int \frac{\cos qx}{e^{i\pi x} - e^{-i\pi x}} \frac{x}{1+x^2} dx = \frac{e^q - e^{-q}}{4} \text{Arctang.} (e^{-q}) + \frac{1}{4} \pi e^{-q} - \frac{1}{2} \quad \text{V. T. 396. N}^\circ. 29.$$

$$23) \int \frac{e^{i\pi x} + e^{-i\pi x}}{e^{i\pi x} - e^{-i\pi x}} \frac{x \cos qx}{1+x^2} dx = -1 + \frac{e^q + e^{-q}}{2} l \frac{1 + e^{-q}}{1 - e^{-q}} \quad \text{V. T. 396. N}^\circ. 31.$$

$$24) \int \frac{e^{i\pi x} - 1}{e^{i\pi x} + 1} \frac{x \cos qx}{1+x^2} dx = -\frac{1}{2} \pi e^q + \frac{e^q + e^{-q}}{2} l \frac{e^q + 1}{e^q - 1} + \frac{e^q - e^{-q}}{2} \text{Arctg.} (e^q) \quad \text{V. T. 396. N}^\circ. 29.$$

$$25) \int \frac{e^{i\pi x} + 1}{e^{i\pi x} - 1} \frac{x \cos qx}{1+x^2} dx = -2 + \frac{1}{2} \pi e^{-q} + \frac{e^q + e^{-q}}{2} l \frac{e^q + 1}{e^q - 1} + \frac{e^q - e^{-q}}{2} \text{Arctg.} (e^{-q}) \quad \text{V. T. 396. N}^\circ. 29.$$

$$26) \int \frac{\cos qx}{e^{\pi x} - e^{-\pi x}} \frac{x}{1+x^2} dx = -\frac{1}{4} + \frac{1}{4} q e^{-q} + \frac{e^q + e^{-q}}{4} l (1 + e^{-q}) \quad \text{V. T. 396. N}^\circ. 31.$$

$$27) \int \frac{e^{\pi x} + e^{-\pi x}}{e^{\pi x} - e^{-\pi x}} \frac{x \cos qx}{1+x^2} dx = -\frac{1}{2} - \frac{1}{2} q e^{-q} - \frac{e^q + e^{-q}}{2} l (1 - e^{-q}) \quad \text{V. T. 396. N}^\circ. 31.$$

$$28) \int \frac{e^{\pi x} + 1}{e^{\pi x} - 1} \frac{x \cos. q x}{1 + x^2} dx = -1 + \frac{e^q + e^{-q}}{2} l \frac{e^q + 1}{e^q - 1} \quad \text{V. T. 396. N°. 29.}$$

$$29) \int \frac{e^{px} + e^{-px}}{e^{i\pi x} - e^{-i\pi x}} \frac{x \cos. q x}{1 + x^2} dx = -1 + \frac{1}{2} \pi e^{-q} \cos. p + \frac{e^q + e^{-q}}{4} \sin. p. l \frac{e^q + 2 \sin. p + e^{-q}}{e^q - 2 \sin. p + e^{-q}} +$$

$$+ \frac{e^q - e^{-q}}{4} \cos. p. \operatorname{Arctang.} \left(\frac{2 \cos. p}{e^q - e^{-q}} \right), \quad p^2 \leq \frac{1}{4} \pi^2; \quad \text{Schlömlich, Beitr. II. § 7.}$$

$$30) \int \frac{e^{px} - e^{-px}}{e^{i\pi x} - e^{-i\pi x}} \frac{\cos. q x}{1 + x^2} dx = \frac{1}{2} \pi e^{-q} \sin. p - \frac{e^q + e^{-q}}{4} \cos. p. l \frac{e^q + 2 \sin. p + e^{-q}}{e^q - 2 \sin. p + e^{-q}} +$$

$$+ \frac{e^q - e^{-q}}{2} \sin. p. \operatorname{Arctang.} \left(\frac{2 \cos. p}{e^q - e^{-q}} \right), \quad p^2 < \frac{1}{4} \pi^2; \quad \text{Schlömlich, Beitr. II. § 9. — Id., Stud. II. 19.}$$

$$31) \int \frac{e^{px} + e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{x \cos. q x}{1 + x^2} dx = \frac{1}{2} e^{-q} (q \cos. p + p \sin. p) - \frac{1}{2} + \frac{e^q + e^{-q}}{4} \cos. p. l (1 + 2 e^{-q} \cos. p + e^{-2q}) +$$

$$+ \frac{e^q - e^{-q}}{2} \sin. p. \operatorname{Arctg.} \left(\frac{\sin. p}{e^q + \cos. p} \right), \quad p^2 \leq \pi^2; \quad \text{V. T. 396 N°. 17.}$$

$$32) \int \frac{e^{px} - e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{\cos. q x}{1 + x^2} dx = \frac{1}{2} e^{-q} (q \sin. p - p \cos. p) + \frac{e^q + e^{-q}}{4} \sin. p. l (1 + 2 e^{-q} \cos. p + e^{-2q}) -$$

$$- \frac{e^q - e^{-q}}{2} \cos. p. \operatorname{Arctang.} \left(\frac{\sin. p}{e^q + \cos. p} \right), \quad p^2 < \pi^2; \quad \text{Schlömlich, Beitr. II. 9. — Id., Stud. II. 19.}$$

$$33) \int \frac{e^{px} + e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{\sin. q x}{r^2 + x^2} dx = \frac{1}{2r^2} - \frac{\pi e^{-qr} \cos. pr}{2r \sin. r\pi} + \sum_1^{\infty} (-1)^{n-1} \frac{e^{-nq} \cos. np}{n^2 - r^2}, \quad \pi \geq p \geq 0; \quad \text{Schlömlich, Beitr. II. 7.}$$

$$34) \int \frac{e^{px} - e^{-px}}{e^{\pi x} - e^{-\pi x}} \frac{\cos. q x}{r^2 + x^2} dx = \frac{\pi e^{-qr} \sin. pr}{2r \sin. r\pi} + \sum_1^{\infty} (-1)^n \frac{e^{-nq} \sin. np}{n^2 - r^2}, \quad \pi > p > 0; \quad \text{Schlömlich, Beitr. II. 9.}$$

$$35) \int \frac{e^{\pi x} + e^{-\pi x}}{e^{\pi x} - e^{-\pi x}} \frac{\cos. q x}{x} \frac{dx}{1 + x^2} = \frac{1}{2} \frac{(1 - q + q e^{-q})}{1 - e^{-q}} + \frac{1}{2} (e^{iq} - e^{-iq})^2 l(1 - e^{-q}) \quad \text{V. T. 396. N°. 4, 27.}$$

$$36) \int \frac{\sin. x}{e^{qx} + 2 \cos. x + e^{-qx}} \frac{x}{x^2 - \pi^2} dx = \frac{1}{2} \operatorname{Arctang.} \left(\frac{1}{q} \right) - \frac{1}{2q}$$

$$37) \int \frac{\sin. x}{e^{qx} - 2 \cos. x + e^{-qx}} \frac{x}{x^2 - \pi^2} dx = \frac{1}{2} \frac{q}{1 + q^2} - \frac{1}{2} \operatorname{Arctang.} \left(\frac{1}{q} \right) \quad \left. \vphantom{\int} \right\} \text{Schlömlich, Beitr. III. § 3.}$$

$$38) \int \frac{\cos. q x - e^{-qx}}{x} \frac{dx}{x^4 + r^4} = \frac{\pi}{2r^4} e^{-iqr} \sqrt[4]{2} \sin. \left(\frac{1}{2} q r \sqrt[4]{2} \right) \quad \text{Cauchy, Exerc. 1826. p. 95.}$$

- 1) $\int e^{-x} \text{Sin. } x. dx \sqrt{x} = \sqrt{\frac{\pi}{2 \sqrt{2}}} \cdot \text{Sin. } \frac{3\pi}{8} \left\{ \begin{array}{l} \text{Fuss, Mém. Pétersb. 1830.} \\ \end{array} \right.$
- 2) $\int e^{-x} \text{Cos. } x. dx \sqrt{x} = \sqrt{\frac{\pi}{2 \sqrt{2}}} \cdot \text{Cos. } \frac{3\pi}{8}$
- 3) $\int e^{-qx} \text{Sin. } q x. dx \sqrt{x} = \frac{1}{4q} \sqrt{(1 + \sqrt{2})} \cdot \sqrt{\frac{\pi}{2q}}$
- 4) $\int e^{-qx} \text{Sin. } q x. x dx \sqrt{x} = \frac{3}{16q^2} \sqrt{(1 + \sqrt{2})} \cdot \sqrt{\frac{\pi}{q}}$
- 5) $\int e^{-qx} \text{Sin. } q x. x^2 dx \sqrt{x} = \frac{15}{32q^3} \sqrt{(-1 + \sqrt{2})} \cdot \sqrt{\frac{\pi}{2q}}$
- 6) $\int e^{-qx} \text{Cos. } q x. dx \sqrt{x} = \frac{1}{4q} \sqrt{(-1 + \sqrt{2})} \cdot \sqrt{\frac{\pi}{2q}}$
- 7) $\int e^{-qx} \text{Cos. } q x. x dx \sqrt{x} = \frac{3}{16q} \sqrt{(-1 + \sqrt{2})} \cdot \sqrt{\frac{\pi}{q}}$
- 8) $\int e^{-qx} \text{Cos. } q x. x^2 dx \sqrt{x} = \frac{15}{32q^3} \sqrt{(1 + \sqrt{2})} \cdot \sqrt{\frac{\pi}{2q}}$
- 9) $\int e^{-qx} \text{Sin. } p x. dx \sqrt{x} = \frac{1}{4} \sqrt{\{-q^3 + 3qp^2 + \sqrt{(q^2 + p^2)^3}\}} \cdot \sqrt{\frac{2\pi}{(q^2 + p^2)^3}}$
- 10) $\int e^{-qx} \text{Sin. } p x. x dx \sqrt{x} = \frac{3}{8} \sqrt{\{-q^5 + 10q^3p^2 - 5qp^4 + \sqrt{(q^2 + p^2)^5}\}} \cdot \sqrt{\frac{2\pi}{(q^2 + p^2)^5}}$
- 11) $\int e^{-qx} \text{Sin. } p x. x^2 dx \sqrt{x} = \frac{15}{16} \sqrt{\{-q^7 + 21q^5p^2 - 35q^3p^4 + 7qp^6 + \sqrt{(q^2 + p^2)^7}\}} \cdot \sqrt{\frac{2\pi}{(q^2 + p^2)^7}}$
- 12) $\int e^{-qx} \text{Cos. } p x. dx \sqrt{x} = \frac{1}{4} \sqrt{\{q^3 - 3qp^2 + \sqrt{(q^2 + p^2)^3}\}} \cdot \sqrt{\frac{2\pi}{(q^2 + p^2)^3}}$
- 13) $\int e^{-qx} \text{Cos. } p x. x dx \sqrt{x} = \frac{3}{8} \sqrt{\{q^5 - 10q^3p^2 + 5qp^4 + \sqrt{(q^2 + p^2)^5}\}} \cdot \sqrt{\frac{2\pi}{(q^2 + p^2)^5}}$
- 14) $\int e^{-qx} \text{Cos. } p x. x^2 dx \sqrt{x} = \frac{15}{16} \sqrt{\{q^7 - 21q^5p^2 + 35q^3p^4 - 7qp^6 + \sqrt{(q^2 + p^2)^7}\}} \cdot \sqrt{\frac{2\pi}{(q^2 + p^2)^7}}$
- Sur ces intégrales 3) à 14) voyez Oettinger, Cr. 38. 216. où 12) et 14) étaient fautives.
- 15) $\int e^{-px^a} x^{4a-1} \text{Cos. } (2q \sqrt{px}) dx = \frac{1}{a} e^{-q^2} \sqrt{\frac{\pi}{p}}$ Boncompagni, Cr. 25. 74. ne vaut que pour $a=1$.

- $$\begin{aligned} 1) \int e^{-qx} \text{Sin. } qx \frac{dx}{\sqrt{x}} &= \frac{1}{2} \sqrt{-1 + \sqrt{2}} \cdot \sqrt{\frac{\pi}{q}} \left\{ \begin{array}{l} \text{Oettinger, Cr. 38. 216.} \\ 2) \int e^{-qx} \text{Cos. } qx \frac{dx}{\sqrt{x}} = \frac{1}{2} \sqrt{1 + \sqrt{2}} \cdot \sqrt{\frac{\pi}{q}} \end{array} \right. \\ 3) \int e^{-x} \text{Sin. } x \frac{dx}{\sqrt{x}} &= \sqrt{\frac{\sqrt{2} - 1}{2}} \cdot \sqrt{\frac{\pi}{2}} \left\{ \begin{array}{l} \text{Fuss, Mém. Pétersb. 1830.} \\ 4) \int e^{-x} \text{Cos. } x \frac{dx}{\sqrt{x}} = \sqrt{\frac{\sqrt{2} + 1}{2}} \cdot \sqrt{\frac{\pi}{2}} \end{array} \right. \\ 5) \int e^{-qx} \text{Sin. } px \frac{dx}{\sqrt{x}} &= \sqrt{\frac{\sqrt{p^2 + q^2} - q}{p^2 + q^2}} \cdot \sqrt{\frac{\pi}{2}} \left\{ \begin{array}{l} \text{Euler, Calc. Int. IV. S. 5. § 136. — Oettinger, Cr. 38. 216.} \\ 6) \int e^{-qx} \text{Cos. } px \frac{dx}{\sqrt{x}} = \sqrt{\frac{\sqrt{p^2 + q^2} + q}{p^2 + q^2}} \cdot \sqrt{\frac{\pi}{2}} \end{array} \right. \\ 7) \int e^{-x} \text{Cos. } (2\sqrt{q}x) \frac{dx}{\sqrt{x}} &= e^{-q} \sqrt{\pi} \quad \text{Kummer, Cr. 17. 210.} \\ 8) \int e^{-px} \text{Cos. } (2q\sqrt{p}x) \frac{dx}{\sqrt{x}} &= e^{-q^2} \sqrt{\frac{\pi}{p}} \quad \text{Boncompagni, Cr. 25. 74.} \\ 9) \int e^{-px+qx \text{Cos. } \lambda} \text{Sin. } (qx \text{Sin. } \lambda) \frac{dx}{\sqrt{x}} &= -\text{Sin. } \frac{1}{2} \beta \cdot \sqrt{\frac{\pi}{r}} \left\{ \begin{array}{l} \text{Dienger, Cr. 46. 119.} \\ \text{, où } r^2 = p^2 + q^2 - 2pq \text{Cos. } \lambda, \\ 10) \int e^{-px+qx \text{Cos. } \lambda} \text{Cos. } (qx \text{Sin. } \lambda) \frac{dx}{\sqrt{x}} = \text{Cos. } \frac{1}{2} \beta \cdot \sqrt{\frac{\pi}{r}} \left\{ \begin{array}{l} \text{Cos. } \beta = \frac{p - q \text{Cos. } \lambda}{r}, \text{Sin. } \beta = \frac{-q \text{Sin. } \lambda}{r}, \\ p > q > 0; \end{array} \right. \\ 11) \int e^{-a^2(x+\frac{1}{x})} \text{Sin. } bx \frac{dx}{\sqrt{x}} = e^{-2qa} \{q \text{Sin. } 2pa + p \text{Cos. } 2pa\} \sqrt{\frac{\pi}{b^2 + a^4}} \\ 12) \int e^{-a^2(x+\frac{1}{x})} \text{Cos. } bx \frac{dx}{\sqrt{x}} = \frac{1}{b} e^{-2qa} \{q \text{Cos. } 2pa - p \text{Sin. } 2pa\} \sqrt{\frac{\pi}{b^2 + a^4}} \end{array} \right. \end{aligned}$$

Des intégrales 11) et 12) voyez Cauchy, P. 28. 147. P. I. § 4. où 12) était fautive, et où l'on a :

$$2p = \sqrt{\sqrt{b^2 + a^4} + a} - \sqrt{\sqrt{b^2 + a^4} - a}$$

$$2q = \sqrt{\sqrt{b^2 + a^4} + a} + \sqrt{\sqrt{b^2 + a^4} - a}$$

- $$1) \int e^{-qx} \text{Sin. } qx \frac{dx}{x \sqrt{x}} = \sqrt{-1 + \sqrt{2}} \cdot \sqrt{2} q \pi$$
- $$2) \int e^{-qx} \text{Sin. } qx \frac{dx}{x^2 \sqrt{x}} = \frac{4}{3} q \sqrt{1 + \sqrt{2}} \cdot \sqrt{q \pi}$$
- $$3) \int e^{-qx} \text{Sin. } qx \frac{dx}{x^3 \sqrt{x}} = \frac{8}{15} q^2 \sqrt{1 + \sqrt{2}} \cdot \sqrt{2} q \pi$$
- $$4) \int e^{-qx} \text{Sin. } qx \frac{dx}{x^4 \sqrt{x}} = \frac{32}{105} q^3 \sqrt{-1 + \sqrt{2}} \cdot \sqrt{q \pi}$$
- $$5) \int e^{-qx} \text{Cos. } qx \frac{dx}{x \sqrt{x}} = -\sqrt{1 + \sqrt{2}} \cdot \sqrt{2} q \pi$$
- $$6) \int e^{-qx} \text{Cos. } qx \frac{dx}{x^2 \sqrt{x}} = \frac{4}{3} q \sqrt{-1 + \sqrt{2}} \cdot \sqrt{q \pi}$$
- $$7) \int e^{-qx} \text{Cos. } qx \frac{dx}{x^3 \sqrt{x}} = \frac{8}{15} q^2 \sqrt{-1 + \sqrt{2}} \cdot \sqrt{2} q \pi$$
- $$8) \int e^{-qx} \text{Cos. } qx \frac{dx}{x^4 \sqrt{x}} = \frac{32}{105} q^3 \sqrt{1 + \sqrt{2}} \cdot \sqrt{q \pi}$$
- $$9) \int e^{-qx} \text{Sin. } px \frac{dx}{x \sqrt{x}} = -\sqrt{-q + \sqrt{p^2 + q^2}} \cdot \sqrt{2} \pi$$
- $$10) \int e^{-qx} \text{Sin. } px \frac{dx}{x^2 \sqrt{x}} = \frac{2}{3} \sqrt{-q^3 + 3p^2 q + \sqrt{(p^2 + q^2)^3}} \cdot \sqrt{2} \pi$$
- $$11) \int e^{-qx} \text{Sin. } px \frac{dx}{x^3 \sqrt{x}} = -\frac{4}{15} \sqrt{-q^5 + 10p^2 q^3 - 5p^4 q + \sqrt{(p^3 + q^2)^5}} \cdot \sqrt{2} \pi$$
- $$12) \int e^{-qx} \text{Sin. } px \frac{dx}{x^4 \sqrt{x}} = \frac{8}{105} \sqrt{-q^7 + 21p^2 q^5 - 35p^4 q^3 + 7p^6 q + \sqrt{(p^3 + q^2)^7}} \cdot \sqrt{2} \pi$$
- $$13) \int e^{-qx} \text{Cos. } px \frac{dx}{x \sqrt{x}} = -\sqrt{q + \sqrt{p^2 + q^2}} \cdot \sqrt{2} \pi$$
- $$14) \int e^{-qx} \text{Cos. } px \frac{dx}{x^2 \sqrt{x}} = \frac{2}{3} \sqrt{q^3 - 3p^2 q + \sqrt{(p^2 + q^2)^3}} \cdot \sqrt{2} \pi$$
- $$15) \int e^{-qx} \text{Cos. } px \frac{dx}{x^3 \sqrt{x}} = -\frac{4}{15} \sqrt{q^5 - 10p^2 q^3 + 5p^4 q + \sqrt{(p^3 + q^2)^5}} \cdot \sqrt{2} \pi$$
- $$16) \int e^{-qx} \text{Cos. } px \frac{dx}{x^4 \sqrt{x}} = \frac{8}{105} \sqrt{q^7 - 21p^2 q^5 + 35p^4 q^3 - 7p^6 q + \sqrt{(p^3 + q^2)^7}} \cdot \sqrt{2} \pi$$

Sur ces intégrales 1) à 16) voyez Oettinger, Cr. 38. 216.

F. Alg. irrat. fract. à autre dén.

Exp.

Circ. Dir.

TABLE 399 suite.

Lim. 0 et ∞ .

$$17) \int e^{-a\sqrt{\frac{1}{2}x}} \frac{\{b + \sqrt{\frac{1}{2}x}\} \text{Cos.} \{a\sqrt{\frac{1}{2}x}\} - \sqrt{\frac{1}{2}x} \cdot \text{Sin.} \{a\sqrt{\frac{1}{2}x}\}}{x + b\sqrt{2x + b^2}} dx = 0$$

$$18) \int e^{-a\sqrt{\frac{1}{2}x}} \frac{\{b + \sqrt{\frac{1}{2}x}\} \text{Cos.} \{a\sqrt{\frac{1}{2}x}\} - \sqrt{\frac{1}{2}x} \cdot \text{Sin.} \{a\sqrt{\frac{1}{2}x}\}}{x + b\sqrt{2x + b^2}} \frac{dx}{c^2 - x^2} =$$

$$= \frac{\{b + \sqrt{\frac{1}{2}c}\} \text{Sin.} \{a\sqrt{\frac{1}{2}c}\} + \sqrt{\frac{1}{2}c} \cdot \text{Cos.} \{a\sqrt{\frac{1}{2}c}\}}{c + b\sqrt{2c + b^2}} \cdot \frac{\pi e^{-a\sqrt{\frac{1}{2}c}}}{2c}$$

Les intégrales 17) et 18) se trouvent: Poisson, Chal. 159.

F. Alg.

Exp.

Circ. Dir.

TABLE 400.

Lim. diverses.

$$1) \int_{-\infty}^{\infty} e^{-px^2 + 2qx \text{Cos.} \lambda} x \text{Sin.} (2qx \text{Sin.} \lambda) dx = \frac{q\pi}{p} e^{p \frac{q^2}{\text{Cos.} 2\lambda}} \cdot \text{Sin.} \left(\lambda + \frac{q^2}{p} \text{Sin.} 2\lambda \right) \cdot \sqrt{\frac{\pi}{p}} \left. \begin{array}{l} \text{Dienger, Cr.} \\ 46. 119. \end{array} \right\}$$

$$2) \int_{-\infty}^{\infty} e^{-px^2 + 2qx \text{Cos.} \lambda} x \text{Cos.} (2qx \text{Sin.} \lambda) dx = \frac{q\pi}{p} e^{p \frac{q^2}{\text{Cos.} 2\lambda}} \cdot \text{Cos.} \left(\lambda + \frac{q^2}{p} \text{Sin.} 2\lambda \right) \cdot \sqrt{\frac{\pi}{p}}$$

$$3) \int_{-\infty}^{\infty} e^{-px^2} x \text{Sin.} qx dx = \frac{q}{2p} e^{-\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}} \quad \text{Ohm, Ausw. 21.}$$

$$4) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{(a+ix)(2 \text{Cos.} x)^{a-1}} x dx = \frac{\pi i}{2a}$$

$$5) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-a \text{Sin.} ix} \sqrt{(2 \text{Cos.} x)} \text{Cos.} \left(a \text{Cos.} \frac{1}{2} x \cdot \sqrt{(2 \text{Cos.} x)} - \frac{1}{2} x \right) \frac{dx}{\sqrt{(2 \text{Cos.} x)}} = \pi \text{Cos.} a \left. \begin{array}{l} \\ \text{Kummer, Cr. 20. 1.} \end{array} \right\}$$

$$6) \int_0^a \left\{ \text{Cos.} x - \frac{e^{bcx} + e^{-bcx}}{2} \right\} \frac{dx}{x} = lbc + \text{Ci.} \left(\frac{a}{c} \right) - \frac{1}{2} \text{Ei.} (ab) - \frac{1}{2} \text{Ei.} (-ab) \quad \text{Arndt, Gr. 11. 70.}$$

F. Algèbr.

Exp.

Circ. Inv.

TABLE 401.

Lim. 0 et ∞ .

$$1) \int e^{-px} \text{Arctang.} \frac{x}{q} dx = \frac{1}{p^2} \left[\text{Ci.} (pq) \cdot \text{Sin.} pq - \left\{ \text{Si.} (pq) - \frac{1}{2} \pi \right\} \text{Cos.} pq - pq \left\{ \text{Ci.} (pq) \cdot \text{Cos.} pq \right. \right.$$

$$\left. \left. + \left(\text{Si.} (pq) - \frac{1}{2} \pi \right) \text{Sin.} pq \right\} \right]$$

$$2) \int e^{-px} \text{Arctang.} \frac{x}{q} x^{2a} dx = \frac{1}{p^{2a+1}} \left[\left\{ Ci.(pq). \sin pq - \left(Si.(pq) - \frac{1}{2} \pi \right) \cos pq \right\} \frac{1^{2a+1}}{1^{2a+1}} \sum_0^a \frac{(-p^2 q^2)^n}{1^{2n+1}} - \right. \\ \left. - pq \left\{ Ci.(pq). \cos pq + \left(Si.(pq) - \frac{1}{2} \pi \right) \sin pq \right\} \frac{1^{2a+1}}{1^{2n+1}} \sum_0^{a-1} \frac{(-p^2 q^2)^n}{1^{2n+1}} + \right. \\ \left. + 3^{2a-2+1} pq \sum_1^a \left\{ \frac{1}{1^{2n+1}} \sum_0^{n-1} 1^{2n-2m+1} (-p^2 q^2)^m \right\} + 4^{2a-3+1} pq \sum_1^a \left\{ \frac{1}{1^{2n+1}} \sum_0^{n-1} 1^{2n-2m-1+1} (-p^2 q^2)^m \right\} \right]$$

$$3) \int e^{-px} \text{Arctang.} \frac{x}{q} x^{2a+1} dx = \frac{1}{p^{2a+2}} \left[\left\{ Ci.(pq). \sin pq - \left(Si.(pq) - \frac{1}{2} \pi \right) \cos pq \right\} \frac{1^{2a+1+1}}{1^{2a+1}} \sum_0^a \frac{(-p^2 q^2)^n}{1^{2n+1}} - \right. \\ \left. - pq \left\{ Ci.(pq). \cos pq + \left(Si.(pq) - \frac{1}{2} \pi \right) \sin pq \right\} \frac{1^{2a+1+1}}{1^{2n+1}} \sum_0^a \frac{(-p^2 q^2)^n}{1^{2n+1}} + \right. \\ \left. + 3^{2a-1+1} pq \sum_1^{a+1} \left\{ \frac{1}{1^{2n+1+1}} \sum_0^{n-1} 1^{2n-2m+1+1} (-p^2 q^2)^m \right\} + 4^{2a-2+1} pq \sum_1^{a+1} \left\{ \frac{1}{1^{2n+1}} \sum_0^{n-1} 1^{2n-2m+1} (-p^2 q^2)^m \right\} \right]$$

$$4) \int e^{-px} \text{Arctang.} \frac{x}{q} \frac{px + pq + 1}{(x+q)^2} dx = \frac{1}{2q} \left[-e^{pq} Ei.(-pq) + Ci.(pq). \sin pq - \right. \\ \left. - \left(Si.(pq) - \frac{1}{2} \pi \right) \cos pq + Ci.(pq). \cos pq + \left(Si.(pq) - \frac{1}{2} \pi \right) \sin pq \right]$$

$$5) \int e^{-px} \text{Arctang.} \frac{x}{q} \frac{px - pq + 1}{(x-q)^2} dx = \frac{1}{2q} \left[-e^{-pq} Ei.(pq) - Ci.(pq). \sin pq + \right. \\ \left. + \left(Si.(pq) - \frac{1}{2} \pi \right) \cos pq + Ci.(pq). \cos pq + \left(Si.(pq) - \frac{1}{2} \pi \right) \sin pq \right]$$

$$6) \int e^{-px} \text{Arctang.} \frac{x(pq+1)x + pq^2 + 2q}{q(x+q)^2} dx = \frac{1}{2p} \left[pq e^{pq} Ei.(-pq) + (pq+2) \left\{ Ci.(pq). \sin pq - \right. \right. \\ \left. \left. - \left(Si.(pq) - \frac{1}{2} \pi \right) \cos pq \right\} - pq \left\{ Ci.(pq). \cos pq + \left(Si.(pq) - \frac{1}{2} \pi \right) \sin pq \right\} \right]$$

$$7) \int e^{-px} \text{Arctang.} \frac{x(pq-1)x - pq^2 + 2q}{q(x-q)^2} dx = \frac{1}{2p} \left[-pq e^{-pq} Ei.(pq) + (pq-2) \left\{ Ci.(pq). \sin pq - \right. \right. \\ \left. \left. - \left(Si.(pq) - \frac{1}{2} \pi \right) \cos pq \right\} + pq \left\{ Ci.(pq). \cos pq + \left(Si.(pq) - \frac{1}{2} \pi \right) \sin pq \right\} \right]$$

$$8) \int e^{-px} \text{Arctang.} \frac{x}{q} \frac{p(q^2+x^2) \text{Arctang.} \frac{x}{q} - 2q}{x^2+q^2} dx = 0$$

$$9) \int e^{-px} \operatorname{Arctang} \frac{x}{q} \frac{px^2 + 2x + pq^2}{(x^2 + q^2)^2} dx = \frac{1}{2q^2} \left[\operatorname{Ci.}(pq) \cdot \operatorname{Sin.}pq - \left(\operatorname{Si.}(pq) - \frac{1}{2}\pi \right) \operatorname{Cos.}pq + \right. \\ \left. + pq \left\{ \operatorname{Ci.}(pq) \cdot \operatorname{Cos.}pq + \left(\operatorname{Si.}(pq) - \frac{1}{2}\pi \right) \operatorname{Sin.}pq \right\} \right]$$

$$10) \int e^{-px} \operatorname{Arctg} \frac{px^3 + x^2 + pq^2x - q^2}{q} dx = \frac{1}{2q} \left[1 - pq \left\{ \operatorname{Ci.}(pq) \cdot \operatorname{Sin.}pq - \left(\operatorname{Si.}(pq) - \frac{1}{2}\pi \right) \operatorname{Cos.}pq \right\} \right]$$

Sur ces intégrales 1) à 10) voyez Bierens de Haan, Verh. d. K. Ak. v. Wet. 1854, bl. 19.

$$11) \int \frac{(2\pi x - 1)e^{2\pi x} + 1}{(e^{2\pi x} - 1)^2} \operatorname{Arctang} \frac{x}{q} dx = -\frac{1}{4} + \frac{q}{2} lq - \frac{1}{2} q Z'(q) \quad \text{V. T. 138. N}^\circ. 9.$$

$$12) \int \frac{(\pi x - 1)e^{\pi x} + (\pi x + 1)e^{-\pi x}}{(e^{\pi x} - e^{-\pi x})^2} \operatorname{Arctang} x dx = \frac{1}{2} \left(l2 - \frac{1}{2} \right) \quad \text{V. T. 138. N}^\circ. 12.$$

$$13) \int \frac{e^{-2\pi x} + 2\pi x - 1}{(e^{\pi x} - e^{-\pi x})^2} \operatorname{Arctang} x dx = \frac{1}{2} A - \frac{1}{4} \quad \text{V. T. 138. N}^\circ. 10.$$

$$14) \int \frac{e^{-2qx} + 2qx - 1}{(e^{qx} - e^{-qx})^2} \operatorname{Arctang} x dx = \frac{1}{2} l \frac{q}{\pi} + \frac{\pi}{4q} - \frac{1}{2} Z' \left(\frac{\pi + q}{\pi} \right) \quad \text{V. T. 138. N}^\circ. 11.$$

$$15) \int \frac{\pi x (e^{l\pi x} + e^{-l\pi x}) - 4(e^{l\pi x} - e^{-l\pi x})}{(e^{l\pi x} - e^{-l\pi x})^2} \operatorname{Arctang} x dx = \pi l' 2 - 4 + \sqrt{2} l \frac{2+1}{\sqrt{2}-1} \quad \text{V. T. 138. N}^\circ. 18.$$

$$16) \int \frac{\pi x (e^{l\pi x} + e^{-l\pi x}) - 2(e^{l\pi x} - e^{-l\pi x})}{(e^{l\pi x} - e^{-l\pi x})^2} \operatorname{Arctang} x dx = \frac{1}{2} \pi - 1 \quad \text{V. T. 138. N}^\circ. 16.$$

$$17) \int \frac{(x^2 - q^2)e^{-2\pi x} + 2\pi x^3 - x^2 + 2\pi q^2 x + q^2}{(e^{2\pi x} - 1)^2} \operatorname{Arctang} \frac{x}{q} \frac{dx}{(q^2 + x^2)^2} = \frac{1}{4q^3} \sum_0^{\infty} \frac{B_{2n+1}}{q^{2n}} \quad \text{V. T. 138. N}^\circ. 20.$$

$$18) \int e^{-(\operatorname{Arctang} x)^2} (\operatorname{Arctang} x)^{2a} \frac{dx}{1+x^2} = \left(\frac{1}{2}\pi \right)^{2a+1} \sum_0^{\infty} \frac{1}{1n! (2a+2n+1)} \left(-\frac{1}{4}\pi^2 \right)^n \quad \text{Helmling, Transf.}$$

$$1) \int e^{-x} \operatorname{li.}(e^x) \cdot x^{p-1} dx = -\pi \operatorname{Cot.} p \pi \cdot \Gamma(p) \quad \left. \vphantom{\int} \right\} , \quad 0 \leq p \leq 1;$$

$$2) \int e^x \operatorname{li.}(e^{-x}) \cdot x^{p-1} dx = -\pi \operatorname{Cosec.} p \pi \cdot \Gamma(p) \quad \left. \vphantom{\int} \right\} \quad \text{Schlömilch, Beitr. III. § 6, 7.}$$

$$\begin{aligned}
 3) \int li.(e^{-x}).x^{p-1} dx &= -\frac{1}{p} \Gamma(p) \\
 4) \int e^{-qx} li.(e^{-x}) \frac{dx}{\sqrt{x}} &= -2 \sqrt{\frac{\pi}{q}} l \left\{ \sqrt{q} + \sqrt{1+q} \right\} \\
 5) \int e^{qx} li.(e^{-x}) \frac{dx}{\sqrt{x}} &= -2 \operatorname{Arcsin}(\sqrt{q}). \sqrt{\frac{\pi}{q}}, \quad q < 1; \quad \text{V. T. 300. N}^\circ 5.
 \end{aligned}
 \left. \vphantom{\int} \right\} \begin{aligned} &, \quad 0 \leq p \leq 1, \quad 0 < q < 1; \\ &\text{Schlömilch, Beitr. III. § 6, 7. où 4) fautive.} \end{aligned}$$

$$\begin{aligned}
 1) \int \operatorname{Sin.}(p l x).x^{q-1} dx &= \frac{-p}{p^2 + q^2} \quad \text{V. T. 278. N}^\circ 8. \\
 2) \int \operatorname{Cos.}(p l x).x^{q-1} dx &= \frac{q}{p^2 + q^2} \quad \text{V. T. 278. N}^\circ 9. \\
 3) \int \operatorname{Sin.}(q l x).(l x)^{a-1}.x^{p-1} dx &= (-1)^a \frac{1^{a-1/1}}{(p^2 + q^2)^{1/2}} \operatorname{Sin.}\left(a \operatorname{Arctang.} \frac{q}{p}\right) \quad \text{V. T. 386. N}^\circ 12 \\
 4) \int \operatorname{Cos.}(q l x).(l x)^{a-1}.x^{p-1} dx &= (-1)^{a-1} \frac{1^{a-1/1}}{(p^2 + q^2)^{1/2}} \operatorname{Cos.}\left(a \operatorname{Arctang.} \frac{q}{p}\right) \quad \text{V. T. 386. N}^\circ 13 \\
 5) \int \operatorname{Sin.}^{2a}(l x).x^{p-1} dx &= \frac{1^{2a/1}}{p^2 + (2a)^2.p^2 + (2a-2)^2 \dots p^2 + 2^2} \frac{1}{p} \quad \text{V. T. 279. N}^\circ 1. \\
 6) \int \operatorname{Sin.}^{2a+1}(l x).x^{p-1} dx &= \frac{-1^{2a+1/1}}{p^2 + (2a+1)^2.p^2 + (2a-1)^2 \dots p^2 + 1^2} \quad \text{V. T. 279. N}^\circ 2. \\
 7) \int \operatorname{Cos.}^{2a}(l x).x^{p-1} dx &= \frac{1}{p} \frac{1^{2a/1}}{p^2 + (2a)^2.p^2 + (2a-2)^2 \dots p^2 + 2^2} \left\{ 1 + \frac{p^2}{1.2} + \frac{p^2}{1.2} \frac{p^2 + 2^2}{3.4} + \right. \\
 &\quad \left. + \dots + \frac{p^2.p^2 + 2^2 \dots p^2 + (2a-2)^2}{1^{2a/1}} \right\} \quad \text{V. T. 279. N}^\circ 3 \\
 8) \int \operatorname{Cos.}^{2a+1}(l x).x^{p-1} dx &= \frac{1}{p} \frac{1^{2a+1/1}}{p^2 + (2a+1)^2.p^2 + (2a-1)^2 \dots p^2 + 1^2} \left\{ 1 + \frac{p^2}{1} + \frac{p^2}{1} \frac{p^2 + 1^2}{2.3} + \right. \\
 &\quad \left. + \dots + \frac{p^2.p^2 + 1^2 \dots p^2 + (2a-1)^2}{1^{2a+1/1}} \right\} \quad \text{V. T. 279. N}^\circ 4. \\
 9) \int \operatorname{Sin.}(q l x).ll \frac{1}{x}.x^{p-1} dx &= \frac{1}{p^2 + q^2} \left\{ -p \operatorname{Arctang.} \frac{q}{p} + \frac{1}{2} q l (p^2 + q^2) + q \Lambda \right\} \quad \text{V. T. 439. N}^\circ 2
 \end{aligned}$$

$$10) \int \cos.(qlx). l \frac{1}{x}. x^{p-1} dx = \frac{1}{p^2 + q^2} \left\{ q \operatorname{Arctang.} \frac{q}{p} + \frac{1}{2} p l (p^2 + q^2) + p A \right\} \quad \text{V. T. 439. N}^\circ. 3.$$

$$11) \int \cot.(qlx). x^{p-1} dx = 4q \sum_1 \frac{n}{p^2 + 4n^2 q^2} \quad \text{V. T. 278. N}^\circ. 12.$$

$$12) \int \sin.\{(qlx)^2\}. x^{2p-1} dx = \frac{1}{4q} \left\{ \cos.\left(\frac{p^2}{q^2}\right) + \sin.\left(\frac{p^2}{q^2}\right) \right\} \sqrt{2\pi} - \frac{p}{q^2} \left\{ \cos.\left(\frac{p^2}{q^2}\right) \cdot \sum_0 \frac{(-1)^n}{(4n+1) 1^{2n+1}} \left(\frac{p}{q}\right)^{4n+1} + \sin.\left(\frac{p^2}{q^2}\right) \cdot \sum_0 \frac{(-1)^n}{(4n-1) 1^{2n-1}} \left(\frac{p}{q}\right)^{4n-2} \right\} \quad \text{V. T. 279. N}^\circ. 18.$$

$$13) \int \cos.\{(qlx)^2\}. x^{2p-1} dx = \frac{1}{4q} \left\{ \cos.\left(\frac{p^2}{q^2}\right) - \sin.\left(\frac{p^2}{q^2}\right) \right\} \sqrt{2\pi} - \frac{p}{q^2} \left\{ \sin.\left(\frac{p^2}{q^2}\right) \cdot \sum_0 \frac{(-1)^n}{(4n+1) 1^{2n+1}} \left(\frac{p}{q}\right)^{4n+1} - \cos.\left(\frac{p^2}{q^2}\right) \cdot \sum_0 \frac{(-1)^n}{(4n-1) 1^{2n-1}} \left(\frac{p}{q}\right)^{4n-2} \right\} \quad \text{V. T. 279. N}^\circ. 19.$$

$$14) \int \sin.\{p^2 - (lx)^2\}. x^{2p-1} dx = -\frac{1}{2} \sqrt{\frac{\pi}{2}} - p^3 \sum_0 \left\{ \frac{p^2 \cos.(2p^2)}{2n(4n+1)} + \frac{\sin.(2p^2)}{4n-1} \right\} \frac{(-p^4)^{n-1}}{1^{2n-1}} \quad \text{V. T. 403. N}^\circ. 12, 13.$$

$$15) \int \cos.\{p^2 - (lx)^2\}. x^{2p-1} dx = \frac{1}{2} \sqrt{\frac{\pi}{2}} - \frac{1}{p} \sum_0 \left\{ \frac{p^2 \sin.(2p^2)}{2n(4n+1)} - \frac{\cos.(2p^2)}{4n-1} \right\} \frac{(-p^4)^n}{1^{2n+1}} \quad \text{V. T. 403. N}^\circ. 12, 13.$$

$$16) \int \sin.^a(lx). x^{b-1} dx = \frac{(-1)^a - e^{b\pi}}{\Gamma\left(\frac{a+bi}{2} + 1\right) \Gamma\left(\frac{a-bi}{2} + 1\right)} \cdot \frac{\pi}{2a} 1^{a/1} e^{b\pi} \quad \text{V. T. 280. N}^\circ. 21.$$

$$17) \int \cos.\left(q l \frac{1}{x}\right). x^{p-1} dx = \frac{1}{p} \sum_0 (-1)^n \frac{1}{(n+1)^{n/1}} \left(\frac{q^2}{p}\right)^n \quad \text{V. T. 388. N}^\circ. 13.$$

$$18) \int \sin.(\sin. \lambda. lx). (lx)^{p-1}. x^{\cos. \lambda - 1} dx = (-1)^p \sin. p \lambda. \Gamma(p) \quad \text{V. T. 390. N}^\circ. 1.$$

$$19) \int \cos.(\sin. \lambda. lx). (lx)^{p-1}. x^{\cos. \lambda - 1} dx = (-1)^{p-1} \cos p \lambda. \Gamma(p) \quad \text{V. T. 390. N}^\circ. 2.$$

$$20) \int l \sin.\left(q l \frac{1}{x}\right). x^{p-1} dx = \frac{1}{2p} l \frac{1}{2} - \frac{p}{2} \sum_1 \frac{1}{n p^2 + n^2 q^2} \quad \text{V. T. 439. N}^\circ. 6.$$

$$21) \int l \cos.(qlx). x^{p-1} dx = -\frac{1}{2p} l 2 + \frac{p}{2} \sum_1 \frac{(-1)^{n-1}}{n} \frac{1}{p^2 + n^2 q^2} \quad \text{V. T. 439. N}^\circ. 7.$$

$$22) \int l \operatorname{Tang.}\left(q l \frac{1}{x}\right). x^{p-1} dx = -p \sum_1 \frac{1}{2n-1} \frac{1}{p^2 + (2n-1)^2 q^2} \quad \text{V. T. 439. N}^\circ. 8.$$

$$1) \int \text{Sin.}(p l x) \frac{dx}{1+x} = \frac{\pi e^{p\pi}}{e^{2p\pi} - 1} - \frac{1}{2p} \quad \text{V. T. 404. N}^\circ. 10, 11.$$

$$2) \int \text{Sin.}(p l x) \frac{dx}{1-x} = -\frac{1}{2} \pi \frac{e^{2p\pi} + 1}{e^{2p\pi} - 1} + \frac{1}{2p} \quad \text{V. T. 281. N}^\circ. 9.$$

$$3) \int \text{Cos.}(p l x) \frac{dx}{1-x} = \infty \quad \text{V. T. 281. N}^\circ. 3.$$

$$4) \int \text{Sin.}(p l x) \frac{x^{n-1} dx}{1-x} = -\frac{1}{2} \pi + \frac{1}{2p} + \frac{\pi}{1-e^{2p\pi}} + \sum_0^{n-2} \frac{p}{p^2 + (n+1)^2} \quad \text{V. T. 281. N}^\circ. 11.$$

$$5) \int \text{Sin.}(p l x) \frac{x^{q-1} dx}{1-x} = q - \frac{1}{2p} \text{Sin. } q - \frac{\infty}{1} \frac{\text{Sin. } 2nq \cdot \text{Sin. } 2np}{2np^{2n}} B_{2n-1}, \text{ où } \text{Cot. } q = \frac{q-1}{p}; \quad \text{V. T. 282. N}^\circ. 15.$$

$$6) \int \text{Sin.}(p l x) \frac{l x}{1+x^2} dx = \frac{1}{4} \pi^2 \frac{e^{4p\pi} - e^{-4p\pi}}{(e^{4p\pi} + e^{-4p\pi})^2} \quad \text{V. T. 391. N}^\circ. 5.$$

$$7) \int \text{Cos.}(p l x) \frac{dx}{1+x^2} = \frac{1}{2} \pi \frac{e^{4p\pi}}{e^{4p\pi} + 1} \quad \text{V. T. 281. N}^\circ. 4.$$

$$8) \int \text{Sin.}(p l x) \frac{x^q - x^{-q}}{1+x^2} dx = \pi \text{Sin. } \frac{1}{2} \pi q \frac{e^{4p\pi} - e^{-4p\pi}}{e^{4p\pi} + e^{-4p\pi} + 2 \text{Cos. } q \pi}, p^2 < 1, q^2 < 1; \quad \text{V. T. 282. N}^\circ. 6.$$

$$9) \int \text{Cos.}(p l x) \frac{x^q + x^{-q}}{1+x^2} dx = \pi \text{Cos. } \frac{1}{2} \pi q \frac{e^{4p\pi} + e^{-4p\pi}}{e^{4p\pi} + e^{-4p\pi} + 2 \text{Cos. } q \pi}, p^2 < 1, q^2 < 1; \quad \text{V. T. 282. N}^\circ. 4.$$

$$10) \int \text{Sin.}(p l x) \frac{dx}{1-x^2} = \frac{1}{4} \pi \frac{1-e^{p\pi}}{1+e^{p\pi}} \quad \text{V. T. 281. N}^\circ. 8.$$

$$11) \int \text{Sin.}(p l x) \frac{x}{1-x^2} dx = \frac{1}{4} \pi \frac{1+e^{p\pi}}{1-e^{p\pi}} + \frac{1}{2p} \quad \text{V. T. 281. N}^\circ. 9.$$

$$12) \int \text{Sin.}(p l x) \frac{x^{q-1}}{1-x^2} dx = -\frac{\infty}{1} \frac{p}{(2n+q)^2 + p^2} \quad \text{V. T. 282. N}^\circ. 13.$$

$$13) \int \text{Sin.}(p l x) \frac{x^q + x^{-q}}{1-x^2} dx = -\frac{\pi}{2} \frac{e^{p\pi} - e^{-p\pi}}{e^{p\pi} + e^{-p\pi} + 2 \text{Cos. } q \pi}, q^2 \leq 1; \quad \text{V. T. 282. N}^\circ. 10.$$

$$14) \int \text{Cos.}(p l x) \frac{l x}{1+x^2} dx = -\frac{1}{2} \pi^2 \frac{e^{p\pi}}{(e^{p\pi} + 1)^2} \quad \text{V. T. 391. N}^\circ. 6.$$

$$15) \int \text{Cos.}(p l x) \frac{x^q - x^{-q}}{1-x^2} dx = -\frac{\pi}{2} \frac{\text{Sin. } \pi q}{e^{p\pi} + e^{-p\pi} + 2 \text{Cos. } q \pi} \quad \text{V. T. 282. N}^\circ. 16.$$

F. Alg. rat. fract. à dén. binôme.

Log.

TABLE 404 suite.

Lim. 0 et 1.

Circ. Dir. de Log.

$$16) \int \text{Sin.}(p l x) \frac{dx}{(1-x^2)x^{q+1}} = - \frac{\pi}{1} \frac{p}{(2n-q)^2 + p^2} \quad \text{V. T. 282. N}^\circ. 14.$$

$$17) \int \text{Sin.}^2(p l x) \frac{dx}{1+x^2} = \frac{\pi}{8} \frac{(e^{p\pi} - 1)^2}{e^{2p\pi} + 1} \quad \text{V. T. 281. N}^\circ. 12.$$

$$18) \int \text{Cos.}^2(p l x) \frac{dx}{1+x^2} = \frac{\pi}{8} \frac{(e^{p\pi} + 1)^2}{e^{2p\pi} + 1} \quad \text{V. T. 281. N}^\circ. 13.$$

$$19) \int \text{Cos.}(p l x).l(1+x) \frac{dx}{x} = \frac{1}{2p^2} - \frac{\pi}{p} \frac{e^{p\pi}}{e^{2p\pi} - 1} \quad \text{V. T. 404. N}^\circ. 1.$$

$$20) \int \text{Cos.}(p l x).l(1-x) \frac{dx}{x} = \frac{1}{2p^2} - \frac{\pi}{2p} \frac{e^{2p\pi} + 1}{e^{2p\pi} - 1} \quad \text{V. T. 404. N}^\circ. 2.$$

$$21) \int \text{Cos.}(p l x).l(1-x^2) \frac{dx}{x} = \frac{1}{p^2} + \frac{\pi}{2p} \frac{1 + e^{p\pi}}{1 - e^{p\pi}} \quad \text{V. T. 404. N}^\circ. 4.$$

F. Alg. fract. à autre dén.

Log.

TABLE 405.

Lim. 0 et 1.

Circ. Dir. de Log.

$$1) \int \text{Sin.}(p l x) \frac{1-x}{1+x} \frac{dx}{x} = \frac{-2\pi}{e^{p\pi} - e^{-p\pi}} \quad \text{V. T. 282. N}^\circ. 2.$$

$$2) \int \text{Sin.}(p l x) \frac{1+x}{1-x} \frac{dx}{x} = -\pi \frac{e^{2p\pi} + 1}{e^{2p\pi} - 1} \quad \text{V. T. 282. N}^\circ. 9.$$

$$3) \int \text{Cos.}(p l x) \frac{1-x}{1+x} \frac{dx}{x} = 2\pi^2 e^{-p\pi} \frac{1 + e^{-2p\pi}}{(1 - e^{-2p\pi})^2} \quad \text{V. T. 391. N}^\circ. 4.$$

$$4) \int \text{Sin.}(p l x) \frac{1+x^2}{1-x^2} \frac{dx}{x} = \frac{1}{2} \pi \frac{1 + e^{p\pi}}{1 - e^{p\pi}} \quad \text{V. T. 282. N}^\circ. 8.$$

$$5) \int \text{Cos.}(p l x) \frac{1+x^2}{1-x^2} \frac{dx}{x} = \pi^2 \frac{e^{p\pi}}{(1 - e^{p\pi})^2} \quad \text{V. T. 391. N}^\circ. 3.$$

$$6) \int \frac{\text{Cos.}(q l x)}{(1+x^p)^2} x^{p-1} dx = \frac{1}{p^2} \frac{\pi q}{e^{\frac{\pi q}{p}} - e^{-\frac{\pi q}{p}}} \quad \text{Euler, N. A. Petr. III. 3.}$$

$$7) \int \frac{\text{Cos.}(p l x)}{1 + 2x \text{Cos.}\lambda + x^2} dx = \frac{1}{2} \pi \text{Cosec.}\lambda \frac{e^{p\lambda} - e^{-p\lambda}}{e^{p\pi} - e^{-p\pi}} \quad \text{V. T. 284. N}^\circ. 1.$$

F. Alg. fract. à autre dén.

Log.

TABLE 405 suite.

Lim. 0 et 1.

Circ. Dir. de Log.

- 8) $\int \text{Sin.}(q l x) \frac{1 - x^{2p}}{1 + 2 x^{2p} \text{Cos.}(2 q l x) + x^{4p}} x^{p-1} dx = -\frac{\pi}{4} \frac{q}{p^2 + q^2}$ V. T. 284. N°. 6.
- 9) $\int \text{Cos.}(q l x) \frac{1 + x^{2p}}{1 + 2 x^{2p} \text{Cos.}(2 q l x) + x^{4p}} x^{p-1} dx = \frac{\pi}{4} \frac{p}{p^2 + q^2}$ V. T. 284. N°. 7.
- 10) $\int \text{Cos.}(q l x) \frac{l x}{x^p + 2 \text{Cos.} \lambda + x^{-p}} \frac{dx}{x} = \frac{\pi}{2 p} \text{Cosec.} \lambda \frac{e^{\frac{q \lambda}{p}} - e^{-\frac{q \lambda}{p}}}{e^{\frac{q \pi}{p}} - e^{-\frac{q \pi}{p}}}, \lambda < \pi$; Euler, N. A. Petr. 1787. 3. (où faut.) — Poisson, P. 18. 295. N°. 28.
- 11) $\int \text{Cos.}(p l x) \frac{1 + x^2}{1 + 2 x \text{Cos.} \lambda + x^2} \frac{dx}{x} = -\pi \text{Cot.} \lambda \frac{e^{p \lambda} - e^{-p \lambda}}{e^{p \pi} - e^{-p \pi}}$ V. T. 284. N°. 4.
- 12) $\int \text{Sin.}(p l x) \frac{1 - x^2}{1 + 2 x \text{Cos.} \lambda + x^2} \frac{dx}{x} = -\pi \frac{e^{p \lambda} + e^{-p \lambda}}{e^{p \pi} - e^{-p \pi}}$ V. T. 284. N°. 5.
- 13) $\int \frac{\text{Cos.}(q l x)}{x^p + \left(a + \frac{1}{a}\right) + x^{-p}} \frac{dx}{x} = \frac{2}{p} \frac{a \pi}{1 - a^2} \frac{\text{Sin.}\left(\frac{q}{p} l a\right)}{\frac{q \pi}{e^p} - e^{-\frac{q \pi}{p}}}$ Euler, N. A. Petr. III. 3.
- 14) $\int \text{Sin.}(q l x) \frac{x^p - x^{1-p}}{1 - x} \frac{dx}{x} = -\pi \frac{e^{2q\pi} - e^{-2q\pi}}{e^{2q\pi} - 2 \text{Cos.} 2 p \pi + e^{-2q\pi}}, p < 1$; V. T. 282. N°. 12.
- 15) $\int \text{Cos.}(p l x) \frac{dx}{(1+x)\sqrt{x}} = \frac{\pi}{e^{p\pi} + e^{-p\pi}}$ V. T. 281. N°. 6.

F. Alg. rat.

Log. en dén. $l x$.

TABLE 406.

Lim. 0 et 1

Circ. Dir. de Log.

- 1) $\int \text{Sin.}(p l x) \frac{x^a}{l x} dx = \text{Arctang.} \left(\frac{p}{a+1} \right)$ Euler, N. C. Petr. 20. 59.
- 2) $\int \text{Sin.} \left(q \sqrt[l]{x} \right) x^{p-1} \frac{dx}{l x} = q \sqrt[l]{\frac{\pi}{p}} \cdot \sum_0^\infty \frac{(-1)^{n-1}}{(2n+1) 1^{n+1}} \left(\frac{q^2}{4p} \right)^n$ V. T. 392. N°. 15.
- 3) $\int \text{Sin.}(l x) \frac{1+x}{l x} x dx = \frac{1}{4} \pi$ Euler, N. C. Petr. 20. 59.
- 4) $\int \frac{x^{p-1} \text{Sin.}(r l x) - x^{q-1} \text{Sin.}(s l x)}{l x} dx = \text{Arctang.} \left(\frac{q r - p s}{p q - r s} \right)$ V. T. 393. N°. 18.
- 5) $\int \text{Cos.}(q l x) x^{p-1} \frac{dx}{l x} = x$ V. T. 392. N°. 4.

$$6) \int \frac{x^{p-1} \text{Cos.}(rlx) - x^{q-1} \text{Cos.}(slx)}{lx} dx = \frac{1}{2} l \frac{p^2 + r^2}{q^2 + s^2} \quad \text{V. T. 393. N}^\circ. 19.$$

$$7) \int \text{Sin.}^2(qlx). x^{a-1} \frac{dx}{lx} = \frac{1}{4} l \frac{a^2}{a^2 + q^2} \quad \text{V. T. 392. N}^\circ. 10.$$

$$8) \int \text{Sin.}(qlx). x^{a-1} \frac{dx}{(lx)^2} = -\infty \quad \text{V. T. 394. N}^\circ. 1.$$

$$9) \int \text{Cos.}(qlx). x^{a-1} \frac{dx}{(lx)^2} = \infty \quad \text{V. T. 394. N}^\circ. 3.$$

$$10) \int \text{Sin.}^2(qlx). x^{p-1} \frac{dx}{(lx)^2} = \frac{1}{2} q \text{Arctang.} \frac{q}{p} - \frac{1}{4} p l \frac{p^2 + q^2}{p^2} \quad \text{V. T. 394. N}^\circ. 4.$$

$$11) \int \text{Sin.}(qlx). x^{b-1} \frac{dx}{(lx)^{a+1}} = (-1)^a (b^2 + q^2)^{1/2} \Gamma(-a) \text{Sin.} \left\{ a \text{Arctang.} \frac{q}{b} \right\} \quad \text{V. T. 394. N}^\circ. 11.$$

$$12) \int \text{Cos.}(qlx). x^{b-1} \frac{dx}{(lx)^{a+1}} = (-1)^{a+1} (b^2 + q^2)^{1/2} \Gamma(-a) \text{Cos.} \left\{ a \text{Arctang.} \frac{q}{b} \right\} \quad \text{V. T. 394. N}^\circ. 15.$$

$$13) \int \frac{x^{p-1} \text{Sin.}(rlx) - x^{q-1} \text{Sin.}(slx)}{(lx)^{a+1}} dx = \\ = (-1)^{a+1} \frac{\Gamma(1-a)}{a} \left[(q^2 + s^2)^{1/2} \text{Sin.} \left(a \text{Arctang.} \frac{s}{q} \right) - (p^2 + r^2)^{1/2} \text{Sin.} \left(a \text{Arctang.} \frac{r}{p} \right) \right] \quad \text{V. T. 394. N}^\circ. 22.$$

$$14) \int \frac{x^{p-1} \text{Cos.}(rlx) - x^{q-1} \text{Cos.}(slx)}{(lx)^{a+1}} dx = \\ = (-1)^{a+1} \frac{\Gamma(1-a)}{a} \left[(q^2 + s^2)^{1/2} \text{Cos.} \left(a \text{Arctang.} \frac{s}{q} \right) - (p^2 + r^2)^{1/2} \text{Cos.} \left(a \text{Arctang.} \frac{r}{p} \right) \right] \quad \text{V. T. 394. N}^\circ. 23.$$

$$15) \int \frac{\text{Sin.}(2plx)}{lx} \frac{dx}{1+x^2} = \text{Arctang.}(e^{p\pi}) \quad \text{V. T. 396. N}^\circ. 1.$$

$$16) \int \frac{\text{Cos.}(2plx)}{lx} \frac{dx}{1-x^2} = \frac{1}{2} l (e^{p\pi} + e^{-p\pi}) \quad \text{V. T. 396. N}^\circ. 2.$$

$$17) \int \frac{\text{Cos.}(2plx)}{xlx} \frac{1-x^2}{1+x^2} dx = l \frac{1-e^{-p\pi}}{1+e^{-p\pi}} \quad \text{V. T. 396. N}^\circ. 3.$$

$$18) \int \frac{\text{Cos.}(2plx)}{xlx} \frac{1+x^2}{1-x^2} dx = l (e^{p\pi} - e^{-p\pi}) \quad \text{V. T. 396. N}^\circ. 4.$$

$$1) \int \text{Sin.} \left(\frac{2 p^2}{l x} \right) \frac{d x}{\sqrt{l \frac{1}{x}}} = - e^{-2 p} \text{Sin.} (2 p) \cdot \sqrt{\pi} \quad \text{V. T. 280. N}^{\circ} 17.$$

$$2) \int \text{Cos.} \left(\frac{2 p^2}{l x} \right) \frac{d x}{\sqrt{l \frac{1}{x}}} = e^{-2 p} \text{Cos.} (2 p) \cdot \sqrt{\pi} \quad \text{V. T. 280. N}^{\circ} 18.$$

$$3) \int \text{Sin.} (p l x) \cdot x^{q-1} \frac{d x}{\sqrt{l \frac{1}{x}}} = - \sqrt{\frac{\pi - q + \sqrt{(p^2 + q^2)}}{2} \frac{1}{p^2 + q^2}} \quad \text{V. T. 398. N}^{\circ} 5.$$

$$4) \int \text{Cos.} (p l x) \cdot x^{q-1} \frac{d x}{\sqrt{l \frac{1}{x}}} = \sqrt{\frac{\pi q + \sqrt{(p^2 + q^2)}}{2} \frac{1}{p^2 + q^2}} \quad \text{V. T. 398. N}^{\circ} 6.$$

$$5) \int \text{Sin.} \left(p \sqrt{l \frac{1}{x}} \right) \cdot x^{q-1} \frac{d x}{\sqrt{l \frac{1}{x}}} = \frac{2}{q} \sum_0^{\infty} \frac{(-1)^n}{(n+2)^{n+1/2}} \left(\frac{p}{q} \right)^{2n+1} \quad \text{V. T. 280. N}^{\circ} 10.$$

$$6) \int \text{Cos.} \left(p \sqrt{l \frac{1}{x}} \right) \cdot x^{q-1} \frac{d x}{\sqrt{l \frac{1}{x}}} = \frac{1}{q} e^{-\frac{p^2}{4 q^2}} \sqrt{\pi} \quad \text{V. T. 280. N}^{\circ} 4.$$

$$7) \int \text{Col.} \left(p \sqrt{l \frac{1}{x}} \right) \frac{d x}{\sqrt{l \frac{1}{x}}} = 2 \sqrt{\pi} \sum_1^{\infty} e^{-n^2 p^2} \quad \text{V. T. 280. N}^{\circ} 22.$$

$$1) \int \frac{\text{Sin.} (2 p l x)}{\pi^2 + (l x)^2} \frac{d x}{1 - x^2} = - \frac{e^{p \pi} + e^{-p \pi}}{\pi} \text{Arctang.} (e^{-p \pi}) + \frac{1}{2} e^{-p \pi} \quad \text{V. T. 396. N}^{\circ} 9.$$

$$2) \int \frac{\text{Sin.} (p l x)}{\pi^2 + (l x)^2} \frac{d x}{1 - x^2} = \frac{1}{4} p e^{-p \pi} - \frac{e^{p \pi} - e^{-p \pi}}{4 \pi} l (1 + e^{-p \pi}) \quad \text{V. T. 396. N}^{\circ} 12.$$

$$3) \int \frac{\text{Sin.} (p l x)}{\pi^2 + (l x)^2} \frac{1 + x^2}{1 - x^2} d x = - \frac{1}{2} p e^{-p \pi} + \frac{e^{p \pi} - e^{-p \pi}}{2 \pi} l (1 - e^{-p \pi}) \quad \text{V. T. 396. N}^{\circ} 13.$$

$$4) \int \frac{\text{Sin.} (p l x)}{\pi^2 + (l x)^2} \frac{x^q + x^{-q}}{1 - x^2} d x = \frac{1}{2} e^{-p \pi} (p \text{Cos.} q \pi + q \text{Sin.} q \pi) -$$

$$- \frac{e^{p \pi} - e^{-p \pi}}{4 \pi} \text{Cos.} q \pi \cdot l (1 + 2 e^{-p \pi} \text{Cos.} q \pi + e^{-2 p \pi}) - \frac{e^{p \pi} + e^{-p \pi}}{2 \pi} \text{Sin.} q \pi \cdot \text{Arctg.} \left(\frac{\text{Sin.} q \pi}{e^{p \pi} + \text{Cos.} q \pi} \right), q^2 \leq 1; \quad \text{V. T. 396 N}^{\circ} 17.$$

- 5) $\int \frac{\text{Sin.}(p l x)}{\pi^2 + (lx)^2} \frac{x^q - x^{-q}}{1 - x^2} l x dx = \frac{1}{2} \pi (p \text{Sin. } q \pi - q \text{Cos. } q \pi) e^{-p\pi} -$
 $-\frac{e^{p\pi} - e^{-p\pi}}{4} \text{Sin. } q \pi l (1 + 2e^{-p\pi} \text{Cos. } q \pi + e^{-2p\pi}) + \frac{e^{p\pi} + e^{-p\pi}}{2} \text{Cos. } q \pi \text{Arctg.} \left(\frac{\text{Sin. } q \pi}{e^{p\pi} + \text{Cos. } q \pi} \right), p^2 < 1; \text{ V. T. 396. N}^\circ 18.$
- 6) $\int \frac{\text{Sin.}(p l x)}{r^2 + (lx)^2} \frac{x^q + x^{-q}}{1 - x^2} dx = -\frac{\pi}{2 r^2} + \frac{\pi e^{-pr} \text{Cos. } q r}{2 r \text{Sin. } r} + \pi \sum_1^\infty (-1)^n \frac{e^{-np\pi} \text{Cos. } n q \pi}{n^2 \pi^2 - r^2}, 0 \leq q \leq 1; \text{ V. T. 396. N}^\circ 33.$
- 7) $\int \frac{\text{Cos.}(p l x)}{\pi^2 + (lx)^2} \frac{l x}{1 - x^2} dx = \frac{1}{4} - \frac{1}{4} p \pi e^{-p\pi} - \frac{e^{p\pi} + e^{-p\pi}}{4} l (1 + e^{-p\pi}) \text{ V. T. 396. N}^\circ 26.$
- 8) $\int \frac{\text{Cos.}(p l x)}{\pi^2 + (lx)^2} \frac{x^q - x^{-q}}{1 - x^2} dx = \frac{1}{2} e^{-p\pi} q \text{Cos. } q \pi - p \text{Sin. } q \pi - \frac{e^{p\pi} + e^{-p\pi}}{4 \pi} \text{Sin. } q \pi l (1 + 2e^{-p\pi} \text{Cos. } q \pi + e^{-2p\pi}) +$
 $+\frac{e^{p\pi} - e^{-p\pi}}{2 \pi} \text{Cos. } q \pi \text{Arctang.} \left(\frac{\text{Sin. } q \pi}{e^{p\pi} + \text{Cos. } q \pi} \right), q^2 < 1; \text{ V. T. 396. N}^\circ 32.$
- 9) $\int \frac{\text{Cos.}(p l x)}{\pi^2 + (lx)^2} \frac{x^q + x^{-q}}{1 - x^2} l x dx = \frac{1}{2} - \frac{\pi}{2} e^{-p\pi} (p \text{Cos. } q \pi + q \text{Sin. } q \pi) -$
 $-\frac{e^{p\pi} + e^{-p\pi}}{4} \text{Cos. } q \pi l \{1 + 2e^{-p\pi} \text{Cos. } q \pi + e^{-2p\pi}\} - \frac{e^{p\pi} - e^{-p\pi}}{2} \text{Sin. } q \pi \text{Arctg.} \left(\frac{\text{Sin. } q \pi}{e^{p\pi} + \text{Cos. } q \pi} \right), p^2 \leq 1; \text{ V. T. 396. N}^\circ 31.$
- 10) $\int \frac{\text{Cos.}(p l x)}{r^2 + (lx)^2} \frac{x^q - x^{-q}}{1 - x^2} dx = -\frac{\pi e^{-pr} \text{Sin. } q r}{2 r \text{Sin. } r} + \pi \sum_1^\infty (-1)^n \frac{e^{-np\pi} \text{Sin. } n q \pi}{r^2 - n^2 \pi^2}, 0 < q < 1; \text{ V. T. 396. N}^\circ 34.$
- 11) $\int \frac{\text{Sin.}(2 p l x)}{\frac{1}{4} \pi^2 + (lx)^2} \frac{1 - x}{1 + x} \frac{dx}{x} = e^{p\pi} + \frac{e^{p\pi} - e^{-p\pi}}{\pi} l \frac{e^{p\pi} - 1}{e^{p\pi} + 1} - 2 \frac{e^{p\pi} + e^{-p\pi}}{\pi} \text{Arctang.}(e^{p\pi}) \text{ V. T. 396. N}^\circ 10.$
- 12) $\int \frac{\text{Sin.}(2 p l x)}{\frac{1}{4} \pi^2 + (lx)^2} \frac{1 + x}{1 - x} \frac{dx}{x} = e^{-p\pi} + \frac{e^{p\pi} - e^{-p\pi}}{\pi} l \frac{e^{p\pi} - 1}{e^{p\pi} + 1} - 2 \frac{e^{p\pi} + e^{-p\pi}}{\pi} \text{Arctang.}(e^{-p\pi}) \text{ V. T. 396. N}^\circ 11.$
- 13) $\int \frac{\text{Sin.}(p l x)}{\pi^2 + (lx)^2} \frac{1 + x}{1 - x} \frac{dx}{x} = \frac{e^{p\pi} - e^{-p\pi}}{2 \pi} l \frac{e^{p\pi} - 1}{e^{p\pi} + 1} \text{ V. T. 396. N}^\circ 14.$
- 14) $\int \frac{\text{Sin.}(p l x)}{\pi^2 + (lx)^2} \frac{1 - x}{1 + x} \frac{dx}{x} = \frac{e^{p\pi} - e^{-p\pi}}{2 \pi} l (1 - e^{-2p\pi}) - p e^{-p\pi} \text{ V. T. 396. N}^\circ 8.$
- 15) $\int \frac{\text{Cos.}(2 p l x)}{\frac{1}{4} \pi^2 + (lx)^2} \frac{1 - x}{1 + x} \frac{dx}{x} = \frac{1}{2} \pi e^{-p\pi} + \frac{e^{p\pi} + e^{-p\pi}}{2} l \frac{e^{p\pi} - 1}{e^{p\pi} + 1} - (e^{p\pi} - e^{-p\pi}) \text{Arctg.}(e^{p\pi}) \text{ V. T. 396. N}^\circ 24.$
- 16) $\int \frac{\text{Cos.}(2 p l x)}{\frac{1}{4} \pi^2 + (lx)^2} \frac{1 + x}{1 - x} \frac{dx}{x} = 2 - \frac{1}{2} \pi e^{-p\pi} + \frac{e^{p\pi} + e^{-p\pi}}{2} l \frac{e^{p\pi} - 1}{e^{p\pi} + 1} - (e^{p\pi} - e^{-p\pi}) \text{Arctg.}(e^{-p\pi}) \text{ V. T. 396. N}^\circ 25.$

F. Alg. rat. fract.

Log. en dën. $q^2 + (lx)^2$.

TABLE 408 suite.

Lim. 0 et 1.

Circ. Dir. de Log.

$$17) \int \frac{\text{Cos.}(plx)}{\pi^2 + (lx)^2} \frac{1+x}{1-x} \frac{lx}{x} dx = 1 + \frac{e^{p\pi} + e^{-p\pi}}{2} l \frac{e^{p\pi} - 1}{e^{p\pi} + 1} \quad \text{V. T. 396. N}^\circ. 23.$$

$$18) \int \frac{\text{Cos.}(plx)}{\pi^2 + (lx)^2} \frac{1+x^2}{1-x^2} \frac{lx}{x} dx = \frac{1}{2} + \frac{1}{2} p \pi e^{-p\pi} + \frac{e^{p\pi} + e^{-p\pi}}{2} l (1 - e^{-p\pi}) \quad \text{V. T. 396. N}^\circ. 27.$$

$$19) \int \frac{\text{Sin.}(lx)}{x^a + 2 \text{Cos.}(lx) + x^{-a}} \frac{lx}{\pi^2 - (lx)^2} \frac{dx}{x} = \frac{1}{2a} - \frac{1}{2} \text{Arccot. } a \quad \text{V. T. 396. N}^\circ. 36.$$

$$20) \int \frac{\text{Sin.}(lx)}{x^a - 2 \text{Cos.}(lx) + x^{-a}} \frac{lx}{\pi^2 - (lx)^2} \frac{dx}{x} = \frac{1}{2} \text{Arccot. } a - \frac{1}{2} \frac{a}{1+a^2} \quad \text{V. T. 396. N}^\circ. 37.$$

$$21) \int \frac{\text{Cos.}(plx)}{\pi^2 + (lx)^2} \frac{1+x^2}{1-x^2} \frac{dx}{x lx} = -\frac{1}{2\pi^2} \frac{1-p\pi + p\pi e^{-p\pi}}{1-e^{-p\pi}} - \frac{(e^{lp\pi} - e^{-lp\pi})^2}{2\pi^2} l (1 - e^{-p\pi}) \quad \text{V. T. 396. N}^\circ. 35.$$

F. Alg. irrat. fract.

Log. en dën. $q^2 + (lx)^2$.

TABLE 409.

Lim. 0 et 1.

Circ. Dir. de Log.

$$1) \int \frac{\text{Sin.}(2plx)}{\frac{1}{4}\pi^2 + (lx)^2} \frac{lx}{1+x} \frac{dx}{1-x} = -\frac{\pi}{2} \frac{e^{-p\pi} - \frac{e^{p\pi} - e^{-p\pi}}{4}}{1-x} l \frac{e^{p\pi} + e^{-p\pi} + 1}{e^{p\pi} + e^{-p\pi} - 1} + \frac{e^{p\pi} + e^{-p\pi}}{2} \text{Arctang.} \left(\frac{1-x}{e^{p\pi} - e^{-p\pi}} \right) \quad \text{V. T. 396. N}^\circ. 5.$$

$$2) \int \frac{\text{Sin.}(plx)}{\pi^2 + (lx)^2} \frac{lx}{1+x} \frac{dx}{1-x} = -\frac{1}{2} p \pi e^{-p\pi} + \frac{e^{p\pi} - e^{-p\pi}}{4} l (1 + e^{-2p\pi}) \quad \text{V. T. 396. N}^\circ. 7.$$

$$3) \int \frac{\text{Sin.}(2plx)}{\frac{1}{4}\pi^2 + (lx)^2} \frac{1}{1-x} \frac{dx}{1-x} = \frac{e^{-p\pi}}{\pi} + \frac{e^{p\pi} - e^{-p\pi}}{2\pi} l \frac{e^{p\pi} - 1}{e^{p\pi} + 1} \frac{2 + e^{-p\pi}}{2 + e^{-p\pi}} - \frac{e^{p\pi} + e^{-p\pi}}{\pi} \text{Arctang.} \left(\frac{1-x}{e^{p\pi} - e^{-p\pi}} \right) \quad \text{V. T. 396. N}^\circ. 6.$$

$$4) \int \frac{\text{Sin.}(plx)}{\pi^2 + (lx)^2} \frac{1}{1-x} \frac{dx}{1-x} = -\frac{e^{p\pi} + e^{-p\pi}}{2\pi} \text{Arctang.}(e^{-p\pi}) + \frac{1}{\pi} e^{-p\pi} \quad \text{V. T. 396. N}^\circ. 9.$$

$$5) \int \frac{\text{Sin.}(plx)}{\pi^2 + (lx)^2} \frac{x^q + x^{-q}}{1-x} \frac{dx}{1-x} = \frac{1}{2} e^{-p\pi} \text{Cos. } q\pi + \frac{e^{p\pi} - e^{-p\pi}}{4\pi} \text{Sin } q\pi l \frac{e^{p\pi} - 2 \text{Sin. } q\pi + e^{-p\pi}}{e^{p\pi} + 2 \text{Sin. } q\pi + e^{-p\pi}} - \frac{e^{p\pi} + e^{-p\pi}}{2\pi} \text{Cos. } q\pi \text{Arctang.} \left(\frac{2 \text{Cos. } q\pi}{e^{p\pi} - e^{-p\pi}} \right), p^2 \leq \frac{1}{4}, \quad \text{V. T. 396. N}^\circ. 13.$$

F. Alg. irrat. fract.

Log. en dén. $q^2 + (lx)^2$.

Circ. Dir. de Log.

TABLE 409 suite.

Lim. 0 et 1.

- $$6) \int \frac{\text{Sin.}(plx) x^q - x^{-q} lx}{\pi^2 + (lx)^2} \frac{1}{1-x} \frac{dx}{\sqrt{x}} = \frac{1}{2} e^{-p\pi} \text{Sin.} q\pi + \frac{e^{p\pi} - e^{-p\pi}}{4\pi} \text{Cos.} q\pi l \frac{1 + 2e^{-p\pi} \text{Sin.} q\pi + e^{-2p\pi}}{1 - 2e^{-p\pi} \text{Sin.} q\pi + e^{-2p\pi}} - \frac{e^{p\pi} + e^{-p\pi}}{2\pi} \text{Sin.} p\pi \text{Arctang.} \left(\frac{2 \text{Cos.} q\pi}{e^{p\pi} - e^{-p\pi}} \right), p^2 < \frac{1}{4}; \text{ V. T. 396. N}^\circ. 16.$$
- $$7) \int \frac{\text{Cos.}(2plx)}{\frac{1}{4}\pi^2 + (lx)^2} \frac{1}{1+x} \frac{dx}{\sqrt{x}} = \frac{1}{2} e^{-p\pi} \sqrt{2} - \frac{e^{p\pi} + e^{-p\pi}}{2\pi \sqrt{2}} l \frac{1 + e^{-p\pi} \sqrt{2} + e^{-2p\pi}}{1 - e^{-p\pi} \sqrt{2} + e^{-2p\pi}} + \frac{e^{p\pi} - e^{-p\pi}}{\pi \sqrt{2}} \text{Arctang.} \left(\frac{\sqrt{2}}{e^{p\pi} - e^{-p\pi}} \right) \text{ V. T. 396. N}^\circ. 19.$$
- $$8) \int \frac{\text{Cos.}(plx)}{\pi^2 + (lx)^2} \frac{1}{1+x} \frac{dx}{\sqrt{x}} = \frac{1}{2} p e^{-p\pi} + \frac{e^{p\pi} + e^{-p\pi}}{4\pi} l (1 + e^{-2p\pi}) \text{ V. T. 396. N}^\circ. 20.$$
- $$9) \int \frac{\text{Cos.}(plx) 1 + \sqrt{x} lx}{\pi^2 + (lx)^2} \frac{1}{1 - \sqrt{x}} \frac{dx}{\sqrt{x}} = 2 - \frac{1}{2} \pi e^{-p\pi} + \frac{e^{-p\pi} + e^{-p\pi}}{2} l \frac{1 - e^{-p\pi}}{1 + e^{-p\pi}} + (e^{p\pi} - e^{-p\pi}) \text{Arctang.}(e^{-p\pi}) \text{ V. T. 396. N}^\circ. 25.$$
- $$10) \int \frac{\text{Cos.}(plx)}{\pi^2 + (lx)^2} \frac{lx}{1-x} \frac{dx}{\sqrt{x}} = - \frac{e^{p\pi} - e^{-p\pi}}{2} \text{Arctang.}(e^{-p\pi}) + \frac{1}{2} - \frac{1}{4} \pi e^{-p\pi} \text{ V. T. 396. N}^\circ. 22.$$
- $$11) \int \frac{\text{Cos.}(plx) 1 - \sqrt{x} lx}{\pi^2 + (lx)^2} \frac{1}{1 + \sqrt{x}} \frac{dx}{\sqrt{x}} = \frac{1}{2} \pi e^{p\pi} + \frac{e^{p\pi} + e^{-p\pi}}{2} l \frac{1 + e^{-p\pi}}{1 - e^{-p\pi}} + (e^{p\pi} + e^{-p\pi}) \text{Arctang.}(e^{p\pi}) \text{ V. T. 396. N}^\circ. 24.$$
- $$12) \int \frac{\text{Cos.}(plx) x^q - x^{-q} lx}{\pi^2 + (lx)^2} \frac{1}{1-x} \frac{dx}{\sqrt{x}} = -e^{-p\pi} \text{Sin.} q\pi + \frac{e^{p\pi} + e^{-p\pi}}{2\pi} \text{Cos.} q\pi l \frac{e^{p\pi} + 2 \text{Sin.} q\pi + e^{-p\pi}}{e^{p\pi} - 2 \text{Sin.} q\pi + e^{-p\pi}} - \frac{e^{p\pi} - e^{-p\pi}}{\pi} \text{Sin.} q\pi \text{Arctang.} \left(\frac{2 \text{Cos.} q\pi}{e^{p\pi} - e^{-p\pi}} \right), p^2 < \frac{1}{4}; \text{ V. T. 396 N}^\circ. 30.$$
- $$13) \int \frac{\text{Cos.}(plx) x^q + x^{-q} lx}{\pi^2 + (lx)^2} \frac{1}{1-x} \frac{dx}{\sqrt{x}} = -\frac{1}{2} \pi e^{-p\pi} \text{Cos.} q\pi + \frac{e^{p\pi} + e^{-p\pi}}{4} \text{Sin.} q\pi l \frac{e^{p\pi} - 2 \text{Sin.} q\pi + e^{-p\pi}}{e^{p\pi} + 2 \text{Sin.} q\pi + e^{-p\pi}} - \frac{e^{p\pi} - e^{-p\pi}}{2} \text{Cos.} q\pi \text{Arctang.} \left(\frac{2 \text{Cos.} q\pi}{e^{p\pi} - e^{-p\pi}} \right), p^2 \leq \frac{1}{4}; \text{ V. T. 396. N}^\circ. 29.$$

F. Alg. rat. ent.

Log. de

Circ. Dir.

TABLE 410.

Lim. 0 et $\frac{\pi}{4}$.

- $$1) \int l \text{Sin.} x. x^{p-1} dx = -\frac{1}{2p} \left(\frac{\pi}{4} \right)^p \left[l 2 - 2 + \sum_{1}^{\infty} \frac{4}{p+2m} \sum_{1}^{\infty} \frac{1}{(4n)^{2m}} \right] \text{ V. T. 238. N}^\circ. 19.$$
- $$2) \int l \text{Tang.} x \frac{x}{\text{Sin.} 2x} dx = -\frac{1}{64} \pi^3 \text{ V. T. 305. N}^\circ. 5.$$

F. Alg. rat. ent.

Log. de
Circ. Dir.

TABLE 410 suite.

Lim. 0 et $\frac{\pi}{4}$.

- 3) $\int (l \operatorname{Tang.} x)^3 \frac{x}{\operatorname{Sin.} 2x} dx = -\frac{5}{512} \pi^5$ V. T. 305. N°. 8.
- 4) $\int (l \operatorname{Tang.} x)^5 \frac{x}{\operatorname{Sin.} 2x} dx = -\frac{61}{3072} \pi^7$ V. T. 305. N°. 9.
- 5) $\int l (\operatorname{Tang.} x)^a \frac{x}{\operatorname{Sin.} 2x} dx = \frac{1^{a+1}}{2(-1)^a} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^{a+2}}$ V. T. 305. N°. 10.
- 6) $\int (l \operatorname{Tang.} x)^{2a+1} \frac{x}{\operatorname{Sin.} 2x} dx = \frac{1^{2a+1}}{2^{2a+3}} \pi^{2a+1} B_{2a+2}$ V. T. 305. N°. 11.
- 7) $\int \operatorname{Sin.} (2p l \operatorname{Tang.} x) \frac{x}{\operatorname{Sin.} 2x} dx = \frac{-\pi (e^{p\pi} - 1)^2}{16p e^{2p\pi} + 1}$ V. T. 329. N°. 3.
- 8) $\int \frac{x}{\sqrt{l \operatorname{Cot.} x} \operatorname{Sin.} 2x} dx = \frac{1}{2} \sqrt{\pi} \sum_0^{\infty} (-1)^n \frac{1}{\sqrt{(2n+1)^3}}$ V. T. 320. N°. 16.
- 9) $\int \frac{x}{\sqrt{(l \operatorname{Cot.} x)^2} \operatorname{Sin.} 2x} dx = \infty$ V. T. 328. N°. 1.
- 10) $\int \frac{x l \operatorname{Tang.} x}{\{\pi^2 + (l \operatorname{Tang.} x)^2\}^2 \operatorname{Sin.} 2x} dx = \frac{\pi - 3}{16\pi}$ V. T. 324. N°. 1.
- 11) $\int \frac{x l \operatorname{Tang.} x}{\{\pi^2 + (l \operatorname{Tang.} x)^2\}^2 \operatorname{Sin.} 2x} dx = \frac{1}{64} (1 - l^2)$ V. T. 324. N°. 2.
- 12) $\int \frac{l \operatorname{Tang.} x}{\{q^2 + (l \operatorname{Tang.} x)^2\}^2 \operatorname{Sin.} 2x} dx = \frac{1}{16q} \left\{ Z' \left(\frac{2q + 3\pi}{4\pi} \right) - Z' \left(\frac{2q + \pi}{4\pi} \right) - \frac{\pi}{q} \right\}$ V. T. 324. N°. 3.

F. Alg. rat. ent.

Log. de
Circ. Dir.

TABLE 411.

Lim. 0 et $\frac{\pi}{2}$.

- 1) $\int l \operatorname{Sin.} x x^{p-1} dx = -\frac{1}{p} \left(\frac{\pi}{2} \right)^p \left\{ 1 - \sum_1^{\infty} \frac{2}{p+2m} \sum_1^{\infty} \frac{1}{(4n^2)^m} \right\}$ V. T. 238. N°. 18.
- 2) $\int l \operatorname{Cos.} x x \operatorname{Tang.} x dx = \infty$ V. T. 332. N°. 7.
- 3) $\int l (1 - \operatorname{Cos.} x) x^{p-1} dx = \frac{1}{2p} \left(\frac{\pi}{2} \right)^p \left[l^2 + 2 - \sum_1^{\infty} \frac{4}{p+2m} \sum_1^{\infty} \frac{1}{(4n^2)^m} \right]$ V. T. 238. N°. 19.
- 4) $\int l \operatorname{Sin.} x \frac{x}{\operatorname{Tang.} x} dx = -\frac{1}{4} \pi \left\{ (l^2)^2 + \frac{1}{12} \pi^2 \right\}$ V. T. 332. N°. 1.

$$5) \int (l \operatorname{Tang}. x)^a \frac{x}{\operatorname{Sin}. 2 x} dx = \infty \quad \text{V. T. 333. N}^\circ. 14, 15.$$

$$6) \int (l \operatorname{Cosec}. x)^{a-1} \frac{\operatorname{Sin}.^2 x. l \operatorname{Cosec}. x + (a + \frac{1}{2}) \operatorname{Cos}.^2 x}{\operatorname{Sin}. x} x dx = \frac{3^{a/2}}{2^a} \sqrt{\pi} \quad \text{V. T. 349. N}^\circ. 2.$$

$$7) \int (l \operatorname{Sec}. x)^{a-1} \frac{\operatorname{Cos}.^2 x. l \operatorname{Cos}. x + (a + \frac{1}{2}) \operatorname{Sin}.^2 x}{\operatorname{Cos}. x} x dx = \infty \quad \text{V. T. 349. N}^\circ. 7.$$

$$8) \int l \operatorname{Tang}. x. \left\{ 1 - q^4 \operatorname{Tang}.^4 x + \frac{4 q^4 x \operatorname{Sin}.^3 x}{\operatorname{Cos}.^5 x} \right\} \frac{dx}{(1 - q^4 \operatorname{Tang}.^4 x)^2} = \frac{1}{8} \pi l \left\{ \frac{q^4}{(1+q)^2 (1+q^2)} \right\} \quad \text{V. T. 241. N}^\circ. 6.$$

$$9) \int \left\{ \frac{1}{2} l(1 - p^2 \operatorname{Sin}.^2 x) - 1 \right\} \frac{x \operatorname{Sin}. 2 x}{\sqrt{(1 - p^2 \operatorname{Sin}.^2 x)^3}} dx = \frac{1}{2 p^2} l(1 - p^2) \cdot \left\{ \frac{\pi}{\sqrt{(1 - p^2)}} - F'(p) \right\} \quad \text{V. T. 348. N}^\circ. 12.$$

$$10) \int \{ 3 l(1 - p^2 \operatorname{Sin}.^2 x) - 2 \} \frac{x \operatorname{Sin}. 2 x}{\sqrt{(1 - p^2 \operatorname{Sin}.^2 x)^5}} dx = \\ = \frac{1}{p^2 (1 - p^2)} \left[2(2 - p^2) F'(p) - \{ 4 + l(1 - p^2) \} E'(p) + \frac{\pi}{1 - p^2} l(1 - p^2) \right] \quad \text{V. T. 348. N}^\circ. 18.$$

$$11) \int \left\{ \frac{p l(1 + p \operatorname{Sin}.^2 x)}{1 - p^2 \operatorname{Sin}.^2 x} + \frac{2}{1 + p \operatorname{Sin}.^2 x} \right\} \frac{x \operatorname{Sin}. 2 x}{\sqrt{(1 - p^2 \operatorname{Sin}.^2 x)}} dx = \\ = \frac{1}{2 p} \left\{ 2 l \left(\frac{\sqrt{p}}{2(1 + p)} \right) \cdot F'(p) + \frac{\pi}{2} F' \{ \sqrt{(1 - p^2)} \} + \frac{2 \pi}{\sqrt{(1 - p^2)}} l(1 + p) \right\} \quad \text{V. T. 348. N}^\circ. 10.$$

$$12) \int \left\{ \frac{p l(1 - p \operatorname{Sin}.^2 x)}{1 - p^2 \operatorname{Sin}.^2 x} - \frac{2}{1 - p \operatorname{Sin}.^2 x} \right\} \frac{x \operatorname{Sin}. 2 x}{\sqrt{(1 - p^2 \operatorname{Sin}.^2 x)}} dx = \\ = \frac{1}{2 p} \left\{ 2 l \left(\frac{\sqrt{p}}{2(1 - p)} \right) \cdot F'(p) + \frac{\pi}{2} F' \{ \sqrt{(1 - p^2)} \} + \frac{2 \pi}{\sqrt{(1 - p^2)}} l(1 - p) \right\} \quad \text{V. T. 348. N}^\circ. 11.$$

$$13) \int \left\{ \frac{l(1 - p^2 \operatorname{Sin}.^2 \lambda. \operatorname{Sin}.^2 x)}{1 - p^2 \operatorname{Sin}.^2 x} - \frac{2 \operatorname{Sin}.^2 \lambda}{1 - p^2 \operatorname{Sin}.^2 \lambda. \operatorname{Sin}.^2 x} \right\} \frac{x \operatorname{Sin}. 2 x}{\sqrt{(1 - p^2 \operatorname{Sin}.^2 x)}} dx = \\ = \frac{1}{p^2} \left[4 F'(p) E(p, \lambda) - 2 E'(p) \{ F(p, \lambda) \}^2 + \frac{\pi}{\sqrt{(1 - p^2)}} l(1 - p^2 \operatorname{Sin}.^2 \lambda) \right] \quad \text{V. T. 348. N}^\circ. 17.$$

$$14) \int \left\{ \frac{l(1 - p^2 \operatorname{Sin}.^4 x)}{1 - p^2 \operatorname{Sin}.^2 x} - \frac{4 \operatorname{Sin}.^2 x}{1 - p^2 \operatorname{Sin}.^4 x} \right\} \frac{x \operatorname{Sin}. 2 x}{\sqrt{(1 - p^2 \operatorname{Sin}.^2 x)}} dx = \\ = \frac{1}{p^2} \left\{ l \left(\frac{p}{2(1 - p^2)} \right) F'(p) + \frac{\pi}{2} F' \{ \sqrt{(1 - p^2)} \} + \frac{\pi}{\sqrt{(1 - p^2)}} l(1 - p^2) \right\} \quad \text{V. T. 348. N}^\circ. 16.$$

- 15) $\int \left\{ \frac{1-q \sqrt{1-p^2 \sin^2 x}}{1+q \sqrt{1-p^2 \sin^2 x}} - \frac{2q \sqrt{1-p^2 \sin^2 x}}{1-q^2+p^2 q^2 \sin^2 x} \right\} \frac{x \sin 2x}{\sqrt{1-p^2 \sin^2 x}^3} dx =$
 $= \frac{\pi}{p^2} F\{\sqrt{1-p^2}, \operatorname{Arcsin} q\} + \frac{\pi}{p^2 \sqrt{1-p^2}} \frac{1-q \sqrt{1-p^2}}{1+q \sqrt{1-p^2}} \quad \text{V. T. 348. N}^\circ 22.$
- 16) $\int \{1+p^2 \sin^2 x (l \sin x - 1)\} \frac{x \cos x}{\sqrt{1-p^2 \sin^2 x}^3} dx = \frac{1}{2} F'(p) l p + \frac{1}{4} \pi F\{\sqrt{1-p^2}\} \quad \text{V. T. 347 N}^\circ 4.$
- 17) $\int \frac{2 \sin^2 x \cdot l \operatorname{Cosec} x - \cos^2 x}{(l \operatorname{Cosec} x)^2} \frac{x}{\sin x} dx = 2 \sqrt{\pi} - \pi \quad \text{V. T. 349. N}^\circ 4.$
- 18) $\int \frac{l \operatorname{Tang} x}{\{q^2 + (l \operatorname{Tang} x)^2\}^2} \frac{x}{\sin 2x} dx = \frac{1}{8q} \left\{ Z' \left(\frac{2q+3\pi}{4\pi} \right) - Z' \left(\frac{2q+\pi}{4\pi} \right) \right\} \quad \text{V. T. 352. N}^\circ 17.$

- 1) $\int \frac{x \operatorname{Tang} x}{x^2 + (l \cos x)^2} dx = \frac{\pi}{2l^2} \quad \text{Poisson, Bull. de la S. Phil. Sept. 1822. — Id., P. 19. 404. N}^\circ 76. — \text{Cauchy, P. 19. 511. — Id., Exerc. 1826. p. 205.}$
- 2) $\int \frac{l \cos x}{x^2 + (l \cos x)^2} dx = \frac{1}{2} \pi \left(1 - \frac{1}{l^2} \right) \quad \text{Poisson, P. 19. 404. N}^\circ 76. — \text{Cauchy, P. 19. 511. — Id., Exerc. 1826. p. 205.}$
- 3) $\int \frac{\cos 2ax \cdot l \cos x + x \sin 2ax}{x^2 + (l \cos x)^2} dx = \frac{1}{2} \pi \quad \text{Poisson, P. 19. 404. N}^\circ 76.$
- 4) $\int \frac{\cos (b \operatorname{Tang} x) \cdot l \cos x + x \sin (b \operatorname{Tang} x)}{x^2 + (l \cos x)^2} dx = -\frac{1}{2} \pi \left(\frac{e^{-b}}{l^2} - 1 \right) \quad \text{V. T. 446. N}^\circ 17.$
- 5) $\int \frac{\sin (b \operatorname{Tang} x) \cdot l \cos x - x \cos (b \operatorname{Tang} x)}{x^2 + (l \cos x)^2} \operatorname{Tang} x dx = -\frac{\pi e^{-b}}{2l^2} \quad \text{V. T. 446. N}^\circ 18.$
- 6) $\int \frac{l \cos x}{x^2 + (l \cos x)^2} \frac{dx}{1 + \cos 2x} = \pi$
- 7) $\int \frac{l \cos 2x}{x^2 + (l \cos x)^2} \frac{dx}{1 - \cos 2x} = \frac{1}{l} \pi$
- 8) $\int \frac{\sin 2x}{x^2 + (l \cos x)^2} \frac{x}{1 - \cos 2x} dx = \pi$
- 9) $\int \frac{l \cos x}{x^2 + (l \cos x)^2} \frac{dx}{1 - 2p \cos 2x + p^2} = \frac{1}{2} \frac{\pi}{p^2 - 1} \left\{ \frac{1}{l^2 - l(1+p)} - \frac{1+p}{1-p} \right\}, p^2 \leq 1.$

Poisson, P.
19. 404.
N^o 76.

F. Alg. rat. fract.	Dén. $x^2 + (l \cos x)^2$. TABLE 412 suite.	Lim. 0 et $\frac{\pi}{2}$.
Log. de		
Circ. Dir.		

$$\begin{aligned}
 10) & \int \frac{x \sin 2x}{x^2 + (l \cos x)^2} \frac{dx}{1 - 2p \cos 2x + p^2} = \frac{\pi}{4p} \left\{ \frac{1}{l2 - l(1+p)} - \frac{1}{l2} \right\}, p^2 \leq 1; \\
 11) & \int \frac{\cos^k x}{x^2 + (l \cos x)^2} \frac{x \sin kx + \cos kx l \cos x}{1 - 2p \cos 2x + p^2} dx = \frac{\pi}{2(1-p)^2} \left. \vphantom{\int} \right\}, p^2 < 1, k = \infty; \\
 12) & \int \frac{\cos^k x \sin 2x}{x^2 + (l \cos x)^2} \frac{\sin kx l \cos x - x \cos kx}{1 - 2p \cos 2x + p^2} dx = 0 \\
 13) & \int \frac{(l \cos x)^2 + 2x \operatorname{Tang} x l \cos x - x^2}{\{x^2 + (l \cos x)^2\}^2} l \cos x dx = \frac{\pi}{2l2} \quad \text{V. T. 412. N}^\circ. 1. \\
 14) & \int \frac{(l \cos x)^2 - 2x \cot x l \cos x - x^2}{\{x^2 + (l \cos x)^2\}^2} x \operatorname{Tang} x dx = \pi \frac{1-l2}{2l2} \quad \text{V. T. 412. N}^\circ. 2.
 \end{aligned}$$

Poisson, P.
19. 404.
N^o. 76.

F. Alg. rat. ent.	TABLE 415.	Lim. 0 et π .
Log. de		
Circ. Dir.		

$$\begin{aligned}
 1) & \int l \sin x \cdot x dx = \frac{1}{2} \pi^2 l \frac{1}{2} \quad \text{Grunert, Gr. 4. 113.} \\
 2) & \quad \quad \quad = -\frac{1}{2} \pi^2 (l2 - 2\alpha\pi i) \quad \text{Arndt, Gr. 6. 187.} \\
 3) & \int l \sin x \cdot (\pi - 2x) dx = 0 \quad \text{Grunert, Gr. 4. 113.} \\
 4) & \int l \cos^2 x \cdot x dx = \pi^2 l \frac{1}{2} \quad \text{V. T. 413. N}^\circ. 3. \\
 5) & \int l \operatorname{Tang}^2 x \cdot x dx = 0 \quad \text{V. T. 413. N}^\circ. 1, 4. \\
 6) & \int l((\sin x)) \cdot x dx = \frac{1}{2} \pi^2 l \frac{1}{2} \pm \alpha \pi^3 i \\
 7) & \int l((- \sin x)) \cdot x dx = \frac{1}{2} \pi^2 l \frac{1}{2} \pm \frac{2\alpha + 1}{1} \pi^3 i \left. \vphantom{\int} \right\} \quad \text{Lindmann, Gr. 16. 94.} \\
 8) & \int l(1 - 2p \cos 2x + p^2) \cdot \cos \{(2a-1)x\} \cdot x^{\pm 2b} dx = 0 \\
 9) & \int l(1 - 2p \cos 2x + p^2) \cdot \sin \{(2a-1)x\} \cdot x^{\pm 2b+1} dx = 0 \left. \vphantom{\int} \right\} \quad \text{Bierens de Haan, Gr. 13. 193.}
 \end{aligned}$$

, $p < 1$;

F. Alg. rat. ent.
Log. de
Circ. Dir.

TABLE 415 suite.

Lim. 0 et ∞ .

$$\begin{aligned}
 10) & \int l(1 - 2p \cos. 2x + p^2). \sin. 3ax. \sin. x. x^{\pm 2b} dx = 0 \\
 11) & \int l(1 - 2p \cos. 2x + p^2). \sin. 2ax. \cos. x. x^{\pm 2b+1} dx = 0 \\
 12) & \int l(1 - 2p \cos. 2x + p^2). \cos. 2ax. \sin. x. x^{\pm 2b+1} dx = 0 \\
 13) & \int l(1 - 2p \cos. 2x + p^2). \cos. 2ax. \cos. x. x^{\pm 2b} dx = 0 \\
 14) & \int l(1 - 2q \cos. x + q^2). \sin. ax. x^{2b+1} dx = \frac{(-1)^{b+1} \pi q^a}{a^{2b+2}} 1^{2b+1,1} \sum_0^{2b+1} \frac{(-a l q)^n}{1^{n/1}} \\
 15) & \int l(1 - 2q \cos. x + q^2). \cos. ax. x^{2b} dx = \frac{(-1)^{b+1} \pi q^a}{a^{2b+1}} 1^{2b,1} \sum_0^{2b} \frac{(-a l q)^n}{1^{n/1}}
 \end{aligned}
 \left. \vphantom{\begin{aligned} 10) \\ 11) \\ 12) \\ 13) \\ 14) \\ 15) \end{aligned}} \right\} \begin{aligned} & , p < 1, q^2 < 1, \\ & \text{Bierens de Haan,} \\ & \text{Gr. 13. 123.} \end{aligned}$$

F. Alg. rat. fract. à dén. x^a .
Log.
Circ. Dir.

TABLE 414.

Lim. 0 et ∞ .

$$\begin{aligned}
 1) & \int l(ax). \sin. ax \frac{dx}{x} = -\frac{1}{2} \pi \Lambda \\
 2) & \int l(bx). \sin. ax \frac{dx}{x} = -\frac{1}{2} \pi \left(\Lambda + l \frac{a}{b} \right) \\
 3) & \int l(1+x^2). \sin. ax \frac{dx}{x} = -\pi li.(e^{-a}) \quad \text{Schlömilch, Beitr. III. § 8.} \\
 4) & \int l \left(\frac{1 + \sin. x}{1 - \sin. x} \right)^2 \frac{dx}{x} = \pi^2 \quad \text{Schlömilch, Gr. 4. 316. où la valeur est } \frac{1}{2} \pi^2 \text{ fautive.} \\
 5) & \int l \left(\frac{1 + \text{Tang. } x}{1 - \text{Tang. } x} \right)^2 \frac{dx}{x} = \frac{1}{2} \pi^2 \quad \text{Schlömilch, Beitr. III. § 5. — Id., Gr. 4. 316.} \\
 6) & \int l \frac{1 + \text{Tang. } px}{1 - \text{Tang. } px} \frac{dx}{x} = \frac{1}{4} \pi^2 \quad \text{Schlömilch, Beitr. II. § 5.} \\
 7) & \int l \frac{1 + 2p \cos. x + p^2}{1 + 2p \cos. ax + p^2} \frac{dx}{x} = l(a^2). l(1+p), p^2 < 1; \\
 8) & \quad \quad \quad = l(a^2). l \left(\frac{1+p}{p} \right), p^2 > 1;
 \end{aligned}
 \left. \vphantom{\begin{aligned} 1) \\ 2) \\ 3) \\ 4) \\ 5) \\ 6) \\ 7) \\ 8) \end{aligned}} \right\} \begin{aligned} & \text{Arndt, Gr. 11. 70.} \\ & \text{Raspe, Gr. 23. 105} \end{aligned}$$

F. Alg. rat. fract. à dén. x^2 .

Log.

Circ. Dir.

TABLE 414 suite.

Lim. 0 et ∞ .

- 9) $\int l \frac{1 + 2p \cos. ax + p^2}{1 + 2p \cos. bx + p^2} \frac{dx}{x} = l(1+p).l\left(\frac{b^2}{a^2}\right), p^2 \leq 1;$
 10) $= l\left(\frac{1+p}{p}\right).l\left(\frac{b^2}{a^2}\right), p^2 \geq 1;$ } Schlömilch, Gr. 5. 152. — Raabe, Cr. 23. 105.
- 11) $\int l(q^2 + x^2) \frac{ax \cot. ax - l \sin. ax}{x^2} dx = \frac{\pi}{q} l \frac{2e^{2aq}}{e^{2aq} - 1}$ V. T. 415. N°. 4.
- 12) $\int l(q^2 + x^2) \frac{ax \text{Tang.} ax + l \cos. ax}{x^2} dx = \frac{\pi}{q} l \frac{e^{2aq} + 1}{2e^{2aq}}$ V. T. 415. N°. 5.
- 13) $\int l(q^2 - x^2) \frac{ax \cot. ax - l \sin. ax}{x^2} dx = p\pi - \frac{\pi^2}{2q}$ V. T. 415. N°. 13.
- 14) $\int l(q^2 - x^2) \frac{ax \text{Tang.} ax + l \cos. ax}{x^2} dx = -p\pi$ V. T. 415. N°. 14.
- 15) $\int l(q^2 + x^2) \frac{2ax - \sin. 2ax.l \text{Tang.} ax}{x^2 \sin. 2ax} dx = \frac{\pi}{q} l \frac{e^{2aq} + 1}{e^{2aq} - 1}$ V. T. 415. N°. 11.
- 16) $\int l(q^2 - x^2) \frac{2ax - \sin. 2ax.l \text{Tang.} ax}{x^2 \sin. 2ax} dx = -\frac{\pi^2}{2q}$ V. T. 415. N°. 17.
- 17) $\int l x \sin. qx \frac{dx}{x^{1-p}} = \frac{1}{q^p} \left\{ \sin. \frac{1}{2} p \pi . Z'(p) - \sin. \frac{1}{2} p \pi . lq + \frac{1}{2} \pi \cos. \frac{1}{2} p \pi \right\} \Gamma(p)$
 18) $\int l x \cos. qx \frac{dx}{x^{1-p}} = \frac{1}{q^p} \left\{ \cos. \frac{1}{2} p \pi . Z'(p) - \cos. \frac{1}{2} p \pi . lq - \frac{1}{2} \pi \sin. \frac{1}{2} p \pi \right\} \Gamma(p)$ } Raabe, Int. 416.
- 19) $\int l(q^2 + x^2) \cdot \left\{ l(1+p^2 \text{Tang.}^2 x) - \frac{2p^2 x \text{Tang.} x}{\cos.^2 x + p^2 \sin.^2 x} \right\} \frac{dx}{x^2} = \frac{2\pi}{q} \left\{ 1 + p \frac{e^q - e^{-q}}{e^q + e^{-q}} \right\}$ V. T. 416. N°. 1.

F. Alg. rat. fract. à dén. $b^2 \pm x^2$.

Log. de

Circ. Dir. monôme.

TABLE 415.

Lim. 0 et ∞ .

- 1) $\int l \sin. qx \frac{dx}{1+x^2} = \frac{1}{2} \pi l \frac{e^{2q} - 1}{2e^{2q}}$
 2) $\int l \cos. qx \frac{dx}{1+x^2} = \frac{1}{2} \pi l \frac{e^{2q} + 1}{2e^{2q}}$
 3) $\int l \text{Tang.} qx \frac{dx}{1+x^2} = \frac{1}{2} \pi l \frac{e^{2q} - 1}{e^{2q} + 1}$ } Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 39.

- 4) $\int l \sin. p x \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} l \frac{1 - e^{-2pq}}{2}$
- 5) $\int l \cos. p x \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} l \frac{1 + e^{-2pq}}{2}$
- 6) $\int l \left(2 \cos. \frac{1}{2} x \right)^2 \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l \frac{e^q + 1}{e^q}$
- 7) $\int l \left(2 \sin. \frac{1}{2} x \right)^2 \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l \frac{e^q - 1}{e^q}$
- 8) $\int l \left(\sin. \frac{1}{2} k x \right) \frac{dx}{q^2 + x^2} = -\frac{\pi}{2q} l 2, k = \infty$; Schlömilch, Beitr. II. 5.
- 9) $\int l \left(2 \sin. \frac{1}{2} q x \right) \frac{dx}{p^2 + x^2} = \frac{\pi}{2p} l (1 - e^{-pq})$
- 10) $\int l \left(2 \cos. \frac{1}{2} q x \right) \frac{dx}{p^2 + x^2} = \frac{\pi}{2p} l (1 + e^{-pq})$
- 11) $\int l \text{Tang. } p x \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} l \frac{e^{2pq} - 1}{e^{2pq} + 1}$
- 12) $\int l \cot. p x \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} l \frac{e^{pq} + e^{-pq}}{e^{pq} - e^{-pq}}$
- 13) $\int l \sin. p x \frac{dx}{q^2 - x^2} = -\frac{1}{4q} \pi^2 + \frac{1}{2} p \pi$
- 14) $\int l \cos. p x \frac{dx}{q^2 - x^2} = \frac{1}{2} p \pi$
- 15) $\int l (2 \sin. p x) \frac{dx}{q^2 - x^2} = \frac{1}{2} p \pi - \frac{1}{4q} \pi^2$
- 16) $\int l (2 \cos. p x) \frac{dx}{q^2 - x^2} = \frac{1}{2} p \pi$
- 17) $\int l \text{Tang. } p x \frac{dx}{q^2 - x^2} = -\frac{1}{4q} \pi^2$
- Bidone, Mém. Turin. 1812. 231. Tableau. — Legendre, Exerc. 4. 133. — Schlömilch, Beitr. II. § 5. — Id., Gr. 10. 410.
- Schlömilch, Stud. II. 18.
- Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 39. — Boncompagni, Cr. 25. 74.
- Bidone, Mém. Turin. 1812. 231. Tableau. — Legendre, Exerc. 4. 133. — Schlömilch, Beitr. II. § 5. — Boncompagni, Cr. 25. 74.
- Schlömilch, Beitr. II. § 5. — Boncompagni, Cr. 25. 74.
- Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 39.

F. Alg. rat. fract. à dén. $b^2 + x^2$.

Log. de
Circ. Dir. polynôme.

TABLE 416.

Lim. 0 et ∞ .

$$\begin{aligned}
 1) & \int l(1 + p^2 \text{Tang.}^2 x) \frac{dx}{q^2 + x^2} = \frac{\pi}{q} \left\{ 1 + p \frac{e^q - e^{-q}}{e^q + e^{-q}} \right\} \\
 2) & \int l(1 + p^2 \text{Cot.}^2 x) \frac{dx}{q^2 + x^2} = \frac{\pi}{q} \left\{ 1 + p \frac{e^q + e^{-q}}{e^q - e^{-q}} \right\} \\
 3) & \int l(1 + p^2 \text{Tang.}^2 r x) \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l \{ 1 + p \text{Tangh} p.(qr) \} \\
 4) & \int l(1 + p^2 \text{Cot.}^2 r x) \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l \{ 1 + p \text{Coth} p.(qr) \} \\
 5) & \int l(1 + 2p \text{Cos.} x + p^2) \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l(1 + p e^{-q}), p^2 \leq 1; \\
 6) & = \frac{\pi}{q} l(p + e^{-q}), p^2 \geq 1; \\
 7) & \int l(1 - 2p \text{Cos.} x + p^2) \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l(1 - p e^{-q}), p^2 \leq 1; \text{ Hoppe, Cr. 40. 139.} \\
 8) & \int l(1 + 2p \text{Cos.} r x + p^2) \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l(p + e^{-qr}), p > 1; \text{ Ohm, Ausw. 26.} \\
 9) & = \frac{\pi}{q} l(1 + p e^{-qr}), p < 1; \\
 10) & \int l(1 - 2p \text{Cos.} r x + p^2) \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l(1 - p e^{-qr}) \left\{ \begin{array}{l} \text{Legendre, Exerc. 4. 133 — Plana, Mém.} \\ \text{Turin. 1818. 7. II. 11. — Boncompagni,} \\ \text{Cr. 25. 74} \end{array} \right. \\
 11) & = \frac{\pi}{q} l(p - e^{-qr}), p > 1; \text{ Ohm, Ausw. 26.} \\
 12) & \int l \frac{1 + 2p \text{Cos.} r x + p^2}{1 - 2p \text{Cos.} r x + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l \frac{1 + p e^{-rq}}{1 - p e^{-rq}} \\
 13) & \int l \frac{1 - 2p \text{Cos.} r x + p^2}{1 + 2p \text{Cos.} r x + p^2} \frac{dx}{q^2 + x^2} = \frac{\pi}{q} l \frac{1 - p e^{-rq}}{1 + p e^{-rq}} \left\{ \begin{array}{l} \text{Boncompagni, Cr. 25. 74. Elles ne valent} \\ \text{que pour } p^2 \leq 1. \end{array} \right.
 \end{aligned}$$

F. Alg. rat. fract. à dén. $b^2 \pm x^2$.

Log. $l(ax)$.
Circ. Dir.

TABLE 417.

Lim. 0 et ∞ .

$$\begin{aligned}
 1) & \int l x \text{Sin.} q x \frac{x}{1 + x^2} dx = -\frac{1}{4} \pi \{ e^q \text{li.}(e^{-q}) + e^{-q} \text{li.}(e^q) \} \\
 2) & \int l x \text{Cos.} q x \frac{dx}{1 + x^2} = \frac{\pi}{4q} \{ e^q \text{li.}(e^{-q}) - e^{-q} \text{li.}(e^q) \}
 \end{aligned}$$

F. Alg. rat. fract. à dén. $b^2 \pm x^2$.

Log. $l(ax)$.

Circ. Dir.

TABLE 417 suite.

Lim. 0 et ∞ .

$$\begin{aligned}
 3) \int l x. \text{Sin. } q x \frac{x dx}{p^2 + x^2} &= \frac{1}{2} \pi e^{-pq} l p - \frac{1}{4} \pi \{ e^{pq} \text{li.}(e^{-pq}) + e^{-pq} \text{li.}(e^{pq}) \} \\
 4) \int l x. \text{Cos. } q x \frac{dx}{p^2 + x^2} &= \frac{\pi}{2 p} e^{-pq} l p + \frac{\pi}{4 p} \{ e^{pq} \text{li.}(e^{-pq}) - e^{-pq} \text{li.}(e^{pq}) \} \\
 5) \int l(r x) \text{Sin. } q x \frac{r dx}{p^2 + x^2} &= \frac{1}{4} \pi e^{-pq} \{ 2 l(p r) - \text{Ei.}(p q) \} - \frac{1}{4} \pi e^{pq} \text{Ei.}(-p q) \\
 6) \int l(r x) \text{Cos. } q x \frac{dx}{p^2 + x^2} &= \frac{\pi}{4 p} e^{-pq} \{ 2 l(p r) - \text{Ei.}(p q) \} + \frac{\pi}{4 p} e^{pq} \text{Ei.}(-p q) \\
 7) \int l\left(\frac{p}{x}\right) \text{Sin. } q x \frac{x dx}{p^2 + x^2} &= \frac{1}{4} \pi \{ e^{-pq} \text{Ei.}(p q) + e^{pq} \text{Ei.}(-p q) \} \\
 8) \int l\left(\frac{p}{x}\right) \text{Cos. } q x \frac{dx}{p^2 + x^2} &= \frac{\pi}{4 q} \{ e^{-pq} \text{Ei.}(p q) - e^{pq} \text{Ei.}(-p q) \} \\
 9) \int l\left(\frac{p}{x}\right) \text{Sin. } q x \frac{x dx}{p^2 - x^2} &= -\frac{1}{2} \pi \left\{ \text{Ci.}(p q) \text{Cos. } p q + \text{Si.}(p q) \text{Sin. } p q - \frac{1}{2} \pi \text{Sin. } p q \right\} \\
 10) \int l\left(\frac{p}{x}\right) \text{Cos. } q x \frac{dx}{p^2 - x^2} &= \frac{\pi}{2 p} \left\{ \text{Ci.}(p q) \text{Sin. } p q - \text{Si.}(p q) \text{Cos. } p q + \frac{1}{2} \pi \text{Cos. } p q \right\}
 \end{aligned}$$

Schlömilch, Gr. 5. 204.
Arndt, Gr. 11. 7c.
Schlömilch, Stud. 11. 21.
où 9) et 10) sont faut. négatives.

F. Alg. rat. fract. à dén. $b^2 \pm x^2$.

Log.

Circ. Dir.

TABLE 418.

Lim. 0 et ∞ .

$$\begin{aligned}
 1) \int l \text{Sin. } q x \frac{dx}{p^2 + x^2} &= \frac{\pi}{2 p^3} l \left[\frac{1}{2} \sqrt{1 - 2 e^{-pq} + 2 \text{Cos.}(p q + 2)} + e^{-2pq} \right] - \\
 &\quad - \frac{\pi}{2 p^3} \text{Arcsin.} \left\{ \frac{e^{-pq} \text{Sin.}(p q + 2)}{1 - 2 e^{-pq} + 2 \text{Cos.}(p q + 2) + e^{-4pq}} \right\} \\
 2) \int l \text{Cos. } q x \frac{dx}{p^2 + x^2} &= \frac{\pi}{2 p^3} l \left[\frac{1}{2} \sqrt{1 + 2 e^{-pq} + 2 \text{Cos.}(p q + 2)} + e^{-2pq} \right] + \\
 &\quad + \frac{\pi}{2 p^3} \text{Arcsin.} \left\{ \frac{e^{-pq} \text{Sin.}(p q + 2)}{1 - 2 e^{-pq} + 2 \text{Cos.}(p q + 2) + e^{-4pq}} \right\} \\
 3) \int l \text{Tang. } q x \frac{dx}{p^2 + x^2} &= \frac{\pi}{2 p^3} l \left[\frac{1 - 2 e^{-pq} + 2 \text{Cos.}(p q + 2) + e^{-2pq}}{1 + 2 e^{-pq} + 2 \text{Cos.}(p q + 2) + e^{-2pq}} \right] - \\
 &\quad - \frac{\pi}{2 p^3} \text{Arcsin.} \left\{ \frac{2 e^{-pq} \text{Sin.}(p q + 2)}{1 - 2 e^{-pq} + 2 \text{Cos.}(p q + 2) + e^{-4pq}} \right\}
 \end{aligned}$$

Piana, Mem. Turin, 1818. 7. 11.

$$\begin{aligned}
 & 4) \int l(1+2p \cos qx + p^2) \frac{dx}{1+x^{2a}} = \frac{\pi}{a} l(1+pe^{-q}) - \frac{\pi^{\frac{1}{2}a-1}}{a} \sum_1^{a-1} \left[\cos \frac{n\pi}{a} \cdot l \left\{ 1+2pe^{-q \cos \frac{n\pi}{a}} \cos \left(q \sin \frac{n\pi}{a} \right) + p^2 e^{-2q \cos \frac{n\pi}{a}} \right\} \right] \\
 & \quad - \frac{2\pi^{\frac{1}{2}a-1}}{a} \sum_1^{a-1} \left\{ \sin \frac{n\pi}{a} \cdot \text{Arcsin} \left[\frac{pe^{-q \cos \frac{n\pi}{a}} \sin \left(q \sin \frac{n\pi}{a} \right)}{\sqrt{\left\{ 1+2pe^{-q \cos \frac{n\pi}{a}} \cos \left(q \sin \frac{n\pi}{a} \right) + p^2 e^{-2q \cos \frac{n\pi}{a}} \right\}}} \right] \right\}, \text{ } a \text{ impair;} \\
 & 5) = \frac{\pi^{\frac{1}{2}a-1}}{a} \sum_0^{a-1} \left[\cos \left(\frac{2n+1}{2a} \pi \right) \cdot l \left\{ 1+2pe^{-q \cos \left(\frac{2n+1}{2a} \pi \right)} \cos \left[q \sin \left(\frac{2n+1}{2a} \pi \right) \right] + p^2 e^{-2q \cos \left(\frac{2n+1}{2a} \pi \right)} \right\} \right] \\
 & \quad + \frac{2\pi^{\frac{1}{2}a-1}}{a} \sum_0^{a-1} \left\{ \sin \left(\frac{2n+1}{2a} \pi \right) \cdot \text{Arcsin} \left[\frac{pe^{-q \cos \left(\frac{2n+1}{2a} \pi \right)} \sin \left[q \sin \left(\frac{2n+1}{2a} \pi \right) \right]}{\sqrt{\left\{ 1+2pe^{-q \cos \left(\frac{2n+1}{2a} \pi \right)} \cos \left[q \sin \left(\frac{2n+1}{2a} \pi \right) \right] + p^2 e^{-2q \cos \left(\frac{2n+1}{2a} \pi \right)} \right\}}} \right] \right\}, \text{ } a \text{ pair;}
 \end{aligned}$$

Sur les intégrales 4), 5) voyez: Plana, Mém. Turin. 1818. 7. III. 15.

$$\begin{aligned}
 & 6) \int l \sin qx \frac{dx}{p^4 - x^4} = \frac{\pi}{8p^3} \left\{ 2pq - \pi + 2l \frac{1 - e^{-2pq}}{2} \right\} \quad \text{V. T. 415. N}^\circ. 4, 13. \\
 & 7) \int l \sin qx \frac{x^2 dx}{p^4 - x^4} = \frac{\pi}{8p} \left\{ 2pq - \pi + 2l \frac{2}{1 - e^{-2pq}} \right\} \quad \text{V. T. 415. N}^\circ. 4, 13. \\
 & 8) \int l \cos qx \frac{dx}{p^4 - x^4} = \frac{\pi}{4p^3} \left\{ pq + l \frac{1 + e^{-2pq}}{2} \right\} \quad \text{V. T. 415. N}^\circ. 5, 14. \\
 & 9) \int l \cos qx \frac{x^2 dx}{p^4 - x^4} = \frac{\pi}{4p} \left\{ pq + l \frac{2}{1 - e^{-2pq}} \right\} \quad \text{V. T. 415. N}^\circ. 5, 14. \\
 & 10) \int l(2 \sin qx) \frac{dx}{p^4 - x^4} = \frac{\pi}{8p^3} \{ 2pq - \pi + 2l(1 - e^{-2pq}) \} \quad \text{V. T. 415. N}^\circ. 9, 15. \\
 & 11) \int l(2 \sin qx) \frac{x^2 dx}{p^4 - x^4} = \frac{\pi}{8p} \{ 2pq - \pi - 2l(1 - e^{-2pq}) \} \quad \text{V. T. 415. N}^\circ. 9, 15. \\
 & 12) \int l(2 \cos qx) \frac{dx}{p^4 - x^4} = \frac{\pi}{4p^3} \{ pq + l(1 + e^{-2pq}) \} \quad \text{V. T. 415. N}^\circ. 10, 16. \\
 & 13) \int l(2 \cos qx) \frac{x^2 dx}{p^4 - x^4} = \frac{\pi}{4p} \{ pq - l(1 + e^{-2pq}) \} \quad \text{V. T. 415. N}^\circ. 10, 16. \\
 & 14) \int l \text{Tang. } qx \frac{dx}{p^4 - x^4} = \frac{\pi}{8p^3} \left\{ -\pi + 2l \frac{1 - e^{-2pq}}{1 + e^{-2pq}} \right\} \quad \text{V. T. 415. N}^\circ. 11, 17.
 \end{aligned}$$

F. Alg. rat. fract. à dén. $b^n \pm x^n$.

Log.

TABLE 418 suite.

Lim. 0 et ∞ .

Circ. Dir.

$$15) \int l \text{Tang. } qx \frac{x^2 dx}{p^4 - x^4} = \frac{\pi}{8p} \left\{ -\pi + 2l \frac{1 + e^{-2pq}}{1 - e^{-2pq}} \right\} \quad \text{V. T. 415. N}^\circ. 11, 17.$$

$$16) \int l \left(\frac{p}{x} \right) \text{Sin. } qx \frac{x dx}{p^4 - x^4} = \frac{\pi}{8p^2} \{ \pi \text{Sin. } pq - 2 \text{Si.}(pq) \cdot \text{Sin. } pq - 2 \text{Ci.}(pq) \cdot \text{Cos. } pq + e^{-pq} \text{Ei.}(pq) + e^{pq} \text{Ei.}(-pq) - lq \} \quad \text{V. T. 417 N}^\circ. 7, 9.$$

$$17) \int l \left(\frac{p}{x} \right) \text{Sin. } qx \frac{x^3 dx}{p^4 - x^4} = \frac{\pi}{8} \{ \pi \text{Sin. } pq - 2 \text{Si.}(pq) \cdot \text{Sin. } pq - 2 \text{Ci.}(pq) \cdot \text{Cos. } pq - e^{-pq} \text{Ei.}(pq) - e^{pq} \text{Ei.}(-pq) \} \quad \text{V. T. 417 N}^\circ. 7, 11.$$

$$18) \int l \left(\frac{p}{x} \right) \text{Cos. } qx \frac{dx}{p^4 - x^4} = \frac{\pi}{8p^3} \{ \pi \text{Cos. } pq - 2 \text{Si.}(pq) \cdot \text{Cos. } pq + 2 \text{Ci.}(pq) \cdot \text{Sin. } pq + e^{-pq} \text{Ei.}(pq) - e^{pq} \text{Ei.}(-pq) \} \quad \text{V. T. 417 N}^\circ. 8, 10.$$

$$19) \int l \left(\frac{p}{x} \right) \text{Cos. } qx \frac{x^2 dx}{p^4 - x^4} = \frac{\pi}{8p} \{ \pi \text{Cos. } pq - 2 \text{Si.}(pq) \cdot \text{Cos. } pq + 2 \text{Ci.}(pq) \cdot \text{Sin. } pq - e^{-pq} \text{Ei.}(pq) + e^{pq} \text{Ei.}(-pq) \} \quad \text{V. T. 417 N}^\circ. 8, 10.$$

F. Alg. rat. fract. à autre dén.

Log.

TABLE 419.

Lim. 0 et ∞ .

Circ. Dir.

$$1) \int l \text{Sin. } qx \frac{dx}{x^4 + 2p^2 x^2 \text{Cos. } 2\lambda + p^4} = \frac{\pi}{4p^3} \text{Sec. } \lambda, l \left[\frac{1}{2} \sqrt{1 - 2e^{-2pq \text{Cos. } \lambda} \text{Cos. } (2pq \text{Sin. } \lambda) + e^{-4pq \text{Cos. } \lambda}} \right] - \frac{\pi}{4p^3} \text{Cosec. } \lambda, \text{Arcsin.} \left\{ \frac{e^{-2pq \text{Cos. } \lambda} \text{Sin. } (2pq \text{Sin. } \lambda)}{1 - 2e^{-2pq \text{Cos. } \lambda} \text{Cos. } (2pq \text{Sin. } \lambda) + e^{-4pq \text{Cos. } \lambda}} \right\}$$

$$2) \int l \text{Cos. } qx \frac{dx}{x^4 + 2p^2 x^2 \text{Cos. } 2\lambda + p^4} = \frac{\pi}{4p^3} \text{Sec. } \lambda, l \left[\frac{1}{2} \sqrt{1 + 2e^{-2pq \text{Cos. } \lambda} \text{Cos. } (2pq \text{Sin. } \lambda) + e^{-4pq \text{Cos. } \lambda}} \right] + \frac{\pi}{4p^3} \text{Cosec. } \lambda, \text{Arcsin.} \left\{ \frac{e^{-2pq \text{Cos. } \lambda} \text{Sin. } (2pq \text{Sin. } \lambda)}{1 + 2e^{-2pq \text{Cos. } \lambda} \text{Cos. } (2pq \text{Sin. } \lambda) + e^{-4pq \text{Cos. } \lambda}} \right\}$$

$$3) \int l \text{Tg. } qx \frac{dx}{x^4 + 2p^2 x^2 \text{Cos. } 2\lambda + p^4} = \frac{\pi}{4p^3} \text{Sec. } \lambda, l \left[\frac{1}{2} \sqrt{\frac{1 - 2e^{-2pq \text{Cos. } \lambda} \text{Cos. } (2pq \text{Sin. } \lambda) + e^{-4pq \text{Cos. } \lambda}}{1 + 2e^{-2pq \text{Cos. } \lambda} \text{Cos. } (2pq \text{Sin. } \lambda) + e^{-4pq \text{Cos. } \lambda}}} \right] - \frac{\pi}{4p^3} \text{Cosec. } \lambda, \text{Arcsin.} \left\{ \frac{2e^{-2pq \text{Cos. } \lambda} \text{Sin. } (2pq \text{Sin. } \lambda)}{1 - 2e^{-2pq \text{Cos. } \lambda} \text{Cos. } (2pq \text{Sin. } \lambda) + e^{-4pq \text{Cos. } \lambda}} \right\}$$

Sur les intégrales 1) à 3) voyez: Plana, Mém. Turin, 1818. 7 II. 9.

$$4) \int l(1 + 2p \text{Cos. } rx + p^2) \frac{dx}{x^2 + q^2} = \frac{\pi}{2q^3} l \left(1 + p e^{-qr} \right) + \frac{\pi}{2q^3} \frac{p e^{-qr}}{1 + p e^{-qr}}, p < 1; \quad \text{Plana, Mém. Turin. 1818. 7. II. 11. où faire.}$$

$$5) \int l(1 + 2p \text{Cos. } rx + p^2) \frac{dx}{x^4 + 2q^2 x^2 \text{Cos. } 2\lambda + q^4} = \frac{\pi}{4q^3} \text{Sec. } \lambda, l \left\{ 1 + 2p e^{-qr \text{Cos. } \lambda} \text{Cos. } (qr \text{Sin. } \lambda) + p^2 e^{-2qr \text{Cos. } \lambda} \right\} + \frac{\pi}{2q^3} \text{Cosec. } \lambda, \text{Arcsin.} \left\{ \frac{p e^{-qr \text{Cos. } \lambda} \text{Sin. } (qr \text{Sin. } \lambda)}{1 + 2p e^{-qr \text{Cos. } \lambda} \text{Cos. } (qr \text{Sin. } \lambda) + p^2 e^{-2qr \text{Cos. } \lambda}} \right\}, p < 1.$$

F. Alg. rat. fract. à autre dén.

Log.

Circ. Dir.

TABLE 419 suite.

Lim. 0 et ∞ .

$$6) \int l(1 + 2p \cos rx + p^2) \frac{dx}{x^4 + 2q^2 x^2 \cos 2\lambda + q^4} = \frac{\pi}{4q^3} \sec \lambda \cdot l \{ p^2 + 2pe^{-qr \cos \lambda} \cos(qr \sin \lambda) + e^{-2qr \cos \lambda} \} \\ + \frac{\pi}{2q^3} \operatorname{Cosec} \lambda \cdot \operatorname{Arcsin} \left\{ \frac{e^{-qr \cos \lambda} \sin(qr \sin \lambda)}{\sqrt{p^2 + 2pe^{-qr \cos \lambda} \cos(qr \sin \lambda) + e^{-2qr \cos \lambda}}} \right\}, p > 1;$$

Les formules 5) et 6) sont trouvées par Plana, Mém. Turin. 1818. 7. II. 8.

$$7) \int l \frac{1 + \operatorname{Tang} q x}{1 - \operatorname{Tang} q x} \frac{dx}{x^2 + x^2} = \frac{\pi}{p^2} \operatorname{Aretang} \left(\frac{e^{pq} - e^{-pq}}{e^{pq} + e^{-pq}} \right) \quad \text{Schlömilch, Beitr. II. § 5.}$$

$$8) \int \frac{\pi(1 - \cos qx) - 2 \sin qx \cdot l x}{\left(\frac{\pi}{2}\right)^2 + (lx)^2} \frac{dx}{x} = 2\pi(1 - e^{-q}) \quad \text{Cauchy, Lim. Imag. Add. 19.}$$

$$9) \int \frac{\cos(qx)}{x^p - 2 \cos \lambda + x^{-p}} \frac{dx}{x} = \frac{\pi}{p \sin \lambda} \frac{e^{-\frac{q}{p}(\pi - \lambda)} - e^{\frac{q}{p}(\pi - \lambda)}}{e^{-\frac{q}{p}} - e^{\frac{q}{p}}} \quad \text{Euler, N. A. Petr. III. 3.}$$

F. Alg. rat. fract.

Log.

Circ. Dir.

TABLE 420.

Lim. — ∞ et ∞ .

$$1) \int l \sin qx \frac{r + sx}{x^2 + 2px \cos \lambda + p^2} dx = -\frac{\pi}{p \sin \lambda} \left(r - \frac{1}{2} s^2 \right) l 2 + \\ + \frac{r - ps \cos \lambda}{p \sin \lambda} \pi l \left[\sqrt{1 - 2e^{-2pq \sin \lambda} \cos(2pq \cos \lambda) + e^{-4pq \sin \lambda}} \right] \\ - s \pi \operatorname{Arcsin} \left\{ \frac{e^{-2pq \sin \lambda} \sin(2pq \cos \lambda)}{\sqrt{1 - 2e^{-2pq \sin \lambda} \cos(2pq \cos \lambda) + e^{-4pq \sin \lambda}}} \right\}$$

$$2) \int l \cos qx \frac{r + sx}{x^2 + 2px \cos \lambda + p^2} dx = -\frac{\pi}{p \sin \lambda} \left(r - \frac{1}{2} s^2 \right) l 2 + \\ + \frac{r - ps \cos \lambda}{p \sin \lambda} \pi l \left[\sqrt{1 + 2e^{-2pq \sin \lambda} \cos(2pq \cos \lambda) + e^{-4pq \sin \lambda}} \right] \\ + s \pi \operatorname{Arcsin} \left\{ \frac{e^{-2pq \sin \lambda} \sin(2pq \cos \lambda)}{\sqrt{1 + 2e^{-2pq \sin \lambda} \cos(2pq \cos \lambda) + e^{-4pq \sin \lambda}}} \right\}$$

$$3) \int l \operatorname{Tang} qx \frac{r + sx}{x^2 + 2px \cos \lambda + p^2} dx = \frac{r - ps \cos \lambda}{2p \sin \lambda} \pi l \frac{1 - 2e^{-2pq \sin \lambda} \cos(2pq \cos \lambda) + e^{-4pq \sin \lambda}}{1 + 2e^{-2pq \sin \lambda} \cos(2pq \cos \lambda) + e^{-4pq \sin \lambda}} \\ - s \pi \operatorname{Arcsin} \left\{ \frac{2e^{-2pq \sin \lambda} \sin(2pq \cos \lambda)}{\sqrt{1 - 2e^{-4pq \sin \lambda} \cos(4pq \cos \lambda) + e^{-8pq \sin \lambda}}} \right\}$$

Ces intégrales se trouvent chez Plana, Mém. Turin. 1818. 7. II. 12.

- 1) $\int_0^{a\pi} l(\sin. x) \cdot x dx = -\frac{1}{2} a^2 \pi^2 \left\{ l 2 - \frac{a \mp 1}{a} a \pi i - \frac{a \pm 1}{a} \frac{2\beta + 1}{2} \pi i \right\} \pm$ pour a pair ou impair; Stegmann, Gr. 7. 103.
 - 2) $\int_0^{a\pi} l(\sin.^2 x) \cdot x dx = -a^2 \pi^2 l 2$ Clausen, Cr. 7. 309.
 - 3) $= a^2 \pi l 2$ (fautive) Hill, Cr. 7. 102.
 - 4) $\int_0^{2a\pi} l \sin. x \cdot x dx = -2 a^2 \pi^2 \left\{ l 2 - a \pi i \frac{2a-1}{2a} - \frac{2\beta+1}{2} \frac{2a+1}{2a} \pi i \right\}$ Arndt, Gr. 6. 187. — Lindmann, Gr. 16. 91.
 - 5) $\int_0^{(2a+1)\pi} l \sin. x \cdot x dx = -\frac{(2a+1)^2}{2} \pi^2 \left\{ l 2 - a \pi i \frac{2a+2}{2a+1} - \frac{2\beta+1}{2} \frac{2a}{2a+1} \pi i \right\}$ Stegmann, Gr. 7. 108. — Lindmann, Gr. 16. 94.
 - 6) $= -\frac{(2a+1)^2}{2} \pi^2 \left\{ l 2 - a \pi i \frac{2a+2}{2a+1} - \frac{2\beta+1}{2} \frac{2a+1}{2a} \pi i \right\}$ (fautive)
 - 7) $\int_{2a\pi}^{(2a+1)\pi} l \sin. x \cdot x dx = -\frac{1}{2} \pi^2 (4a+1) l 2 + a \pi^3 i (4a+1)$
 - 8) $\int_{(2a-1)\pi}^{2a\pi} l \sin. x \cdot x dx = -\frac{1}{2} \pi^2 (4a-1) l 2 + \frac{2a+1}{2} \pi^3 i (4a-1)$
 - 9) $\int_{(2a-2)\pi}^{2a\pi} l \sin. x \cdot x dx = -\frac{1}{2} \pi^2 (2a)^2 l 2 + a \pi^3 i (4a-3) + \frac{2\beta+1}{2} (4a-1) \pi^2 i$ (fautive).
 - 10) $= -2 \pi^2 (2a-1) l 2 + a \pi^3 i (4a-3) + \frac{2\beta+1}{2} (4a-1) \pi^2 i$ Stegmann, Gr. 7. 108.
 - 11) $\int_0^{2a\pi} l(1+2p \cos. x + p^2) \cdot x^b dx = 2 \sum_0^{b-1} \left\{ 1^{n_1} \binom{b}{n} 2a\pi \right\}^{b-n} \cos. \left\{ \frac{n+1}{2} \pi \right\} \cdot \frac{x}{1} \frac{p^n}{m^{n+2}} \right\}, p^2 < 1;$ Hoppe, Cr. 10. 139.
 - 12) $\int_0^\lambda \left\{ 2x + l \frac{1 + \sin. x}{1 - \sin. x} \right\} \frac{dx}{1 / (\cos.^2 x - \cos.^2 \lambda) (1 - \cos.^2 \lambda \cdot \cos.^2 x)} = \pi \operatorname{Cosec}. q \cdot F(p, q)$
 - 13) $\int_0^\lambda \left\{ 2x \cos. x - l \frac{1 + \sin. x}{1 - \sin. x} \right\} \frac{\cos. x}{\sin.^2 x \cdot (\cos.^2 x - \cos.^2 \lambda) (1 - \cos.^2 \lambda \cdot \cos.^2 x)} dx =$
- $$= \frac{\pi \cos.^2 \lambda}{\sin. \lambda \cdot \sin. q} F(p, q) - \frac{\pi \sin. q}{\sin.^4 \lambda} E(p, q) + \frac{\pi \cos. \lambda}{\sin.^2 \lambda}$$

$\left. \begin{array}{l} \cos. q = \cos.^2 \lambda, \\ p = \sin. \lambda \operatorname{Cosec}. q; \\ \text{Legendre, Exerc.} \\ \text{Suppl. 49} \end{array} \right\}$

- 1) $\int \text{Arcsin. } x. (2lx + 1) x dx = \frac{1}{4} \pi \left(l2 - \frac{1}{2} \right)$ V. T. 163. N°. 4.
- 2) $\int \text{Arcsin. } x. (3lx + 1) x^2 dx = \frac{2}{3} \left(\frac{5}{6} - l2 \right)$ V. T. 163. N°. 5.
- 3) $\int \text{Arcsin. } x. (4lx + 1) x^3 dx = \frac{3}{16} \pi \left(l2 - \frac{7}{12} \right)$ V. T. 163. N°. 6.
- 4) $\int \text{Arcsin. } x. (5lx + 1) x^4 dx = \frac{8}{15} \left(\frac{47}{60} - l2 \right)$ V. T. 163. N°. 7.
- 5) $\int \text{Arcsin. } x. lx \frac{dx}{x} = -\frac{1}{4} \pi \left\{ (l2)^2 + \frac{1}{12} \pi^2 \right\}$ V. T. 164. N°. 1.
- 6) $\int \text{Arcsin. } x. \left\{ xl(1-p^2x^2) - \frac{p^2x^3}{1-p^2x^2} \right\} dx = \frac{1}{8} \pi \left\{ 2l \frac{2(1-p^2)}{1+\sqrt{1-p^2}} + \frac{1-\sqrt{1-p^2}}{1+\sqrt{1-p^2}} \right\}, p < 1; \text{ V. T. 165. N°. 17.}$
- 7) $\int \text{Arcsin. } x. \left\{ l(1+q^2x^2) + \frac{q^2x^2}{1+q^2x^2} \right\} x dx = \frac{\pi}{4} l \left\{ \frac{2(1+q^2)}{1+\sqrt{1+q^2}} - \frac{1-\sqrt{1+q^2}}{q^2} - \frac{1}{2} \right\}, q < 1; \text{ V. T. 165. N°. 8.}$
- 8) $\int \text{Arcsin. } x. \left\{ l(px+1) - \frac{px}{px+1} \right\} \frac{dx}{x} = \frac{1}{8} \pi^2 - \frac{1}{2} (\text{Arccos. } p)^2 - \frac{1}{2} \pi l(1+p), p^2 < 1; \text{ V. T. 165. N°. 3.}$
- 9) $\int \text{Arcsin. } x. \left\{ l \frac{1+qx}{1-qx} - \frac{2qx}{1-q^2x^2} \right\} \frac{dx}{x^2} = \frac{\pi}{2} l \frac{1-q}{1+q} + \pi \text{Arcsin. } q$ V. T. 166. N°. 6.
- 10) $\int \text{Arcsin. } x. \left\{ l \frac{1+x \text{Sin. } \lambda}{1-x \text{Sin. } \lambda} - \frac{2x \text{Sin. } \lambda}{1-x^2 \text{Sin.}^2 \lambda} \right\} \frac{dx}{x^2} = \pi l \text{Cot. } \left(\frac{\pi}{4} - \frac{1}{2} \lambda \right) + \pi \lambda$ V. T. 166. N°. 7.
- 11) $\int \text{Arcsin. } x. \left\{ \frac{1}{x} l \frac{1+x}{1-x} + \frac{2}{1-x^2} \right\} \frac{dx}{x} = \infty$ V. T. 166. N°. 5.
- 12) $\int \text{Arcsin. } x. \left\{ \frac{1+qx^2}{(1-qx^2)^2} l \frac{1+px}{1-px} + \frac{2p}{1-qx^2} \frac{x}{1-p^2x^2} \right\} dx = \frac{\pi}{2(1-q)} l \frac{1+p}{1-p} +$
 $+ \frac{\pi}{\sqrt{q}(1-q)} l \frac{p\sqrt{q} - \{1-\sqrt{1-q}\}}{p\sqrt{q} + \{1-\sqrt{1-q}\}} \frac{\{1-\sqrt{1-p^2}\}}{\{1-\sqrt{1-p^2}\}}$ V. T. 166. N°. 17.
- 13) $\int \text{Arccos. } x. \left\{ \frac{1+qx^2}{(1-qx^2)^2} l \frac{1+px}{1-px} + \frac{2p}{1-qx^2} \frac{x}{1-p^2x^2} \right\} dx =$
 $\frac{\pi}{\sqrt{q}(1-q)} l \frac{p\sqrt{q} + \{1-\sqrt{1-q}\}}{p\sqrt{q} - \{1-\sqrt{1-q}\}} \frac{\{1-\sqrt{1-p^2}\}}{\{1-\sqrt{1-p^2}\}}$ V. T. 166. N°. 17.

$$14) \int \operatorname{Arccos}.x. \{1 + l(x^2)\} x dx = \frac{\pi}{4} \left(\frac{1}{2} - l 2 \right) \quad \text{V. T. 163. N}^{\circ}. 4.$$

$$15) \int \operatorname{Arccos}.x. \{1 + l(x^3)\} x^2 dx = -\frac{2}{3} \left(\frac{5}{6} - l 2 \right) \quad \text{V. T. 163. N}^{\circ}. 5.$$

$$16) \int \operatorname{Arccos}.x. \{1 + l(x^4)\} x^3 dx = -\frac{3\pi}{16} \left(l 2 - \frac{7}{12} \right) \quad \text{V. T. 163. N}^{\circ}. 6.$$

$$17) \int \operatorname{Arccos}.x. \{1 + l(x^5)\} x^4 dx = -\frac{8}{15} \left(\frac{17}{60} - l 2 \right) \quad \text{V. T. 163. N}^{\circ}. 7.$$

$$18) \int \operatorname{Arccos}.x. \left\{ l(1 + q^2 x^2) + \frac{q^2 x^2}{1 + q^2 x^2} \right\} x dx = \frac{\pi}{4} \left\{ l \frac{1 + \sqrt{1 + q^2}}{2} + \frac{1 - \sqrt{1 + q^2}}{q^2} + \frac{1}{2} \right\}, q < 1; \quad \text{V. T. 165. N}^{\circ}. 8.$$

$$19) \int \operatorname{Arccos}.x. \left\{ xl(1 - p^2 x^2) - \frac{p^2 x^3}{1 - p^2 x^2} \right\} dx = \frac{1}{8} \pi \left\{ 2l \frac{1 + \sqrt{1 - p^2}}{2} - \frac{1 - \sqrt{1 - p^2}}{1 + \sqrt{1 - p^2}} \right\}, p < 1; \quad \text{V. T. 165. N}^{\circ}. 17.$$

$$20) \int \operatorname{Arccos}.x. \left\{ l(px + 1) - \frac{px}{px + 1} \right\} \frac{dx}{x^2} = \frac{1}{2} (\operatorname{Arccos}.p)^2 - \frac{1}{8} \pi^2 + \frac{1}{2} p \pi, p < 1; \quad \text{V. T. 165. N}^{\circ}. 3.$$

$$21) \int \operatorname{Arccos}.x. \left\{ l \frac{1 + x}{1 - x} - \frac{2x}{1 - x^2} \right\} \frac{dx}{x^2} = -\pi \frac{2 + \pi}{2} \quad \text{V. T. 166. N}^{\circ}. 5.$$

$$22) \int \operatorname{Arccos}.x. \left\{ l \frac{1 + qx}{1 - qx} - \frac{2qx}{1 - q^2 x^2} \right\} \frac{dx}{x^2} = -\pi \{q + \operatorname{Arcsin}.q\} \quad \text{V. T. 166. N}^{\circ}. 6.$$

$$23) \int \operatorname{Arccos}.x. \left\{ l \frac{1 + x \operatorname{Sin}. \lambda}{1 - x \operatorname{Sin}. \lambda} - \frac{2x \operatorname{Sin}. \lambda}{1 - x^2 \operatorname{Sin}.^2 \lambda} \right\} \frac{dx}{x^2} = -\pi \{\lambda + \operatorname{Sin}. \lambda\} \quad \text{V. T. 166. N}^{\circ}. 7.$$

$$24) \int \operatorname{Arctang}.x. lx \frac{dx}{x} = -\frac{1}{32} \pi^3 \quad \text{V. T. 154. N}^{\circ}. 1.$$

$$25) \int \operatorname{Arctang}.x. (lx)^3 \frac{dx}{x} = -\frac{5}{256} \pi^5 \quad \text{V. T. 155. N}^{\circ}. 1.$$

$$26) \int \operatorname{Arctang}.x. (lx)^5 \frac{dx}{x} = -\frac{61}{1536} \pi^7 \quad \text{V. T. 155. N}^{\circ}. 8.$$

$$27) \int \operatorname{Arctang}.x. (lx)^{q-1} \frac{dx}{x} = \frac{(-1)^{q-1}}{q} \Gamma(q+1) \frac{\pi}{1} \frac{(-1)^n}{(2n+1)^{q+1}} \quad \text{V. T. 158. N}^{\circ}. 1.$$

$$28) \int \operatorname{Arctang}.x. \frac{(5 - 3x^2)lx + 1 - x^2}{(1 - x^2)^2} x^4 dx = \frac{\pi^2 - 4\pi - 8}{32} \quad \text{V. T. 152. N}^{\circ}. 13.$$

F. Alg. rat.

Log. en num.

Circ. Inv.

TABLE 422 suite.

Lim. 0 et 1.

$$29) \int \text{Arctang. } x \frac{(1+x^2)lx + 1-x^2}{(1+x^2)^2} dx = \frac{1}{4}l2 \quad \text{V. T. 153. N}^\circ. 15.$$

$$30) \int \text{Arctang. } x \frac{(3-x^2)lx + 1-x^2}{(1-x^2)^2} x^2 dx = \pi \frac{\pi-12}{96} \quad \text{V. T. 152. N}^\circ. 17.$$

$$31) \int \text{Arctang. } x \frac{(1+x^2)lx - x^2 + 1}{(1-x^2)^2} dx = -\pi \frac{4-\pi}{32} \quad \text{V. T. 152. N}^\circ. 16.$$

$$32) \int \text{Arctang. } (x^2) \frac{(1+x^2)lx + 1-x^2}{(1-x^2)^2} dx = \frac{\pi^2}{8(2+\sqrt{2})} - \frac{1}{16}\pi \quad \text{V. T. 153. N}^\circ. 16.$$

$$33) \int \frac{\text{Arctang. } (lx)}{x} \frac{dx}{1-x^{-2\pi}} = \frac{1}{4}l2\pi - \frac{1}{2} \quad \text{V. T. 299. N}^\circ. 2.$$

$$34) \int \frac{\text{Arctang. } \left(\frac{1}{p}lx\right)}{x} \frac{dx}{1-x^{-2\pi}} = l \left[\left(\frac{p}{e}\right)^p \frac{1}{\Gamma(p)} \sqrt[p]{\frac{2\pi}{p}} \right] \quad \text{V. T. 299. N}^\circ. 3.$$

F. Alg. irrat.

Log. en num.

Circ. Inv.

TABLE 425.

Lim. 0 et 1.

$$1) \int \text{Arcsin. } x \frac{1+x^2+lx}{\sqrt{(1+x^2)^3}} dx = \frac{\pi}{8}l2 \quad \text{V. T. 163. N}^\circ. 12.$$

$$2) \int \text{Arcsin. } x \frac{2-l(1-p^2x^2)}{\sqrt{(1-p^2x^2)^3}} x dx = \frac{1}{2p^2}l(1-p^2) \cdot \left\{ F'(p) - \frac{\pi}{\sqrt{(1-p^2)}} \right\}, p < 1; \quad \text{V. T. 165. N}^\circ. 18.$$

$$3) \int \text{Arcsin. } x \frac{3l(1-p^2x^2)-2}{\sqrt{(1-p^2x^2)^5}} x dx = \frac{1}{2p^2(1-p^2)} \left[\frac{\pi}{\sqrt{(1-p^2)}} l(1-p^2) + \right. \\ \left. + (2-p^2)2F'(p) - \{4+l(1-p^2)\}E'(p) \right] \quad \text{V. T. 165. N}^\circ. 20.$$

$$4) \int \text{Arcsin. } x \cdot \left(\frac{p^2x^2lx}{1-p^2x^2} + 1 \right) \frac{dx}{x\sqrt{(1-p^2x^2)}} = \frac{1}{2}lp \cdot F'(p) + \frac{1}{4}\pi F' \{ \sqrt{(1-p^2)} \}, p < 1; \quad \text{V. T. 163. N}^\circ. 14.$$

$$5) \int \text{Arcsin. } x \cdot \left\{ \frac{p^l(1+px^2)}{1-p^2x^2} + \frac{2}{1+px^2} \right\} \frac{x}{\sqrt{(1-p^2x^2)}} dx = -\frac{1}{2p} \left[l \left\{ \frac{2(1+p)}{\sqrt{p}} \right\} \cdot F'(p) - \right. \\ \left. - \frac{1}{4}\pi F' \{ \sqrt{(1-p^2)} \} - \frac{\pi}{\sqrt{(1-p^2)}} l(1+p) \right], p^2 < 1; \quad \text{V. T. 165. N}^\circ. 9.$$

- 6) $\int \text{Arcsin. } x. \left\{ \frac{p l (1 - p x^2)}{1 - p^2 x^2} - \frac{2}{1 - p x^2} \right\} \frac{x}{\sqrt{1 - p^2 x^2}} dx = -\frac{1}{2p} \left[l \left\{ \frac{2(1-p)}{\sqrt{p}} \right\} \cdot F'(p) - \right.$
 $\left. - \frac{1}{4} \pi F' \{ \sqrt{1 - p^2} \} - \frac{\pi}{\sqrt{1 - p^2}} l(1 - p) \right], p < 1; \text{ V. T. 165. N}^\circ. 19.$
- 7) $\int \text{Arcsin. } x. \left\{ l(1 - p^2 x^2 \text{ Sin.}^2 \lambda) - \frac{1 - p^2 x^2}{1 - p^2 x^2 \text{ Sin.}^2 \lambda} \cdot 2 \text{ Sin.}^2 \lambda \right\} \frac{x dx}{\sqrt{1 - p^2 x^2}^3} =$
 $= \frac{1}{p^2} \left[\frac{\pi}{2 \sqrt{1 - p^2}} l(1 - p^2 \text{ Sin.}^2 \lambda) - E'(p) \cdot \{ F(p, \lambda) \}^2 + 2 F'(p) \cdot \gamma(p, \lambda) \right], p < 1; \text{ V. T. 165. N}^\circ. 13.$
- 8) $\int \text{Arcsin. } x. \left\{ \frac{l(1 - p^2 x^2)}{1 - p^2 x^2} - \frac{1 x^2}{1 - p^2 x^2} \right\} \frac{x}{\sqrt{1 - p^2 x^2}} dx =$
 $= \frac{1}{2p^2} \left[l \left\{ \frac{4(1-p^2)}{p} \right\} \cdot F'(p) - \frac{1}{2} \pi F' \{ \sqrt{1 - p^2} \} - \frac{\pi}{\sqrt{1 - p^2}} l(1 - p^2) \right], p < 1; \text{ V. T. 165. N}^\circ. 24.$
- 9) $\int \text{Arcsin. } x. \left\{ \frac{1}{\sqrt{1 - p^2 x^2}} l \frac{1 + q \sqrt{1 - p^2 x^2}}{1 - q \sqrt{1 - p^2 x^2}} - \frac{2q}{1 - q^2 (1 - p^2 x^2)} \right\} \frac{x}{(1 - p^2 x^2)^2} dx =$
 $= \frac{\pi}{p^2} \left[\frac{1}{2 \sqrt{1 - p^2}} l \frac{1 + q \sqrt{1 - p^2}}{1 - q \sqrt{1 - p^2}} - F' \{ \sqrt{1 - p^2}, \text{Arcsin. } q \} \right] \text{ V. T. 166. N}^\circ. 11.$
- 10) $\int \text{Arccos. } x \frac{1 + x^2 + l x}{\sqrt{1 + x^2}^3} dx = \frac{\pi}{8} l \frac{1}{2} \text{ V. T. 163. N}^\circ. 12.$
- 11) $\int \text{Arccos. } x \frac{2 - l(1 - p^2 x^2)}{\sqrt{1 - p^2 x^2}^3} x dx = \frac{1}{2p^2} l(1 - p^2) \cdot F'(p), p < 1; \text{ V. T. 165. N}^\circ. 18.$
- 12) $\int \text{Arccos. } x \frac{3l(1 - p^2 x^2) - 2}{\sqrt{1 - p^2 x^2}^5} x dx = \frac{1}{p^2(1 - p^2)} \left[(p^2 - 2)F'(p) + \left\{ 2 + \frac{1}{2} l(1 - p^2) \right\} E'(p) \right] \text{ V. T. 165. N}^\circ. 20.$
- 13) $\int \text{Arccos. } x. \left\{ \frac{p^2 l(1 - x^2)}{1 - p^2 x^2} - \frac{2}{1 - x^2} \right\} \frac{x dx}{\sqrt{1 - p^2 x^2}} = \frac{1}{2} l \left\{ \frac{1 - p^2}{p^2} \right\} \cdot F'(p) - \frac{1}{2} \pi F' \{ \sqrt{1 - p^2} \}, p < 1; \text{ V. T. 165. N}^\circ. 12.$
- 14) $\int \text{Arccos. } x. \left\{ \frac{p l (1 + p x^2)}{1 - p^2 x^2} + \frac{2}{1 + p x^2} \right\} \frac{x}{\sqrt{1 - p^2 x^2}} dx =$
 $= \frac{1}{2p} \left[\frac{1}{4} \pi F' \{ \sqrt{1 - p^2} \} - l \left\{ \frac{2(1+p)}{\sqrt{p}} \right\} \cdot F'(p) \right], p < 1; \text{ V. T. 165. N}^\circ. 9.$
- 15) $\int \text{Arccos. } x. \left\{ \frac{p l (1 - p x^2)}{1 - p^2 x^2} - \frac{2}{1 - p x^2} \right\} \frac{x}{\sqrt{1 - p^2 x^2}} dx =$
 $= \frac{1}{2p} \left[\frac{1}{4} \pi F' \{ \sqrt{1 - p^2} \} - l \left\{ \frac{2(1-p)}{\sqrt{p}} \right\} \cdot F'(p) \right], p < 1; \text{ V. T. 165. N}^\circ. 19.$

$$16) \int \operatorname{Arccos} x. \left\{ l(1 - p^2 x^2 \operatorname{Sin}^2 \lambda) - \frac{1 - p^2 x^2}{1 - p^2 x^2 \operatorname{Sin}^2 \lambda} 2 \operatorname{Sin}^2 \lambda \right\} \frac{x dx}{\sqrt{(1 - p^2 x^2)^3}} =$$

$$= \frac{1}{p^2} [E'(p). \{F(p, \lambda)\}^2 - 2 F'(p). \gamma(p, \lambda)], p < 1; \quad \text{V. T. 165. N}^\circ. 13.$$

$$17) \int \operatorname{Arccos} x. \left\{ \frac{l(1 - p^2 x^4)}{1 - p^2 x^2} - \frac{4 x^2}{1 - p^2 x^4} \right\} \frac{x}{\sqrt{(1 - p^2 x^2)}} dx =$$

$$- \frac{1}{2 p^2} \left[\frac{1}{2} \pi F' \{ \sqrt{(1 - p^2)} \} - l \left\{ \frac{4(1 - p^2)}{p} \right\} . F'(p) \right], p < 1; \quad \text{V. T. 165. N}^\circ. 23.$$

$$18) \int \operatorname{Arccos} x. \left\{ \frac{1}{\sqrt{(1 - p^2 x^2)}} l \frac{1 + q \sqrt{(1 - p^2 x^2)}}{1 - q \sqrt{(1 - p^2 x^2)}} - \frac{2 q}{1 - q^2 (1 - p^2 x^2)} \right\} \frac{x}{(1 - p^2 x^2)^2} dx =$$

$$= \frac{\pi}{p^2} \left[\frac{1}{2} l \frac{1 - q}{1 + q} + F \{ \sqrt{(1 - p^2)}, \operatorname{Arcsin} q \} \right] \quad \text{V. T. 166. N}^\circ. 11.$$

$$19) \int \operatorname{Arcsin} x. l x \frac{x dx}{\sqrt{(1 - x^2)^3}} = \frac{1}{8} \pi^2 - 2 \sum_0^{\infty} \frac{(-1)^n}{(2n + 1)^2} \quad \text{V. T. 261. N}^\circ. 6 \text{ et T. 152. N}^\circ. 13.$$

$$20) \int (\operatorname{Arcsin} x)^{p-1}. l x \frac{dx}{\sqrt{(1 - x^2)}} = - \frac{1}{p} \left(\frac{1}{2} \pi \right)^p \left\{ 1 - \sum_1^{\infty} \frac{2}{p + 2m} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\} \quad \text{V. T. 257. N}^\circ. 2.$$

$$21) \int (\operatorname{Arccos} x)^{p-1}. l(1 + x) \frac{dx}{\sqrt{(1 - x^2)}} = \frac{1}{p} \left(\frac{\pi}{2} \right)^p \sum_1^{\infty} \left\{ \frac{2^{2m} - 1}{4^{m-1}} \frac{1}{p + 2m} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\} \quad \text{V. T. 259. N}^\circ. 3.$$

$$22) \int (\operatorname{Arccos} x)^{p-1}. l(1 - x) \frac{dx}{\sqrt{(1 - x^2)}} = \frac{1}{p} \left(\frac{\pi}{2} \right)^p \left[-2 + \sum_1^{\infty} \left\{ \frac{1}{4^{m-1}} \frac{1}{p + 2m} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\} \right] \quad \text{V. T. 259. N}^\circ. 4.$$

$$23) \int (\operatorname{Arccos} x)^{p-1}. l(1 - x^2) \frac{dx}{\sqrt{(1 - x^2)}} = \frac{2}{p} \left(\frac{1}{2} \pi \right)^p \left\{ -1 + \sum_1^{\infty} \frac{2}{p + 2m} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\} \quad \text{V. T. 259. N}^\circ. 8.$$

$$24) \int l x \left\{ \frac{x}{\sqrt{(1 - x^2)}} \operatorname{Arcsin} x + p \right\} (\operatorname{Arcsin} x)^{p-1} \frac{dx}{1 - x^2} = \left(\frac{\pi}{2} \right)^p \left[1 + \sum_1^{\infty} \left\{ \frac{1}{4^{m-1}} \frac{2^{2m} - 1}{p + 2m} \sum_1^{\infty} \frac{1}{(2n)^{2m}} \right\} \right] \quad \text{V. T. 261. N}^\circ. 24.$$

$$25) \int l(1 + x^2). \left\{ \frac{x}{\sqrt{(1 + x^2)^3}} \operatorname{Arcsin} x - \frac{1}{\sqrt{(1 - x^4)}} \right\} dx = - \frac{\pi}{2} \sqrt{2} - \frac{\pi}{2 \sqrt{2}} l 2 + \sqrt{2} F \left(\operatorname{Sin} \frac{\pi}{4} \right) \quad \text{V. T. 261. N}^\circ. 7.$$

$$26) \int l(1 + x^2). \left\{ 2 \operatorname{Arctang} x - \frac{x}{1 + x^2} \right\} \frac{dx}{x^3} = \sum_0^{\infty} \frac{(-1)^n}{(2n + 1)^2} \quad \text{V. T. 260. N}^\circ. 4.$$

$$27) \int l(1 + x^2). \left\{ 2 \operatorname{Arctg} x - \frac{p x}{1 + x^2} \right\} (\operatorname{Arctg} x)^{p-1} \frac{dx}{x^3} = \left(\frac{\pi}{4} \right)^p \left[2 - l 2 - \sum_1^{\infty} \frac{4}{p + 2m} \sum_1^{\infty} \frac{1}{(4n)^{2m}} \right] \quad \text{V. T. 260. N}^\circ. 20.$$

F. Alg. irrat.

Log. en num.

Circ. Inv.

TABLE 425 suite.

Lim. 0 et 1.

$$28) \int l(1-x^2) \cdot \left\{ \operatorname{Arccos}.x + \frac{x}{\sqrt{1-x^2}} \right\} \frac{dx}{x^2} = -4 \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2} \quad \text{V. T. 253. N}^\circ. 13.$$

$$29) \int l(\operatorname{Sin}^2 \lambda - x^2) \cdot \left\{ \operatorname{Arccos}.x + \frac{x}{\sqrt{1-x^2}} \right\} \frac{dx}{x^2} = -4 \operatorname{Cosec}.\lambda \sum_0^{\infty} \frac{\operatorname{Sin}.\{(2n+1)\lambda\}}{(2n+1)^2} \quad \text{V. T. 253. N}^\circ. 15.$$

$$30) \int l(1-x^2 \operatorname{Sin}^2 \mu) \frac{1-x^2 \operatorname{Sin}^2 \lambda - 2x \operatorname{Sin}^2 \lambda \cdot \operatorname{Arccos}.x \cdot \sqrt{1-x^2}}{(1-x^2 \operatorname{Cos}^2 \lambda)^2} \frac{dx}{\sqrt{1-x^2}} =$$

$$= \frac{2\pi}{\operatorname{Cos}^2 \lambda - \operatorname{Cos}^2 \mu} l \operatorname{Cos}.\frac{1}{2}\mu \cdot \operatorname{Sec}.\frac{1}{2}\lambda \quad \text{V. T. 260. N}^\circ. 11.$$

$$31) \int l(\operatorname{Cos}^2 \mu + x^2 \operatorname{Sin}^2 \mu) \frac{\operatorname{Cos}^2 \lambda + x^2 \operatorname{Sin}^2 \lambda - 2x \operatorname{Sin}^2 \lambda \cdot \operatorname{Arcsin}.x \cdot \sqrt{1-x^2}}{(\operatorname{Cos}^2 \lambda + x^2 \operatorname{Sin}^2 \lambda)^2} \frac{dx}{\sqrt{1-x^2}} =$$

$$= \frac{2\pi \operatorname{Sin}^2 \mu}{\operatorname{Sin}.\{\lambda + \mu\} \cdot \operatorname{Sin}.\{\lambda - \mu\}} l \operatorname{Cos}.\frac{1}{2}\lambda \cdot \operatorname{Sec}.\frac{1}{2}\mu \quad \text{V. T. 260. N}^\circ. 6.$$

F. Alg.

Log. en dén.

Circ. Inv.

TABLE 424.

Lim. 0 et 1.

$$1) \int \operatorname{Arctang}.x \frac{1-x \cdot 2x \operatorname{Lx} - x + 1}{x (lx)^2} dx = l \frac{1}{\pi} \quad \text{V. T. 172. N}^\circ. 1.$$

$$2) \int \operatorname{Arctang}.x \frac{x^q - x^{-q} - q \{x^q + x^{-q}\} \operatorname{Lx}}{x (lx)^2} dx = l \operatorname{Tang}.\left\{ \frac{1+q}{4} \right\} \quad \text{V. T. 174. N}^\circ. 5.$$

$$3) \int \operatorname{Arctang}.x \frac{lx}{\{x^2 + (lx)^2\}^2} \frac{dx}{x} = \frac{3-\pi}{8\pi} \quad \text{V. T. 173. N}^\circ. 7.$$

$$4) \int \operatorname{Arctang}.x \frac{lx}{\{x^2 + 4(lx)^2\}^2} \frac{dx}{x} = \frac{l2-1}{32\pi} \quad \text{V. T. 173. N}^\circ. 8.$$

$$5) \int \operatorname{Arctang}.x \frac{lx}{\{q^2 + (lx)^2\}^2} \frac{dx}{x} = \frac{1}{8q} \left\{ -\frac{\pi}{q} + Z\left(\frac{2q+3\pi}{4\pi}\right) - Z\left(\frac{2q+\pi}{4\pi}\right) \right\} \quad \text{V. T. 173. N}^\circ. 9.$$

$$6) \int \operatorname{Arccot}.x \frac{lx}{\{x^2 + (lx)^2\}^2} \frac{dx}{x} = \frac{\pi-5}{8\pi} \quad \text{V. T. 173. N}^\circ. 7.$$

$$7) \int \operatorname{Arccot}.x \frac{lx}{\{x^2 + (lx)^2\}^2} \frac{dx}{x} = -\frac{1+l2}{32\pi} \quad \text{V. T. 173. N}^\circ. 8.$$

$$8) \int \operatorname{Arccot}.x \frac{lx}{\{q^2 + (lx)^2\}^2} \frac{dx}{x} = \frac{1}{8q} \left\{ -\frac{\pi}{q} + Z\left(\frac{2q+\pi}{4\pi}\right) - Z\left(\frac{2q+3\pi}{4\pi}\right) \right\} \quad \text{V. T. 173. N}^\circ. 9.$$

F. Alg.

Log. en dén.

Circ. Inv.

TABLE 424 suite.

Lim. 0 et 1.

- 9) $\int \frac{\text{Arccos. } x}{(\text{Arccos. } x)^2 + (lx)^2} \frac{dx}{x} = \frac{\pi}{2l2} \quad \text{V. T. 412. N}^\circ. 1.$
- 10) $\int \frac{\text{Arccos. } x}{(\text{Arccos. } x)^2 + (lx)^2} \frac{x}{1-x^2} dx = \infty \quad \text{V. T. 412. N}^\circ. 8.$
- 11) $\int \frac{lx}{(\text{Arccos. } x)^2 + (lx)^2} \frac{lx}{(1-p)^2 - 4px^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2} \frac{\pi}{p^2-1} \left\{ \frac{1}{l2-l(1+p)} - \frac{1+p}{1-p} \right\}, p^2 \leq 1; \quad \text{V. T. 412. N}^\circ. 9.$
- 12) $\int \frac{\text{Arccos. } x}{(\text{Arccos. } x)^2 + (lx)^2} \frac{x}{(1-p)^2 - 4px^2} dx = \frac{\pi}{8p} \left\{ \frac{1}{l2-l(1+p)} - \frac{1}{l2} \right\} \quad \text{V. T. 412. N}^\circ. 10.$
- 13) $\int \frac{lx}{(\text{Arccos. } x)^2 + (lx)^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2} \pi \left(1 - \frac{1}{l2} \right) \quad \text{V. T. 412. N}^\circ. 2.$
- 14) $\int \frac{l(1+2x^2)}{(\text{Arccos. } x)^2 + (lx)^2} \frac{dx}{\sqrt{(1-x^2)^3}} = \frac{1}{2} \pi \quad \text{V. T. 412. N}^\circ. 7.$
- 15) $\int \frac{lx}{(\text{Arccos. } x)^2 + (lx)^2} \frac{dx}{x^2 \sqrt{(1-x^2)}} = \infty \quad \text{V. T. 412. N}^\circ. 6.$
- 16) $\int \frac{(\text{Arccos. } x)^2 + 2 \text{Arccos. } x \cdot lx \frac{x}{\sqrt{(1-x^2)}} - (lx)^2}{\{(\text{Arccos. } x)^2 + (lx)^2\}^2} \frac{\text{Arccos. } x}{x} dx = \pi \frac{l2-1}{2l2} \quad \text{V. T. 424. N}^\circ. 13.$
- 17) $\int \frac{(\text{Arccos. } x)^2 - 2 \text{Arccos. } x \cdot lx \frac{\sqrt{(1-x^2)}}{x} - (lx)^2}{\{(\text{Arccos. } x)^2 + (lx)^2\}^2} \frac{lx}{\sqrt{(1-x^2)}} dx = -\frac{\pi}{2l2} \quad \text{V. T. 424. N}^\circ. 9.$

F. Alg.

Log.

Circ. Inv.

TABLE 425.

Lim. 0 et ∞ .

- 1) $\int \text{Arctang. } x \cdot (lx)^{2a-1} \frac{dx}{x} = \infty \quad \text{V. T. 180. N}^\circ. 5.$
- 2) $\int \text{Arctang. } x \cdot lx \cdot \left\{ \text{Arctang. } x - \frac{2x}{1+x^2} \right\} \frac{dx}{x^2} = \pi l2 \quad \text{V. T. 264. N}^\circ. 4.$
- 3) $\int \text{Arctang. } x \cdot \left\{ lx + \frac{1+x^2}{1-x^2} \right\} \frac{1-x^2}{(1+x^2)^2} dx = 0 \quad \text{V. T. 182. N}^\circ. 2.$
- 4) $\int \text{Arctang. } \frac{x}{q} \cdot \left\{ \frac{q^2-x^2}{q^2+x^2} l(1+x) + \frac{x}{1+x} \right\} \frac{dx}{q^2+x^2} = -\frac{1}{2(1+q^2)} \left\{ \frac{\pi}{2} + qlq \right\} \quad \text{V. T. 182. N}^\circ. 8.$

$$5) \int \text{Arctang.} \frac{x}{q} \cdot \left\{ \frac{q^2 - x^2}{q^2 + x^2} l(1-x)^2 - \frac{2x}{1-x} \right\} \frac{dx}{q^2 + x^2} = \frac{1}{1+q^2} \left\{ \pi - q l q \right\} \quad \text{V. T. 182. N}^\circ. 9.$$

$$6) \int \text{Arctang.} \frac{x}{q} \cdot \left\{ \frac{q^2 - x^2}{q^2 + x^2} l(1-x^2)^2 - \frac{4x^2}{1-x^2} \right\} \frac{dx}{q^2 + x^2} = - \frac{2q}{1+q^2} l q \quad \text{V. T. 182. N}^\circ. 20.$$

$$7) \int \text{Arctg.} x \frac{\{(p-1)x^p - (q-1)x^q\} l x - x^p + x^q}{x^2 (l x)^2} dx = \pi l \left(\text{Tg.} \frac{1}{4} p \pi. \text{Cot.} \frac{1}{4} q \pi \right), p < 1, q < 1; \quad \text{V. T. 180. N}^\circ. 7.$$

$$8) \int l x. \text{Arctang.} x \frac{x}{(1+x^2)^2} dx = \frac{\pi}{4} l 2 \quad \text{V. T. 266. N}^\circ. 1 \text{ et T. 182. N}^\circ. 2.$$

$$9) \int l x. \left\{ 2x \text{Arctang.} x - \frac{q^2 + x^2}{1+x^2} \right\} \frac{dx}{(q^2 + x^2)^2} = \frac{\pi}{2q^2} l(1+q) \quad \text{V. T. 266. N}^\circ. 2.$$

$$10) \int l x. \left\{ 2x \text{Arctang.} \frac{x}{p} - p \frac{q^2 + x^2}{p^2 + x^2} \right\} \frac{dx}{(q^2 + x^2)^2} = \frac{1}{2q^2} \pi l \frac{p+q}{p} \quad \text{V. T. 266. N}^\circ. 5.$$

$$11) \int l x. \left\{ 2x \text{Arctang.} \frac{x}{p} + p \frac{q^2 - x^2}{p^2 + x^2} \right\} \frac{dx}{(q^2 - x^2)^2} = \frac{\pi}{4q^2} l \frac{p^2 + q^2}{p^2} \quad \text{V. T. 266. N}^\circ. 6.$$

$$12) \int l x. \left\{ \frac{\text{Arctg.} \frac{x}{a} \cdot \text{Arctg.} \frac{x}{b}}{x^2} - \frac{a}{x(a^2 + x^2)} \text{Arctg.} \frac{x}{b} - \frac{b}{x(b^2 + x^2)} \text{Arctg.} \frac{x}{a} \right\} dx = \frac{1}{2} \pi \left\{ \frac{1}{a} l \frac{a+b}{b} + \frac{1}{b} l \frac{a+b}{a} \right\} \quad \text{V. T. 264. N}^\circ. 14.$$

$$13) \int l x. (1 - 2x \text{Arctang.} x) \frac{dx}{\sqrt{(1+x^2)^3}} = 2 \sum_0^\infty \frac{(-1)^{n-1}}{(2n+1)^2} \quad \text{V. T. 268. N}^\circ. 2.$$

$$14) \int l x. \text{Arctang.} \left(x \sqrt{\frac{p^2 - 1}{1+x^2}} \right) \frac{x}{\sqrt{(1+x^2)^3}} dx = \frac{\pi}{2} l \{ p + 1 \sqrt{p^2 - 1} \} - \frac{\pi}{2} l \frac{(p^2 - 1)}{p} l p, p \geq 1; \quad \text{V. T. 269. N}^\circ. 10 \text{ et T. 180. N}^\circ. 10.$$

$$15) \int l(1+x). \left\{ \text{Arctang.} x - \frac{2x}{1+x^2} \right\} \frac{dx}{x \sqrt{x}} = \frac{1}{2} \pi^2 \quad \text{V. T. 268. N}^\circ. 1.$$

$$16) \int l(1+x). \left\{ \text{Arccot.} x + \frac{2x}{1+x^2} \right\} \frac{dx}{x \sqrt{x}} = \frac{1}{2} \pi^2 \quad \text{V. T. 268. N}^\circ. 9.$$

$$17) \int l(1+x^2). \text{Arccot.} x \frac{x}{(1+x^2)^2} dx = \frac{\pi}{4} (1-l 2) \quad \text{V. T. 267. N}^\circ. 17 \text{ et T. 182. N}^\circ. 12.$$

$$18) \int l(1+x^2). \text{Arctang.} x \frac{dx}{x^2} = \frac{1}{3} \pi^2 \quad \text{V. T. 265. N}^\circ. 1 \text{ et T. 184. N}^\circ. 14.$$

F. Alg.	TABLE 425 suite.	Lim. 0 et ∞ .
Log.		
Circ. Inv.		

- 19) $\int l(1+x^2). \text{Arctang. } x \frac{x}{(1+x^2)^2} dx = \frac{1}{4} \pi l 2$ V. T. 267. N°. 2 et T. 182. N°. 12.
- 20) $\int l(1+x^2). \text{Arccot. } x \frac{dx}{x^2} = \frac{1}{6} \pi^2$ V. T. 265. N°. 9 et T. 184. N°. 14.
- 21) $\int l(1+x^2). \left\{ 2 \text{Arctang. } x - \frac{x}{1+x^2} \right\} \frac{dx}{x^3} = \pi l 2$ V. T. 266. N°. 1.
- 22) $\int l(1+x^2). \left\{ \text{Arccot. } x + \frac{px}{1+x^2} \right\} (\text{Arccot. } x)^{p-1} \frac{dx}{x^2} = \frac{\pi^{p+1}}{2^p(p+1)}$ V. T. 265. N°. 27.
- 23) $\int l(q^2+x^2). \left\{ 2 \text{Arctang. } x - \frac{x}{1+x^2} \right\} \frac{dx}{x^3} = \frac{\pi}{q^2} l(1+q)$ V. T. 266. N°. 2.
- 24) $\int l(q^2+x^2). \left\{ 2 \text{Arctang. } \frac{x}{p} - \frac{px}{p^2+x^2} \right\} \frac{dx}{x^3} = \frac{\pi}{q^2} l \frac{p+q}{p}$ V. T. 266. N°. 5.
- 25) $\int l(q^2+x^2). \left\{ 2x \text{Arctang. } x - \frac{q^2+x^2}{1+x^2} \right\} \frac{dx}{(q^2+x^2)^2} = \frac{\pi}{2q(q+1)}$ V. T. 267. N°. 3.
- 26) $\int l(q^2+x^2). \left\{ 2x \text{Arctang. } \frac{x}{p} - p \frac{q^2+x^2}{p^2+x^2} \right\} \frac{dx}{(q^2+x^2)^2} = \frac{\pi}{2q(p+q)}$ V. T. 267. N°. 5.
- 27) $\int l(q^2+x^2). \left\{ \frac{2x}{x^2-q^2} \text{Arctang. } px - \frac{p}{1+p^2x^2} \right\} \frac{dx}{x^2-q^2} = \frac{\pi}{4q^2} l \frac{(pq+1)^2}{p^2q^2+1}$ V. T. 265. N°. 4.
- 28) $\int l(1+x^2). \{ 2 \text{Arctg. } x - px \} (\text{Arctg. } x)^{p-1} \frac{dx}{(p^2+x^2)x^3} = \left(\frac{\pi}{2}\right)^p \left[1 - \sum_{1}^{\infty} \frac{2}{p+2m} \sum_{1}^{\infty} \frac{1}{(2n)^{2m}} \right]$ V. T. 266. N°. 10.
- 29) $\int l(q^2-x^2). \left\{ 2 \text{Arctang. } px - \frac{px}{1+p^2x^2} \right\} \frac{dx}{x^3} = \frac{\pi}{2q^2} l(p^2q^2+1)$ V. T. 266. N°. 6.

F. Alg.	TABLE 426.	Lim. 1 et ∞ .
Log.		
Circ. Inv.		

- 1) $\int \text{Arctang. } x. (lx)^p \frac{dx}{x} = \infty$ V. T. 187. N°. 6.
- 2) $\int \text{Arctang. } x. \left(\frac{1-x^2}{1+x^2} lx + 1 \right) \frac{dx}{1+x^2} = -\frac{1}{4} l 2$ V. T. 187. N°. 4.
- 3) $\int \text{Arctang. } (lx) \frac{1}{x^{2\tau}-1} \frac{dx}{x} = \frac{1}{2} - \frac{1}{4} l 2\pi$ V. T. 299. N°. 2.

F. Alg.	TABLE 426 suite.	Lim. 1 et ∞ .
Log.		
Circ. Inv.		

- 4) $\int \text{Arctang.}(p l x) \frac{1}{x^{2\pi}-1} \frac{dx}{x} = \frac{1}{2} l \left[\left(\frac{e}{p} \right)^p \Gamma(p) \vee \frac{p}{2\pi} \right]$ V. T. 299. N°. 3.
- 5) $\int \text{Arccot.} x. l x \frac{dx}{x^2} = \frac{1}{4} \pi - \frac{1}{2} (l 2)^2 - \frac{1}{2^{\frac{1}{2}}} \pi^2 - \frac{1}{2} l 2$ V. T. 270. N°. 5 et T. 187. N°. 8.
- 6) $\int \text{Arccot.} x. \left\{ \frac{1-x^2}{1+x^2} l x + 1 \right\} \frac{dx}{1+x^2} = \frac{1}{4} l 2$ V. T. 187. N°. 4.
- 7) $\int l x. \{ 2 x \text{Arctang.} x - 1 \} \frac{dx}{(1+x^2)^2} = \frac{3}{8} \pi l 2 - \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 2.
- 8) $\int l x. \left\{ \text{Arccot.} \frac{x}{p} + \frac{p x}{p^2+x^2} \right\} \frac{dx}{x^2} = \text{Arctang.} p - \frac{1}{2p} l(1+p^2)$ V. T. 270. N°. 6.
- 9) $\int l x. \text{Arccoscc.} x \frac{dx}{x^2} = l 2 + \frac{\pi}{2} - 2$ V. T. 270. N°. 9 et T. 187. N°. 5.
- 10) $\int l(1+x^2). \left\{ 2 \text{Arctang.} x - \frac{x}{1+x^2} \right\} \frac{dx}{x^3} = \frac{3\pi}{8} l 2 - \frac{1}{2} \sum_0^{\infty} \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 2.

F. Alg.	TABLE 427.	Lim. diverses.
Log.		
Circ. Inv.		

- 1) $\int_0^1 (x^{\frac{1}{2}} \text{Arcsin.} x)^{p-1} l x \frac{dx}{\vee(1-x^2)} = \frac{1}{2p} \left(\frac{\pi}{4} \right)^p \left[-l 2 - 2 + \sum_1^{\infty} \frac{4}{p+2m} \sum_1^{\infty} \frac{1}{(4n)^{2m}} \right]$ V. T. 271. N°. 5.
- 2) $\int_{1-\frac{1}{4}}^1 (\text{Arccos.} x)^{p-1} l(1-x^2) \frac{dx}{\vee(1-x^2)} = \frac{1}{p} \left(\frac{\pi}{4} \right)^p \left[l 2 - 2 + \sum_1^{\infty} \frac{4}{p+2m} \sum_1^{\infty} \frac{1}{(4n)^{2m}} \right]$ V. T. 271. N°. 7.

F. Alg.	TABLE 428.	Lim. diverses.
Log.		
Autres Fonctions.		

- 1) $\int_0^1 li.(x). \left(l \frac{1}{x} \right)^{p-1} \frac{dx}{x} = -\frac{1}{p} \Gamma(p), 1 \geq p > 0;$ V. T. 402. N°. 3.
- 2) $\int_0^1 li.(x). \left(l \frac{1}{x} \right)^{p-1} \frac{dx}{x^2} = -\pi \text{Coscc.} p \pi. \Gamma(p), 1 \geq p > 0;$ V. T. 402. N°. 2.
- 3) $\int_0^1 li.(x) \frac{x^{p-1}}{\vee l \frac{1}{x}} dx = -2 \vee \frac{\pi}{p}. l \{ \vee p + 1 - (1+p) \}, p < 1;$ V. T. 300. N°. 4.

$$4) \int_0^1 li.(x) \frac{dx}{x^{p+1} \sqrt{1-\frac{1}{x}}} = -2 \sqrt{\frac{\pi}{p}} \text{Arcsin}(\sqrt{p}), p < 1; \quad \text{V. T. 300. N}^\circ. 5.$$

$$5) \int_1^\infty li.(x) (lx)^{p-1} \frac{dx}{x^2} = -\pi \text{Cot. } p \pi, \Gamma(p) \quad \text{V. T. 402. N}^\circ. 1.$$

$$1) \int \frac{x}{\text{Sin. } x \cdot \text{Tang. } x} \left\{ \text{Arctg.}(p \text{Sin. } x) - \frac{p \text{Sin. } x}{1+p^2 \text{Sin.}^2 x} \right\} dx = \frac{\pi}{2} \left[-\text{Arctg. } p + l\{p + \sqrt{1+p^2}\} \right] \quad \begin{matrix} \text{V. T. 369.} \\ \text{N}^\circ. 3. \end{matrix}$$

$$2) \int \frac{x}{\text{Cos. } x} \left\{ \text{Arctang.}(p \text{Cos. } x) - \frac{p \text{Cos. } x}{1+p^2 \text{Cos.}^2 x} \right\} \text{Tang. } x dx = \frac{\pi}{2} \left[p - l\{p + \sqrt{1+p^2}\} \right] \quad \begin{matrix} \text{V. T. 369.} \\ \text{N}^\circ. 6. \end{matrix}$$

$$3) \int \left[\text{Arctang.} \left\{ \frac{\text{Cot. } \lambda}{\sqrt{1-p^2 \text{Sin.}^2 x}} \right\} - \frac{1}{2} \frac{\text{Sin. } 2 \lambda \sqrt{1-p^2 \text{Sin.}^2 x}}{1-p^2 \text{Sin.}^2 \lambda \cdot \text{Sin.}^2 x} \right] \frac{x \text{Sin. } 2 x}{\sqrt{1-p^2 \text{Sin.}^2 x}} dx =$$

$$= \frac{\pi}{p^2} \left[\text{E}(p, q) - \frac{\text{Cot. } \lambda}{\sqrt{1-p^2}} \{ \sqrt{1-p^2 \text{Sin.}^2 q} - \sqrt{1-p^2} \} - q \sqrt{1-p^2} \right] \quad \begin{matrix} \text{V. T. 368.} \\ \text{N}^\circ. 13. \end{matrix}$$

$$4) \int \left[\text{Arctang.} \left\{ \frac{\text{Cot. } \lambda}{\sqrt{1-p^2 \text{Sin.}^2 x}} \right\} + \frac{1}{2} \frac{\text{Sin. } 2 \lambda \sqrt{1-p^2 \text{Sin.}^2 x}}{1-p^2 \text{Sin.}^2 \lambda \cdot \text{Sin.}^2 x} \right] \frac{x \text{Sin. } 2 x}{\sqrt{1-p^2 \text{Sin.}^2 x}^3} dx =$$

$$= \frac{\pi}{p^2} \left[\frac{q}{\sqrt{1-p^2}} - \text{F}(p, q) \right] \quad \text{V. T. 369. N}^\circ. 15.$$

$$5) \int \left[\text{Arctang.} \left\{ \frac{\text{Cot. } \lambda}{\sqrt{1-p^2 \text{Sin.}^2 x}} \right\} + \frac{1}{6} \frac{\text{Sin. } 2 \lambda \sqrt{1-p^2 \text{Sin.}^2 x}}{1-p^2 \text{Sin.}^2 \lambda \cdot \text{Sin.}^2 x} \right] \frac{x \text{Sin. } 2 x}{\sqrt{1-p^2 \text{Sin.}^2 x}^5} dx =$$

$$= \frac{\pi}{3 p^2} \left[\frac{q}{\sqrt{1-p^2}^3} - \frac{1}{1-p^2} \text{E}(p, q) + \frac{\text{Tang. } \lambda}{\sqrt{1-p^2}} \{ 1 - \sqrt{1-p^2 \text{Sin.}^2 q} \} \right] \quad \begin{matrix} \text{V. T. 369.} \\ \text{N}^\circ. 17. \end{matrix}$$

Dans les formules 3) à 5) on a $\text{Cot. } q = \text{Tang. } \lambda \sqrt{1-p^2}$.

$$6) \int \left[\text{Arctang.} \left\{ \text{Tang. } \lambda \sqrt{1-p^2 \text{Sin.}^2 x} \right\} + \frac{1}{2} \frac{\text{Sin. } 2 \lambda \sqrt{1-p^2 \text{Sin.}^2 x}}{1-p^2 \text{Sin.}^2 \lambda \cdot \text{Sin.}^2 x} \right] \frac{x \text{Sin. } 2 x}{\sqrt{1-p^2 \text{Sin.}^2 x}} dx =$$

$$= \frac{\pi}{p^2} \left[\text{E}(p, \lambda) - \text{Cot. } \lambda \cdot \{ 1 - \sqrt{1-p^2 \text{Sin.}^2 \lambda} \} - \sqrt{1-p^2} \cdot \text{Arctg.} \{ \text{Tang. } \lambda \sqrt{1-p^2} \} \right] \quad \begin{matrix} \text{V. T. 368.} \\ \text{N}^\circ. 11. \end{matrix}$$

F. Alg.

Circ. Dir.

Circ. Inv.

TABLE 429 suite.

Lim. 0 et $\frac{\pi}{2}$.

$$7) \int \left[\text{Arctang.} \left\{ \text{Tang.} \lambda \sqrt{1-p^2 \text{Sin.}^2 x} \right\} - \frac{1}{2} \frac{\text{Sin.} 2\lambda \sqrt{1-p^2 \text{Sin.}^2 x}}{1-p^2 \text{Sin.}^2 \lambda \text{Sin.}^2 x} \right] \frac{x \text{Sin.} 2x}{\sqrt{1-p^2 \text{Sin.}^2 x}^3} dx =$$

$$= \frac{\pi}{p^2} \left[\frac{1}{\sqrt{1-p^2}} \text{Arctang.} \left\{ \text{Tang.} \lambda \sqrt{1-p^2} \right\} - \text{E}(p, \lambda) \right] \quad \text{V. T. 369. N}^\circ. 14.$$

$$8) \int \left[\text{Arctang.} \left\{ \text{Tang.} \lambda \sqrt{1-p^2 \text{Sin.}^2 x} \right\} - \frac{1}{6} \frac{\text{Sin.} 2\lambda \sqrt{1-p^2 \text{Sin.}^2 x}}{1-p^2 \text{Sin.}^2 \lambda \text{Sin.}^2 x} \right] \frac{x \text{Sin.} 2x}{\sqrt{1-p^2 \text{Sin.}^2 x}^5} dx =$$

$$= \frac{\pi}{3p^2} \left[\frac{1}{\sqrt{1-p^2}^3} \text{Arctg.} \left\{ \text{Tg.} \lambda \sqrt{1-p^2} \right\} - \frac{1}{1-p^2} \text{E}(p, \lambda) + \frac{\text{Tg.} \lambda}{1-p^2} \left\{ \sqrt{1-p^2 \text{Sin.}^2 \lambda} - \sqrt{1-p^2} \right\} \right] \quad \text{V. T. 369. N}^\circ. 16.$$

F. Alg.

Circ. Dir.

Circ. Inv.

TABLE 450.

Lim. 0 et π .

$$1) \int \text{Arctg.} \left\{ \frac{p \text{Sin.} x}{1-p \text{Cos.} x} \right\} \cdot \text{Sin.} a x \cdot x^{2b} dx = \frac{(-1)^b \pi p^a}{2 a^{2b+1}} 1^{2b/1} \sum_0^{2b} \frac{(-a l p)^n}{1^{n/1}}$$

$$2) \int \text{Arctg.} \left\{ \frac{p \text{Sin.} x}{1-p \text{Cos.} x} \right\} \cdot \text{Cos.} a x \cdot x^{2b+1} dx = \frac{(-1)^{b+1} \pi p^a}{2 a^{2b+2}} 1^{2b+1/1} \sum_0^{2b+1} \frac{(-a l p)^n}{1^{n/1}}$$

$$3) \int \text{Arctg.} \left\{ \frac{2p \text{Sin.} x}{1-p^2} \right\} \cdot \text{Sin.} 2ax \cdot x^{\pm 2b} dx = 0$$

$$4) \int \text{Arctg.} \left\{ \frac{2p \text{Sin.} x}{1-p^2} \right\} \cdot \text{Sin.} \{(2a-1)x\} \cdot x^{2b} dx = \frac{(-1)^b \pi p^{2a-1}}{2^{2b} (2a-1)^{2b+1}} 1^{2b/1} \sum_0^{2b} \frac{\{-(2a-1)lp\}^n}{1^{n/1}}$$

$$5) \int \text{Arctg.} \left\{ \frac{2p \text{Sin.} x}{1-p^2} \right\} \cdot \text{Cos.} 2ax \cdot x^{\pm 2b+1} dx = 0$$

$$6) \int \text{Arctg.} \left\{ \frac{2p \text{Sin.} x}{1-p^2} \right\} \cdot \text{Cos.} \{(2a-1)x\} \cdot x^{2b+1} dx = \frac{(-1)^b \pi p^{2a-1}}{2^{2b+1} (2a-1)^{2b+2}} 1^{2b+1/1} \sum_0^{2b+1} \frac{\{-(2a-1)lp\}^n}{1^{n/1}}$$

$$7) \int \text{Arctg.} \left\{ \frac{2p \text{Sin.} x}{1-p^2} \right\} \cdot \text{Sin.} \{(2a-1)x\} \cdot \text{Sin.} x \cdot x^{\pm 2b+1} dx = 0$$

$$8) \int \text{Arctg.} \left\{ \frac{2p \text{Sin.} x}{1-p^2} \right\} \cdot \text{Sin.} \{(2a-1)x\} \cdot \text{Cos.} x \cdot x^{\pm 2b} dx = 0$$

$$9) \int \text{Arctg.} \left\{ \frac{2p \text{Sin.} x}{1-p^2} \right\} \cdot \text{Cos.} \{(2a-1)x\} \cdot \text{Sin.} x \cdot x^{\pm 2b} dx = 0$$

$$10) \int \text{Arctg.} \left\{ \frac{2p \text{Sin.} x}{1-p^2} \right\} \cdot \text{Cos.} \{(2a-1)x\} \cdot \text{Cos.} x \cdot x^{\pm 2b+1} dx = 0$$

, $p^2 < 1$, $0 \leq q < 1$ Bierens de Haan.
Gr. 13. 193.

F. Alg.
Circ. Dir.
Circ. Inv.

TABLE 450 suite.

Lim. 0 et π .

$$\begin{aligned}
 11) & \int \text{Arctg.} \left\{ \frac{q \sin. 2x}{1 - q \cos. 2x} \right\} \cdot \sin. 2ax \cdot x^{2b} dx = \frac{(-1)^b \pi q^a}{2^{2b} a^{2b+1}} 1^{2b/1} \sum_0^{2b} \frac{(-a l q)^n}{1^{n/1}} \\
 12) & \int \text{Arctg.} \left\{ \frac{q \sin. 2x}{1 - q \cos. 2x} \right\} \cdot \sin. \{(2a-1)x\} \cdot x^{\pm 2b} dx = 0 \\
 13) & \int \text{Arctg.} \left\{ \frac{q \sin. 2x}{1 - q \cos. 2x} \right\} \cdot \cos. 2ax \cdot x^{2b+1} dx = \frac{(-1)^{b+1} \pi q^a}{2^{2b+1} a^{2b+2}} 1^{2b+1/1} \sum_0^{2b+1} \frac{(-a l q)^n}{1^{n/1}} \\
 14) & \int \text{Arctg.} \left\{ \frac{q \sin. 2x}{1 - q \cos. 2x} \right\} \cdot \cos. \{(2a-1)x\} \cdot x^{\pm 2b+1} dx = 0 \\
 15) & \int \text{Arctg.} \left\{ \frac{q \sin. 2x}{1 - q \cos. 2x} \right\} \cdot \sin. 2ax \cdot \sin. x \cdot x^{\pm 2b+1} dx = 0 \\
 16) & \int \text{Arctg.} \left\{ \frac{q \sin. 2x}{1 - q \cos. 2x} \right\} \cdot \sin. 2ax \cdot \cos. x \cdot x^{\pm 2b} dx = 0 \\
 17) & \int \text{Arctg.} \left\{ \frac{q \sin. 2x}{1 - q \cos. 2x} \right\} \cdot \cos. 2ax \cdot \sin. x \cdot x^{\pm 2b} dx = 0 \\
 18) & \int \text{Arctg.} \left\{ \frac{q \sin. 2x}{1 - q \cos. 2x} \right\} \cdot \cos. 2ax \cdot \cos. x \cdot x^{\pm 2b+1} dx = 0 \\
 19) & \int (a^2 + 2ab \cos. x + b^2)^{1/2} p \cos. \left\{ cx - p \text{Arctg.} \left(\frac{a \sin. bx}{b + a \cos. x} \right) \right\} dx = \frac{\pi a^c b^{p-c} \Gamma(1+p)}{\Gamma(1+c) \Gamma(1+p-c)}, b > a;
 \end{aligned}$$

$p^2 < 1, 0 < q < 1;$
 Bierens de Haan,
 Gr. 13. 193.

Hill, Cr. 7. 102.
 (faut.) — Clausen,
 Cr. 7. 309.

F. Alg. rat. fract.
Circ. Dir.
Circ. Inv.

TABLE 451.

Lim. 0 et ∞ .

$$\begin{aligned}
 1) & \int \text{Arctang.} x \cdot \cos. ax \frac{dx}{x} = -\frac{1}{2} \pi \text{li.}(e^{-a}) \text{ Schlömilch, Beitr. III. § 8.} \\
 2) & \int \text{Arctang.} \frac{x}{a} \cdot \cos. x \frac{dx}{x} = -\frac{1}{2} \pi \text{li.}(e^{-a}) \\
 3) & \int \text{Arctang.} \left(\frac{q - \frac{1}{q}}{1 + x^2} x \right) \cdot \cos. x \frac{dx}{x} = \frac{1}{2} \pi \{ \text{li.}(e^{-q}) - \text{li.}(e^{-\frac{1}{q}}) \} \\
 4) & \int \text{Arctg.}(cx) \cdot \sin. px \frac{dx}{x^2 + q^2} = \frac{\pi}{4q} e^{-pq} \left\{ \frac{1}{2} \left(\frac{1+qe}{1-qe} \right)^2 + \text{Ei.} \left(-\frac{p}{e} + pq \right) \right\} - \frac{\pi}{4q} e^{pq} \text{Ei.} \left(-\frac{p}{e} - pq \right) \\
 5) & \int \text{Arctang.} \left(\frac{x}{q} \right) \cdot \sin. px \frac{dx}{x^2 + q^2} = \frac{\pi}{4q} e^{-pq} \{ \Lambda + \text{li.}(2pq) \} - \frac{\pi}{4q} e^{pq} \text{Ei.}(-2pq)
 \end{aligned}$$

Schlömilch, Gr. 9. 307.
 Arndt,
 Gr. 11.
 70.

F. Alg. rat. fract.
Circ. Dir.
Circ. Inv.

TABLE 451 suite.

Lim. 0 et ∞

- $$6) \int \text{Arctg.}(cx) \cdot \text{Cos.} px \frac{x}{x^2 + q^2} dx = -\frac{1}{4} \pi e^{-pq} \left\{ \frac{1}{2} l \left(\frac{1+qc}{1-qc} \right)^2 + \text{Ei.} \left(-\frac{p}{c} + pq \right) \right\} - \frac{1}{4} \pi e^{pq} \text{Ei.} \left(-\frac{p}{c} - pq \right) \left\{ \begin{array}{l} \text{Arndt,} \\ \text{Gr. 11.} \\ 70. \end{array} \right.$$
- $$7) \int \text{Arctg.} \left(\frac{x}{q} \right) \cdot \text{Cos.} px \frac{x}{x^2 + q^2} dx = -\frac{1}{4} \pi e^{-pq} \{ \Lambda + l(2pq) \} - \frac{1}{4} \pi e^{pq} \text{Ei.} (-2pq)$$
- $$8) \int \text{Arctang.} \left(\frac{p \text{Sin.} x}{1 + p \text{Cos.} x} \right) \frac{x}{x^2 + q^2} dx = \frac{1}{2} \pi l(1 + p e^{-q}), p^2 \leq 1; \left. \begin{array}{l} 9) \int \text{Arctang.} \left(\text{Tang.} \frac{1}{2} x \right) \frac{x}{x^2 + q^2} dx = \frac{1}{2} \pi l \frac{e^q + 1}{e^q} \\ 10) \int \text{Arctang.} \left(\text{Cot.} \frac{1}{2} x \right) \frac{x}{x^2 + q^2} dx = \frac{1}{2} \pi l \frac{e^q}{e^q - 1} \end{array} \right\} \text{Schlömlich, Stud. II. 1-}$$
- $$11) \int \text{Arctang.} \left(\frac{p \text{Sin.} rx}{1 + p \text{Cos.} rx} \right) \frac{x}{x^2 + q^2} dx = \frac{\pi}{2} l(1 + p e^{-qr}) \text{ Boncompagni, Cr. 25. 74; où il y a faut. } dx.$$

F. Alg. irrat. fract. à dén. binôme.
Circ. Dir.
Circ. Inv.

TABLE 452.

Lim. 0 et ∞

- $$1) \int \text{Sin.} \left\{ q \text{Arctg.} \left(\frac{a \text{Sin.} cx}{1 + a \text{Cos.} cx} \right) \right\} \cdot (1 + 2a \text{Cos.} cx + a^2)^{\frac{1}{2}q} \frac{x}{p^2 + x^2} dx = \frac{1}{2} \pi (1 + a e^{-cp})^q - \frac{1}{2} \pi$$
- $$2) \int \text{Cos.} \left\{ q \text{Arctg.} \left(\frac{a \text{Sin.} cx}{1 + a \text{Cos.} cx} \right) \right\} \cdot (1 + 2a \text{Cos.} cx + a^2)^{\frac{1}{2}q} \frac{dx}{p^2 + x^2} = \frac{\pi}{2p} (1 + a e^{-cp})^q$$
- $$3) \int \text{Sin.} \left\{ q \text{Arctg.} \left(\frac{a \text{Sin.} cx}{1 + a \text{Cos.} cx} \right) \right\} \cdot \frac{(1 + 2a \text{Cos.} cx + a^2)^{\frac{1}{2}q} + (1 + 2a \text{Cos.} cx + a^2)^{-\frac{1}{2}q}}{p^2 + x^2} x dx =$$
- $$= \frac{1}{2} \pi \{ (1 + a e^{-cp})^q - (1 + a e^{-cp})^{-q} \} \left\{ \begin{array}{l} \text{Boncompagni,} \\ \text{Cr. 25. 74;} \\ \text{où 3, 4, 5 sont} \\ \text{fautes.} \end{array} \right.$$
- $$4) \int \text{Sin.} \left\{ q \text{Arctg.} \left(\frac{a \text{Sin.} cx}{1 + a \text{Cos.} cx} \right) \right\} \cdot \frac{(1 + 2a \text{Cos.} cx + a^2)^{\frac{1}{2}q} - (1 + 2a \text{Cos.} cx + a^2)^{-\frac{1}{2}q}}{p^2 + x^2} x dx =$$
- $$= \frac{1}{2} \pi \{ (1 + a e^{-cp})^q + (1 + a e^{-cp})^{-q} - 2 \}$$
- $$5) \int \text{Cos.} \left\{ q \text{Arctg.} \left(\frac{a \text{Sin.} cx}{1 + a \text{Cos.} cx} \right) \right\} \cdot \frac{(1 + 2a \text{Cos.} cx + a^2)^{\frac{1}{2}q} + (1 + 2a \text{Cos.} cx + a^2)^{-\frac{1}{2}q}}{p^2 + x^2} dx =$$
- $$= \frac{\pi}{2p} \{ (1 + a e^{-cp})^q + (1 + a e^{-cp})^{-q} \}$$

$$6) \int \cos. \left\{ q \operatorname{Arctg.} \left(\frac{a \sin. cx}{1 + a \cos. cx} \right) \right\} \frac{(1 + 2a \cos. cx + a^2)^{\frac{1}{2}q} - (1 + 2a \cos. cx + a^2)^{-\frac{1}{2}q}}{p^2 + x^2} dx =$$

Boncompagni,
Cr. 25. 74.
qui est fautive.

$$= \frac{\pi}{2p} \{ (1 + a e^{-cp})^q - (1 + a e^{-cp})^{-q} \}$$

$$7) \int \sin. \left\{ (a+1) \operatorname{Arctang.} \frac{x}{b} \right\} \cdot \sin. cx \frac{dx}{(b^2 + x^2)^{\frac{a+1}{2}}} = \frac{\pi e^{-bc} e^a}{2 \Gamma(a+1)}$$

$$8) \int \cos. \left\{ (a+1) \operatorname{Arctang.} \frac{x}{b} \right\} \cdot \cos. cx \frac{dx}{(b^2 + x^2)^{\frac{a+1}{2}}} = \frac{\pi e^{-bc} e^a}{2 \Gamma(a+1)}$$

$$9) \int \sin. \left\{ (a+1) \operatorname{Arctg.} \frac{x}{b} \right\} \cdot \sin. \left\{ x\lambda + c \operatorname{Arctg.} \left(\frac{\sin. x}{\cos. x - e^{-b}} \right) \right\} \cdot (e^{2b} - 2e^b \cos. x + 1)^{\frac{1}{2}c} \frac{dx}{(b^2 + x^2)^{\frac{1}{2}(a+1)}} =$$

$$= \frac{\pi}{2 e^{b\lambda} \Gamma(a+1)} \Delta^{b, \lambda a}$$

$$10) \int \cos. \left\{ (a+1) \operatorname{Arctg.} \frac{x}{b} \right\} \cdot \cos. \left\{ x\lambda + c \operatorname{Arctg.} \left(\frac{\sin. x}{\cos. x - e^{-b}} \right) \right\} \cdot (e^{2b} - 2e^b \cos. x + 1)^{\frac{1}{2}c} \frac{dx}{(b^2 + x^2)^{\frac{1}{2}(a+1)}} =$$

$$= \frac{\pi}{2 e^{b\lambda} \Gamma(a+1)} \Delta^{b, \lambda a}$$

$$11) \int \cos. \left[x\lambda + c \operatorname{Arctg.} \left(\frac{\sin. x}{\cos. x - e^{-b}} \right) - (a+1) \operatorname{Arctg.} \frac{x}{b} \right] \cdot (e^{2b} - 2e^b \cos. x + 1)^{\frac{1}{2}c} \frac{dx}{(b^2 + x^2)^{\frac{1}{2}(a+1)}} =$$

$$= \frac{\pi}{e^{b\lambda} \Gamma(a+1)} \Delta^{b, \lambda a}$$

$$12) \int \cos. \left[x\lambda + c \operatorname{Arctg.} \left(\frac{\sin. x}{\cos. x - e^{-b}} \right) + (a+1) \operatorname{Arctg.} \frac{x}{b} \right] \cdot (e^{2b} - 2e^b \cos. x + 1)^{\frac{1}{2}c} \frac{dx}{(b^2 + x^2)^{\frac{1}{2}(a+1)}} = 0$$

Sur ces intégrales 7) à 12) voyez: Cauchy, P. 28. 147. P. III. § 2.

$$13) \int \cos. (px - p \operatorname{Arctang.} x) \frac{dx}{(1 + x^2)^{\frac{1}{2}p}} = \frac{\pi}{\Gamma(p+1)} \left(\frac{p}{e} \right)^p \quad \text{V. T. 61. N° 1.}$$

$$14) \int \cos. (qx - p \operatorname{Arctang.} x) \frac{dx}{(1 + x^2)^{\frac{1}{2}p}} = \frac{\pi q^{p-1} e^{-q}}{\Gamma(p)} \quad \text{V. T. 61. N° 5.}$$

$$15) \int \cos. (qx + p \operatorname{Arctang.} x) \frac{dx}{(1 + x^2)^{\frac{1}{2}p}} = 0 \quad \text{V. T. 61. N° 3.}$$

$$16) \int \cos. (qx + p \operatorname{Arctang.} x) \frac{dx}{(1 + x^2)^{\frac{1}{2}p-1}} = \frac{\pi e^{-q}}{2^{p+1}} \quad \text{V. T. 61. N° 4.}$$

- 1) $\int \text{Sin.} \left(b \text{ Arctang.} \frac{x}{a} \right) \frac{dx}{x(x^2 + a^2)^{\frac{1}{2}b}} = \frac{1}{2} \pi a^{-b}$ Liouville, Cr. 13. 209.
 - 2) $\int \text{Sin.} (p \text{ Arctang.} x) \frac{dx}{x^q(1+x^2)^{\frac{1}{2}p}} = \frac{1}{2} \pi \text{Coscc.} \frac{1}{2} q \pi \frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)}, 2 > q > 0;$
 - 3) $\int \text{Cos.} (p \text{ Arctang.} x) \frac{dx}{x^q(1+x^2)^{\frac{1}{2}p}} = \frac{1}{2} \pi \text{Secc.} \frac{1}{2} q \pi \frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)}, 1 > q > 0;$
 - 4) $\int \text{Sin.} \left(p \text{ Arctg.} \frac{x}{r} \right) \frac{dx}{x^q(r^2+x^2)^{\frac{1}{2}p}} = \frac{\pi}{2 r^{p+q-1}} \text{Coscc.} \frac{1}{2} q \pi \frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)}, 2 > q > 0;$
 - 5) $\int \text{Cos.} \left(p \text{ Arctg.} \frac{x}{r} \right) \frac{dx}{x^q(r^2+x^2)^{\frac{1}{2}p}} = \frac{\pi}{2 r^{p+q-1}} \text{Secc.} \frac{1}{2} q \pi \frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)}, 1 > q > 0;$
 - 6) $\int \text{Sin.} (ax + p \text{ Arctang.} x) \frac{dx}{x(1+x^2)^{\frac{1}{2}p}} = \frac{1}{2} \pi$ V. T. 70. N°. 22.
 - 7) $\int \frac{\text{Sin.} (p \text{ Arctang.} x) + \text{Sin.} (bx - p \text{ Arctang.} x)}{x(1+x^2)^{\frac{1}{2}p}} dx = \frac{1}{2} \pi$ V. T. 70. N°. 23.
 - 8) $\int \frac{\text{Sin.} (p \text{ Arctang.} x)}{(1+x^2)^{\frac{1}{2}p}} \frac{x^{2b+1}}{c^2+x^2} dx = (-1)^{\frac{1}{2}b} \frac{c^{2b}}{(1+c)^p} \frac{\pi}{2}$
 - 9) $\int \frac{\text{Cos.} (p \text{ Arctang.} x)}{(1+x^2)^{\frac{1}{2}p}} \frac{x^{2b}}{c^2+x^2} dx = (-1)^{\frac{1}{2}b} \frac{c^{2b-1}}{(1+c)^p} \frac{\pi}{2}$
 - 10) $\int \frac{\text{Sin.} (p \text{ Arctang.} x)}{(1+x^2)^{\frac{1}{2}p}} \text{Sin.} \left\{ (q+1) \text{ Arctang.} \frac{x}{r} \right\} \frac{x^a}{(r^2+x^2)^{\frac{1}{2}(q+1)}} dx =$
 - 11) $\int \frac{\text{Sin.} (p \text{ Arctang.} x)}{(1+x^2)^{\frac{1}{2}p}} \text{Cos.} \left\{ (q+1) \text{ Arctang.} \frac{x}{r} \right\} \frac{x^{a-1}}{(r^2+x^2)^{\frac{1}{2}(q+1)}} dx =$
 - 12) $\int \frac{\text{Cos.} (p \text{ Arctang.} x)}{(1+x^2)^{\frac{1}{2}p}} \text{Sin.} \left\{ (q+1) \text{ Arctang.} \frac{x}{r} \right\} \frac{x^{a-1}}{(r^2+x^2)^{\frac{1}{2}(q+1)}} dx =$
- Schlömilch, Stud. I. 15. — Id., Cr. 33. 353.
Schlömilch, Gr. 6 200.
Schlömilch, Cr. 33. 353. ne valent que pour $b = 0$.
Schlömilch, Cr. 33. 353. ne valent que pour $a = 0$.

$$\begin{aligned}
 13) & \int \frac{\cos.(p \operatorname{Arctang}.x)}{(1+x^2)^{\frac{1}{2}p}} \cos.\left\{(q+1) \operatorname{Arctang}.\frac{x}{r}\right\} \frac{x^a}{(r^2+x^2)^{\frac{1}{2}(q+1)}} dx = \\
 & = (-1)^{\frac{1}{2}a} \frac{\pi}{2} \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q+1)} \frac{r^a}{(1+r)^{p+q}} \sum_0^{\infty} (-1)^n \binom{a}{n} \frac{q^{n/-1}}{(p+q-1)^{n/-1}} \left(\frac{1+r}{r}\right)^n \left\{ \begin{array}{l} \text{Schl}^{\circ} \\ \text{milch,} \\ \text{Cr. 33.} \\ \text{353} \end{array} \right. \\
 14) & \int \frac{\sin.(p \operatorname{Arctg}.x)}{(1+x^2)^{\frac{1}{2}p}} \frac{x}{(r^2+x^2)^{a+1}} dx = \frac{\pi \Gamma(p+a)}{\Gamma(p)\Gamma(a+1)(2r)^{a+1}(1+r)^{p+a}} \sum_0^{\infty} \frac{(a+n-1)^{2n-1}}{2^{n/2}(p+a-1)^{n/-1}} \left(\frac{1+r}{r}\right)^n \left\{ \begin{array}{l} \text{13) ne} \\ \text{vaut que} \\ \text{pour} \\ \text{a = 0.} \end{array} \right. \\
 15) & \int \frac{\cos.(p \operatorname{Arctg}.x)}{(1+x^2)^{\frac{1}{2}p}} \frac{dx}{(r^2+x^2)^{a+1}} = \frac{\pi \Gamma(p+a)}{\Gamma(p)\Gamma(a+1)(2r)^{a+1}(1+r)^{p+a}} \sum_0^{\infty} \frac{(a+n)^{2n/-1}}{2^{n/2}(p+a-1)^{n/-1}} \left(\frac{1+r}{r}\right)^n \\
 16) & \int \frac{\sin.\left\{p \operatorname{Arctang}.x + (q+1) \operatorname{Arctang}.\frac{x}{r}\right\}}{(1+x^2)^{\frac{1}{2}p}} \frac{x^{a-1}}{(r^2+x^2)^{\frac{1}{2}(q+1)}} dx = 0 \quad \left\{ \begin{array}{l} \text{V. T. 433. N}^{\circ}. 11, 12. \\ \text{ne vaut que pour } a = 0. \end{array} \right. \\
 17) & \int \frac{\cos.\left\{p \operatorname{Arctang}.x + (q+1) \operatorname{Arctang}.\frac{x}{r}\right\}}{(1+x^2)^{\frac{1}{2}p}} \frac{x^{a-1}}{(r^2+x^2)^{\frac{1}{2}(q+1)}} dx = 0 \quad \left\{ \begin{array}{l} \text{V. T. 433. N}^{\circ}. 10, 13. \\ \text{ne vaut que pour } a = 1. \end{array} \right. \\
 18) & \int \sin.\left\{p \operatorname{Arctg}.\left(\frac{q \sin.ax}{1+q \cos.ax}\right)\right\} \frac{x}{r^2+x^2} \frac{dx}{(1+2q \cos.ax+q^2)^{\frac{1}{2}p}} = \frac{1}{2}\pi - \frac{1}{2}\pi(1+qe^{-ar})^{-p} \left\{ \begin{array}{l} \text{Boncom-} \\ \text{pagni, Cr.} \\ \text{25. 74.} \\ \text{(fautes).} \end{array} \right. \\
 19) & \int \cos.\left\{p \operatorname{Arctg}.\left(\frac{q \sin.ax}{1+q \cos.ax}\right)\right\} \frac{1}{r^2+x^2} \frac{dx}{(1+2q \cos.ax+q^2)^{\frac{1}{2}p}} = \frac{1}{2}\pi \{1+qe^{-ar}\}^{-p} \\
 20) & \int \left[\cos.x - \frac{\cos.(p \operatorname{Arctang}.x)}{(1+x^2)^{\frac{1}{2}p}} \right] \frac{dx}{x} = Z'(p) \quad \text{Arndt, Gr. 10. 225.}
 \end{aligned}$$

$$\begin{aligned}
 1) & \int_{-\infty}^{\infty} \frac{\cos.(p \operatorname{Arctang}.qx)}{1+x^2} \frac{dx}{(1+q^2x^2)^{\frac{1}{2}p}} = \frac{\pi}{(1+q)^p} \quad \text{Cauchy, C. R. 11. 1008.} \\
 2) & \int_0^{\frac{\pi}{2}} \operatorname{Arctang} \left(\frac{1}{q} \operatorname{Tang}.x \right) \frac{x}{\sin.^2x} dx = \frac{\pi}{2q} \left\{ \ell(1+q) + q \ell \frac{1+q}{q} \right\} \quad \text{V. T. 264. N}^{\circ}. 13.
 \end{aligned}$$

- 1) $\int \text{Sin. } ax. \text{Si. } (cx) \frac{dx}{b^2 + x^2} = \frac{\pi}{4b} e^{-ab} \{ \text{li. } (e^{bc}) - \text{li. } (e^{-bc}) \} \quad , a > c; \left\{ \begin{array}{l} \text{Schlömilch,} \\ \text{Gr. 11.174.} \end{array} \right.$
- 2) $= \frac{\pi}{4b} \{ e^{-ab} \text{li. } (e^{ab}) - e^{ab} \text{li. } (e^{-ab}) + (e^{ab} - e^{-ab}) \text{li. } (e^{-bc}) \} , a < c;$
- 3) $\int \text{Sin. } ax. \text{Si. } (cx) \frac{dx}{b^2 + x^2} = \frac{\pi}{4b} e^{-ab} \{ \text{Ei. } (bc) - \text{Ei. } (-bc) \} \quad , a \geq c;$
- 4) $= \frac{\pi}{4b} [e^{-ab} \{ \text{Ei. } (ab) - \text{Ei. } (-bc) \} - e^{ab} \{ \text{Ei. } (-ab) - \text{Ei. } (-bc) \}] , a \leq c;$
- 5) $\int \text{Cos. } ax. \text{Ci. } (cx) \frac{dx}{b^2 + x^2} = \frac{\pi}{4b} (e^{ab} + e^{-ab}) \text{Ei. } (-bc) \quad , a \geq c;$
- 6) $= \frac{\pi}{4b} [e^{-ab} \{ \text{Ei. } (bc) + \text{Ei. } (-bc) - \text{Ei. } (ab) \} + e^{ab} \text{Ei. } (-ab)] , a \leq c;$
- 7) $\int \text{Sin. } ax. \text{Ci. } (cx) \frac{x dx}{b^2 + x^2} = -\frac{1}{4} \pi (e^{ab} - e^{-ab}) \text{Ei. } (-bc) \quad , a \geq c;$
- 8) $= \frac{1}{4} \pi [e^{-ab} \{ \text{Ei. } (bc) + \text{Ei. } (-bc) - \text{Ei. } (ab) \} - e^{ab} \text{Ei. } (-ab)] , a \leq c;$
- 9) $\int \text{Cos. } ax. \text{Si. } (cx) \frac{x dx}{b^2 + x^2} = -\frac{1}{4} \pi e^{-ab} \{ \text{Ei. } (bc) - \text{Ei. } (-bc) \} \quad , a \geq c;$
- 10) $= -\frac{1}{4} \pi [e^{-ab} \{ \text{Ei. } (ab) - \text{Ei. } (-bc) \} - e^{ab} \{ \text{Ei. } (-ab) - \text{Ei. } (-bc) \}] , a \leq c;$
- 11) $\int [\text{Cos. } ax. \{ \text{Ci. } (cx) - \text{Ci. } (ax) \} + \text{Sin. } ax. \{ \text{Si. } (cx) - \text{Si. } (ax) \}] \frac{dx}{b^2 + x^2} = \frac{\pi}{2b} e^{-ab} \{ \text{Ei. } (bc) - \text{Ei. } (ab) \} , a \geq c;$
- 12) $\int [\text{Sin. } ax. \{ \text{Ci. } (cx) - \text{Ci. } (ax) \} - \text{Cos. } ax. \{ \text{Si. } (cx) - \text{Si. } (ax) \}] \frac{x dx}{b^2 + x^2} = \frac{\pi}{2} e^{-ab} \{ \text{Ei. } (bc) - \text{Ei. } (ab) \} , a \geq c;$
- 13) $\int \left[\text{Sin. } ax. \text{Ci. } (ax) + \text{Cos. } ax. \left\{ \frac{1}{2} \pi - \text{Si. } (ax) \right\} \right] \frac{x dx}{b^2 + x^2} = -\frac{1}{2} \pi e^{ab} \text{Ei. } (-ab)$
- 14) $\int \left[\text{Sin. } ax. \text{Ci. } (cx) - \text{Cos. } ax. \left\{ \frac{1}{2} \pi - \text{Si. } (cx) \right\} \right] \frac{x dx}{b^2 + x^2} = \frac{1}{2} \pi e^{-ab} \text{Ei. } (-bc)$
- 15) $\int \left[\text{Cos. } ax. \text{Ci. } (ax) - \text{Sin. } ax. \left\{ \frac{1}{2} \pi - \text{Si. } (ax) \right\} \right] \frac{dx}{b^2 + x^2} = \frac{\pi}{2b} e^{ab} \text{Ei. } (-ab)$
- 16) $\int \left[\text{Cos. } ax. \text{Ci. } (cx) + \text{Sin. } ax. \left\{ \frac{1}{2} \pi - \text{Si. } (cx) \right\} \right] \frac{dx}{b^2 + x^2} = \frac{\pi}{2b} e^{-ab} \text{Ei. } (-bc)$

Sur ces intégrales (3) à (16) voyez: ARNDT, Gr. 11. 70.

F. Exp. monôme.
Log.
Circ. Dir. ent.

TABLE 456.

Lim. 0 et $\frac{\pi}{2}$.

$$1) \int dx \, l(1 - e^{-2a\pi \text{Tang.} x}) = \pi \left\{ a(la - 1) + \frac{1}{2} l 2a\pi - l \Gamma(a + 1) \right\} \quad \text{V. T. 378. N}^\circ. 4.$$

$$2) \int e^{-2a \text{Sec.} x} l(2 \text{Sec.} x - 1) \cdot \text{Tang.} x \, dx = \frac{1}{2} \{li.(e^{-a})\}^2 \quad \text{V. T. 383. N}^\circ. 3.$$

$$3) \int e^{p \text{Cos.} 2x} l \text{Sin.} x \cdot \text{Cos.} (p \text{Sin.} 2x + 2x) \, dx = \pi \frac{1 - e^{-p}}{4p} \quad \text{V. T. 296. N}^\circ. 9.$$

$$4) \int e^{p \text{Cos.} 2x} l \text{Cos.} x \cdot \text{Cos.} (p \text{Sin.} 2x + 2x) \, dx = \pi \frac{1 - e^p}{4p} \quad \text{V. T. 296. N}^\circ. 10.$$

$$5) \int e^{p \text{Cos.} 2x} l \text{Tang.} x \cdot \text{Cos.} (p \text{Sin.} 2x + 2x) \, dx = \pi \frac{e^{-p} - e^p}{4p} \quad \text{V. T. 296. N}^\circ. 11.$$

$$6) \int e^{p \text{Cos.} 2x} l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \cdot \text{Sin.} (p \text{Sin.} 2x + 2x) \, dx = \pm \infty \quad \text{V. T. 296. N}^\circ. 12.$$

F. Exp. monôme.
Log.
Circ. Dir. fract.

TABLE 457.

Lim. 0 et $\frac{\pi}{2}$.

$$1) \int e^{-q \text{Cot.} x} l \text{Sin.} x \frac{dx}{\text{Sin.}^2 x} = \frac{1}{q} \left[Ci.(q) \cdot \text{Cos.} q - \text{Sin.} q \cdot \left\{ \frac{1}{2} \pi - Si.(q) \right\} \right] \quad \text{V. T. 290. N}^\circ. 10.$$

$$2) \int e^{-p \text{Tang.}^2 x} \text{Tang.}^{2a} x \cdot l \text{Tang.} x \frac{2p \text{Sin.}^2 x - (2a - 1) \text{Cos.}^2 x}{\text{Sin.}^2 2x} dx = \frac{1}{8(2p)^{a-1}} 1^{a-1/2} \sqrt{\frac{\pi}{p}} \quad \text{V. T. 289. N}^\circ. 5.$$

$$3) \int e^{-p \text{Tang.}^2 x} \text{Tang.}^{2a+1} x \cdot l \text{Tang.} x \frac{p \text{Sin.}^2 x - a \text{Cos.}^2 x}{\text{Sin.}^2 2x} dx = \frac{1}{2^{a+3} p^a} 1^{a-1/4} \quad \text{V. T. 289. N}^\circ. 4.$$

$$4) \int e^{-q(\text{Tang.}^2 x + \text{Cot.}^2 x)} l \text{Tang.} x \cdot \text{Tang.}^{2a+1} x \frac{(2a + 1) \text{Sin.} 2x + 2q \text{Cos.} 2x}{\text{Sin.}^3 2x} dx = \\ - \frac{1}{32} e^{-2q} \sqrt{\frac{\pi}{q}} \cdot \sum_0^a \frac{1}{(2q)^n} \frac{(a - n + 1)^{2n/4}}{2^n 1^{n/4}} \quad \text{V. T. 289. N}^\circ. 11.$$

$$5) \int e^{-\text{Tang.}^{2p} x} l \text{Tang.} x \cdot \text{Tang.}^{2p} x \frac{2 \text{Sin.}^{2p} x - \text{Cos.}^{2p} x}{\text{Sin.}^{p+1} 2x} dx = \frac{1}{2^{p+2} p^2} \sqrt{\pi} \quad \text{V. T. 289. N}^\circ. 7.$$

$$6) \int e^{-q \text{Tang.} x} l \text{Cos.} x \frac{dx}{\text{Cos.}^2 x} = \frac{1}{q} \left[Ci.(q) \cdot \text{Cos.} q - \text{Sin.} q \cdot \left\{ \frac{1}{2} \pi - Si.(q) \right\} \right] \quad \text{V. T. 288. N}^\circ. 5.$$

$$7) \int e^{-p \text{Tang.} x} l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{dx}{\text{Cos.}^2 x} = \pm \frac{2}{p} \{ e^p Ei.(-p) - e^{-p} Ei.(p) \} \quad \text{V. T. 290. N}^\circ. 12.$$

- 8) $\int e^{-p \text{Tang. } x} l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{p \text{Sin. } x - \text{Cos. } x}{\text{Cos.}^3 x} dx = \mp 2 \{ e^{-p} \text{Ei.}(p) + e^p \text{Ei.}(-p) \}$ V. T. 290. N°. 13.
- 9) $\int e^{-p \text{Tang. } x} l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{\text{Tang. } x}{\text{Cos.}^2 x} dx = \mp \{ (1+p) e^{-p} \text{Ei.}(p) - (1-p) e^p \text{Ei.}(-p) \}$ V. T. 437. N°. 7, 8.
- 10) $\int l \text{Tang. } x. (p e^{-p \text{Tang. } x} - q e^{-q \text{Tang. } x}) \frac{dx}{\text{Cos.}^2 x} = l \frac{q}{p}$ V. T. 289. N°. 14.
- 11) $\int e^{-\text{Tang.}^{ac} x} \text{Tang.}^{ab-1} x. l \text{Tang. } x \frac{c \text{Sin.}^{ac} x - b \text{Cos.}^{ac} x}{\text{Cos.}^{ac+2} x} dx = \frac{1}{a^2 b} l^{\frac{b}{c}+1}$ V. T. 289. N°. 9.
- 12) $\int e^{-2a \text{Cosec. } x} l (2 \text{Cosec. } x - 1) \frac{dx}{\text{Tang. } x} = \frac{1}{2} \{ li.(e^{-a}) \}^2$ V. T. 383. N°. 3.
- 13) $\int e^{-p \text{Cot. } x} l \text{Tang.}^2 \left(\frac{\pi}{4} \pm x \right) \frac{p \text{Cot. } x - 1}{\text{Sin.}^2 x} dx = \pm 2 \{ e^{-p} \text{Ei.}(p) + e^p \text{Ei.}(-p) \}$ V. T. 291. N°. 1.
- 14) $\int e^{-\text{Tg.}^2 x} l \text{Tang. } x \frac{1 - \text{Cos. } 2x. \text{Sin.}^2 x}{\text{Cos.}^2 x. \text{Sin.}^2 2x} dx = \frac{3}{8} \sqrt{\pi}$ V. T. 289. N°. 15.
- 15) $\int e^{-\text{Tg.}^2 x} l \text{Sin. } 2x \frac{1 - \text{Cos. } 2x. \text{Sin.}^2 x}{\text{Cos.}^2 x. \text{Sin.}^2 2x} dx = \frac{1}{8} \sqrt{\pi}$ V. T. 289. N°. 16.
- 16) $\int e^{-q \text{Tang. } x} l \text{Tang. } x \frac{q \text{Sin. } x - p \text{Cos. } x}{\text{Sin. } 2x} \frac{\text{Tang.}^p x}{\text{Cos. } x} dx = \frac{1}{2 q^p} \Gamma(p)$ V. T. 289. N°. 3.
- 17) $\int e^{-p \text{Tang. } x} l \text{Cos. } x \frac{2 \text{Tang. } 2x. \text{Cos.}^2 x - p}{\text{Cos. } 2x. \text{Cos.}^2 x} dx = \frac{1}{2} \{ e^{-p} \text{Ei.}(p) + e^p \text{Ei.}(-p) \}$ V. T. 290. N°. 13.
- 18) $\int e^{-\text{Cot.}^{2p} x} l \text{Tang. } x. (\text{Sin.}^{2p} x - 2 \text{Cos.}^{2p} x) \frac{dx}{\text{Sin.}^{3p-1} x. \text{Cos.}^{1-p} x} = \frac{1}{2 p^2} l^2 \pi$ V. T. 291. N°. 7.
- 19) $\int e^{-\text{Cot.}^{ac} x} l \text{Tang. } x \frac{b \text{Sin.}^{ac} x - c \text{Cos.}^{ac} x}{\text{Sin.}^{ab+ac+1} x. \text{Cos.}^{1-ab} x} dx = \frac{1}{a^2 b} l^{\frac{b}{c}+1}$ V. T. 291. N°. 10.
- 20) $\int e^{-p \text{Cot.}^2 x} l \text{Tang. } x \frac{(2a+1) \text{Sin.}^2 x - 2p \text{Cos.}^2 x}{\text{Sin.}^2 2x. \text{Tang.}^{2a+2} x} dx = \frac{1}{8 (2p)^a} l^{a/2+1} \frac{\pi}{p}$ V. T. 291. N°. 5.
- 21) $\int e^{-\text{Cot.}^2 x} l \text{Tang. } x \frac{a \text{Sin.}^2 x - \text{Cos.}^2 x}{\text{Sin.}^4 x. \text{Tang.}^{2a-1} x} dx = \frac{1}{2^{a+1}} l^{a-1/2}$ V. T. 291. N°. 4.

F. Exp. monôme.
Log.
Circ. Dir. fract.

TABLE 457 suite.

Lim. 0 et $\frac{\pi}{2}$.

$$22) \int e^{-q(Tang.^2x+Cot.^2x)} l Tang.x \frac{(2a+1) Sin.^2x.Cos.^2x-2qCos.2x}{Tang.^{2a+1}x.Sin.^32x} dx = \\ = -\frac{1}{32} e^{-2q} \sqrt{\frac{\pi}{q}} \cdot \sum_0^a \frac{1}{(2q)^n} \frac{(a-n+1)^{2n/1}}{2^n 1^{n/1}} \quad \text{V. T. 291. N}^\circ. 12.$$

$$23) \int e^{-qCot.x} l Tang.x \frac{p Sin.x-q Cos.x}{Sin.2x.Sin.x.Tang.^px} dx = -\frac{1}{2q^p} \Gamma(p) \quad \text{V. T. 291. N}^\circ. 3.$$

$$24) \int e^{-aTang.x} l Tang.x \frac{dx}{Cos.x.\sqrt{Sin.2x}} = -\{lq+2l2+\Lambda\} \sqrt{\frac{\pi}{2a}} \quad \text{V. T. 381. N}^\circ. 6.$$

$$25) \int e^{-pTang.x} l (qCos.x)^{\frac{pq l (qCos.x)+2Cos.^2x}{Cos.^2x}} dx = -\frac{1}{16q} (lq^4)^2 \quad \text{V. T. 378. N}^\circ. 7.$$

$$26) \int e^{-pTang.x} l \left(q^2 \frac{Cos.2x}{Cos.^2x} \right)^{\frac{pqCos.2x.l(q^2Cos.2x.Sec.^2x)-4Cos.^2x}{Cos.2x.Cos.^2x}} dx = \frac{1}{4q} (lq^4)^2 \quad \text{V. T. 378. N}^\circ. 8.$$

F. Exp. en dén. binôme.
Log.
Circ. Dir. fract.

TABLE 458.

Lim. 0 et $\frac{\pi}{2}$.

$$1) \int \frac{l Cos.x}{(e^{2\pi Tang x}-1)^2} e^{2\pi Tang.x} \frac{dx}{Cos.^2x} = \frac{1}{8\pi} (1-2\Lambda) \quad \text{V. T. 292. N}^\circ. 13.$$

$$2) \int \frac{l Cos.x}{(e^q Tang x-1)^2} e^q Tang x \frac{dx}{Cos.^2x} = -\frac{1}{2q} \left\{ -l \frac{2\pi}{q} + \frac{\pi}{q} - Z \left(\frac{q+2\pi}{2\pi} \right) \right\} \quad \text{V. T. 292. N}^\circ. 14.$$

$$3) \int \frac{e^{\frac{1}{2}\pi Tang x} + e^{-\frac{1}{2}\pi Tang.x}}{(e^{\frac{1}{2}\pi Tang.x} - e^{-\frac{1}{2}\pi Tang.x})^2} \frac{l Cos.x}{Cos.^2x} dx = \frac{1}{2\pi} (2-\pi) \quad \text{V. T. 292. N}^\circ. 6.$$

$$4) \int \frac{e^{\frac{1}{2}\pi Tang.x} + e^{-\frac{1}{2}\pi Tang.x}}{(e^{\frac{1}{2}\pi Tang.x} - e^{-\frac{1}{2}\pi Tang.x})^2} \frac{l Cos.x}{Cos.^2x} dx = \frac{4}{\pi} \left\{ 1 - \frac{\pi}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} l \sqrt{\frac{2-1}{2+1}} \right\} \quad \text{V. T. 292. N}^\circ. 5.$$

$$5) \int \frac{(\pi+p)(e^{(\pi-p)Tang x} + e^{(p-\pi)Tang.x}) + (\pi-p)(e^{(p+\pi)Tang x} + e^{-(p+\pi)Tang.x})}{(e^{\pi Tang x} - e^{-\pi Tang x})^2} \frac{l Cos.x}{Cos.^2x} dx = \\ = \frac{1-p Sin.p}{2} - \frac{1}{2} Cos.p. l \{ 2-1+Cos.p \}, 0 < p < \pi; \quad \text{V. T. 292. N}^\circ. 9.$$

$$6) \int \frac{(p-q)(e^{(p+q)Tyx} + e^{-(p+q)Tyx}) - (p+q)(e^{(p-q)Tyx} + e^{(q-p)Tyx})}{(e^{pTang.x} + e^{-pTang.x})^2} l Ty.x \frac{dx}{Cos.^2x} = l Ty. \left(\frac{p+q}{4p} \pi \right) \quad \text{V. T. 293. N}^\circ. 15.$$

$$7) \int \frac{q(e^{pTyx} - e^{-pTyx}) e^q Tyx - e^{-qTyx} - p(e^q Tyx + e^{-qTyx} - 2\sqrt{e^q Tyx + e^{-pTyx}})}{(e^{pTang.x} - e^{-pTang.x})^2} l Ty.x \frac{dx}{Cos.^2x} = l Cos. \frac{q\pi}{2p} \quad \text{V. T. 293. N}^\circ. 16.$$

$$1) \int e^{-x} l x . \operatorname{Sin} . x d x = \frac{1}{2} \left(\frac{1}{4} \pi - \frac{1}{2} l 2 - \Lambda \right) \quad \text{Schlömilch, Cr. 33. 316. — Id., Cr. 33. 325. (faul.)}$$

$$\left. \begin{aligned} 2) \int e^{-ax} l \frac{1}{x} . \operatorname{Sin} . b x d x &= \frac{1}{a^2 + b^2} \left\{ \frac{1}{2} b l (a^2 + b^2) - a \operatorname{Arctang} . \frac{b}{a} + b \Lambda \right\} \\ 3) \int e^{-ax} l \frac{1}{x} . \operatorname{Cos} . b x d x &= \frac{1}{a^2 + b^2} \left\{ \frac{1}{2} a l (a^2 + b^2) + b \operatorname{Arctang} . \frac{b}{a} + a \Lambda \right\} \end{aligned} \right\} \text{Schlömilch, Stud. I. 14.}$$

$$4) \int e^{-px} \operatorname{Sin} . 2 q x . l x . (p \operatorname{Tang} . q x - q) d x = \frac{1}{2} l \frac{p^2 + 4 q^2}{p^2} \quad \text{V. T. 392. N°. 10.}$$

$$5) \int e^{-px} \operatorname{Cos} . q x . l x . (p \operatorname{Tang} . q x - q) d x = \operatorname{Arctang} . \frac{q}{p} \quad \text{V. T. 392. N°. 3.}$$

$$\left. \begin{aligned} 6) \int e^{-px} l \operatorname{Sin} . (q x) d x &= -\frac{1}{2 p} l 2 - \frac{p}{2} \sum_1^{\infty} \frac{1}{n p^2 + n^2 q^2} \\ 7) \int e^{-px} l \operatorname{Cos} . (q x) d x &= -\frac{1}{2 p} l 2 - \frac{p}{2} \sum_1^{\infty} \frac{(-1)^n}{n} \frac{1}{p^2 + n^2 q^2} \\ 8) \int e^{-px} l \operatorname{Tang} . (q x) d x &= -p \sum_1^{\infty} \frac{1}{2 n - 1} \frac{1}{p^2 + (2 n - 1)^2 q^2} \end{aligned} \right\} \text{Schlömilch, Beitr. II. 5.}$$

$$\left. \begin{aligned} 9) \int e^{-x^2} l \operatorname{Sin} . (a x) d x &= \frac{1}{2} \sqrt{\pi} . \left\{ -l 2 + \sum_1^{\infty} \frac{e^{-(na)^2}}{n} \right\} \\ 10) \int e^{-x^2} l \operatorname{Cos} . (a x) d x &= \frac{1}{2} \sqrt{\pi} . \left\{ -l 2 + \sum_1^{\infty} (-1)^n \frac{e^{-(na)^2}}{n} \right\} \\ 11) \int e^{-x^2} l \operatorname{Tang} . (a x) d x &= \sqrt{\pi} . \sum_1^{\infty} \frac{e^{-(2n-1)^2 a^2}}{2 n - 1} \end{aligned} \right\} \text{Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 38.}$$

$$\left. \begin{aligned} 12) \int e^{-x^2} l (2 \operatorname{Sin} . a x)^2 d x &= \sqrt{\pi} . \sum_1^{\infty} \frac{e^{-a^2 n^2}}{n} \\ 13) \int e^{-x^2} l (2 \operatorname{Cos} . a x)^2 d x &= \sqrt{\pi} . \sum_1^{\infty} (-1)^n \frac{e^{-a^2 n^2}}{n} \\ 14) \int e^{-x^2} l (1 - 2 p \operatorname{Cos} . 2 a x + p^2) d x &= \sqrt{\pi} . \sum_1^{\infty} \frac{1}{n} p^n e^{-a^2 n^2} \end{aligned} \right\} \text{Schlömilch, Stud. I. 25.}$$

$$15) \int l x \frac{(e^{\pi x} + e^{-\pi x}) p \operatorname{Cos} . p \pi - (e^{\pi x} - e^{-\pi x}) \pi \operatorname{Sin} . p \pi}{(e^{\pi x} + e^{-\pi x})^2} d x = - \operatorname{Arctang} . (e^{\frac{1}{2} p}) \quad \text{V. T. 396. N°. 1.}$$

F. Exp.	TABLE 459 suite.	Lim. 0 et ∞ .
Log.		
Circ. Dir.		

- 16) $\int l x \frac{(e^{\pi x} - e^{-\pi x}) p \sin. p x + (e^{\pi x} + e^{-\pi x}) \pi \cos. p x}{(e^{\pi x} - e^{-\pi x})^2} dx = \infty$ V. T. 396. N°. 2.
- 17) $\int l(1+x^2) \frac{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}) 4 q \cos. q x - (e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}) \pi \sin. q x}{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x})^2} dx =$
 $= 2 \pi e^{-q} \sqrt{2 + (e^q - e^{-q})} \sqrt{2} \cdot l \frac{e^q + \sqrt{2 + e^{-q}}}{e^q - \sqrt{2 + e^{-q}}} - 2(e^q + e^{-q}) \sqrt{2} \cdot \text{Arctg.} \left(\frac{\sqrt{2}}{e^q - e^{-q}} \right)$ V. T. 396. N°. 5.
- 18) $\int l(1+x^2) \frac{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}) 2 q \cos. q x - (e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}) \pi \sin. q x}{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x})^2} dx = 2 q e^{-q} - (e^q - e^{-q}) l(1 + e^{-2q})$ V. T. 396. N°. 7.
- 19) $\int l(1+x^2) \frac{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}) 2 q \sin. q x + (e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}) \pi \cos. q x}{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})^2} dx =$
 $= 2(e^q - e^{-q}) \text{Arctang.}(e^{-q}) + \pi e^{-q} - 2$ V. T. 396. N°. 22.
- 20) $\int l(1+x^2) \frac{(e^{\pi x} - e^{-\pi x}) q \sin. q x + (e^{\pi x} + e^{-\pi x}) \pi \cos. q x}{(e^{\pi x} - e^{-\pi x})^2} dx = -\frac{1}{2} + \frac{1}{2} q e^{-q} + \frac{e^q + e^{-q}}{2} l(1 + e^{-q})$ V. T. 396. N°. 26.

F. Exp.	TABLE 440.	Lim. diverses.
Log.		
Circ. Dir.		

- 1) $\int_0^{\pi} e^{2\pi a x} l \sin. \pi x dx = -\frac{1}{2a}$ Schaar, Mém. Cour. Brux. T. 23.
- 2) $\int_0^{\pi} e^{-2a \cot x} l(2 \cot. x - 1) \frac{dx}{\sin. 2x} = \frac{1}{4} \{l i. (e^{-q})\}^2$ V. T. 383. N°. 3.
- 3) $\int_0^{\pi} e^{p \cos. x} l \left(\frac{1}{2} \sin. x \right) \cdot \cos. (p \sin. x + x) dx = -\frac{\pi}{4p} (e^{\frac{1}{2}p} - e^{-\frac{1}{2}p})^2$ V. T. 436. N°. 3, 4.
- 4) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (e^{px} + e^{-px}) \sin. (p l \cos. x) dx = -2 \pi \sin. (p l 2)$ V. T. 447. N°. 15.
- 5) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (e^{px} + e^{-px}) \cos. (p l \cos. x) dx = 2 \pi \cos. (p l 2)$ V. T. 447. N°. 16.

- 1) $\int \text{Arctang. } x \frac{(e^{l\pi x} + e^{-l\pi x}) 4q \text{Sin. } qx + (e^{l\pi x} - e^{-l\pi x}) \pi \text{Cos. } qx}{(e^{l\pi x} + e^{-l\pi x})^2} dx = \pi e^{-q} \sqrt{2} +$
 $+ \frac{e^q + e^{-q}}{\sqrt{2}} l \frac{e^q - \sqrt{2} + e^{-q}}{e^q + \sqrt{2} + e^{-q}} + \frac{e^q - e^{-q}}{\sqrt{2}} 2 \text{Arctang.} \left(\frac{\sqrt{2}}{e^q - e^{-q}} \right)$ V. T. 396 N°. 19.
- 2) $\int \text{Arctg. } x \frac{(e^{l\pi x} + e^{-l\pi x}) 2q \text{Sin. } qx + (e^{l\pi x} - e^{-l\pi x}) \pi \text{Cos. } qx}{(e^{l\pi x} + e^{-l\pi x})^2} dx = qe^{-q} + \frac{e^q + e^{-q}}{2} l (1 + e^{-2q})$ V. T. 396. N°. 20.
- 3) $\int \text{Arctang. } x \frac{(e^{l\pi x} - e^{-l\pi x}) 4q \text{Cos. } qx - (e^{l\pi x} + e^{-l\pi x}) \pi \text{Sin. } qx}{(e^{l\pi x} - e^{-l\pi x})^2} dx = e^{-q} \sqrt{2} +$
 $+ \frac{e^q - e^{-q}}{\sqrt{2}} l \frac{e^q - \sqrt{2} + e^{-q}}{e^q + \sqrt{2} + e^{-q}} - \frac{e^q + e^{-q}}{\sqrt{2}} 2 \text{Arctang.} \left(\frac{\sqrt{2}}{e^q - e^{-q}} \right)$ V. T. 396. N°. 6.
- 4) $\int \text{Arctg. } x \frac{(e^{l\pi x} - e^{-l\pi x}) 2q \text{Cos. } qx - (e^{l\pi x} + e^{-l\pi x}) \pi \text{Sin. } qx}{(e^{l\pi x} - e^{-l\pi x})^2} dx = \frac{\pi}{2} e^{-q} - (e^q + e^{-q}) \text{Arctg. } e^{-q}$ V. T. 396. N°. 9.
- 5) $\int \text{Arctg. } x \frac{(e^{\pi x} - e^{-\pi x}) q \text{Cos. } qx - (e^{\pi x} + e^{-\pi x}) \pi \text{Sin. } qx}{(e^{\pi x} - e^{-\pi x})^2} dx = \frac{1}{4} q e^{-q} - \frac{e^q - e^{-q}}{4} l (1 + e^{-q})$ V. T. 396. N°. 12.

- 1) $\int_0^{\pi} \text{li.}(e^{-\text{Tang. } x}). \text{Tang. }^p x \frac{dx}{\text{Sin. } 2x} = -\frac{1}{2p} \Gamma(p)$ V. T. 402. N°. 3.
 - 2) $\int_0^{\infty} \text{li.}(e^{-x}). \text{Sin. } qx dx = -\frac{1}{2q} l(1 + q^2)$
 - 3) $\int_0^{\infty} \text{li.}(e^{-x}). \text{Cos. } qx dx = -\frac{1}{q} \text{Arctang. } q$
 - 4) $\int_0^{\infty} \text{li.}(e^{-x}). e^x \text{Sin. } qx dx = -\frac{1}{1 + q^2} \left(\frac{1}{2} \pi + q l q \right)$
 - 5) $\int_0^{\infty} \text{li.}(e^x). e^{-x} \text{Sin. } qx dx = \frac{1}{1 + q^2} \left(\frac{1}{2} \pi - q l q \right)$
 - 6) $\int_0^{\infty} \text{li.}(e^{-x}). e^x \text{Cos. } qx dx = -\frac{1}{1 + q^2} \left(\frac{1}{2} q \pi - l q \right)$
- Schlömilch, Beitr. III. § 8.
Schlömilch, Gr. 5. 204.

F. Exp.

Circ. Dir.

Autres Fonctions.

TABLE 442 suite.

Lim. diverses.

$$\begin{aligned}
 & \left. \begin{aligned}
 7) \int_0^\infty li.(e^x).e^{-x} \cos qx dx &= -\frac{1}{1+q^2} \left(\frac{1}{2} q \pi + l q \right) \\
 8) \int_0^\infty \{e^x li.(e^{-x}) + e^{-x} li.(e^x)\} \sin qx dx &= -\frac{2q}{1+q^2} l q \\
 9) \int_0^\infty \{e^x li.(e^{-x}) - e^{-x} li.(e^x)\} \sin qx dx &= -\frac{\pi}{1+q^2} \\
 10) \int_0^\infty \{e^x li.(e^{-x}) + e^{-x} li.(e^x)\} \cos qx dx &= -\frac{q \pi}{1+q^2} \\
 11) \int_0^\infty \{e^x li.(e^{-x}) - e^{-x} li.(e^x)\} \cos qx dx &= \frac{2}{1+q^2} l q
 \end{aligned} \right\} \text{Schlömlich, Gr. 5. 204.} \\
 & 12) \int_0^\infty li.(e^{-x}).e^{-px} \sin qx dx = -\frac{1}{p^2+q^2} \left[q l \sqrt{\{(1+p)^2+q^2\}} - p \operatorname{Arctg}.\left(\frac{q}{1+p}\right) \right] \\
 & 13) \int_0^\infty li.(e^{-x}).e^{-px} \cos qx dx = -\frac{1}{p^2+q^2} \left[p l \sqrt{\{(1+p)^2+q^2\}} + q \operatorname{Arctg}.\left(\frac{q}{1+p}\right) \right]
 \end{aligned}$$

Schlömlich,
Beitr. III.
§ 8.

F. Log.

Circ. Dir.

Circ. Inv.

TABLE 445.

Lim. diverses.

$$\begin{aligned}
 1) \int_0^\pi l \operatorname{Tang}.x. \left\{ \cos x. \operatorname{Arctg}.(p \cos x) - \frac{p \sin^2 x}{1+p^2 \cos^2 x} \right\} dx &= -\frac{1}{2} \pi l \{p + \sqrt{1+p^2}\} \quad \begin{matrix} \text{V. T. 369.} \\ \text{N}^\circ. 3 \end{matrix} \\
 2) \int_0^\pi l \operatorname{Tang}.x. \left\{ \sin x. \operatorname{Arctg}.(p \sin x) - \frac{p \cos^2 x}{1+p^2 \sin^2 x} \right\} dx &= \frac{1}{2} \pi l \{p + \sqrt{1+p^2}\} \quad \begin{matrix} \text{V. T. 369.} \\ \text{N}^\circ. 3. \end{matrix} \\
 3) \int_0^\pi \left[\sin x. l(1+2p \cos x + p^2) + 2 \cos x. \operatorname{Arcsin}.\left(\frac{p \sin x}{\sqrt{1+2p \cos x + p^2}}\right) \right] dx &= \\
 &= \frac{2}{p} l \frac{1+p}{1-p} + 2l(1-p^2) - 4 \quad \begin{matrix} \text{Dienger, Cr.} \\ 37. 363. \end{matrix} \\
 & \quad , p^2 < 1; \\
 4) \int_0^\pi \left[\cos x. l(1+2p \cos x + p^2) - 2 \sin x. \operatorname{Arcsin}.\left(\frac{p \sin x}{\sqrt{1+2p \cos x + p^2}}\right) \right] dx &= 0
 \end{aligned}$$

F. Log. Circ. Dir. Autres Fonctions.	TABLE 444.	Lim. diverses.
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- 1) $\int_0^1 li.\left(\frac{1}{x}\right).Sin.(qlx) dx = \frac{1}{1+q^2} \left(qlq - \frac{1}{2}\pi\right)$ V. T. 442. N°. 5.
- 2) $\int_0^1 li.\left(\frac{1}{x}\right).Cos.(qlx) dx = \frac{-1}{1+q^2} \left(lq + \frac{1}{2}q\pi\right)$ V. T. 442. N°. 7.
- 3) $\int_0^1 l\Gamma(x).Sin.(2a\pi x) dx = \frac{1}{2a\pi} (la + A + l2\pi)$
- 4) $\int_0^1 l\Gamma(x).Cos.(2a\pi x) dx = \frac{1}{4a}$
- 5) $\int_0^x li.\left(\frac{1}{x}\right).Sin.(qlx) dx = -\frac{\pi}{1+q^2}$ V. T. 444. N°. 1 et 7.
- 6) $\int_0^x li.\left(\frac{1}{x}\right).Cos.(qlx) dx = -\frac{q\pi}{1+q^2}$ V. T. 444. N°. 2 et 8.
- 7) $\int_1^x li.\left(\frac{1}{x}\right).Sin.(qlx) dx = -\frac{1}{1+q^2} \left\{\frac{1}{2}\pi + qlq\right\}$ V. T. 442. N°. 4.
- 8) $\int_1^x li.\left(\frac{1}{x}\right).Cos.(qlx) dx = \frac{1}{1+q^2} \left\{lq - \frac{1}{2}q\pi\right\}$ V. T. 442. N°. 6.
- 9) $\int_0^{\frac{\pi}{2}} l Sin. Amp. \left(\frac{2x F'(p)}{\pi}\right) dx = \frac{-\pi}{4 F'(p)} \left[F'(p).lp + \frac{1}{2}\pi F\{1-(1-p^2)\}\right]$
- 10) $\int_0^{\frac{\pi}{2}} l Cos. Amp. \left(\frac{2x F'(p)}{\pi}\right) dx = \frac{\pi}{4 F'(p)} \left[F'(p).l\frac{1-(1-p^2)}{p} - \frac{1}{2}\pi F\{1-(1-p^2)\}\right]$

Roberts,
L. 12.
449.

F. Alg. rat. ent. Plusieurs Fonctions.	TABLE 445.	Lim. diverses.
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- 1) $\int_0^1 li.(x).Sin.(qlx).x^{p-1} dx = \frac{-1}{p^2+q^2} \left[p Arctg.\left(\frac{q}{1+p}\right) - \frac{1}{2}ql\{(1+p)^2+q^2\}\right]$ V. T. 442
N°. 12.
- 2) $\int_0^1 li.(x).Cos.(qlx).x^{p-1} dx = \frac{-1}{p^2+q^2} \left[q Arctg.\left(\frac{q}{1+p}\right) + \frac{1}{2}pl\{(1+p)^2+q^2\}\right]$ V. T. 442.
N°. 13.

$$3) \int_0^1 \text{Sin.}(q \text{ Arccos. } x). l x. x^{q-1} dx = \frac{\pi}{2q+2} \left(\Delta + Z'(q) - \frac{1}{q} - 2 l 2 \right) \quad \text{V. T. 331. N}^\circ. 12.$$

$$4) \int_0^\infty e^{-x} \text{Sin. } x. l x. x^{p-1} dx = \frac{1}{2^{1/2} p} \Gamma(p) \left[\frac{1}{4} \pi \text{Cos. } \frac{1}{4} p \pi - \frac{1}{2} \text{Sin. } \frac{1}{4} p \pi. l 2 + \text{Sin. } \frac{1}{4} p \pi. Z'(p) \right] \quad \text{Schlömlich, Cr. 33. 316.}$$

$$5) \int_0^\infty e^{-ax} \text{Sin. } b x. l x. x^{p-1} dx = \frac{-\Gamma(p)}{\sqrt{(a^2 + b^2)^p}} \left\{ \frac{1}{2} l (a^2 + b^2). \text{Sin.} \left(p \text{ Arctang. } \frac{b}{a} \right) - \right. \\ \left. - \text{Arctang. } \frac{b}{a}. \text{Cos.} \left(p \text{ Arctang. } \frac{b}{a} \right) - \text{Sin.} \left(p \text{ Arctang. } \frac{b}{a} \right). Z'(p) \right\}$$

$$6) \int_0^\infty e^{-ax} \text{Cos. } b x. l x. x^{p-1} dx = \frac{-\Gamma(p)}{\sqrt{(a^2 + b^2)^p}} \left\{ \frac{1}{2} l (a^2 + b^2). \text{Cos.} \left(p \text{ Arctang. } \frac{b}{a} \right) + \right. \\ \left. + \text{Arctang. } \frac{b}{a}. \text{Sin.} \left(p \text{ Arctang. } \frac{b}{a} \right) - \text{Cos.} \left(p \text{ Arctang. } \frac{b}{a} \right). Z'(p) \right\}$$

Sur les intégrales (5) à (6) voyez: Legendre, Exerc. 3. 56. — Cauchy, P. 28. 147. I. § 6. — Schlömlich, Stud. I. 14.

$$7) \int_0^\infty e^{-px} \text{Cos. } q x. l x. \{ p x \text{Tang. } q x - q x - a \text{Tang. } q x \} x^{a-1} dx = \frac{1^{a-1/1}}{(p^2 + q^2)^{1/2} a} \text{Sin.} \left(a \text{ Arctg. } \frac{q}{p} \right) \quad \text{V. T. 386. N}^\circ. 12.$$

$$8) \int_0^\infty e^{-px} \text{Cos. } q x. l x. \{ p x + q x \text{Tang. } q x - a \} x^{a-1} dx = \frac{1^{a-1/1}}{(p^2 + q^2)^{1/2} a} \text{Cos.} \left(a \text{ Arctg. } \frac{q}{p} \right) \quad \text{V. T. 386. N}^\circ. 13.$$

$$9) \int_0^\infty e^{-px} \{ p l \text{Sin. } q x - q \text{Cos. } q x \} x dx = -\frac{1}{2p} l 2 - \frac{1}{2} p \sum_1 \frac{1}{n} \frac{1}{p^2 + n^2 q^2} \quad \text{V. T. 439. N}^\circ. 6.$$

$$10) \int_0^\infty e^{-px} \{ p l \text{Cos. } q x + q \text{Tang. } q x \} x dx = -\frac{1}{2p} l 2 - \frac{1}{2} p \sum_1 \frac{1}{n} \frac{(-1)^n}{p^2 + n^2 q^2} \quad \text{V. T. 439. N}^\circ. 7.$$

$$11) \int_0^\infty e^{-px} \{ p l \text{Tang. } q x - 2q \text{Coscc. } 2q x \} x dx = -p \sum_1 \frac{1}{2n-1} \frac{1}{p^2 + (2n-1)^2 q^2} \quad \text{V. T. 439. N}^\circ. 8.$$

$$12) \int_0^\infty e^{-bx} \text{Sin.} \left(c x - \text{Arctang. } \frac{c}{b} \right). l x. dx = \frac{1}{\sqrt{(b^2 + c^2)}} \text{Arctang. } \frac{c}{b} \left. \vphantom{\int_0^\infty} \right\} \quad \text{Legendre, Exerc. 3. 56.}$$

$$13) \int_0^\infty e^{-bx} \text{Sin.} \left(c x - a \text{Arctang. } \frac{c}{b} \right). l x. x^{a-1} dx = \frac{1^{a-1/1}}{(b^2 + c^2)^{1/2} a} \text{Arctang. } \frac{c}{b}$$

$$14) \int_0^\infty e^{-hcx} (e^{-2cx} - 2e^{-cx} \cos. bx + 1)^{\frac{1}{2}g} \sin. \left\{ b h x + g \operatorname{Arctang.} \left(\frac{e^{-cx} \sin. bx}{e^{-cx} \cos. bx - 1} \right) \right\} x^{q-1} dx =$$

$$= \frac{\Gamma(q)}{(b^2 + c^2)^{\frac{1}{2}g}} \sin. \left(q \operatorname{Arctang.} \frac{b}{c} \right) \Delta^g. h^{-q}$$

$$15) \int_0^\infty e^{-hcx} (e^{-2cx} - 2e^{-cx} \cos. bx + 1)^{\frac{1}{2}g} \cos. \left\{ b h x + g \operatorname{Arctang.} \left(\frac{e^{-cx} \sin. bx}{e^{-cx} \cos. bx - 1} \right) \right\} x^{q-1} dx =$$

$$= \frac{\Gamma(q)}{(b^2 + c^2)^{\frac{1}{2}g}} \cos. \left(q \operatorname{Arctang.} \frac{b}{c} \right) \Delta^g. h^{-q}$$

Les intégrales (14), (15) se trouvent chez Cauchy, P. 28. 147. P. III. § 1.

$$16) \int_0^\infty e^{-px} \{lx + Z'(q)\} x^{q-1} dx = -\Gamma(q) \frac{l p}{p^q}$$

$$17) \int_0^\infty e^{-px} (e^{-x} - 1)^a \{lx + Z'(q)\} x^{q-1} dx = -\Gamma(q) \Delta^a. \frac{l p}{p^q}$$

Cauchy, P. 28. 147. 1. § 7.

$$18) \int_0^{\frac{1}{a}} l x. \sin. (b \operatorname{Arccos.} ax). x^{b-1} dx = \frac{\pi}{2^{b+2} a^b} \left[\Lambda + Z'(a) - \frac{1}{a} - 2l(2a) \right] \quad \text{Lindmann, Stockh. Handl. 1850. III.}$$

$$1) \int_0^1 \operatorname{Arctang.} x. \sin. (p l x) \frac{dx}{x} = -\frac{\pi}{4p} \frac{(e^{\frac{1}{2}p\pi} - 1)^2}{e^{p\pi} + 1} \quad \text{V. T. 404. N°. 7.}$$

$$2) \int_0^1 \operatorname{Arctang.} x \frac{p l x. \cos. (p l x) - \sin. (p l x)}{(l x)^2} \frac{dx}{x} = \frac{1}{2} p \pi - \operatorname{Arctang.} (e^{\frac{1}{2}p\pi}) \quad \text{V. T. 406. N°. 15.}$$

$$3) \int_0^1 li. (x). \sin. (q l x) \frac{dx}{x} = \frac{1}{2q} l (1 + q^2) \quad \text{V. T. 442. N°. 2.}$$

$$4) \int_0^1 li. (x). \sin. (q l x) \frac{dx}{x^2} = \frac{1}{1 + q^2} \left(q l q + \frac{1}{2} \pi \right) \quad \text{V. T. 442. N°. 4.}$$

$$5) \int_0^1 li. (x). \cos. (q l x) \frac{dx}{x} = -\frac{1}{q} \operatorname{Arctang.} q \quad \text{V. T. 442. N°. 3.}$$

$$6) \int_0^1 li. (x). \cos. (q l x) \frac{dx}{x^2} = \frac{1}{1 + q^2} \left(l q - \frac{1}{2} q \pi \right) \quad \text{V. T. 442. N°. 6.}$$

$$\begin{aligned}
 7) \int_0^1 \left\{ li.(x) + x^2 li.\left(\frac{1}{x}\right) \right\} Sin.(qlx) \frac{dx}{x^2} &= \frac{2q}{1+q^2} lq \quad \text{V. T. 442. N}^\circ. 8. \\
 8) \int_0^1 \left\{ li.(x) - x^2 li.\left(\frac{1}{x}\right) \right\} Sin.(qlx) \frac{dx}{x^2} &= \frac{\pi}{1+q^2} \quad \text{V. T. 442. N}^\circ. 9. \\
 9) \int_0^1 \left\{ li.(x) + x^2 li.\left(\frac{1}{x}\right) \right\} Cos.(qlx) \frac{dx}{x^2} &= -\frac{q\pi}{1+q^2} \quad \text{V. T. 442. N}^\circ. 10. \\
 10) \int_0^1 \left\{ li.(x) - x^2 li.\left(\frac{1}{x}\right) \right\} Cos.(qlx) \frac{dx}{x^2} &= \frac{2lq}{1+q^2} \quad \text{V. T. 442. N}^\circ. 11. \\
 11) \int_0^\infty e^{-x^2} (lCos.ax + ax Tang.ax) \frac{dx}{x^2} &= \sqrt{\pi} \cdot \left\{ -l2 + \sum_1^\infty (-1)^{n-1} \frac{e^{-(na)^2}}{n} \right\} \quad \text{V. T. 431. N}^\circ. 10. \\
 12) \int_0^\infty Sin. \{ pl(1 + 2q Cos.ax + q^2) \} \cdot \left[e^{p Arctg.\left(\frac{q Sin.ax}{1+q Cos.ax}\right)} + e^{-p Arctg.\left(\frac{q Sin.ax}{1+q Cos.ax}\right)} \right] \frac{x}{c^2 + x^2} dx &= \\
 &= \pi Sin. \{ pl(1 + qe^{-ac}) \} \\
 13) \int_0^\infty Sin. \{ pl(1 + 2q Cos.ax + q^2) \} \cdot \left[e^{p Arctg.\left(\frac{q Sin.ax}{1+q Cos.ax}\right)} - e^{-p Arctg.\left(\frac{q Sin.ax}{1+q Cos.ax}\right)} \right] \frac{x}{c^2 + x^2} dx &= \\
 &= -\pi Sin. \{ pl(1 + qe^{-ac}) \} \\
 14) \int_0^\infty Cos. \{ pl(1 + 2q Cos.ax + q^2) \} \cdot \left[e^{p Arctg.\left(\frac{q Sin.ax}{1+q Cos.ax}\right)} + e^{-p Arctg.\left(\frac{q Sin.ax}{1+q Cos.ax}\right)} \right] \frac{dx}{c^2 + x^2} &= \\
 &= \frac{\pi}{c} Cos. \{ pl(1 + qe^{-2ac}) \} \\
 15) \int_0^\infty Cos. \{ pl(1 + 2q Cos.ax + q^2) \} \cdot \left[e^{p Arctg.\left(\frac{q Sin.ax}{1+q Cos.ax}\right)} - e^{-p Arctg.\left(\frac{q Sin.ax}{1+q Cos.ax}\right)} \right] \frac{dx}{c^2 + x^2} &= \\
 &= -\frac{\pi}{2c} + \frac{\pi}{c} Cos. \{ pl(1 + qe^{-2ac}) \}
 \end{aligned}$$

Sur les intégrales (12) à (15) voyez : Boncompagni Cr. 25. 74; elles sont fautives.

$$\begin{aligned}
 16) \int_0^\infty \frac{Cos.(\frac{1}{2}a\pi - bx).l(1+x^2) + 2 Sin.(\frac{1}{2}a\pi - bx).Arctg.x}{\left\{ \frac{1}{2}l(1+x^2) \right\}^2 + (Arctg.x)^2} \frac{x^a}{x^2 + c^2} dx &= \frac{\pi c^{a-1}}{l(1+c)} e^{-bc} \left\{ \begin{array}{l} \text{Cauchy,} \\ \text{P.19.511.} \end{array} \right. \\
 17) \int_0^\infty \frac{Cos.bx.l(1+x^2) - 2 Sin.bx.Arctg.x}{\left\{ \frac{1}{2}l(1+x^2) \right\}^2 + (Arctg.x)^2} \frac{dx}{x^2 + c^2} &= \frac{\pi}{c} \left\{ \frac{e^{-bc}}{l(1+c)} - \frac{1}{c} \right\}
 \end{aligned}$$

$$18) \int_0^\infty \frac{\text{Sin. } a x . l(1+p^2 x^2) + 2 \text{Cos. } a x . \text{Arctg. } p x}{\left\{ \frac{1}{2} l(1+p^2 x^2) \right\}^2 + (\text{Arctg } p x)^2} \frac{x}{1+x^2} dx = \frac{\pi e^{-a}}{l(1+p)} \quad \text{Cauchy, Cours. Lec. 39.}$$

$$19) \int_0^\infty \text{li.}(x) . \text{Sin.}(qlx) \frac{dx}{x^2} = -\frac{\pi}{1+q^2} \quad \text{V. T. 446. N}^\circ. 4, 24.$$

$$20) \int_0^\infty \text{li.}(x) . \text{Cos.}(qlx) \frac{dx}{x^2} = -\frac{q\pi}{1+q^2} \quad \text{V. T. 446. N}^\circ. 6, 25.$$

$$21) \int_0^\infty e^{-px} (e^{-x} - 1)^a \frac{l x + Z'(a)}{x^{b+1}} dx = \frac{\pi}{\Gamma(b+1) \text{Sin. } b\pi} \Delta^a . (p^b l p), b < a; \quad \text{Cauchy, P. 28. 147. I. § 7.}$$

$$22) \int_0^\infty e^{-px} (e^{-x} - 1)^a \frac{l x - \frac{Z''(q+1) - \pi \text{Cot.}\{(q+1)\pi\}}{x^{q+1}}}{x^{q+1}} dx = -\frac{\pi}{\Gamma(q+1)} \text{Cosec.}\{(q+1)\pi\} . \Delta^a . (p^a l p), q < a;$$

$$23) \quad = -\frac{\pi}{\Gamma(q+1)} \text{Cosec.}\{(q+1)\pi\} . \Delta^a . (p^a l p) \quad \begin{matrix} \text{valeur extra-} \\ \text{ord. } q > a; \end{matrix}$$

Les intégrales 22) et 23) se trouvent chez Cauchy, Exerc. 1826. p. 58.

$$24) \int_1^\infty \text{li.}(x) . \text{Sin.}(qlx) \frac{dx}{x^2} = \frac{-1}{1+q^2} \left\{ q l q - \frac{1}{2} \pi \right\} \quad \text{V. T. 442. N}^\circ. 5.$$

$$25) \int_1^\infty \text{li.}(x) . \text{Cos.}(qlx) \frac{dx}{x^2} = \frac{-1}{1+q^2} \left\{ l q + \frac{1}{2} q \pi \right\} \quad \text{V. T. 442. N}^\circ. 7.$$

$$26) \int_{-\infty}^\infty \text{Cos.}(p \text{Arctang. } ax) \frac{l(1+a^2 x^2)}{(1+a^2 x^2)^p} \frac{dx}{1+x^2} = \frac{2\pi}{(1+a)^p} l(1+a)$$

$$\left. \begin{aligned} 27) \int_{-\infty}^\infty \frac{e^{p \text{Arctg. } ax} + e^{-p \text{Arctg. } ax}}{1+x^2} \text{Sin.} \left\{ \frac{1}{2} p l(1+a^2 x^2) \right\} dx &= 2\pi \text{Sin.} \{p l(1+a)\} \\ 28) \int_{-\infty}^\infty \frac{e^{p \text{Arctg. } ax} - e^{-p \text{Arctg. } ax}}{1+x^2} \text{Cos.} \left\{ \frac{1}{2} p l(1+a^2 x^2) \right\} dx &= 2\pi \text{Cos.} \{p l(1+a)\} \end{aligned} \right\} \begin{matrix} \text{Cauchy, C. R. 11.} \\ \text{1008.} \\ \text{où 28) était fau-} \\ \text{tive.} \end{matrix}$$

$$1) \int_0^1 \{e^{ql'(1-x^2)} - e^{-ql'(1-x^2)}\} \text{Sin. } qx . \text{Sin. } (2c \text{Arccos. } x) \frac{dx}{1-(1-x^2)} = \frac{1}{2} \pi \frac{(-1)^{c-1} q^{2c}}{\Gamma(2c)} \quad \text{V. T. 288. N}^\circ. 6.$$

$$2) \int_0^1 \{e^{ql'(1-x^2)} + e^{-ql'(1-x^2)}\} \text{Sin. } qx . \text{Cos.} \{(2c-1) \text{Arccos. } x\} \frac{dx}{1-(1-x^2)} = \frac{1}{2} \pi \frac{(-1)^{c-1} q^{2c-1}}{\Gamma(2c-1)} \quad \text{V. T. 288. N}^\circ. 8.$$

- $$3) \int_0^1 \{e^{q\sqrt{1-x^2}} - e^{-q\sqrt{1-x^2}}\} \cos qx \sin \{(2c-1) \operatorname{Arccos} x\} \frac{dx}{\sqrt{1-x^2}} = \frac{1}{2} \pi \frac{(-1)^{c-1} q^{2c-1}}{1^{2c-1}} \quad \text{V. T. 288. N}^\circ. 7.$$
- $$4) \int_0^1 \{e^{q\sqrt{1-x^2}} + e^{-q\sqrt{1-x^2}}\} \cos qx \cos \{(2c) \operatorname{Arccos} x\} \frac{dx}{\sqrt{1-x^2}} = \frac{1}{2} \pi \frac{(-1)^c q^{2c}}{1^{2c}} \quad \text{V. T. 288. N}^\circ. 9.$$
- $$5) \int_0^\infty \sin \{q \operatorname{Arctg} x\} \frac{lx}{x(1+x^2)^{\frac{1}{2}q}} dx = -\frac{1}{2} \pi \{\Lambda + Z'(q)\} \quad \text{V. T. 333. N}^\circ. 9.$$
- $$6) \int_0^\infty \cos \{q \operatorname{Arctg} x\} \cdot lx \frac{dx}{(1+x^2)^{\frac{1}{2}q}} = -\frac{\pi}{2(q-1)} \quad \text{V. T. 333. N}^\circ. 8.$$
- $$7) \int_0^\infty \sin \{q \operatorname{Arccot} x\} \cdot lx \frac{x^{q-1}}{(1+x^2)^{\frac{1}{2}q}} dx = \frac{1}{2} \pi \{\Lambda + Z'(q)\} \quad \text{V. T. 447. N}^\circ. 5.$$
- $$8) \int_0^\infty \cos \{q \operatorname{Arccot} x\} \cdot lx \frac{x^q}{(1+x^2)^{\frac{1}{2}q}} dx = \frac{\pi}{2(q-1)} \quad \text{V. T. 447. N}^\circ. 6.$$
- $$9) \int_0^\infty \sin \left\{ (c+1) \operatorname{Arctg} \frac{a}{bx} \right\} \cdot lx \frac{x^c}{\sqrt{(a^2+b^2x^2)^{c+1}}} dx = \frac{\pi}{2b^{c+1}} \left\{ l \frac{a}{b} + \Lambda + Z'(c+1) \right\}$$
- $$10) \int_0^\infty \cos \left\{ (c+1) \operatorname{Arctg} \frac{a}{bx} \right\} \cdot lx \frac{x^{c-1}}{\sqrt{(a^2+b^2x^2)^{c+1}}} dx = \frac{\pi}{2acb^c}$$

Lindmann,
Stockh.
Handl.
1850. II.

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