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A PRACTICAL METHOD FOR DETERMINING OCEAN CURRENTS

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TREASURY DEPARTMENT
UNITED STATES COAST GUARD

A PRACTICAL METHOD FOR DETERMINING OCEAN CURRENTS

BY

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TABLE OF CONTENTS

	Page
Foreword.....	v
The origin of currents.....	1
Static consideration of a water mass.....	2
Three general static conditions.....	3
Dynamic consideration of a water mass.....	4
Three variables in the sea.....	6
Gravity.....	7
Pressure.....	9
Application of dynamic units.....	12
Depth at which greatest obliquity of isobaric surfaces occur.....	13
Specific volume.....	14
Given temperature and salinity—a graphic method to find density.....	16
Tables for converting densities into specific volumes in situ.....	18
Distribution of mass.....	19
Effect of earth rotation on ocean currents.....	20
Resolution of forces in gradient currents.....	23
The practical methods and form of computations generally followed in dynamic physical oceanography.....	24
Determination of dynamic depth, stations 205 and 206.....	28
Velocity of a current—how determined.....	31
Direction of flow.....	35
General suggestions for a program of hydrographical survey.....	36
Description of a dynamic topographical chart (current map).....	37
Friction.....	41
Effect of bottom configuration on currents.....	43
Tides.....	44
Variations in atmospheric pressure.....	45
Winds.....	46

FOREWORD

The following paper has been compiled from a series of lecture notes made by the writer when he took an advanced course on oceanography under Prof. Björn Helland-Hansen, Geo-Physical Institute, Bergen, Norway. Writers of textbooks on oceanography, fail from time to time, due to the rapid growth of this science, to keep pace in print with the newest methods in practice. The need for the appearance of the present treatise is emphasized when it is realized that a complete exposition of the methods elucidated herein has never before, to the writer's knowledge, been collected in a single publication, and the particular hydrographical information, prior to this, has been unavailable short of personal instruction in Europe. Although the illustrations to be found throughout the paper are in most cases examples taken from observations of the International Ice Patrol off Newfoundland, and although the bulletin is intended especially to assist the prosecution of Ice Patrol service, the application of the text is, nevertheless, quite broad in its scope. It is therefore recommended to the attention of all students interested in the subject of physical oceanography.

The foundation upon which this paper rests was first laid down by Prof. V. Bjerknes, (see "Dynamic Meteorology and Hydrography," Carnegie Institution publications, Washington, 1910-11). In the lines of history which record attempts to apply mathematics to the natural sciences this treatise by Bjerknes stands out as one of the most successful and progressive. A perusal of the book can not fail to impress one with the infinite care and exactitude with which the theories have been presented and the exposition developed. It is a model of scientific treatment, but he who is searching for a practical method directly applicable to a hydrographical problem is bound to note the absence of just this sort of pertinent information. Since the time when Bjerknes' theories became recognized by scientists there have been a few oceanographers, especially Helland-Hansen, Nansen, Ekman, and Sandstrom, who have done much to give the formulæ of motion a practical application to the sea. As a result of such development we are now supplied with a scientific method whereby if the temperature and salinity of the ocean are given from several known depths and stations the direction and velocity of the currents even in the deep water off soundings can be computed and mapped. In this connection it may be of interest to know that the currents calculated from the observational data collected in 1922 off the Grand Banks have been found to agree very closely with the drifts of the icebergs of that same year and region.

This paper endeavors to encompass in a general way the foregoing subject with its various aspects. The contents deal with the following: The causes of currents; static consideration of a water mass; dynamics and Bjerknes' theory; and a practical method for mapping currents. Other related subjects discussed are friction; effect of bottom configuration; tides; variations in atmospheric pressures; and the winds. The writer has tried to present a rather technical scientific subject in such a manner that it may easily be understood by the ordinary student. Always there has been the hope that the methods elucidated herein would serve some practical economic service.

I wish to recognize with appreciation the advice and suggestions made with regard to this paper by the curator of the Museum of Comparative Zoology, Harvard University, and the hydrographic engineer, United States Hydrographic Office.

The foreword is not complete unless this place is reserved to express a sincere appreciation and acknowledgment of the untiring, generous assistance and instruction given me in this work by the director of the Geo-Physical Institute, Bergen, Norway. He has in many instances placed even his personal notes at my disposal, and in a hundred other ways has shown an unselfish spirit of cooperation and friendship. As I leave Norway I bid him a fond farewell.

E. H. S.

AUGUST 13, 1925.

A PRACTICAL METHOD FOR DETERMINING OCEAN CURRENTS

EDWARD H. SMITH

THE ORIGIN OF CURRENTS

In order to make a systematic exposition of the circulation taking place in the oceans with especial regard to the origin of currents, we have found it convenient to divide the forces into two general classes: (1) Internal and (2) external.

(1) *Internal forces* appear in an ocean mass whenever any change takes place in the physical character of the water itself; that is, if either the temperature or the salinity varies in the sea then the dynamic equilibrium is upset and a tendency to readjust must follow. The internal system of forces in an ocean are disturbed whenever that mass radiates or absorbs heat; evaporates from the surface; receives additions of fresh water; or suffers internal physical transformation as a result of its turbulent activity. Radiation is simply a gain or loss of heat by the ocean, which tends to vary the temperature of the surface layers. Evaporation tends to vary the salinity of the surface. The ocean receives fresh water from rain, snow, or melting ice. When an ocean mixes internally it alters its physical character within the region of mixing.

(2) *Forces classified as external* and provocative of currents are winds, tides, and variations in atmospheric pressure. The winds we shall divide into two groups, determined primarily by their extent and duration: (a) Those winds which by a tangential pressure on the surface of the sea frictionally propagate a pure wind current only; and (b) those winds which by virtue of friction drive water particles against boundary surfaces in the sea and give rise to gradient currents. Winds classified as (b) are by far the most important of the external forces assisting to maintain the more or less prevailing system of circulation in the oceans.

There are, however, two other forces which are classified as secondary, but only in so far as they tend to deform the components established by (1) and (2). They are, nevertheless, of the utmost importance in the consideration of currents, namely, (a) the quasi force due to terrestrial rotation which acts simultaneously as soon as a movement as described in paragraph (1) or (2) begins; and (b), fric-

tion, that due primarily to land and bottom configuration as it tends to guide and shape the direction as well as to effect the velocity of ocean currents. Friction also is an important factor, arising whenever water particles of dissimilar motions interact among one another. A well-known example of this process is contained in the waters of a mixing zone which lies adjacently inshore of the Gulf Stream and stretches along the American continental slope.

It is difficult, even in such a well-known current as the Gulf Stream, to state which class of forces, internal or external, is the fundamental cause of movement, yet the subsequent forces tending toward alterations of the movements spring from two influences—friction and rotation of the earth. A discussion of some of the foregoing features will assist to a clearer understanding of the entire subject.

STATIC CONSIDERATION OF A WATER MASS

Let us imagine that we can pass a plane vertically downwards through the ocean and can regard a cross section of the water in profile, with a view to studying its static condition, or distribution of mass. If now the water particles could be colored with reference to their relative weights, we would find the lightest water in the surface layers, and the heaviest particles on the bottom. The two fundamental essentials usually determined and which lead to hydrostatic examination are temperature and salinity; once they are found the specific gravity (density) follows as a dependent from convenient hydrographical tables. It is often desirable to speak in terms of specific volume, it being the volume of a body per unit mass, or the reciprocal of the density. If d = density, and v = specific volume, then $v = \frac{1}{d}$. As an example of the contractions which are customarily adopted by practical hydrographers, we may have given, $d = 1.02711$; this is written, for the sake of brevity, 27.11. The corresponding value of v in this case is 0.97361, and this is often shortened to a numeral of only three digits, viz, 361. The greater the specific volume at any point the lighter the water is there.

If now we return to our vertical section in the sea and connect all points wherein the water particles have the same specific volume for differences of every 10 units of the latter, we obtain a number of lines called isosteres running throughout the profile. An isoster is a line all points along which represent like values of specific volume; an isosteric surface merely increases the consideration to the two dimensions of an area. An isosteric surface may be visualized as spread out beneath the surface of the sea—an undulating floor whose depth can be determined with the same reality as the more tangible floor of the ocean is sounded out by the hydrographer.

THREE GENERAL STATIC CONDITIONS

There are three general static conditions revealed by vertical sections of the ocean arranged in accordance with a grouping of relative positions of the isosteric surfaces, and with reference strictly to the vertical. (1) The water may be found to have the same density throughout its column when compression is disregarded—i. e., homogeneous as to temperature and salinity. The specific volume in such cases, due to pressure, will necessarily decrease downward, thus it follows that the isosteric surfaces will be arranged solely in dependence with pressure. Such conditions may prevail at the end of winter when vertical convection has attained a maximum influence, or in the cases of strong winds which mix the surface layers, sometimes to a considerable depth. Such a water mass is homothermal and homohaline, and thus presents a consequent neutral

FIG. 1.—A type of stratified water mass found over the Grand Banks south of Newfoundland. The boundary of discontinuity between the two distinct layers is shown by the closely spaced parallel lines

equilibrium vertically. (2) When one homogeneous mass of water lies over another, then the water is in layers and is said to be stratified; it will be found that there are few isosteric surfaces in each layer compared with the number between two adjacent layers. An example of stratification often occurs in the column lying over the Grand Banks, when a cover of heavy water from the slopes is spread over the bottom; above this, and extending to the surface, is a layer of lighter, coastal water, maintained more or less homogeneous by the turbulent effect of the winds. (3) But the most common distribution in the sea is where the density increases proportionally and more or less regularly with the depth. The water in such cases is characterized by numerous isosteric surfaces lying in greater abundance at those levels where transitions of density occur; and this condition is termed stable. A direct measure of the stability of any water column is to be found in the number of isosteric surfaces in excess of that contained in homogeneous water per unit increase

in depth. The sea above the abyssal water, furthermore (with the exception of comparatively restricted places, such as a turbulent mixing zone during a gale), is in a condition pronouncedly stable. Winter cooling of the surface layers, it is true, sets up temporary, vertical, convectional currents, but this condition is short lived when we consider the entire year's span.

DYNAMIC CONSIDERATION OF A WATER MASS

In support of what has just been remarked, we might continue by regarding a vertical section of a stable water mass devoid of circulation. We will find the densest water rests on the bottom of the basin; the lightest water on the surface; and the isosteric surfaces will be exactly horizontal. If now a water particle from a bottom layer be shifted to the surface it will begin to sink to the isosteric sheet from which it was removed. A surface particle, just as truly, if submerged to the bottom will tend to rise and return to its former level. But if a sample be taken from one position to another position, all within the same layer, then there is no force giving rise to its return. It is obvious from this that water particles resist any tendency toward removal from their own particular isosteric sheet, but may move freely within such, if friction does not hinder the motion.

Every motion may be regarded simply as a displacement of masses, therefore a study of various types of distribution of mass in the sea is bound to reveal a vast deal regarding the currents, and in this respect the extreme importance of isosteric bounds governing the movements of the water particles can not be over emphasized. It will be seen, therefore, in the light of further remarks that once we have determined the general contour of the isosteric surfaces we have gained an insight, not only of the direction in which the water is moving, but also a measure of its relative rate of flow. The well-known principle of Archimedes is of great assistance in clarifying the components of the forces due to varying densities.

Let us again regard in profile a vertical section of any body of sea water wherein a distribution of density prevails from which dynamic variations may easily follow. Such a case may arise, as we have pointed out, as an effect of either one of two classes of forces. (See internal and external forces, page 1.) For example, imagine that the ocean has absorbed and mixed heat unevenly during the summer, causing the water to become lighter in a zone over a shallow coastal shelf than the water farther offshore; or perhaps an abnormal percentage of onshore winds have amassed a quantity of light water from the surface layers against a coast. Here, then, class (1) or class (2) forces have produced similar results which can best be examined by recourse to a vertical section normal to the coastal trend.

In Figure 2 the oblique lines are isosteres which have been formed by the intersection of the vertical plane of the section with the isosteric surfaces running through the water mass. The space between any two isosteric surfaces is called an isosteric sheet. The uppermost isosteric sheet on the left-hand side of Figure 2, in wedge-shaped form, bounds the body of lightest water that has accumulated against the coast. Now the water in the deepest portion of this isosteric sheet "A" is specifically lighter than the water *at the same level* in any of the other isosteric sheets, so according to the Archimedian principle this portion of sheet "A" will tend to be driven bodily upwards. The water in the highest portion of sheet "B" is specifically heavier than the water *at the same level* of the inshore sheet, and thus it will be dragged downwards. It is plain to see that

FIG. 2.—A vertical profile of a water mass showing a distribution of light and heavy water and dynamic tendencies which would prevail in such a state

there are forces tending to turn all the isosteric sheets into a horizontal position, and the greater the obliquity of the isosteric surfaces the greater the forces tending toward the leveling process. The water particles themselves, however, as a result of these stresses, will be forced from the thicker portion of the isosteric sheet to the thinner portion of it, the particles tending to keep, for reasons as pointed out in a previous paragraph, wholly within their own respective layer. When the sheets have attained a mean uniform thickness, then the isosteric surfaces have resumed the horizontal, dynamic equilibrium is established and circulation ceases.

The causes provoking currents were divided, it will be recalled, as being due to two classes of forces—viz, internal and external. The distinction between the two rests mainly on the manner in which energy is transmitted to the sea. This conception should be clearly understood.

(1) *Internal class of forces* refers to those agencies, the effects from which appear forthwith to alter the internal character of the water mass itself. This results in varying the distribution of density.

An example has been given when, by the absorption of heat, the water becomes lighter over a coastal shelf in summer.

(2) *External class of forces* can not possibly produce the slightest physical change in the character of the water particles themselves (when the turbulent effect of the wind is disregarded), but either they directly drive the water particles in a current or they deform a water mass that is qualified by boundary conditions. The latter type, similar to (1), tends to vary the distribution of density in the sea; an example has been given in the case of an onshore wind piling up the lighter surface water against a coast.

Thus we may sum up the distinction between the two classified origins of currents—viz, class (1) forces tend to alter the physical character of the sea water while class (2) forces tend either (a) to move the water particles in a current or (b) to deform eventually a given water mass.

THREE VARIABLES IN THE SEA

It is best to begin by treating the distribution of density in the light of mechanics and physics. We may regard each type as being a field of strain inherent to the mass itself, an effect of stresses, the fields of which in the sea can be treated when expressed in terms of three variables classified as follows: (1) *Gravity*, (2) *pressure*, (3) *specific volume*. Let us examine each one of the three variables separately and their combinations as they lead to dynamic measurement of currents.

First, however, it will be helpful to review some of the fundamentals elementary to a physical science. The three fundamentals in physics are MASS, LENGTH, and TIME, represented by the letters M , L , and T , respectively, and in these terms we may express any form of physical phenomena belonging to the sea. If a length, which is the most tangible of the three, be squared, the result is an area; if cubed, a volume. L =length, L^2 =area, L^3 =volume. If we consider any mass with respect to unit volume we then are determining density, or $q = \frac{M}{L^3} = ML^{-3}$. But inversely, if we contemplate a volume with respect to unit mass, the result is termed specific volume, or $v = \frac{L^3}{M} = L^3 M^{-1}$. If we divide a length by a time then it gives rise to a consideration of motion called velocity, or $c = \frac{L}{T} = LT^{-1}$; continuing to divide a velocity by a time (rate of rate of motion) is called acceleration or $a = \frac{c}{T} = \frac{L}{T^2} = LT^{-2}$. A force is that agent which gives motion to a mass. It is expressed in a measurement which considers the mass relative to its rate of change of motion—i. e., acceleration. $K = Ma$; but substituting $a = LT^{-2}$, we get $k = MLT^{-2}$. If M is

unity then we see that the force is equal to the acceleration. The force per unit mass is called the accelerating force. The most common natural force is that of gravity, and is expressed, of course, like other forces, in relation to a mass—e. g., $k = M g$ —where g is the rate of change of motion (acceleration) of a falling body. Work is consideration of a force and length; $w = k L$, but substituting for k its value MLT^{-2} , we get $w = ML^2T^{-2}$. Work may also be spoken of in other forms as energy or potential—viz, the ability to do work. There is another force which enters hydrodynamics—namely, pressure—and it is defined as a force with respect to an area, or $p = \frac{k}{L^2} = ML^{-1}T^{-2}$. The pressure at any depth in the sea is equal to the weight of a column of water of unit depth h with respect to unit area, or $p = qgh$. But substituting $q = ML^{-3}$, $g = LT^{-2}$, and $h = L$, we get $p = ML^{-1}T^{-2}$.

The distribution in space of the value of the variables in the sea—viz., *gravity*, *pressure*, and *specific volume*—may be represented by a series of equiscalar surfaces. Those of gravity are known as equipotential surfaces; those of pressure are called isobaric surfaces; and those of specific volume, isosteric surfaces. The space between two successive equiscalar surfaces is called an equiscalar sheet. If we construct the equiscalar surfaces for unit differences in numerical value of the quantities in question, then we obtain unit scalar sheets. For example, the differences between equiscalar surfaces of potential corresponds to equiscalar units of work.

GRAVITY

Let us contemplate this force apart and alone with respect especially to the envelope of water which surrounds the earth. We may imagine that all the equipotential surfaces throughout an ocean's mass are level, then the surface of such a sea must also be exactly level, and a line to the center of the earth, with an attractive force to that point, called gravity, will plumb exactly perpendicular. Everywhere in such a sea gravity will exert a pull at right angles to the equiscalar surfaces, and the sea surface itself will be an example of a level equipotential plane. Such a motionless state is represented by Figure 3, (a), page 8. For the purposes of measuring and coordinating the accelerating force exerted by gravity in the hydrosphere, we shall endeavor to construct a series of concentric equipotential spheroid surfaces, each one separated by equipotential unit sheets. The thickness of such sheets will vary with the latitude, and in our particular subject (the sea) with the depth. The fundamental basis for fixing the relative position of equipotential surfaces in the sea, rests, of course, upon the presence of an attractive force which exists between the earth and the water masses on it.

A free-falling body, regardless of time or its velocity of descent, will be continuously accelerated at the constant rate of about 10 meters per second. For the purpose of measuring forces in the sea we wish to construct a series of coordinate equipotential surfaces, not merely a linear distance apart, but separated by a difference equal to 1 unit of work. Since gravity accelerates a free falling mass about 10 meters, it performs a unit amount of work, not in 10 meters, or even 1 meter, but in one-tenth of a meter, and this unit is recognized as the unit distance fixing equipotential gravity surfaces, always measured along the plumb. A unit of work, therefore, is definitely fixed and unalterable, it being, in the meter-ton-second system of units, the amount of work equivalent to raising 1 ton vertically $\frac{1}{g}$, or about one-tenth of a meter.

The unit work-length—viz, one-tenth of a meter (decimeter)—has been called by V. Bjerknes, who first used it, the dynamic decimeter; the other multiples being named dynamic meter, dynamic centimeter, etc. It is obvious that this new measure has all the equivalents of linear measure but is restricted in its use solely to the vertical. The dynamic depth of any point is not the common linear distance of this point below the surface of the sea, but it is a direct statement regarding the amount of potential or work inherent to that point relative to the sea surface.

FIG. 3.—The two states of "rest" and "motion" considered with regard to the position of the sea surface. (a), "rest," all equiscalar surfaces, including the sea surface are level, and the entire force of gravity is directed as a component perpendicularly downward; (b), "motion," the equiscalar surfaces, including the sea surface, are tilted, which gives rise in such surfaces to a component of the force of gravity and causes a movement of the water particles

We have considered a motionless sea, and its equipotential surface. Suppose, on the other hand, we regard a sea surface not level; let us say, raised near the coast by a wind pressing the water masses up the inclined, continental slope. Now the sea surface being no longer level is, by definition, no longer of equal value potentially, and gravity exerts a component in the plane of the sea. Here we have the birth of a current. The size of the component force is directly proportional to the obliquity of the surface, the two conditions, "rest" and "motion," being graphically illustrated in Figure 3.

If D in Figure 3 (*b*) is the distance in dynamic decimeters between two points in the sea, and h is the unit vertical distance in common meters, then $D = g h$, where g is the acceleration of gravity. In (*b*), if we know the difference in dynamic depth units (the number of dynamic decimeters) between any two points, A and B in the sea, this number will be the same as the gravity potential released by a unit water mass flowing from A to B . Expressed geometrically we have from the figure, two points A and B between two level surfaces M and N , the two latter of which are h decimeters apart. The angle between line L and the planes M and N is called α

$$w = L \sin \alpha \quad \bar{g} = h \quad \bar{g} = D.$$

where D = difference between M and N in dynamic decimeters, and α is so small in all cases that $\sin \alpha$ may be put equal to α .

Let us, before passing on to a discussion of pressure, glance at the more exact values of acceleration due to gravity at various points on the earth, and also determine the corresponding values of potential expressed in dynamic measure. The attractive force of the earth, g , increases both with the latitude and with the depth in the sea, therefore the distances between equipotential unit surfaces—i. e., the dynamic decimeters—will be longer at the equator and near the surface of the sea, where g is comparatively small, than at the pole and near the bottom where g is comparatively large. In the meter-ton-second system of units a free falling body will accelerate approximately 9.8 meters in one second, therefore the dynamic decimeter, or unit of gravity potential, will be equal numerically to the reciprocal of this value, or 1.02 common decimeters. Stated inversely one common decimeter equals 0.98 dynamic decimeters. Simply multiplying units by 10 give results in terms of ordinary meters and dynamic meters, both of which are of a magnitude most convenient for practical investigations in hydrodynamics.

PRESSURE

Pressure is defined as a force, the intensity of which may be represented at any depth by the weight of a column of water of unit area extended vertically upwards to the surface. The force of pressure, though present at every point in the ocean, does not actually manifest itself as an active agent until we extend our consideration to two points and the difference of pressure arising. This statement, of course, holds true more or less for all forces, but it seems worth remarking here, as sea pressure, to most people, is an effect difficult to comprehend; yet a difference in pressure, such as exists when a hollow sphere is submerged in the sea, immediately becomes tangible.

Let us take, for example, the motionless ocean in which we constructed a system of equipotential surfaces 1 dynamic decimeter

apart (about one-tenth of a meter) and calculate the pressure per unit area on such a plane at a depth of about 1 decimeter. The pressure of the atmosphere being subject to comparatively slight and compensating variations can be totally disregarded throughout hydrodynamic works. (See p. 45.) We have given by definition values of pressure = weight per unit area =

$$\frac{\text{area, height, density, acceleration of gravity.}}{\text{area}}$$

Since the area values cancel, we have

$$\text{pressure} = h \ g \ \bar{q}.$$

But it has been determined that $g \ h = D$, where D equals 1 dynamic decimeter.

Substituting:

$$p = \bar{q} \ D$$

Now it remains to find a suitable system of units of pressure based upon the value equal to a water column 1 dynamic decimeter high and possessing a mean density q .

The most common example of natural pressure with which we are familiar is that of the atmosphere. It has been a practice, long established, to balance the perpendicular column of the atmospheric envelope against an equal cross-sectional area of mercury. This is a well-known experiment of any physics laboratory in which mercury has come to be adopted because of its great density; other liquids being forced to too great a height by the balance. We employ exactly the same equation, of course, as evolved in the case of a motionless ocean; in fact, we might imagine finding the pressure at various depths in the sea, theoretically, by means of a balanced column of mercury.

It has been found that at 0° C. and 45° latitude at sea level, the normal height to which mercury is forced by the ever pressing air envelope, is 0.76 meters, sometimes termed an "atmosphere." Since the acceleration of gravity at 45° latitude is known, viz, 9.8 meters, and the density of mercury at 0° C. is 13.59, let us calculate the pressure p in meter-ton-second units—i. e., the system upon which previous dynamic figures have been based. Substituting in

$$p = q \ g \ h, \text{ we have } p = 13.59 \times 9.8 \times 0.76 = 101.218.$$

V. Bjerknes has used this quantity of 101.218 as a guide in deciding upon the value ascribable to p . He has selected as a unit suitable for hydrodynamic computations, the nearest integral number of 10 to 101.218, viz, 100, and has called this a bar. A bar is approximately the pressure exerted by a column of water 10 meters in height; therefore the pressure of 1 meter of water is very nearly equal to the

pressure of 1 decibar. We should note the coincidence that 1 meter below the surface the gravity potential is very nearly 1 dynamic meter less, and the pressure 1 decibar more.

$$p = \bar{q} D \dots \text{in terms of decibar units} \dots \dots \dots (a)$$

$$D = \bar{v} p \dots \text{in terms of dynamic meter units} \dots \dots \dots (b)$$

In order to show the close coincidence existing between dynamic units and pressure units of this system for increasing depth, we may regard the various values for the three arguments, viz, common meters, dynamic meters, and decibars, as they exist in a sea of 0° C. temperature, and 35 per mille salinity.

Decibars	100	200	300	400	500	600	700	800	900	1,000	1,200	1,400	1,600	1,800	2,000
Meters	99	198	298	397	496	595	693	792	891	990	1,187	1,385	1,582	1,779	1,975
Dynamic meters	97	194	292	389	486	583	680	777	874	970	1,164	1,357	1,551	1,744	1,936

It will be seen from the foregoing that under conditions as specified there is a difference of about 1 per cent between a depth expressed in pressure decibars and that expressed in common meters. This difference becomes even smaller under natural conditions prevailing on the earth, and thus being so insignificant, when contemplating the horizontal extension of ordinary sea areas, permits us, with the same number, to express a depth either in common meters or in decibars. The difference between dynamic meters and common meters averages about 2 per cent, and between dynamic meters and decibars about 3 per cent, and these are of a magnitude that can not be disregarded.

The two foregoing equations (a) and (b), in the case of equilibrium, expresses as simply as possible the relation existing between gravity potential, pressure, and specific volume. Thus it follows that we may by (a) find the pressure in decibars at a given dynamic depth, or by (b) the dynamic depth of a certain given pressure.

We have already described the equipotential gravity surfaces and the potential sheets with a thickness of 1 dynamic meter. Now the surfaces of equal pressure are given, called isobaric surfaces, which are separated by isobaric sheets 1 decibar thick. It is seldom that we have under natural conditions a motionless water mass, and so then it will usually be found that isobaric and level surfaces intersect. In other words, an isobaric surface contains varying potentials of gravity, and a level surface, in like manner, contains many baric variations. The intersections of these two surfaces may be considered as lines of the one inscribed on the plane of the other, accordingly as we employ equation (a) or (b). If the lines of intersection are considered inscribed on the level surfaces, they are isobars, and the chart is similar to the ordinary meteorological charts

which show the distribution of pressure. But if we employ equation (b) we must represent the result as dynamic isobaths inscribed on an isobaric surface and drawn—e. g., for unit differences of 5 dynamic millimeters. Such a method of representation corresponds to that of a common topographical chart, but the contour lines on a dynamic chart instead of showing ordinary, linear heights, show levels of equal potential. A dynamic topographical chart of a certain isobaric surface is the most approved method employed in modern dynamic oceanography to map ocean currents.

APPLICATION OF DYNAMIC UNITS

The number of unit equipotential sheets found in an isobaric sheet between two different station verticals represents a certain amount of potential energy existing between the two verticals.

FIG. 4.—A vertical section through a sea basin and including the two stations A and B, with the respective points C and D separated by the distance L . C and D are at a depth of p decibars below the surface

Figure 4 shows a section through a sea basin which includes two stations, A and B. The horizontal lines represent the intersections with some equipotential surfaces, and the oblique lines the intersections with some isobaric surfaces. The dynamic distance from the sea surface to the isobaric surface of p decibars is d_a at station A, and d_b at station B. According to equation (b) we have:

$$d_b = p_b \bar{v}_b$$

$$d_a = p_a \bar{v}_a$$

$$\text{But } p_a = p_b$$

and therefore

$$d_a - d_b = p (\bar{v}_a - \bar{v}_b) \dots \dots \text{in terms of dynamic meters} \dots \dots (c)$$

$d_a - d_b$ represents the difference of potential energy, due to gravity, between the points D and C in Figure 4. This energy may be converted into work, $d_a - d_b = k L$, where k is a force and L is the distance between the two points. Hence the force per unit mass due to gravity may be expressed

$$k = \frac{d_a - d_b}{L} = \frac{p(\bar{v}_a - \bar{v}_b)}{L}$$

DEPTH AT WHICH GREATEST OBLIQUITY OF ISOBARIC SURFACES OCCUR

It is important to distinguish where the greatest obliquity of the isobaric surfaces prevail in an ocean mass. Dynamic measurements and pressures have been considered as being laid off from the surface of the sea downwards on the assumption that the sea surface is always level—an equipotential surface. This premise demands considerable revision, as we shall see, in the light of the following facts:

As a result of compiled oceanographic observations, it is well known to-day that the greatest variations in temperature and salinity of the water take place in the upper levels of the sea. In the North Atlantic, for example, below depths of 3,000 meters there is little variation, as we proceed from place to place, in the temperature or the salinity. Now, if we regard two stations with widely differing specific volumes, we shall generally find that their difference decreases more or less rapidly with an increase in depth, and gradually approaches a constant or zero. Where the water is light we shall observe a relatively low pressure in decibars at a certain dynamic depth, or conversely at a given observed pressure in decibars, the dynamic depth will be least where the water is heaviest. In view of this natural state of the ocean, if the sea surface be level, then the obliquity of the isobaric surfaces must increase downwards and the maximum of forces and currents would be relegated to the greater depths, a condition which we know is contrary to fact. It follows alternatively that at an *appreciable* depth below the surface there will generally be a sheet where motion most nearly approaches zero and where isobaric, isosteric, and equipotential surfaces are parallel. It follows, furthermore, that above such a motionless plane, the water, over any given horizontal extent, lies at the greatest height (the surface of the sea highest) at that place where the water is the lightest—i. e., the specific volume the greatest.

We should endeavor to select from a group of observations indicative of a surveyed area an isobaric surface which in itself has the most nearly equal dynamic depths, thereby sounding out a level or motionless plane and which as stated before generally will be found to lie at a relatively great depth beneath the surface of the sea. When employed as a "bench mark" this surface provides a means of measuring the currents which usually are present in the upper levels. The velocities are determined by a comparison of any two dynamic heights measured upwards from the level, isobaric plane to the surface of the sea. Figure 5, page 14, shows in exaggerated form the obliquity of the sea surface, and also the other isobaric surfaces of observation as they lay May 5-7, 1922, south of the Grand Banks, between stations 206 and 201. The state of relative obliquity is based upon the assumption that the maximum depth of observation, the 750 decibar

surface, was a level plane. Other observations in this locality indicate that the 750 isobaric plane, however, is not always level, but a motionless state probably lies at some greater depth. The depth of 750 decibars, nevertheless, approaches most nearly to the level where absence of motion may prevail of any depth of which the International Ice Patrol records; therefore, it has been employed in this paper as an illustration of the most accurate base upon which to calculate surface currents in the vicinity of the Grand Banks.

FIG. 5.—The decrease in obliquity of observed isobaric surfaces with the observed increase in depth and based upon the assumption that the depth of 750 decibars was a level plane in which no motion prevailed. The figure includes a line of stations, 206 to 201, taken by the International Ice Patrol south of Newfoundland May 5-7, 1922

The position of a level surface depends solely on the acceleration of gravity. Also, it has been pointed out that the depth to an isobaric surface depends not only upon gravity, but upon the specific volume of the overlying masses. Since we have already discussed gravity, let us now turn to the remaining term, specific volume.

SPECIFIC VOLUME

Pressure per unit area depends upon two variables, gravity and specific volume, but gravity being a more or less constant force, the agency which exerts the greatest influence to vary the pressure throughout the sea is specific volume. Specific volume has been defined as the volume of unit mass of any body. It is simply the

reciprocal of the specific gravity and is chosen in preference to the density because its value leads to the simplest method of dynamic calculations. It is, in such cases, combined directly with other parts of the term pressure, and furnishes a result in terms of gravity potential. (See equation (b), p. 11.)

In the depths of the sea observations are not made directly of specific volume, but it is obtained only after first finding the temperature, salinity, or density at a given temperature, and then correcting for the particular depth below the surface at which the observation was made. The temperature is, of course, a direct instrumental observation. The salinity is calculated ordinarily by determining the chlorine content of the sample and substituting in Knudsen's formula:

$$s = 0.30 + 1.805 \, Cl$$

The two foregoing characters of sea water have been tabulated by Knudsen with regard to corresponding values of density, within the range of that normally met in the oceans.

It is vitally necessary in the course of dynamic computations, moreover, to know the specific volume in situ—that is, the actual specific volume as it existed at the particular depth at which it was found. Thus, after the specific volume has been determined from the temperature and salinity, it must be corrected for a third variable, viz, compression. It is easy to appreciate that a mass, even such as water, becomes more and more compressed the deeper down we penetrate beneath the surface. Naturally, the more compressed a body becomes, the denser it grows—i. e., its specific volume becomes increasingly less. The compressibility of sea water is not entirely dependent upon the depth below the surface, but it is also influenced by the temperature (and to a much slighter degree by the salinity) prevailing in the water itself. Generally speaking, the warmer and saltier is a water mass, the less it can be compressed. Investigations have been made regarding the compressibility of sea water at various depths under different combinations of temperature and salinity by Ekman. (cf. *Die Zusammendrückbarkeit des Meerwassers*, etc. Pub. de Constance, Copenhagen, 1908.

In order to construct tables for specific volume in situ, it is necessary to combine the two previous tables—namely, those of Knudsen for temperature and salinity with those of Ekman for compressibility. It is impossible, however, to arrange one convenient and accurate table for specific volume in situ, because of the multitudinous combinations arising between the three variables, viz, temperature, salinity, and compression, as they commonly range in the sea. Direct tabulation, according to V. Bjerknes, would require something like 256,000 pages of 500 numbers each, if intervals of 0.1 degree temperature, 0.01 per mille salinity, and 10 decibars pressure were em-

ployed as arguments. Helland-Hansen and Sandstrom in "Report on the Norwegian Fishery Investigations, Volume II, No. 4, Bergen, 1903," first provided a way to avoid such a ponderous, unwieldy work by calculating, as an initial step, the values of specific volume at frequent depths, and covering the normal range of change in compressibility in an ocean of 0° C., and a salinity of 35 per mille. A correction called the anomaly of specific volume is then added to this first figure, representing the specific volume of any characterized water, but under a similar pressure. According to these arrangements all corrections are embodied in a total of four small handy tables. The details of this ingenious method of tabulation are also described in Bjerknes', "Dynamic Meteorology and Hydrography," Carnegie Institution Publications, 1910-11. Later table groupings have been made and published by Hesselberg and Sverdrup, "Beitrag zur Berechnung der Druckund Massenverteilung im Meere," Bergens Museums Aarbok, 1914-15.

GIVEN TEMPERATURE AND SALINITY—A GRAPHIC METHOD TO FIND DENSITY

The specific volume in situ, as determined by the foregoing tables, is based upon an initial given density, usually found by means of Knudsen's Hydrographical Tables with addendum. There is considerable labor attached to interpolating when there are perhaps several hundred observational records of temperature and salinity which require conversion into density form. The Geo-Physical Institute, Bergen, where the writer spent some time, finds it convenient to facilitate such work by the construction of a graph based upon the three arguments of temperature, salinity, and density, within the range which prevails for the first two in the temperate zones. The method possesses such great advantages over the use of the tables that it is set forth here for the benefit of future investigators who may have to deal with a large number of field observations.

The construction of the graph is based upon the three formulæ of Knudsen:

$$(1) s = 0.30 + 1.805 \, Cl.$$

$$(2) \delta_0 = -0.069 + 1.4708 \, Cl - 0.001570 \, Cl^2 + 0.0000398 \, Cl^3.$$

$$(3) \delta_t = \Sigma_t + (\delta_0 + 0.1324) [1 - A_t + B_t (\delta_0 - 0.1324)].$$

(For d_0 , d_t , Σ_t , A_t , and B_t see Martin Knudsen's, "Hydrographical Tables," Copenhagen, 1901.) Density values are plotted as abscissæ, salinity values as ordinates, and isotherm curves, determined in accordance with the fixed relation existing between the three variables, run diagonally across the graph. In order to determine the latter with a sufficient degree of accuracy, it is necessary to fix definitely a

certain minimum number of points, by substituting values in the three given equations. As a first step let us substitute a value of Cl in the first equation, which will result in the lowest value in the range of salinities which it is desired to span. A few trials indicate that a value of Cl equal to 17.5 gives a desirable value of s equal to 31.618 per mille, which in turn is substituted in equation (2) furnishing the numeral 25.4025 as the value of δ_o . This again is substituted in equation (3) along with the values for A_t , B_t , and Σ_t for every two degrees change in the range of temperature known to prevail in the particular region which is under investigation. Thus we obtain a series of values for a salinity of 31.618 per mille, and in the same manner, another series of temperature points on the graph using other values of salinity, as shown in Table I:

TABLE I

	Cl	s_1	δ_o	δ_o-1324	δ_o+1324
a-----	17.5000	31.618	25.4025	25.2701	25.5349
b-----	18.000	32.520	26.1267	25.9943	26.2591
c-----	18.700	33.783	27.1462	27.0138	27.2786
d-----	19.287	34.843	28.0000	-----	-----
e-----	19.928	36.000	28.9310	28.7990	29.0638
f-----	20.482	37.000	29.7390	26.6069	28.8717

The values as tabulated in Table I, upon further substitution, result in the following values as given in Table II:

TABLE II

Temperature (degrees) 2	Densities 3					
	a	b	c	d	e	f
-2	25.458	26.190	27.216	28.076	-----	-----
0	25.402	26.127	27.146	28.000	-----	-----
2	25.290	26.008	27.020	27.868	-----	-----
4	25.124	25.836	26.842	27.863	-----	-----
6	24.907	25.616	26.614	27.450	-----	-----
8	24.644	25.348	26.340	27.171	-----	-----
10	24.337	25.037	26.023	26.849	-----	-----
12	23.987	24.685	25.665	26.486	-----	-----
14	23.600	24.292	25.267	26.084	-----	-----
16	23.173	23.862	24.833	25.646	-----	-----
18	22.709	23.395	24.362	25.171	-----	-----
20	-----	-----	23.856	24.662	-----	-----
21	-----	-----	-----	-----	25.272	26.034
22	-----	-----	-----	24.119	24.995	25.755
23	-----	-----	-----	-----	24.710	25.469
24	-----	-----	-----	23.544	24.417	25.175
25	-----	-----	-----	-----	24.116	24.872

Having determined arguments 1, 2, and 3, we are now provided with data sufficient to fix the construction of a graph which in turn furnishes a rapid means of obtaining density values from given temperatures and salinities.

TABLES FOR CONVERTING DENSITIES INTO SPECIFIC VOLUMES
IN SITU

In order to obtain values of specific volume in situ, Hesselberg and Sverdrup have arrived at the following formula:

$$V_{s, t, p} = 1 - \frac{\delta_t \cdot 10^{-3}}{1 + \delta_t \cdot 10^{-3}} + \delta_p + \delta_{s, p} + \delta_{t, p}.$$

where δ_t = density; δ_p = density correction for pressures; $\delta_{s, p}$ = correction for salinity under various pressures; $\delta_{t, p}$ = correction for temperature under various pressures; $V_{s, t, p}$ = specific volume in situ. These four arguments have been arranged in an equal number of tables by Hesselberg and Sverdrup, "Beitrag zur Berechnung der Druckund Masserverteilung im Meere." Bergens Museums Aarbok,

1914-15, Nr. 4. The first table representing the value $10^5 \cdot \frac{\delta_t \cdot 10^{-3}}{1 + \delta_t \cdot 10^{-3}}$,

is reprinted herewith, and the three tables for the last three terms have been combined in one table (Table IV), which makes reference much easier and without sacrifice of the requirements of practicality. When employing Tables III and IV in the method of computations as followed on page 32 it is well to note that the values as carried in Table III are from the nature of the equation (shown at the head of this page) of a negative character. In Table IV the corrections for the three factors, viz, pressure, salinity, and temperature, as they affect the specific volume in situ, are combined in accordance with signs as indicated at the head of Table IV. An example of the application of Tables III and IV is given in the computations for two oceanographic stations, 205 and 206, page 28.

TABLE III

$$10^5 \cdot \frac{\delta_t \cdot 10^{-3}}{1 + \delta_t \cdot 10^{-3}}$$

δ_t	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
24	2344	2353	2363	2372	2387	2391	2401	2410	2420	2430
25	2439	2449	2458	2468	2477	2487	2496	2506	2515	2525
26	2534	2544	2553	2563	2572	2582	2591	2601	2610	2620
27	2629	2638	2648	2657	2667	2676	2686	2695	2705	2714
28	2724	2733	2743	2752	2762	2771	2780	2790	2800	2809

TABLE IV

-		-		+																				
D	Σ_D	-2°	-1°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°			
0	0	0	0																		0			
10	5																				0			
15	7																				0			
20	9																				1			
25	11																				1			
30	14																				1			
35	16																				1			
40	19																				1			
45	21	0																			1			
50	23	1						0	1	1	1	1	1	1	1	1	1	1	1	1	1			
55	25	1						0	1	1	1	1	1	1	1	1	1	1	1	1	1			
60	27	1						0	1	1	1	1	1	1	1	1	1	1	1	1	1			
65	29	1						0	1	1	1	1	1	1	1	1	1	1	1	1	1			
70	31	1				0	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2			
75	34	1				0	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2			
80	36	1				0	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2			
85	38	1				0	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2			
90	40	1				0	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3			
100	45	1				1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3			
125	56		0			1	1	1	2	2	2	2	2	2	3	3	3	3	3	4	4			
150	68	1	0			0	1	1	2	2	3	3	3	3	3	4	4	4	5	5	5			
200	90	1	1			1	1	2	3	3	4	4	4	5	5	5	5	6	6	6	6			
250	112	1	1			1	1	2	3	3	4	4	5	5	6	6	6	7	7	8	8			
300	135	2	1			1	2	2	3	4	4	5	5	6	7	7	8	8	9	9	9			
350	157	2	1			1	2	2	3	4	4	5	6	7	8	8	9	10	10	11	11			
400	180	2	1			1	2	2	3	4	5	6	6	7	8	9	10	11	12	12	12			
450	202	2	1			1	3	3	4	5	6	7	8	9	10	11	11	12	13	13	13			
500	225	3	1			1	3	4	5	6	7	8	9	10	11	12	13	14	15	15	15			
600	269	3	2			2	3	5	6	7	8	10	11	12	13	14	15	16	17	20	20			
700	313	4	2			2	4	5	7	8	10	11	13	14	15	16	17	18	19	20	21			
750	336	4	2			2	4	5	7	9	10	12	13	15	16	17	18	19	20	21	22			
800	358	4	2			2	4	6	8	10	11	13	14	16	17	18	19	20	21	23	23			
900	402	5	2			2	5	7	9	11	13	14	16	18	19	20	22	23	24	26	26			
1,000	446	6	3			3	5	7	10	12	14	16	18	20	21	22	23	25	27	29	29			
1,100	491	6	3			3	6	8	11	13	16	17	19	21	23	24	25	27	29	31	31			
1,200	533	7	3			3	6	9	12	14	17	19	21	23	25	26	28	30	32	34	34			
1,400	620	8	4			4	7	10	13	16	19	22	25	27	29	31	33	35	37	40	40			
1,600	706	9	4			4	8	12	15	19	22	25	28	31	33	35	37	39	42	45	45			
1,800	791	10	5			5	9	13	17	21	24	28	31	34	37									
2,000	876	11	5			5	10	14	19	23	27	31	34	38	41									
																					32°	33°	34°	36°
																					-1	-1	0	0
																					-1	-1	0	0
																					-1	-1	0	0
																					-1	-1	0	0
																					-1	-1	0	0
																					-2	-1	-1	+1
																					-2	-1	-1	+1
																					-2	-1	-1	+1
																					-3	-2	-1	+1
																					-3	-2	-1	+1
																					-3	-2	-1	+1
																					-4	-3	-1	+1
																					-4	-3	-1	+1
																					-5	-3	-1	+1
																					-5	-3	-1	+1
																					-5	-4	-2	+2
																					-6	-4	-2	+2
																					-7	-5	-2	+2
																					-8	-5	-3	+3
																					-9	-6	-3	+3

DISTRIBUTION OF MASS

Information as to the distribution of mass in a free-moving media, such as in an ocean, furnishes a direct insight of the dynamic conditions there. Representation of mass distribution is clearly shown by isosteric lines, which in profile form, after all corrections have been made, including that of compressibility, is called a dynamic section. An example of such is to be seen in Figure 12, page 30, which has been constructed from a group of stations taken by the International Ice Patrol, 1922, and extended in a line across two currents in the ice regions south of Newfoundland.

The importance of the position of isosteric surfaces as an indicator of the motions taking place in a water mass, was pointed out in a previous paragraph. Enlargement of this exposition can be continued hereby regarding such a vertical section where a system of isobaric and isosteric surfaces, by intersection with the vertical plane, divide the latter into a set of parallelograms. If we extend the vision to three dimensions, then the parallelograms take form as a set of tubes.

Since they lie between adjacent isobaric surfaces their continuation must cease only by turning on themselves or by meeting the sides of the basin. V. Bjerknes has given the name "solenoid" to an isobaric-isosteric tube. It is convenient to select as a unit tube one included by isosteric surfaces constructed for intervals of 10^{-5} of specific volume, and isobaric surfaces constructed for intervals of one centibar. Bjerknes has also called attention to the significance of solenoids by stating that the measure of the intensity of forces in a given vertical sectional area is in direct proportion to the number of solenoids running through it. This number depends upon the degree of stability and inclination; the greater the stability and the inclination, the greater the number of solenoids per unit cross-sectional area.

EFFECT OF EARTH ROTATION ON OCEAN CURRENTS

Dynamic tendencies of water particles have been discussed purely as indicated by mass distribution; now the behavior of such phenomena are traced in the form of actual motion on, and as qualified by, the veering surface of a rotating sphere.

In order to understand the effect of earth rotation on currents, we might begin by studying very closely the absolute movement of a fixed body at the pole of a rotating sphere and another similar body on the equator. It will soon be perceived that the former enjoys a pure centric movement while the latter has a pure translatory motion, and any intervening point partakes a centric-translatory path. Bodies at rest relatively to the globe, as also the surface of the earth itself, are, strictly speaking, under a phase of centric and translatory motion, the relation between the two depending upon the geographical latitude. This phenomenon is very difficult to comprehend, since all of our senses are trained to accept the earth and resting bodies as a stationary base, and these remarks in so short a space, can hope to touch generalities only. Those who are interested in a detailed exposition of the subject are referred to Krummel (cf., *Handbuch der Ozeanographie*, vol. 2). Also Humphreys (cf., "Physics of the Air." 1920).

As long as all bodies remained in fixed relations, a state of "rest" may be said to prevail, by virtue of the fact that no variations from the relative positions exist. But distinction immediately arises whenever any free motion whatsoever, relatively to the earth, is introduced. At any other point on the earth's surface than along the equator, due to the element of centricity previously described, divergence takes place between the straight path of a particle due solely to inertia, and the movement of other particles held fast to the surface of the earth and carried around with it as it rotates. This fact was proven years ago when the straight line of motion possessed by Foucault's pendulum swinging to and fro soon revealed

that the surface of the earth was veering to the left in the Northern Hemisphere. It is more natural to regard the inverse perspective—that is, the earth and resting bodies as stationary—then the paths of inertia are apparently being continuously deflected to the right. Earth rotation exerts no effect on a water mass free from circulation relatively to the earth, but on the other hand no true conception of free-moving currents can be had unless this great influence is considered. In this connection it should be realized, from the foregoing remarks on motion on a rotating sphere, that currents can not be traced solely to a provocative force at their source, but they are only to be observed as a resultant of a force, the effect of which is constantly being deformed by the earth “sliding” beneath it. If a water particle moves solely due to inertia, without being acted upon by any force, it will follow a course “cum sole” (clockwise with the sun). As the latitude increases the tendency which drives a water particle to the right of its course becomes more and more intensified, and the faster it moves, the greater becomes the quasi force tending to deflect it.

In order to study this quasi force in detail, it is convenient, similar to the procedure employed in the investigation of varying mass and pressure (see fig. 4, p. 12) to regard the circulation of the curve in a plane between any two verticals. We may take, for example, stations A and B (fig. 6, p. 22), with their verticals AC and BD forming the plane ABDC. The development of an equation for expressing the rotation effect demands too great a digression into mathematics and is not warranted here, but it has been evolved by V. Bjerknes as equal to

$$2\omega \frac{ds}{dt}$$

where ω represents the angular velocity of the earth, viz, 0.0000729; and is the projection of the closed curve of the circulation, as illustrated here by the rectangle ABDC, on the equatorial plane; and $\frac{ds}{dt}$ represents the rate of change of the projection on the plane of the equator.

In Figure 6, page 22, if the curve of circulation ABDC, which is being investigated, is projected upon the equatorial plane, it is evident that a change of the proportional area is effected only by components normal to the plane and not by those tangential to it. Also the vertical movements can be considered negligible, since they are insignificant as compared with horizontal magnitudes. Helland-Hansen and Sandstrom have, by this means, found the value for Bjerknes' equation in terms of the projection on the plane of the sea surface

$$\frac{ds}{dt} = \frac{d\sigma}{dt} \sin \phi$$

where σ is the projection on the sea surface, and ϕ is the geographical latitude. Substituting this new value for $\frac{ds}{dt}$ we have

$$2\omega \frac{d\sigma}{dt} \sin \phi \text{ ----- (d)}$$

$$\text{but } \frac{d\sigma}{dt} = A'B'D''C'' = (c_0 - c_1)L \text{ where}$$

c_0 = velocity per second in a given horizontal plane.

c_1 = velocity per second in another horizontal plane.

L = distance between stations.

Substituting in (d) for the new value $\frac{d\sigma}{dt}$ we have

$$2\omega \sin \phi (c_0 - c_1)L \text{ ----- (e)}$$

FIG. 6.—Lines AA' and BB' represent the velocity of the surface current, or c_0 , per unit T; CC' and DD' indicate the velocity of the current at a greater depth, or c_1 . The difference in the velocity of the two movements is equal to D''B', or $c_0 - c_1$. The movement is assumed to be normal to the vertical plane ABDC, which is passed through the two stations A and B. Area C''D''B'A', indicated by the symbol σ , represents the difference in the change of areas per change of T, projected on the sea surface and developed by the progression of the two lines AB and CD with the respective velocities c_0 and c_1 .

Thus by (e) we are furnished with an expression for the effect of terrestrial rotation in terms of the latitude; the distance between stations; and the difference in velocity of the current between any two levels. It is easy to see that if we are able to find some point along the verticals where zero velocity prevails, then we have a means of expressing the real velocity. It is customary to extend the investigations to depths where it is believed motionless water lies, and then $c_1 = 0$, and c_0 is the true velocity on the surface. (See p. 13 regarding the obliquity of isobaric surfaces.)

RESOLUTION OF FORCES IN GRADIENT CURRENTS

It has been pointed out in the previous section that the effect of rotation tended to deflect currents to the right in the Northern Hemisphere. This quasi force can be represented by a vector of a certain magnitude which lies 90° to the right of the current. If we let the line AB, Figure 7, represent a more or less steady current of sufficient size to give the water particles a translatory path, then the effect of terrestrial rotation may be shown by the line AC, Figure 7. Since the rotation effect is always present, as represented by the line AC, it follows that the only condition under which a current can flow, stream, and be preserved, is fulfilled by a force or system of forces (when friction is disregarded) which acts equal and opposite to AC, and is represented in Figure 7 as the line AE. AE illustrates the force characterized as

due to varying mass and pressure, and is measured by the equated values of the three variables—*gravity, pressure, and specific volume*. It is, moreover, the impelling force of such gradient currents (i. e., currents resulting from an obliquity of equiscalar surfaces); and it should be remarked here that this driving force is to be sought not back along the current's course to a river-like source, but it always lies on the right hand stretching along the entire extent of flow. The Gulf Stream, for example, as it follows a general path

FIG. 7.—A diagrammatic front view showing the relative positions of the major elements belonging to a steady gradient current. AB represents the path of flow of the water particles; AE, the position of the forces due to Archimedean tendencies which impel the current; and AC the position of the quasiforce of earth rotation in a plane 90° to the right (in the northern hemisphere) of the direction of the current

around the periphery of the North Atlantic basin, is energized along the shores of Europe (a fact which is just as vital for its propagation) as well as receiving propulsion in the Caribbean. Where the velocity is relatively great, there the dynamic gradient is correspondingly steep, and without such energy distributed around Atlantic slopes, the Gulf Stream would directly disintegrate. If we divide gradient currents into the forces which combine to give flow to the water particles we have (1) dynamic inequalities due to varying densities, and (2) the effect of earth rotation, each one of which acts in a plane perpendicular to the path of the moving water particles. Since (1) and (2) lie in the same plane, and inasmuch as the acceleration of the closed curve ABDC (see fig. 6, p. 22) (represented by the line AC, fig. 7) has been determined, let us now regard the rectangle ABDC with respect to acceleration tending in the opposite direction. (Shown as line AE, fig. 7.)

It will be recalled that the force of varying mass and pressure tending to produce acceleration by equation (b) is equal to

$$D = p \bar{v}$$

and the accelerating force in a closed curve ABDC between stations A and B, in the plane formed by the verticals AC and BD, is equal to

$$d_a - d_b = p (\bar{v}_a - \bar{v}_b).$$

Since AC equals AE, (fig. 7, p. 23), we may substitute (e) and obtain the following:

$$d_a - d_b = p (\bar{v}_a - \bar{v}_b) = 2 \omega \sin \phi (c_0 - c_1) L \dots \dots \dots (f)$$

Thus finally we are furnished with an expression which includes the forces due to the distribution of mass and pressure tending to accelerate a current moving on a rotating earth, and moreover, it is formed of terms which readily lend themselves to the requirements of practical oceanography.

THE PRACTICAL METHODS AND FORM OF COMPUTATIONS GENERALLY FOLLOWED IN DYNAMIC PHYSICAL OCEANOGRAPHY

We may now continue by describing the manner in which the unknown terms of (f) are determined by observational data secured from a closed curve ABDC in a plane formed by verticals AC and BD, between stations A and B. First we shall regard the forces tending to

FIG. 8.—Two verticals A and B at stations 206 and 205, respectively, and with the observed values of v and p at depths expressed in decibars or meters as follows: 0, 50, 125, 250, 450, and 750

accelerate the particles as a result of varying degrees of stability in the water columns of any given area. The abstract exposition, furthermore, has been supplemented by a practical example wherein stations A and B are replaced by stations 206 and 205, respectively. (See computations, p. 28.) These stations were taken by the International Ice Patrol in 1922, the sectional line forming approximately a right angle with the northern edge of the Gulf Stream south of the Grand Banks.

It is assumed that the specific volumes in situ have been calculated from the tables, as based upon the temperature and salinity records from the observed depths, viz, 0, 50, 125, 250, 450, and 750 meters. The values of p are, for all practical purposes, equal to the depth in meters—that is, at a depth of 750 meters the pressure is 750 decibars (see p. 11). In order to compute as accurately as possible the value of D (the dynamic depth) for the vertical AC, it is necessary to consider the change at frequent depths which occurs in the value of v (the specific volume). In order to comprehend the method of mathematical computation customarily followed, e. g., p. 28, it will be helpful to regard Figure 9.

FIG. 9.—A graphic means of illustrating the mathematical integration customarily employed in computing the dynamic depth to a given observed pressure beneath the sea surface

The area MNPO (assumed equal numerically to D_a) is formed partly by the curve NP, which represents varying values of v , and the side MO, which indicates the scale of pressure. The value of D_a , therefore, represented by the area MNPO is equal in value to the sum of all the smaller areas d_1 , d_2 , d_3 , d_4 , and d_5 . The area d_1 (and the values of all the other small rectangles in similar manner) may, with sufficient accuracy, be put equal to

$$d_1 = \left(\frac{v_0 + v_1}{2} \right) dp_1$$

or

$$D_a = \left(\frac{v_0 - v_1}{2} \right) dp_1 + \left(\frac{v_1 + v_2}{2} \right) dp_2 + \left(\frac{v_2 + v_3}{2} \right) dp_3 + \left(\frac{v_3 + v_4}{2} \right) dp_4 + \left(\frac{v_4 + v_5}{2} \right) dp_5$$

which equals the value of the gravity potential, relatively to the sea surface, expressed as a depth in dynamic meters at which the pressure

of 750 decibars prevails. Other verticals are computed in a like manner and

$$D_a - D_b = \Delta D,$$

which represents the value of the force tending to produce acceleration of the water particles in the plane of the closed rectangular curve ABDC. The direction in which the force tends to act may be deter-

FIG. 10.—Distribution of specific volume in a section south of Newfoundland May 5-7, 1922, and also a cross-sectional view of the solenoidal tubes, each complete parallelogram of which represents the presence of 400 solenoids. Horizontal scale, 1:2,000,000; vertical scale, 1:5,000

mined by the relative values of v , the water being forced upwards most pronouncedly at that vertical where the values of v are a maximum.

It is customary and of assistance in dynamic investigation of an ocean mass to represent the distribution of specific volume, as found by actual observations, graphically in vertical projection, the pressure in decibars being shown as ordinates in the graph. Figure 10 includes a line of stations, 201 to 206, taken from the records of the International Ice Patrol, and furnishes an example of such a method of illustration.

In this particular figure the horizontal lines have been drawn for every 20 decibars. The curved lines represent lines of equal specific volume and are drawn for every 20 units in the fifth decimal place of v . In Figure 10, v^1 means $10^5 (v - 0.97000)$. Since a solenoid is formed by the intersection of isobaric surfaces with isosteric surfaces, it is not difficult to see that a vertical plane, such as illus-

FIG. 11.—An effective illustration of the distribution of solenoids and the dynamic tendencies in a vertical section taken approximately at right angles to both the Gulf Stream and the Labrador Current south of the Grand Banks, May 5-7, 1922. Each whole dot represents 400 solenoids, which gives approximately 50,000 solenoids for the Gulf Stream as represented by the black circles, and about 12,200 solenoids for the Labrador Current as shown by the white circles. The current of the Gulf Stream is represented as flowing perpendicular to the plane of the paper toward the reader and the Labrador Current in the opposite direction. Horizontal scale, 1:2,000,000; vertical scale, 1:5,000

trated in Figure 10, page 26, will intersect the solenoidal tubes forming a number of parallelograms, each one of which indicates the presence of 400 solenoids.

The distribution of forces tending to produce acceleration in such a vertical section may be further emphasized by erasing all the isobaric-isosteric lines after the location of the centers of the parallelograms have been marked out. This method of illustration is shown in Figure 11.

Where the isosteres lie deepest and the inclination is greatest, there is indicated at that place a tendency to push the water upwards with a maximum strength, and where the isosteres lie highest, there the force is at a maximum tending to drive the water downwards. But whatever the position of the isosteres may be, it is well to bear in mind that when the section lies at right angles to the direction of the current, there is no actual movement of the water particles within the vertical plane whatsoever. The value of the solenoids lies in the fact that they express the *presence* of a force or forces tending to cause circulation around the rectangle. The Ferrelian force (effect of earth rotation) precludes *actual* movements restricted solely parallel to the current path AB, Figure 7, page 23, as previously described.

DETERMINATION OF DYNAMIC DEPTH, STATIONS 205 AND 206

We may continue to treat the Ice Patrol records of stations 201 to 206 dynamically, by computing the values of specific volume from the given station data and correcting the same to specific volume in situ; then, by means of the equation on page 25, determine the dynamic depth of the successive isobaric surfaces of observation. In order simply to illustrate the methods customarily employed, we have selected two stations only, stations 205 and 206 located on the northern edge of the Gulf Stream south of Newfoundland. Similar procedure and similar results follow, of course, in like manner from other given station data.

dp	Meter depth	t	s	d ₁	Table III	Table IV	Σ_1	Mean Σ_1	$\Sigma_{1m-\delta p}$	Σ_2	E	$(E - E_1)10^5$	$V10^5$	$(V - V_1)10^5$
STATION 205														
50	0	5.7	33.93	26.77	2607	0	2607	2622.5	131125	-----	0	0	.97393	129
75	50	12.0	35.31	26.85	2615	22	2637	2662.5	199687	131125	48.68875	.06225	.97363	121
125	125	10.1	35.16	27.07	2635	53	2688	2735.5	341938	330812	121.69188	.14676	.97312	104
200	250	6.7	35.00	27.48	2674	109	2783	2837.0	567400	672750	243.27250	.25251	.97217	65
300	450	5.4	35.04	27.68	2694	197	2891	2959.0	887700	1240150	437.59850	.36501	.97119	47
	750	4.6	35.01	27.75	2700	327	3027			2127850	728.72150	.50201	.96973	44
STATION 206														
50	0	18.1	36.21	26.18	2551	0	2551	2568.5	128425	-----	0	0	.97449	185
75	50	18.0	36.33	26.31	2564	22	2586	2611.5	195862	128425	48.71750	.09100	.97414	172
125	125	16.3	36.11	26.55	2587	50	2637	2679.5	334937	334287	121.75713	.21201	.97363	155
200	250	12.9	35.56	26.86	2616	106	2722	2781.5	555500	659224	243.40776	.38777	.97275	126
300	450	9.2	35.12	27.20	2648	193	2841	2918.0	875400	1215724	437.84276	.60927	.97159	97
	750	6.6	34.95	27.46	2672	323	2995			2091524	729.08476	.86527	.97005	76
Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11	Col. 12	Col. 13	Col. 14	Col. 15

The abbreviations appearing at the top of the columns in the preceding compilation of computations are explained as follows:

Column 1 (dp) represents the difference of pressure in decibars of successive observed depths, which for all practical purposes is equal to the differences in depths of observation in meters.

Column 2 contains the depths at which observations were made.

Column 3 (t) contains the observed temperatures.

Column 4 (s) contains the determined salinity.

Column 5 (d_t) contains the density as found directly from the temperature and the salinity. (Contraction adopted, see p. 2.)

Column 6 (Table III) is a form of inversion table combined with other corrections (see p. 18).

Column 7 (Table IV) contains the combined corrections with due regard to signs for the three factors, pressure with depth, with temperature, and with salinity (see p. 19).

Column 8, contains the values of $(1-v) 10^5$, where v represents the specific volumes in situ.

Column 9, contains the mean values between the successive depths of observation as they appear in column 8.

Column 10, contains the product of the values as contained in column 9 and the difference of pressure in decibars as shown by column 1.

Column 11 (Σ_2) contains the results obtained by adding progressively the successive ciphers as contained in column 10. Values from columns 6 to 11, inclusive, are negative throughout.

Column 12 (E) contains the calculation of the dynamic depths of the observed isobaric surfaces. Found by combining values in column 2 with those in column 11.

Column 13 ($E - E_1$) 10^5 contains the anomaly of the dynamic depth of observation—i. e., it represents the difference in dynamic depth between the isobaric surface actually observed and the position of the same isobaric surface in a sea of 0.0° C. and 35 per mille salinity.

Column 14 (V) 10^5 contains the specific volume in situ, or one minus the value as contained in column 8 multiplied by 10^5 .

Column 15 ($V - V_1$) 10^5 contains the anomaly of specific volume in situ, or, in other words, the difference in specific volume in situ as found from that at a similar depth in a sea of zero degrees Centigrade and 35 per mille salinity. The values for the dynamic depth (D_1) and the specific volume in situ, (V_1), as found in an ocean of zero degrees Centigrade and 35 per mille salinity, are given in the following table, Table V. The selected depths recorded therein are the same as those previously carried in Table IV.

TABLE V*

E_1 = dynamic depth in sea 0° C., 35 per mille. V_1 = specific volume in sea 0° C., 35 per mille.

Deci- bars	E_1	V_1	Deci- bars	E_1	V_1	Deci- bars	E_1	V_1	Deci- bars	E_1	V_1
0	0	.97264	50	48.62650	.97242	125	121.54512	.97208	700	679.74949	.96951
5	4.86315	.97262	55	53.48854	.97240	150	145.84574	.97197	750	728.21949	.96929
10	9.72620	.97260	60	58.35045	.97237	200	194.43849	.97174	800	776.67849	.96907
15	14.58914	.97257	65	63.21225	.97235	250	243.01999	.97152	900	873.56349	.96863
20	19.45195	.97255	70	68.07395	.97233	300	291.59024	.97129	1000	970.40449	.96819
25	24.31465	.97253	75	72.93554	.97231	350	340.14924	.97107	1200	1163.95549	.96732
30	29.17725	.97251	80	77.79700	.97228	400	388.69699	.97084	1400	1357.33249	.96645
35	34.03974	.97249	85	82.65835	.97266	450	437.23349	.97062	1600	1550.53649	.96559
40	38.90210	.97246	90	87.51960	.97224	500	485.75899	.97040	1800	1743.56849	.96473
45	43.76435	.97244	100	92.24175	.97219	600	582.77649	.96995	2000	1936.42949	.96388

* The values shown are based upon those contained in Table 8II, Bjerknes' "Dynamic Meteorology and Hydrography," Carnegie Inst. Pub., 1910.

drawn is the type most commonly employed for purposes of illustration. A dynamic section, Figure 12, formed by stations 201 to 206, International Ice Patrol, 1922, is shown on page 30.

VELOCITY—HOW DETERMINED

We may now return to a consideration of equation (f), page 24, in order to find the velocity of the current between stations 206 and 205, by substituting at the same time for d_a and d_b the values as found at the six levels of observation of the two foregoing stations. Since the velocity is the term desired, equation (f), page 24, may be written in the following form:

$$c_0 - c_1 = \frac{(d_a - d_b)10^5}{2\omega \sin \phi_m L 10^5} \dots \dots \dots (g)$$

where $\phi_m = 41^\circ - 10'$, the mean latitude of stations 206 and 205. The value of $10^5 \cdot 2\omega \sin \phi$, by Table VI, page 32, is found to be equal to 9.60. The multiple 10^5 is introduced simply to bring the velocity values into centimeter-gram-second terms. L , which is the distance between stations, is equal to 32 miles, or 59 kilometers. (See Table VII, p. 33.)

Stations	Depth in dynamic meters				
	50	125	250	450	750
205.....	48.71750	121.75713	243.40776	437.84276	729.08476
206.....	48.68875	121.69188	243.27250	437.59850	728.72150
$d_a - d_b$	0.02875	0.06525	0.13526	0.24426	0.36326

If we treat the values of $d_a - d_b$ as whole numbers and divide them by the value of $2\omega \sin \phi_m L 10^5$, the latter of which is found equal to 569.28, we obtain the following:

	50	225	250	450	750
$C_0 - C_1$	5.0	11.5	23.8	42.9	63.6

If it is assumed that c_1 is equal to zero at a depth of 750 decibars (meters), then the following velocities are furnished at the various levels of observation, from the surface downwards. (If it is desired to express velocities in terms of knots per hour, 10 cm./sec. is equal approximately to 0.2 knots per hour.)

Meters or decibars	0	50	125	250	450	750
C_0 —in cm./sec.....	64	59	52	40	21	0
C_0 —in Kt./Hr.....	1.3	1.2	1.0	0.8	0.4	0

It is often desirable to express the velocities at the various levels of observation graphically in the form of a current diagram, in which case it is customary to measure the velocity units along the abscissa and the depths along the ordinate. The graphic representation of the current between stations 205 and 206 is shown by Figure 13.

Table VI is a copy of a table (Table 12), which first appeared in a contribution by Sandstrom and Helland-Hansen, in "Report on Norwegian Fishery and Marine Investigations," Volume II, No. 4, Bergen, 1903. It contains the computed values of $2 \omega \sin \phi 10^5$ for each degree of latitude.

FIG. 13.—Current velocity diagram, northern edge of Gulf Stream between stations 205 and 206, May 5-7, 1922

TABLE VI

$$2\omega \sin \phi 10^5 = 14.58 \sin \phi$$

ϕ	0	1	2	3	4	5	6	7	8	9
0.....	0.00	00.26	00.51	00.76	01.02	01.27	01.52	01.78	02.03	02.28
10.....	02.53	02.78	03.03	03.28	03.53	03.77	04.02	04.26	04.51	04.75
20.....	04.99	05.23	05.46	05.70	05.93	06.16	06.39	06.62	06.85	07.07
30.....	07.29	07.51	07.73	07.94	08.15	08.36	08.57	08.78	08.98	09.18
40.....	09.37	09.57	09.76	09.95	10.12	10.31	10.49	10.67	10.81	11.01
50.....	11.17	11.33	11.49	11.65	11.80	11.95	12.09	12.23	12.37	12.50
60.....	12.63	12.75	12.87	12.99	13.11	13.22	13.32	13.42	13.52	13.61
70.....	13.70	13.79	13.87	13.95	14.02	14.09	14.15	14.21	14.26	14.31
80.....	14.35	14.40	14.44	14.47	14.50	14.53	14.55	14.56	14.57	14.58

Since the value of L , the distance between stations, is expressed in kilometers, a conversion table of nautical miles to kilometers is included herewith:

TABLE VII

Nautical miles	Kilometers									
	0	1	2	3	4	5	6	7	8	9
0-----	0.0	1.9	3.7	5.6	7.4	9.3	11.1	13.0	14.8	16.7
10-----	18.5	20.4	22.2	24.1	25.9	27.8	29.6	31.5	33.3	35.2
20-----	37.0	38.9	40.7	42.6	44.4	46.3	48.2	50.0	51.9	53.7
30-----	55.6	57.4	59.3	61.1	63.0	64.8	66.7	68.5	70.4	72.2
40-----	74.1	75.9	77.8	79.6	81.5	83.3	85.2	87.0	88.9	90.7
50-----	92.6	94.5	96.3	98.2	100.0	101.9	103.7	105.6	107.4	109.3
60-----	111.1	113.0	114.8	116.7	118.5	120.4	122.2	124.1	125.9	127.8
70-----	129.6	131.5	133.3	135.1	137.1	138.9	140.8	142.6	144.5	146.3
80-----	148.2	150.0	151.9	153.7	155.6	157.4	159.3	161.1	163.0	164.8
90-----	166.7	168.5	170.4	172.2	174.1	175.9	177.8	179.6	181.5	183.3
100-----	185.2	187.1	188.9	190.8	192.6	194.5	196.3	198.2	200.0	201.8

The velocity values as they are usually finally shown, represented by c_0 , page 31, are the differences between the movement on the surface and that at a level where it is believed motionless water lies. But it is important to bear in mind that as a result of dynamic computations, the values of velocities are expressed in terms NORMAL TO THE VERTICAL SECTION which may include any two stations. Another step is necessary if it is desired to obtain the value of the real velocity. Let us assume that the direction of flow but not the rate is known. In Figure 14 suppose the direction of the current is represented by the parallel lines AM and BM₁, between the two stations A and B. Furthermore, let it be given that the velocity normal to the section has been computed by means of equation (g), page 31, and that it is given on the figure as the line K. We now wish to determine the true velocity, V, which lies in a direction parallel to the lines AM and BM₁, and which forms the angle α with the computed velocity. The value of

FIG. 14.—Lines AM and BM₁ indicate the known direction of the circulation. K represents the computed velocity normal to the sectional line AB. To find: The true velocity (V) of the current

V , it is easy to see from the figure, is equal to $\frac{K}{\cos \alpha}$. The same results may be obtained graphically by laying off the angle α and dropping a perpendicular from the end point of the known side K upon the unknown side V, and then measuring the length of the latter in units the same as K.

In Figure 15, AB represents the path of the current; AE, the resultant of the real physical forces; and AC, the quasi force due to earth rotation, acting with the same magnitude but in the opposite direction. The two vectors AE and AC lie in one and the same plane, EAC, which is perpendicular to the course of the current. The fields of forces may be investigated by regarding them graphically in either

FIG. 15.—Resolution of the two principal forces in a gradient current AB; AE, the forces due to Archimedean tendencies; AC, the Ferrelian force

a vertical view, called a dynamic section (see fig. 12, p. 30) or in a horizontal view called a dynamic topographical chart (see fig. 19, p. 39). Both vertical and horizontal projections, it will be found, assist to reveal particular knowledge regarding the types of forces involved. It will lead also to a clearer understanding of the relative position of the forces, and, moreover, to the course of the current, if we now review some of

the fundamental notions pertaining to such representations of forces. The well-known method of regarding a force as represented by equiscalar surfaces and unit scalar sheets is especially applicable here. The potential value of an irregularly formed equiscalar surface, obviously, can be traced by its intersections with a series of unit parallel horizontal planes. The rate of variation of contours (intersections) measured along a normal vector called the gradient is a direct expression of the acceleration of the scalar force. In Figure 16

FIG. 16.—A diagram of forces similar to that shown in Fig. 15, but with the addition of dynamic isobaths MN, OP, etc., which show the position of such contours (when friction is disregarded) relative to the actual movement of the water particles

the vector AE representing the acceleration due to primary forces in the sea provoking currents may be regarded as a gradient force due to the variations in level. These variations are in horizontal projection shown by a series of horizontal lines (dynamic isobaths) MN, OP, etc., all perpendicular to line AE, and inscribed on the scalar field of the force. But MN, OP, etc., are also parallel to the line AB, the stream line of the current. Lines MN, OP, etc., then, correspond to the

dynamic contours of a given isobaric surface as illustrated by the dynamic topographical chart described on page 37 (see fig. 19). Therefore it follows that the dynamic isobaths recorded on such charts possess a tremendous significance in as much as they delineate the courses of the water particles over any given area that has been investigated. Not only may the paths of the currents be traced on such charts, but the degree of compactness of the dynamic isobaths along the gradient at right angles to the current, is a measure of the relative velocity of the current. The closer together the contours lie in any given latitude indicates the more rapidly the current is flowing at that time and place.

DIRECTION OF FLOW

The direction toward which the water moves is, of course, requisite information which may be obtained best perhaps by reference to a vertical section—i. e., the dynamic section. If a plane be passed

FIG. 17.—Showing the two types of distribution of specific volume in vertical section and the resultant tendency toward flow of the water in consequence

vertically downward through the plane of the forces AC and AE (fig. 16 p. 34) and a distribution of specific volume be secured, a dynamic section similar to Figure 12, page 30, will result. Consideration of the closed curve formed by the two verticals at stations 206 and 205 will reveal the fact that the water being lighter to a greater depth at 206 than at 205 tends to be forced upward at 206 and downward at 205. But when the current is constant there is no actual movement of the water particles in these planes, as the real forces are exactly counter-balanced by the Ferrelian force (effect of earth rotation), the latter of which acts in a direction opposite to the tendency of the Archimedean forces. The real movement of the water particles, as represented by the foregoing figure, takes the form of a current which flows at right angles to the plane of the dynamic section. In such a distribution of forces as shown by Figure 17 the current would run in a direction through the paper, either toward or away from the

eye of the reader. Provided that the water is moving faster in the surface layers than in the depths, the rule follows: **LOOK IN THE DIRECTION TOWARD WHICH THE CURRENT IS RUNNING, IN THE NORTHERN HEMISPHERE, AND THE LIGHTEST WATER WILL ALWAYS LIE ON THE RIGHT HAND.** The vertical differences of velocity may be calculated from equation (f), see page 24, which is affected fundamentally by the values of temperature, salinity, and depth, at any two verticals in a plane and which it is important to note lies at right angles across the path of flow.

GENERAL SUGGESTIONS FOR A PROGRAM OF HYDROGRAPHICAL SURVEY

Ocean currents, it has been pointed out, may be determined by a distribution of temperature and salinity in a plane, the position of which is perpendicular to the flow of the water. Conversely, if no forces are found as represented by the position of the isosteres and isobars in vertical section, then there is no current at right angles to the plane of the section. It is easy to see, on the other hand, that a section parallel with the course of a current contains no information whatsoever regarding its movement. Hydrographical survey of extensive ocean surfaces involves in any event a large program of time and expense, and the task grows to considerable magnitude, especially when the work devolves upon the efforts of one vessel. An ideal program, of course, includes a maximum number of oceanographic stations distributed netlike over the area to be investigated, and wherein the promulgation of the work most nearly approaches a simultaneousness of observation. Unfortunately, the ideal survey rarely occurs, and it is usual that resort is made to lines of stations along a vessel's track. Under such conditions it is apparent that before commencing the observational work the particular area should be studied carefully with respect to all previous, available knowledge of a hydrographical nature, remembering that the lines of stations in a program of dynamic investigation should run in such a manner that the sections secured approach most nearly to right angles across known or suspected currents. As an example let us take the region around the tail of the Grand Banks where there are two main movements. (1) The Labrador Current is the inshore set, which flows southward along the east side of the Grand Banks and to a variable distance around the "Tail." (2) Offshore in the Atlantic basin the easterly moving masses of the Gulf Stream, guided by the trend of the bottom configuration, progress in a generally opposite direction to the cold water inshore. A program of dynamic investigation in this region should be based upon a series of lines of stations running offshore in a direction normal to the Grand Banks' slopes as shown by Figure 18. Stations should be taken as close together (and repeated as often) as practicable in order that the influence belonging to tem-

porary boundary waves and vortex movements be disassociated from a representative picture of prevailing conditions. (cf. Helland-Hansen and Nansen, "The Norwegian Sea," Bergen, 1909.) To such an end the dynamic features of modern physical oceanography are best carried out by several craft cooperating in one systematic program of investigation, which may or may not extend over great expanses of the ocean. Here is an important requirement which would appear to demand certain revisions in the program of expeditions, which in the past have usually been performed by one vessel sailing under a more or less roving commission. These modern methods in dynamic

FIG. 18.—A series of station lines radiating from the Grand Banks is an example of the correct methods to employ in order to obtain the best collection of material leading to an investigation of the currents in this region

oceanography, particularly the graphic representation as embodied in the dynamic topographical chart, provide, furthermore, an easy and efficient means of mapping currents over extensive ocean surfaces—advantages which are bound to guarantee a great employment for this science in future hydrographical surveys.

DESCRIPTION OF A DYNAMIC TOPOGRAPHICAL CHART (CURRENT MAP)

We now come to the description of a dynamic topographical chart, a subject which has been reserved until the close of the various methods of illustration, because, from its practical importance, it merits especial emphasis. The basis for the construction of such a projection depends fundamentally on the dynamic computations

previously discussed on page 28. It possesses great practical advantages in that it presents two pertinent, desirable pieces of information, viz, (1) the direction of movement, and (2) the relative rate of flow of the current, over any given area. Let us suppose that the temperature and salinity data, surface to 750 decibars, have been collected from a sufficient number of stations in the region south of the Grand Banks. This, as a matter of fact, corresponds to an actual oceanographical investigation carried out by the International Ice Patrol in these waters during the spring of 1922. Dynamic treatment of these data leads through the accepted methods of calculation as shown on page 28. Column 12 on that page contains the dynamic depths of the successive surfaces of observation, and also the material for the construction of a dynamic topographical chart, of which Figure 19, page 39, is an example.

An isobaric surface, the dynamic topography of which is the subject of interest, may be visualized as spread out beneath the surface of the sea, an undulating floor, the depth of which we plumb with the same reality as the more tangible floor of the ocean is sounded out by the hydrographer. As a first step toward the mapping of currents, let us investigate any one of the standard isobaric planes of observation adopted by the International Ice Patrol, viz, 50, 125, 250, 450, and 750 decibars, by plotting its dynamic soundings on a map at those positions in latitude and longitude where the respective stations have been located. This procedure, it is plainly seen, is identical to that in which depths to the bottom are fixed on any ordinary navigational chart. If, as a next step, equipotential (level) planes are passed at frequent heights through the selected isobaric surface which is under investigation, a number of lines of intersection are formed, which for convenience may be called dynamic isobaths. If now we recall the fact that when the accelerating force of friction is disregarded the movement of water particles on an isobaric surface tends along such a surface, as well as along the same equipotential surface (see p. 34, fig. 16), it is not difficult to appreciate the significance of dynamic isobaths. The small sketch in the lower left-hand corner of Figure 19, page 39, shows a series of dynamic isobaths and the direction of the two forces which are always present wherever there prevails translatory movements of water particles in a steady current. Friction, for all practical purposes, may be disregarded. (See p. 43). (1) AE illustrates the resultant of the forces which impel and maintain gradient flow; (2) AC represents the Ferrelian force acting in a plane 90° to the right of the current; and (3) AB is the path of the actual established movement following along the dynamic isobaths. When these latter are recorded on an ordinary geographical map, as a series of dynamic contours, it permits the reader, at a glance, to picture the course followed by a water particle throughout the region which is under survey. Figure 19 is shown as an example of a dynamic

topographical chart or ocean current map. This particular illustration delineates the dynamic form of the 750-decibar surface as it was found to lie by the International Ice Patrol, May 5-7, 1922, south of the Grand Banks.

It was pointed out on page 13 that the greatest variations in the physical state of oceans and consequently the obliquity of the isobaric surfaces take place in the upper levels; constant conditions and a level position of isobaric surfaces, on the other hand, are most nearly approached in the greater depths. Therefore, in order to compare dynamic measurements with the same level plane, and ultimately to record the obliquity of the sea surface at several points of observation, it is necessary to measure upwards from a relatively deep-seated, level, isobaric plane, instead of downwards from the usually tilted surface of the sea. It is obvious, then, that the best representation of the real movements taking place on a sea surface is contained in a dynamic topographical chart of a relatively deep isobaric surface. Since the 750-decibar surface was the maximum depth to which the Ice Patrol's investigations extended, this has been taken as representing the surface of minimum motion. The record of its form, as shown by Figure 19, approximates very closely to a surface current chart of the southern Grand Banks region.

It is customary on dynamic topographical charts to record the dynamic contours of an isobaric surface for multiples of dynamic millimeters difference in depth, but sometimes, such as the present example, due to the great inclination of the baric plane, only every 5 dynamic centimeters is graphically possible. It will be noted that the 750-decibar surface was found at a depth of 729.0848 dynamic meters at station 206 in the northern edge of the Gulf Stream (see column 12, p. 28), but at station 205, 32 miles nearer the continental slope, it was recorded at a depth of only 728.7215 dynamic meters. If it is assumed in conformity with previous remarks, that the 750-decibar surface most nearly approached a position level, then there remains but one alternative, namely, that the sea surface was approximately 36 centimeters higher at station 206 than at 205. This obliquity of the sea surface, and to a less degree in the other depths of observation, south of the Grand Banks, May 5-7, 1922, is shown in greatly exaggerated (but proportionate) profile, Figure 5, page 14. In order to give the reader an idea of the slight inclination necessary in the sea surface to drive a current, we call attention to the difference in actual dynamic height recorded, viz, about 1 centimeter per mile, which, according to computations, page 31, represents a gradient that impelled the surface layers at a velocity of 1.3 knots per hour.

The direction in which the water is flowing also may be determined by comparing the dynamic heights of any two points of observation. If it is assumed that the current is more rapid in the upper levels than in the depths we shall be facing, in the direction of flow in the

Northern Hemisphere when the greatest dynamic measurements lie on the right hand. (See p. 35.)

It is customary to chart the dynamic form of several isobaric surfaces of observation—i. e., at several standard decibar depths. Such methods permit one to follow the movements in several different layers, each one of which contains supplementary information regarding the combined movements of the entire mass. It is often desirable to chart the current at a depth below the surface where temporary fluctuations, such as those due to variable winds, become eliminated. For example, if surface conditions are shown on a 2,000-decibar chart, then 2,000–100 decibars will reveal the movement prevailing at a depth of about 100 meters below the surface.

FRICITION

In the resolution of current forces it was shown that the primary force, AE , giving rise to a constant current was equally opposed by the effect of terrestrial rotation; provided there were no accelerating force of friction (see fig. 7, p. 23). Friction, it was pointed out, was of no practical importance to the discussion of gradient currents, but was, however, a factor of considerable magnitude in dealing with the tangential effect of winds upon surface layers, and thus from a theoretical view, at least, the entire subject merits discussion here. There are two kinds of friction, viz, (1) molecular and (2) virtual. Molecular friction, or viscosity in the sea, is of insignificant consequence and may be completely disregarded. Virtual friction or real friction not only considers the viscosity existing between any two layers of different velocities, but it also includes the resistance due to turbulence. Virtual friction exerts itself to a varying degree in the sea; for example, a homogeneous water mass mixed by the turbulent effect of the winds possesses greater friction dynamically than prevails in a region of more pronouncedly stratified water. Since we consider movements in a horizontal direction only, the friction force is consequently confined to that direction. The particular form given to the velocity diagram of a current contains direct information bearing upon the degree of friction prevailing at the time of observation, and thus wherever constant currents are established and flowing in the same direction at different levels, the acceleration or retardation of friction acts either in the same or an opposite direction to the flow of the water particles.

If we examine several velocity diagrams we find that there are three general types as shown by Figure 20, page 42. In "A" the velocity of particle "a" is equal to the mean of the velocities of a water particle, or water layer, adjacently above and adjacently below; then the accelerating effect of the former is equal to the retarding effect of the latter, and the resulting value of friction in such currents is zero. This is a form often observed in a voluminous current such as the

Gulf Stream, and is well illustrated by the velocity diagram (fig. 13, p. 32). In "B," Figure 20, the velocity of "a" being greater than adjacent particles, or adjacent sheets above and below, is thereby retarded and friction acts to hinder the translatory progress of particle "a." In "C," the velocity of particle "a" is less than either of its immediate neighbors, above or below, and friction therefore

FIG. 20.—Three general types of current velocity diagrams

tends to accelerate the velocity of "a." If the water particles in a current be retarded by a constant accelerating force of friction throughout the depth, then the velocity diagram will assume the form of a parabola.

Cases as shown in "B" and "C" (fig. 20) may also be illustrated by components and force diagrams in horizontal projection as follows:

The dotted lines in Figure 21 represent the direction of flow of the current, and the solid lines are equipotential lines inscribed on the scalar field of the force tending to produce a movement in the sea,

FIG. 21.—The two types of force diagrams belonging to gradient currents when friction is included either as (1) a retarding or as (2) an accelerating force

The gradient AE being perpendicular, of course, represents the force due to variations in gravity potential. AC is the force due to terrestrial rotation lying 90° to the right of the direction of the current. By vector analysis we may find the force AF due to friction, where in "A" it retards the current AB, and in "B" it accelerates the same. If all but one of the parallel lines of flow and all but one of the parallel

equipotential projections be removed, the angle between these two may be more easily seen and designated as α , in the figure. It follows that:

$$AF = AC \tan \alpha = 2\omega \sin \varphi V \tan \alpha \dots\dots\dots (h)$$

It is seen from this that the force due to virtual friction varies directly as the tangent of α and the velocity of the current. There is, however, very little data bearing on the value of this angle α for various currents at different places and under different conditions. In order to determine various values of α (contemplating of course that there are several assumptions), simultaneous observations should be made regarding the direction and the rate of flow of the current with the aid of current meters, and at the same time observations for temperature and salinity should be taken in a section at right angles

FIG. 22.—An illustration similar to Fig. 21, but with all but one of the dynamic isobaths erased and all but one of the parallel lines of flow erased

across the current. By such a means the virtual friction, as represented by the line AF, can be tabulated directly. The value of this effect, however, as it enters the present discussion, may be safely stated, rarely exceeds a magnitude of 0.05 to 0.07 knots per hour; either to accelerate or to retard the flow of a steady current, and such ciphers being relatively insignificant can be disregarded in the practical determination of currents.

EFFECT OF BOTTOM CONFIGURATION ON CURRENTS

Frictional retardation attains considerable proportions, however, between flowing water particles in contact with the fixed configurations of an ocean basin. Ekman has suggested as a result of mathematical investigations that if an ocean current proceeding along in a steady manner moves in over a gradually shallowing shelf, it will tend to be deflected more and more to the right in the Northern Hemisphere. On the other hand, if by a continuance the bottom begins to recede deeper and deeper from the surface, then the current will be proportionately deflected to the left. This phenomenon

is revealed by reference to certain current charts (dynamic topography) of which the Ice Patrol has record. As an example, we might call attention to Figure 19, page 39, wherein the position of the Gulf Stream relative to the 4,000-meter contour, clearly indicates that the current flooded in up the grade occasioned by the southeasterly extension of the Grand Banks (about 53° west longitude), but meeting increased resistance in the constantly shallowing depths it was deflected to the right, offshore. The current, proceeding along in this new direction where the depths increase, tended to swing to the left, inshore; and so in this fashion its course may be traced as it flowed along the continental slope in a serpentine path.

Inshore, over continental shelves, it has been found that the coastal waters are in a slow primary circulation which Huntsman believes due to the pumping action of the tides combined with the effect of terrestrial rotation. It has been expressed in a general statement, viz, bottom configuration in the Northern Hemisphere tends to deflect currents to the right (*cum sole*), around islands and shoals, and to the left (*contra solem*), around basins and deeps.

TIDES

One of the external forces provoking currents in the sea was ascribed to the tides (see p. 1). Such currents rotate in their movement either clockwise or counterclockwise one complete cycle, when unaffected by other influences, in a period of 12.4 hours. The theory of semidiurnal tides is based upon a series of waves which are known sometimes to be propagated great distances, but the discussion of the form of such waves, and the theory of orbital motion given to the water particles, is too lengthy to be included in this paper. Those interested in a more detailed exposition are referred to Darwin (cf. *Tides and their Kindred Phenomena in the Solar System*.) We may remark, however, a few generalities regarding the various tidal phenomena as they affect certain oceanographical problems. Tidal currents in the deep ocean basins are of comparatively subordinate importance, but near continental slopes and over shelves they may attain great magnitude. Even when such slopes and shelves extend far out into an ocean basin, tidal effects remain quite prominent. The sheet of water lying over the Grand Banks, for example, averaging a thickness of about 35 fathoms (65 meters), receives a regular tidal clockwise rotation which attains velocities ranging from 0.2 to 0.5 knots per hour. Well-marked rippling on the surface during periods of calm sea, moreover, has been observed along the eastern edge of the Grand Banks, which it is easy to believe were caused by the semidiurnal tidal wave meeting the rise as presented by the eastern face of the bank. Icebergs around the Grand Banks have often been carried inshore and grounded temporarily during calm weather when no other cause seemed so plausible as that of a favorable tidal set at the time and place.

Various places, as a result of land and shoal formations, may be under the influence of two tidal waves in varying phases. If these latter are similar and both at a maximum, then follows a maximum range of level and a minimum velocity of current. On the other hand, where two tidal waves meet in opposite phases, a minimum range of level results but a maximum strength of current is attained. Tidal currents on soundings are usually determined by the aid of current meters, which in the open sea are often illustrated by an elliptical form of diagram when the current is purely tidal. The water over shelves and near continental slopes is usually in progressive movement as well as being under a tidal influence, and in those cases the current diagram will be a resultant of the two different types of movement. Another method of illustration of current observations is that of a number of vectors radiating from a common center and where the position and relative length of the successive vectors indicates the direction and velocity respectively of the current reckoned in moon hours.

VARIATIONS IN ATMOSPHERIC PRESSURE

Among the causes of currents ascribed to external forces, there was included (see p. 1) that of the atmosphere as it pressed down upon a sea surface unequally. Some very interesting observations have been collected which deal with this subject where the bodies of water are partially inclosed by basins and the currents thus produced are forced. An example of such geographical qualifications is illustrated by the Mediterranean Sea and its connection—the Strait of Gibraltar—with the Atlantic. Knudsen has found that atmospheric pressure differences between the Baltic and the North Sea can be traced in the acceleration (or retardation) of the current through the Belts and Öresund. Since a difference in atmospheric pressure of 1 centimeter of mercury is equal to about 13 centimeters of sea water, it is not difficult to appreciate that the volume of a water mass which has this dimension as a thickness and an area equal to the Baltic will cause a considerable current when forced through such an opening as Öresund. Apparent natural difficulties have prevented the collection of scientific observations which will throw light upon the degree of this influence in the open sea. Because of (a) the absence of boundary surfaces against which the variations in atmospheric pressure may react; (b) the compensating effect which results from the progressive movement of such maxima and minima areas over the sea's surface; and (c) a counteracting drift current which tends to flow as a result of the accompanying system of winds make it safe to state that in general the relative importance of variations in atmospheric pressure causing currents in the open sea is small indeed. This phenomenon can be totally disregarded in hydrodynamic computations of the ocean.

WINDS

The discussion of the most important of all forces classified as external and provoking currents in the sea—the winds—has been reserved until the last. Winds, as they are treated in this connection, are divided into two groups: (a) Those winds the effects from which impel the surface layers—propagate frictionally downward—and produce a drift current only; and (b) those winds which by virtue of (a) drive water particles against boundary surfaces in the sea and give rise to gradient currents.

Let us first examine (a) the current caused by the wind and the earth's rotation alone. Nansen, on board the *Fram* while held fast in the Arctic pack, observed that the drift of his ship was 20° to 40° to the right of the direction of the wind. More recently Ekman has made some very interesting theoretical investigations regarding a wind blowing tangentially over a water surface, taking the rotation of the earth into account. (See Ekman's "Earth Rotation and Ocean Currents," Arkiv för Matematik, Astronomi, och Fysik, Band II, No. 11, Uppsala, 1905.) To begin with, Ekman has made the following assumptions:

1. The ocean be unlimited in a horizontal direction.
2. The depth to the bottom be great.
3. The water mass be homogeneous.

The following deductions are then made:

(a) The surface water particles are driven 45° cum sole (to the right in the Northern Hemisphere) of the direction of the wind.

(b) This effect is propagated frictionally downward. The direction of the current will alter cum sole in direct ratio to the increase in depth, and the velocities will decrease in geometrical progression.

(c) At a certain variable depth, called the frictional depth (or depth of wind current), the water will flow in a direction exactly opposite to that of the surface current, but its velocity will be only about one-twenty-third of that on the surface.

In order to obtain a clearer conception of the penetration of wind currents in the upper levels of an ocean, relative values of c (the velocity of the drift current throughout the vertical range of frictional depth) and D (the frictional depth) may easily be calculated by means of Ekman's formulæ and placed in table form:

TABLE VIII

Direction			Velocity
Meters	α°	α'	C
0	0	0	C_0
0.2 D	0.2π	36°	$0.53 C_0$
0.4 D	0.4π	72°	$0.28 C_0$
0.6 D	0.6π	108°	$0.15 C_0$
0.8 D	0.8π	144°	$0.08 C_0$
D	π	180°	$0.04 C_0$

(*the direction of the wind current at the very surface is assumed to be 45° to the right of the wind). Angles α and α' are the differences between the direction of the surface current and that prevailing at the given fractions of the frictional depth. The same results were first shown graphically by Ekman in the form of a diagram, a copy of which is shown herewith. As an example of the use of Table VIII, and as further illustrated by Figure 23, if the frictional depth be 50 meters, then at a depth of 10 meters the water particles will flow in a direction 36° to the right of the current on the surface. So it is seen that if the surface velocity and the frictional depth be known, the velocity and the direction of the pure drift current throughout the vertical range may be determined. Ekman has found that for practical purposes the equation may be simplified to

$$D = \frac{7.6 W}{\sqrt{\sin \phi}} \text{----- (j)}$$

(j) is based upon the value of q equal to 1.025; and W represents the wind velocity in meters per second. It is easy to see from (j) that the greater the wind velocity the deeper downward in an ocean will its effect penetrate. Also since $\sin \phi$ is

zero at the Equator and 1 at the pole, it follows that given winds will exert a maximum influence at the Equator and the least effect at the pole. The density of the water is of some importance; a given wind current will be stronger and penetrate to a greater depth in a region of light water than in a region of heavy water. Table IX gives

FIG. 23.—Ekman's diagram in which the position and length relatively of the successive arrows represent the direction and velocity, respectively, of the pure drift current, down to the frictional depth, set up as a result of wind and earth rotation alone

TABLE IX

Wind velocity		Latitude										
Cm. sec.	Beau- fort	1°	5°	10°	20°	30°	40°	50°	60°	70°	80°	90°
5	3	238	129	91	65	54	47	43	41	39	38	38
10	5	575	257	182	130	107	95	87	82	78	77	76
15	7	863	386	274	195	161	142	130	123	118	115	114
20	8	1,151	515	367	260	215	190	174	163	157	153	152

the frictional depth for different wind velocities at different latitudes and is computed by means of equation (j). The table is also based upon several assumptions, two important ones of which are, (1) the water mass be homogeneous, and (2) the depth to the bottom be great compared with the frictional depth. Such restrictions, as a matter of fact, differ from conditions actually met in the ocean, but the table gives an idea, nevertheless, of the depth of the pure wind current in the open sea, bearing in mind that when great variations in the density are found, the frictional depths will be less. At such places where abrupt transitions in the density of a water column takes place (e. g., a well-pronounced condition during the summer when the superficial layers become relatively light), then the boundary between the surface water and the heavier underlying mass acts as a virtual bottom in determining the development of the pure wind current. The density curves for the Grand Banks column, for example, often reveal an abrupt transition at a depth of about 20 to 30 meters, and such a discontinuity layer probably indicates the lower limit of the drift current at the time. If the depth of the upper layer be less than the frictional depth, as found by Table IX, then the effect of earth rotation is small and the direction of the wind current will more or less parallel the direction of the wind. On the other hand, at those places where homogeneous water is found extended downward 200 or 300 meters (e. g., in an ocean which has been subjected to the effect of an entire winter's cooling), then we may expect wind currents (after a day or so outside of the Tropics) to develop in characteristic form. (See fig. 23. p. 47.)

A distinction has been made between (a) the direct frictional effect of the wind blowing over a surface in the deep open sea, as it sets in motion a pure wind current, and (b) a similar effect of the wind but under the influence of coast lines or other hinderances, by which the water becomes amassed against (or sucked out from) such boundary surfaces in the sea. Winds classified as (b) bring to the problem a consideration of two subsequent movements known as "the mid-water current" and the "bottom current." The bottom current compensation for the surface current as the latter flows toward (or away from) boundaries. As an example of the building up ability of far reaching currents by a system of prevailing winds, we might regard the northwestern North Atlantic region along the North American coast, stretching from Baffin Land to the southern reaches of Newfoundland. Normal atmospheric distribution, especially well marked in this region during the December-March period, furnishes a strong northwesterly wind component, which a glance at the map will show lies approximately parallel to the coastal trend, Baffin Bay to Cape Race. Ekman has pointed out (see "Earth Rotation and Ocean Currents," *Arkiv för Matematik, Astronomi och*

FIG. 24.—The northwestern North Atlantic region illustrated in such a manner as to present one of the fundamental factors causing the gradient Labrador Current. The long curved arrows indicate the average wind direction December-March, and the short double arrows show the general movement of the water as a result. The diagram AB—CE represents the relative positions of Archimedean and Ferrelian forces and the direction of gradient flow

Fysik, Band II, No. 11, Uppsala, 1905) that the influence of a coast line upon currents set up by the wind is to produce a general movement of the water along in the direction of the coastal trend. The bulk of the current—i. e., the mid-water current—flows more or less parallel to the coast line, but as we approach the shore the mid-water current disappears and the surface and bottom systems merge into one with a consequent loss of character. This last-mentioned movement also tends to flow parallel to the shore line. A wind of given strength will produce a maximum effect, states Ekman, when directed 13° to the left of the coast line, which conditions, it is interesting to note, accord closely with relative directions of winter wind and coast line in the northwestern North Atlantic region. Figure 24, page 49, illustrates this particular phenomenon. The long curved arrows represent the average direction of the wind during the December-March period, and the short double arrows indicate the general movement of the surface layers. Offshore, where the depth to the bottom is relatively great, the wind current at the surface will be deflected nearly 45° to the right of the wind, and the mean transport of the water layer of the wind current will most nearly approach a direction 90° to the right of the wind. As we near the shore and shallow water the direction of the wind and movement of the surface water tend to become parallel. The gradient current, which is the most voluminous of the movements, arises whenever the sea surface, which has been deformed into a state of obliquity by the wind, tends to return to the level. A regular state of motion is soon established with a steady flow perpendicular to the pressure gradient, and we are permitted to draw a diagram showing the position of impelling and Ferrelian forces, AE and AC, respectively, with resultant direction of the gradient current as line AB. The time required for such a development must be calculated in weeks or months depending upon the magnitude of the wind and the width of the current. Gradient currents are more or less independent of slight variations in the winds such as affect surface currents, but it is quite probable that variations in the circulation of the atmosphere are followed by corresponding variations in the gradient currents. As an example of such a phenomenon as described in the foregoing we point to the Labrador Current, which may be due to the melting of polar ice, but which, nevertheless, is controlled to a great degree by a system of seasonal winds, December-March, tending to aid the transport of cold water and ice out of the Arctic along the western side of the North Atlantic basin.