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ON WIND GENERATED OCEAN WAVES
WITH SPECIAL REFERENCE TO THE PROBLEM OF WAVE FORECASTING

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ON WIND GENERATED OCEAN WAVES
WITH SPECIAL REFERENCE TO THE PROBLEM OF WAVE FORECASTING

By
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Notation

x	Horizontal coordinate in direction of wave propagation.
z	Vertical coordinate, positive upward from undisturbed sea surface.
t	Time.
g	Acceleration of gravity (980 cm sec^{-2}).
ρ	Density of sea water (1.028 g cm^{-3}).
ρ'	Density of air ($1.25 \times 10^{-3} \text{ g cm}^{-3}$).
ξ	Horizontal component of displacement of a water particle.
η	Vertical component of displacement of a water particle (elevation of sea surface relative to undisturbed level).
u_0	Horizontal component of particle velocity at the surface.
w_0	Vertical component of particle velocity at the surface.
u_0'	Mass transport velocity at the surface.
v	Wind velocity at "anemometer height" (about 8-10 m above the surface). (v_0 wind velocity immediately over the surface.)
$H = 2a$	Wave height (a = amplitude).
λ	Wave length ($\kappa = 2\pi/\lambda$ = wave number).
σ	Velocity of propagation of wave (phase velocity).
c	Group velocity.
E	Mean total energy of wave per unit area.
τ_{eff}	Effective stress of the wind at sea surface.
τ_n, τ_t	Normal and tangential component of wind stress.
f_n, f_t	Dimensionless factors of proportionality.
$\delta = H/\lambda$	Wave steepness.
$\beta = \sigma/v$	Wave age ($\beta_0 = \sigma/v_0$).
$\beta_m, \beta(1), \beta_m^*$	Different stages in characteristic wave ages.

$\gamma_{on} = \frac{1}{2} \pi \delta f_n$ $\gamma_{ot} = 2\pi^2 \delta^2 f_t$	Resistance factors as functions of $\beta_o = \sigma / v_o$
γ_n, γ_t	Resistance factors as functions of $\beta = \sigma / v$.
$\gamma^2(\beta)$	Effective friction coefficient.
A_n	Mean rate of energy transfer to wave due to normal wind pressure.
A_t	Mean rate of energy transfer to wave due to tangential wind stress.
$A = A_n + A_t = \rho' v^3 \beta \cdot C(\beta)$	Effective energy transfer to wave due to wind action.
$C(\beta)$	Effective resistance coefficient (dimensionless) as function of β .
$s = 0.095$	Sheltering coefficient; $s^* = 2s$; $s' = s/2$.
$c_d = s\pi\delta = sa\alpha$	
$n = 0.1075$ $r = 1.667$ $p = 0.062$	Constants used in empirical relationship $\delta = f(\beta)$.
D_μ	Mean rate of energy dissipation due to viscosity.
μ	Viscosity of sea water.
D_M, D	Mean rate of energy dissipation due to eddy viscosity in turbulent wave motion.
M	Eddy viscosity coefficient at fully developed sea.
$M(\beta)$	Eddy viscosity coefficient at growing wave motion as function of wave age β .
$B(\beta)$	Effective factor of energy dissipation due to eddy viscosity (dimensionless quantity) as function of β .
gx/v^2	Dimensionless fetch parameter.
gt/v	Dimensionless duration parameter.
gH/v^2	Dimensionless wave height parameter.

Abstract

Sea state forecasts are based on a combination of empirical knowledge and theoretical relationships of wind and ocean waves. Because the sea surface wave pattern usually consists of a locally produced sea and superimposed swell, two separate procedures of forecasting are necessary, both generally based on oceanic weather maps: forecasts of waves generated by the direct action of wind, and sea swell forecasting. After combination of these two independent phenomena, one may estimate the actual state of the sea surface pattern at a given locality.

Chapter I of this report communicates some results obtained by observations on the composite nature of wind generated waves. The characteristics of dominating waves in the fully developed state at different wind velocities are discussed.

A conspicuous feature of the rough sea is the phenomenon of interference. Observed periods, lengths and heights of waves vary through wide ranges, and in this complex wave motion outsize waves occur, as a consequence of interference patterns. An attempt is made to explain some striking features of composite wave motion by principles of interference.

Chapter II deals with the growth of the dominating waves. Relationships are derived between the waves, the wind velocity and the area of water over which the wind blows (fetch), or the length of time that the wind has blown (duration). The numerical application of these relationships requires the knowledge of the amount of energy available for the growth of waves under various conditions. In this report a first attempt is made to estimate the difference

between the energy supply by wind and the energy dissipation by eddy viscosity at different stages of wave development. These relationships have been developed on the basis of theoretical considerations and application of empirical laws. The results are presented in graphs and tables for practical use, by means of which the height, length, period and velocity of dominating waves at different wind velocities, fetches or durations can be estimated. The comparison of theoretical results with observations shows satisfactory agreement.

ON WIND GENERATED OCEAN WAVES

WITH SPECIAL REFERENCE TO THE PROBLEM OF WAVE FORECASTING

Introduction

The most familiar cause of sea surface waves is the wind, and it is on these wind generated waves that this report dwells. In recent years, questions about the growth of waves under the action of wind, and the dimensions of waves in the fully developed state have been brought into the foreground of special oceanographic work. It was due to necessity that this recent period of sea wave investigations was begun. Nobody who follows the sea or depends in any way on the state of the sea can ignore the behavior of waves. Not only navigators, sailors and fishermen, but naval personnel, naval engineers and seaside dwellers who have to protect their coast against the attacks of the sea, are deeply interested in this problem.

The state of the sea, or the sea surface roughness pattern, is a complicated combination of many waves often including swell. The rather steep breaking waves of irregular appearance result from local winds, whereas the more regular undulating swell will in general have been generated in an area far away from the region of observation. For analyzing quantitatively the state of the sea at different wind velocities, it is required to separate these two independent phenomena. When describing or forecasting the state of the sea under different conditions, independent forecasts of local wind generated waves and of swell are necessary. After combination of the two separated procedures we may get the actual state of the sea surface pattern.

Many new ideas from recent progress on this subject have proven to be useful for important practical work. Present methods of fore-

casting sea state are based essentially on the earliest American report on a method for forecasting sea and swell by H. U. Sverdrup and collaborators in 1942, later revised and published by H. U. Sverdrup and W. H. Munk [1]. Its publication stimulated further investigations of wind and sea state relationships, so that more comprehensive empirical data were soon available.

Upon surveying the results of investigations up to the present, we find a lack of knowledge with respect to certain aspects, sometimes as it seems, to basic problems. There is the question of the "characteristic" waves, or those waves existing at a certain wind velocity when the sea is fully arisen, and also on the period, wave length and height of waves. First, we must know the characteristic pattern under different conditions at a sea surface which has complex wave motion. The definition of the "significant wave" given by Sverdrup and Munk is not very satisfactory. It is a statistical definition which takes merely a certain average for the highest waves and approximately the wave length of the longest wave present at the sea surface. The steepness of these waves, when fully developed, is about 1:46 (ratio of wave height to wave length), but there are much steeper waves present with a velocity of propagation smaller than the wind velocity. These characteristic "seas" cause the broken appearance of the sea surface and obscure to a large degree the presence of longer and flatter waves.

Furthermore, there are some questions about the energy transfer from wind to waves and about the growth of the waves under the action of wind. What type of pattern of sea surface (wave heights and lengths) is to be expected when a wind of a certain mean velocity has blown over the sea surface for one, two, three or more hours?

Or, what type of pattern may we expect to find when observing the waves at different distances from the windward shore?

This report is concerned with questions, which aim primarily at the practical ends of the problem of wave forecasting. It deals merely with wind driven waves, while the problem of swell forecasting will be considered in a later report.

Chapter I.

The composite nature of wind generated waves according to observations

1) Comments on observations

It seems very difficult to define a distinct wave motion at the sea surface, because of the composite pattern of the "sea," as the undulatory motion of the sea surface in the case of wind-driven waves is called by seamen. Besides the wind-driven sea it is possible that one or more different types of swell are running across or in the same direction as the waves generated by the local wind, which fact further complicates the resultant wave motion and therefore the observations at a given locality. The swell is not causally connected with the local wind, and this report does not take into account its behavior when travelling over long distances at sea. The discussion at first pertains only to waves under the direct action of wind, considering the matured state as well as the state of wave development. Therefore when trying to observe wind generated waves we have to eliminate, if necessary, in the best possible manner any swell that appears. Sometimes this separation seems very difficult, especially when taking mechanical wave records, like the recent wave records of H. R. Seiwell [10]. They were obtained by automatic recordings of wave pressure variations at the sea bottom. The results of H. R. Seiwell obtained by application of the principles of generalized harmonic analysis to such oceanic wave observations, and the conclusions drawn from the mathematical treatments--as far as they have been published to this date--may be summarized after H. R. Seiwell and G. P. Wadsworth [11] as follows:

"1. Periodogram analyses performed on oceanic wave records do not appear to give correct geophysical information. The numerous wave periods, and bands of periods, indicated by this type of analysis do not necessarily possess physical significance.

2. Application of the hypothesis of generalized harmonic analyses to western North Atlantic wave records indicates that ocean wave patterns are not complex interference patterns resulting from combinations of many wave frequencies, but frequently consist of a single sinusoidal wave ("cyclical component") on which is superimposed an "oscillatory component." (In the case of the published wave record 53-X [11] the latter component appeared with the same period as the cyclical component.)

3. The cyclical component appears to be that generated under the influence of a dominating oceanic meteorological situation, and the oscillatory component by local winds and other local disturbances tending to change the basic ocean wave pattern."

The physical meaning of these mathematical results seems somewhat obscure. But there is a fact to which we have to pay attention when considering the results of Seiwel's investigations: the records of Seiwel, as far as published, comprise sea bottom pressure observations and the results mentioned above are based on these pressure-recordings. However the pattern of pressure-fluctuations at the sea bottom does not agree with the actual pattern of complex surface waves. Shorter waves are filtered out, and in his papers of 1948, H. R. Seiwel [10] calls special attention to the fact that surface wave lengths less than 240 feet were not registered by the Bermuda wave recorder at a depth of 20 fathoms. The average bottom period

of waves at Bermuda was about three times that at the surface and at the Cuttyhunk observations the average period of bottom waves was about two times that at the surface. Furthermore, the longer waves of the complex wave motion are influenced by shallow water effects when approaching the location of recordings from the deep open sea. The results obtained by pressure-records are therefore not strictly comparable with observations at the surface of the open sea. The pressure-recordings evaluated by H. R. Seiwel indicated that ocean wave patterns are not complex interference patterns resulting from combinations of several (two or more) distinct waves, but frequently consist of a single wave. This fact is perhaps the consequence of filter-effects: the superimposed shorter waves disappear, because they are filtered out by damping.

During his stay at the Woods Hole Oceanographic Institution in the summer of 1951, the author became aware of new unpublished wave recordings made by Seiwel in 1950 at the sea surface. The director of the Woods Hole Oceanographic Institution was kind enough to permit the use of these data and to look into the results of mathematical analysis as far as these data were available.* Unfortunately most of the original data and the results of analysis were lent out, but the remaining part of these observations already seems to indicate important differences compared with the former observations and results, which were obtained at small water depths by pressure recordings. This new material, collected in 1950 in the vicinity of the Bermudas, comprises sea surface measurements taken by photographic

*The author is very much obliged to Admiral E. H. Smith and Dr. C. O. D'Iselin for permission to make use of these unpublished data of H. R. Seiwel.

recordings of sea surface wave heights from which discrete values are scaled at equidistant time intervals. The most striking feature of the results of the mathematical analysis seems to be that the wave patterns at the sea surface are dominated by more than one "cyclical component" in most cases, which agrees with the findings of the author. But in his notes on his work, H. R. Seiwel points out that the autocorrelation structure of data containing more than one cyclical component becomes complicated and usually does not permit identification of the periods involved. Its use is limited to a means of revealing the presence of more than one period, and further information concerning the identification need be obtained by some form of a Fourier Transform of the autocorrelations into a power spectrum. One may look forward with interest to the results of analysis of Seiwel's surface observations in 1950.

Another method for observing the state of the sea under different conditions, particularly from ships under way, is based on stereophotogrammetric pictures. This method [2] clearly reveals the complicated configuration of the wind affected sea surface, and makes possible exact morphological measurements, though the present possibilities of this method are limited too. It is a rather expensive method and there are not many observations available at present. A recent summary of the results obtained and an outlook on future developments with respect to stereophotogrammetric wave pictures in rapid sequence by A. Schumacher [3] shows that the improvements of this method may be expected to prove very useful in future work.

Besides these more expensive methods, there are several simple methods of measuring waves from ships under way by direct observations. Compared with some highly developed technical methods

for modern geophysical observations, these "old fashioned" simple "observations by eye" may perhaps appear too primitive today. Most of them were proposed and have already been used in the past century. But a great deal of our empirical knowledge on waves today is based on such direct observations, and up to date these simple observations by eye have proved very useful when taken carefully and utilized critically. There are descriptions of these methods in several textbooks on waves (V. Cornish [4], O. Krümmel [5], H. Thorade [6]) and more recently they are briefly discussed in H. O. Pub. No. 602 by H. B. Bigelow and W. T. Edmondson [7].

The time intervals between succeeding crests of waves at a fixed locality* can be estimated from shipboard with a fair degree of accuracy by observing the rise and fall of the water, observing for example, foam patches, seaweed or other bodies drifting just beneath the sea surface. This method was shown to be very useful by V. Cornish. But in taking such measurements, it is necessary to observe a large number of single vertical oscillations under the given conditions, because the single values of periods and heights of succeeding waves differ over a large range. Furthermore, it should be warned against simply averaging these widely scattered observations. There is much confusion in our empirical knowledge of the relation between wind and waves due to the fact that one has considered only some individual observations under certain conditions, or that one has summarized the measurements by taking "average values" over a large range of variation.

In the fall of 1950 and the winter of 1950-51, while crossing

*This quantity will be referred to hereafter as "wave period."

the North Atlantic Ocean with a slow-speed freighter, the author took the opportunity to make systematic wave observations. The voyage started October 14, 1950, at Hamburg, aboard the "Heidelberg." She docked at ports in the Caribbean Sea, in the West Indies and in the Gulf of Mexico, and returned through the Straits of Florida to Hamburg again, where she arrived February 3, 1951. Crossing the Caribbean, the Gulf of Mexico and round the West Indies there was often a good opportunity to observe the state of wave development due to different fetches. Besides systematic observations of wave periods and heights together with exact wind measurements, the author provided for special studies on wave groups and for the making of moving pictures of sea waves. Altogether, about 27,000 observations were collected under different conditions in the North Atlantic, the Caribbean Sea and the Gulf of Mexico, the values ranging up to wind velocities of 22 m/sec. In order to get an idea of the daily North Atlantic weather situation and its history, two weather maps were drawn daily on the basis of ship reports. The methods of observation are described in more detail in "Deutsche Hydrographische Zeitschrift" [9], where the observations themselves are also published.

2) Characteristics of waves in the fully developed state at different wind velocities

The observations aboard the "Heidelberg" related mainly to measurements of time intervals between succeeding crests ("periods") and heights of succeeding waves in the complex wave motion, simultaneous with wind measurements by cup-anemometers. Between 70 and 150 or more single measurements of periods at a given locality under nearly

steady wind conditions were collected and related to the measured wind speed. In this way, about 27,000 single observations have been placed together in a set of more than 250 "series," comprising the range of wind velocity between 2 and 22 m/sec. Separated from these series, further sets of wave measurements at limited fetches were gathered which will be used in Chapter II.

The single measurements of each series were utilized by counting the frequency of the observed periods, taking period intervals of 0.5 seconds, and then representing them in diagrams showing the frequency distribution of periods at a certain locality and at a certain wind speed. Observations under conditions with rapid wind changes are here omitted. This method of counting time intervals between succeeding crests naturally provides no exact mathematical analysis of water level fluctuations or of time series in general. But it seems to be useful just for practical purposes. The result provides a realistic picture of sea surface state, showing the periods which are to be expected in the characteristic composite wave pattern, the range of periods under different conditions, and the more or less frequent appearance of periods in certain "bands."

In the composite wave pattern the rather steep "breaking sea" was the most conspicuous undulation as long as the sea surface was under the influence of generating winds, whereas the superimposed smaller waves, as random components, did not disturb this pattern or influence it essentially. But the heights and periods of succeeding waves at a fixed locality mostly scattered through a large range, and in the ever-changing wave patterns of the sea surface longer waves with greater periods appeared occasionally, clearly marked by smaller

steepness, but running in the same direction as the wind and even as high as the steeper "tumultuous sea." Following these longer waves by eye from a look-out position on the mast, they appeared to change their shape rather soon and disappeared, being replaced by steeper and shorter waves.

In figures 1 to 3, three series of observations are represented as examples of frequency distributions for observed periods at nearly steady wind conditions. The characteristic periods cover a more or less broad interval depending on the wind velocity. At lower values we find mostly a rather sharp limitation of the observed period interval. This means that waves with periods lower than a certain value have only minor significance, and do not visibly influence the striking pattern of sea surface roughness.

Figure 1 shows a scattering of observed periods between about 2.5 and 8.5 seconds, and some distinct peaks in this interval. Besides a frequency maximum between 3.0 and 4.5 seconds, maximum peaking occurs between 5.0 and 6.5 seconds, and at 7.5 to 8.0 seconds. The single observations of this series were obtained when the ship drifted with stopped engines about 150 nautical miles southwest of the Azores. The wind was steady easterly with a velocity of 9 m/sec.

In figure 2 are represented the observations of a series at 13.5 m/sec wind velocity. The periods scatter between about 4 and 13 seconds. Maximum peaking occurs at about 6.6, 8.7, and 11.8 seconds. Similar distributions are to be found in other series. At a wind velocity of 16 m/sec (figure 3) the range of periods is between 6.5 and 15 seconds with maximum frequencies at 8.3 seconds for the shortest characteristic waves, between 10.0 and 11.5 seconds for the next

Fig. 1

Fig. 2

Fig. 3

Fig. 1-3 Frequency distribution of observed wave periods T (sec) at different wind velocities. For explanation of T_1 , T_2 , and T_3 , see text. (Steady wind conditions).

longer, and between 12.5 and 14.5 seconds for the longest periods of waves present.

To present a simple interpretation it seems that two or three frequency bands could be defined as characterizing the observed data in the case where the sea was generated by a quasistationary and quasi-homogeneous wind field. Hence the usual conclusions drawn from the observations would be that the sea surface roughness pattern consists of discrete bands which indicate dominating periods in the composite wave motion.

It may be pointed out here that a nearly continuous distribution of observed periods is found in series of observations under more complicated wind conditions; for example, where a stronger wind at the windward part of the fetch generates a fully arisen sea, while the wind decreases to the leeward, where the sea is observed (see page 26).

The prediction of certain characteristics of period patterns may be based on the prediction of the distribution function for individual wave periods. Statistical data for individual wave periods at given wind velocities indicate regularities in the distribution, namely, the range of characteristic periods, the most frequent periods, and the asymmetry of the distribution. Some examples for fully developed sea are presented in figure 4, covering the range of wind velocities between 5.5-6.4 m/sec, 7.5-8.4 m/sec, 10.5-11.4 m/sec, 11.5-12.4 m/sec, and 14.5-15.4 m/sec, respectively. In these distribution graphs all data are collected from observations at fully developed sea within the given wind velocity interval. The graphs show the percentages of observed characteristic periods at

Fig.4. Frequency distribution of characteristic periods at different wind velocity v (m/sec) (Fully developed sea).

the given range of wind velocity. In a later report ("Instructions for practical wave forecasting") certain "standard types" of period distributions will be presented for practical use, covering the wind velocities up to 24 m/sec.

The results of all measurements are summarized in figure 5, taking the periods of maximum frequencies out of each individual series and plotting them as a function of wind velocity. This diagram shows the range of characteristic periods between wind velocities of 2 m/sec and 20 m/sec. At low wind velocities of about 4 m/sec we find a scattering of observed periods at fully developed sea between 1 and 4 seconds, at moderate wind of about 10 m/sec between 4 and 10 seconds, and at strong wind of about 16 m/sec between 7 and 15 seconds.

In most cases the lowest characteristic period T_1 appeared clearly separated from the highest period T_2 , but very often secondary maxima were indicated in the frequency distribution between two limiting periods, as shown in figures 1 to 3. These different periods, indicated by peaks in the diagrams of the different observational series, are marked in figure 5 by dots, circles and crosses. At low wind velocities the intermediate period T_3 seems to approach the period T_2 , whereas at higher wind speeds it seems to approach T_1 . At winds lower than 4 m/sec the occurrence of higher periods was less striking and at light winds less than 3 m/sec no observations of characteristic periods higher than T_1 are made. But in all cases the observed periods scattered over a certain interval, as is generally to be expected in interference patterns (see section 3).

The results of observations represented in figure 5 indicate

Fig.5. Periods of characteristic waves (T) as functions of wind velocity v (m/sec). (Observations are indicated by symbols.)

regularities in the relation between wind and sea state at different wind speeds, which do not stand out if only single observations under different conditions are taken at random, or when the observed periods are averaged over the whole range of scattering values. This perhaps explains the fact that the older attempts to determine empirical relationships between wave periods or wave lengths and wind velocity led to less satisfactory results.

The most conspicuous feature of the sea surface pattern at all wind velocities is, as already mentioned, the occurrence of the steepest characteristic waves, which are slower than the wind velocity. They are indicated in Fig. 1-3, 4 and 5 by the period T_1 . The curve T_1 in Fig. 5 is based on previous calculations of the relationship between the dimensions of fully developed wind waves and the wind velocity in the composite wave motion (G. Neumann [8]). The condition that the energy supply by wind to waves equals the energy dissipation by turbulence at fully developed sea leads to a theoretical relation

$$T_1(\text{sec}) = \frac{2\pi}{g} \frac{2r\left(\frac{\sigma_2}{v} - 1\right)}{\ln 182.5 - \frac{1}{2} \ln v} \cdot v \quad (1a)$$

where v is the wind velocity in cm/sec, σ_2 the propagation velocity of the fully developed "longer waves" in cm/sec (see [8]), g the acceleration of gravity, and the dimensionless constant $r = 1.667$. $\ln$ denotes the natural logarithm. With the value $\sigma_2/v = \beta_m^* = 1.37$ formula (1a) leads to the same periods as evaluated formerly [8], except at wind velocities lower than 3-4 m/sec, where the periods given by (1a) are slightly larger than the periods given in the paper

of 1950. But the difference does not seem to be very important. Formula (1a) will be derived in Chapter II.

The periods of the fully developed "longer waves" are, as it seems, fairly well approximated by a linear relation

$$T_2(\text{sec}) = \frac{2\pi}{g} \sigma_2 = \frac{2\pi}{g} \beta_m^* \cdot v = 0.877v \quad (v \text{ given in m/sec}) \quad (2a)$$

This relation is based on the assumption $\sigma_2 = 1.37 v$ as suggested by empirical evidence and already used in the papers of Sverdrup and Munk [1], and of Neumann [8]. At present, it is difficult to say whether this relation is exactly a linear one, or whether there is a slight curvature. But possible deviations from the linear proportionality as given by (2a) seem to be of the second order.

The period T_3 in Fig. 5, although not always clearly separated from T_1 or T_2 , indicates an intermediate wave with a velocity of phase propagation very near to the wind velocity. The broken line T_3 in Fig. 5 is calculated by taking $\sigma_3 = v$ and represents the relation

$$T_3(\text{sec}) = \frac{2\pi}{g} v = 0.64 v \quad (v \text{ given in m/sec}) \quad (3a)$$

These three waves which presumably dominate the composite sea surface pattern in fully or nearly fully developed state may be called

- 1) the "sea" (T_1), or β_m -wave ($\sigma_1/v = \beta_m$)
- 2) the "longer wave" (T_2), or β_m^* -wave ($\sigma_2/v = \beta_m^*$)
- 3) the "intermediate wave" (T_3), or $\beta(1)$ -wave ($\sigma_3/v = 1$)

The periods and wave-lengths of these three waves are given in Table 1 at different wind velocities up to 24 m/sec.

Table 1.

Periods T and wave lengths λ of dominating waves at different wind velocities.

$v(\text{m/sec})$	2	4	6	8	10	12	14	16	18	20	24
$T_1(\text{sec})$	0.615	1.42	2.36	3.37	4.5	5.7	7.0	8.3	9.7	11.3	14.5
$\lambda_1(\text{m})$	0.58	3.16	8.7	17.8	31.5	50.8	75.1	107	148	198	327
$T_2(\text{sec})$	1.76	3.51	5.25	7.03	8.8	10.5	12.3	14.0	15.8	17.6	21.1
$\lambda_2(\text{m})$	4.8	19.2	43.2	76.8	120	172	236	307	387	481	693
$T_3(\text{sec})$	1.28	2.56	3.84	5.12	6.4	7.7	9.0	10.2	11.5	12.8	15.4
$\lambda_3(\text{m})$	2.56	10.2	23.0	41.0	64	92	126	163	207	256	370

The results of some measurements of wave heights are represented in figure 6. These measurements relate to observations in the open sea, and as much as possible to stable weather conditions. They were taken by estimating the eye-height above the water line when the ship was on an even keel in the trough of the waves, and leveling the wave crest with the horizon. During the time between October 25 and 30, 1950, in the region SW of the Azores more reliable measurements have been made from the drifting ship at low wind velocities between 4 and 10 m/sec. The ship drifted slowly athwart to wind and sea, and accurate measurements could be made from the overhanging bow by means of readings at a sounding line with marks hanging down into the water. The opportunity for making such measurements from the drifting ship arose several times during the voyage.

The variation of heights of succeeding waves is the most striking feature of fully developed sea, and it seems impossible to decide which height is the characteristic one. A single measured height has only little significance. It was attempted to determine

Figure 6. Observed heights of characteristic waves at different wind velocities.

the average height and the upper limit of the height of succeeding characteristic waves as accurately as possible. These limits are indicated in figure 6 by vertical lines. A single cross (x) means average heights without estimation of the upper limit. We find at a wind velocity of 15-16 m/sec a variation in the height of characteristic seas between about 4 and 8 m, which may be interpreted as an interference phenomenon. But at higher wind velocities it is difficult to determine the upper limit exactly if the long rolling sea interferes suddenly and the steepened high crests fall forward, often forming breakers of several meters in height.

In the open sea, successive waves always differ considerably

in height and length, and from time to time a wave comes that is considerably higher than the common run, or its time interval from the preceding crest to the following crest observed at a fixed locality is much longer than the time intervals of the waves before and after this particular wave. These longer or higher waves disappear on their track after some undulations, changing their pattern continuously. If it were possible to observe these conspicuous waves long enough from a high point it would seem that they are replaced by others not so conspicuous, while waves with uncommon heights and lengths are growing up at another place. Thus waves with a height of 3.0 to 3.5 m were observed on the "Heidelberg" when the usual height was about 1.5 to 2.5 m (wind velocity 5-6 Beaufort). During the storm in the eastern North Atlantic on 27 January, 1951, at 20 m/sec wind velocity from time to time the wave height attained 12-13 m while the average of the common run was about 9 m. On 13 January, 1951, at noon, still larger variations occurred at 14-15 m/sec wind velocity. On the average, the common run was about 4 m in height, while some individual waves attained a height of 7 m or more. In the evening of this day, the wind increased to about 8 Bft. with squalls to 10 Bft. and the wave height of single waves arose to 10-11 m while the average was about 8 m (see Fig. 6).

The curve in figure 6 represents the relation

$$H = 0.215 \frac{g}{2\pi} T^2 e^{-1.667 \frac{g}{2\pi} \frac{T}{v}} \quad (4a)$$

with $T = T_1$. Formula (4a) is given by the empirical relationship

$$H = 0.215 \lambda e^{-1.667\beta}$$

used in Chapter II (see also [8]). In (4a) $\beta = \sigma/v$, and σ , T and λ are the propagation velocity, period, and wave length of the considered wave given by (1a), (2a) or (3a) as a function of v . If v is given in m/sec, we get H in meters by means of (4a). Table 2 shows the mean wave heights at different wind velocities calculated by (4a) with $T = f(v)$ according to (1a), (2a) and (3a) for the three dominating waves. At moderate and high wind velocities the difference between the height of these waves is small, and especially H_3 nearly equals H_2 .

Table 2.

Mean wave heights (meters) at different
wind velocities v (m/sec).

v m/sec	2	4	6	8	10	12	14	16	18	20	24
H_1	0.05	0.26	0.66	1.3	2.1	3.2	4.4	5.9	7.8	9.8	14.6
H_2	0.11	0.43	0.96	1.7	2.7	3.8	5.3	6.8	8.6	10.7	15.4
H_3	0.106	0.41	0.93	1.66	2.6	3.7	5.1	6.6	8.4	10.4	15.0

It must be pointed out that the height of characteristic waves, if taken as a mean and if considered without respect to the actual wave length is less typical for the state of the sea than the steepness of the waves, and the everchanging sequence of wave heights and periods at the rough sea surface with its composite wave motion. This feature also seems to be of more practical importance than any average values. Steep and high waves may be dangerous for ships and amphibious aircraft, aircraft carriers, etc., especially when the waves run in groups and form high breakers, whereas flat swell or "dead sea" even as high or perhaps higher than the steep breaking waves is for the most part less dangerous or not dangerous at all

in the open sea. But these longer and flatter waves achieve significance and practical importance when entering shallower water or when approaching the shores (surf).

3) Composite wave patterns. Interference phenomena. Groups of waves.

Everchanging patterns and single outsize waves are striking features of the sea surface roughness, and we have to consider these facts in practical wave forecasts. Just these single waves which often cause heavy breakers in storm areas have to be taken into account for practical purposes.

The source of outsize waves of this sort is the merging of those waves that advance with different velocities, because longer and faster waves are constantly overtaking the slower running ones. This always happens when there are two or more series of characteristic waves present, and the phenomenon is called interference. More complicated patterns of outsize waves may occur when waves are running in different directions, combining a series of characteristic waves produced by the local wind and "dead sea" (very young swell) running from another direction. This happens under certain meteorological conditions, for example when passing cold fronts with pronounced "wind-jumps," or in some sectors of hurricanes, especially in the "eye" where "dead sea" of different directions and locally produced sea interfere. The so-called "cross-sea" is a well known phenomenon, where waves are piled up in single interference patterns to considerable height, often forming irregular dangerous breakers in the fierce sea. But it seems that this tumultuous cross sea is usually dissipated in a rather short time.

More complicated sea surface patterns also may be encountered when the wind blows in violent squalls. The effect of violent squalls in gales is--as every seaman or everyone who has observed the sea in the more stormy parts of the oceans knows--a rather rapid increase of the waves. There are more breakers to be observed and the sea surface is covered with more white foam patches taking on the appearance of more fierceness. But it seems that at first mainly the smaller superimposed waves are increased in their size. Overtaken by longer and faster running waves they pile up the crests and fall forward in foaming breakers (see Chapter II, section 6). The sizes of the largest waves produced by squally wind may correspond to a wind velocity somewhat higher than the mean wind velocity or about the average velocity of the wind within the squalls.

Furthermore, we have to take into account that the time average of the wind velocity for the most part is not constant over the whole fetch. This leads to further complications and makes the practical wave forecast much more difficult, especially if there is an increase of wind velocity to the windward of the fetch. Consider, for example, a locality of observation on the western part of a low in the North Atlantic Ocean within the region of NW or NNW winds. It occurs that this region westward of the cold front has a large extension and the wind direction is approximately the same in this sector, but the wind velocity may be 2 Bft. degrees or more higher in the northern part than at a given point of observation in the southern part. From this northern region "dead sea" arrives at the point of observation, perhaps not even as high as in its region of origin, but high enough to disturb the interference patterns of local sea at the southern end of this fetch. It may happen that these more

complicated interference patterns produce a higher and longer sea than that which would be the normal state with respect to the wind conditions in the middle and southern parts of the fetch. Perhaps it may give the impression that the (local) waves have been developed to larger size at the utmost end of the fetch as a result of increasing. On the growth of the sea under the action of wind, see Chapter II.

In the following, an attempt is made to explain some striking features of complex wave motion by principles of interference, considering at first steady-state conditions at the end of a fetch over which a steady wind is blowing and assuming infinitely wide waves.

Let $\lambda_1, \lambda_2, \lambda_3$ be the wave lengths of the three characteristic waves mentioned in Chapter II, $\sigma_1, \sigma_2, \sigma_3$ their velocities of propagation, and a_1, a_2, a_3 the amplitudes of these waves. Assuming sine wave profiles with first approximation, we have

$$y = a_1 \sin 2\pi \left(\frac{\sigma_1 t - x}{\lambda_1} \right) + a_2 \sin 2\pi \left(\frac{\sigma_2 t - x}{\lambda_2} \right) + a_3 \sin 2\pi \left(\frac{\sigma_3 t - x}{\lambda_3} \right).$$

This equation contains both of the variables, the time t and the fetch x . We chose $x = 0$ and $t = 0$ so that at this place and time all three waves have the same phase and $y = 0$. With regard to an observer at sea, who is measuring the waves at a fixed locality as a function of time, we consider the variations of the resulting wave motion at a given place $x = \text{const.}$

Let us take as an example a wind velocity of 16 m/sec. There we have to expect the following wave lengths, periods, wave velocities and heights, supposing the wave motion is fully developed

according to (1a), (2a), (3a), (4a)

$\lambda_1 = 107 \text{ m}$	$\lambda_2 = 307 \text{ m}$	$\lambda_3 = 163 \text{ m}$
$T_1 = 8.3 \text{ sec}$	$T_2 = 14.0 \text{ sec}$	$T_3 = 10.2 \text{ sec}$
$\sigma_1 = 12.95 \text{ m/sec}$	$\sigma_2 = 21.9 \text{ m/sec}$	$\sigma_3 = 16.0 \text{ m/sec}$
$a_1 = 3.0 \text{ m}$	$a_2 = 3.4 \text{ m}$	$a_3 = 3.3 \text{ m}$

Fig. 7 shows the resultant wave patterns at a wind velocity of 16 m/sec. The upper wave train (a) represents this theoretically constructed "wave record" at a locality $x = 0$ for a time interval of 260 sec. The lower wave train (b) represents a "wave record" during the same time at a locality $x = 550 \text{ m}$, which means about $1/3$ nautical miles away from the locality of "wave record" (a) in the direction of wave propagation. These theoretically constructed "wave records" indicate that groups of large waves are to be expected only intermittently at a certain locality. At $x = 0$ in our example they occur at the beginning, at $x = 550 \text{ m}$ at the end of the "wave record." The groups follow with a time interval of about 45 sec between each group. Within these groups some waves are growing up to considerable height. But in nature these waves presumably do not attain their exact theoretical height. When approaching a certain maximal steepness which depends upon the wind velocity, they become unstable and the crests break over. Because of a certain regularity in the occurrence of these outsize waves in groups it is to be expected that there is a certain regularity in the occurrence of high breakers too.

When a "family" of distinct groups has passed the locality of observation, the sea surface patterns gradually change their characteristic appearance. By and by the distinct groups disappear and

Fig. 7 Wave pattern of fully developed waves at a wind velocity of 16 m/sec. Computed "wave records" for a time interval of 260 seconds at two different localities
 "Wave record" (b) is located 550 m downwind from (a).

the striking contrast between extremely high waves and lower ones is smoothed out. The waves at the sea surface now show a more irregular pattern without distinct groups, until after some time new groups of waves may arise.

When discussing the results of observations in Section 2, a large scattering of periods over a certain interval was pointed out. Scattering periods are to be expected in any way when considering the resultant interference patterns of waves, and the example in Fig. 7 shows these different time intervals between succeeding crests by horizontal arrows. Thus at the place $x = 0$ after passing the high crest at $t = 2.8$ sec, we would observe the next crest after 9.2 sec and the following crests after 7.0; 14.9 (double-wave," 7.7; 7.2); 10.2; 9.0; 8.0; 14.0; 10.1; 8.6; 8.4 seconds and so on. These periods are to be compared directly with the representation of the observations in Fig. 3, obtained at a wind velocity of 16 m/sec in fully developed wave motion.

Fig. 8 represents some profiles of wave patterns, showing the variations of dominant waves with space and time at $v = 16$ m/sec. The wave profiles are computed for time intervals of 3 seconds and extend over a distance of 1600 m as shown by the scale of distance at the upper horizontal line.

By means of these profiles some outstanding features of complex wave motion may be explained. For example, we find the striking wave a_1 at $t = 0$ at the place $x = 350$ m, but, when progressing, this wave decreases in height very rapidly as indicated by the dashed line which is drawn from A downwards to the right hand side. After 15 seconds this wave has moved about 200 m farther and is to be found

Fig. 8. Profiles of wave patterns, showing the variations of dominant waves with space and time at a wind velocity of 16 m/sec.

at B only as an unimportant elevation, whereas the following crest a_2 has risen to a wave of considerable height. We find similar rapid variations at other places too, when examining the changing wave patterns with regard to space and time, for example, between G and H at a distance of 1400 m. This rapid decay on the one end

and growing up of single waves on the other end is a fact well known from observation of the sea. But besides this, there are other waves which change their form much more slowly, as indicated by the wave placed at $x = 550$ m (B) at the time $t = 0$. When it progresses a distance of 280 m, we find this wave after 21 seconds only slightly changed in the crest b_1 .

If we should observe the wave motion at the point B ($x = 550$ m on the distance scale in Fig. 8) we should get the "wave record" (b) represented in Fig. 7 (the lower one). After passing the wave crest b_1 at $t = 0$ the next crest (b_1') follows 8 seconds later, slightly higher than the preceding one. The next wave would be the wave with the crest a_1 , but this wave almost disappears in the following broad trough and in many cases it will not be possible to recognize this intermittent wave. At the time $t = 23$ sec the next high wave crest, a_2 , passes the locality of observation. The "period" between b_1' and a_2 or the time interval between these two striking waves is 14.8 seconds. Waves like these occur rather often in the composite surface pattern of waves and may be called "double-waves" [9]. In all cases the scattering of periods of succeeding waves and the continuous variations of the shape of wave profiles in rather short time intervals are striking features of ocean waves.

Similar wave profiles like those in Fig. 8 are represented in Fig. 9 at a wind velocity of 19 m/sec. Within the relatively short time of 21 seconds the "wave group" between points A and B changes its shape considerably when progressing a distance of about 350 m. At $t = 0$ we find a group of waves between $x = 500$ m and

Fig. 9. Profiles of wave patterns, showing the variations of dominant waves with space and time at a wind velocity of 19 m/sec.

$x = 1000$ m with the crests a_1 , b_2 , b_1 . After a time interval of 12 seconds the height of the wave b_1 has decreased while the small elevation in the trough of the "double-wave" between points B and C has grown up to c_2 , forming a long stretched crest. During the same time another wave a_2 , grows up at the rear edge of the "group."

This wave now plays a role similar to the wave a_1 at $t = 0$, whereas the wave a_1 after about 21 seconds is to be considered as the "middle wave" of this group. The former front wave b_1 has almost disappeared after 21 seconds, while it seems as if this wave is replaced in the group by the former "middle wave" b_2 . The "group" as a whole remains behind the waves. This well-known property of wave groups is observed very often at sea and the wave profiles in Figs. 8 and 9 show these variations by superposing the three characteristic waves evaluated in Chapter 2. The displacement of the highest (not individual) waves in Fig. 9 would lead to a "group velocity" of about 8 m/sec, whereas the steepest characteristic wave, the "sea" at a wind velocity of 19 m/sec has a velocity of phase propagation of 16.4 m/sec ($T_1 = 10.5$ sec, according to formula (1a), $\sigma_1 = (g/2\pi)T_1$).

Theoretical "wave records" or "wave profiles" like those in Figs. 8 and 9, are based on the assumption of three characteristic waves, and are valid in the fully developed state of wave motion. If the wave motion is not fully developed, it is first necessary to calculate the dimensions of the characteristic waves in their different stages of generation at given wind velocities. In any case, it seems possible after these computations, to evaluate characteristic wave profiles or "wave records" for practical purposes. These profiles allow us to predict some striking features of composite wave motion as required by practice, for example:

- (1) the periods of succeeding waves, the heights of succeeding waves and their steepness. The range of these elements and their average and maximum values.

- (2) The time interval between the steepest or highest waves in the complex pattern of wave motion observed at a fixed locality.
- (3) The occurrence and the behavior of wave groups and if possible the appearance of "high breakers."

Chapter II

The growth of waves under the action of wind

1) Introduction and general remarks

Any attempt to calculate the growth of the waves under the action of wind requires the knowledge of both the energy transfer from wind to waves and the rate of dissipation of wave energy in different phases of wave development. The sea may grow only in the case where the supply of energy by wind exceeds the loss of energy by friction and turbulence. At a given wind velocity the waves attain their fully developed state when the energy transfer by wind A equals the energy dissipation by frictional forces D , which are connected with turbulent wave motion. This means that the sea is "mature" when the energy balance $A - D = 0$.

As to the question of energy transfer by wind it seems to be necessary to take into account the complex pattern of sea surface roughness, because the stress or the total drag of the wind blowing over a rough and wavy air-sea interface depends to a large extent on the "short-wavy" roughness, which is always superimposed on the main profile of larger, striking waves. The energy transfer from wind to waves is due partly to normal pressure components and partly to tangential pressure components (tangential stress, drag), and to do a net amount of work both of these wind force components should be regarded as acting on the particle velocity of water due to the wave motion. Therefore, the actual velocity difference between the particle speed and the wind speed immediately at the sea surface must be considered. But at the present state of our know-

ledge it seems very difficult to estimate with the necessary degree of accuracy the value of the single components of wind force under different conditions, especially when considering the complicated sea surface pattern with its complex wave motion. Further difficulties arise in the calculation of the work done by the wind on the wave motion, when the actual wind speed at the sea surface or the actual velocity difference between the motion of water particles and the wind is sought.

Under these circumstances it seems more expedient at present, to make use of empirical relationships between the "effective wind stress" at the rough sea surface and the wind velocity. The resultant action of tangential stresses and normal pressures may be estimated by certain observations and related to the average wind speed at "anemometer-height." "Anemometer-height" is defined as a height of about 10 m above the sea surface, where the vertical increase of wind velocity with height is relatively small.

The second question which becomes quite important in this problem is the question of energy dissipation with turbulent wave motion. The energy dissipation may be due to viscosity or molecular friction, in the absence of turbulence. But in the case of ocean waves, the energy dissipation by eddy viscosity (which Ekman calls turbulent-friction or virtual friction) has to be taken into account. This turbulence in the uppermost layers of the sea seems to be caused mainly by the breaking of larger and smaller waves, associated with whirling and stirring up of the water masses (eddy) due to unstable wave motion. If there are any other more efficient causes which lead to an additional eddy of the sea surface water, that is, to an additional turbulent motion, the wave motion or wave-

formation will be damped out faster. It is well known that a rough sea can be calmed to a certain extent by whirling water which is upwelling on the windward side of a drifting ship. This "natural" kind of wave decay is often used by captains of ships which have to lie to when caught in storms. Lying athwart or nearly athwart to wind and sea, the ship slowly drifts to leeward, causing by its motion a zone of upwelling water to windward. This whirling, upwelling water is an additional source of turbulence and energy dissipation with regard to the local wave motion and acts - as experience shows - as a damper on waves, and at first the steep breaking waves are damped. It is this whirling water caused by the drift of the ship which destroys to a certain extent high and heavily breaking seas mostly far away from the ship to windward, or far enough not to become too dangerous for the ship. At lower wind velocities the same effect of upwelling water is to be observed, and during the voyage with the "Heidelberg" these additional effects of turbulence could be studied when the ship drifted southwest of the Azores for several days with engines stopped. At a wind velocity of 8 to 10 m/sec the relatively small but steep waves of the common run were breaking about 50 m to 100 m windward of the ship and the upwelling water in the drifting path near the ship was almost free of larger breaking waves.

Similar effects of additional turbulence are also to be observed in the wake of ships under way. This is most striking in the case where a slight breeze causes smaller waves or ripples at the sea surface and the sun shines on the water. One can follow the wake of the ship over rather long distances as a nearly smooth,

bright path across the rippled surrounding surface which, with its rough wave motion, appears darker.

With a very weak wind, when the first small wavelets grow up in the initial state of wave formation, the energy dissipation apparently is determined only by viscosity or molecular friction (if the water is not disturbed by strong currents, for example, strong tidal currents or other turbulent motion). Since surface tension is of decisive influence on small wavelets (besides gravity and viscosity), a minimum wind velocity for the generation of initial waves is found (Neumann [14]). This limit is a wind speed of about 70 cm/sec, and the first ripples that will be generated are those with a wave length of 1.75 cm. If the wind speed is 90 cm/sec the wave length of the ripples has increased to 5.2 cm, provided we disregard capillary waves which are generated together with the longer (gravity) waves. (The wave length of these capillary waves at a wind velocity of 90 cm/sec would be about 0.57 cm.) These "initial waves" grow in such a way, that their steepness increases with increasing wind speed. At a wind velocity of about 123 cm/sec these primary waves attain the steepest form possible for stable waves. The ratio of wave height to wave length is in this case $H/\lambda = 1/7$, which is the maximum steepness computed by Michell based on the theory of Stokes. The wave length of the ripples in this state of development generated by a wind of 123 cm/sec velocity is about 10.5 cm.*

If the wind becomes a little stronger, the pointed crests of

*In shallow water layers, where the bottom friction is an influencing factor, the wind velocity for generating waves of maximum steepness is higher than in the case of "deep" water, and the wave length of the ripples with maximum steepness is shorter than in the case where the waves "do not feel" the bottom [14].

the wavelets take on a "glassy" appearance. The wavelets have exceeded the initial state. They become unstable and "break up" at the crests. This "break up" rather soon changes to a distinct "break over," when the wind speed increases, and observations show that at a wind velocity of about 150 cm/sec the crests of the fully developed ripples clearly fall forward. The wave length of these small "breaking" waves is about 20-25 cm. In the further development of wind generated waves into the state of real ocean waves, the turbulence which is now present and connected with the breaking unstable waves in all phases plays an important role. This turbulence (which implies energy dissipation) increases rapidly with increasing wind velocity.

The first attempt to calculate the growth of ocean waves due to the action of wind at different stretches of water over which the wind has blown (the fetches) and at different durations of wind action, was made by H. U. Sverdrup and W. H. Munk [1]. It was the first approach to a scientific basis for this special problem of great practical importance. Until this remarkable work, the results of which have partly been used in practice since 1942 and which was published in its final form in 1947, no one had attacked the complicated problem of wave forecasting. This first attempt incited further investigations both on theoretical and empirical bases, so that new ideas and more comprehensive observations were soon forthcoming. Even if we know today that some of the assumptions made in this treatise do not hold or are an oversimplification of the mechanism of wave generation, this first attempt has its value as a pioneer work because it stimulated further research and new approaches, like this report, and presumably other scientific work

which will follow.

H. U. Sverdrup and W. H. Munk compute the work done by wind to waves ("significant waves") by normal pressures and tangential stresses separately. But there are some uncertainties in their assumptions and the forthcoming calculations seem inconsistent with the degree of accuracy which is required in the energy balance, when considering the growth of ocean wave motion under the action of wind. For estimating the tangential stresses and the work done by this wind force component on the wave motion, Sverdrup-Munk had to assume a "resistance coefficient," γ^2 , which was constant at all wind velocities, although they mentioned that this would not be true in the case where the wind velocity differs too much from the wave velocity. Under these conditions the value of γ^2 would probably be greater. In the present state of our knowledge we have to take into account the fact that there is really substantial evidence that γ^2 varies with the wind velocity ([8], [13]) and with the stage of wave development. The assumption of a constant γ^2 -value is only a rough approximation. A similarly insufficient approximation is represented by the assumption that the tangential stress (drag) over the wave profile is constant at different parts of the wave even if we assume a constant resistance-(drag-) coefficient γ^2 over the wave.

Besides these important questions of energy transfer from wind to waves, which in the meantime were also discussed by Schaaf and Sauer [15] with respect to the tangential transfer, it must be mentioned that Sverdrup-Munk do not include the dissipated energy in the energy budget of growing and fully risen waves. After mentioning the idea, they disregard turbulent friction, arguing that turbulent

friction or eddy viscosity would give too rapid a decrease of wave height, and that it would be necessary to introduce a smaller coefficient applicable only to wave motion. These conclusions are not quite clear and perhaps not admissible. If in the upper layers of the water where wave motion takes place, a certain state of turbulence is present, this "disordered motion" superimposes itself upon the regular wave motion too, without regard to the causes of this "disordered motion." (See the notes on observations in the wake of ships underway and in the windward upwelling water of drifting ships.) In comparison with this "turbulence" all secondary effects of eddy viscosity, which are smaller than this "turbulence" (including molecular viscosity) may be neglected. L. Prandtl [16] states: "Man kann annehmen, dass wenn zwei Ursachen vorhanden sind, die einen Austausch hervorbringen, der wirklich eintretende Austausch ungefähr mit dem grösseren von beiden Austauschbeträgen übereinstimmt."

Our present knowledge of turbulence in the surface layers of the ocean at different stages of wave development is very meager, and we have to suppose that this turbulence depends to a certain degree on other oceanographical and meteorological conditions. Oceanographic observations indicate that the coefficients of eddy viscosity are about 1,000 to 100,000 times as large as the ordinary viscosity coefficients. Assuming that dissipation takes place by ordinary viscosity only, the effect of friction is neglected in the energy balance by Sverdrup-Munk. These authors explain the observed decay of waves only as the effect of air resistance against the advancing wave. When wind-generated waves spread out from the

fetch area into a region of calm or into a region where the wind velocity is small compared to the wave velocity, the waves naturally meet an air resistance. Due to this air resistance a loss of energy takes place and a decay of waves will be observed. In certain cases, turbulence may play an unimportant part in damping the wave motion, or may be absent altogether, which happens perhaps when the waves travel through a region of smooth sea. But this fact does not exclude the significance of eddy viscosity when the waves are growing in the wind area or when they are maintained by wind action. In this case, the turbulent state of real "wind sea" and the energy dissipation by eddy viscosity is not to be neglected in the energy budget. Similarly, eddy viscosity has to be taken into account when considering the decay of waves (swell), if the waves travel through a region of turbulent sea.

Wind generated waves present themselves to the observer as a series of more or less "hill-like" irregular crests separated by intervening troughs. The formation of the typical "short-crested wind-waves" may be explained partly by the turbulence of the wind and the different wind pressures on the windward and leeward slope of the wave profile. In any event, it is to be expected that irregularities of the air current counteract the formation of long-stretched wave crests. This especially seems to be the case where very "young sea," with rather short wave lengths, but characterized by a great steepness, is generated. If once arisen, these shortcrested waves will themselves disturb the air motion and react on the state of atmospheric turbulence over the sea surface. On the other hand, especially in stormy weather, the fully developed sea with its

characteristic waves has more long stretched wave crests, as already pointed out by Cornish (see Thorade, [17]). This fact must be founded in the nature of ocean waves and perhaps it corroborates theoretical results. Taking short crested waves into account, Jeffreys [18] could show mathematically that the waves with long stretched crests require a smaller amount of energy from wind for growing than the short crested waves. The long crested waves therefore have the better chance to grow than the other ones.

The intensity of turbulence or the irregular fluctuations of the wind perpendicular to its average direction may be assumed to be nearly of the same order of magnitude as the fluctuations in the average wind direction superimposed on the mean wind velocity. For a sufficiently long time interval, the fluctuations perpendicular to the average wind direction and their effects on irregular wave motion probably cancel out. A characteristic wave motion and a significant "sea" therefore will be developed only in the average direction of wind, and this fact seems to be proved by experience at sea. The characteristic "seas" propagate in the direction of the average wind, if we consider undisturbed "wind-seas." The irregularities of wave motion, mostly concerning smaller superimposed waves, may be considered as "perturbation effects" when dealing with the growth of the sea under the action of wind and when trying to comprehend the essential features of ocean waves. But nevertheless, we have to keep in mind that the ocean waves with their limited crests are to be considered strictly as a "three dimensional phenomenon," when going into details. This was shown by Jeffreys (see Thorade [17], p. 34).

When considering the growth of the sea under the influence of wind action, many possibilities of generation have to be taken into account. It happens very often that the wind encounters an "old sea," has to destroy it (or partly destroy it) and to generate a new wave motion. Therefore, allowances must be made for waves that are present when the wind starts blowing. Or, in another case, the wind may increase with time and the "sea" grows slowly with this increasing wind speed. However the simplest case is where the wind in the generating area is constant in time and space and begins to blow over an undisturbed water surface. First we shall assume a wind field of constant velocity and direction in the following considerations.

The present approach to wind waves forecasting deals with the growth of the complex sea in the area of wave formation depending upon the wind velocity, the stretch of water over which the wind has blown (the fetch), and the length of time the wind has blown over the fetch (the duration). A first attempt was made to take into account the composite nature of wind generated waves as indicated in Chapter I of this paper. It seems that the development of composite wind generated wave motion from small steep waves to the case of fully arisen sea is not a continuous process. Discontinuities are to be expected in certain states of composite wave formation, because the relatively short "sea" is to be found as a characteristic steep, breaking wave in all further stages of ocean wave patterns up to fully developed sea. Probably we have to assume these rather steep, breaking waves in all higher stages of development so that longer waves with a higher amount of energy may be

generated at the rough sea surface, until the net energy supply by wind equals the energy losses by dissipation in the composite ocean wave pattern. Until now, it has not been possible to observe a continuous growth of certain waves after they have attained a certain height, length and maximum steepness. These waves broke heavily in stormy weather and they did not disappear or increase when the fetch increased or the duration of wind became longer. But when exceeding a certain fetch longer and flatter waves rather soon emerged at this rough sea surface with its steep wave motion, and these waves probably grew independently of the rough sea as individual waves. The superposition of these longer waves with a phase velocity greater than the wind velocity, and the characteristic "sea" with a phase velocity smaller than the wind velocity finally led in the fully developed sea to significant fluctuations of wave periods (time intervals between succeeding crests at a fixed place), heights, and to the occurrence of outsize waves, groups of waves and other phenomena, as described in Chapter I.

This first attempt to consider the growth of complex ocean wave motion under the action of wind, and the ideas involved in theoretical calculations are based to a large extent on observations and on empirical relationships. Both of them are incomplete today, and it is quite possible that more comprehensive information on complex sea wave motion in the future will lead to better approximations and theoretical treatments. This approach does not claim to have the desirable completeness. At present, it seems that we are still rather far away from a complete understanding of the problems of ocean wave generation and behavior. At any rate, it seems

necessary for wave forecasts to consider the complex nature of ocean waves as they present themselves to an observer at sea. Further theoretical work and observations well suited to support new ideas are needed to compile the knowledge which may lead us step by step to a satisfactory solution of all the problems about ocean waves and their generation.

2) Energy transfer from wind to waves

The energy transfer A from wind to rough ocean waves depends at a given wind velocity upon the pushing and dragging forces which act at the sea surface, and therefore it depends upon the present state of wave development itself. In this way, the actual roughness conditions of the air-sea interface become important with regard to the problem. The effective wind force τ may always be split up into a component τ_n acting normally to the "wavy" interface, and into a component τ_t acting in the tangential direction, as represented in Fig. 10 which shows a schematic profile of a rough wavy surface.

At present, it seems very difficult to estimate the real distribution of both of these components (τ_n and τ_t) over the wave profile with the necessary degree of accuracy, especially when the rough irregular forms of ocean waves are considered. Therefore, an attempt has been made to split up the resultant effective resistance of the actual rough sea-surface into a "pressure-resistance" and a "friction-resistance." This has been done for practical reasons by considering the smooth form of the wavy air-sea interface and approximating it by a "general wave profile" ("Hauptprofil," G. Neumann [8]). The resistance of the "rough" superpositions on this general profile, which would have to be counted as "pressure-

Fig. 10. Schematic wave profile with a rough surface (η), and wind force components (τ_n , τ_t) of the effective wind stress τ . (σ = velocity of wave propagation in direction of the wind velocity v).

resistances" in the exact meaning of the definition, will be attributed to an effective "frictional-resistance" with regard to the general profile (L. Prandtl, [16], p. 160). In this way, the wind force acting on each surface element of the rough sea surface may be divided into an effective pressure force and an effective frictional force, where the first one may be considered as a normal component and the second one as an effective tangential component with respect to the general profile. The work done by these single components on the waves at present cannot be estimated with the necessary degree of accuracy, if we consider their actions separately. It only seems possible to examine these effects in a merely formal manner, if we consider a very simple main profile of waves, as is shown, for example, in Fig. 11. We assume a simple sine wave, for which the elevation relative to the undisturbed level may be given by

Fig. 11. Streamlines of the air over a wavy surface, and distribution of wind force components τ_n and τ_t in schematic representation.

$$\eta = a \sin \kappa(x - \sigma t), \quad (1)$$

where a is the wave amplitude, $\kappa = 2\pi/\lambda$ the wave number and σ the velocity of wave propagation (phase velocity). The horizontal component of the displacement of a water particle at the surface is

$$\xi = a \cos \kappa(x - \sigma t). \quad (2)$$

With these expressions, we have the vertical (w_0) and horizontal (u_0) component of partial velocity as given by

$$w_0 = \partial \eta / \partial t = -a \kappa \sigma \cos \kappa(x - \sigma t), \quad (3)$$

and

$$u_0 = \partial \xi / \partial t = a \kappa \sigma \sin \kappa(x - \sigma t). \quad (4)$$

When considering waves of finite amplitude, Stokes' theory concerning irrotational waves leads to the important result that upon the completion of each nearly circular orbital motion the water particles have advanced a short distance in the direction of wave propagation. The average velocity of this forward motion at the sea surface during one wave period is $u_0' = a^2 \kappa^2 \sigma$. Thus Stokes' waves with finite amplitude are accompanied by a horizontal mass transport of water. Taking this into account, the horizontal component of particle velocity at the water surface may be written

$$U_0 = u_0 + u_0' = a \kappa \sigma \sin \kappa (x - \sigma t) + a^2 \kappa^2 \sigma. \quad (4a)$$

The average rate at which energy is transmitted to the wave by normal pressure is

$$A_n = - \frac{1}{\lambda} \int_0^\lambda \tau_n w_0 dx, \quad (5)$$

and by tangential stress, considering (4a)

$$A_t = \frac{1}{\lambda} \int_0^\lambda \tau_t U_0 dx. \quad (6)$$

Whether these forces do a net amount of work on the wave motion or not depends upon the distribution of τ_n and τ_t along the wave profile. In order to do a positive amount of work on the wave, the wind force components have to be in phase with the components of the particle velocity. Because the normal pressure on the windward slope of the wave profile is on the average greater than on the leeward slope, these pressure forces in general will do a positive amount of work, as long as the phase velocity of the wave is smaller than the wind velocity. In the case where the phase velocity σ exceeds the wind velocity v , the wave form encounters an air resistance,

because the wind relative to the wave acts like an opposing wind. When $\sigma > v$, the vertical particle velocity and the normal wind force component are out of phase by a difference of π , and energy is given off from the wave to the air. In this case the wave motion is slowed down.

There will be no objection to considering the action of this normal pressure component τ_n , and H. Jeffreys [18] took it into account as the main source of wave energy. He assumed the effective pressure component to be proportional to the product of the density of the air ρ' , the square of wind velocity relative to the wave velocity $(v_0 - \sigma)^2$, and the slope of the wave profile $\partial \eta / \partial x$. These assumptions are the most plausible which may be made with regard to the action of normal wind force components as considered here, but some uncertainties still exist when going into details. For example, there is the question of the accurate definition of the difference $v_0 - \sigma$, where v_0 is the wind velocity immediately at the sea surface, and questions about the so called "sheltering coefficient" under different conditions (wave form).

However, it seems that there are more difficulties encountered, and opinions differ when the action of tangential wind stress components is considered. If we assume that the wind blows over a general wave profile which may be considered really as an ideal "smooth" surface in the hydrodynamic meaning of this word, then only viscosity stresses would act. The work done by these viscosity stresses would probably be small compared with the effect of normal pressure components. This perhaps may be the reason that Jeffreys did not take into account a transfer of energy by tangential stress and considered

this process as negligible. This really seems to be the case when the generation of initial waves or wavelets at very low wind velocities is considered ([18], [14]). But if we refer to actual ocean waves, we have to regard a certain general profile with all the smaller superimposed waves, including ripples, as a "rough" wavy surface, where the "effective frictional forces" are determined not by viscosity but by the "roughness" of the wave profile. Therefore the possible work done by these effective stresses, τ_t , on the wave motion may be of the same order of magnitude as the work done by normal components acting on the main wave profile.

The distribution of the effective stress τ_t over the wave profile probably depends upon several factors (roughness, wave form, wind distribution over waves, etc.), all of which may be different under different conditions. If we assume with Sverdrup-Munk [1] that τ_t is constant along the whole wave profile, then in the integral (6) the periodic term would vanish. This means that this force would not do a net amount of work at the horizontal component u_0 (in (4a)) of particle velocity. But the assumption $\tau_t = \text{const}$ seems to be an oversimplification which annuls wind effects of the same order of magnitude as the effects considered with the normal pressure components. Even though it seems very difficult at the present state of knowledge to estimate the accurate distribution of τ_t along the wave profile, we have to assume that τ_t is different at different parts of the wave. The horizontal stress component may be written

$$\tau_t = \rho' f_t v_0^2 ,$$

where ρ' is the density of the air, v_0 the wind velocity immediately

over the "surface" (the wave), and f_t a "resistance coefficient." This dimensionless number depends upon certain surface characteristics and its value has to be determined under different conditions. Here, the same question arises for the actual or "effective" wind velocity v_0 . But we may leave this question open at first and regard only the relative distribution of v_0 over the rough wavy surface (Fig. 11). Then we have to expect that v_0 will be greater over the crests than over the troughs, and τ_t will be greater over the crests too, even if we assume a constant "resistance coefficient" over the wavy surface. Therefore, energy is also transferred by the tangential stress which the wind exerts on the wavy surface. The effect of this drag is to speed up the motion of particles at the wave crests and to slow down the motion of particles at the trough; but the speed-up is greater than the slowdown, so that a net increase in wave energy results not only by "normal pressures" but also by "frictional forces."

It is seen that the attempt to consider separately the effects of the single wind force components encounters many difficulties and uncertainties, even if these effects may be written in a merely formal way.

Let us assume that τ_n on the windward slope of the waves is relatively greater than on the leeward slope and $\tau_n \sim \partial \eta / \partial x$. Then we may write for the distribution of the normal pressure component over the wave profile

$$\tau_n = \overline{\tau}_n + \tau' \cos \kappa(x - \sigma t) \quad (7)$$

where $\overline{\tau}_n$ represents a constant pressure value over the wave profile.

If a relatively higher wind velocity over the wave crests is considered, the distribution of wind velocity over the wave may be

written in the form

$$v_o = \bar{v}_o [1 + \pi \delta \sin \kappa(x - \sigma t)] ,$$

where $\delta = 2a/\lambda$, and $\bar{v}_o$ means a constant average value of the wind velocity over the wave profile. Dropping terms of higher order in δ , we have

$$v_o^2 = \bar{v}_o^2 [1 + 2\pi \delta \sin \kappa(x - \sigma t)] ,$$

and

$$\tau_t = \rho' f_t v_o^2 = \rho' f_t \bar{v}_o^2 [1 + 2\pi \delta \sin \kappa(x - \sigma t)] \quad (8)$$

Here, $\rho' f_t \bar{v}_o^2 = \bar{\tau}_t$ means a constant value of tangential stress over the wave. Fig. 11 represents these simple assumptions, where in the upper part the streamlines of the air over the wave are shown in a schematic distribution. This distribution may or may not be symmetrical to the wave profile. But in any case it is to be expected that the streamlines are crowded over each crest, so that over the crests a bundle of streamlines with higher velocity rushes ahead towards the lee. An asymmetrical distribution of the air current in this case may perhaps affect the wave in such a way that a flatter windward slope and a steeper leeward slope of the wave profile may result. In each individual case the distribution of air currents may be more or less asymmetrical and complicated. It would be sufficient to consider the effective wind force components to be composed of a series of harmonic terms of wave lengths $\lambda, 2\lambda, 3\lambda, \dots$, but of these terms only the one in phase with u_o and w_o does a net amount of work.

For the average rate of work done on a wave of length λ and of phase velocity σ , we get together with the formal expressions (7) and (8) according to (3), (4a), (5) and (6)

$$A_n = \frac{1}{\lambda} \pi \delta \sigma \int_0^\lambda [\bar{\tau}_n \cos \kappa(x - \sigma t) + \tau'_n \cos^2 \kappa(x - \sigma t)] dx$$

$$A_t = \frac{1}{\lambda} \rho' f_t \bar{v}_0^2 \int_0^\lambda [\pi \delta \sigma \sin \kappa(x - \sigma t) + 2\pi^2 \delta^2 \sigma \sin^2 \kappa(x - \sigma t) + \pi^2 \delta^2 \sigma] dx$$

after the term with δ^3 in the expression for A_t is dropped. The terms with sine and cosine do not contribute to an energy transfer from wind to waves, and on the average after integrating over one wave length, the result is

$$\begin{aligned} A_n &= \frac{1}{2} \pi \delta \tau'_n \cdot \sigma \\ A_t &= 2\pi^2 \rho' \delta^2 f_t \bar{v}_0^2 \cdot \sigma \end{aligned} \quad (9)$$

The accuracy to which A_n and A_t can be separately evaluated is perhaps not sufficient, because the effective wind force components depend upon several unknown factors of the hydrodynamical character of the sea surface (G. Neumann [12]). But one might see that both of the components may contribute to an energy transfer, and only in the case where in (8) $\tau_t = \text{const.}$ over the wave, does the average work of this drag at the particle velocity u_0 become zero. H. U. Sverdrup and W. H. Munk [1] therefore only consider the work done by the stress $\tau_t = \text{const.}$ at the mass transport velocity u_0' , which accompanies Stokes' waves of finite amplitude.

Let

$$\tau'_n = \pm \rho' f_n (\bar{v}_0 - \sigma)^2,$$

where $\tau'_n > 0$ for $\sigma < \bar{v}_0$, and $\tau'_n < 0$ for $\sigma > \bar{v}_0$,

then
$$A_n = \pm \frac{1}{2} \pi \delta \rho' f_n (\bar{v}_0 - \sigma)^2 \cdot \sigma$$

The dimensionless coefficients of proportionality f_n and f_t in (9) probably depend upon the actual conditions at the air-sea interface (stage of wave development, wave form, relative wind velocity, stability of the air above the interface, etc.), and therefore they will not be constants.

Let $\beta_0 = \sigma/\bar{v}_0$, and collect the dimensionless factors by putting

$$\gamma_{on} = \frac{1}{2} \pi \delta f_n \quad \text{and} \quad \gamma_{ot} = 2\pi^2 \delta^2 f_t .$$

We then have

$$A_n = \pm \rho' \gamma_{on}(\beta_0) \cdot \bar{v}_0^3 (1 - \beta_0)^2 \beta_0 \quad (10)$$

$$A_t = \rho' \gamma_{ot}(\beta_0) \bar{v}_0^3 \beta_0 . \quad (11)$$

The unknown "conditions" of the rough surface are now involved in the dimensionless quantities γ_{on} and γ_{ot} . We may call these quantities "resistance factors" or "frictional factors." They are given as functions of $\beta = \sigma/v$ (or $\beta_0 = \sigma/v_0$ resp.), because the hydrodynamic "roughness" of a surface not only depends upon the height of roughness elements (in a geometrical sense), but also upon the steepness δ of the waves and their form, which highly determines the roughness conditions of the sea surface. Because $\delta = f(\beta)$, and the roughness involved in f_n and f_t may be expressed as a function of β , it is to be expected that the quantities γ_{on} and γ_{ot} are functions of β too (G. Neumann [8]).

In the formal equations (10) and (11) neither the single resistance factors, nor the effective wind velocity v_0 are known with the necessary degree of accuracy. But it seems possible to estimate the effective wind force - the "effective stress" - by means of emp-

irical relationships, which may lead to a satisfactory approximation.

Let

$$A = A_n + A_t = \rho' v^3 \beta [\gamma_t(\beta) + \gamma_n(\beta)(1 - \beta)^2], \quad (12)$$

the combined action of both wind force components or

$$A = \rho' v^3 \beta \cdot C(\beta) . \quad (13)$$

The effective factor $C(\beta)$ with the meaning of a resistance coefficient may be estimated empirically and determined as a function of β . The hydrodynamical characteristics of the rough wavy sea surface are now implied in the dimensionless quantity $C(\beta)$, and its value is related empirically to the wind velocity at "anemometer height," say, 10 m above the mean sea surface level. This means that C is determined by observations in such a way that it refers to the wind velocity at a certain height by definition. This empirical method is not a very satisfactory one, but steady pursuit in this direction may in the future yield a means of determining better approximations. At present this empirical way seems to be the only approach.

For estimating the resistance coefficients, several different methods have been tried. As already mentioned, H. Jeffreys [18] considers only the normal wind force components, and assumes for the effective wind pressure a priori a certain distribution. Let the effective pressure component be p^* ; then we have according to H. Jeffreys

$$p^* = \bar{s} \rho' (v - \sigma)^2 \partial \eta / \partial x$$

H. Jeffreys calls the coefficient of proportionality, $\bar{s}$, the "sheltering coefficient" ("streamlining coefficient" according to Sverdrup-Munk [1]). The work done by this force per unit surface area and unit time on a wave, will be

$$A = \frac{1}{\lambda} \bar{s} \rho' (v - \sigma)^2 \cdot \int_0^\lambda \frac{\partial \eta}{\partial x} \frac{\partial \eta}{\partial t} dx ,$$

or with (1) and (3)

$$A = \frac{1}{2} \rho' c_d (v - \sigma)^2 \cdot \sigma . \quad (14)$$

The dimensionless factor

$$c_d = \bar{s} \pi^2 \left(\frac{2a}{\lambda} \right)^2 = \bar{s} \pi^2 \delta^2 \quad (15)$$

corresponds to the resistance coefficient. (14) may be written again

$$A = \rho' v^3 \beta \cdot \overline{C(\beta)},$$

with

$$\overline{C(\beta)} = \frac{1}{2} \bar{s} \pi^2 \delta^2 (1 - \beta)^2$$

in analogy to (13). Based on observations of waves in their initial state, Jeffreys evaluated $\bar{s} \approx 0.30$ (0.27).

Another attempt for estimating the resistance coefficient was made by H. Motzfeld [19]. The distribution of the pressure p over wooden wave profiles of different form was determined by experiments and measurements in a wind tunnel. For the width $b = 1$ of a given profile, the pressure resistance

$$W_d = \int_0^S p \sin \alpha \, ds,$$

where S is the "length of unrolling" of the wave, ds an element of the wave profile, S , and α the angle between the direction x of flow and ds . With $d\eta = ds \cdot \sin \alpha$, the integral has the value

$$W_d = \int_0^\lambda p(\eta) d\eta .$$

The pressure resistance over a wave length λ is given as the plane area bounded by the curve $p(\eta)$. If we put

$$W_d = \frac{1}{2} \rho' c_d (v - \sigma)^2 \cdot \lambda, [g \text{ sec}^{-2}] \quad (26a)$$

for the surface area of the width one and the length λ , c_d may be evaluated by the values of p determined by experiments. The results of Motzfeld's measurements seem to indicate the proportionality $c_d \sim (\pi\delta)^{3/2}$, but it seems uncertain whether these results of measurements over smooth, rigid wave profiles are applicable to the conditions at the actual sea surface.

Using an empirical relationship [13] between the wind force exerted at the rough sea surface and the wind velocity, $\tau_{\text{eff}} = f(v)$, the author [8], [13] made an attempt to estimate the effective value of the wind force as a function of wave development at different wind velocities. By means of a relationship between δ and β , as suggested by Sverdrup-Munk [1], it was possible to relate $\tau_{\text{eff}} = F(\beta(v))$ where, according to a previous paper [14]

$$c_d = s\alpha x = s\pi\delta, \quad (16)$$

with $s = 0.095$ was used as suggested by empirical evidence. Both assumptions (15) and (16) lead to nearly the same numerical values c_d , when considering initial waves with the steepness $\delta = 1/10$. This follows with Jeffreys' assumption from (15) $\bar{s}\pi^2\delta^2 = 2.96 \cdot 10^{-2}$, and from (16) $s\pi\delta = 2.98 \cdot 10^{-2}$. But when considering the generation and the growth of initial waves with increasing wind speed it seems that the relation (16) holds good. The determination of the c_d -value at different conditions of the sea surface is necessary for an exact evaluation of energy transfer from wind to waves, and further investigation of this point is indicated.

Another important empirical relationship deduced from experience,

which we shall use to a large extent in the following considerations, is that the steepness of the waves, $\delta = H/\lambda$, depends on the stage of wave development. Such a relationship has already been suggested by O. Krümmel [5], but H. U. Sverdrup and W. H. Munk [1] first related the ratio H/λ to the "age" of the waves and used this relationship for theoretical discussions. The stage of development or the "wave age" can be conveniently expressed by the ratio of velocity of wave propagation σ to wind speed v , that is $\beta = \sigma/v$. The relationship $\delta = f(\beta)$ is shown in Fig. 12 where the observed corresponding values of H/λ and σ/v were plotted. While Sverdrup-Munk did not fit an empirical curve $\delta = f(\beta)$ to the observed data and chose another way to the solution of the problem, we put, in accordance with a previous paper [8]

$$\delta = 2n e^{-r\beta} \quad \text{for} \quad 1/3 \leq \beta \leq \beta_m^*, \quad (17)$$

where $n = 0.1075$, $r = 1.667$, $\beta_m^* = 1.37$, and later carry out the integrations numerically.

For the initial state of wave development of very young waves, we put

$$\delta = 2p = \text{const.} \quad \text{for} \quad \beta \leq 1/3 \quad (18)$$

with $p = 0.062$. These empirical relationships are represented in Fig. 3 by full lines. While for $\beta > 1/3$ a definite relationship between the steepness and the wave age already appeared fairly well established by older observations (see [1] and [8]), this is not the case for the "initial state" where $\beta < 1/3$. Based on the collection of data by Sverdrup-Munk only four not very reliable measurements in the state $\beta < 1/3$ were available. In Fig. 12 some further

Fig. 12. Wave steepness $\delta = H/\lambda$ plotted against the ratio $\sigma/v = \beta$. Assumed relationship shown by full line.

observations are plotted, obtained during the voyage with M. S. "Heidelberg" to the West Indies. The observations were taken near the coast of San Miguel (Azores) and along different coasts of the Caribbean Sea. These observations indicate a rather rapid development of wave steepness in the stage of very young waves up to a maximum value of $\delta = 1/10$ to $\delta = 1/7$. This steepness seems to remain nearly constant until the wave age attains a value of about $\beta = 1/3$. In this early stage of development a rather intensive breaking of these very young waves is to be observed. But this state of wave development with $\beta < 1/3$ passes so rapidly, especially when the wind is stronger, that the exact relationship is of very little consequence to the later development of the sea. At present, all observations of very young waves seem to indicate that it would be a good approximation to assume rather steep waves and $\delta = \text{const}$ for the stage of wave development $\beta < 1/3$, and to neglect the initial stage (perhaps $\beta < 0.05$) where the first disturbances grow with increasing steepness.

In the first stages of wave generation let us consider a general wave profile with the steepness $\delta = 2p$. If we assume that normal pressure forces on this steep wave are the effective forces needed to do a net amount of work, we get from (10)

$$A = \rho' s \pi p v^3 (1 - \beta)^2 \beta, \quad (19)$$

where the resistance coefficient has been replaced by (16). With the dimensionless quantity

$$C_1(\beta) = s \pi p (1 - \beta)^2 \quad (20)$$

we have

$$A = \rho' v^3 \beta \cdot C_1(\beta) \quad \text{for } \beta \leq 1/3. \quad (21)$$

With further development of the waves the effective wind forces become more complicated. They depend upon the stage of development of complex wave motion, and in addition we have to consider effective drag forces. The resistance for the flow of the air over the rough wavy surface is composed of form resistances and frictional stresses. From the sum of the two a total resistance results, which was called "effective wind stress" [8]. The smaller superimposed waves on the general profile may be considered to act as roughness elements with regard to the air which flows over the general profile of the largest waves (Fig. 10). With reference to a previous paper [8], where an attempt was made to evaluate the effective wind stress under these assumptions, we put

$$W_{\text{eff}} = \rho' F \gamma^2(\beta_m) v^2 + \frac{1}{2} \rho' F s' \pi \delta_m (1 - \beta_m)^2 v^2 - \frac{1}{2} \rho' F s^* \pi \delta_m^* (1 - \beta_m^*)^2 v^2, \quad (22)$$

where

$$\gamma^2(\beta_m) = [1.75 + \frac{16.2}{(\beta_m - \frac{1}{3})} (e^{-r\beta} \{0.48(\beta_m + 0.6) - 0.6(1 + \beta_m^2)\} + 0.126)] \cdot 10^{-3} \quad (23)$$

has the meaning of a resistance coefficient or friction coefficient depending on β . The factors of proportionality s' and s^* are assumed to be constant, and $s' = s/2$, $s^* = 2s$, where $s = 0.095$ as defined by (16).

W_{eff} is the effective resistance of the rough wavy sea surface per area $F = b \cdot \lambda$, and β_m^* means the ratio $\sigma/v = 1.37$, δ_m^* the steepness for fully developed " β_m^* -waves" with this ratio β_m^* (see Chapter I). Correspondingly β_m is the ratio σ/v for fully developed " β_m -waves", as explained in Chapter I, and δ_m their maximum steepness

given by the empirical relationship (17).

The estimates which led to (22) and (23) are based partly on theoretical considerations and partly on empirical evidence. It follows from (23) that the more the wave velocity differs from the wind velocity, the greater is the value of γ^2 . The relation $\gamma^2 = f(\beta)$ seems to be a better approximation than the assumption of a constant value γ^2 at all wind velocities and all stages of wave development. In their paper [1], Sverdrup-Munk call special attention to the fact that the value of γ^2 would probably be greater than the assumed constant value $\gamma^2 = 2.6 \times 10^{-3}$, if the wave velocity differs too much from the wind velocity. By the relations (23) and (22), an attempt has been made to account for these circumstances, and in the following table some numerical values for γ^2 are given as calculated by means of (23):

β	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
$\gamma^2 \cdot 10^{-3}$	5.62	4.55	4.05	3.64	3.27	3.01	2.78	2.68	2.57

Per unit area we have

$$\frac{W_{\text{eff}}}{F} = \tau_{\text{eff}} = \rho' C(\beta) v^2, \quad (24)$$

if we introduce the abbreviated form

$$C(\beta) = \gamma^2(\beta_m) + \frac{s'}{2} \pi \delta_m (1 - \beta_m)^2 - \frac{s^*}{2} \pi \delta_m^* (1 - \beta_m^*)^2 \quad (25)$$

for the effective coefficient of resistance.

It may be mentioned in this connection that the determination of the sheltering coefficient from experiments by Sir Thomas Stanton, according to Sverdrup-Munk [1], leads to a numerical value which apparently is in good agreement with our assumption $s = 0.095$. The sheltering coefficient s can be evaluated from these measurements

by using the equation

$$s = \frac{\Delta p L}{\rho' U^2 \pi H}$$

in the notation of Sverdrup-Munk, where Δp is the wind pressure against different portions of the wave, L the wave length, H the wave height and U the wind velocity relative to the wave (the measurements were taken with small wooden models of waves, placed in a wind tunnel). With this notation equation (26a) per unit area (page 59) is written

$$W_d = \frac{1}{2} \rho' s \pi \delta (v - \sigma)^2 = |\tau| ; [cm^{-1} g sec^{-2}] \text{ or } [dyn cm^{-2}], \quad (26)$$

taking (16) into account. Therefore

$$\frac{1}{2} s = \frac{|\tau|}{\rho' (v - \sigma)^2 \pi \delta} .$$

s in our notation equals $2s$ in the notation of Sverdrup-Munk as given above. The average value of s evaluated by these authors from Stanton's measurements is $s = 0.049$.

The effective wind stress is given by (24) as a function of wind velocity and $C(\beta)$, where the latter value takes into account the different "friction conditions" of the rough sea surface depending on the stage of wave development. Because $\beta = \beta_m$ in the fully arisen sea only depends upon the wind velocity v (see formula (60)), the frictional coefficient $C(\beta_m)$ is given as a function of v , too. By this reason a quadratic relation between the effective stress and the wind velocity v is not to be expected at the sea surface with its changing roughness conditions, and in fact not observed at all [13]. When the assumption is made that the vertical distribution of the wind velocity in the lower layer of the atmosphere immediately over the sea surface may be represented by a logarithmic law, then

according to L. Prandtl [20]:

$$\tau = \rho' \frac{k_o^2}{\left(\ln \frac{z + z_o}{z_o} \right)^2} v^2 ,$$

where z_o depends on the roughness of the surface over which the air is flowing, and is called the roughness parameter, or the "roughness length." The nondimensional quantity k_o is approximately constant and has the value 0.40. z is the "anemometer height" where the wind velocity v is measured. The dimensionless factor $k_o^2 / [\ln(z + z_o)/z_o]^2$ corresponds to the frictional factor or effective resistance coefficient in (24).

If we consider the effective resistance coefficient for $\beta \geq 1/3$ to be given by (25) as a function of the stage of wave development or the wave age β , it follows with (17) from equation (12) that

$$A = \rho' v^3 \beta [\gamma^2(\beta) + s' \pi n e^{-r\beta} (1 - \beta)^2] , \quad 1/3 \leq \beta \leq 1 , \quad (27)$$

is the rate at which work is being done by the wind on a wave, which is characterized by its wave age β .

With

$$C_2(\beta) = \gamma^2(\beta) + s' \pi n e^{-r\beta} (1 - \beta)^2 ; \quad 1/3 \leq \beta \leq 1 , \quad (28)$$

equation (27) can be written

$$A = \rho' v^3 \beta C_2(\beta) . \quad (27a)$$

When the waves proceed in development and longer waves are generated with $\beta^* > 1$

$$A = \rho' v^3 \beta^* C_3(\beta) ; \quad 1 \leq \beta^* \leq \beta_m^* , \quad (29)$$

where

$$C_3(\beta) = \gamma^2(\beta_m) + s'\pi n e^{-r\beta_m} (1 - \beta_m)^2 - s^*\pi n e^{-r\beta^*} (1 - \beta^*)^2 \quad (30)$$

In the stage of fully developed complex wave motion $C_3(\beta)$ is given by the expression (25).

3) Energy dissipation due to turbulent wave motion

Energy may be dissipated by viscosity (D_μ) or by turbulent motion in the wave (D_M). The viscosity of the water is so slight that this effect can be neglected when dealing with the growth of real ocean waves. Only the process of generation of primary wavelets, which are not breaking, is apparently influenced by viscosity [14]. But the "sea" evidently is a turbulent wave motion, and when considering its growth under the action of wind the effect of turbulence has to be taken into account by introduction of an "eddy viscosity" or "virtual friction."

The dissipation of wave energy due to a viscosity coefficient μ is given (H. Lamb [21]) by

$$D_\mu = 2\mu \left(\frac{2\pi}{\lambda}\right)^3 \sigma^2 a^2. \quad (31)$$

When dealing with turbulent ocean waves which are characterized by a phase velocity σ , or at a given wind velocity v by the ratio $\sigma/v = \beta$, and the steepness $2a/\lambda = \delta$, we may write in analogy to (31)

$$D_m = 2M(\beta)\pi^2 g \delta^2, \quad (32)$$

where the ordinary viscosity coefficient μ is replaced by a turbulence coefficient or coefficient of eddy viscosity $M[\text{cm}^{-1}\text{g sec}^{-1}]$. This coefficient M naturally is not to be regarded as a physical constant characteristic of the fluid like μ (at a given salinity and temperature of sea water), but it will depend upon the state of the "sea" and on the wind velocity. Therefore M is written in (32)

as a function of β .

Formula (32) follows from (31) when putting $2a/\lambda = \delta$, and eliminating the phase velocity of the wave by

$$\sigma^2 = \frac{g\lambda}{2\pi} . \quad (33)$$

In fully developed sea, where $A = D$, and where the " β_m^* -wave" (β_m^*, δ_m^*) and the " β_m -wave" (the "sea") (β_m, δ_m) (see Chapter I) are fully developed, we have with (29) and (32), considering (25) and (17)

$$\frac{8M\pi^2 g n^2 e^{-2r\beta_m^*}}{\rho' v^3 \beta_m^*} = \gamma^2(\beta_m) + s'\pi n e^{-r\beta_m}(1 - \beta_m)^2 - s^*\pi n e^{-r\beta_m^*}(1 - \beta_m^*)^2. \quad (34)$$

Since $\beta_m = f(v)$ in the fully developed sea, it is to be expected that M is only a function of wind velocity, $M(v)$, whereas in the case of not fully arisen sea, M depends also on the stage of wave development. With reference to Chapter I, 2, we put $\beta_m^* = 1.37$.

From (34), or with the aid of an empirical formula for the effective stress given in a previous paper [13],

$$\tau_{\text{eff}} = \rho' k(v) v^2; \quad k(v) = \frac{1}{10} \left(\frac{1 \text{ m/sec}}{v \text{ m/sec}} \right)^{1/2}, \quad (35)$$

which agrees with equation (24), when putting $\beta_m = f(v)$ as given by equation (60) in this paper, we get

$$M = \frac{1}{8} \frac{\rho' k(v) v^3 \beta_m^*}{\pi^2 g n^2 e^{-2r\beta_m^*}} .$$

From this expression, and with $\rho' = 1.25 \cdot 10^{-3}$, $g = 980$, $n = 0.1075$ and $r = 1.667$

$$M = 0.1825 \cdot 10^{-4} v^{5/2} [\text{cm}^{-1} \text{ g sec}^{-1}]. \quad (36)$$

The coefficients of eddy viscosity computed by this formula are of the same order of magnitude as the values of the "Austausch-

coefficients" in the upper layers of the ocean, as far as they are known. But the values given by (36) are to be expected only in the case where the complex wave motion is fully developed. Therefore, they depend upon only the wind velocity, and the coefficients M represent maximum values at a given wind velocity. In the stages where the sea is not fully developed, that is, when the sea is still growing, the M-values will be smaller. We denote these smaller values by $M(\beta)$, and relate them to the "age" β of the longest wind-generated wave present in the composite sea. The information currently available on the state of turbulence and on the eddy viscosity in the surface layers of the ocean at different wind velocities and at different stages of wave development is very meager. With respect to the nature of the phenomenon it seems reasonable to assume that the state of turbulence increases with increasing development of the sea by obeying an exponential law. Evidence of this will be presented later but here it will be assumed that the increase of turbulence, and of eddy viscosity in layers with turbulent wave motion at a given wind velocity obeys the law

$$M(\beta) = M e^{-2r \left(\frac{\beta_m^* - \beta}{\beta_m} \right)} \text{ for } 1 \leq \beta \leq \beta_m^* , \quad (37)$$

and

$$M(\beta) = M(1) e^{-2r(1-\beta)} \text{ for } 0.1 \leq \beta \leq 1 , \quad (38)$$

where M is the coefficient of eddy viscosity of the fully developed sea as given by formula (36). β_m means the ratio $\bar{\sigma}/v$ for fully arisen " β_m -waves," which were considered to be the first characteristic waves developed in the complex wave pattern. β_m is given by computations in a previous paper [8] or by equation (60) in this report as a function of wind velocity. β_m^* and r have the same mean-

ing as in the preceding formulas, and $M(1)$ is the value of the eddy viscosity coefficient in the case where $\beta = 1$ in (37). By putting $\beta = \beta_m^*$ in (37) it follows $M(\beta_m^*) = M$ as given by (36) for the fully developed state.

The formula (38) is not applicable to the very earliest stages of wave development, but it seems to be sufficiently accurate for $\beta \geq 0.1$. It has already been mentioned in connection with (18) that these earliest stages of turbulent wave generation are so rapid that the exact form of the relationship is of very little consequence to the later development of the sea. It is to be expected that the change from ordinary viscosity to turbulent eddy viscosity does not occur continuously. This change to turbulent wave motion probably will take place when the maximum steepness of the "ripples" is attained with the ratio $H/\lambda \cong 1/7$, that is at a wind velocity of about 123 cm/sec [14]. At a wind velocity of $v = 125$ cm/sec, formula (36) would result in a value of $M = 3.2$ [$\text{cm}^{-1} \text{g sec}^{-1}$] for the "coefficient of turbulence" (eddy viscosity) in the uppermost layers of the sea where wave motion takes place. The wave length of the ripples at this wind velocity is 11-12 cm, and these wavelets are already breaking up at the crests. Below this limit of about $v = 123$ cm/sec the initial waves have a steepness less than $1/7$. They are stable, and the dissipation takes place only by ordinary viscosity.

In the case of fully developed (not breaking) initial waves it follows from the condition $A = D$, according to (21) and (32), that

$$\rho' v^3 \beta \frac{\pi \delta}{2} (1 - \beta)^2 = 2 \mu \pi^2 g \delta^2, \quad (39)$$

where μ is the coefficient of ordinary viscosity, substituted in (32) for $M(\beta)$, and where in (21)

$$C_1(\beta) = \frac{5\pi\delta}{2} (1 - \beta)^2 .$$

With the condition (39) we get

$$v^3 = \frac{4\pi\mu g\delta}{\rho's(1 - \beta)^2\beta} . \quad (40)$$

If $\mu = 0.018$, $g = 980$, $\rho' = 1.25 \cdot 10^{-3}$, $s = 0.095$, $\beta = 1/3$ and $\delta = 0.138$, then $v = 120.3$ cm/sec for the wind velocity necessary to generate and maintain this type of initial wave motion with a steepness of $\delta = 0.138$ and a phase velocity $\sigma = \frac{1}{3} v$. This result agrees fairly well with the more exact computations on the generation of initial waves given in a previous paper [14]. In this special case a slightly higher wind velocity would result from more accurate considerations, that is, $v = 122.7$ cm/sec. The small discrepancy of 2.4 cm/sec is due to the fact that in (39) or (40) the effect of capillarity is neglected. For practical purposes, capillarity effects only become important if the wave lengths are smaller than, say, 10 cm. Therefore only when considering the generation and maintenance of primary wavelets, does surface tension have to be taken into account.

In Table 3, some numerical values of the coefficient of eddy viscosity (coefficients of turbulence) are given for different wind velocities and for three distinct stages of sea development. M is the value for fully developed sea as given by (36), $M(1)$ is the value given by (37) for the state $\beta = 1$ and $M(\beta_m)$ the value for β_m as given by (38), when $\beta = \beta_m$. For very weak winds (1-2 m/sec), the coefficients of eddy viscosity approach the value of ordinary viscosity in the first stages of wave formation. But with increasing wind velocity and increasing sea, the coefficients of eddy viscosity increase very rapidly, and at moderate wind velocities of about 8-10 m/sec

their numerical values are between about 50 and 500 $\text{cm}^{-1} \text{g sec}^{-1}$.

Table 3. Coefficients of eddy viscosity (turbulence coefficients) [$\text{cm}^{-1} \text{g sec}^{-1}$] at different wind velocities and $\beta_m = f(v)$.

$v(\text{m/sec})$	2	4	6	8	10	12	14	16	18	20	24
β_m	0.425	0.53	0.60	0.66	0.70	0.74	0.78	0.81	0.85	0.88	0.94
M	10.3	58	161	332	577	912	1350	1860	2520	3260	5120
$M(1)$	0.57	5.2	20.5	51	99	174	274	409	583	800	1382
$M(\beta_m)$	0.08	1.1	5.4	16.4	36	73	132	216	355	538	1134

Figure 13 represents the relationship between M , $M(1)$, $M(\beta_m)$ and v , respectively, where for comparison the "Austausch coefficients" or "Koeffizienten der Scheinreibung" are plotted, as given by A. Defant in "Dynamische Ozeanographie" (p. 76) or by Sverdrup, Johnson and Fleming in "The Oceans," according to the results of H. Thorade and W. Schmidt. These values show a rather good agreement with the curve for M , that means with formula (36). Only for strong winds are the values given by Thorade and Schmidt somewhat lower than the M -values given by (36). Their numerical values are between M and $M(1)$, and this probably indicates that at very strong winds the stage of fully developed sea was not attained in all cases, when the observations for the determination of the "Austausch-Koeffizienten" by Schmidt or Thorade were taken. As an example of the increase of the coefficient of eddy viscosity with rising sea, Table 4 shows the $M(\beta)$ values at a wind velocity of $v = 10 \text{ m/sec}$ as a function of increasing "wave age" β .

Fig. 13. Relationship between the coefficients of eddy viscosity $M(\text{cm}^{-1} \text{ g sec}^{-1})$ and the wind velocity $v(\text{m sec}^{-1})$ at different stages of sea development (wave age β). Values of coefficients of eddy viscosity shown by symbols.)

Table 4. Coefficients of eddy viscosity (turbulence coefficients) $M(\beta)$ [$\text{cm}^{-1} \text{ g sec}^{-1}$] in different stages of wave development at a wind velocity of $v = 10 \text{ m/sec}$.

β	0.15	0.25	0.317	0.367	0.45	0.55	0.65	0.7	1.0	1.15	1.25	1.37
$M(\beta)$	5.85	8.15	10.1	12.0	15.8	22.1	30.9	36.5	99	202	326	577

4) Energy equations

A steady supply of energy by wind is necessary to cover the losses of energy by virtual friction in turbulent wave motion. Only in the case where the energy supply exceeds the energy dissipation may the sea grow. Let E be the mean energy of the wave motion per unit area of the sea surface, A the supplied energy and D the dissipated energy per unit surface area. Then, for the total energy $E \cdot \lambda$ per unit crest width of a wave with wave length λ ,

$$\frac{d}{dt} (E \lambda) = (A - D) \quad (41)$$

This equation states that the individual change of wave energy with time equals the difference between the supplied energy and the energy lost by friction and turbulence. Associated with the wave motion is a flow of energy in the direction of wave propagation. This energy flow per unit time across a vertical "control-section" of unit width and a depth below which the wave motion is negligible, is

$$cE = \frac{1}{4} \rho g a^2 \sigma, \quad (42)$$

where σ is the velocity of wave propagation (phase velocity) if we consider deep water waves. The average energy per unit area of a wave with the amplitude $a = H/2$ equals

$$E = \frac{1}{2} g \rho a^2. \quad (43)$$

Equation (42) can be interpreted either to mean that half the energy ($E/2$) is propagated with the phase velocity, or that the rate of transmission of total energy E is equal to the group velocity $c = \sigma/2$.

From (41)

$$\lambda \frac{dE}{dt} + E \frac{d\lambda}{dt} = (A - D)\lambda \quad (44)$$

or

$$\lambda \left(\frac{\partial E}{\partial t} + c \frac{\partial E}{\partial x} \right) + E \left(\frac{\partial \lambda}{\partial t} + c \frac{\partial \lambda}{\partial x} \right) = (A - D)\lambda \quad (45)$$

Let us consider two cases:

Case A: If a constant wind with mean velocity v blows over an unlimited sea room (fetch), and if the energy added is the same everywhere so that the waves grow at all localities at the same rate with time t (duration), then

$$\frac{\partial E}{\partial x} = 0 ; \quad \frac{\partial \lambda}{\partial x} = 0 ,$$

and equation (45) reduces to

$$\frac{\partial E}{\partial t} + \frac{E}{\lambda} \frac{\partial \lambda}{\partial t} = A - D. \quad (46)$$

With (43) and the substitution (33) we have

$$\rho g a \frac{\partial a}{\partial t} + \frac{\rho g a^2}{\sigma} \frac{\partial \sigma}{\partial t} = A - D. \quad (47)$$

Case B: If, on the other hand, the duration, t , of wind action is unlimited or in practice long enough to produce a steady state, but the fetch x is limited, then for local steady state conditions

$$\frac{\partial E}{\partial t} = 0 ; \quad \frac{\partial \lambda}{\partial t} = 0 .$$

In this case,

$$\frac{\partial E}{\partial x} \text{ or } \frac{\partial a}{\partial x} \text{ and } \frac{\partial \lambda}{\partial x}$$

has to be determined.

From equation (45) we have

$$c \left(\frac{\partial E}{\partial x} + \frac{E}{\lambda} \frac{\partial \lambda}{\partial x} \right) = A - D . \quad (48)$$

Again replacing λ by σ and considering

$$\frac{1}{\lambda} \frac{\partial \lambda}{\partial x} = \frac{2}{\sigma} \frac{\partial \sigma}{\partial x}$$

it follows with (43) that

$$c \left(gpa \frac{\partial a}{\partial x} + \frac{gpa^2}{\sigma} \frac{\partial \sigma}{\partial x} \right) = A - D , \quad (48a)$$

or, putting $c = \sigma/2$, we get

$$\frac{1}{2} gpa^2 \frac{\partial \sigma}{\partial x} + \frac{\sigma}{2} gpa \frac{\partial a}{\partial x} = A - D . \quad (49)$$

Since $\beta = \sigma/v$ is given by the empirical relationships (17) and (18) as a function of $\delta = 2a/\lambda$, we get by this substitution for

Case A:

$$\begin{aligned} \frac{12\rho p^2 \pi^2}{g} \sigma^3 \frac{\partial \sigma}{\partial t} &= A - D ; \quad \beta \leq 1/3 , \\ \rho \frac{4\pi^2}{g} n^2 \sigma^3 e^{-2r\frac{\sigma}{v}} \left(3 - r\frac{\sigma}{v} \right) \frac{\partial \sigma}{\partial t} &= A - D ; \quad 1/3 \leq \beta \leq \beta_m^* \end{aligned} \quad (50)$$

or

$$\begin{aligned} \frac{12\rho p^2 \pi^2}{g} v^4 \beta^3 \frac{\partial \beta}{\partial t} &= A - D ; \quad \beta \leq 1/3 , \\ \rho \frac{4\pi^2}{g} n^2 v^4 \beta^3 e^{-2r\beta} (3 - r\beta) \frac{\partial \beta}{\partial t} &= A - D ; \quad 1/3 \leq \beta \leq \beta_m^* , \end{aligned} \quad (51)$$

and similarly for

Case B:

$$\frac{6\rho p^2 \pi^2}{g} \sigma^4 \frac{\partial \sigma}{\partial x} = A - D; \quad \beta \leq 1/3, \quad (52)$$

$$\rho \frac{2\pi^2}{g} n^2 \sigma^4 e^{-2r\frac{\sigma}{v}} \left(3 - r \frac{\sigma}{v}\right) \frac{\partial \sigma}{\partial x} = A - D; \quad 1/3 \leq \beta \leq \beta_m^*$$

or

$$\frac{6\rho p^2 \pi^2}{g} v^5 \beta^4 \frac{\partial \beta}{\partial x} = A - D; \quad \beta \leq 1/3 \quad (53)$$

$$\rho \frac{2\pi^2}{g} n^2 v^5 \beta^4 e^{-2r\beta} (3 - r\beta) \frac{\partial \beta}{\partial x} = A - D; \quad 1/3 \leq \beta \leq \beta_m^* .$$

5) The growth of the "sea" ("β_m-wave")

Case A: The growth of the waves over an unlimited fetch as a function of duration t of wind action.

For the "initial" state $\beta \leq 1/3$ of wave development it follows from (51) according to (21), (32) and (38)

$$\frac{12\rho p^2 \pi^2}{g} v^4 \beta^3 \frac{d\beta}{dt} = \rho' v^3 \beta C_1(\beta) - 8gp^2 \pi^2 M(\beta)$$

and

$$dt = \frac{\rho}{\rho'} \frac{12\pi^2 p^2}{g} v \frac{\beta^2}{C_1(\beta) - B_1(\beta)} d\beta; \quad \beta \leq 1/3, \quad (54)$$

where the dimensionless quantity $B_1(\beta)$ is given by

$$B_1(\beta) = \frac{8\pi^2 gp^2}{\rho'} \frac{M(\beta)}{v^3 \beta} . \quad (55)$$

If the waves continue to grow and exceed the stage $\beta = 1/3$, the second equation in (51) with consideration of (27a), (28) and (17) leads to

$$\rho \frac{4\pi^2}{g} n^2 v^4 \beta^3 e^{-2r\beta} (3 - r\beta) \frac{d\beta}{dt} = \rho' v^3 C_2(\beta) - 8gn^2 \pi^2 M(\beta) e^{-2r\beta}$$

or

$$dt = \frac{\rho}{\rho'} \frac{4\pi^2}{g} n^2 v \frac{\beta^2 e^{-2r\beta} (3 - r\beta)}{C_2(\beta) - B_2(\beta)} d\beta ; \quad 1/3 \leq \beta \leq \beta_m. \quad (56)$$

Here the dimensionless quantity $B_2(\beta)$, which takes into account the effect of dissipation, is given by

$$B_2(\beta) = \frac{8\pi^2 g n^2}{\rho'} \frac{M(\beta) e^{-2r\beta}}{v^3 \beta}. \quad (57)$$

Case B: The growth of waves at limited fetches as a function of the fetch x .

Similarly as in Case A equations (53) lead to the following expressions

$$dx = \frac{\rho}{\rho'} \frac{6\pi^2 p^2}{g} v^2 \frac{\beta^3}{C_1(\beta) - B_1(\beta)} d\beta ; \quad \beta \leq 1/3 \quad (58a)$$

and

$$dx = \frac{\rho}{\rho'} \frac{2\pi^2}{g} n^2 v^2 \frac{\beta^3 e^{-2r\beta} (3 - r\beta)}{C_2(\beta) - B_2(\beta)} d\beta ; \quad 1/3 \leq \beta \leq \beta_m. \quad (58b)$$

These equations have to be evaluated by numerical integration.

The most important quantity in these equations on which the evaluation of the wave development depends, is the difference $C(\beta) - B(\beta)$. In this paper, a first attempt is made to estimate this difference which is determined by the energy supply by wind and the energy losses by virtual friction. At present such an estimate seems possible only with the aid of empirical relationships. It is to be expected that these relationships will be improved in the future as a result of more comprehensive observations, and that theoretical knowledge will increase especially with regard to the problem of turbulence. The state of the sea and of turbulence in the upper layers of the ocean are seemingly closely related, and the problem

of the growth of ocean waves under the action of wind probably cannot be solved without considering turbulent energy dissipation.

Special difficulties are encountered at present when the earliest stages of wave development are considered. Here the initial waves which form as wavelets or ripples at very weak winds are not meant, but rather the small, steep, turbulent waves which are generated by stronger winds from short fetches or wind gusts, and which grow rapidly with increasing fetch or duration of wind action. These stages of wave development are to be described by "wave ages" β of about 0.1 to 0.2, that is very "young sea." Our knowledge of the exact empirical relationship $\delta = f(\beta)$ for $\beta < 1/3$ is still very weak, and when dealing with equation (18) it was pointed out that the assumption $\delta = \text{const}$ represents only a first approximation.

The dimensionless quantities $C(\beta)$ and $B(\beta)$ are represented in Fig. 14 as functions of β . Because of the different empirical relationships for $\beta < 1/3$ and $\beta > 1/3$ and the assumptions at the earliest stages of wave development, a discontinuous change results at $\beta = 1/3$. In nature, we have to expect a certain continuity in slope of the steepness δ when β increases above the value $1/3$. It would have been possible to establish another empirical relationship for $\beta < 1/3$ to approach a more continuous change between $\beta = 0.2$ and $\beta = 0.4$, perhaps as indicated by the broken line in figure 12, but such attempts would not contribute to a better understanding of the mechanism of wave generation. Therefore it seems to be more expedient to wait for more complete empirical knowledge. For the practical aim in question, it seems adequate to smooth the numerical values by assuming a continuous change over $\beta = 1/3$. The smoothing is

Fig. 14. Dimensionless quantities $C(\beta)$ and $B(\beta)$ as functions of wave age.

indicated in Fig. 14 by a broken line. But if the original values of C and B (full lines) were used, the results would not be changed essentially for larger "fetch parameters" or "duration parameters" (see section 7). The reason for this is that the earliest stages of sea development are so rapid in the state $\beta < 1/3$, that the relatively short fetches (or durations) are of very little consequence to the later development of the sea. Table 5 gives the values of $C_1(\beta)$, $C_2(\beta)$ and $B_1(\beta)$, $B_2(\beta)$ as they are used for the numerical computation. The original values are given in parenthesis, if they differ from the used values.

Table 5. Values of "resistance-factors" $C(\beta)$ and "dissipation-factors" $B(\beta)$ for different values of β in the stages $\beta \leq 1.0$.

β	0.05	0.15	0.25	0.30	0.40	0.45	0.55	0.65	0.75	0.85	0.95	1.00
$C_1(\beta)$.10 ³	16.75	13.38	10.65 (10.42)	9.48 (9.06)								
$C_2(\beta)$.10 ³					7.50 (8.58)	6.71 (7.30)	5.39 (5.50)	4.42	3.71	3.21	2.88	2.78
$B_1(\beta)$.10 ³	(12.50)	10.25 (7.75)	8.60 (8.00)	7.85 (7.87)								
$B_2(\beta)$.10 ³					6.38 (6.50)	5.75 (5.78)	4.73	4.01	3.47	3.06	2.75	2.60

The very rapid development of the sea in the earliest stages at $\beta < 1/3$ may be shown by an example for a wind velocity $v = 16$ m/sec.

The results given in section 7 demonstrate that the state $\beta = 1/3$ is attained from a fetch of only 8.3 km. But a fetch of 300 km is necessary to attain the state $\beta = 0.81 = \beta_m$. That means that at any rate 300 km is necessary for the development of the sea to the stage

$\beta = \beta_m$, that is, for the development of the first characteristic wave in the sea, which was called the "short sea" or "sea" and characterized by a phase velocity smaller than the wind velocity. From a fetch of 300 km, this "sea" or " β_m -wave" would have attained its maximum height, length and final steepness. But beyond this state, the growth of the sea continues, and longer waves are generated by the wind. The complex sea will be nearly fully arisen when the fetch is 500 km. These results will be discussed later on in more detail, but they show that the uncertainties involved in our assumptions for the state $\beta < 1/3$ play only a minor role, when the development of the complex sea with "ages" of the waves $\beta > 1/3$ is considered.

The condition for fully developed sea at any given wind velocity is $A = D$. With this and the equations (34) and (36), β_m is related in a fixed manner to the wind velocity v (see also [8]). But from equations (37)

$$M(1) = M e^{-2r \left(\frac{\beta_m^* - 1}{\beta_m} \right)},$$

and according to (38)

$$M(1) = M(\beta) e^{2r(1-\beta)}.$$

If we introduce this in (57), we have

$$B_2(\beta) = \frac{8\pi^2 g n^2}{\rho'} \frac{M(1)}{v^3 \beta} e^{-2r},$$

where $M(1)$ according to (36) may be written

$$M(1) = 0.1825 \cdot 10^{-4} v^{5/2} e^{-2r \left(\frac{\beta_m^* - 1}{\beta_m} \right)}.$$

Because $\beta_m = f(v)$, $M(1)$ must be a function of v too, if we assume β_m^* to be constant at all wind velocities. If $M(1)$ represents a function of v only, it has to be determined in such a way that there are no contradictions to the relation $\beta_m = f(v)$ as determined for example, in a previous paper [8].

With

$$\beta_m = \frac{2r(\beta_m^* - 1)}{\ln 182.5 - \ln \sqrt{v}} = f(v) , \quad (60)$$

it follows from (59) that

$$M(1) = 10^{-7} v^3 [\text{cm}^{-1} \text{g. sec}^{-1}] . \quad (61)$$

If numerical values for constants are introduced, and (61) is considered, we get from (57a)

$$B_2(\beta) = 2.6 \cdot 10^{-3} \beta^{-1} . \quad (62)$$

Thus with (60), $B_2(\beta)$ is represented as a function of β . Since $C_2(\beta)$ according to (28) only is a function of β , the difference $C_2 - B_2 = F(\beta)$.

Equation (60) relates the phase velocity $\sigma = \beta_m v$ of fully developed "seas" (β_m -waves) to the wind velocity (v given in cm/sec), and yields practically the same values for $v \geq 2$ m/sec as calculated previously [8]. Table 6 shows the values β_m given by (60) and the values β_m as published in [8].

Table 6. $\beta_m = f(v)$ at different wind velocities given by formula (60) and in an earlier report [8].

v m/sec	2	4	6	8	10	12	16	20	24	28
$\beta_m(60)$	0.48	0.56	0.61	0.66	0.70	0.74	0.81	0.88	0.94	1.00
$\beta_m[8]$	0.425	0.53	0.60	0.66	0.70	0.74	0.81	0.88	0.94	0.995

It is possible that the ratio β_m^* is not strictly constant for all wind velocities. Probably, it is smaller at lower wind speeds, but this question has to be checked by observations. In this connection, Chapter I of this report, where this particular question has already been discussed, may be referred to. With very weak winds, a value $\beta_m = 1/3$ is to be expected when $v = 1.23$ m/sec. If at a wind velocity $v = 1.5$ m/sec the value $\beta_m = 0.38$ as given in [8], it follows from equation (60) that $\beta_m^* = 1.30$. The differences are perhaps only of minor significance.

6) Generation and growth of longer waves.

Within the fetch area where the waves are generated there always exists a large number of wave trains with different lengths and heights, traveling with the wind or at small angles to the wind direction. Fluctuations of both the wind velocity and direction may thereby contribute to the formation of short crested irregular waves, and it does not seem astonishing that a complicated pattern of irregular wave motion results from interference and criss-crossing. But experience shows that special systems of larger waves always can be recognized in the wave mixture. They dominate the sea surface as characteristic waves and are the striking features of the wind driven undulations.

So far the discussion of wave generation has dealt with waves called "sea" or " β_m -waves." When these waves have attained a certain maximum length and height, or a certain "age" β_m and steepness δ_m , they apparently do not continue to develop, but remain constant as relatively steep waves, breaking from time to time. However, the complex sea in this stage is not fully developed. If the wind con-

tinues, the sea surface pattern changes its appearance considerably. Symptoms of fully or nearly fully developed sea are striking fluctuations of wave heights and periods, groups of waves and the occurrence of outsize waves with long stretched crests. In stormy weather the sea begins to "roll."

To all appearance longer waves emerge at the rough sea surface probably independent of the fully arisen β_m -waves, and distinguished by lesser steepness but with propagation velocities which may exceed the wind velocity. By interference of these wave systems typical phenomena of complex wave motion result.

In Chapter I, an attempt was made to explain the observed striking fluctuations of wave periods and heights in the complex wave pattern by coincidence of three characteristic waves, called the "sea," the "intermediate wave" and the "longer wave." In the following discussions these dominating waves may also be noticed by their "ages," that is, by the ratio "phase velocity : wind velocity" in fully arisen state, that is " β_m -wave," " $\beta(1)$ -wave" and " β_m^* -wave." It seems probable that in the mixture of wind generated ocean waves certain undulations are favored by the wind with respect to the energy transfer. They have a maximum growth, and having attained their maximum values of height and length, they are maintained by a steady energy supply as long as the wind remains unaltered.

The first characteristic wave, the relatively steep β_m -wave travels with a phase velocity, that is always slower than the wind velocity. Probably we have to presuppose the existence of these waves, in order that longer waves with a higher amount of energy, and faster than the wind can be generated. The short but steep β_m -waves cause, with superimposed smaller waves, the broken appearance

of the sea, and a definite roughness of the sea surface. Thus they contribute essentially to the resistance coefficient and therefore to the effective horizontal stress, which acts at the rough air-sea interface. If longer waves emerge at this rough interface the total horizontal stress will do a net amount of work even if the longer waves travel faster than the wind (see equation (6) with (4a), or A_t in (9)). Thus, a certain "waviness" of the air-sea interface will be maintained against dissipation as long as the wind velocity remains constant.

However, the mechanism governing the tangential transfer of energy to waves which may travel with $\sigma \geq v$ is not yet completely explained. If the surface mass transport velocity $\pi^2 \delta^2 \sigma$ from Stokes' theory is taken into account, (equation (42)), a transfer of energy from wind to waves would be possible even if the waves move faster than the wind (equation (6) and A_t in (9)). This idea, first used by Sverdrup-Munk [1] seems plausible, and is fit to overcome the difficulties. But if the difference between the wind velocity and the horizontal component of particle velocity is introduced in the expression for τ_t , and if the variation in shear due to the variation of wind velocity and due to the variation of the water particle velocity at the wavy surface is considered, apparently no energy is added even on the basis of Stokes' theory, if the waves move with $\sigma \geq v$. The result of an analysis of Schaaf and Sauer [15] would limit the growth of the waves to the region where the wave velocity is less than the wind velocity. From the expression for A_t given by these authors, it follows that no energy is added by tangential shear stress, if the wave velocity exceeds about 75% of

the wind velocity.

But, in fact, observations show that longer waves with $\sigma > v$ are generated in the wind region (see [23]). Probably these waves have been generated under conditions not covered in the analysis of Schaaf and Sauer. Special attention must be called again to the fact that our knowledge of the actual distribution of τ_t (and τ_n), or of the actual difference of wind velocity-water particle velocity at the real ocean surface with its complicated wave pattern is too meager, and it seems very difficult at present to estimate the effect of normal stress components and tangential stress components separately. The actual rough sea surface may offer entirely different conditions than are assumed in mathematical treatments of the problem, where only a simple sine-wave profile and simple water particle displacements are considered.

A special mechanism therefore may be considered in this connection, which probably plays no unimportant part in the generation of long waves which are faster than the wind. The idea is that short period waves in the generating area may disappear, transferring their energy to other waves. This special kind of energy transfer seems to be possible only in complex wave motion, and its mechanism perhaps may be explained in the following way.

Consider the fully arisen steep "sea" or β_m -wave traveling always with $\sigma < v$, and constantly overtaken by flat disturbances traveling in the same or nearly the same direction but with a phase velocity, $\sigma = v$ or $\sigma > v$. Every time the low crest of the long wave disturbance overtakes a crest of the steep β_m -wave, the water particle velocities of both wave motions are added, and according to the dimensions of the two waves, the horizontal particle speed will increase

more at the surface, than in deeper water. Thus, the horizontal particle speed will not only increase at the crest of the β_m -wave, but at the same time the vertical velocity gradient will become steeper too. Both effects support an extensive breaking of the crest of the superimposed steep β_m -wave. Now the long wave overtakes the breaking crest and approaches the next β_m -crest in the leeward direction, where after some time the same thing happens, and so on. Thus, as long as the faster traveling long wave overtakes a train of undisturbed β_m -waves (with originally maximum steepness δ_m) breakers may occur at the crest of the long wave disturbance, and every new breaker is placed to the leeward of the previous one, looking in the direction of wave propagation. It may be mentioned that perhaps this may be offered as an explanation for the "law of breakers" by K. Wegener [24], who states: "Die See bricht so, dass sich eine brechende See immer vor die vorhergehende setzt." Furthermore, the foam patches (at higher velocities) will orient themselves in rows, extending to leeward and gradually disappearing at the windward end.

The particle speed of the β_m -waves at a given wind velocity is greater than the particle speed of any other wave with $\sigma \geq v$ and a maximum steepness given by (17), as shown in the following Table 7 by the values u_0 at different wind velocities. The maximum horizontal component of particle speed at the surface is given by

$$u_0 = \pi \delta \sigma .$$

The table shows δ_m and σ_m at different wind velocities for the fully arisen β_m -wave, as well as for the $\beta(1)$ -wave and β_m^* -wave. From this, amplitudes of particle speed u_0 are computed. At a wind

Table 7. Steepness (δ), phase velocity (σ), and amplitude of horizontal velocity component u_0 at the sea surface for three characteristic waves.

v m/sec		5	10	16	20	24	28
β_m -wave	δ_m	0.083	0.067	0.0555	0.049	0.0445	0.0407
	σ_m m/sec	.284	7.00	12.95	17.6	22.6	27.9
	u_0 m/sec	0.74	1.48	2.26	2.71	3.16	3.56
$\beta(1)$ -wave	$\sigma(1)$ m/sec	5	10	16	20	24	28
	$\delta(1)=0.0406$ u_0 m/sec	0.64	1.28	2.05	2.56	3.06	3.56
β_m^* -wave	σ_m m/sec	6.84	12.3	21.9	27.4	32.9	38.4
	$\delta_m=0.0222$ u_0 m/sec	0.48	0.86	1.54	1.92	2.3	2.7

velocity of $v = 16$ m/sec, for example, the sum of u_0 for the β_m -wave and the $\beta(1)$ -wave would be 4.31 m/sec.

When the crests of the β_m -waves break, the waves lose energy and diminish in height. Part of this energy is dissipated and lost with respect to wave motion. But it seems reasonable to assume that another part of the energy, given off by the β_m -wave will be transferred to the underlying long wave. Because these breakers always occur at the crests of the long waves where the particle displacement is in the direction of wind and wave propagation, the particle velocity originally connected with long wave motion will be speeded up by the forward rushing water masses of the breakers.* Here the breakers do a net amount of work analogous to the work done by the wind in the case of waves with $\sigma < v$. The degenerated β_m -wave becomes regenerated by energy supply from wind, and in this way energy may

*It also seems possible that the wind will do a net amount of work directly on the long wave, by accelerating the spraying water masses of breakers at the crests and the upper windward slope of the long waves.

be transferred from the wind over the shorter waves to longer waves with $\sigma \geq v$, as long as in the energy balance $A - D > 0$.

When in the generating process of complex wave motion the first characteristic waves (β_m -waves) attain their maximum steepness, the wave length of these waves, or their propagation velocity respectively, is given by β_m (equation (60)) and their height H by (17). In this stage of wave development, a certain amount of energy in the difference $A - D$ is left, and this remainder is used for generation of longer waves until an equilibrium state is attained between the acting forces, that is, between the drag, the normal pressure forces and the dissipative forces. Without additional increase of the effective resistance of the sea surface, and therefore without an additional increase of the total drag at a given wind velocity, this stage $A = D$ will be approached by nature in the easiest way when waves which proceed with a phase velocity equal to the wind velocity ($\sigma = v$; $\beta(1) = 1$) are generated.

Let us assume that the sea surface takes on a waviness which corresponds to the wave length of $\beta(1)$ -waves, originally beginning with very small disturbances which may always be present. By taking up energy from wind, the height $H(1)$ of these disturbances will increase until it attains its maximum value. This maximum height, $H(1)_m$, is given by the maximum steepness, according to (17)

$$\delta(1)_m = 2n e^{-r} = 0.0406 . \quad (63)$$

Thus

$$H(1)_m = \lambda(1) 2n e^{-r} . \quad (63a)$$

The total wind forces at the sea surface have not changed during the generation of these "intermediate waves" or $\beta(1)$ -waves. As

long as there is a positive amount of energy $A - D$ left, the waves will grow further even when reaching the state $\delta(1)_m$. But they cannot exceed the maximum steepness given by (63), and therefore they will grow by increasing their wave length. Later on, we shall see that the further development of waves beyond the state $\beta(1)$, $\delta(1)_m$, manifests itself in an increase of wave length, whereas the height remains nearly the same. Thus, the longer waves (β^* -waves) rather soon increase in length, and attain propagation velocities that exceed the wind velocity. With this newly generated waviness the normal pressure components of wind force are out of phase by the phase difference π . The further development of the waves therefore will be increasingly delayed, the more their wave lengths increase and the faster these waves travel. Finally the state $A = D$ will be reached approximately when the longer waves travel with a phase velocity of $\sigma^*_m = 1.37v$. The work done by these normal pressure forces, which act with a negative sign at the long β_m^* -waves, together with the dissipation, balance the work done by tangential stresses on complex wave motion in this state.

It is to be expected that with increasing height of the $\beta(1)$ -waves, interference phenomena appear between these waves and the β_m -waves, which depend upon the wind velocity and upon the state of development of the $\beta(1)$ -waves. As a consequence, from time to time, particularly high waves with considerable steepness will occur, forming higher and more spacious "breakers" than in the preceding stages of wave development. These increasing breakers imply an eddying of more extended and deeper water masses of the surface layer, and therefore will be succeeded by an increase of the

turbulent state of surface layers. It seems reasonable to assume that the turbulence, that means the eddying of the surface layers as far as they are concerned in the wave motion, increases with the development of height of the $\beta(1)$ -wave. By (38) the coefficient of eddy viscosity in the stage of fully arisen β_m -waves was given as $M(\beta_m)$. In the stage of fully developed $\beta(1)$ -wave with a height given by (63), we have $M(1)$ as given by (37) when $\beta = 1$. For the increase of eddy viscosity during the stage of $\beta(1)$ -wave development, we assumed $M(1)$ to be a function of wave height $H(1) = 2a(1)$, and put

$$M(1) = M(\beta_m) e^{2r(1-\beta_m) \frac{a}{a(1)_m}} \quad (64)$$

where $a(1)_m$ is the maximum amplitude of the $\beta(1)$ -wave.

As an example, the following table shows the increase of the coefficient of turbulence (eddy viscosity) with increasing $\beta(1)$ -wave at a wind velocity of $v = 10$ m/sec:

$a(1)/a(1)_m$	0	0.25	0.5	0.75	1.00
$M(1)$ cgs	36.5	46.9	60.2	77.1	99.0

The increase of the $\beta(1)$ -wave with increasing fetch for stationary wind conditions (Case B) will be, considering

$$\frac{\partial E}{\partial t} = 0, \quad \text{and} \quad \lambda(1) = \text{const.},$$

$$\frac{\sigma}{2} \frac{dE}{dx} = A - D$$

or

$$\frac{\sigma}{2} g \rho a \frac{da}{dx} = A - D. \quad (65)$$

Taking $\beta(1) = 1$, we have with (27)

$$A = [\gamma(\beta_m) + s' \pi n e^{-r\beta_m(1-\beta_m)^2}] \rho' v^3 \quad (66)$$

$$A = C_2(\beta_m) \rho' v^3$$

The dissipated energy according to (64) and (32) is

$$D = 2M(1)\pi^2 g \delta^2 = 2M(\beta_m)\pi^2 g \frac{4a^2}{\lambda(1)^2} e^{2r(1-\beta_m) \cdot a/a(1)_m} \quad (67)$$

where a means the amplitude of the increasing $\beta(1)$ -wave, $a(1)_m$ its maximum value given by (63) and

$$\lambda(1) = \frac{2\pi}{g} v^2 \quad (68)$$

the wave length. With

$$a_m = \lambda(1)n e^{-r} = \lambda(1)\bar{n} \quad (69)$$

(67) can be written

$$D = 8\pi^2 g \frac{a^2}{\lambda(1)^2} M(\beta_m) e^{2r(1-\beta_m) \frac{a}{\lambda(1)\bar{n}}} \quad (70)$$

Since $\sigma = v$ and $\beta(1) = 1$, equation (65) takes the form

$$\frac{1}{2} \cdot g \rho v a \frac{da}{dx} = C_2(\beta_m) \rho' v^3 - 8\pi^2 g \frac{a^2}{\lambda(1)^2} M(\beta_m) e^{\frac{2r}{\lambda(1)\bar{n}}(1-\beta_m)a} \quad (71)$$

according to (66) and (70). By division with $\rho' v^3$ and considering (68) we get

$$\frac{1}{2} \cdot \frac{\rho g}{\rho' v^2} a \frac{da}{dx} = C_2(\beta_m) - \frac{2g^3 M(\beta_m) a^2}{\rho' v^7} e^{\frac{rg}{\pi v^2 \bar{n}}(1-\beta_m)a} \quad (72)$$

or

$$dx = \frac{1}{2} \frac{\rho g}{\rho' v^2} \cdot \frac{a da}{C_2(\beta_m) - \frac{2g^3 M(\beta_m)}{\rho' v^7} a^2 \exp\left[\frac{rg}{\pi v^2 \bar{n}}(1-\beta_m)a\right]} \quad (73)$$

With regard to the later numerical calculation, it is more convenient

to introduce the dimensionless wave steepness

$$\delta(1) = \frac{2a}{\lambda(1)}$$

in place of the amplitude a . From equation (68) we have

$$\frac{g^2}{v^4} = \frac{4\pi^2}{\lambda(1)^2} ,$$

and therefore

$$dx = \frac{1}{2} \frac{\rho}{\rho'} \frac{v^2}{g} \pi^2 \frac{\delta(1) d\delta(1)}{C_2(\beta_m) - \frac{2\pi^2 g}{\rho' v^3} \delta(1)^2 M(\beta_m) \exp \left[\frac{r}{n} (1 - \beta_m) \delta(1) \right]} \quad (74)$$

In an analogous manner we get in the case of an unlimited fetch with increasing duration of wind action (Case A) according to equation (45), and the condition

$$\frac{d(\lambda(1))}{dx} = 0 ,$$

the differential relation

$$\frac{v^4}{g} g\pi^2 \delta(1) \frac{d\delta(1)}{dt} = C_2(\beta_m) \rho' v^3 - 2\pi^2 g \delta(1)^2 M(\beta_m) e^{\frac{r}{n}(1-\beta_m)\delta(1)} \quad (75)$$

or

$$dt = \frac{\rho}{\rho'} \frac{v}{g} \pi^2 \frac{\delta(1) d\delta(1)}{C_2(\beta_m) - \frac{2\pi^2 g}{\rho' v^3} \delta(1)^2 M(\beta_m) \exp \left[\frac{r}{n} (1 - \beta_m) \delta(1) \right]} . \quad (76)$$

With

$$D = 8M\pi^2 g n^2 e^{-2r\beta_m^*}$$

the fully developed state of complex sea is given by equation (34). This happens when the long waves (β_m^* -waves) are fully arisen and $A = D$. When considering the growth of the β^* -waves up to the state $\beta_m^* = 1.37$, it is necessary to take into account that with increasing

wave length the height of these waves may also grow. But the results will show that the increase in wave height is only of minor significance, and this result confirms experience at sea.

For Case A where the waves are considered to grow with the duration of wind action along an unlimited fetch, and for Case B where the waves are considered to grow along the fetch under the action of a wind of sufficiently long duration, we get analagous to (57) and (58) for the state $\beta^* > 1$

Case A

$$dt = \frac{\rho}{\rho'} \frac{4\pi^2 n^2 v \cdot \beta^{*2} e^{-2r\beta^*} (3 - r\beta^*)}{C_3(\beta) - B_3(\beta)} d\beta^* ; \quad 1 \leq \beta^* \leq \beta_m^* \quad (77)$$

Case B

$$dx = \frac{\rho}{\rho'} \frac{2\pi^2 n^2 v^2 \cdot \beta^{*3} e^{-2r\beta^*} (3 - r\beta^*)}{C_3(\beta) - B_3(\beta)} d\beta^* ; \quad 1 \leq \beta^* \leq \beta_m^*, \quad (78)$$

where $C_3(\beta)$ is given by equation (30), or

$$C_3(\beta) = C_2(\beta_m) - s^* \pi n e^{-r\beta^*} (1 - \beta^*)^2 \quad (79)$$

and

$$B_3(\beta) = \frac{8\pi^2 n^2 g \cdot M(\beta^*) e^{-2r\beta^*}}{\rho' v^3 \beta^*} \quad (80)$$

$M(\beta^*)$ is to be determined by (37).

7) The growth of complex sea under the action of wind. Numerical results, and comparison with observations

The differential relations derived in Sections 5 and 6 give the increase of fetch x or the increase of duration t of wind action necessary for the development of certain stages of complex wave motion. So far, the growth of the waves has been examined only in two special cases, Case A and Case B. Under natural conditions,

both fetch and duration are to be taken into account, and often the increase of wind-speed with time has also to be considered. It is possible to give numerical solutions for the case of a variable wind $v = v(x, t)$, but this report shall be confined to evaluations of the two special cases considered in the preceding sections, which may give some help in wave forecasting. An attempt to consider more complicated weather conditions at sea for practical wave forecasts will be made in a following report.

For the purpose of numerical evaluation, the derived differential relations are written in non-dimensional form using the "fetch parameter" gx/v^2 , and gt/v as the "duration parameter." Integrations must be carried out numerically.

Case A: Growth of the sea over an unlimited fetch as a function of wind duration t .

$$1) \frac{gt}{v} = \frac{\rho}{\rho'} 12\pi^2 p^2 \sum_{\beta=0}^{\beta=1/3} \frac{\beta^2}{C_1(\beta) - B_1(\beta)} \Delta \beta ; \quad \beta \leq 1/3 \quad (81)$$

$$2) \frac{gt}{v} = \frac{\rho}{\rho'} 4\pi^2 n^2 \sum_{\beta=1/3}^{\beta_m} \frac{\beta^2 e^{-2r\beta} (3 - r\beta)}{C_2(\beta) - B_2(\beta)} \Delta \beta + K_1 ; \quad 1/3 \leq \beta \leq \beta_m \quad (82)$$

$$3) \frac{gt}{v} = \frac{\rho}{\rho'} \pi^2 \sum_{\delta(1)=0}^{\delta(1)_m} \frac{\delta(1) \Delta \delta(1)}{C_2(\beta_m) - \frac{2\pi^2 g}{\rho' v^3} \delta(1)^2 M(\beta_m) \exp[(r/\bar{n})(1-\beta_m)\delta(1)]} + K_2 ;$$

$$\beta(1) ; \quad 0 \leq \delta(1) \leq \delta(1)_m ; \quad (83)$$

$$4) \frac{gt}{v} = \frac{\rho}{\rho'} 4\pi^2 n^2 \sum_{\beta^*=1}^{\beta_m^*} \frac{\beta^{*2} e^{-2r\beta^*} (3 - r\beta^*)}{C_3(\beta) - B_3(\beta)} \Delta \beta^* + K_3 ; \quad 1 \leq \beta^* \leq \beta_m^* ; \quad (84)$$

The "factors of energy supply" $C_1(\beta)$, $C_2(\beta)$ and $C_3(\beta)$ are given by (20), (28) and (30), "the factors of energy dissipation"

$B_1(\beta)$, $B_2(\beta)$ and $B_3(\beta)$ are given by (55), (57), and (80). Let the density of the air be $\rho' = 1.25 \cdot 10^{-3}$ and the density of the sea water $\rho = 1.028$. The other constants, which act in the formulas are already explained and their numerical values are given in the preceding text. K_1 , K_2 and K_3 in (82), (83) and (84) are constants of integration, taking into account a duration parameter gt/v as an additional value at the lower limit of integration.

Case B: Growth of the sea with a constant wind of unlimited duration as a function of fetch x.

$$1) \frac{gx}{v^2} = \frac{\rho}{\rho'} 6\pi^2 p^2 \sum_{\beta=1/3}^{\beta=1/3} \frac{\beta^3}{C_1(\beta) - B_1(\beta)} \Delta\beta ; \quad \beta \leq 1/3 ; \quad (85)$$

$$2) \frac{gx}{v^2} = \frac{\rho}{\rho'} 2\pi^2 n^2 \sum_{\beta=1/3}^{\beta_m} \frac{\beta^3 e^{-2r\beta} (3 - r\beta)}{C_2(\beta) - B_2(\beta)} \Delta\beta + K_1' ; \quad 1/3 \leq \beta \leq \beta_m ; \quad (86)$$

$$3) \frac{gx}{v^2} = \frac{1}{2} \frac{\rho}{\rho'} \pi^2 \sum_{\delta(1)=0}^{\delta(1)_m} \frac{\delta(1) \Delta\delta(1)}{C_2(\beta_m) - \frac{2\pi^2 g \delta(1)^2 M(\beta_m) \exp[(r/\bar{n})(1-\beta_m)\delta(1)]}{\rho' v^3}} + K_2' ;$$

$$\beta(1) ; \quad 0 \leq \delta(1) \leq \delta(1)_m ; \quad (87)$$

$$4) \frac{gx}{v^2} = \frac{\rho}{\rho'} 2\pi^2 n^2 \sum_{\beta^*=1}^{\beta_m^*} \frac{\beta^{*3} e^{-2r\beta^*} (3 - r\beta^*) \Delta\beta}{C_3(\beta) - B_3(\beta)} + K_3' ; \quad 1 \leq \beta^* \leq \beta_m^* , \quad (88)$$

where K_1' , K_2' , and K_3' are constants of integration.

The equations relate

$$\beta \quad \text{or} \quad \beta^* = \varphi\left(\frac{gx}{v^2}\right), \quad \text{and} \quad \beta \quad \text{or} \quad \beta^* = F\left(\frac{gt}{v}\right) \quad (89)$$

for the phase velocity ($\sigma = v \cdot \beta$) considering the state of wave development

either for $\beta \leq \beta_m$ or for $1 \leq \beta^* \leq \beta_m^*$,

as well as $\delta(1) = 2a/\lambda(1)$ as a function of the fetch parameter gx/v^2 , or the duration parameter gt/v , when considering the growth of amplitude of the $\beta(1)$ -wave. $\lambda(1)$ as given by (68) is a function of the wind velocity.

The wave height is

$$H = \delta \cdot \lambda = \frac{2\pi}{g} \delta v^2 \beta^2 .$$

Therefore H will be represented at a given wind velocity by the non-dimensional parameter

$$\frac{gH}{v^2} = 2\pi\delta\beta^2 . \quad (90)$$

Because $\delta = f(\beta)$ as given by (17) and (18), and β is related to the fetch parameter or duration parameter by (89), the relation (90) may be written

$$\frac{gH}{v^2} = \Phi\left(\frac{gx}{v^2}\right), \quad \text{or} \quad \frac{gH}{v^2} = f\left(\frac{gt}{v}\right) . \quad (89a)$$

With all this, the wave elements (β , δ or σ , λ , T and H) are related in a nondimensional form to the fetch parameter or the duration parameter.

From (18),

$$\frac{gH}{v^2} = 4\pi p \beta^2 = 0.78\beta^2 ; \quad \beta \leq 1/3 , \quad (91)$$

and from (17)

$$\frac{gH}{v^2} = 4\pi n \cdot e^{-r\beta} \beta^2 = 1.35e^{-r\beta} \beta^2 ; \quad 1/3 \leq \beta \leq \beta_m , \quad (92a)$$

and

$$\frac{gH}{v^2} = 4\pi n e^{-r\beta^*} \beta^{*2} = 1.35e^{-r\beta^*} \beta^{*2} ; \quad 1 \leq \beta^* \leq \beta_m^* \quad (92b)$$

Equation (92b) has a maximum value when $\beta^* = 1.191$. That means,

in the state of development of complex wave motion at sea, the long wave at a certain stage $\sigma^* = 1.191v$ grows up to a height a little larger than its height in the stage of fully developed sea, where $\sigma_m^* \sim 1.37v$. But the differences are not very important, likewise the growth of the wave height from the fully developed $\beta(1)$ -wave to the state of fully arisen β_m^* -wave is of minor importance. Hence, it follows that in the stage of "long wave"-generation the energy supply by wind is used almost completely for an increase of the wave length. (Compare the heights H in Table 10 for these stages of wave development.)

Tables 8 and 9 represent some numerical results of the theory which may be supplemented by the graphs in figures 15 through 19, and figure 26. These tables and graphs are suitable for determining the characteristics of complex wave generation by means of data from adequate, synoptic weather maps, for given wind and fetch conditions.

Numerical examples:

From Tables 8 and 9 or graphs, at wind velocities of $v = 5$ m/sec, 10 m/sec, 16 m/sec, 20 m/sec, and 24 m/sec, the following characteristics of complex sea, represented in Table 10 can be found.

These examples show that the minimum fetch (x_m) or duration (t_m) needed for generating fully arisen sea rapidly increases with increasing wind velocity. At $v = 5$ m/sec, the complex sea with the three characteristic waves would be fully developed over a fetch $x_m = 13.8$ km, or over an unlimited fetch it would take $t_m = 2.25$ hours from the time the wind starts to blow (with constant velocity). At $v = 10$ m/sec, the minimum fetch would be $x_m = 106$ km and the duration

Table 8.

1) gx/v^2 , gt/v and gH/v^2 for different values of $B = \sigma/v$ in the stages of development $B \leq 1/3$ and $1/3 \leq B \leq 1$.

B	0.10	0.20	0.30	0.35	0.40	0.45	0.50	0.55	0.60
gx/v^2	0.55	23.5	180	385	657	1040	1560	2260	3160
gt/v	22	309	1520	2750	4240	6020	8270	10880	14000
gH/v^2	0.0078	0.0311	0.0700	0.0925	0.1108	0.129	0.147	0.163	0.1785

B	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
gx/v^2	4270	6100	8050	10800	14050	17500	21200	26000
gt/v	17600	23000	28400	35400	43500	52000	60000	70000
gH/v^2	0.1920	0.206	0.217	0.227	0.236	0.243	0.248	0.254

2) $a = f(t)$ and $a = f(x)$ for the growth of $B(1)$ -waves at different wind velocities v .

($a = H/2$ = wave amplitude)

$v = 2$ m/sec : $H = 0.105$ m

a (cm)	0	1	2	3	4	5	5.25	For $H = 0.105$ m
x (m)	0	7.1	28.3	64.5	117.0	190	214	gx/v^2 525
t (min)	0	0.12	0.47	1.08	1.92	3.2	3.6	gt/v 1050
								gH/v^2 0.254

$v = 5$ m/sec : $H = 0.65$ m

a (cm)	0	5	10	15	20	25	32.5	For $H = 0.65$ m
x (m)	0	39	157	358	652	1062	2047	gx/v^2 819
t (min)	0	0.26	1.05	2.4	4.2	7.1	13.7	gt/v 1650
								gH/v^2 0.254

$v = 10 \text{ m/sec} : H = 2.6 \text{ m.}$

$a(\text{cm})$	20	40	60	80	100	120	130	For $H = 2.6 \text{ m}$	
$x(\text{ m})$	200	810	1850	3420	5710	9280	12110	$\frac{gx}{v^2}$	1211
$t(\text{min})$	0.67	2.7	6.2	11.4	19.1	31.0	40.5	$\frac{gt}{v}$	2400
								$\frac{gH}{v^2}$	0.254

$v = 16 \text{ m/sec} : H = 6.64 \text{ m}$

$a(\text{cm})$	50	100	150	200	250	300	332	For $H = 6.64 \text{ m}$	
$x(\text{ m})$	590	2600	5760	10620	18000	29900	43700	$\frac{gx}{v^2}$	1710
$t(\text{min})$	1.23	5.4	12.0	22.2	37.5	63	92	$\frac{gt}{v}$	3360
								$\frac{gH}{v^2}$	0.254

$v = 20 \text{ m/sec} : H = 10.34 \text{ m.}$

$a(\text{cm})$	100	200	250	300	350	400	450	517	For $H = 10.34 \text{ m}$	
$x(\text{ m})$	1650	6970	11140	16640	23940	33790	47990	86000	$\frac{gx}{v^2}$	2150
$t(\text{min})$	2.75	11.6	19.0	27.7	39.9	56.5	80	144	$\frac{gt}{v}$	4300
									$\frac{gH}{v^2}$	0.254

$v = 24 \text{ m/sec} : H = 15 \text{ m.}$

$a(\text{cm})$	100	200	300	400	500	600	700	750	For $H = 15 \text{ m}$	
$x(\text{ m})$	1210	4950	11570	21770	37170	61370	105600	150000	$\frac{gx}{v^2}$	2500
$t(\text{min})$	1.7	6.9	16.0	30.5	52	86	147	209	$\frac{gt}{v}$	5280
									$\frac{gH}{v^2}$	0.254

$v = 28 \text{ m/sec} : H = 20.4 \text{ m.}$

$a(\text{cm})$	200	400	500	600	700	800	900	1020	For $H = 20.4 \text{ m}$	
$x(\text{ m})$	3800	15900	26100	40000	59200	86800	130500	247500	$\frac{gx}{v^2}$	3100
$t(\text{min})$	4.53	18.9	31.0	47.7	71	104	156	295	$\frac{gt}{v}$	6200
									$\frac{gH}{v^2}$	0.254

3) gx/v^2 , gt/v and gH/v^2 for different values of $\beta^* > 1$
at different wind velocities between $v = 2$ m/sec
and $v = 28$ m/sec.

$v = 2$ m/sec.

β^*	1.0	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.37
gx/v^2	1355	1433	1513	1598	1688	1790	1920	2435	
gt/v	6250	6402	6553	6702	6855	7022	7228	7995	
gH/v^2	0.254	0.2585	0.261	0.2625	0.263	0.2625	0.2615	0.259	0.258

$v = 5$ m/sec.

β^*	1.0	1.05	1.10	1.15	1.20	1.25	1.30	1.35
gx/v^2	3469	3611	3764	3929	4110	4329	4619	5429
gt/v	13650	13927	14210	14500	14810	15170	15590	16885
gH/v^2	0.254	0.2585	0.261	0.2625	0.263	0.2625	0.2615	0.259

$v = 10$ m/sec.

β^*	1.0	1.05	1.10	1.15	1.20	1.25	1.30	1.35
gx/v^2	7311	7591	7889	8201	8530	8930	9440	10610
gt/v	25400	25945	26500	27050	27630	28260	29070	30860
gH/v^2	0.254	0.2585	0.261	0.2625	0.263	0.2625	0.2615	0.259

$v = 16$ m/sec.

β^*	1.0	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.37
gx/v^2	13210	13720	14215	14700	15220	15800	16560	18060	
gt/v	40560	41558	42470	43350	44220	45190	46380	48610	
gH/v^2	0.254	0.2585	0.261	0.2625	0.263	0.2625	0.2615	0.259	0.258

$v = 20$ m/sec.

B^*	1.0	1.05	1.10	1.15	1.20	1.25	1.30	1.35
gx/v^2	18550	19274	19945	20600	21280	22030	22940	24620
gt/v	52800	54210	55450	56600	57750	58940	60400	62950
gH/v^2	0.254	0.2585	0.261	0.2625	0.263	0.2625	0.2615	0.259

$v = 24$ m/sec.

B^*	1.0	1.05	1.10	1.15	1.20	1.25	1.30	1.35
gx/v^2	23700	24750	25640	26460	27230	28080	29130	31000
gt/v	63780	65820	67480	68960	70300	71660	73310	76130
gH/v^2	0.254	0.2585	0.261	0.2625	0.263	0.2625	0.2615	0.259

$v = 28$ m/sec.

B^*	1.0	1.05	1.10	1.15	1.20	1.25	1.30	1.35
gx/v^2	29100	30540	31690	32700	33660	34700	35950	37900
gt/v	76200	79020	81150	82930	84600	86300	88200	91200
gH/v^2	0.254	0.2585	0.261	0.2625	0.263	0.2625	0.2615	0.259

Table 9.

Minimum fetch and minimum duration at different wind velocities for generating the characteristic wave motion to the stage of fully arisen B_m - waves, $B(1)$ - waves, and fully arisen complex sea (B_m^*). (Fetch x and duration t are given by

gx/v^2 and gt/v)

B_m -wave

v (m/sec)	2	5	7.5	10	12	14	16	18	20	24	28
B_m	0.425	0.57	0.65	0.70	0.74	0.775	0.81	0.845	0.88	0.94	0.995
gx/v^2	830	2650	4270	6100	7600	9350	11500	14000	16400	20700	26000
gt/v	5200	12000	17600	23000	27500	32200	37200	43000	48500	58500	70000

$B(1)$ - wave (complex).

v (m/sec)	2	5	7.5	10	12	14	16	18	20	24	28
gx/v^2	1355	3469	5290	7311	8980	10900	13210	15920	18550	23200	29100
gt/v	6250	13650	19650	25400	30180	35200	40560	46800	52800	63780	76200

B_m^* ($B^* = 1.35$) - wave (complex).

v (m/sec)	2	5	7.5	10	12	14	16	18	20	24	28
gx/v^2	2435	5429	7940	10611	12780	15200	18060	21320	24620	30500	37900
gt/v	7995	16885	24000	30860	36480	42380	48610	55800	62950	76130	91200

Table 10.

Characteristics of complex sea at different wind velocities.

H = wave height, λ = wave length, T = period.

1) $v = 5$ m/sec : $B_m = 0.57$

	H (m)	λ (m)	T (sec)	gx/v^2	gt/v	x (kilomet.)	t (hrs)
B_m -wave	0.43	5.2	1.82	2650	12000	6.75	1.7
$B(1)$ -wave	0.65	16.0	3.20	3469	13650	8.8	1.9
$B^* = 1.35$	0.66	29.2	4.33	5429	16885	13.8	2.25

2) $v = 10$ m/sec : $B_m = 0.70$

	H (m)	λ (m)	T (sec)	gx/v^2	gt/v	x (km)	t (h)
B_m -wave	2.1	31.3	4.48	6100	23000	61	6.4
$B(1)$ -wave	2.6	64	6.4	7311	25400	73	7.1
$B^* = 1.35$	2.65	117	8.65	10611	30860	106	8.6

3) $v = 16 \text{ m/sec}$: $B_m = 0.81$

	H (m)	λ (m)	T (sec)	gx/v^2	gt/v	x (km)	t (hrs)
B_m -wave	5.9	107	8.3	11500	37200	300	16.8
$B(1)$ -wave	6.6	163	10.2	13210	40560	345	18.4
$B^* = 1.35$	6.75	298	13.8	18060	48610	472	22.1

4) $v = 20 \text{ m/sec}$: $B_m = 0.88$

	H (m)	λ (m)	T (sec)	gx/v^2	gt/v	x (km)	t (hrs)
B_m - wave	9.8	198	11.3	16400	48500	670	27.6
$B(1)$ -wave	10.4	256	12.8	18550	52800	758	30.0
$B^* = 1.35$	10.6	467	17.3	24620	62950	1050	35.6

5) $v = 24 \text{ m/sec}$: $B_m = 0.94$

	H (m)	λ (m)	T (sec)	gx/v^2	gt/v	x (km)	t (hrs)
B_m - wave	14.6	327	14.5	20700	58500	1220	40.0
$B(1)$ -wave	14.9	370	15.4	23200	63780	1360	43.3
$B^* = 1.35$	15.2	673	20.7	30500	76130	1800	51.8

Fig 15

Fig. 16

Fig.17

Fig.18

Fig. 19

$t_m = 8.6$ hours. But for the development of fully arisen storm waves, fetches of $x_m > 1000$ km are necessary. Thus, example 4 (Table 10) shows that at $v = 20$ m/sec (about 8 to 9 Beaufort) the minimum fetch for the generation of complex sea is $x_m = 1050$ km or nearly 600 nautical miles; at $v = 24$ m/sec (example 5) $x_m = 1800$ km or about 1000 nautical miles.

These computed fetches are in agreement with the statements of experienced wave and sea observers (V. Cornish, Graf von Larisch). V. Cornish estimates the minimum fetch for the generation of fully arisen "storm waves" to be about 600 to 1000 nautical miles (see H. Thorade [22], page 293). If the wind blows constantly, the duration t_m for developing these storm waves would be at $v = 20$ m/sec, $t_m = 1.5$ days ($t_m = 35.6$ hours); and at $v = 24$ m/sec, it would take $t_m = 2.2$ days ($t_m = 51.8$ hours).

Furthermore, the examples represented in Table 10 show that the development of complex sea after the generation of fully arisen β_m -waves continues relatively quickly. It has already been stated in Chapter I that the observations at sea indicate a rapid development of longer waves at the rough sea surface when the fetches exceed certain minimum stretches. Under these conditions characteristic interference patterns of complex wave motion emerged as the striking features of the sea.

To get an idea of the results of the theoretical computation, conditions at a constant wind velocity of $v = 16$ m/sec are considered in more detail. The application of the presented graphs and tables leads to the following characteristics of complex sea generation at this wind velocity: The first dominating wave (β_m -wave) appears

fully arisen with $\sigma_m = 0.81 \cdot v = 13 \text{ m/sec}$ at the end of a fetch $x = 300 \text{ km}$ (Table 10). (Considering the growth with time t over an unlimited fetch, or over a fetch long enough, it would take 16.8 hours, if the wind starts to blow over an undisturbed water surface.) The development of these β_m waves with increasing fetch and duration is illustrated in figures 20 and 22. Fig. 20 shows the height and the period of the β_m -waves, as functions of the distance from the coast, for various wind durations. When the wind has blown, say, for 5 hours, a very rapid increase in wave height out to a distance of 62 km from the coast is found. The steepness of these waves is considerable, and the steepness graph in Fig. 21 gives for a duration of 5 hours and a fetch of 62 km $H/\lambda = 0.0855$. The wave height of the waves is 4.28 m, and their period (see curve T in Fig. 22) is $T = 5.65 \text{ sec}$. ($\lambda = 50 \text{ m}$.) Beyond 62 km the waves are similar at a duration of 5 hours, but still in the growing state, whereas inside of 62 km a steady state has been reached. That means, at any given point along the fetch from $x = 0$ to $x = 62 \text{ km}$ the waves do not change with increasing duration of wind action, while beyond 62 km the waves continue to grow for a length of time which depends upon the fetch. Thus, for example, a steady state is to be found after a duration of 10 hours inside of a fetch of about 150 km. If the wind continues to blow with constant velocity (16 m/sec), the fully arisen β_m -wave with $\sigma_m = 0.81 v$, $T = 8.3 \text{ sec}$, $\lambda = 107 \text{ m}$, $H = 5.9 \text{ m}$ appears after 16.8 hours, and inside of the fetch $x = 300 \text{ km}$ a steady state is attained as given by the curves H and T in figures 20 and 22. Correspondingly, the other graphs may be used for determining the state of the sea, that is, the steepness graph (Fig. 21) and the wave age graph (Fig. 23). (Similar graphs and tables for other

Fig. 20. Wave height (H) and period (T) in the first stages of complex sea development (β_m -wave) as functions of distance from coast line (fetch x) at different hours of duration after a wind of $v = 16 \text{ m/sec}$ started to blow over an undisturbed water surface.

Fig. 21. Wave steepness, H/λ , as a function of x and duration (hours) at a wind velocity $v = 16$ m/sec, corresponding to figure 20.

wind velocities will be given in a following report.)

If the duration of wind with $v = 16$ m/sec exceeds 16.8 hours, and if the fetch is longer than 300 km, the development of complex sea will continue, but for the β_m -wave a "steady state" is attained over any fetch or for any duration. Longer waves develop, but the β_m -waves remain always present as the first dominating waves, breaking from time to time, and being continuously regenerated by energy supply from the wind.

Table 10 (example 3) for $v = 16$ m/sec shows that the fully developed $\beta(1)$ -wave would appear already at a fetch of $x = 345$ km (minimum duration $t = 18.4$ hours). Its dimensions are

$$T(1) = 10.2 \text{ sec}, \quad \lambda(1) = 163 \text{ m}; \quad H(1)_m = 6.6 \text{ m}.$$

The growth of this wave up to fully arisen state is given in Table 8, 2) for $v = 16$ m/sec.

Fig. 22. Wave height (H) and period (T) in the first stages of complex wave development (β_m -wave) as functions of duration t after a wind of $v = 16 \text{ m/sec}$ started to blow over an unlimited fetch and an undisturbed water surface.

Fig. 23. Wave age, σ/v , as a function of fetch x and duration (hours) after a wind of $v = 16$ m/sec started to blow over an undisturbed water surface.

By interference of the β_m -wave and the $\beta(1)$ -wave, characteristic variations in wave height and period occur in complex wave motion, but the form of these interference phenomena is only temporary if the wind continues to blow over longer fetches, because the relative dimensions of the dominating waves will be changed with further development of the sea.

If the fetch is as long as $x = 472$ km, the "long wave" with

$\beta^* = 1.35$ arises after a duration $t = 22.1$ hours (example for $v = 16$ m/sec in Table 8, 3), and Table 10). With this, the sea has almost attained its final stage, and this stage is not signified only by this β^* -wave, but also by the presence of the other two dominating waves, that is, by their interference pattern. Strictly speaking, the final state is attained when $\beta_m^* = 1.37$ has developed, but in this state $A - D = 0$ and this happens theoretically for x (or t) $\rightarrow \infty$. For practical purposes $\beta^* = 1.35$ will approximate the fully arisen state, because the variations from $\beta^* = 1.35$ to $\beta_m^* = 1.37$ may be neglected. For the growth of the long waves with increasing fetch and duration see figures 15-19 and figure 26.

In the fully arisen state of (wind-driven) complex sea three characteristic waves dominate the surface pattern. The smaller waves which are superimposed contribute only to a certain roughness of this main pattern. But this roughness is very important for energy transfer by wind, and therefore for maintaining the main pattern of the sea.

In our example, $v = 16$ m/sec, the fully arisen complex sea is characterized by the following dominating waves:

β_m -wave	:	$\lambda_1 = 107$ m	$H_1 = 5.9$ m	$(T_1 = 8.3$ sec)
$\beta(1)$ -wave	:	$\lambda_3 = 163$ m	$H_3 = 6.6$ m	$(T_2 = 10.2$ sec)
β_m^* -wave	:	$\lambda_2 = 307$ m	$H_2 = 6.8$ m	$(T_3 = 14.0$ sec)

Some theoretical interference patterns of complex wave motion for fully developed sea at $v = 16$ m/sec are represented in Figures 7 and 8 in Chapter I. They show theoretical "wave records" and the variations in space and time as they could be forecasted for a fixed point on the sea surface. The computed variations of time intervals

between succeeding crests ("periods" in complex sea) at a fixed point may be compared with the diagram in Fig. 3 of Chapter I, which represents the results of observations. At a wind velocity of $v = 16$ m/sec the characteristic "periods" vary over a range of about 8.5 seconds, that is, between 6.5 sec and 15 sec. With the appearance of typical interference phenomena and groups of waves, a certain regularity is to be expected in the occurrence of high breakers (see Chapter I).

8) Comparison of theoretical results with observations

Special attention was paid to observations on waves when the "Heidelberg" sailed or anchored under the protection of windward shores. This opportunity arose during a longer stay at San Miguel (Azores) and in many cases at the coasts of Venezuela and Colombia, in the northern parts of the Caribbean Sea, and in the Straits of Florida, and in the Gulf of Mexico.

The "Heidelberg"-observations at limited fetches comprise the range of wind velocity from 1.7 m/sec to 13 m/sec with two single measurements at 16 and 18 m/sec wind velocity and fetches of 400 and 450 m, respectively. There are included some observations at low wind speeds made at the German coast of the North Sea (Islands of Sylt and Amrum) in the summer of 1950, and an observation at $v = 7$ m/sec made in Vineyard Sound during the author's stay at Woods Hole in the summer of 1951. The results of these observations are presented in an appendix to this report, because measurements of the growth of waves are still rather scarce. The table contains in some cases measurements of the wave height H , besides the observed characteristic period (or periods) T at different fetches and

and wind velocities. The propagation velocity ϕ and the wave length λ are computed, by means of Gerstner's formulas, from the period T . The fetch is given by the distance from the coast to the ship's position in bearing the direction of wind.

Fig. 24 represents these new observations, augmented by older estimates which were collected and used by H. U. Sverdrup and W. U. Munk [1], and by two observations of V. Cornish (see H. Thorade [6]). For comparison with theoretical results, these data are plotted into the fetch-graph $\beta = \varphi \left(\frac{gx}{v^2} \right)$. The observations are in fair agreement with the theoretical curves. In the region of low fetch parameters, the observed values β for all wind velocities fit the theoretical curve very well, but for higher fetch parameters, the single observations spread out into the region on the right hand of the curve, as it is to be expected when the β_m -waves have attained their fully arisen state. Theoretically, this is indicated by the straight lines which branch off at certain fetch parameters depending upon the wind velocity. Thus, in this region we have observations at fully developed β_m -waves, but, besides these waves, $\beta(1)$ -waves and finally β_m^* -waves may be observed, if the fetch is long enough. The growth of these longer waves at different wind velocities is represented in the fetch-graph by curves which branch off to the top of the graph. In the fully developed state, the observations of β values (computed from observed "periods" at given wind velocities) range between the upper straight line and the lower straight line for the given wind-velocity, but a "piling up" of observations is to be expected around the lower line (β_m), the upper line with $\beta_m^* = 1.37$, and $\beta = 1$. The distribution of β -values as computed from observed "periods" T in fully developed

Fig. 24 Wave velocity σ as a function of fetch x and wind velocity v , using nondimensional parameters. Theoretical relationships shown by curves, observations by symbols.

sea (represented in Fig. 5, Chapter I) is in fair agreement with the results for high fetch parameters, but these observations are not plotted on the diagram. The data used for comparison of theoretical results in the graph are special observations made aboard the "Heidelberg" at well known fetches as mentioned above, and also some other data used by Sverdrup-Munk. But the difficulty with the latter data is that it is not known what the given β -values, or periods, mean (average values or maximum values of observed σ (or T) at a given v). Most of them are placed into the part of the graph with $gx/v^2 > 10^4$, where, depending on the wind velocity, a fully arisen or nearly fully arisen sea is to be expected. The observations of Stanton and Gibson at low fetches are separately indicated. The "Heidelberg"-observations, and well defined other data for higher fetch parameters in the region of the straight lines are stated together with the wind velocity in m/sec.

The curves in figure 24 give the stage of growth, and the straight lines the fully arisen state of dominating waves. Thus, on the right hand of the curves fully developed waves are to be expected. The other graphs (figures 25 and 26) are to be interpreted similarly. The representation of data at low fetches by means of $\sigma/\sigma_m = \varphi(gx/v^2)$ in Fig. 25 shows the different relations at different wind velocities.* The complete set of "Heidelberg"-observations of partially developed waves was divided into three parts, comprising the observations at wind velocities lower than 7.5 m/sec, between 7.5 m/sec and 15 m/sec

* T is the period of waves observed at limited fetches, and T_m is the period of the fully developed β_m -wave as given by formula (1a) in Chapter I. The ratio T/T_m corresponds to the ratio σ/σ_m , where $\sigma = gT/2\pi$ and $\sigma_m = gT_m/2\pi$.

Fig.25 Representation of observed data at low fetches. The three curves show the theoretical relationship $\frac{\sigma}{\sigma_m} = \varphi\left(\frac{gx}{v^2}\right)$ at $v=5$ m/sec, 10 m/sec and 20 m/sec wind velocity. Observations are indicated by symbols.

Fig. 26 Wave height H as a function of fetch x and wind velocity v , using nondimensional parameters. Theoretical relationships shown by curves, observations by symbols

and higher than 15 m/sec. They are marked in Fig. 25 by different signs. The observations at a given fetch-parameter scatter over a certain interval of the ratio σ/σ_m , but there is a distinct separation between lower and higher wind speeds of such a kind, so that at a given value gx/v^2 , the ratios σ/σ_m at light winds are higher than at moderate and strong winds. The three curves are the theoretical relationships between $\sigma/\sigma_m = \varphi(gx/v^2)$ at 5, 10, and 20 m/sec wind velocity. When the fetch parameter has attained a certain value, depending upon the wind velocity, the ratio σ/σ_m or T/T_m becomes unity. That means that these waves have attained their maximum value of σ or T as given by σ_m or T_m for the β_m -wave in the fully developed state.

Fig. 26 represents a comparison of measured wave heights H at different fetches and wind velocities with computed heights using the dimensionless form $gH/v^2 = \Phi(gx/v^2)$. At low fetch parameters the computed heights appear a little too high compared with the observations, but it must be kept in mind that the computed values for the stage $\beta < 1/3$ are maximum values. By the empirical relationship $\delta = 2p = 0.124$ (equation (18)), the steepness of the waves in their earliest stages of development probably has been assumed a little too high (the value $2p = 0.10$ seems to be more representative). But these early stages of wave development are only of minor significance with respect to the later development of the sea.*

* Supplement after completion of this report: New observations by U. Roll [25] also indicate a very rapid increase of wave steepness at very low fetches, similar to the broken curve of Fig. 12 of this report, and in agreement with the observations of the "Heidelberg" at low β -values. Probably it would be a better approximation to use the broken curve in Fig. 12 when considering the relationship $\delta = f(\beta)$. It

It may be called to the user's mind that these results refer to pure "wind sea" which is under the influence of a steady wind. In practical wave forecasts, intervening "dead sea" or swell has to be taken into account, if necessary. It may be mentioned in this connection, that the estimates of the decay of the waves require a consideration of the turbulent state of the sea surface layers. For similar conditions with respect to air resistance, the decay of the waves (including swell) is different depending on whether these waves travel through a calm region of the ocean with a smooth sea surface or whether they have to cross storm areas where heavily breaking wind-generated seas and a considerable state of turbulence are to be expected.

is to be expected that the results of this report will not be changed essentially by taking this slightly changed empirical relationship between the steepness and the ages of the waves, but the discontinuities at about $\beta = 1/3$ may disappear (see the remarks on page 79). In the following report on methods of practical wave forecasting, this slightly changed empirical relationship can be taken into account.

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Appendix

Observations of wave periods and heights at limited fetches.

Island of Amrum (depth of water 20 - 40 cm).

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/λ
4.5	25	0.60	0.935	0.207	0.02	12.3	0.010	0.54	3.7
5.4	100	0.75	1.17	0.216	0.05	34.2	0.017	0.88	5.7
5.0	150	0.83	1.30	0.260		60.0			
4.5	150	0.82	1.28	0.285	0.06	74.0	0.029	1.05	5.7
4.8	200	0.83	1.30	0.271		86.6			
3.8	10	0.50	0.78	0.205	0.02	6.9	0.0135	0.39	5.1

South coast of San Miguel (Azores)

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/λ
5.4	18000	2.1	3.27	0.605		6160.			
12.0	5200	3.3	5.15	0.43	1.5	361	0.102	17.0	8.9
12.5	31000	4.06	6.35	0.51		1990			
18.0	450	1.6	2.5	0.139		13.9			
16.0	400	1.4	2.19	0.137	0.30	15.6	0.015	3.00	10.0
5.2	2000	1.5	2.34	0.45		738			
5.2	2000	2.1	3.27	0.63		738			
11.0	380	1.4	2.19	0.20	0.25	31.5	0.020	3.00	8.3
6.0	380	0.9	1.40	0.233	0.15	106	0.041	1.26	11.9
2.6	260	0.6	0.93	0.358		385			
4.7	360	0.62	0.97	0.206		163			
5.6	350	0.9	1.40	0.25		112			
6.0	310	0.65	1.02	0.17		86			
2.8	1300	0.9	1.40	0.50		1660			
3.8	1100	0.88	1.37	0.36		760			

South coast of San Miguel (Azores). continued.

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/λ
1.7	390	0.4	0.62	0.364		1350		0.25	
2.5	5200	0.85	1.32	0.527		8320			
2.1	2000	0.5	0.78	0.371		4540		0.39	
3.7	480	0.8	1.25	0.337	1.10	350	0.072	1.00	10.0
3.2	430	0.72	1.12	0.261		420			
4.3	550	0.92	1.43	0.333	0.15	298	0.080	1.30	11.5
2.9	550	0.6	0.93	0.321		652			
4.4	500	0.88	1.37	0.311		259			
5.4	3500	2.0	3.12	0.578		1200			
10.2	305000	5.0	7.80	0.763		28800			
9.8	285000	4.8 (8.0)	7.5 12.4	0.765 1.26		29200			
8.0	250000	3.6 (7.6)	5.61 11.8	0.70 1.47		39400			
6.5	7300	2.6 (4.8)	4.05 7.5	0.622 1.16		1730			
7.0	11100	2.8 (5.0)	4.36 7.8	0.623 1.12		2260			

Caribbean Sea (approaching La Guesira).

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	H/λ
10.2	315000	3 5 7	4.67 7.8 10.9	0.459 0.765	1.07	29700		
10.2	300000	4.85 6.2 8.5	7.56 9.7 13.2	0.74 0.95	1.295	28300		

Caribbean Sea (approaching La Guaira). continued.

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	H/λ	
9.8	285000	4.75 6.3 8.0	7.4 9.8 12.5	0.755 1.00	1.28	29000			
9.7	370000	4.75 6.3 7.8	7.4 9.8 12.2	0.765 1.01	1.26	38600			
7.3	250000	3.8 6.0 7.8	5.92 9.35	0.81 1.28		45800			
6.5	7400	2.6 4.8	4.05 7.5	0.622	1.155	1720			
8.0	33200	2.8 5.0	4.86 7.8	0.608	0.975	5100			

Off La Guaira (Venezuela).

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/λ
6.4	9250	2.0	3.12	0.488	0.40	2260	0.096	6.2	6.5
6.3	9250	1.8 (4.8)	2.81 7.5	1.19 0.446		2330			
4.9	9250	1.65	2.57	0.525		3850			
8.7	9250	2.2 3.4 5.0	3.43 5.3	0.394 0.61	0.60	1220	0.078	7.5	8.0
8.0	9250	2.6 (6.0)	4.05 9.35	0.505 1.17		1440			
10.3	9250	3.35	5.22	0.51		870			

Off Port au Prince (Haiti)

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	H/ λ	
4.7	3000	1.25	1.95	0.415		1360			
4.5	3000	1.30	2.03	0.452		1480			
5.2	50000	2.15	3.35	0.643		18400			
3.3	5300	1.20	1.87	0.567		4860			
3.5	37000	0.85	1.32	0.377		30200			

Off Cape Haitien (Haiti)

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/ λ
12.2	4600	2.6	4.06	0.333	1.2	310	0.079	10.6	11.3
13.0	222000	7.18	11.2	0.86		13200			
8.0	3100	2.93	4.56	0.57	0.7	485	0.092	13.4	5.2
9.5	295000	4.2	6.55	0.69		32600			
		6.1	9.5	1.0					
		(8.5)	13.2	1.38					
9.9	295000	4.75	7.4	0.75	2.0	30200	0.200	35.0	5.7
10.1	295000	4.8	7.45	0.735	2.0 -	29000	0.200	35.8	5.85
		(9.5)	14.8	1.46	2.2				

Bahama Channel

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	H/ λ	
9.6	222000	4.15	6.47	0.675		32000			
		5.8	9.0	0.938					
		(8.1)	12.6	1.31					
9.5	111000	4.3	6.71	0.705		12300			
		7.0	10.9	1.145					
		(9.0)	14.0	1.47					
9.0	111000	4.0	6.32	0.692		13700			
		5.5	8.6	0.96					
		(8.5)	13.2	1.46					

Bahama Channel (continued).

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/ λ
12.6	40700	5.4 (11.0)	8.42 17.1	0.670 1.36	2.0 - 3.0	2500	0.124- 0.185	45.5	4.4- 6.6
10.7	78000	5.07	7.90	0.760	2 - 3	6800	0.17 - 0.255		
9.0	260000	3.65	5.70	0.632		31500			
10.1	260000	4.60	7.18	0.710		25000			

Habana (Harbor)

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/ λ
5.5	700	1.1	1.72	0.312		231			
5.8	300	0.9	1.40	0.241		89			
5.0	300	0.8	1.25	0.250	0.15	120	0.059	1.00	15.0
6.0	300	1.0	1.56	0.26	0.15- 0.20	83.5	0.049	1.56	11.3
5.0	300	0.85	1.33	0.266	0.10- 0.15	120	0.049	1.15	10.8
3.5	350	0.72	1.12	0.32	0.10	285	0.080	0.81	12.4
6.3	1500	1.30	2.03	0.322		378			
5.8	50	0.64	1.00	0.173	0.08	14.8		0.64	12.5

Puerto Mariel (Cuba) (Bay and Harbor)

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	H/ λ
3.9	650	0.66	1.03	0.264		427		
5.0	700	0.95	1.48	0.296		280		
4.2	600	0.88	1.37	0.326		340		

Puerto Mariel (Cuba) (Bay and Harbor). continued.

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/ λ
8.2	1150	1.30	2.03	0.248					
8.2	1250	1.42	2.21	0.270	0.40	186	0.058	3.15	12.7
4.0	2000	1.30	2.03	0.510		1250			
5.0	2000	1.30	2.03	0.410	0.25	805	0.098	2.63	9.5
3.7	3100	1.18	1.84	0.496		2260			
10.5	2000	2.00	3.12	0.297		182			
7.3	2000	1.98	3.09	0.423		375			
8.3	2350	1.85	2.89	0.348		341			

Off Puerto Mariel

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/ λ
4.5	4100	1.55	2.41	0.536		2020		3.75	
4.5	7500	1.80	2.81	0.625	0.35	3700	0.170	5.05	6.9
5.4	18000	2.1	3.27	0.607		6160			

Gulf of Mexico

v	F	T	σ	σ/v	H	gF/v^2	gH/v^2	λ	H/ λ
10.6	370000	4.5	7.00	0.660		32900			
12.2	400000	5.77	9.00	0.738	2.8	26400	0.184	51.5	5.4

Notation: v = wind velocity in m/sec F = fetch in meters

T = observed periods in seconds. H = wave height in meters.

H/ λ = Steepness in per cent.

σ = phase velocity in m/sec; $\sigma = \frac{g}{2\pi} T$.

λ = wave length in meters; $\lambda = \frac{g}{2\pi} T^2$.

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