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# PERIODICITIES IN THE SOLAR-CONSTANT MEASURES

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## INTRODUCTION

This paper, based on over 40 years of observations of solar radiation, ties together the following conclusions:

1. The sun's output of radiation varies.
2. It varies in at least 23 regular periodicities, all proceeding simultaneously.
3. The periods of solar variation are integral submultiples of 22¾ years.
4. Synthesis of curves representing the 23 periodicities reproduces the original observations of the "solar constant" to within about 0.1 percent.
5. Synthesis of these curves for 12 years as a prediction, prior to the observations on which they depend, shows rough agreement with Mount Wilson observations of the solar constant, in the years 1908 to 1920.
6. A much more satisfactory agreement is found between this predicted synthetic solar-constant curve and the Mount Wilson determinations of the march of contrast along the east-west diameter of the sun, of 1913 to 1920.
7. Higher contrast attends higher solar-constant values.

In several former publications[2] I have discussed the periodic changes in observed values of the solar constant of radiation.

For several years I have been investigating the effect on terrestrial weather of these periodic changes in the sun's emission. I had become convinced by the earlier solar-constant studies, just cited, that the sun's radiation varies simultaneously in many regular periods, all

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[1] I wish to express my sincere acknowledgments to L. B. Aldrich, Director of the Astrophysical Observatory, who made the data available for this paper and gave highly valuable criticisms; to Frederick E. Fowle, deceased, whose careful measurements of solar contrast appear in table 6; to Mrs. A. M. Bond, deceased, whose critical judgment and accurate computations aided in the preparation of the data; to the many observers on high mountains in distant lands who sacrificially kept up this long campaign of measurement; to Mrs. I. W. Windom, who assisted in preparing this text; and to Miss M. A. Neill, who continuously over many years greatly assisted me in keeping the observing stations in operation.

[2] Annals Astrophys. Obs., Smithsonian Inst., vol. 5, p. 250 et seq., 1932; vol. 6, p. 178 et seq., 1942. Smithsonian Misc. Coll., vol. 111, No. 7, 1949.

aliquot parts of $22\frac{3}{4}$ years. I hoped, by using a long interval of scores of years of an unbroken series of monthly weather records, that I could discover from them all the submultiples of $22\frac{3}{4}$ years which yield effective periodic variations of the solar radiation.

But I found that the variations of the atmospheric conditions from time to time, some associated with the seasons and some with the sunspot cycle, so badly confuse the phases of responses to solar variation that I could not be certain that all the suspected solar periodicities, inferred from weather records, are real. Hence I felt constrained to reinvestigate the observed fluctuations of the solar constant, to determine directly which of the submultiples of $22\frac{3}{4}$ years are truly periods in solar variation.

In former papers I have used 273 months as the master period, of which the others are integral submultiples. My present work leads me to prefer 272 months. All the periods which I have found lie within less than 1 percent of being integral fractions of 272 months.

## ADVANTAGES OF METHOD

Some investigators would prefer to submit the available solar-constant data to a Fourier analysis based on 272 months. I prefer to tabulate the data according to each suspected possible period. There are several advantages in this method. In so doing, I divide the total interval covered by the data into several parts, if periods are short enough to furnish a large number of repetitions. In this way the phases of features may be compared in the several independent tabulations of one period. Graphs showing this procedure are given in figure 1. Slight shifts,[3] from one to another of the successive tabulations, indicate small corrections to the assumed period. The form of the curve of fluctuation is determined by the tabulations. Also the amplitude of the periodic variation is found. If it is too small to be certainly exceeding the probable error, then the periodicity is to be rejected altogether. Proceeding in this way, I found 23 periodicities in solar-constant results which meet the tests of veridity just indicated. Fifteen other periods were tabulated, but rejected. Each search involved tabulating more than a thousand decade mean values of the solar constant. The results appear in table 1.[4]

[3] See the curves, 6 1/30, of figure 1, in comparison with table 1C, below.

[4] In tabulating any one periodicity, all the others exercise confusing influences, which are not wholly eliminated, because of the small numbers of repetitive columns going to make up the tables. Hence, irregularities in the curves of figure 1 are caused by conflicting periodicities, in addition to the effects of accidental errors of observation.

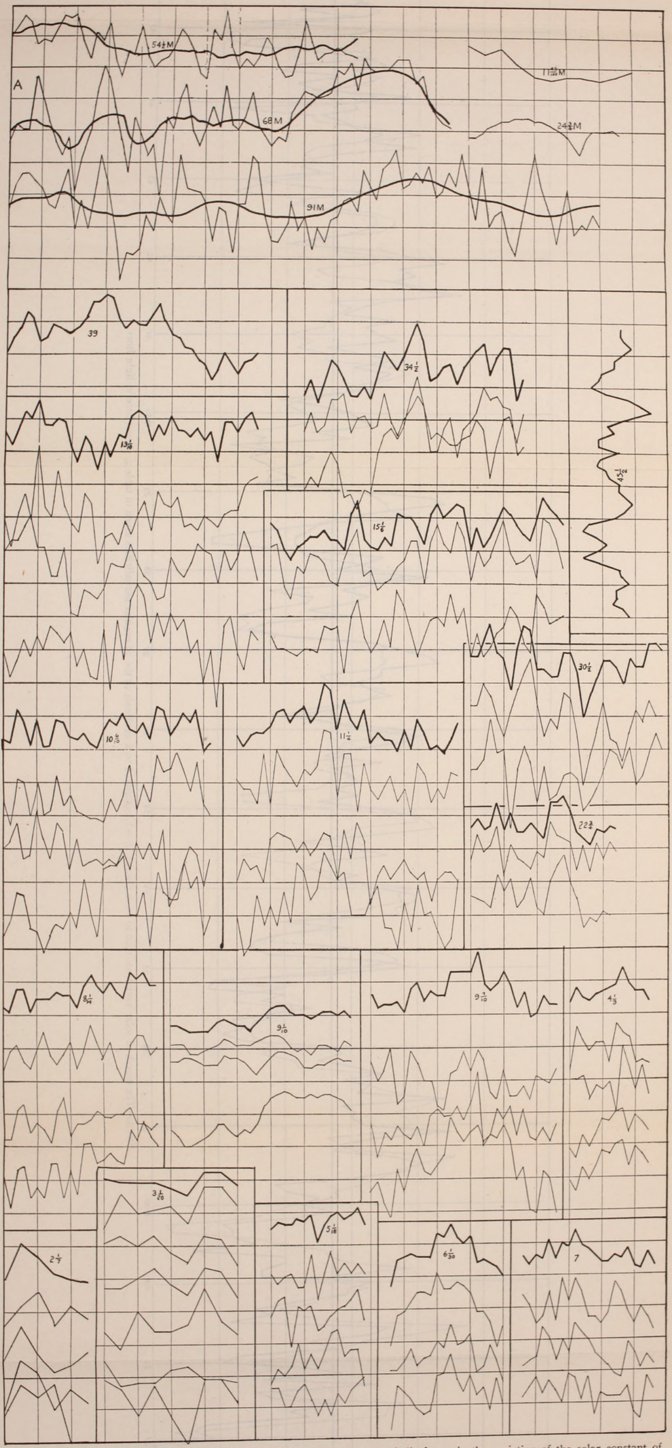


FIG. 1.—Consecutive partial determinations and general means of periodic forms in the variation of the solar constant of radiation as observed 1920 to 1950. Periods are indicated in months. Spaces in ordinates represent 1/10 percent variation in solar radiation.

It may aid to fix ideas on the method of tabulation to give an example. Table 1C is a facsimile of the computation for the period 6 1/30 months. I select it as indicating how fractional parts of months and of 10-day means are treated, so as to preserve the exact average period. I had at first assumed that 6 1/15 months was the proper length of period. The data were separated into three groups. The assumed period corresponds with 18 1/5 10-day intervals. When the mean values for the three groups were computed, they were plotted, superposed. It was then apparent that the maximum ordinates shifted progressively toward earlier dates, as time went on. This indicated that the assumed period is too long by 4/700 of itself. Making this correction, the true period is 6 1/30 months.

## PREPARATION OF DATA

L. B. Aldrich, Director of the Astrophysical Observatory, and his associates had painstakingly considered every circumstance affecting every daily solar-constant observation, at all the Smithsonian mountain stations in various lands. By consensus of three individual opinions, they had assigned to every observed day its most probable solar-constant value, as indicated by the checked results of all stations. Many days were not observed at all. However, there was no decade of any month, from 1920 to 1950, which did not have at least more than one observation.

Mr. Aldrich having been good enough to place these daily solar-constant results in my hands, I computed 10-day and monthly mean values from them for the 31 years 1920 to 1950. To have them in most convenient form for my use, I took their departures from the value 1.900 calories per square centimeter per minute and divided these departures by 1.940. Thus the results became expressed in percentage departures of the solar constant from 1.900 calories. In that form any well-evidenced periodic change resulting from a tabulation shows at once its amplitude in percentage of the solar constant. All values are positive as thus treated, which is convenient in tabulation. These data are given in table 4, appendix I.

## PERIODS FOUND AND NOT FOUND

With these clarifying remarks, I now introduce the results. The following periodic changes in the solar constant were found well evidenced. Their approximate relation to 272 months and their amplitudes in percentage of the solar constant are given in table 1A.

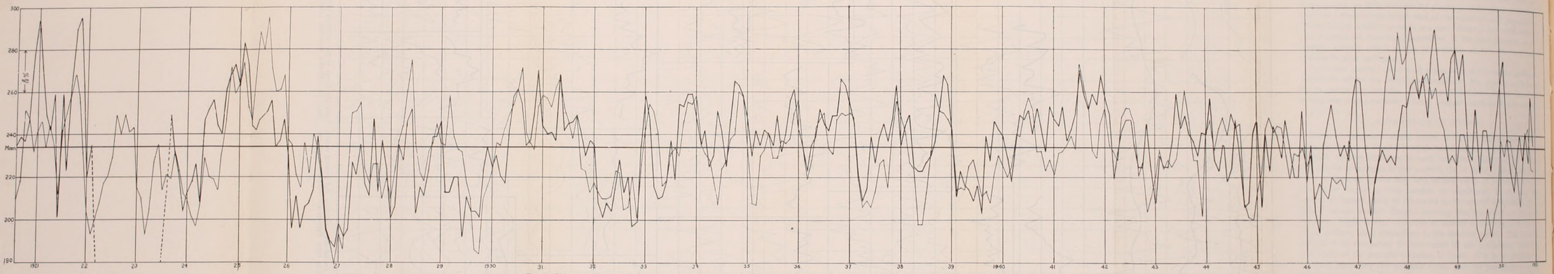


FIG. 4.—The march of solar variation, 1920 to 1950 (heavy lines), compared to a synthesis of the 23 regular periodicities given in table 2, shown here in light lines.

The following periodic changes, given in table 1B, if real, are too small in percentage to be verified.

TABLES 1A, 1B.—*Periodicities in solar-constant observations*

| A. Periodicities confirmed * | | | B. Periodicities sought but not found | |
|---|---|---|---|---|
| Period Months | Amplitude Percent | Period Fraction of 272 | Period Months | Period Fraction of 272 |
| 2 1/7 | 0.05 | 1/127 | 4 1/2 | 1/60 |
| 3 1/20 | 0.05 | 1/90 | 5 1/2 | 1/50 |
| 4 1/3 | 0.06 | 1/63 | 6 1/2 | 1/42 |
| 5 1/18 | 0.05 | 1/54 | 7 5/6 | 1/35 |
| 6 1/30 | 0.12 | 1/45 | 8 1/2 | 1/32 |
| 7 | 0.08 | 1/39 | 10 1/9 | 1/27 |
| 8 1/14 | 0.06 | 1/34 | 10 9/10 | 1/25 |
| 9 1/10 | 0.08 | 1/30 | 13 6/10 † | 1/20 |
| 9 7/10 | 0.10 | 1/28 | 14 4/10 | 1/19 |
| 10 6/10 | 0.06 | 1/26 | 17 | 1/16 |
| 11 1/5 | 0.17 | 1/24 | 18 1/5 | 1/15 |
| 11.43 | 0.11 | 1/24 | 19 1/2 | 1/14 |
| 12.0 | 0.20 | .... | 21 | 1/13 |
| 13 1/10 | 0.11 | 1/21 | 24 8/10 | 1/11 |
| 15 1/6 | 0.09 | 1/18 | 136 | 1/2 |
| 22 3/4 | 0.07 | 1/12 | | |
| 24 3/4 | 0.12 | 1/11 | | |
| 30 1/3 | 0.13 | 1/9 | | |
| 34 1/2 | 0.15 | 1/8 | | |
| 39 | 0.20 | 1/7 | | |
| 45 1/2 | 0.13 | 1/6 | | |
| 54 1/2 † | 0.13 | 1/5 | | |
| 68 | 0.25 | 1/4 | | |
| 91 | 0.12 | 1/3 | | |
| 272 | ... | 1 | | |

* The periodicities of 11.43, 12.0 (the periodicity of 12 months is not used in preparing figure 4; if it were, that figure would present closer accord between the curves), and 24¾ months were added to the list after search among the departures of the synthetic values, found by summing 21 periodicities, from the observed solar-constant values. It is indeed curious to find *two* periodicities both within 1 percent of 1/24 of 272 months. Both of them are excellently evidenced and of good amplitude. The 12-month period is of terrestrial, not solar, causation. When one reflects that the pyrheliometer observes only about 70 percent of the solar constant, the remaining 30 percent being supplied by our estimates of atmospheric transmission, it is perhaps not surprising that the yearly (terrestrial) periodic error in the solar-constant values is as large as 0.2 percent in amplitude. The periodicity of 24¾ months was the only other one which could be discerned in a residual plot of differences, smoothed by 7-month running means.

† After this work was done, I computed a table of the periodicity 54 8/10 months in the precipitation of Peoria, Ill., 1856 to 1939. It showed *no* periodicity of 54 8/10 months, but *four* strong, well-shaped periodicities of 54 8/10 ÷ 4 = 13 7/10 months. Hence I think the sun's radiation has a periodic variation of one-twentieth of 22¾ years, though it did not impress me as real in the tabulation of the solar constant.

All periods of these two lists were separately sought for by tabulating over 1,000 solar-constant 10-day means for each suspected periodicity. The investigation does not cover entirely the years 1922 and

1923. I have elsewhere discussed the large solar change observed in those years.[5] I still think it was a real one. But it may be either a very unusual sporadic solar change, or it may be a periodic change related to a longer period than 272 months.

## CONCERNING DOUBTS OF SOLAR VARIATION

For those who do not have intimate association with the Smithsonian observations of the solar constant of radiation, it seems difficult to accept the results as having the high degree of accuracy claimed for them. Observers, familiar with the clouds, dust, and water-vapor load which the lower atmosphere bears to make it milky, do not readily visualize a sky so clear that, if one holds his little finger at arm's length before the sun, the sky seems deep blue right down to the sun's edge. But even if the superior excellence of stations like Montezuma, Table Mountain, and St. Katherine be granted, it still seems incredible to many that the fraction, amounting to about 30 percent of the solar constant, cut off by the atmosphere, can be so correctly estimated that variations of the order of 1/10 percent of the solar constant can be evaluated.

Still more doubtful does it appear to many that, lacking any theoretical support, it can be proved from the observations that the solar variation consists of 23 simultaneously operating regular periodicities, all aliquot parts of $22\frac{3}{4}$ years. Yet it seems to me this cannot longer be doubted. I have tried to demonstrate by a couple of examples that it is necessary to use integral fractions of $22\frac{3}{4}$ years, rather than any other intervals, to represent the the sun's periodic variation. The two periods I have chosen to experiment upon are those which are 1/7 and 1/45 of $22\frac{3}{4}$ months. In figure 1 the longer period is plotted as 39 months.

I made a new tabulation in four parts for a period lying between 1/45 and 1/44 of $22\frac{3}{4}$ years. It was assumed to be $6\frac{1}{6}$ months, or 19 10-day intervals. In each of the four groups tabulated there are 14 columns. Taking the mean values, they are as plotted in figure 2,A. Evidently, if the four mean results were combined directly, they would so contradict each other that the general mean would show no periodicity at all. But the principal feature, marked A at its right-hand edge in each plot, is equally displaced from curve to curve toward the left by about 6 10-day intervals. The displacement is 19

[5] Monthly Weather Rev., U. S. Weather Bureau, February 1923. Proc. Nat. Acad. Sci., vol. 9, No. 6, pp. 194-198, 1923. Smithsonian Misc. Coll. vol. 77, No. 5, 1925 (see fig. 11); vol. 80, No. 2, 1927.

10-day intervals, in all, from curve I to curve IV. Between these curves I and IV lies a stretch of time of about 800 10-day intervals. Hence the period should have been taken less than $6\frac{1}{6}$ months by $19/800 \times 6\frac{1}{6} = 0.146$. Subtracting from 6.163, this yields a corrected period of 6.017 months. Within the error of determination, this checks

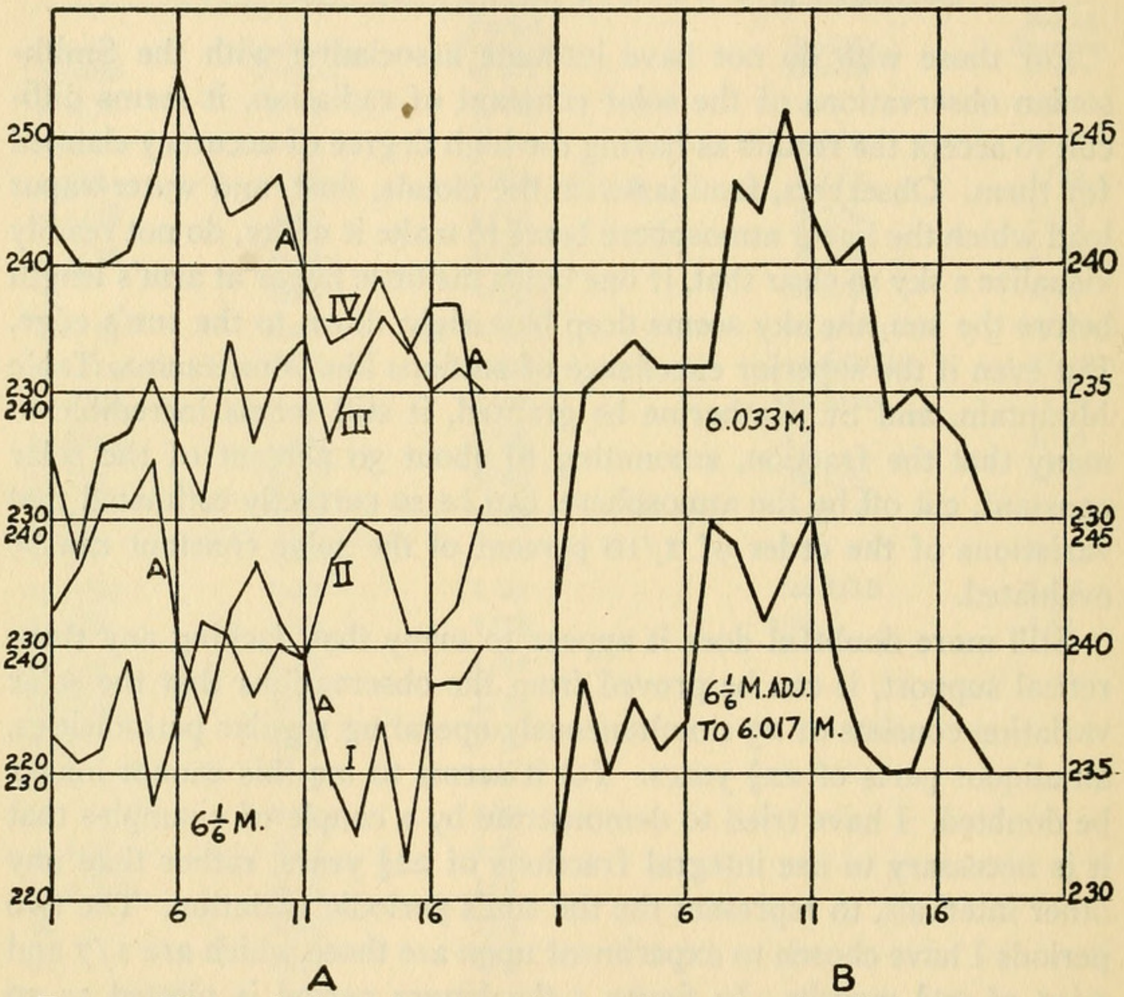


FIG. 2.—The periodicity 6.033 months, confirmed by the displacement of the feature A gradually from I to IV, when the period is assumed to be $6\frac{1}{6}$ months, as shown in figure A. In figure B this displacement is adjusted to a period of 6.017 months, which nearly agrees with the true period, 6.033 months.

with 6.003, which is the period given in table 1C. Having displaced curves II, III, and IV by 6, 12, and 19 10-day intervals respectively, and having taken the general mean of the four and plotted it, the result appears in figure 2,B. It is to be compared with the curve of 6.033 months above it, representing the mean value as given in table 1C. It must be admitted that the agreement is striking.

Proceeding similarly, I computed two curves [6] for the seventh of

[6] There being but four columns in these part computations for 39 and 37 months, the plots of the results are very ragged, owing to the disturbing influences of 22 other periodic factors superposed.

| | | | | | | | | | | | | | | | | | | | Σ | ÷ 18 | Adjusted places | | Mean | Δ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 254 | 237 | 211 | 206 | 201 | 232 | 288 | 273 | 180 | 252 | 154 | 237 | 237 | 201 | 242 | 191 | 211 | 227 | 3998 | 222 | 231 | 229 | 227 | —10 |
| 2 | 239 | 288 | 263 | 211 | 232 | 211 | 299 | 263 | 211 | 216 | 216 | 258 | 247 | 237 | 191 | 211 | 216 | 232 | 4241 | 236 | 231 | 238 | 235 | — 2 |
| 3 | 265 | 278 | 196 | 252 | 252 | 232 | 263 | 263 | 232 | 216 | 216 | 247 | 221 | 196 | 206 | 211 | 221 | 227 | 4205 | 234 | 234 | 239 | 236 | — 1 |
| 4 | 263 | 299 | 227 | 252 | 211 | 278 | 258 | 242 | 191 | 227 | 247 | 221 | 216 | 227 | 191 | 216 | 206 | 232 | 4178 | 232 | 238 | 242 | 237 | 0 |
| 5 | 227 | 206 | 263 | 242 | 216 | 252 | 263 | 232 | 170 | 201 | 232 | 206 | 227 | 232 | 242 | 175 | 227 | 247 | 4115 | 229 | 233 | 245 | 236 | — 1 |
| 6 | 227 | 242 | 268 | 237 | 196 | 268 | 252 | 237 | 201 | 206 | 201 | 211 | 242 | 211 | 227 | 206 | 232 | 242 | 4080 | 227 | 235 | 245 | 236 | — 1 |
| 7 | 227 | 242 | 268 | 221 | 206 | 263 | 216 | 232 | 216 | 180 | 206 | 232 | 227 | 227 | 216 | 227 | 252 | 242 | 4105 | 228 | 236 | 244 | 236 | — 1 |
| 8 | 278 | 242 | 294 | 237 | 221 | 263 | 221 | 216 | 201 | 185 | 242 | 232 | 263 | 252 | 216 | 237 | 242 | 263 | 4305 | 239 | 240 | 251 | 243 | + 6 |
| 9 | 206 | 242 | 309 | 221 | 237 | 268 | 258 | 252 | 206 | 185 | 216 | 247 | 247 | 237 | 227 | 237 | 242 | 268 | 4321 | 240 | 238 | 246 | 242 | + 5 |
| 10 | 278 | 267 | 294 | 227 | 252 | 273 | 247 | 242 | 211 | 201 | 253 | 242 | 247 | 237 | 206 | 237 | 273 | 278 | 4434 | 246 | 245 | 246 | 246 | + 9 |
| 11 | 258 | 247 | 283 | 211 | 252 | 283 | 237 | 247 | 196 | 170 | 221 | 227 | 278 | 227 | 175 | 227 | 258 | 263 | 4241 | 238 | 239 | 248 | 242 | + 5 |
| 12 | 201 | 263 | 309 | 221 | 297 | 263 | 247 | 237 | 216 | 201 | 227 | 201 | 232 | 252 | 196 | 211 | 232 | 216 | 4212 | 234 | 240 | 247 | 240 | + 3 |
| 13 | 294 | 185 | 273 | 216 | 247 | 273 | 258 | 258 | 211 | 191 | 232 | 221 | 232 | 242 | 206 | 232 | 278 | 247 | 4311 | 239 | 242 | 245 | 242 | + 5 |
| 14 | 263 | 206 | 247 | 175 | 263 | 242 | 263 | 237 | 221 | 206 | 216 | 196 | 221 | 258 | 211 | 232 | 273 | 268 | 4188 | 233 | 230 | 239 | 234 | — 3 |
| 15 | 278 | 211 | 268 | 216 | 247 | 263 | 278 | 258 | 211 | 196 | 232 | 216 | 216 | 237 | 216 | 211 | 242 | 247 | 4238 | 235 | 232 | 239 | 235 | — 2 |
| 16 | 299 | 258 | 165 | 211 | 268 | 283 | 206 | 196 | 211 | 191 | 211 | 237 | 191 | 232 | 201 | 232 | 268 | 258 | 4102 | 228 | 239 | 236 | 234 | — 3 |
| 17 | 304 | 268 | 247 | 211 | 252 | 278 | 258 | 201 | 232 | 175 | 221 | 211 | 227 | 211 | 206 | 247 | 252 | 216 | 4284 | 238 | 234 | 228 | 233 | — 4 |
| 18 | 278 | 252 | 247 | 221 | 278 | 283 | 221 | 206 | 232 | 242 | 232 | 201 | 227 | 196 | 206 | 232 | 247 | 227 | 4146 | 230 | 227 | 234 | 230 | — 7 |
| 19 | | | | | 221 | | | | | 160 | | | | | 211 | | | | | | | | | |

Mean 237

| | | | | | | | | | | | | | | | | | | | | ÷ 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 237 | 232 | 206 | 237 | 206 | 263 | 258 | 263 | 237 | 237 | 232 | 252 | 211 | 227 | 206 | 247 | 227 | 227 | 4205 | 234 |
| 2 | 258 | 263 | 191 | 211 | 211 | 263 | 232 | 268 | 242 | 242 | 237 | 252 | 216 | 242 | 237 | 263 | 201 | 258 | 4289 | 238 |
| 3 | 237 | 263 | 221 | 180 | 227 | 237 | 221 | 263 | 227 | 242 | 237 | 242 | 180 | 247 | 237 | 268 | 211 | 258 | 4198 | 233 |
| 4 | 237 | 221 | 211 | 201 | 191 | 263 | 221 | 268 | 247 | 247 | 237 | 268 | 227 | 247 | 227 | 268 | 221 | 221 | 4223 | 235 |
| 5 | 237 | 237 | 232 | 211 | 196 | 242 | 227 | 263 | 232 | 268 | 232 | 273 | 237 | 247 | 206 | 268 | 211 | 227 | 4246 | 236 |
| 6 | 288 | 221 | 227 | 185 | 206 | 263 | 242 | 258 | 247 | 252 | 247 | 258 | 237 | 258 | 211 | 258 | 211 | 258 | 4327 | 240 |
| 7 | 258 | 232 | 154 | 201 | 232 | 273 | 221 | 268 | 237 | 247 | 258 | 278 | 237 | 252 | 227 | 273 | 196 | 242 | 4286 | 238 |
| 8 | 258 | 237 | 206 | 258 | 206 | 268 | 242 | 258 | 237 | 263 | 252 | 263 | 242 | 278 | 232 | 258 | 221 | 237 | 4416 | 245 |
| 9 | 247 | 221 | 221 | 237 | 216 | 252 | 258 | 247 | 247 | 273 | 247 | 247 | 221 | 232 | 227 | 242 | 221 | 242 | 4298 | 239 |
| 10 | 247 | 247 | 221 | 211 | 232 | 258 | 252 | 242 | 247 | 237 | 252 | 247 | 227 | 268 | 232 | 237 | 227 | 237 | 4321 | 240 |
| 11 | 232 | 242 | 263 | 258 | 247 | 258 | 258 | 268 | 227 | 263 | 242 | 273 | 232 | 206 | 221 | 242 | 201 | 227 | 4360 | 242 |
| 12 | 232 | 242 | 206 | 247 | 242 | 237 | 232 | 232 | 232 | 227 | 242 | 242 | 242 | 221 | 227 | 216 | 201 | 227 | 4147 | 230 |
| 13 | 252 | 242 | 216 | 268 | 221 | 247 | 232 | 237 | 247 | 242 | 232 | 247 | 232 | 237 | 232 | 185 | 180 | 232 | 4181 | 232 |
| 14 | 247 | 221 | 196 | 258 | 221 | 263 | 211 | 237 | 263 | 263 | 252 | 237 | 242 | 273 | 232 | 232 | 227 | 232 | 4307 | 239 |
| 15 | 247 | 227 | 227 | 242 | 221 | 216 | 237 | 216 | 237 | 196 | 242 | 252 | 252 | 268 | 237 | 221 | 252 | 211 | 4201 | 234 |
| 16 | 258 | 232 | 216 | 242 | 216 | 227 | 227 | 221 | 232 | 201 | 232 | 211 | 247 | 237 | 232 | 216 | 232 | 211 | 4090 | 227 |
| 17 | 232 | 165 | 191 | 237 | 252 | 247 | 232 | 242 | 237 | 237 | 252 | 221 | 237 | 242 | 242 | 232 | 232 | 227 | 4157 | 231 |
| 18 | 252 | 175 | 232 | 206 | 247 | 221 | 247 | 263 | 227 | 232 | 262 | 227 | 242 | 232 | 247 | 221 | 221 | 211 | 4165 | 231 |
| 19 | | 221 | | | | | 263 | | | | | 201 | | | | | 237 | | | |

Adjusting period

$$\frac{4}{700}\times 6 = 0.034$$

6.067 — 0.034

is 6.033

or 6 1/30

| | | | | | | | | | | | | | | | | | | | | | Σ | ÷ 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 268 | 252 | 221 | 247 | 227 | 237 | 242 | 237 | 242 | 237 | 237 | 242 | 252 | 237 | 216 | 247 | 263 | 283 | 242 | 263 | 4892 | 245 |
| 2 | 242 | 252 | 247 | 268 | 242 | 242 | 232 | 247 | 237 | 232 | 242 | 252 | 227 | 232 | 211 | 258 | 283 | 278 | 232 | 242 | 4898 | 245 |
| 3 | 237 | 227 | 216 | 247 | 257 | 257 | 237 | 206 | 227 | 237 | 232 | 221 | 258 | 304 | 247 | 237 | 273 | 283 | 191 | 288 | 4882 | 244 |
| 4 | 268 | 216 | 263 | 247 | 257 | 232 | 252 | 232 | 237 | 268 | 227 | 216 | 247 | 273 | 242 | 263 | 273 | 278 | 242 | 288 | 5021 | 251 |
| 5 | 247 | 242 | 263 | 263 | 237 | 211 | 263 | 227 | 221 | 232 | 227 | 216 | 258 | 221 | 237 | 278 | 299 | 278 | 237 | 268 | 4925 | 246 |
| 6 | 252 | 258 | 216 | 258 | 247 | 227 | 263 | 263 | 227 | 201 | 258 | 221 | 237 | 278 | 232 | 247 | 288 | 247 | 242 | 268 | 4930 | 246 |
| 7 | 242 | 258 | 268 | 283 | 252 | 206 | 252 | 252 | 237 | 211 | 242 | 227 | 258 | 258 | 211 | 263 | 283 | 273 | 247 | 227 | 4950 | 248 |
| 8 | 258 | 232 | 258 | 288 | 257 | 206 | 221 | 242 | 227 | 206 | 242 | 258 | 221 | 258 | 216 | 273 | 283 | 309 | 242 | 237 | 4934 | 247 |
| 9 | 252 | 268 | 268 | 237 | 232 | 211 | 257 | 227 | 237 | 242 | 247 | 211 | 221 | 216 | 221 | 268 | 278 | 294 | 263 | 247 | 4897 | 245 |
| 10 | 242 | 242 | 283 | 247 | 232 | 216 | 247 | 257 | 263 | 247 | 232 | 201 | 216 | 237 | 252 | 258 | 237 | 232 | 221 | 227 | 4789 | 239 |
| 11 | 242 | 216 | 283 | 247 | 237 | 242 | 242 | 263 | 216 | 232 | 221 | 201 | 252 | 242 | 221 | 278 | 252 | 242 | 227 | 227 | 4783 | 239 |
| 12 | 237 | 247 | 252 | 252 | 237 | 232 | 257 | 252 | 206 | 232 | 232 | 191 | 252 | 185 | 237 | 247 | 278 | 252 | 237 | 211 | 4726 | 236 |
| 13 | 242 | 273 | 242 | 247 | 247 | 247 | 206 | 216 | 206 | 232 | 216 | 185 | 232 | 206 | 221 | 247 | 278 | 206 | 206 | 221 | 4561 | 228 |
| 14 | 263 | 258 | 278 | 221 | 196 | 247 | 247 | 242 | 221 | 206 | 211 | 206 | 221 | 216 | 227 | 293 | 258 | 221 | 232 | 211 | 4675 | 234 |
| 15 | 252 | 232 | 263 | 221 | 232 | 221 | 242 | 227 | 191 | 232 | 232 | 252 | 216 | 191 | 237 | 252 | 258 | 206 | 221 | 206 | 4584 | 229 |
| 16 | 242 | 268 | 216 | 216 | 232 | 211 | 232 | 216 | 232 | 253 | 221 | 242 | 232 | 227 | 263 | 258 | 253 | 258 | 263 | 232 | 4767 | 238 |
| 17 | 247 | 247 | 273 | 227 | 227 | 227 | 247 | 227 | 206 | 247 | 206 | 211 | 237 | 232 | 273 | 258 | 278 | 242 | 268 | 206 | 4786 | 239 |
| 18 | 227 | 242 | 258 | 237 | 216 | 237 | 232 | 227 | 185 | 237 | 258 | 252 | 247 | 232 | 242 | 252 | 268 | 273 | 247 | 247 | 4816 | 242 |
| 19 | | | | 232 | | | | | 247 | | | | | 232 | | | | 7 | 237 | | | |

Mean values when plotted show progressive displacement. Total displacement, 4 places on 700 places.

$22\frac{3}{4}$ years, assumed as 39 months. In this new tabulation I used monthly mean values, instead of 10-day means, as had been done in computing for the curve shown in figure 1. I also computed two curves for a period of 37 months. They show opposition rather than similarity. It now appeared that in both the 39-month and the 37-month computations, the principal features were displaced toward the right in the second half of the 31-year interval. The corrected interval from the 39-month tabulation is $39\frac{1}{2}$ months. Plots of the 37-month tabulation shown in figure 3,A indicated a displacement toward the right of 8 months in an interval of 180 months of time. This gives a positive correction of $\frac{8}{180} \times 37 = 1.6$ months. Thus combined, the contrary curves of figure 3,A yield the lower curve of figure 3,B. Thus the 37-month tabulation yields an adjusted period of 39.6 months, closely agreeing with that yielded by the adjusted 39-month tabulation which was 39.5 months. This later period agrees within slightly more than 1 percent of being $\frac{273}{7}$, or 39.0 months. (See figure 3,B.)

If critics feel that still more evidence is needed to prove that only integral fractions of $22\frac{3}{4}$ years are to be found in the solar variation, I will remind them that many of the periodicities plotted in figure 1 show integral fractions of the periods in question superposed upon them. Conspicuous examples in figure 1 are periodicities of $15\frac{1}{6}$, $34\frac{1}{2}$, 39, $45\frac{1}{2}$, and $54\frac{1}{2}$ months.

## ACCURACY OF DATA

As shown in Annals of the Astrophysical Observatory of the Smithsonian Institution (vol. 6, p. 163), the comparison of daily solar-constant values, independently measured at stations thousands of miles apart, in opposite hemispheres of the earth, extending over many years, yields a probable error for a well-observed solar-constant value, resulting from work of two stations on a single day, of $\frac{0.164}{\sqrt{2}}$ percent or $\frac{1}{8}$ percent. Using the familiar relation (the probable error of a mean is that of the individual divided by the square root of the number of values), this indicates that a 10-day mean of good quality should be assigned a probable error of 1/25 percent. Then if nine such 10-day means are tabulated in searching for a solar periodicity, the probable error of their mean becomes only 1/75 percent. These considerations indicate not only that real solar variations of 1/10 percent of the solar constant might be detected, but that the features of the march of a periodic variation of this small amplitude would appear well delineated from a tabulation.

To be sure, these optimum conditions do not always prevail. Not infrequently no more than three or five days of a decade yielded solar-constant observations. Often no more than one station reported. Dur-

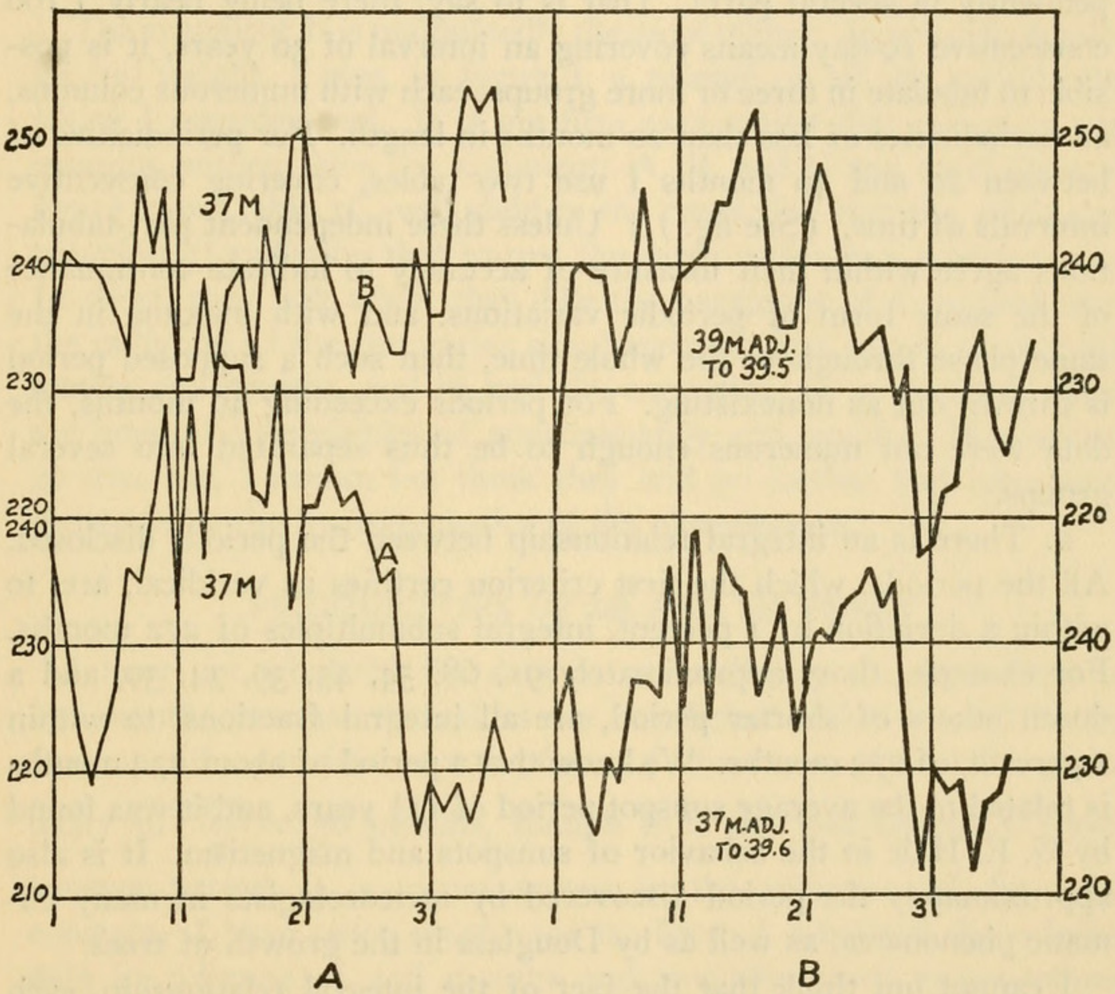


FIG. 3.—The periodicity of approximately $\frac{1}{7} \times 272$ months, tested just as the periodicity of approximately $1/45 \times 272$ months was tested in figure 2.

ing parts of the year less favorable conditions prevailed at one or other of the stations. Such is the case at Table Mountain from March through June, and at Montezuma from November through January. (See figs. 7, 8, pp. 70, 71, Annals, vol. 5.)

On these accounts it need not surprise us that, as shown below, while the sum of periodic variations represents the variation of monthly mean solar-constant results to within an average deviation of 1/10 percent, much larger departures sometimes occur. However, divergences depend not only on accidental errors of the observations, but, in part also on imperfect determination of the form, amplitude, and period of the periodicities, for reasons explained above.

## SUPPORTING EVIDENCES OF VERIDITY OF PERIODICITIES

There are several indications, not flowing from a consideration of probable errors, that strongly support the veridity of periodicities here disclosed:

1. In tabulating periodicities, the data have been treated independently in several parts. That is to say, there being nearly 1,100 consecutive 10-day means covering an interval of 30 years, it is possible to tabulate in three or more groups, each with numerous columns, all periodicities of less than 20 months in length. For periodicities of between 20 and 40 months I use two tables, covering consecutive intervals of time. (See fig. 1.) Unless these independent part-tabulations agree within their measure of accuracy to indicate continuance of the same form of periodic variations, and with maxima in the same phase throughout the whole time, then such a supposed period is thrown out as nonexisting. For periods exceeding 40 months, the data were not numerous enough to be thus separated into several groups.

2. There is an integral relationship between the periods disclosed. All the periods, which the first criterion certifies as veridical, are, to within a deviation of 1 percent, integral submultiples of 272 months. For example, those approximately 91, 68, 54, 45, 39, 34, 30, and a dozen others of shorter period, are all integral fractions, to within 1 percent, of 272 months. We know that a period of about 272 months is related to the average sunspot period of 11⅓ years, and it was found by G. E. Hale in the behavior of sunspots and magnetism. It is also approximately the period discovered by meteorologists in many climatic phenomena, as well as by Douglass in the growth of trees.

I cannot but think that the fact of the integral relationship, each to each, of the solar-radiation periodicities here disclosed, and the relationship of all of them to a master period of 272 months, well known in other solar and terrestrial phenomena, strengthens the case for validity of these periodicities. If that be granted, surely the existence of these integral solar-radiation relationships, so reminiscent of the overtones of the vibrations of musical instruments, is a phenomenon well worth investigating by astronomers and by students of hydrodynamics.

I have just stated three arguments for the reality of numerous regularly periodic variations of the output of radiation from the sun as follows: A. Measurements whose small probable error is consistent with the amplitudes of the apparent periodicities display them. B. Tabulations of a chosen periodicity, with the data separated into independent groups, covering successive time intervals, show separately the periodicity in similar amplitudes, forms, and phases. C. The periods are integrally related, each to each, and all are approximately exact integral submultiples of 272 months, itself a well-known period in other solar and terrestrial phenomena. A fourth supporting evidence is to be referred to later.

The argument B is undoubtedly the most telling. In order to display its full weight, I give, in figure 1, a résumé of all the periodicities which I consider real. It is my firm expectation that scientists who examine without bias the arguments A, B, and C and carefully scan figure 1 and table 1C, will yield to the conviction that the sun's contribution of radiation that warms the earth varies in a complex way. In short, they will admit that, like the overtones of a musical note, the radiation of the sun varies simultaneously in a period of approximately 272 months, and in periods, exceeding 20 in number, which are integral submultiples of approximately 272 months. If scientists go thus far, I cannot but think they will go farther and investigate theoretically the hydrodynamics of the phenomenon.

## PERIODICITIES OF $22\frac{2}{3}$ AND $11\frac{1}{3}$ YEARS

I have not tabulated the data so as to display the periodicity of 272 months, because the values are insufficient. There would be too few repetitions to fairly fix the form of this curve. As for the periodicity of $\frac{272}{2}=136$ months, though it is the well-known $11\frac{1}{3}$-year sunspot period, it is inconspicuous in the variation of the solar constant. I have twice sought for it. First, I tabulated the original data in columns of 136 months and smoothed their mean values. Second, I smoothed by 7-month running means the residual departures, which separate the original data from the synthetic reproduction of them in figure 4 by 23 periodic terms. Neither treatment gave conclusively a periodicity of 136 months. Its well-evidenced weather influence, I think, is attributable to fluctuation of the intensity of the bombardment of the atmosphere by electric ions, acting as centers of condensation of water vapor and dust, as sunspot numbers wax and wane.

## GRAPHS OF RESULTS

Figure 1 is introduced to emphasize the force of the argument B by a graphical appeal to the eye. The figure shows the mean result of every partial tabulation of the values used to compute table 1A, and also the general mean of these partial tabulations for almost all periodicities included in table 1A. Curves for periodicities of 2 1/7 and 3 1/20 months are given on a scale of abscissae 2½ times as great as the other curves. Horizontal lines in figure 1 are separated by 1/10 percent of the solar constant. The curves for periodicity 2 1/7 months are given on a scale of ordinates twice as great as that used for all others. Up to a periodic length of 22¾ months, all the curves are plotted at 10-day intervals. Periodicities of 22¾ months and longer are plotted in monthly intervals. Of periodicities less than 22¾ months in length, one, that of 9 1/10-months period, is shown smoothed throughout by 5-decade running means. It has a small amplitude and would perhaps have seemed doubtful to many had not running means of 5-decade values been shown, instead of the separate 10-day mean values. This smoothing brings out plainly the similarity of the partial tabulations.

The amplitudes of the 23 periodicities plotted in figure 1 may seem to some critics too small to be of any significance. Not so. For it is shown in figure 4 that the synthesis of these 23 periodic fluctuations produces a curve closely matching, and of the same amplitude of variation as, the curve of original observation. A 12-month period of terrestrial origin with amplitude of 0.2 percent is not introduced into figure 4. Its inclusion would improve the agreement there. No additional regular periodicities were discernible. The analysis appears to be exhaustive.

As the periods grow longer, they are apt to display integral submultiples riding upon the period under examination. This is strongly marked with the period of 15⅙ months. It shows seven subperiods of 2 1/7 months very plainly. Similarly the 30½-month curve shows also the 6 1/30-month influence. The 34½-month curve shows influence of the 11¼-month period. Other examples are obvious. Note the curves for periodicities of 54½, 68, and 91 months shown in figure 1. Owing to superposed periods of less length, these long periodicities had to be smoothed by 5- or 7-month running means.

In addition to the direct mean results for each period, I give in a few cases also the smoothed mean, resulting from taking 5-value or 7-value running means for the entire length of the periodicity under consideration. These smooth curves give a more convincing and truer idea of the periodicities, thought to be real, than do the rougher direct means, affected by accidental errors of observation and influences of extraneous periods. Readers should bear in mind that the knicks in the broken lines, which look so large, really average less than 1/10 percent of the solar constant. This bears witness to the high accuracy of the Smithsonian solar-constant observing. Its probable error has been discussed above.

## INTEGRAL RELATIONSHIPS

I had long been of the opinion that the regular periodicities of solar variation are all integrally related to approximately 272 months. This impression is supported by the fact, so obvious in figure 1, that the longer periods shown, themselves being integrally related to 272 months, have in several instances shorter periodicities riding on their backs, which are integral submultiples of them. Further proof of the integral relationships is shown in figures 2 and 3, already described.

Assuming that this integral relationship to 272 months is a condition necessary to the real existence of a regular period in solar variation, the number of such periods that are of considerable amplitudes seems not to exceed 23. At least a rather extensive search has not yielded others strong enough to be certainly real. If these be all, and their forms and amplitudes are as shown in figure 1, then a synthesis of them ought to represent the march of solar variation from 1920 to 1950, except for the interval of 1922 and 1923, when exceptionally large solar variations were observed and which is excluded from this analysis. I have made such a synthesis, and compare it with the march of the solar variation in figure 4.

## SYNTHESIS OF PERIODICITIES

To determine the quantities plotted in figure 4, I have computed the departures, plus and minus, from the mean ordinate for each smoothed periodicity, as expressed monthly, which together fix the form of its curve. This gives, in each case, a short series of small monthly departures suitable to the form of each periodicity. All the tabulations begin with August 1920 as zero time. In table 2 they are all tabulated in the smoothed form actually used in preparing the synthetic curve shown in figure 4. In computing the mean periodic forms, and afterward in using them for synthesizing the solar-constant values, I allow for fractions of a decade, or of a month, by adding or withdrawing a value from certain columns, or at appropriate intervals in synthesizing, so as to preserve the correct period.

I tabulate these series, end to end, over the whole interval of more than 30 years. Thus I make a great table of 23 columns and 367 lines. Adding algebraically the plus and minus values of the lines across the table, I find the total synthesized monthly departures, in ten-thousandths of the solar constant, from the mean solar constant 1.94 calories. The results, covering 367 months, are compared in figure 4 with the monthly observational values recorded in table 4.

## CLOSE AGREEMENT BETWEEN SYNTHESIS AND OBSERVATION

Table 3, below, shows the high degree of accuracy with which the synthesis of the original 21 periodicities (before those of 11.43 and $24\frac{3}{4}$ months were found) corresponded to the observations.

These results came from the comparison of observation with the synthesis of 21 periodicities. The average departures are reduced below these figures when periodicities of 11.43, 12.0,[7] and $24\frac{3}{4}$ months are introduced. The value for the best 233 months then becomes 1.00-tenths percent. The larger average departures prior to July 1926 are attributable to the then imperfect development of the "short method" of solar-constant work. The larger departures after 1945 are thought by Mr. Aldrich to be caused by temporary errors in the scales of pyrheliometers used in the field. He hopes to correct this discrepancy.

Some minds may still prefer to think that the solar-constant observations do not prove the variability of solar radiation. They may point out that the average deviation of the observations from their mean is 0.15 percent, and the average deviation of the synthetic curve from that of observation is still 0.10 percent. They may urge that this amount of improvement is not sufficient to warrant belief in the thesis that the sun's radiation varies in the discovered 23 regular periods, all integral submultiples of 272 months.

Such critics may be reminded that the "weight" of any measurement, that is, its claim to respectful recognition, is proportional to the number of observations that enter into the result; but the probable error (proportional to the average deviation from the mean) is proportional to the square root of the number of observations. It follows that the "weight," or credibility of a solution, is proportional to the square of the average deviation of its components. Hence the weight of the solution here advocated is $\left(\frac{15}{10}\right)^2 = 2.25$ times the weight of the conclusion of an invariable sun.

But it must also be considered that a certain irreducible minimum of accidental error, comparable in a graph to the teeth of a saw, adheres to the solar-constant observations. Whatever excursions from the mean value may be produced by real solar variations, these acci-

[7] The 12-month period is not used in preparing figure 4; its use would improve the agreement of the curves.

TABLE 2.—*Twenty-three solar periodicities in ten-thousandths of the solar constant, based on August 1920. Also the 12-month terrestrial period, same unit*

2 1/7 M: +2 −2. 3 1/20 M: 0 −2 +2. 4 1/3 M: −1 −2 +3 ±0.

5 1/18 M: −1 ±0 −2 +2 +2. 6 1/30 M: −4 −1 +3 +6 ±0 −5.

7 M: −1 +1 +5 +2 −1 −1 −2. 8 1/14 M: −2 −2 −1 −1 +1 +1 +3 +2.

9 1/10 M: −2 −4 −3 −1 ±0 +2 +3 +1. ±0.

9 7/10 M: −4 −3 −1 +1 +5 +5 +2 −1 −4 −3.

10 6/10 M: −1 −1 −1 −1 −3 +1 +1 +2 +3 +1 −1.

11 1/5 M: −4 −2 ±0 +3 +1 +9 +3 −1 +4 −2 −8.

11.43 M: +7 +4 +6 +1 −3 −4 −3 −3 −4 −3 −1.

13 1/10 M: +1 +4 +3 −2 −6 −4 +2 +2 +1 ±0 −2 +1 +3.

15 1/6 M: −3 −6 −6 −1 ±0 +2 +1 +2 +3 +2 ±0 ±0 +2 +1 +1.

22 3/4 M: −1 +1 ±0 +1 +1 +1 +1 +1 ±0 ±0 +1 +2 +3 +3 +2 +2 +1 −1 −2 −3 −3 −2 −1.

24 3/4 M: −2 −2 −1 +1 +2 +3 +3 +4 +4 +4 +3 +3 +2 +1 ±0 −2 −5 −7 −2 ±0 ±0 ±0 ±0 −1 −1.

30 1/3 M: +6 +5 +4 +3 +3 +4 +3 +1 +1 ±0 ±0 ±0 −1 −3 −5 −6 −6 −5 −5 −6 −6 −4 −3 −2 −1 −1 ±0 +3 +3 +4.

34 1/2 M: −5 −6 −4 −3 −3 −2 −3 −5 −7 −6 −3 −1 −1 +2 +5 +6 +8 +7 +6 +4 +1 −1 ±0 +1 +2 +3 +3 +4 +5 +5 +2 +1 −1 −3.

39 M: −4 +1 +2 +2 +2 +2 +1 +1 +1 +2 +2 +4 +6 +8 +10 +10 +8 +7 +5 +3 +4 +5 +5 +4 +3 +3 +1 −1 −4 −6 −8 −10 −10 −10 −9 −9 −9 −8 −6.

45 1/2 M: −3 −4 −3 −3 −2 −1 ±0 +1 +1 +3 +4 +6 +6 +3 +2 +1 ±0 −2 −3 −1 −1 +1 +1 +2 +3 +2 +2 ±0 −1 −3 −4 −5 −4 −3 −2 ±0 +1 +1 ±0 −2 −3 −4 −2 −2 −1.

54 1/2 M: +4 +4 +5 +6 +6 +7 +7 +7 +7 +6 +6 +6 +5 +3 ±0 −1 −1 −1 −2 −4 −4 −3 −3 −2 −2 −2 −2 −3 −2 −3 −2 −4 −5 −4 −3 −4 −2 −3 −3 −4 −3 −2 −1 −1 −1 −2 −1 ±0 ±0 −2 −2 −1 +1 +2.

68 M: −7 −5 −4 −4 −4 −6 −6 −8 −12 −13 −12 −9 −5 −4 −2 −3 −2 −2 −8 −11 −11 −10 −6 −6 −4 −3 −4 −4 −3 −5 −5 −6 −5 −4 −4 −4 −4 −6 −7 −8 −7 −8 −6 −4 −2 ±0 +2 +4 +5 +6 +7 +8 +9 +10 +10 +11 +11 +12 +12 +11 +11 +10 +8 +5 +2 −2 −3 −7.

91 M: ±0 +1 +2 +2 +2 +2 +2 +3 +4 +2 +1 −1 −2 −3 −3 −3 −4 −4 −4 −3 −3 −3 −3 −4 −4 −4 −4 −3 −2 −1 ±0 ±0 ±0 ±0 ±0 −1 −2 −3 −3 −4 −4 −4 −4 −4 −4 −4 −4 −4 −3 −2 −1 ±0 +1 +2 +2 +3 +4 +5 +6 +6 +7 +7 +7 +7 +7 +6 +5 +4 +3 +2 +2 +1 +1 +1 ±0 ±0 −1 −2 −2 −3 −4 −4 −4 −4 −3 −2 −1 −1 ±0 ±0 ±0.

*The 12-month period of terrestrial causation*

| Jan. | Feb. | Mar. | Apr. | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| +0.1 | +0.6 | −2.1 | −6.7 | −0.9 | +1.7 | +1.4 | +2.1 | +4.3 | +6.2 | +13.2 | +13.5 |

dental errors of observation will still load the curve with their sawtoothlike vibrations about its true course. No system of periodicities, which may truly represent the true courses of the solar variation, can possibly follow these small accidental errors of observation. It is therefore unreasonable to demand that such a system of periodicities, even though the true one, can be expected to reduce the average deviation of its curve from the curve of observation below the one-tenth

TABLE 3.—*Average departures of synthetic from observational curve*

| | | | | |
|---|---|---|---|---|
| Aug. 1920—Mar. 1922, | 20 months, | 2.01 | tenths | percent. |
| Aug. 1923—July 1926, | 36 months, | 1.82 | " | " |
| Aug. 1926—Dec. 1945, | 233 months, | 1.10 | " | " |
| Jan. 1945—Dec. 1950, | 60 months, | 2.38 | " | " |
| Aug. 1920—Dec. 1950, | 349 months, | 1.45 | " | " |

of a percent found. For though, as stated, the probable error of *first-rate* 10-day means, as found by comparing the simultaneous observations of two solar-constant observations, is 1/25 percent, very many 10-day means are not first rate, as explained above. Moreover the "average deviation" is 5/4 of the "probable error," as is well known, raising the figure to 1/19 percent for the average deviation of first-rate 10-day means.

The real crux of the question, as between the hypothesis of constant solar radiation, and solar radiation varying in 23 regular periods, painstakingly determined and tested by several criteria of reality, lies in considering the large excursions of the curve of observation from its mean. Examples of such methodically marching excursions are found from 1924 to 1927, from 1929 to 1933, from 1937 to 1942, and from 1947 to 1949. The hypothesis of a constant solar radiation offers no explanation for them. On the other hand, the synthetic curve follows these large, methodically marching excursions with some fidelity.

Yet notwithstanding this striking harmony in the principal features between the curve of observation and the synthetic curve of regular periodicities, there are limited intervals of substantial disagreement. Among these the major one occurs in 1922 and 1923, regarding which I have already written. The disagreement in 1920 and 1921 may be attributed to the incomplete development of the short method of solar-constant determination in those earliest years. The same perhaps applies to the disagreement in the years 1924 and 1925, for even then the short method was not fully developed, as now used. As for the period 1946 to 1950, Mr. Aldrich inclines to think the scales of pyrheliometry may have varied a little in those years. There is also a possibility that, in carrying the computations so far forward as 1950 from their base in 1920, slight errors in the length of the periods have accumulated so as to mar the results of synthesis.

Brief intervals of unusually large divergence between the synthetic and the observed curves occur in 1927, 1929, 1934-1935, 1938, 1940-1941, and 1944. Nearly all these cases occur at the times of the year when sky conditions for observing are inferior at one or both stations, as indicated by figures 7 and 8, pages 70 and 71, Annals, volume 5, already cited. It is not probable, however, that regular periods of variation include *all* the variations of solar radiation. We know, indeed, that outbursts of sunspots and flares cause changes in the sun's output of radiation. Some of the discrepancies referred to are doubtless due to such causes.

I hope the reader will agree that the synthesis of 23 independently and separately computed periodic terms has represented, to within the error of observation, the march of the solar constant as given by the monthly means of the original observations from 1920 to 1950, excluding the extraordinary values of 1922 and 1923. This close agreement in form and amplitude between the observed and the synthetic curve seems to me a fourth kind of evidence supporting the existence of a complex of over 20 regular periods all approximately integral submultiples of 272 months in the observed variation of the sun's output of radiation.

It will occur to the reader that curves of solar observation should tend to repeat their features after 272 months, or approximately 23 years. There is a slight indication that the curve of 1921 in figure 4 is similar to that of 1944, but the work of 1921, as mentioned elsewhere, is too inaccurate to prove it. In the years 1922 and 1923 occurred a unique large depression of the curve of observation. A real test must begin with the year 1924. Unfortunately, as stated elsewhere, there appears to have been a change of scale of about $\frac{1}{3}$ percent in 1948. To correct for it, I subtract 32 units from all the monthly means, July 1948 to February 1950.

In figure 4A, I superpose the corrected curve 1947 to 1950 (light line) upon the observed curve of observation 1924 to 1927 (heavy line). The similarity is striking. During 48 months there are five large divergencies: 0.55, 0.50, and three of 0.45 percent. The extreme range of the great feature shown in figure 4A is 0.9 percent, and the average deviation between the curves is but 0.19 percent—less than the expected combined probable errors of observing. One regrets that the interval, 276 months, exceeds the expected interval, 272 months. But as solar conditions modify the lengths of the sunspot cycles, they may also slightly modify that of the 272-month cycle from time to time.

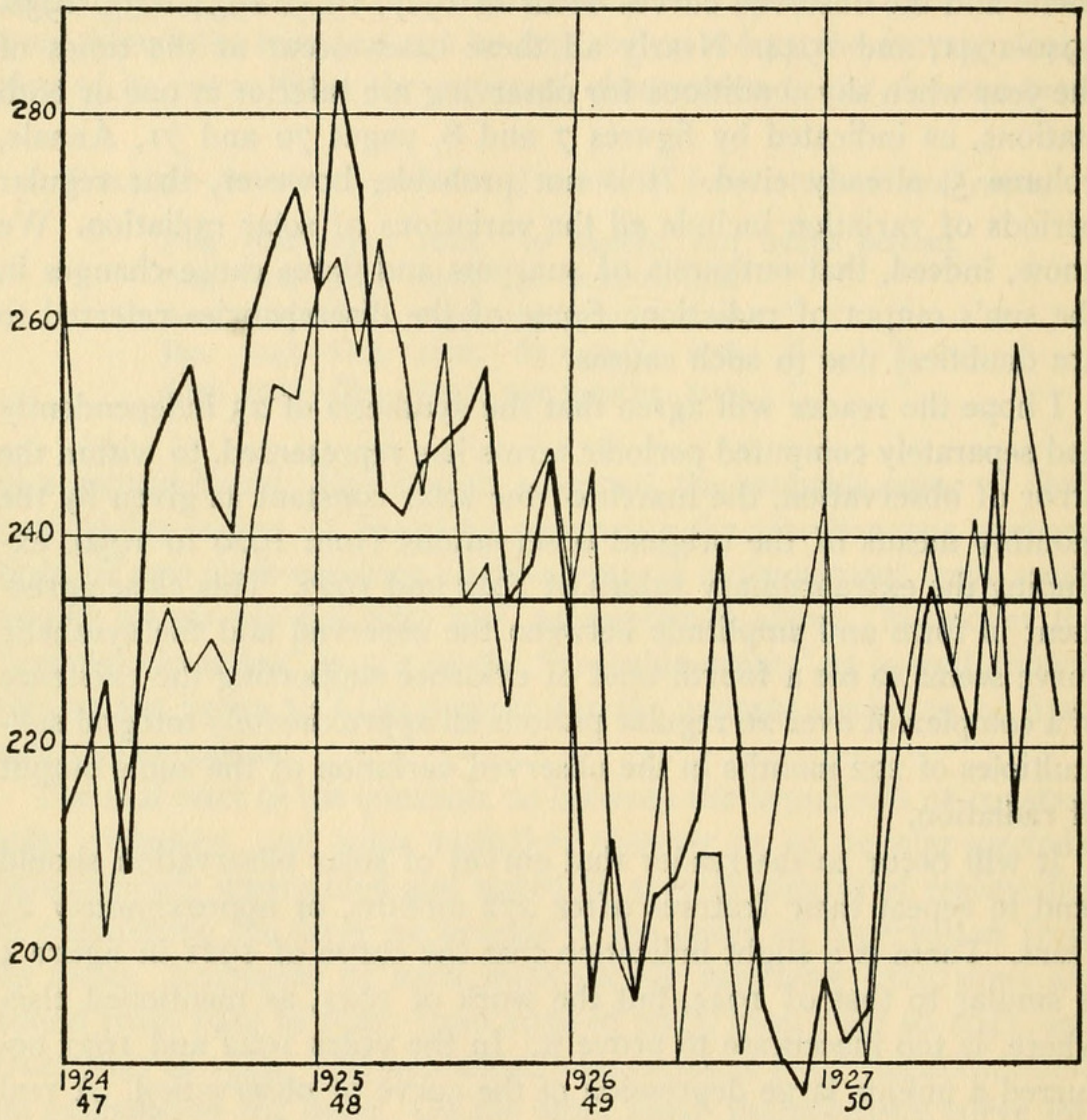


FIG. 4A.—Comparison of solar constants 1924-1927 (heavy lines) and 1947-1950 (light lines).

## SCALE OF SOLAR CONSTANT NEARLY UNCHANGED IN 30 YEARS

It is very pleasing that the comparison of synthesized and original curves shows the features generally with equal amplitudes in the two curves. The comparison gives no indication that the scale of observation has changed in 30 years, except perhaps for a rise of 3/10 percent from June 1948 to January 1950. This is remarkable in view of many changes of instruments and of procedures that have taken place meanwhile.

## APPENDIX 1

### SOLAR-CONSTANT MONTHLY AND 10-DAY MEANS, 1920–1950

Doubtless there are those who are engaged in research on cycles in various lines who may wish to know the Smithsonian results on solar variability as nearly as possible up to date. Mr. Aldrich kindly permits me to publish the following table (table 4) giving the percentage excesses of solar-constant values above 1.900 calories from 1920 to 1950. These percentage excesses are in the form of means of 10 days (i.e., decades of months) and means of months. Taking the first trio of values, given here for illustration, the table may be explained as follows. We have:

2, 8, I, 0, 1, 154
2, 8, II, 0, 2, 139, 153[8]
2, 8, III, 0, 3, 165

The above figure 2, with the figure 0, makes 20, meaning the year 1920. The figure 8 means August, the eighth month of 1920. The Roman numerals I, II, III stand for the first, second, and third decades of August. That is: August 1-9, 10-19, 20-31. The values 154, 139, 165 represent decade-means of the daily excesses of the solar constant by which these observations exceeded in ten-thousandth parts of the mean solar constant (taken as 1.94 calories) the value 1.9000 calories. Thus the value 154 signifies that the mean solar constant for the first decade of August 1920 was 1.54 percent of 1.94 or 0.0299 calorie above 1.90 calories. Finally, the value 153 is the mean of the three decade values and signifies that the average solar constant for August 1920 was 1.90+1.53 percent of 1.94 calories, or 1.930 calories.[8] As stated above, the percentages of excess over 1.90 calories was chosen to suit my investigation because, first, all values are positive, and second, results come out in percentages of the solar constant.

## APPENDIX 2

### PROBABLE SOLAR-CONSTANT VALUES BEFORE 1920

Smithsonian solar-constant observations were made in the summers on Mount Wilson, Calif., in most years from 1905 to 1920. But partly because of experimental crudity, and partly from the variability of sky transparency, and mainly because those measurements were all made by the fundamental "long method," which requires constant sky transparency for hours, the results were wide-ranging, from about

[8] This result is far out of line, and indicates experimental error. In drawing figure 4 I have assumed, instead, 235, given in parenthesis in table 4.

TABLE 4.—*Ten-day and monthly means*

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2, 8 | I, 0 | 1 | 154 | | 2, 11 | I, 2 | 82 | 134 | | 2, 2 | I, 5 | 163 | 278 | |
| | II | 2 | 139(235) | | | II | 83 | 154 | | | II | 164 | 283 | |
| | III | 3 | 165 | 153 | | III | 84 | 98 | 129 | | III | 165 | 288 | 283 |
| 2, 9 | I, 0 | 4 | 263 | | 2, 12 | I, 2 | 85 | 124 | | 2, 3 | I, 5 | 166 | 299 | |
| | II | 5 | 227 | | | II | 86 | 113 | | | II | 167 | 263 | |
| | III | 6 | 227 | 239 | | III | 87 | 118 | 118 | | III | 168 | 258 | 273 |
| 2, 10 | I, 0 | 7 | 227 | | 2, 1 | I, 3 | 88 | 232 | | 2, 4 | I, 5 | 169 | 263 | |
| | II | 8 | 278 | | | II | 89 | 185 | | | II | 170 | 252 | |
| | III | 9 | 206 | 237 | | III | 90 | 154 | 190 | 2, 4 | III, 5 | 171 | 216 | 244 |
| 2, 11 | I, 0 | 10 | 278 | | 2, 2 | I, 3 | 91 | 160 | | 2, 5 | I, 5 | 172 | 221 | |
| | II | 11 | 258 | | | II | 92 | 142 | | | II | 173 | 258 | |
| | III | 12 | 201 | 246 | | III | 93 | 77 | 126 | | III | 174 | 247 | 242 |
| 2, 12 | I, 0 | 13 | 294 | | 2, 3 | I, 3 | 94 | 160 | | 2, 6 | I, 5 | 175 | 237 | |
| | II | 14 | 263 | | | II | 95 | 175 | | | II | 176 | 247 | |
| | III | 15 | 278 | 278 | | III | 96 | 160 | 165 | | III | 177 | 258 | 247 |
| 2, 1 | I, 1 | 16 | 299 | | 2, 4 | I, 3 | 97 | 175 | | 2, 7 | I, 5 | 178 | 263 | |
| | II | 17 | 304 | | | II | 98 | 134 | | | II | 179 | 278 | |
| | III | 18 | 278 | 294 | | III | 99 | 165 | 158 | | III | 180 | 206 | 249 |
| 2, 2 | I, 1 | 19 | 237 | | 2, 5 | I, 3 | 100 | 175 | | 2, 8 | I, 5 | 181 | 258 | |
| | II | 20 | 288 | | | II | 101 | 180 | | | II | 182 | 221 | |
| | III | 21 | 278 | 268 | | III | 102 | 191 | 182 | | III | 183 | 273 | 251 |
| 2, 3 | I, 1 | 22 | 299 | | 2, 6 | I, 3 | 103 | 118 | | 2, 9 | I, 5 | 184 | 263 | |
| | II | 23 | 206 | | | II | 104 | 170 | | | II | 185 | 263 | |
| | III | 24 | 242 | 249 | | III | 105 | 165 | 151 | | III | 186 | 242 | 256 |
| 2, 4 | I, 1 | 25 | 242 | | 2, 7 | I, 3 | 106 | 180 | | 2, 10 | I, 5 | 187 | 232 | |
| | II | 26 | 242 | | | II | 107 | 144 | | | II | 188 | 237 | |
| | III | 27 | 242 | 242 | | III | 108 | 227 | 184 | | III | 189 | 232 | 234 |
| 2, 5 | I, 1 | 28 | 267 | | 2, 8 | I, 3 | 109 | 216 | | 2, 11 | I, 5 | 190 | 216 | |
| | II | 29 | 247 | | | II | 110 | 206 | | | II | 191 | 252 | |
| | III | 30 | 263 | 259 | | III | 111 | 211 | 211 | | III | 192 | 242 | 237 |
| 2, 6 | I, 1 | 31 | 185 | | 2, 9 | I, 3 | 112 | 252 | | 2, 12 | I, 5 | 193 | 247 | |
| | II | 32 | 206 | | | II | 113 | 252 | | | II | 194 | 237 | |
| | III | 33 | 211 | 201 | | III | 114 | 242 | 249 | | III | 195 | 258 | 247 |
| 2, 7 | I, 1 | 34 | 258 | | 2, 10 | I, 3 | 115 | 237 | | 2, 1 | I, 6 | 196 | 237 | |
| | II | 35 | 268 | | | II | 116 | 221 | | | II | 197 | 258 | |
| | III | 36 | 252 | 259 | | III | 117 | 237 | 232 | | III | 198 | 196 | 230 |
| 2, 8 | I, 1 | 37 | 211 | | 2, 11 | I, 3 | 118 | 221 | | 2, 2 | I, 6 | 199 | 201 | |
| | II | 38 | 263 | | | II | 119 | 227 | | | II | 200 | 206 | |
| | III | 39 | 196 | 223 | | III | 120 | 211 | 220 | | III | 201 | 180 | 196 |
| 2, 9 | I, 1 | 40 | 227 | | 2, 12 | I, 3 | 121 | 221 | | 2, 3 | I, 6 | 202 | 211 | |
| | II | 41 | 263 | | | II | 122 | 216 | | | II | 203 | 232 | |
| | III | 42 | 268 | 253 | | III | 123 | 175 | 204 | | III | 204 | 191 | 211 |
| 2, 10 | I, 1 | 43 | 268 | | 2, 1 | I, 4 | 124 | 216 | | 2, 4 | I, 6 | 205 | 170 | |
| | II | 44 | 294 | | | II | 125 | 211 | | | II | 206 | 201 | |
| | III | 45 | 309 | 290 | | III | 126 | 211 | 213 | | III | 207 | 216 | 196 |
| 2, 11 | I, 1 | 46 | 294 | | 2, 2 | I, 4 | 127 | 221 | | 2, 5 | I, 6 | 208 | 201 | |
| | II | 47 | 283 | | | II | 128 | 201 | | | II | 209 | 206 | |
| | III | 48 | 309 | 295 | | III | 129 | 232 | 218 | | III | 210 | 211 | 206 |
| 2, 12 | I, 1 | 49 | 273 | | 2, 3 | I, 4 | 130 | 252 | | 2, 6 | I, 6 | 211 | 196 | |
| | II | 50 | 247 | | | II | 131 | 211 | | | II | 212 | 216 | |
| | III | 51 | 268 | 263 | | III | 132 | 216 | 226 | | III | 213 | 211 | 208 |
| 2, 1 | I, 2 | 52 | 165 | | 2, 4 | I, 4 | 133 | 196 | | 2, 7 | I, 6 | 214 | 221 | |
| | II | 53 | 247 | | | II | 134 | 206 | | | II | 215 | 211 | |
| | III | 54 | 247 | 220 | | III | 135 | 221 | 208 | | III | 216 | 211 | 214 |
| 2, 2 | I, 2 | 55 | 206 | | 2, 5 | I, 4 | 136 | 237 | | 2, 8 | I, 6 | 217 | 232 | |
| | II | 56 | 252 | | | II | 137 | 252 | | | II | 218 | 232 | |
| | III | 57 | 247 | 235 | | III | 138 | 252 | 247 | | III | 219 | 252 | 239 |
| 2, 3 | I, 2 | 58 | 221 | | 2, 6 | I, 4 | 139 | 247 | | 2, 9 | I, 6 | 220 | 216 | |
| | II | 59 | 201 | | | II | 140 | 247 | | | II | 221 | 216 | |
| | III | 60 | 154 | 192 | | III | 141 | 263 | 252 | | III | 222 | 227 | 220 |
| 2, 4 | I, 2 | 61 | 165 | | 2, 7 | I, 4 | 142 | 247 | | 2, 10 | I, 6 | 223 | 201 | |
| | II | 62 | 165 | | | II | 143 | 268 | | | II | 224 | 206 | |
| | III | 63 | 139 | 156 | | III | 144 | 252 | 256 | | III | 225 | 180 | 196 |
| 2, 5 | I, 2 | 64 | 139 | | 2, 8 | I, 4 | 145 | 278 | | 2, 11 | I, 6 | 226 | 185 | |
| | II | 65 | 160 | | | II | 146 | 221 | | | II | 227 | 185 | |
| | III | 66 | 165 | 156 | | III | 147 | 232 | 244 | | III | 228 | 201 | 190 |
| 2, 6 | I, 2 | 67 | 129 | | 2, 9 | I, 4 | 148 | 211 | | 2, 12 | I, 6 | 229 | 170 | |
| | II | 68 | 72 | | | II | 149 | 232 | | | II | 230 | 201 | |
| | III | 69 | 72 | 91 | | III | 150 | 278 | 240 | | III | 231 | 191 | 187 |
| 2, 7 | I, 2 | 70 | 21 | | 2, 10 | I, 4 | 151 | 252 | | 2, 1 | I, 7 | 232 | 206 | |
| | II | 71 | 88 | | | II | 152 | 268 | | | II | 233 | 196 | |
| | III | 72 | 72 | 60 | | III | 153 | 263 | 261 | | III | 234 | 191 | 198 |
| 2, 8 | I, 2 | 73 | 98 | | 2, 11 | I, 4 | 154 | 263 | | 2, 2 | I, 7 | 235 | 175 | |
| | II | 74 | 124 | | | II | 155 | 268 | | | II | 236 | 242 | |
| | III | 75 | 103 | 108 | | III | 156 | 273 | 268 | | III | 237 | 160 | 192 |
| 2, 9 | I, 2 | 76 | 160 | | 2, 12 | I, 4 | 157 | 283 | | 2, 3 | I, 7 | 238 | 154 | |
| | II | 77 | 52 | | | II | 158 | 263 | | | II | 239 | 216 | |
| | III | 78 | 88 | 100 | | III | 159 | 273 | 273 | | III | 240 | 216 | 195 |
| 2, 10 | I, 2 | 79 | 144 | | 2, 1 | I, 5 | 160 | 242 | | 2, 4 | I, 7 | 241 | 247 | |
| | II | 80 | 129 | | | II | 161 | 263 | | | II | 242 | 232 | |
| | III | 81 | 93 | 122 | | III | 162 | 283 | 263 | | III | 243 | 201 | 227 |

TABLE 4.—Continued

| | | | | |
|---|---|---|---|---|
| 2, 5 | I, 7 | 244 | 206 | |
| | II | 245 | 242 | |
| | III | 246 | 216 | 221 |
| 2, 6 | I, 7 | 247 | 258 | |
| | II | 248 | 221 | |
| | III | 249 | 227 | 235 |
| 2, 7 | I, 7 | 250 | 232 | |
| | II | 251 | 216 | |
| | III | 252 | 232 | 227 |
| 2, 8 | I, 7 | 253 | 211 | |
| | II | 254 | 221 | |
| | III | 255 | 232 | 227 |
| 2, 9 | I, 7 | 256 | 237 | |
| | II | 257 | 258 | |
| | III | 258 | 247 | 247 |
| 2, 10 | I, 7 | 259 | 221 | |
| | II | 260 | 206 | |
| | III | 261 | 211 | 213 |
| 2, 11 | I, 7 | 262 | 232 | |
| | II | 263 | 232 | |
| | III | 264 | 247 | 237 |
| 2, 12 | I, 7 | 265 | 242 | |
| | II | 266 | 227 | |
| | III | 267 | 201 | 223 |
| 2, 1 | I, 8 | 268 | 221 | |
| | II | 269 | 196 | |
| | III | 270 | 216 | 211 |
| 2, 2 | I, 8 | 271 | 237 | |
| | II | 272 | 211 | |
| | III | 273 | 201 | 216 |
| 2, 3 | I, 8 | 274 | 237 | |
| | II | 275 | 247 | |
| | III | 276 | 221 | 235 |
| 2, 4 | I, 8 | 277 | 216 | |
| | II | 278 | 227 | |
| | III | 279 | 242 | 228 |
| 2, 5 | I, 8 | 280 | 227 | |
| | II | 281 | 263 | |
| | III | 282 | 247 | 246 |
| 2, 6 | I, 8 | 283 | 247 | |
| | II | 284 | 278 | |
| | III | 285 | 232 | 252 |
| 2, 7 | I, 8 | 286 | 232 | |
| | II | 287 | 221 | |
| | III | 288 | 216 | 223 |
| 2, 8 | I, 8 | 289 | 191 | |
| | II | 290 | 227 | |
| | III | 291 | 227 | 215 |
| 2, 9 | I, 8 | 292 | 201 | |
| | II | 293 | 237 | |
| | III | 294 | 196 | 211 |
| 2, 10 | I, 8 | 295 | 227 | |
| | II | 296 | 232 | |
| | III | 297 | 211 | 223 |
| 2, 11 | I, 8 | 298 | 227 | |
| | II | 299 | 252 | |
| | III | 300 | 237 | 239 |
| 2, 12 | I, 8 | 301 | 237 | |
| | II | 302 | 227 | |
| | III | 303 | 252 | 239 |
| 2, 1 | I, 9 | 304 | 242 | |
| | II | 305 | 258 | |
| | III | 306 | 237 | 246 |
| 2, 2 | I, 9 | 307 | 232 | |
| | II | 308 | 211 | |
| | III | 309 | 196 | 213 |
| 2, 3 | I, 9 | 310 | 242 | |
| | II | 311 | 191 | |
| | III | 312 | 206 | 213 |
| 2, 4 | I, 9 | 313 | 191 | |
| | II | 314 | 242 | |
| | III | 315 | 227 | 220 |
| 2, 5 | I, 9 | 316 | 216 | |
| | II | 317 | 216 | |
| | III | 318 | 227 | 220 |
| 2, 6 | I, 9 | 319 | 206 | |
| | II | 320 | 175 | |
| | III | 321 | 196 | 192 |
| 2, 7 | I, 9 | 322 | 206 | |
| | II | 323 | 211 | |
| | III | 324 | 216 | 211 |
| 2, 8 | I, 9 | 325 | 201 | |
| | II | 326 | 206 | |
| | III | 327 | 206 | 204 |
| 2, 9 | I, 9 | 328 | 211 | |
| | II | 329 | 191 | |
| | III | 330 | 211 | 204 |
| 2, 10 | I, 9 | 331 | 211 | |
| | II | 332 | 216 | |
| | III | 333 | 175 | 201 |
| 2, 11 | I, 9 | 334 | 206 | |
| | II | 335 | 227 | |
| | III | 336 | 237 | 223 |
| 2, 12 | I, 9 | 337 | 237 | |
| | II | 338 | 237 | |
| | III | 339 | 227 | 234 |
| 3, 1 | I, 0 | 340 | 211 | |
| | II | 341 | 232 | |
| | III | 342 | 232 | 225 |
| 3, 2 | I, 0 | 343 | 211 | |
| | II | 344 | 232 | |
| | III | 345 | 247 | 230 |
| 3, 3 | I, 0 | 346 | 232 | |
| | II | 347 | 211 | |
| | III | 348 | 216 | 220 |
| 3, 4 | I, 0 | 349 | 221 | |
| | II | 350 | 206 | |
| | III | 351 | 227 | 218 |
| 3, 5 | I, 0 | 352 | 232 | |
| | II | 353 | 252 | |
| | III | 354 | 242 | 242 |
| 3, 6 | I, 0 | 355 | 242 | |
| | II | 356 | 273 | |
| | III | 357 | 258 | 258 |
| 3, 7 | I, 0 | 358 | 232 | |
| | II | 359 | 278 | |
| | III | 360 | 273 | 261 |
| 3, 8 | I, 0 | 361 | 242 | |
| | II | 362 | 268 | |
| | III | 363 | 252 | 254 |
| 3, 9 | I, 0 | 364 | 247 | |
| | II | 365 | 227 | |
| | III | 365 | 232 | 235 |
| 3, 10 | I, 0 | 367 | 227 | |
| | II | 368 | 237 | |
| | III | 369 | 247 | 237 |
| 3, 11 | I, 0 | 370 | 242 | |
| | II | 371 | 242 | |
| | III | 372 | 263 | 249 |
| 3, 12 | I, 0 | 373 | 268 | |
| | II | 374 | 278 | |
| | III | 375 | 263 | 270 |
| 3, 1 | I, 1 | 376 | 216 | |
| | II | 377 | 247 | |
| | III | 378 | 268 | 244 |
| 3, 2 | I, 1 | 379 | 247 | |
| | II | 380 | 258 | |
| | III | 381 | 216 | 240 |
| 3, 3 | I, 1 | 382 | 227 | |
| | II | 383 | 237 | |
| | III | 384 | 258 | 241 |
| 3, 4 | I, 1 | 385 | 237 | |
| | II | 386 | 237 | |
| | III | 387 | 237 | 237 |
| 3, 5 | I, 1 | 388 | 288 | |
| | II | 389 | 258 | |
| | III | 390 | 258 | 268 |
| 3, 6 | I, 1 | 391 | 247 | |
| | II | 392 | 247 | |
| | III | 393 | 232 | 242 |
| 3, 7 | I, 1 | 394 | 237 | |
| | II | 395 | 252 | |
| | III | 396 | 247 | 245 |
| 3, 8 | I, 1 | 397 | 247 | |
| | II | 398 | 258 | |
| | III | 399 | 232 | 246 |
| 3, 9 | I, 1 | 400 | 252 | |
| | II | 401 | 232 | |
| | III | 402 | 263 | 249 |
| 3, 10 | I, 1 | 403 | 263 | |
| | II | 404 | 221 | |
| | III | 405 | 237 | 240 |
| 3, 11 | I, 1 | 406 | 221 | |
| | II | 407 | 232 | |
| | III | 408 | 237 | 230 |
| 3, 12 | I, 1 | 409 | 221 | |
| | II | 410 | 247 | |
| | III | 411 | 242 | 237 |
| 3, 1 | I, 2 | 412 | 242 | |
| | II | 413 | 242 | |
| | III | 414 | 221 | 235 |
| 3, 2 | I, 2 | 415 | 227 | |
| | II | 416 | 232 | |
| | III | 417 | 165 | 208 |
| 3, 3 | I, 2 | 418 | 175 | |
| | II | 419 | 221 | |
| | III | 420 | 206 | 201 |
| 3, 4 | I, 2 | 421 | 191 | |
| | II | 422 | 221 | |
| | III | 423 | 211 | 208 |
| 3, 5 | I, 2 | 424 | 232 | |
| | II | 425 | 227 | |
| | III | 426 | 154 | 204 |
| 3, 6 | I, 2 | 427 | 206 | |
| | II | 428 | 221 | |
| | III | 429 | 221 | 216 |
| 3, 7 | I, 2 | 430 | 263 | |
| | II | 431 | 206 | |
| | III | 432 | 216 | 228 |
| 3, 8 | I, 2 | 433 | 196 | |
| | II | 434 | 227 | |
| | III | 435 | 216 | 213 |
| 3, 9 | I, 2 | 436 | 191 | |
| | II | 437 | 232 | |
| | III | 438 | 237 | 220 |
| 3, 10 | I, 2 | 439 | 211 | |
| | II | 440 | 180 | |
| | III | 441 | 201 | 197 |
| 3, 11 | I, 2 | 442 | 211 | |
| | II | 443 | 185 | |
| | III | 444 | 201 | 199 |
| 3, 12 | I, 2 | 445 | 258 | |
| | II | 446 | 237 | |
| | III | 447 | 211 | 235 |
| 3, 1 | I, 3 | 448 | 258 | |
| | II | 449 | 247 | |
| | III | 450 | 268 | 258 |
| 3, 2 | I, 3 | 451 | 258 | |
| | II | 452 | 242 | |
| | III | 453 | 242 | 247 |
| 3, 3 | I, 3 | 454 | 237 | |
| | II | 455 | 206 | |
| | III | 456 | 206 | 216 |
| 3, 4 | I, 3 | 457 | 211 | |
| | II | 458 | 227 | |
| | III | 459 | 191 | 210 |
| 3, 5 | I, 3 | 460 | 106 | |
| | II | 461 | 206 | |
| | III | 462 | 232 | 211 |
| 3, 6 | I, 3 | 463 | 206 | |
| | II | 464 | 216 | |
| | III | 465 | 232 | 218 |
| 3, 7 | I, 3 | 466 | 247 | |
| | II | 467 | 242 | |
| | III | 468 | 221 | 237 |
| 3, 8 | I, 3 | 469 | 221 | |
| | II | 470 | 221 | |
| | III | 471 | 216 | 219 |
| 3, 9 | I, 3 | 472 | 252 | |
| | II | 473 | 247 | |
| | III | 474 | 263 | 254 |
| 3, 10 | I, 3 | 475 | 263 | |
| | II | 476 | 237 | |
| | III | 477 | 263 | 253 |
| 3, 11 | I, 3 | 478 | 242 | |
| | II | 479 | 263 | |
| | III | 480 | 273 | 259 |
| 3, 12 | I, 3 | 481 | 268 | |
| | II | 482 | 252 | |
| | III | 483 | 258 | 259 |
| 3, 1 | I, 4 | 484 | 258 | |
| | II | 485 | 237 | |
| | III | 486 | 247 | 247 |

TABLE 4.—Continued

| | | | | |
|---|---|---|---|---|
| 3, 2 | I, 4 | 487 | 263 | |
| | II | 488 | 216 | |
| | III | 489 | 227 | 235 |
| 3, 3 | I, 4 | 490 | 247 | |
| | II | 491 | 221 | |
| | III | 492 | 258 | 242 |
| 3, 4 | I, 4 | 493 | 232 | |
| | II | 494 | 221 | |
| | III | 495 | 221 | 225 |
| 3, 5 | I, 4 | 496 | 227 | |
| | II | 497 | 242 | |
| | III | 498 | 221 | 230 |
| 3, 6 | I, 4 | 499 | 242 | |
| | II | 500 | 258 | |
| | III | 501 | 252 | 251 |
| 3, 7 | I, 4 | 502 | 258 | |
| | II | 503 | 232 | |
| | III | 504 | 232 | 241 |
| 3, 8 | I, 4 | 505 | 211 | |
| | II | 506 | 237 | |
| | III | 507 | 227 | 225 |
| 3, 9 | I, 4 | 508 | 232 | |
| | II | 509 | 247 | |
| | III | 510 | 263 | 247 |
| 3, 10 | I, 4 | 511 | 263 | |
| | II | 512 | 268 | |
| | III | 513 | 263 | 265 |
| 3, 11 | I, 4 | 514 | 268 | |
| | II | 515 | 263 | |
| | III | 516 | 258 | 263 |
| 3, 12 | I, 4 | 517 | 268 | |
| | II | 518 | 258 | |
| | III | 519 | 247 | 258 |
| 3, 1 | I, 5 | 520 | 242 | |
| | II | 521 | 268 | |
| | III | 522 | 232 | 247 |
| 3, 2 | I, 5 | 523 | 237 | |
| | II | 524 | 237 | |
| | III | 525 | 216 | 230 |
| 3, 3 | I, 5 | 526 | 221 | |
| | II | 527 | 242 | |
| | III | 528 | 263 | 242 |
| 3, 4 | I, 5 | 529 | 237 | |
| | II | 530 | 242 | |
| | III | 531 | 227 | 235 |
| 3, 5 | I, 5 | 532 | 247 | |
| | II | 533 | 232 | |
| | III | 534 | 247 | 242 |
| 3, 6 | I, 5 | 535 | 237 | |
| | II | 536 | 237 | |
| | III | 537 | 247 | 240 |
| 3, 7 | I, 5 | 538 | 247 | |
| | II | 539 | 227 | |
| | III | 540 | 232 | 235 |
| 3, 8 | I, 5 | 541 | 247 | |
| | II | 542 | 263 | |
| | III | 543 | 237 | 249 |
| 3, 9 | I, 5 | 544 | 232 | |
| | II | 545 | 237 | |
| | III | 546 | 227 | 232 |
| 3, 10 | I, 5 | 547 | 237 | |
| | II | 548 | 242 | |
| | III | 549 | 242 | 240 |
| 3, 11 | I, 5 | 550 | 247 | |
| | II | 551 | 268 | |
| | III | 552 | 252 | 256 |
| 3, 12 | I, 5 | 553 | 247 | |
| | II | 554 | 263 | |
| | III | 555 | 273 | 261 |
| 3, 1 | I, 6 | 556 | 237 | |
| | II | 557 | 263 | |
| | III | 558 | 227 | 242 |
| 3, 2 | I, 6 | 559 | 242 | |
| | II | 560 | 263 | |
| | III | 561 | 196 | 234 |
| 3, 3 | I, 6 | 562 | 201 | |
| | II | 563 | 237 | |
| | III | 564 | 232 | 223 |
| 3, 4 | I, 6 | 565 | 232 | |
| | II | 566 | 237 | |
| | III | 567 | 237 | 235 |
| 3, 5 | I, 6 | 568 | 237 | |
| | II | 569 | 232 | |
| | III | 570 | 247 | 239 |
| 3, 6 | I, 6 | 571 | 258 | |
| | II | 572 | 252 | |
| | III | 573 | 247 | 252 |
| 3, 7 | I, 6 | 574 | 252 | |
| | II | 575 | 242 | |
| | III | 576 | 242 | 245 |
| 3, 8 | I, 6 | 577 | 232 | |
| | II | 578 | 252 | |
| | III | 579 | 242 | 242 |
| 3, 9 | I, 6 | 580 | 232 | |
| | II | 581 | 252 | |
| | III | 582 | 263 | 249 |
| 3, 10 | I, 6 | 583 | 252 | |
| | II | 584 | 252 | |
| | III | 585 | 242 | 249 |
| 3, 11 | I, 6 | 586 | 268 | |
| | II | 587 | 273 | |
| | III | 588 | 258 | 266 |
| 3, 12 | I, 6 | 589 | 278 | |
| | II | 590 | 263 | |
| | III | 591 | 247 | 263 |
| 3, 1 | I, 7 | 592 | 247 | |
| | II | 593 | 273 | |
| | III | 594 | 242 | 254 |
| 3, 2 | I, 7 | 595 | 247 | |
| | II | 596 | 237 | |
| | III | 597 | 252 | 245 |
| 3, 3 | I, 7 | 598 | 211 | |
| | II | 599 | 221 | |
| | III | 600 | 227 | 220 |
| 3, 4 | I, 7 | 601 | 201 | |
| | II | 602 | 211 | |
| | III | 603 | 216 | 209 |
| 3, 5 | I, 7 | 604 | 180 | |
| | II | 605 | 227 | |
| | III | 606 | 237 | 215 |
| 3, 6 | I, 7 | 607 | 237 | |
| | II | 608 | 237 | |
| | III | 609 | 242 | 239 |
| 3, 7 | I, 7 | 610 | 221 | |
| | II | 611 | 227 | |
| | III | 612 | 232 | 227 |
| 3, 8 | I, 7 | 613 | 242 | |
| | II | 614 | 232 | |
| | III | 615 | 242 | 239 |
| 3, 9 | I, 7 | 616 | 252 | |
| | II | 617 | 247 | |
| | III | 618 | 237 | 245 |
| 3, 10 | I, 7 | 619 | 242 | |
| | II | 620 | 227 | |
| | III | 621 | 242 | 237 |
| 3, 11 | I, 7 | 622 | 247 | |
| | II | 623 | 247 | |
| | III | 624 | 247 | 247 |
| 3, 12 | I, 7 | 625 | 258 | |
| | II | 626 | 252 | |
| | III | 627 | 278 | 263 |
| 3, 1 | I, 8 | 628 | 232 | |
| | II | 629 | 268 | |
| | III | 630 | 206 | 235 |
| 3, 2 | I, 8 | 631 | 221 | |
| | II | 632 | 237 | |
| | III | 633 | 273 | 244 |
| 3, 3 | I, 8 | 634 | 268 | |
| | II | 635 | 237 | |
| | III | 636 | 242 | 249 |
| 3, 4 | I, 8 | 637 | 232 | |
| | II | 638 | 206 | |
| | III | 639 | 237 | 225 |
| 3, 5 | I, 8 | 640 | 237 | |
| | II | 641 | 227 | |
| | III | 642 | 206 | 223 |
| 3, 6 | I, 8 | 643 | 211 | |
| | II | 644 | 227 | |
| | III | 645 | 232 | 223 |
| 3, 7 | I, 8 | 646 | 227 | |
| | II | 647 | 232 | |
| | III | 648 | 221 | 227 |
| 3, 8 | I, 8 | 649 | 227 | |
| | II | 650 | 232 | |
| | III | 651 | 232 | 230 |
| 3, 9 | I, 8 | 652 | 237 | |
| | II | 653 | 232 | |
| | III | 654 | 242 | 237 |
| 3, 10 | I, 8 | 655 | 247 | |
| | II | 656 | 247 | |
| | III | 657 | 263 | 252 |
| 3, 11 | I, 8 | 658 | 268 | |
| | II | 659 | 268 | |
| | III | 660 | 268 | 268 |
| 3, 12 | I, 8 | 661 | 258 | |
| | II | 662 | 273 | |
| | III | 663 | 258 | 263 |
| 3, 1 | I, 9 | 664 | ... | |
| | II | 665 | 237 | |
| | III | 666 | 242 | 240 |
| 3, 2 | I, 9 | 667 | 216 | |
| | II | 668 | 185 | |
| | III | 669 | 232 | 211 |
| 3, 3 | I, 9 | 670 | 221 | |
| | II | 671 | 216 | |
| | III | 672 | 232 | 223 |
| 3, 4 | I, 9 | 673 | 221 | |
| | II | 674 | 227 | |
| | III | 675 | 201 | 216 |
| 3, 5 | I, 9 | 676 | 211 | |
| | II | 677 | 221 | |
| | III | 678 | 211 | 214 |
| 3, 6 | I, 9 | 679 | 211 | |
| | II | 680 | 196 | |
| | III | 681 | 221 | 209 |
| 3, 7 | I, 9 | 682 | 221 | |
| | II | 683 | 227 | |
| | III | 684 | 201 | 216 |
| 3, 8 | I, 9 | 685 | 201 | |
| | II | 686 | 180 | |
| | III | 687 | 227 | 203 |
| 3, 9 | I, 9 | 688 | 252 | |
| | II | 689 | 232 | |
| | III | 690 | 232 | 239 |
| 3, 10 | I, 9 | 691 | 221 | |
| | II | 692 | 237 | |
| | III | 693 | 227 | 228 |
| 3, 11 | I, 9 | 694 | 258 | |
| | II | 695 | 258 | |
| | III | 696 | 221 | 246 |
| 3, 12 | I, 9 | 697 | 227 | |
| | II | 698 | 258 | |
| | III | 699 | 242 | 242 |
| 4, 1 | I, 0 | 700 | 237 | |
| | II | 701 | 242 | |
| | III | 702 | 237 | 239 |
| 4, 2 | I, 0 | 703 | 227 | |
| | II | 704 | 227 | |
| | III | 705 | 232 | 229 |
| 4, 3 | I, 0 | 706 | 232 | |
| | II | 707 | 211 | |
| | III | 708 | 211 | 218 |
| 4, 4 | I, 0 | 709 | 227 | |
| | II | 710 | 211 | |
| | III | 711 | 268 | 235 |
| 4, 5 | I, 0 | 712 | 242 | |
| | II | 713 | 237 | |
| | III | 714 | 268 | 249 |
| 4, 6 | I, 0 | 715 | 247 | |
| | II | 716 | 252 | |
| | III | 717 | 242 | 247 |
| 4, 7 | I, 0 | 718 | 258 | |
| | II | 719 | 252 | |
| | III | 720 | 242 | 251 |
| 4, 8 | I, 0 | 721 | 242 | |
| | II | 722 | 237 | |
| | III | 723 | 242 | 240 |
| 9 | I | 724 | 263 | |
| | II | 725 | 252 | |
| | III | 726 | 242 | 252 |
| 10 | I | 727 | 247 | |
| | II | 728 | 227 | |
| | III | 729 | 252 | 242 |

TABLE 4.—Continued

| | | | | |
|---|---|---|---|---|
| 11 | I | 730 | 252 | |
| | II | 731 | 227 | |
| | III | 732 | 216 | 232 |
| 12 | I | 733 | 242 | |
| | II | 734 | 258 | |
| | III | 735 | 258 | 253 |
| 4, 1 | I, 1 | 736 | 232 | |
| | II | 737 | 268 | |
| | III | 738 | 242 | 247 |
| 2 | I | 739 | 216 | |
| | II | 740 | 247 | |
| | III | 741 | 273 | 245 |
| 3 | I | 742 | 258 | |
| | II | 743 | 232 | |
| | III | 744 | 268 | 253 |
| 4 | I | 745 | 247 | |
| | II | 746 | 242 | |
| | III | 747 | 221 | 237 |
| 5 | I | 748 | 247 | |
| | II | 749 | 216 | |
| | III | 750 | 263 | 242 |
| 6 | I | 751 | 263 | |
| | II | 752 | 216 | |
| | III | 753 | 268 | 249 |
| 7 | I | 754 | 258 | |
| | II | 755 | 268 | |
| | III | 756 | 283 | 270 |
| 8 | I | 757 | 283 | |
| | II | 758 | 252 | |
| | III | 759 | 242 | 259 |
| 9 | I | 760 | 278 | |
| | II | 761 | 263 | |
| | III | 762 | 216 | 252 |
| 10 | I | 763 | 273 | |
| | II | 764 | 258 | |
| | III | 765 | 247 | 259 |
| 11 | I | 766 | 268 | |
| | II | 767 | 247 | |
| | III | 768 | 247 | 254 |
| 12 | I | 769 | 263 | |
| | II | 770 | 258 | |
| | III | 771 | 283 | 268 |
| 4, 1 | I, 2 | 772 | 288 | |
| | II | 773 | 237 | |
| | III | 774 | 247 | 257 |
| 2 | I | 775 | 247 | |
| | II | 776 | 252 | |
| | III | 777 | 247 | 249 |
| 3 | I | 778 | 221 | |
| | II | 779 | 221 | |
| | III | 780 | 216 | 219 |
| 4 | I | 781 | 227 | |
| | II | 782 | 237 | |
| | III | 783 | 232 | 232 |
| 5 | I | 784 | 227 | |
| | II | 785 | 242 | |
| | III | 786 | 257 | 242 |
| 6 | I | 787 | 257 | |
| | II | 788 | 237 | |
| | III | 789 | 247 | 247 |
| 7 | I | 790 | 252 | |
| | II | 791 | 257 | |
| | III | 792 | 232 | 247 |
| 4, 8 | I, 2 | 793 | 232 | |
| | II | 794 | 237 | |
| | III | 795 | 237 | 235 |
| 9 | I | 796 | 247 | |
| | II | 797 | 196 | |
| | III | 798 | 232 | 225 |
| 10 | I | 799 | 232 | |
| | II | 800 | 227 | |
| | III | 801 | 216 | 225 |
| 11 | I | 802 | 237 | |
| | II | 803 | 242 | |
| | III | 804 | 257 | 245 |
| 12 | I | 805 | 232 | |
| | II | 806 | 211 | |
| | III | 807 | 227 | 223 |
| 4, 1 | I, 3 | 808 | 206 | |
| | II | 809 | 206 | |
| | III | 810 | 211 | 208 |
| 2 | I | 811 | 216 | |
| | II | 812 | 242 | |
| | III | 813 | 232 | 230 |
| 3 | I | 814 | 206 | |
| | II | 815 | 247 | |
| | III | 816 | 221 | 225 |
| 4 | I | 817 | 211 | |
| | II | 818 | 227 | |
| | III | 819 | 237 | 225 |
| 5 | I | 820 | 242 | |
| | II | 821 | 232 | |
| | III | 822 | 237 | 237 |
| 6 | I | 823 | 252 | |
| | II | 824 | 263 | |
| | III | 825 | 263 | 259 |
| 7 | I | 826 | 252 | |
| | II | 827 | 221 | |
| | III | 828 | 257 | 243 |
| 8 | I | 829 | 247 | |
| | II | 830 | 242 | |
| | III | 831 | 257 | 249 |
| 9 | I | 832 | 232 | |
| | II | 833 | 247 | |
| | III | 834 | 242 | 240 |
| 10 | I | 835 | 232 | |
| | II | 836 | 247 | |
| | III | 837 | 232 | 237 |
| 11 | I | 838 | 237 | |
| | II | 839 | 247 | |
| | III | 840 | 206 | 230 |
| 12 | I | 841 | 232 | |
| | II | 842 | 227 | |
| | III | 843 | 263 | 241 |
| 4, 1 | I, 4 | 844 | 252 | |
| | II | 845 | 242 | |
| | III | 846 | 227 | 240 |
| 2 | I | 847 | 257 | |
| | II | 848 | 263 | |
| | III | 849 | 252 | 257 |
| 3 | I | 850 | 216 | |
| | II | 851 | 242 | |
| | III | 852 | 227 | 228 |
| 4 | I | 853 | 216 | |
| | II | 854 | 227 | |
| | III | 855 | 227 | 223 |
| 5 | I | 856 | 242 | |
| | II | 857 | 237 | |
| | III | 858 | 227 | 235 |
| 6 | I | 859 | 237 | |
| | II | 860 | 221 | |
| | III | 861 | 227 | 228 |
| 7 | I | 862 | 237 | |
| | II | 863 | 227 | |
| | III | 864 | 237 | 234 |
| 4, 8 | I, 4 | 865 | 263 | |
| | II | 866 | 216 | |
| | III | 867 | 206 | 228 |
| 9 | I | 868 | 206 | |
| | II | 869 | 221 | |
| | III | 870 | 191 | 206 |
| 10 | I | 871 | 232 | |
| | II | 872 | 206 | |
| | III | 873 | 185 | 208 |
| 11 | I | 874 | 247 | |
| | II | 875 | 237 | |
| | III | 876 | 232 | 239 |
| 12 | I | 877 | 237 | |
| | II | 878 | 268 | |
| | III | 879 | 232 | 246 |
| 4, 1 | I, 5 | 880 | 201 | |
| | II | 881 | 211 | |
| | III | 882 | 206 | 206 |
| 2 | I | 883 | 242 | |
| | II | 884 | 247 | |
| | III | 885 | 232 | 240 |
| 3 | I | 886 | 232 | |
| | II | 887 | 232 | |
| | III | 888 | 206 | 223 |
| 4 | I | 889 | 232 | |
| | II | 890 | 253 | |
| | III | 891 | 247 | 244 |
| 5 | I | 892 | 237 | |
| | II | 893 | 237 | |
| | III | 894 | 242 | 239 |
| 6 | I | 895 | 232 | |
| | II | 896 | 227 | |
| | III | 897 | 227 | 229 |
| 7 | I | 898 | 258 | |
| | II | 899 | 242 | |
| | III | 900 | 242 | 247 |
| 8 | I | 901 | 247 | |
| | II | 902 | 232 | |
| | III | 903 | 221 | 233 |
| 9 | I | 904 | 232 | |
| | II | 905 | 216 | |
| | III | 906 | 211 | 220 |
| 10 | I | 907 | 232 | |
| | II | 908 | 221 | |
| | III | 909 | 206 | 220 |
| 11 | I | 910 | 258 | |
| | II | 911 | 242 | |
| | III | 912 | 252 | 251 |
| 12 | I | 913 | 221 | |
| | II | 914 | 216 | |
| | III | 915 | 216 | 218 |
| 4, 1 | I, 6 | 916 | 221 | |
| | II | 917 | 227 | |
| | III | 918 | 258 | 235 |
| 2 | I | 919 | 211 | |
| | II | 920 | 201 | |
| | III | 921 | 201 | 204 |
| 3 | I | 922 | 191 | |
| | II | 923 | 185 | |
| | III | 924 | 206 | 194 |
| 4 | I | 925 | 252 | |
| | II | 926 | 242 | |
| | III | 927 | 211 | 235 |
| 5 | I | 928 | 252 | |
| | II | 929 | 252 | |
| | III | 930 | 227 | 244 |
| 6 | I | 931 | 258 | |
| | II | 932 | 247 | |
| | III | 933 | 258 | 254 |
| 7 | I | 934 | 237 | |
| | II | 935 | 258 | |
| | III | 936 | 221 | 239 |
| 4, 8 | I, 6 | 937 | 221 | |
| | II | 938 | 216 | |
| | III | 939 | 252 | 230 |
| 9 | I | 940 | 252 | |
| | II | 941 | 232 | |
| | III | 942 | 221 | 235 |
| 10 | I | 943 | 216 | |
| | II | 944 | 232 | |
| | III | 945 | 237 | 228 |
| 11 | I | 946 | 247 | |
| | II | 947 | 258 | |
| | III | 948 | 263 | 256 |
| 12 | I | 949 | 304(?) | |
| | II | 950 | 273(?) | |
| | III | 951 | 221(?) | 266 |
| 4, 1 | I, 7 | 952 | 278 | |
| | II | 953 | 258 | |
| | III | 954 | 258 | 265 |
| 2 | I | 955 | 216(?) | |
| | II | 956 | 237 | |
| | III | 957 | 242 | 232 |
| 3 | I | 958 | 185 | |
| | II | 959 | 206 | |
| | III | 960 | 216 | 202 |
| 4 | I | 961 | 191 | |
| | II | 962 | 227 | |
| | III | 963 | 232 | 217 |
| 5 | I | 964 | 232 | |
| | II | 965 | 232 | |
| | III | 966 | 216 | 227 |
| 6 | I | 967 | 211 | |
| | II | 968 | 247 | |
| | III | 969 | 242 | 233 |
| 7 | I | 970 | 237 | |
| | II | 971 | 232 | |
| | III | 972 | 211 | 227 |

Table 4.—Concluded

| | | | | |
|---|---|---|---|---|
| 8 | I | 973 | 216 | |
| | II | 979 | 221 | |
| | III | 975 | 252 | 230 |
| 9 | I | 976 | 221 | |
| | II | 977 | 237 | |
| | III | 978 | 221 | 226 |
| 10 | I | 979 | 227 | |
| | II | 980 | 237 | |
| | III | 981 | 263 | 242 |
| 11 | I | 982 | 273 | |
| | II | 983 | 242 | |
| | III | 984 | 247 | 254 |
| 12 | I | 985 | 258 | |
| | II | 986 | 237 | |
| | III | 987 | 263 | 253 |
| 4, 1 | I, 8 | 988 | 278 | |
| | II | 989 | 247 | |
| | III | 990 | 263 | 263 |
| 2 | I | 991 | 273 | |
| | II | 992 | 268 | |
| | III | 993 | 258 | 266 |
| 3 | I | 994 | 278 | |
| | II | 995 | 247 | |
| | III | 996 | 247 | 257 |
| 4 | I | 997 | 293(?) | |
| | II | 998 | 252 | |
| | III | 999 | 258 | 268 |
| 5 | I | 1000 | 258 | |
| | II | 1001 | 252 | |
| | III | 1002 | 263 | 258 |
| 6 | I | 1003 | 283 | |
| | II | 1004 | 273 | |
| | III | 1005 | 273 | 276 |
| 7 | I | 1006 | 299 | |
| | II | 1007 | 288 | |
| | III | 1008 | 283 | 290 |
| 4, 8 | I, 8 | 1009 | 283 | |
| | II | 1010 | 278 | |
| | III | 1011 | 237 | 266 |
| 9 | I | 1012 | 252 | |
| | II | 1013 | 278 | |
| | III | 1014 | 278 | 269 |
| 10 | I | 1015 | 258 | |
| | II | 1016 | 258 | |
| | III | 1017 | 253 | 356 |
| 11 | I | 1018 | 278 | |
| | II | 1019 | 268 | |
| | III | 1020 | 283 | 276 |
| 12 | I | 1021 | 278 | |
| | II | 1022 | 283 | |
| | III | 1023 | 278 | 280 |
| 4, 1 | I, 9 | 1024 | 278 | |
| | II | 1025 | 247 | |
| | III | 1026 | 273 | 266 |
| 2 | I | 1027 | 304 | |
| | II | 1028 | 294 | |
| | III | 1029 | 232 | 278 |
| 3 | I | 1030 | 242 | |
| | II | 1031 | 252 | |
| | III | 1032 | 206 | 233 |
| 4 | I | 1033 | 221 | |
| | II | 1034 | 206 | |
| | III | 1035 | 258 | 228 |
| 5 | I | 1036 | 242 | |
| | II | 1037 | 273 | |
| | III | 1038 | 242 | 252 |
| 6 | I | 1039 | 232 | |
| | II | 1040 | 191 | |
| | III | 1041 | 242 | 222 |
| 7 | I | 1042 | 237 | |
| | II | 1043 | 242 | |
| | III | 1044 | 247 | 242 |
| 8 | I | 1045 | 242 | |
| | II | 1046 | 263 | |
| | III | 1047 | 221 | 242 |
| 9 | I | 1048 | 227 | |
| | II | 1049 | 237 | |
| | III | 1050 | 206 | 223 |
| 10 | I | 1051 | 232 | |
| | II | 1052 | 221 | |
| | III | 1053 | 263 | 239 |
| 11 | I | 1054 | 268 | |
| | II | 1055 | 247 | |
| | III | 1056 | 237 | 251 |
| 12 | I | 1057 | 263 | |
| | II | 1058 | 242 | |
| | III | 1059 | 288 | 264 |
| 5, 1 | I, 0 | 1060 | 288 | |
| | II | 1061 | 268 | |
| | III | 1062 | 268 | 275 |
| 2 | I | 1063 | 227 | |
| | II | 1064 | 237 | |
| | III | 1065 | 247 | 237 |
| 3 | I | 1066 | 227 | |
| | II | 1067 | 227 | |
| | III | 1068 | 211 | 222 |
| 4 | I | 1069 | 221 | |
| | II | 1070 | 211 | |
| | III | 1071 | 206 | 213 |
| 5 | I | 1072 | 232 | |
| | II | 1073 | 206 | |
| | III | 1074 | 247 | 228 |
| 6 | I | 1075 | 242 | |
| | II | 1076 | 232 | |
| | III | 1077 | 247 | 240 |
| 7 | I | 1078 | 211 | |
| | II | 1079 | 242 | |
| | III | 1080 | 232 | 228 |
| 5, 8 | I, 0 | 1081 | 253 | |
| | II | 1082 | 253 | |
| | III | 1083 | 216 | 241 |
| 9 | I | 1084 | 206 | |
| | II | 1085 | 237 | |
| | III | 1086 | 237 | 227 |
| 10 | I | 1087 | 243 | |
| | II | 1088 | 249 | |
| | III | 1089 | 283 | 258 |
| 11 | I | 1090 | 263 | |
| | II | 1091 | 227 | |
| | III | 1092 | 252 | 247 |
| 12 | I | 1093 | 232 | |
| | II | 1094 | 247 | |
| | III | 1095 | 227 | 235 |

1.9 to 2.0 calories, or even more. Still, by forming these less-accurate solar-constant values into large groups of days, according to magnitude, H. H. Clayton was able to correlate solar changes with weather elements.[9]

It now occurs to me that since the periodicities now discovered in the solar emission have been expressed as to form and amplitude, and since 1920 seem to be permanent as far as known in period, amplitude, and form, it may be worth while to synthesize monthly mean solar variation *backward* from 1920. This done, it would be possible to compare the values synthesized with monthly mean solar-constant values observed on Mount Wilson. If, on the whole, high, medium, and low solar constants as synthesized correspond to high, medium, and low Mount Wilson values, it will be a confirmatory evidence of the sun's real variability, of the constancy of periodicities, of their comprising nearly the total solar variation, and of the value of Clayton's work on the correlation of solar variation with weather.

Table 5 gives the synthesized monthly solar-constant values from

[9] Smithsonian Misc. Coll., vol. 68, No. 3, 1917.

August 1908 to December 1920. These results are given graphically in figure 5,C. These are actual estimated solar constants in calories per square centimeter per minute, not, as in table 4, percentage departures from 1.90 calories.

## COMPARISON OF SYNTHETIC WITH MOUNT WILSON SOLAR-CONSTANT VALUES

From table 53, page 193, volume 4, Annals of the Astrophysical Observatory, I take monthly solar-constant values determined from Mount Wilson observations in the months May to November, 1908 to 1920. I omit four values, July and August 1912, because the sky was then very much fouled by dust from the volcano, Mount Katmai.[10] I also omit July values of 1910 and 1917 because they are very wild indeed, far beyond the limits of dispersal of the others.

Having plotted the Mount Wilson values and such parts of the synthetic series as corresponded in time with them, I saw that there was a gradual rise in values in both observed and synthetic series from 1908 to 1914. I drew straight lines best following this trend to represent the means of the values over that interval, and read off the departures of the individual solar-constant values on the plot from these lines. For the rest of the total interval, that is 1915 to 1920, I read departures from straight horizontal lines drawn in the mean of ordinates. The plot was in arbitrary units, with the units for ordinates in the synthetic plot twice as large as those for the Mount Wilson data. These departure values follow in table 6.

Taking the sums of the data in the columns of table 6 they yield: Mount Wilson $\div$ synthetic $= \frac{503}{284} = 1.77$. Recalling the ratio of units, 2 to 1, it appears that the dispersal of Mount Wilson data is 3.54 times as great as that of the synthetic data. The synthetic curve 1920-1950, however, as plotted in figure 4, shows practically the same range of variation as does the curve of original modern observations. Hence it appears that the Mount Wilson solar-constant observations of 1908 to 1920 are probably $3\frac{1}{2}$ times less accurate than the modern work set forth in table 4.

Taking account of the numbers of departures of the same sign in the columns of table 6, and the numbers of them of opposite signs, the sums are 28 and 21.

Taking the sums of departures that are of the same sign in both columns, the results are 324 for Mount Wilson and 170 for the syn-

[10] See Smithsonian Misc. Coll., vol. 60, No. 29, 1913.

Table 5.—*Synthesized solar constant, 1908-1920*

Values to be prefixed by 1.9

| | | |
|---|---|---|
| 1908 | Aug. | 49 |
| | Sept. | 49 |
| | Oct. | 48 |
| | Nov. | 46 |
| | Dec. | 45 |
| 1909 | Jan. | 45 |
| | Feb. | 44 |
| | Mar. | 43 |
| | Apr. | 40 |
| | May | 39 |
| | June | 39 |
| | July | 42 |
| | Aug. | 43 |
| | Sept. | 42 |
| | Oct. | 45 |
| | Nov. | 42 |
| | Dec. | 40 |
| 1910 | Jan. | 40 |
| | Feb. | 41 |
| | Mar. | 43 |
| | Apr. | 49 |
| | May | 47 |
| | June | 47 |
| | July | 46 |
| | Aug. | 47 |
| | Sept. | 46 |
| | Oct. | 46 |
| | Nov. | 44 |
| | Dec. | 42 |
| 1911 | Jan. | 45 |
| | Feb. | 45 |
| | Mar. | 46 |
| | Apr. | 48 |
| | May | 52 |
| | June | 48 |
| | July | 47 |
| | Aug. | 46 |
| | Sept. | 45 |
| | Oct. | 44 |
| | Nov. | 40 |
| | Dec. | 41 |
| 1912 | Jan. | 45 |
| | Feb. | 46 |
| | Mar. | 48 |
| | Apr. | 45 |
| | May | 46 |
| | June | 45 |
| | July | 43 |
| | Aug. | 44 |
| | Sept. | 42 |
| | Oct. | 44 |
| | Nov. | 47 |
| | Dec. | 46 |
| 1913 | Jan. | 45 |
| | Feb. | 47 |
| | Mar. | 46 |
| | Apr. | 48 |
| | May | 45 |
| | June | 46 |
| | July | 47 |
| | Aug. | 49 |
| | Sept. | 48 |
| | Oct. | 46 |
| | Nov. | 45 |
| | Dec. | 43 |
| 1914 | Jan. | 46 |
| | Feb. | 48 |
| | Mar. | 48 |
| | Apr. | 52 |
| | May | 51 |
| | June | 44 |
| | July | 40 |
| | Aug. | 41 |
| | Sept. | 41 |
| | Oct. | 41 |
| | Nov. | 41 |
| | Dec. | 43 |
| 1915 | Jan. | 45 |
| | Feb. | 50 |
| | Mar. | 48 |
| | Apr. | 51 |
| | May | 48 |
| | June | 45 |
| | July | 42 |
| | Aug. | 38 |
| | Sept. | 40 |
| | Oct. | 42 |
| | Nov. | 43 |
| | Dec. | 45 |
| 1916 | Jan. | 43 |
| | Feb. | 51 |
| | Mar. | 53 |
| | Apr. | 52 |
| | May | 47 |
| | June | 42 |
| | July | 40 |
| | Aug. | 36 |
| | Sept. | 39 |
| | Oct. | 43 |
| | Nov. | 42 |
| | Dec. | 44 |
| 1917 | Jan. | 43 |
| | Feb. | 44 |
| | Mar. | 47 |
| | Apr. | 46 |
| | May | 44 |
| | June | 44 |
| | July | 42 |
| | Aug. | 44 |
| | Sept. | 43 |
| | Oct. | 46 |
| | Nov. | 50 |
| | Dec. | 48 |
| 1918 | Jan. | 47 |
| | Feb. | 46 |
| | Mar. | 43 |
| | Apr. | 44 |
| | May | 43 |
| | June | 46 |
| | July | 45 |
| | Aug. | 46 |
| | Sept. | 47 |
| | Oct. | 48 |
| | Nov. | 50 |
| | Dec. | 51 |
| 1919 | Jan. | 52 |
| | Feb. | 49 |
| | Mar. | 46 |
| | Apr. | 47 |
| | May | 48 |
| | June | 46 |
| | July | 44 |
| | Aug. | 44 |
| | Sept. | 48 |
| | Oct. | 47 |
| | Nov. | 44 |
| | Dec. | 41 |
| 1920 | Jan. | 43 |
| | Feb. | 46 |
| | Mar. | 45 |
| | Apr. | 42 |
| | May | 44 |
| | June | 43 |
| | July | 42 |
| | Aug. | 41 |
| | Sept. | 42 |
| | Oct. | 48 |
| | Nov. | 48 |
| | Dec. | 46 |

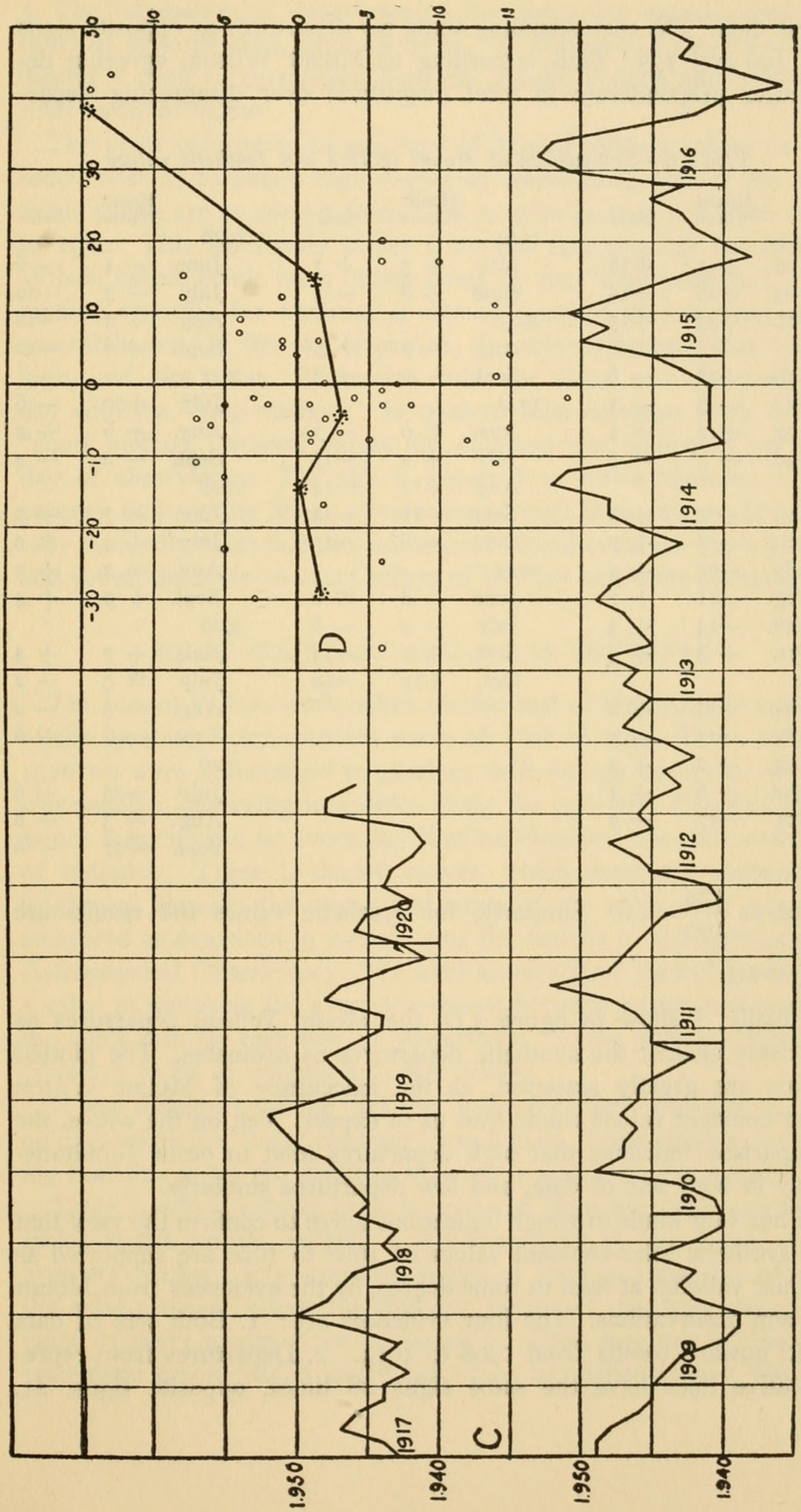


FIG. 5.—C, Solar-constant values in calories, as synthesized from 23 periodicities, 1908 to 1920. D, A comparison of mean monthly solar-constant departures from the mean, Mount Wilson (abscissae) vs. synthetic (ordinates).

thetic data. The corresponding sums for departures of opposite signs are 199 and 135. Thus, according to Mount Wilson, agreeing departures preponderate in total magnitude over disagreeing depar-

Table 6.—*Comparison of Mount Wilson and synthetic values*

| | Mount Wilson | Syn. | | Mount Wilson | Syn. | | Mount Wilson | Syn. |
|---|---|---|---|---|---|---|---|---|
| *1908* | | | *1912* | | | *1916* | | |
| Aug. | +45 | +16 | May | + 5 | + 1 | June | − 1 | − 6 |
| Sept. | +28 | +15 | June | − 8 | − 1 | July | − 3 | −10 |
| Oct. | +43 | +13 | *1913* | | | Aug. | + 2 | −18 |
| *1909* | | | Aug. | − 7 | + 5 | Sept. | − 8 | −12 |
| June | +17 | − 6 | Sept. | −30 | + 3 | *1917* | | |
| July | − 3 | − 1 | *1914* | | | July | +20 | − 6 |
| Aug | +12 | + 1 | June | ± 0 | − 7 | Aug. | + 6 | − 2 |
| Sept. | − 7 | − 1 | July | + 4 | −15 | Sept. | − 2 | − 4 |
| *1910* | | | Aug. | +11 | −14 | *1918* | | |
| May | +12 | + 8 | Sept. | −11 | −14 | June | − 7 | + 2 |
| June | − 5 | + 7 | Oct. | − 6 | −15 | July | + 4 | ± 0 |
| July | −24 | + 5 | *1915* | | | Aug. | − 3 | + 2 |
| Aug. | −11 | + 7 | June | − 8 | ± 0 | Sept. | + 9 | + 4 |
| Sept. | −13 | + 5 | July | − 3 | − 6 | *1919* | | |
| Oct. | + 3 | + 5 | Sept. | + 1 | −14 | June | + 7 | + 4 |
| *1911* | | | Oct. | +17 | −10 | July | ± 0 | − 2 |
| June | +15 | + 8 | | | | Aug. | − 5 | − 2 |
| July | −13 | + 5 | | | | Sept. | − 6 | + 6 |
| Aug. | − 2 | + 3 | | | | *1920* | | |
| Sept. | + 6 | + 1 | | | | July | −25 | − 6 |
| Oct. | −17 | − 2 | | | | Aug. | − 3 | − 8 |
| | | | | | | Sept. | −37 | − 6 |

tures as $\frac{324}{199} = 1.6$. Similarly, for synthetic values the results are $\frac{170}{135} = 1.3$.

Finally, I show in figure 5,D, the Mount Wilson departures as abscissae against the synthetic departures as ordinates. The plotted points are greatly scattered, as the inaccuracy of Mount Wilson solar-constant values would lead us to expect. Yet, on the whole, the comparison indicates that high departures tend to occur simultaneously in both sets of data, and low departures similarly.

Thus four kinds of rough indications agree to confirm the view that the synthetic solar-constant values of 1908 to 1920 are supported as to their validity, at least in some degree, by the evidences from Mount Wilson observations. The four evidences are: 1. Both sets of data yield upward trends from 1908 to 1914. 2. Departures from representative lines have the same signs 28 times, opposite signs, 21.

3. The summation of departures of the same sign exceeds that for those of opposite sign about 1½ times. 4. The plot of departures indicates a positive correlation between Mount Wilson and synthetic solar-constant values.

The great inferiority in accuracy of Mount Wilson values of the solar constant forbids a high degree of correlation, even if the synthetic values are as correct from 1908 to 1920 as they are from 1920 to 1950. This inferiority arises from the fact that all the Mount Wilson values result from observations by the "long method." That method requires for accuracy a sky of constant transparency over several hours. If the sky improves, the solar-constant value is too high, and vice versa. Moreover, only one value was obtained per day with the "long method." In modern solar-constant work by the "short method," several values are obtained and combined on each day of observation. The sky is required to retain uniform transparency only during about 10 minutes of each observation. It might vary decidedly from one determination to another of the day's group, and yet all the solar-constant values of the day be closely agreeing.

## SOLAR CONSTANT AND SOLAR CONTRAST

The Mount Wilson work offers another test of the probable validity of the synthetic solar-constant curve of 1908 to 1920. From 1913 to 1920 we were accustomed to produce drift energy curves in several wavelengths, observing intensities along the east-west diameter of an 8-inch solar image, on every day that we observed the solar constant of radiation. These U-shaped curves, which show the contrast in brightness between the center and edges of the sun's disk, were all measured as described in volume 4 of the Annals of the Smithsonian Astrophysical Observatory. We used an empirical formula to obtain a value to represent the average contrast between center and edge of the sun's disk on each day of observation. These data are given in tables 75 to 82 of volume 4 of the Annals.

It was thought probable that the "solar contrast" would be greater on days when the "solar constant" was higher. Some figures, indicating that this is so, are given in volumes 3 and 4 of the Annals.

Table 7, which follows here, is prepared from the "solar contrast" tables of the Annals, volume 4, and from table 6, just given, which presents synthetic solar-constant values of 1908 to 1920. To prepare the solar-contrast values for this use, means of the daily values are taken of every month given in Annals 4. Then, in order to eliminate systematic errors which might introduce inconsistencies, a separate mean value is computed for the available months of each year, 1913 to 1920. Differences from these yearly means are given in column 2 of table 7. To make the synthetic solar-constant values entirely com-

TABLE 7.—*Comparison of synthetic solar-constant departures with solar-contrast values of 1913-1920*

Solar-constant departures in thousandths of a calorie.

| Solar constant | Solar contrast |
|---|---|
| +17 | +19 |
| − 3 | −32 |
| −13 | +14 |
| +36 | −35 |
| + 6 | −24 |
| −34 | −18 |
| −14 | +28 |
| + 6 | +49 |
| +20 | +10 |
| 0 | −29 |
| −40 | −75 |
| −10 | 0 |
| +30 | + 4 |
| −17 | −18 |
| + 3 | +15 |
| − 7 | +14 |
| −14 | −23 |
| − 4 | −12 |
| + 6 | + 8 |
| +16 | +16 |
| + 5 | − 8 |
| −15 | −13 |
| −15 | +13 |
| +25 | +40 |
| + 3 | +46 |
| − 7 | +18 |
| + 3 | −70 |
| +23 | −13 |
| − 4 | + 9 |

parable to these contrast values, separate means of them are taken for each year of the comparison, including only the months used in obtaining the separate contrast means. Differences from these synthetic solar-constant means, expressed in thousandths of a calorie, form column 1 of table 7.

Counting the numbers of months when values in columns 1 and 2 have the same sign and opposite signs, the numbers (counting zero values into each group) are 18 and 13, respectively. So here is another straw pointing to the reliability of the synthetic solar-constant values. But more convincing, and more informing, is figure 6. Here the

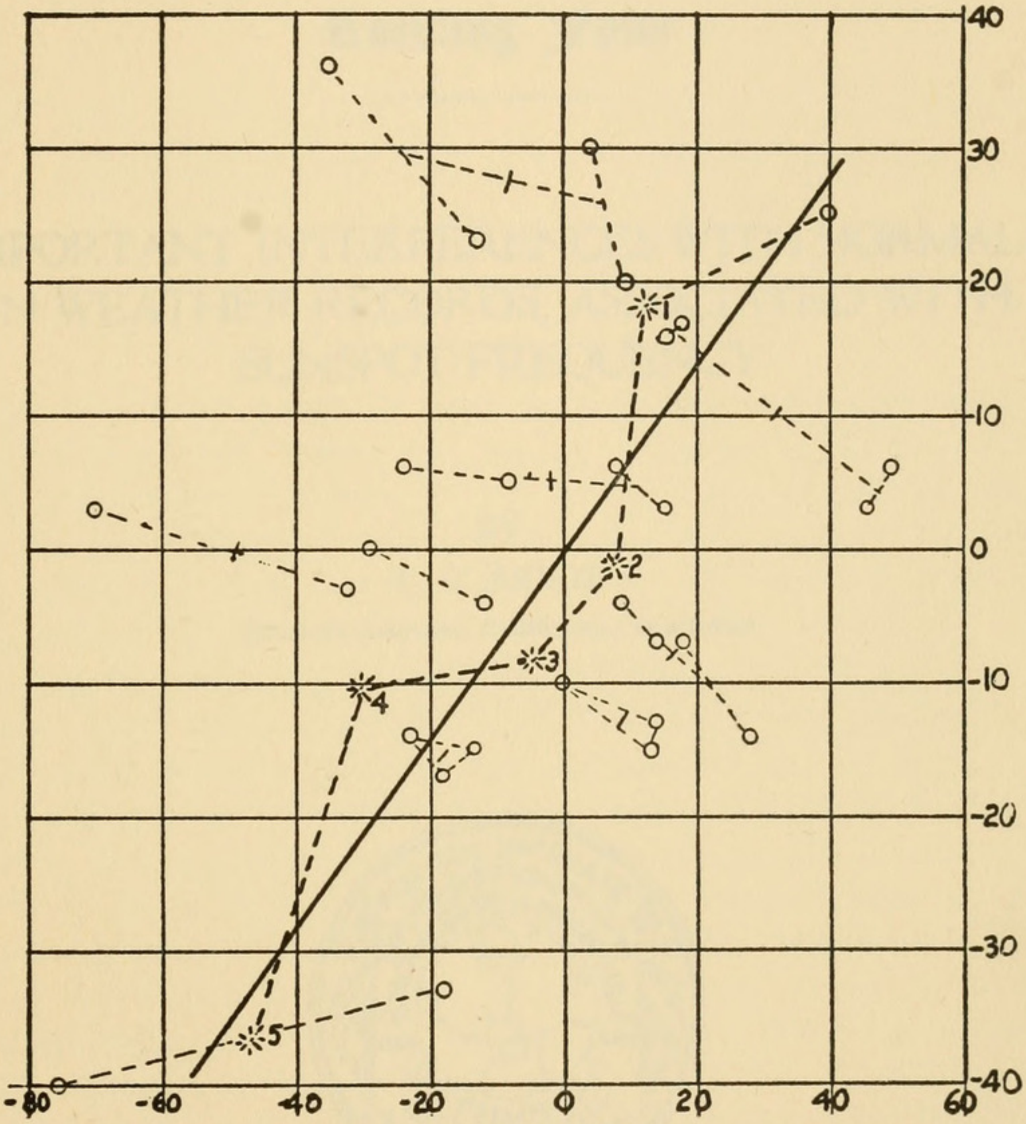


FIG. 6.—Mount Wilson solar contrast (abscissae) vs. synthetic solar constants (ordinates).

values in the columns of table 7 are plotted against each other, solar constants as ordinates, solar contrasts as abscissae. In order to bring out plainly the fact that higher contrast values attend higher synthetic solar-constant values, stars 1, 2, 3, 4, 5, have been plotted to give the centers of gravity of groups of 8, 8, 5, 5, and 2 months, respectively. A full heavy line has been drawn to show the trend of the results.