Brahe's 'Mechanica,' which he immediately began to print at Hamburg, whilst he remained at Ranzovius's, and that there may not have been time to have it engraved; or, may be, it was too self-glorious.

The residence of Ranzovius, half a mile from Hamburg, existed till ten or fifteen years ago, when it was pulled down, and a full-length portrait of Brahe disappeared, representing him with an astronomical instrument in one hand.

Dr. C. exhibited a photograph from his picture, taken by an able photographer, Mr. White, Crescent, Salford.

Note.—In the description of Brahe's dress there is a serious omission, viz. that he is represented in a cloak with a high collar. Further particulars respecting this picture will be found in 'Nature,' October 11, 1877.

XII. *On the Types of Compound Statement involving four Classes.* By Professor W. K. Clifford, M.A., F.R.S. Communicated by Professor W. S. Jevons, M.A., F.R.S.

Read January 9th, 1877.

Professor Stanley Jevons has enumerated* the types of compound statement involving three classes, among which the premises of a syllogism appear as a type of fourfold statement. He propounded at the same time the corresponding problem of enumeration for four classes, which is solved in the present communication. The reader is referred to the paper or the book just mentioned for further

* 'Proceedings of the Manchester Philosophical Society' vol. vi. pp. 65–68, and 'Memoirs,' Third Series, vol. v. pp. 119–130. 'The Principles of Science,' vol. i. pp. 154-164.

explanation of the nature and purpose of the problem than is to be found in art. 1. It may, however, be premised that the letters A, B, C, D denote four *classes* or *terms* (for example, hard, wet, black, nice), and that, according to a convenient notation of De Morgan's, the small letters *a, b, c, d* denote the complementary classes or contrary terms (not hard, not wet, not black, not nice). A *simple* statement is of the form $ABCD=0$ (no hard, wet, black, nice things exist, or, which is the same thing, all hard, wet, black things are nasty). The statement $ABC=0$ (no hard, wet, black things exist, or all hard, black things are dry) is to be regarded as made of these two, $ABCD=0$, $ABCd=0$ (no hard, wet, black, nice things exist, and no hard, wet, black, nasty things exist) and so is called a *compound* (in this case a *twofold*) statement. The notion of *types* is defined in art. 1.

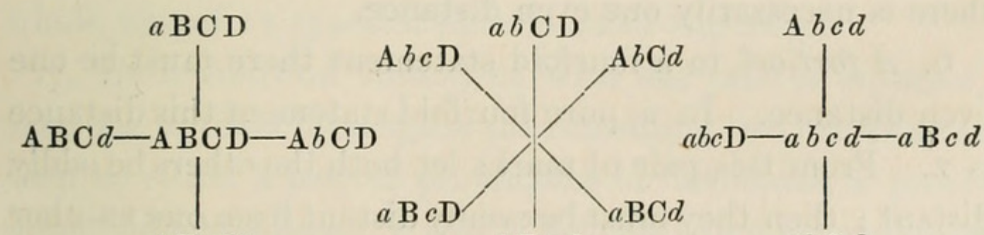
1. Four classes or terms A, B, C, D, give rise to sixteen cross divisions or *marks*, such as $AbCd$. A denial of the existence of one of these cross divisions, or of any thing having its mark (such as $AbCd=0$), is called a simple statement. A denial of two or more cross divisions is called a compound statement, and, moreover, twofold, threefold, etc., according to the number denied.

When two compound statements can be converted into one another by interchange of the classes A, B, C, D with each other or with their complementary classes *a, b, c, d*, they are called *similar*; and all similar statements are said to belong to the same *type*. The problem before us is to enumerate all the types of compound statement that can be made with four terms.

2. Two statements are called *complementary* when they deny between them all the sixteen marks without both denying any mark, or, which is the same thing, when each denies just those marks which the other permits to exist. It is obvious that when two statements are similar, the

complementary statements will also be similar; and, consequently, for every type of n-fold statement there is a *complementary type* of $\overline{16-n}$-fold statement. It follows that we need only enumerate the types as far as the eighth order; for the types of more than eightfold statement will already have been given as complementary to types of lower orders. Every eightfold statement is complementary to an eightfold statement; but these are not necessarily of the same type.

3. One mark ABCD may be converted into another A*b*C*d* by interchanging one or more of the classes A, B, C, D with its complementary class. The number of such changes is called the *distance* of the two marks. Thus in the example given the distance is 2. In two similar compound statements the distances of the marks denied must be same; but it does not follow that when all the distances are the same the two statements are similar. There is, however, as we shall see, only one example of two dissimilar statements having the same distances. When the distance is 4, the two marks are said to be *obverse* to one another, and the statements denying them are called *obverse* statements—as ABCD, *abcd*, or, again, A*b*C*d*, *a*B*c*D. When any one mark is given (called the *origin*), all the others may be grouped in respect of their relations to it as follows:—Four are at distance *one* from it, and may be called *proximates*; six at distance *two*, and may be called *mediates*; four at distance *three*, and may be called *ultimates*. Finally, the obverse is at distance *four*.



Origin and 4 proximates. 6 mediates. Obverse and 4 ultimates.

It will be seen from the above table that the four proximates are respectively obverse to the four ultimates, and that the mediates form three pairs of obverses. Every proximate or ultimate is distant 1 and 3 respectively from such a pair of mediates. Thus each proximate or ultimate divides the mediates into two classes; three of them are at distance 1 from it, and three at distance 3. Two mediates which are not obverse are at distance 2. Two proximates or two ultimates, or an ultimate and a proximate which are not obverse, are also at distance 2.

This view of the mutual relations of the marks is the basis of the following enumeration of types.

4. There is clearly only one type of simple statement. But of twofold statements there are four types; viz. the distance may be 1, 2, 3, or 4; and so, in general, with n classes there are n types of twofold statement.

5. A compound statement containing no pair of obverses is called *pure*. In a threefold statement there are three distances; one of these must be not less than either of the others. If this be 2, the remaining mark must be at odd distance from both of these or at even distance from both; thus we get the types 1, 1, 2, and 2, 2, 2. If the not-less distance be 3, the remaining distances must be one even and the other odd; the even distance must be 2, the odd one either 1 or 3; and the types are 1, 2, 3; 2, 3, 3. Thus there are 4 pure threefold types. With a pair of obverses, the remaining mark must be at odd or even distance from them; 1, 3, 4; 2, 2, 4. In all six threefold types observe that there is necessarily one even distance.

6. *A fortiori*, in a fourfold statement there must be one even distance. In a pure fourfold statement this distance is 2. From this pair of marks let both the others be oddly distant; then they must be evenly distant from one another *i. e.* at distance 2, obverses being excluded. The odd

distances are 1 or 3; and it will be easily seen that the following are all the possible cases :—

$$\begin{array}{c|c}1&1\\\hline 1&1\end{array}\quad\begin{array}{c|c}1&1\\\hline 1&3\end{array}\quad\begin{array}{c|c}1&1\\\hline 3&3\end{array}\quad\begin{array}{c|c}1&3\\\hline 3&1\end{array}\quad\begin{array}{c|c}1&3\\\hline 3&3\end{array}\quad\begin{array}{c|c}3&3\\\hline 3&3\end{array}$$

In these figures the dots indicate the four marks, the cross lines indicate distance 2, and the other figures the distances between the marks on either side of them. Next, from the pairs of marks at distance 2 let one of the others at least be evenly distant, *i. e.* at distance 2. Then we have three marks which are all at distance 2 from one another; and it is easy to show that they are all proximates of a certain other mark. For, select one of them as origin; then the other two are mediates which are not obverse, and which consequently are at distance 1 from some one proximate. With this proximate as origin, therefore, all three are proximates. We have therefore only to inquire what different relations the fourth mark can bear to these three. It may be the origin, its obverse, the remaining proximate, its obverse, or one of two kinds of mediates, viz. at distance 1 or 3 from the remaining proximate. Thus we have 6 types, in which the distances of the fourth mark from the triad are respectively 1 1 1, 3 3 3, 2 2 2, 2 2 2, 1 3 3, 1 1 3. The third and fourth of these are especially interesting, as being distinct types with the same set of distances; I call them *proper* and *improper groups* respectively: viz., a proper group is the four proximates of any origin; an improper group is three proximates with the obverse of the fourth. On the whole we get 12 types of pure fourfold statement

7. In a fourfold statement with *one* pair of obverses, take one of them for origin; the remaining two marks must then be either a pair of proximates or ultimates, a proximate and an ultimate, a pair of mediates, or a proximate or ultimate with one of two kinds of mediate—in all, 5 types,

with the distances $1\,3^2$, $1\,3$; $1\,3^2$, $3\,1$; $2\,2^2$, $2\,2$; $1\,3^1$, $2\,2$; $1\,3^3$, $2\,2$. With *two* pairs of obverses they must be either at odd or even distances from one another; two types. Altogether $12+5+2=19$ fourfold types.

8. In a *pure* fivefold statement there is always a triad of marks at distance 2 from one another. For there is a pair evenly distant; if there is not another mark evenly distant from these, the remaining three are all oddly distant, and therefore evenly distant from one another. First, then, let the remaining two marks be both oddly distant from the triad. In regard to the origin of which these are proximates, the two to be added must be either two mediates, like (of two kinds) or unlike, or a mediate of either kind with the origin or the obverse; 7 types. Next, if one of the two marks be evenly distant from the triad, it must form with the triad either a proper or an improper group of four. To a proper group we may add the origin, the obverse, or a mediate; to an improper group, the origin or the obverse (the mediates give no new type), 5 types; or, in all, 12 pure fivefold types.

9. In a fivefold statement with one pair of obverses there must be another pair of marks at distance 2. We have therefore to add one mark to each of the following three types of fourfold statement,—a pair of obverses together with (1) two proximates, (2) a proximate and an ultimate, (3) two mediates. To the first we may add another proximate, an ultimate or a mediate of three kinds, viz. at distances 1 1, 1 3, 3 3 from the two proximates; 5 types. To the second if we add a proximate or an ultimate, we fall back on one of the previous cases; but there are again three kinds of mediates, at distances 1 1, 3 3, 1 3 from the proximate and ultimate; 3 types. To the third we may add another mediate, whereby the type becomes a proper group together with the obverse of one of its marks, which is the same thing as an improper group together with the

obverse of one of its marks—or a proximate or ultimate which are of three kinds, at distances 1 1, 1 3, 3 3 from the two mediates; 4 types. Thus there are 12 fivefold types with one pair of obverses. With two pairs of obverses at odd distances, there is only one type, all the remaining marks being similarly related to them; at even distance the remaining mark may be evenly or oddly distant from them; 2 types. On the whole we have $12+12+3=27$ types of fivefold statement.

It is to be remarked that there is no pure fivefold statement in which all the distances are even, and that, if there is only one pair of obverses with all the distances even, the type is a proper group together with the obverse of one of its marks.

10. We may now prove, as a consequence of the last remark, that a pure sixfold statement either contains a group of four with a pair oddly distant from it or consists of two triads oddly distant from one another. For there must be a pair at distance 2: if the other four are all oddly distant from these, they form a group; if one is evenly distant and three oddly distant, we have the case of the two triads; if two are evenly distant, we again have a group. We must add, then, first to a proper group, and then to an improper group, a pair oddly distant from it. To a proper group consisting of the proximates to a certain origin we may add the origin or its obverse with a mediate, or two mediates; 3 types. An improper group is symmetrical; that is to say, if we substitute for any one of its marks the obverse of that mark, we shall obtain a proper group. In this way we shall get four origins distant 1 1 1 3 from the group, and four obverses distant 1 3 3 3; if we add to these the obverses of the marks in the group itself, we have described the relation of the twelve remaining marks to the group. To form, therefore, a pure sixfold statement we may add either two origins or two obverses or an origin and an obverse; 3 types.

In the case of the two triads, since they are oddly distant from one another their origins must be oddly distant; that is, they must be distant either 1 or 3. If they are distant 1, neither, both, or one of the origins may appear in the statement; if they are distant 3, neither, both, or one of the obverses: 6 types. Thus we obtain 12 types of purely sixfold statement.

11. If a sixfold statement contains one pair of obverses, the remaining four marks cannot all be evenly distant from this pair. For in that case they would constitute a group; and it is easy to see that the marks evenly distant from a group, whether proper or improper, do not contain a pair of obverses. We have therefore only these four cases to consider:—

(1) The four marks are all oddly distant from the obverses.
(2) One is evenly distant and three oddly distant.
(3) Two are evenly distant and two oddly.
(4) Three are evenly distant and one oddly.

In the first case the four marks form a group. If this is a proper group, the pair of obverses must be either the origin and obverse of the group, or a pair of mediates; 2 types. If the group is improper, the pair must be an origin and an obverse; 1 type. In the second case we have an origin, an obverse, and a mediate, to which we must add 3 marks taken out of the proximates and ultimates. We may add 3 proximates distant respectively 1 1 3 or 1 3 3 from the mediates (2 types),—or 2 proximates distant respectively 1 1, 1 3, 3 3 from the mediate, and with each of these combinations an ultimate distant either 1 or 3 (6 types). To interchange proximates with ultimates clearly makes no difference; so that in reckoning the cases of 1 proximate and 2 ultimates or 3 ultimates, we should find no new types. In the third case we have an origin, an obverse, and two mediates distant 2 from each other; and to these we have to add either two proximates or a proximate and an ultimate. The two proximates may be distant from the two mediates 1 1, 1 3, or 1 1, 3 3, or 1 3, 1 3, or 1 3, 3 3; 4 types. The proximate or ultimate must not be respectively distant 1 1, 3 3, or 3 3, 1 1; for then they would form a pair of obverses; there remain the cases 1 1 with 1 1 or 1 3, 1 3 with 1 3, and 3 3 with 1 3 or 3 3; 5 types. In the fourth case we have an origin, obverse, and three mediates distant 2 from one another; the remaining mark must be distant either 1 or 3 from these mediates; 2 types. This makes twenty-two types of sixfold statement with one pair of obverses.

12. If a sixfold statement contains two pairs of obverses, these must be either evenly or oddly distant. If they are evenly distant we have an origin, obverse, and two obverse mediates, to which two other marks are to be added. These may be both evenly distant; taking one of them as origin, it is associated with 5 mediates, so that there is 1 type only. Or both oddly distant; here there are two cases, according as the distances are 1 1, 3 3, or 1 3, 1 3. Or one oddly and one evenly distant; the latter is any one of the four remaining mediates, and then the former is distant 1 or 3 from it; 2 types. If the two pairs of obverses be oddly distant they form an aggregate which is related in the same way to all the remaining twelve marks; viz. any one of these being taken as origin, we have a pair of mediates and a proximate with its obverse ultimate. The thing to be considered, therefore, is the distance between the two marks to be added, which may be 1, 2, or 3, and each in two ways; 6 types.

A sixfold statement with three pairs of obverses is one of two types only; viz. these are all evenly distant when they are the mediates to one origin, or two evenly distant and one oddly distant from both of them.

13. A pure sevenfold statement must consist of a group

and a triad; for it must contain a triad, by the same reasoning by which this was proved for a fivefold statement; and then either all the other four marks are oddly distant from this, and so form a group by themselves, or else one of them is evenly distant from the triad and so forms a group with it. If the group is proper, being the proximates to a certain origin, the triad must consist of two mediates and either the origin, the obverse, or another mediate; and in the latter case the three mediates are distant 1 1 1 or 3 3 3 from some proximate; 4 types. If the group is improper, the triad is either all origins or all obverses, or two origins and an obverse, or an origin and two obverses; 4 types. In all, 8 types of pure sevenfold statement.

14. A sevenfold statement with one pair of obverses must consist either of four marks evenly distant from one another and three oddly distant from them; or of five marks evenly distant from one another and two oddly distant from them. In the former case the pair of obverses may be in the four or in the three. If they are in the four, the three form a triad which are proximates to one origin; and then the pair may be the origin and obverse or a pair of mediates. If the pair are origin and obverse, the other two (at distance 2) are mediates, distance 1 1, 1 3 or 3 3 from the proximate which is not in the triad; if the pair are mediates, the two may be the origin or obverse with a mediate distant 1 or 3 from that proximate (4 types), or two mediates distant 1 1, 1 3, 3 3 from it (3 types). If the pair of obverses are in the set of three marks, the four form a group, which may be proper or improper. If proper, the three may be origin and obverse with a mediate, or a pair of mediates with origin, obverse, or another mediate; 4 types. If improper, the three must be two origins and an obverse, or an origin and two obverses; 3 types.

Five marks evenly distant containing only one pair of obverses, must be a proper group with the obverse of one of its marks; see end of art. 9. To these we may add the origin or obverse of the proper group with a mediate distant 1 or 3 from the extra mark, or else two mediates distant 1 1, 1 3, or 3 3 from that mark; 7 types.

15. A sevenfold statement with two pairs of obverses may have six marks evenly distant from one another and one oddly distant from them; in this case the six are an origin and five mediates in two different ways, or say two pairs and a two; the remaining mark may be distant 1 1, 1 3, or 3 3 from the two, which gives 3 types.

Otherwise the sevenfold statement must subdivide (as in the last case) into five and two or into four and three. If it subdivide into five and two, the two may be a pair or not. In the first case we have a proper group and the obverse of one of its marks, together with the origin and obverse of the group or a pair of mediates; 2 types. In the second case we have five mediates of an origin or its obverse, to which we may add two proximates distant 1 1, 1 3 or 3 3 from the odd mediate, or a proximate and an ultimate distant 1 1, 1 3 or 3 3 respectively from the odd mediate; 6 types.

If the sevenfold statement subdivide into four and three, the two pairs may be both in the four, or one in the four and one in the three. In the former case we have a triad, to which may be added the origin and obverse and a pair of mediates or two pairs of mediates; 2 types. In the latter case the four consist of an origin and obverse and two mediates; we must add a pair consisting of a proximate and an ultimate, which may be distant 1 1, 3 3 or 1 3, 1 3 from the two mediates, and then another proximate or ultimate which may be distant 1 1, 1 3, or 3 3 from the two mediates; 6 types.

16. Three pairs of obverses in a sevenfold statement

may be all evenly distant, or two evenly and the other pair oddly distant from each. If they are all evenly distant they are the mediates to a certain origin or its obverse, and the seventh mark may be the origin or a proximate, 2 types. In the other case we have an origin, obverse, and pair of mediates together with a proximate and its obverse ultimate; we may add a proximate or a mediate, 2 types.

17. A pure eightfold statement must consist of two groups, either both proper or both improper, or one of each. Two proper groups may have their origins distant 1 or 3; 2 types. To an improper group we may add a proper group made of one origin and three obverses or of three origins and one obverse, or an improper group made of four origins or four obverses or two origins and two obverses; 5 types. Altogether there are 7 types of pure eightfold statement.

18. An eightfold statement with one pair of obverses must subdivide into four and four, or into five and three. In the former case we have a pair of obverses, viz. an origin and its obverse, and two mediates; to which we must add a group formed out of the proximates and ultimates. This group may be proper, 1 type, or improper, the mediates being in regard to it two origins, two obverses, or an origin and an obverse; 3 types. In the latter case the five marks must be a proper group with the obverse of one mark, to which we must add a triad made out of the origin, obverse, and mediates of the group. This triad may be the origin or obverse together with two mediates distant 1 1, 1 3, 3 3 from the ultimate, 6 types; or else it may be three mediates distant 1 1 1, 1 1 3, 1 3 3, 3 3 3 from the ultimate, 4 types.

19. An eightfold statement with two pairs of obverses must subdivide into four and four, or into five and three, or into six and two. In the first case the two pairs of ob-

verses may be evenly distant, when the remaining marks form a group either proper, with its origin, obverse, and pair of mediates, or two pairs of mediates, or else improper, 3 types; or oddly distant, when the remainder form one of the six pure fourfold statements enumerated art. 6. Two marks distant 2 from each other may be distant 1 1, 3 3 or 1 3, 1 3 from the pair of obverses which are oddly distant from them; thus each of the six fourfold statements gives 3 types of eightfold statement, except the third, which gives 4; in all, 19. In the second case the three may be a triad or may contain a pair of obverses. If it is a triad, the five are mediates to one origin and its obverse, and we may add three proximates distant 1 1 3 or 1 3 3 or two proximates distant 1 1, 1 3, or 3 3 with an ultimate distant respectively 1 1 or 3 3 from the odd mediate; 6 types. If the three contain a pair of obverses, the five make a proper group with obverse of one mark; to this we may add the origin and obverse of the group with mediate distant 1 or 3 from the ultimate, or a pair of obverse mediates with a mediate distant 1 or 3 as before; 4 types. In the third case the six must be an origin and five mediates, and we may add two proximates distant 1 1, 1 3, 3 3 from the odd mediate, or a proximate and an ultimate, or two ultimates, distant as before; 9 types.

20. In an eightfold statement with three pairs of obverses these may be either all evenly distant, or two of them evenly distant and the other oddly distant from both. In the first case they are mediates to a certain origin and its obverse, and we may add the origin with a proximate or ultimate, two proximates, or a proximate and ultimate; 4 types. In the second case take the oddly distant pair for origin and obverse; then these are associated with two proximates and their obverse ultimates, and we may add the two other proximates, a proximate and an ultimate, a proximate and a mediate (distant 1 1, 1 3, 3 1, 3 3 from this proximate and

the remaining one), or two mediates distant 1 1, 3 3 or 1 3, 1 3 from the two proximates; 8 types.

Lastly, in an eightfold statement with four pairs of obverses they may be all evenly distant, or the statement may subdivide into six and two, or into four and four; in the latter case there are 2 types.

21. To obtain the whole number of types, we observe that for every less-than-eightfold type there is a complementary more-than-eightfold type (art. 2); so that we must add the number of eightfold types (78) to twice the number of less-than-eightfold types (159); the result is 396.

TABLE.

Art.				
4.	1-fold			1
	2-fold, distance 1, 2, 3, 4			4
5.	3-fold, pure, distance 1 1 2, 2 2 2, 1 2 3, 2 3 3		4	
	1 pair obv., dist. 1 3 4, 2 2 4		2	
			6	6
6.	4-fold, pure, two and two:—			
	$\frac{1\,\mid\,1}{1\,\mid\,1}$ $\frac{1\,\mid\,1}{1\,\mid\,3}$ $\frac{1\,\mid\,1}{3\,\mid\,3}$ $\frac{1\,\mid\,3}{3\,\mid\,1}$ $\frac{1\,\mid\,3}{3\,\mid\,3}$ $\frac{3\,\mid\,3}{3\,\mid\,3}$	6		
	three and one	4		
	group, proper or improper	2		
		12	12	
7.	" 1 pair obv.		5	
	2 pair obv., dist. odd or even		2	
			19	19
8.	5-fold, pure, three and two	7		
	four and one	5		
			12	
9.	" 1 pair obv. + two prox.	5		
	+ prox. + ult.	3		
	+ two med.	4		
		12		
			12	
	2 pair obv., odd dist., 1; even, 2		3	
			27	27

TABLE (*continued*).

Art.				
10.	6-fold, pure, three and three	6		
	four and two	6		
		12		
			12	
11.	„ 1 pair obv., two and four	3		
	three and three	8		
	four and two	9		
	five and one	2		
		22		
			22	
12.	„ 2 pair obv., odd dist., 6 ; even, 5		11	
	3 pair obv.		2	
			47	47
13.	7-fold, pure ; proper group, 4 ; improper, 4		8	
14.	„ 1 pair obv., four and three	10		
	three and four	7		
	five and two	7		
		24		
			24	
15.	„ 2 pair obv., six and one	3		
	five and two	8		
	four and three	8		
		19		
			19	
16.	„ 3 pair obv.		4	
			55	55
	Total of less-than-eightfold statements			159
	Complementary more-than-eightfold statements			159
17.	8-fold, pure		7	
18.	„ 1 pair obv., four and four	4		
	five and three	10		
		14	14	
19.	„ 2 pair obv., four and four	22		
	five and three	10		
	six and two	9		
		41	41	

TABLE.

Art.			
20. 8-fold, 3 pair obv., all evenly dist.	4		
two evenly dist.	8		
	12	12	
4 pair obv.		4	
		78	78
Grand Total			396

XIII. *On the Law of Force when a Thin, Homogeneous, Spherical Shell exerts no Attraction on a Particle within it.* By J. H. Poynting, B.A., B.Sc.

Read March 6th, 1877.

If a homogeneous, thin, spherical shell of uniform thickness exert no attraction on a particle within it, then the law of the force is the law of nature.

Professor Maxwell uses this proposition ('Electricity,' vol. i. § 74) to deduce the law of the force between electrified bodies, and shows that it proves, far more conclusively than any direct measurements of electrical forces, that the law is that of the inverse square. It would therefore be an advantage to have a simpler proof of such an important proposition than that given by Laplace (Méc. Céleste, liv. ii. ch. 2) and followed by Maxwell. The following seems more simple, as it requires neither integration nor the solution of a functional equation :—

Let P be any point inside the spherical shell, C the