The Probability of Annual Extreme Winter Temperatures in Kentucky

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ABSTRACT

The minimum winter temperatures were examined for several different locations in Kentucky having long periods of record. The cumulative frequency distribution of the temperatures was compared with the normal and the Fisher–Tippett Type I extreme value distributions to determine which provided the best approximation to the data. The Kolmogorov–Smirnov test for goodness of fit did not allow either distribution to be rejected at the .10 probability level.

Further tests were made for asymmetry in the data using the coefficient of skew. Five locations tested failed to exhibit significant skew at the .10 probability level which would be characteristic of the extreme value distribution. In the absence of significant skew, the normal distribution was then selected for use in the estimation of temperatures to be expected for various return periods. Observed temperatures from 33 locations were used to estimate the parameters of the normal distribution for each and predict coldest temperatures to be expected with return periods of once in 2, 5, 10, 25, 50, and 100 years.

INTRODUCTION

An analysis of the probability of extreme winter temperatures has far-reaching implications for anyone involved in long-range planning. The agriculturist must select varieties of winter grain or fruit trees for their ability to withstand a particular degree of cold. Builders must be able to anticipate the extent of cold their structures will have to withstand and still provide a measure of comfort. The coldest temperature ever recorded at official observing stations in Kentucky, –37 C at Cynthiana and at Bonnieville in Hart County, may be an extremely rare event or it might be expected to occur several times in a normal lifetime. A study of the record of extreme cold temperatures can reveal the frequency of occurrence of any particular value.

The climate of Kentucky is characterized by 2 factors which determine the coldest temperatures usually experienced during the winter season. The first is the movement of cold air masses across the state which provide the conditions necessary for occasional frigid readings. The second is a more local factor characterized by the state's irregular terrain, which allows uneven nighttime cooling and the drainage of cold air into low-lying areas. Because of the local effects, it is difficult to generalize and assume the coldest temperatures will always occur in the northern portions of the state or in the higher elevations of the eastern sections. The probability of extreme values must be investigated separately for each location where temperatures have been recorded.

SOURCE OF DATA

Climatological weather observing stations in Kentucky have been established by the National Weather Service for the purpose of gathering daily temperature data at about 80 locations and rainfall data at about 180 locations. The instruments are installed either at the home of a volunteer observer or at a cooperating organization such as a water plant, radio station, etc. Maximum and minimum thermometers are read once a day in order to determine the extremes during the preceding 24-hour period. These thermometers are mounted inside a standard, ventilated instrument shelter at a height of 5 feet above a grass surface. While sites are selected to minimize any modifying influences, the location may occasionally be near a body of water or prone to cold air drainage.

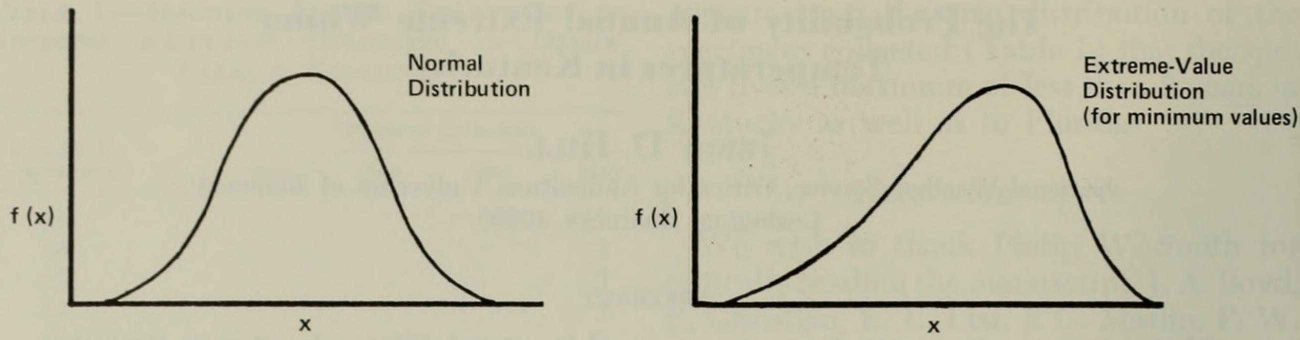

FIG. 1. Comparative shape of normal and extreme value frequency distributions.

A total of 33 well-distributed temperature observing stations were chosen for this study, all of which have periods of record extending from 20 to more than 75 years. The coldest single temperature reached during each year was used to define a set of extreme temperature observations for the period of record at every location.

METHODS

In any study of probability, it is necessary to find a theoretical frequency distribution which represents the data under consideration. In a study of extreme temperatures in Ohio (Miller and Weaver 1970), the well-known normal distribution was used to prepare tables of temperatures occurring at selected probability levels. In many design problems where annual extremes are being considered, the Fisher–Tippett Type I extreme value distribution (Thom 1966) has been widely used. While the normal distribution is symmetrical with equal probability on either side of the mode, the extreme value distribution is skewed and, in the case of minimum values, has a greater probability of values below the mode than above (Fig. 1). The extreme value distribution was used to determine the probability of extreme winter temperature in Tennessee (Bailey 1965).

TABLE 1.—RESULTS OF KOLMOGOROV–SMIRNOV TESTS AT 3 LOCATIONS

Location	Length of Record	Value of D: Normal	Value of D: Extreme Value	Critical Value of D at Level .10
Greensburg	75 years	.07	.11	.139
Murray	41 years	.09	.05	.187
Beaver Dam	36 years	.06	.06	.199

To prepare tables of probabilities or return periods of extreme winter temperatures for Kentucky, it is necessary to determine which of these frequency distributions best represents the population from which the observed data have been drawn. As a preliminary test, data from Greensburg, Beaver Dam, and Murray were analyzed, and the Kolmogorov–Smirnov test (Feller 1948) applied to determine if either the normal or extreme value distribution could be rejected. In order to reject the hypothesis that the data are from a particular distribution under consideration the test requires that D, the maximum deviation between the theoretical cumulative distribution function and the sample cumulative distribution function, exceed a critical value. The value depends upon the number of observations and the level of confidence to be placed in the test. For this investigation, a rather liberal 10 percent confidence level was chosen for all statistical tests. The results of the test using both the normal distribution and the extreme value distribution functions proved inconclusive in differentiating between the two possibilities (Table 1).

Further testing was done to determine if the samples exhibited certain qualities unique to the assumed theoretical frequency distributions. An obvious test is for symmetry since the normal distribution is symmetrical with the mode, mean, and the median values equal while the extreme value distribution is asymmetrical. A useful measure of symmetry is the coefficient of skew, sk (Panofsky 1965) defined in standard statistical notation as:

TABLE 2.—RESULTS OF TESTS FOR SYMMETRY

Location	Mean	Median	Skew	t	Critical t at Level .10
Bowling Green	–17.8 C	–17.8 C	–0.258	.912	1.289
Greensburg	–19.6	–20.0	–0.335	1.184	1.289
Anchorage	–20.8	–20.8	–0.054	.190	1.289
Richmond	–18.9	–19.2	–0.083	.293	1.289
Williamsburg	–17.8	–17.8	–0.015	.053	1.289

$$sk = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^3}{N\sigma^3}$$

A sample of data with more values below the mode than above will be skewed to the left and have a negative coefficient of skew. The normal distribution has a coefficient of skew equal to 0.

Tests of symmetry were made using data from 5 stations each with periods of record extending over 70 years or more. All exhibited a negative coefficient of skew, however, most of the differences from 0 were small. In order to test whether the differences were significant, a t-test was made (Snedecor 1956) where the t statistic was formed as:

$$t = \frac{sk - 0}{\text{Standard error of sk}}$$

The standard error of sk can be estimated (Brooks and Carruthers 1953) as approximately $\sqrt{6/N}$. To evaluate whether the skew is significant, the hypothesis can be formed that the data were drawn from a sample with a coefficient of skew equal to 0. If the value of t exceeds the appropriate tabled values at the selected level of significance, the hypothesis can be rejected. Table 2 gives the results of these tests.

Since the t values all fail to exceed the critical limit, the hypothesis cannot be rejected. This would imply that the normal distribution reasonably represents the population although the data for Greensburg exhibit a near significant skew at the 10 percent level.

As a further check on the distributional assumption, temperatures corresponding to various return periods were estimated using the normal and extreme value distributions, then compared to observed data. The results are shown in Table 3 where only Greensburg exhibited evidence which would question the use of the normal distribution function. In the data from Greensburg, temperatures occurring with intermediate return periods of once in 10 to 25 years agree more closely with the normal probabilities while less frequent occurrences on the order of once in 50 to 75 years are skewed considerably and favor the extreme-value distribution.

DISCUSSION

None of the data presented here provides overwhelming conclusive proof for the selection of one distribution function or the other; however, the absence of pronounced

TABLE 3.—COLDEST WINTER TEMPERATURES °C PREDICTED AND OBSERVED FOR GIVEN RETURN PERIODS. N = NORMAL, EV = EXTREME VALUE, O = OBSERVED VALUE

Coldest Temperature Reached or Exceeded	Greensburg			Bowling Green			Anchorage			Richmond			Williamsburg		
One Year in:	N	EV	O	N	EV	O	N	EV	O	N	EV	O	N	EV	O
5	–23	–23	–23	–22	–21	–21	–24	–24	–24	–23	–22	–23	–22	–21	–23
10	–26	–26	–25	–23	–24	–24	–26	–27	–26	–24	–26	–24	–23	–24	–24
20	–27	–29	–27	–25	–27	–26	–27	–28	–28	–26	–28	–26	–25	–27	–24
25	–28	–30	–27	–26	–28	–27	–28	–29	–29	–27	–29	–26	–26	–28	–25
40	–28	–32	–32	–27	–29	–29	–29	–32	–29	–28	–31	–29	–27	–29	–26
50	–29	–33	–33	–27	–31	–29	–29	–32	–30	–28	–32	–31	–27	–31	–26
75	–31	–33	–34	–28	–32	–29	–29	–33	–	–29	–33	–31	–28	–32	–28

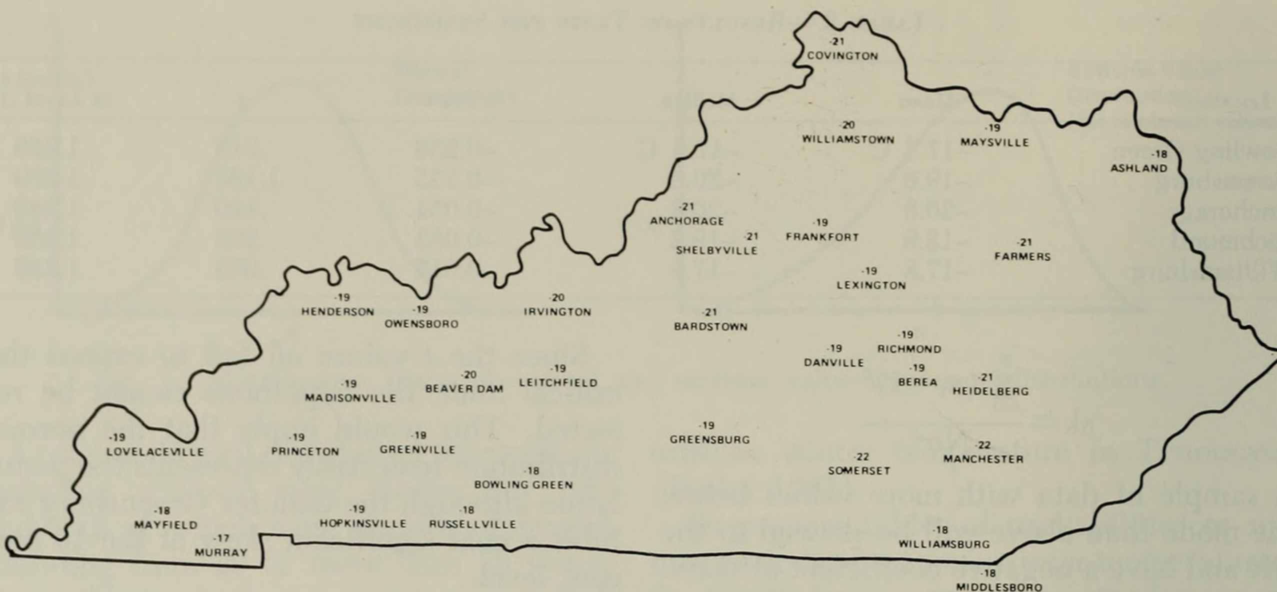

FIG. 2. Locations used in the study and their mean annual minimum temperature, °C.

skew in most sets of data favor the normal distribution. Therefore, the normal distribution has been used to estimate the coldest annual temperature expected to occur with frequencies ranging from once in 2 years (the mean) to once in 100 years for the locations shown in Fig. 2. Those temperatures are presented in Table 4.

It is interesting to note that the coldest mean annual minimum temperature among all stations shown is at Somerset, in southeastern Kentucky at an elevation of approximately 330 m. The weather instruments are located about 2 km north of the city in a broad valley favorable for cold air drainage. The observing station at Farmers has a similar exposure and also shows a tendency to favor cold temperatures.

Data from Ashland have been collected since 1916 by the U.S. Corps of Engineers at their dam on the Ohio River. The moderating effect of the large body of water is evident in reducing the temperature extremes expected at the dam. The values are slightly more temperate than those indicated for Russellville which is at approximately the same elevation but a full 2 degrees of latitude further south.

SUMMARY

The coldest temperature expected during the winter at any location in Kentucky is determined not only by the latitude but also

TABLE 4.—COLDEST ANNUAL TEMPERATURE EXPECTED FOR SELECTED RETURN PERIODS AT SELECTED LOCATIONS IN KENTUCKY

	Coldest Temperature °C Reached or Exceeded One Year in:					
Location	2	5	10	25	50	100
Anchorage	–21	–24	–26	–28	–29	–30
Ashland	–18	–21	–23	–24	–26	–27
Bardstown	–21	–24	–26	–28	–29	–31
Beaver Dam	–20	–24	–26	–28	–30	–31
Berea	–19	–22	–24	–26	–27	–28
Bowling Green	–18	–22	–23	–26	–27	–28
Covington	–21	–23	–25	–27	–28	–29
Danville	–19	–22	–23	–26	–27	–27
Farmers	–21	–25	–27	–29	–31	–32
Frankfort	–19	–23	–24	–26	–27	–28
Greensburg	–19	–23	–26	–28	–29	–31
Greenville	–19	–23	–24	–26	–28	–29
Heidelberg	–21	–24	–26	–28	–29	–30
Henderson	–19	–22	–24	–26	–27	–28
Hopkinsville	–19	–22	–24	–27	–28	–29
Irvington	–20	–23	–25	–27	–28	–29
Leitchfield	–19	–22	–24	–26	–28	–29
Lexington	–19	–23	–24	–26	–27	–28
Lovelaceville	–19	–23	–24	–27	–28	–29
Madisonville	–19	–23	–24	–27	–28	–29
Manchester	–22	–25	–27	–28	–29	–31
Mayfield	–18	–21	–23	–25	–26	–27
Maysville	–19	–23	–24	–27	–28	–29
Middlesboro	–18	–21	–23	–25	–26	–27
Murray	–17	–20	–22	–24	–25	–26
Owensboro	–19	–23	–24	–26	–28	–29
Princeton	–19	–23	–26	–28	–29	–31
Richmond	–19	–23	–24	–27	–28	–29
Russellville	–18	–22	–24	–26	–27	–28
Shelbyville	–21	–24	–26	–28	–29	–31
Somerset	–22	–26	–28	–30	–31	–33
Williamsburg	–18	–22	–23	–26	–27	–28
Williamstown	–20	–23	–25	–27	–28	–29

the topography of the location. For most locations the long-term weather records show that extreme low temperatures occur with a frequency which can be approximated by the normal distribution function. Tables of the normal distribution can be used to calculate the extreme winter temperatures expected to occur with any given frequency.

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