# PROOF OF THE LAWS OF TWIN-BIRTHS.

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SYNOPSIS. 1. Formulæ giving separately the relative numbers of uniovular and diovular twins. 2. Applicability to total twins. 3. Applicability to cases of two males, male and female, and two females. Limitations of the formulæ.

1. *Formulae giving separately the relative numbers of uniovular and diovular twins.* In an earlier paper, this Journal, Vol. 61, pp. 190-217, I showed that the frequency of twins born through the subdivision of a single fertilised ovum was sensibly the same for each age of the mothers, being always about 0.00300 of the cases of maternity. I showed, also, that the numbers born from two ova increased sensibly linearly up to age, say, 37 or 38, then decreased nearly linearly to the end of the child-bearing period (see formulae (2) and (2a), p. 200; and (3) and (3a), p. 202, this Journal). In formulae (3) the sign should be +. From this it is evident that the numbers for each age for the two types of cases can be given by the following type of formulae, in which $t$ denotes the ratio of the number of twins to the number of cases of maternity:—

| | Uniovular cases. | + Diovular cases. |
|---|---|---|
| Up to age 37 .. .. | $t=0.00300$ | $+0.00058\ (x-15)$ |
| Beyond age 37 .. .. | $t=0.00300$ | $+0.01276-0.00128\ (x-37)$ |

The constants of these formulae have been computed to fit approximately with few figures, the Australian results for the six years 1920 to 1925. The number for each age

of cases of maternity (mothers) on which these are based are given in Table VI., column (ii), pp. 197-8, of this Journal.

2. *Applicability to total twins.* In order to obtain a definitive idea of how these expressions do actually represent the data, the values of the two parts of $Ct$ are given as against the data. In making the comparisons, however, it has to be remembered that the ages are merely *as stated* by the mothers (see remarks p. 191, this Journal). The actual ages probably differ slightly, for the ages are not given exactly.

Between the ages 12 and 49 inclusive there were 809,126 mothers (cases of maternity) and 8,403 cases of twins, viz., MM 2,792, MF 2,989, and FF 2,622. This would indicate that the probable number of uniovular cases was 2,792 + 2,622 — 2,989, that is 2,425. The calculated sum with the adopted approximate coefficient 0.00300, was 2,427.3.

It has already been shown (pp. 211-2 of this Journal) that this ratio is the same for each age during the child-bearing period.

Having regard to the number of cases, the rough formulae, given above, are seen from the table to represent closely the laws enunciated in the beginning of this paper, inasmuch as the observed and calculated results differ from each other for any one age, never more than — 32 and + 30 (ages 38 and 40 respectively), and the differences of the sums of these from age 12 upwards never disagree by more than — 58 to + 23 (ages 39 and 19 respectively).

The paucity of the numbers of cases suggests that it is not worth while attempting at present to furnish formulae of higher numerical precision.

The results thus established are:—(a) *three twins are produced, each from a single ovum, in every thousand cases*

**Table I. Showing calculated and observed numbers of twins, for each age of mother from 13 to 49, and the sums of their differences.**

| Ages. | Uniov. | Diov. | Calc. | Obsd. | Diff. | Sum of Diff. |
|---|---|---|---|---|---|---|
| i | ii | iii | iv | v | vi | vii |
| 13 .. .. .. | 0.1 | 0.0 | 0 | 1 | + 1 | + 1 |
| 14 .. .. .. | 0.5 | 0.0 | 1 | 1 | 0 | + 1 |
| 15 .. .. .. | 1.5 | 0.0 | 2 | 3 | + 1 | + 2 |
| 16 .. .. .. | 6.3 | 1.2 | 7 | 11 | + 4 | + 6 |
| 17 .. .. .. | 17.1 | 6.6 | 24 | 22 | — 2 | + 4 |
| 18 .. .. .. | 35.2 | 20.4 | 56 | 54 | — 2 | + 2 |
| 19 .. .. .. | 56.4 | 43.6 | 100 | 121 | +21 | +23 |
| 20 .. .. .. | 75.2 | 72.7 | 148 | 134 | —14 | + 9 |
| 21 .. .. .. | 96.6 | 112.1 | 209 | 205 | — 4 | + 5 |
| 22 .. .. .. | 114.1 | 154.5 | 270 | 266 | — 4 | + 1 |
| 23 .. .. .. | 129.5 | 200.3 | 330 | 337 | + 7 | + 8 |
| 24 .. .. .. | 138.5 | 240.9 | 379 | 349 | —30 | —22 |
| 25 .. .. .. | 142.2 | 274.9 | 417 | 437 | +20 | — 2 |
| 26 .. .. .. | 144.6 | 307.5 | 452 | 443 | — 9 | —11 |
| 27 .. .. .. | 141.9 | 329.2 | 471 | 469 | — 2 | —13 |
| 28 .. .. .. | 141.3 | 355.2 | 497 | 466 | —31 | —44 |
| 29 .. .. .. | 135.7 | 367.3 | 503 | 521 | +18 | —26 |
| 30 .. .. .. | 137.8 | 399.5 | 537 | 545 | + 8 | —18 |
| 31 .. .. .. | 114.2 | 353.2 | 467 | 458 | — 9 | —27 |
| 32 .. .. .. | 116.0 | 381.3 | 497 | 486 | —11 | —38 |
| 33 .. .. .. | 103.2 | 359.0 | 462 | 483 | +21 | —17 |
| 34 .. .. .. | 96.2 | 353.3 | 450 | 456 | + 6 | —11 |
| 35 .. .. .. | 86.9 | 336.1 | 423 | 419 | — 4 | —15 |
| 36 .. .. .. | 79.4 | 322.4 | 402 | 403 | + 1 | —14 |
| 37 .. .. .. | 68.1 | 289.7 | 358 | 362 | + 4 | —10 |
| 38 .. .. .. | 62.8 | 240.2 | 303 | 271 | —32 | —42 |
| 39 .. .. .. | 52.0 | 176.7 | 229 | 213 | —16 | —58 |
| 40 .. .. .. | 42.1 | 125.2 | 167 | 197 | +30 | —28 |
| 41 .. .. .. | 29.1 | 74.1 | 103 | 98 | — 5 | —33 |
| 42 .. .. .. | 25.0 | 52.9 | 78 | 84 | + 6 | —27 |
| 43 .. .. .. | 16.9 | 28.7 | 46 | 41 | — 5 | —32 |
| 44 .. .. .. | 10.1 | 12.8 | 23 | 29 | + 6 | —26 |
| 45 .. .. .. | 6.0 | 5.1 | 11 | 11 | 0 | —26 |
| 46 .. .. .. | 2.7 | 1.1 | 4 | 6 | + 2 | —24 |
| 47 .. .. .. | 1.3 | — | 1 | 0 | — 1 | —25 |
| 48 .. .. .. | 0.6 | — | 1 | 1 | 0 | —25 |
| 49 .. .. .. | 0.2 | — | 0 | 0 | 0 | —25 |

Column (iv) is the sum of columns (ii) and (iii) cast to the nearest unit. Column (vi) is column (v) less column (iv). Column (vii) is the algebraic sums of the figures in column (vi).

*of maternity, whatever the age of the mother;* and (b) *two ova are effectively fertilised* (i.e., result in living twins), *with a frequency increasing linearly with age till, say, age* 37, *but afterwards diminishing linearly till, say, about age* 49 (or the end of the child-bearing period).

3. *Applicability of formulae to cases of two males, a male and a female, and two females.* In order to divide the calculated number of twins into MM, MF, and FF cases, it is necessary to know how the masculinity over all varies, if at all, with the age of the mother, and how the masculinity in uniovular cases compares with that of diovular cases.

It has already been shown in this Journal, sections 8 and 9 of former paper, p. 202, that the changes in masculinity are complex and irregular. Its general trend over all (m-f)/(m-f), see Table IX., may be approximately indicated by the following linear equations, viz:—

Ages 17 to 30; $\mu' = -0.0330 + 0.0074\ (x - 17)$.
Ages 30 to 38; $\mu' = +0.0632 - 0.0104\ (x - 30)$.
Ages 38 to 46; $\mu' = -0.0200 + 0.0046\ (x - 38)$.

The masculinity over all, however, is 0.02023, and is thus not very different from that of first-births, and of all births, since the former for 1921 to 1925 was 0.0278, and that for all births was 0.0250 (see Table IV., p. 195, and also column (viii.), Table VI., p. 197, this Journal).

While it will not be possible to compute exactly the MM, MF, and FF cases for each age, they can be closely approximated on the assumption that the masculinity over all, was for each age $\lambda = 0.02023$, and that the masculinity was $2\lambda = 0.04046$ for the MM and FF diovular cases, the former applying to the uniovular cases (see p. 203 of this Journal).

On the assumptions indicated we obtain from columns (ii.) and (iii.) in the preceding Table, the numbers of MM, MF, and FF cases. The respective results are:—

| | | | | |
|---|---|---|---|---|
| $U$; | Uniovular cases, age $x$, | $\frac{1}{2}U_x(1+\lambda)$ | | $\frac{1}{2}U_x(1+\lambda)$ |
| $D$: | Diovular cases, age $x$, | $\frac{1}{4}D_x(1+2\lambda)$ | $\frac{1}{2}D_x$ | $\frac{1}{4}D_x(1+2\lambda)$ |
| $U+D$; | Total cases, age $x$. | MM: | MF; | FF. |

For example for age 37 we have from the tabular results, 68.1 and 289.7 in Table I., the following:—

| | MM | MF | FF | Totals |
|---|---|---|---|---|
| Masc.* 0.0202; Uniovular | 34.7 | 0.0 | 33.4 | 68.1 |
| Masc.* 0.0405; Diovular | 75.3 | 144.9 | 69.5 | 289.7 |
| Masc. 0.0202; Totals | 110.0 | 144.9 | 102.9 | 357.8 |
| According to the data ... | 105 | 148 | 109 | 362 |
| Difference (obsd.-calc.) .. | —5 | +3 | +6 | +4 |

The masculinities marked * are for the MM and FF cases alone. The discrepancies —5, +3 and +6 are not large considering the paucity in the number of cases.

Table II. below gives the calculated results for each age to the nearest whole number, and also the observed results, for the MM, MF, and FF cases; it discloses, therefore, how far the formulae, which are general, agree with the data. It will be seen that it establishes the fact that (c) *over the entire child-bearing period,* excepting, perhaps, say the first three or four commencing and closing years thereof, *the uniovular cases for each year of age are equal to the sum of the cases of two males and two females, less the cases of male and female.*

The measure of the goodness of fit of the formulae as representative of the data is not merely the closeness of the MM, MF, and FF individual results for each age, but also the aggregate of such results, viz., the sums of MM, MF, and FF up to each age. It is evident from the table giving these that the dictum is confirmed, viz., that the number of diovular cases increases sensibly linearly to age 37, and diminishes very nearly linearly afterwards. This is clearly the general law.

**Table II. Showing Calculated and Observed Distribution of Twins, Australia, 1920 to 1925, for each age and their sums up to each age.**

| Mother's Age. | Ca. MM | Ob. MM | Ca. MF | Ob. MF | Ca. FF | Ob. FF | Ca. Sum MM | Ob. Sum MM | Ca. Sum MF | Ob. Sum MF | Ca. Sum FF | Ob. Sum FF |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 .. | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 14 .. | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 2 |
| 15 .. | 1 | 1 | 0 | 0 | 1 | 2 | 2 | 1 | 0 | 0 | 1 | 4 |
| 16 .. | 4 | 4 | 0 | 1 | 3 | 6 | 6 | 5 | 0 | 1 | 4 | 10 |
| 17 .. | 11 | 8 | 3 | 4 | 10 | 10 | 17 | 13 | 3 | 5 | 14 | 20 |
| 18 .. | 23 | 25 | 10 | 7 | 22 | 22 | 40 | 38 | 13 | 12 | 36 | 42 |
| 19 .. | 40 | 39 | 22 | 33 | 38 | 49 | 80 | 77 | 35 | 45 | 74 | 91 |
| 20 .. | 57 | 56 | 36 | 36 | 54 | 42 | 137 | 133 | 71 | 81 | 128 | 133 |
| 21 .. | 78 | 73 | 56 | 42 | 74 | 90 | 215 | 206 | 127 | 123 | 202 | 223 |
| 22 .. | 98 | 95 | 77 | 73 | 93 | 98 | 313 | 301 | 204 | 196 | 295 | 321 |
| 23 .. | 118 | 120 | 100 | 99 | 112 | 118 | 431 | 421 | 304 | 295 | 407 | 439 |
| 24 .. | 133 | 127 | 120 | 108 | 126 | 114 | 564 | 548 | 424 | 403 | 533 | 553 |
| 25 .. | 144 | 149 | 137 | 144 | 136 | 144 | 708 | 697 | 561 | 547 | 669 | 697 |
| 26 .. | 154 | 152 | 154 | 152 | 145 | 139 | 862 | 849 | 715 | 699 | 814 | 836 |
| 27 .. | 158 | 152 | 165 | 166 | 149 | 151 | 1020 | 1001 | 880 | 865 | 963 | 987 |
| 28 .. | 164 | 156 | 178 | 171 | 154 | 139 | 1184 | 1157 | 1058 | 1036 | 1117 | 1126 |
| 29 .. | 165 | 171 | 184 | 192 | 155 | 158 | 1349 | 1328 | 1242 | 1228 | 1272 | 1284 |
| 30 .. | 174 | 197 | 200 | 209 | 163 | 139 | 1523 | 1525 | 1442 | 1437 | 1405 | 1423 |
| 31 .. | 150 | 145 | 177 | 170 | 141 | 143 | 1673 | 1670 | 1619 | 1607 | 1546 | 1566 |
| 32 .. | 158 | 166 | 191 | 195 | 148 | 125 | 1819 | 1836 | 1810 | 1802 | 1694 | 1691 |
| 33 .. | 146 | 154 | 179 | 185 | 137 | 144 | 1965 | 1990 | 1989 | 1987 | 1831 | 1835 |
| 34 .. | 141 | 148 | 177 | 164 | 132 | 144 | 2106 | 2138 | 2166 | 2151 | 1963 | 1979 |
| 35 .. | 132 | 128 | 168 | 172 | 123 | 119 | 2238 | 2266 | 2334 | 2323 | 2086 | 2098 |
| 36 .. | 124 | 118 | 161 | 162 | 116 | 123 | 2362 | 2384 | 2495 | 2485 | 2202 | 2221 |
| 37 .. | 110 | 105 | 145 | 148 | 103 | 109 | 2472 | 2489 | 2640 | 2633 | 2305 | 2330 |
| 38 .. | 95 | 95 | 120 | 89 | 88 | 87 | 2567 | 2584 | 2760 | 2722 | 2393 | 2417 |
| 39 .. | 72 | 62 | 88 | 97 | 68 | 54 | 2639 | 2646 | 2848 | 2819 | 2461 | 2471 |
| 40 .. | 54 | 58 | 63 | 73 | 51 | 66 | 2693 | 2704 | 2911 | 2892 | 2512 | 2537 |
| 41 .. | 34 | 33 | 37 | 33 | 32 | 32 | 2727 | 2737 | 2948 | 2925 | 2544 | 2569 |
| 42 .. | 27 | 28 | 26 | 33 | 25 | 23 | 2754 | 2765 | 2974 | 2958 | 2569 | 2592 |
| 43 .. | 16 | 11 | 14 | 18 | 15 | 12 | 2770 | 2776 | 2988 | 2976 | 2584 | 2604 |
| 44 .. | 9 | 11 | 6 | 8 | 8 | 10 | 2779 | 2787 | 2994 | 2984 | 2592 | 2614 |
| 45 .. | 4 | 3 | 3 | 5 | 2 | 3 | 2783 | 2790 | 2997 | 2989 | 2594 | 2617 |
| 46 .. | 2 | 2 | 1 | 0 | 1 | 4 | 2785 | 2792 | 2998 | 2989 | 2595 | 2621 |
| 47 .. | 1 | 0 | 0 | 0 | 0 | 0 | 2786 | 2792 | 2998 | 2989 | 2595 | 2621 |
| 48 .. | 1 | 0 | 0 | 0 | 0 | 1 | 2787 | 2792 | 2998 | 2989 | 2595 | 2622 |
| 49 .. | 0 | 0 | 0 | 0 | 0 | 0 | 2787 | 2792 | 2998 | 2989 | 2595 | 2622 |

Why there should be this sudden change after age 37, in the frequency of the effective fertilisation of two ova, is not apparent. The changes in the frequency of mater-

nity, column (ii.), pp. 197-8, of this Journal, throw no light upon it; and so far as the writer is aware, there are no other related statistical progressions which give any indication of a possible cause.

4. *Limitations of the formulae.* Obviously, it is practically certain that the number of uniovular cases is not represented by $0.00300C$ at the very beginning and end of the child-bearing period, although otherwise it is sensibly constant throughout. This needs investigation with a much larger number of cases. It also seems probable that the straight lines, representing the changes of frequency with age, are terminated by short curves concave upwards, and also that the two lines are joined by a curve convex upwards. To determine whether these are so or not, also requires as data a far larger number of cases. So far as I am aware, the necessary data are not yet available. It is to be hoped in future the ages of the mothers can be more accurately ascertained. An examination of the data will show that the numbers attributed to certain ages should probably be assigned to adjoining ages.

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