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Galaxies, Clusters and Invisible Mass*

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INTRODUCTION

I am happy to be associated with the James Pollock Memorial Lectures, especially since they provide a tie with my formative years at Sydney University: My last year as a student here, 1945, was about when these lectureships were started. The topic of my talk also provides a link - theoretical physics and radioscience made the strongest impression on me as a Sydney student and these two topics are interwoven in the subject of my talk.

Since a memorial lecture should appeal to a fairly broad audience, I usually like starting off qualitatively and getting mathematical only towards the end. However, in the present case, I have to start off with an equation, Kepler's Law, even before I state the topic: For a circular orbit at a distance r from the centre of a mass M(r) the rotation velocity is given by

$$V_{rot}^2 = \frac{GM(r)}{r} \qquad (1)$$

A recurring theme of my talk is the use of observed velocities V of "test particles" to infer the total (gravitational) mass M of a system, whether "invisible" or not. The first topic (Rotation Curves of Spiral Galaxies) will be the rotation curve of a spiral galaxy, where orbits are fairly accurately circular and the gravitational mass inside radius r is given almost directly by equation (1). However, I will touch on various other topics which, at first sight, sound like a different phenomenon and yet the governing equation is essentially (to within a factor of two) the same:

For an equilibrated gravitationally bound system of N particles, the Virial Theorem can be used, which is essentially the "statistical average" equivalent of equation (1). At least for the *core* of a rich cluster of galaxies the Virial Theorem works quite well; for the outer layers of a galaxy cluster I will describe (Dynamical Masses for Galaxy Clusters) more intricate dynamical calculations, but they are merely a quantitative refinement to the Virial Theorem. For the formation of a galaxy cluster out of a local density enhancement in an expanding cosmological model an important question is whether the proto-cluster is gravitationally bound or not. This question can be rephrased by asking whether the initial expansion velocity was less than V_{esc} or not, where

$$V_{esc} = \left(2\frac{GM}{r}\right)^{\frac{1}{2}} \qquad (2)$$

is the escape velocity at a distance r from mass M (which differs from equation (1) by merely a factor of 2). Finally, for cosmological models an important question is whether the universe is open or closed. This is usually expressed by asking whether the mean density of the universe is smaller or larger than the cosmological "critical density".

$$\rho_{crit} \equiv (3/8\pi G)H_0^2 \sim 10^{-29}\text{gm cm}^{-3} \qquad (3)$$

where H_0 is the Hubble constant. However, this critical density is essentially that required to make the "mass of the universe" such that V_{esc} in equation (2) equals the speed of light c.

Instead of giving masses or mass densities in absolute terms, it will be more convenient to give mass-to-light ratios (expressed in units of M_0/L_0 where M_0 is solar mass and L_0 is solar luminosity). For many considerations, the numerical value of H_0 is unimportant, but I will use a value of 70 kms^{-1} Mpc^{-1} (as a compromise between two "fashionable" values of ∿50 and ∿100). The dimensionless cosmological density parameter Ω can be re-expressed as

$$\Omega \equiv \frac{\langle\rho\rangle}{\rho_{crit}} \sim \frac{\langle M/L\rangle}{1500} \qquad (4)$$

where ⟨ρ⟩ is the mean density of the universe (averaged over volumes large compared with our Local Supercluster of galaxies). To give a preview of Sections II and III: Ordinary stars give an average mass-to-light ratio of M/L ∿5 or 10, whereas rotation curves for galaxies and velocity dispersions in galaxy clusters give larger value - hence the inference of "invisible mass" somewhere.

ROTATION CURVES OF SPIRAL GALAXIES

Optical measurements of shifts in spectral lines from stellar distributions in nearby spiral galaxies can give, at least in principle, the velocity (component along the line of sight) as a function of distance r from the galaxy centre. Equation (1) then gives M(r), the total mass contained inside a sphere of radius r, if the mass distribution is spherical; if the mass is distributed in a disk the numerical factor in the equation only changes by π/2 with even smaller changes for other distributions. Such mass determinations were already carried out more than 50 years ago (Öpik, 1922) and much optical data exists on rotation curves $V_{rot}(r)$ for the *inner* galactic disks of spiral galaxies (Burbidge and Burbidge, 1975). Since the mass density is finite at the centre, $M(r) \propto r^3$

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at small radii r and $V_{rot}(r)$ increases linearly at first. Since the mass density decreases outwards the linear increase in $V_{rot}(r)$ stops soon, but the behaviour at large r is of greatest interest. The optical surface brightness $\sigma_{opt}(r)$ decreases exponentially with r (Freeman, 1970) and so would the mass surface density $\sigma_M(r)$ *IF* most mass were contributed by ordinary stars, so that the mass to light ratio M/L were constant everywhere. In that case, M(r) would approach the total mass M_{tot} rapidly and the rotation curve would approach the Kepler Law, $V_{rot} \propto r^{-\frac{1}{2}}$, in the outer regions.

Because of the exponential decrease of optical surface brightness it is difficult to extend optical rotation curves to the outer regions, but fortunately the neutral hydrogen in the galactic disk extends about 2 or 3 times further out than most of the starlight. The direct contribution to M_{tot} of the hydrogen gas is very small, but modern radiotelescopes are very sensitive and can detect neutral hydrogen and measure its velocity (or, rather, its component along the line of sight) through the λ21cm hyperfine structure line. By a peculiar quirk of history, one of the earliest galaxies for which accurate 21cm data were taken, M81, showed the expected turnover in the rotation curve and the approach to the Kepler Law and seemed to corroborate the assumption of a constant mass to light ratio. Troubles with this assumption soon surfaced, in particular our nearest large galaxy, M31 (Andromeda), seemed to show a flat rotation curve. A trigonometric conversion factor has to be applied to the measured line-of-sight velocity component to derive $V_{rot}(r)$ and there could be uncertainties if the outer disk of the galaxy is warped. Fortunately, these uncertainties are quite unimportant when the galaxy is viewed almost edge-on. The most recent 21cm radiotelescopes are sensitive enough to be able to observe a large number of galaxies and one can select those which are close to edge-on. The Westerbork array (Sancisi, 1976) has particularly good angular resolution and the Arecibo dish (Krumm and Salpeter, 1979) has particularly good sensitivity and a lot of reliable data is now available (including also newer optical data). The situation has been reviewed by Bosma and van der Kruit (1979) and by Rubin (1979) and it is now clear that most spiral galaxies (with a quarter or less of the galaxies, including M81, forming an exception) have flat rotation curves and M(r) must increase linearly with r - at least as far as the observations can be carried out.

The 21cm observations peter out at two or three times the optical radius of a typical galaxy, because the hydrogen signal becomes too weak, and the rotation curves are usually still flat there. The last values for M(r) provide lower limits for M_{tot} and are two or three times larger than the old optical estimates, giving a lower limit to the overall mass to light ratio of about $20M_\odot/L_\odot$. This is not spectacular in itself, but the *local* mass to light ratios are: The mass surface density decreases only as r^{-1} whereas the optical surface brightness decreases exponentially, so that $\sigma_M(r)/\sigma_L(r)$ increases very drastically, as shown in Figure 1, up to about $500M_\odot/L_\odot$. We therefore have the tantalizing situation of knowing about the existence of "almost invisible" matter but not knowing how much further out it extends and how much larger M_{tot} is than our lower limit for it.

We also do not know in what physical form the matter in such an "invisible galaxy halo" is. Some form of stars seems the most conservative hypothesis, but "ordinary stars" in the mass range $0.2M_\odot$ to $2M_\odot$ have too small values for M/L (shown in Figure 2). M/L increases as stellar mass decreases and stars of about $0.1M_\odot$ are sufficiently faint for present-day data, although they might be detectable with improved optical sensitivity; objects less massive than about $0.08M_\odot$ cannot burn hydrogen at all, cool off rapidly and these "overgrown Jupiters" are essentially invisible. By analogy with the well-studied "stellar population II halo", it is usually assumed that a "stellar population III" was formed *before* stellar populations I and II, more than 10^{10} years ago. If that is true, a population of very massive stars - initially - is also a possibility ($\geq 4M_\odot$, say), since they will have ended their main sequence life a long time ago and their compact remnants (white dwarfs, neutron stars or black holes) are very faint optically (Salpeter, 1977).

Before we can say if it is reasonable that the masses of the earliest stars to form were either very small or very large, we should look at the situation of "ordinary" stars. For this purpose one should look not at the present-day observed luminosity function, but the extrapolation back to the birthrate or "initial mass function IMF" (I am again happy to talk on this topic in this continent since my first work in "real" astronomy was done in Australia on this subject). This IMF, when expressed in the appropriate logarithmic form as shown schematically in Figure 2, is still slightly uncertain (Salpeter, 1955; Lequeux, 1979; Miller and Scalo, 1979), but the interesting thing is that it is rather flat. A similar empirical fact holds in a more sociological realm - roughly as many people live in cities between 1 and 2 million population as in towns between 10,000 and 20,000 population, etc. ... The causes are not really understood in either realm, but it is known that the demographic law sometimes fails such as in the megalopolis on the Eastern U.S. seabord or in highly rural populations. We should perhaps not be too surprised if a similar thing happened to tilt the earliest IMF towards very small or very large masses.

DYNAMICAL MASSES FOR GALAXY CLUSTERS

We saw that we can only get a lower limit to the total gravitational mass of an individual galaxy and that there might be more "invisible" mass further out, either bound to the galaxy or elsewhere. Fortunately, an appreciable fraction of all galaxies lives in large clusters and we can investigate the total gravitational mass of such a cluster by studying the dynamics of the galaxies in it. There are different types of clusters, from loose ones containing few galaxies to very "rich" ones, containing thousands of galaxies with very high central number density. The very richest clusters show signs of relaxation subsequent to their formation due to dissipative galaxy - galaxy interaction. Fortunately, we are situated relatively close to a medium-rich cluster, the Virgo cluster, which is large enough to give good statistics but has not suffered much relaxation, so that we can now treat individual galaxies as point particles.

As I mentioned in the Introduction, the Virial Theorem is the statistical equivalent of Kepler's Law, relating the mean-squared velocity of particles in a cluster (the systemic velocity of each galaxy relative to the cluster centre) to total gravitational mass of the cluster (divided by its radius. De Vaucouleurs (1960) already applied the Virial Theorem to velocity data for the Virgo cluster and found a surprisingly large cluster mass, corresponding to a mass-to-light ratio of about 500. Some objections have been raised to the use of the Virial Theorem, which strictly speaking applies only to an isolated system at equilibrium, to a galaxy cluster which is not fully isolated and still has galaxies falling in on it from the outside. It is important to settle this point, since it has a bearing on two interesting questions:

1. How much invisible mass is there in the Virgo cluster core?

2. What is the value of the cosmological density parameter Ω?

Equation (4) relates Ω to the mean mass-to-light ratio $\langle M/L \rangle$ and one might think that questions 1. and 2. above are identical. Until a few years ago this view was prevalent and there was optimism that a reliable value for Ω would be found soon. Developments since have illustrated an unfortunate (but fascinating) aspect of observational cosmology: As more observational data becomes available on some topic, the claimed accuracy for determining some interesting number often gets *worse* for a while - not better - because some systematic cause for error has been found but not yet eliminated. We have seen that some invisible matter is associated with individual galaxies, but it is becoming more and more likely that M/L is even larger for the core of a cluster like Virgo than for an individual galaxy. We therefore have to expect the possibility that M/L either increases or decreases radically with distance from the centre of a large cluster, so that the two questions above become decoupled. We shall see that question 1. can be answered quite accurately, but question 2. is wide open.

Since the pioneering work on the Virgo cluster and its surrounding by de Vaucouleurs (much of it carried out in Australia) there have been advances both on the observational and theoretical side. Observationally there has been a veritable information explosion, both from optical spectroscopy and 21cm-line work, and accurate systemic velocities are now available for more than a thousand galaxies in the general Virgo cluster vicinity. Instead of merely getting one velocity dispersion for the whole cluster, one can get observational values for velocity dispersion as a function of angular distance θ from the Virgo cluster centre. Part of this data is shown in Figure 3 for the cluster core (nominally defined as the sphere inside $\theta \sim 6°$, where the number density of galaxies has a steep gradient) and some distance outside. The cluster is by no means isothermal, but the velocity dispersion decreases with increasing θ (it becomes almost constant again at slightly larger θ). On the theoretical side the main advance has been the possibility to carry out a large series of dynamical model calculations (Peebles, 1970; Gott, 1975; Gunn, 1977; Silk and Wilson, 1979; Hoffman, Olson and Salpeter, 1980), which can eliminate the controversy surrounding the (much simpler) use of the Virial Theorem.

Although the supercluster surrounding the Virgo cluster is flattened, the cluster itself is not, so that spherically symmetric dynamical models are sufficiently accurate. Figure 4 is a schematic illustration of such a model for an open universe which already contained one spherically symmetric density enhancement at early times. Because of the enhanced gravitational force, the total energy per particle is negative for all spherical shells inside "marginally bound surface" containing mass m^*. At early times all shells take part in the general cosmological expansion but at some epoch (labelled as time $t = 1$) shells far inside of m^* come to rest, start collapsing and hit the origin at approximately $t = 2$. Shells further out (but still inside m^*) reach zero velocity at later times and, whatever the present epoch t_{now} is, there exists some zero velocity surface. At any finite time the cluster is never fully isolated from its surrounding, but at times later than about $t \sim 3$, there is a substantial cluster core which does not change its internal density much although the matter outside m^* keeps expanding and decreasing its density.

There are at least two dimensionless parameters characterizing a particular model - the present epoch t_{now} (relative to the first turnaround of the proto-cluster), and the present value of the cosmological density parameter Ω. A scaling factor can be adjusted to make the overall mean velocity dispersion of the model agree with the observed mean, but there is still the question whether the shape of the curve for velocity dispersion as a function of distance θ from the cluster centre fits the observed curve (the histogram) in Figure 3. A number of curves for different models are also shown in Figure 3 and they do in fact fit the observed shape quite well. That is gratifying from one point of view - the basic assumptions (growth of an original approximately spherical density enhancement, neglect of dissipation, etc.) cannot be badly off - but disappointing from another: these curves (taken from Hoffman, Olson and Salpeter, 1980) cover a range of Ω from 0.03 to 0.7 (and we now have models covering an even wider range) and since they all fit equally well, one of the questions I asked above has a negative answer - if the mass-to-light ratio is allowed to vary with distance from the centre (or with density) but if we do not know the sign of the variation then we can say nothing at the moment about the cosmological density parameter Ω!

The other question, however, has a very positive answer: all the models which fit the observed velocity dispersion curve give almost the same mass and mass-to-light ratio for a sphere of radius which subtends an angle of 6°. We get

$$M/L \sim 500 M_0/L_0$$

and $M_{6°} \simeq (3.8 \pm 0.4) \times 10^{12} M_{\odot}$

if we assume $H_0 \simeq 70 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$ (in fact the largest uncertainty in $M_{6°}$ at the moment is due to the uncertainty in the Hubble constant H_0). Since Ω varies from model to model, M/L for much larger volumes than the 6°-sphere is uncertain (and the same is true to a lesser extent for much smaller volumes) but for the volume picked by optical astronomers as "the Virgo cluster proper" the Virial Theorem is vindicated pretty well. The most important result, though, is the fact that M/L is at least 20 times larger still than we obtained for individual galaxies (out to where the neutral hydrogen begins to peter out) so that we have striking evidence for even more invisible mass - possible of a different form and probably distributed differently: if the invisible mass is mainly low-mass stars from large proto-haloes they are likely to have been "rubbed off" in the dense cluster core more than further out and M/L decreases with increasing distance from the cluster centre; if the invisible mass is due mainly to massive neutrinos forming the background cluster (Sato and Takara, 1980), the neutrinos will have suffered even less dissipation than proto-galaxies, have a larger cluster radius and M/L increases.

SUPERCLUSTERS AND FUTURE WORK

I almost ended by saying that we know the mass of a cluster core, such as the inner 6° of the Virgo cluster, quite well but know nothing about the mean mass-density (or the dimensionless parameter Ω): the masses of the cluster cores alone would give only $\Omega \sim 0.02$, a constant mass-to-light ratio M/L everywhere would give $\Omega \sim 0.3$, but an increased M/L outside of cluster cores could easily give a closed universe, $\Omega \geq 1$. There is some hope to get more information in the future from the study of "superclusters", which come either in the form of tenuous surroundings of a single cluster core of about 10 or 20 times its radius (which is the case for the Local Supercluster which surrounds the Virgo cluster), or as a collection of 4 or 5 clusters (de Vaucouleurs, 1956; Jôeveer, Einasto and Tago, 1978).

The attempts made at the moment for getting some measure of the mass of the Local Supercluster mainly centre around measuring the "Virgocentric deviation velocity ΔV from pure Hubble flow". By this is meant the additional recession velocity (above the observed one) we would have relative to the Virgo cluster if we had not been decelerated by the gravitational pull of the Virgo cluster. This requires precision measurements of the relative distances from us to galaxies in different directions, which is a difficult task at the moment. Consequently, present estimates for ΔV cover a rather wide range from about 125 to 500 km/sec, but hopefully the accuracy will improve. Unfortunately it is not clear whether even an accurate observed value for ΔV will be able to pin down Ω. The reason is that ΔV does not depend so much on the total mass contained in the Local Supercluster (or in a sphere centred on the Virgo Cluster and passing through our location), but on the *excess* mass contained over what the mean cosmological density would give. We thus can have the paradoxical situation that of two models (with M/L varying differently), both giving the velocity dispersion of the cluster core correctly, the one with the *larger* Ω may predict a *smaller* ΔV.

Some attempts have also started to measure velocities equivalent to ΔV in a supercluster containing several clusters (Ford, et al., 1981). I am equally sceptical that these measurements can give Ω directly, but I am confident that we will learn much about the formation, structure and evolution of superclusters from such studies. From such understanding will eventually also come a value for the elusive parameter Ω and I hope there will be a more definitive Pollock Memorial Lecture on this subject before the end of the millenium.

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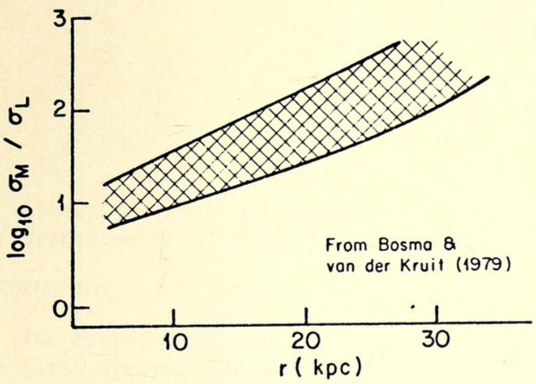

Fig. 1

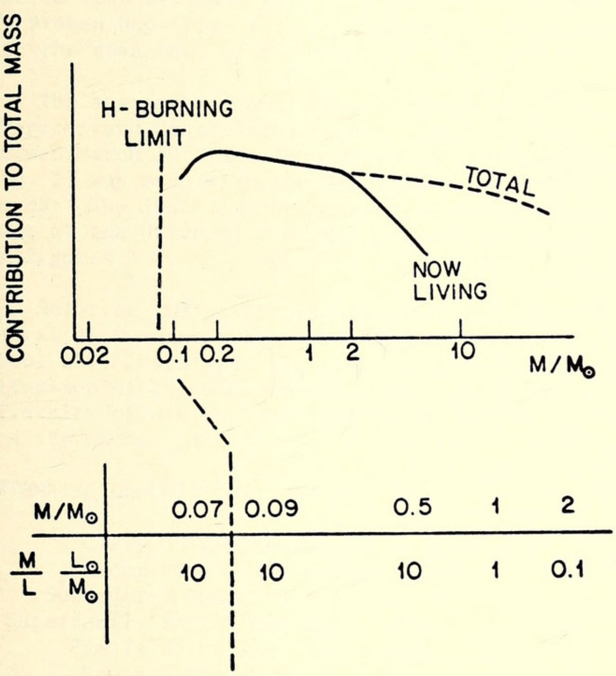

Fig. 2

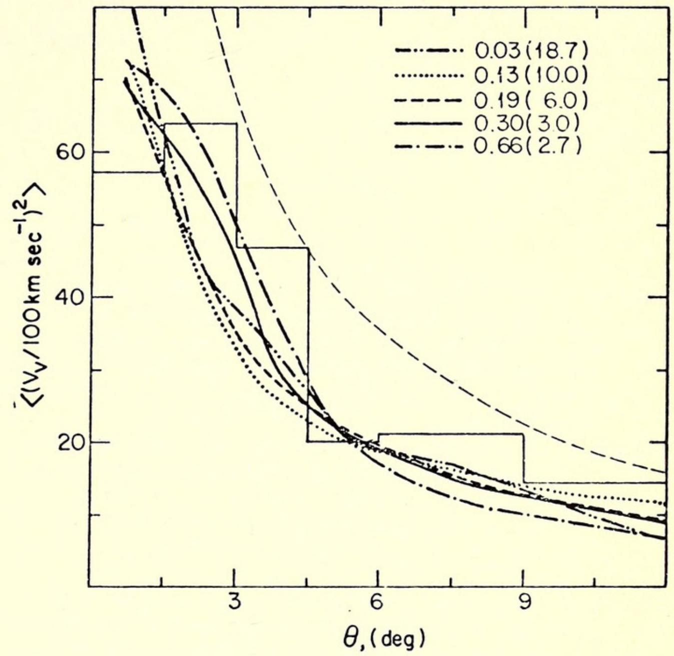

Fig. 3

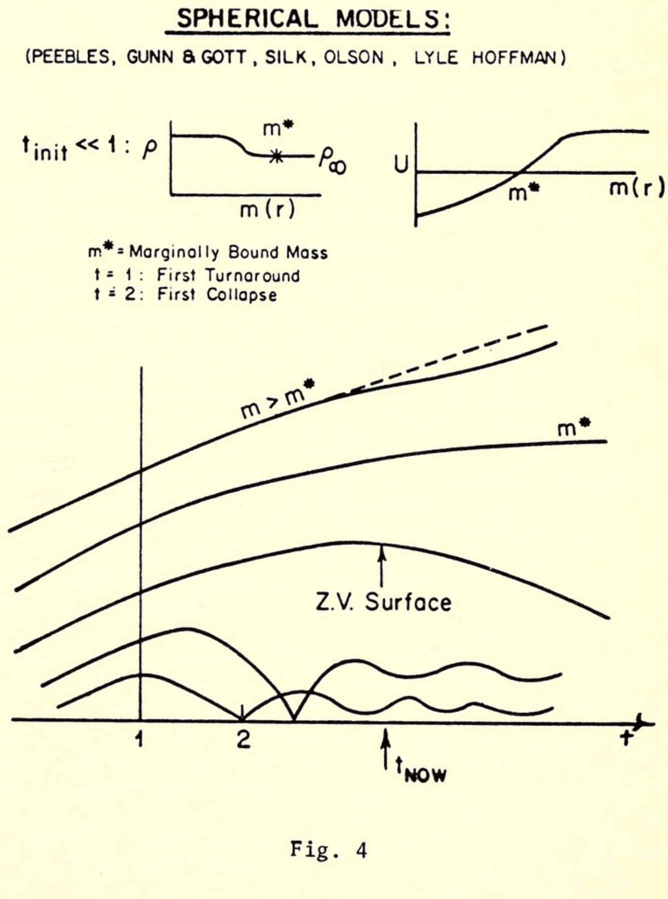

Fig. 4

Figure 1: The ratio of local surface mass density σ_M to optical surface brightness σ_L as a function of radial distance r from the centre of an individual galaxy.

Figure 2: A schematic picture of the "initial mass function" IMF in logarithmic units as a function of stellar mass M. Typical mass-to-light ratios are also shown.

Figure 3: The velocity dispersion of systemic galaxy velocities as a function of angular distance θ from the centre of the Virgo cluster. The histogram is the observational data, the curves are for various models (labelled by Ω and by t_{now} in brackets).

Figure 4: A schematic picture for the time development of a density enhancement into a cluster.